11/II-2019-11/X-2021

Campus

Microperforado ■ Bandas de color

Escaneado con CamScanner

11/II 2019

План работы: (см. 11/XI 2018)

1) Sk-mass calculation via SW - $M_s(T)$
2) AF - DW dynamics с Ruben, Ricardo Окс.
3) $\Phi_0(s)$ Sk - static, $N \rightleftarrows B$ - Sk transform. $\Phi_0(s,t)$ - breath. mode, Verba-MS
4) Sk $M(H_x)$ in dots - Arantxa
5) ERC - Adv. grant appl.
6) Sk-pair/chain, PSS - Zakhra, Q_ℓ^m
7) HMM'19 - Crete (hotel, fee, tickets), JEMS'19, Uppsala (hotel, fee, tickets)
8) ~~NE tarjetas renewal - K., N., A. - photos, tasas~~
9) UPV/EHU, Pais Vasco travel grants „Movilidad" - skyrobs link (see May '18 mail) subm. Nov.
10) Drum modes MS - calc. $m_n(s)$, ω_n finite β - CoFe dots
11) ~~Pun. order trip Poznan - Jan. 19~~ subm. 13/II
12) ~~Abs., program SolSkyMag'19~~
13) ~~Final report MagIC project~~ ← 28/II → Maciej
14) ~~MINECO project report - work hours~~ go 15/III
15) ~~Review Polish project, review Skyrmion RRL~~, Ruben-COST-report
16) ~~COST travel appl. to Porto - subm. 27/II~~
17) Gipuzkoa postdoc, new grants Emilia
18) ~~tax return, cgeuzo cita within May 8-16, IOT gue nara, N. u 10.~~)
19) Rev. European project 3D Skyrmions" go 5/V or 3/VI ↪ cita 29/V 10^45 ~~Tolosa~~.
20) Prorroga MINECO „Skyrmion" project.
21) Ирбеграз „Chase" bank, coord. 6/V
22) Rev. Vortex/Skyrmion gue S.K. Kim, go Oct. 2019

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(8-9/II) Recent papers By Eleonskii VM et al. on 1D - LLG equation. Nonlinear $E_{kin} \sim (\dot\Phi)^2$ vs $E_{kin}(\Phi) \sim (1-\cos\Phi)\Phi$ by ansatz "skyrmion" $S_x \sim \dot\Phi$

(10/II) Travel report Poznan' Jan. '79. Eleonskii '78 - exact solution - large 1 soliton.

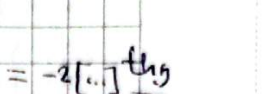
(11/II) Omatsu Takas NTE trajectory invariable - K., Π., A. Сделано фото Fuji C 0.
Обзор skyrmion papers. Обзор papers By Eleonskii et al., related papers.
Eleonskii Sov. Phys. JETP, 42, 1113 (1978), предполагает to use $\xi = x - vt$, $\operatorname{tg}\frac{\Theta}{2} = e^y$
Вместо $\Theta(\xi), \Phi(\xi)$ будут $y(\xi), \Phi(\xi)$ подобно имеющ подходу 5/II.

$\Theta_\xi \Rightarrow \sin\Theta = \frac{1}{\operatorname{ch} y}$, $\cos\Theta\,\Theta_\xi = -\frac{1}{\operatorname{ch}^2 y}\operatorname{sh} y\, y_\xi = -\frac{1}{\operatorname{ch} y}\operatorname{th} y\, y_\xi = +\sin\Theta\cos\Theta\, y_\xi$, $y_\xi = \frac{\partial y}{\partial \xi}$

$\Theta_\xi = +\sin\Theta\cdot y_\xi$, $\cos\Theta = \operatorname{th} y$, $y(\xi=-\infty) = \infty$, $\Theta(\xi=-\infty) = 0$, $\Theta_\xi > 0$, $\Theta(x)$ нарастает с $x\uparrow$.

$\cos\Theta = \frac{1-\operatorname{tg}^2\Theta/2}{1+\operatorname{tg}^2\Theta/2} = \frac{1+e^{2y}}{1+e^{2y}} = \frac{e^{-y}-e^{y}}{e^{-y}+e^{y}} = -\frac{\operatorname{sh} y}{\operatorname{ch} y} = -\operatorname{th} y$, $\operatorname{tg}\frac{\Theta}{2} = e^y$

При $\operatorname{tg}\frac{\Theta}{2} = e^y$ надо $\sin\Theta = 1/\operatorname{ch} y$, $\cos\Theta = -\operatorname{th} y$ .. знаки как у Eleonskii '78.



$L(y,\Phi) = \left[\left(y_\xi\right)^2 + \left(\Phi_\xi\right)^2 + 1 + \varepsilon\cos^2\Phi - 2u\, y_\xi\Phi\right]\frac{1}{\operatorname{ch}^2 y}$ – Lagrangian

$\frac{\partial}{\partial\xi}\frac{\partial L}{\partial y_\xi} = \frac{\partial}{\partial\xi}\frac{1}{\operatorname{ch}^2 y}(2y_\xi) = \frac{\partial L}{\partial y} = [\dots]\frac{\partial}{\partial y}\frac{1}{\operatorname{ch}^2 y} = [\dots]\frac{(-2)}{\operatorname{ch}^3 y}\operatorname{sh} y = -2[\dots]\frac{\operatorname{th} y}{\operatorname{ch}^2 y}$

$2y_{\xi\xi}\frac{(-2)}{\operatorname{ch}^3 y}\operatorname{sh} y\cdot y_\xi + \frac{1}{\operatorname{ch}^2 y}2y_{\xi\xi\xi} = -2[\dots]\frac{\operatorname{th} y}{\operatorname{ch}^2 y} = -2\frac{\operatorname{th} y}{\operatorname{ch}^2 y}\left[y_\xi^2 + \Phi_\xi^2 + 1 + \varepsilon\cos^2\Phi - 2u y_\xi\Phi\right]$

$2\frac{\operatorname{sh} y}{\operatorname{ch} y}y_\xi^2 - y_{\xi\xi\xi} = \operatorname{th} y\left[y_\xi^2 + \Phi_\xi^2 + 1 + \varepsilon\cos^2\Phi - 2u y_\xi\Phi\right]$

$-y_{\xi\xi\xi} = \operatorname{th} y\left[\Phi_\xi^2 - y_\xi^2 + 1 + \varepsilon\cos^2\Phi - 2u\Phi y_\xi\right]$, $\dot\Phi = \frac{d}{dt}\Phi(\xi) = \Phi_\xi u$, $\xi = x - vt$.

У меня 5/II $\dot\Phi = -u\Phi_\xi = y_{\xi\xi\xi} + \operatorname{th} y\left[1 + \lambda\cos^2\Phi + \Phi_\xi^2 - y_\xi^2\right]$, $\frac{\partial}{\partial x} = \frac{\partial}{\partial\xi}\cdot\lambda = \varepsilon$.

Должно $-u\Phi_\xi = +2u\Phi y_\xi$, $-\Phi_\xi = 2\Phi y_\xi\operatorname{th} y$, $L(\Theta,\Phi) = \dots - 2u y_\xi\Phi\frac{1}{\operatorname{ch}^2 y}$ определен с тем, до членов $\operatorname{th} y$ включительно, т.к. $\mathcal{L} = \int d\xi\, L(\Theta,\Phi)$ имеет смысл и члены в $L(\Theta,\Phi)$ не сокр. с членами решения 5/II.

$\frac{\partial}{\partial\xi}\frac{\partial L}{\partial\Phi_\xi} = \frac{\partial}{\partial\xi}\frac{1}{\operatorname{ch}^2 y}2\Phi_\xi = \frac{\partial L}{\partial\Phi} = \frac{1}{\operatorname{ch}^2 y}\varepsilon(-)\sin 2\Phi = 2\Phi_\xi\frac{(-2)}{\operatorname{ch}^2 y}\operatorname{sh} y\, y_\xi + \frac{1}{\operatorname{ch}^2 y}2\Phi_{\xi\xi}$

$\frac{1}{\operatorname{ch}^2 y}\left[-\varepsilon\sin 2\Phi - 2u y_\xi\right] = \frac{2}{\operatorname{ch}^2 y}\left[\Phi_{\xi\xi} - 2\Phi_\xi y_\xi\operatorname{th} y\right]$, $-\frac{\varepsilon}{2}\sin 2\Phi - u y_\xi =$

$= \Phi_{\xi\xi} - 2\Phi_\xi y_\xi\operatorname{th} y$, $u y_\xi = -\frac{\varepsilon}{2}\sin 2\Phi - \Phi_{\xi\xi} + 2\Phi_\xi y_\xi\operatorname{th} y$. Тоже совп. с 5/II

по $\xi = x - vt$, $\dot y = \frac{d}{dt}y(\xi) = y_\xi\dot\xi = -u y_\xi$. При $\dot y = u y_\xi$.

Вычислим член в $\frac{\partial L}{\partial y_\xi}$. Надо $\frac{\partial L}{\partial y_\xi} = \frac{1}{\operatorname{ch}^2 y}\left[2y_\xi - 2u\Phi\right]$

$\frac{\partial}{\partial\xi}\frac{\partial L}{\partial y_\xi} = \frac{\partial}{\partial\xi}2\frac{1}{\operatorname{ch}^2 y}(y_\xi - u\Phi) = 2\frac{(-2)}{\operatorname{ch}^3 y}\operatorname{sh} y\, y_\xi(y_\xi - u\Phi) + 2\frac{1}{\operatorname{ch}^2 y}(y_{\xi\xi} - u\Phi_\xi) =$

$= -2\left[y_\xi^2 + \Phi_\xi^2 + 1 + \varepsilon\cos^2\Phi - u y_\xi\Phi\right]\frac{\operatorname{th} y}{\operatorname{ch}^2 y}$, $2\operatorname{th} y\, y_\xi(y_\xi - u\Phi) - y_{\xi\xi} + u\Phi_\xi =$

$= \left(y_\xi^2 + \Phi_\xi^2 + 1 + \varepsilon\cos^2\Phi - 2u y_\xi\Phi\right)\operatorname{th} y$, $-y_{\xi\xi} + u\Phi_\xi = \operatorname{th} y\left[\Phi_\xi^2 - y_\xi^2 + 1 + \varepsilon\cos^2\Phi\right]$

$u\Phi_\xi = y_{\xi\xi} + \operatorname{th} y\left[\Phi_\xi^2 - y_\xi^2 + 1 + \varepsilon\cos^2\Phi\right]$, Тоже совп. с 5/II если $\dot\Phi = u\Phi_\xi$.

Разницу знаков $\dot\Phi \to -\dot\Phi$, $\Theta \to -\Theta$, $\dot y \to -\dot y$ с Eleonskii '78 можно объяснить wrong sign of γ-term у Eleonski. У меня $-\frac{M_S}{\gamma}\sin\Theta\dot\Theta = \frac{\delta W}{\delta\Phi}$, $\frac{M_S}{\gamma}\sin\Theta\dot\Phi = \frac{\delta W}{\partial\Theta}$ – знаки верные и $\gamma > 0$. $J = M_S/\gamma > 0$.

Т.е. точная система ур-ий с $\xi = x - vt$, $\operatorname{tg}\frac{\Theta}{2} = e^y$, $W_a = K_x m_x^2 - K_z m_z^2$

$$\left.\begin{aligned} u y_\xi &= \frac{\lambda}{2}\sin 2\Phi + \Phi_{\xi\xi} - 2\Phi_\xi y_\xi\operatorname{th} y \\ -u\Phi_\xi &= y_{\xi\xi} + \left[1 + \lambda\cos^2\Phi + \left(\Phi_\xi\right)^2 - \left(y_\xi\right)^2\right]\operatorname{th} y \end{aligned}\right\} \quad \varepsilon = \lambda = \frac{K_x}{K_z},\ \operatorname{tg}\frac{\Theta}{2} = e^y$$

② Trivial solution $\Phi_\xi = 0$, $y_{\xi\xi} = 0$, $(y_\xi)^2 = 1 + \lambda\cos^2\Phi$, $u y_\xi = \frac{\lambda}{2}\sin 2\Phi$, $y_\xi = \text{const}(\xi)$.

$1 + \lambda\cos^2\Phi = \left(\frac{\varepsilon}{2u}\right)^2\sin^2 2\Phi$, $u^2(\Phi) = \frac{\varepsilon^2}{4}\frac{\sin^2 2\Phi}{1 + \varepsilon\cos^2\Phi}$ $\varepsilon = \lambda$ даёт $u = u(\varepsilon)$ $\Phi = \text{const}(\xi)$

$u(\varepsilon) = \sqrt{1+\varepsilon} - 1$.

1st integral of motion $\frac{1}{\operatorname{ch}^2 y}\left[\left(y_\xi\right)^2 + \left(\Phi_\xi\right)^2 - 1 - \varepsilon\cos^2\Phi\right] = C$. $C = 0$ при $\operatorname{ch} y(\pm\infty) \to \infty$.

$\to y_\xi^2 + \Phi_\xi^2 = 1 + \varepsilon\cos^2\Phi$ подбором нашл. y_ξ, или Φ_ξ, или Φ из y_ξ.

$\left[u + 2\Phi_\xi\operatorname{th} y\right]y_\xi = \frac{1}{2}\lambda\sin 2\Phi + \Phi_{\xi\xi}$, $\left(u + 2\Phi_\xi\operatorname{th} y\right)\sqrt{1 + \lambda\cos^2\Phi - \left(\Phi_\xi\right)^2} = \frac{1}{2}\lambda\sin 2\Phi + \Phi_{\xi\xi}$ –

– одно нелинейное дифф. ур. для $\Phi(\xi)$. При $\Phi_\xi = 0$ $u^2(1 + \lambda\cos^2\Phi) = \frac{1}{4}\lambda^2\sin^2 2\Phi$

– напомним ур. типа Walker. Легко исключить Φ, оставив ур-ие для y_ξ, Φ_ξ, y.

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Trivial/ultimate solution $y_3=0$ $\Phi_{33}+\frac{\lambda}{2}\sin 2\Phi=0$ осил. ур sin-Gordon $\Phi_{33}=\frac{\lambda}{2}\sin 2\Phi$

$y_3=\frac{\Theta_3}{\sin\Theta}$, $L(\Theta,\Phi)=\left[\frac{\Theta_3^2}{\sin^2\Theta}+(\Phi_3)^2+1+\varepsilon\cos^2\Phi-2u\frac{\Theta_3}{\sin\Theta}\Phi\right]\sin^2\Theta=$

$=(\Theta_3)^2+\sin^2\Theta(\Phi_3)^2+\sin^2\Theta+\varepsilon\sin^2\Theta\cos^2\Phi-2u\Theta_3\sin\Theta\Phi$, $-u\Theta_3=\dot\Theta$

$\dot\Theta\sin\Theta\Phi=-\Phi\frac{d}{dt}(\cos\Theta)=\frac{d\Phi}{dt}\cos\Theta-\frac{d}{dt}(\Phi\cos\Theta)=\dot\Phi\cos\Theta$

$L(\Theta,\Phi)=(\Theta_3)^2+\sin^2\Theta(\Phi_3)^2+\sin^2\Theta+\varepsilon\sin^2\Theta\cos^2\Phi+2\dot\Phi\cos\Theta$

Верный Lagrangian $L'(\Theta,\Phi)=-\frac12 L(\Theta,\Phi)=-\cos\Theta\dot\Phi-E$, $E=\frac12\Theta_3^2+\frac12\sin^2\Theta(\Phi_3)^2+\frac12\sin^2\Theta(1+\varepsilon\cos^2\Phi)$

или $L(\Theta,\Phi)=L_{kin}-E(\Theta,\Phi)$. Да, см. 2/w

т.е. $L(\Theta,\Phi)$ у Eleonskii'78 отличается с точностью до множителя.

(b) trivial solution $y_3=0$ $\Phi_{33}+\frac{\lambda}{2}\sin 2\Phi=0$ получен $\Phi=\frac{\pi}{2}-\varphi$ substitution

$\sin 2\Phi=\sin(\pi-2\varphi)=-\cos\pi\cdot\sin 2\varphi=\sin 2\varphi\Rightarrow\varphi_{33}=\frac{\lambda}{2}\sin 2\varphi$ – sin-Gordon

$\varphi_{xx}-\frac{1}{c^2}\varphi_{tt}=\frac{\lambda}{2}\sin 2\varphi$, осил $\varphi(x,t)=\varphi[\gamma(x-vt)]$, $\gamma=(1-\frac{v^2}{c^2})^{-\frac12}$. $\varphi(\xi)$, $\xi=x-vt$

$\varphi_{xx}=\gamma^2\varphi_{33}$, $\varphi_{tt}=\gamma^2v^2\varphi_{33}$, $\gamma^2\varphi_{33}-\frac{1}{c^2}\gamma^2v^2\varphi_{33}=\gamma^2(1-\frac{v^2}{c^2})\varphi_{33}=\varphi_{33}$

$\varphi_{33}=\frac{\lambda}{2}\sin 2\varphi\to$ sin-Gordon вида Sklyanin'79: $\frac{1}{\lambda}\varphi_{xx}-\frac{1}{c_0^2\lambda}\varphi_{tt}=\frac12\sin 2\varphi$

$\frac{1}{\lambda}\varphi_{xx}=\frac{1}{\lambda}\frac{\partial^2\varphi}{\partial x^2}=\frac{\partial^2\varphi}{\partial y^2}$, $y=\sqrt\lambda x$. у Sklyanin'79 $y=\frac{x}{\sqrt{\lambda}}$ – наоборот.

$c^2=c_0^2\lambda$, $c=c_0\sqrt\lambda$, $c=c_0\sqrt\lambda$ (разм. ед. $c_0=1$) $=\frac{2\gamma}{M_3}\sqrt{AK_2}\sqrt{\frac{K_x}{K_2}}=\frac{2\gamma}{M_3}\sqrt{AK_x}$

$c=v_m(K_x/K_2\gg1)$, $v_m(\lambda)=\sqrt{1+\lambda}-1\propto\sqrt\lambda$ при $\lambda\gg1$.

(12/II) Hamog. УД (Θ,Φ) (см. 18/I, 1/II)

$$\begin{cases}-\sin\Theta\dot\Theta=-\lambda\sin^2\Theta\sin 2\Phi-2\Phi_{xx}\sin^2\Theta-2\Phi_x\Theta_x\sin 2\Theta\\ \sin\Theta\dot\Phi=(1+\lambda\cos^2\Phi)\sin 2\Theta+(\Phi_x)^2\sin 2\Theta-2\Theta_{xx}\end{cases}$$

x в ед. $\Delta_0=\sqrt{A/K_2}$, t в ед. $t_0=1/\omega_0$, $\omega_0=2K_2/M_3$. Отыщем $\Theta(\xi)$, $\Phi(\xi)$, $\xi=x-vt$

$\dot\Theta=\frac{d}{dt}\Theta(\xi)=\Theta_\xi\dot\xi=-v\Theta_\xi$, $\dot\Phi=-v\Phi_\xi$

$$\begin{cases}v\Theta_\xi\sin\Theta=\lambda\sin^2\Theta\sin 2\Phi+2\Phi_{\xi\xi}\sin^2\Theta+2\Phi_\xi\Theta_\xi\sin 2\Theta\\ v\Phi_\xi\sin\Theta=2\Theta_{\xi\xi}-(1+\lambda\cos^2\Phi)\sin 2\Theta-(\Phi_\xi)^2\sin 2\Theta\end{cases}$$ (*)

Надо $u=\frac{v}{2}$ малое по u^2. УД Eleonskii'76

(a) Walker solution $\Phi_\xi=0$ $\Phi=\text{const}$ даст $\Theta_{\xi\xi}=(1+\lambda\cos^2\Phi)\frac12\sin 2\Theta$, $-v\Theta_\xi\sin\Theta=\lambda\sin^2\Theta\sin 2\Phi$ вычтем + $\frac12$

Def $\frac{1}{\Delta^2(\Phi)}=1+\lambda\cos^2\Phi\to\Theta_{\xi\xi}=\frac{1}{2\Delta^2}\sin 2\Theta$, отсюда $\Theta=\Theta(\xi)=2\operatorname{arctg}e^{\pm\xi/\Delta}$, $y=\frac{\xi}{\Delta}$.

$\operatorname{tg}\frac{\Theta}{2}=\exp(\pm y)$, сообщ $\sin\Theta=\frac{1}{\operatorname{ch}y}$, $\cos\Theta=\mp\operatorname{th}y$, $\Theta_\xi=\pm\sin\Theta\cdot y_\xi$, $\Theta_{\xi\xi}\to$

Берем $(-y)$, $\Theta_\xi=-\sin\Theta y_\xi$, $-v\sin\Theta(-\sin\Theta y_\xi)=\lambda\sin^2\Theta\sin 2\Phi$, $vy_\xi=\lambda\sin 2\Phi$

$y=\frac{\xi}{\Delta}$ из рем. $\Theta_{\xi\xi}=\frac{1}{2\Delta^2}\sin 2\Theta$, $y_\xi=\frac{1}{\Delta}$, $v=2u$, $\frac{u}{\Delta}=\frac{\lambda}{2}\sin 2\Phi$, $y_{\xi\xi}=0\to y_\xi=\frac{1}{\Delta}$

$\Phi=\text{const}$, $\Phi(u,\lambda)$ определен. из соотн. $\frac{1}{\Delta^2}=1+\lambda\cos^2\Phi$, $\frac{u}{\Delta}=\frac{\lambda}{2}\sin 2\Phi$. $y=\xi/\Delta$.

Но $\Theta_{\xi\xi}=\frac{1}{2\Delta^2}\sin 2\Theta$ можно трактовать как sin-Gordon для $\Theta(x,t)$ без всякого $\lambda\to\infty$ интеграла.

Assume $\Theta(x,t)=\Theta[\gamma(x-vt)]$, $\Theta_{xx}-\frac{1}{c^2}\Theta_{tt}=\gamma^2(1-\frac{v^2}{c^2})\Theta_{\xi\xi}=\Theta_{\xi\xi}$ при $\gamma=(1-v^2/c^2)^{-1/2}$.

$\Theta(x,t)=\Theta(\gamma\xi)$, с другой стороны $\Theta(x,t)=\Theta(\frac{\xi}{\Delta})\to$ допускается $\Delta=\frac{1}{\gamma}=\sqrt{1-v^2/c^2}$, матем. (?)

Δ в ед. $\Delta_0=\sqrt{A/K_2}$. Возможно ли физически $\Delta=\sqrt{1-v^2/c^2}$ – ? Нет

$\frac{u^2}{\Delta^2}=\frac{\lambda^2}{4}\sin^2 2\Phi=u^2(1+\lambda\cos^2\Phi)$ – попробуем $x=\cos^2\Phi$, $\sin^2 2\Phi=4\sin^2\Phi\cos^2\Phi=4(1-x^2)x$

$\lambda^2(1-x^2)x=u^2(1+\lambda x)$ – кубич. ур. для $x=\cos^2\Phi$

$\sin^2 2\Phi=1-\cos^2 2\Phi=1-x^2$, $x=\cos 2\Phi$, $\cos^2\Phi=\frac12(1+\cos 2\Phi)=\frac12(1+x)$

$\frac{\lambda^2}{4}(1-x^2)=u^2\left(1+\frac{\lambda}{2}(1+x)\right)$, $y=\frac12\lambda x$, $\frac{\lambda^2}{4}-y^2=u^2(1+\frac{\lambda}{2}+y)$, $y=\frac12\lambda\cos 2\Phi$

$y^2+u^2y+u^2(1+\frac{\lambda}{2})-\frac{\lambda^2}{4}=0$, $2y_{1,2}=-u^2\pm\sqrt{u^4-4\left[u^2(1+\frac{\lambda}{2})-\frac{\lambda^2}{4}\right]}$

$2y_{1,2}=-u^2\pm\sqrt{(u^2-\lambda)^2-4u^2}$, $u_m=\frac12 v_m(\lambda)=\frac12\left[\sqrt{1+\lambda}-1\right]$.

$\sqrt{(\cdot)}$, $(\cdot)\ge0$. $\frac{1}{\Delta^2}=1+\lambda\cos^2\Phi=1+\frac{\lambda}{2}+y$, $y=\frac12\left[-u^2\pm\sqrt{(u^2-\lambda)^2-4u^2}\right]\Rightarrow\Delta(u)$.

при $u=0$ $\frac{1}{\Delta^2(0)}=1+\frac{\lambda}{2}+\frac12\left[\pm\lambda\right]$, надо брать "+", т.к. $\Delta(u=0)=1=\Delta_0$ – верно.

$\frac{1}{\Delta^2(u)}=1+\frac{\lambda}{2}+\frac12\left[u^2+\sqrt{(u^2-\lambda)^2-4u^2}\right]$, сокр. $u=\frac{v}{2}$ неверно. Обезразмеривание

УД (Θ,Φ) 1/II в $x\frac{1}{2A}$ и ξ как $x\Delta_0^2$. $\frac{1}{2A}\frac{A}{K_2}=\frac{1}{2K_2}$, а не $\frac{1}{K_2}$. Надо $v=u$, и $x(\frac12)$ см. (*)

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(*) $\begin{cases} -v\theta_\xi \sin\theta = \frac{\lambda}{2}\sin^2\theta\sin 2\Phi + \Phi_{\xi\xi}\sin^2\theta + \theta_\xi \Phi_\xi \sin 2\theta \\ v\Phi_\xi \sin\theta = \theta_{\xi\xi} - [1+\lambda\cos^2\Phi + (\Phi_\xi)^2]\sin\theta\cos\theta \end{cases}$ точная - верная система, совп. с Eleonskii '78, $u=v$.

$u_m = v_m = \sqrt{1+\lambda} - 1$, $\lambda = (u_m+1)^2 - 1 = u_m^2 + 2u_m$

$\frac{1}{\Delta^2(u)} = 1 + \frac{\lambda}{2} - \frac{1}{2}\left[u^2 + \sqrt{(u_m^2-u^2)\{(u_m+2)^2-u^2\}}\right]$, искали $\frac{1}{\Delta^2(u)} = \gamma^2 = \frac{1}{(1-u^2/u_m^2)}$

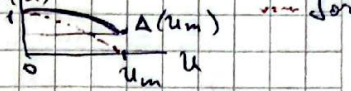
$\frac{1}{\Delta^2(u_m)} = 1 + \frac{\lambda}{2} - \frac{1}{2}u_m^2 = 1 + \frac{\lambda}{2} - \frac{1}{2}\left[\sqrt{1+\lambda}-1\right]^2 = 1 + \frac{\lambda}{2} - \frac{1}{2}(1+\lambda+1-2\sqrt{1+\lambda}) = \sqrt{1+\lambda}$

$\Delta^2(u_m) = \frac{1}{\sqrt{1+\lambda}}$, сов. с $E_{DW}(u_m) = E_0(1+\lambda)^{1/4}$ ст/п, т.к. $E_{DW}(u) = \frac{E_0}{\Delta(u)}$.

т.е. наш $\Delta(u)$ из Walker-Schlöman solution $\Phi(x,t) = \text{const}$.

$A - \Delta^2(u) = (\lambda - u^2/u_m^2)$ при $\Phi = \text{const}$ невозможно, хотя формально допускало из sin-Gordon $\theta_{\xi\xi} = \frac{1}{2\Delta^2}\sin 2\theta$ (при $\Phi = \text{const}$).

но при $u^2 \to 0$, $\Delta^2(u)$-real и $\Delta^2(u) = (1 - u^2/u_m^2)$ совпадают.



--- formal sin Gordon

$\frac{1}{\Delta^2(u)} = u_m + 1 + \frac{1}{2}(u_m^2 - u^2) - \frac{1}{2}\sqrt{(u_m^2-u^2)[(u_m+2)^2-u^2]}$, $u_m + 1 = \sqrt{1+\lambda}$.

DW width vs. DW velocity и для $\Phi = \text{const}$ - Walker solution (θ, Φ) УД.

// Можно и проще анзатц $\operatorname{tg}\frac{\theta}{2} = e^y$ в $(y\Phi)$-УД потребовать еще Walker-like eqs. $(y_\xi)^2 = 1 + \lambda\cos^2\Phi$, и $y_\xi = \frac{\lambda}{2}\sin 2\Phi$, тогда

$\begin{cases} \Phi_{\xi\xi} - 2\Phi_\xi y_\xi \operatorname{th} y = 0 \\ -u\Phi_\xi = y_{\xi\xi\xi} + (\Phi_\xi)^2 \operatorname{th} y \end{cases}$ $\Phi_\xi = \omega(\xi)$ $\begin{cases} \omega_\xi - 2\omega y_\xi \operatorname{th} y = 0 \\ -u\omega = y_{\xi\xi\xi} + \omega^2 \operatorname{th} y \end{cases}$, $u^2(1+\lambda\cos^2\Phi) = \frac{\lambda^2}{4}\sin^2 2\Phi$

- это может быть монотонное движение (only) при $\Phi(x,t) = \text{const}$, $\Phi = \Phi(u,\lambda)$, $\omega = 0$, $y_{\xi\xi\xi} = 0$, сводится к Walker-Solution $\theta(\xi)$, $\Phi(\xi) = \text{const}$.

// Eleonskii ZETF '1978 дает при крит. скорости $\begin{cases} v_\pm = \sqrt{1+\varepsilon} \pm 1 \\ v_0 = 2(1+\varepsilon)^{1/4} \end{cases}$, $\varepsilon = \frac{K_z}{K_x}$

$v_- = \sqrt{1+\varepsilon} - 1 = v_m$ (max velocity DW), $v_0^2 = v_+^2 - v_-^2$

Решение $\theta(\xi)$, $\Phi(\xi) = \text{const}$ существует при $v < v_-$ (Walker type solution).

При $v_- < v < v_+$ есть solitary waves (soliton) solutions with $\Phi = \Phi(\xi) \neq \text{const}$

при $v > v_+$ — нет soliton solutions, а есть "nonlinear sw" — это не граница.

Неясно верен ли rigid anzatz $\theta(\xi)$, $\Phi(\xi)$ при $v > v_-$, $\xi = x - vt$ (?). Eleonskii '78 считает, что YES.

// Eleonskii JETP '1985 дает решение LL точное, осциллирующее с $\omega = \text{const}$ (Hirota method)

$\operatorname{tg}\frac{\theta}{2}e^{i\Phi} = \frac{g_1(x,t) + g_3(x)}{1 + g_2(x,t)}$, $\theta = \theta(x,t)$, $\Phi = \Phi(x,t)$ не сводящиеся, в общем случае к $\theta(\xi)$, $\Phi(\xi)$, $\xi = x - vt$.

(3/II) Meeting с Ruben: привели поправки large block comp. for Ricardo. Обещал updated AF-DW paper (где Ricardo нужен analytic part addition?). Я обещал провести boundary sin-Gordon и из S-eq of two-sublattice AFM и perturbation theory at $v > v_m$ (где $\Phi = \text{const}$). Обратили, что в точке $\Phi = \text{const}$, $\theta(\xi)$, $v < v_m$ других реш. в нат. нет. // Полиция N.D. - 10 kg. Visit to Policia - negan pais tasas, photo и сделали fingerprints. New tarjetas NIE не ранее March 13, 2019.

Допущение и анализ УД для (θ,Φ) или (y,Φ) решение $\omega = \Phi_\xi = \text{const}$? sin 2Φ

$\omega = \Phi_\xi = \text{const}$, $\Phi_{\xi\xi} = 0$ дает $\begin{cases} -v\theta_\xi \sin\theta = \frac{\lambda}{2}\sin^2\theta + \omega\theta_\xi \sin 2\theta \\ v\omega\sin\theta = \theta_{\xi\xi} - [1 + \lambda^2\cos^2\Phi + \omega^2]\sin\theta\cos\theta \end{cases}$

$(y_\xi)^2 + \omega^2 = 1 + \varepsilon\cos^2\Phi$

$-u\Phi_\xi = -u\omega = y_{\xi\xi\xi} + \left[(y_\xi)^2 + \omega^2 + \omega^2 - (y_\xi)^2\right]\operatorname{th} y = y_{\xi\xi\xi} + 2\omega^2 \operatorname{th} y$ $\operatorname{tg}\frac{\theta}{2} = e^y$

$y_{\xi\xi\xi} = -u\omega - 2\omega^2 \operatorname{th} y = f(y)$ должно решаться. $\times y_\xi$, $y_\xi y_{\xi\xi\xi} = \frac{d}{d\xi}(y_\xi)^2\frac{1}{2}$

$\frac{d}{d\xi}\frac{1}{2}(y_\xi)^2 = f(y)y_\xi$, $\frac{1}{2}(y_\xi)^2 = \int d\xi\, y_\xi f(y) + C = \int dy\, f(y) + C$, $\int dy f(y) = \int dy\left[-u\omega - 2\omega^2\operatorname{th} y\right] = -u\omega y - 2\omega^2 \ln\operatorname{ch} y + C$, $\frac{1}{2}(y_\xi)^2 = C - u\omega y - 2\omega^2 \ln\operatorname{ch} y$. $y_\xi = \sqrt{2}\sqrt{(\ldots)}$

$\frac{1}{\sqrt{2}}\int \frac{dy}{\sqrt{C - u\omega y - 2\omega^2 \ln\operatorname{ch} y}} = \xi - \xi_0$. $y(\xi) = \ln\operatorname{tg}\frac{\theta(\xi)}{2}$, $y_\xi = \frac{1}{\operatorname{tg}\frac{\theta}{2}}\frac{1}{\cos^2\frac{\theta}{2}}\frac{1}{2}\theta_\xi = \frac{1}{\sin\theta}\theta_\xi$.

при $\omega = 0$ $\int \frac{dy}{\sqrt{2}\sqrt{C}} = \xi - \xi_0$, $\frac{1}{\sqrt{2C}}y = \xi - \xi_0$, $C = 1/2$. Тогда $y = \frac{1}{\Delta}\xi$ при $\omega = 0$, $\Delta(\Phi) = \frac{1}{\sqrt{2C}}$.

$y(\xi)$ - странное solitary wave, не похожее на DW.

(*)

Escaneado con CamScanner

Бывают решения LLG $\omega = \Phi_t = const$ - это precession solitons by Kosevich's SW easy plane/easy axis (Schlomann'71) : $\omega^2 = (1+k^2)(1+\beta+k^2)$, $v = v_{ph} = \omega/k$

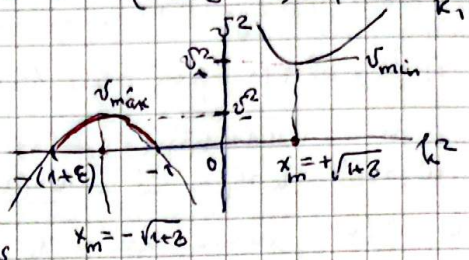
$v^2(k) = \left(1+\frac{1}{k^2}\right)(1+\beta+k^2) = \frac{(1+\beta)}{k^2} + 2 + \beta + k^2$, $v^2(x)$ находим min $v^2(k^2)$

при real k. $v^2(x) = \frac{(1+\beta)}{x} + x + 2 + \beta$, $\frac{dv^2}{dx} = -\frac{(1+\beta)}{x^2} + 1 = 0$, $x_m = \sqrt{1+\beta} = k_m^2 > 0$

$v_m^2 = v^2(x_m) = \frac{(1+\beta)}{\sqrt{1+\beta}} + \sqrt{1+\beta} + 2 + \beta = 2\sqrt{1+\beta} + 2 + \beta = \left(\sqrt{1+\beta}+1\right)^2$, $\beta = \frac{2\pi M_s}{K_1}$

т.е. v_m (real k) $= \sqrt{1+\varepsilon} + 1$, $\varepsilon = \frac{K_x}{K_z} > 0$

$\begin{cases} v_-(\varepsilon) = v_{max}\ (imag.\ k) = \sqrt{1+\varepsilon} - 1 \\ v_+(\varepsilon) = v_{min}\ (real\ k) = \sqrt{1+\varepsilon} + 1 \end{cases}$



Ранее 19/I used $k_m^2 = x_m = -\sqrt{1+\varepsilon}$, $k_m^2 < 0$, сейчас $k_m^2 = +\sqrt{1+\varepsilon}$

Phase velocity SW обращается в нуль скорости DW/solitons

$v < v_- = v_{max}$ (imag. k) – DW motion с $\Phi = const$, Walker DW

$v_- < v < v_+ = v_{min}$ (real k) – precession solitons (solitary waves-Eleonskii), $\Phi \neq const$, $\omega = \Phi_t = const$

$v > v_+$ – no solitons

Precession solitons (ω, V)

// Kosevich et al. Phys. Rep. 1990, Chapt. 7.

(!) Есть точное решение m_z, m_x, m_y для non-linear SW in biaxial ferromagnet. I.M. Babich, A.M. Kosevich, ZETF 82, 1277 (1982) — газет. ЖЭТФ (7.17).

(!) точное решение для two parametric solitons (ω, V) in biaxial ferromagnet, Eqs. (7.20)-(7.21), $tg\frac{\theta}{2} = f(x,t)$, $\Phi = g(x,t)$ — было решение совпадающее с Hirota-решением by Eleonskii 1985 (12/II).

Основа: complex SW dispersion eq. $\Omega^2 = (1+K^2)(1+\varepsilon+K^2)$, $\Omega = \tilde{\omega} + i\varkappa V$, $K = k + i\varkappa$

$w = tg\frac{\theta}{2} e^{i\Phi} \rightarrow \boxed{w = -\frac{2\varkappa}{\sqrt{\varepsilon}} \frac{1}{sh 2d} \frac{sh(\xi - id)}{sh(\eta + i\gamma)}}$, d, γ из $1+K^2 = -\varepsilon ch^2(d - i\gamma)$, $d > 0$

$\theta = \theta(\xi, \eta)$, $\Phi(\xi,\eta) = \Phi\left[(x - Vt), (kx - \tilde{\omega}t)\right]$, $\xi = \varkappa(x - Vt)$, $\eta = kx - \tilde{\omega}t$, $\gamma \leq \frac{\pi}{2}$

$V = \frac{1}{\varkappa} Im\,\Omega$ – скорость soliton, $kx - \omega t = k\left(x - \frac{\omega}{k}t\right) = k(x - vt)$

$v = \frac{\tilde{\omega}}{k}$ – phase velocity SW.

т.е. $\Phi(x,t)$ не сводится к $\Phi(\xi)$, $\xi = x - Vt$, как назвал Eleonskii 1978, $\Phi(x,t)$ более сложная.

при $\varkappa \ll 1$, $\gamma = \frac{\pi}{2}$ получим linear SW $w = i\varkappa\frac{\sqrt{\varepsilon}}{\omega} \sin(kx - \tilde{\omega}t)$ из общей $w(\eta, \xi)$

$\tilde{\omega}$ – precessional frequency in the laboratory frame.

Reference frame is moving with the group velocity of linear SW $\vec{v}_g = \frac{\partial\tilde{\omega}}{\partial\vec{k}}$, в the moving frame $\omega = \tilde{\omega} - \vec{k}\cdot\vec{v}_g$. The velocity of soliton pattern: $v = \frac{\tilde{\omega}}{k}$, V - soliton velocity

$\frac{\tilde{\omega}}{k} = v = V + \frac{\omega}{k}$, при $\omega > 0$ modulation moves more rapidly than the soliton itself. ($\omega < 0$)

At $\omega = 0$, $v = V$ ($V = v_g$ [Kosevich'90]) (slowly) → localized wave of stationary profile, „quick wave" [Eleonskii]

$\xi = x(x - Vt)$, $\eta = kx - \tilde{\omega}t = k(x - vt)$. Если $\omega = 0$, $v = V$ и $\eta = k(x - Vt)$, $\Theta, \Phi = f(\xi)$, $\xi = x - Vt$

т.е. при $\omega = 0$ (покоится в moving frame) $\Theta = \Theta(\xi)$, $\Phi = \Phi(\xi)$ и все сводится к 1D-Eleonskii'78.

$V = k(2 + \varepsilon + 2k^2)/\tilde{\omega}$ - group velocity of SW. $\tilde{\omega} = \vec{k}\cdot\vec{v}_g + \omega = kV + \omega$, $\frac{\tilde{\omega}}{k} = v = V + \frac{\omega}{k}$.

(14/II) // Базисные авт. (Kosevich'90) зависимость $\omega(V)$, ω - oscillat. frequency in moving frame, V - soliton velocity.

1) Для „easy" axis FM, $\tilde{\omega} = \omega_0\left[1 + (kl_0)^2\right]$, $\vec{v}_g = \frac{\partial\tilde{\omega}}{\partial k} = 2\omega_0(kl_0)l_0 = V_m(kl_0)\hat{k}$

$v_g = V_m(kl_0)$, V_m - min phase velocity of SW. $v_{ph} = \frac{\tilde{\omega}}{k} = \omega_0\left[\frac{1}{k} + kl_0^2\right] = v_{ph}(k)$

$\frac{\partial v_{ph}}{\partial k} = \omega_0\left[-\frac{1}{k^2} + l_0^2\right] = 0 \rightarrow k_m^2 l_0^2 = 1$, $v_{ph}(k_m) = \omega_0 l_0\left[\frac{1}{k_m l_0} + k_m l_0\right] = 2\omega_0 l_0$

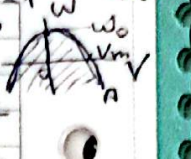
$V_m = v_{ph}(k_m) = \min v_{ph}(k) = 2\omega_0 l_0$. $\omega = \tilde{\omega} - \vec{k}\cdot\vec{v}_g = \tilde{\omega} - V_m l_0 k^2 = \omega_0\left[1 + k^2 l_0^2\right] - V_m l_0 k^2$. $k = v_g/V_m l_0$, $\omega = \omega_0\left[1 + k^2 l_0^2 - \frac{V_m l_0 \cdot k^2}{\omega_0}\right] = \omega_0\left[1 + k^2 l_0^2 - 2k^2 l_0^2\right] =$

$= \omega_0\left[1 - k^2 l_0^2\right] = \omega_0\left[1 - \left(\frac{v_g}{V_m l_0}\right)^2 l_0^2\right] = \omega_0\left[1 - (v_g/V_m)^2\right]$. Нужное soliton velocity $V = v_g$ (SW)

Тогда $\omega = \omega_0\left[1 - (V/V_m)^2\right]$ - frequency of oscillations in moving frame, $V_m = \min v_{ph}(k)$ of SW.

Escaneado con CamScanner

$\frac{V}{V_m} = \frac{v_{gr}(k)}{V_m} = \frac{v_{gr}(k)}{\min v_{ph}(k)}$. Для $\tilde\omega = ck$, $v_{gr} = \frac{\partial\tilde\omega}{\partial k} = c$, $v_{ph} = \frac{\tilde\omega}{k} = c$, $\frac{V}{V_m} = 1$



Т.е. linear SW dispersion relation $\tilde\omega(k)$ should not be trivial.

Нелинейная SW $\tilde\omega = \omega_0[1 + k^2 l_0^2]$, $\cos\Theta_0 \to \omega = \omega_0\left[1 - (V/V_m)^2\right]\cos\Theta_0 = \omega(V)$ немагнитная

магнитная $\omega = \omega_0[1 - V^2/V_m^2]$, в зависимости от $\cos\Theta_0$ меняется, $\cos\Theta_0$ меняется, неизменность скорости

A-A магнитная $\omega(V) = \omega_0\left[1 - (V/V_m)^2\right]$

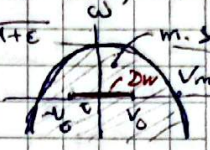
Согласно Косевичу солитон velocity V не произвольна, $V = v_g(k)$ - group velocity of lin. SW

2) biaxial FM, "easy" plane / "easy" axis, $\varepsilon = K_x/K_z$

$\tilde\omega = \sqrt{(1+k^2)(1+\varepsilon+k^2)}$, $v_g = \frac{\partial\tilde\omega}{\partial k} = \frac{1}{2\sqrt{(\cdot)}}\left[2k(1+\varepsilon+k^2) + (1+k^2)\cdot 2k\right] = \frac{k(2+\varepsilon+2k^2)}{\tilde\omega(k)}$

$\omega = \tilde\omega - k\cdot v_g = \tilde\omega - \frac{k^2(2+\varepsilon+k^2)}{\tilde\omega}$, $v_{ph} = \frac{\tilde\omega(k)}{k} = v_{ph}(k)$, $V_m = \min v_{ph}(k) = \sqrt{1+\varepsilon}+1$.

$\omega = \omega(k^2)$, можно найти $k^2 = k^2(v_g) \to \omega = \omega(v_g) = \omega(V)$



$\Theta(x,t) = \Theta(\xi)$, $\Phi(x,t) = \omega t + \Psi(\xi)$, $\xi = x - Vt$

Solitons с $\omega \neq 0$ - precession solitons (Косевич'90).

Preces. solitons с $\omega = 0$, $V \le V_m$ - stationary Solitons (solitary waves) - Лисовский Елеонский '78-79.
"quick waves"

Solitons с $|V| < V_0$, $\Phi(\xi) = \text{const}$, $V_0 = V_- = v_{ph}^{max}(\text{imag. } k)$ - DW, $V = v_g(k)$ - soliton velocity

DW: topol. Solitons, $\Theta(\pm\infty) = 0, \pi$ с $\Phi(\xi) = \text{const}$ и $V \le V_- = \sqrt{1+\varepsilon} - 1$ (Walker velocity)

Даже если $\omega = 0$ и $V < V_-$ профиль DW есть еще решение с $\Phi(\xi) \neq \text{const}$.

При $\omega = 0$, $\tilde\omega = kV$, $\Omega = \tilde\omega + i\varkappa V = (k + i\varkappa)V = KV$, $\xi = \varkappa(x - Vt)$, $\varphi = kx - \tilde\omega t = k(x - Vt)$

$\xi = x - Vt$

$\operatorname{tg}\frac{\Theta}{2}e^{i\Phi} = W(\xi) = -\frac{2\varkappa}{\sqrt{\varepsilon}}\frac{1}{\operatorname{sh}2d}\frac{\sin(k\xi - id)}{\operatorname{sh}(\varkappa\xi + i\gamma)}$, $K^2V^2 = (1+K^2)(1+\varepsilon+K^2)$, $K = k + i\varkappa$

- задает $k(V)$ и $\varkappa(V)$, V - parameter - soliton velocity

$\sin(k\xi - id) = \sin(k\xi)\operatorname{ch}d - i\cos(k\xi)\operatorname{sh}d$, $\operatorname{sh}(\varkappa\xi + i\gamma) = \operatorname{sh}(\varkappa\xi)\cos\gamma + i\operatorname{ch}(\varkappa\xi)\sin\gamma$

$m_x \pm im_y = \frac{2W}{1+|W|^2}$, $m_z = \frac{1-|W|^2}{1+|W|^2}$, $W(\xi)$ - complex function by Hirota.

$\operatorname{tg}\frac{\Theta(\xi)}{2} = |W(\xi)|$, $|W|^2 = \frac{4\varkappa^2}{\varepsilon}\frac{1}{\operatorname{sh}^2 2d}\frac{\sin^2(k\xi)\operatorname{ch}^2 d + \cos^2(k\xi)\operatorname{sh}^2\varkappa}{\left[\operatorname{sh}^2(\varkappa\xi)\cos^2\gamma + \operatorname{ch}^2(\varkappa\xi)\sin^2\gamma\right]} = \operatorname{tg}^2\frac{\Theta}{2}$, $Q = -\frac{2\varkappa}{\sqrt{\varepsilon}}\frac{1}{\operatorname{sh}2d}$

$1 + K^2 = 1 + (k+i\varkappa)^2 = 1 + k^2 - \varkappa^2 + 2ik\varkappa = -\varepsilon\operatorname{ch}^2(d - i\gamma) = -\varepsilon\left[\operatorname{ch}^2 d\cos^2\gamma - \operatorname{sh}^2 d\sin^2\gamma - 2i\operatorname{ch}d\operatorname{sh}d\cos\gamma\sin\gamma\right]$

$1 + k^2 - \varkappa^2 = \varepsilon\left[\operatorname{sh}^2 d\sin^2\gamma - \operatorname{ch}^2 d\cos^2\gamma\right]$, $2k\varkappa = 2\varepsilon\left[\operatorname{ch}d\operatorname{sh}d\cos\gamma\sin\gamma\right]$, $\gamma \in [0, \pi]$, $d > 0$

$\operatorname{tg}^2\frac{\Theta(0)}{2} = \frac{4\varkappa^2}{\varepsilon}\frac{1}{\operatorname{sh}^2 2d}\frac{\operatorname{sh}^2 d}{\sin^2\gamma}$. Видно $\Theta(\xi = 0) \neq \pi/2$ как для DW с $\cos\Theta = \pm\operatorname{th}(\lambda\xi)$, $m_z(0) \neq 0$, $\Theta_{DW}(0) = \pi/2$.

$\operatorname{tg}\frac{\Theta}{2}\left[\cos\Phi + i\sin\Phi\right] = a\frac{\sin(k\xi - id)}{\operatorname{sh}(\varkappa\xi + i\gamma)} = \frac{a}{|\operatorname{sh}(\varkappa\xi + i\gamma)|^2}\left[(\sin k\xi\operatorname{ch}d - i\cos k\xi\operatorname{sh}d)(\operatorname{sh}\varkappa\xi\cos\gamma - i\operatorname{ch}\varkappa\xi\sin\gamma)\right]$

$\operatorname{tg}\frac{\Theta(\xi)}{2}\cos\Phi(\xi) = \frac{a}{|\operatorname{sh}(\varkappa\xi + i\gamma)|^2}\left(\sin k\xi\operatorname{ch}d\cdot\operatorname{sh}\varkappa\xi\cos\gamma - \cos k\xi\operatorname{sh}d\operatorname{ch}\varkappa\xi\sin\gamma\right)$, $a = -\frac{2\varkappa}{\sqrt{\varepsilon}}\frac{1}{\operatorname{sh}2d}$.

$\operatorname{tg}\frac{\Theta(\xi)}{2} = |a|\frac{1}{|\operatorname{sh}(\varkappa\xi + i\gamma)|}\sqrt{\sin^2 k\xi\operatorname{ch}^2 d + \cos^2 k\xi\cdot\operatorname{sh}^2 d}$, $\cos\Phi(\xi) = \frac{a}{|a|}\frac{1}{|\operatorname{sh}(\varkappa\xi + i\gamma)|}\frac{\left(\sin k\xi\operatorname{sh}\varkappa\xi\operatorname{ch}d\cos\gamma - \cos k\xi\operatorname{ch}\varkappa\xi\operatorname{sh}d\sin\gamma\right)}{\sqrt{\sin^2 k\xi\operatorname{ch}^2 d + \cos^2 k\xi\operatorname{sh}^2 d}}$

- явно $\Phi(\xi)$. Видно что $\Phi(\xi) = \text{const}$ (для DW) очень не очевидно.

$K^2V^2 = V^2\left[1 + k^2 - \varkappa^2 + 2ik\varkappa\right] = \left[1 + k^2 - \varkappa^2 + 2ik\varkappa\right]\left[1 + \varepsilon + k^2 - \varkappa^2 + 2ik\varkappa\right] =$

$= (1 + k^2 - \varkappa^2)(1 + \varepsilon + k^2 - \varkappa^2) - 4k^2\varkappa^2 + 2ik\varkappa\left(1 + \varepsilon + k^2 - \varkappa^2 + 1 + k^2 - \varkappa^2\right)$

$V^2(1 + k^2 - \varkappa^2) = (1 + k^2 - \varkappa^2)(1 + \varepsilon + k^2 - \varkappa^2) - 4k^2\varkappa^2$, $V^2 = 2 + \varepsilon + 2k^2 - 2\varkappa^2$, $k^2 - \varkappa^2 = \frac{1}{2}(V^2 - 2 - \varepsilon)$

$V^2\left[1 + \frac{1}{2}(V^2 - 2 - \varepsilon)\right] = \left[1 + \frac{1}{2}(V^2 - 2 - \varepsilon)\right]\left[1 + \varepsilon + \frac{1}{2}(V^2 - 2 - \varepsilon)\right] - 4k^2\varkappa^2$

$V^2\frac{1}{2}(V^2 - \varepsilon) = \frac{1}{2}(V^2 - \varepsilon)\frac{1}{2}V^2 - 4k^2\varkappa^2$, $4k^2\varkappa^2 = -\frac{1}{4}V^2(V^2 - \varepsilon)$, $k^2\varkappa^2 = \frac{1}{16}V^2(\varepsilon - V^2)$

$V < \sqrt{\varepsilon}$?

Иначе $K^2V^2 = (1+K^2)(1+\varepsilon+K^2) \to K^2V^2 = 1 + \varepsilon + K^2(2+\varepsilon) + K^4$

$K^4 + (2 + \varepsilon - V^2)K^2 + 1 + \varepsilon = 0$, $2K_{1,2}^2 = V^2 - 2 - \varepsilon \pm \sqrt{(V^2 - 2 - \varepsilon)^2 - 4(1+\varepsilon)} = V^2 - 2 - \varepsilon \pm \sqrt{V^4 - 2V^2(2+\varepsilon) + \varepsilon^2}$

$K_{1,2}^2 = \frac{1}{2}V^2 - 1 - \frac{\varepsilon}{2} \pm \frac{1}{2}\sqrt{(V_0^2 - V^2)(V_m^2 - V^2)}$, $V_0 = \sqrt{1+\varepsilon} - 1$, $V_m = \sqrt{1+\varepsilon} + 1$, см. $\frac{1}{\Delta^2(u)} = +\varkappa^2(u)$ от 12/11

Escaneado con CamScanner

$K^2_{1,2}(V) = -1 - V_0 + \frac{1}{2}(V^2 - V_0^2) \pm \frac{1}{2}\sqrt{(V_0^2 - V^2)(V_m^2 - V^2)} = k^2 - x^2 + 2ixk$, $V_0 < V_m$

(a) $V \le V_0$, $K^2 < 0$, K^2 - real, $x(V)k(V) = 0$; (b) $V_0 < V < V_m$ — K^2 = complex

$K^2_{1,2}(V) = -1 - V_0 + \frac{1}{2}(V^2 - V_0^2) \pm \frac{1}{2} i \sqrt{|V_0^2 - V^2|(V_m^2 - V^2)} = k^2 - x^2 + 2ixk$

$xk = \frac{1}{4}\sqrt{(V^2 - V_0^2)(V_m^2 - V^2)}\,(\pm 1)$, $k^2 - x^2 = -1 - V_0 + \frac{1}{2}(V^2 - V_0^2)$, $xk = a$, $k^2 - x^2 = b \to$

$2k^2 = b + \sqrt{b^2 + 4a^2}$, знак a совпадает (берем $a > 0$), $k^2(V) = \frac{1}{2}b + \frac{1}{2}\sqrt{b^2 + 4a^2}$, $x = \frac{a}{k}$, $a > 0$

(c) $V > V_m$, $K^2(V)$ - real, $x(V)k(V) = 0$. Значит $k(V) = 0$, $V < V_0$ и $x(V) = 0$, $V > V_m$

(a) $V \le V_0$, $k(V) = 0$ $\quad 1 + K^2 = 1 - x^2 = -\varepsilon \mathrm{ch}^2(d - i\gamma) = -\frac{\varepsilon}{2}\left[1 + \mathrm{ch}2(d - i\gamma)\right] =$

$= -\frac{\varepsilon}{2}\left[1 + \mathrm{ch}2d\cos 2\gamma - i\,\mathrm{sh}2d \cdot \sin 2\gamma\right]$ - real, $\Rightarrow \sin 2\gamma \cdot \mathrm{sh}2d = 0$, откуда $d = 0$ (т.к. $d > 0$)

$2\gamma = 0, \pi$; $\gamma = 0, \frac{\pi}{2}$

$1 - x^2 = -\frac{\varepsilon}{2}\left(1 + \mathrm{ch}2d\,(\pm 1)\right)$, $x^2 = 1 + \frac{\varepsilon}{2}(1 \pm \mathrm{ch}2d)$, $\gamma = 0$ ($k = 0$) дает $\cos\Phi(\xi) = 0$

$\cos\Phi(\xi) = \frac{a}{|a|}\frac{1}{\sqrt{\mathrm{sh}^2 x\xi \cos^2\gamma + \mathrm{ch}^2 x\xi \sin^2\gamma}}$ $\left(\sin k\xi \sin x\xi \,\mathrm{ch}d \cdot \cos\gamma - \cos k\xi \,\mathrm{ch}x\xi \,\mathrm{sh}d \cdot \sin\gamma\right)$

$\sqrt{\sin^2 k\xi \,\mathrm{ch}^2 d + \cos^2 k\xi \,\mathrm{sh}^2 d}$

при $k = 0$, $\cos\Phi(\xi) = \frac{a}{|a|}\frac{1}{\sqrt{\mathrm{sh}^2 x\xi \cos^2\gamma + \mathrm{ch}^2 x\xi \sin^2\gamma}}$ $\quad \frac{(-1)\,\mathrm{ch}x\xi \,\mathrm{sh}d \sin\gamma}{\mathrm{sh}d} = \begin{cases} 0, & \gamma = 0 \\ \cos\Phi(\xi, \frac{\pi}{2}), & \gamma = \frac{\pi}{2} \end{cases}$

при $\gamma = \frac{\pi}{2}$ $\cos\Phi(\xi) = \frac{a}{|a|}\frac{1}{\mathrm{ch}x\xi}(-1)\,\mathrm{ch}x\xi = -\frac{a}{|a|} = 1$, $\Phi(\xi) = \begin{cases} \pm\pi/2, & \gamma = 0, \text{ Bloch DW} \\ 0, \pi, & \gamma = \frac{\pi}{2}, \text{ Neel DW} \end{cases}$

т.е. получаем из общей $\Phi(\xi)$: $\Phi(\xi) = \text{const}$ при $V \le V_0$ (!!)

DW + Solitons - нет, т.к. $\Theta(\pm\infty) = 0$ - soliton

(b) $V_0 < V < V_m$, $k(V) \neq 0$, $x(V) \neq 0$ - осциллирующий режим ядра при $\omega = 0$ - Soliton region.

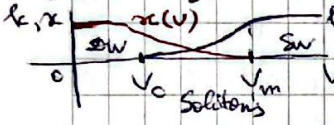
(c) $V > V_m$, $x(V) = 0$, $K = k$ real $\quad k^2 V^2 = (1 + k^2)(1 + \varepsilon + k^2)$ - SW режим.

$\omega = kV$ - real

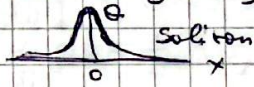
// DW с Φ - const при $V \le V_0 = V_- = \sqrt{1 + \varepsilon} - 1$ и Solitons с $W(\xi) = \mathrm{tg}\frac{\Theta(\xi)}{2} e^{i\Phi(\xi)}$ Kosevich'90 (2.25)

$V_m = V_+ = \sqrt{1 + \varepsilon} + 1$ - $\min V_{ph}(k)$ or real (ω, k) spin waves. при $V_0 < V < V_m$

Walker solution $\Phi(\xi) = \Phi_0 = \text{const}$ должен не только $\Phi_0 = \pm\frac{\pi}{2}$ (B-DW) и $(0, \pi)$ N-DW



- схема $k(V)$, $x(V)$, $K = k + ix$ - wave vector дает stationary solitons ($\omega = 0$) by Kosevich'90.



Solitons при $V_0 < V < V_m$ удовл. b. conditions $\Theta(x \to \pm\infty) = 0$ и ограничатся от DW $\Theta(\pm\infty) = 0$, $\Theta(\mp\infty) = \pi$.

если $\omega < 0$, то soliton velocity $V > V_m$.

// Eleonskii et al. ZETF 89, 2174 (1985) находят "wobbly" DW solution, которое не сохран. $\Phi(\xi) = \text{const}$ и дает $\Phi(x,t)$, $\Theta(x,t)$ не сводятся к $\Theta(\xi)$, $\Phi(\xi)$, $\xi = x - Vt$.

Где асимптотика при $x \to \infty$? $\Theta(x \to -\infty, t) = 0$, $\Theta(x \to +\infty, t) = \pi$ - DW. Internal DW degrees of freedom? Комментарии по поводу Fil'z - нет. Sin-Gordon на фоне осциллирующих $\Theta(t, x \to \pm\infty)$ граничных условий.

(15/II) Звонил Rase, обсуждали LL → Sin-Gordon mapping, послал main Refs по теме.

Note: если Kosevich'90 solitons $\Theta(\xi)$, $\Phi(\xi)$ при $\omega = 0$, то нарушает ли решение Sklyanin'79 (1/II) в класс засевших решений Kosevich'90? Удовлетворяет ли решение $\Theta(\xi)$, $\Phi(\xi)$, $\xi = x - vt$, по $S_x \ll 1$, $\varepsilon \to \infty$, удовлетв. ли Sklyanin'79 решение to the first integral by Eleonskii'78?

// Вопрос. $E_{sol}(V)$ на решении Kosevich'90. Продолжаем $E_s(V)$ зависим $E_{DW}(V)$?

при $V = V_0$, $k(V_0) = 0$, $\cos\Phi(\xi) = \frac{a}{|a|}\frac{1}{|\mathrm{sh}(x\xi + iy)|}$ $\quad \frac{(-1)\,\mathrm{ch}x\xi \,\mathrm{sh}d \sin\gamma}{\mathrm{sh}d} = \frac{(-\theta)}{|a|}\,\mathrm{ch}x\xi \sin\gamma \times$

$\times \frac{1}{\sqrt{\mathrm{sh}^2 x\xi \cos^2\gamma + \mathrm{ch}^2 x\xi \sin^2\gamma}} = \frac{(-\theta)}{|a|}\frac{\mathrm{ch}x\xi \sin\gamma}{\sqrt{\mathrm{sh}^2 x\xi + \sin^2\gamma}} = \frac{\mathrm{ch}x\xi \cdot \sin\gamma}{\sqrt{\mathrm{sh}^2 x\xi + \sin^2\gamma}} = \cos\Phi(\xi)$

$\mathrm{tg}\frac{\Theta(\xi)}{2} = |a|\frac{\mathrm{sh}d}{|\mathrm{sh}(x\xi + iy)|} = |a|\frac{\mathrm{sh}d}{\sqrt{\mathrm{sh}^2 x\xi + \sin^2\gamma}} = \frac{2x}{\sqrt{\varepsilon}\,\mathrm{sh}2d}\frac{\mathrm{sh}d}{\sqrt{\mathrm{sh}^2 x\xi + \sin^2\gamma}} = \frac{x}{\sqrt{\varepsilon}\,\mathrm{ch}d}\frac{1}{\sqrt{\mathrm{sh}^2 x\xi + \sin^2\gamma}}$

$\cos\Phi(\xi = 0) = 1$, $\Phi(0) = 0$, $\mathrm{tg}\frac{\Theta(\pm\infty)}{2} = 0$, $\Theta(\pm\infty) = 0$ - т. soliton, не DW.

$\mathrm{tg}\frac{\Theta(0)}{2} = \frac{x}{\sqrt{\varepsilon}}\frac{1}{\mathrm{ch}x\sin\gamma}$. 1st integral $I = (\Theta_\xi)^2 + (\Phi_\xi)^2 \sin^2\Theta - (1 + \varepsilon\cos^2\Phi)\sin^2\Theta$

$\mathrm{tg}\frac{\Theta(\xi)}{2} = f(\xi)$, $\Theta(\xi) = 2\,\mathrm{arctg}\,f(\varepsilon)$, $\Theta'_\xi = 2\frac{1}{1 + f^2(\xi)}\frac{\partial f}{\partial\xi}$, $\frac{\partial f}{\partial\xi} = \frac{\partial}{\partial\xi}\frac{b}{\sqrt{\mathrm{sh}^2 x\xi + \sin^2\gamma}} = b\left(-\frac{1}{2}\right)\frac{x\,\mathrm{sh}2x\xi}{(\ldots)^{3/2}}$

$\Theta_\xi(\xi \to +\infty) \sim \frac{e^{2x\xi}}{(e^{2x\xi})^{3/2}} \sim e^{-x\xi} = 0$. при $\Theta(\pm\infty) = 0$, $\Theta_\xi(\pm\infty) = 0$ будет $I = 0$

Escaneado con CamScanner

$E_{sol} = \frac{1}{2} M_3 H_a \int dx \left[(\theta_x)^2 + \sin^2\theta (\varphi_x)^2 + \sin^2\theta (1 + \varepsilon \cos^2\varphi) \right]$, $\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}$, $\xi = x - vt$

$\vec{M}_3 \cdot \vec{M}_3 = 1 = 0$ $(\theta_\xi)^2 + \sin^2\theta (\varphi_\xi)^2 = (1 + \varepsilon \cos^2\varphi) \sin^2\theta$, $E_{sol} = \frac{1}{2} M_3 H_a \int d\xi \sin^2\theta (1 + \varepsilon \cos^2\varphi) \cdot 2$

$\int d\xi \sin^2\theta (1 + \varepsilon \cos^2\varphi)$, $\sin\theta = \frac{2 \operatorname{tg}\theta/2}{1 + \operatorname{tg}^2\theta/2} = \frac{2 f(\xi)}{1 + f^2(\xi)}$, $\cos^2\varphi(\xi) = \frac{\operatorname{ch}^2 \alpha\xi \sin^2\gamma}{\operatorname{sh}^2 \alpha\xi + \sin^2\gamma}$

$f(\xi) = \frac{b}{\sqrt{\operatorname{sh}^2 \alpha\xi + \sin^2\gamma}}$, $\operatorname{sh}^2 \alpha\xi + \sin^2\gamma = \operatorname{sh}^2 \alpha\xi + 1 - \cos^2\gamma = \operatorname{ch}^2 \alpha\xi - \cos^2\gamma$

$J_1 = \int_{-\infty}^{\infty} d\xi \sin^2\theta = 4 \int d\xi \frac{f^2(\xi)}{[1 + f^2(\xi)]^2}$, $1 + f^2(\xi) = 1 + \frac{b^2}{\operatorname{sh}^2 \alpha\xi + \sin^2\gamma} = \frac{b^2 + \operatorname{sh}^2 \alpha\xi + \sin^2\gamma}{\operatorname{sh}^2 \alpha\xi + \sin^2\gamma}$

$J_1 = 4 \int d\xi \frac{b^2}{(\operatorname{sh}^2 \alpha\xi + \sin^2\gamma)} \frac{(\operatorname{sh}^2 \alpha\xi + \sin^2\gamma)^2}{(b^2 + \operatorname{sh}^2 \alpha\xi + \sin^2\gamma)^2} = 4b^2 \int d\xi \frac{(\operatorname{sh}^2 \alpha\xi + \sin^2\gamma)}{[b^2 + \operatorname{sh}^2 \alpha\xi + \sin^2\gamma]^2} =$

$= 4b^2 \int_{\xi} \frac{(\operatorname{ch}^2 \alpha\xi - \cos^2\gamma)}{[b^2 + \operatorname{ch}^2 \alpha\xi - \cos^2\gamma]^2} = \left| \alpha\xi = z \right| = \frac{4b^2}{\alpha} \int dz \frac{(\operatorname{ch}^2 z - \cos^2\gamma)}{[b^2 + \operatorname{ch}^2 z - \cos^2\gamma]^2} = \frac{4b^2}{\alpha} \int \frac{dz}{\operatorname{ch}^2 z} \times$

$\times \frac{\operatorname{ch}^2 z (\operatorname{ch}^2 z - \cos^2\gamma)}{[\cdots]^2} = \left| \frac{dz}{\operatorname{ch}^2 z} = d(\operatorname{th} z) = dx, \quad x = \operatorname{th} z, \quad \operatorname{th}^2 z + \frac{1}{\operatorname{ch}^2 z} = 1, \quad \operatorname{ch}^2 z = \frac{(-1)}{\operatorname{th}^2 z - 1} = \frac{(-1)}{x^2 - 1} = \frac{1}{1 - x^2} \right| =$

$= \frac{4b^2}{\alpha} \int_{-1}^{+1} dx \frac{\frac{1}{(1-x^2)} \left[\frac{1}{(1-x^2)} - \cos^2\gamma \right]}{\left[b^2 + \frac{1}{(1-x^2)} - \cos^2\gamma \right]^2} = \frac{4b^2}{\alpha} \int dx \frac{\left[1 - (1-x^2)\cos^2\gamma \right]}{\left[1 + (b^2 - \cos^2\gamma)(1-x^2) \right]^2} = \frac{4b^2}{\alpha} \int dx \frac{\sin^2\gamma + x^2 \cos^2\gamma}{\left[\sin^2\gamma + x^2 \cos^2\gamma + b^2 (1-x^2) \right]^2}$

$J_1 = \frac{4b^2}{\alpha} \int_{-1}^{+1} dx \frac{\sin^2\gamma + x^2 \cos^2\gamma}{[b^2 + \sin^2\gamma + x^2 (\cos^2\gamma - b^2)]^2}$, $b = \frac{x}{\sqrt{\varepsilon}\, \operatorname{ch} d}$ считаем $b^2 \ll 1$ (это верно или $\varepsilon \gg 1$)

$\cos^2\gamma - b^2 \geq 0$

$J_2 = \int_{-1}^{+1} dx \frac{1}{[b^2 \sin^2\gamma + (\cos^2\gamma - b^2) x^2]^2} = \frac{1}{(\cos^2\gamma - b^2)^2} \int dx \frac{1}{\left[\frac{b^2 + \sin^2\gamma}{\cos^2\gamma - b^2} + x^2 \right]^2} = \frac{1}{(\cdot)^2} \int dx \frac{1}{[\tilde{a}^2 + x^2]^2}$

$\tilde{a}^2 = \frac{b^2 + \sin^2\gamma}{(\cos^2\gamma - b^2)} = \frac{b^2 + \sin^2\gamma}{1 - \sin^2\gamma - b^2} = (-1) \frac{1 - b^2 - \sin^2\gamma - 1}{1 - b^2 - \sin^2\gamma} = \frac{1}{1 - b^2 - \sin^2\gamma} - 1 \approx \operatorname{tg}^2\gamma > 0$ при $b^2 \to 0$

Вообще-то $x^2 = -k^2$ (Schlomann) $\in [1, 1+\varepsilon]$ где DW, то $x_m^2 = (1+\varepsilon)^{1/2}$ при $v = v_-$

$v_- = v_m(\alpha)$

$b^2 = \frac{x^2}{\varepsilon \operatorname{ch}^2 d} \simeq \frac{x_m^2}{\varepsilon} \frac{1}{\operatorname{ch}^2 d} = \frac{(1+\varepsilon)^{1/2}}{\varepsilon} \frac{1}{\operatorname{ch}^2 d} \ll 1$ при $\varepsilon \gg 1$.

$J_2 = \frac{1}{(\cos^2\gamma - b^2)^2} \int_{-1}^{+1} \frac{dx}{[\tilde{a}^2 + x^2]^2} = \frac{1}{(\cdot)^2} \left[\frac{1}{2\tilde{a}^2} \frac{x}{(\tilde{a}^2 + x^2)} + \frac{1}{2\tilde{a}^3} \operatorname{arctg} \frac{x}{\tilde{a}} \right]_{-1}^{+1} = \frac{1}{(\cos^2\gamma - b^2)^2} \times \frac{1}{\tilde{a}^2} \times$

$\times \left[\frac{1}{(\tilde{a}^2 + 1)} + \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right] = \frac{1}{(\cos^2\gamma - b^2)^2} \frac{(\cos^2\gamma - b^2)}{(b^2 + \sin^2\gamma)} \left[\frac{1}{(1 + \tilde{a}^2)} + \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right]$

$J_3 = \int dx \frac{x^2}{[b^2 + \sin^2\gamma + (\cos^2\gamma - b^2) x^2]^2} = \Big| = \frac{1}{(\cos^2\gamma - b^2)^2} \int dx \frac{x^2}{[\tilde{a}^2 + x^2]^2} = \frac{1}{(\cos^2\gamma - b^2)^2} \left[-\frac{1}{2} \frac{x}{\tilde{a}^2 + x^2} + \right.$

$\left. + \frac{1}{2\tilde{a}} \operatorname{arctg} \frac{x}{\tilde{a}} \right]_{-1}^{+1} = \frac{1}{(\cos^2\gamma - b^2)^2} \left[-\frac{1}{1 + \tilde{a}^2} + \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right]$

$J_1 = \frac{4b^2}{\alpha} \left[\sin^2\gamma\, J_2 + \cos^2\gamma\, J_3 \right] = \frac{4b^2}{\alpha} \left[\frac{\sin^2\gamma}{(b^2 + \sin^2\gamma)(\cos^2\gamma - b^2)} \left\{ \frac{1}{(1 + \tilde{a}^2)} + \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right\} + \right.$

$\left. + \cos^2\gamma \frac{1}{(\cos^2\gamma - b^2)^2} \left\{ \frac{(-1)}{1 + \tilde{a}^2} + \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right\} \right] = \frac{4b^2}{\alpha} \frac{1}{(b^2 + \sin^2\gamma)(\cos^2\gamma - b^2)^2} \times$

$\times \left[\sin^2\gamma (\cos^2\gamma - b^2) \left\{ \frac{1}{(1 + a^2)} + \frac{1}{a} \operatorname{arctg} \frac{1}{a} \right\} + \cos^2\gamma (b^2 + \sin^2\gamma) \left\{ -\frac{1}{1 + a^2} + \frac{1}{a} \operatorname{arctg} \frac{1}{a} \right\} \right] =$

$= \frac{4b^2}{\alpha} \frac{1}{(b^2 + \sin^2\gamma)(\cos^2\gamma - b^2)^2} \left[-\frac{b^2}{(1 + \tilde{a}^2)} + (2 \sin^2\gamma \cos^2\gamma + b^2 \cos 2\gamma) \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right]$, $\tilde{a}^2 = \frac{(b^2 + \sin^2\gamma)}{(\cos^2\gamma - b^2)}$

$J_1(\gamma) = \int_{-\infty}^{\infty} d\xi \sin^2\theta$, сим. 14/II $\gamma = 0, \pi/2$ при $v \leq v_0$, $J_1(0) = \frac{4b^2}{\alpha} \frac{1}{b^2 (1 - b^2)^2} \times$

$\times \left[-\frac{b^2}{\left[1 + \frac{b^2}{(1 - b^2)} \right]} + \frac{b^2}{b \sqrt{1 - b^2}} \operatorname{arctg} \frac{1}{b} \sqrt{1 - b^2} \right] = \frac{4b^2}{\alpha} \frac{1}{(1 - b^2)^2} \left[b^2 - 1 + \frac{1}{b} \sqrt{1 - b^2} \operatorname{arctg} \frac{1}{b} \sqrt{1 - b^2} \right]$

$J_1(\pi/2) = \frac{4b^2}{\alpha} \frac{1}{(b^2 + 1)} \frac{1}{b^4} \left[-\frac{b^2}{\left[1 + \left(-1 - \frac{1}{b^2} \right) \right]} + (-b^2) \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right]$, $\tilde{a}^2 (\gamma = \frac{\pi}{2}) = -1 - \frac{1}{b^2} < 0$ и

в вычислении J_1 при $\gamma = \pi/2$ не проходит – надо пересчитать с $\tilde{a}^2 = -a^2 < 0$.

$J_1(x, \gamma)$, $x = x(v)$ – надо еще найти. Видно $x = x(v_0) = (1+\varepsilon)^{1/4}$, т.к. $v = v_0 = v_- = v_m$

Осталось еще найти $d = d(v_0) \Rightarrow J_1 = J_1(\varepsilon)$ только. 6/II показать $E_{sol}(v_0) = E_0 (1+\varepsilon)^{1/4}$

Осталось найти $J_4 = \int_{-\infty}^{\infty} d\xi \sin^2\theta \cos^2\varphi$ чтоб получить $E_{sol} = M_3 H_a \int d\xi \sin^2\theta (1 + \varepsilon \cos^2\varphi)$

Не нужно, что $E_{sol}(v_0) = E_{sw}(v_0)$. Проверить это.

Ананьев file 4 → sin-Gordon eq. sent by Ricardo. Там неверно, т.к. все при $\varphi = \text{const}$.

Escaneado con CamScanner

$E_{sol} = \frac{1}{2} M_3 H_a \int dx \left[(\theta_x)^2 + \sin^2\theta (\phi_x)^2 + \sin^2\theta (1 + \varepsilon \cos^2\phi) \right]$, $\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}$, $\xi = x - vt$

$\omega_3 \cdot 2\pi \; T = 0$ $(\theta_\xi)^2 + \sin^2\theta (\phi_\xi)^2 = (1 + \varepsilon \cos^2\phi) \sin^2\theta$, $E_{sol} = \frac{1}{2} M_3 H_a \int d\xi \sin^2\theta (1 + \varepsilon \cos^2\phi) \cdot 2$

$\int d\xi \sin^2\theta (1 + \varepsilon \cos^2\phi)$, $\sin\theta = \frac{2 \operatorname{tg}\theta/2}{1 + \operatorname{tg}^2\theta/2} = \frac{2 f(\xi)}{1 + f^2(\xi)}$, $\cos^2\phi(\xi) = \frac{\operatorname{ch}^2 x\xi \sin^2\gamma}{\operatorname{sh}^2 x\xi + \sin^2\gamma}$

$f(\xi) = \frac{b}{\sqrt{\operatorname{sh}^2 x\xi + \sin^2\gamma}}$, $\operatorname{sh}^2 x\xi + \sin^2\gamma = \operatorname{sh}^2 x\xi + 1 + \cos^2\gamma = \operatorname{ch}^2 x\xi - \cos^2\gamma$

$J_1 = \int_{-\infty}^{\infty} d\xi \sin^2\theta = 4 \int d\xi \frac{f^2(\xi)}{[1 + f^2(\xi)]^2}$, $1 + f^2(\xi) = 1 + \frac{b^2}{\operatorname{sh}^2 x\xi + \sin^2\gamma} = \frac{b^2 + \operatorname{sh}^2 x\xi + \sin^2\gamma}{\operatorname{sh}^2 x\xi + \sin^2\gamma}$

$J_1 = 4 \int d\xi \frac{b^2}{(\operatorname{sh}^2 x\xi + \sin^2\gamma)} \frac{(\operatorname{sh}^2 x\xi + \sin^2\gamma)^2}{(b^2 + \operatorname{sh}^2 x\xi + \sin^2\gamma)^2} = 4b^2 \int d\xi \frac{(\operatorname{sh}^2 x\xi + \sin^2\gamma)}{[b^2 + \operatorname{sh}^2 x\xi + \sin^2\gamma]^2} =$

$= 4b^2 \int d\xi \frac{(\operatorname{ch}^2 x\xi - \cos^2\gamma)}{[b^2 + \operatorname{ch}^2 x\xi - \cos^2\gamma]^2} = \left| x\xi = z \right| = \frac{4b^2}{x} \int dz \frac{(\operatorname{ch}^2 z - \cos^2\gamma)}{[b^2 + \operatorname{ch}^2 z - \cos^2\gamma]^2} = \frac{4b^2}{x} \int \frac{dz}{\operatorname{ch}^2 z} \times$

$\times \frac{\operatorname{ch}^2 z (\operatorname{ch}^2 z - \cos^2\gamma)}{[\cdots]^2} = \left| \frac{dz}{\operatorname{ch}^2 z} = d(\operatorname{th} z) = dx, \; x = \operatorname{th} z, \; \operatorname{th}^2 z + \frac{1}{\operatorname{ch}^2 z} = 1, \; \operatorname{ch}^2 z = \frac{(-1)}{\operatorname{th}^2 z - 1} = \frac{(-1)}{x^2 - 1} = \frac{1}{1 - x^2} \right| =$

$= \frac{4b^2}{x} \int_{-1}^{+1} dx \frac{\frac{1}{(1-x^2)} \left[\frac{1}{(1-x^2)} - \cos^2\gamma \right]}{\left[b^2 + \frac{1}{(1-x^2)} - \cos^2\gamma \right]^2} = \frac{4b^2}{x} \int dx \frac{\left[1 - (1-x^2)\cos^2\gamma \right]}{\left[1 + (b^2 - \cos^2\gamma)(1-x^2) \right]^2} = \frac{4b^2}{x} \int dx \frac{\sin^2\gamma + x^2 \cos^2\gamma}{\left[\sin^2\gamma + x^2\cos^2\gamma + b^2(1-x^2) \right]^2}$

$J_1 = \frac{4b^2}{x} \int_{-1}^{+1} dx \frac{\sin^2\gamma + x^2\cos^2\gamma}{\left[b^2 + \sin^2\gamma + x^2(\cos^2\gamma - b^2) \right]^2}$, $b = \frac{x}{\sqrt{\varepsilon}\,\operatorname{ch}\alpha}$, считаем $b^2 \ll 1$ (это верно или $\varepsilon \gg 1$)

$\cos^2\gamma - b^2 > 0$

$J_2 = \int_{-1}^{+1} dx \frac{1}{\left[b^2 + \sin^2\gamma + (\cos^2\gamma - b^2)x^2 \right]^2} = \frac{1}{(\cos^2\gamma - b^2)^2} \int dx \frac{1}{\left[\frac{b^2 + \sin^2\gamma}{\cos^2\gamma - b^2} + x^2 \right]^2} = \frac{1}{(\cdot)^2} \int dx \frac{1}{[\tilde{a}^2 + x^2]^2}$

$\tilde{a}^2 = \frac{b^2 + \sin^2\gamma}{(\cos^2\gamma - b^2)} = \frac{b^2 + \sin^2\gamma}{1 - \sin^2\gamma - b^2} = (-1) \frac{1 - b^2 - \sin^2\gamma - 1}{1 - b^2 - \sin^2\gamma} = \frac{1}{1 - b^2 - \sin^2\gamma} - 1 \approx \operatorname{tg}^2\gamma > 0$ при $b^2 \to 0$

Вообще-то $x^2 = -k^2$ (Schlomann) $\in [1, 1+\varepsilon]$ где Dw, то $x_m^2 = (1+\varepsilon)^{1/2}$ при $v = v_-$, $v_- = v_m(x)$

$b^2 = \frac{x^2}{\varepsilon \operatorname{ch}^2\alpha} \approx \frac{x_m^2}{\varepsilon} \frac{1}{\operatorname{ch}^2\alpha} = \frac{(1+\varepsilon)^{1/2}}{\varepsilon} \frac{1}{\operatorname{ch}^2\alpha} < 1$ при $\varepsilon \gg 1$.

$J_2 = \frac{1}{(\cos^2\gamma - b^2)^2} \int_{-1}^{+1} \frac{dx}{[\tilde{a}^2 + x^2]^2} = \frac{1}{(\cdot)^2} \left[\frac{1}{2\tilde{a}^2} \frac{x}{(\tilde{a}^2 + x^2)} + \frac{1}{2\tilde{a}^3} \operatorname{arctg} \frac{x}{\tilde{a}} \right]_{-1}^{+1} = \frac{1}{(\cos^2\gamma - b^2)^2} \times \frac{1}{2\tilde{a}^2} \times$

$\times \left[\frac{1}{(\tilde{a}^2 + 1)} + \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right] = \frac{1}{(\cos^2\gamma - b^2)^2} \frac{(\cos^2\gamma - b^2)}{(b^2 + \sin^2\gamma)} \left[\frac{1}{(1 + \tilde{a}^2)} + \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right]$

$J_3 = \int dx \frac{x^2}{\left[b^2 + \sin^2\gamma + (\cos^2\gamma - b^2)x^2 \right]^2} = \frac{1}{(\cos^2\gamma - b^2)^2} \int dx \frac{x^2}{[\tilde{a}^2 + x^2]^2} = \frac{1}{(\cos^2\gamma - b^2)^2} \left[-\frac{1}{2} \frac{x}{\tilde{a}^2 + x^2} + \right.$

$\left. + \frac{1}{2\tilde{a}} \operatorname{arctg} \frac{x}{\tilde{a}} \right]_{-1}^{+1} = \frac{1}{(\cos^2\gamma - b^2)^2} \left[-\frac{1}{1 + \tilde{a}^2} + \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right]$

$J_1 = \frac{4b^2}{x} \left[\sin^2\gamma J_2 + \cos^2\gamma J_3 \right] = \frac{4b^2}{x} \left[\frac{\sin^2\gamma}{(b^2 + \sin^2\gamma)(\cos^2\gamma - b^2)} \left\{ \frac{1}{(1 + \tilde{a}^2)} + \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right\} + \right.$

$\left. + \cos^2\gamma \frac{1}{(\cos^2\gamma - b^2)^2} \left\{ \frac{(-1)}{1 + \tilde{a}^2} + \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right\} \right] = \frac{4b^2}{x} \frac{1}{(b^2 + \sin^2\gamma)(\cos^2\gamma - b^2)^2} \times$

$\times \left[\sin^2\gamma (\cos^2\gamma - b^2) \left\{ \frac{1}{(1 + \tilde{a}^2)} + \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right\} + \cos^2\gamma (b^2 + \sin^2\gamma) \left\{ -\frac{1}{1 + \tilde{a}^2} + \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right\} \right] =$

$= \frac{4b^2}{x} \frac{1}{(b^2 + \sin^2\gamma)(\cos^2\gamma - b^2)^2} \left[-\frac{b^2}{(1 + \tilde{a}^2)} + (2\sin^2\gamma \cos^2\gamma + b^2 \cos 2\gamma) \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right]$, $\tilde{a}^2 = \frac{(b^2 + \sin^2\gamma)}{(\cos^2\gamma - b^2)}$

$J_1(\gamma) = \int_{-\infty}^{\infty} d\xi \sin^2\theta$, Сим. 14/III $\gamma = 0, \pi/2$ при $v \le v_0$, $J_1(0) = \frac{4b^2}{x} \frac{1}{b^2} \frac{1}{(1 - b^2)^2} \times$

$\times \left[-\frac{b^2}{\left[1 + \frac{b^2}{(1 - b^2)} \right]} + \frac{b^2}{\frac{b}{\sqrt{1 - b^2}}} \operatorname{arctg} \frac{1}{b} \sqrt{1 - b^2} \right] = \frac{4b^2}{x} \frac{1}{(1 - b^2)^2} \left[b^2 - 1 + \frac{1}{b} \sqrt{1 - b^2} \operatorname{arctg} \frac{1}{b} \sqrt{1 - b^2} \right]$

$J_1(\pi/2) = \frac{4b^2}{x} \frac{1}{(b^2 + 1)} \frac{1}{b^4} \left[-\frac{b^2}{\left[1 + \left(-1 - \frac{1}{b^2} \right) \right]} + (-b^2) \frac{1}{\tilde{a}} \operatorname{arctg} \frac{1}{\tilde{a}} \right]$, $\tilde{a}^2(\gamma = \frac{\pi}{2}) = -1 - \frac{1}{b^2} < 0$ и

в вычислении J_1 при $\gamma = \pi/2$ не проходит – надо пересчитать с $\tilde{a}^2 = -a^{*2} < 0$.

$J_1(x, \gamma)$, $x = x(v)$ – надо еще найти. Ясно что $x = x(v_0) = (1+\varepsilon)^{1/4}$, т.к. $V = V_c = v_- = v_m$

Осталось еще найти $\alpha = \alpha(v_0) \Rightarrow J_1 = J_1(\varepsilon)$ только. б/п показан $E_{sol}(v_0) = E_0 (1+\varepsilon)^{1/4}$

Осталось найти $J_4 = \int_{-\infty}^{\infty} d\xi \sin^2\theta \cos^2\phi$ этот интеграл $E_{sol} = M_3 H_a \int d\xi \sin^2\theta (1 + \varepsilon \cos^2\phi)$

Не нужно, что $E_{sol}(v_0) = E_{sol}(v_0)$. Проверить это.

Аналог file 4 → sin-Gordon eq. sent by Ricardo. Там неверно, т.к. все при $\Phi = \text{const}$.

Escaneado con CamScanner

(16/II) Сегодня 14/II при $V \le V_0$, $k=0$ и $\gamma = 0, \pi/2$.

$$W\left[\xi, \gamma=0, \underset{\omega=0}{k=0}\right] = -\frac{2\varkappa}{\sqrt{\varepsilon}} \frac{1}{\operatorname{sh} 2\alpha} \frac{\sin(-i\alpha)}{\operatorname{sh}(\varkappa\xi + i\gamma)} = -\frac{2\varkappa}{\sqrt{\varepsilon}} \frac{1}{\operatorname{sh}2\alpha} \frac{-i\operatorname{sh}\alpha}{[\operatorname{sh}\varkappa\xi\cos\gamma + i\operatorname{ch}\varkappa\xi\sin\gamma]}$$

$k=0$ $W(\xi) = \frac{i\varkappa}{\sqrt{\varepsilon}} \frac{1}{\operatorname{ch}\alpha} \frac{1}{[\operatorname{sh}\varkappa\xi\cos\gamma + i\operatorname{ch}\varkappa\xi\sin\gamma]}$, $\operatorname{tg}^2\frac{\Theta(\xi)}{2} = \frac{\varkappa^2}{\varepsilon\operatorname{ch}^2\alpha} \frac{1}{[\operatorname{sh}^2\varkappa\xi\cos^2\gamma + \operatorname{ch}^2\varkappa\xi\sin^2\gamma]}$

Видно $\Theta(\xi \to \pm\infty) = 0$ $\Theta(\xi)$ – even function of ξ. Это soliton, а не DW

с $\Theta(\xi \to \pm\infty) = (0, \pi)$. Хотя при $V \le V_0$ $\Phi(\xi) = \text{const}$, $\Theta = \Theta(\xi)$

$1 - \varkappa^2 = -\frac{\varepsilon}{2}(1 + \cos 2\gamma \operatorname{ch}2\alpha)$ дает $\alpha(\varkappa)$. $\operatorname{ch}2\alpha = \frac{1}{\cos 2\gamma}\left[\frac{2}{\varepsilon}(\varkappa^2 - 1) - 1\right]$

$\operatorname{tg}\frac{\Theta(\xi)}{2} = \frac{\varkappa}{\sqrt{\varepsilon}\operatorname{ch}\alpha} \frac{1}{\sqrt{\operatorname{sh}^2\varkappa\xi\cos^2\gamma + \operatorname{ch}^2\varkappa\xi\sin^2\gamma}}$, при $\cos\gamma = 0$, $\operatorname{tg}\frac{\Theta(\xi)}{2} = \frac{\varkappa}{\sqrt{\varepsilon}\operatorname{ch}\alpha} \frac{1}{(\operatorname{sh}\varkappa\xi)}$

11/II аналогия для точного уравнения $y(\xi, \Phi)$, $\operatorname{tg}\frac{\Theta}{2} = e^y$, которое при $\Phi = \text{const}$

$y_\xi = \frac{\varepsilon}{2}\sin 2\Phi$ и $y_{\xi\xi} + [1 + \varepsilon\cos^2\Phi - (y_\xi)^2]\operatorname{th} y = 0 \to (y_\xi)^2 = 1 + \varepsilon\cos^2\Phi$, $y_{\xi\xi} = 0$

Soliton [Kosevich'90]: $\operatorname{tg}\frac{\Theta}{2} = \frac{\delta}{\operatorname{sh}\varkappa\xi}$, $\delta = \frac{\varkappa}{\sqrt{\varepsilon}\operatorname{ch}\alpha}$

$\operatorname{tg}\frac{\Theta}{2} = e^y$, $y = \ln\operatorname{tg}\frac{\Theta}{2}$, $y(\xi) = \ln\delta - \ln\operatorname{sh}(\varkappa\xi)$, $y_\xi = -\varkappa\operatorname{cth}(\varkappa\xi)$,

$y_{\xi\xi} = \varkappa^2 \frac{1}{\operatorname{sh}^2\varkappa\xi} \neq 0$, $y_{\xi\xi}(\xi)$ дает симметрию при $\xi = 0$.

// Т.е. soliton solutions by Kosevich'90, $\operatorname{tg}\frac{\Theta}{2}(\xi) \sim \frac{1}{\operatorname{sh}\varkappa\xi}$ $(\gamma = 0)$, $\operatorname{tg}\frac{\Theta}{2}(\xi) \sim \frac{1}{\operatorname{ch}\varkappa\xi}$ $(\gamma = \frac{\pi}{2})$

не удовлетворяют ур. eq. (!?). Это решение при $V \le V_0$, $k=0$, $\omega = 0$.

Найдем $k(V)$, $\varkappa(V)$. $K = k + i\varkappa$, $\Omega = \tilde\omega + i\varkappa V$, $\Omega^2 = (1+K^2)(1+\varepsilon+K^2)$

Для stationary solitons $\omega = 0$ и $\tilde\omega = kV \to \Omega = kV + i\varkappa V = KV$

$K^2V^2 = (1+K^2)(1+\varepsilon+K^2)$, $z = 1+K^2$, $(z-1)V^2 = z(z+\varepsilon)$

$z^2 + \varepsilon z - (z-1)V^2 = 0$, $z^2 + (\varepsilon - V^2)z + V^2 = 0$, $2z_{1,2} = V^2 - \varepsilon \pm \sqrt{(V^2-\varepsilon)^2 - 4V^2}$

$D(V) = (V^2-\varepsilon)^2 - 4V^2$, $V_0 = \sqrt{1+\varepsilon} - 1$, $V_m = \sqrt{1+\varepsilon} + 1$ (max, min relk ph. velocity)

$(V^2 - V_0^2)(V^2 - V_m^2) = V^4 - (V_0^2 + V_m^2)V^2 + V_0^2V_m^2$, $V_0^2 V_m^2 = \left[\sqrt{1+\varepsilon} - 1\right]^2\left[\sqrt{1+\varepsilon}+1\right]^2 =$

$= \left[(\sqrt{1+\varepsilon}-1)(\sqrt{1+\varepsilon}+1)\right]^2 = \left[(\sqrt{1+\varepsilon})^2 - 1^2\right]^2 = \varepsilon^2$, $V_0^2 + V_m^2 = \left[\sqrt{1+\varepsilon} - 1\right]^2 + \left[\sqrt{1+\varepsilon}+1\right]^2 =$

$= 1 + \varepsilon + 1 - 2\sqrt{1+\varepsilon} + 1 + \varepsilon + 1 + 2\sqrt{1+\varepsilon} = 2(2+\varepsilon)$

$(V^2 - V_0^2)(V^2 - V_m^2) = V^4 - 2(2+\varepsilon)V^2 + \varepsilon^2$, $D(V) = V^4 + \varepsilon^2 - 2\varepsilon V^2 - 4V^2$

$D(V) = (V^2 - V_0^2)(V^2 - V_m^2) \to 2z_{1,2} = 2(1 + K_{1,2}^2) = V^2 - \varepsilon \pm \sqrt{(V^2 - V_0^2)(V^2 - V_m^2)}$

$K_{1,2}^2 = -1 + \frac{1}{2}(V^2 - \varepsilon) \pm \frac{1}{2}\sqrt{(V^2 - V_0^2)(V^2 - V_m^2)} = k^2 - \varkappa^2 + 2ik\varkappa$, $V_0 < V_m \; \forall\, \varepsilon$ – finite

Видно, что при $V \le V_0$, $V \ge V_m$ $D(V) > 0$ и $K_{1,2}^2(V)$ is pure real, т.к. $k(V)\varkappa(V) = 0$. При $V_0 < V < V_m$, $K_{1,2}^2$ имеет real и imaginary parts, $k(V) \neq 0$, $\varkappa(V) \neq 0$

$K_{1,2}^2(V \to 0) = -1 - \frac{\varepsilon}{2} \pm \frac{1}{2}\sqrt{V_0^2 V_m^2} = -1 - \frac{\varepsilon}{2} \pm \frac{1}{2}\varepsilon = \begin{cases} -1 \\ -(1+\varepsilon) \end{cases}$, это совпад. $\gamma^2(k^2)$ Schlömann

Вывод с "+" дает совпад. stable DW см. Schlömann'11 при $V \le V_0$. расчет. 19/I, 13/II

При $V \le V_0$, надо брать $\varkappa(V) \neq 0$, $k(V) = 0$, это и дает

// Soliton solution $W(\xi)$ если и верно, то при $V > V_0$ (!). $W = \operatorname{tg}\frac{\Theta}{2}e^{i\Phi} = e^{y + i\Phi}$ Kosevich'90

Делая подстановку $\operatorname{tg}\frac{\Theta}{2} = e^y \to W = e^{y+i\Phi}$, и отражение $|W| = 1$ решение

$m_z = \frac{1 - |W|^2}{1 + |W|^2} = 0$ (!). Но это не так, верно $\cos\Theta = \pm\operatorname{th} y$. $\cos\Theta = \frac{1 - \operatorname{tg}^2\Theta/2}{1 + \operatorname{tg}^2\Theta/2} = -\operatorname{th} y$

Нет, $|W|^2 = e^{2y}$.

$y + i\Phi = \ln W$ – можно записать eqs. for $W(\xi) = W'(\xi) + iW''(\xi)$.

При $V \ge V_m$ $K_{1,2}^2$ снова is pure real, $\varkappa(V)k(V) = 0$, но уже $\varkappa(V) = 0$, $k(V) \neq 0$

$\to V \ge V_m$, $k \neq 0$, $\varkappa = 0 \to W(\xi) = -\frac{2\varkappa}{\sqrt{\varepsilon}} \frac{1}{\operatorname{sh}2\alpha} \frac{\sin(k\xi - i\alpha)}{\operatorname{sh}(i\gamma)} = -\frac{2\varkappa}{\sqrt{\varepsilon}} \frac{1}{\operatorname{sh}2\alpha} \frac{[\sin k\xi \operatorname{ch}\alpha - i\cos k\xi \operatorname{sh}\alpha]}{i\sin\gamma}$, $\varkappa = 0$

Видно $\frac{\varkappa}{\sin\gamma} \to$ finite при $\varkappa \to 0$. $1 + K^2 = 1 + k^2 = -\frac{\varepsilon}{2}\left[1 + \operatorname{ch}2\alpha\cos 2\gamma - i\sin 2\gamma \operatorname{sh}2\alpha\right]$ - real

$\sin 2\gamma \operatorname{sh}2\alpha = 0$, $\gamma = 0$ или $\gamma = \pi/2$. $1 + k^2 - \varkappa^2 + 2i\varkappa k = -\frac{\varepsilon}{2}\left[1 + \operatorname{ch}2\alpha\cos 2\gamma - i\operatorname{sh}2\alpha\sin 2\gamma\right]$

$\varkappa k = \frac{\varepsilon}{4}\operatorname{sh}2\alpha \cdot 2\sin\gamma\cos\gamma$, $\frac{\varkappa}{\sin\gamma} = \frac{\varepsilon}{2k}\operatorname{sh}2\alpha\cos\gamma \to W(\xi) = -\frac{2\varkappa(-i)}{\sqrt{\varepsilon}\operatorname{sh}2\alpha\sin\gamma}[\cdots]$

$W(\xi) = i\frac{\sqrt{\varepsilon}}{k}\cos\gamma\left[\sin k\xi\operatorname{ch}\alpha - i\cos k\xi \cdot \operatorname{sh}\alpha\right]$, $\operatorname{tg}\frac{\Theta(\xi)}{2} = |W(\xi)| = \frac{\sqrt{\varepsilon}}{k}\cos\gamma\sqrt{\sin^2 k\xi\operatorname{ch}^2\alpha + \cos^2 k\xi\operatorname{sh}^2\alpha}$

$\cos\Phi(\xi)_{\varkappa=0} = \frac{a}{|a|}\frac{1}{\sin\gamma}\frac{(-\cos k\xi \operatorname{sh}\alpha\sin\gamma)}{\sqrt{(\cdot)}} = \cos k\xi \cdot \operatorname{sh}\alpha(\sin^2 k\xi\operatorname{ch}^2\alpha + \cos^2 k\xi\operatorname{sh}^2\alpha)^{-1/2}$

Escaneado con CamScanner

Для $V \geq V_m$ Косевич'го дал Soliton solutions $\Theta(\xi)$, $\Phi(\xi) = F(\sin k\xi, \cos k\xi)$ осуществл-
с ξ change ($\xi = x - Vt$) с периодом $T_\xi = \frac{2\pi}{k}$ → Nonlinear Sw (?).

$1 + k^2 = 1 + k^2 - \varkappa^2 + 2ik\varkappa = -\frac{\varepsilon}{2}\left[1 + \operatorname{ch}2\alpha\cos 2\gamma - i\operatorname{sh}2\alpha\cdot\sin 2\gamma\right]$

При $\varkappa$ - finite, $k \to 0$, $2ik\varkappa = i\frac{\varepsilon}{2}\operatorname{sh}2\alpha\sin 2\gamma$, $k = \frac{\varepsilon}{4\varkappa}\operatorname{sh}2\alpha\sin 2\gamma = \frac{\varepsilon}{2\varkappa}\operatorname{sh}2\alpha\sin\gamma\cos\gamma$

При $V \leq V_0$, $k \to 0$ (единственное $\cos\Phi(\xi) = \frac{a}{|a|}\frac{1}{\operatorname{sh}\alpha}\frac{[k\xi\cdot\operatorname{sh}\varkappa\xi\operatorname{ch}\alpha\cos\gamma - \operatorname{ch}\varkappa\xi\operatorname{sh}\alpha\sin\gamma]}{\sqrt{\operatorname{sh}^2\varkappa\xi\cos^2\gamma + \operatorname{ch}^2\varkappa\xi\sin^2\gamma}} =$ ($\sin k\xi \to k\xi$)

$= \frac{a}{|a|}\frac{1}{\operatorname{sh}\alpha}\frac{1}{\sqrt{(\dots)}}\left[\frac{\varepsilon}{2\varkappa}\operatorname{sh}2\alpha\sin\gamma\cos\gamma\cdot\xi\operatorname{sh}\varkappa\xi\operatorname{ch}\alpha\cos\gamma - \operatorname{ch}\varkappa\xi\operatorname{sh}\alpha\cdot\sin\gamma\right] = \frac{a}{|a|}\frac{1}{\operatorname{sh}\alpha}\frac{1}{\sqrt{(\dots)}} \times$

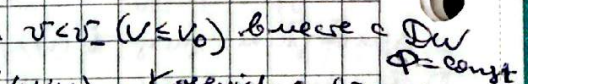
$\times \operatorname{sh}\alpha\sin\gamma\left[\frac{\varepsilon\xi}{\varkappa}\operatorname{ch}^2\alpha\cos^2\gamma\operatorname{sh}\varkappa\xi - \operatorname{ch}\varkappa\xi\right] = \cos\Phi(\xi)$, $\cos\Phi(\xi) \sim \sin\gamma$, $\sin\gamma \sim k(V) \to 0$ при $k \to 0$

$\Phi(\xi) \to \pm\frac{\pi}{2}$, $\cos\Phi(\xi) \to 0$, $k \to 0$

Т.е. все же $\Phi(\xi) \to \pm\frac{\pi}{2}$, $\Phi(\xi) = \text{const}$ при $V \leq V_0$, $k \to 0$, решение Косевич'го не применимо при $V \leq V_0$ ($v < v_m$ в обозн. Schlömann'71)

Sklyanin'79: Sin-Gordon $\varphi_{xx} - \frac{1}{c^2}\varphi_{tt} = \frac{1}{2\Delta_0^2}\sin 2\varphi$

Sklyanin'79



$c = \frac{2\gamma}{M_s}\sqrt{AK_x} = \frac{2\gamma}{M_s}\sqrt{AK_z}\sqrt{\frac{K_x}{K_z}} = v_0\sqrt{\varepsilon}$, $v_0 = \frac{2\gamma}{M_s}\sqrt{AK_z}$. c in units v_0: $c = \sqrt{\varepsilon}$, $\varepsilon \gg 1$.

$v_\pm(V_0, V_m) = \sqrt{1+\varepsilon} \pm 1$. При $\varepsilon \gg 1$ $v_+ \approx v_- = \sqrt{\varepsilon} = c$. $\Delta(v) = \Delta_0\sqrt{1 - v^2/c^2}$, $\Delta_0^2 = A/K_z$.

$v < c$ неочевидно. Но $c = \sqrt{\varepsilon}$ это $v_-(V_0)$ или $v_+(V_m)$?

а) Если $c = v_-$, то Sklyanin solution существует при $v < v_-$ ($V \leq V_0$) вместе с Sw $\Phi = \text{const}$.

в) Если $c = v_+$, то Sklyanin solution сосуществует с Sw ($V < V_0$) и Kosevich solitons — тогда Kosevich-solution не единственное при $V \in [V_0, V_m]$. ($V > V_0$, $V \leq V_m$)

Sklyanin solution: $S_x(x,t) = -\frac{\sqrt{\gamma}}{R}\frac{v}{\sqrt{1-v^2}}\frac{1}{\operatorname{ch}u}$, $S_y = \frac{1}{\operatorname{ch}u}$, $S_z = \operatorname{th}u$, $u = \frac{\sqrt{\gamma}}{\sqrt{1-v^2}}(x/\sqrt{R} - vt)$ $R \to \infty$

$u = \frac{\sqrt{\gamma}}{\sqrt{R}}\frac{(x - Vt)}{\sqrt{1 - V^2/V_m^2}}$, $V = \sqrt{R}v$, $V_m = \sqrt{R}v_0 = \sqrt{R}\cdot\frac{2\gamma}{M_s}\sqrt{AK_z}$. $v^2 = \frac{V^2}{R}$ → in v_0 units $V_m = \sqrt{R}$, $v^2 = V^2/V_m^2$

$S_x = \sin\Theta\cos\Phi$, $S_y = \sin\Theta\sin\Phi$, $S_z = \cos\Theta$. $y = x/\sqrt{R}$

Отождествление $\cos\Theta = \operatorname{th}u$, $\sin\Theta = \frac{1}{\operatorname{ch}u}$, тогда $\sin\Phi \approx 1$, $S_x = -\frac{\sqrt{\gamma}}{R}\frac{v}{\sqrt{1-v^2}}$ $\sin\Theta = \cos\Phi\sin\Theta$

$\cos\Phi = -\frac{\sqrt{\gamma}}{R}\frac{v}{\sqrt{1-v^2}} \ll 1$. $\Phi = \pi/2 - \varphi$ где $\sin\Phi = \cos\varphi \approx 1$, $\cos\Phi = \sin\varphi \ll 1$, $\varphi \ll 1$.

Но $\cos\Phi = -\frac{\sqrt{\gamma}}{R}\frac{v}{\sqrt{1-v^2}}$ и $\Phi = \frac{\pi}{2} - \varphi$ несовместимы с $\dot{\varphi} = -\dot{\Phi} \neq 0$, $\dot{\Phi} \neq 0$
$\Phi(\xi) = \text{const}$

Применительно у Sklyanin $S_x = -\frac{1}{R}\dot{\varphi}$, $S_y = \sqrt{1 - \dot{\varphi}^2/R^2}\sin\varphi$, $S_z = \sqrt{1 - \dot{\varphi}^2/R^2}\cos\varphi$

$-\varphi_{yy} - \varphi_{tt} = \frac{\gamma}{2}\sin 2\varphi$ ср. с $\varphi_{xx} - \frac{1}{c^2}\varphi_{tt} = \frac{1}{2\Delta_0^2}\sin 2\varphi \to \varphi(x,t) = \frac{\sin\varphi(x,t)}{}$

$\varphi(y,t) \to \sin\varphi(y,t) = \frac{1}{\operatorname{ch}\left[\sqrt{\gamma}\frac{(y - vt)}{\sqrt{1-v^2}}\right]}$ $\gamma = 1/\Delta_0^2$, $c^2 = 1$, $y \to x$. $\sin\varphi(\xi) = \frac{1}{\operatorname{ch}(\frac{\xi}{\Delta_0})}$, $\varphi_{\xi\xi} = \frac{1}{2\Delta^2}\sin 2\varphi$ $\operatorname{ch}\left(\frac{x - vt}{\Delta_0\sqrt{1 - v^2/c^2}}\right)$

$\eta = \sqrt{\gamma}\frac{(y - vt)}{\sqrt{1-v^2}}$, $\sin\varphi(y,t) = \frac{1}{\operatorname{ch}\eta}$, $\cos\varphi = \operatorname{th}\eta$. $\cos\varphi\,\dot{\varphi} = -\frac{1}{\operatorname{ch}^2\eta}\dot{\eta}\operatorname{sh}\eta = -\frac{\operatorname{sh}\eta}{\operatorname{ch}^2\eta}\dot{\eta} = -\sin\varphi\cos\varphi \times \dot{\eta}$

$\dot{\varphi} = -\sin\varphi\,\dot{\eta}$. $\dot{\eta} = \frac{d}{dt}\sqrt{\gamma}\frac{(y - vt)}{\sqrt{1-v^2}} = -\sqrt{\gamma}\frac{v}{\sqrt{1-v^2}}$, $S_x = -\frac{1}{R}\dot{\varphi} = -\frac{1}{R}(-\sin\varphi)(-1)\sqrt{\gamma}\frac{v}{\sqrt{1-v^2}} = -\frac{\sqrt{\gamma}}{R}\frac{v\sin}{\sqrt{1-v^2}}$

$S_x = -\frac{\sqrt{\gamma}}{R}\frac{v}{\sqrt{1-v^2}}\sin\varphi$, $S_y = \sqrt{1 - S_x^2}\sin\varphi$, $S_z = \sqrt{1 - S_x^2}\cos\varphi$, $S_x = -\frac{1}{R}\dot{\varphi}$

$S_x = \sin\Theta\cos\Phi$, $S_y = \sin\Theta\sin\Phi$, $S_z = \cos\Theta$

$\cos\Theta = \sqrt{1 - S_x^2}\cos\varphi$, $\sin\Theta\sin\Phi = \sqrt{1 - S_x^2}\sin\varphi$, $\sin\Theta\cos\Phi = S_x$

$\cos^2\Theta + \sin^2\Theta\sin^2\Phi = (1 - S_x^2) = 1 - \sin^2\Theta\cos^2\Phi$ — тождество $S^2 = 1$.

$S_x = -\frac{\sqrt{\gamma}}{R}\frac{v}{\sqrt{1-v^2}}\sin\varphi = -\varepsilon\sin\varphi$. Имеем для Θ, Φ: $\left\{\begin{array}{l}\cos\Theta = \sqrt{1 - \varepsilon^2\sin^2\varphi}\cos\varphi \\ \sin\Theta\sin\Phi = \sqrt{1 - \varepsilon^2\sin^2\varphi}\sin\varphi \\ \sin\Theta\cos\Phi = -\varepsilon\sin\varphi\end{array}\right\}$

$\Theta(\varphi)$ опр. $\sqrt{1 - \varepsilon^2\sin^2\varphi}\cos\varphi = \cos\Theta(\varphi)$, $\varepsilon = \frac{\sqrt{\gamma}}{R}\frac{v}{\sqrt{1-v^2}}$

$\cos\Phi(\varphi) = \frac{1}{\sin\Theta}(-\varepsilon\sin\varphi)$, $\cos\Phi(\varphi) = \frac{-\varepsilon\sin\varphi}{\sqrt{1 - \cos^2\Theta}}$, $1 - \cos^2\Theta = 1 - \cos^2\varphi(1 - \varepsilon^2\sin^2\varphi)$

$\cos\Phi(\varphi) = -\frac{\varepsilon\sin\varphi}{\sqrt{\sin^2\varphi(1 + \varepsilon^2\cos^2\varphi)}} = -\frac{\varepsilon}{\sqrt{1 + \varepsilon^2\cos^2\varphi}}$, $= \sin^2\varphi + \varepsilon^2\cos^2\varphi\sin^2\varphi$

$\cos\Phi(\xi) = -\frac{\varepsilon}{\sqrt{1 + \varepsilon^2\cos^2\varphi(\xi)}}$, $\cos\Phi(\xi) = -\varepsilon + O(\varepsilon^3)$, $\varepsilon \to 0$. $S_x = -\varepsilon\sin\varphi \ll 1$ не из-за $\varphi \ll 1$ а из-за $\varepsilon \ll 1$

т.е. решение Sklyanin все же $\Phi(\xi) = \text{const}$ с точностью $\varepsilon^3 = \left(\frac{\sqrt{\gamma}}{R}\frac{v}{\sqrt{1-v^2}}\right)^3 \ll 1$, которая заведомо нарушается при $v \to c$ ($v \to 1$). $\Phi = \pi/2 - \varphi$ — нежизнь ($\varepsilon\varphi \ll 1$).

Escaneado con CamScanner

$\sin\Phi = \frac{\sin\varphi\sqrt{1-\varepsilon^2\sin^2\varphi}}{\sqrt{\sin^2\varphi(1+\varepsilon^2\cos^2\varphi)}} = \frac{\sin\varphi\sqrt{1-\varepsilon^2\sin^2\varphi}}{\sin\varphi\sqrt{1+\varepsilon^2\cos^2\varphi}} = \frac{\sqrt{1-\varepsilon^2\sin^2\varphi}}{\sqrt{1+\varepsilon^2\cos^2\varphi}} = O(\varepsilon^2)+1$

по теор. ε^2 надо учитывать $\sqrt{1-\cos^2\Theta}$

$\sin^2\Phi = \frac{1-\varepsilon^2\sin^2\varphi}{1+\varepsilon^2\cos^2\varphi} = (1-\varepsilon^2\sin^2\varphi)(1-\varepsilon^2\cos^2\varphi) = 1-\varepsilon^2+O(\varepsilon^4)$, $\Phi(3)=\text{const}$

$\begin{cases}\Theta_{33} = (\lambda+\lambda\cos^2\Phi)\sin\Theta\cos\Theta \\ -v\Theta_3 = \frac{\lambda}{2}\sin\Theta\sin 2\Phi\end{cases}$, $-v\Theta_3\sin\Theta = \frac{\lambda}{2}\sin^2\Theta\sin 2\Phi$, $+O(\varepsilon^4)$ с точн. $\sim\varepsilon^4$.

$(+\dot\Theta\sin\Theta)$ при $\Phi(3)=\text{const}$.

$\varphi(x,t)$: $\sin\varphi = \frac{1}{\operatorname{ch}y}$, $\cos\varphi = \operatorname{th}y$, $y = \frac{\sqrt{\gamma}(x-vt)}{\sqrt{1-v^2}} = \frac{(x-vt)}{\sqrt{1-v^2/c^2}}\frac{1}{\Delta_0} = \frac{3}{\Delta(v)}$, $3 = x-vt$, $\xi = \frac{x}{\sqrt{R}}$

$\cos\Theta = \cos\varphi\sqrt{1-\varepsilon^2\sin^2\varphi}$

$-\sin\Theta\Theta_3 = \frac{\partial}{\partial 3}\cos\varphi\sqrt{1-\varepsilon^2\sin^2\varphi} = \frac{\partial}{\partial 3}\operatorname{th}y\sqrt{1-\varepsilon^2\frac{1}{\operatorname{ch}^2y}} = \frac{1}{\operatorname{ch}^2y}\frac{1}{\Delta}\sqrt{1-\frac{\varepsilon^2}{\operatorname{ch}^2y}} +$

$+\operatorname{th}y\frac{1}{2\sqrt{1-\varepsilon^2/\operatorname{ch}^2y}}\frac{2\varepsilon^2}{\operatorname{ch}^3y}\operatorname{sh}y\frac{1}{\Delta} = \frac{1}{\Delta\operatorname{ch}^2y}\left[\sqrt{1-\frac{\varepsilon^2}{\operatorname{ch}^2y}} + \operatorname{th}^2y\frac{\varepsilon^2}{\sqrt{1-\varepsilon^2/\operatorname{ch}^2y}}\right] \approx$ (до ε^2) $=$

$= \frac{1}{\Delta\operatorname{ch}^2y}\left[1-\frac{1}{2}\frac{\varepsilon^2}{\operatorname{ch}^2y}+\varepsilon^2\operatorname{th}^2y\right] = \frac{1}{\Delta\operatorname{ch}^2y\sqrt{1-\varepsilon^2/\operatorname{ch}^2y}}\left[1-\frac{\varepsilon^2}{\operatorname{ch}^2y}+\operatorname{th}^2y\cdot\varepsilon^2\right]$

$\operatorname{th}^2y + \frac{1}{\operatorname{ch}^2y} = 1$, $\operatorname{th}^2y - \frac{1}{2}\frac{1}{\operatorname{ch}^2y} = \cos^2\varphi - \frac{1}{2}\sin^2\varphi = 1-\frac{3}{2}\sin^2\varphi = -\frac{1}{2}+\frac{3}{2}\cos^2\varphi =$

$\operatorname{th}^2y - \frac{1}{\operatorname{ch}^2y} = \cos^2\varphi - \sin^2\varphi = \cos 2\varphi^2 = \frac{1}{2}\left[2\cos^2\varphi-\sin^2\varphi\right] = \frac{1}{2}\left[\cos^2\varphi+\cos 2\varphi\right]$

$-\sin\Theta\Theta_3 = \frac{\sin^2\varphi}{\Delta\sqrt{1-\varepsilon^2\sin^2\varphi}}\left[1+\varepsilon^2\cos 2\varphi\right]$, $\sin^2\Theta = 1-\cos^2\Theta = 1-\cos^2\varphi(1-\varepsilon^2\sin^2\varphi) =$

$= \sin^2\varphi(1+\varepsilon^2\cos^2\varphi)$, $\sin\Theta = \sin\varphi\sqrt{1+\varepsilon^2\cos^2\varphi}$

$-\Theta_3 = \frac{1}{\Delta}\frac{1}{\sin\varphi\sqrt{1+\varepsilon^2\cos^2\varphi}}\frac{\sin^2\varphi}{\sqrt{1-\varepsilon^2\sin^2\varphi}}(1+\varepsilon^2\cos 2\varphi) = \frac{1}{\Delta}\frac{\sin\Theta}{(1+\varepsilon^2\cos^2\varphi)(1-\varepsilon^2\sin^2\varphi)^{1/2}}(1+\varepsilon^2\cos 2\varphi)$

Должно $-v\Theta_3 = \lambda\sin\Theta\sin\Phi\cos\Phi$, $\sin\Phi\cos\Phi = -\varepsilon\frac{\sqrt{1-\varepsilon^2\sin^2\varphi}}{\sqrt{1+\varepsilon^2\cos^2\varphi}\sqrt{1+\varepsilon^2\cos^2\varphi}} =$

$= -\varepsilon\frac{\sqrt{1-\varepsilon^2\sin^2\varphi}}{(1+\varepsilon^2\cos^2\varphi)}$, $\varepsilon = \frac{\sqrt{\gamma}}{R}\frac{v}{\sqrt{1-v^2}} = \frac{\sqrt{\gamma}}{R}$ (?), $y = \sqrt{\gamma}\frac{(x/R - vt)}{\sqrt{1-v^2}} = \frac{\sqrt{\gamma}}{\sqrt{R}}\frac{(x-vt)}{\sqrt{1-v^2/c^2}}$

Было $y = \frac{1}{\Delta_0}\frac{x-vt}{\sqrt{1-v^2/c^2}} = \frac{1}{\Delta(v)}(x-vt)$, теперь $y = \frac{\sqrt{\gamma}}{\sqrt{R}}\frac{(x-vt)}{\sqrt{1-v^2/c^2}} = \frac{\sqrt{\gamma}}{\sqrt{R}}\frac{3}{\sqrt{1-v^2/c^2}}$

$\frac{1}{\Delta(v)} = \frac{1}{\sqrt{1-v^2/c^2}\sqrt{R}}\sqrt{\gamma} = \frac{1}{\Delta_0}\frac{1}{\sqrt{1-v^2/c^2}}$, $\Delta_0 = \frac{\sqrt{R}}{\sqrt{\gamma}}$, $\varepsilon = \frac{\sqrt{\gamma}}{R}\frac{v}{\sqrt{1-v^2}} = \frac{\sqrt{\gamma}}{R^{3/2}}\frac{V}{\sqrt{1-v^2/c^2}}$, $V = \sqrt{R}v$.

$\frac{1}{\Delta(v)} = \frac{\sqrt{\gamma}}{\sqrt{R}}\frac{1}{\sqrt{1-v^2/c^2}}$, $\varepsilon = \frac{1}{R}\frac{V}{\Delta(v)}$, $R = \frac{K_x}{K_z} = \lambda$, $\varepsilon = \frac{1}{\lambda}\frac{V}{\Delta(v)}$, $y = \frac{1}{\Delta(v)}(x-Vt) = \frac{\xi}{\Delta}$

$\sin\Phi\cos\Phi = -\varepsilon + O(\varepsilon^3)$, $-\Theta_3 = \frac{1}{\Delta}\sin\Theta\frac{1}{\sqrt{1-\varepsilon^2\sin^2\varphi}} \approx \frac{1}{\Delta}\sin\Theta$

Из исх. LL eq. $-v\Theta_3 = \lambda\sin\Theta\sin\Phi\cos\Phi = \lambda\sin\Theta(-\varepsilon)$, $\Theta_3 = \frac{\lambda\varepsilon}{V}\sin\Theta = \frac{1}{\Delta}\sin\Theta$ (!)

V и Δ в eq. $v_0 = \delta/t_0$, $t_0 = 1/\omega_a$, $\delta = \sqrt{A/K_2}$. с точн. до знака V.

$\varepsilon = \frac{1}{\lambda}\frac{V}{\Delta(v)} = \frac{1}{\lambda}\frac{V}{\Delta_0\sqrt{1-v^2/c^2}} << 1$, $\frac{V^2}{\lambda^2\Delta_0^2} << 1-v^2/c^2$, $\left(\frac{1}{\lambda^2\Delta_0^2}+\frac{1}{c^2}\right)V^2 << 1$

$\lambda\Delta_0 = \lambda\frac{\sqrt{R}}{\sqrt{\gamma}} = \frac{R^{3/2}}{\sqrt{\gamma}}$, $c^2 = R$, $\left(\frac{\gamma}{R^3}+\frac{1}{R}\right)v^2 << 1 \xrightarrow[R^3>>1]{} \frac{v^2}{R} << 1$, $\frac{v^2}{c^2} << 1$.

Т.о. применимость eqs by Selyanin предвар. $\varepsilon << 1$, что оправдан. $V << c$, $c = V_0 = v$.

$S_x = -\varepsilon\sin\varphi << 1$ из-за $\varepsilon << 1$. Может и $\sin\varphi << 1$, $\frac{1}{\operatorname{ch}y} = \frac{1}{\operatorname{ch}\left[\frac{\sqrt{\gamma}}{\sqrt{R}}\frac{(x-vt)}{\sqrt{1-v^2/c^2}}\right]} << 1$

при $y = \frac{3}{\Delta} >> 1$ – не имеет смысла, т.к. если $\xi = x-vt = 0$.

Вывод: // Т.е. Сделаем подстановку $\cos\Theta = \cos\varphi\sqrt{1-\varepsilon^2\sin^2\varphi}$, $\sin\Theta = \sin\varphi\sqrt{1+\varepsilon^2\cos^2\varphi}$

$\sin^2\Theta + \cos^2\Theta = \cos^2\varphi(1-\varepsilon^2\sin^2\varphi) + \sin^2\varphi(1+\varepsilon^2\cos^2\varphi) = \cos^2\varphi + \sin^2\varphi = 1$

② $\cos\Phi = +\varepsilon$, $\sin\Phi = 1+O(\varepsilon^2)$, $\sin\Phi\cos\Phi = +\varepsilon$ и $S_x = \varepsilon\sin\varphi$

LL-eq. $-v\Theta_3 = \lambda\sin\Theta\sin 2\Phi = \lambda\varepsilon\sin\Theta$, $-\Theta_3 = \frac{\lambda\varepsilon}{V}\sin\Theta$

③ Считаем Θ_3 из $\cos\Theta = \cos\varphi\sqrt{1-\varepsilon^2\sin^2\varphi}$ и находим $-\Theta_3 = \frac{1}{\Delta}\sin\Theta\left(1+O(\varepsilon^2)\right)$,

если $\sin\Phi = 1/\operatorname{ch}y$, $\cos\Phi = \operatorname{th}y$, где $y = \frac{3}{\Delta(v)}$, т.ч. $\varphi(x,t)$ находится

из sin-Gordon $\varphi_{xx} - \frac{1}{c^2}\varphi_{tt} = \frac{1}{2\Delta_0^2}\sin 2\varphi$, $3 = x-vt$, $\Delta(v) = \Delta_0(1-v^2/c^2)^{1/2}$, $c = v$.

$S_z = \sqrt{1-S_x^2}\cos\varphi$ предполагаем и если $S_x = \varepsilon\sin\varphi$, с $\varepsilon = \frac{V}{\lambda\Delta}$, φ из sin-Gordon

$\varphi_{xx} - \frac{1}{c^2}\varphi_{tt} = \frac{1}{2\Delta_0^2}\sin 2\varphi$, $\Delta(v) = \Delta_0\sqrt{1-v^2/c^2}$, то LL eq. для $\Theta(3)$ решается при $\Phi = \text{const}$.

Формально $E_{SW}(v) = E_0/\sqrt{1-v^2/c^2}$, но если оценивать $V << c$, а когда sin Gordon решение сводится к $E_{SW}(v^2)$ – Schlömann'ей с $\Phi = \text{const}$.

→ предположение ① интересно, а в ③ достаточно $\cos\Theta = \cos\varphi$.

УД II/II - форма $\begin{cases} v y_3 = \lambda \sin\Phi\cos\Phi + \Phi_{33} - 2\Phi_3 y_3 \operatorname{th} y \\ -v\Phi_3 = y_{33} + [1 + \lambda\cos^2\Phi + (\Phi_3)^2 - (y_3)^2]\operatorname{th} y \end{cases}$ $\operatorname{tg}\frac{\theta}{2} = e^y$, $\sin\theta = \frac{1}{\operatorname{ch} y}$, $\cos\theta = -\operatorname{th} y$

Частный $\Phi = \text{const}$, $S_x = \sin\theta\cos\Phi \ll 1$ из-за $\lambda = \frac{K_x}{K_z} \gg 1$.

$\Phi = \text{const}$ дает $v y_3 = \lambda \sin\Phi\cos\Phi = \text{const}$, $y_{33} = 0$, $1 + \lambda\cos^2\Phi = (y_3)^2$

$\cos\Phi = \frac{S_x}{\sin\theta} = \varepsilon$, $\sin\Phi\cos\Phi = \sqrt{1-\varepsilon^2}\,\varepsilon \propto \varepsilon$ при $\varepsilon \ll 1$.

$v y_3 = \lambda\varepsilon$, $y_3 = \frac{\lambda\varepsilon}{v}$, $1 + \lambda\varepsilon^2 = \left(\frac{\lambda\varepsilon}{v}\right)^2$; $1 = \lambda\varepsilon^2\left[\frac{\lambda}{v^2} - 1\right]$

$\varepsilon^2 = \frac{1}{\lambda}\frac{1}{\left(\frac{\lambda}{v^2} - 1\right)} = \frac{1}{\lambda^2}\frac{v^2}{(1 - v^2/\lambda)}$; $\varepsilon = \frac{1}{\lambda}\frac{v}{\sqrt{1 - v^2/\lambda}}$, $c^2 = \lambda$, $c = \sqrt{\lambda}$ в ед. v_0.

$\varepsilon = \frac{1}{\lambda}\frac{v}{\sqrt{\lambda - v^2/c^2}}$, $y_3 = \frac{\partial}{\partial\xi} y = \frac{\lambda}{v}\varepsilon = \frac{1}{\sqrt{1 - v^2/c^2}}$, $y = \frac{\xi}{\sqrt{1 - v^2/c^2}} = \frac{\xi}{\Delta(v)}$

$\xi = x - vt$, $\Delta(v) = \sqrt{\lambda - v^2/c^2}$, в размер. переменных $\Delta(v) = \Delta_0\sqrt{1 - v^2/c^2}$.

$E_{sw} = E_0\frac{1}{\Delta(v)} = \frac{E_0}{\sqrt{1 - v^2/c^2}}$. $\varepsilon \ll 1$ – означает $\lambda \gg 1$, $v \ll c$.

$\sin\theta = \frac{1}{\operatorname{ch} y}$, $y = \frac{\xi}{\Delta(v)} = \frac{(x - vt)}{\sqrt{1 - v^2/c^2}}\frac{1}{\Delta_0}$, $\varepsilon = \frac{1}{\sqrt{\lambda}}\frac{u}{\sqrt{1 - u^2}}$, $u = v/c$.

$\theta(x,t)$ с данным $\sin\theta(\xi)$, $\cos\theta(\xi)$

уравн. Sin-Gordon $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{2\Delta_0^2}\sin 2\theta$, $c = \sqrt{\lambda}$, $c = \sqrt{\frac{K_x}{K_z}}\frac{2\gamma}{M_s}\sqrt{AK_z} = \frac{2\gamma}{M_s}\sqrt{AK_x}$

$v_- = \sqrt{1+\lambda} - 1$. Очевидно $c = v_-(\lambda \to \infty) = \sqrt{\lambda}$.

Единственное предположение $S_x \ll 1$, $S_x = \varepsilon\sin\theta$, $\cos\Phi = \frac{S_x}{\sin\theta} = \varepsilon = \text{const}$ $\varepsilon \ll 1$.

$S_y = \sin\theta\sin\Phi = (1 - \varepsilon^2/2)\sin\theta \approx \sin\theta$, $S_z = \cos\theta$. $\Phi = \frac{\pi}{2} - \varepsilon$, $\cos\Phi = \sin\varepsilon \approx \varepsilon$

$\sin\Phi = \sin(\pi/2 - \varepsilon) = \cos\varepsilon = 1 - \varepsilon^2/2 \approx 1$ – формально $\Phi = \frac{\pi}{2} - \varepsilon$ подходит, но это излишне. Достаточно $S_x = \varepsilon\sin\theta \ll 1$, $\varepsilon \ll 1$.

Вывод: доме $\theta(x,t)$ можно записать sin-Gordon при $\varepsilon \ll 1$. Но релятивистские $\Delta(v) = \Delta_0\sqrt{1 - v^2/c^2}$, $E_{sw}(v) = \frac{E_0}{\sqrt{1 - v^2/c^2}}$ – функции, т.к. $\varepsilon \ll 1$ означает $v \ll c$.

$\chi^2(v) = 1 - \frac{1}{2}(v^2 - \lambda) - \frac{1}{2}\sqrt{(v^2 - v_-^2)(v^2 - v_+^2)}$, $E_{sw}(v) = E_0\,\chi(v)$, $v_\pm = \sqrt{1+\lambda} \pm 1$.

$E_{sw}^{rel}(v) = E_0(1 - v^2/v_-^2)^{-\frac{1}{2}}$, при $v \to v_- - 0$, $E_{sw}^{rel} \to \infty$, $E_{sw}(v_-) = E_0(1+\lambda)^{1/4}$ – конечна. Хотя при $\lambda \to \infty$ $E_{sw}(v_-) \sim \lambda^{1/4} \to \infty$, $\chi(v) = \frac{1}{\Delta(v)}$.

$\varepsilon = \frac{1}{\sqrt{\lambda}}\frac{v/c}{\sqrt{1 - v^2/c^2}}$ при $\lambda \to \infty$, $v = c = v_-$ неопределенно.

Ansatz - Switching $S_x = -\frac{1}{R}\dot{\varepsilon}$ тоже верен, но более громоздок, и хочется в $\vec{S}(S_x, S_y, S_z)$ представлении. $B(\theta, \Phi)$ - представление $\cos\Phi = \varepsilon$ достаточно.

(17/II) $\frac{d}{dt}(\cos\theta\Phi) = \cos\theta\dot{\Phi} - \sin\theta\dot{\theta}\Phi$, $\mathcal{L}_{kin} = -J\cos\theta\dot{\Phi} = -J\sin\theta\dot{\theta}\Phi$, $J = M_s/\gamma$

УД(θ,Φ) V/II, S/II при $\Phi_x = 0$, $J\dot{\theta} = K_x\sin\theta\sin 2\Phi = K_x\sin\theta\sin 1(\pi/2 - \varepsilon) =$

$= K_x\sin\theta\sin 2\varepsilon \approx 2\varepsilon K_x\sin\theta$, $\mathcal{L}_{kin} = -J\sin\theta\dot{\theta}(\pi/2 - \varepsilon) = J\sin\theta\dot{\theta}\varepsilon$

$\sin\theta = \frac{J}{2\varepsilon K_x}\dot{\theta}$, $\mathcal{L}_{kin} = J\varepsilon\dot{\theta}\frac{J}{2\varepsilon K_x}\dot{\theta} = \frac{1}{2}\frac{J^2}{K_x}(\dot{\theta})^2 = \frac{1}{2}m_{sw}(\dot{\theta})^2$. Где взяли ε-нем $\varepsilon(\text{old}) = \lambda = \frac{K_x}{K_z}$

$m_{sw} = \frac{J^2}{K_x} = \frac{M_s^2}{\gamma^2 K_x}$, при $K_x = 2\pi M_s^2$, $m_{sw} = \frac{1}{2\pi\gamma^2}$ - нет. Должно (22/I) $m_{sw} = \frac{1}{2\pi\gamma^2\Delta_0}$.

В безразмерных variables (t_0, Δ_0) - см. 12/II $\sin\theta\dot{\theta} = \frac{\lambda}{2}\sin^2\theta\sin 2\Phi$ при $\Phi = \text{const}$

$\sin 2\Phi = \sin\lambda(\pi/2 - \varepsilon) = \sin 2\varepsilon \approx 2\varepsilon$

$\dot{\varepsilon} = \frac{\lambda}{2}\sin\theta\sin 2\Phi \approx \varepsilon\lambda\sin\theta$, $\sin\theta = \frac{1}{\varepsilon\lambda}\dot{\theta}$, $\mathcal{L}_{kin} = -\cos\theta\dot{\Phi} = \sin\theta\dot{\theta}\varepsilon = \varepsilon\dot{\theta}\frac{\dot{\varepsilon}}{\varepsilon\lambda} = \frac{1}{\lambda}(\dot{\theta})^2$

$\mathcal{L} = -\frac{M_s}{\gamma}\cos\theta\dot{\Phi} - \left[A(\nabla\vec{m})^2 + K_x m_x^2 - K_z m_z^2\right] = -\frac{M_s}{\gamma}\cos\theta\dot{\Phi} - \left[A\{(\nabla\theta)^2 + \sin^2\theta(\nabla\Phi)^2\} + \right.$

$\left. + K_x\sin^2\theta\cos^2\Phi + K_z\sin^2\theta\right] =$ | t в ед. $t_0 = 1/\gamma H_a$, x в ед. $\Delta_0 = \sqrt{A/K_z}$ |

$\frac{d}{dt} = \frac{1}{t_0}\frac{d}{d\tau} = \gamma H_a\frac{d}{d\tau}$, $\frac{M_s}{\gamma}\frac{d\Phi}{dt} = \frac{M_s}{\gamma}\gamma H_a\frac{d\Phi}{d\tau} = M_s H_a\frac{d\Phi}{dt}$, $M_s H_a = M_s\frac{2K_z}{M_s} = 2K_z$

$\mathcal{L} = -2K_z\cos\theta(\Phi_\tau) - 2K_z\frac{1}{2}\left[\frac{A}{K_z}(\nabla\vec{m})^2 + \lambda\sin^2\theta\cos^2\Phi + \sin^2\theta\right]$, $x \to \frac{x}{\Delta_0}$

$\mathcal{L}/2K_z = -\cos\theta(\Phi_\tau) - \frac{1}{2}\left[(\nabla\theta)^2 + \sin^2\theta(\nabla\Phi)^2 + \sin^2\theta(1 + \lambda\cos^2\Phi)\right]$

$E/2K_z = \frac{1}{2}\left[(\nabla\theta)^2 + \sin^2\theta(\nabla\Phi)^2 + \sin^2\theta(1 + \lambda\cos^2\Phi)\right]$ energy density $\tilde{E}$ in units $2K_z$

$\mathcal{L}_{kin}/2K_z = -\cos\theta\dot{\Phi} = -\sin\theta\Phi\dot{\theta}$. Верные только УД в безразмерных перем.

См. (*) 12/II $\sin\theta\dot{\theta} = -v\theta_3\sin\theta = \frac{\lambda}{2}\sin 2\theta\sin 2\Phi + \Phi_{33}\sin^2\theta + \theta_3\Phi_3\sin 2\theta$, $\xi = x - vt$

Escaneado con CamScanner

$\dot\theta = \frac{\lambda}{2}\sin\theta\sin 2\varphi$, $\sin\theta = \frac{2}{\lambda\sin 2\varphi}\dot\theta = \frac{2}{\lambda\cdot 2\varepsilon}\dot\theta$, $\mathcal{L}_{kin}/2K_2 =$

$= -\cos\theta\dot\varphi = -\sin\theta\dot\theta\varphi = \sin\theta\dot\theta\varepsilon = \varepsilon\dot\theta\cdot\frac{1}{\lambda\varepsilon}\dot\theta = \frac{1}{\lambda}(\dot\theta)^2$, $\dot\theta = \frac{d\theta}{dt}$ здесь

$\mathcal{L}_{kin} = 2K_2\frac{1}{\lambda}t_0^2\left(\frac{d\theta}{dt}\right)^2 = 2K_2\frac{K_2}{K_x}\frac{1}{(\gamma H_a)^2}\left(\frac{d\theta}{dt}\right)^2 = \frac{1}{2}m_{DW}\left(\frac{d\theta}{dt}\right)^2$ $(\varphi = \pi/2 - \varepsilon)$

$m_{DW} = 4K_2\frac{K_2}{K_x}\frac{1}{\gamma^2}\frac{1}{(2K_2/M_S)^2} = \frac{M_S^2}{\gamma^2}\frac{1}{K_x}$, тогда при $K_x = 2\pi M_S^2$ дает $m_{DW} = \frac{1}{2\pi\gamma^2}$

$L_{kin} = \int dx\,\mathcal{L}_{kin} = \Delta_0\int d\left(\frac{x}{\Delta_0}\right) \to \Delta_0\int dx\,\mathcal{L}_{kin} = \int dx\,\Delta_0\cdot 2K_2\frac{1}{\lambda}\left(\frac{d\theta}{dt}\right)^2$, (erg/cm^2)

$L_{kin} = \int dx\,\frac{2K_2\Delta_0}{\lambda}\left(\frac{d\theta}{dt}\right)^2$ дает $m_{DW} = \frac{\Delta_0}{2\pi\gamma^2}$, $m_{DW} = \frac{1}{\gamma^2}\frac{M_S^2}{K_x}\Delta_0$ если

Отсюда $\mathcal{L}_{kin} = \frac{1}{2}m_{DW}\left(\frac{d\theta}{dt}\right)^2$, m_{DW} in $[g]$. , $\mathcal{L}_{kin} = 2K_2\Delta_0\frac{1}{\lambda}\left(\frac{d\theta}{dt}\right)^2$ in erg/cm^2

Доменная масса $m_{DW} = \frac{E_0}{c^2} = \frac{4\sqrt{AK_2}}{(2\gamma/M_S)^2 AK_x} = \frac{1}{\gamma^2}\frac{1}{\Delta_0}\frac{M_S^2}{K_x}$. Другие варианты

коэф. при $\left(\frac{d\theta}{dt}\right)^2$ в $\mathcal{L}_{kin}$ m_θ. $m_\theta = \frac{1}{\gamma^2}\frac{M_S^2}{K_x}\Delta_0$, $\left(m_{DW} = \frac{1}{\Delta_0^2}m_\theta\right)$ GW mass

$\mathcal{L}_{kin} = \frac{1}{2}m_\theta\left(\frac{d\theta}{dt}\right)^2 = 2K_2\Delta_0\frac{1}{\lambda}\left(\frac{d\theta}{dt}\right)^2$ в ед. $[erg/cm^2]$.

$\mathcal{L}/(2K_2\Delta_0) = \mathcal{L}_{kin}/2K_2\Delta_0 - \frac{1}{2}\left[(\nabla\theta)^2 + \sin^2\theta(\nabla\varphi)^2 + \sin^2\theta(1+\lambda\cos^2\varphi)\right]$

$\mathcal{L}/2K_2\Delta_0 = \frac{1}{\lambda}\left(\frac{d\theta}{dt}\right)^2 - \frac{1}{2}\left[(\nabla\theta)^2 + \sin^2\theta(\nabla\varphi)^2 + \sin^2\theta(1+\lambda\cos^2\varphi)\right]$

$\nabla\varphi = 0$, $(\nabla\theta)^2 = (\theta_x)^2 = (\theta_\xi)^2$, 1st integral $(\theta_\xi)^2 + \sin^2\theta(\varphi_\xi)^2 = \sin^2\theta(1+\lambda\cos^2\varphi)$

$[\dots] = 2\left[(\nabla\theta)^2 + \sin^2\theta(\nabla\varphi)^2\right] = 2(\theta_\xi)^2 \to \mathcal{L}/2K_2\Delta_0 = \frac{1}{\lambda}\left(\frac{d\theta}{dt}\right)^2 - \left(\frac{d\theta}{d\xi}\right)^2$

$\mathcal{L}/2K_2\Delta_0 = \frac{1}{\lambda}\left(\frac{d\theta}{dt}\right)^2 - \frac{1}{2}\left[(\theta_\xi)^2 + \sin^2\theta(1+\lambda\cos^2\varphi)\right]$

В силу УД $1+\lambda\cos^2\varphi = (\eta_\xi)^2$, $\eta_\xi = \frac{d}{d\xi}\frac{\xi}{\Delta} = \frac{1}{\Delta} \to \frac{\mathcal{L}}{(2K_2\Delta_0)} = \frac{1}{\lambda}\left(\frac{d\theta}{dt}\right)^2 - \frac{1}{2}\left[\theta_\xi^2 + \frac{\sin^2\theta}{\Delta^2}\right]$

$\mathcal{L} = \frac{1}{\lambda}\left(\frac{d\theta}{dt}\right)^2 - \frac{1}{2}\left[(\theta_\xi)^2 + \frac{1}{\Delta^2}\sin^2\theta\right]$, $\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial(\theta_t)} + \frac{\partial}{\partial\xi}\frac{\partial\mathcal{L}}{\partial\theta_\xi} = \frac{\partial\mathcal{L}}{\partial\theta}$

$\frac{2}{\lambda}\theta_{tt} - \theta_{\xi\xi} = -\frac{1}{2\Delta^2}\sin 2\theta$, $\theta_{\xi\xi} - \frac{2}{\lambda}\theta_{tt} = \frac{1}{2\Delta^2}\sin 2\theta$

Устав „2" из $\frac{2}{\lambda}$ и $\Delta \to \Delta_0$. тогда $\theta_{\xi\xi} - \frac{1}{\lambda}\theta_{tt} = \frac{1}{2\Delta_0^2}\sin 2\theta$, $c^2 = \lambda$.

New Ansatz // $\cos\theta = \cos\varphi\sqrt{1-\varepsilon^2\sin^2\varphi}$, $\cos\varphi = \frac{\varepsilon}{\sqrt{1+\varepsilon^2\cos^2\varphi}}$ можно расчет как

переход к new variables $\varepsilon(x,t)$ и $\varphi(x,t)$ с сохранением $\vec m^2 = 1$.

$\vec m^2 = \sin^2\theta\cos^2\Phi + \sin^2\theta\sin^2\Phi + \cos^2\theta = \sin^2\varphi\left(\sqrt{1+\varepsilon^2\cos^2\varphi}\right)^2\frac{\varepsilon^2}{(1+\varepsilon^2\cos^2\varphi)} +$

$+ \sin^2\varphi(1+\varepsilon^2\cos^2\varphi)\left[1 - \frac{\varepsilon^2}{1+\varepsilon^2\cos^2\varphi}\right] + \cos^2\varphi(1-\varepsilon^2\sin^2\varphi) =$

$= \sin^2\varphi(1+\varepsilon^2\cos^2\varphi) + \cos^2\varphi(1-\varepsilon^2\sin^2\varphi) = \sin^2\varphi + \cos^2\varphi = 1$

$\cos^2\Phi + \sin^2\Phi = 1$, $\cos\Phi = \frac{\varepsilon}{\sqrt{1+\varepsilon^2\cos^2\varphi}}$, $\sin\Phi = \frac{\sqrt{1-\varepsilon^2\sin^2\varphi}}{\sqrt{1+\varepsilon^2\cos^2\varphi}}$

$\frac{\varepsilon^2}{1+\varepsilon^2\cos^2\varphi} + \frac{1-\varepsilon^2\sin^2\varphi}{1+\varepsilon^2\cos^2\varphi} = \frac{1+\varepsilon^2\cos^2\varphi}{1+\varepsilon^2\cos^2\varphi} = 1$, $s_x = \cos\Phi\sin\theta = \varepsilon\sin\varphi$

$\sin\theta = \sin\varphi\sqrt{1+\varepsilon^2\cos^2\varphi}$ дает $\sin^2\theta + \cos^2\theta = 1$.

$m_x = \varepsilon\sin\varphi$, $m_y = \sin\varphi\sqrt{1+\varepsilon^2\cos^2\varphi}\,\frac{\sqrt{1-\varepsilon^2\sin^2\varphi}}{\sqrt{1+\varepsilon^2\cos^2\varphi}} = \sin\varphi\sqrt{1-\varepsilon^2\sin^2\varphi}$

$m_z = \cos\varphi\sqrt{1-\varepsilon^2\sin^2\varphi}$. Очевидно $m_x = \varepsilon\sin\varphi$, $m_y = \sin\varphi\sqrt{1-m_x^2}$.

m_y, m_z для any m_x. Но $m_x = \varepsilon\sin\varphi$ specific, $m_z = \cos\varphi\sqrt{1-m_x^2}$

при $\varepsilon(x,t) \ll 1$ можно считать $\varepsilon = const$ (?) Но в силу $\Phi = \frac{\pi}{2} - \varepsilon$, $\theta \approx \varphi$. 2008 $\varepsilon \ll 1$ нарежем будет.

Но при $\varepsilon(x,t) = const$, $\Phi(x,t)$ имеем УД (θ,Φ) или (η,Φ) имеет точное решение УД

типа Walker (и не надо $\varepsilon \ll 1$) $\eta_\xi = \lambda\sin\Phi\cos\Phi$, $(\eta_\xi)^2 = 1+\lambda\cos^2\Phi$, $\eta_{\xi\xi} = 0$.

Написание file of the paper „LL → Sine-Gordon mapping".

(13/II) $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{2}\sin 2\theta$, при $\theta_t = 0$, $\theta(x) = 2\,arctg\,e^y$, $y = \frac{x}{\Delta}$, $\Delta(v) = \sqrt{1-v^2/c^2}$, $\Delta(0) = 1$

$\theta_x = \frac{\partial}{\partial x}2\,arctg\,e^y = 2\frac{1}{1+e^{2y}}e^y\frac{\partial y}{\partial x} = \left|y = \frac{x}{\Delta},\ \frac{\partial y}{\partial x} = \frac{1}{\Delta} = 1 \text{ при } v = 0\right| = \frac{1}{ch\,y} = \sin\theta$

$\theta_{xx} = \frac{\partial}{\partial x}\sin\theta = \cos\theta\frac{\partial\theta}{\partial x} = \sin\theta\cos\theta$, $\theta_{xy} = \frac{1}{2}\sin 2\theta$. $x \to \frac{x-vt}{\sqrt{1-v^2/c^2}} = \frac{x}{\Delta}$, $\theta\left(\frac{x}{\Delta}\right)$

$\theta_t = \frac{\partial}{\partial x}\theta(x)\dot x = \sin\theta\frac{d}{dt}\frac{x}{\Delta} = -\frac{v}{\Delta}\sin\theta$, $\theta_{tt} = -\frac{v}{\Delta}\cos\theta\,\theta_t = -(v/\Delta)\cos\theta(-v/\Delta)\sin\theta$

$\theta_{tt} = +\frac{v^2}{\Delta^2}\sin\theta\cos\theta$, $\theta_{xx} = \frac{1}{\Delta^2}\sin\theta\cos\theta$, $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{\Delta^2}\sin\theta\cos\theta - \frac{1}{c^2}\frac{v^2}{\Delta^2}\sin\theta\cos\theta =$

$= \frac{1}{\Delta^2}(1 - v^2/c^2)\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$ где $\Delta(v) = \sqrt{1 - v^2/c^2}$; Лоренц.

$\mathcal{L} = \frac{1}{2}(\theta_t)^2 - \frac{1}{2}(\theta_x)^2 - \frac{1}{2}\sin 2\theta$, $\frac{\partial \mathcal{L}}{\partial\theta} = \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial(\theta_t)} + \frac{\partial}{\partial x}\frac{\partial \mathcal{L}}{\partial(\theta_x)}$

$-\frac{1}{2}\sin 2\theta = \frac{\partial}{\partial t}\theta_t + \frac{\partial}{\partial x}(-\theta_x)$, $\theta_{xx} - \theta_{tt} = \frac{1}{2}\sin 2\theta$.

Надо $\mathcal{L} = \frac{1}{2}\frac{1}{c^2}(\theta_t)^2 - \frac{1}{2}(\theta_x)^2 - \frac{1}{2}\sin^2\theta \to \theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{2}\sin 2\theta$. $\theta = \theta(\frac{x}{\Delta})$

но если $\theta = \theta(z)$, $\theta_z = \sin\theta$, $\theta_{zz} = \frac{1}{2}\sin 2\theta \to \theta = \theta(z)$, $\theta_{zz} = \frac{1}{2}\sin 2\theta$

$z = \frac{x}{\Delta} = \frac{(x - vt)}{\Delta}$, $\theta(x,t) = \theta\left(\frac{x-vt}{\Delta}\right)$, $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{2}\sin 2\theta$ где $\Delta^2 = 1 - v^2/c^2$.

$\mathcal{L}_{kin} = -\cos\theta\dot{\Phi} = -\Phi\sin\theta\dot{\theta}$, $\dot{\theta} = -\frac{v}{\Delta}\sin\theta$, если $\theta = \theta(\frac{x-vt}{\Delta}) = \theta(\frac{z}{\Delta})$ и

$\theta(z) \Rightarrow$ теория Toda $\theta_{zz} = \frac{1}{2}\sin 2\theta$, т.е. $\ln\frac{\theta}{2} = e^z$, $\theta = 2\arctg e^z$, $\theta_z = \sin\theta(z)$.

Ebconski'78 (11/II): $\mathcal{L}_{kin} = -2v\, y_3\Phi\frac{1}{ch^2 y} = \left|\dot{y} = y_{3}\dot{z} = -v y_3\right| = 2\dot{y}\Phi\frac{1}{ch^2 y}$

$y = \ln\tg\frac{\theta}{2}$, $\dot{y} = \frac{1}{\tg\frac{\theta}{2}}\frac{1}{\cos^2\frac{\theta}{2}}\frac{1}{2}\dot{\theta}$, $\dot{y}\frac{1}{ch^2 y} = \frac{1}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\dot{\theta}\sin^2\theta = \sin\theta\dot{\theta}$

$\mathcal{L}_{kin}(\text{Ebcon}) = 2\Phi\dot{y}\frac{1}{ch^2 y} = 2\Phi\sin\theta\dot{\theta}$, надо $\mathcal{L}_{kin} = -\Phi\sin\theta\dot{\theta}$, $\mathcal{L}_{kin}(\text{Ebc})\times(-\frac{1}{2})$

Вернемся $\mathcal{L}(y,\Phi) = -\frac{1}{2}\frac{1}{ch^2 y}\left[(y_3)^2 + (\Phi_3)^2 + 1 + \lambda\cos^2\Phi - 2v\, y_3\Phi\right] = -\Phi\sin\theta\dot{\theta} -$

$-\frac{1}{2}\sin^2\theta\left[(y_3)^2 + (\Phi_3)^2 + 1 + \lambda\cos^2\Phi\right]$, $\mathcal{L} = \mathcal{L}_{kin} - E$, $E = \frac{1}{2}\sin^2\theta\left[y_3^2 + \Phi_3^2 + 1 + \lambda\cos^2\Phi\right]$

$\frac{y_3}{ch^2 y} = \sin\theta\cdot\theta_3$, $\sin^2\theta\, y_3^2 = \sin^2\theta\frac{(\theta_3)^2}{\sin^2\theta} = (\theta_3)^2 \Rightarrow E = \frac{1}{2}\left[(\theta_3)^2 + \sin^2\theta(\Phi_3)^2 + \sin^2\theta(1 + \lambda\cos^2\Phi)\right]$

$y_3 = \theta_3\frac{1}{\sin\theta}$

Но моя dimensionless energy $e(\theta,\Phi) = (\theta_x)^2 + \sin^2\theta(\Phi_x)^2 + \sin^2\theta(1 + \lambda\cos^2\Phi)$, почему разница в (1/2) множителе?

Может надо $\mathcal{L}(\theta,\Phi) = v y_3\Phi\frac{1}{ch^2 y} - \frac{1}{ch^2 y}\left[y_3^2 + \Phi_3^2 + 1 + \lambda\cos^2\Phi\right] = \mathcal{L}_{kin} - E$, ~~да~~ ! но 2/IV

$\mathcal{L}_{kin} = v y_3\Phi\frac{1}{ch^2 y} = -\dot{y}\Phi\sin^2\theta = -\Phi\sin\theta\dot{\theta}$ – тогда верно. И в пределе $\Phi = \text{const}$, $\lambda\to\infty$

Ещё $\mathcal{L}(\theta) = (\theta_t)^2\frac{1}{c^2} - (\theta_x)^2 - \sin^2\theta = \mathcal{L}_{kin} - E$, $E = (\theta_x)^2 + \sin^2\theta$ $(\Phi = \frac{\pi}{2})$, $\Phi = \frac{\pi}{2} - \varepsilon$

$\mathcal{L}_{kin}(\theta,\Phi) = -\Phi\sin\theta\dot{\theta}$, $\mathcal{L}_{kin}(\theta) = (\theta_t)^2\frac{1}{c^2}$ при $\lambda\to\infty$, $\Phi = \pi/2 - \varepsilon$, $\varepsilon \ll 1$

↳ при $\lambda\to\infty$ $\mathcal{L}_{kin} = \varepsilon\sin\theta\dot{\theta}$, надо $\varepsilon\sin\theta = \ddot{\theta}$; где $\mathcal{L}_{kin}(\lambda\gg 1) = (\dot{\theta})^2\frac{1}{c^2}$. $c^2 = \lambda$

VI/12/II $-v\theta_3\sin\theta = \frac{1}{2}\sin^2\theta\sin 2\Phi + \Phi_{33}\sin^2\theta + \theta_3\Phi_3\sin 2\theta = \dot{\theta}\sin\theta$, $\dot{\theta} = \frac{d}{dt}\theta(z) = \theta_z\dot{z} = -v\theta_3$

где $\dot{\theta}\sin\theta = \frac{1}{\lambda}\sin^2\theta\sin\Phi\cos\Phi$, $\ddot{\theta} = \lambda\sin\theta\cdot\varepsilon$, $\sin\theta = \frac{1}{\lambda\varepsilon}\ddot{\theta}$

$\mathcal{L}_{kin}(\theta,\Phi) = -\Phi\sin\theta\ddot{\theta}$ – при $\lambda\to\infty$, $\varepsilon\ll 1$, $\Phi = \frac{\pi}{2} - \varepsilon$ где $\mathcal{L}_{kin}(\theta) = \frac{1}{\lambda}(\dot{\theta})^2 = \frac{1}{c^2}(\dot{\theta})^2$ (!)

Если $\mathcal{L}_{kin} = v y_3\Phi\frac{1}{ch^2 y}$, то из УД $v y_3 = \lambda\sin\Phi\cos\Phi + \Phi_{333} - 2\Phi_3 y_3\,\text{th}\,y$ при $\Phi = \frac{\pi}{2} - \varepsilon$

получим $v y_3 = \lambda\varepsilon$, $\mathcal{L}_{kin} = \lambda\varepsilon\Phi\sin^2\theta = \lambda\varepsilon\Phi\frac{(\dot{\theta})^2}{\lambda^2\varepsilon^2} = \frac{1}{\lambda\varepsilon}\left(\frac{\pi}{2} - \varepsilon\right)(\dot{\theta})^2$ и не

нужно выбирать dimensionless derivative $\frac{d}{dt}(\ldots)$ как в $\mathcal{L}_{kin} = -\Phi\sin\theta\ddot{\theta} = -(\frac{\pi}{2} - \varepsilon)\sin\theta\ddot{\theta} = \varepsilon\sin\ddot{\theta} =$

Условие $\dot{\theta} = \lambda\varepsilon\sin\theta$ нужно получать из (θ,Φ) метода. $= \frac{1}{\lambda}(\ddot{\theta})^2$

Интеграл от motion $(y_3)^2 + (\Phi_3)^2 = 1 + \lambda\cos^2\Phi \to -v\Phi_3 = y_{33} + 2(\Phi_3)^2\,\text{th}\,y$, дано где $y_{33} = 0$ при $\Phi_3 = 0$.

$\mathcal{L}(\lambda\to\infty) = (\dot{\theta})^2\frac{1}{c^2} - E(\theta,\Phi) = \frac{1}{c^2}(\dot{\theta})^2 - \left[(\theta_x)^2 + \sin^2\theta(\Phi_x)^2 + \sin^2\theta(1 + \lambda\cos^2\Phi)\right] =$

$\Phi_x = 0$, $\Phi = \pi/2 - \varepsilon \subseteq +\frac{1}{c^2}(\dot{\theta})^2 - (\theta_x)^2 - \sin^2\theta\left[1 + \lambda\varepsilon^2\right]$, $\lambda\varepsilon^2 = \lambda\frac{1}{\lambda^2}\frac{v^2}{(1 - v^2/c^2)} = \frac{v^2/c^2}{(1 - v^2/c^2)}$

$1 + \lambda\varepsilon^2 = 1 + \frac{u^2}{1 - u^2} = \frac{1}{1 - u^2}$, $u = v/c$, $1 + \lambda\varepsilon^2 = \frac{1}{\Delta^2(v)}$, $\Delta(v) = \sqrt{1 - v^2/c^2}$

$\mathcal{L}(\lambda\to\infty) = \frac{1}{c^2}(\dot{\theta})^2 - (\theta_x)^2 - \frac{1}{\Delta^2(v)}\sin^2\theta$, $\theta = \theta\left(\frac{x - vt}{\Delta(v)}\right)$, $(\dot{\theta})^2 = (\theta_\xi)^2\frac{v^2}{\Delta^2}$, $\theta_x^2 = \theta_\xi^2\frac{1}{\Delta^2}$

$\frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial\dot{\theta}} + \frac{\partial}{\partial x}\frac{\partial \mathcal{L}}{\partial(\theta_x)} = 2\frac{1}{c^2}\frac{\partial}{\partial t}\dot{\theta} + \frac{\partial}{\partial x}(-2\theta_x) = \frac{\partial \mathcal{L}}{\partial\theta} = -\frac{1}{\Delta^2}\sin 2\theta$, $y = \frac{x}{\Delta}$.

$\frac{\partial^2\theta}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\theta}{\partial t^2} = \frac{1}{2\Delta^2}\sin 2\theta$, $\frac{\partial\theta}{\partial x} = \theta_\xi\frac{1}{\Delta}$, $\theta_{xx} = \frac{1}{\Delta^2}\theta_{yy}$, $\theta_{tt} = \theta_{yy}\left(\frac{y}{\partial t}\right)^2 = \theta_{yy}\frac{v^2}{\Delta^2}$

$\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{\Delta^2}\theta_{yy} - \left(\frac{v}{c}\right)^2\frac{v^2}{c^2}\theta_{yy}\frac{1}{\Delta^2} = \theta_{yy}\left[1 - v^2/c^2\right]\frac{1}{\Delta^2} = \theta_{yy} = \frac{1}{2}\sin 2\theta$

$\theta_{yy} = \frac{1}{2}\sin 2\theta$ cos-b. $\ln\tg\frac{\theta}{2} = e^y$. Надо более $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{2}\sin 2\theta$, $\mathcal{L} = \frac{1}{c^2}(\dot{\theta})^2 - (\theta_x)^2 - \sin^2\theta$,

Без всякого $\Delta(v)\cdot\Delta(v) \to \frac{1}{\Delta^2}$ обращается при $\frac{\partial}{\partial t}\theta(y)$, $\frac{\partial}{\partial x}\theta(y)$, $y = (x - vt)/\Delta(v)$ см. 23/II

Это слабое место, т.к. подстановка $\varepsilon = \frac{1}{\lambda}\frac{v}{\Delta}$ в $\mathcal{L}$ даст $\mathcal{L} = \ldots - \frac{1}{\Delta^2}\sin 2\theta$

Escaneado con CamScanner

Решение Walker $v y_3 = \lambda \sin\Phi_0 \cos\Phi_0$, $1 + \lambda\cos^2\Phi_0 = (y_3)^2$, $\Phi_0 = const$, $y_{33} = 0$, $\eta = \frac{3}{\Delta}$.

$v^2(\Phi_0) = \lambda^2 \sin^2\Phi_0 \cos^2\Phi_0$, найдем $\Phi_0(v)$ явно. $\cos^2\Phi_0 = x$, $v^2(1+\lambda x) = \lambda^2 x(1-x)$

$\lambda^2 x^2 + v^2 + \lambda v^2 - \lambda^2 x = 0$, $x^2 + \left(\frac{v^2}{\lambda} - 1\right)x + \frac{v^2}{\lambda^2} = 0$, $2x_{1,2} = 1 - \frac{v^2}{\lambda} \pm \sqrt{\left(\frac{v^2}{\lambda}-1\right)^2 - 4\frac{v^2}{\lambda^2}}$

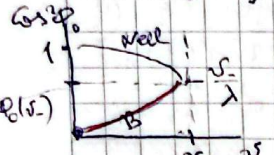
$\cos^2\Phi_0 = \frac{1}{2} - \frac{1}{2}\frac{v^2}{\lambda} \pm \frac{1}{2}\sqrt{\left(\frac{v^2}{\lambda}-1\right)^2 - 4\frac{v^2}{\lambda^2}}$, $\cos^2\Phi_0(v=0) = \frac{1}{2} \pm \frac{1}{2} = \begin{cases} 1, & \Phi_0 = 0, \pi \text{ - Neel DW} \\ 0, & \Phi_0 = \pm\frac{\pi}{2} \text{ - Bloch DW} \end{cases}$

$\cos^2\Phi_0(v_-) = \frac{1}{2} - \frac{1}{2}\frac{v_-^2}{\lambda} \pm \frac{1}{2\lambda}\sqrt{(v_-^2-\lambda)^2 - 4v_-^2} = \frac{1}{2} - \frac{1}{2\lambda}\left(2+\lambda - 2\sqrt{1+\lambda}\right) \pm$

$\pm \frac{1}{2\lambda}\sqrt{\left(\lambda - 2\sqrt{1+\lambda}\right)^2 - 4\left(2+\lambda-2\sqrt{1+\lambda}\right)} = (-) \pm \sqrt{4 + 4(1+\lambda) - 8\sqrt{1+\lambda} - 8 - 4\lambda + 8\sqrt{1+\lambda}}\,\frac{1}{2\lambda}$

$\to \cos^2\Phi_0(v_-) = -\frac{1}{2\lambda}\left(2 - 2\sqrt{1+\lambda}\right) \pm \frac{1}{2\lambda}\sqrt{4+4-8} = \frac{1}{\lambda}\left(\sqrt{1+\lambda} - 1\right) = v_-/\lambda$. $v_-/\lambda \begin{matrix} \ll 1 \\ \gg 1 \end{matrix}$

Domain $v_- < \lambda$, $v_-^2 = 2 + \lambda - 2\sqrt{1+\lambda} \le \lambda^2$, $\lambda^2 - \lambda - 2 + 2\sqrt{1+\lambda} \ge 0$, что выполнено



$\cos^2\Phi_0(v_-) = \frac{v_-}{\lambda} \ll 1$ для $\lambda \gg 1$, $\cos^2\Phi_0(v_-) \le \frac{1}{2}$ $\forall\lambda$. Как находить что Neel-DW metastable or unstable

$\frac{v_-}{\lambda} = \frac{\sqrt{1+\lambda}-1}{\lambda} \approx \frac{1}{2}$, $\lambda \to 0$

Можно найти $E(\Phi_0)$ для двух видов "+" - Neel-DW, и "-" B-DW

$E(v) = E_0 x(v)$, $x(v)$ известна. $E_\pm(v) = E_0 x_\pm(v^2)$, $v^2 = \frac{\lambda^2 \sin^2\Phi_0 \cos^2\Phi_0}{1+\lambda\cos^2\Phi_0}$

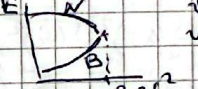
$E_\pm = E_0^\pm + \frac{m_\pm}{2} v^2$, $m_+ = m_N < 0$. $E_\pm(\Phi) = const + \frac{1}{2} m_\pm \frac{\lambda^2 \sin\Phi\cos\Phi}{1+\lambda\cos^2\Phi}$

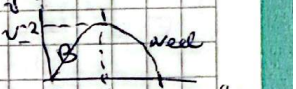
$\frac{\partial E_\pm}{\partial x} = \frac{1}{2} m_\pm \lambda^2 \frac{\partial}{\partial x}\frac{x(1-x)}{(1+\lambda x)} = \frac{1}{2} m_\pm \frac{(1-2x)(1+\lambda x) - x(1-x)\lambda}{(1+\lambda x)^2} \gtrless 0$, $x = \cos^2\Phi$

$(1-2x)(1+\lambda x) - x(1-x)\lambda = 0$, $1 + \lambda x - 2x - 2\lambda x^2 - x\lambda + x^2\lambda = 0$

$\lambda x^2 + 2x - 1 = 0$, $x_{1,2} = \frac{1}{2\lambda}\left[-2 \pm \sqrt{4+4\lambda}\right] = \frac{1}{\lambda}\left[-1 \pm \sqrt{1+\lambda}\right]$

$x = \cos^2\Phi \ge 0$, $x_0 = \frac{1}{\lambda}\left[\sqrt{1+\lambda} - 1\right] = v_-/\lambda$, $v^2(x_0) = v_-^2$





$E_\pm[v(x)]$, $\frac{\partial E_\pm}{\partial x} = \frac{\partial E_\pm}{\partial v}\frac{\partial v}{\partial x}$, $\frac{\partial E}{\partial x} = 0 \to \frac{\partial v}{\partial x} = 0$, $x = x_0$

$\frac{\partial^2 E_\pm}{\partial x^2} = \frac{\partial}{\partial x}\left[\frac{\partial E_\pm}{\partial v}\frac{\partial v}{\partial x}\right] = \frac{\partial^2 E_\pm}{\partial v^2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{\partial E_\pm}{\partial v}\frac{\partial^2 v}{\partial x^2}$ — минимо не дает

Надо устойчивость (θ_1, Φ_1) solution из анализа $\frac{\delta^2 E}{\delta\theta^2}$, $\frac{\delta^2 E}{\delta\Phi^2}$, $\frac{\delta^2 E}{\delta\theta\delta\Phi}$ at $\theta = \theta_1, \Phi = \Phi_1$

13/II // Оже AFM (21/I): $\omega^2 = \omega_0^2 + c^2k^2$, $\omega_0^2 = (\gamma M_S)^2 A\beta$, $c = \gamma M_S\sqrt{A\alpha}$

AF $W\left(\frac{\vec{L}}{M}\right) = \frac{1}{2}A\vec{M}^2 + \frac{1}{8}\alpha\left(\frac{\partial\vec{L}}{\partial x_j}\right)^2 + W_a(\vec{L})$, $\vec{L} = \vec{M}_1 - \vec{M}_2$, $\vec{M} = \vec{M}_1 + \vec{M}_2$, $\vec{l} = \frac{\vec{L}}{2M_0}$, $\vec{L}^2 + \vec{M}^2 = 4M_0^2$

$K = k + ix$, $v_- = v_+ = c$, $x(v) = x_0\frac{1}{\sqrt{1-v^2/c^2}}$, $x_0 = \omega_0/c$, $\Delta(v) = \frac{1}{x_0}\sqrt{1-v^2/c^2}$

$E(v) = E_0 x(v) = E_0\frac{x_0}{\sqrt{1-v^2/c^2}}$, $x(v)$ релятивистское. $x_0 = \frac{\gamma M_S\sqrt{A\beta}}{\gamma M_S\sqrt{A\alpha}} = \sqrt{\frac{\beta}{\alpha}}$

Domcan's file 1D DW motion in biaxial FM: mapping to sine Gordon" и послано по "Ruben и Ricardo. Оказалась публикация papers by B. Ивенов и книжку Kosevich et al. "Нелинейные волны намагниченности" динамические и топологические солитоны" 1983 Просмотр the papers и Refs в них. Чтение Косевич 1983 книги, которая оказалась немного от Kosevich. Phys. Rep. 1990. Или разослала Solomyagr 2019 reminder - Abstracts (на 18/II - submitted 8 Abstracts)

Ruben project

Чтение project Ruben (Sept. 2017): "Fast excitations in AFM domain walls" и Suppl. Mat. к manuscript (Feb. 2018): "Supermagnonic regime of AF domain wall" в Suppl. Mat. приводятся θ и Φ соответственно углы $\vec{M}_1$ и $\vec{M}_2$ - model of 2 FM layers coupled by strong AFM exch. interaction. Model: две Neel DW in 1st, 2nd layers, in-plane xoy, OMH $\parallel Ox$, $Ox \perp$ DW planes, $\vec{H}_{so} \parallel Oy$. Пусть знак inter. coupling: $J\vec{m}_1\vec{m}_2 \to -J\vec{m}_1\vec{m}_2$, $\Phi = \varphi_1 = \varphi_2$ и одновременно $\varphi = \Phi_1 - \Phi_2$

$\Phi_1(x,t)$, $\Phi_2(x,t)$ считаются не зависящими от x в интервале $x \in [0, \Delta]$, где DW расположена. Хотя $E_{int} = J\int_{-\infty}^{\infty} dx \cos[\Phi_1(x) - \Phi_2(x)]$ будет было бы не $\int_0^\Delta dx \cos[-]$. Идея Ruben $\varphi = \Phi_1(t) - \Phi_2(t) = \varphi(t)$, $F(t) = \cos\varphi(t)$

$F(t) = F(t_0) + \Delta t\,\dot{F}(t_0) + \frac{1}{2}(\Delta t)^2\ddot{F}(t_0)$, $\ddot{F}(t) = -\partial_{tt}\varphi\sin\varphi - \cos\varphi(\varphi_t)^2$, $\dot{F}(t) = -\sin\varphi\,\dot{\varphi}$

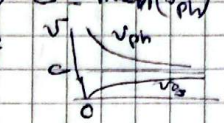
$\varphi \to 0$, $F(t) = F(t_0) + \frac{1}{2}(\Delta t)^2(-1)(\varphi_t)^2 \to E_{int} = \frac{1}{2}J\Delta\cos\varphi(t) = \frac{1}{2}(\Delta t)^2[-(\varphi_t)^2]J\Delta = -\varkappa(\dot{\varphi})^2$

$E_{int} = -\varkappa(\dot\varphi)^2$ минимумът $L_{kin}^{(AF)} = \varkappa(\dot\varphi)^2$ и даже $\Delta(v) = \Delta_0\sqrt{1-v^2/c^2}$ и limiting DW velocity c, которую Рубен называет max group velocity v_g (= min v_{ph}).

Walker solution $v = \frac{\gamma}{d}\Delta_0 H_{30}$ можно применить за оно $\Delta_0 \to \Delta_0\sqrt{1-v^2/c^2}$.

Из DW $\omega^2 = \omega_0^2 + c^2k^2$ [Kosevich 90] $v_{ph}^2 = \left(\frac{\omega}{k}\right)^2 = \frac{\omega_0^2}{k^2} + c^2$ видно, что $c = \min(v_{ph})$ – phase velocity

$v_g = \frac{\partial\omega}{\partial k} = \frac{1}{2\sqrt{\omega_0^2 + c^2k^2}}\,2c^2k = \frac{c^2k}{\sqrt{\omega_0^2+c^2k^2}}$, $\max v_g\,(k\to\infty) = c$



(AF)

Kosevich $v_{soliton} = v_g$, $\max v_g = \max v_{sol} = c$

Kosevich $V(k^2) = v_{gr}(k^2) = v_{ph}(k^2) = \frac{\omega(k)}{k}$, $Vk = \omega(k) \to k = k_1 + ik_2 = f(v)$,

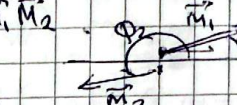
$\varkappa(v) = \frac{1}{\Delta(v)}$ – определяет DW width $\Delta(v)$ и energy DW $E_{DW} = E_0\varkappa(v)$

→ Steady state soliton (DW) motion с $\Theta(\xi), \Phi(\xi), \xi = x - Vt$ определ. eq. $v_{gr}(k) = v_{ph}(k)$ для линейных SW. Schlömann'61: $V = v_{ph}$, Kosevich'90: $V \equiv v_{gr}$ (!) definition.

Предположение Рубен $E_{int} = J\cos(\Phi_1 - \Phi_2)$ с $J\to\infty$, $\Phi_1 - \Phi_2 = \varphi \ll 1$ ложно, и $E_{int} = -\varkappa(\dot\varphi)^2$ – ложно „two coord. model", где $M^2 \ll L^2$

$W(\vec M, \vec L) = AM^2 + W_{AF}(\vec L)$, при $A = J_{int} \to \infty$, $M^2 \to 0$ и $M^2 \ll L^2$, $\vec M = \vec M_1 + \vec M_2$, $\vec L = \vec M_1 - \vec M_2$

$W_F(\vec M) = A\vec M^2 = A(M_1^2 + M_2^2 + 2\vec M_1\vec M_2) = \text{const} + 2A\vec M_1\vec M_2$



только в 2-mode φ – угол $\vec L$, а не $\varphi = \Phi_1 - \Phi_2$.

$E_{int}(\vec M_1, \vec M_2) = J\vec m_1\cdot\vec m_2 = J\cos(\Phi_1 - \Phi_2)$, $\Phi_1 = \Phi_1$, $\Phi_2 = \pi + \Phi_1 + \varepsilon$, $J>0$

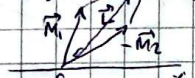
$\cos(\Phi_1 - \Phi_2) = \cos(\Phi_1 - \pi - \Phi_1 - \varepsilon) = \cos(\pi + \varepsilon) = -\cos\varepsilon$, $E_{int} = -J\cos\varepsilon$,

$\varepsilon = \Phi_1 - \Phi_2 - \pi$, а не $\varepsilon = \varphi = \Phi_1 - \Phi_2$. $\varepsilon = \varphi - \pi$, $\dot\varphi = \dot\varepsilon$ – это неважно. min при $\varepsilon \to 0$,

$E_{int} = -J\cos\varepsilon = -J\cos(\varphi - \pi) = J\cos\varphi$, $\cos\varphi(t) = \cos\varphi(t_0) - \frac{1}{2}(\Delta t)^2(\dot\varphi_t)^2$, $J>0$

$E_{int} = -\frac{1}{2}(\Delta t)^2 J(\dot\varphi)^2 = \varkappa(\dot\varphi)^2 \to J_{kin} = \varkappa(\dot\varphi)^2$, $\varkappa > 0$, $\varphi = \Phi_1 - \Phi_2$.





$\varphi_{\vec L} = \frac{1}{2}(\varphi_{\vec M_1} + \varphi_{-\vec M_2}) = \frac{1}{2}(\Phi_1 + \Phi_2 - \pi) = \frac{1}{2}(\Phi_1 + \Phi_2) - \frac{\pi}{2}$

Азим. угол $\vec L \neq \Phi_1 - \Phi_2$

Lorentz invariance появилась именно для DW для $\vec L$.

Разложение $F(t) = F(\tau) + (\frac{d}{dt})F(\tau) + \frac{1}{2}(\Delta t)^2\ddot F(\tau)$ – неочевидно, $\Delta t = t - \tau$.

20/II Мы обзор по ссылкам на Kosevich'1990 (наши интерес. работы). С. Л. и Ю. перевели книгу для нашего YSO Е (где ISE). С Л. и Ю. „да Перла", Чинза и Ингланде. Скирра с Б. Иванов по DW-solutions с $\Phi(\xi) = \text{const}$. при $v > v_+$ нет DW solutions (Shilnikov), а при $v < v_-$ нелинейно осц. по variable $\Phi(\xi)$ – DW solutions, $\Phi(\xi) = \text{const}$ $v_- < v < v_+$.

$$\begin{cases} v\eta_\xi = \lambda\sin\Phi\cos\Phi + \Phi_{\xi\xi} - 2\Phi_\xi\eta_\xi\,\text{thy} \\ -v\Phi_\xi = \eta_{\xi\xi} + \left[1 + \lambda\cos^2\Phi + (\Phi_\xi)^2 - (\eta_\xi)^2\right]\text{thy} \end{cases}$$

$\text{tg}\frac{\Theta}{2} = e^y$. Это будет при $\Phi = \frac{\pi}{2} - \varepsilon$, $\varepsilon \ll 1$ это $\underline{\varepsilon = \varepsilon(\xi)}$, $\xi = x - vt$, $\Phi_\xi = -\varepsilon_\xi$

$$\begin{cases} v\eta_\xi = \lambda\varepsilon - \varepsilon_{\xi\xi} + 2\varepsilon_\xi\eta_\xi\,\text{thy} \\ v\varepsilon_\xi = \eta_{\xi\xi} + \left[1 + \lambda\varepsilon^2 - (\eta_\xi)^2\right]\text{thy} \end{cases}$$

Assume $\lambda\varepsilon^2 \sim \varepsilon$, $\eta_\xi \sim \varepsilon$, $\eta_\xi^2 \sim \varepsilon^2$ бесполезно?

$\lambda\varepsilon \gg \varepsilon_{\xi\xi}, \varepsilon_\xi$, тогда $\varepsilon \sim \frac{1}{\lambda} \to \underline{v\eta_\xi = \lambda\varepsilon}$, $\eta_\xi = \frac{\lambda}{v}\varepsilon_\xi$

$v\varepsilon_\xi = \eta_{\xi\xi} + \left[1 + \lambda\varepsilon^2 - \frac{\lambda^2\varepsilon^2}{v^2}\right]\text{thy}$, $\left(v - \frac{\lambda}{v}\right)\varepsilon_\xi = \left[1 + \lambda\varepsilon^2\left(1 - \frac{\lambda}{v^2}\right)\right]\text{thy}$

$v\left(1 - \frac{\lambda}{v^2}\right)\varepsilon_\xi = \left[1 + \lambda\varepsilon^2\left(1 - \frac{\lambda}{v^2}\right)\right]\text{thy}$ или $v\varepsilon_\xi = v\frac{v}{\lambda}\eta_{\xi\xi} = \eta_{\xi\xi} + \left[1 + \lambda\left(\frac{v}{\lambda}\eta_\xi\right)^2 - \eta_\xi^2\right]\text{thy}$

$\left(\frac{v^2}{\lambda} - 1\right)\eta_{\xi\xi} = \left[1 + \left(\frac{v^2}{\lambda} - 1\right)\eta_\xi^2\right]\text{thy}$, $\frac{v^2}{\lambda} - 1 = \frac{v^2}{c^2} - 1 = -\left(1 - \frac{v^2}{c^2}\right) = -\gamma^2$, $\gamma^2 = (1 - v^2/c^2) > 0$

$-\gamma^2\eta_{\xi\xi} = \left[1 - \gamma^2\eta_\xi^2\right]\text{thy}$ – ур-е на $\eta(\xi)$, $\varepsilon(\xi)$ исчезающе, $\varepsilon = \frac{v}{\lambda}\eta_\xi$.

$\frac{\eta_{\xi\xi}}{-\frac{1}{\gamma^2} + \eta_\xi^2} = \frac{\eta_{\xi\xi}}{\eta_\xi^2 - a^2} = \frac{1}{\eta_\xi}\cdot\frac{\eta_{\xi\xi}\cdot\eta_\xi}{\eta_\xi^2 - a^2} = \frac{1}{\eta_\xi}\frac{1}{2}\frac{d(\eta_\xi^2)/d\xi}{\eta_\xi^2 - a^2}$, $\frac{1}{2}\frac{1}{\eta_\xi^2 - a^2}\frac{d(\eta_\xi^2)}{d\xi} = \text{thy}\cdot\eta_\xi$, $a^2 = \frac{1}{\gamma^2}$

$\eta_\xi^2 = u$, $\frac{1}{2}\frac{du}{u - a^2} = \text{thy}\cdot\eta_\xi d\xi = \text{thy}\cdot dy$, $\frac{1}{2}\ln(u - a^2) = \int\text{thy}\cdot dy = \ln\text{ch}y + \ln c\cdot\frac{1}{2}$

$\ln[u - a^2] = 2\ln\text{ch}y + \ln C$, $u - a^2 = C\text{ch}^2y$, $\eta_\xi^2 = a^2 + C\text{ch}^2y$, $\eta_\xi = \sqrt{a^2 + C\text{ch}^2y}$

$\int\frac{dy}{\sqrt{a^2 + C\text{ch}^2y}} = \xi + \xi_0$, $C>0$. $\frac{C}{a^2} = \beta^2$, $\int\frac{dy}{\sqrt{1 + \beta^2\text{ch}^2y}} = a(\xi - \xi_0)$, $\int\frac{dx}{\sqrt{1 + \frac{k^2}{k'^2}\text{ch}^2x}} = kF(\text{arcsin}\,\text{th}x, k)$

$\frac{1 - k^2}{k^2} = \beta^2$, $1 - k^2 = \beta^2k^2$, $1 = k^2(\beta^2 + 1)$, $k^2 = \frac{1}{\beta^2 + 1}$, $k = \frac{1}{\sqrt{1 + \beta^2}}$ $k'^2 = 1 - k^2$,

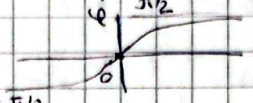
Escaneado con CamScanner

$k = \frac{1}{\sqrt{1+\frac{c}{a^2}}} = \frac{a}{\sqrt{c+a^2}}$, $a(\xi-\xi_0) = k F(\arcsin(th\psi), k)$, $F(\varphi,k) = \int_0^x \frac{dx}{\sqrt{1-k^2\sin^2 x}}$

эл. интеграл $F(-\varphi,k) = -F(\varphi,k)$. При $\xi\to+\infty$, $th\psi\to 1$, $a\xi = kF(\frac{\pi}{2},k) \to \infty$

$F(\frac{\pi}{2},k) = K(k)$, $kK(k)\to\infty$ при $k' = \sqrt{1-k^2}\to 0$, $K(k) \underset{k\to 1}{\approx} \ln\frac{4}{\sqrt{1-k^2}}$

$k^2\to 1$ или грубо $c\to 0$.



$\varphi(y) = \arcsin(th\psi)$

Но если $c\to 0$, $u = a^2$, $y_\xi = a$

$\eta(\xi) = \frac{\xi}{\gamma} = \frac{\xi}{\sqrt{1-v^2/c^2}} = \frac{\xi}{\Delta(v)}$ — но это я получал при $\varepsilon_0 = const$, $y_{\xi\xi} = 0$

По def. $F(x,k) = u \to x = sn(u,k)$; $F(\varphi,k) = \frac{a}{k}(\xi-\xi_0)$, $\varphi = sn(u,k)$, $u = \frac{a}{k}(\xi-\xi_0)$

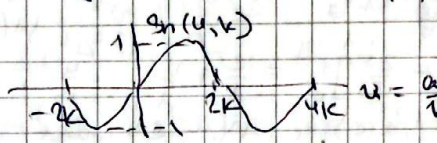
$\arcsin(th\psi) = gd(u,k)$, $th\psi \neq$ нет, не так. $F(\varphi,k) = F(x|k) = \int_0^x \frac{dt}{\sqrt{(1-t^2)(1-t^2k^2)}}$ $x = \sin\varphi$

$x = \sin\varphi = sn(u,k)$, $\sin\varphi = \sin[\arcsin(th\psi)] = th\psi$,

$th\psi = sn(u,k)$, где $u = \frac{a}{k}(\xi-\xi_0)$, $k = \frac{a}{\sqrt{c+a^2}}$, $a^2 = \frac{1}{(1-v^2/c^2)}$, $u = \sqrt{c+a^2}(\xi-\xi_0)$

$sn(u,k)$ периодическая с периодом $4K(k)$. $sn(u,k)$, $sn(u,0) = \sin u$, $sn(u,1) = th u$

$m_2 = \cos\theta(\xi) = th\psi(\xi) = sn\left(\frac{a}{k}\xi, k\right)$



$\varepsilon(\xi) = \frac{v}{\lambda}\eta_\xi$, $\psi(\xi) = \text{arcth}\, sn(u,k)$, $y_\xi = \frac{1}{1-sn^2(u,k)}\frac{d}{du}sn(u,k)\frac{du}{d\xi} = \frac{1}{cn(u,k)}\,ch(u,k)\,du\cdot\frac{a}{k}$

$y_\xi = dn(u,k)\cdot\frac{a}{k}$, $\varepsilon(\xi) = \frac{av}{\lambda k}dn(u,k)$, $\varepsilon(\xi) = \frac{v}{\lambda}\sqrt{c+a^2}\cdot dn(u,k)$, $k = \frac{a}{\sqrt{c+a^2}}$

$dn(u,k) = \begin{cases} 1, & k=0 \\ \frac{1}{ch u}, & k=1 \end{cases}$, $a = \frac{1}{\sqrt{1-v^2/c^2}}$, $v\to c$, $a\to\infty$, $k\to 1$, $\varepsilon(\xi) = \frac{v}{\lambda}a\frac{1}{ch u}$

т. е. при $v\to c-0$, $\varepsilon(\xi) = \frac{v}{\lambda}\frac{1}{\sqrt{1-v^2/c^2}}\frac{1}{ch(a\xi)} = \frac{v}{\lambda}\frac{a}{ch(a\xi)} \to 0$ при $a\xi\gg 1$.

При $v\to 0$, $a\to 1$, $\varepsilon(\xi) = \frac{v}{\lambda}\sqrt{1+c}\,dn\left(\sqrt{c+1}\,\xi, k\right)\to 0$, т.к. $dn(u,k)\approx 1$, при $k<1$.

по смыслу $\cos\theta(\pm\infty) = \pm 1$, надо $k^2\to 1$, $c\to 0$, $dn(u,k\to 1)\approx \frac{1}{ch u}$

$th\psi = sn(u,k\to 1) = th u$, $\psi(\xi) = u = \sqrt{c+a^2}\,\xi \approx a\xi = \frac{\xi}{\Delta}$, $\Delta = (1-v^2/c^2)^{1/2}$

$\varepsilon(\xi) = \frac{v}{\lambda}a\cdot dn(u,k) = |k\to 1| = \frac{v}{\lambda}\frac{a}{ch(a\xi)}$ и нет сингулярности ε при $v\to c$

Если сохранить $\dot\theta = \lambda\sin\theta\cdot\varepsilon$, $\sin\theta = \frac{1}{\lambda\varepsilon}\dot\theta$, $\mathcal{L}_{kin} = -(\frac{\pi}{2}-\varepsilon)\sin\theta\dot\theta$ $a\to\infty$.

$\mathcal{L}_{kin} = \varepsilon\sin\theta\dot\theta = \varepsilon\frac{1}{\lambda\varepsilon}\dot\theta^2 = \frac{1}{\lambda}(\dot\theta)^2$ сохраняется. $\Phi(\xi) = \frac{\pi}{2} - \varepsilon(\xi)$, $a = \frac{1}{\sqrt{1-v^2/c^2}}$

$\varepsilon(\xi) = \frac{v}{\lambda\Delta}\frac{1}{ch\left(\frac{\xi}{\Delta}\right)}$, $\Delta = \Delta(v) = \sqrt{1-v^2/c^2}$. $a = \frac{1}{\Delta(v)}$

→ **Вывод**: mapping LL → sin-Gordon можно делать при $v\to c$ если уже $\varepsilon(\xi)$ $\varepsilon(\xi)\ll 1$.

Более точно $sn(u,k\to 1) = th u + \frac{1}{4}(1-k^2)\frac{(sh u\,ch u - u)}{ch^2 u}$, $1-k^2 = 1-\frac{a^2}{c+a^2} = \frac{c}{c+a^2}$

(21/II) Прислал Глеб. Работа над текст „Drum" modes. Отдых. Блины.

(22/II) Работа над „Drum" modes MS. Решили Ph. Rev. Applied, но подобрать PRL. План: 1) Глебу переписать Introduction под PRL (+ single dot FMR и determination A, M_S) и послать FMR; 2) Воспользоваться Vouk 200 если по simulations и насколько они верны и нужны; 3) мне — обработать eqs. $\omega_n(\beta)$ APL'04 и new ones и пересчитать μ_n^{res} A, M_S для $R = 100, 150$ нм (с $\beta = 0.3 - 0.4$). „Eraski" с Л. и Глебом.

(23/II) с Л.Н., Глебом в downtown и обед в „Nam", с Л., Г. в Safeway. Market и Safeway" Olaberria (Mission Bay).

(24/II) Все же 13/II при $\varepsilon = const$ надо $E(\theta,\Phi) = (\theta_x)^2 + \frac{1}{\Delta^2}\sin^2\theta$ при $\lambda\to\infty$ Надо $E\to\frac{1}{2}E$. 2/IV

и $\mathcal{L} = \frac{1}{c^2}(\dot\theta)^2 - (\theta_x)^2 - \frac{1}{\Delta^2}\sin^2\theta$, $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{2\Delta^2}\sin 2\theta$, $\Delta = \Delta(v) = (1-v^2/c^2)^{1/2}$.

$\theta_\xi = \sin\theta\,y_\xi$, $y_\xi = \frac{1}{\Delta}$. $E = \frac{2}{\Delta^2}\sin^2\theta$, $W = \int dx\,E = \frac{2}{\Delta^2}\int dx\sin^2\theta$, $\sin\theta = \frac{1}{ch y}$

$E(w\to E) = \frac{2}{\Delta^2}\int_{-\infty}^{\infty}\frac{dx}{ch^2(x/\Delta)} = \frac{2}{\Delta^2}\Delta\int_{-\infty}^{\infty}\frac{dy}{ch^2 y} = \frac{2}{\Delta}th y\Big|_{-\infty}^{\infty} = \frac{4}{\Delta}$ или $E(v) = E_0/\Delta(v)$

Escaneado con CamScanner

Если $\varepsilon \neq \text{const}$, то $1+\lambda\varepsilon^2 = 1+\lambda\cos^2\Phi$ уже не $= \frac{1}{\Delta^2}$, с $\varepsilon = \frac{1}{\lambda}\frac{v}{\Delta}$, $\Phi = \frac{\pi}{2} - \varepsilon$.

Нов $\varepsilon(\xi) = \frac{v}{\lambda\Delta}\frac{1}{\operatorname{ch} y}$, $y = \xi/\Delta$. $1+\lambda\varepsilon^2 = 1+\lambda\frac{v^2}{\lambda^2\Delta^2}\frac{1}{\operatorname{ch}^2 y}$ и $E(\Theta,\Phi)$ другая.

$E = (\Theta_x)^2 + \sin^2\Theta(\Phi_x)^2 + \sin^2\Theta(1+\lambda\cos^2\Phi) = E_{ex} + E_a$, 1st int. $\to E_{ex} = E_a$

$E_{ex} = (\Theta_x)^2 + \sin^2\Theta(\varepsilon_x)^2 + \sin^2\Theta(1+\lambda\varepsilon^2)$, $\varepsilon_\xi = \frac{v}{\lambda\Delta}\frac{(-1)}{\operatorname{ch}^2 y}\operatorname{sh} y \cdot y_\xi =$

$= \frac{v}{\lambda\Delta^2}\sin\Theta\cos\Theta$, $E_{ex} = \sin^2\Theta\frac{1}{\Delta^2} + \sin^2\Theta\frac{v^2}{\lambda^2\Delta^4}\sin^2\Theta\cos^2\Theta$ — ?

$E_a = \sin^2\Theta\left[1+\frac{1}{\lambda}\frac{v^2}{\Delta^2}\frac{1}{\operatorname{ch}^2 y}\right] = \frac{1}{\operatorname{ch}^2 y}\left[1+\frac{1}{\lambda}\frac{v^2}{\Delta^2}\frac{1}{\operatorname{ch}^2 y}\right]$, $W_a = \int_{-\infty}^{\infty} d\varphi\, E_a =$

$= \int\frac{1}{\operatorname{ch}^2 y}d\varphi + \frac{1}{\lambda}\frac{v^2}{\Delta^2}\int\frac{d\varphi}{\operatorname{ch}^4 y}$, $\int\frac{d\varphi}{\operatorname{ch}^2(\xi/\Delta)} = \Delta\int_{-\infty}^{\infty}\frac{dy}{\operatorname{ch}^2 y} = 2\Delta$.

$\int\frac{d\varphi}{\operatorname{ch}^4(\xi/\Delta)} = \Delta\int dy\frac{1}{\operatorname{ch}^4 y} = \Delta\int d(\operatorname{th} y)\frac{1}{\operatorname{ch}^2 y} = \left|\frac{1}{\operatorname{ch}^2 y} + \operatorname{th}^2 y = 1\right| = \Delta\int_{-1}^{1} dz\,(1-z^2) =$

$= \Delta\left[z - \frac{1}{3}z^3\right]_{-1}^{1} = \Delta\cdot 2\left[1-\frac{1}{3}\right] = \frac{4}{3}\Delta$, $W_a = 2\Delta + \frac{4}{3}\Delta\frac{1}{\lambda}\frac{v^2}{\Delta^2} = 2\Delta\left[1+\frac{2}{3}\frac{1}{\lambda}\frac{v^2}{\Delta^2}\right]$

$\varepsilon = \frac{v}{\lambda\Delta}\sin\Theta$, $\varepsilon_\xi = \frac{v}{\lambda\Delta}\cos\Theta\,\Theta_\xi = \frac{v}{\lambda\Delta}\cos\Theta\cdot\sin\Theta\, y_\xi = \frac{v}{\lambda\Delta^2}\sin\Theta\cos\Theta$.

$E_{ex} = \frac{1}{\Delta^2}\sin^2\Theta + \frac{v^2}{\lambda^2\Delta^4}\sin^4\Theta\cos^2\Theta$, $W_{ex} = \frac{1}{\Delta^2}\int\frac{d\xi}{\operatorname{ch}^2(\xi/\Delta)} + \frac{v^2}{\lambda^2\Delta^4}\int d\xi\frac{1}{\operatorname{ch}^4(\frac{\xi}{\Delta})}\left(1-\frac{1}{\operatorname{ch}^2 y}\right) =$

$= \frac{1}{\Delta}\cdot 2 + \frac{v^2}{\lambda^2\Delta^3}\int dy\frac{\operatorname{th}^2 y}{\operatorname{ch}^4 y} = \frac{2}{\Delta} + \frac{v^2}{\lambda^2\Delta^3}\int d(\operatorname{th} y)\frac{\operatorname{th}^2 y}{\operatorname{ch}^2 y} = \frac{2}{\Delta} + \frac{v^2}{\lambda^2\Delta^3}\int_{-1}^{1} dz\, z^2(1-z^2) =$

$= \frac{2}{\Delta} + \frac{v^2}{\lambda^2\Delta^3}\cdot 2\left[\frac{z^3}{3} - \frac{z^5}{5}\right]_0^1 = \frac{2}{\Delta} + \frac{v^2}{\lambda^2\Delta^3}\cdot\frac{4}{15}$. $W_{ex} = W_a$ выполняется за счет

Оказывается $\Delta(v) = 1/\varkappa(v)$. Итак $\varepsilon = \text{const}$ $\Delta(v) = (1-v^2/c^2)^{1/2}$, $c^2 = \lambda$.

$W_{ex} = W_a \Rightarrow 2\Delta\left[1+\frac{2}{3}\frac{1}{\lambda}\frac{v^2}{\Delta^2}\right] = \frac{2}{\Delta}\left[1+\frac{2}{15}\frac{v^2}{\lambda^2\Delta^2}\right]$ — биквадр. eq. на $\Delta(v)$.

$\varkappa^2(v) = 1 - \frac{1}{2}(v^2-\lambda) - \frac{1}{2}\sqrt{(v^2-v_-^2)(v^2-v_+^2)}$. При $\lambda\to\infty$, $v_+ = v_- = c = \sqrt{\lambda}$

$\varkappa^2(v) = 1 - \frac{1}{2}(v^2-\lambda) - \frac{1}{2}(v^2-\lambda) = 1+\lambda - v^2 \approx \lambda - v^2$, $\varkappa^2(v) = c^2 - v^2$

$\varkappa^2(v) = c^2(1-v^2/c^2)$, $\Delta(v) = \frac{1}{\varkappa(v)} = \frac{1}{c}\frac{1}{\sqrt{1-v^2/c^2}}$, $c^2 = \lambda$.

$\Delta^2 + \frac{2}{3}\frac{v^2}{\lambda} = 1 + \frac{2}{15}\frac{v^2}{\lambda^2\Delta^2}$, $\Delta^4 + \frac{2}{3}\frac{v^2}{\lambda}\Delta^2 - \Delta^2 - \frac{2}{15}\frac{v^2}{\lambda^2} = 0$

$2\Delta^2_{1,2} = 1 - \frac{2}{3}\frac{v^2}{\lambda} \pm \sqrt{\left(1-\frac{2}{3}\frac{v^2}{\lambda}\right)^2 + 4\frac{2}{15}\frac{v^2}{\lambda^2}}$, $D = 1 - \frac{4}{3}\frac{v^2}{\lambda} + \frac{4}{9}\frac{v^4}{\lambda^2} + \frac{8}{15}\frac{v^2}{\lambda^2}$

Не похоже на $\frac{1}{\Delta^2(v)} = \lambda(1-v^2/\lambda)$ которое вычислялось при $\Phi = \text{const}$ [Schlomann]

$v_- v_+ = \left(\sqrt{1+\lambda}+1\right)\left(\sqrt{1+\lambda}-1\right) = (1+\lambda) - 1 = \lambda$, $\varkappa_s^2(v\to 0) = \frac{1}{2}\lambda + 1 - \frac{1}{2}\lambda = 1$, ang λ.

Но если $\lambda\to\infty$, то $\varkappa_s^2(v) = \lambda(1-v^2/\lambda)$. $\varkappa_s^2(v\to 0) = \lambda$.

Как, все же? $\varkappa_s^2(v, \lambda\to\infty) = 1 - \frac{1}{2}(v^2-\lambda) - \frac{1}{2}\sqrt{(v^2-\lambda)^2} = 1 - \frac{1}{2}(v^2-\lambda) - \frac{1}{2}(\lambda - v^2) = 1$.

К Сообщ. max velocity v_-. $x_m = (1+\lambda)^{1/4}$, $v_- \approx c$ при $\lambda\to\infty$.

Sin-Gordon $\Delta(v)$ при $\lambda\to\infty$ $2\Delta^2 = \lambda - \frac{2}{3}\frac{v^2}{\lambda} \pm \left(1-\frac{2}{3}\frac{v^2}{\lambda}\right) = \sqrt{2\left(1-\frac{2}{3}\frac{v^2}{\lambda}\right)}$

$\Delta^2_{SG}(v) = 1 - \frac{2}{3}\frac{v^2}{\lambda}$, $\lambda\to\infty$. $v_m^2 = \frac{3}{2}\lambda$ — ? Но $\varepsilon(\xi)$ дает рост v_m?

$\varepsilon(\xi) \to v_m = \sqrt{\frac{3}{2}}c$, $c = \sqrt{\lambda}$, т.е. $\varepsilon = \text{const}$ Walker-Pearson следует $v_m = c$. Проверить. см. 3/III

Обед с Окс., Раф., Ишма, Гиеб, Л., Ю., А. у нас дома — Cordero y esterion.

25/II Meeting с Ишби и Окс.: Dw/Soliton/Sw решения в зависимости от v. Ricardo прислал Figs. для new MS по Dw dynamics — отлично! Нужно.

R. Verbon прислал MS draft по Stagnation с variable $\Phi_0(\xi)$ static.

M. Krawczyk — запрос на magIC report. Ужин в рес. „Londres“: Окс., Раф., Ишма, Julian с женой, Гиеб, Л., Ю., Julian'a и я — день рождения 60 лет.

26/II Meeting с Окс.: разбор new Dw MS по $U \to$ sin-Gordon mapping. Доказательство и продолжение анализа. работа в этом направлении может быть solitons с $v_- < v < v_+$

Ужасно. Нобрев. Cost — Porto application. Запрос review Stagnation MS — RRL.

27/II Запрос в A. giant Black comp. Данные не решают. Подготовка рез. плана по FM Dw motions и Subm. Cost appl. trip to Porto. Запрос to review Polish project.

→ Sobolev 1995: max Walker velocity $v_m = \frac{\lambda}{\sqrt{2(2+\lambda)}}$, $\lambda = \frac{2\pi M_s^2}{K_2}$, d-H$_2$ method.

$v_m(\lambda) \le v_-(\lambda)\ \forall\lambda$. Но $v_m(\lambda)$ значительно меньше $v_+(\lambda) = \sqrt{1+\lambda} + 1$ при $\lambda < 1$.

$v_m(\lambda) \simeq \frac{\lambda}{2}$, $v_+ \approx 2$ при $\lambda\to 0$. v in units $v_0 = \frac{2\gamma}{M_s}\sqrt{AK_2}$. При $\lambda \gg 1$

$v_m(\lambda) \simeq \frac{1}{\sqrt{2}}\sqrt{\lambda}$, $v_+(\lambda) \approx \sqrt{\lambda}$ — разница всего в $\sqrt{2} \simeq 1.4$ раза.

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(28/II) Новые airtickets to Kiev. Booked hotel на горе в Porto на 10/III. Transfers to Ногог. и посылка to Maciej techn. report MagIC (extended reports Tasks 1.2, 1.5, 2.5). "Bankix"

(1/III) П. Вдела dinner в клеб 29/III - 7/IV "lufthensa". переписка SolSkyMag и COST - Occ; Ruben

Lemme Helicity $\gamma(\vec{s}) = \Phi_0(\vec{s})$ - static MS by Verba. Minor changes Sec. 1 - III.

Не согласен с Eq. (8) $\langle B_s(\vec{s},z)\rangle_z = B_s(\vec{s}) \sim f_\beta(\vec{s}) = F(t_z)\big|_{z=0}^{z=t_{HL}}$, $t_z^2 = \frac{4sR}{z^2+(s+R)^2}$

Надо не $\tilde{t}_{HL} = t_{HL} - t_d$, а t_{HL} (hard matter thickness) здесь.

$B_s(s,z)$ описывает s - component of stray field of hard matrix $\vec{M}_{HL} = pM_{HL}\hat{z}$

в районе soft dot с t_d - thickness и radius R.

$W_{Zeem} \sim \int_0^{t_d} dz \int ds_s\, B_s(s,z) M_s(s)$, $\vec{M}$ - magnetization of soft dot, Fig. 1a, 1e.

$W_z \sim \int dz\, B_s(s,z)$, $B_s(\vec{r}) = \int dz' \int ds's' \int d\varphi' G_{sz}(\vec{r},\vec{r}')\, M_{HL}^z(\vec{r}')$, $\vec{M}_{HL}(\vec{r}) = const$

$W_z \sim \int_0^{t_d} dz \int_0^{t_{HL}} dz' \int ds's'\, g_{sz}(s,z,s',z')$, $g_{sz} = \int d\varphi \int d\varphi' G_{sz}(\vec{r},\vec{r}')$, $\vec{r} = (s,\varphi,z)$

$g_{sz}(s,z,s',z') = (2\pi)^2 \frac{\partial}{\partial s}\frac{\partial}{\partial z'} \int dk\, e^{-k|z-z'|} J_0(ks) J_0(ks')$

$W_z \sim \int dz \int dz' \int ds's' \frac{\partial}{\partial s}\frac{\partial}{\partial z'} \int dk e^{-k|z-z'|} J_0(ks) J_0(ks')$, $\frac{\partial J_0(ks)}{\partial s} = -k J_1(ks)$

$\int_0^R ds's' J_0(ks') = |ks' = x| = \frac{1}{k^2}\int_0^{kR} dx\, x J_0(x) = \frac{1}{k^2} x J_1(x)\big|_0^{kR} = \frac{R}{k} J_1(kR)$

$W_z \sim \int_0^{t_d} dz \int_0^{t_{HL}} dz' \frac{\partial}{\partial z'} \int dk e^{-k|z-z'|} J_1(ks) J_1(kR) = \int dk\, J_1 J_1 \int_0^{t_d} dz\, e^{-k|z-z'|}\Big|_{z'=0}^{z'=t_{HL}} =$

$= \int dk\, J_1 J_1 \int_0^{t_d} dz \left[e^{-k(t_{HL}-z)} - e^{-kz}\right] = \int dk\, J_1 J_1 \frac{1}{k}\left[e^{-kt_{HL}}\left(e^{kt_d}-1\right) + e^{-kt_d} - 1\right]$

Если $kt_d \ll 1$, то $W_z \sim \int dk\, J_1(ks) J_1(kR)\left[e^{-kt_{HL}} - 1\right] = F(x)\big|_{z=0}^{z=t_{HL}}$, $x^2 = \frac{4sR}{z^2+(s+R)^2}$ - нечувствительно. Eq. (S2) Suppl. Mat. Navas - submitted.

$W_z \sim \int_0^\infty \frac{dk}{k} J_1(ks) J_1(kR)\left[e^{-k(t_{HL}-t_d)} + e^{-kt_d} - e^{-kt_{HL}} - 1\right] = F_1(x) =$

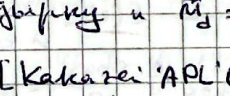
$= F_1(t_{HL}-t_d) + F_1(t_d) - F_1(t_{HL}) - F_1(0)$ видно $W_z = 0$ при $t_{HL} = t_d$.

Все выражается через $I_1(z) = \int_0^\infty \frac{dk}{k} J_1(ks) J_1(kR) e^{-kz}$, тогда как если интеграл $\int dz'(\dots)$, то все выражается через $I(z) = \int dk\, J_1(ks) J_1(kR) e^{-kz}$, $x^2 = \frac{4sR}{z^2+(s+R)^2}$

$z = z - z'$ ($z' = 0, t_{HL}$) и $\int dz(\dots)$ не будет. Verba: $\int dz(\dots)$, здесь $kt_d \ll 1$ и $\int dz'(\dots)$ - тогда $z' = (t_{HL}, 0)$.

Все это в предположении, что dip. field внутри фольги от ∞-film с $\vec{M} \parallel \hat{z}$, также не как от dot развернем с фольгу и $\vec{M}_d = -\vec{M}$.

Проверка SW 1-нанесли. circular dot [Kakazei 'APL' 04].



Тут коэффициент $\beta = L/R$ описывает pinning $d(\beta) \sim \frac{1}{\beta[\dots]}$ и корни $x J_1(x) = d J_0(x)$ при finite β стремятся меньше корней для $J_0(\alpha) = 0$ при $d \to \infty$. Численная проверка by MathCad ("Radial_SW_eqt_dot.mcd") дала при $R = 100$ nm, $\beta \le 0.4$ что $\omega_n(\beta)$ - real d лишь немного ниже $\omega_n^{d\to\infty}(\beta)$ при $n = 1,2$ и linear SW dispersion relation: $\omega_n/\omega_M = h_z + h_{nn} + \left(\frac{l_e}{R}\right)^2 x_n^2 + \frac{1}{2} f(\beta x_n)$, все в eq. $4\pi M_s$, $l_e^2 = A/2\pi M_s^2$

Ошибка $\le 5\%$ даже при $\beta = 0.4$ ($L = 40$ nm, $R = 100$ nm).

Видимо главным источником ошибки здесь не неоднородный pinning "d", а неверное eq. Slavin из Kakaz' APL'04: $\omega_k^2 = [\omega_H + d\omega_M k^2][\omega_H + \omega_M f(kL) + d\omega_M k^2]$

$d = l_e^2 = A/2\pi M_s^2$. $\left(\frac{\omega_k}{\omega_M}\right)^2 = \left[h_z - N(s) + l_e^2 k^2\right]\left[h_z - N(s) + f(kL) + l_e^2 k^2\right]$

$\omega_H = \gamma\left[H_z - 4\pi M_s N(s)\right]$

$N(s) \to N_{nn}$, $N_{nn} = \frac{1}{A_n}\int_0^1 ds\, s\, N(s) J_0^2(x_n s)$, $A_n = \frac{1}{2} J_1^2(x_n)$, $d \to \infty$, $\frac{L}{R} \to 0$.

Сом. 1/I $H_z^s(s) = -\frac{4\pi M_s}{\beta}\int_0^\infty \frac{dx}{x}\left(1 - e^{-\beta x}\right) J_0(sx) J_1(x) = -4\pi M_s N(s)$

$N(s) = \frac{1}{\beta}\int_0^\infty \frac{dx}{x}\left(1 - e^{-\beta x}\right) J_0(sx) J_1(x)$, $h_0(s) = \frac{1}{4\pi M_s} H_z^s(s) = -\frac{1}{\beta}\int \frac{dx}{x}\left(1 - e^{-\beta x}\right) J_0(sx) J_1(x) =$

$= \int_0^\infty dx\, J_0(sx) J_1(x)\left[f(\beta x) - 1\right] = -1 + \int_0^\infty dx\, J_0(sx) J_1(x) f(\beta x)$, $h_0(s) = -N(s)$.

$\langle n | h_0(s) | n \rangle = h_{nn} = -1 + \int ds\, s\, e_n^2(s) \int dx\, f(\beta x) J_0(sx) J_1(x) = -1 + \Gamma_{nn}$ (static), $e_n(s) = \frac{1}{\sqrt{A_n}} J_0(x_n s)$

Сом. 1-4/II - general.

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$-N_{nn} = h_{nn} = -1 + \frac{1}{A_n}\int_0^\infty d\vartheta\, J_0^2(x_n\vartheta)\int_0^\infty f(\beta x) J_0(\vartheta x) J_1(x)\,dx$, $A_n = \frac{1}{2}\frac{(x_n^2+d^2)}{x_n^2} J_0^2(x_n)$

Slavin $\left(\frac{\omega_n}{\omega_M}\right)^2 = \left[h_z + h_{nn} + \left(\frac{l_e}{R}\right)^2 x_n^2\right]\left[h_z + h_{nn} + f(\beta x_n) + \left(\frac{l_e}{R}\right)^2 x_n^2\right]$ (x_n из $dJ_0(x)=xJ_1(x)$; при $d\to\infty$ $A_n = \frac{1}{2}[J_0^2(x_n)+J_1^2(x_n)]$, x_n из $J_0(x)=0$)

Moe дисперс. eq. $h_z + h_{nn} + \frac{1}{2}f(\beta x_n) + \left(\frac{l_e}{R}\right)^2 x_n^2 = \frac{\omega_n}{\omega_M}$.

$R = 100$ nm: $n=1$) ω_1 (real d) $\approx \omega_1(d\to\infty)$ - lin. SW; $n=2$) ω_2^{lin} (real d) немного $\omega_2^{lin}(d\to\infty) \approx \omega_2^{slav}$; $n=3$) ω_3^{lin} (real) $\approx \omega_3^{slav}$, $\omega_3^{lin}(\infty) > \omega_3^{lin}$ (real d)

$\omega_{SE}^{(1)} < \omega_1$

Вторые чувствительности $\omega_n(\beta)$ $\beta \le 0.4$ при $H/4\pi M_S = h_z = 1$.

$R = 150$ nm: $n = 1,2,3$, ω_n^{lin} (real d) $\approx \omega_n^{lin}(d\to\infty)$ $\forall\beta$, $\omega_n^{slav} < \omega_n^{lin}$ $\forall\beta$, при $\beta = 0.2$–0.4 eq. Slavin занижает частоту SW на 10–15% (!)

А. е. $\omega_n(\beta)$ частоты (linear) сильно зависят от ширины d, но eq. Slavin - APL'04 сильно занижает частоты при finite $\beta = 0.1$–0.4.

3/III 1D DW motion

Предполагаем вывод sin-Gordon при $\lambda \gg 1$, $\varepsilon = \text{const}$ (18/II) и $\varepsilon = \varepsilon(\xi)$ (20/II).

a) $\varepsilon = \text{const}$, $\Phi_\xi = 0$, $\Phi(\xi) = \frac{\pi}{2} - \theta(\xi)$ - решение системы УД - LL от 11/II (normal) при $\varepsilon \ll 1$ даст $y(\xi) = \xi/\Delta(v)$, $\Delta(v) = \sqrt{1-v^2/c^2}$, $\varepsilon = \frac{v}{\lambda}\frac{1}{\Delta(v)}$. $\operatorname{tg}\frac{\theta}{2} = e^y$, $y = \xi/\Delta$, $\xi = x - vt$.

Если считать $\theta = \theta(\xi)$, $\theta(\xi) = 2\operatorname{arctg} e^{\xi/\Delta}$, то $\theta_\xi = \theta_x$, $\theta_{\xi\xi} = \theta_{xx}$, $\theta_t = \theta_\xi \dot\xi = -v\theta_\xi$, $\theta_{tt} = -v\theta_{\xi\xi}\dot\xi = v^2\theta_{\xi\xi}$, $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \theta_{\xi\xi} - \frac{1}{c^2}v^2\theta_{\xi\xi} = \left(1-\frac{v^2}{c^2}\right)\theta_{\xi\xi} = \frac{1}{\Delta^2}\theta_{\xi\xi} = \frac{1}{2\Delta^2}\sin 2\theta$

$\theta_{\xi\xi} = \frac{1}{2}\sin 2\theta$, $\theta_{xx} = \frac{1}{2}\sin 2\theta$ - static sin-Gordon. $\theta(x) = 2\operatorname{arctg}(e^x)$.

$\theta_x = 2\frac{1}{1+e^{2x}}e^x = \frac{1}{\operatorname{ch} x}$, $\theta_{xx} = -\frac{1}{\operatorname{ch}^2 x}\operatorname{sh} x = \frac{1}{\operatorname{ch} x}(-\operatorname{th} x) = \sin\theta\cos\theta$, $\theta_{xx} = \frac{1}{2}\sin\theta\cos\theta$

Static $\theta(x) = 2\operatorname{arctg}(e^x)$. Дин. θ при $x \to \frac{x-vt}{\Delta(v)} = \frac{\xi}{\Delta}$, $\theta(x) \to \theta(x')$, $x' = \frac{x-vt}{\sqrt{1-v^2/c^2}}$.

$\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{2\Delta^2}\sin 2\theta$ - такое sin-Gordon не обеспечивает и бу $E(\theta,\Phi)$ при $\lambda\to\infty$.

$E(\theta,\Phi) = (\theta_x)^2 + \sin^2\theta\left[1+\lambda\cos^2\Phi\right] = \left|\begin{matrix}\cos^2\Phi = \varepsilon^2 \\ \lambda\varepsilon^2 = \frac{1}{\Delta^2(v)}\end{matrix}\right| = (\theta_x)^2 + \frac{1}{\Delta^2(v)}\sin^2\theta$

$\Phi(\xi) = \text{const}$

$\mathcal{L}(\lambda\to\infty) = \frac{1}{c^2}(\theta_t)^2 - (\theta_x)^2 - \frac{1}{\Delta^2(v)}\sin^2\theta$, $\frac{\partial}{\partial t}\frac{\partial\mathcal{L}}{\partial\dot\theta} + \frac{\partial}{\partial x}\frac{\partial\mathcal{L}}{\partial\theta_x} = \frac{\partial\mathcal{L}}{\partial\theta}$ даст $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{2\Delta^2}\sin 2\theta$

б) $\varepsilon = \varepsilon(\xi)$, $\varepsilon(\xi) \ll 1$, см. 20/II.

$\left.\begin{cases} v y_\xi = \lambda\varepsilon - \varepsilon_{\xi\xi} + 2\varepsilon_\xi y_\xi \operatorname{th} y \\ v\varepsilon_\xi = y_{\xi\xi} + \left[1 + \lambda\varepsilon^2 - (y_\xi)^2\right]\operatorname{th} y \end{cases}\right\}$ - линеаризованная по $\varepsilon(\xi) \ll 1$ система УД

Если принять $\lambda\varepsilon \gg \varepsilon_{\xi\xi}, \varepsilon_\xi$, то $y_\xi = \frac{\lambda}{v}\varepsilon$ и $\varepsilon(\xi)$ можно исключить из системы, решив ее и найти $y(\xi) = \frac{\xi}{\Delta}$, $\varepsilon(\xi) = \frac{v}{\lambda\Delta}\frac{1}{\operatorname{ch}(\xi/\Delta)}$

Согласовывая их решение $\varepsilon(\xi) = \frac{v}{\lambda}\frac{a}{\operatorname{ch}(a\xi)}$, $a = \frac{1}{\Delta}$? $\Delta = \sqrt{1-v^2/c^2}$

$\varepsilon_\xi/\varepsilon = -\frac{va^2}{\lambda}\frac{\operatorname{th} a\xi}{\operatorname{ch} a\xi}\frac{\lambda}{v}\frac{\operatorname{ch} a\xi}{a} = -a\operatorname{th}(a\xi)$, $a = \frac{1}{\sqrt{1-v^2/c^2}}$, $\varepsilon_\xi = \frac{va^2}{\lambda}(-1)\frac{a\cdot\operatorname{sh}(a\xi)}{\operatorname{ch}^2(a\xi)} = -\frac{va^2}{\lambda}\frac{\operatorname{th} a\xi}{\operatorname{ch} a\xi}$

$\frac{\varepsilon_\xi}{\lambda\varepsilon} \sim \frac{a}{\lambda} = \frac{1}{\lambda\Delta}$, если $v/c = \text{const}$, $\lambda\to\infty$, все ОК. $\frac{\varepsilon_\xi}{\lambda\varepsilon} = -\frac{a}{\lambda}\operatorname{th} a\xi$, $|\operatorname{th} a\xi| \le 1$.

и $v\to c-0$, $\frac{\varepsilon_\xi}{\lambda\varepsilon} \to \infty$, т.е. Но at any finite λ, решение similar to approach $\varepsilon = \text{const}$.

Что делает для 1st integrals of motion (Elechnskii)? $\Phi(\xi) = \pi/2 - \theta(\xi)$

Сначала 11/II 1st integral $y_\xi^2 + \Phi_\xi^2 = 1 + \lambda\cos^2\Phi$, $(y_\xi)^2 + (\varepsilon_\xi)^2 = 1 + \lambda\varepsilon^2$

$(y_\xi)^2 = 1 + \lambda\varepsilon^2 - (\varepsilon_\xi)^2$. а 20/II а сейчас $y_\xi = \frac{\lambda}{v}\varepsilon$. $\left[1 + \lambda\varepsilon^2 - (y_\xi)^2\right] = (\varepsilon_\xi)^2$ и поскольку $\varepsilon_\xi \ne 0$.

→ Вторые 1st integral (Elechnskii '79): $\left\{\begin{matrix} m_x = \sin\theta\cos\Phi = \sin\theta\cdot\varepsilon + O(\varepsilon^3) \\ m_y = \sin\theta\sin\Phi = \sin\theta\left[1 - \frac{1}{2}\varepsilon^2\right]\end{matrix}\right.$

$(1+\lambda)(m_x')^2 + (m_y')^2 - (1+\lambda)m_x^2 - m_y^2 - \lambda(1+\lambda)m_x^2 + \left[(1+\lambda)m_x^2 + m_y^2\right]^2 = Q$, при H (1st integral) $= 0$.

Используем наш основной $\theta(\xi) = 2\operatorname{arctg} e^{y(\xi)}$, $\sin\theta = \frac{1}{\operatorname{ch} y}$, $\cos\theta = -\operatorname{th} y$.

при $\lambda\to\infty$: $\lambda(m_x')^2 + (m_y')^2 - \lambda m_x^2 - m_y^2 - \lambda^2 m_x^2 + \left[\lambda m_x^2 + m_y^2\right]^2 = Q$

$m_x' = \frac{\partial m_x}{\partial\xi} = \frac{\partial}{\partial\xi}\sin\theta\cos\Phi = \cos\theta\,\theta_\xi\cos\Phi + \sin\theta(-\sin\Phi)\Phi_\xi = \varepsilon\theta_\xi\cos\theta + \sin\theta\,\varepsilon_\xi$

$m_y' = \frac{\partial}{\partial\xi} m_y = \frac{\partial}{\partial\xi}\sin\theta\sin\Phi = \cos\theta\theta_\xi\sin\Phi + \sin\theta\cos\Phi\,\Phi_\xi = \cos\theta\theta_\xi\left(1-\frac{1}{2}\varepsilon^2\right) - \sin\theta\cdot\varepsilon\varepsilon_\xi$

$m_y' = \theta_\xi\cos\theta - \frac{1}{2}\varepsilon^2\cos\theta\theta_\xi - \varepsilon\varepsilon_\xi\sin\theta = \theta_\xi\cos\theta + O(\varepsilon^2)$, $m_x' = O(\varepsilon) + O(\varepsilon^3)$, пренебрег.

$\lambda\left[\varepsilon^2\theta_\xi^2\cos^2\theta + \sin^2\theta\,\varepsilon_\xi^2 + 2\varepsilon\varepsilon_\xi\theta_\xi\cos\theta\sin\theta\right] + \theta_\xi^2\cos^2\theta - \theta_\xi\cos\theta(\varepsilon^2\cos\theta\theta_\xi + 2\varepsilon\varepsilon_\xi\sin\theta) -$

$- \sin^2\theta(1-\varepsilon^2) - \lambda^2\sin^2\theta\,\varepsilon^2 + \lambda^2\sin^4\theta\,\varepsilon^4 + \sin^4\theta(1-2\varepsilon^2) + 2\lambda\sin^2\theta\,\varepsilon^2\sin^2\theta = Q$

$\lambda\varepsilon^2\theta_\xi^2\cos^2\theta + \sin^2\theta\,\varepsilon_\xi^2 + 2\varepsilon\varepsilon_\xi\theta_\xi\cos\theta\sin\theta\right] + \theta_\xi^2\cos^2\theta - \sin^2\theta(1-\varepsilon^2) - \lambda^2\varepsilon^2\sin^2\theta + \sin^4\theta(1-2\varepsilon^2) +$

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$+2\lambda\varepsilon^2\sin^4\theta = Q$; $\lambda\left[\varepsilon^2\theta_3^2\cos^2\theta + \sin^2\theta\,\varepsilon_3^2 + 2\varepsilon\varepsilon_3\theta_3\cos\theta\sin\theta\right] + \theta_3^2\cos^2\theta -$

$-\lambda^2\varepsilon^2\sin^2\theta - \sin^2\theta\cos^2\theta + 2\lambda\varepsilon^2\sin^4\theta = Q$, $\theta = 2\operatorname{arctg} e^{y(\xi)}$, $\theta_3 = 2\frac{1}{e^{2y}+1}e^y y_3 = \frac{1}{\operatorname{ch} y}y_3$

$\theta_3 = \sin\theta\, y_3$, $\lambda\sin^2\theta\left[\varepsilon^2 y_3^2\cos^2\theta + \varepsilon_3^2 + 2\varepsilon\varepsilon_3 y_3\cos\theta\right] + y_3^2\sin^2\theta\cos^2\theta - \lambda^2\varepsilon^2\sin^2\theta -$

$-\sin^2\theta\cos^2\theta + 2\lambda\varepsilon^2\sin^4\theta = Q$, Гр. условие $m_x, m_y\,(x\to\pm\infty) = 0$ дают $Q=0$

$\lambda\left[\varepsilon^2 y_3^2\cos^2\theta + \varepsilon_3^2 + 2\varepsilon\varepsilon_3 y_3\cos\theta\right] + (y_3^2-1)\cos^2\theta - \lambda^2\varepsilon^2 + 2\lambda\varepsilon^2\sin^2\theta = 0$

$\lambda\left[\varepsilon y_3\cos\theta + \varepsilon_3\right]^2 = (1-y_3^2)\cos^2\theta + \lambda^2\varepsilon^2$. При $\varepsilon_3 \to 0$ ($\varepsilon y_3 \gg \varepsilon_3$):

$\lambda\varepsilon^2 y_3^2\cos^2\theta + (y_3^2-1)\cos^2\theta = \lambda^2\varepsilon^2$, можно найти $\varepsilon(\xi)$ через y, y_3.

$\varepsilon^2\left[\lambda\left[y_3^2\cos^2\theta - \lambda\right]\right] = (1-y_3^2)\cos^2\theta$, $\lambda\varepsilon^2 = \frac{(1-y_3^2)\cdot\operatorname{th}^2 y}{\left[y_3^2\operatorname{th}^2 y - \lambda\right]}$

При $\varepsilon_3 = 0$, $y_3 = \frac{1}{\Delta}$, $1-y_3^2 = 1-\frac{1}{\Delta^2} = \frac{(\Delta^2-1)}{\Delta^2} = -\frac{v^2}{c^2}\frac{1}{\Delta^2}$

$\Delta^2 = (1-v^2/c^2)$ $\varepsilon^2 = \frac{1}{\lambda}(-1)\frac{v^2}{c^2}\frac{1}{\Delta^2}\frac{-\operatorname{th}^2 y}{\left[\frac{1}{\Delta^2}\operatorname{th}^2 y - \lambda\right]} = \frac{v^2}{\lambda^2}\frac{\operatorname{th}^2 y}{\left[\lambda\Delta^2 - \operatorname{th}^2 y\right]}$ даст $\varepsilon(\xi)$ через $y(\xi)$.

$\varepsilon(\xi) = \frac{v}{\Delta}\frac{\operatorname{th} y}{\sqrt{\lambda\Delta^2 - \operatorname{th} y}}$, если $\operatorname{th} y(\xi) \to 1$, $\varepsilon(\xi) = \varepsilon = \frac{v}{\lambda\Delta}\frac{1}{\lambda^{1/2}}$ отсюда $\varepsilon = \frac{v}{\lambda\Delta}$.

т.е. second fit integral насколько сложен, что нельзя как из него извлечь полезную info о $y(\xi)$, $\varepsilon(\xi)$. Система при $\varepsilon(\xi)\ll 1$ $\left\{\begin{array}{l} v y_3 = \lambda\varepsilon - \varepsilon_{\xi\xi} + 2\varepsilon_3 y_3 \operatorname{th} y \\ v\varepsilon_3 = y_{\xi\xi} + 2(\varepsilon_3)^2\operatorname{th} y \end{array}\right\}$ (*)

Возможность из $1+\lambda\varepsilon^2 - y_3^2 = \varepsilon_3^2$ на модель решения $y(\xi) = \frac{\xi}{\Delta}$, $\varepsilon(\xi) = \frac{v}{\lambda\Delta}\frac{1}{\operatorname{ch} y}$? Проверим $1+\lambda\frac{v^2}{\lambda^2\Delta^2}\frac{1}{\operatorname{ch}^2 y} - \frac{1}{\Delta^2} = \frac{v^2}{\lambda^2\Delta^4}\frac{\operatorname{th}^2 y}{\operatorname{ch}^2 y}$

$\varepsilon_3 = -\frac{v}{\lambda\Delta^2}\frac{\operatorname{th} y}{\operatorname{ch} y}$. $1-\frac{v^2}{\lambda} + \frac{v^2}{\lambda}\frac{1}{\operatorname{ch}^2 y} - 1 = \frac{v^2}{\lambda^2\Delta^2}\frac{\operatorname{th}^2 y}{\operatorname{ch}^2 y}$, $\operatorname{th}^2 y + \frac{1}{\operatorname{ch}^2 y} = 1$

$\frac{v^2}{\lambda}\left(\frac{1}{\operatorname{ch}^2 y} - 1\right) = -\operatorname{th}^2 y = \frac{v^2}{\lambda^2\Delta^2}\frac{\operatorname{th}^2 y}{\operatorname{ch}^2 y}$, $\frac{1}{\operatorname{ch}^2 y}\frac{v^2}{\lambda^2\Delta^2} = -1$ — не выполняется.

т.е. $y(\xi) = \xi/\Delta$, $\varepsilon(\xi) = \frac{v}{\lambda\Delta}\frac{1}{\operatorname{ch} y}$ не удовлетв. 1st integral eq., и не есть решение (*).

но $\varepsilon(\xi) = \frac{v}{\lambda\Delta}\frac{1}{\operatorname{ch} y}$ — решение типа Sklyanin $S_x = m_x = -\frac{1}{R}\dot\varphi = -\frac{\sqrt{\gamma}}{R}\frac{1}{\operatorname{ch} u}\frac{v}{\sqrt{1-v^2}}$ $(m_x = \varepsilon\sin\psi \approx \varepsilon\sin\theta)$

у Sklyanin'79 $u = \frac{\sqrt{\gamma_s}}{\sqrt{1-v^2}}(x - vt)$, $x = \frac{x}{\sqrt{R}}$, $u = \frac{\sqrt{\gamma_s}}{\sqrt{R}}\frac{(x-\sqrt{R}vt)}{\sqrt{1-v^2}} = \frac{(x-Vt)}{\Delta(v)}$

$\Delta(v) = \frac{\sqrt{R}}{\sqrt{\gamma_s}}\sqrt{1-v^2}$, $V = \sqrt{R}v$. $\Delta_0 = \sqrt{\frac{R}{\gamma_s}}$. Если оценка скорости v в eq. $v_0 = \frac{2\gamma}{M_s}\sqrt{AK_2}$

$v^2 \to v^2/v_0^2 = \frac{Rv^2}{Rv_0^2} = \frac{V^2}{V_m^2}$, $V_m = \sqrt{R}\,v_0$, $R = \sqrt{\frac{\gamma_s}{K_2}}\sqrt{\frac{K_x}{K_2}}$ — у оценки Sklyanin'79.

$V_m = \sqrt{R}\,v_0 = \gamma_s^{1/4}\left(\frac{K_x}{K_2}\right)^{1/4}\frac{2\gamma}{M_s}\sqrt{AK_2}$ — ? Склянин'79 неверен. Должно $V_m = \frac{2\gamma}{M_s}\sqrt{AK_x}$

Другие следовали Kivshar'89: $S_x = -\frac{1}{R}\dot\varphi$, $\varphi_{xy} - \varphi_{tt} = \frac{1}{2}\sin 2\varphi$, $x = \frac{x}{\sqrt{R}}$, $\lambda = R = \frac{K_x}{K_2}$, $\lambda = \xi = R$.

Static $\varphi_{yy} = \frac{1}{2}\sin 2\varphi$, $\varphi(y) = 2\operatorname{arctg} e^y$.

Dyn. $\varphi(y)$, $y \to y' = \frac{y - vt}{\sqrt{1-v^2}}$, $\sin\varphi = \frac{1}{\operatorname{ch}\Lambda}$, $\Lambda = \frac{y - vt}{\sqrt{1-v^2}}$, v в eq. v_0.

$\cos\varphi = -\operatorname{th}\Lambda$. $\cos\varphi\dot\varphi = \frac{1}{\operatorname{ch}^2\Lambda}(-1)\operatorname{sh}\Lambda\cdot\dot\Lambda = \cos\varphi\sin\varphi\frac{(-v)}{\sqrt{1-v^2}}$, $\dot\varphi = -\frac{v}{\sqrt{1-v^2}}\sin\varphi$

$S_x = -\frac{1}{R}\dot\varphi = \frac{1}{R}\frac{v}{\sqrt{1-v^2}}\frac{1}{\operatorname{ch}^2\Lambda}$, $\Lambda = \frac{1}{\sqrt{1-v^2}}\left(\frac{x}{\sqrt{R}} - vt\right) = \frac{1}{\sqrt{R}}\frac{(x-\sqrt{R}vt)}{\sqrt{1-v^2}}$

$\frac{v}{v_0} \to \frac{v}{v_0} = \frac{\sqrt{R}v}{\sqrt{R}v_0} = V/V_m$, $V_m = \sqrt{R}\,v_0 = \sqrt{\frac{K_x}{K_2}}\frac{2\gamma}{M_s}\sqrt{AK_2} = \frac{2\gamma}{M_s}\sqrt{AK_x}$ — верно.

Или $V_m/v_0 = \sqrt{R} = \sqrt{\lambda}$. $S_x(x,t) = \frac{1}{R}\frac{v}{\sqrt{1-v^2}}\frac{1}{\operatorname{ch}\Lambda} = \frac{1}{R^{3/2}}\frac{\sqrt{R}v}{\sqrt{1-v^2}}\frac{1}{\operatorname{ch}\Lambda} = \frac{1}{\lambda^{3/2}}\frac{V}{\sqrt{1-\frac{V^2}{\lambda}}}\cdot\frac{1}{\operatorname{ch}\Lambda}$

Надо перенорм. скорость $V = \sqrt{R}\,v$.

Sklyanin-решение в редакции Kivshar'89: $S_x(x,t) = \frac{1}{\lambda^{3/2}}\frac{V\sin\varphi}{\sqrt{1-V^2/c^2}}$, $c^2 = \lambda$,

$\lambda = R = \frac{K_x}{K_2}$ $\left(R = \frac{|\beta_1|}{\beta_3}\right)$, $\sin\varphi = \frac{1}{\operatorname{ch}\Lambda}$, $\Lambda(x,t) = \frac{1}{\sqrt{R}}\frac{(x-Vt)}{\sqrt{1-V^2/c^2}} = \frac{1}{\sqrt{\lambda}}\frac{(x-Vt)}{\sqrt{1-V^2/\lambda}}$

$S_x(x,t) = m_x(x,t) = \frac{1}{\lambda^{3/2}}\frac{V}{\sqrt{1-V^2/\lambda}}\frac{1}{\operatorname{ch}\left[\frac{1}{\sqrt{\lambda}}\frac{(x-Vt)}{\sqrt{1-V^2/\lambda}}\right]}$, введем $\Delta(V) = \sqrt{\lambda}\sqrt{1-V^2/\lambda}$

$S_x(x,t) = \frac{V}{\lambda\Delta(V)}\cdot\operatorname{ch}^{-1}\left[\frac{x-Vt}{\Delta(V)}\right]$. У меня в ε-методе Sw width $\Delta(v) = \sqrt{1-v^2/\lambda}$

У меня $m_x(x,t) = \varepsilon\cdot\sin\psi$. Пусть $\varepsilon = \frac{V}{\lambda\Delta(V)}$, $\sin\psi(x,t) = 1/\operatorname{ch}\left[\frac{x-Vt}{\Delta(V)}\right]$, $\psi = \theta$, $\eta = \frac{x-Vt}{\Delta(V)}$

Проверим $1+\lambda\varepsilon^2 - y_3^2 = \varepsilon_3^2 = 0$, $y = \frac{\xi}{\Delta}$, $y_3 = \frac{1}{\Delta}$, $1+\lambda\frac{v^2}{\lambda^2\Delta^2} = \frac{1}{\Delta^2}$, $\Delta(1) = \sqrt{1-v^2/\lambda}$

т.е. если реш. Sklyanin–Kivshar верно, то нельзя $\varepsilon = V/\lambda\Delta(V)$ и $\psi = \theta(x,t)$.

Надо $\varepsilon = \varepsilon(\xi)$.

Escaneado con CamScanner

1st integral $(m_x')^2 + (m_y')^2 + (m_z')^2 = (1+\lambda) m_y^2 + m_z^2 + \gamma l - \gamma l = 0$ give Dv

Проверить выкладки и дать ссылку решение Sklyanin-Kirshner. $m_i' = \frac{\partial m_i}{\partial \xi}$, $\xi = x - vt$

$S_x = \frac{v}{\sqrt{\lambda}\Delta} \frac{1}{\operatorname{ch}\Lambda}$, $S_y = \sqrt{1-S_x^2}\frac{1}{\operatorname{ch}\Lambda}$, $S_z = \sqrt{1-S_x^2}(-1)\operatorname{th}\Lambda$

$\Lambda = \frac{1}{\Delta(v)}(x-vt) = \xi/\Delta(v)$, $\Delta(v) = \sqrt{\lambda}\sqrt{1-v^2/\lambda}$. Note: все $S_i = S_i(\xi)$.

$v/\sqrt{\lambda}\Delta(v) = d$, $S_x(\xi) = d\frac{1}{\operatorname{ch}\Lambda}$, $S_x' = d\frac{(-1)}{\operatorname{ch}^2\Lambda}\operatorname{sh}\Lambda\cdot\frac{1}{\Delta} = -\frac{d}{\Delta}\frac{1}{\operatorname{ch}\Lambda}\operatorname{th}\Lambda$

$S_y' = \frac{\partial}{\partial\xi}\sqrt{1-S_x^2}\frac{1}{\operatorname{ch}\Lambda} = \frac{1\cdot(-2S_x)}{2\sqrt{1-S_x^2}}S_x'\frac{1}{\operatorname{ch}\Lambda} + \sqrt{1-S_x^2}\frac{(-1)}{\operatorname{ch}^2\Lambda}\operatorname{sh}\Lambda\frac{1}{\Delta} = -\frac{S_xS_x'}{\sqrt{1-S_x^2}\operatorname{ch}\Lambda}\frac{1}{\Delta} - \frac{\sqrt{1-S_x^2}}{\Delta}\frac{\operatorname{th}\Lambda}{\operatorname{ch}\Lambda}$

$S_z' = \frac{\partial}{\partial\xi}\sqrt{1-S_x^2}(-\operatorname{th}\Lambda) = +\frac{S_xS_x'}{\sqrt{1-S_x^2}}\operatorname{th}\Lambda + \sqrt{1-S_x^2}(-1)\frac{1}{\operatorname{ch}^2\Lambda}\frac{1}{\Delta}$.

$\frac{d^2}{\Delta^2}\frac{\operatorname{th}^2\Lambda}{\operatorname{ch}^2\Lambda} + \frac{1}{\operatorname{ch}^2\Lambda}\left[\frac{S_xS_x'}{\sqrt{1-S_x^2}} + \frac{\sqrt{1-S_x^2}}{\Delta}\operatorname{th}\Lambda\right]^2 + \left[\frac{S_xS_x'}{\sqrt{1-S_x^2}}\operatorname{th}\Lambda - \frac{\sqrt{1-S_x^2}}{\Delta}\frac{1}{\operatorname{ch}^2\Lambda}\right]^2 =$

$= (S_x')^2 + \frac{1}{\operatorname{ch}^2\Lambda}\left[\frac{S_x^2S_x'^2}{(\sqrt{1-S_x^2})^2} + \frac{(1-S_x^2)}{\Delta^2}\operatorname{th}^2\Lambda + \frac{2S_xS_x'}{\Delta}\operatorname{th}\Lambda\right] + \left[\frac{S_x^2S_x'^2}{(1-S_x^2)}\operatorname{th}^2\Lambda + \frac{(1-S_x^2)}{\Delta^2}\frac{1}{\operatorname{ch}^2\Lambda} + 2\frac{S_xS_x'}{\Delta}\frac{\operatorname{th}\Lambda}{\operatorname{ch}^2\Lambda}\right] = (S_x')^2 + \frac{S_x^2S_x'^2}{(1-S_x^2)} + \frac{(1-S_x^2)}{\Delta^2}\frac{1}{\operatorname{ch}^2\Lambda}$ и это должно равняться

$(1+\lambda)S_x^2 + S_y^2 = (1+\lambda)\frac{d^2}{\operatorname{ch}^2\Lambda} + \left(1-\frac{d^2}{\operatorname{ch}^2\Lambda}\right)\frac{1}{\operatorname{ch}^2\Lambda}$, $\sum_\alpha (S_\alpha')^2 = \frac{S_x'^2}{(1-S_x^2)} + \frac{(1-S_x^2)}{\Delta^2}\frac{1}{\operatorname{ch}^2\Lambda} =$

$= (1+\lambda)\frac{d^2}{\operatorname{ch}^2\Lambda} + \left(1-\frac{d^2}{\operatorname{ch}^2\Lambda}\right)\frac{1}{\operatorname{ch}^2\Lambda}$; $S_x' = -\frac{d}{\Delta}\frac{\operatorname{th}\Lambda}{\operatorname{ch}\Lambda}$

$\frac{d^2}{\Delta^2}\frac{\operatorname{th}^2\Lambda}{\operatorname{ch}^2\Lambda} + \frac{1}{\Delta^2}\frac{1}{\operatorname{ch}^2\Lambda}\left(1 - 2\frac{d^2}{\operatorname{ch}^2\Lambda} + \frac{d^4}{\operatorname{ch}^4\Lambda}\right) = \frac{1}{\operatorname{ch}^2\Lambda}\left[(1+\lambda)d^2 + 1 - \frac{d^2}{\operatorname{ch}^2\Lambda}\right]\left[1 - \frac{d^2}{\operatorname{ch}^2\Lambda}\right]$

$d^2\operatorname{th}^2\Lambda + 1 - 2\frac{d^2}{\operatorname{ch}^2\Lambda} + \frac{d^4}{\operatorname{ch}^4\Lambda} = \Delta^2\left[\lambda d^2 + d^2\operatorname{th}^2\Lambda + 1\right]\left[1 - d^2\frac{1}{\operatorname{ch}^2\Lambda}\right]$

$\frac{1}{\operatorname{ch}^2\Lambda} = 1 - \operatorname{th}^2\Lambda$, $\frac{1}{\operatorname{ch}^4\Lambda} = 1 - 2\operatorname{th}^2\Lambda + \operatorname{th}^4\Lambda$, $d^2\operatorname{th}^2\Lambda + 1 - 2d^2(1-\operatorname{th}^2\Lambda) +$

$+ d^4(1 - 2\operatorname{th}^2\Lambda + \operatorname{th}^4\Lambda) = \Delta^2\left[\lambda d^2 + \operatorname{th}^2\Lambda\cdot d^2 + 1\right]\left[1 - d^2(1-\operatorname{th}^2\Lambda)\right]$

$\operatorname{th}^4\Lambda$: $d^4 = \Delta^2 d^4$; $\operatorname{th}^2\Lambda$: $d^2 + 2d^2 - 2d^4 = \Delta^2\left[(\lambda d^2 + 1)d^2 + (1-d^2)d^2\right]$

$\operatorname{th}^0\Lambda$: $1 - 2d^2 + d^4 = \Delta^2(\lambda d^2 + 1)(1-d^2) \to 1 - d^2 = \Delta^2(\lambda d^2 + 1)$

$(1-d^2) = \Delta^2\left(\lambda\frac{v^2}{\lambda^2\Delta^2} + 1\right)$, $1 - \frac{v^2}{\lambda\Delta^2} = \frac{v^2}{\lambda} + \Delta^2$, $\Delta^2 = \lambda(1 - v^2/\lambda) = \lambda - v^2$.

$\Delta^2 - \frac{v^2}{\lambda\Delta^2} = \frac{v^2}{\lambda}\Delta^2 + \Delta^4$, $\lambda - v^2 - \frac{v^2}{\lambda} = \frac{v^2}{\lambda}(\lambda - v^2) + (\lambda^2 - 2\lambda v^2 + v^4)$ — не было.

$\Delta^2 = 1$ - nonsense, i.e. если выше нет ошибок то решение Sklyanin-Kirshner не удовл. 1st integral of LL eq. (1) Проверить.

Из системы (*) дано ε, y : $2(\varepsilon_\xi)^2\operatorname{th} y - v\varepsilon_\xi + y_{\xi\xi} = 0$ найдем ε_ξ. Проверить умн. 2

$(\varepsilon_\xi)_{1,2} = \frac{1}{2\operatorname{th} y}\left[v \pm \sqrt{v^2 - 4y_{\xi\xi}\operatorname{th} y}\right]$, 2-ое подвешено перейти к диф. ур. для $y(\xi)$.

$\text{при } \xi \to 0$, $\operatorname{th} y \to 0$ $\varepsilon_\xi(\xi) = \frac{1}{2\operatorname{th} y}\left[v \pm \sqrt{v}\sqrt{1 - \frac{4}{v^2}y_{\xi\xi}\operatorname{th} y}\right] = \frac{1}{2\operatorname{th} y}\left[v \pm v\left(1 - \frac{2}{v^2}y_{\xi\xi}\operatorname{th} y\right)\right]$

Надо $= -$, $\varepsilon_\xi(\xi) = \frac{2^\circ}{v}y_{\xi\xi}$ при $\xi \to 0$. т.е $\varepsilon_\xi \neq 0$ из-за $y_{\xi\xi} \neq 0$, $y(\xi) = \xi/\Delta$ - много.

$y(\xi) = \xi/\Delta$, $y_{\xi\xi} = 0$ означает и $\varepsilon_\xi = 0$, $\varepsilon(\xi) = \text{const}$. $\varepsilon_\xi = \frac{1}{v}y_{\xi\xi}$ при $\xi \to 0$, $y \to 0$.

$vy_\xi = \lambda\varepsilon - \varepsilon_{\xi\xi} + 2\varepsilon_\xi y_\xi\operatorname{th} y \approx |y \to 0| = \lambda\varepsilon - \varepsilon_{\xi\xi}$, $vy_{\xi\xi} = \lambda\varepsilon_\xi - \varepsilon_{\xi\xi\xi}$

$vy_{\xi\xi} = \lambda\frac{1}{v}y_{\xi\xi} - \frac{1}{v}y_{\xi\xi\xi\xi}$, $y_{\xi\xi} = \mu(\xi)$, $-\mu_{\xi\xi} = (v^2 - \lambda)\mu$, $\mu_{\xi\xi} = \lambda\left(1 - \frac{v^2}{\lambda}\right)\mu = \Delta^2(v)\mu$

$\mu(\xi) = a\operatorname{ch}(\Delta\xi) + b\operatorname{sh}(\Delta\xi)$, $y_{\xi\xi} = a\operatorname{ch}(\Delta\xi) + b\operatorname{sh}(\Delta\xi)$, $y(\xi)$ - нечет., $y_{\xi\xi}(\xi)$ - нечет.

$y_{\xi\xi} = b\operatorname{sh}(\Delta\xi) \approx b\Delta\xi$, $y(\xi) = b\Delta\frac{\xi^3}{3} + c\xi \approx c\xi$, т.к. $\xi \to 0$. $y_\xi = \frac{b}{\Delta}\operatorname{ch}(\Delta\xi)$

$y(\xi) \approx \frac{b}{\Delta^2}\operatorname{sh}(\Delta\xi)$ и т.к. $y(\xi) = b\frac{\xi}{\Delta}$ при $\xi \to 0$. $\varepsilon_\xi = \frac{1}{v}b\Delta\xi \approx \frac{1}{v}b\operatorname{sh}(\Delta\xi)$, $\varepsilon = \frac{b}{v\Delta}\operatorname{ch}(\Delta\xi)$

1st integral $(y_\xi^2) + (\varepsilon_\xi)^2 = 1 + \lambda\varepsilon^2$, $\frac{b^2}{\Delta^2}\operatorname{ch}^2(\Delta\xi) + \frac{b^2}{v^2}\operatorname{sh}^2(\Delta\xi) = 1 + \lambda\frac{b^2}{v^2\Delta^2}\operatorname{ch}^2\Delta\xi$

$\frac{b^2}{\Delta^2}\operatorname{ch}^2 x + \frac{b^2}{v^2}[\operatorname{ch}^2 x - 1] = 1 + \lambda\frac{b^2}{v^2\Delta^2}\operatorname{ch}^2 x$, $\frac{b^2}{v^2} = -1$, $\frac{b^2}{\Delta^2} + \frac{b^2}{v^2} = \lambda\frac{b^2}{v^2\Delta^2}$

$\frac{1}{\Delta^2} + \frac{1}{v^2} = \frac{\lambda}{v^2\Delta^2}$, $\frac{1}{\Delta^2}\left(\frac{\lambda}{v^2} - 1\right) = \frac{1}{v^2} \Rightarrow \Delta^2 = \lambda - v^2$, $\Delta = \sqrt{\lambda}\sqrt{1 - v^2/\lambda}$.

Т. е. $\text{при } \xi \to 0$, $y \to 0$ $\varepsilon(\xi) = \frac{b}{v\Delta}\operatorname{ch}(\Delta\xi)$, $y(\xi) = \frac{b}{\Delta^2}\operatorname{sh}(\Delta\xi)$, $\Delta^2 = \lambda(1 - v^2/\lambda)$ для λ, решения LL eq. удовлетв. 1st integral. Далее $\varepsilon(\xi) = \frac{v}{\sqrt{\lambda}(1-v^2/\lambda)^{1/2}}\frac{1}{\operatorname{ch}(\Delta\xi)} = |\xi \to 0| = \frac{v}{\Delta(v)\sqrt{\lambda}}$ (2o/II)

Новое $\varepsilon(\xi \to 0) = \frac{b}{\Delta(v)v}$, $\Delta(v) = \sqrt{\lambda}\sqrt{1 - v^2/\lambda}$. Далее и новое $\varepsilon(\xi)$ сильно различаются.

Видимо далее $\varepsilon(\xi) = \frac{v}{\sqrt{\lambda}\Delta(v)}\operatorname{ch}^{-1}\left(\xi/(1 - v^2/\lambda)\right)$ неверно.

Escaneado con CamScanner

Note: Schlomann eq. $\gamma^2(v) = 1 - \frac{1}{2}(v^2 - \lambda) - \frac{1}{2}\sqrt{(v^2 - v_-^2)(v^2 - v_+^2)}$ $v_- = v_+ = \sqrt{\lambda}$ $\lambda \to \infty$

24/II gaer $x^2(v) = \frac{1}{\Delta_s^2(v)} = \lambda(1 - v^2/\lambda)$ и $m = F\left(\frac{\xi}{\Delta_s}\right) = F\left[\Delta_K(v)\xi\right]$, $\Delta_K^2(v) = \frac{1}{\Delta_s^2(v)}$

$\Delta_K^2(v) = \lambda(1 - v^2/\lambda)$ – DW width у Kivshar '89. $v/m\ \xi \to 0$, $\varepsilon = \varepsilon(\Delta_K \xi)$, $y = y(\Delta_K \xi)$

у Sklyanin-Kivshar $m_d = m_d\left(\frac{\xi}{\Delta_K}\right)$, $\xi = x - vt$.

Решение $\varepsilon(\xi) = \frac{v}{\lambda \Delta_s(v)} \frac{1}{ch(\xi/\Delta)}$, $y(\xi) = \xi/\Delta_s(v)$ от 20/II можно расск. как профиль (form) функции и определить $\Delta(v)$ из 1st integral $(y_\xi)^2 + (\varepsilon_\xi)^2 = 1 + \lambda\varepsilon^2$. Или описав 2го $\varepsilon(\xi)$ есть решение параметрами $(\varepsilon_\xi)^2 << 1$ и считая $\lambda\varepsilon >> \varepsilon_{\xi\xi}$, $\lambda\varepsilon >> \varepsilon_\xi y_\xi$. Тогда 1 eq. дает $y_\xi = \frac{1}{v}\varepsilon(\xi)$ и $\varepsilon(\xi)$ можно исключить из системы, напр. ур. на $y(\xi)$ и решить его как 20/II

$E(y,\varepsilon) = \frac{1}{ch^2 y}\left[(y_\xi)^2 + (\varepsilon_\xi)^2 + 1 + \lambda\varepsilon^2\right]$, 1 st integral $(y_\xi)^2 + (\varepsilon_\xi)^2 = 1 + \lambda\varepsilon^2$, $y = \frac{\xi}{\Delta}$

$\varepsilon_\xi = \frac{\partial}{\partial\xi}\frac{v}{\lambda\Delta}\frac{1}{chy} = \frac{v}{\lambda\Delta}\frac{(-1)}{ch^2 y} shy \frac{\partial y}{\partial\xi} = -\frac{v}{\lambda\Delta^2}\frac{shy}{ch^2 y}$, $(y_\xi)^2 + (\varepsilon_\xi)^2 = \frac{1}{\Delta^2} + \frac{v^2}{\lambda^2\Delta^4}\frac{th^2 y}{ch^4 y}$

$\int d\xi E_{ex} = \int d\xi \frac{1}{ch^2 y}\left[\frac{1}{\Delta^2} + \frac{v^2}{\lambda^2\Delta^4}\frac{th^2 y}{ch^2 y}\right] = \frac{1}{\Delta^2}\int_{-\infty}^{\infty} dy \Delta \frac{1}{ch^2 y} + \frac{v^2}{\lambda^2\Delta^4}\Delta\int_{-\infty}^{\infty}\frac{dy}{ch^2 y}\frac{th^2 y}{ch^2 y} =$

$= \frac{2}{\Delta} + \frac{v^2}{\lambda^2\Delta^3}\int d(thy)\, th^2 y\,(1 - th^2 y) = \left|thy = z\right| = \frac{2}{\Delta} + \frac{v^2}{\lambda^2\Delta^3}\int_{-1}^{1} dz\, z^2(1 - z^2) =$

$= \frac{2}{\Delta} + \frac{v^2}{\lambda^2\Delta^3}\, 2\left(\frac{z^3}{3} - \frac{z^5}{5}\right)\Big|_0^1 = \frac{2}{\Delta} + \frac{2v^2}{\lambda^2\Delta^3}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2}{\Delta} + \frac{2v^2}{\lambda^2\Delta^3}\frac{2}{15}$

$\int d\xi E_a = \int d\xi \frac{1}{ch^2 y}(1 + \lambda\varepsilon^2) = \Delta\int dy \frac{1}{ch^2 y}\left[1 + \lambda\frac{v^2}{\lambda^2\Delta^2}\frac{1}{ch^2 y}\right] = 2\Delta + \frac{v^2}{\lambda\Delta}\int d(thy)\frac{1}{ch^2 y} =$

$= 2\Delta + \frac{v^2}{\lambda\Delta}\int dz(1 - z^2) = 2\Delta + \frac{2v^2}{\lambda\Delta}\left(z - \frac{1}{3}z^3\right)\Big|_0^1 = 2\Delta + \frac{4}{3}\frac{v^2}{\lambda\Delta}$

Требуем $\int d\xi E_a = \int d\xi E_{ex}$ выполнение 1st integral & среднем $\Rightarrow$ eq. на $\Delta(v)$.

$\frac{2}{\Delta} + \frac{4}{15}\frac{v^2}{\lambda^2\Delta^3} = 2\Delta + \frac{4}{3}\frac{v^2}{\lambda\Delta}$, $\Delta^2 + \frac{2}{15}\frac{v^2}{\lambda^2} = \Delta^4 + \frac{2}{3}\frac{v^2}{\lambda}\Delta^2$, $\Delta^4 + \left(\frac{2}{3}\frac{v^2}{\lambda} - 1\right)\Delta^2 - \frac{2}{15}\frac{v^2}{\lambda^2} = 0$

$D = \left(\frac{2}{3}\frac{v^2}{\lambda} - 1\right)^2 + 4\frac{2}{15}\frac{v^2}{\lambda^2}$; $2\Delta^2_{1,2}(v) = 1 - \frac{2}{3}\frac{v^2}{\lambda} \pm \sqrt{D(v)}$, $D(v=0) = 1$, $\Delta^2(0) = 1$ взяли "+"

$\Delta_s^2(v) = \frac{1}{2}\left(1 - \frac{2}{3}\frac{v^2}{\lambda}\right) + \frac{1}{2}\sqrt{D(v)}$. $v/m\ \lambda \to \infty$ $\Delta_s^2(v) = 1 - \frac{2}{3}\frac{v^2}{\lambda} = 1 - \frac{v^2}{v_m^2}$ – Lorentz contract.

$v_m = \sqrt{\frac{3}{2}}\lambda^{1/2} \simeq 1{,}225\,\lambda^{1/2}$. Т.е. решение такого типа $\varepsilon(\xi) = \frac{v}{\lambda\Delta_s(v)}\frac{1}{chy}$, $y = \frac{\xi}{\Delta_s(v)}$

существует только до $v \le v_m = \sqrt{\frac{3}{2}}c$, $c = \sqrt{\lambda}$.

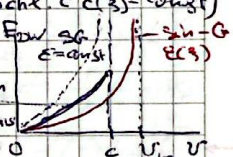
$E(v) = \int d\xi E_{ex} + \int d\xi E_a = 2\int d\xi E_a = 2\left[2\Delta + \frac{4}{3}\frac{v^2}{\lambda\Delta}\right] = \frac{4}{\Delta}\left[\Delta^2 + \frac{2}{3}\frac{v^2}{\lambda}\right] =$

$= \frac{4}{\Delta}\left[1 - \frac{2}{3}\frac{v^2}{\lambda} + \frac{2}{3}\frac{v^2}{\lambda}\right] = \frac{4}{\Delta_s(v)}$, $E_{DW}(v) = \frac{4}{\sqrt{1 - v^2/v_m^2}}$, $v_m = \sqrt{\frac{3}{2}}\sqrt{\lambda}$.

$E_{DW}(v << v_m) = 4\left[1 + \frac{1}{2}\frac{v^2}{v_m^2}\right] = E_0 + \frac{1}{2}\frac{E_0}{v_m^2}v^2$, $m_{DW} = \frac{E_0}{v_m^2} = \frac{2}{3}\frac{E_0}{c^2} = \frac{2}{3}m_{DW}$ (Döring)

т.к. $v_m > c$, то $m_{DW} < m_{DW}$(Döring) и E_{DW} (new $\varepsilon(\xi)$) $< E_{DW}$ (Schl. с $\varepsilon(\xi) = const$)

$E_{DW}(v=c) = \frac{E_0}{\sqrt{1 - \frac{c^2}{(3/2)c^2}}} = \frac{E_0}{\sqrt{1 - \frac{2}{3}}} = \sqrt{3}E_0$, $E_{DW}(Schl) \simeq E_0\lambda^{1/4}$ $v/m\ \lambda \to \infty$



→ Вывод: хотя $\varepsilon(\xi) = \frac{v}{\lambda\Delta(v)}\frac{1}{chy}$, $y = \frac{\xi}{\Delta(v)}$ не есть точное решение LL (a fixed function), учитывая 1st integral дает $\Delta(v)$ такое, что $E_{DW}(v) < E_{DW}^{Schl.}$ ($\varepsilon = const$), а max velocity $v_m > c = \sqrt{\lambda} = v_+$ $\lambda \to \infty$

DW с $\Phi(\xi)$ существует и при $v > v_- = \sqrt{1 + \lambda} - 1 \approx \sqrt{\lambda}$

Уточнение trial $y(\xi)$, $\Phi(\xi) = \pi/2 - \varepsilon(\xi)$ может дать $v_{DW}^{max} > \sqrt{\frac{3}{2}}c$. $c = v_+ = \sqrt{\lambda}$.

$v/m\ v = c$ $E_{Schl} = E_{DW}^{Schl}(v_-) = E_0(1 + \lambda)^{1/4}$, $E_{DW}(new) = \sqrt{3}E_0 < E_{DW}^{Schl}$ $v/m\ \lambda > 8$.

(4/III) Корнилова Sol-Sky Mag 19. Корнилова с Цао по COST-travel Action (конец 2008) Analysing new projects (Emilia „INVARIMA"). Пред: IBero-Am. project не стоит внимания: ≤ 25 k€/год и 6 команд (4 из Latin America). Бюдж. Modified COST appl.

(5/III) yD LL $\begin{cases} \dot{v}y_\xi = \lambda\varepsilon - \varepsilon_{\xi\xi} + 2\varepsilon_\xi y_\xi thy \\ \dot{v}\varepsilon_\xi = y_{\xi\xi} + (\lambda\varepsilon^2 + 1 - y_\xi^2)thy \end{cases}$ + 1st integral $1 + \lambda\varepsilon^2 = y_\xi^2 + \varepsilon_\xi^2$ v. note pen. (fluid fmic.) $\varepsilon(\xi)$ - 3/III и решение Sklyanin-Kivshar это упрощение $y_\xi = const$, (y, ε) $y_{\xi\xi} = 0$, $y(\xi) = \xi/\Delta(v)$, где $\Delta(v)$ определяется однозначно.

Ищем решение класса $y_{\xi\xi\xi}=0$, $\nu\varepsilon_\xi = (1+\lambda\varepsilon^2 - y_\xi^2)\operatorname{th} y = 2(\varepsilon_\xi)^2 \operatorname{th} y \to \varepsilon_\xi = \frac{\nu}{2}\operatorname{cth} y$

$\nu y_\xi = \lambda\varepsilon - \varepsilon_{\xi\xi} + 2\varepsilon_\xi y_\xi \operatorname{th} y = \lambda\varepsilon - \varepsilon_{\xi\xi} + 2 y_\xi \operatorname{th} y \cdot \frac{\nu}{2}\operatorname{cth} y = \lambda\varepsilon - \varepsilon_{\xi\xi} + \frac{2}{2}\nu y_\xi - \nu y_\xi$ - ок

$\varepsilon_{\xi\xi} = \lambda\varepsilon + c$ - ? νy_ξ eq. выполняется. $\frac{d\varepsilon(\xi)}{d\xi} = \frac{\nu}{2}\operatorname{cth} y = \frac{\nu}{2}\operatorname{cth}\frac{\xi}{\Delta}$

$\varepsilon(\xi) = \frac{\nu}{2}\int d\xi \operatorname{cth}\frac{\xi}{\Delta} = \frac{1}{2}\nu\Delta \ln|\operatorname{sh}\frac{\xi}{\Delta}|$; $y(\xi) = \xi/\Delta$.

Проверим 1st integral $(\varepsilon_\xi)^2 + y_\xi^2 = 1 + \lambda\varepsilon^2$, $\frac{1}{4}\nu^2 \operatorname{cth}^2\frac{\xi}{\Delta} + \frac{1}{\Delta^2} = 1 + \frac{\lambda}{4}\nu^2\Delta^2 \operatorname{sh}^2(\xi/\Delta)$

$\varepsilon(\xi) = \frac{1}{2}\nu\Delta \ln|\operatorname{sh}\frac{\xi}{\Delta}| + C$, $\varepsilon(\xi) \underset{\xi\to 0}{\sim} \ln|\xi|$ singular. $\varepsilon(\xi\to\pm\infty) = \frac{1}{2}\nu|\xi| \to \infty$.

Т.е. требование $y_{\xi\xi\xi}=0$ приводит к singular $\varepsilon(\xi)$ $\Rightarrow$ надо $\varepsilon(\xi)=\text{const}$.

$\varepsilon_{\xi\xi} = \lambda\varepsilon$, $\varepsilon(\xi) = a\operatorname{ch}(\sqrt{\lambda}\xi) + b\operatorname{sh}(\sqrt{\lambda}\xi)$ не совместимо с $\varepsilon_\xi = \frac{\nu}{2}\operatorname{th} y$, $\varepsilon(\xi)\sim \ln\operatorname{sh}\frac{\xi}{\Delta}$

Причина? Поскольку 1st integral надо $\ell\ell \neq 0$. Сам. Elsgolts '78 $\ell(y,\Phi)$ упрощает.

$\frac{1}{\operatorname{ch}^2 y}\left[(y_\xi)^2 + (\Phi_\xi)^2 - 1 - \lambda\cos^2\Phi\right] = \ell\ell$ и если $\Phi_\xi = -\varepsilon_\xi \sim \frac{\operatorname{ch} y}{\operatorname{sh} y}$, то $\ell\ell \neq 0$.

$(y_\xi)^2 + (\varepsilon_\xi)^2 = 1 + \lambda\cos^2\Phi + \ell\ell_1 = 1 + \lambda\varepsilon^2 + \ell\ell\operatorname{ch}^2 y$, $1 + \lambda\varepsilon^2 - y_\xi^2 = \varepsilon_\xi^2 - \ell\ell\operatorname{ch}^2 y$

$\nu\varepsilon_\xi = y_{\xi\xi\xi} + \left[1 + \lambda\varepsilon^2 + \varepsilon_\xi^2 - y_\xi^2\right]\operatorname{th} y = y_{\xi\xi\xi} + \left[2\varepsilon_\xi^2 - \ell\ell\operatorname{ch}^2 y\right]\operatorname{th} y$

Решение типа $y_{\xi\xi\xi}=0$, $y_\xi = \frac{\xi}{\Delta}$, $y(\xi) = \frac{\xi}{\Delta}$, $2\varepsilon_\xi^2 - \nu\varepsilon_\xi - \ell\ell\operatorname{ch}^2 y\operatorname{th} y = 0$

$\varepsilon_\xi^2 - \frac{\nu}{2}\operatorname{cth} y\,\varepsilon_\xi - \frac{1}{2}\ell\ell\operatorname{ch}^2 y = 0$

$2(\varepsilon_\xi)_{1,2} = \frac{\nu}{2}\operatorname{cth} y \pm \sqrt{\frac{1}{4}\nu^2\operatorname{cth}^2 y + 2\ell\ell\operatorname{ch}^2 y}$

$\varepsilon_\xi = \frac{\nu}{4}\operatorname{cth} y \pm \frac{1}{4}\sqrt{\nu^2\operatorname{cth}^2 y + 8\ell\ell\operatorname{ch}^2 y}$, если $y = \frac{\xi}{\Delta}$, $y(\xi=0)=0$ и $\varepsilon_\xi(\xi\to 0) \to \infty$ $\operatorname{cth} y \to \infty$

Возможен "-", тогда singularity at $\xi=0$ нет, $\varepsilon_\xi(\xi\to 0)\to 0$.

при $\xi\to\pm\infty$ $\varepsilon_\xi(\xi) = \frac{\sqrt{2}}{2}\sqrt{\ell\ell}\operatorname{ch} y$.

$\nu y_{\xi\xi} = \lambda\varepsilon - \varepsilon_{\xi\xi} + 2\varepsilon_\xi y_\xi \operatorname{th} y$

$\nu y_{\xi\xi\xi} = 0 = \lambda\varepsilon_\xi - \varepsilon_{\xi\xi\xi} + 2\frac{d}{d\xi}(\varepsilon_\xi \operatorname{th} y)\, y_\xi$, $\varepsilon_\xi \operatorname{th} y = \frac{1}{4}\nu - \frac{1}{4}\sqrt{\nu^2 + 8\ell\ell\operatorname{sh}^2 y}$

$\lambda\varepsilon_\xi - \varepsilon_{\xi\xi\xi} + 2y_\xi\left[\varepsilon_{\xi\xi}\operatorname{th} y + \varepsilon_\xi\frac{1}{\operatorname{ch}^2 y}\frac{1}{\Delta}\right] = 0$ или $\lambda\varepsilon = \frac{\nu}{\Delta} + \varepsilon_{\xi\xi} - 2\varepsilon_\xi\frac{1}{\Delta}\operatorname{th} y$

$\varepsilon_{\xi\xi} = \frac{1}{4}\nu\frac{\epsilon(1)}{\operatorname{sh}^2 y} y_\xi - \frac{1}{4}\frac{1}{2\sqrt{(\dots)}}\left[\nu^2\operatorname{cth} y\frac{\epsilon(1)}{\operatorname{sh}^2 y} + 16\ell\ell\operatorname{ch} y\operatorname{sh} y\right]\frac{1}{\Delta}$

$\lambda\varepsilon(\xi) = \frac{\nu}{\Delta} - \frac{2}{\Delta}\left[\frac{1}{4}\nu - \frac{1}{4}\sqrt{\nu^2 + 8\ell\ell\operatorname{sh}^2 y}\right] - \frac{1}{4}\frac{\nu}{\Delta}\frac{1}{\operatorname{sh}^2 y} - \frac{1}{4\Delta}\frac{\left[-\nu^2\operatorname{cth} y\frac{1}{\operatorname{sh}^2 y} + 8\ell\ell\operatorname{ch} y\operatorname{sh} y\right]}{\sqrt{\nu^2\operatorname{cth}^2 y + 8\ell\ell\operatorname{ch}^2 y}}$

$\lambda\Delta\varepsilon(\xi) = \frac{1}{2}\nu + \frac{1}{2}\sqrt{\nu^2 + 8\ell\ell\operatorname{sh}^2 y} - \frac{1}{4}\frac{\nu}{\operatorname{sh}^2 y} - \frac{1}{4}\left[8\ell\ell\operatorname{ch} y\operatorname{sh} y - \nu^2\operatorname{cth} y/\operatorname{sh}^2 y\right]$

$\lambda\Delta\varepsilon(\xi) \xrightarrow[\nu\to 0]{} \frac{1}{2}\frac{\nu^2}{4}\cdot 2\sqrt{\ell\ell\operatorname{sh}^2 y} - \frac{4}{2}\ell\ell\operatorname{ch} y\operatorname{sh} y$ $\quad$ $\sqrt{\nu^2\operatorname{cth}^2 y + 8\ell\ell\operatorname{ch}^2 y}$

$\lambda\Delta\varepsilon(\xi) \xrightarrow[\nu\to 0]{} \sqrt{2}\sqrt{\ell\ell}\operatorname{sh} y - \frac{1}{\sqrt{2}}\sqrt{\ell\ell}\operatorname{sh} y = \frac{\sqrt{2}}{2}\sqrt{\ell\ell}\operatorname{sh} y$. $\varepsilon(\xi) \xrightarrow[\nu\to 0]{} 0$ за счет $\ell\ell(\nu=0)=0$.

Поэтому и обычное solution не дает. 1st integral (3/III) потому что $\ell\ell \neq 0$ нам.

$\lambda\Delta\varepsilon(\xi\to 0) = \nu - \frac{1}{4}\frac{\nu}{\operatorname{sh}^2 y} - \frac{1}{4}(-\nu^2)\frac{\operatorname{cth} y}{\operatorname{sh}^2 y}\frac{1}{\nu\operatorname{cth} y} = \nu - \frac{1}{4}\frac{\nu}{\operatorname{sh}^2 y} + \frac{1}{4}\nu\frac{1}{\operatorname{sh}^2 y}$

$\varepsilon(\xi\to 0) = \frac{\nu}{\lambda\Delta}$ (!) - самое не нам trivial $\varepsilon(\xi) = \frac{\nu}{\lambda\Delta}\frac{1}{\operatorname{ch}^3\xi/\Delta}$ при $\xi\to 0$.

$\lambda\Delta\varepsilon(\xi) = \frac{1}{2}\nu + \frac{1}{2}\sqrt{\nu^2 + 8\ell\ell\operatorname{sh}^2 y} - \frac{1}{4}\frac{\nu}{\operatorname{sh}^2 y} + \frac{1}{4}\left[\nu^2\frac{1}{\operatorname{sh}^2 y} + 8\ell\ell\operatorname{sh} y\right]$

$\lambda\Delta\varepsilon(\xi) = \frac{1}{2}\nu + \frac{1}{2}\sqrt{\nu^2 + 8\ell\ell\operatorname{sh}^2 y} - \frac{1}{4}\frac{\nu}{\operatorname{sh}^2 y} + \frac{1}{4}\sqrt{\nu^2/\operatorname{sh}^2 y + 8\ell\ell}\left[\nu^2/\operatorname{sh}^2 y + 8\ell\ell\operatorname{sh}^2 y\right]$

$\operatorname{ch}^2 y - \frac{1}{\operatorname{sh}^2 y} = 1$, $\frac{1}{\operatorname{sh}^2 y} = \operatorname{cth}^2 y - 1$

$\nu^2\frac{1}{\operatorname{sh}^2 y} - 8\ell\ell\operatorname{sh}^2 y\sqrt{\nu^2 + 8\ell\ell\operatorname{sh}^2 y} = \nu^2\operatorname{cth}^2 y - \nu^2 - 8\ell\ell\operatorname{sh}^2 y$

$\lambda\Delta\varepsilon(\xi) = \frac{1}{2}\nu + \frac{1}{4}\sqrt{\nu^2 + 8\ell\ell\operatorname{sh}^2 y} - \frac{1}{4}\frac{\nu}{\operatorname{sh}^2 y} + \frac{1}{4}\frac{\nu^2\operatorname{cth}^2 y}{\sqrt{\nu^2 + 8\ell\ell\operatorname{sh}^2 y}}$, $y = \xi/\Delta$.

Проверим вывод $\varepsilon(\xi)$ из LL eq. Note $\varepsilon(\xi)\sim\operatorname{sh}\frac{\xi}{\Delta}$ at $\xi\to\pm\infty$ и надо $\ell\ell\neq 0$.

If $\ell\ell = 0$ $\lambda\Delta\varepsilon(\xi) = \frac{1}{2}\nu + \frac{1}{4}\nu - \frac{1}{4}\frac{\nu}{\operatorname{sh}^2 y} + \frac{1}{4}\frac{\nu^2}{\nu}\operatorname{cth}^2 y = \frac{1}{2}\nu + \frac{1}{4}\nu + \frac{1}{4}\nu = \nu$

$\varepsilon(\xi) = \frac{\nu}{\lambda\Delta} = \text{const}(\xi)$ Walker solution с $\Delta(\nu) = \sqrt{1-\nu^2/4}$, $y_{\xi\xi\xi}=0$, $y=\xi/\Delta$

т.е. надо решение $\varepsilon(\xi)=\text{const}$ при $\ell\ell=0$ и ищ $\varepsilon=\varepsilon(\xi)$ при $\ell\ell\neq 0$. ($\varepsilon(\xi)\sim\operatorname{sh}\frac{\xi}{\Delta}$ неверно $\frac{\xi}{\Delta}\to\pm\infty$.

Проверка $\nu\to 0$ в ход. eqs. дает y,ε $\lambda\varepsilon - \varepsilon_{\xi\xi} + \frac{2}{\Delta}\varepsilon_\xi\operatorname{th} y = 0$ $1 + \lambda\varepsilon^2 + \varepsilon_\xi^2 = \frac{1}{\Delta^2}$

$(y_\xi)^2 + \varepsilon_\xi^2 = 1 + \lambda\varepsilon^2 + \ell\ell\operatorname{ch}^2 y$, $1 + \lambda\varepsilon^2 + \varepsilon_\xi^2 = y_\xi^2 + 2\varepsilon_\xi^2 - \ell\ell\operatorname{ch}^2 y$, $\varepsilon_\xi^2 = \frac{1}{2}\ell\ell\operatorname{ch}^2 y$

$\varepsilon_\xi(\xi, \nu=0) = \frac{\sqrt{2}}{2}\sqrt{\ell\ell}\operatorname{ch} y$, $\varepsilon(\xi, \nu=0) = (\cdot)\int d\xi\operatorname{ch}\frac{\xi}{\Delta} = \frac{\sqrt{2}}{2}\sqrt{\ell\ell}\Delta\operatorname{sh}(\xi/\Delta)$. У нас

$\varepsilon\neq 0$ $\varepsilon(\xi, \nu=0) = \frac{\sqrt{2}}{2}\sqrt{\ell\ell}\operatorname{sh} y\frac{1}{\lambda\Delta}$. Надо при $\nu=0$ $\Delta = \frac{1}{\lambda\Delta}$, $\Delta(\nu=0) = \frac{1}{\sqrt{\lambda}}$.

$\ell\ell$ (1st integral) может дать интересные решения, будучи metastable.

$\neq 0$ $\quad$ $\ell\ell\neq 0$

Escaneado con CamScanner

(6/III) COST appl. approved (2,000€). Julian G. Zanfocus the list of secretary invited with affiliations - Barcelona, Orocovis. E-mail to Antxon (Robin's reimb.). Pasenyuk Sb. dynamics Singapore MS for Rep. Res. Lett. (reject, simul. - невероятно нормально)

(7/III) E-mails reminders to invited (Jungwirth, Kent Nostoroy, Mochizuki, Suess). Report on COST visit of Ruben Ochoa to UPV/EHU. Bienvenida positive info.

Вернемся к $(y,\Phi)\to(y,\varepsilon)$ сначала УД от 5/III $y_{\xi\xi}=0$, $y_\xi=\text{const}$, $y=\xi/\Delta(v)$

$$\begin{cases}\frac{v}{\Delta}=\lambda\varepsilon-\varepsilon_{\xi\xi}+\frac{2}{\Delta}\varepsilon_\xi\,\text{thy}\\ v\varepsilon_\xi=\left(1+\lambda\varepsilon^2+\varepsilon_\xi^2-y_\xi^2\right)\text{thy}\end{cases}\qquad y_\xi^2+\Phi_\xi^2-1-\lambda\cos^2\Phi=\text{fl}\,\text{ch}^2y-1\ \text{fl integral}$$

$$\frac{1}{\Delta^2}+\varepsilon_\xi^2-1-\lambda\varepsilon^2=\text{fl}\,\text{ch}^2y\qquad \Phi(\xi)\to\varepsilon(\xi)$$

$1+\lambda\varepsilon^2+\varepsilon_\xi^2-\frac{1}{\Delta^2}=\left(\frac{1}{\Delta^2}+\varepsilon_\xi^2-\text{fl}\,\text{ch}^2y\right)+\varepsilon_\xi^2-\frac{1}{\Delta^2}=2\varepsilon_\xi^2-\text{fl}\,\text{ch}^2y$

$v\varepsilon_\xi=(2\varepsilon_\xi^2-\text{fl}\,\text{ch}^2y)\,\text{thy}$ и $\frac{v}{\Delta}=\lambda\varepsilon-\varepsilon_{\xi\xi}+\frac{2}{\Delta}\varepsilon_\xi\,\text{thy}$ - fla fup fp на $\varepsilon(\xi)$ - геометр. вывод дифференциальное!

$2\varepsilon_\xi^2\,\text{thy}-v\varepsilon_\xi-\text{fl}\,\text{sh}y\,\text{ch}y=0$.

$\varepsilon_\xi^2-\frac{1}{2}v\,\text{cth}y\,\varepsilon_\xi-\frac{1}{2}\text{fl}\,\text{ch}^2y=0$, $y=\frac{\xi}{\Delta}$, $2\varepsilon_\xi=\frac{1}{2}v\,\text{cth}y\pm\sqrt{\frac{1}{4}v^2\,\text{cth}^2y-4\left(-\frac{1}{2}\text{fl}\,\text{ch}^2y\right)}$

$\varepsilon_\xi=\frac{1}{4}v\,\text{cth}y\pm\frac{1}{4}\sqrt{v^2\,\text{cth}^2y+8\,\text{fl}\,\text{ch}^2y}$ - можно численно $\int d\xi(\dots)$, а можно искать $\varepsilon(\xi)$ из 1-го УД как 5/III. $\varepsilon(\xi)=\int d\xi\left[\frac{1}{4}v\,\text{cth}y\pm\frac{1}{4}\sqrt{v^2\,\text{cth}^2y+8\,\text{fl}\,\text{ch}^2y}\right]$

$I_1(\xi)=\int d\xi\,\text{cth}y=\int d\xi\,\text{cth}\frac{\xi}{\Delta}=\Delta\int dx\cdot\text{cth}x=\Delta\ln|\text{sh}x|+C_1$.

$I_2(\xi)=\int d\xi\sqrt{v^2\,\text{cth}^2y+8\,\text{fl}\,\text{ch}^2y}=\int d\xi\,\text{ch}y\sqrt{v^2\frac{1}{\text{sh}^2y}+\text{fl}8}=\Delta\int dx\,\text{ch}x\sqrt{\frac{v^2}{\text{sh}^2x}+8\,\text{fl}}=$

$=\left|\begin{matrix}u=\text{sh}x\\ du=\text{ch}x\,dx\end{matrix}\right|=\Delta\int du\sqrt{\frac{v^2}{u^2}+8\,\text{fl}}=\Delta\int du\frac{1}{|u|}\sqrt{v^2+8\,\text{fl}\,u^2}$

$I_2(\xi)$ зависит от $\text{sng}(\text{fl})$. a) $\text{fl}<0$ $I_2(\xi)=\Delta\int\frac{du}{u}\sqrt{v^2-8|\text{fl}|u^2}=\Delta\sqrt{8\,\text{fl}}\int\frac{du}{|u|}\sqrt{\frac{v^2}{8\,\text{fl}}-u^2}=$

$=\Delta\sqrt{8|\text{fl}|}\left[\sqrt{\frac{v^2}{8\,\text{fl}}-u^2}-\frac{v}{\sqrt{8\,\text{fl}}}\ln\left|\frac{v/\sqrt{8\,\text{fl}}+\sqrt{v^2/8\,\text{fl}-u^2}}{u}\right|\right]$ при $u^2<\frac{v^2}{8|\text{fl}|}$, т.к. иначе подкоренное выраж. <0.

$a^2=\frac{v^2}{8|\text{fl}|}$, $I_2(\xi)=\Delta\sqrt{8|\text{fl}|}\int\frac{du}{|u|}\sqrt{a^2-u^2}$, $u=\text{sh}y$, $\xi\in[-\infty,\infty]$, $u\in[-\infty,\infty]$

$\frac{1}{\Delta\sqrt{8|\text{fl}|}}I_2(\xi)=\int_{-\infty}^{\infty}\frac{du}{|u|}\sqrt{a^2-u^2}=\int_{-a}^{a}\frac{du}{|u|}\sqrt{a^2-u^2}+\int_{-\infty}^{-a}du(\dots)+\int_{a}^{\infty}du(\dots)=$

$=2\int_0^{\infty}\frac{du}{u}\sqrt{a^2-u^2}=2\int_0^{a}\frac{du}{u}\sqrt{a^2-u^2}+2\int_a^{\infty}\frac{du}{u}\sqrt{a^2-u^2}=2\int_0^{a}\frac{du}{u}\sqrt{a^2-u^2}+2i\int_a^{\infty}\frac{du}{u}\sqrt{u^2-a^2}$ -

- тогда $\varepsilon(\xi)$ - is complex. Это нереально. Остается б) $\text{fl}>0$.

$I_2(\xi)=\Delta\sqrt{8\,\text{fl}}\int_{-\infty}^{\infty}\frac{du}{|u|}\sqrt{a^2+u^2}=2\Delta\sqrt{8\,\text{fl}}\int_0^{\infty}\frac{du}{u}\sqrt{a^2+u^2}=(*)\left[\sqrt{a^2+u^2}-a\ln\left(\frac{a+\sqrt{a^2+u^2}}{u}\right)\right]=$

$=(*)\left[\sqrt{a^2+u^2}-a\,\text{Arsh}\left(\frac{a}{u}\right)\right]_0^{\infty}=2\Delta\sqrt{8\,\text{fl}}\left[\underset{u\to\infty}{\sqrt{u}}-\left(a-a\ln\frac{2a}{u}\right)\right]$ расходится и на верхнем и на нижнем пределе $u=\text{sh}y=(0,\infty)$ если $\text{fl}>0$.

c) $\text{fl}=0$, $I_2(\xi)=\Delta v\int_{-\infty}^{\infty}\frac{du}{|u|}=2\Delta v\int_0^{\infty}\frac{du}{u}=2\Delta v\ln u\Big|_0^{\infty}$ — расходится.

Eq. $\varepsilon_\xi^2-\frac{1}{2}v\varepsilon_\xi-\frac{1}{2}\text{fl}\,\text{ch}^2y=0$ при $\text{fl}=0$ имеет тривиальное решение $\varepsilon_\xi=0$, $\varepsilon_1(\xi)=\text{const}$ — Walker solution и $\varepsilon_\xi=\frac{1}{2}v$, $\varepsilon_2(\xi)=\frac{1}{2}v\xi$. $\frac{1}{\Delta^2}+\frac{1}{4}v^2=1+\lambda\frac{1}{4}v^2\xi^2$ — это невероятно.

// Т.е. при $y_{\xi\xi}=0$, если только Walker solution $\varepsilon(\xi)=\text{const}$ при $\text{fl}=0$. При $\varepsilon(\xi)$ sin/sinh/ch

One new 1st integral $\frac{1}{\Delta^2}=1+\lambda\varepsilon^2$, $\lambda\varepsilon^2=\frac{1}{\Delta^2}-1=\frac{(1-\Delta^2)}{\Delta^2}$, $\varepsilon=\frac{1}{\sqrt{\lambda}}\frac{\sqrt{1-\Delta^2}}{\Delta}$

$\frac{v}{\Delta}=\lambda\varepsilon$, $\frac{1}{\Delta^2}=1+\lambda\frac{v^2}{\Delta^2}\frac{1}{\lambda^2}$, $\frac{1}{\Delta^2}\left(1-\frac{v^2}{\lambda}\right)=1$, $\Delta^2=1-\frac{v^2}{\lambda}$, $\lambda\varepsilon^2=\frac{\Delta}{\Delta^2}-1=\frac{v^2}{\lambda\Delta^2}$

$\varepsilon^2=\frac{v^2}{\lambda^2\Delta^2}$, $\varepsilon=\frac{1}{\lambda}\frac{v}{\Delta}$, $\Delta=\sqrt{1-v^2/\lambda}$ - Sine-Gordon решение с $\varepsilon=\text{const}$.

Если $\varepsilon_\xi\neq0$, то $\varepsilon(\xi)=\frac{1}{4}vI_1(\xi)\pm\frac{1}{4}I_2(\xi)$, $\frac{d\varepsilon}{d\xi}=\frac{v}{4}\text{cth}y\pm\frac{1}{4}\sqrt{v^2\,\text{cth}^2y+8\,\text{fl}\,\text{ch}^2y}$

и вычисляя $\varepsilon(\xi)$ надо $\int d\xi\,\varepsilon_\xi+C$ - indefinite $d\xi$ integrals.

$I_1(\xi)=\Delta\ln|\text{sh}y|+C_1$, $I_2(\xi)=\int d\xi\sqrt{v^2\,\text{cth}^2y+8\,\text{fl}\,\text{ch}^2y}=|\text{при}\ \text{fl}>0|=$

$=\Delta\sqrt{8\,\text{fl}}\int dy\sqrt{a^2\,\text{cth}^2y+\text{ch}^2y}=(*)\int dy\,\text{ch}y\sqrt{a^2/\text{sh}^2y+1}=(*)\int d(\text{sh}y)\sqrt{1+a^2/\text{sh}^2y}=$

$=|u=\text{sh}y|=(*)\int du\sqrt{1+a^2/u^2}=(*)\int\frac{du}{|u|}\sqrt{a^2+u^2}$ - odd function of $u=\text{sh}y$, $y=\xi/\Delta$.

$I_2(\xi)=\sqrt{8\,\text{fl}}\,\Delta\,\text{sign}(u)\int\frac{du}{u}\sqrt{a^2+u^2}=(*)\left[\sqrt{a^2+u^2}-a\,\text{Arsh}\frac{a}{u}\right]$, где $a^2=v^2/8\,\text{fl}$, $u=\text{sh}y$.

$\varepsilon(\xi)=\frac{1}{4}v\left[\Delta\ln|\text{sh}y|+C_1\right]\pm\frac{1}{4}\Delta\sqrt{8\,\text{fl}}\left[\sqrt{a^2+\text{sh}^2y}-a\,\text{Arsh}\frac{a}{\text{sh}y}+C_2\right]\to\varepsilon(\xi,v,\text{fl})$. $I_2\to I_2+C_2$.

При $\frac{v=0}{\text{fl}\neq0}$, $\varepsilon(\xi)=\pm\frac{1}{4}\Delta\sqrt{8\,\text{fl}}\left[|\text{sh}y|+C_2\right]$ — это что? $\varepsilon(\xi)\to\infty$ при $\xi\to\pm\infty$. $\text{sign}(u)$

При $\text{fl}=0$ $\varepsilon(\xi)=\frac{1}{4}v\left[\Delta\ln|\text{sh}y|+C_1\right]\pm\frac{1}{4}v\Delta\left[\ln|\text{sh}y|+\frac{C_2}{\Delta}\right]$, надо, чтобы $\varepsilon(\xi)$ finite при $\xi\to\pm\infty$

$\varepsilon(\xi,\text{fl}=0)=\frac{1}{4}v(C_1\pm C_2)=\text{const}$, $\varepsilon(\xi,\text{fl}=0)=\frac{1}{\lambda}\frac{v}{\Delta}$ — Walker solution → sine-Gordon.

Escaneado con CamScanner

→ Предположение $y_{\xi\xi}=0$, $y=\frac{\xi}{\Delta}$ даст нам $\varepsilon(\xi)=\text{const}$ - Walker, нам $\varepsilon(\xi)\sim sh\frac{\xi}{\Delta}$ singular at $\xi\to\infty$

$\varepsilon(\xi)=\frac{1}{4}v\Delta\ln|sh y|+\frac{1}{4}\Delta\sqrt{2\mu}\sqrt{a^2+sh^2y}+\frac{a}{4}\Delta\sqrt{2\mu}\,\text{Arsh}\frac{a}{|sh y|}+\frac{1}{4}vC_1-\frac{1}{4}\Delta\sqrt{2\mu}C_2$

$a^2=v^2/2\mu$. Singularity $\varepsilon(\xi)$ можно избежать если потребовать дополнительно сокращение

при $\xi\to\infty$, $y\to\pm\infty$. $\sqrt{a^2+sh^2y}\to|sh y|$ при $y\to\pm\infty$. Нет, сингулярность

$\sqrt{v^2cth^2y+8\mu ch^2y}=\sqrt{v^2}\,ch y\sqrt{1+\frac{8\mu}{v^2}\frac{ch^2y}{cth^2y}}=v\,cth y\sqrt{1+\frac{8\mu}{v^2}sh^2y}\overset{|\mu\to0|}{=}$ тоже только при $\mu\to0$

$=v\,cth y\left[1+\frac{1}{2}\frac{8\mu}{v^2}sh^2y\right]$, $I_2(\xi,\mu\to0)=\Delta\int dy\sqrt{v^2cth^2y+8\mu ch^2y}=$

$=\Delta v\int dy\,cth y\left[1+\frac{4\mu}{v^2}sh^2y\right]=\Delta v\ln|sh y|+\Delta v\frac{4\mu}{v^2}\int dy\,ch y\,sh y=$

$=\Delta v\ln|sh y|+\Delta v\left(\frac{4\mu}{v^2}\right)\frac{1}{2}\int dy\,sh(2y)=\Delta v\ln|sh y|+\Delta v\frac{\mu}{v^2}ch2y+C_2$

$\varepsilon(\xi)=\frac{1}{4}v\left[\Delta\ln|sh y|+C_1\right]+\frac{1}{4}\left[\Delta v\ln|sh y|+\Delta v\frac{\mu}{v^2}ch2y+C_2\right]$,

при $\mu=0$ все ок с $\varepsilon(\xi)=\text{const}$, а при μ - finite, $\mu\to0$ $\varepsilon(\xi)\sim\mu ch2y\to\infty$ при $\xi\to\pm\infty$.

Предположение $y_{\xi\xi\xi}=0$ с $\varepsilon(\xi)\neq\text{const}$ не проходит:

либо $y_{\xi\xi}=0$, $\varepsilon_\xi=0$ - Walker solution, либо $y_{\xi\xi\xi}\neq0$, $\varepsilon(\xi)$ конечна $\neq0$, $\varepsilon(\xi)\neq\text{const}$.

$$\begin{cases} v y_\xi=\lambda\varepsilon-\varepsilon_{\xi\xi}+2\varepsilon_\xi y_\xi thy \\ v\varepsilon_\xi=y_{\xi\xi\xi}+(1+\lambda\varepsilon^2+\varepsilon_\xi^2-y_\xi^2)thy \end{cases} \qquad y_\xi^2+\varepsilon_\xi^2=1+\lambda\varepsilon^2+\mu ch2y \text{ - 1st integral}$$

$$(1+\lambda\varepsilon^2+\varepsilon_\xi^2-y_\xi^2)=2\varepsilon_\xi^2-\mu ch2y.$$

$y(\xi)$ - odd function ξ, $y_{\xi\xi}(\xi)$ - odd, $\varepsilon_\xi(\xi)$ - odd, $\varepsilon(\xi)$ - even function of ξ, y_ξ - even $f(\xi)$.

если $\xi\to0$ $v\varepsilon_\xi=y_{\xi\xi}$, $vy_\xi=\lambda\varepsilon-\varepsilon_{\xi\xi}$, $\varepsilon_\xi=\frac{1}{v}y_{\xi\xi}=\frac{1}{v}\frac{d}{d\xi}\frac{1}{v}(\lambda\cdot\varepsilon-\varepsilon_{\xi\xi})$

$\varepsilon_\xi=\frac{\lambda}{v^2}\varepsilon_\xi-\frac{1}{v^2}\varepsilon_{\xi\xi\xi}$; $\mu=\varepsilon_\xi$, $\frac{1}{v^2}\mu_{\xi\xi}=\left(\frac{\lambda}{v^2}-1\right)\mu$, $\mu_{\xi\xi}=\lambda\left(1-\frac{v^2}{\lambda}\right)\mu$

$\mu(\xi)=\varepsilon_\xi\sim sh(\Delta_K\xi)$, $\Delta_K^2=\lambda(1-v^2/\lambda)$ при $\xi\to0$.

при $v=0$ $\lambda\varepsilon-\varepsilon_{\xi\xi}+2\varepsilon_\xi y_\xi thy=0$, $y_{\xi\xi}=-(2\varepsilon_\xi^2-\mu ch^2y)thy$

если $\varepsilon_\xi^2\ll\mu ch^2y$ $\to$ $y_{\xi\xi}=\mu sh y ch y$ - sine-Gordon $\to$ $y_{\xi\xi}=\frac{1}{2}\mu sh2y$

если $\mu=0$, $y_{\xi\xi}=-2\varepsilon_\xi^2thy$, и при $\varepsilon_\xi^2\ll1$, $y_{\xi\xi}\cong0$, $\lambda\varepsilon-\varepsilon_{\xi\xi}+\frac{2}{\Delta}\varepsilon_\xi thy=0$

Sine-Gordon $y_{\xi\xi}\cdot y_\xi=\frac{1}{2}\mu sh2y\cdot y_\xi$, $\frac{1}{2}(y_\xi)^2=\frac{1}{4}\mu ch2y+C$, $\int\frac{dy}{\sqrt{\frac{\mu}{2}ch2y+C}}=\xi-\xi_0$.

$\frac{1}{2}\frac{2}{\sqrt{\mu/2+C}}F(\alpha,k)=\xi-\xi_0$, $\sin\delta=thy$, $k^2=\frac{\mu/2-C}{\mu/2+C}$.

$thy=f(\xi)\to$ нужно $thy(\xi)=\sin f(\xi)$ — но это $v\to0$ limit.

Нужно как-то находить ε_ξ если (пока $\mu=0$): $v\varepsilon_\xi=y_{\xi\xi}+(2\varepsilon_\xi^2-\mu ch2y)thy$, $\mu=0$

$thy2(\varepsilon_\xi)^2-v\varepsilon_\xi+y_{\xi\xi}=0$, $(\varepsilon_\xi)^2+\frac{1}{2}v\,cthy\cdot\varepsilon_\xi+\frac{1}{2}cthy\cdot y_{\xi\xi}=0$ (т.е. вместо $\mu\neq0$ оказывается $y_{\xi\xi}\neq0$)

$2(\varepsilon_\xi)_{1,2}=\frac{v}{2}cthy\pm\sqrt{\frac{v^2}{4}cth^2y-4\cdot\frac{1}{2}cthy\,y_{\xi\xi}}=\frac{v}{2}cthy\pm cthy\sqrt{\frac{v^2}{4}-2thy\cdot y_\xi}$

$\varepsilon_\xi=\frac{1}{4}v\,cthy\pm\frac{1}{4}cthy\sqrt{v^2-8thy\,y_{\xi\xi}}$, $\varepsilon_\xi(y_{\xi\xi}=0)=\frac{1}{4}v\,cthy\pm\frac{1}{4}v\,cthy=\begin{cases}\frac{v}{2}cthy\\0\text{ - более реш.}\end{cases}$

$\varepsilon_\xi(\xi)=\frac{1}{4}v\,cthy\left[1-\sqrt{1-\frac{8}{v^2}thy\,y_{\xi\xi}}\right]$, $vy_\xi=\lambda\varepsilon-\varepsilon_{\xi\xi}+2y_\xi\varepsilon_\xi thy$

$\varepsilon_\xi thy=\frac{1}{4}v\left[1-\sqrt{1-\frac{8}{v^2}thy\cdot y_{\xi\xi}}\right]$. Видно $y_{\xi\xi}=0\to\varepsilon_\xi=0$ автомат. (т.к. $\mu=0$).

$v-2\varepsilon_\xi thy=v-\frac{v}{2}\left[1-\sqrt{1-\frac{8}{v^2}thy\,y_{\xi\xi}}\right]$. Чтобы $\frac{8}{v^2}thy\,y_{\xi\xi}\ll1$. $1-\sqrt{1-\alpha}=1-\left(1-\frac{\alpha}{2}\right)=\frac{\alpha}{2}$ упрощение

$\varepsilon_\xi thy=\frac{1}{4}v\cdot\frac{1}{2}\frac{8}{v^2}thy\,y_{\xi\xi}$, $\varepsilon_\xi=\frac{1}{v}y_{\xi\xi}$, это согласуется $\varepsilon_\xi^2\ll1$.

$vy_{\xi\xi}=\lambda\varepsilon_\xi-\varepsilon_{\xi\xi\xi}+\frac{d}{d\xi}\left[2y_\xi thy\frac{1}{v}y_{\xi\xi}\right]$ - нелин. eq. для $y_{\xi\xi}$. $\Delta^2=1-v^2/\lambda$

$\varepsilon(\xi)=\frac{1}{v}y_\xi+C\to vy_\xi=\lambda\left[\frac{1}{v}y_\xi+C\right]-\frac{1}{v}y_{\xi\xi\xi}+2y_\xi thy\frac{1}{v}y_{\xi\xi}$ $\quad v^2-\lambda=-\Delta_K^2$

$\left(v-\frac{\lambda}{v}\right)y_\xi=\lambda C-\frac{1}{v}y_{\xi\xi\xi}+\frac{2}{v}y_{\xi\xi}y_\xi thy$, $y_{\xi\xi\xi}+(v^2-\lambda)y_\xi=\lambda Cv$. $v^2<\lambda=c^2$.

$y_\xi=a\,ch\left(\sqrt{-v^2+\lambda}\,\xi\right)$, $\sqrt{v^2-\lambda}\Rightarrow\sqrt{\lambda-v^2}=\sqrt{\lambda}\sqrt{1-v^2/\lambda}=\Delta_K(v)$. $y(\xi)=a\,sh(\Delta_K\xi)\frac{1}{\Delta_K}$

$y(\xi)=y_\xi$, $y_{\xi\xi}-\Delta_K^2y=\lambda Cv$, $y(\xi)=a\,ch(\Delta_K\xi)+y_1$, $a\Delta_K^2ch(\Delta\xi)-\Delta^2\left[a\,ch(\Delta\xi)+y_1\right]=\lambda Cv$

$-\Delta^2y_1=\lambda Cv$, $y_1=-\frac{\lambda Cv}{\Delta^2}$. $y_\xi(\xi)=a\,ch(\Delta_K\xi)-\frac{\lambda Cv}{\Delta_K^2}$, $y(\xi)=\frac{a}{\Delta_K}sh(\Delta_K\xi)-\frac{\lambda Cv}{\Delta_K^2}\xi$

$\varepsilon_\xi=\frac{1}{v}y_{\xi\xi}$, $\varepsilon(\xi)=\frac{1}{v}y_\xi+C=\frac{a}{v}ch(\Delta_K\xi)-\frac{\lambda C}{\Delta_K^2}+C$, $\varepsilon(\xi)_{a\to0}=C-\frac{\lambda C}{\Delta_K^2}$ Walker limit.

$\varepsilon=\frac{v}{\lambda}y_\xi$ in Walker limit. y_ξ (Walker lim.) $=-\frac{\lambda Cv}{\Delta_K^2}$, $\varepsilon=\frac{v}{\lambda}(-)\frac{\lambda Cv}{\Delta_K^2}=C\left(1-\frac{\lambda}{\Delta_K^2}\right)$

$-\frac{Cv^2}{\Delta_K^2}=C\frac{1}{\Delta_K^2}(\Delta_K^2-\lambda)$, $-v^2=\Delta_K^2-\lambda$, $\Delta_K^2=\lambda+v^2$ - верно.

In Walker limit $y_W(\xi)=\frac{\xi}{\Delta}$ и $y_{a\to0}(\xi)=-\frac{\lambda C}{\Delta_K^2}\xi$, $\frac{1}{\Delta}=-\frac{\lambda Cv}{\Delta_K^2}$, $C=-\frac{\Delta_K^2}{\lambda v}\frac{1}{\Delta}$

$C=-\frac{\lambda(1-v^2/\lambda)}{\sqrt{1-v^2/\lambda}}\frac{1}{\lambda v}=\frac{-1}{v}\sqrt{1-v^2/\lambda}$, $\varepsilon(\xi)=\frac{1}{v}y_\xi-\frac{1}{v}\sqrt{1-v^2/\lambda}$, $y(\xi)=\frac{a}{\Delta_K}sh(\Delta_K\xi)+\frac{\xi}{\Delta}$, $y_\xi=a\,ch(\Delta_K\xi)+\frac{1}{\Delta}$

Escaneado con CamScanner

$\mathcal{E}(\zeta) = \frac{a}{v}\,ch(\Delta_k\zeta) + \frac{v}{\lambda\Delta}\,\eta(\zeta) = \frac{a}{\Delta_k}\,sh(\Delta_k\zeta) + \frac{\zeta}{\Delta}$, $\Delta^2 = 1 - v^2/\lambda$, $\Delta_k^2 = \lambda(1 - v^2/\lambda) = \lambda\Delta^2$

Walker limit $a \to 0$, $\varepsilon_w = \frac{v}{\lambda\Delta}$, $\eta_w = \zeta/\Delta$, $\mathcal{H} = 0$.

$\varepsilon_\zeta = \frac{a\Delta_k}{v}\,sh(\Delta_k\zeta)$, $\eta_\zeta = a\,ch(\Delta_k\zeta) + \frac{1}{\Delta}$, проверим $\eta_\zeta^2 + \varepsilon_\zeta^2 = 1 + \lambda\varepsilon^2$

$\eta_\zeta^2 + \varepsilon_\zeta^2 = \frac{a^2\Delta_k^2}{v^2}\,sh^2x + a^2 ch^2x + \frac{1}{\Delta^2} + \frac{2a}{\Delta}\,ch x = 1 + \lambda\left[\frac{a^2}{v^2}ch^2x + \frac{v^2}{\lambda^2\Delta^2} + \frac{2a}{\lambda\Delta}\,ch x\right]$

$\frac{a^2\Delta_k^2}{v^2}(ch^2x - 1) + a^2 ch^2x + \frac{1}{\Delta^2} = \frac{\lambda a^2}{v^2}\,ch^2x + 1 + \frac{v^2}{\lambda\Delta^2}$

$(ch x)^0$: $\frac{1}{\Delta^2} = 1 + \frac{v^2}{\lambda\Delta^2}$, $\Delta^2 + \frac{v^2}{\lambda} = 1$, $\Delta^2 = 1 - v^2/\lambda$ - верно.

Но надо еще $-\frac{a^2\Delta_k^2}{v^2}$ в $(ch x)$: $ch^2 x$: $\frac{a^2\Delta_k^2}{v^2} + a^2 = \frac{\lambda a^2}{v^2}$, $\Delta_k^2 + v^2 = \lambda$

Не сходится еще член $-\frac{a^2\Delta_k^2}{v^2}$ в $a(ch x)v^2$: $\frac{a^2}{v^2}\Delta_k^2 = \frac{a^2}{v^2}(\lambda - v^2)$ $\Delta_k^2 = \lambda - v^2$ - верно.

$a \to 0$ можно, надо $v \to c - 0$. $\lambda - v^2 = c^2 - v^2 \to 0$, это сходится. $\frac{8}{v^2}\,th\gamma\,\eta_{3\zeta} \ll 1$

$\eta_{3\zeta} = a\Delta_k\,sh(\Delta_k\zeta)$, $\frac{8}{v^2}\,th\gamma\,\eta_{3\zeta} = \frac{8}{v^2}\,a\Delta_k\,sh(\Delta_k\zeta)\,th\gamma \ll 1$ - условие самосогл.

либо при $\zeta \to 0$, либо $v \to c - 0$ ($\Delta_k \to 0$), либо $a \to 0$. Фиксим $v \to c - 0$.

$\mathcal{E}(\zeta) \ll 1$ по смыслу $\to (a/v) \ll 1$. $v \sim c = \sqrt{\lambda}$, $\frac{a^2}{\lambda} \ll 1$.

Хотя и нашли решение $\mathcal{E} = \mathcal{E}(\zeta)$, но оно (при $\mathcal{H}=0$) применимо при $v \le c = \sqrt{\lambda}$.

Решение симметрично при $\zeta \to \pm\infty$, хотя 1st integral и было инвариантен.

$\theta(\zeta) = 2\,arctg\,\exp[\eta(\zeta)]$, $\theta(\zeta \to \pm\infty) = 2\,arctg\exp[\pm\infty] = \begin{cases}\pi, & \zeta \to +\infty\\ 0, & \zeta \to -\infty\end{cases}$ - это DW.

Скорость max of DW, не возрастая и $v_{max} = c = \sqrt{\lambda}$ далее при $\varepsilon = \mathcal{E}(\zeta)$.

(8/III) Meeting с Ricardo. обсуждение DW-ей Gordon MS. Ответы на его вопросы.
Определение Figs for the MS (1- geometry, 2- $k(v)$, $x(v)$, 3 - $E(v)$, $\Delta(v)$).
Meeting с Julian. Тема, бюджет обсуждение 3.000€, а CAF-бюджет 6.000€ to Solskymag.
Michal прислал simulation on Edge modes" eg dots y "rings (ответил).
COST - Confirm. letter sent to "Ruben y Oksana.

(9/III) Supermercados с Л. перемерка. Solskymag и подготовка conf. Program.

(10 - 18/III) поездка с Л. на машине в Porto within COST action „Magnetophore"
Оформление conf. Program. Обсуждение - Fres и сервиз. расходов
Hres(n) - drum modes и Cole Cyl. dots. Кардио Simul. DW dyn. - stripes.
перемерка Sol-Skymag, Arantxa (w. hours) и Ricardo (DW - MS).

18/III перепрезервирован „Londres" to A. Fert с June 23-27 на June 24-28 по его
Запросу. Reminders to Blügel и Suess-invited (no response) - 11/III → Black list.

(19/III) E-mail to Орс., Julian - табель w. hours за 2018 - FIS2016 project.
→ Сделано: 1) получил NIE tarjetas; 2) Hres(n) - drum modes MathCad; 3) w. hours FIS2016;
4) review Polish project; 5) Verba - MS new; 6) SolSkyMag Program.

(20/III) Л. и А. получили new tarjetas NIE на 5 лет. Л., А., Enoski" Urbil.
Revision SolSkyMag-2019 Program - A. Fert talk - Wed., June 26, Jeju AM.
Получил и submitted review на Polish proposal - Topol-SW" By M. Krawczyk
(чудо PLN - монограф). Тема резюме. Art. Skyrmions MS" to APL Materials.

(21/III) Reminder horas 2018 Cristina UPV/EHU. E-mails Орс. (horas 2018, Bergenza PRX).
Создала и послала Horas - 2018 FIS2016 дело сдала и Julian (добавила Arantxa).
Перемерка с Орс. о Bergenza PRX. Решили подать appeal на 3rd referee -
а ответили не позже monday, 25/III подать. Letter Editor draft.

(22/III) Вызов в „Bankia" и „Santander" с new NIE tarjetas. Снял 300€ (п.
снял 500€ 21/III). С Л. в „Nam". Принесли A. Hernando на SolSkyMag 19.
Оксана прислала simul. тезю SK ΔE barriers в Pt/Co/Pt, Pt/Co/Pt. Sent comments

(23/III) Radial SW - cylinder, $\vec{M}_0 \perp$ dot plane, Прин. FMR - Dobrovolskij
SW cylinder - „drum" modes
Уточнила расчет „drum" modes Kakazei APL'2004 - См. 2/I 2018:

(*) $\frac{\omega_n}{\omega_M} = h + h_{nn} + \frac{1}{2}f(\beta x_n) + \left(\frac{l_e}{R}\right)^2 x_n^2$, $l_e^2 = A/2\pi M_s^2$, x_n из $xJ_1(x) = dJ_0(x)$

Assuming $d = d(\beta)$ из Gus' PRB'2005, $\beta = L/R$. $h = H_z/4\pi M_s$, $h_{nn} = (h_{dm})_{nn}/4\pi M_s$ -
- static dipolar fields matrix elements. Применил. СФ $\varphi_n(\xi) = \frac{1}{\sqrt{N_n}}J_0(x_n\xi)$,

$N_n = \frac{(x_n^2 + d^2)}{2x_n^2}J_0^2(x_n)$, $h_{nn} = \frac{1}{N_{nn}}\int_0^\infty d\eta f(\beta x)J_1(x)\int_0^1 d\xi\,\xi J_0^2(x_n\xi)J_0(\xi x) - 1$

$h_{nn} = \int d\xi\,\varphi_n^2(\xi)\,h_0(\xi)$, $h_0(\xi) = -\frac{1}{\beta}\int\frac{dx}{x}(1 - e^{-\beta x})J_0(\xi x)J_1(x)$

Escaneado con CamScanner

SW теория $\omega_n(*)$ – это diagonal approximation для dipolar field на $\perp$ СФ $\varphi_n(\rho) = \frac{1}{\sqrt{N_n}} J_0(\chi_n \rho)$ с усреднением угла θ_H dipolar field $\sim f(\beta\chi_n)$

Отличие SW $\omega_n(\chi)$ от Kalinikos' AFL'ов: 1) линейное $\omega_n(\chi)$; 2) reduced pinning $d(\beta)$.
См. Comments о точности $\omega_n(\chi)$ 2/I. Задачу ω_n можно решать более точно для finite β.

$(*) \to \frac{\omega_n}{\gamma} = H_z + H_{nn} + 2\pi M_s f(\beta\chi_n) + \frac{2A}{M_s}\frac{\chi_n^2}{R^2}$. FMR, $\frac{\omega}{2\pi} = 9.86$ GHz is fixed,

нелинейное $H_{res}(n) = \frac{\omega}{\gamma} - H_{nn} - 2\pi M_s f(\beta\chi_n) - \frac{2A}{M_s}\frac{\chi_n^2}{R^2}$, χ_n – безразмерное $\chi J_1(\chi) = d J_0(\chi)$

$H_{nn} = 4\pi M_s h_{nn} = 4\pi M_s \left[-1 + \frac{1}{N_n}\int_0^\infty d\chi f(\beta\chi) J_1(\chi) \int_0^1 d\rho\rho J_0^2(\chi_n\rho) J_0(\rho\chi)\right]$

$H_{nn} = 4\pi M_s[-1 + O(\beta)]$ при $\beta \to 0$. Верное утверждение.

Mcd file: Radial_SW_Cyl-dot2.mcd* даёт набор $\chi_n(\beta_j) = \chi_{nj}$, n – mode number, j нумерует dot radius R_j = 1000, 700, 500, 400, 300, 200, 150, 100 nm

$H_{res}(n) = \frac{\omega}{\gamma} - 4\pi M_s\left[-1 + \frac{1}{N_n}\int_0^\infty d\chi f(\beta\chi) J_1(\chi) \int_0^1 d\rho\rho J_0^2(\chi_n\rho) J_0(\rho\chi) + \frac{1}{2} f(\beta\chi_n) + \frac{l_e^2}{R^2}\chi_n^2\right]$

Внешнее поле $\gamma/2\pi = 3.1$ GHz/kOe (типично для CoFe alloys), $\frac{\omega}{2\pi} = 9.85$ GHz.
$\omega/\gamma = 3.177$ kOe.

$H_{res}(\chi_n) = \frac{\omega}{\gamma} + 4\pi M_s - 4\pi M_s\left[\frac{1}{N_n}\int_0^\infty d\chi f(\beta\chi) J_1(\chi) \int_0^1 d\rho\rho J_0^2(\chi_n\rho) J_0(\rho\chi) + \frac{1}{2} f(\beta\chi_n) + \frac{l_e^2}{R^2}\chi_n^2\right]$

$I(\chi_n) = \frac{1}{N_n(\chi_n)}\int_0^\infty d\chi f(\beta\chi) J_1(\chi) \int_0^1 d\rho\rho J_0^2(\chi_n\rho) J_0(\rho\chi)$

$N_n(\chi_n) = \frac{1}{2}\chi_n^2 (\chi_n^2 + d^2) J_0^2(\chi_n) \to H_{res}(\chi_n) = \frac{\omega}{\gamma} + 4\pi M_s - 4\pi M_s\left[I(\chi_n) + \frac{1}{2} f(\beta\chi_n) + \frac{l_e^2}{R^2}\chi_n^2\right]$

$H_{res}(n) = \frac{\omega}{\gamma} + 4\pi M_s - 4\pi M_s F(\chi_n)$, $F(\chi_n) = I(\chi_n) + \frac{1}{2} f(\beta\chi_n) + \frac{l_e^2}{R^2}\chi_n^2$.

$F(\chi_n) > 0$ и растет с ростом χ_n (ростом mode number "n"). $H_{res}(n)$ падает с n↑.

(24/III) Для R = 1000 nm, $\beta = 40$ nm/1000 nm = 0.04 $H_{res}^{calc}(n)$ заметно ниже $H_{res}^{exp}(n)$ при n > 1.
$H_0(n=1)$ – SWR res. field находим M_s – приближение? H_z(n) exp, calc; 1, 8, n

$\vec{M}_0 + \vec{\mu} = \vec{M}$, SW $\vec{\mu} = (\mu_x, \mu_y, 0)$, $\vec{M}_0 \| Oz$, $\mu_\pm(\rho, t) = \mu_x \pm i\mu_y$

$\mu_\pm(\rho, t) = e^{\pm}(\rho, t) e^{\pm i\Phi_0}$, $\Theta = \Theta_0 + e\theta$, $\Phi = \Phi_0 + \psi$, $\mu_\pm$, УФ для μ_x, μ_- рассчит.

из-за axial symmetry, $\vec{M}_0 \| Oz$, $\vec{H} \| Oz$, $m = 0$ – radial modes – см. 1-2/I.

Точное УФ для $\mu = \mu_+(\rho)$: $\left[\frac{\omega}{\omega_M} - h - h_0(\rho) + l_e^2\nabla^2\right]\mu(\rho) = \frac{1}{2}\int_0^\infty d\rho' \rho' g(\rho\rho')\mu(\rho')$

$h = H_z/4\pi M_s$, $h_0(\rho) = H_z^0/4\pi M_s = -\frac{1}{\beta}\int_0^\infty \frac{d\chi}{\chi}(1 - e^{-\beta\chi}) J_0(\rho\chi) J_1(\chi)$, $l_e^2 = A/2\pi M_s^2$

$g(\rho\rho') = \int_0^\infty d\chi\chi f(\beta\chi) J_0(\chi\rho) J_0(\chi\rho')$. Точка поворота $\rho = \rho^*$, $\frac{\omega}{\omega_M} = h + h_0(\rho^*)$ – Abe '74

$h_0(\rho) = -1 + \int_0^\infty d\chi f(\beta\chi) J_0(\rho\chi) J_1(\chi) = -1 + h_1(\rho)$, $\frac{\omega}{\omega_M} = h - 1 + h_1(\rho^*)$, $h_1 > 0$

$h - 1 = \frac{1}{4\pi M_s}(H_z - 4\pi M_s) = \frac{1}{15.6 \text{ kOe}}[(18 \div 16.3)\text{ kOe} - 15.6\text{ kOe}]$ при R = 1000 nm, $h_1 \sim \beta$

$h - 1 = (0.154 \div 0.045)$, ω/ω_M, $\frac{\omega}{\gamma} = 3.18$ kOe, $\frac{\omega}{\omega_M} = \frac{\omega}{\gamma \cdot 4\pi M_s} = \frac{3.18}{15.6} = 0.568$

Exp. $0.568 = (0.154 \div 0.045) + h_1(\rho^*)$, $h_0^{max} = h_0(\rho=1) = -\frac{1}{2}$, $h_1^{max}(\rho) = h_0^{max} + 1 = 0.5$,

т.е. точка поворота волновода дана для R = 1000 nm с $\rho^* \to 1 - 0$. (при $\beta \to 0$)

Уточненное точное решение (из-за 2-й exch. $\beta = 0.04$, $l_e^2\nabla^2 = h_{ex}$).

Базисные функции $\varphi_\nu(\rho) = \frac{1}{\sqrt{N_\nu}} J_0(\chi_\nu\rho)$, χ_ν из B.C. $d J_0(\chi) = \chi J_1(\chi)$

$N_\nu = \frac{1}{2}\left(1 + \frac{d^2}{\chi_\nu^2}\right) J_0^2(\chi_\nu) = \frac{1}{2}[J_0^2(\chi_\nu) + J_1^2(\chi_\nu)]$, $\int_0^1 d\rho\rho\,\varphi_\nu(\rho)\varphi_{\nu'}(\rho) = (N_\nu)\delta_{\nu\nu'} = \delta_{\nu\nu'}$

$\frac{\omega}{\omega_M} - h - h_0(\rho) = \left(\frac{\omega}{\omega_m} - h + 1\right) - h_1(\rho) = \Omega - h_1(\rho)$, $\Omega = \frac{\omega}{\omega_M} + 1 - h = \lambda$ ($\lambda \geq 0$?)

$\to \left[\lambda - h_1(\rho) + l_e^2\nabla^2\right]\mu(\rho) = \frac{1}{2}\int_0^\infty d\rho'\rho' g(\rho\rho')\mu(\rho')$, $h_1(\rho) = \int_0^\infty d\chi f(\beta\chi) J_0(\rho\chi) J_1(\chi)$

при $\beta \to 0$, $h_1 \to 0$, $g(\rho\rho') \to 0$ $\left(\lambda + l_e^2\nabla^2\right)\mu(\rho) = 0$, $\nabla^2 = \nabla_\rho^2 = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho}$

$\left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{\lambda}{l_e^2}\right]\mu(\rho) = 0$, уравнение Бесселя $\frac{\partial^2 u}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial\rho} + \left(\chi^2 - \frac{m^2}{\rho^2}\right)u = 0$, $J_m(\chi\rho)$

Очевидно, если $m = 0$, $\chi^2 = \lambda/l_e^2$, то $\mu(\rho) = J_0(\chi\rho)$ (масштаб $\lambda = l_e^2\chi^2$ или

$\frac{\omega}{\omega_M} = h - 1 + \frac{l_e^2}{R^2}\chi^2$, $\frac{\omega}{\gamma} = H_z - 4\pi M_s + \frac{2A}{M_s^2}\frac{1}{R^2}\chi^2$, χ – dimensionless (in units $1/R$).

$\beta \to 0$ нулевое Finite $\beta \to$ finite dip. fields $h_1 \ldots g(\rho\rho')$

Имеем $\mu(s) = \sum_\nu a_\nu J_0(x_\nu s)$, $x J_0'(x) + d J_0(x) = 0$ дает корни $x_\nu \to$ Dini series.

$a_\nu = \frac{1}{N_\nu} \int_0^1 ds\, s\, \mu(s) J_0(x_\nu s)$, $N_\nu = \frac{1}{2}\left(1 + \frac{d^2}{x_\nu^2}\right) J_0^2(x_\nu) = \frac{1}{2}\left[J_0^2(x_\nu) + J_1^2(x_\nu)\right]$

Нормировал. базисные функции $\varphi_\nu(s) = \frac{1}{\sqrt{N_\nu}} J_0(x_\nu s)$, $\mu(s) = \sum_\nu a_\nu \varphi_\nu(s)$

$a_\nu = \int_0^1 ds\, s\, \mu(s) \varphi_\nu(s)$. $\varphi_\nu(s) = |\nu\rangle$, см. 2/I.

Задача на СЗ/СФ: $\hat\Gamma \mu = \left(\frac{\omega}{\omega_M}\right)\mu$, $\sum_\nu \Gamma_{\nu\nu'} a_{\nu'} = \frac{\omega}{\omega_M} a_\nu$, $\Gamma_{\nu\nu'} = \langle\nu|\hat\Gamma|\nu'\rangle$

$\Gamma_{\nu\nu'} = \left(h + \ell_e^2 x_\nu^2\right)\delta_{\nu\nu'} + \langle\nu|h_0|\nu'\rangle + \langle\nu|h_{syn}|\nu'\rangle$

$\hat\Gamma\mu = \left[h + h_0(s) - \ell_e^2 \nabla^2\right]\mu(s) + \frac{1}{2}\int ds' s' g(ss')\mu(s')$, $\hat\Gamma\mu = \frac{\omega}{\omega_M}\mu$, $\frac{\omega}{\omega_M} \to \omega$

$\mu = \sum_\nu a_\nu |\nu\rangle$, $\hat\Gamma\mu = \hat\Gamma\sum_\nu a_\nu|\nu\rangle = \omega\sum_\nu a_\nu|\nu\rangle$. умн. на $\langle\nu'|$

$\langle\nu'|\hat\Gamma\sum_\nu a_\nu|\nu\rangle = \omega\sum_\nu a_\nu\langle\nu'|\nu\rangle$, $\sum_\nu \langle\nu'|\hat\Gamma|\nu\rangle a_\nu = \omega\sum_\nu a_\nu\delta_{\nu\nu'} = \omega a_{\nu'}$

$\sum_{\nu'} \Gamma_{\nu\nu'} a_{\nu'} = \omega a_\nu$ – сек. ур-е. $\nu' \rightleftarrows \nu$

$\Gamma_{\nu\nu'} = \langle\nu|\hat\Gamma|\nu'\rangle = \int ds\, s\, \varphi_\nu(s)\varphi_{\nu'}(s)\hat\Gamma(s)$, $\hat\Gamma\varphi_{\nu'}(s) = \hat\Gamma|\nu'\rangle = (h_s + h_0 - \ell_e^2\nabla^2)|\nu'\rangle +$

$+ \frac{1}{2}\int ds' s' g(ss')\varphi_{\nu'}(s')$, $\Gamma_{\nu\nu'} = \langle\nu|\cdot\hat\Gamma|\nu'\rangle = \int ds\, s\, \varphi_\nu(s)\hat\Gamma|\nu'\rangle = \int ds\, s\, \varphi_\nu(s) \times$

$\times\left[(h + h_0 - \ell_e^2\nabla^2)\varphi_{\nu'}(s) + \frac{1}{2}\int ds' s' g(ss')\varphi_{\nu'}(s')\right]$, $\nabla^2\varphi_\nu(s) = -x_\nu^2\varphi_\nu(s)$

$\Gamma_{\nu\nu'} = \int ds\, s\, \varphi_\nu(s)\left[(h + h_0 + \ell_e^2 x_{\nu'}^2)\varphi_{\nu'}(s) + \frac{1}{2}\int ds' s' g(ss')\varphi_{\nu'}(s')\right]$, $\int ds\, s\, \varphi_\nu\varphi_{\nu'} = \delta_{\nu\nu'}$

$\Gamma_{\nu\nu'} = (h + \ell_e^2 x_\nu^2)\delta_{\nu\nu'} + \int ds\, s\, \varphi_\nu(s) h_0(s)\varphi_{\nu'}(s) + \frac{1}{2}\int ds\, s\, \varphi_\nu(s)\int ds' s' g(ss')\varphi_{\nu'}(s')$

$\Gamma_{\nu\nu'} = (h - 1 + \ell_e^2 x_\nu^2)\delta_{\nu\nu'} + \int ds\, s\, \varphi_\nu(s) h_1(s)\varphi_{\nu'}(s) + \frac{1}{2}\int ds\, s\int ds' s'\, \varphi_\nu(s) g(ss')\varphi_{\nu'}(s')$

$\Gamma_{\nu\nu'} = \omega_\nu^0\delta_{\nu\nu'} + \Gamma'_{\nu\nu'}$, $\Gamma'_{\nu\nu'} = \int ds\, s\, \varphi_\nu\varphi_{\nu'} h_1(s) + \frac{1}{2}\int ds\, s\int ds' s'\, \varphi_\nu(s) g(ss')\varphi_{\nu'}(s')$

$\omega_\nu^0 = h - 1 + \ell_e^2 x_\nu^2$ (частота ω_M при $\beta \to 0$).

$\sum_{\nu'}\Gamma_{\nu\nu'}a_{\nu'} = \sum_{\nu'}\left[\omega_\nu^0\delta_{\nu\nu'} + \Gamma'_{\nu\nu'}\right]a_{\nu'} = \omega a_\nu$, $\omega - \omega_\nu^0$

$\sum_{\nu'}\left[(h - 1 + \ell_e^2 x_\nu^2)\delta_{\nu\nu'} + \Gamma'_{\nu\nu'}\right]a_{\nu'} = \omega a_\nu$, $\lambda = \omega + 1 - h \to \sum_{\nu'}\left[\ell_e^2 x_\nu^2\delta_{\nu\nu'} + \Gamma'_{\nu\nu'}\right]a_{\nu'} = \lambda a_\nu$

$\tilde\Gamma_{\nu\nu'} = \ell_e^2 x_\nu^2\delta_{\nu\nu'} + \Gamma'_{\nu\nu'} = \Gamma_{\nu\nu'}(\omega_\nu) + \Gamma'_{\nu\nu'}(\text{dip})$, $\tilde\Gamma\mu = \lambda\mu$, $\tilde\Gamma \to B$

$\hat B\mu(s) = \lambda\mu(s)$, $B_{\nu\nu'} = \ell_e^2 x_\nu^2\delta_{\nu\nu'} + \Gamma'_{\nu\nu'}$, $\sum_{\nu'} B_{\nu\nu'}a_{\nu'} = \lambda a_\nu$.

т.е. спектр в λ-представлении определяется диаг. exchange и диагон./недиагон. магнитостатики $\sim\beta$. $\beta = 0.04 \div 0.4$ велико, поэтому diagon. approximation $B_{\nu\nu'} \to B_{\nu\nu}\delta_{\nu\nu'}$, $B_{\nu\nu} = \ell_e^2 x_\nu^2 + \Gamma'_{\nu\nu}$ см 2/I

может работать много далее при $\beta = 0{,}04$, $R = 1000$ mm.

$\Gamma'_{\nu\nu} = \int_0^\infty dx\, f(\beta x) J_1(x)\left[\int_0^1 ds\, s\, \varphi_\nu^2(s) J_0(sx) + \frac{1}{2}\int dx\, f(\beta x) I_\nu^2(x_\nu, x)\right]$, $I_\nu(x_\nu, x) = \int_0^1 ds\, s\, \varphi_\nu(s) J_0(xs)$

Спектр в diag. approximation $\lambda_\nu = B_{\nu\nu} = \ell_e^2 x_\nu^2 + \Gamma'_{\nu\nu}$ или $\varphi_\nu(s) = N_\nu^0 - \nu_2\, J_0(x_\nu s)$

$\frac{\omega_\nu}{\omega_M} = h - 1 + \ell_e^2 x_\nu^2 + \Gamma'_{\nu\nu}$, если еще предположить $I_\nu^2(x_\nu, x) \approx \frac{1}{x}\delta(x_\nu - x)$, то

$\frac{\omega_\nu}{\omega_M} = h - 1 + (h_1)_{\nu\nu} + \frac{1}{2}f(\beta x_\nu) + \frac{\ell_e^2}{R^2}x_\nu^2$ – получим ω_ν см 2/III, которое работает много для $R = 1000$ mm. Если R уменьшить, то точность ω_ν или $h_{res}(n)$ – диагональ невысокая, т.к. exch. $\sim\frac{1}{R^2}$ возрастает. И/I показано, что d-функция приведет к росту $\Gamma'_{\nu\nu}(d) = \left(\frac{x^2 + d^2}{d^2}\right)\Gamma'_{\nu\nu}(d \to \infty)$ и уменьшению $h_{res}(\nu)$.

$\to h_{res}(n) = \frac{\omega}{\omega_M} + 1 - (h_1)_{nn} - \frac{1}{2}f(\beta x_\nu) - \frac{\ell_e^2}{R^2}x_\nu^2 = \frac{\omega}{\omega_M} + 1 - \frac{\ell_e^2 x_\nu^2}{R^2} - \Gamma'_{\nu\nu}(\beta)$, $n \to \nu$. при $\omega_\nu = \omega = \omega_M$

$h_{res}(n) \simeq \frac{\omega}{\omega_M} + 1 - \left(\frac{\ell_e}{R}\right)^2 x_n^2 - \Gamma'_{nn}(\beta)$, $\Gamma'_{nn} = (h_1)_{nn} + \Gamma'_{nn}(dyn)$

Finite d $\to$ рост $\Gamma'_{nn}(dyn) \to$ падение $h_{res}(n)$. Видимо $\Gamma'_{nn}(dyn)$ велико, $(h_1)_{nn}$, которое относительно мало и сильно зависит от mode number "n".

Сравнить числ. $\Gamma'_{nn}(dyn) = \frac{1}{2}\int dx\, f(\beta x) I_\nu^2(x_\nu, x) \times c\ \frac{1}{2}f(\beta x_\nu)$ при $d \to \infty$ d-функция.

$I_{(x_\nu, x)} = \frac{1}{\sqrt{N_\nu}}\int_0^1 ds\, s\, J_0(x_\nu s) J_0(xs) = \frac{\sqrt{2}\,x}{\sqrt{x^4 + d^2}}\underbrace{\left[d J_0(x) - x J_1(x)\right]}_{(x^2 - x^2)}$, $\Gamma'_{nn}(dyn) = \frac{x^2}{(x^2 + d^2)}\int_{xx} f(\beta x)\frac{[dJ_0(x) + x]}{(x^2 - x^2)^2}$

и в.с. $d J_0(x) = x J_1(x)$ делаем Γ'_{nn} несингулярным

Если брать $\lambda \to \infty$ в diag. approximation и $I^2_\nu(x,x) = \frac{1}{x}\delta(x-x)$ approximation то $h_{res}(n)$ остается практически таким же как для real pinning $\delta(\beta)$-finite $(h_1)_{nn}$ немного увеличен, а $\Gamma'_{nn}(dyn) = \frac{1}{2}\delta(\beta x_n)$ немного возрастает за счет роста $x_n(\lambda\to\infty)$, сумма $(h_1)_{nn} + \frac{1}{2}\delta(\beta x_n)$ практически не изменится. Это для (численно) $R = 1000$ nm.

Вывод: два приближения - diagonal и $I^2_\nu = \frac{1}{x}\delta(x-x)$ дающее $\delta(\beta x_n)$

$I_\nu(x,x) = \frac{1}{\sqrt{N(x)}}\int_0^1 d\rho\rho\, J_0(x\rho)J_0(x\rho) \to I^2_\nu(x,x)$ имеет резкий max при $x = x, x \gg 1$

приближение $\Gamma'_{\nu\nu}(dyn) = \frac{1}{2}\int_0^\infty dx\, x\,\delta(\beta x)\, I^2_\nu(x,x) \approx \frac{1}{2}\delta(\beta x_\nu)$ хорошо работает при $x_\nu = x \gg 1$ $\forall\beta$. Причина $I^2_\nu(x,x) \overset{0}{\approx} \frac{1}{x}\delta(x-x)$ для $x \gg 1$.

при $R = 1000$ nm $x_\nu > 1$ начиная с $\nu = n = 3$. Как раз с $n=3$ $H^{calc}_{res}(n)$ начинает заметно отличаться от $H^{exp}_{res}(n)$.

$\Gamma'^{(stat)}_{nn} = (h_1)_{nn} = \int d\rho\, \varphi_n^2(\rho)\rho\, h_1(\rho) = \int d\rho\rho\,\varphi_n^2(\rho)\int dx\, x\,\delta(\beta x) J_0(\rho x) J_1(x)$ Считается точно.

$\Gamma'_{nn}(dyn) \overset{0}{\approx} \frac{1}{2}\delta(\beta x_n)$ - хорошо выполн. при $x_n > 1$, $h_{ex}(n) = \frac{l_e^2}{R^2}x_n^2$ Считается точно.

// Т.е. ни $\Gamma'_{nn}(stat)$ ни $\Gamma'_{nn}(dyn)$ не дают ошибки при $x_n \gg 1$ ($n \geq 3$) для $h_{res}(n)$. Но $h^{calc}_{res}(n)$ заметно меньше $h^{exp}_{res}(n)$ для $n \geq 3$ $\Rightarrow$ Выводы:

- 1) diagonal-approximation для ω_ν (λ_ν) не работает для $R = 1000$ nm.
- 2) M_s fitted по $n=1$ - peak, $x_1 = 2{,}1 \pm \frac{h_{res}(\nu)}{1{,}5}$ для 1st peak $h_{res}(1)$, для данных малых x, $\Gamma'_{nn}(dyn) = \frac{1}{2}\delta(\beta x_n)$ не работает (надо $x \gg 1$).
- 3) Turning fails

Замена $\frac{1}{2}\delta(\beta x_n)$ на $\Gamma'_{nn}(dyn)$ - точное с подгонкой M_s по $H_{res}(1)$, $x_1 = 2{,}13$ дает не сильно $H^{calc}_{res}(n) < H^{exp}_{res}(n)$ при $n \geq 3$. $\longrightarrow$ Остается отклонение от diagonal approxim.

[Хотя $\Delta H_{res} \approx 1$ kOe при $n=8$].

$R = 1000$ nm, $M_s = 1243$ G. При $R = 1000$ nm, $H_{ex} \leq 0{,}2$ kOe - неважно.

// $R = 700$ nm, подгонка M_s по $H_{res}(1)$ дает $M_s = 1182$ G, $H^{exp}_{res}(9) = 14{,}76$ kOe, $H^{calc}_{res}(9) = 13{,}33$ kOe даже при $l_e = 0$ - diagonal approxim.

// $R = 500$ nm, -//- $M_s = 1{,}134$ kG, $H^{exp}_{res}(10) = 13{,}12$ kOe, $H^{calc}_{res}(10) = 11{,}75$ kOe при $l_e = 0$.

// $R = 400$ nm, -//-, $M_s = 1099$ G, $H^{exp}_{res}(10) = 11{,}22$ kOe, $H^{calc}_{res}(10) = 10{,}36$ kOe при $l_e = 0$.

// $R = 300$ nm -//- $M_s = 1028$ G, $H^{exp}_{res}(10) = 9{,}19$ kOe, $H^{calc}_{res}(10) = 9{,}23$ kOe при $l_e = 0$

$H_{res}(10) = 9{,}19$ kOe при $l_e = 2$ nm

Diagonal approxim. Разработано с $R = 300$ nm.

// $R = 200$ nm, -//- $M_s = 965$ G, $H^{exp}_{res}(7) = 7{,}31$ kOe, $H^{calc}_{res}(7) = 8{,}21$ $l_e = 0$

$H^{calc}_{res}(7) = 7{,}31$ при $l_e = 12{,}7$ nm.

// $R = 150$ nm -//- $M_s = 928$ G, $H^{exp}_{res}(4) = 7{,}89$ kOe $H^{calc}_{res}(4) = 8{,}36$ kOe при $l_e = 0$.

$H^{calc}_{res}(4) = 7{,}88$ kOe при $l_e = 0{,}4$ nm

// $R = 100$ nm -//- $M_s = 886$ G, $H^{exp}_{res}(2) = 7{,}91$ kOe, $H^{calc}_{res}(2) = 7{,}98$ kOe при $l_e = 0$.

$H^{calc}_{res}(2) = 7{,}90$ kOe при $l_e = 7$ nm

→ Вывод: diagonal approximation $\omega_\nu = \omega^0_\nu + \Gamma'_{\nu\nu}$ дает при точных $\Gamma'_{\nu\nu}(stat), \Gamma'_{\nu\nu}(dyn)$ не работает для $R = 1000, 700, 500, 400$ nm, сильно занижая $H^{calc}_{res}(n)$ для больших mode numbers "n". Но для $R = 300, 200, 150, 100$ nm diag. approxim. начинает работать, давая разумные M_s и l_e.

(25/III) A. Hernando прислал preg. invited talk Seeskymag 2019. Drum modes

Отклонение от diagonal approxim. при расчете ω_ν или $H_{res}(n)$.

Решим $\sum_{\nu'} B_{\nu\nu'} a_{\nu'} = \lambda a_\nu$ численно при $\nu = 1, 2, \dots, N$ $N \geq 10$.

$\lambda = \frac{\omega}{\omega_M} + 1 = h$, $B_{\nu\nu'} = \left(\frac{l_e}{R}\right)^2 x_\nu^2 \delta_{\nu\nu'} + \Gamma'_{\nu\nu'}$, $\Gamma'_{\nu\nu'}$ см. 24/III, это матрица $n\times n$.

→ $H_{res}(n) = \frac{\omega}{\gamma} + 4\pi M_s(1-\lambda_n)$, где λ_n - СЗ оператора $\hat{B}$, заданного матрицей $B_{\nu\nu'}$.

$h_1(\beta) = \int_0^\infty dx\, \delta(\beta x) J_0(\beta x) J_1(x)$, $\int_0^1 d\rho\rho\, J_0(x_\nu\rho) J_0(x_{\nu'}\rho) \int_0^\infty dx\, \delta(\beta x) J_0(\beta x) J_1(x)$ - вычислить.

$h_1(\beta)$ можно через эти функции: $\int_0^1 d\rho\rho\, J_0(x_\nu\rho)\int_0^1 d\rho'\rho'\, J_0(x_{\nu'}\rho')\int_0^\infty dx\, x\,\delta(\beta x) J_0(\rho x) J_0(\rho' x) = \int_0^\infty dx\, x\,\delta(\beta x) J_\nu(x) J_{\nu'}(x)$

$J_\nu(x) = \int_0^1 d\rho\rho\, J_0(x_\nu\rho) J_0(x\rho) = \frac{\rho}{(x_\nu^2 - x^2)}\left[x_\nu J_1(x_\nu\rho) J_0(x\rho) - x J_0(x_\nu\rho) J_1(x\rho)\right]_0^1 = \frac{x_\nu J_1(x_\nu) J_0(x) - x J_0(x_\nu) J_1(x)}{(x_\nu^2 - x^2)}$

Escaneado con CamScanner

$J_\nu(x) = \frac{1}{(x_\nu^2 - x^2)} \left[x_\nu J_1(x_\nu) J_0(x) - x J_0(x_\nu) J_1(x) \right]$, в.с. $x J_1(x) = d J_0(x)$ для $x = x_\nu$.

$y_\nu(x) = \frac{J_0(x_\nu)}{(x_\nu^2 - x^2)} \left[d J_0(x) - x J_1(x) \right]$, $J_\nu(x \to x_\nu)$ - finite из-за в.с.

$$\Gamma'_{\nu\nu'}(dyn) = \frac{1}{1} \frac{1}{\sqrt{N_\nu N_{\nu'}}} \int_0^\infty dx\, x\, f(\beta x) \frac{J_0(x_\nu) J_0(x_{\nu'})}{(x_\nu^2 - x^2)(x_{\nu'}^2 - x^2)} \left[d J_0(x) - x J_1(x) \right]^2$$

$$\Gamma'_{\nu\nu'}(dyn) = \frac{1}{2} \frac{J_0(x_\nu) J_0(x_{\nu'})}{\sqrt{N_\nu N_{\nu'}}} \int_0^\infty dx\, x\, f(\beta x) \frac{\left[d J_0(x) - x J_1(x) \right]^2}{(x_\nu^2 - x^2)(x_{\nu'}^2 - x^2)}$$

MathCad маш. элементы $B_{\nu\nu'}(h, g)$ считает очень медленно.

Введем $J_\nu(x) = \int_0^1 d\varrho\varrho\, J_0(x_\nu \varrho) J_0(\varrho x)$ явно в dyn. - $\Gamma'_{\nu\nu'}$. $\varphi_\nu(\varrho) = \frac{1}{\sqrt{N_\nu}} J_0(x_\nu \varrho)$

$$\Gamma'_{\nu\nu'} = \int d\varrho\varrho\, \varphi_\nu \varphi_{\nu'} h_1(\varrho) + \frac{1}{2} \int d\varrho\varrho \int d\varrho'\varrho' \varphi_\nu(\varrho) g(\varrho\varrho') \varphi_{\nu'}(\varrho') = (h_1)_{\nu\nu'} + \frac{1}{2\sqrt{N_\nu N_{\nu'}}} \int dx\, x f(\beta x) J_\nu(x) J_{\nu'}(x)$$

$\hat{B}\vec{a} = \lambda \vec{a}$, $\mu(\varrho) = \sum_\nu a_\nu \varphi_\nu(\varrho)$, $\vec{a} = (a_1, \dots, a_N)$. Зная СЗ λ_n, подставим их в $\hat{B}\vec{a} = \lambda\vec{a}$ находим $\vec{a}^{(n)}_\nu = \{a_\nu^{(n)}\}$ - сод. вектора и $\mu_n(\varrho) = \sum_\nu a_\nu^{(n)} \varphi_\nu(\varrho)$

File „Radial_sw_cyl_dot.mcd". Quan. racer при $R = 1000$ nm, $M_S = 1243$ G дает $H_z(8) = 15,88$ kOe с $l_e = 0$

Non-dig. racer $R = 1000$ nm дает $M_S = 1239$ G, $H_z(8) = 15,34$ kOe при $l_e = 0$ - или $H_z^{exp}(8) = 16,26$ kOe

Т.е. для $R = 1000$ nm non-diagonal racer не спасает $H_z(8)$, давая даже худшие results, чем diag.-racer (при $l_e = 0$, при l_e - finite $H_z^{calc}(8)$ еще меньше). Это при $B_{\nu\nu'}$, $N = 10$ (10×10)

Non-diagonal racer $R = 700$ nm ($j = 1$) $M_S = 1172$ G $H_z^{calc}(9) = 13,81$ kOe при $l_e = 0$

Quan.-racer $R = 700$ nm дает $M_S = 1182$ G $H_z^{calc}(9) = 13,33$ kOe $H_z^{exp}(9) = 14,76$ kOe

Т.е. для $R = 700$ nm заметно лучше, чем quan. racer ($l_e = 0$).

Non-diagonal racer $R = 500$ nm ($j = 2$): $M_S = 1120$ G, $H_z^{calc}(10) = 11,71$ kOe ($l_e = 0$), $H_z^{exp}(10) = 13,12$ kOe

Quan.-racer $R = 500$ nm дает $M_S = 1134$ G, $H_z^{calc}(10) = 11,75$ kOe ($l_e = 0$)

Для $R = 500$ nm diag.- и nondiag.-расчеты практически совпадают.

Non-diag racer для $R = 150$ nm: $M_S = 878$ G, $H_z^{calc}(4) = 7,88$ kOe дает $l_e = 9,93$ nm

Quan.-racer дает $M_S = 928$ G, $l_e = 10,4$ nm - близкие значения.

В MathCad хоть и задано $\nu = 1, \dots N$, $\nu' = 1, \dots N$, $\Gamma_{\nu\nu'}$, $B_{\nu\nu'}$ и мелат $\Gamma_{0\nu'}$, $\Gamma_{\nu 0}$, Γ_{00} elements. исправили индексы ν, ν' (i, j) так чтобы $\nu' = 0, 1, \dots N$, $\nu' = 0, 1, \dots N$

→ Исходно СЗ $\lambda = 0$, но $\det(\lambda_n)$ не в порядке возрастания и не убывания ряда.

Fitted M_S не изменились. $\mu(\varrho) = \sum_{\nu=0}^{N} a_\nu \frac{1}{\sqrt{N_\nu}} J_0(x_\nu \varrho)$, x_ν, $\nu = 0, 1, \dots N$ из $dJ_0(x) = x J_1(x)$

$N = 10$ - разумно, MathCad считает.

→ // при $\omega/\gamma = 3,177$ подгонять $H_{res}^{exp}(n)$ не выходит даже и non-diagonal racer.

$f = 9,25$ GHz, $\gamma/2\pi = 3,1$ GHz/kOe.

$H_{res}(n) = \frac{\omega}{\gamma} + 4\pi M_S(1 - \lambda_n)$, $\lambda_n \approx B_{nn} = \left(\frac{l_e}{R}\right)^2 x_n^2 + \Gamma'_{nn}$, $\lambda_n = F(\beta, 1/R^2)$

$x_n = x_n(\beta, L) \approx x_n(\beta)$

Dimensionless $\lambda_n = F(L, R, l_e)$, $\lambda_n \approx F(\beta)$ для R - разных ($R = 1000, 700, 500$ nm)



ΔH_{res} определяется M_S. M_S fitted из $H_{res}(1)$ и сильно зависит от ω/γ. (ω/γ) увел. → M_S снижается, ΔH_{res} ↓.

при $R = 1000$ nm $\gamma/2\pi = 2,9$ GHz/kOe, $\frac{\omega}{\gamma} = 3,397$ → $M_S = 1220$ G, $H_z^{calc}(8) = 15,32$ kOe, $H_z^{exp}(8) = 16,26$ kOe

Т.е. ($\gamma/2\pi$) уменьшение не спасает. Видимо при $R = 400 - 1000$ nm либо R, L другие, либо dot shape отличается от cylinder (variable thickness?)

Может есть uniaxial anisotropy, $H_\perp$ и $H_z \to H_z + H_\perp$, $H_z(n) = \frac{\omega}{\gamma} - H_\perp + 4\pi M_S(1 - \lambda_n)$

$H_\perp \approx -1$ kOe (easy plane) приведет ли к M_S ↓ fitted и ΔH_z ↓ - сужение спектра res. fields $\Delta H_z(n)$.

(26/III) Срочно: 1) ERC-project review; ~~2) Возврат в PRX-lett.~~; ~~3) Обзор-перевод~~

Обсужд. Skype с Глебом „D'yn modes" расчета. Сказ. ему что учитывая dot thicknesses, dot shape и нет ли $H_\perp$ ко. Итоге $R = 400 - 1000$ nm не объяснить на основе наших точных формул. Разобрался какие $\omega(H)$-ки использовались и кем для подгонки N.i/Rev в APL'2004 и почему они фразовано Разобрался почему $G(k)$ - eq. Slavin объясняет $H_{res}(n)$ Gue does for R. письма Oec. и Suess. Написал и послал Augustina/Oec. letter Editor draft не возвращая PRX. // Принял на рев. ERC project „3D solitons" - deadline May 5.

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Л. 4 посещения Киев: 3 ночи, 1 Л.Д. (ранно сосед) — 37 €.

Due dipolar Skyrmions $E_m = -E_0 p C$ (Tuna Bergamo), где DMI-Skyrmions $E_{DMI} = D_p C$, где p - Sk polarity, C - chirality. Dip. Skyrmion are chiral, как и DMI-Skyrmions, но chirality-dependence $E(\Phi)$ у них чуда magnetostatics, а не чуда DMI как у traditional Neel skyrmions in ultrathin films/dots.

(27/III) Confirm. letter of Гнева на COST travel. С Л. в Eroski Urbil. Solskyrmag'19 program updating. Бинц.

(28/III) Обучение SolSkyrmag gen (ann. Julian restaurant 100 € receipt). Julian introduced new group postdoc - Karin Garcia (согр. FERT по скирмионам) - nonfunded 3 years Ophelia Fellows stipend. Предложил ей m. simulations. Обучение с Ruben DW-pair nucleation initial stage via „gyrofield" в $\vec{m}$. Планы COST. Полусезд report и посещ Оссане. С Л. в „hidi" Olaberria.

(29/III) Л. поехала в Киев на ужас. зубов (30% доп. за имплантацию - тит., 5300 €) и 14 new коронок metalloceramics. С собой - 3.300 €.
Review new papers по Skyrmions. Разгрузка COST-report (see report + Glob letter) to COST-web system на утверждение Оссане, Бинц. Bank transfers c.card, soc.sec.

(30/III) Срочно: ~~1) email Ruben/Ricardo about DW-dyn. paper;~~ ~~2) MS art Skyrmions Verba.~~ 3) ERC grant help from UR/SKEL; 4) „num" modes calcs.; 5) HMM'19 Crete - fee, tickets; 6) SW - Skyrmion simul. Mruczkiewicz; 7) Qm-Sk зигза.

(31/III) переделка Sol-Skyrmag'19 program (Hillebrands, Suess). Бинц.

(1/IV) E-mails to Hillebrands у Tomasello (HMM'19 conf fee). DW dyn.

→ 1D DW dynamics MS: ~~1) Проверить нормализацию energy Eq. (5);~~ 2) $v \lessgtr v_-$, check that "+" in $\chi_\pm(v)$ corresponds to unstable DW (Walker-type?), а "-" corresp. „stable DW"; 3) Проверить достаточно ли $\Theta = \Psi$ точно при разных energy ϵ; 4) check Eq. (8) для Sine-Gordon; 5) убедиться в Skyrmion/Kivshar solution $\lambda \to \infty$ to the 1st integral of motion? 6) проверить $\epsilon(3)$ variable решение, $\epsilon_{DW}[\epsilon(3)] < \epsilon_{DW}[\text{const}]$

Комиссии заплатить HMM'19 conf. fee (275 €). IBAN не работает. Bank system „Santander" в/ведет DC(?) и IBAN отдельно. Riccardo Tomasello предложил заплатить Cash 300 € на конф.

(2/IV) E-mails to HMM'2019 Temnov, Ricardo (ESM'19). Оплата „Aranburu" 370,16 € за June 2018 - March 2019.

Second variational derivative, см. I.M. Gelfand, S.V. Fomin „Calculus of variations" Prentice-Hall, Inc. Englewood Cliffs, USA 1963 (достаточное условие minimum S)

$$J[\vec{y}(x)] = \int_a^b F(\vec{y}, \vec{y}_x, x)\,dx, \quad \vec{y} = (y_1, y_2, \dots y_n) \text{ n-functions of one variable } x$$

Если $y_1(x), y_2(x)$ удовл. L. Euler eqs. $\frac{\partial F}{\partial y_i} - \frac{d}{dx}\frac{\partial F}{\partial y_{ix}} = 0$ и matrix $\hat{P} = \frac{1}{2}\hat{F}_{y'y'}$ $\hat{F}_{y'y'} = (F_{y_i' y_j'})$ is positive definite, then $F[\vec{y}]$ has a weak minimum.

Сом. 18/III $\mathcal{L}(y, \Phi) = \frac{1}{ch^2 y}\left[v y_3 \Phi\right] - \frac{1}{ch^2 y}\left[y_3^2 + \Phi_3^2 + 1 + \lambda \cos^2\Phi\right]$ - лагранжиан $\operatorname{tg}\frac{\theta}{2} = e^y$, $y(\xi), \Phi(\xi), \xi = x - vt$.

$$J_{kin} = \frac{1}{ch^2 y} v y_3 \Phi = -\dot{y}\Phi \sin\theta = \left|\dot{y} = \frac{1}{\sin\theta}\dot{\theta}\right| = -\Phi\dot{\theta}\sin\theta = -\cos\theta\dot{\Phi}$$

$$\frac{y_3^2}{ch^2 y} = y_3^2\left(\frac{1}{\sin^2\theta}\right)^{-1} = \left(\frac{1}{\sin^2\theta}\right)^{-1}\frac{1}{\sin^2\theta}\theta_3^2 = \theta_3^2 \to \mathcal{L}(\theta, \Phi) = -\Phi\dot{\theta}\sin\theta - \left[\theta_3^2 + \sin^2\theta(\Phi_3^2) + \sin^2\theta(1 + \lambda\cos^2\Phi)\right]$$

$$J_{kin} = v\Phi\, y_3 \sin^2\theta = v\Phi\theta_3 \sin\theta = v\Phi_3 \cos\theta$$

$$\frac{\partial \mathcal{L}}{\partial \theta} + \frac{\partial}{\partial \xi_3}\frac{\partial \mathcal{L}}{\partial \theta_3} = 0 \text{ и.е. } \frac{\partial \mathcal{L}}{\partial \theta} - \frac{\partial}{\partial x}\frac{\partial \mathcal{L}}{\partial \theta_x} - \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \theta_t} = 0, \quad \frac{\partial \mathcal{L}}{\partial \Phi} - \frac{\partial}{\partial \xi_3}\frac{\partial \mathcal{L}}{\partial \Phi_3} = 0$$

$$\mathcal{L}(\theta, \Phi) = v\Phi\sin\theta\,\theta_3 - \frac{1}{2}\left[\theta_3^2 + \sin^2\theta(\Phi_3^2) + \sin^2\theta(1 + \lambda\cos^2\Phi)\right], \quad \dot{\Phi} = -v\Phi_3$$

$$0 = -v\sin\theta\Phi_3 - \frac{\partial}{\partial \xi_3}\frac{\partial \mathcal{L}}{\partial \theta_3} - \frac{\partial}{\partial \theta}\left[\sin^2\theta\left(\Phi_3^2 + 1 + \lambda\cos^2\Phi\right)\right], \quad v\sin\theta\Phi_3 = 2\theta_{3\xi_3} - \frac{\partial}{\partial\theta}\sin^2\theta\left[\Phi_3^2 + 1 + \dots\right]$$

$$0 = v\sin\theta\theta_3 - \sin^2\theta\left[\frac{\partial}{\partial\theta}\cos^2\Phi + \frac{\partial}{\partial\xi_3}\sin^2\theta\cdot 2\Phi_3\right], \quad -v\sin\theta\theta_3 = -\lambda\sin^2\theta\frac{\partial\cos^2\Phi}{\partial\Phi} + 2\frac{\partial}{\partial\xi_3}\left(\sin^2\theta\Phi_3\right)$$

$$-v\sin\theta\,\theta_3 = 2\lambda\sin^2\theta\sin\Phi\cos\Phi + 2\left[\sin^2\theta\Phi_{3\xi_3} + \Phi_3 2\sin\theta\cos\theta\Phi_3\right]. \text{ Вычт. надо } \times\frac{1}{2}$$

energy part of $\mathcal{L}$: $\mathcal{L}(\theta, \Phi) = v\Phi\sin\theta\theta_3 - \frac{1}{2}\left[\theta_3^2 + \sin^2\theta(\Phi_3)^2 + \sin^2\theta(1 + \lambda\cos^2\Phi)\right]$

$$\mathcal{L}(\theta, \Phi) = J_{kin} - \epsilon(\theta, \Phi), \quad \epsilon(\theta, \Phi) = \frac{1}{2}\left[\theta_3^2 + \sin^2\theta(\Phi_3)^2 + \sin^2\theta(1 + \lambda\cos^2\Phi)\right]$$

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$$\mathcal{L} = -\frac{M_s}{\gamma}\cos\Phi\dot{\Phi} - \left[A\left(\theta_x^2 + \sin^2\theta\,(\Phi_x)^2\right) + K_z\sin^2\theta + K_x\sin^2\theta\cos^2\Phi\right]$$

$$\frac{1}{2K_z}\mathcal{L}(\theta,\Phi) = -\frac{M_s}{2\gamma K_z}\cos\theta\dot{\Phi} - \frac{1}{2}\left[\frac{A}{K_z}\left(\theta_x^2+\sin^2\theta\Phi_x^2\right) + (1+\lambda\cos^2\Phi)\sin^2\theta\right],\quad \frac{A}{K_z} = \Delta_0^2$$

$$\frac{1}{2K_z}\mathcal{L}(\theta,\Phi) = -\frac{1}{\gamma H_a}\cos\theta\dot{\Phi} - \frac{1}{2}\left[\Delta_0^2\left(\theta_x^2+\sin^2\theta\Phi_x^2\right) + (1+\lambda\cos^2\Phi)\sin^2\theta\right],\quad \gamma H_a = t_0^{-1},\ \Delta_0\frac{\partial}{\partial x}\to\frac{\partial}{\partial x}$$

$$\frac{1}{2K_z}\mathcal{L}(\theta,\Phi) = -\cos\theta\dot{\Phi} - \frac{1}{2}\left[\theta_x^2+\sin^2\theta\Phi_x^2 + \sin^2\theta(1+\lambda\cos^2\Phi)\right],\quad t_0\frac{\partial}{\partial t}\to\frac{\partial}{\partial t}$$

$$\frac{1}{2K_z}\mathcal{L}(\theta,\Phi) \to \mathcal{L}(\theta,\Phi) = -\cos\theta\dot{\Phi} - \frac{1}{2}\left[\theta_x^2+\sin^2\theta\Phi_x^2+\sin^2\theta(1+\lambda\cos^2\Phi)\right]$$

e (в ед $2K_z$) $= \frac{1}{2}\left[\theta_x^2+\sin^2\theta\Phi_x^2+\sin^2\theta(1+\lambda\cos^2\Phi)\right]$, $e_{(\text{density})} = 2K_z\cdot e_{(\text{dimensionless})}$

$E_{DW} = \int_{-\infty}^{\infty} dx\, e_{DW} = \Delta_0\int dx\, e$ в ед. $\Delta_0\cdot 2K_z = 2\sqrt{\frac{A}{K_z}}K_z = 2\sqrt{AK_z}$ $E = \int dx\, e$ в ед. $2K_z\Delta_0$

$E_{DW} = 2E_{DW}^{ex} = 2\cdot(\Delta_0)^2\int dx\, e_{ex} = 2\int dx\,\frac{1}{2}\theta_x^2 = \frac{1}{2}\int dx\,(\sin\theta)^2 = \frac{1}{2}\int dx\,\frac{1}{ch^2 y} = \int dx\,\frac{1}{ch^2 x} = 2$

Сом. 1 st integral $\theta_x^2+\sin^2\theta\Phi_x^2 = \sin^2\theta(1+\lambda\cos^2\Phi)$ или $e = e_{ex}+e_a$, $e_{ex} = e_a$

$e = 2e_{ex}$, $E_{DW}^{ex} = \int dx\, e_{ex}$, $e_{ex} = \frac{1}{2}\left[\theta_x^2+\sin^2\theta\Phi_x^2\right]$, $E_{DW} = 2E_{DW}^{ex}$.

Для Walker solution $\Phi_x = 0$, получим $E_{DW} = 2$, $E_{DW} = 2\cdot 2\sqrt{AK_z} = 4\sqrt{AK_z}$ в ед. $2K_z\Delta_0$

(стационарная DW $v=0$), $E_{DW}(v=0) = E_0 = 4\sqrt{AK_z}$, or $E_0 = 2$ in Δ_0, $2K_z$ units. ascon. eg.

Energy density $\left(\frac{erg}{cm^3}\right)$ $e(x)$ в ед. $2K_z$, energy per unit area $E = \int dx\, e$ in units $2K_z\Delta_0$. $\left(\frac{erg}{cm^2}\right)$

Lagrangian density $\mathcal{L} = -\cos\theta\dot{\Phi} - \frac{1}{2}\left[\theta_x^2+\sin^2\theta\Phi_x^2+\sin^2\theta(1+\lambda\cos^2\Phi)\right]$, $\xi = x - vt$

$\mathcal{L}(\theta(\xi),\Phi(\xi)) = \mathcal{L}(\xi) = v\cos\theta\Phi_\xi - \frac{1}{2}\left[\theta_\xi^2+\sin^2\theta\Phi_\xi^2+\sin^2\theta(1+\lambda\cos^2\Phi)\right]$ — in units $2K_z$

$(v\Phi\sin\theta\theta_\xi)$

Action $S = \int d\xi\,\mathcal{L}(\xi) = \int_{\xi_1}^{\xi_2}\mathcal{L}(\theta,\theta_\xi,\Phi,\Phi_\xi) = S[\theta,\Phi]$, $\min S \to$ LLG for $\theta(\xi),\Phi(\xi)$

$\mathcal{L}_{\theta_\xi\theta_\xi} = \frac{\partial^2\mathcal{L}}{\partial\theta_\xi^2} = \frac{\partial^2}{\partial\theta_\xi^2}\left(-\frac{1}{2}\theta_\xi^2\right) = -1$, $\mathcal{L}_{\Phi_\xi\Phi_\xi} = \frac{\partial^2\mathcal{L}}{\partial\Phi_\xi^2} = -\sin^2\theta$, $\mathcal{L}_{\theta_\xi\Phi_\xi} = \frac{\partial^2\mathcal{L}}{\partial\theta_\xi\partial\Phi_\xi} = 0$

$\hat{P} = \frac{1}{2}\left(\mathcal{L}_{y_i'y_j'}\right) = \frac{1}{2}\begin{pmatrix}-1 & 0\\ 0 & -\sin^2\theta\end{pmatrix}$ явно отрицательно определена (?).

Если $\Phi(\xi) = \Phi_0 = const$ $\mathcal{L}(\theta) = v\Phi\sin\theta\theta_\xi - \frac{1}{2}\left[\theta_\xi^2+\sin^2\theta(1+\lambda\cos^2\Phi_0)\right]$

$\hat{P} = \frac{1}{2}\mathcal{L}_{\theta_\xi\theta_\xi} = \frac{1}{2}\frac{\partial^2}{\partial\theta_\xi^2}\left(-\frac{1}{2}\theta_\xi^2\right) = -1/2$, т.е. any $\Phi(\xi) = \Phi_0$ solution - это Walker

maximum of action? применим ли принцип $\min S$ к field-Lagrangians

$\mathcal{L} = \mathcal{L}(\theta,\Phi,\theta_\xi,\Phi_\xi)$? может надо $\min E$?

$e(\theta,\Phi) = \frac{1}{2}\left[\theta_\xi^2+\sin^2\Phi_\xi^2+\sin^2\theta(1+\lambda\cos^2\Phi)\right]$, $E = \int d\xi\, e$, $\min E$ имеем

$\hat{P} = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & \sin^2\theta\end{pmatrix}$ is positive definite, т.е. любые $\theta(\xi),\Phi(\xi)$ удовлетвор.

(3/IV) $\frac{\partial e}{\partial\theta} - \frac{\partial}{\partial\xi}\frac{\partial e}{\partial\theta_\xi} = 0$, $\frac{\partial e}{\partial\Phi} - \frac{\partial}{\partial\xi}\frac{\partial e}{\partial\Phi_\xi} = 0$ дают $\min E = \int d\xi\, e(\xi)$. т.е. Neel-DW at $\xi\to\pm\infty$ is metastable.

Max $S = \int d\xi\,\mathcal{L}(\xi) = L$ не сущ-ет! Schlömann 1972 тем и нужен $\zeta = x - vt$

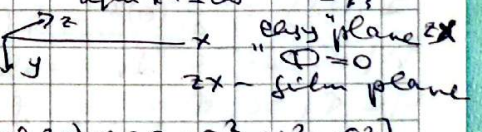
„Maximizing the Lagrangian $L = T - E$ ..." $\mathcal{L} = T - e$, $T = \int d\xi\left[-v(\cos\theta)_\xi'\Phi\right]$

$T_1 = -\frac{M_s}{\gamma}v\frac{\partial}{\partial x}(\cos\theta)\Phi = \frac{M_s}{\gamma}v\sin\theta\theta_\xi\Phi = -\frac{M_s}{\gamma}\Phi\sin\theta\dot{\theta}$ или

$T_2 = -\frac{M_s}{\gamma}v\frac{\partial}{\partial\xi}(\cos\theta)\Phi = \frac{M_s}{\gamma}v\cos\theta\Phi_\xi = -\frac{M_s}{\gamma}\cos\theta\dot{\Phi}$. T_1 нужно, т.к. $\frac{\partial\mathcal{L}}{\partial\theta_\xi}\neq 0$ при $x\to\pm\infty$

$e(\theta,\Phi) = \frac{1}{2}M_s\left[(H_a+4\pi M_s\sin^2\Phi)\sin^2\theta + D(\nabla\vec{m})^2\right]$

$e(\theta,\Phi) = K_z\sin^2\theta + K_y\sin^2\Phi\sin^2\theta + A(\nabla\vec{m})^2$



$T = \int d\xi\, T = -\frac{M_s}{\gamma}v\int d\xi\,\frac{\partial}{\partial\xi}(\cos\theta)\Phi$, $E = \frac{1}{2}\int d\xi\left[(1+\lambda\sin^2\Phi)\sin^2\theta + \theta_\xi^2 + \sin^2\theta\,\Phi_\xi^2\right]$

$\alpha = 4\pi M_s/H_a = K_y/K_z$, $\alpha \gg 1$ может быть.

Очевидно, что для Walker solution $\Phi(\xi) = \Phi_0 = const$, $T = -\frac{M_s}{\gamma}v\Phi_0\cos\theta\Big|_{-\infty}^{+\infty} = \frac{M_s}{\gamma}2\Phi_0 v$

$L_{(Walker)} = \frac{M_s}{\gamma}2v\Phi_0 - \frac{1}{2}\int d\xi\left[\theta_\xi^2 + (1+\lambda\sin^2\Phi_0)\sin^2\theta\right]$, $\sin\theta = \frac{1}{ch(a\xi)}$, $\cos\theta = -th(a\xi)$, $\theta_\xi > 0$

где $a = \frac{1}{\Delta} = 1+\lambda\sin^2\Phi_0$, $\sqrt{a} = \lambda\sin\Phi_0\cos\Phi_0$.

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1D-DW

4/IV Написание 1D-DW paper. Проверка eqs. сам. Надо $e(\theta,\Phi)=\frac{1}{2}\left[(\theta_x)^2+\dots\right]$.

$\operatorname{tg}\frac{\theta}{2}=e^y$, $\theta=2\operatorname{arctg}e^y$, $\theta_y=\frac{1}{\operatorname{ch}y}=\sin\theta$, $\theta_{yy}=\sin\theta\cos\theta=\frac{1}{2}\sin 2\theta$

переход к sin-Gordon ($\lambda\to\infty$) при $\varepsilon=\text{const}$, $v_x=\varepsilon\sin\psi\ll 1$:

$\Phi_\xi=0\to v y_\xi=\lambda\sin\Phi_0\cos\Phi_0$, $1+\lambda\cos^2\Phi_0=(y_\xi)^2$. Если $\Phi_0=\frac{\pi}{2}-\varepsilon$, $\varepsilon\to 0$

$v y_\xi=\lambda\sin\left(\frac{\pi}{2}-\varepsilon\right)\cos\left(\frac{\pi}{2}-\varepsilon\right)=\lambda\cos\varepsilon\sin\varepsilon=\lambda\left[\varepsilon-\frac{1}{3!}\varepsilon^3\right]\left[1-\frac{1}{2}\varepsilon^2\right]=\lambda\varepsilon+O(\lambda\varepsilon^3)$

При этом $v y_\xi=\lambda\varepsilon$. $\lambda\cos^2\Phi_0+1=\lambda\left[\varepsilon-\frac{1}{3!}\varepsilon^3\right]^2+1=\lambda\varepsilon^2+O(\lambda\varepsilon^4)+1=(y_\xi)^2$

при $\lambda\cos^2\Phi_0=\lambda\varepsilon^2$, что $\lambda\varepsilon^4$ и не мало при $v\to c$.

$y_\xi=\frac{\lambda}{v}\varepsilon$, $1+\lambda\varepsilon^2=(y_\xi)^2\to\varepsilon=\frac{v}{\lambda}\frac{1}{\Delta'(v)}$, $\Delta'(v)=\sqrt{1-v^2/c^2}$, $c=\sqrt{\lambda}$.

$y_\xi=\frac{\lambda}{v}\varepsilon=\frac{\lambda}{v}\frac{v}{\lambda}\frac{1}{\Delta'}=\frac{1}{\Delta'}$, $y=\frac{\xi}{\Delta'}$. Решение $\Phi_\xi=0$ (Walker) всегда, даже $y=\frac{\xi}{\Delta(v)}$,

но частное реш. Walker $\Phi_\xi=0$, $\Phi=\Phi_0=\pi/2+\varepsilon$ при $\varepsilon\ll 1$ даст $y=\frac{\xi}{\Delta'(v)}$, с частным

$\Delta'(v)=\sqrt{1-v^2/c^2}$. Это с учетом 1st integral $y_\xi^2+\Phi_\xi^2=1+\lambda\cos^2\Phi$, который при $\Phi_\xi=0$

дает $y_\xi^2=1+\lambda\cos^2\Phi$ и $-v\Phi_\xi=y_{\xi\xi}+\left[1+\lambda\cos^2\Phi+(\Phi_\xi)^2-(y_\xi)^2\right]\operatorname{th}y\to y_{\xi\xi}=0$, $y_\xi=\text{const}$

$\Phi_\xi=0\to y=\xi/\Delta$. $\theta(\xi)=2\operatorname{arctg}e^{\xi/\Delta}$, $\theta_y=\sin\theta$, $\theta_\xi=\theta_y\frac{dy}{d\xi}=\frac{1}{\Delta}\theta_y$, $\theta_{\xi\xi}=\frac{1}{\Delta^2}\theta_{yy}$

$\theta_{yy}=\sin\theta\cos\theta\to\theta_{\xi\xi}=\frac{1}{\Delta^2}\sin\theta\cos\theta$ — эквивалентно ли это sin Gordon для $\theta(x,t)$?

$\theta(x,t)=\theta(\xi)=\theta(x-vt)$, θ_{xx} из $\theta_x=\theta_\xi\frac{d\xi}{dx}=\theta_\xi$, $\theta_{xx}=\frac{d}{dx}\theta_\xi=\theta_{\xi\xi}\frac{d\xi}{dx}=\theta_{\xi\xi}$

$\theta_t=\theta_\xi\frac{d\xi}{dt}=-v\theta_\xi$, $\theta_{tt}=\frac{d}{dt}(-v)\theta_\xi(\xi)=-v\theta_{\xi\xi}\frac{d\xi}{dt}=(-v)(-v)\theta_{\xi\xi}=v^2\theta_{\xi\xi}$

$\theta_{xx}-\frac{1}{c^2}\theta_{tt}=\theta_{\xi\xi}-\frac{1}{c^2}v^2\theta_{\xi\xi}=\left(1-\frac{v^2}{c^2}\right)\theta_{\xi\xi}=\left(1-\frac{v^2}{c^2}\right)\frac{1}{\Delta^2}\theta_{yy}=\frac{1}{\Delta^2}\left(1-\frac{v^2}{c^2}\right)\frac{1}{2}\sin 2\theta$

т.е. для любого $\theta(\xi)=2\operatorname{arctg}e^{\frac{\xi}{\Delta}}$, $y=\frac{\xi}{\Delta}$ получим $\theta_{xx}-\frac{1}{c^2}\theta_{tt}=\frac{1}{\Delta^2}\left(1-\frac{v^2}{c^2}\right)\frac{1}{2}\sin 2\theta$ — SG

$\Delta_e=\frac{\Delta}{\sqrt{1-v^2/c^2}}$, $\theta_{xx}-\frac{1}{c^2}\theta_{tt}\Rightarrow\theta_{xx}-\frac{1}{v_m^2}\theta_{tt}=\frac{1}{\Delta_e^2}\frac{1}{2}\sin 2\theta$, $\Delta_e=\frac{\Delta}{\sqrt{1-v^2/v_m^2}}$ — SG

$\theta(x,t)=\theta(\xi)$

v_m не обязательно $=c=\sqrt{\lambda}$, но $y=\xi/\Delta$, $\theta=2\operatorname{arctg}e^y$.

$\mathcal{L}(\theta,\Phi)=v\Phi\sin\theta\theta_\xi-\frac{1}{2}\left[\theta_\xi^2+\sin^2\theta\Phi_\xi^2+\sin^2\theta(1+\lambda\cos^2\Phi)\right]=\mathcal{L}(\xi)$

Случай $\Phi_\xi=0$, $\Phi=\Phi_0\to\mathcal{L}(\theta,\Phi_0)=v\Phi_0\sin\theta\theta_\xi-\frac{1}{2}\left[\theta_\xi^2+\sin^2\theta(1+\lambda\cos^2\Phi)\right]$

учтем 1st integral $\theta_\xi^2+\sin^2\theta(\Phi_\xi)^2=\sin^2\theta(1+\lambda\cos^2\Phi)\to\mathcal{L}(\theta,\Phi_0)=v\Phi_0\sin\theta\theta_\xi-\frac{1}{2}\theta_\xi^2$

$\mathcal{L}(\theta,\Phi_0)=v\Phi_0\sin\theta\theta_\xi-\theta_\xi^2$, $\theta_\xi=\frac{d\theta(\xi)}{d\xi}$. $\theta_y=\sin\theta$, $\frac{d\theta}{d\xi}=\frac{d\theta}{dy}\frac{dy}{d\xi}=\frac{1}{\Delta}\sin\theta$, $y=\frac{\xi}{\Delta}$,

$\mathcal{L}(\theta,\Phi_0)=(\Delta v\Phi_0-1)\theta_\xi^2=\frac{v}{\Delta}\Phi_0\sin^2\theta-\theta_\xi^2$ — вариационная на решении $\theta(\xi)=2\operatorname{arctg}e^{\xi/\Delta}$, $y=\frac{\xi}{\Delta}$

частный случай (*) $v_m=c$, $\Delta=\Delta'=\sqrt{1-v^2/c^2}$, $\Delta_e=1$ при $\Phi_0=\frac{\pi}{2}-\varepsilon$, $\varepsilon\to 0$ $\Rightarrow\theta_{xx}-\frac{1}{c^2}\theta_{tt}=\frac{1}{2}\sin 2\theta$.

$v\theta_\xi=-\dot\theta$, $\mathcal{L}(\theta,\Phi_0)=-\Phi_0\sin\theta\dot\theta-\theta_x^2$, $\dot\theta=-v\theta_\xi$, $\sin\theta=\theta_y$

$\dot\theta=-v\theta_\xi=-\frac{v}{\Delta}\sin\theta$. Из (θ,Φ)-ур при $\Phi=\Phi_0=\text{const}$ $\sin\theta\dot\theta=\lambda\sin^2\theta\sin\Phi_0\cos\Phi_0$

$\dot\theta=\lambda\sin\Phi_0\cos\Phi_0\sin\theta$, $\lambda\sin\Phi_0\cos\Phi_0=v y_\xi=\frac{v}{\Delta}$ и $\dot\theta=\frac{v}{\Delta}\sin\theta$ — имеет ли физический смысл?

$\Phi_0=\frac{\pi}{2}-\varepsilon$, $\mathcal{L}(\theta,\varepsilon)=-\left(\frac{\pi}{2}-\varepsilon\right)\sin\theta\dot\theta-\theta_x^2$, $\dot\theta=\lambda\sin\Phi_0\cos\Phi_0=\lambda\varepsilon\sin\theta$

$\mathcal{L}_{kin}(\theta,\varepsilon)=-\left(\frac{\pi}{2}-\varepsilon\right)\sin\theta\dot\theta=\varepsilon\sin\theta\dot\theta=\varepsilon\frac{1}{\lambda\varepsilon}\dot\theta\dot\theta=\frac{1}{\lambda}(\dot\theta)^2$, что только при $\varepsilon=\varepsilon(\xi)$

иначе $\sin\theta\dot\theta=\frac{d}{dt}(\cos\theta)\to 0$

$e(\theta,\varepsilon)=\frac{1}{2}\left[\theta_x^2+\sin^2\theta(\xi_x)^2+\sin^2\theta(1+\lambda\varepsilon^2)\right]=\sin^2\theta(1+\lambda\varepsilon^2)=\sin^2\theta(y_\xi)^2=\frac{1}{\Delta'^2}\sin^2\theta$

т.е. при $\Phi(\xi)=\Phi_0=\pi/2-\varepsilon$ $\mathcal{L}(\theta,\varepsilon)=\frac{1}{\lambda}(\dot\theta)^2-e(\theta,\varepsilon)=\frac{1}{c^2}(\dot\theta)^2-\frac{1}{\Delta'^2}\sin^2\theta$

или $e(\theta,\xi)=\left|\varepsilon_\xi^2\to 0\right|=\frac{1}{2}\theta_x^2+\frac{1}{2}\sin^2\theta(1+\lambda\varepsilon^2)=\frac{1}{2}(\theta_+)^2+\frac{1}{2\Delta'^2}\sin^2\theta$

$\mathcal{L}(\theta,\varepsilon)=\frac{1}{c^2}(\dot\theta)^2-\frac{1}{2}(\theta_x)^2-\frac{1}{2\Delta'^2}\sin^2\theta$ отличается от SG-Lagrangian.

Проблема с $\mathcal{L}_{kin}=v\Phi\sin\theta\theta_\xi=-\Phi\sin\theta\dot\theta$ или $\mathcal{L}_{kin}=-\cos\theta\dot\Phi=v\cos\theta\Phi_\xi$

при $\Phi_\xi=0$, $\Phi=\Phi_0$, $\mathcal{L}_{kin}(\theta_0^\xi,\Phi_0)=0$ с точностью до полной t-производной.

Поэтому Lagrangian с-нибудь другие не имеет, $\mathcal{L}(\theta,\varepsilon)$ из Eq. (6'), Eq (8) другие не писать.

Если $\varepsilon(\xi)=\varepsilon=\text{const}$ то нет $\mathcal{L}_{kin}=\frac{1}{\lambda}(\dot\theta)^2$. Если $\varepsilon(\xi)$, то $e(\theta,\varepsilon)$ неположена (не дает SG) из-за $\varepsilon_\xi\neq 0$

$e(\theta,\Phi_0)=\frac{1}{2}\left[\theta_\xi^2+\sin^2\theta(1+\lambda\cos^2\Phi_0)\right]$, $1+\lambda\cos^2\Phi_0=y_\xi^2=1/\Delta^2\to e(\theta,\Phi_0)=\frac{1}{2}\left[\theta_\xi^2+\frac{1}{\Delta^2}\sin^2\theta\right]$

в 1st integral $\theta_\xi^2=\sin^2\theta(1+\lambda\cos^2\Phi_0)=\frac{1}{\Delta^2}\sin^2\theta$, $\theta_\xi=\frac{1}{\Delta}\sin\theta$, что верно для $\frac{\theta}{2}=e^y$

и $\Phi(\xi)=\Phi_0$ case с $y=\xi/\Delta$.

Escaneado con CamScanner

$\mathcal{L}(\theta,\Phi_0) = -\Phi_0 \sin\theta\,\dot\theta - \frac{1}{2}\left[(\theta_x)^2 + \frac{1}{\Delta^2}\sin^2\theta\right]$ при $\dot\theta = -\frac{v}{\Delta}\sin\theta$

где $\mathcal{L}_{kin}(\theta,\Phi_0) = -\Phi_0\dot\theta\,(-1)\frac{\Delta}{v}\dot\theta = \frac{\Phi_0\Delta}{v}(\dot\theta)^2$ формально. $\mathcal{L}(\theta,\Phi_0) = \frac{\Phi_0\Delta}{v}(\dot\theta)^2 - \frac{1}{2}\left[\theta_x^2 + \frac{\sin^2\theta}{\Delta^2}\right]$

$\frac{\partial}{\partial t}\frac{\partial\mathcal{L}}{\partial\dot\theta} + \frac{\partial}{\partial x}\frac{\partial\mathcal{L}}{\partial\theta_x} = \frac{\partial\mathcal{L}}{\partial\theta}$, $\frac{2\Phi_0\Delta}{v}\ddot\theta - \theta_{xx} = -\frac{1}{\Delta^2}\sin\theta\cos\theta$ из $\mathcal{L}(\theta,\Phi_0)$.

Тогда как из LLG $\frac{1}{v_m^2}\theta_{tt} - \theta_{xx} = -\frac{1}{\Delta_e^2}\sin\theta\cos\theta$, уравнение $\theta(\xi) = 2\arctg e^{\xi/\Delta}$.

Далее $v_m^2 = \frac{v}{2\Phi_0\Delta}$? Нет. Можно $\dot\theta = \lambda\sin\Phi_0\cos\Phi_0\sin\theta$ и $\mathcal{L}_{kin} = -\Phi_0\dot\theta\frac{\dot\theta}{\lambda\sin\Phi_0\cos\Phi_0}$

$\mathcal{L}_{kin} = -\frac{1}{\lambda}\frac{\Phi_0}{\sin\Phi_0\cos\Phi_0}(\dot\theta)^2$, $\mathcal{L}(\theta,\Phi_0) = -\frac{1}{\lambda}\frac{\Phi_0}{\sin\Phi_0\cos\Phi_0}(\dot\theta)^2 - \frac{1}{2}\left[\theta_x^2 + \frac{\sin^2\theta}{\Delta^2}\right]$

Но из $\mathcal{L}(\theta,\Phi_0)$ ничего ... не те. Но при $\Phi_0 = \frac{\pi}{2} - \varepsilon$, $\sin\Phi_0\cos\Phi_0 \approx \varepsilon$

$\mathcal{L}_{kin} = -\frac{1}{\lambda}\frac{(\pi/2 - \varepsilon)}{\varepsilon}(\dot\theta)^2 = \frac{1}{\lambda}(\dot\theta)^2$ если вообразить член $-\frac{1}{\lambda}\frac{\pi}{2\varepsilon}(\dot\theta)^2$.

Итого $\mathcal{L}_{kin} = -\Phi_0\sin\theta\dot\theta = -(\frac{\pi}{2} - \varepsilon)\sin\theta\dot\theta = \varepsilon\sin\theta\dot\theta = \varepsilon\frac{\dot\theta}{\lambda\sin\Phi_0\cos\Phi_0}\dot\theta = \frac{1}{\lambda}(\dot\theta)^2$

Но $e(\theta,\varepsilon) = \frac{1}{2}\left[\theta_x^2 + \frac{1}{\Delta^2}\sin^2\theta\right] \to \frac{\partial\mathcal{L}}{\partial\theta} = -\frac{\partial e}{\partial\theta} = -\frac{1}{2}\frac{1}{\Delta^2}2\sin\theta\cos\theta$

$\mathcal{L}(\theta,\varepsilon) = \frac{1}{\lambda}(\dot\theta)^2 - \frac{1}{2}\theta_x^2 - \frac{1}{2\Delta^2}\sin^2\theta$ → $\frac{2}{\lambda}\ddot\theta - \theta_{xx} = -\frac{1}{\Delta^2}\sin\theta\cos\theta$, $\lambda = c^2$,

коэффициент при $\ddot\theta$ должен быть $\frac{1}{\lambda} = \frac{1}{c^2}$.

→ Раньше $e(\theta,\Phi)$ была (неверно) ξ_3 Y_2 и $\frac{2}{\lambda}\ddot\theta - 2\theta_{xx} = -\frac{2}{\Delta^2}\sin\theta\cos\theta$, действительно $\mathcal{L}(\theta,\varepsilon)$ давал Sin-Gordon $\frac{1}{c^2}\theta_{tt} - \theta_{xx} = -\frac{1}{\Delta^2}\sin\theta\cos\theta$, $\Delta\to\Delta'$,

$\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{2\Delta'^2}\sin 2\theta$. Однако оба Sin-Gordon были неверны, надо было Δ'.

Ведь в наш Sin-Gordon $\Phi_0 = \text{const}$, $\Phi_0 = \pi/2 - \varepsilon$ (в теории) $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{2}\sin 2\theta$, $\Delta_e = 1$, Δ' нет.

(*) Это все для $\Phi_0 = \text{const}$, включая $\Phi_0 = \pi/2 - \varepsilon$.

Для $\varepsilon = \varepsilon(\xi)$ надо все исследовать заново, может $\mathcal{L}(\theta,\varepsilon(\xi))$ безмонтно построить.

Проверим DW energy at $\lambda\to\infty$. $e(\theta,\varepsilon) = \frac{1}{2}\left[\theta_\xi^2 + \frac{1}{\Delta^2}\sin^2\theta\right] = \theta_\xi^2$

$E_{DW} = \int d\xi\, e(\theta,\varepsilon) = \int d\xi(\theta_\xi)^2 = \int d\xi\left[\frac{1}{\Delta'}\sin\theta\right]^2 = \frac{1}{\Delta'^2}\int_{-\infty}^{\infty} d\xi\frac{1}{ch^2\xi/\Delta'} = \frac{1}{\Delta'}\int_{-\infty}^{\infty}dx\frac{1}{ch^2x} = \frac{2}{\Delta'}$

$E_{DW}(v) = \frac{2}{\Delta'(v)} = \frac{2}{\sqrt{1-v^2/c^2}} = \frac{E_0}{\sqrt{1-v^2/c^2}}$ — верно, $E_0 = 2$ in units $2K_2\Delta_0$

Как классифицировать решение Sklyanin'79? Это $\Phi(\xi)$, $\Phi(\xi) = \text{const}$ или еще что-то?

(5/iv) A. de Laire et al. Ann. I. H. Poincaré – AN 35 (2018), 1885–1945 Sin-Gordon, см. 3/III

$\dot{\vec m} + \vec m\times(\Delta\vec m - \hat J(\vec m)) = 0$, $\hat J = \text{diag}(J_1, J_2, J_3)$ $E_{LL}(\vec m) = \frac{1}{2}\int dR\left[|\nabla m|^2 + \lambda_1 m_1^2 + \lambda_3 m_3^2\right]$

$0 < \lambda_1 \ll 1$, $\lambda_3 \gg 1$, $\lambda_3 > 0$ - easy plane $(x_1 - x_2)$, $\lambda_1 > 0$ → hard axis $\hat x_1$ (надо easy axis x_1 или x_2 как у Sklyanin'79, Kivshar'89).

$\lambda_1 = 2\varepsilon$, $\lambda_3 = \frac{1}{\varepsilon}$, $\varepsilon\to 0$. Решение $\vec m = \sqrt{1-m_3^2}[\sin\varphi + i\cos\varphi]$, $u = m_3 \ll 1$ $m = m_1 + im_2$

$u(x,t) = \varepsilon u_\varepsilon(\sqrt{\varepsilon}x, t)$, $\varphi(x,t) = \Phi_\varepsilon(\sqrt{\varepsilon}x, t)$ ansatz подставь в LLG для (m_3, φ)

и $\varepsilon\to 0$ дает $\dot u = \Delta\psi - \frac{2}{2}\sin 2\psi$, $\dot\psi = u$, $\ddot\psi - \Delta\psi + \frac{2}{2}\sin 2\psi = 0$ Sin-Gordon для $\psi(x,t) = \Phi_{\varepsilon\to 0}(\sqrt{\varepsilon}x, t)$. $\psi_{xx} - \psi_{tt} = \frac{2}{2}\sin 2\psi$, здесь $\psi_{xx}\to\psi_{yy}$, $y = \frac{1}{\sqrt{R}}x$.

$\frac{\lambda_3}{\lambda_1} = \frac{1}{\varepsilon}(2\varepsilon)^{-1} = \frac{1}{2}\frac{1}{\varepsilon^2}$ Cf. Sklyanin $\frac{K_x}{K_z} = \frac{R}{2}\frac{2R}{\gamma} = \frac{R^2}{\gamma} \gg 1$ при $R = \frac{1}{\varepsilon}$. надо $y = \sqrt{\varepsilon}x$.

Sklyanin'79: $S_x = -\frac{1}{R}\dot\psi$, $S_y = \sqrt{1-S_x^2}\sin\psi$, $S_z = \sqrt{1-S_x^2}\cos\psi$, $y = x/\sqrt{R}$.

$\psi_{yy} - \psi_{tt} = \frac{\gamma}{2}\sin 2\psi$. / $\theta_{xx} - \frac{1}{v_m^2}\theta_{tt} = \frac{1}{2\Delta_e^2}\sin 2\theta$, $\theta_{\xi\xi} = \frac{1}{2\Delta^2}\sin 2\theta$ equi

имеет решение $\theta(\xi) = 2\arctg(\exp(\xi/\Delta))$, $\Delta_e = \frac{\Delta}{\sqrt{1-v^2/v_m^2}}$

Т.е. также $\theta(\xi) = 2\arctg e^{\xi/\Delta}$ соотв. SG equation $\theta_{xx} - \frac{1}{v_m^2}\theta_{tt} = \frac{1}{2\Delta_e^2}\sin 2\theta$, $\xi = x - vt$

Сделаем eqs. for $\psi(x,t)$ и $\theta(x,t)$ эквивалентными, см. 1/II.

$\psi\to\theta$, $y\to x$, $\frac{1}{v_m^2} = 1$, $\gamma^{-1} = \Delta_e^2$, $\theta(x,t)\to\psi(y,t) = 2\arctg\exp\left[\frac{y - vt}{\Delta}\right]$

$\Delta = \Delta_e\sqrt{1-v^2/v_m^2} = (\sqrt{\gamma})\sqrt{1-v^2}$, $\psi(y,t) = 2\arctg\exp\left[\frac{1}{(\sqrt{\gamma})}\frac{(y-vt)}{\sqrt{1-v^2}}\right]$ реш. SG Sklyanin.

$\psi(\xi,t) = 2\arctg\exp\left[\frac{s}{\Delta_s}\right]$, $s = y - vt$, $\Delta_s = (\sqrt{\gamma})\sqrt{1-v^2}$, см. 1/II. $\Delta_s = \frac{1}{\sqrt{\gamma}}\sqrt{1-v^2}$

Escaneado con CamScanner

$\Psi(\xi) = 2\arctg e^{\xi/\Delta_s}$, $\xi = y - vt$, $\Delta_s = \frac{1}{\sqrt{\gamma}}\sqrt{1-v^2}$, $y = \frac{1}{\sqrt{R}}x$. $\cos\psi = -\th(\xi/\Delta_s)$.

$\dot\psi = 2\frac{1}{1+e^{2\xi/\Delta_s}} e^{\xi/\Delta_s}\frac{1}{\Delta_s}\dot\xi = \frac{1}{ch(\xi/\Delta_s)}\frac{1}{\Delta_s}\dot\xi = -\frac{v}{\Delta_s}\frac{1}{ch(\xi/\Delta_s)}$, $S_x = \frac{v}{R\Delta_s}\frac{1}{ch(\xi/\Delta_s)} = \frac{v}{R\Delta_s}\sin\psi$

Sklyanin

$S_y = \sqrt{1-S_x^2}\sin\psi$, $S_z = \sqrt{1-S_x^2}\cos\psi$, $S_x = \frac{v}{R\Delta_s}\sin\psi$. $\sin\psi = \frac{1}{ch(\xi/\Delta_s)}$

$\vec S = (S_x, S_y, S_z)$ выражается через $\Psi(\xi)$, $\xi = y - vt = \left(\frac{x}{\sqrt{R}} - vt\right)$, а через $\xi = x - vt$.

Я сразу ansatz $m_x = \varepsilon\sin\psi$, $\psi = \psi(\xi)$ с $\varepsilon = const$ и $\varepsilon(\xi)$.

Note мое ε ($\varepsilon(\xi) = const$) $\varepsilon = \frac{v}{\lambda}\frac{1}{\sqrt{1-v^2/c^2}}$, $c^2 = \lambda$ похоже на $\varepsilon_s = \frac{v}{R\Delta_s} = \frac{v\sqrt{\gamma}}{R\sqrt{1-v^2}}$ Sklyanin ($c=1$).

Моя $E = \frac{1}{2}\int dx\left[\theta_x^2 + \sin^2\theta\,\Phi_x^2 + \sin^2\theta(1+\lambda\cos^2\Phi)\right] = \frac{1}{2}\int dx\left[\left(\frac{\partial\vec m}{\partial x}\right)^2 + m_z^2 + \lambda m_x^2\right]$

Sklyanin $\mathcal{H} = \frac{1}{2}\int dx\left[\left(\frac{\partial\vec S}{\partial x}\right)^2 - \varkappa_z S_z^2 + \varkappa_x S_x^2\right]$, $\varkappa_z = J_3 - J_2 > 0$, $\varkappa_x = J_2 - J_1 > 0$.

$\lambda = \frac{\varkappa_x}{\varkappa_z} \gg 1$, $\frac{R^2}{\gamma} = \frac{\varkappa_x}{\varkappa_z}$ у Sklyanin, $R = \sqrt{\frac{\gamma\varkappa_x}{\varkappa_z}} \gg 1$. Моя $\lambda = \frac{R^2}{\gamma}$. $\sqrt{\lambda} = \frac{R}{\sqrt{\gamma}}$

$tg\Phi_s = \frac{S_y}{S_x}$ для $\vec S(\theta,\Phi) = \sqrt{1-S_x^2}\,\frac{R\Delta_s}{v} \approx \frac{R}{v\sqrt{\gamma}}\sqrt{1-v^2} = \frac{\sqrt{\lambda}}{v}\sqrt{1-v^2}$. (!) $\Phi_s(\xi) = const$ $\xi\to\rho$

У меня $\Phi = \pi/2 - \varepsilon$. $\Phi_s = \arctg\left[\frac{\sqrt{\lambda}}{v}\sqrt{1-v^2}\right] = \frac{\pi}{2} - \frac{v}{\sqrt{\lambda}}\frac{1}{\sqrt{1-v^2}} = \frac{\pi}{2} - \varepsilon_s$.

$\varepsilon_s = \frac{v}{\sqrt{\lambda}}\frac{1}{\sqrt{1-v^2}} = \frac{\sqrt{\gamma}}{R}\frac{v}{\sqrt{1-v^2}}$. У меня $\varepsilon = \frac{v}{\lambda}\frac{1}{\sqrt{1-v^2/c^2}}$. $\varepsilon_s \sim \frac{1}{\sqrt{\lambda}}$, $\varepsilon \sim \frac{1}{\lambda}$.

$\varepsilon_s \Rightarrow$ Sklyanin похожа на мою $\varepsilon \sim 1/\lambda$.

Anyway главное предположение Sklyanin $S_x \ll 1$, но $S_x \sim \frac{1}{\sqrt{1-v^2}}$ и не мало при $v\to 1$.

У Sklyanin $\vec S = \vec S(\xi) = \vec S(x-vt) = \vec S\left(\frac{x}{\sqrt{R}} - vt\right)$, у меня $\vec m = \vec m(x-vt)$

$\frac{\xi}{\Delta_s} = \frac{(y-vt)}{\Delta_s} = \frac{\left(\frac{x}{\sqrt{R}} - vt\right)}{\Delta_s} = \sqrt{\gamma}\,\frac{(x - v\sqrt{R}t)}{\sqrt{R}\sqrt{1-v^2}}$. Решение $\vec S(\xi)$ Sklyanin не нужно будет искать классу $\vec m(\xi)$. $\xi = x - vt$.

Ведь нельзя просто $v\sqrt{R}\to v$ и считать $\xi' = x - Vt$, $\vec e(\xi')$, ведь v входит в Δ_s.

$\dot S_{kin} = -\Phi\sin\theta\dot\theta = +\Phi\frac{d}{dt}\cos\theta = \Phi\frac{dm_z}{dt} = \Phi\frac{d}{dt}\left[\sqrt{1-m_x^2}\cos\psi(\xi)\right] \approx \Phi\frac{d}{dt}\cos\psi(\xi) =$

$= -\Phi\sin\psi\,\psi_\xi(-v) = v\sin\psi\,\psi_\xi\cdot\Phi$. $\frac{\xi}{\Delta_s} = \frac{(x-Vt)}{\Delta_s'}$, $\Delta_s' = \frac{\sqrt{R}}{\sqrt{\gamma}}(1-v^2) \to \infty$ при $R\to\infty$ (v - finite)

В переменных $\xi' = x - Vt$, $V = \sqrt{R}v$.

т.е. решение $\Psi(\xi)$, $\vec S(\Psi)$ - Sklyanin'79 за пределами ($\xi = x - vt$) ansatz (?) и соответ. $\Phi_s(\xi) = const$

1st integral $(\theta_x)^2 + \sin^2\theta(\Phi_x)^2 = \sin^2\theta(1+\lambda\cos^2\Phi) + \mathcal{H}$ или $\sum_{j=1,2,3}\left(\frac{\partial m_j}{\partial x}\right)^2 = \lambda m_x^2 + 1 - m_z^2 + \mathcal{H}$

$S_z = \sqrt{1-S_x^2}\cos\psi = \cos\theta$, $-\sin\theta\,\theta_x = \frac{2}{\partial x}\sqrt{1-S_x^2}\cos\psi = \frac{1}{2\sqrt{(\cdot)}}(-2)S_x\frac{\partial S_x}{\partial x}\cos\psi +$

$+\sqrt{1-S_x^2}(-\sin\psi)\frac{\partial\psi}{\partial x} = -\frac{S_x\cos\psi}{\sqrt{(\cdot)}}\frac{\partial S_x}{\partial x} - \sqrt{(\cdot)}\sin\psi\,\psi_x = \left|S_x = \frac{v}{R\Delta_s}\sin\psi = \mu\sin\psi\right| =$

$= -\frac{S_x\cos\psi}{\sqrt{(\cdot)}}\mu\cos\psi\,\psi_x - \sqrt{(\cdot)}\sin\psi\,\psi_x = \left|\psi_x = \frac{\partial}{\partial x}\psi(\xi) = \psi_\xi'\frac{\partial\xi}{\partial x} = \frac{1}{\sqrt{R}}\psi_\xi'\right| =$

$= -\frac{\psi_x}{\sqrt{(\cdot)}}\left[\mu S_x\cos^2\psi + (1-S_x^2)\sin\psi\right] = -\frac{\psi_x}{\sqrt{(\cdot)}}\left[\mu^2\cos^2\psi + 1 - \mu^2\sin^2\psi\right]\sin\psi$

$-\sin\theta\,\theta_x = -\frac{\psi_x}{\sqrt{(\cdot)}}\sin\psi\left[1+\mu^2\cos2\psi\right]$, $\sin^2\theta = 1-\cos^2\theta = 1-(1-S_x^2)\cos^2\psi =$

$= 1 - (1-\mu^2\sin^2\psi)\cos^2\psi = \sin^2\psi + \mu^2\sin^2\psi = (1+\mu^2)\sin^2\psi$, $\sin\theta = \sqrt{1+\mu^2}\sin\psi$

$\theta_x = \frac{1}{\sin\theta}\frac{\psi_x}{\sqrt{(\cdot)}}\sin\psi(1+\mu^2\cos2\psi) = \frac{\psi_x}{\sqrt{1+\mu^2}}\frac{1}{\sqrt{1-S_x^2}}(1+\mu^2\cos2\psi)$, $\Phi_x = \Phi_\xi\frac{\partial\xi}{\partial x} = 0$. $\Phi_s = \frac{\partial\Phi}{\partial\xi} = 0$

$(\theta_x)^2 = \frac{(1+\mu^2\cos2\psi)^2}{(1+\mu^2)(1-\mu^2\sin^2\psi)}(\psi_x)^2 = (1+\mu^2)\sin^2\psi\left[1+\lambda\cos^2\Phi\right] + \mathcal{H}$ $\mathcal{H} = 0$.

$\psi_x = \frac{1}{\sqrt{R}}\frac{\partial\psi}{\partial\xi}$, $\frac{\partial\psi}{\partial\xi} = \frac{\partial}{\partial\xi}2\arctg\left(e^{\xi/\Delta_s}\right) = 2\frac{1}{1+e^{2\xi/\Delta_s}}e^{\xi/\Delta_s}\frac{1}{\Delta_s} = \frac{1}{ch(\xi/\Delta_s)}\frac{1}{\Delta_s} = \frac{1}{\Delta_s}\sin\psi$

$(1+\mu^2\cos2\psi)^2\frac{1}{\Delta_s^2}\sin^2\psi = (1+\mu^2)^2(1-\mu^2\sin^2\psi)\sin^2\psi(1+\lambda\cos^2\Phi)$

$(1+\mu^2\cos2\psi)^2\frac{1}{\Delta_s^2} = (1+\mu^2)^2(1-\mu^2\sin^2\psi)(1+\lambda\cos^2\Phi)$, $\cos\Phi = \cos\Phi_s = \cos\left(\frac{\pi}{2} - \varepsilon_s\right) = \varepsilon_s$

$1+\lambda\cos^2\Phi = 1+\lambda\varepsilon_s^2 = 1+\lambda\frac{v^2}{\lambda}\frac{1}{1-v^2} = 1+\frac{v^2}{1-v^2} = \frac{1}{(1-v^2)}$, $\Delta_s^2 = \frac{1}{\gamma}(1-v^2)$

$(1+\mu^2\cos2\psi)^2\frac{\gamma}{(1-v^2)} = (1+\mu^2)^2(1-\mu^2\sin^2\psi)\frac{1}{(1-v^2)}$, $\mu^2+1 = \frac{v^2}{R^2\Delta_s^2} + 1$, $\mu = \frac{v}{R\Delta_s} = \frac{v}{R}\frac{\sqrt{\gamma}}{\sqrt{1-v^2}}$

$(1+\mu^2\cos2\psi)^2\gamma = (1+\mu^2)^2(1-\mu^2\sin^2\psi)$ $\mu = \frac{\sqrt{\gamma}}{R}\frac{v}{\sqrt{1-v^2}} = \varepsilon_s$.

Если $\mu^2\to 0$, то $\gamma = 1$, 1st integral is Ok с norm $\Theta(\mu^2)$. $S_x = \varepsilon_s\sin\psi$

Я сразу $m_x = \varepsilon\sin\psi$. т.е. мой $(\theta,\Phi)\to(\varepsilon,\psi)$ замена coord. $(\varepsilon_s, \psi(\xi))$ - Sklyanin'79

Escaneado con CamScanner

Hoi ansatz: $m_x = \varepsilon \sin\Psi$, $m_y = \sqrt{1-m_x^2}\sin\Psi$, $m_z = \sqrt{1-m_x^2}\cos\Psi$, $\vec{m} = \vec{m}(x,t)$, $\varepsilon(x,t)$, $\Psi(x,t)$

Sklyanin-solution: $S_x = \varepsilon_s \sin\Psi(\vartheta)$, $S_y = \sqrt{1-S_x^2}\sin\Psi$, $S_z = \sqrt{1-S_x^2}\cos\Psi$, $\varepsilon_s(x,t) = \text{const}$ $\Psi = \Psi(\vartheta)$

$\Psi(\vartheta) = 2\operatorname{arctg} e^{\vartheta/\Delta_s}$, $\frac{\vartheta}{\Delta_s} = \frac{\sqrt{\gamma}}{\sqrt{R}}\frac{(x-Vt)}{\sqrt{1-v^2}} = \frac{\sqrt{\gamma}}{\sqrt{R}}\frac{(x-Vt)}{\sqrt{1-V^2/R}} = \frac{\sqrt{\gamma}}{\sqrt{R}}\frac{(x-Vt)}{\sqrt{1-V^2/V_0^2}}$ $\vartheta = \frac{x}{\sqrt{R}} - \sqrt{\gamma}t$

$= \xi'/\Delta_s'$

! $\vartheta/\Delta_s = \xi'/\Delta_s' \to \xi' = x - Vt$, $\Delta_s' = \sqrt{\frac{R}{\gamma}}\sqrt{1-V^2/V_0^2}$, $V_0 = \sqrt{R}$.

Sklyanin solution можно в виде $S_x = \varepsilon_s \sin\Psi$, $S_y = \sqrt{1-S_x^2}\sin\Psi$, $S_z = \sqrt{1-S_x^2}\cos\Psi$, где $\Psi = \Psi(\xi')$, $\xi' = x - Vt$, $\Psi(\xi') = 2\operatorname{arctg} e^{\xi'/\Delta_s'}$ и $\varepsilon_s(\xi') = \varepsilon_s = \text{const}$

$\vec{S} = \vec{S}(\xi'/\Delta_s')$ – неподвижное Sklyanin'89 solution: 1) $\varepsilon_s(\xi') = \text{const}$, $\varepsilon_s \ll 1$ не выполнено при $V \to V_0$; 2) $\Delta_s' = \sqrt{\frac{R}{\gamma}}\sqrt{1-V^2/V_0^2} \to \infty$ при $R \to \infty$ бесконечно широкий

$\Psi(\xi) = 2\operatorname{arctg} e^{\xi/\Delta_s'} \to \Psi_{\xi\xi} = \frac{1}{2(\Delta_s')^2}\sin 2\Psi$ или $\Psi_{xx} - \frac{1}{V_m^2}\Psi_{tt} = \frac{1}{2\Delta_e^2}\sin 2\Psi$, с

$\Delta_e = \frac{\Delta_s'}{\sqrt{1-v^2/V_m^2}} = \frac{\sqrt{R}\sqrt{1-V^2/V_0^2}}{\sqrt{\gamma}\sqrt{1-V^2/V_m^2}} = \sqrt{\frac{R}{\gamma}}$, $V_0 = V_m$, $\Psi_{xx} - \frac{1}{V_m^2}\Psi_{tt} = \frac{\gamma}{2R}\sin 2\Psi$, $V_m^2 = R$.

или $\Psi_{yx} - \Psi_{tt} = \frac{\gamma}{2}\sin 2\Psi$, $\frac{\partial}{\partial y} = \sqrt{R}\frac{\partial}{\partial x}$, $\frac{\partial^2}{\partial y^2} = R\frac{\partial^2}{\partial x^2}$, $\Psi_{xx} - \frac{1}{R}\Psi_{tt} = \frac{\gamma}{2R}\sin 2\Psi$

$E(\text{Sklyanin}) = \frac{1}{2}\int dx\left[\left(\frac{\partial \vec{S}}{\partial x}\right)^2 - S_z^2 + \lambda S_x^2\right]$ $\gamma=1$, $\varkappa_2=1$ совп. с моей $E = \frac{1}{2}\int dx\left[\left(\frac{\partial \vec{m}}{\partial x}\right)^2 - m_z^2 + \lambda m_x^2\right]$,

т.е. скорость в eq. $\Delta_0/t_0 = \frac{2\gamma}{M_0}\sqrt{AK_2}$, $V_m = \sqrt{R}$, $V_m = \sqrt{\sqrt{\gamma\lambda}} = \lambda^{1/4}$, $\lambda = \frac{R^2}{\gamma}$

V_m это $V_- = \sqrt{\lambda}$ можно. Можно это corrected Kivshar'89 defining $\gamma = 1$ и $R = \lambda$ ($\Leftrightarrow V_m = \sqrt{R} = \sqrt{\lambda} = V_-$), $R = \lambda = \frac{K_x}{K_z} \gg 1$. Т.е. Kivshar заменил $R^2 \to R$ и $\gamma=1$, $R=\lambda$.

sin-Gordon Kivshar'89: $\Psi_{xx} - \frac{1}{\lambda}\Psi_{tt} = \frac{1}{2\lambda}\sin 2\Psi$, $V_m = c = \sqrt{\lambda} = V_-$.

Corrected Sklyanin solution $\varepsilon_s = \frac{\sqrt{\gamma}}{R}\frac{v}{\sqrt{1-v^2}} = \frac{\sqrt{\gamma}}{R^{3/2}}\frac{V}{\sqrt{1-V^2/V_m^2}} \to \varepsilon_s = \frac{V}{\lambda^{3/2}\sqrt{1-V^2/c^2}}$

$\Delta_s' = \sqrt{\frac{R}{\gamma}}\sqrt{1-V^2/c^2} \to \Delta_s' = \sqrt{\lambda}\sqrt{1-V^2/c^2}$. $\varepsilon_s = \frac{V}{\lambda\Delta_s'}$ вообще можно $\varepsilon = \frac{v}{\lambda\Delta}$, $\Delta = \sqrt{1-v^2/c^2}$

Вывод: Sklyanin'89 corrected by Kivshar'89 это $\vec{S} = \vec{S}(\xi'/\Delta_s')$, $\xi' = x - Vt$ с SW width $\Delta_s' = \sqrt{\lambda}\sqrt{1-V^2/c^2} \to \infty$ и $\varepsilon_s = \frac{V}{\lambda\Delta_s'} = \text{const}$. SG: $\Psi_{xx} - \frac{1}{\lambda}\Psi_{tt} = \frac{1}{2\lambda}\sin 2\Psi$, $\lambda = c^2$.

Energy of Sklyanin solution ($\Phi_x = 0$) $E = \frac{1}{2}\int dx\left[(\Theta_x)^2 + \sin^2\Theta(1+\lambda\cos^2\Phi)\right] = \int dx\,(\Theta_x')^2$

$\Theta_x = \frac{1}{\sqrt{1+\mu^2}}\frac{(1+\mu^2\cos 2\Psi)}{\sqrt{1-S_x^2}}\frac{1}{\Delta_s}\sin\Psi$, $\Theta_x^2 = \frac{(1+\mu^2\cos 2\Psi)^2}{(1+\mu^2)}\frac{1}{\Delta_s^2}\frac{\sin^2\Psi}{[1-\mu^2\sin^2\Psi]} \simeq \frac{1}{\Delta_s^2}\sin^2\Psi + O(\mu^2)$

$\Delta_s = \frac{1}{\sqrt{\gamma}}\sqrt{1-v^2} = \frac{1}{\sqrt{\gamma}}\sqrt{1-V^2/V_0^2}$, $\sin\Psi = \frac{1}{\operatorname{ch}(\xi'/\Delta_s')}$, $\xi' = x - Vt$.

$E_{sw} = \int dx\,(\Theta_x')^2 = \frac{1}{\Delta_s^2}\int dx\frac{1}{\operatorname{ch}^2(\xi'/\Delta_s')} = \frac{\Delta_s'}{\Delta_s^2}\int_{-\infty}^{\infty}\frac{dz}{\operatorname{ch}^2 z} = 2\frac{\Delta_s'}{\Delta_s^2} = 2\frac{\sqrt{R}\sqrt{1-V^2/V_0^2}}{\left[\sqrt{1-V^2/V_0^2}\right]^2}$

$E_{sw}(\text{Sklyanin}) = 2\frac{\sqrt{R}}{\sqrt{1-V^2/c^2}}$. Сравн. Kivshar'89 $R=\lambda$, $E_{sw}(\text{skl}) = \frac{2\sqrt{\lambda}}{\sqrt{1-V^2/c^2}}$

$\Psi_x = \frac{1}{\sqrt{R}}\frac{\partial\Psi}{\partial\vartheta} \Rightarrow \frac{1}{\Delta_s}\frac{\sin\Psi}{\sqrt{R}} \to E_{sw}(\text{Sklyanin}) = \frac{2\sqrt{\lambda}}{\sqrt{1-V^2/c^2}} = \frac{E_0}{\sqrt{1-V^2/c^2}}\sqrt{1-V^2/V_0^2}$, $E_{sw}(\text{skl}) = \sqrt{\lambda}E_{sw}(\text{mod})$

Наоборот, $\Psi_x = \frac{1}{\sqrt{R}}\frac{1}{\Delta_s}\sin\Psi$, $\Delta_s = \sqrt{1-V^2/V_0^2}$, $\Delta_s' = \sqrt{R}\sqrt{1-V^2/V_0^2} = \sqrt{R}\Delta_s$.

Надо $\Psi_x = \frac{1}{\Delta_s'}\sin\Psi$ и $\Theta_x = \frac{(1+\mu^2\cos 2\Psi)}{\sqrt{1+\mu^2}\sqrt{1-\mu^2\sin^2\Psi}}\Psi_x \simeq \Psi_x = \frac{1}{\Delta_s'}\sin\Psi$

$E_{sw}(\text{skl}) = \int dx(\Theta_x')^2 = \frac{1}{(\Delta_s')^2}\int dx\frac{1}{\operatorname{ch}^2(\xi'/\Delta_s')} = \frac{1}{\Delta_s'}\int dz\frac{1}{\operatorname{ch}^2 z} = \frac{2}{\Delta_s'} = \frac{2}{\sqrt{R}\sqrt{1-V^2/V_0^2}}$

$E_{sw}(\text{skl}) = \frac{1}{\sqrt{R}}E_{sw}(\text{mod})$, $E_{sw}(\text{mod}) = \frac{E_0}{\sqrt{1-V^2/c^2}}$, $E_0 = 2$. $R = \lambda$ (Kivshar), $E_{sw}(\text{skl.}) = \frac{1}{\sqrt{\lambda}}E_{sw}(\text{mod})$

Чтобы 1st integral: $1 + \lambda\cos^2\Phi = 1 + \lambda\varepsilon_s^2 = 1 + \lambda\frac{V^2\gamma}{R^2(1-V^2)}$. что значит $\lambda = \varkappa_1 = \frac{R^2}{\gamma}$

$\varepsilon_s = \frac{\sqrt{\gamma}}{R^{3/2}}\frac{V}{\sqrt{1-V^2/V_0^2}}$, $1 + \lambda\cos^2\Phi = 1 + \frac{R^2}{\gamma}\frac{\gamma}{R^3}\frac{V^2}{(1-V^2/V_0^2)} = 1 + \frac{1}{R}\frac{V^2}{(1-V^2/V_0^2)} = 1 + \frac{V^2/V_0^2}{1-V^2/V_0^2} = \frac{1}{(1-V^2/V_0^2)}$

$\cos\Phi = \cos(\sigma\pi/2 - \varepsilon_s) = \varepsilon_s$. $\Delta_s'^2 = R(1-V^2/V_0^2)$

Подобрание $\frac{1}{(\Delta_s')^2}$ с $\Delta_s' = \sqrt{R}\sqrt{1-V^2/V_0^2}$ даёт $\frac{1}{R}$ коэф. и $(\Theta_x)^2$ и $E_{ex} \to 0$ при $R \to \infty$

Тогда 1st integral не выполнено (!!). $E_{sw}(\text{skl.}) = \frac{1}{2}\int dx\left[(\Theta_x)^2 + \sin^2\Theta(1+\lambda\cos^2\Phi)\right] =$

$= \frac{1}{2}\int dx\left[\frac{1}{(\Delta_s')^2}\sin^2\Psi + \sin^2\Psi\frac{1}{(1-V^2/V_0^2)}\right] \simeq \left|R\to\infty\right| = \frac{1}{2}\frac{1}{(1-V^2/V_0^2)}\int dx\sin^2\Psi = \frac{1}{2}\frac{1}{(\cdot)}\int dx\frac{1}{\operatorname{ch}^2\xi'/\Delta_s'} =$

$= \frac{1}{2}\frac{1}{(1-V^2/V_0^2)}\Delta_s'\int dz\frac{1}{\operatorname{ch}^2 z} = \frac{\Delta_s'}{(\cdot)} = \frac{\sqrt{R}\sqrt{1-V^2/V_0^2}}{\sqrt{\gamma}} = \frac{\sqrt{R}}{\sqrt{\gamma}}\frac{1}{\sqrt{1-V^2/V_0^2}}$. Нельзя с $E_{sw} \to \infty$

Escaneado con CamScanner

Новая revised 1D-DW paper to Ricardo и получил ответ от Ricardo (overleaf)

6/IV С А. В. Егорки - только. Вторая домашняя. Ruben. Проверка SG Sklyanin'79

7/IV Теме Vorba - my - artificial Skyrmions с $\Phi_0(z)$ - variable helicity. Запрос leaf-file. Также 1D-DW из leaf-file By Ricardo. Синяков и много неортогональных Regs и немагнитные eqs. Извинился. Я послал Ruben the last outputs (?)

- SG из Sklyanin'79 : $\varphi_{yy} - \varphi_{tt} = \frac{\gamma}{J}\sin 2\varphi$, $\gamma = \frac{x}{\sqrt{R}}$, $R \gg 1$
- SG из Kivshar'89 : $\varphi_{tt} = \varphi_{yy} - \frac{1}{2}\sin 2\varphi$, $y = \frac{x}{\sqrt{R}}$, $\lambda = R = \frac{K_x}{K_z} \gg 1$, $J = 1$

Sklyanin $\mathcal{H} = \frac{1}{2}\int dx \left[\left(\frac{\partial \vec S}{\partial x}\right)^2 + (J_3 - J_1) S_x^2 + (J_3 - J_2) S_y^2 \right]$, $J_1 < J_2 < J_3$

$S_y^2 = 1 - S_x^2 - S_z^2$, $(J_3 - J_1) S_x^2 + (J_3 - J_2)(1 - S_x^2 - S_z^2) = \text{const} + (J_2 - J_1) S_x^2 - (J_3 - J_2) S_z^2$

$\varkappa_z = J_3 - J_2 > 0$, $\varkappa_x = J_2 - J_1$, $\lambda = \frac{\varkappa_x}{\varkappa_z}$, $R(J_3 - J_2) = \gamma$, $(J_2 - J_1) = R$ (Sklyanin)

$\frac{(J_3 - J_2)}{(J_2 - J_1)} = \frac{\varkappa_z}{\varkappa_x} = \frac{\gamma}{R}\frac{1}{R}$, $\frac{\varkappa_x}{\varkappa_z} = \lambda = \frac{R^2}{\gamma}$. У Kivshar $R \gg \lambda$, $R = \lambda$.

$\mathcal{H} = \frac{1}{2}\int dx \left[\left(\frac{\partial \vec S}{\partial x}\right)^2 + \varkappa_x S_x^2 - \varkappa_z S_z^2 \right]$, $\dot{\vec S} = \vec S \times \vec S_{xx} + \vec S \times \hat J \vec S$, $\dot{\vec M} = -\gamma \vec M \times \vec H$, $\gamma > 0$

$\dot{\vec S} = -\vec S \times \vec h$, $\vec h = -\frac{\delta \mathcal{H}}{\delta \vec S}$, $\dot{\vec S} = \vec S \times \frac{\delta \mathcal{H}}{\delta \vec S}$, $\frac{\delta \mathcal{H}}{\delta \vec S} = \frac{\partial}{\partial x}\frac{\partial \mathcal{H}}{\partial (\partial \vec S/\partial x)} - \frac{\partial \mathcal{H}}{\partial \vec S} = \frac{\partial}{\partial x}\frac{1}{2}\cdot 2\frac{\partial \vec S}{\partial x} - \frac{1}{2}\frac{\partial}{\partial \vec S}\left[\varkappa_x S_x^2 - \varkappa_z S_z^2\right] = \vec S_{xx} - \varkappa_x S_x \hat x + \varkappa_z S_z \hat z$.

$\mathcal{H} = \int dx \left[A \left(\frac{\partial \vec m}{\partial x}\right)^2 + K_x S_x^2 - K_z S_z^2 \right]$, $\dot{\vec m} = -\gamma \vec m \times \vec H = \gamma \vec m \times \frac{\delta \mathcal{H}}{\delta \vec m}\frac{1}{M_s}$

$\frac{\delta \mathcal{H}}{\delta \vec m} = \frac{\partial}{\partial x}\frac{\partial \mathcal{H}}{\partial (\partial \vec m/\partial x)} - \frac{\partial \mathcal{H}}{\partial \vec m} = 2A\vec m_{xx} + 2K_x m_x \hat x + 2K_z m_z \hat z$.

$\dot{\vec m} = \gamma \vec m \times \frac{\delta \mathcal{H}}{\delta \vec m}\frac{1}{M_s} = 2\gamma \vec m \times \left[A\vec m_{xx} - \lambda K_x m_x + K_z m_z \hat z \right]\frac{1}{M_s}$, $\frac{A}{K_z} = \Delta_0^2$, $\frac{K_x}{K_z} = \lambda$

$\frac{\partial \vec m}{\partial t} = \frac{2\gamma K_z}{M_s}\vec m \times \left[\Delta_0^2 \vec m_{xx} - \lambda m_x \hat x + m_z \hat z \right]$, $\frac{2\gamma K_z}{M_s} = \gamma H_a = \frac{1}{t_0}$

$-\frac{\partial \vec m}{\partial x} = \left(\vec m_{xx} - \lambda m_x \hat x + m_z \hat z\right) \times \vec m$, $\frac{\partial \vec m}{\partial t} = \vec m \times \left[\vec m_{xx} - \lambda m_x \hat x + m_z \hat z \right]$, x - в ед Δ_0, t в ед t_0

$\mathcal{H}(\vec m) = \int dx \left[A(\vec m_x')^2 + K_x m_x^2 - K_z m_z^2 \right] = \int dx \left[\Delta_0^2 (\vec m_x')^2 + \lambda m_x^2 - m_z^2 \right]$

$e(\vec m) = \left(\frac{1}{K_z}\right)^{-1}\left[(\vec m_x')^2 + \lambda m_x^2 - m_z^2 \right]$, $\mathcal{H}(\vec m) \frac{1}{K_z} = \left(\frac{1}{K_z}\right)^{-1}\Delta_0 \int dx \left[(\vec m')^2 + \lambda m_x^2 - m_z^2 \right]$

$\mathcal{L} = \mathcal{L}_{kin} - e(\vec m) = -\cos\theta\dot\Phi \frac{M_s}{\gamma} - K_z\left[(\vec m')^2 + \lambda m_x^2 - m_z^2 \right]$, $\frac{d\Phi}{dt} = \frac{1}{t_0}\frac{d\Phi}{d(t/t_0)} = \frac{1}{t_0}\dot\Phi$

$\mathcal{L} = -\cos\theta\dot\Phi \frac{2\gamma K_z}{M_s}\frac{M_s}{\gamma} - K_z\left[(\vec m')^2 + \lambda m_x^2 - m_z^2 \right]$, $\vec m' = \frac{\partial}{\partial x}\vec m$

$\mathcal{L} = -\cos\theta\cdot\dot\Phi \cdot 2K_z - K_z\left[(\vec m')^2 + \lambda m_x^2 - m_z^2 \right]$, $\frac{1}{2K_z}\mathcal{L} = -\cos\theta\dot\Phi - \frac{1}{2}\left[(\vec m')^2 + \lambda m_x^2 - m_z^2 \right]$

$\mathcal{H}[\vec m] = 2K_z\Delta_0 \cdot \frac{1}{2}\int dx \left[(\vec m')^2 + \lambda m_x^2 - m_z^2 \right] \to \mathcal{H}(\vec S)$ в ед. $2K_z\Delta_0 = 2\sqrt{AK_z}$.

$L = \int dx\, \mathcal{L} = \Delta_0 \int dx \cdot 2K_z \left\{ -\cos\theta\dot\Phi - \frac{1}{2}\left[(\vec m')^2 + \lambda m_x^2 - m_z^2 \right] \right\} = 2K_z\Delta_0 \int dx\, \mathcal{L}$

т. е. $\mathcal{H}[\vec S]$ - Sklyanin'79 записана в units $2K_z\Delta_0$, x в ед Δ_0, t в ед $t_0 = 1/\gamma H_a$.

Ur. $\dot{\vec S} = \vec S \times \left[\vec S_{xx} - \lambda S_x \hat x + S_z \hat z \right] = \vec S \times \left[\vec S'' - \lambda S_x \hat x + S_z \hat z \right]$, см. 6/II.

Ansatz - Sklyanin $S_x = -\frac{1}{R}\dot\varphi$, $\dot S_x = -\frac{1}{R}\ddot\varphi$ с гр. спрямо $\dot S_x = \vec S \times \left[\vec S'' - \lambda S_x \hat x + \hat z S_z \right]\cdot\hat x$

$(\vec S \times \vec S'')_x = \begin{vmatrix} \hat x & \hat y & \hat z \\ S_x & S_y & S_z \\ S_x'' & S_y'' & S_z'' \end{vmatrix}$, $\dot x = S_y S_z'' - S_z S_y''$, $S_y = \sqrt{1 - S_x^2}\sin\varphi$, $S_z = \sqrt{1 - S_x^2}\cos\varphi$

$(\vec S \times \vec S'')_x = \sqrt{1 - S_x^2}\sin\varphi\, S_z'' - \sqrt{1 - S_x^2}\cos\varphi\, S_y''$, $S_y' = \frac{\partial}{\partial x}\sqrt{1 - S_x^2}\sin\varphi =$

$= \frac{1}{2\sqrt{(\cdot)}}(-2S_x)S_x'\sin\varphi + \sqrt{1 - S_x^2}\cos\varphi\,\varphi' = -\frac{S_x S_x' \sin\varphi}{\sqrt{(\cdot)}} = -\frac{S_x S_x'}{(1 - S_x^2)} S_y + \sqrt{1 - S_x^2}\cos\varphi\varphi' =$

$= -\frac{S_x S_x'}{(1 - S_x^2)} S_y + S_z\varphi'$, $S_y'' = -\frac{\partial}{\partial x}\frac{S_x S_x'}{(1 - S_x^2)}\cdot S_y + \frac{(-S_x S_x')}{1 - S_x^2} S_y' + S_z\varphi' + S_z\varphi''$

Нужно $S_y = F(S_x)\sin\varphi$, $S_z = F(S_x)\cos\varphi$, $S_y' = F' S_x' \sin\varphi + F\cos\varphi\varphi'$

$S_y'' = F''(S_x')^2\sin\varphi + F' S_x''\sin\varphi + F' S_x'\cos\varphi\varphi' + F' S_x'\cos\varphi\varphi' + F(-\sin\varphi)\varphi'^2 +$

$+ F\cos\varphi\varphi''$, где $F' = \frac{d}{dz}F(z)$. Нужно $F' = \frac{\partial}{\partial x}F(S_x)$ не рассчитано.

$S_y = F\sin\varphi$, $S_y' = F'\sin\varphi + F\cos\varphi\varphi'$, $S_y'' = F''\sin\varphi + F'\cos\varphi\varphi' +$

$+ F'\cos\varphi\varphi' + F(-\sin\varphi)\varphi'^2 + F\cos\varphi\varphi''$, $S_z' = F'\cos\varphi + F(-\sin\varphi)\varphi'$,

$S_z'' = F''\cos\varphi + F'(-\sin\varphi)\varphi' - F'\sin\varphi\varphi' - F\cos\varphi\varphi'^2 - F\sin\varphi\varphi''$

$S_y S_z'' - S_z S_y'' = F\sin\varphi\left[F''\cos\varphi - 2F'\sin\varphi\varphi' - F\cos\varphi\varphi'^2 - F\sin\varphi\varphi'' \right] - F\cos\varphi \times$

$\times \left[F''\sin\varphi + 2F'\cos\varphi\varphi' - F\sin\varphi\varphi'^2 + F\cos\varphi\varphi'' \right] = -F\sin^2\varphi\left[2F'\varphi' + F\varphi'' \right] - F\cos^2\varphi\left[2F'\varphi' + F\varphi'' \right] =$

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$= -F\left[2F'\varphi' + F\varphi''\right]$, $S_y S_z'' - S_z S_y'' = -F\left[2F'\varphi' + F\varphi''\right]$, $F = F(S_x) = \sqrt{1-S_x^2}$

$\vec{S} \times \left[-\lambda S_x \hat{x} + S_z \hat{z}\right] \cdot \hat{x} = -\lambda \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{vmatrix} \hat{x} + \left(S_x S_z \hat{z}\right) \cdot \hat{x} = S_z \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ S_x & S_y & S_z \\ 0 & 0 & S_z \end{vmatrix} \cdot \hat{x} =$ (=0)

$= S_z \hat{x} \cdot \left(S_y S_z \hat{x} - \hat{y} S_x S_z\right) = S_y S_z^2$.

$\dot{S}_x = -\frac{1}{R}\dot{\varphi} = -F\left[2F'\varphi' + F\varphi''\right] + S_y S_z = -\left(1-S_x^2\right)\varphi'' - 2FF'\varphi' +$

$+ \left(1-S_x^2\right)\sin\varphi\cos\varphi$, $F' = \frac{2}{\partial x}\sqrt{1-S_x^2} = -\frac{S_x S_x'}{\sqrt{1-S_x^2}}$

$-\frac{1}{R}\dot{\varphi} = -\left(1-S_x^2\right)\varphi'' + \left(1-S_x^2\right)\sin\varphi\cos\varphi - 2\sqrt{1-S_x^2}\,\varphi'(-1)\frac{S_x S_x'}{\sqrt{1-S_x^2}}$

$\left(1-S_x^2\right)\varphi'' - \frac{1}{R}\dot{\varphi} = \left(1-S_x^2\right)\sin\varphi\cos\varphi + 2\varphi' S_x S_x'$. Инвариантна $\sim S_x^2$

(*) $\varphi'' - \frac{1}{R}\ddot{\varphi} = \sin\varphi\cos\varphi$, $R=1$. Sklyanin'79 gaer $\varphi_{xt} - \frac{1}{R}\ddot{\varphi} = \frac{\gamma}{2R}\sin 2\varphi$ Eq.(5.1)

Стат Sine-Gordon обладает с расщеп./проверкой Sklyanin $\alpha = 6/\pi$.

т.е. у Sklyanin'79 – ошибка. В размерных переменных $\varphi_{xx}'' - \frac{1}{c^2}\ddot{\varphi} = \frac{1}{2\Delta_0^2}\sin 2\varphi$,

если $R = \lambda = \frac{K_x}{K_z}$, как у Kivshar'89. Уд дно S_y, S_z вычисляются если $1+\lambda = \lambda$, $\lambda \to \infty$

Eq. (*) – corrected Sklyanin'79 при $R = \lambda = c^2$, $\varphi_{xx} - \frac{1}{c^2}\varphi_{tt} = \frac{1}{2}\sin 2\varphi$ совпадает с SG Eq.(6) при $\Delta = \Delta' = \sqrt{1-v_m^2/c^2}$, $v_m = c$ (когда $\Delta_0 = 1$) когда $E(z) = \text{const}$, $\varepsilon \ll 1$, что не удивительно ведь Sklyanin'79 solution для перем. типа $E(z) = \text{const}$, $\lambda \to \infty$.

Конечно нового Sklyanin solution (5.1) не удовлетворяет 1st integral of motion, но самому LL eq. не удовлетворяет.

E-mail с comments on 1D-DW dyn. MS to Ricardo, Ruben (новая Sklyanin preprint?)

(8/IV) переделка SolSkyMag'2019 program (вчера сюда Kim + 2 new posters B. Kim, Wagner) Теме Second variation, Buldygin book (Roman прислал). Ёлки.

(9/IV) Встреча с Julian/Arcady – совм. project с A. Fert. Обсуждение с Karin Garcia ее skyrmion research (general CoFeB/Pt/Pt ML образцы в Nano GUNE, Sugar (Arcady) перед MFM и transport topol. Hall effect in Thales, Paris). Karin сдала, послала мне several sk-papers (Nayak-Parkin, Nature-2017; thales presentation 2018; Kanazawa PRB, 2015, Lemaitre, THE, arXiv 2018; Legrand arXiv'2018). Просмотр этих files. Future skyrmion joint research с Fert и Garcia. Спросить: какое направление предпочитает A. Fert?

Вопросы: отчет/продолжение FIS2016 project; ① ~~Отчет MagIC-krasnoyarsk-Сделано~~, ② ~~Recom. letter Ricardo to EFSM, Brno~~, ③ ~~Refs. 2018 для Angel Alegria~~, ④ ~~Abstracts to JEMS~~ (Apr. 24), ⑤ ~~Recom. letter Ricardo~~ до 15/IV – сделано, ⑥ ~~Birkhäuser Heraklion, HMM 2019~~.

(10/IV) переписка с Dec. о FIS2016 extension project и Dodd book. Собрать и послать Recom. letter Ricardo to EFSM, Brno. Ответ на comments/questions by Ricardo по 1D-DW manuscript.

(11/IV) Meeting с Julian: progress с SolSkyMag'2019 и joint research с A. Fert (а создание сотрудничества с A. Fert через exp. MS by Karin, или A. Fert предложит a new direction). Обсуждение с Ricardo 1D-DW manuscript: получил ему вчера проверил Sklyanin ansatz и финал подстановки в LL eq. и найти 2nd variation of functional to check max/min of F,

Check parameters in Kivshar'89: $E = \frac{1}{2}\int dx \left[\alpha(\nabla \vec{m})^2 - \beta_1 m_1^2 + \beta_3 m_3^2\right]$

$E = \beta_3 M_s^2 \frac{1}{2} l_0 \int \frac{dx}{l_0}\left[\left(\frac{\partial \vec{m}}{\partial x}\right)^2 + \xi m_x^2 - m_z^2\right] S$, x in units of $l_0 = \sqrt{\alpha/\beta_3}$, t in units $\omega_0^{-1} = (\gamma M_s \beta_3)^{-1}$

$E/S = \frac{1}{2}\beta_3 M_s^2 l_0 \int\left[\left(\frac{\partial \vec{m}}{\partial x}\right)^2 + \xi m_x^2 - m_z^2\right] dx$, $\xi = \frac{|\beta_1|}{\beta_3} = \frac{K_x}{K_z} = \lambda$, $A = \frac{1}{2}\alpha M_s^2$

$\frac{1}{2}|\beta_x| M_s^2 = K_x$, $\frac{1}{2}\beta_z M_s^2 = K_z$, $\frac{K_x}{K_z} = \frac{|\beta_1|}{\beta_3}$, $\beta_x < 0$, $\beta_z > 0$. $l_0^2 = \frac{2A}{M_s^2 \beta_z} = \frac{A}{K_z} = \Delta_0^2$

$E/S = K_z l_0 \int dx \left[\left(\frac{\partial \vec{m}}{\partial x}\right)^2 + \xi m_x^2 - m_z^2\right]$, $t_0^{-1} = \omega_0 = \gamma M_s \beta_3 = \gamma \frac{1}{2}\frac{\beta_z M_s^2}{M_s} 2 = \gamma H_a$, $H_a = 2K_z/M_s$.

$\mathcal{H}(\text{Sklyanin}) = \frac{1}{2}\int dx \left[\left(\frac{\partial \vec{S}}{\partial x}\right)^2 + \varkappa_x S_x^2 - \varkappa_z S_z^2\right]$, $\varkappa_x = J_2 - J_1$, $\varkappa_z = J_3 - J_2$

$\mathcal{H}(\text{Skl}) = \frac{1}{2}\varkappa_z \int dx \left[\left(\frac{\partial \vec{S}}{\partial x}\right)^2 + \lambda S_x^2 - 1 \cdot S_z^2\right] \to \dot{\vec{S}} = \vec{S} \times \vec{S}_{xx} + \vec{S} \times \left[-\lambda S_x \hat{x} + S_z \hat{z}\right]$

Или $\dot{\vec{S}} = \vec{S} \times \vec{S}_{xx} + \vec{S} \times \left[-\varkappa_x S_x \hat{x} + \varkappa_z S_z \hat{z}\right]$ если $\varkappa_z$ не вычесть.

Escaneado con CamScanner

У Кишвара $\hat{J} = \mathrm{diag}(J_1, J_2, J_3) = (0, \xi, 1+\xi)$, $\hat{S}\times\hat{J}\hat{S} = x_z\left[S_z S_y \hat{x} + \lambda' S_z S_x \hat{y} + \lambda S_x S_y \hat{z}\right]$

$\lambda' = (J_1 - J_3)/(J_3 - J_2)$, $\lambda' = \frac{1}{x_z}(J_1 - J_3)$, $\lambda = \frac{1}{x_z}(J_2 - J_1)$ (3/II). Но $J_1 = 0$, $J_2 = \xi$, $J_3 = 1+\xi$

$\lambda' = -\frac{(1+\xi)}{\xi} = -(1+\xi)$, $\lambda = \frac{\xi}{1} = \xi$. $\lambda = \frac{(J_2 - J_1)}{(J_3 - J_2)} = \frac{x_x}{x_z}$. $x_z = 1$.

$x_x = J_2 - J_1 = \xi$.

С другой стороны $R(J_3 - J_2) = 1$, $R\xi = 1$; $(J_2 - J_1) = R \to x_x = R$, при $R \to \infty$.

$(J_3 - J_2) = \frac{1}{R}$, $(J_2 - J_1) = R$, $x_x/x_z = \frac{(J_2 - J_1)}{(J_3 - J_2)} = \frac{R}{1/R} = R^2 = \lambda$, т.е. $\lambda = R^2$, $\xi = R^2$ Kivshar'89

Видимо $J_2 = \xi + \frac{d_2}{R^n}$, $J_3 = 1 + \xi + \frac{d_3}{R^n}$, $(J_3 - J_2)R = |R\to\infty| = \left(1 + \xi + \frac{d_3}{R^n} - \xi - \frac{d_2}{R^n}\right)R =$

$= \left[1 + \frac{(d_3 - d_2)}{R^n}\right]R = 1$, $R(R\to\infty) = 1$ - так Да ерунда?

$-\lambda' = \frac{1}{x_z}(J_3 - J_1) = 1 + \xi$ и $\lambda = \frac{1}{x_z}(J_2 - J_1) = \xi$, т.е. $(J_1 = 0)$ $\frac{J_3}{x_z} = 1 + \xi$, $\lambda = \frac{J_2}{x_z} = \xi$,

а не $J_3 = 1 + \xi$, $J_2 = \xi$ как у Kivshar'89. $R(J_3 - J_2) = R\left[x_z(1+\xi) - x_z\xi\right] = R x_z = 1$

$(J_2 - J_1) \underset{R\to\infty}{=} R$ или $J_2 \cong R$ и $(J_3 - J_1) \underset{R\to\infty}{=} R$, $J_3 \cong R$ at $R \to \infty$.

$x_z = (J_3 - J_2) \to 0$ при $R \to \infty$ т.к. $J_3 \to R$, $J_2 \to R$. Indeed $x_z = \frac{1}{R}$ at $R \to \infty$

$\lambda = \xi = \frac{J_2}{x_z} = \frac{R}{1/R} = R^2$ следует из Kivshar'89 при верной корреляции

$\frac{J_2}{x_z} = \xi$, $\frac{J_3}{x_z} = 1 + \xi$. Но у Sklyanin более $\lambda = \frac{K_x}{K_z} = \frac{R^2}{\gamma} \Rightarrow$ Kivshar дал только $\gamma = 1$.

Change the 1D-DW manuscript [у Кишвара $\lambda = R^2$, а не $\lambda = R$ как раньше у нас.]

$\varepsilon_s = \frac{V}{R\Delta_s}$, $\Delta_s = \sqrt{\frac{R}{\gamma}}\sqrt{1 - V^2/V_0^2}$, $V_0 = \sqrt{R}$, $\frac{R^2}{\gamma} = \lambda$ (Sklyanin), $R = \sqrt{\gamma\lambda}$.

$V_0 = \sqrt{\sqrt{\gamma\lambda}} = (\gamma\lambda)^{1/4}$. // Решение Sklyanin'79 это неверное решение класса $\Phi(\xi) = \mathrm{const}$ при $\lambda \to \infty$

E-mail to my secondment-visitors - request to fill in questionnaria MagIC project.

Ананду какие MagIC-related papers нужно чтобы в open (free) access. Verba'PRB'16, Verba'PRA'2018, Gus'JMMM'2017, Aranda'JMMM'2017, Ganeeva'JPh.Con'2018.

Послал email to Krawczyk с этими papers - PDF. Осталось - open access, arXiv или ResGate.

(12/IV) Editing 1D-DW manuscript. Сегодня послал to Angel Aragon список my papers-2018. Нужны file от Ricardo negro. Bulgy eq. SC by Sklyanin. Неудобный order.

Лагр. пл. (dimensionless) $= \frac{1}{2}\int dx\left[\left(\frac{\partial\vec{S}}{\partial x}\right)^2 + \lambda S_x^2 - S_z^2\right]$, $\lambda = \frac{x_x}{x_z} = \frac{(J_2 - J_1)}{(J_3 - J_2)} > 0$, а не $\mathcal{H} = \frac{1}{2}\int dx\left[\left(\frac{\partial\vec{S}}{\partial x}\right)^2 + x_x S_x^2 - x_z S_z^2\right]$ как у Sklyanin'79

$x_x = (J_2 - J_1)$, $x_z = (J_3 - J_2)$

$\mathcal{H}$ in units $2K_z\Delta_0 = 2\sqrt{AK_z}$.

$\mathcal{H}(\vec{m}) = \int dx\left[A\left(\frac{\partial\vec{m}}{\partial x}\right)^2 + K_x m_x^2 - K_z m_z^2\right] = 2K_0\frac{1}{2}\int dx\left[\left(\frac{\partial\vec{m}}{\partial x}\right)^2\frac{A}{K_0} + \frac{K_x}{K_0}m_x^2 - \frac{K_z}{K_0}m_z^2\right]$, $\Delta_0^2 = \frac{A}{K_0}$

$\mathcal{H}(\vec{m}) = 2K_0\Delta_0\frac{1}{2}\int dx\left[\left(\frac{\partial\vec{m}}{\partial x}\right)^2 + x_x m_x^2 - x_z m_z^2\right]$, $\mathcal{H}(\vec{S}) = \frac{1}{2}\int dx\left[\left(\frac{\partial\vec{S}}{\partial x}\right)^2 + x_x S_x^2 - x_z S_z^2\right]$

$x_x = J_2 - J_1$, $x_z = J_3 - J_2$. K_0 - any parameter, $[K_0] = \mathrm{erg/cc}$. Θ eg. $2K_0\Delta_0$. $x_x = \frac{K_x}{K_0}$, $x_z = \frac{K_z}{K_0}$

$\dot{\vec{S}} = \frac{2\gamma K_0}{M_s}\vec{S}\times\left[\frac{\partial^2\vec{S}}{\partial x^2} - x_x S_x\hat{x} + x_z m_z\hat{z}\right] \to \dot{\vec{S}} = \vec{S}\times\left[\frac{\partial^2\vec{S}}{\partial x^2} - x_x S_x\hat{x} + x_z S_z\hat{z}\right]$, $t_0 = \frac{1}{\gamma H_a}$, $H_a = \frac{2K_0}{M_s}$

$x_x = (J_2 - J_1) = R$, $x_z = (J_3 - J_2) = \frac{1}{R}$. $(\dot{\vec{S}})_x = (\vec{S}\times\vec{S}'')_x + \vec{S}\times\left[-x_x S_x\hat{x} + x_z S_z\hat{z}\right]\cdot\hat{x}$

$\left(\vec{S}\times x_z S_z\hat{z}\right)\cdot\hat{x} = x_z S_z\hat{x}\cdot\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ S_x & S_y & S_z\\ 0 & 0 & 1\end{vmatrix} = x_z S_z\hat{x}\left(\hat{x}S_y - \hat{y}S_x\right) = \overset{0}{x_z} S_z S_y$

$\dot{S}_x = S_y S_z'' - S_z S_y'' + x_z S_y S_z \to \varphi'' - \frac{\dot{\varphi}}{R} = x_z\sin\varphi\cos\varphi$, $x_z = \frac{1}{R}$ assumption, $x_x = R$ assumption

Тогда $\mathcal{H}(\vec{S}) = \frac{1}{2}\int dx\left[(\vec{S}')^2 + R S_x^2 + \frac{1}{R}S_z^2\right] \approx \frac{1}{2}R\int dx\, S_x^2$, $R \to \infty$.

S_x из Sklyanin SG $S_x(\xi') = \varepsilon_S\sin\varphi(\xi')$, $\varphi(\xi') = 2\arctg e^{\xi'/\Delta_S}$, $\sin\varphi = \frac{1}{\mathrm{ch}\,\xi'/\Delta_S}$, $\xi' = x - Vt$

$\mathcal{H}_{DW}(\vec{S}) \approx \frac{1}{2}R\int dx\, S_x^2 = \varepsilon_S^2\frac{1}{2}R\int dx\frac{1}{\mathrm{ch}^2(\xi'/\Delta_S)} = \frac{1}{2}R\varepsilon_S^2\Delta_S\int_{-\infty}^{\infty}dz\frac{1}{\mathrm{ch}^2 z} = R\varepsilon_S^2\Delta_S$, $V = \sqrt{R}v$.

$E_{DW}(\mathrm{Sklyanin}) = R\Delta_S\varepsilon_S^2 = R\sqrt{R}\sqrt{1 - V^2/V_0^2}\frac{V^2}{R^2\cdot R(1 - V^2/V_0^2)} = \frac{\sqrt{R}V^2}{R^2}\frac{1}{\sqrt{1 - V^2/V_0^2}} = \frac{1}{\sqrt{R}}\frac{v^2}{\sqrt{1 - v^2/v_0^2}}$, $v_0^2 = R$

$E_{DW}(\mathrm{Skl}) \sim v^2/\sqrt{R} \to 0$ при $R \to \infty$ в нашем случае.

$(\vec{S}')^2 = (\psi')^2 + \sin^2\psi(\Phi')^2 = (\psi')^2$ $\psi = \varphi = 2\arctg e^{\xi'/\Delta_S}$, $\psi' = \frac{d\psi}{d\xi'} = \frac{d\psi}{dx} = \frac{1}{\Delta_S}\sin\psi$

$\mathcal{H}_{DW}(\mathrm{Sklyanin}) = \frac{1}{2}\int dx\,(\vec{S}')^2 = \frac{1}{2}\int dx\,(\psi')^2 = \frac{1}{2}\frac{1}{\Delta_S^2}\int dx\sin^2\psi = \frac{1}{2}\frac{1}{\Delta_S^2}\int dx\frac{1}{\mathrm{ch}^2\xi'/\Delta_S} = \frac{1}{2\Delta_S^2}\Delta_S\int_{-\infty}^{\infty}dz\frac{1}{\mathrm{ch}^2 z}$

$\mathcal{H}_{DW}(\mathrm{Skl}) = \frac{1}{\Delta_S} = \frac{1}{\sqrt{R}\sqrt{1 - V^2/V_0^2}}$, $\mathcal{H}_{DW}(\mathrm{Skl}) = \frac{1}{\sqrt{R}}\frac{1}{\sqrt{1 - V^2/V_0^2}} + \frac{1}{\sqrt{R}}\frac{v^2}{\sqrt{1 - V^2/V_0^2}} = \frac{1}{\sqrt{R}\sqrt{1 - V^2/V_0^2}}\left[1 + v^2\right]$

$\mathcal{H}_{DW}(\mathrm{Sklyanin}) \sim \frac{1}{\sqrt{R}} \to 0$ при $R \to \infty$.

1DW width (Sklyanin) $\Delta_S = \sqrt{R}\sqrt{1 - V^2/V_0^2} \to \infty$ при $R \to \infty$ (нереально).

Escaneado con CamScanner

При перенормировке $\mathcal{H}(S)$ на K_0, $E_{DW} \sim \frac{1}{\sqrt{R}} \to 0$, DW width $\Delta_S \sim \sqrt{R} \to \infty$, т.е. $\vec{m}(x,t)$ при $R \to \infty$, это $\vec{m} = \text{const}$ в плоскости yoz, DW исчезает у Sklyanin.

$K_z \sim \varkappa_z = (J_3 - J_2) \sim \frac{1}{R}$ и $E_{DW} \sim \sqrt{K_z}$, т.е. $\frac{E_{DW}}{\sqrt{AK_z}} = \text{const}(R)$ – finite in units of $\sqrt{AK_z}$ при нормировке $\mathcal{H}$ на K_z.

DW width $\Delta_S \sim \sqrt{R} \sim \frac{1}{\sqrt{K_z}}$ и при нормировке Δ_S на $\sqrt{\frac{A}{K_z}}$, $\Delta_z \to \Delta$ бывает finite.

// Ошибка Sklyanin '79 в неверной нормировке $\mathcal{H}$ (на K_0, а надо на K_z)

Kivshar '89 used correct normalization $\mathcal{H}$ на $\beta_3 \sim K_z$, $\Delta_0 = \sqrt{A/\beta_3}$ units of x и $\hat{J} = (0, \gamma, 1+\gamma) = (J_1, J_2, J_3)$, $\beta_3 \sim \varkappa_z = (J_2 - J_1)$

$$\vec{S} \times \hat{J}\vec{S} = \varkappa_z \left[S_z S_y \hat{x} + \frac{(J_1 - J_3)}{(J_3 - J_2)} S_z S_x \hat{y} + \frac{(J_2 - J_1)}{(J_3 - J_2)} S_x S_y \hat{z} \right] = \varkappa_z \left[S_z S_y \hat{x} - \frac{J_3}{\varkappa_z} S_z S_x \hat{y} + \frac{J_2}{\varkappa_z} S_x S_y \hat{z} \right] =$$

$$= \varkappa_z \left[S_z S_y \hat{x} - (1+\gamma) S_z S_x \hat{y} + \gamma S_x S_y \hat{z} \right]$$

$\dot{S}_a = \vec{S} \times \frac{\hat{J}}{\varkappa_z} \vec{S} = S_z S_y \hat{x} - (1+\gamma) S_z S_x \hat{y} + \gamma S_x S_y \hat{z}$ — это $\frac{\hat{J}}{\varkappa_z} = (0, \gamma, 1+\gamma)$ при-водящее к такому $(\dot{\vec{S}})_a$ с точн. $= 1$ при $S_z S_y \hat{x}$ не использован, а использован Lleq. и Sine-Gordon in Sklyanin's form (not normalized to $\varkappa_z$).

Если принять $(J_3 - J_2) = \varkappa_z = \frac{1}{R}$, $(J_2 - J_1) = \varkappa_x = R$, то получим $\gamma = \lambda = \frac{(J_2 - J_1)}{(J_3 - J_2)} = \frac{J_2}{\varkappa_z} = \gamma$, то получим correct Lleq. и correct Sine-Gordon at $\lambda \to \infty$.

(13/IV) переезд с A. new Gas/El. Contract. Updating Conf. Program. Бланк BBVA airtickets (12/IX) Madrid – Heraklion – Madrid на 18–23/V, HMM '2019.

(14/IV) E-mails to Tenner и Окс. (Dodd и Sklyanin's solutions). Sine-Gordon eq.

Dodd et al. in the book „Solitons and NL wave equations" нашли Sine-Gordon $\theta(x,t)$ для $\Phi = \frac{\pi}{2} + \varepsilon$ вблизи ЕЛ biaxial ferromagnet, $\varepsilon \ll 1$, то $\varepsilon_x = 0$, $\varepsilon_t \neq 0$.

Lleq. (θ, Φ): $\left\{ \begin{array}{l} \sin\theta\dot{\theta} = \lambda \sin^2\theta \sin\Phi\cos\Phi + \Phi_{xx}\sin^2\theta + \theta_x\Phi_x \sin 2\theta \\ -\sin\theta\dot{\Phi} = \theta_{xx} - [1 + \lambda\cos^2\Phi + (\Phi_x)^2] \sin\theta\cos\theta \end{array} \right\}$

Пусть $\lambda \gg 1$, $\underset{\Phi_x = 0}{\Phi = \pi/2 - \varepsilon} \Rightarrow \dot{\theta} = \lambda \sin\theta\,\varepsilon$, $\sin\theta\,\dot{\varepsilon} = \theta_{xx} - (1 + \lambda\varepsilon^2)\sin\theta\cos\theta$

$\ddot{\theta} = \lambda\dot{\varepsilon}\sin\theta + \lambda\varepsilon\cos\theta\,\dot{\theta} = \lambda\left[\theta_{xx} - (1+\lambda\varepsilon^2)\sin\theta\cos\theta\right] + \lambda\varepsilon\cos\theta \cdot \lambda\sin\theta\varepsilon$ и формально $\ddot{\theta} = \lambda(\theta_{xx} - \sin\theta\cos\theta)$, $\theta_{xx} - \frac{1}{\lambda}\ddot{\theta} = \frac{1}{2}\sin\theta\cos\theta$ – верное Sine-Gordon $(\lambda \to \infty, \lambda = c^2)$

но из SG $\dot{\theta}/\sin\theta = \dot{\zeta} = -v = \text{const}(t)$ и $\varepsilon(t) = \frac{1}{\lambda}\frac{\dot{\theta}}{\sin\theta} = \text{const}$, $\dot{\varepsilon} = 0$ и преобразование через $\dot{\varepsilon} \neq 0$ делать нельзя. Надо $\varepsilon(t) = \text{const}$.

(15/IV) Editing/changing Verba NS по artificial Skyrmions. Hocian revised version to Roman y Glob при. Session chairs „SolSkyMag 2019". Update Conf. Program.

(16/IV) Встреча с Julian (Conf.) и Ricardo (1D-DW, Sklyanin solution, куда сдать NS?).

Вдали сидим по 2го из-за праздников 28-29/IV и 1-2/V не удается прогр. до 7/V.

„Super Amara". Hocian Report to Ann Marthor (не нашл. отпуск аудиты FIS2016).

(17/IV) переделка SolSkyMag'2019 program. Invit. letters to Session chairs. BBVA-award? Финансы с A.

(18/IV) Разбор над 1D-DW manuscript (Sklyanin's solution). Min action → LLG

Action $\mathcal{S} = \int dt \int dx\, \mathcal{L}(x,t)$. Можно ли LLG вывести из $\mathcal{S} \to \min$ для travelling DW с $\xi = x - vt$? Какая форма of Lagrangian density $\mathcal{L}$?

• E. Magyari, H. Thomas, PRB 25, 531 (1982): предлагают искать min of functional

$$K[\theta, \Phi] = E(\vec{m}) + v\int d\xi\, \Phi'_\xi \cos\theta, \quad \theta \perp \hat{z}, \hat{x} \perp DW, \quad \xi = z - vt, \quad \underset{(S)}{E} = \int \frac{dz}{2}\left[(\vec{S}')^2 + S_z^2 - bS_x^2\right]$$

$b = B_x/B_a$, $B_a = 2K_z S/g\mu_B$ – anisotropy field „easy plane" $\perp Oy$.

не говорит, что $K[\theta,\Phi] = \int d\xi\,[e(\theta,\Phi) + v\cos\theta\,\Phi'_\xi]$ есть action/действие.

• E. Magyari, H. Thomas, Z. Phys. B – Cond. Mat. 59, 167 (1985): $\Delta_{1,2}(v)$ – DW width acc. Schlömann '71. Neel-branch с большой $E_{DW}(v) \sim \frac{1}{\Delta(v)}$ – неадиабатично относительно a non-uniform localized perturbation („corrugating" mode), и $m_{DW} < 0$, see Slonczewski AIP Conf. 5, 170 (1972)

Если явно $\Delta_{1,2}(v)$ acc. Schlömann '71 и $\omega_{1,2}(q)$ DW instability.

Escaneado con CamScanner

$\varphi_{tt} - \varphi_{xx} = -V'(\varphi)$ Klein-Gordon eq. with potential $V(\varphi)$ допускает солитон

traveling solutions $\varphi(x,t) = \varphi_s(x - vt)$, $\varphi_s(\pm\infty) = \varphi_\pm$, $\varphi_\pm$ are local minima of $V(\varphi)$.

Доказано no static case $v=0$, т.к. Klein-Gordon dynamics $x \to \frac{(x-vt)}{\sqrt{1-v^2}}$, $\varphi(x,t) = \varphi(\xi)$.

→ E. Magyari, H. Thomas, Z. Phys. B 53, 141 (1984): DW instability in biaxial ferromagnets

$E[\vec{S}] = \frac{1}{2}\int dz \left[(\vec{S}')^2 + a S_z^2 - S_x^2\right]$, $\dot{\vec{S}} = \vec{S} \times (\vec{S}'' + S_x \hat{x} - a S_z \hat{z})$

$a = \frac{K_2}{K_x}$ uniaxial pos. mode $\lambda = K_y/K_z$ ($x \parallel z$, $\hat{z} \perp$ DW, DW plane $\perp Oy$).

Обз. integrals of motions see Y. Eleonskii '78, '79. $\xi = z - ut$, уд. ур.

$\min K[\theta,\Phi] = E(\theta,\Phi) + u\int d\xi \cos\theta \Phi'_\xi$. Schlömann solution $S_x = -\text{th}(q\xi)$, $S_{y,z} = k_{y,z}\frac{1}{\text{ch}(q\xi)}$

где $(q_\pm)^2 = 1 + \frac{1}{2}(a - u^2) \pm \frac{1}{2}\sqrt{(a-u^2)^2 - 4u^2}$ - для $\varkappa_\pm(v)$ из 1D-DW manuscript.

$u < u_0 = \sqrt{1+a} - 1$, $u_0 = v_-$. $E_{DW}(v) = 2q(v)$ - получено авто. DW $\varphi(x=\pm\infty) = 0, \pi$.

If $a \gg 1$, LL eqs. yield by "adiabatic elimination" of $\theta = \frac{1}{a}\ddot{\varphi}$, $\left(\varphi'' - \frac{1}{a}\ddot{\varphi} = \frac{1}{2}\sin 2\varphi\right)$ -

- Bernoni sine-Gordon for angle $\varphi(x,t)$. [Ref. Mikeska, JAP 52, 1950 (1981)].

Далее SG $\varphi(x,t) \overset{\sin}{=} \pm \frac{1}{\text{ch}(\gamma\xi)}$, $\gamma = \frac{1}{\sqrt{1 - u^2/a}}$ and for $a \gg 1$, $u^2 \ll a = v_-^2$ the "slowly moving" DW is given in "sine-Gordon approximation" $S_x = -\text{th}(\gamma\xi)$, $S_y = \frac{(\pm 1)}{\text{ch}(\gamma\xi)}$, $S_z = \frac{u\gamma}{a}\frac{(\pm 1)}{\text{ch}(\gamma\xi)}$

$S_z = \frac{u}{a\sqrt{1-u^2/a}}\frac{1}{\text{ch}(\gamma\xi)}$ Сравн. новому решению $m_x = \varepsilon \sin\theta$ где $\varepsilon = \text{const}$, $\Phi(\xi) = \text{const}$

$S_z = \frac{\gamma u}{a}\sin\varphi(\xi)$ и $\varepsilon = \frac{v}{\lambda}\frac{1}{\sqrt{1-v^2/\lambda}}$.

• H.J. Mikeska, JAP 52, 1950 (1981): кинк ОПН в DW-plane, есть B_x-field.

есть sine-Gordon $\Phi''_{zz} - \frac{1}{c^2}\Phi_{tt} = m^2 \sin\Phi$, $m^2 = \frac{2M_s B_x}{JS}$ получается из LL eqs. при

$\theta_{zz} \to 0$ и $v^2/c^2 \ll 1$ (Видимо $v \to 0$ limit Magyari '84 и называл "adiabatic elimination")

• H.J. Mikeska, Z. Phys. B Cond. Mat. 52, 111 (1983): $\lambda = \frac{2AS}{\mu B_x}$, A - easy plane anis $A\sum_n (S_n^z)^2$

SG при $\lambda \to \infty$ дает $\Phi(x,t)$. Static SG $\left[\Phi = \Phi_0 = 4\arctg e^{\pm\frac{\mu B_x}{c^2}\theta} = 0\right]$ is unstable at $\lambda < 3$ (!).

Это при m. field B_x должен ОПН в "плоском DW" $\perp Oy$. и это дает kink, not DW.

• E. Magyari, H. Thomas, PRB 29, 6358 (1984): kink, B_x — dyn SG отличается от LL eq. даже при $u \to 0$, крайне слаб. условий на B_x, $B_x << B_c(u)$.

Противоречит SG-Mikeska JAP '81 и approximate solution LL eqs at $u \to 0$. Это спорно

говорит что SG-Mikeska '81 is wrong.

Sin-Gordon, $\mathcal{E}(\xi)$

→ уравнение sine-Gordon дает $\theta(x,t)$ описывает $\eta(\xi) = \xi/\Delta$. Сом. 20/II решение

системы дает $\theta(\xi)$, $\mathcal{E}(\xi)$ которая $\eta(\xi) = \xi/\Delta$ и $\mathcal{E}(\xi) = \frac{v}{\lambda\Delta}\frac{1}{\text{ch}(\xi/\Delta)}$, т.е. $\theta(x,t) \in$ SG,

несмотря на variable $\mathcal{E}(\xi)$ Сом. 20/II, можно 3-7/III и показать, что $\mathcal{E}(\xi) = \left(\frac{v}{\lambda\Delta}\right)\frac{1}{\text{ch}(\xi/\Delta)}$ -

не есть решение LL eq, даже при $\lambda \to \infty$.

7/III и показать что deviation от $\mathcal{E}(\xi) = \mathcal{E}_0 = \frac{v}{\lambda\Delta}$ и $\eta_0(\xi) = \xi/\Delta \sim e^{\pm\Delta_k \xi}$ экспоненциально

велики при $\xi \to \pm\infty$. только $\mathcal{E}(\xi) = \text{const}$ дает $\eta(\xi) = \xi/\Delta$ и SG-sine $\theta(x,t)$.

(*) $\begin{cases} v\eta_\xi = \lambda\mathcal{E} - \mathcal{E}_{\xi\xi} + 2\mathcal{E}_\xi \eta_\xi \text{th}\eta \\ v\mathcal{E}_\xi = \eta_{\xi\xi} + \left[1 + \lambda\mathcal{E}^2 + (\mathcal{E}_\xi)^2 - (\eta_\xi)^2\right]\text{th}\eta \end{cases}$ - LL eq. + 1st integral $\eta_\xi^2 + \mathcal{E}_\xi^2 = 1 + \lambda\mathcal{E}^2 + \text{const}\cdot\text{ch}^2\eta$.

при $\mathcal{E}(\xi) = \text{const} \Rightarrow \eta_\xi = \frac{\lambda}{v}\mathcal{E}$, $\eta = \frac{\xi}{\Delta}$, $1 + \lambda\mathcal{E}^2 = \frac{1}{\Delta^2}$, $\Delta^2 = 1 - v^2$. $\mathcal{E} = \sqrt{\delta}/\lambda\Delta$.

Это если $\mathcal{E}(\xi) = v\mu(\xi)$, $\eta(\xi) = \frac{\xi}{\Delta} + \nu(\xi)v$, $\eta_\xi = \frac{1}{\Delta} + \nu_\xi v$, $\eta_{\xi\xi} = v\nu_{\xi\xi}$ и $v \to 0$, $v/c \ll 1$.

$\begin{cases} \delta\left(\frac{1}{\Delta} + v\nu_\xi\right) = \lambda v\mu - v\mu_{\xi\xi} + 2v\mu_\xi\left(\frac{1}{\Delta} + v\nu_\xi\right)\text{th}\eta \\ v^2\mu_\xi = v\nu_{\xi\xi} + \left[1 + \lambda v^2\mu^2 + v^2\mu_\xi^2 - \left(\frac{1}{\Delta} + v\nu_\xi\right)^2\right]\text{th}\eta \end{cases}$ Assume $\mu(\xi) \sim \frac{1}{\lambda} << 1$, $\nu(\xi) \sim 1$

$\frac{v}{\Delta} + v^2\nu_\xi = \lambda v\mu + 2v\mu_\xi\frac{1}{\Delta}\text{th}(\xi/\Delta) \to \frac{1}{\Delta} = \lambda\mu$ и $v\nu_\xi = 2\mu_\xi\frac{1}{\Delta}\text{th}(\xi/\Delta)$, $\mu(\xi) = \frac{1}{\lambda\Delta}$ и $\nu_\xi = 0$.

$0 = v\nu_{\xi\xi} + \left[1 - \left(\frac{1}{\Delta} + v\nu_\xi\right)^2\right]\text{th}\eta \cdot \left[1 + \lambda^2 v^2\mu^2 - \left(\frac{1}{\Delta} + v\nu_\xi\right)^2\right] = 0$, $1 + \lambda^2 v^2\frac{1}{\lambda^2\Delta^2} = \frac{1}{\Delta^2}$, $\Delta^2 = 1 - v^2/\lambda$.

- ТВ по $v << 1$, не работает. Сравнить k $\mathcal{E}(\xi) = v/\lambda\Delta = \text{const}$, $\nu(\xi) = \nu_0 = \text{const}$.

$\lambda + \lambda\mathcal{E}^2 + \mathcal{E}_\xi^2 - \eta_\xi^2 = \left|\mathcal{H} = 0\right| = (\eta_\xi^2 + \mathcal{E}_\xi^2) + \mathcal{E}_\xi^2 - \eta_\xi^2 = 2\mathcal{E}_\xi^2$, $v\mathcal{E}_\xi = \eta_{\xi\xi} + 2\mathcal{E}_\xi^2\text{th}\eta \to$

$\mathcal{E}_\xi = \frac{1}{4}\text{cth}\eta\left[v \pm \sqrt{v^2 - 8\eta_{\xi\xi}\text{th}\eta}\right]$, $v^2\mu_\xi = v\nu_{\xi\xi} + 2v^2\mu_\xi^2\text{th}\eta$, $\nu_{\xi\xi} = v\mu_\xi = \mathcal{E}_\xi = \frac{1}{v}\eta_{\xi\xi}$

$\mathcal{E}_\xi = \frac{1}{4}\text{cth}\eta\left[v \pm \sqrt{-8\eta_{\xi\xi}\text{th}\eta}\right]_{v\to 0} = \pm\frac{1}{4}\text{cth}\eta\sqrt{-8\eta_{\xi\xi}}\sqrt{\text{th}\eta} = \pm\frac{1}{4}\sqrt{-8}\frac{1}{\sqrt{\text{th}\eta}}\sqrt{\eta_{\xi\xi}}$ при $v \to 0$. $\mathcal{E}_\xi^2 = -\frac{1}{2}\frac{\eta_{\xi\xi}}{\text{th}\eta}$

Escaneado con CamScanner

(19/IV) Bug Lagrangian of ferromagnet and action.

· E. Schlömann JAP 1972: $\delta \int dx\, \mathcal{L} = 0$ (max), $\mathcal{L} = \mathcal{T} - \mathcal{E}$, $\mathcal{T}$ - kinetic energy density, $\mathcal{E}$ - potential

$\mathcal{T} = -\frac{M_s}{\gamma} v (\cos\theta)' \Phi$ или $\mathcal{T} = \frac{M_s}{\gamma} \cos\theta \Phi' v$, $(\cdot)' = \frac{\partial}{\partial x}(\cdot)$, $\mathcal{E} = A(\nabla \vec{m})^2 + E_a$

$\mathcal{T} = \frac{M_s}{\gamma} v \sin\theta\, \theta_\xi \Phi$ или $\mathcal{T} = \frac{M_s}{\gamma} \cos\theta \Phi_\xi v$, $\frac{\partial}{\partial \xi} = \frac{\partial}{\partial x}$, $\xi = x - vt$.

LL eq. $\frac{M_s}{\gamma} \sin\theta \dot{\Phi} = \frac{\delta E}{\delta \theta}$, $-\frac{M_s}{\gamma} \sin\theta \dot{\theta} = \frac{\delta E}{\delta \Phi}$, $\dot{q} = -v q_\xi$, $q = \theta, \Phi$. $\max L = \int dx\, \mathcal{L}$

$\frac{\delta E}{\delta \theta} = \frac{\partial E}{\partial \theta} - \nabla \cdot \frac{\partial E}{\partial (\nabla \theta)}$, $\frac{\delta E}{\delta \Phi} = \frac{\partial E}{\partial \Phi} - \nabla \cdot \frac{\partial E}{\partial (\nabla \Phi)}$, $\gamma > 0$.

In dimensionless units $L = T - E = \int d\xi \left[+ v \sin\theta\, \theta_\xi \Phi - \frac{1}{2}\left\{ (\theta')^2 + \sin^2\theta (\Phi')^2 + \varepsilon_a \right\} \right] = \int d\xi\, \mathcal{L}$

$\varepsilon_a(\theta, \Phi) = \sin^2\theta (1 + \lambda \cos^2\Phi)$, $\mathcal{L} = -\cos\theta \dot{\Phi} + v(\cos\theta \Phi_\xi) - \mathcal{E}(\theta, \Phi)$, $\mathcal{E}(\theta, \Phi) = \frac{1}{2}\left[(\theta')^2 + \sin^2\theta (\Phi')^2 + \varepsilon_a \right]$

- в такой форме $\mathcal{L}$ и использовал в 1D. 2w manuscript.

$\frac{d}{d\xi}(\cos\theta \Phi) = -\sin\theta \theta_\xi \Phi + \cos\theta \Phi_\xi$, $\int_{-\infty}^{+\infty} d\xi \frac{d}{d\xi}(\cos\theta \Phi) = \cos\theta\Phi \Big|_{\rho=-\infty}^{+\infty} = \text{const}$ (0 для DW)

Поэтому под $\int d\xi (\dots)$ $\sin\theta \theta_\xi \Phi = \cos\theta \Phi_\xi$, см. 2/IV. $T(\Phi_0)$ walker $= \frac{M_s}{\gamma} 2\Phi_0 v$, $\Phi(\xi) = \Phi_0$.

$\mathcal{L} = \mathcal{T} - \mathcal{E}$ дает верное LLeq., но $L = \int d\xi\, \mathcal{L} \to \max$.

Приходилось $L = \int d\xi\, \mathcal{L} \to \max$ чтобы решить Eleonskii '78: $L_{Eleon} = -2L$, и $L_{Eleon} \to \min$.

• H.-B. Braun, D. Loss, PRB 1996: Classical Lagrangian density $\mathcal{L} = -\frac{SN}{a} \cos\theta \dot{\Phi} + \mathcal{H}$

$\mathcal{H} = N \int dx \left\{ J\left[(\theta_x)^2 + \sin^2\theta (\Phi_x)^2 \right] + K_z \cos^2\theta - K_y \sin^2\theta \sin^2\Phi \right\}$ - the energy $\mathcal{H}$.

Т.е. Braun-Loss заменили $-\mathcal{E}$ на $\mathcal{H}$, исказив signs in LL eq. Braun-Loss cited Brown'63.

→ W.F. Brown, Micromagnetics 1963: $G = \int dV \left[A(\nabla \vec{m})^2 + W_a \right]$, $W_a = K_1 (\vec{m} \cdot \vec{n})^2 + \dots$

Lagrangian $L = \frac{M_s}{\gamma_0} \int dV \cos\theta \dot{\Phi} - G = T - G = \int dV \left[\frac{M_s}{\gamma_0} \cos\theta \dot{\Phi} - \left\{ A(\nabla \vec{m})^2 + W_a \right\} \right]$

Hamilton's principle $\delta \int dt\, L = 0$ - min action. Angular momnt $\vec{G} = \frac{1}{\gamma_0} \vec{M}$, $\gamma_0 = -\gamma < 0$ где $\gamma > 0$

$\frac{d\vec{M}}{dt} = \gamma_0 [\vec{M} \times \vec{H}]$, $\frac{1}{M_s}\left[-\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\vec{m}}} + \frac{\partial \mathcal{L}}{\partial \vec{m}} \right] = \frac{1}{M_s}(-)\frac{M_s}{\gamma_0} \frac{\dot{\vec{m}} \times \vec{k}}{\gamma}$, (α, β, γ) - направ. cos $\vec{m}$.

$-\gamma_0^{-1} \frac{\dot{\vec{m}} \times \vec{k}}{\gamma} + \vec{H} = \lambda \vec{M}$, $-\frac{1}{\gamma_0} \vec{M} \times \left(\frac{\dot{\vec{m}} \times \vec{k}}{\gamma} \right) + \vec{M} \times \vec{H} = 0$, $\vec{m} \times (\dot{\vec{m}} \times \vec{k}) = \gamma \dot{\vec{m}}$

$-\frac{M_s}{\gamma_0} \dot{\vec{m}} + \vec{M} \times \vec{H} = 0$, $\frac{d\vec{M}}{dt} = \gamma_0 (\vec{M} \times \vec{H})$. Сравн. с [Мелихов, Магн. колебания] $\dot{\vec{M}} = -\gamma \vec{M} \times \vec{H}$, $\gamma > 0$.

Braun-Lagrangian $L = \int dV \left[-\frac{M_s}{\gamma} \cos\theta \dot{\Phi} - \mathcal{E} \right]$, $\mathcal{E} = A(\nabla \vec{m})^2 + \varepsilon_a(\vec{m})$. $L = \int dV \mathcal{L}$.

$\mathcal{L}(\text{Schl}) = -\frac{M_s}{\gamma} \cos\theta \dot{\Phi} - \mathcal{E}(\theta, \Phi)$, $\mathcal{E} = A(\nabla \vec{m})^2 + \varepsilon_a$. $\mathcal{L}(\text{Brown}) = \mathcal{L}(\text{Schl})$, т.е.

Schlömann'72 следует Brown '1963, а Braun-Loss'96 произвольно $-\mathcal{E} \to \mathcal{H}$ (energy)

$\delta S = \delta \int dt L = 0$ - max, если сначала $S = \int dt L$ - action. поменяли на max energy.

(20/IV) С Ю.В. Ветоки" Tolosa. Уборка в доме. Написание 1D. DW manuscript.

Mikeska'1991 ссылается на Long-Bishop'79, якобы они подвергли Boberg SG eq. by Enz'1964. Paper Long-Bishop'79 я разобрал 26/I, они приняли $\Phi \ll 1$, а надо $\Phi_e = \frac{\pi}{2} - \Phi \ll 1$ как у Enz'1964.

• Long-Bishop'79: minimize $I(\theta, \Phi, \Omega_1, v) = \int dx\, \mathcal{H}(x) - \Omega_1 (M - M_0) - v(P - P_0)$, Ω_1, v - are Lagrangian multipliers to account conservation of momentum $P = S \int dx (1 - \cos\theta) \frac{\partial \Phi}{\partial x}$ and angular momentum (M_z-comp.) $M = S \int dx\, (\cos\theta - 1)$, $P = P_0 = \text{const}$, $M = M_0 = \text{const}$, $\vec{S} = S(\sin\theta\cos\Phi, \sin\theta\sin\Phi, \cos\theta)$

$\cos\theta(x,t) = \cos\theta(x - vt)$ и $\Phi(x,t) = \Omega_1 t + \bar{\Phi}(x - vt)$, т.е. $\Omega_1 = \bar{\omega}$ - soliton precession frequency in moving frame [Kosevich'90]

$I(v) = -vP = -v \int dx (1 - \cos\theta) \frac{\partial \Phi}{\partial x} S$, $-vP = -vS(1 - \cos\theta)\frac{\partial \Phi}{\partial x} = vS\cos\theta \frac{\partial \Phi}{\partial x} = -\dot{S}\cos\theta\dot{\Phi}$

min

Т.е. Long'79 suggest to minimize $I = \int dx \left[\mathcal{H}(x) - \dot{S}\cos\theta\dot{\Phi} \right]$, Lagrangian $L = \int dx \left[-\frac{M_s}{\gamma} \cos\theta\dot{\Phi} - \mathcal{E}(x) \right]$

Надо $I = \int dx \left[\mathcal{H}(x) + \dot{S}\cos\theta\dot{\Phi} \right] \to \min$, $L \to \max$ (сравн. Schlömann'72, Brown'63).

U. Enz' Helv. Phys. Acta '1964: γD $\varphi = \varepsilon = \frac{\pi}{2} - \Phi \ll 1$

$\left\{ \begin{array}{l} \theta_{xx} - \frac{1}{2}\sin 2\theta = \dot{\varepsilon} \\ \lambda\varepsilon = \dot{\theta} \end{array} \right\}$ Enz' 64 $\quad \left\{ \begin{array}{l} \sin\theta\, \dot{\varepsilon} = \theta_{xx} - (1 + \lambda\varepsilon^2)\sin\theta\cos\theta \\ \dot{\theta} = \lambda \sin\theta\, \varepsilon, \quad \varepsilon_x = 0 \end{array} \right\}$ или γD, $(\varepsilon_x)^2 \to 0$

$\left\{ \begin{array}{l} 2A \frac{\partial^2 \theta}{\partial x^2} - K \sin 2\theta = \frac{M}{\gamma} \frac{\partial \varphi}{\partial t} \\ 4\pi M^2 \varphi = \frac{M}{\gamma} \frac{\partial \theta}{\partial t} \end{array} \right\}$. В моих переменных $K = K_2$, $2\pi M_s^2 = K_x$, $A_0^2 = A/K_2$, $t_0 = \frac{1}{\gamma H_a}$, $H_a = 2K_2/M_s$, $\lambda = K_y/K_2$

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УД Энг'64 (невернее) дает $\theta_{xx} - \frac{1}{\lambda}\theta_{tt} = \frac{1}{2}\sin 2\theta$ вернее sine-Gordon, $\lambda = c^2 \gg 1$.

$c = |Eнг'64| = 2\beta\sqrt{2\pi A}$, ибо $c = \sqrt{\lambda} = \sqrt{\frac{K_x}{K_z}}$ in abs. units, $c = \sqrt{\frac{K_x}{K_z}}\frac{2\beta}{M_s}\sqrt{AK_z} = 2\beta\sqrt{2\pi A}$

невернее eq. УД (14'), (12') значит, что они не следуют из LLeq. при $\varepsilon = \pi/2 - \varphi \ll 1$. где $K_x = 2\pi M_s^2$.

Мак УД (см. 14/IV) также дает SG: $\theta_{xx} - \frac{1}{\lambda}\theta_{tt} = \frac{1}{2}\sin 2\theta$, но не выдерживает self-consistency check, ведь $\varepsilon(t) = \frac{1}{\lambda}\frac{\dot\theta}{\sin\theta} = \text{const}$, если $\theta(x,t)$ удовл. SG, и $\dot\varepsilon = 0$

$\theta(x,t) = 2\arctg e^{\xi/\Delta}$, $\xi = x - vt$, $\dot\theta = 2\frac{1}{e^{2\xi/\Delta}+1}e^{\xi/\Delta}\frac{1}{\Delta}\dot\xi = \frac{1}{\operatorname{ch}(\xi/\Delta)}\frac{1}{\Delta}(-v) = -\frac{v}{\Delta}\sin\theta$

$\cos\sin\theta = \frac{1-\operatorname{tg}^2\theta/2}{1+\operatorname{tg}^2\theta/2} = \frac{1-e^{2\xi}}{1+e^{2\xi}} = -\operatorname{th}(\frac{\xi}{\Delta})$ $\quad \sin\theta = \frac{2\operatorname{tg}\theta/2}{1+\operatorname{tg}^2\theta/2} = \frac{2e^{\xi}}{1+e^{2\xi}} = \frac{1}{\operatorname{ch}(\xi/\Delta)}$

→ (θ,φ) УД при $\varepsilon_x \neq 0$, $\varepsilon^2 \ll 1$ (*) $\begin{cases} \dot\theta = -v\theta_\xi = \lambda\sin\theta\,\varepsilon - \varepsilon_{\xi\xi}\sin\theta - 2\theta_\xi\varepsilon_\xi\cos\theta \\ \sin\theta\,\dot\varepsilon = -v\sin\theta\,\varepsilon_\xi = \theta_{\xi\xi} - (1+\lambda\varepsilon^2)\sin\theta\cos\theta \end{cases}$ при $(\varepsilon_\xi)^2 \ll 1$.

$-v\theta_{\xi\xi} = \lambda\sin\theta\varepsilon_\xi + \lambda\varepsilon\cos\theta\theta_\xi - \varepsilon_{\xi\xi\xi}\sin\theta - \varepsilon_{\xi\xi}\cos\theta\theta_\xi - 2\theta_{\xi\xi}\varepsilon_\xi\cos\theta - 2\theta_\xi\varepsilon_{\xi\xi}\cos\theta + 2\sin\theta\,\varepsilon_\xi(\theta_\xi)^2 =$

$= \lambda\left(\frac{-1}{v}\right)\left[\theta_{\xi\xi} - (1+\lambda\varepsilon^2)\sin\theta\cos\theta\right] + \lambda\varepsilon\cos\theta\frac{(-1)}{v}\left[\lambda\sin\theta\,\varepsilon - \varepsilon_{\xi\xi}\sin\theta - 2\theta_\xi\varepsilon_\xi\cos\theta\right] - \frac{2}{2\xi}\{\dots\}$

$\left(\frac{\lambda}{v} - v\right)\theta_{\xi\xi} = \frac{\lambda}{v}\sin\theta\cos\theta + \frac{\lambda}{v}\varepsilon\cos\theta\left[\varepsilon_{\xi\xi}\sin\theta + 2\theta_\xi\varepsilon_\xi\cos\theta\right] - \varepsilon_{\xi\xi\xi}\sin\theta - \varepsilon_{\xi\xi}\theta_\xi\cos\theta -$

$- 2\theta_{\xi\xi}\varepsilon_\xi\cos\theta - 2\theta_\xi\varepsilon_{\xi\xi}\cos\theta + 2\sin\theta\,\varepsilon_\xi(\theta_\xi)^2$, если пренебречь $\lambda\varepsilon\varepsilon_{\xi\xi}$, $\lambda\varepsilon\varepsilon_\xi$ и $\varepsilon_\xi\varepsilon_{\xi\xi}$, $\varepsilon_\xi\varepsilon_{\xi\xi\xi}$,

то $\frac{1}{v}(\lambda - v^2)\theta_{\xi\xi} = \frac{\lambda}{v}\sin\theta\cos\theta$, $\left(1 - \frac{v^2}{c^2}\right)\theta_{\xi\xi} = \sin\theta\cos\theta$, $\theta_{\xi\xi} = \frac{1}{2\Delta^2(v)}\sin 2\theta$, $\Delta^2 = 1 - v^2/c^2$.

$\theta_{\xi\xi} = \frac{1}{2\Delta^2}\sin 2\theta$, $\Rightarrow y = \xi/\Delta$. $\operatorname{tg}\theta/2 = e^{-y}$. $\theta(x,t) = \theta(\xi) = \theta(x - vt)$.

$\theta_{\xi\xi} = \frac{1}{2\Delta^2}\sin 2\theta \to \theta_{xx} - \frac{1}{v_m^2}\theta_{tt} = \frac{1}{2\Delta_e^2}\sin 2\theta$, $\Delta_e = \frac{\Delta}{\sqrt{1 - v^2/v_m^2}} = \frac{\sqrt{1-v^2/c^2}}{\sqrt{1-v^2/v_m^2}} = 1$ при $v_m = c$ и $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{2}\sin 2\theta$ — вернее SG $\varepsilon = \text{const}$

т.е. будем $\varepsilon = \varepsilon(\xi)$ и пренебрежем всеми $\lambda\varepsilon\varepsilon_\xi$, ε_ξ, $\varepsilon_{\xi\xi}$, $\varepsilon_{\xi\xi\xi}$ и получим такой же

SG $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{2}\sin 2\theta$ как при $\varepsilon = \text{const}(\xi)$ и $y = \xi/\Delta$, $\varepsilon \to 0$, $\lambda \to \infty$ (!)

Если сразу $\varepsilon = \text{const}$ в УД, то $-v\theta_\xi = \lambda\sin\theta\,\varepsilon$, $\theta_{\xi\xi} = (1+\lambda\varepsilon^2)\sin\theta\cos\theta = \frac{1}{\Delta^2}\sin\theta\cos\theta$

$\frac{1}{\Delta^2} = 1 + \lambda\varepsilon^2$, $\theta_\xi = \frac{1}{\operatorname{ch}(\xi/\Delta)}\frac{1}{\Delta} = \frac{1}{\Delta}\sin\theta$ $\quad \theta_\xi = -\frac{1}{\Delta}\sin\theta$ при $\operatorname{tg}\theta/2 = e^{-y}$, $\cos\theta = \operatorname{th} y$,

$-v\theta_\xi = -v\left(-\frac{1}{\Delta}\sin\theta\right) = \frac{v}{\Delta}\sin\theta = \lambda\varepsilon\sin\theta$, $\frac{v}{\Delta} = \lambda\varepsilon$, $\varepsilon = \frac{v}{\lambda\Delta}$, $\frac{1}{\Delta^2} = 1 + \lambda\frac{v^2}{\lambda^2\Delta^2}$

$\frac{1}{\Delta^2}\left(1 - \frac{v^2}{\lambda}\right) = 1$, $\Delta^2 = (\lambda - v^2/\lambda)$ и $\varepsilon = \frac{v}{\lambda\sqrt{1 - v^2/\lambda}}$ — вернее $\varepsilon = \text{const}$.

Вопрос: если $\varepsilon(\xi)$, то какое оно? Зная, что $\theta(\xi)$ удовл. SG $\theta_{\xi\xi} = \frac{1}{2\Delta^2}\sin 2\theta$, $\varepsilon \to 0$

$-v\sin\theta\,\varepsilon_\xi = \theta_{\xi\xi\xi} - (1+\lambda\varepsilon^2)\sin\theta\cos\theta = \left[\frac{1}{\Delta^2} - (1+\lambda\varepsilon^2)\right]\sin\theta\cos\theta$, $\cos\theta = \operatorname{th}(\xi/\Delta)$ и $\operatorname{tg}\theta/2 = e^{-y}$.

$-v\varepsilon_\xi = \left(\frac{1}{\Delta^2} - 1 - \lambda\varepsilon^2\right)\cos\theta$. $\Delta^2 = 1 - v^2/\lambda$, $\varepsilon_\xi = \frac{1}{v}\left(\lambda\varepsilon^2 + 1 - \frac{1}{\Delta^2}\right)\operatorname{th}(\xi/\Delta)$ — диф. ур. на $\varepsilon(\xi)$.

$1 - \frac{1}{\Delta^2} = \frac{1}{\Delta^2}(\Delta^2 - 1) = \frac{1}{\Delta^2}\left(1 - \frac{v^2}{\lambda} - 1\right) = -\frac{v^2}{\lambda\Delta^2}$.

$-v\theta_\xi = \lambda\sin\theta\,\varepsilon - \varepsilon_{\xi\xi}\sin\theta - 2\theta_\xi\varepsilon_\xi\cos\theta$, $\theta_\xi = -\frac{1}{\Delta}\sin\theta \to \frac{v}{\Delta}\sin\theta = \lambda\sin\theta\,\varepsilon -$

$- \varepsilon_{\xi\xi}\sin\theta - 2\left(-\frac{1}{\Delta}\sin\theta\right)\varepsilon_\xi\cos\theta$, $\frac{v}{\Delta} = \lambda\varepsilon - \varepsilon_{\xi\xi} + \frac{2}{\Delta}\varepsilon_\xi\operatorname{th}(\xi/\Delta)$, $\varepsilon_{\xi\xi} - \frac{2}{\Delta}\operatorname{th}\frac{\xi}{\Delta}\cdot\varepsilon_\xi - \lambda\varepsilon = \frac{v}{\Delta}(-1)$

— лин. неоднород. диф. ур. на $\varepsilon(\xi)$. $\varepsilon'' - 2\Delta^{-1}\operatorname{th}(y)\varepsilon' - \lambda\Delta^2\varepsilon = -v\Delta$, $\varepsilon' = \frac{d\varepsilon}{dy}$, $y = \xi/\Delta$.

Будет ли $\varepsilon(y) = \frac{a}{\operatorname{ch} y}$ решением? $\varepsilon' = -\frac{a}{\operatorname{ch}^2 y}\operatorname{sh} y = -\varepsilon\operatorname{th} y$, $\varepsilon'' = -\varepsilon'\operatorname{th} y - \varepsilon\frac{1}{\operatorname{ch}^2 y}$

$\varepsilon'' = \varepsilon\operatorname{th}^2 y - \frac{\varepsilon}{\operatorname{ch}^2 y}$, $\operatorname{th}^2 y + \frac{1}{\operatorname{ch}^2 x} = 1$, $\operatorname{th}^2 x - \frac{1}{\operatorname{ch}^2 x} = \frac{\operatorname{sh}^2 x - 1}{\operatorname{ch}^2 x} = 1 - \frac{2}{\operatorname{ch}^2 x}$

$\varepsilon\left(\lambda - \frac{2}{\operatorname{ch}^2 y}\right) - 2\Delta^{-1}\operatorname{th} y(-\varepsilon)\operatorname{th} y - \lambda\Delta^2\varepsilon = -v\Delta$, $\varepsilon\left[1 - \frac{2}{\operatorname{ch}^2 y} + 2\Delta^{-1}\operatorname{th}^2 y - \lambda\Delta^2\right] = -v\Delta$.

$\operatorname{th}^2 y - \frac{1}{\operatorname{ch}^2 y} = \operatorname{th}^2 y - (1 - \operatorname{th}^2 y) = 2\operatorname{th}^2 y - 1$, $\varepsilon\left[2\operatorname{th}^2 y + 2\Delta^{-1}\operatorname{th}^2 y - 1 - \lambda\Delta^2\right] = -v\Delta$

$\frac{d\varepsilon}{dy} = \frac{\Delta}{v}\left[\lambda\varepsilon^2 + 1 - \frac{1}{\Delta^2}\right]\operatorname{th} y$, $\varepsilon = \frac{a}{\operatorname{ch} y}$, $\varepsilon' = -\frac{a}{\operatorname{ch} y}\operatorname{th} y$, $\varepsilon^2 = \frac{a^2}{\operatorname{th}^2 y} = a^2(1 - \operatorname{th}^2 y)$

Видно, что $\varepsilon(\xi) = \frac{a}{\operatorname{ch}(\xi/\Delta)}$ не удовлетв. ни одному из диф. ур. на $\varepsilon(\xi)$.

$\varepsilon_{\xi\xi} - \frac{2}{\Delta}\operatorname{th}\left(\frac{\xi}{\Delta}\right)\varepsilon_\xi - \lambda\varepsilon = -\frac{v}{\Delta}$ → Л.О.Ду. $\varepsilon'' - \frac{2}{\Delta}\operatorname{th}\left(\frac{\xi}{\Delta}\right)\varepsilon' - \lambda\varepsilon = 0$. $\varepsilon(\xi) = y(\xi) + \frac{v}{\lambda\Delta}$

$y'' - 2\Delta^{-1}\operatorname{th}(y)y' - \lambda\Delta^2 y(x) = 0$, $x = \xi/\Delta$. $y'' - 2\operatorname{th} x\cdot y' - \lambda\Delta^2 y = 0$ $\quad y = y(x)$ — Л.О.Ду.

Но $\lambda \to \infty$ и $\approx \lambda\Delta^2 y = 0$, $y(x) = 0$ — реш. ЛОДу. Следуя A.D. Polyanin et al. Handbook ODE

сделаем подстановку $w = y\operatorname{ch} x$, $y = \frac{w(x)}{\operatorname{ch} x}$, $y' = -\frac{1}{\operatorname{ch}^2 x}\operatorname{th} x\cdot w + \frac{1}{\operatorname{ch} x}w = -\operatorname{th} x\cdot y + \frac{w'}{\operatorname{ch} x}$

$y'' = -\frac{1}{\operatorname{ch}^2 x}y - \operatorname{th} x\,y' - \frac{1}{\operatorname{ch}^2 x}\operatorname{sh} x\,w' + \frac{1}{\operatorname{ch} x}w'' = -\frac{1}{\operatorname{ch}^3 x}w - \operatorname{th} x\left[-\operatorname{th} x\frac{w}{\operatorname{ch} x} + \frac{w'}{\operatorname{ch} x}\right] - \frac{\operatorname{th} x}{\operatorname{ch} x}w' + \frac{1}{\operatorname{ch} x}w'$

$y'' - 2\operatorname{th} x\,y' = \frac{1}{\operatorname{ch} x}\left(-\frac{1}{\operatorname{ch}^2 x} + \operatorname{th}^2 x\right)w - 2\frac{\operatorname{th} x}{\operatorname{ch} x}w' + \frac{1}{\operatorname{ch} x}w'' - 2\operatorname{th} x\left[-\operatorname{th} x\frac{w}{\operatorname{ch} x} + \frac{w'}{\operatorname{ch} x}\right] =$

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$= \frac{w}{ch x}\left(-\frac{1}{ch^2 x} + th^2 x + 2th^2 x\right) - 2\left(\frac{th x}{ch x} + \frac{th x}{ch x}\right)w' + \frac{1}{ch x}w'' \to \frac{1}{ch x}(w'' - w)$ если $-2th x\, y' \to +2th x\, y'$.

$-\vartheta\Theta_{\xi} = \lambda \sin\theta\, \varepsilon - \varepsilon_{\xi\xi} - 2\Theta_{\xi}\varepsilon_{\xi}\cos\theta$, $+\Theta_{\xi}\cos\theta = \frac{\partial}{\partial\xi}(\sin\theta) = \frac{\partial}{\partial\xi}\frac{1}{ch^{3/2}\frac{\xi}{\Delta}} = -\frac{1}{ch^2\frac{\xi}{\Delta}}\frac{1}{\Delta}sh\frac{\xi}{\Delta} =$

$= -\frac{1}{ch\frac{\xi}{\Delta}}th\frac{\xi}{\Delta}\cdot\frac{1}{\Delta} = -\frac{1}{\Delta}\sin\theta\, th\frac{\xi}{\Delta}$, $-\frac{\vartheta}{\sin\theta}\Theta_{\xi} = \lambda\varepsilon - \varepsilon_{\xi\xi} - 2\varepsilon_{\xi}\left(-\frac{1}{\Delta}\right)th\frac{\xi}{\Delta}$ и

значит при $\varepsilon_{\xi\xi}$ и ε_{ξ} разные, подставляем $y(x) = w(x)/ch x$ не выходит.

$y'' + 2th x\, y' + ay = 0$ имеет реш. $y(x) = \frac{1}{ch x}\begin{pmatrix} c_1\cos bx + c_2\sin bx, & b^2 = a-1 > 0 \\ c_1 ch bx + c_2 sh bx, & -b^2 = a+1 < 0 \end{pmatrix}$

у нас $a = -\lambda\Delta^2$, $a - 1 = -\lambda\Delta^2 - 1 < 0$.

$\frac{d\varepsilon}{dy} = \frac{\Delta}{\vartheta}\left[\lambda\varepsilon^2 + 1 - \frac{1}{\Delta^2}\right]th y$, $\frac{d\varepsilon}{\left[\lambda\varepsilon^2 + 1 - \frac{1}{\Delta^2}\right]} = \frac{\Delta}{\vartheta}th y\, dy$, $\frac{1}{\lambda}\left(1 - \frac{1}{\Delta^2}\right) = -\frac{\vartheta^2}{\lambda^2\Delta^2} = -a^2 < 0$

$\frac{d\varepsilon}{(\varepsilon^2 - a^2)} = \frac{\Delta}{\vartheta\lambda}th y\, dy$, $-\frac{1}{a}Arcth\frac{\varepsilon}{a}$. $\int\frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| = \frac{1}{a}\begin{cases} Arth\frac{x}{a}, & x^2 < a^2 \\ Arcth\frac{x}{a}, & x^2 > a^2 \end{cases}$

$-\frac{1}{2a}\ln\left|\frac{a+\varepsilon}{a-\varepsilon}\right| = \frac{\lambda\Delta}{\vartheta}\ln ch y + C$, $\frac{\lambda\Delta}{\vartheta} = \frac{1}{a}$, $a = \frac{\vartheta}{\lambda\Delta} > 0$.

$-\frac{1}{2}\ln\left|\frac{a+\varepsilon}{a-\varepsilon}\right| = \ln ch y + Ca = \ln\left[ch y\, ce^{a}\right]$, $\frac{a+\varepsilon}{a-\varepsilon} = \frac{1}{ch^2 y\, c^2 e^{2a}}$

Берем $\varepsilon < a = \frac{\vartheta}{\lambda\Delta}$, $\varepsilon = a\left(\frac{x-1}{x+1}\right)$, $x = \frac{1}{c^2 e^{2a} ch^2 y}$. $\frac{1}{2a}\ln\left|\frac{a+\varepsilon}{a-\varepsilon}\right| = \frac{1}{a}Arth\frac{\varepsilon}{a}$, $\varepsilon < a$

(21/IV) не обозначался. $\Theta_{\xi\xi} = \frac{1}{2\Delta^2}\sin 2\theta + O(\varepsilon)$ и $y = \frac{\xi}{\Delta} + O(\varepsilon)$ верны при $\varepsilon \to 0$

при поиске $\varepsilon(\xi)$ надо $y = \frac{\xi}{\Delta} + O(\varepsilon)$, а не просто $y = \xi/\Delta$ as above.

$\frac{d\varepsilon}{\varepsilon^2 - a^2} = \frac{1}{a}th y\, dy$, $\frac{1}{\varepsilon^2 - a^2} = \frac{c_1}{\varepsilon - a} + \frac{c_2}{\varepsilon + a} = \frac{c_1(\varepsilon+a) + c_2(\varepsilon - a)}{(\varepsilon^2 - a^2)}$, $c_1 + c_2 = 0$

$c_2 = -c_1$, $2c_1 a = 1$, $c_1 = \frac{1}{2a}$. $\int\frac{d\varepsilon}{(\varepsilon^2 - a^2)} = \frac{1}{2a}\int d\varepsilon\left[\frac{1}{\varepsilon - a} - \frac{1}{\varepsilon + a}\right] =$

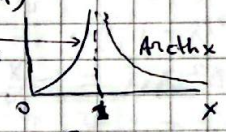
$= \frac{1}{2a}\left[\ln|\varepsilon - a| - \ln(\varepsilon + a)\right] = |\varepsilon < a| = \frac{1}{2a}\left[\ln(a - \varepsilon) - \ln(a + \varepsilon)\right] = \frac{1}{2a}\ln\left(\frac{a-\varepsilon}{a+\varepsilon}\right)$

$\frac{1}{2a}\ln\left(\frac{a-\varepsilon}{a+\varepsilon}\right) = \frac{1}{a}\ln ch y + C$, $\ln\left(\frac{a-\varepsilon}{a+\varepsilon}\right) = 2\ln ch y + 2Ca = 2\left[\ln ch y + \ln e^{Ca}\right]$

$\ln\left(\frac{a-\varepsilon}{a+\varepsilon}\right) = 2\ln\left[ch y \cdot e^{Ca}\right]$, $\frac{a-\varepsilon}{a+\varepsilon} = ch^2 y\, e^{2Ca} \to \infty$ при $\xi \to \pm\infty$, но $\frac{a-\varepsilon}{a+\varepsilon} < 1$.

Пусть $\varepsilon > a$, $\int\frac{d\varepsilon}{(\varepsilon^2 - a^2)} = \frac{1}{2a}\left[\ln(\varepsilon - a) - \ln(\varepsilon + a)\right] = \frac{1}{2a}\ln\left(\frac{\varepsilon - a}{\varepsilon + a}\right) = \frac{1}{a}\ln ch y + C$

$\left(\frac{\varepsilon - a}{\varepsilon + a}\right) = ch^2 y\, e^{2Ca} \to \infty$, $\xi \to \pm\infty$, $\frac{\varepsilon - a}{\varepsilon + a} = 1 - \frac{2a}{\varepsilon + a} < 1$.



$\frac{1}{2a}\ln\left|\frac{\varepsilon - a}{\varepsilon + a}\right| = \frac{1}{a}\ln ch y + C$, $\frac{1}{2}\ln\left|\frac{\varepsilon - a}{\varepsilon + a}\right| = \ln ch y + Ca$.

$-\frac{1}{2}\ln\left|\frac{\varepsilon + a}{\varepsilon - a}\right| = -F\left(\frac{x}{a}\right) = +2\ln ch y + Ca$, $F(\varepsilon/a) = -\ln ch y - Ca$.

$F(x) = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| = \begin{cases} Arth x, & |x| < 1 \\ Arcth x, & |x| > 1 \end{cases}$, $F(x) > 0$ при $x > 0$, $F(x)$ – нечет. $x = \frac{\varepsilon}{a}$.

$F(\varepsilon/a) = -\ln\left[ch y\, e^{Ca}\right]$? $\ln ch(y) = \ln ch\frac{\xi}{\Delta} = \ln\frac{1}{2}e^{|\xi|/\Delta} = \frac{|\xi|}{\Delta} - \ln 2$

при $|\xi|/\Delta \to \infty$, $\ln ch y > 0$, и $F\left(\frac{\varepsilon}{a}\right) = -\ln ch y + const < 0$, $F(x) < 0$ не может выполн.

Пусть $\varepsilon < 0$. Пусть $|\varepsilon| < a$. $\frac{1}{2}\ln\left|\frac{\varepsilon - a}{\varepsilon + a}\right| = \ln ch y + Ca$, $\left|\frac{\varepsilon - a}{\varepsilon + a}\right| = |\varepsilon = -|\varepsilon| < 0| =$

$= \left|\frac{-|\varepsilon| - a}{a - |\varepsilon|}\right| = \left(\frac{a + |\varepsilon|}{a - |\varepsilon|}\right)$, $\frac{1}{2}\ln\left|\frac{\varepsilon - a}{\varepsilon + a}\right| = \frac{1}{2}\ln\left(\frac{a + |\varepsilon|}{a - |\varepsilon|}\right) = F\left(\frac{|\varepsilon|}{a}\right) = Arth\left(\frac{|\varepsilon|}{a}\right)$, $|\varepsilon| < a$

$\ln\left(\frac{1+x}{1-x}\right) = |x = |\varepsilon|/a < 1, \varepsilon < 0| = 2\ln ch y + 2Ca = 2\ln\left[ch y\, e^{Ca}\right]$, $\frac{1+x}{1-x} = ch^2 y\, e^{2Ca}$

$x = \frac{|\varepsilon|}{a} = \frac{y-1}{y+1}$, $y = ch^2 y\, e^{2Ca}$, $|\varepsilon| = a\frac{ch^2 y\, e^{2Ca} - 1}{ch^2 y\, e^{2Ca} + 1}$, при $\xi \to \pm\infty$, $|\varepsilon| \to a - 0$.

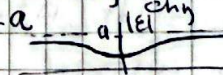
$\varepsilon(\xi) = -|\varepsilon(\xi)| = -a\frac{ch^2 y\, e^{2Ca} - 1}{ch^2 y\, e^{2Ca} + 1}$, $ch y = e^{f(y)}$, $f(y) = \ln ch y$

$\varepsilon(\xi) = -a\frac{e^{2f + 2Ca} - 1}{e^{2f + 2Ca} + 1}$, $\frac{e^{2y} - 1}{e^{2y} + 1} = \frac{e^{y} - e^{-y}}{e^{y} + e^{-y}} = th y$, $\varepsilon(\xi) = -a\, th\left[f(y) + Ca\right]$

$\varepsilon(\xi) = -a\, th\left[\ln ch\frac{\xi}{\Delta} + Ca\right]$, $th(\ln ch y) = \frac{e^{\ln ch y} - e^{-\ln ch y}}{e^{\ln ch y} + e^{-\ln ch y}} = \frac{ch y - \frac{1}{ch y}}{ch y + \frac{1}{ch y}} = \frac{ch^2 y - 1}{ch^2 y + 1} =$

$= \frac{sh^2 y}{ch^2 y + 1}$, $th(x+y) = \frac{th x + th y}{1 + th x\, th y}$. $\varepsilon(0) = -a\, th(Ca)$, $\varepsilon(\pm\infty) = -a$

что это все при $y = \xi/\Delta$. $\left(\frac{\lambda}{\vartheta} - \vartheta\right)\Theta_{\xi\xi} - \frac{\lambda}{\vartheta}\sin\theta\cos\theta = O(\lambda\varepsilon\varepsilon_{\xi}, \varepsilon_{\xi})$



$\frac{\lambda}{\vartheta}\Delta^2\left[\Theta_{\xi\xi} - \frac{1}{2\Delta^2}\sin\theta\cos\theta\right] = O(\lambda\varepsilon\varepsilon_{\xi}, \varepsilon_{\xi})$, $\frac{\vartheta}{\lambda\Delta^2} = \frac{\varepsilon_0}{\Delta}$, $\varepsilon_0 = \frac{\vartheta}{\lambda\Delta}$ при $\varepsilon(\xi) = const\ \Delta^2 = 1 - \frac{\vartheta^2}{\lambda}$

$\Theta_{\xi\xi} - \frac{1}{2\Delta^2}\sin\theta\cos\theta = \frac{1}{\Delta}\varepsilon_0\, O(\lambda\varepsilon\varepsilon_{\xi}, \varepsilon_{\xi})$, $O(\lambda\varepsilon_{\xi}\varepsilon, \varepsilon_{\xi}) = \frac{\lambda}{\vartheta}\varepsilon\cos\theta\left(\varepsilon_{\xi\xi}\sin\theta + 2\Theta_{\xi}\varepsilon_{\xi}\cos\theta\right) - \varepsilon_{\xi\xi\xi}\sin\theta -$

$- \varepsilon_{\xi\xi}\Theta_{\xi}\cos\theta - 2\Theta_{\xi\xi}\varepsilon_{\xi}\cos\theta - 2\Theta_{\xi}\varepsilon_{\xi\xi}\cos\theta + 2\sin\theta\,\varepsilon_{\xi}(\Theta_{\xi}^2)$

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$O(\lambda\varepsilon\varepsilon_\xi\varepsilon_\xi) = O(\varepsilon)$. Уравнение, очевидно $\theta(\xi)$ удовл. СГ $\theta_{\xi\xi} = \frac{1}{2\Delta^2}\sin 2\theta$, $\theta_\xi = -\frac{1}{\Delta}\sin\theta$

$$O(\varepsilon) = \frac{\lambda}{v}\varepsilon\cos\theta\left[\varepsilon_{\xi\xi}\sin\theta + 2\varepsilon_\xi\cos\theta\left(-\frac{1}{\Delta}\sin\theta\right)\right] - \varepsilon_{\xi\xi\xi}\sin\theta - \varepsilon_{\xi\xi}\cos\theta\left(-\frac{1}{\Delta}\right)\sin\theta - 2\varepsilon_\xi\cos\theta\cdot\frac{1}{\Delta^2}\sin\theta\cos\theta -$$

$$- 2\varepsilon_{\xi\xi}\cos\theta\left(-\frac{1}{\Delta}\right)\sin\theta + 2\sin\theta\varepsilon_\xi\frac{1}{\Delta^2}\sin^2\theta = \sin\theta\left[\frac{\lambda}{v}\varepsilon\cos\theta\left(\varepsilon_{\xi\xi} - \frac{2}{\Delta}\varepsilon_\xi\cos\theta\right) - \varepsilon_{\xi\xi\xi} + \right.$$

$$\left. + 3\frac{1}{\Delta}\varepsilon_{\xi\xi}\cos\theta - \frac{2}{\Delta^2}\cos^2\theta\,\varepsilon_\xi + \frac{2}{\Delta^2}\sin^2\theta\,\varepsilon_\xi\right] = \sin\theta\left[\frac{\lambda}{v}\varepsilon\cos\theta\left(\varepsilon_{\xi\xi} - \frac{2}{\Delta}\varepsilon_\xi\cos\theta\right) - \right.$$

$$\left. - \varepsilon_{\xi\xi\xi} + \frac{3}{\Delta}\cos\theta\,\varepsilon_{\xi\xi} - \frac{2}{\Delta^2}\varepsilon_\xi\cos 2\theta\right], \quad \theta_{\xi\xi} - \frac{1}{\Delta^2}\sin\theta\cos\theta = \frac{1}{\Delta}\varepsilon_0 O(\varepsilon), \quad \varepsilon_0 = \frac{v}{\lambda\Delta}$$

$$O(\varepsilon) = \sin\theta\left[\frac{\lambda}{v}\varepsilon\cos\theta\left(\varepsilon_{\xi\xi} - \frac{2}{\Delta}\varepsilon_\xi\cos\theta\right) - \varepsilon_{\xi\xi\xi} + \frac{3}{\Delta}\cos\theta\,\varepsilon_{\xi\xi} - \frac{2}{\Delta^2}\cos 2\theta\,\varepsilon_\xi\right]$$

УД: $-v\varepsilon_\xi\sin\theta = \theta_{\xi\xi} - (1+\lambda\varepsilon^2)\sin\theta\cos\theta = \frac{1}{\Delta^2}\sin\theta\cos\theta + \frac{1}{\Delta}\varepsilon_0 O(\varepsilon) - (1+\lambda\varepsilon^2)\sin\theta\cos\theta$

$$-v\varepsilon_\xi = \left[\frac{1}{\Delta^2} - 1 - \lambda\varepsilon^2\right]\frac{\sin\theta\cos\theta}{\sin\theta} + \frac{\varepsilon_0}{\Delta}\frac{1}{\sin\theta}O(\varepsilon) = \left[\frac{1}{\Delta^2} - 1 - \varepsilon^2\lambda\right]\cos\theta +$$

$$+ \frac{\varepsilon_0}{\Delta}\left[\frac{\lambda}{v}\varepsilon\cos\theta\left(\varepsilon_{\xi\xi} - \frac{2}{\Delta}\varepsilon_\xi\cos\theta\right) - \varepsilon_{\xi\xi\xi} + \frac{3}{\Delta}\cos\theta\,\varepsilon_{\xi\xi} - \frac{2}{\Delta^2}\cos 2\theta\,\varepsilon_\xi\right], \text{ вместо } v\varepsilon_\xi = \left[\lambda\varepsilon^2 + 1 - \frac{1}{\Delta^2}\right]\cos\theta$$

т.е. (θ, Φ) УД на самом деле достаточно для нахождения $\varepsilon(\xi)$. Вернемся к (y, Φ) УД (*)

$$-v\varepsilon_\xi\sin\theta = \theta_{\xi\xi} - \left[1 + \lambda\varepsilon^2 + \varepsilon_\xi^2\right]\sin\theta\cos\theta \quad \text{1-й интеграл} \quad \theta_\xi^2 + \sin^2\theta(\varepsilon_\xi)^2 = \sin 2\theta\,(1+\lambda\varepsilon^2) + \text{const}$$

$\varepsilon \to 0$ $\quad 1 + \lambda\varepsilon^2 = \frac{1}{\sin^2\theta}\theta_\xi^2 + \varepsilon_\xi^2 \approx \frac{1}{\sin^2\theta}\theta_\xi^2$ $\quad -v\varepsilon_\xi\sin\theta = \theta_{\xi\xi} - \theta_\xi^2\operatorname{ctg}\theta$

$$-v\theta_\xi = \lambda\sin\theta\,\varepsilon - \varepsilon_{\xi\xi}\sin\theta - 2\theta_\xi\varepsilon_\xi\cos\theta, \quad \theta_\xi\left(2\varepsilon_\xi\cos\theta - v\right) = \sin\theta\left(\lambda\varepsilon - \varepsilon_{\xi\xi}\right)$$

$$-v\varepsilon_\xi\sin\theta = \theta_{\xi\xi} - \operatorname{ctg}\theta\sin^2\theta\left(\frac{\lambda\varepsilon - \varepsilon_{\xi\xi}}{2\varepsilon_\xi\cos\theta - v}\right)^2 = \theta_{\xi\xi} - \cos\theta\sin\theta\frac{(\lambda\varepsilon - \varepsilon_{\xi\xi})^2}{(2\varepsilon_\xi\cos\theta - v)^2}$$

$$-v\varepsilon_\xi\sin\theta \simeq \theta_{\xi\xi} - \frac{\lambda^2}{v^2}\varepsilon^2\sin\theta\cos\theta, \quad \theta_\xi \simeq -\frac{1}{v}\lambda\varepsilon\sin\theta, \quad \theta_{\xi\xi} = -\frac{\lambda}{v}\varepsilon_\xi\sin\theta - \frac{\lambda}{v}\varepsilon\cos\theta\,\theta_\xi$$

Не то.

(*) $\begin{cases} vy_\xi = \lambda\varepsilon - \varepsilon_{\xi\xi} + 2\varepsilon_\xi y_\xi\,\text{th}y \\ v\varepsilon_\xi = y_{\xi\xi} + \left[1 + \lambda\varepsilon^2 + \varepsilon_\xi^2 - y_\xi^2\right]\text{th}y \end{cases}$ $\quad y_\xi^2 + \varepsilon_\xi^2 = 1 + \lambda\varepsilon^2 + \text{const}\,\text{ch}^2 y$

$$vy_{\xi\xi} = \lambda\varepsilon_\xi - \varepsilon_{\xi\xi\xi} + 2\varepsilon_{\xi\xi}y_\xi\,\text{th}y + 2\varepsilon_\xi y_{\xi\xi}\,\text{th}y + 2\varepsilon_\xi\frac{y_\xi^2}{\text{ch}^2 y} =$$

$$= \lambda\frac{1}{v}\left[y_{\xi\xi} + \left(1 + \lambda\varepsilon^2 + \varepsilon_\xi^2 - y_\xi^2\right)\text{th}y\right] - \varepsilon_{\xi\xi\xi} + 2\frac{\partial}{\partial\xi}\left(\varepsilon_\xi y_\xi\,\text{th}y\right), \quad 1 + \lambda\varepsilon^2 + \varepsilon_\xi^2 - y_\xi^2 = \left(y_\xi^2 + \varepsilon_\xi^2 - \text{const}\,\text{ch}^2 y\right) +$$

$$\left(v - \frac{\lambda}{v}\right)y_{\xi\xi} = \frac{\lambda}{v}\text{th}y\left(2\varepsilon_\xi^2 - \text{const}\,\text{ch}^2 y\right) + \frac{\partial}{\partial\xi}\left[-\varepsilon_{\xi\xi} + 2\varepsilon_\xi y_\xi\,\text{th}y\right], \quad + \varepsilon_\xi^2 - y_\xi^2 = 2\varepsilon_\xi^2 - \text{const}\,\text{ch}^2 y$$

$(v^2/\lambda - 1)y_{\xi\xi} = -\text{const}\,\text{sh}y\,\text{ch}y$, $\quad + \Delta^2 y_{\xi\xi} = \text{const}\,\text{sh}y\,\text{ch}y$, $\quad \Delta^2 = 1 - v^2/\lambda$. $\quad \varepsilon \to 0$ и нужна $\text{const} \neq 0$

$$-\frac{\lambda}{v}\Delta^2 y_{\xi\xi} = -\frac{\lambda}{v}\text{const}\,\text{ch}^2 y\,\text{th}y + \frac{2\lambda}{v}\text{th}y\,\varepsilon_\xi^2 + \frac{\partial}{\partial\xi}\left[-\varepsilon_{\xi\xi} + 2\varepsilon_\xi y_\xi\,\text{th}y\right], \quad v\varepsilon_\xi = y_{\xi\xi} + \left(2\varepsilon_\xi^2 - \text{const}\,\text{ch}^2 y\right)\text{th}y$$

$$v\varepsilon_\xi = -\frac{v}{\lambda\Delta^2}\left[-\frac{\lambda}{v}\text{const}\,\text{sh}y\,\text{ch}y + \frac{2\lambda}{v}\text{th}y\,\varepsilon_\xi^2 + \frac{\partial}{\partial\xi}(\dots)\right] + \left(2\varepsilon_\xi^2 - \text{const}\,\text{ch}^2 y\right)\text{th}y$$

Если $\text{const} = 0$, $v\varepsilon_\xi = y_{\xi\xi} + 2\varepsilon_\xi^2\,\text{th}y$, $v\varepsilon_\xi = y_{\xi\xi} + O(\varepsilon^2)$, т.е. $\varepsilon_\xi \neq 0$ и $y_{\xi\xi} \neq 0$.

$v\varepsilon = y_\xi + C \to vy_\xi = \lambda\varepsilon - \varepsilon_{\xi\xi} + 2\varepsilon_\xi y_\xi\,\text{th}y = \frac{\lambda}{v}(y_\xi + C) - \frac{1}{v}y_{\xi\xi\xi} + \frac{2}{v}y_{\xi\xi}y_\xi\,\text{th}y$

— исключили $\varepsilon(\xi)$. $\quad v^2 y_\xi = \lambda(y_\xi + C) - y_{\xi\xi\xi} + 2y_{\xi\xi}y_\xi\,\text{th}y$, $\quad -\lambda\Delta^2 y_\xi = -y_{\xi\xi\xi} + 2y_{\xi\xi}y_\xi\,\text{th}y + \lambda C$

$\lim \varepsilon \to 0$, $y_{\xi\xi} = 0$, $\quad -\lambda\Delta^2 y_\xi = \lambda C$, $\quad y_\xi = -\frac{C}{(+\Delta^2)} = \frac{1}{\Delta}$, $\quad C = -\Delta$ $(\varepsilon \to 0)$.

$$y_{\xi\xi\xi} - 2y_{\xi\xi}y_\xi\,\text{th}y - \lambda\Delta^2 y_\xi = -\lambda\Delta$$

$$\int d\xi\left[y_{\xi\xi\xi} - 2y_{\xi\xi}y_\xi\,\text{th}y - \lambda\Delta^2 y_\xi\right] = -\lambda\Delta\xi + C_1, \quad \int d\xi\,2y_{\xi\xi}y_\xi\,\text{th}y = \int d\xi\frac{d}{d\xi}(y_\xi^2)\,\text{th}y =$$

$$= y_\xi^2\,\text{th}y - \int y_\xi^2\,d(\text{th}y) = y_\xi^2\,\text{th}y - \int y_\xi^2\frac{d\xi}{\text{ch}^2 y}y_\xi \to y_{\xi\xi} - \left[y_\xi^2\,\text{th}y - \int d\xi\frac{y_\xi^3}{\text{ch}^2 y}\right] - \lambda\Delta^2 y = \dots$$

$y_\xi^2 + \varepsilon_\xi^2 = 1 + \lambda\varepsilon^2$, $\quad (v\varepsilon - C)^2 + \varepsilon_\xi^2 = \lambda\varepsilon^2 + 1$, $\quad v^2\varepsilon^2 - 2Cv\varepsilon + C^2 + \varepsilon_\xi^2 = \lambda\varepsilon^2 + 1$

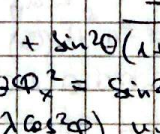
$\dots C = -\lambda\Delta\xi + C_1$

$\varepsilon_\xi^2 = (\lambda - v^2)\varepsilon^2 + 2Cv\varepsilon - C^2 + 1$, $\quad \lambda - v^2 = \lambda(1 - v^2/\lambda) = \lambda\Delta^2$

$\lim C = -\Delta?$, $\quad \varepsilon_\xi^2 = \lambda\Delta^2\varepsilon^2 - 2\Delta v\varepsilon + 1 - \Delta^2 = F(\varepsilon)$, $\quad F(\varepsilon) = 0$, $\quad \lambda\Delta^2\varepsilon^2 - 2\Delta v\varepsilon + 1 - \Delta^2 = 0$

$$\varepsilon_{1,2} = \frac{1}{2\lambda\Delta^2}\left[2v\Delta \pm \sqrt{4v^2\Delta^2 - 4\lambda\Delta^2(1-\Delta^2)}\right], \quad \det(v) = 2\Delta\sqrt{v^2 - \lambda\frac{v^2}{\lambda}} = 0$$

$\varepsilon_{1,2} = \frac{v}{\lambda\Delta}$, $\quad F(\varepsilon) = \lambda\Delta^2(\varepsilon - \varepsilon_1)^2$, $\quad \varepsilon_\xi = \pm\sqrt{\lambda\Delta^2}(\varepsilon - \varepsilon_1)$. $\quad \varepsilon(\xi) = \varepsilon_1 + [\varepsilon(0) - \varepsilon_1]e^{-\sqrt{\lambda\Delta^2}|\xi|}$

$\varepsilon(\xi) = \varepsilon_1 + [\varepsilon(0) - \varepsilon_1]e^{-\Delta\sqrt{\lambda}|\xi|}$ $\quad$  $\quad$ надо $\varepsilon(0) = \varepsilon(\xi = 0) < \varepsilon_1$, тогда $\varepsilon(\xi) \leq \varepsilon_1$

$$E_{\text{дин}} = \frac{1}{2}\int d\xi\left[\theta_\xi^2 + \sin^2\theta\,\Phi_\xi^2 + \sin^2\theta(1 + \lambda\cos^2\Phi)\right]^3 = \frac{1}{2}\int d\xi\left[\theta_\xi^2 + \sin^2\theta\,\varepsilon_\xi^2 + \sin^2\theta(1+\lambda\varepsilon^2)\right]$$

1-й интеграл $\theta_x^2 + \sin^2\theta\Phi_x^2 = \sin 2\theta\,[1 + \lambda\cos^2\Phi]$ или $y_\xi^2 + \Phi_\xi^2 = 1 + \lambda\cos^2\Phi$

$E_{\text{дин}} = \int d\xi\sin^2\theta\,(1 + \lambda\cos^2\Phi)$ или $E_{\text{дин}} = \int\frac{d\xi}{\text{ch}^2 y}(1 + \lambda\cos^2\Phi)$, $\quad \Phi(\xi) = \frac{\pi}{2} - \varepsilon(\xi)$

В $\theta(\xi)$ или $y(\xi)$ не знаем членов $\sim \varepsilon$. $\quad y(\xi) = \frac{\xi}{\Delta} + O(\varepsilon)$, $\quad \theta_{\xi\xi} = \frac{1}{\Delta^2}\sin\theta\cos\theta + \frac{\varepsilon_0}{\Delta}O(\varepsilon)$

$y_\xi = v\varepsilon - C = v\varepsilon(\xi) + \Delta$, $\quad \varepsilon(\xi) = \varepsilon_1 + \delta\varepsilon\,e^{-\Delta\sqrt{\lambda}|\xi|}$, $\quad \varepsilon_1 = \frac{v}{\lambda\Delta}$, $\quad v\varepsilon_1 + \Delta = \frac{v^2}{\lambda\Delta} + \Delta = \frac{1}{\Delta}\left(\frac{v^2}{\lambda} + \Delta^2\right) = 1/\Delta$

Escaneado con CamScanner

$$y(\xi) = v\int d\xi\,\varepsilon(\xi) + \Delta\xi + y_0 = v\int d\xi\left[\varepsilon_1 + \delta\varepsilon\, e^{-\varkappa|\xi|}\right] + \Delta\xi + y_0 = \frac{\xi}{\Delta} + y_0 + v\int d\xi\, e^{-\varkappa|\xi|}\delta\varepsilon$$

$$\int_{-\infty}^{\infty} d\xi e^{-\varkappa|\xi|} = 2\int_0^{\infty} d\xi e^{-\varkappa\xi} = 2\frac{(-1)}{\varkappa} e^{-\varkappa\xi}\Big|_0^{\infty} = 2\left(-\frac{1}{\varkappa}\right)[0-1] = \frac{2}{\varkappa}, \quad y_0 = 0$$

$$y(\xi) = \frac{\xi}{\Delta} + v\delta\varepsilon\int_{-\infty}^{\infty} d\xi e^{-\varkappa|\xi|} = \frac{\xi}{\Delta} + v\delta\varepsilon\frac{2}{\varkappa} = \left|\varkappa = \Delta\sqrt{\lambda}\right| = \frac{\xi}{\Delta} + 2v\delta\varepsilon\frac{1}{\Delta\sqrt{\lambda}}$$

$\delta\varepsilon = \varepsilon(\xi=0) - \varepsilon_1$ – undefined. Но $\delta\varepsilon \sim \varepsilon_1 = \frac{v}{\lambda\Delta}$, $y(\xi) = \frac{\xi}{\Delta} + y_1$, $y_1 = \frac{2v\delta\varepsilon}{\Delta\sqrt{\lambda}}$

$$E_{DW} = \int_{-\infty}^{\infty} d\xi \frac{1}{ch^2 y}\left(1 + \lambda\varepsilon^2\right) = \int d\xi \frac{1}{ch^2\left[\frac{\xi}{\Delta} + y_1\right]}\left[1 + \lambda\varepsilon^2(\xi)\right] = \int_{-\infty}^{\infty}\frac{d\xi}{ch^2\left[\frac{\xi}{\Delta} + y_1\right]} + \lambda\int d\xi\frac{\varepsilon^2(\xi)}{ch^2 y}$$

y_1 уменьшается из 1-го интеграл заменой ξ, и $y_1 = 0$ во 2-ом интеграле if E_{DW} go ε^2.

$$E_{DW} = \int_{-\infty}^{\infty} d\xi \frac{1}{ch^2\frac{\xi}{\Delta}} + \lambda\int_{-\infty}^{\infty}\frac{d\xi}{ch^2(\xi/\Delta)}\left[\varepsilon_1 + \delta\varepsilon\, e^{-\varkappa|\xi|}\right]^2, \quad \varepsilon^2(\xi) = \varepsilon_1^2 + 2\varepsilon_1\delta\varepsilon e^{-\varkappa|\xi|} + \delta\varepsilon^2 e^{-2\varkappa|\xi|}$$

$$J_1 = \int_0^{\infty} d\xi \frac{e^{-\frac{\varkappa}{\Delta}\xi}}{ch^2\xi/\Delta} = \Delta\int_0^{\infty} dx \frac{e^{-kx}}{ch^2 x}, \text{ где } k = \varkappa\Delta, \ x = \xi/\Delta. \quad J_1(k) = \Delta\left[k\beta\left(\frac{k}{2}\right) - 1\right]$$

$$E_{DW} = 2\Delta + 2\lambda\Delta\int_0^{\infty}\frac{dx}{ch^2 x}\left[\varepsilon_1^2 + 2\varepsilon_1\delta\varepsilon e^{-kx} + \delta\varepsilon^2 e^{-2kx}\right] = 2\Delta + 2\lambda\Delta\left[\varepsilon_1^2 + 2\varepsilon_1\delta\varepsilon\left\{k\beta\left(\frac{k}{2}\right) - 1\right\} + \right.$$

$$\left. + \delta\varepsilon^2\left\{2k\beta(k) - 1\right\}\right], \quad \beta(x) = \int_0^1 \frac{t^{x-1}}{1+t}dt$$ – "beta-function". $k = \varkappa\Delta = \Delta\sqrt{\lambda}\frac{1}{\Delta} = \sqrt{\lambda\Delta^2} \gg 1$

$$\beta(x) = \sum_{n=0}^{\infty}\frac{(-1)^n}{(x+n)} = \frac{1}{x} - \frac{1}{x+1} + \frac{1}{x+2} - \dots. \quad 2\beta(x) = \psi\left(\frac{x+1}{2}\right) - \psi\left(\frac{x}{2}\right).$$

$\lim_{x\gg 1}$

Abramowitz, p 263 $\psi(z) \sim \ln z - \frac{1}{2z} - \frac{1}{12z^2} + \dots$ at $z\to\infty$, $|\arg z| < \pi$.

$x\gg 1$, $2\beta(x) = \psi\left(\frac{1}{2}(x+1)\right) - \psi\left(\frac{x}{2}\right) = \ln\frac{1}{2}(x+1) - \ln\frac{x}{2} = \ln\left(\frac{1+x}{x}\right) \approx \frac{1}{x}$, $x\to\infty$

$$\psi\left(\frac{x+1}{2}\right) - \psi\left(\frac{x}{2}\right) = \ln\left(\frac{1+x}{x}\right) - \frac{1}{2}\frac{2}{x+1} + \frac{1}{2}\frac{1}{x/2} = \frac{1}{x} + \frac{1}{x} - \frac{1}{(x+1)} = \frac{1}{x} + \frac{1}{x(x+1)}$$

$\beta(x\gg 1) = \frac{1}{2}\frac{1}{x}$. $k\beta\left(\frac{k}{2}\right) - 1 = k\frac{1}{2}\frac{2}{k} - 1 \approx 0$, $2k\beta(k) - 1 = 2k\frac{1}{2}\frac{1}{k} - 1 = 0$

$\beta(x\gg 1) = \frac{1}{2}\frac{1}{x} + \frac{1}{2}\frac{1}{(x+1)x} \approx \frac{1}{2}\frac{1}{x} + \frac{1}{2}\frac{1}{x^2}$, $k\beta\left(\frac{k}{2}\right) - 1 = k\cdot\frac{1}{2}\left[\frac{1}{k/2} + \frac{1}{(k/2)^2}\right] - 1 = \frac{2}{k}$

$2k\beta(k) = 2k\cdot\frac{1}{2}\left[\frac{1}{k} + \frac{1}{k^2}\right] - 1 = 1/k$.

$$\varepsilon_1^2 + 2\varepsilon_1\delta\varepsilon\left\{k\beta(k/2) - 1\right\} + \delta\varepsilon^2\left\{2k\beta(k) - 1\right\} = \varepsilon_1^2 + 2\varepsilon_1\delta\varepsilon\frac{2}{k} + \delta\varepsilon^2\frac{1}{k}, \quad k = \sqrt{\lambda\Delta^2} \gg 1$$

$$\frac{1}{2\Delta}E_{DW} = 1 + \lambda\left[\varepsilon_1^2 + \dots\right] = 1 + \lambda\left[\varepsilon_1^2 + \frac{4}{\sqrt{\lambda}}\varepsilon_1\frac{\delta\varepsilon}{\Delta^2} + \frac{\delta\varepsilon^2}{\sqrt{\lambda}\Delta^2}\right]$$

$$1 + \lambda\varepsilon^2 = 1 + \lambda\left(\frac{v}{\lambda\Delta}\right)^2 + \sqrt{\lambda}\left[4\varepsilon_1\delta\varepsilon + \delta\varepsilon^2\right] = \frac{1}{\Delta^2} + 4\sqrt{\lambda}\,\varepsilon_1\frac{\delta\varepsilon}{\Delta^2} + \sqrt{\lambda}\frac{\delta\varepsilon^2}{\Delta^2}, \quad \sqrt{\lambda}\varepsilon_1 = \frac{v}{\sqrt{\lambda}\Delta}$$

$$E_{DW}(v) = \frac{2}{\Delta} + 8\Delta\sqrt{\lambda}\varepsilon_1\delta\varepsilon + 2\Delta\sqrt{\lambda}\,\delta\varepsilon^2 = \frac{2}{\Delta} + 8\frac{v}{\sqrt{\lambda}}\frac{\delta\varepsilon}{\Delta^2} + 2\Delta\frac{\sqrt{\lambda}}{\Delta^2}\delta\varepsilon^2, \quad \Delta = \sqrt{1 - v^2/\lambda}.$$

Нужно $\delta\varepsilon = \varepsilon(\xi=0) - \varepsilon_1$. $y(\xi) = \xi/\Delta + y_1$, $y_1 = \frac{1}{\Delta}$, $v\varepsilon = y_1 + C = \frac{1}{\Delta} - \Delta = \frac{v^2}{\lambda\Delta}$

$\varepsilon = \frac{v}{\lambda\Delta} = \varepsilon_1$ – надо точнее. $\varepsilon(0) < \varepsilon_1$, $\delta\varepsilon = \varepsilon(0) - \varepsilon_1 = -d\varepsilon_1$, $d>0$, $d<1$. ← (физически)

$$\frac{v}{\sqrt{\lambda}}\delta\varepsilon = \frac{v}{\sqrt{\lambda}}(-d)\varepsilon_1 = \frac{v}{\sqrt{\lambda}}(-d)\frac{v}{\lambda\Delta} = -\frac{d}{\lambda^{3/2}}\frac{v^2}{\Delta}, \quad \Delta\sqrt{\lambda}\delta\varepsilon^2 = \Delta\sqrt{\lambda}\,d^2\varepsilon_1^2 = d^2\sqrt{\lambda}\Delta\frac{v^2}{\lambda^2\Delta^2} = \frac{d^2}{\lambda^{3/2}}\frac{v^2}{\Delta}$$

$$E_{DW}(v) = \frac{2}{\Delta} + 8\frac{(-d)}{\lambda^{3/2}}\frac{v^2}{\Delta}\frac{1}{\Delta^2} + 2\frac{d^2}{\lambda^{3/2}}\frac{v^2}{\Delta\Delta^2} = \frac{2}{\Delta}\left[1 - d(4-d)\frac{v^2}{\lambda^{3/2}\Delta^2}\right]$$ – поправка к

Энергия DW $E_{DW}^{\varepsilon=\text{const}}(v) = \frac{2}{\Delta}$ для ст $\varepsilon(\xi) = \frac{v}{\lambda\Delta} + \delta\varepsilon e^{-\sqrt{\lambda}\Delta|\xi|}$ – зависимость $\delta\varepsilon = -d\varepsilon_1$, $d<1$.

$\Delta(v) = \sqrt{1 - v^2/\lambda}$ здесь, это поправка не для ст $\Delta_{sch}(v)$ или $\Phi(\xi) = \text{Const}$, $\varepsilon(\xi) = \text{const}$

а именно для ст variable $\varepsilon(\xi)$ при $\lambda\to\infty$ // т.е. уже $\varepsilon = \varepsilon(\xi)$ при вводим

к поправочной энергии SG-DW $E_{DW} = \frac{E_0}{\sqrt{1 - v^2/c^2}}$, $c^2 = \lambda$.

$$\delta E_{DW} = -\frac{2}{\Delta}d(4-d)\frac{v^2}{\lambda^{3/2}} = -\frac{2}{\Delta}d(4-d)\frac{1}{\lambda^{1/2}}\left(\frac{v^2}{c^2}\right), \quad \text{т.е. } \varepsilon = \varepsilon(\xi) \text{ соответствует DW при } \lambda\to\infty$$

$\varepsilon(\xi) = \varepsilon_1\left[1 - de^{-\sqrt{\lambda}\Delta|\xi|}\right]$, $y(\xi) = \frac{\xi}{\Delta} + \frac{2v\delta\varepsilon}{\Delta\sqrt{\lambda}}$, $y(\xi) = -2d\frac{v^2}{\lambda^{3/2}\Delta^2} + \frac{\xi}{\Delta}$ – DW shifted to $\xi_0 \neq 0$

$y(\xi) = \frac{1}{\Delta}(\xi - \xi_0)$, $\xi_0 = 2d\frac{v^2}{\lambda^{3/2}\Delta}$ – propulsion of DW (!?)

Нем $\varepsilon(\xi)$ отлич. от $\varepsilon(\xi) = (v/\lambda\Delta)\frac{1}{ch(\xi/\Delta)}$ 20/II, нужно оба расчета, см. 7/III $\varepsilon(\xi)$.

(22/IV) Sokolsky Mag'19 переписка. Тоо статьи на JEMS'19: 1) Nevay – APLMat? 2) Verba – Arxt. Sk – PRA(?), 3) Dmitriev/Dobrovolskiy; 4) Quirin – Skyrmion/Zakharov, 5) 1D-DW/Ricardo? 6) Sk – MPD/APEX – Mater. 7) SW-DW interaction/ARL. Нужен IVA "Constanto" IkB Suhom. Sk-statics Afs. JEMS

(23/IV) переписка с Бухрой о dyn. Sk – abstract JEMS. Refs: Verba – MS to PRB после публикации APL-Mat. Yuk. Тоо eq. Hres(k) – Search Omelchenko ε_1^2. Nic. Nikie – APL'2004

21/IV исследован $y_3^2 + \varepsilon_3^2 = 1 + \lambda\varepsilon^2$ и $v\varepsilon_3 = y_{33}$, $y(3) = \xi/\Delta + O(\varepsilon)$, $\varepsilon_3^2 \ll v\varepsilon_3$, y_{33}, $\delta\varepsilon = 0$.

20/II исследован $vy_3 = \lambda\varepsilon$, $y_{33} = \frac{\lambda}{v}\varepsilon_3$ и решена граф. пр. для $y(\xi)$, $y(\xi) = \xi/\Delta$, но $\varepsilon(\xi) = v/\lambda y_3 = (v/\lambda\Delta)\frac{1}{ch(\xi/\Delta)}$, а не $\varepsilon(\xi) = (v/\lambda\Delta)$. Исправление только 20/II

Escaneado con CamScanner

Ищем $y(\xi) = \frac{\xi}{\Delta} + y_1(\xi)$, $y_1 \sim \varepsilon \ll 1$. $\begin{cases} v y_3 = \lambda\varepsilon - \varepsilon_{\xi\xi} + 2\varepsilon_\xi y_\xi \operatorname{th} y \\ v\varepsilon_\xi = y_{\xi\xi\xi} + [1 + \lambda\varepsilon^2 + \varepsilon_\xi^2 - y_3^2]\operatorname{th} y \end{cases}$ $y_3^2 + \varepsilon_\xi^2 = 1 + \lambda\varepsilon^2 + \text{ff}\operatorname{ch}^2 y$

$y_\xi = y_1'$, $v y_1' = \lambda\varepsilon - \varepsilon'' + 2\varepsilon' y_1' \operatorname{th} y \approx \lambda\varepsilon - \varepsilon''$, $v\varepsilon' = y_1'' + [2(\varepsilon')^2 - \text{ff}[\operatorname{ch}^2 y]$

$\begin{cases} v y_1' = \lambda\varepsilon - \varepsilon'' \\ v\varepsilon' = y_1'' - \text{ff}\operatorname{ch}^2 y \end{cases}$ тогда $y(\xi) = \frac{\xi}{\Delta} + \mu(\xi)$, $y_\xi = \frac{1}{\Delta} + \mu_\xi$, $y_{\xi\xi} = \mu_{\xi\xi}$

$v\left(\frac{1}{\Delta} + \mu_\xi\right) = \lambda\varepsilon - \varepsilon_{\xi\xi} + 2\varepsilon_\xi \operatorname{th}\left(\frac{1}{\Delta} + \mu_\xi\right) = \lambda\varepsilon - \varepsilon_{\xi\xi} + \frac{2}{\Delta}\varepsilon_\xi \operatorname{th} y$

$\begin{cases} \frac{v}{\Delta} + v\mu_\xi = \lambda\varepsilon - \varepsilon_{\xi\xi} + \frac{2}{\Delta}\varepsilon_\xi \operatorname{th} y \\ v\varepsilon_\xi = \mu_{\xi\xi} - \text{ff}\operatorname{ch}^2 y \end{cases}$ при $\text{ff} = 0$, $v\varepsilon_\xi = \mu_{\xi\xi}$ как 21/IV.

$v\mu_{\xi\xi} = \lambda\varepsilon_\xi + \frac{\partial}{\partial\xi}\left[-\varepsilon_{\xi\xi} + \frac{2}{\Delta}\varepsilon_\xi \operatorname{th} y\right]$

$\varepsilon_\xi = \frac{1}{v}\mu_{\xi\xi} = \frac{1}{v^2}\left[\lambda\varepsilon_\xi + \frac{\partial}{\partial\xi}\left(-\varepsilon_{\xi\xi} + \frac{2}{\Delta}\varepsilon_\xi \operatorname{th} y\right)\right]$, $\left(1 - \frac{\lambda}{v^2}\right)\varepsilon_\xi = \frac{\partial}{\partial\xi}\left(-\varepsilon_{\xi\xi} + \frac{2}{\Delta}\varepsilon_\xi \operatorname{th} y\right) =$

$= -\varepsilon_{\xi\xi\xi} + \frac{2}{\Delta}\varepsilon_{\xi\xi}\operatorname{th} y + \frac{2}{\Delta}\varepsilon_\xi \frac{1}{\operatorname{ch}^2 y} y_\xi = -\varepsilon_{\xi\xi\xi} + \frac{2}{\Delta}\varepsilon_{\xi\xi}\operatorname{th}\left(\frac{\xi}{\Delta}\right) + \frac{2}{\Delta^2}\varepsilon_\xi \frac{1}{\operatorname{ch}^2(\xi/\Delta)}$

$\left(1 - \frac{\lambda}{v^2}\right) = \left(\frac{v^2}{\lambda} - 1\right) = -\frac{\lambda}{v^2}\Delta^2$, $\left(1 - \frac{\lambda}{v^2} - \frac{2}{\Delta^2}\frac{1}{\operatorname{ch}^2 \xi/\Delta}\right)\varepsilon_\xi = -\varepsilon_{\xi\xi\xi} + \frac{2}{\Delta}\varepsilon_{\xi\xi}\operatorname{th}\left(\frac{\xi}{\Delta}\right)$

$\varepsilon_\xi = y(\xi) = y(\xi)$. $\left[1 - \frac{\lambda}{v^2} - \frac{2}{\Delta^2}\frac{1}{\operatorname{ch}^2 \xi/\Delta}\right] y(\xi) = -y_{\xi\xi\xi} + \frac{2}{\Delta} y_\xi \operatorname{th}\left(\frac{\xi}{\Delta}\right)$

$y'' - \frac{2}{\Delta}\operatorname{th}\left(\frac{\xi}{\Delta}\right) y' + \left[1 - \frac{\lambda}{v^2} - \frac{2}{\Delta^2}\frac{1}{\operatorname{ch}^2 \xi/\Delta}\right] y(\xi) = 0$ — нахожу на $y(\xi)$ как 20/IV.

Л.О.Д.У. на $y(\xi)$ с переменными коэффициентами. $\Delta^2 = 1 - \frac{v^2}{\lambda}$

$y(\xi)$ — нечет. $f(\xi)$, а $\varepsilon(\xi)$ — чет. функция ξ. $\cos\theta = m_z(\xi) = \operatorname{th} y(\xi) > 0$

$y(-\xi) = -y(\xi)$, $m_z(-\xi) = \operatorname{th} y(-\xi) = -\operatorname{th} y(\xi) < 0$, $m_z(\xi)$ — нечет. функция.

$y(\xi) = \frac{\xi}{\Delta} + \mu(\xi)$, $\mu(\xi)$ — нечет., $\mu_\xi(\xi)$ — чет. В ин., по ε-инвариантам. $v\varepsilon_\xi = \mu_{\xi\xi}$.

Если $y(\xi)$ — нечет., то расчет 21/IV $y(\xi) = \frac{\xi}{\Delta} + 2\frac{v\delta\varepsilon}{\Delta\sqrt{\lambda}}$ не проходит. $\operatorname{th}^2 y + \frac{1}{\operatorname{ch}^2 y} = 1$

$1 - \frac{\lambda}{v^2} - \frac{2}{\Delta^2} = -\frac{\lambda}{v^2}\Delta^2 - \frac{2}{\Delta^2} = \frac{1}{\Delta^2}\left[-\frac{\lambda}{v^2}\Delta^4 - 2\right] = \frac{1}{\Delta^2}\left[-\frac{\lambda}{v^2}\left(\frac{v^4}{\lambda^2} + 1 - \frac{2v^2}{\lambda}\right) - 2\right] = -\frac{\lambda}{v^2}\frac{(1 + v^4/\lambda^2)}{\Delta^2}$

При $\xi \to \pm\infty$ $y'' \pm \left(\frac{2}{\Delta}\right) y' - \frac{\lambda}{v^2}\Delta^2 y(\xi) = 0$ $y \sim e^{kx}$, $k^2 \mp \frac{2}{\Delta}k - \frac{\lambda}{v^2}\Delta^2 = 0$

$\text{Дет} = \frac{4}{\Delta^2} - 4\left(-\frac{\lambda}{v^2}\Delta^2\right) = \frac{4}{\Delta^2} + \frac{4\lambda}{v^2}\left(1 - \frac{v^2}{\lambda}\right) = \frac{4}{\Delta^2} + \frac{4\lambda}{v^2} - 4 = \frac{4}{\Delta^2}\left[1 + \frac{\lambda}{v^2}\left(1 - \frac{v^2}{\lambda}\right) - \Delta^2\right] =$

$= \frac{4}{\Delta^2}\left[\frac{\lambda}{v^2} - \left(1 - \frac{v^2}{\lambda}\right)\right] = \frac{4}{\Delta^2}\left(\frac{\lambda}{v^2} + \frac{v^2}{\lambda} - 1\right)$.

$\begin{cases} \frac{v}{\Delta} + v\mu_\xi = \lambda\varepsilon - \varepsilon_{\xi\xi} + \varepsilon_\xi \frac{2}{\Delta}\operatorname{th}(\xi/\Delta) \\ v\varepsilon_\xi = \mu_{\xi\xi} \end{cases}$ $\varepsilon(\xi) = \frac{1}{v}\mu_\xi + C$, $\mu_\xi = v\varepsilon - C\cdot v$

$\frac{v}{\Delta} + v\mu_\xi = \lambda\left(\frac{1}{v}\mu_\xi + C\right) - \frac{1}{v}\mu_{\xi\xi\xi} + \frac{1}{v}\mu_{\xi\xi}\frac{2}{\Delta}\operatorname{th}(\xi/\Delta)$

$-\frac{1}{v}\mu_{\xi\xi\xi} + \frac{2}{\Delta v}\mu_{\xi\xi}\operatorname{th} y + \left(\frac{\lambda}{v} - v\right)\mu_\xi = \frac{v}{\Delta} - \lambda C$, $\lambda - v^2 = \lambda(1 - v^2/\lambda) = \lambda\Delta^2$, $\mu_\xi = f(\xi)$

$f_{\xi\xi} - \frac{2}{\Delta}\operatorname{th} y \cdot f_\xi + \lambda\Delta^2 f(\xi) = v(\lambda C - \frac{v}{\Delta})$, $y = \frac{\xi}{\Delta}$ задача. $y_3^2 + \varepsilon_3^2 = 1 + \lambda\varepsilon^2$

Eq. for $f(\xi) = \mu_\xi$ иначе eq. for $y(\xi) = \varepsilon_\xi$, $y(\xi) = \frac{\xi}{\Delta} + \mu(\xi)$. $\frac{1}{\Delta^2} + 2\frac{\mu_\xi}{\Delta} + \mu_\xi^2 + \varepsilon_\xi^2 = 1 + \lambda\varepsilon^2$

Видно реш. $f(\xi) = 0$, $\mu_\xi = \text{const}$ если имеет при $v = 0$. $\frac{\xi}{\Delta} = \frac{1}{\Delta}\frac{1}{\Delta\sqrt{\lambda}}x$

Л.О.Д.У. $f_{\xi\xi\xi} - \frac{2}{\Delta}\operatorname{th}\left(\frac{\xi}{\Delta}\right) f_{\xi\xi} - \lambda\Delta^2 f(\xi) = 0$, $x = \Delta\sqrt{\lambda}\,\xi$, $\frac{\partial^2}{\partial\xi^2} = (\sqrt{\lambda}\Delta)^2\frac{\partial^2}{\partial x^2}\cdot\frac{2}{\Delta}\frac{1}{\lambda\Delta^2} = \frac{2}{\lambda\Delta^3}$

$f_{xx} - \frac{2}{\sqrt{\lambda}\Delta^2}\operatorname{th} y \cdot f_x - f(x) = 0$. при $\sqrt{\lambda}\Delta^2 \gg \frac{1}{v^2}$ ест. $\frac{2}{\Delta}\frac{1}{\lambda\Delta^2}\frac{\partial}{\partial\xi} = \frac{2}{\lambda\Delta^3}\sqrt{\lambda}\Delta\frac{\partial}{\partial x} = \frac{2}{\sqrt{\lambda}\Delta^2}\frac{\partial}{\partial x}$

$f_{xx} - f(x) = 0$, $f(x) = e^{\pm x} \to f(\xi) = \mu_\xi$ — чет. $\Rightarrow f(x) = C_1\operatorname{ch} x$, $\mu_\xi = C_1\operatorname{ch} x$, $\mu(x) = C_1\operatorname{sh} x$

$y(x) \sim \frac{\xi}{\Delta} + C_1\operatorname{sh} x$, $\varepsilon_x(x) \approx \frac{1}{v}\mu_{xx}$, $\varepsilon(x) \sim \operatorname{ch} x + C_2$

Решение такого типа $y(\xi) = \frac{a}{\sqrt{\lambda}\Delta}\operatorname{sh}(\sqrt{\lambda}\Delta\xi) + \frac{\xi}{\Delta}$, $\varepsilon(\xi) = \frac{a}{v}\operatorname{ch}(\sqrt{\lambda}\Delta\xi) + \frac{v}{\lambda\Delta}$

и нашли 21/III) Физически разумно $f(x) = \begin{cases} e^{-x}, & x > 0 \\ e^{+x}, & x < 0 \end{cases}$, т.е. $f(x) \sim e^{-|x|}$.

Л.Н.О.У. $f_{xx} - \frac{2}{\sqrt{\lambda}\Delta^2}\operatorname{th} y\, f_x - f(x) = \frac{v}{\lambda\Delta^2}(\lambda C - \frac{v}{\Delta})$. Реш. неодн. $f_0 = \frac{v}{\lambda\Delta^2}\left(\frac{v}{\Delta} - \lambda C\right)$

$f(x) = ae^{\pm x} + f_0$, $f(\xi) = ae^{\pm\Delta\sqrt{\lambda}\xi} + f_0 = \mu_\xi$, $\varepsilon_\xi = \frac{1}{v}\mu_{\xi\xi} = \pm\sqrt{\lambda}\Delta\, e^{\pm\sqrt{\lambda}\Delta\xi} = \pm\frac{\partial x}{v}e^{\pm x\xi}\cdot\frac{a}{v}$

$\varepsilon(\xi) = \pm\frac{\partial x}{v}a\int d\xi e^{\pm x\xi} + C_\varepsilon = \frac{a}{v}e^{\pm x\xi} + C_\varepsilon$ — имеет вид $\varepsilon(\xi) = \varepsilon_1 + \delta\varepsilon e^{-\alpha|\xi|}$ — 21/IV.

$y_\xi = \frac{1}{\Delta} + \mu_\xi = \frac{1}{\Delta} + ae^{\pm x\xi} + f_0$, $y(\xi) = \frac{\xi}{\Delta} + a\left(\pm\frac{1}{x}\right)e^{\pm x\xi} + f_0\xi$ — нечет. функция ξ.

$y(\xi) = \frac{\xi}{\Delta} + f_0\xi - \frac{a}{x}\operatorname{sign}(\xi)e^{-x|\xi|}$ — отличается от $y(\xi) = \frac{\xi}{\Delta} + 2\frac{v\delta\varepsilon}{\sqrt{\lambda}\Delta}$ от 21/IV

$\varepsilon(\xi) = \frac{a}{v}e^{-x|\xi|} + C_\varepsilon$. Найти f_0 (или C), a и C_ε.

$\varepsilon = \frac{1}{v}\mu_\xi + C \to \frac{a}{v}e^{-x|\xi|} + C_\varepsilon = \frac{1}{v}\left[ae^{-x|\xi|} + f_0\right] + C$, $C_\varepsilon = \frac{1}{v}f_0 + C = \frac{1}{\lambda\Delta^2}\left(\frac{v}{\Delta} - \lambda C\right) + C$

$C_\varepsilon = \frac{v}{\lambda\Delta^3} + C\left(1 - \frac{1}{\Delta^2}\right) = \frac{v}{\lambda\Delta^3} - \frac{v^2}{\lambda\Delta^2}C = \frac{v}{\lambda\Delta^3}(1 - v\Delta^2 C)$.

Escaneado con CamScanner

(24/IV) 1D-DW dynamics. $\lambda \gg 1$. Решение (Φ, y)-УД as $y(\xi) = \frac{\xi}{\Delta} + \mu(\xi)$, $\Phi(\xi) = \frac{\pi}{2} - \varepsilon(\xi)$, $\mu, \varepsilon \ll 1$.

$y(\xi) = \frac{\xi}{\Delta} + f_0 \xi - \frac{a}{\nu} s(\xi) e^{-\varkappa|\xi|}$, $\mu_\xi = a e^{-\varkappa|\xi|} + f_0$, $\varepsilon(\xi) = \frac{a}{\nu} e^{-\varkappa|\xi|} + C_\varepsilon$, $s = \mathrm{sign}$.

$\varepsilon_\xi = \frac{1}{\nu}\mu_{\xi\xi} \to \varepsilon = \frac{1}{\nu}\mu_\xi + C$, $C_\varepsilon = \frac{\nu}{2\Delta^3}(1 - \nu\Delta C)$, $f_0 = \frac{\nu}{\lambda\Delta^2}\left(\frac{\nu}{\Delta} - \lambda C\right)$, $a(\nu) - ?$

перенормировка: $\xi_3 = \frac{1}{\nu}\mu_{\xi\xi}$, $\varepsilon = \frac{1}{\nu}\mu_\xi + C$ и $\varepsilon_\xi = \pm \frac{\varkappa}{\nu} a e^{\pm \varkappa\xi}$, $\varepsilon(\xi) = \frac{a}{\nu} e^{\pm\varkappa\xi} + C_\varepsilon$

$C = C_\varepsilon$, $C_\varepsilon = \frac{1}{\nu} f_0 + C \to f_0 = 0$, $C = \nu/(\lambda\Delta)$.

$y(\xi) = \frac{\xi}{\Delta} - \frac{a}{\nu} s(\xi) e^{-\varkappa|\xi|}$, $\varepsilon(\xi) = \frac{a}{\nu} e^{-\varkappa|\xi|} + \frac{\nu}{\lambda\Delta}$ — совпал $\varepsilon(\xi)$ 21/IV с $\varepsilon_1 = \frac{\nu}{\lambda\Delta}$.

Но это нашли ЛНДУ для $f(\xi) = \mu_\xi$ или $\sqrt{\lambda}\Delta^2 \gg 1$, иначе д.ур. с f'_ξ не решить.

$\Delta^2 = 1 - \nu^2/\lambda$, $\varkappa = \sqrt{\lambda}\Delta$, $x\xi = \sqrt{\lambda}\Delta\xi = \sqrt{\lambda}\Delta^2(\xi/\Delta)$, $x\Delta = \sqrt{\lambda}\Delta^2 \gg 1$. $x\xi = (x\Delta)\left(\frac{\xi}{\Delta}\right)$

т.е. $y(\xi)$, $\varepsilon(\xi)$ — малые отклонения от $\varepsilon(\xi) = \mathrm{const}$ case, где $y_0(\xi) = \xi/\Delta$, $\varepsilon_0 = \nu/\lambda\Delta$.

$E_{DW} = \int_{-\infty}^{+\infty} d\xi \frac{1}{\mathrm{ch}^2 y(\xi)}\left[1 + \lambda\varepsilon^2(\xi)\right] = \int_{-\infty}^{\infty} \frac{d\xi}{\mathrm{ch}^2 y(\xi)} + \lambda\int d\xi \frac{\varepsilon^2(\xi)}{\mathrm{ch}^2(\xi/\Delta)}$, см. 21/IV.

$y(\xi) = \frac{\xi}{\Delta} + y_1(\xi)$, $y_1(\xi) = \mu(\xi) \neq \mathrm{const}$ как 21/IV. Но 21/IV надо $y(\xi) = \nu\int_{\text{indefinite int.}} \varepsilon(\xi) + \dots$

$y(\xi) = y_0 + y_1$, $\frac{1}{\mathrm{ch}^2 y} = \frac{1}{\mathrm{ch}^2(y_0 + y_1)}$, $2\mathrm{ch}^2(y_0 + y_1) = \mathrm{ch}\,2(y_0 + y_1) + 1 = 1 + f'(y)\Delta y + \dots =$

$\frac{1}{\mathrm{ch}^2(y_0+y_1)} = \frac{2}{1 + 2\mathrm{sh}\,2y_0 \cdot y_1} = 2(1 - 2\mathrm{sh}\,2y_0 \cdot y_1) - \dots$ $= 1 + 2\,\mathrm{sh}\,2y_0 \cdot y_1 + \mathrm{ch}\,2y_0$

$2\mathrm{ch}^2(y_0 + y_1) = 2\mathrm{ch}^2 y_0 + 2\mathrm{sh}\,2y_0 \cdot y_1$, $\frac{1}{\mathrm{ch}^2 y} = \frac{1}{\mathrm{ch}\,2y_0 + \mathrm{sh}\,2y_0 y_1} = \frac{1}{\mathrm{ch}^2 y_0}\frac{1}{1 + 2\,\mathrm{th}\,y_0 \cdot y_1}$

$\int d\xi \frac{1}{\mathrm{ch}^2 y(\xi)} = \int d\xi \frac{1}{\mathrm{ch}^2 y_0}\left[1 - 2\,\mathrm{th}\,y_0 \cdot y_1\right]$, $\int_{-\infty}^{\infty} d\xi \frac{\mathrm{th}(\xi/\Delta)}{\mathrm{ch}^2(\xi/\Delta)} e^{-\varkappa|\xi|} = 0$ как инт. от нечет. $f(\xi)$.

$\mathrm{ch}\,2y = \mathrm{ch}\,2y_0 + 2\,\mathrm{sh}\,2y\Big|_{y=y_0}\Delta y + \frac{1}{2}\cdot 2\cdot 2\mathrm{ch}\,2y\Big|_{y_0}\Delta y^2 = \mathrm{ch}\,2y_0 + 2\,\mathrm{sh}\,2y_0 y_1 + 2\mathrm{ch}\,2y_0 y_1^2$

$\mathrm{ch}^2 y = \frac{1}{2}(1 + \mathrm{ch}\,2y) = \frac{1}{2}\left[1 + \mathrm{ch}\,2y_0 + 2\,\mathrm{sh}\,2y_0 \cdot y_1 + 2\mathrm{ch}\,2y_0 y_1^2\right] = \mathrm{ch}^2 y_0 + \mathrm{sh}\,2y_0 y_1 + \mathrm{ch}\,2y_0 y_1^2$

$\frac{1}{\mathrm{ch}^2 y} = \frac{1}{\mathrm{ch}^2 y_0}\left[1 + \frac{\mathrm{sh}\,2y_0}{\mathrm{ch}^2 y_0} y_1 + \frac{\mathrm{ch}\,2y_0}{\mathrm{ch}^2 y_0} y_1^2\right] = \frac{1}{\mathrm{ch}^2 y_0}\frac{1}{(1+\varepsilon)} = \frac{1}{\mathrm{ch}^2 y_0}\left[1 - \varepsilon + \varepsilon^2\right] =$

$= \frac{1}{\mathrm{ch}^2 y_0}\left[1 - 2\,\mathrm{th}\,y_0 \cdot y_1 - \frac{\mathrm{ch}\,2y_0}{\mathrm{ch}^2 y_0} y_1^2 + 4\,\mathrm{th}^2 y_0\, y_1^2\right]$, $4\,\mathrm{th}^2 y_0 - \frac{\mathrm{ch}\,2y_0}{\mathrm{ch}^2 y_0} = 4\,\mathrm{th}^2 y - \left(2 - \frac{1}{\mathrm{ch}^2 y}\right) =$

$= 4\,\mathrm{th}^2 y + \frac{1}{\mathrm{ch}^2 y} - 2 = 3\,\mathrm{th}^2 y - 1$. $\frac{1}{\mathrm{ch}^2 y} = \frac{1}{\mathrm{ch}^2 y_0}\left[1 - 2\,\mathrm{th}\,y_0 \cdot y_1 + (3\,\mathrm{th}^2 y_0 - 1) y_1^2\right]$

$\int_{-\infty}^{\infty} d\xi \frac{1}{\mathrm{ch}^2 y(\xi)} = \int d\xi \frac{1}{\mathrm{ch}^2(\xi/\Delta)}\left[1 - 2\,\mathrm{th}\frac{\xi}{\Delta} \cdot y_1 + \left(3\,\mathrm{th}^2\frac{\xi}{\Delta} - 1\right) y_1^2\right] = \int \frac{d\xi}{\mathrm{ch}^2 \xi/\Delta} + \int d\xi \frac{1}{\mathrm{ch}^2 \frac{\xi}{\Delta}}\left(3\,\mathrm{th}^2\frac{\xi}{\Delta} - 1\right) y_1^2(\xi)$

$y_1(\xi) = -\frac{a}{\nu} e^{-\varkappa|\xi|}$, $\int_{-\infty}^{\infty} d\xi \frac{1}{\mathrm{ch}^2 \frac{\xi}{\Delta}}\left(3\,\mathrm{th}^2\frac{\xi}{\Delta} - 1\right) e^{-2\varkappa|\xi|} \frac{a^2}{\nu^2} = 2\frac{a^2}{\nu^2}\Delta \int_0^{\infty} \frac{dx}{\mathrm{ch}^2 x}\left(3\,\mathrm{th}^2 x - 1\right) e^{-2x\Delta|x|}$

$3\,\mathrm{th}^2 x - 1 = 3\frac{\mathrm{sh}^2 x}{\mathrm{ch}^2 x} - 1 = \frac{3}{\mathrm{ch}^2 x}(\mathrm{ch}^2 x - 1) - 1 = 2 - \frac{3}{\mathrm{ch}^2 x}$, $\mu = 2x\Delta$

$\int_0^{\infty} dx \frac{e^{-\mu x}}{\mathrm{ch}^q(x/2)} = \frac{2^{2q-2}}{\mu^q} B(q,q) - \frac{1}{2q}\frac{1}{\mu^q}$, $2q+1 = 4$, $q = 3/2$, $\int_0^{\infty} dx \frac{e^{-\mu x}}{\mathrm{ch}^4(x/2)} = \frac{2}{\mu} B\left(\frac{3}{2},\frac{3}{2}\right) - \frac{1}{3\mu}$

$\int_{-\infty}^{\infty} d\xi \frac{1}{\mathrm{ch}^2 y(\xi)} = 2\Delta\int_0^{\infty} \frac{dx}{\mathrm{ch}^2 x} + 2\Delta\frac{a^2}{\nu^2}\int_0^{\infty} dx\left[\frac{2}{\mathrm{ch}^2 x} - \frac{3}{\mathrm{ch}^4 x}\right] e^{-\mu x} = 2\Delta + 2\Delta\frac{a^2}{\nu^2}\left[2\int_0^{\infty}\frac{dx\, e^{-\mu x}}{\mathrm{ch}^2 x} - 3\int_0^{\infty}\frac{dx\, e^{-\mu x}}{\mathrm{ch}^4 x}\right] =$

$= 2\Delta + 2\Delta\frac{a^2}{\nu^2}\left[2\left(\mu\beta\left(\frac{\mu}{2}\right) - 1\right) - 3\left(\frac{2}{\mu}B\left(\frac{2}{2},\frac{2}{2}\right) - \frac{1}{3\mu}\right)\right]$. Интеграл с $\int \frac{dx}{\mathrm{ch}^4 x}$ (!) упростило, так $e^{\mu x}$, $\mathrm{ch}\,\mu x$ — аргументы одного

$\int_{-\infty}^{\infty} d\xi \frac{1}{\mathrm{ch}^2 y(\xi)} = 2\Delta + 2\Delta\frac{a^2}{\nu^2}\left[2\left(p\beta\left(\frac{p}{2}\right) - 1\right) - 3\,I_4(p)\right]$, $I_4(p) = \int_0^{\infty} dx \frac{e^{-px}}{\mathrm{ch}^4 x}$ — нет в гр. рынке. $p = 2x\Delta \gg 1$.

$\int_{-\infty}^{\infty} \frac{d\xi}{\mathrm{ch}^2 y(\xi)} = 2\Delta + 2\Delta\frac{a^2}{\nu^2}\left[2\,I_2(p) - 3\,I_4(p)\right]$, $I_n(p) = \int_0^{\infty} dx \frac{e^{-px}}{\mathrm{ch}^n x}$, $p = 2x\Delta$, $k = x\Delta$.

$\int_{-\infty}^{\infty} \frac{d\xi}{\mathrm{ch}^2(\xi/\Delta)}\varepsilon^2(\xi) = \int d\xi \frac{1}{\mathrm{ch}^2 \xi/\Delta}\left[\varepsilon_1 + \delta\varepsilon e^{-\varkappa|\xi|}\right]^2 = 2\Delta\int_0^{\infty}\frac{dx}{\mathrm{ch}^2 x}\left[\varepsilon_1^2 + 2\varepsilon_1\delta\varepsilon e^{-kx} + \delta\varepsilon^2 e^{-2kx}\right] =$

$= 2\Delta\left[\varepsilon_1^2 + 2\varepsilon_1\delta\varepsilon\, I_2(k) + \delta\varepsilon^2 I_2(2k)\right] \to E_{DW} = 2\Delta + 2\Delta\frac{a^2}{\nu^2}\left[2I_2(p) - 3I_4(p)\right] +$

$\varepsilon(\xi) = \frac{\nu}{\lambda\Delta} + \frac{a}{\nu} e^{-\varkappa|\xi|} = \varepsilon_1 + \delta\varepsilon e^{-\varkappa|\xi|}$, $\delta\varepsilon = \frac{a}{\nu}$. $+ \lambda\cdot 2\Delta\left[\varepsilon_1^2 + 2\varepsilon_1\delta\varepsilon\, I_2(p/2) + \delta\varepsilon^2 I_2(p)\right]$

$I_n(p) = \int_0^{\infty} dx \frac{e^{-px}}{\mathrm{ch}^n x}$ найдем как интегралы Лапласа (оператор). $I(p) = \int_0^{\infty} dx e^{-px} q(x) = \sum_{s=0}^{\infty} \frac{q^{(s)}(0)}{p^{s+1}}$

$p \gg 1$,

В нашем приближ. $I(p) = \frac{q(0)}{p}$, $I_n = \int_0^{\infty} dx e^{-px}\frac{1}{\mathrm{ch}^n x} = \frac{1}{p}\left(\frac{1}{\mathrm{ch}^n x}\right)_{x=0} = \frac{1}{p}$.

$E_{DW} = 2\Delta + 2\Delta\frac{a^2}{\nu^2}\left[\frac{2}{p} - \frac{3}{p}\right] + \lambda 2\Delta\left[\varepsilon_1^2 + 2\varepsilon_1\delta\varepsilon\frac{2}{p} + \delta\varepsilon^2\frac{1}{p}\right]$, $\varepsilon_1 = \nu/(\lambda\Delta)$, $\delta\varepsilon = \frac{a}{\nu}$.

т.к. eq. for $f(\xi) = \mu_\xi \to f_{\xi\xi} - \frac{2}{\Delta}\mathrm{th}\frac{\xi}{\Delta} f_\xi - \lambda\Delta^2 f(\xi) = 0$ linear homogeneous, то $f(\xi) = a e^{\pm\sqrt{\lambda}\Delta\xi}$

Как и 21/IV [$\delta\varepsilon = -\varepsilon_1\alpha, \alpha < 1, \alpha > 0$] все сводится к свойствам $\delta\varepsilon = \frac{a}{\nu}$ и ε_1. Оставим $\delta\varepsilon = -\alpha\varepsilon_1 = \frac{a}{\nu}$ (a - произвольная)

$\frac{E_{DW}}{2\Delta} = 1 - \frac{1}{p}\frac{a^2}{\nu^2} + \lambda\left[1 - 4\frac{\alpha}{p} + \frac{\alpha^2}{p}\right]\varepsilon_1^2 = 1 + \lambda\varepsilon_1^2 - \frac{1}{p}\frac{a^2}{\nu^2} + \frac{\lambda}{p}\alpha(\alpha - 4)\varepsilon_1^2$, $1 + \lambda\varepsilon_1^2 = 1 + \lambda\frac{\nu^2}{\lambda^2\Delta^2} = \frac{1}{\Delta^2}$

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$$\frac{1}{2\Delta}E_{dw} = \frac{1}{\Delta^2} - \frac{1}{\rho}\frac{a^2}{x^2} + \frac{\lambda}{\rho}(\alpha-4)\alpha\frac{a^2}{\alpha^2 v^2},\quad \rho = 2\varkappa\Delta,\quad E_{dw} = \Delta^2 + \left[-\frac{1}{\varkappa^2} + \frac{\lambda}{v^2}\frac{\alpha(4-\alpha)}{\alpha^2}\right]\frac{a^2}{\rho}$$

$$\frac{1}{2\Delta}E_{dw} = \frac{1}{\Delta^2} - \left[\frac{1}{\varkappa^2} + \frac{\lambda}{v^2}\frac{(\alpha-4)\alpha}{\alpha^2}\right]\frac{a^2}{\rho} = \frac{1}{\Delta^2} - \left[\frac{1}{\lambda\Delta^2} + \frac{\lambda^2\Delta}{v^2}\frac{\alpha(\alpha-4)}{\alpha^2}\right]\frac{a^2}{2\Delta\sqrt{\lambda}\Delta}$$

$$E_{dw}(v) = \frac{2}{\Delta} - \frac{1}{\sqrt{\lambda}\Delta}\left[\frac{1}{\lambda\Delta^2} + \frac{\alpha(\alpha-4)}{\alpha^2}\frac{\lambda}{v^2}\right]a^2,\quad E_{dw}(v) < E^0_{dw} = \frac{2}{\Delta},\ \text{т.к. } \alpha<1,\ \alpha>0$$

$$a = -\alpha v\varepsilon_1 = -\alpha v\frac{v}{\lambda\Delta} = -\frac{\alpha}{\lambda}\frac{v^2}{\Delta},\quad E_{dw} = \frac{2}{\Delta} - \frac{1}{\sqrt{\lambda}\Delta}\left[\frac{1}{\lambda\Delta^2} + \frac{(\alpha-4)\alpha}{\alpha^2}\frac{\lambda}{v^2}\right]\frac{\alpha^2}{\lambda^2}\frac{v^4}{\Delta^2}$$

$$E_{dw}(v) = \frac{2}{\Delta} - \frac{1}{\sqrt{\lambda}\Delta^3\lambda}\left[\frac{v^2\alpha^2}{\lambda^2\Delta^2} + \alpha(\alpha-4)\right]v^2$$

— но другими $E_{dw}(v)$ похожа $2\sqrt{|v|}$, но с доп. членом ур-я $y(\xi) = \frac{\xi}{\Delta} + \mu(\xi)$

$$a(v) = -\frac{\alpha}{\lambda}\frac{v^3}{\Delta},\quad a(v\to c) = -\frac{\alpha}{\lambda}\frac{\lambda}{\Delta} = -\frac{\alpha}{\sqrt{1-v^2/c^2}} \to \infty,\quad a(v\to 0)\sim v^2\to 0.\quad \varkappa = \sqrt{\lambda}\Delta.$$

т.е. при конечных a и b $y(\xi) = \frac{\xi}{\Delta} - \frac{a}{\varkappa}\operatorname{sign}(\xi)e^{-\varkappa|\xi|}$, $\varepsilon(\xi) = \frac{v}{\lambda\Delta} + \frac{a}{v}e^{-\varkappa|\xi|}$, E_{dw} монотонна,

$E_{dw} = E^0_{dw} - |\dots|a^2$ монотонна. Возмущение: $y_1(\xi) = -\frac{a}{\varkappa}\operatorname{sign}(\xi)e^{-\varkappa|\xi|}$ — чётное. → linear term on a^4

$$\int_{-\infty}^{\infty} d\xi \frac{1}{ch^2(\xi/\Delta)} th\left(\frac{\xi}{\Delta}\right) y_1(\xi) = \left|\text{чётная}\right| = -\frac{a}{\varkappa}\int_{-\infty}^{\infty} d\xi \frac{th(\xi/\Delta)\operatorname{sign}(\xi)e^{-\varkappa|\xi|}}{ch^2(\xi/\Delta)} = -\frac{a}{\varkappa}2\Delta\int_0^{\infty} dx \frac{th x}{ch^2 x}e^{-x\Delta x},\quad k = \varkappa\Delta$$

$$J(k) = \int_0^{\infty} dx \frac{th x}{ch^2 x}e^{-kx},\quad \text{Olver}_2,\ \text{p. 96}\quad \int_0^{\infty} e^{-kx}q(x)dx = \sum_{s=0}^{\infty}\Gamma\left(\frac{s+\lambda}{\mu}\right)\frac{b_s}{k^{(s+\lambda)/\mu}},\ \text{если}$$

$$q(x)\sim\sum_{s=0}^{\infty} b_s x^{(s+\lambda-\mu)/\mu}\ \text{при } x\to 0.\quad q(x) = \frac{th x}{ch^2 x} \sim x \text{ при } x\to 0,\ b_0 = 1,\ \frac{\lambda-\mu}{\mu} = 1$$

$$q(x) = b_0 x^{(\lambda-\mu)/\mu} + O(x^2).\ \text{Итого } \lambda = 2\mu,\ \mu = 1,\quad \lambda/\mu = 2$$

$$\int_0^{\infty} dx\, e^{-kx}q(x) = \Gamma\left(\frac{\lambda}{\mu}\right)\frac{b_0}{k^{\lambda/\mu}} = \Gamma(2)\frac{b_0}{k^2},\quad J(k) = \frac{\Gamma(2)}{k^2}\ \text{при } k\gg 1,\ \Gamma(2) = 1$$

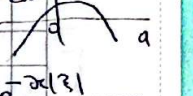
$$\int_{-\infty}^{\infty}\frac{d\xi}{ch^2\xi/\Delta} th(\xi/\Delta)y_1(\xi) = -2\Delta\frac{a}{\varkappa}J(k) = -2\Delta\frac{a}{\varkappa}\frac{1}{k^2} = -2\Delta\frac{a}{\varkappa}\frac{1}{\varkappa^2\Delta^2}$$

и в E_{dw} есть член $\sim a$

$$\int_{-\infty}^{\infty}\frac{d\xi}{ch^2 y(\xi)} = -2\int\frac{d\xi}{ch^2(\xi/\Delta)}th(\xi/\Delta)y_1(\xi) + (\text{вторые равные}) = +4\Delta\frac{a}{\varkappa^3\Delta^2} - \text{доп. вклад в } E_{dw}.$$

$$E_{dw}(a) = \frac{2}{\Delta} + \frac{4}{\varkappa^3\Delta}a - \frac{1}{\sqrt{\lambda}\Delta}\left[\frac{1}{\lambda\Delta^2} + \frac{\alpha(4-\alpha)}{\alpha^2}\frac{\lambda}{v^2}\right]a^2,\quad \frac{\partial^2 E_{dw}}{\partial a^2} = -[\dots]$$

нет min $E_{dw}(a)$



$a = -\alpha v\varepsilon_1 < 0$ по смыслу, если $\alpha>0$ $\quad \delta\varepsilon = -\alpha\varepsilon_1 < 0$.

$$F(y) = \frac{1}{ch^2 y},\quad F'(y) = \left[ch^{-2}y\right]' = -\frac{2}{ch^3 y}shy = -2\frac{thy}{ch^2 y}$$

$$F(y_0+y_1) = F(y_0) + F'(y_0)y_1 = \frac{1}{ch^2 y_0} - 2\frac{thy_0}{ch^2 y_0}y_1,\quad y_1 = y_1(\xi) = -\frac{a}{\varkappa}\operatorname{sign}(\xi)e^{-\varkappa|\xi|} - \text{чётно.}$$

Видимо, решение SG с $\varepsilon = \text{const}$ $\left[\varepsilon_0 = v/\lambda\Delta,\ y_0 = \frac{\xi}{\Delta},\ \Delta^2 = 1 - v^2/\lambda\right]$ неустойчиво по отношению к $y(\xi) = \frac{\xi}{\Delta} + \mu(\xi)$, $\varepsilon = \varepsilon_0 + \frac{a}{v}e^{-\varkappa|\xi|} = \varepsilon(\xi)$ деформация.

Смысл L_{kin}? $L = \int d\xi\left[-\cos\theta\dot\Phi - e(\xi)\right] = -\int d\xi\cos\theta\dot\Phi - E_1,\ E = \int d\xi\, e(\xi)$.

$$L_{kin} = -\int_{-\infty}^{\infty} d\xi\cos\theta\dot\Phi = -\int d\xi\, thy(-\dot\xi),\quad tg\theta/2 = e^{-y}\ \text{связь},\ \cos\theta = +thy,\ \dot\xi = \xi_{\dot\xi}(-v)$$

$$L_{kin} = -\int d\xi\, thy\cdot\varepsilon_{\xi},\quad \int d\xi\,\varepsilon_{\xi}thy = \int d(\varepsilon)thy = \left|\begin{matrix}\varepsilon = v\\ thy = u,\ \frac{1}{ch^2 y}\frac{dy}{d\xi} = \frac{du}{d\xi}\end{matrix}\right| =$$

$$= \varepsilon thy\Big|_{-\infty}^{\infty} - \int\varepsilon\frac{dy}{ch^2 y} = \varepsilon(+\infty)\cdot 1 - \varepsilon(-\infty)(-1) = 2\varepsilon_0 - \int\varepsilon\frac{dy}{ch^2 y},\quad \frac{dy}{d\xi} = \frac{1}{\Delta} + \mu_{\xi},\ y_0 = \frac{\xi}{\Delta},\ k = \varkappa\Delta = \sqrt{\lambda}\Delta^2$$

$$L_{kin} = -2\varepsilon_0 + \int d\xi\frac{\varepsilon}{ch^2 y} = -2\varepsilon_0 + \int\frac{\varepsilon}{ch^2 y}\left(\frac{1}{\Delta} + \mu_{\xi}\right)d\xi,\quad J_1 = \int d\xi\frac{\varepsilon}{ch^2 y} = \int d\xi\,\varepsilon\times$$

$$\times\left[\frac{1}{ch^2 y_0} - 2\frac{thy_0}{ch^2 y_0}y_1\right] = \int\frac{d\xi}{ch^2 y_0}\left[\varepsilon_0 + \frac{a}{v}e^{-\varkappa|\xi|}\right] - 2\int d\xi\left[\varepsilon_0 + \frac{a}{v}e^{-\varkappa|\xi|}\right]\frac{thy_0}{ch^2 y_0}\left(-\frac{a}{\varkappa}\right)\varsigma(\xi)e^{-\varkappa|\xi|} =$$

$$= 2\Delta\int_0^{\infty}\frac{dx}{ch^2 x}\left[\varepsilon_0 + \frac{a}{v}e^{-kx}\right] + \frac{2a}{\varkappa}2\Delta\int_0^{\infty} dx\frac{th x}{ch^2 x}\left[\varepsilon_0 + \frac{a}{v}e^{-kx}\right]e^{-kx} = 2\Delta\varepsilon_0 + 2\Delta\frac{a}{v}\int_0^{\infty}\frac{dx}{ch^2 x}e^{-kx} +$$

$$+ 4\frac{a\Delta}{\varkappa}\left[\varepsilon_0\int_0^{\infty} dx\frac{th x}{ch^2 x}e^{-kx} + \frac{a}{v}\int_0^{\infty}\frac{dx}{ch^2 x}th x\, e^{-2kx}\right] = 2\Delta\varepsilon_0 + 2\Delta\frac{a}{v}\frac{1}{k} + 4\frac{a\Delta}{\varkappa}\left[\varepsilon_0\frac{1}{k^2} + \frac{a}{v}\frac{1}{(2k)^2}\right] =$$

$$= 2\Delta\varepsilon_0 + 2\Delta\frac{a}{v}\frac{1}{\sqrt{\lambda}\Delta^2} + 4a\frac{\Delta}{\sqrt{\lambda}\Delta}\left[\varepsilon_0\frac{1}{\lambda\Delta^4} + \frac{1}{4}\frac{a}{v}\frac{1}{\lambda\Delta^4}\right] = 2\Delta\varepsilon_0 + \frac{2a}{\sqrt{\lambda}v\Delta} + \frac{4a}{\sqrt{\lambda}}\frac{1}{\lambda\Delta^4}\left(\varepsilon_0 + \frac{1}{4}\frac{a}{v}\right) = J_1$$

$$J_2 = \int d\xi\frac{\mu_{\xi}\varepsilon}{ch^2 y} = \int d\xi\frac{\mu_{\xi}\varepsilon}{ch^2 y_0} = 2\Delta\int_0^{\infty}\frac{dx}{ch^2 x}\left[ae^{-kx}\right]\left[\varepsilon_0 + \frac{a}{v}e^{-kx}\right] = 2\Delta a\int_0^{\infty}\frac{dx}{ch^2 x}\left[\varepsilon_0 e^{-kx} + \frac{a}{v}e^{-2kx}\right] =$$

$$= 2\Delta a\left[\varepsilon_0\frac{1}{k} + \frac{a}{v}\frac{1}{2k}\right] = \frac{2\Delta a}{\sqrt{\lambda}\Delta^2}\left(\varepsilon_0 + \frac{1}{2}\frac{a}{v}\right),\quad L_{kin} = -2\varepsilon_0 + \frac{1}{\Delta}J_1 + J_2,\quad \varepsilon_0 = v/\lambda\Delta$$

$$L_{kin} = -2\varepsilon_0 + 2\varepsilon_0 + \frac{2a}{\sqrt{\lambda}v\Delta^2} + \frac{4a}{\sqrt{\lambda}}\frac{1}{\lambda\Delta^5}\left(\varepsilon_0 + \frac{1}{4}\frac{a}{v}\right) + \frac{2a}{\sqrt{\lambda}\Delta}\left(\varepsilon_0 + \frac{1}{2}\frac{a}{v}\right) = \frac{2a}{v\sqrt{\lambda}\Delta}\left[\frac{1}{v\Delta} + \frac{2\varepsilon_0}{\lambda\Delta^4} + \varepsilon_0\right] +$$

$$+ \frac{a^2}{\sqrt{\lambda}v\Delta}\left[1 + \frac{1}{\lambda\Delta^4}\right],\quad E_{dw}(a) = \frac{2}{\Delta} + \frac{4}{\lambda^{3/2}\Delta^4}a - \frac{1}{\sqrt{\lambda}\Delta}\left[\frac{1}{\lambda\Delta^2} + \frac{\alpha(4-\alpha)}{\alpha^2}\frac{\lambda}{v^2}\right]a^2,\quad \alpha = -\frac{a}{v\varepsilon_0}.$$

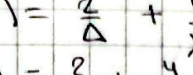
Escaneado con CamScanner

С d в $E_{DW}(a)$ переставляю. $\frac{4d}{d^2}\frac{\lambda}{v^2}a^2 = \frac{4}{d}\frac{\lambda}{v^2}a^2 = -\frac{4}{a}\nu\varepsilon_0\frac{\lambda}{v^2}a^2 = -4a\varepsilon_0\frac{\lambda}{v}$

$E_{DW}(a) = \frac{2}{\Delta} + \frac{4a}{\lambda^{3/2}\Delta^4} - \frac{1}{\sqrt{\lambda}\Delta}(4a)(-1)\frac{\lambda\varepsilon_0}{v} - \frac{1}{\sqrt{\lambda}\Delta}\left[\frac{1}{\lambda\Delta^2} - \frac{\lambda}{v^2}\right]a^2$, $\varepsilon_0 = \frac{v}{\lambda\Delta}$,

$E_{DW}(a) = \frac{2}{\Delta} + \frac{4a}{\lambda^{3/2}\Delta^4} + \frac{4a}{\sqrt{\lambda}\Delta^2} - \frac{1}{\sqrt{\lambda}\Delta\lambda\Delta^2}\left[1 - \frac{\lambda}{v^2}\lambda\Delta^2\right]a^2$

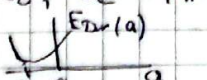
$E_{DW}(a) = \frac{2}{\Delta} + \frac{4}{\sqrt{\lambda}\Delta^2}\left(\frac{1}{\lambda\Delta^2} + 1\right)a - \frac{1}{\lambda^{3/2}\Delta^3}\left[1 - \frac{\lambda^2}{v^2}\Delta^2\right]a^2$, $1 - \frac{\lambda^2}{v^2}\Delta^2 = 1 + \lambda\left(1 - \frac{\lambda}{v^2}\right)$

Sign of $1 - \frac{\lambda^2}{v^2}\Delta^2 = f(v)$ — 

$1 - \frac{\lambda^2}{v^2}\Delta^2(v) = 0$ при $v = v_c$, $v_c = \frac{c}{\sqrt{1+\nu_\lambda}} < c$, $v_c \to c$

$1 - \frac{\lambda^2}{v^2}\Delta^2(v) < 0$ при $v < v_c$ $\Rightarrow$ коэф. при $a^2 > 0$, при $v < v_c$, $v_c \to c$, $\lambda \to \infty$

$E_{DW}(a) = E_{DW}(0) + Aa + Ba^2$, $\frac{\partial E_{DW}}{\partial a} = A + Ba_0 = 0$, $a_0 = -\frac{A}{B} < 0$, $\frac{\partial^2 E_{DW}}{\partial a^2} = B > 0$



Equil. $a_0 = +\frac{4}{\sqrt{\lambda}\Delta^2}\frac{(1+\nu_\lambda\Delta^2)}{\lambda^{3/2}\Delta^{-3}\left[1 - \frac{\lambda^2}{v^2}\Delta^2\right]}$, $a_0 = +4\lambda\Delta\frac{(1+\nu_\lambda\Delta^2)}{(1-\frac{\lambda^2}{v^2}\Delta^2)}$

Проверить коэф. $E_{DW}(a)$ при a^2.

$L_{min}(a) = \frac{2a}{\sqrt{\lambda}\Delta}\left[\frac{1}{v\Delta} + \frac{2}{\lambda\Delta^4}\frac{v}{\lambda\Delta} + \frac{v}{\lambda\Delta}\right] + \frac{a^2}{\sqrt{\lambda}v\Delta}\left(1 + \frac{1}{\lambda\Delta^4}\right) = \frac{2a}{\sqrt{\lambda}v\Delta^2}\left(1 + \frac{2}{\lambda^2}\frac{v^2}{\Delta^4} + \frac{v^2}{\lambda}\right) + \frac{a^2}{\sqrt{\lambda}v\Delta}\left(1 + \frac{1}{\lambda\Delta^4}\right)$

→ Проверить и $E_{DW}(a)$ и $L_{min}(a)$ от $E(\xi)$ - approach.

(25/IV) Meeting с Ricardo: обсуждение 1D DW manuscript. Наруш-ли ему передавал Introduction, получил Refs и обсудил DW dyn. experiments in thin stripes с 1D DWs. E-mail/Fedex how to finish MS to Ruben и Ricardo.

В "Lukas" и "Todo-todo" - upgrade.

(26/IV) переписка с Ruben о 1D DW paper! (кто со-авторы, куда и когда сдать, что доделать). Сделали ему про Letters / Brief Commun. без Pierre, который пока не сделал свой вклад и я - the last coauthor as project coordinator, who understands all details of calculations. Идея о Sine-Gordon применительно до $E_{tr}\sim 1/\cosh(x)$ (а не Ruben), все calculations сделал я.

SW — Проверить eqs. by Slavin для расчета "дискретных" modes, SW radial in H⊥ plane cylindrical dot. Глеб у Бернардо, что Slavin's eqs $H_{res}(n)$ использовались в Kakazei APL'2004 для подгонки FMR $H_{res}^{exp}(n)$ для Ni и FeNi. См. "Radial_SW_Cyl_dot_Slavin.mcd":

СФ $m(\beta,k) = J_0(k\beta)$, нормировка

СФ $A_n(k) = \frac{1}{2}\left[J_0^2(k) + J_1^2(k)\right]$. Gaussian dip. field $H_d^{nn} = \frac{4\pi M_s}{A_n(k)}\int d\beta\beta\, m^2(\beta,k)\, H_d(\beta,\beta)$

$H_d(\beta,\beta) = -\frac{1}{\beta}\int_0^\infty \frac{dx}{x}(1 - e^{-\beta x})J_0(\beta x)J_1(x)$

Частота f in GHz, γ in GHz/kOe ($\gamma/2\pi = 3.1$ для CoFe dots), M_s in kG, $10^{-3}\frac{emu}{cm^3}$.

$\tilde{\omega} = \frac{f}{(\gamma/2\pi)} = \frac{2\pi f}{\gamma} = \frac{\omega}{\gamma}$ used in the file. $H_{ex} = \frac{2A}{M_s}k^2 = \frac{2A}{M_s}\frac{x_n^2}{R^2}$, см. 23/III

x - wave vector in units YR. $xJ_1(x) = dJ_0(x)$.

$H_n(n) = \frac{\omega}{\gamma} - H_d^{nn} - 2\pi M_s f(\beta k)$ и $H_{re}(n) = \frac{\omega}{\gamma} - H_d^{nn} - 2\pi M_s f(\beta k) - H_{ex}(n)$ -

- diagonal approximation, linear dispersion relation $\omega(H)$.

Slavin's eqs. $h_z(k) = G(k) - H_d^{nn}(k)$, $h_{re}(k) = G(k) - H_d^{nn}(k) - H_{ex}(n)$

$H_{ex}(n) = \frac{2A}{M_s}\frac{x_n^2}{R^2}$, R in nm, M_s in kG, A in abs. units 10^{-6} erg/cm, H in kOe

$R_a = (R\cdot 10^{-7})$ cm, $(M_s)_{abs} = (M_s\cdot 10^3)$ G, $H_{ex}(n) = \frac{2A}{10^3 M_s}\frac{x_n^2}{10^{-14}R^2}\cdot 10^{-3}$ kOe

В file Slavina $H_{ex}(n)$ верное. $k \to x_n$.

Вопрос: как у Slavina вместо $\frac{\omega}{\gamma} - 2\pi M_s f(\beta k)$ появилось $G(k)$?

$G(k) = -2\pi M_s f(\beta k) + \sqrt{(\omega/\gamma)^2 + [2\pi M_s f(\beta k)]^2}$. Дает ли $G(k)$ подгонку exp. Ni, NiFe APL'2004 и дает ли подгонку new exp. на CoFe dots?

$\frac{1}{4\pi}G^2 + 2\cdot 2\pi f M_s G - (\omega/\gamma)^2 = 0$ - примерно такое. И откуда оно?

$G = H = H_z + H_d + H_{ex}$, $H^2 + 4\pi M_s f H = (\frac{\omega}{\gamma})^2$; $(\frac{\omega}{\gamma})^2 = H(H + 4\pi M_s f)$ —

это eq. Herring-Kittel $\frac{1}{\gamma}\omega_k^2 = (H + H_{ex})\times(H + H_{ex} + 4\pi M_s f(\beta k))$ из APL'2004.

Но $\omega(k)$ должно быть не Herring-Kittel (верное при $k\to 0$) а линейное дисперс. eq. $\omega/\gamma = H + H_{ex}$ как я использовал для fitting Ni dots - Jan. 2003

Eq. by Slavin $H_z^s(n) = G(x_n) - H_d^{nn} - H_{ex}(x_n)$, $G = H_{stat} = H_z + H_d + H_{ex}$ – уравнение для static fields, далее подробно изучено. Описание exp. M-APL'2004, там linear $H_z(n) = \frac{\omega}{\gamma} - 2\pi M_s f(\beta x_n) - H_d^{nn} - H_{ex}(n)$ при $x_n = d_n$, $J_0(d_n) = 0$ – strong pinning. Уравнение $G(x) = -2\pi M_s f(\beta x) + \sqrt{(\omega/\gamma)^2 + (2\pi M_s f(\beta x))^2}$ нагадает с ростом $x = kR$, поскольку, там $\omega/\gamma - 2\pi M_s f(\beta x) = H(x)$

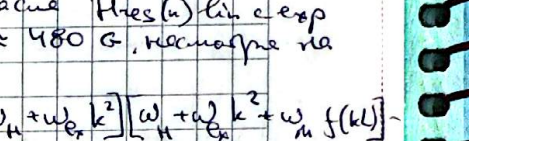
$G(x) = -2\pi M_s f + \frac{\omega}{\gamma}\sqrt{1 + \frac{(2\pi M_s f)^2}{\omega^2}} = \frac{\omega}{\gamma} - 2\pi M_s f(\beta x) + \frac{1}{2}\frac{[2\pi M_s f(\beta x)]^2}{(\omega/\gamma)}$ при $\beta x \ll 1$

Разница linear $H(x) = \frac{\omega}{\gamma} - 2\pi M_s f(\beta x)$ и $G(x)$ с увеличением M_s и βx.

$\beta x = \beta x_n$ растет с β и n mode number "n".

$g(x) = \frac{1}{4\pi M_s} G(x) = -\frac{1}{2} f + \sqrt{\left(\frac{\omega}{\omega_M}\right)^2 + \frac{1}{4} f^2}$, $h(x) = \frac{1}{4\pi M_s} H(x) = \frac{\omega}{\omega_M} - \frac{1}{2} f$, $f = f(\beta x)$.

$f(x) = 1 - \frac{1}{x}(1 - e^{-x})$. $g(x) \approx \frac{\omega}{\omega_M} - \frac{1}{2} f + \frac{1}{8}\frac{\omega_M}{\omega} f^2(x) = h(x) + \frac{1}{8}\frac{\omega_M}{\omega} f^2(\beta x)$



$\beta = 0.1$ - typical, $x_n = d_n$ уравн. до $d_8 = 24.35 \to \beta x$ up to 2.5. $\frac{\omega_A}{\omega_M} \simeq 0.2(G)$ $0.5(Ni)$ при $f \approx 10$ GHz

от 2.40

Abs-ness $G(x) - H(x) \approx 2\pi^2 \frac{M_s^2}{(\omega/\gamma)} f(\beta x) \sim M_s^2$ at $\omega = const$, т.к. $f(x) \to 1$ при $x \geq 1$, large mode number "n".

$G(x) - H(x) \sim M_s^2 f(\beta x)$, для которых linear $H_z(n)$ не работало где $R \geq 400$ nm (см. стр. 24/III)

При $R \geq 400$ nm $M_s \simeq 1100 - 1200$ G (по числу $n = 1$) Вычисл. и lin $H_{res}(n)$ с $H(x)$ сравнимо отличается от Slavin $H_{res}(n)$ с $G(x)$. $G(x)$ больше $> H(x) \Rightarrow H_{res}^{Slav}(n) > H_{res}^{lin}(n)$.

(27/IV) Исследование $G(x)$, $H(x)$. Изображение APL'2004 fitting $H_{res}(n)$ для FeNi.

Parameters M_s, L, γ, A как в APL'04. Slavin $H_z(n)$ хорошо по границам exp. $H_z(n)$, $H_z(n)$ - lin больше чем $H_z(n)$ - Slavin. Для $n = 8$ lin чуть H_z - Slav на $\simeq 1.3$ kOe. Для Ni ($n = 8$) было H_{res} - lin < H_z - Slavin на 0.7 kOe, согласно $H_{res}(n)$ lin с exp и $H_z(n)$ Slavin было лучше, что объясняется малым M_s (Ni) $\simeq 480$ G, насколько на рост β от 0.10 (FeNi) to 0.14 (Ni).



Вывод: fitting SW R Kakazei - APL'04 генерация по Eq. $\omega_k^2 = [\omega_H + \omega_{ex} k^2][\omega_H + \omega_{ex} k^2 + \omega_M f(kL)]$ - Slavin (тут using $G(x) = -2\pi M_s f + \sqrt{(\omega/\gamma)^2 + [2\pi M_s f]^2}$). $G(x)$ - fitting Slavin хорошо лучше fitting в $H_z(n)$ - lin с $\omega/\gamma = H_z^{(n)} + H_d^{nn} + H_{ex}(n) + 2\pi M_s f(\beta x_n)$. Это при $x_n = d_n$

Reducing pinning d при $d \to \infty$ pinning усиливается, diagonal approximation linear $\frac{\omega}{\gamma} = H_{st} + 2\pi M_s f(\beta x)$ усиливается совпадение с экспер., уменьшение $H_z(n)$, но не принципиально. Только linear - $H_{res}(n)$ всегда заметно лучше $H_z(n)$ - Slavin (several %) в поведении $\omega(k)$.

Valinikos - Slavin'1986: Eq. (45) $\omega_k^2 = [\omega_H + d\omega_M k^2][\omega_H + d\omega_M k^2 + \omega_M F(k)]$

k - in plane and antinormal, $F \to F_{nn}$, n - number of $x_n = k_z(n)$ - quantization along film thickness.

$F = P + \sin^2\Theta\left[1 - P(1 + \cos^2\Phi) + \omega_M \frac{P(1-P)\sin^2\Phi}{(\omega_H + d\omega_M k^2)}\right]$, $P_{00} = P_{nn}(n=0) = f(kL)$.

(Θ, Φ) - polar angles of $\vec{M}_0(\Theta, \Phi)$ с $oz \perp$ film surface/plane. Φ angle $(\vec{M}_0)_{xy}$ и $\vec{k}$, $\vec{k}$ in plane xy.

Out-of-plane (perpendicularly) magnetized film $\Theta = 0$, $F = P$, $F_{00} = P_{00} = f(kL)$.

$\omega_k^2 = [\omega_H + d\omega_M k^2][\omega_H + d\omega_M k^2 + \omega_M f(kL)]$ used in Kakazei APL'04.

Но интересно ω_k все же Eq. (44). При $\Theta = 0$, $B = 0$, $D = 0$, $A = \cos^2\Phi$, $C = \sin\Phi\cos\Phi$, $E = \sin^2\Phi$

$\left(\frac{\omega_n}{\omega_M}\right)^2 (T_{nn})^2 = \left[\frac{\Omega_n}{\omega_M} + \cos^2\Phi P_{nn}\right]\left[\frac{\Omega_n}{\omega_M} + \sin^2\Phi P_{nn}\right] - \sin^2\Phi\cos^2\Phi (P_{nn})^2$

$\Omega_n = \omega_H + d\omega_M k^2$. $T_{nn} = \frac{1}{L}\int_{-L/2}^{L/2} d\zeta\, \Phi_n^2(\zeta) = 1$ for normalized functions $\Phi_n(\zeta)$.

In ω_M units $\omega_n^2 = [\Omega + P_{nn}\cos^2\Phi][\Omega + P_{nn}\sin^2\Phi] - (P_{nn})^2\sin^2\Phi\cos^2\Phi$

$\omega_n^2 = \Omega(\Omega + P_{nn})$. При $P_{nn} \to P_{00} = f(kL) \to \omega_k^2 = \Omega(\Omega + f(kL))$

Это все для dynamic $\vec{m}(\xi, \zeta, t) = \vec{m}(\zeta) e^{i(\omega t - k_x x)} = \vec{m}(\zeta) e^{i(\omega t - \vec{k}\vec{s})}$ - plane waves с $\vec{k}(k_x, k_y)$ in the film plane xOy. Plane waves одновременно для ∞ - film. $z \to \zeta$

Для finite cylindrical dot $\vec{m} = \vec{m}(\rho, \varphi, t) = \vec{m}_0(\rho) e^{i(\omega t - m\varphi)}$ вместо plane waves $\vec{m}(x,y) \sim e^{i\vec{k}\vec{s}}$, $\vec{m}_0(\rho)$ вопрос. через Bessel functions $J_m(k\rho)$, где k is radial wave number, k is not k_z, где z - arbitrary direction in film plane.

Возвращаемся к plane-wave eq. $\omega_k^2 = \Omega[\Omega + \omega_M f(kL)]$, с $\vec{m} \sim e^{i(\omega t - k_z z)}$ не учитывается. $k = k_z$

Но даже Slavin $\omega_k^2 = \Omega[\Omega + \omega_M f(kL)]$ нет ограничения $kL \ll 1$, any kL, $\omega = \omega_\perp$ $kL \gg 1$!
Для cylindrical waves $\omega_k = (\Omega + \ldots)$ is linear due to radial symmetry ($m=0$ case).
ω_k^2 - plane waves на контрах radial symmetry, когда $\vec{k} = k_z \hat{z}$ задают axial symmetry
с осью $\| \vec{k}$. Замена $k_z \to k_\beta = x_n/R$ без сильных предположений.

Видимо для cylindrical dot с $\vec{M}_0 \perp$ plane есть тоже „exact" solution, а eq. Slavin $\omega_k = \Omega[\Omega + \omega_M f(\rho x)]$ к нему сводится вплоть до linear $\omega_k = \Omega + \frac{A}{\mu_0} k^2$ eq.
Pinning at $\rho = R$, diagonal/nondiagonal approximation within linear ω_k - approach имеют воспроизведение значения. Ищем тоже „exact" solution сначала для dots with finite R и $H_d(\rho)$. Может точки поворота $\frac{\omega}{\gamma} = H_{int}(\rho)$, $\rho = \rho_0$ важны?

24/III → $\left[\frac{\omega}{\omega_M} - h - h_0(\rho) + l_e^2 \nabla^2\right] \mu(\rho) = \frac{1}{2}\int_0^1 d\rho' \rho' g(\rho\rho') \mu(\rho')$ - quasi-uniform eq for $\mu_\pm(\rho,t) = e\psi(\rho) e^{\pm i\omega t}$

$h_0(\rho) = -\frac{1}{\beta}\int_0^\infty \frac{dx}{x}(1 - e^{-\beta x}) J_0(\rho x) J_1(x)$, $g(\rho\rho') = \int_0^\infty dx\, x f(\beta x) J_0(x\rho) J_0(x\rho')$, $h_0(\rho) < 0$,
$\max h_0(\rho) = h_0(1) = -\frac{1}{2}$
$\min h_0(\rho) = h_0(0) \simeq -1 + O(\beta)$.

24/III я искал решение $\mu(\rho) = \sum_\nu a_\nu \psi_\nu(\rho)$, $\psi_\nu(\rho) = \frac{1}{\sqrt{N_\nu}} J_0(x_\nu \rho)$, $x_\nu J_0'(x_\nu) + d J_0(x_\nu) = 0$
$a_\nu = \frac{1}{2}[J_0^2(x_\nu) + J_1^2(x_\nu)]$
Без обмена: $F(\rho)\mu(\rho) = \frac{1}{2}\int d\rho' \rho' g(\rho\rho') \mu(\rho')$, $F(\rho) = \frac{\omega}{\omega_M} - h - h_0(\rho)$, $F(\rho)$ может $= 0$ at $\rho \in [0,1]$.
Int-equation для $\mu(\rho)$ с variable $F(\rho)$ не есл Fredholm equation, как обычно для SW.
Int-eq. for $\mu(\rho)$ - homogeneous Fredholm of third kind - нетрадиц.
Вернемся к T. Yukawa, K. Abe, JAP 45, 3146 (1974) и Y. Ishikawa, T. Abe, JAP 48, 3601 (1977).
Damon et al. JAP 33, 3453 (1965) для film с $\vec{M}_0$: $\rho^{1/2} \operatorname{tg}\left[\frac{1}{2}\rho^{1/2} kL\right] = 1$

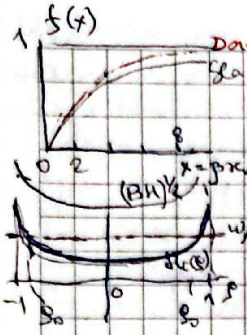
$\rho = \frac{BH_i - (\omega/\gamma)^2}{(\omega/\gamma)^2 - H_i^2}$, $H_i = H_z + H_d(\rho)$, $B_i = H_i + 4\pi M_s$, k - in-plane wave number
Volume MSW exist at $\rho \geq 0$ $H_i \leq \frac{\omega}{\gamma} \leq (BH)^{1/2}$, $\frac{1}{2}kL = \left(\frac{\omega^2 - H^2}{BH - \omega^2}\right)^{1/2} \operatorname{arctg}\left[\left(\frac{\omega^2 - H^2}{BH - \omega^2}\right)^{1/2}\right] \to k(\rho)$
Standing wave $2\int_0^{z_1} dz \frac{1}{(\lambda/2)} = n$, $k = \frac{2\pi}{\lambda}$, $n = \frac{2R}{\pi}\int_0^{\rho_1} d\rho\, k(\rho)$, $\rho = z/R$, $z_1 \to \frac{\omega}{\gamma} = H_i(z_1)$ turn. point.
$dH = \frac{\omega}{\gamma} - (BH)^{1/2}_{\rho=0} \leq 0$ - условие bulk MSW localized at $\rho \in [0, \rho_1]$.
Если $kL \ll 1$, $\frac{1}{2}\rho kL = 1$, $\left(\frac{1}{2}kL\right)[BH - \omega^2] = \omega^2 - H^2$, $\omega^2 = H\left(H + 4\pi M_s \frac{1}{2}\rho kL\right)$ - верно.
$\int_{-z_1}^{+z_1} dz\, k(z) = \pi n$, $n = 2N-1$ или $2N$ ($N = 1,2,3\ldots$). Исикава $\int_0^R k(z) dz = d_{n-1,N}$, $J_{n-1}(d) = 0$, N-root number
$\int_0^{z_1} dz\, k(z) = \frac{\pi}{2}\frac{(2N-1)}{2N}$ или $\int_0^R dz\, k(z) = d_{n,N}$, $J_n(d) = 0$ - quantization.

Гур.-Мелков, p. 170, ∞-film с $\vec{M}_0 \perp$ plane $\omega^2 = \omega_H\left[\omega_H + \omega_M \frac{1}{1 + z_n^2/(kL)^2}\right]$, $\operatorname{tg} z = \frac{2zkL}{z^2 - k^2L^2}$, z_n-root
$z = k_z L = \sqrt{-\mu}\, kL$, $k_z = \sqrt{-\mu}\, k$. $\mu = 1 + \chi_{xx} = \frac{\omega_H(\omega_H + \omega_M) - \omega^2}{\omega_H^2 - \omega^2} = \frac{BH - \omega^2}{H^2 - \omega^2}$, MSW at $\mu(\omega) \leq 0$
$\rho = \frac{BH - \omega^2}{\omega^2 - H^2} = -\mu$. $\frac{2zkL}{z^2 - (kL)^2} = \frac{2(z/kL)}{[z^2/kL^2 - 1]} = 2\frac{\sqrt{-\mu}}{-1-1} = -2\frac{\sqrt{-\mu}}{1+\mu}$, $\operatorname{tg}\left(\sqrt{-\mu}\, kL\right) = -2\frac{\sqrt{-\mu}}{(1+\mu)}$
$\operatorname{tg}\left(\rho^{1/2} kL\right) = \frac{2\operatorname{tg}\left(\frac{1}{2}\rho^{1/2} kL\right)}{1 - \operatorname{tg}^2(\frac{1}{2}\ldots)} = \frac{2\rho^{-1/2}}{1 - \rho^{-1}} = \frac{2\rho^{1/2}}{\rho - 1} = \frac{2\sqrt{-\mu}}{-\mu - 1} = -2\frac{\sqrt{-\mu}}{(1+\mu)}$
т.е. eq. for z - Гур. Мелков совпадает с eq. $\rho^{1/2}\operatorname{tg}\left(\rho^{1/2}\frac{1}{2}kL\right) = 1$ Damon'1965 или $\rho = -\mu$.
$\omega_n^2 = \omega_H\left[\omega_H + \frac{\omega_M}{1 + z_n^2/(kL)^2}\right]$, ω_n изменен от ω_H до $\omega_\perp = \sqrt{\omega_H(\omega_H + \omega_M)}$ с ростом kL
$x_n = z_n/kL$, $\omega_n^2 = \omega_H\left[\omega_H + \frac{\omega_M}{1 + x_n^2}\right]$, $\operatorname{tg}(x kL) = \frac{2x}{(x^2 - 1)} \to x_n$. $x\operatorname{tg}\left(\frac{1}{2}x kL\right) = 1 \to x_n$.
Damon-Standard $\omega_n^2 = \omega_H[\omega_H + \omega_M F_n(kL)]$, $F_n(kL) = \frac{1}{1 + x_n^2}$, $x_n = x_n(kL)$ - n-root $x\operatorname{tg}\left(\frac{1}{2}x kL\right) = 1$.
Диагональное приближение Slavin $\omega_k^2 = \omega_H[\omega_H + \omega_M f(kL)]$ для the lowest $x_n = k_z/k$ обычно
$x_1(kL)$
приближает to eq. Damon'1965 - точному решению MSW.

Check $dH = \frac{\omega}{\gamma} - (BH)^{1/2}_{\rho=0}$, $BH = H_i(H_i + 4\pi M_s)$, $H_i = H_z + H_d(\rho)$, $\frac{H_d(0)}{4\pi M_s} = -\frac{1}{\beta}\int_0^\infty \frac{dx}{x}(1 - e^{-\beta x}) J_1(x) =$
$= -\frac{1}{\beta}\int \frac{dx}{x}\left[e^{-\beta_0 x} - e^{-\beta x}\right] J_1(x) = -\frac{1}{\beta}\left[\frac{\sqrt{\beta_0^2 + 1} - \beta_0}{1} - \frac{\sqrt{\beta^2 + 1} - \beta}{1}\right] = -\frac{1}{\beta}\left[-\sqrt{\beta^2+1} + \beta + 1\right] = \frac{1}{\beta}\left[1 + \beta - \sqrt{1+\beta^2}\right]$ (-1)
$b_i h_i = h_i(h_i + 1) = \left[h - \frac{1}{\beta}(1 + \beta - \sqrt{1+\beta^2})\right]\left[h - \frac{1}{\beta}(1 - \sqrt{1+\beta^2})\right]$, $dH = \frac{\omega}{\omega_M} - (b_i h_i)_0^{1/2} = F(\omega, h, \beta) = F(\frac{\omega}{\omega_M}, h, \beta)$
$h = h_{res}(n)$ зависит от mode number n. Надо проверить $\operatorname{sign} dH_n$ для n - всех модов.

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$f_{Dam}(x) = \frac{1}{1+x_1^2(kL)}$ → x_1 – first root $x\,tg(\frac{1}{2}xkL)=1$ – Damon'1965

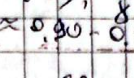
$f(x) = 1-\frac{1}{x}(1-e^{-x})$ – Slavin Pos(x). $f(x)$ немного меньше $f_{Dam}(x)$ при $x>1$.

$\frac{\Delta f}{f} \approx 4\%$ при $x=\beta x_1 = \beta$. $\omega_{Slav} < \omega_{Dam}(H) \Rightarrow H_{res}^{Dam} < H_{res}^{Slav}(\omega)$ at $\omega=const$.

Две FeNi пленки (APL'1980) с $\beta=0.1$ $\frac{\omega}{\gamma} > H_i(\theta)$, $\frac{\omega}{\gamma} - (BH_i)^{1/2} = dH < 0$, если толщина подбирается $d_n = 1,-8$ при $\theta_0 \approx 0.90 - 0.95$. $H_n(n)$ по Slavin eq со strong pinning.

Damon'65: $\omega^2 = \omega_H[\omega_H + \omega_M f_D(kL)]$, Slavin'86: $\omega^2 = \omega_H[\omega_H + \omega_M f(kL)]$

$\omega_D(kL) > \omega_S(kL)$ at $kL \gg 1$ ($kL = \beta x_n$) и при $\omega = const$ $H_n^{Dam}(n) < H_n^{Slav}(n)$ для large n (точнее при $\beta x_n \gg 1$). По Damon $\omega_D(kL)$ – чисто magnetostatic SW – nondiagonal, а Slavin $\omega_S(kL)$ – diagonal magnetostatic SW, где учтено упругое exchange.



(28/IV) С то. зр перпов на нач. окружн. Оформ.

В Damon'65 $\omega_D(kL)$ и Slavin'86 $\omega_S(kL)$ удовлетворяют MSW – direct volume MSW rule $\omega_H \le \omega(kL) \le \omega_\perp$, $\omega_\perp^2 = \omega_H(\omega_H+\omega_M)$ для infinite film. Но и $\omega_D(kL)$ и $\omega_S(kL)$ имеют невзрачно симметрию, их симметрия не свойственна 2D – radially sym. spin waves. Симметрия ω_D, ω_S – это uniaxial, соответствующая plane wave e^{ikr} с $\vec{k}=(k_x k_y)$ (along $\vec{H}$) ch-plane.

→ Вопрос: $\omega(k)$ дает radial cylindrical waves в ∞-film, или нельзя Refl (sparks?) root и symmetry была верной, $\omega(kL\to\infty)$ предел был верным.

$f_D(x) = \frac{1}{1+z_1^2(x)}$, z_1 – first root $z\,tg(\frac{1}{2}zx)=1$, $x=kL$, $z=p^{1/2}=\sqrt{f_M(\omega)} = \left(\frac{BH-\omega^2}{\omega^2-H^2}\right)^{1/2}$

$p = z^2 = \frac{\omega_\perp^2-\omega^2}{\omega^2-\omega_H^2} \ge 0 \to \omega_H \le \omega \le \omega_\perp$, $\omega_\perp^2 = BH = \omega_H(\omega_H+\omega_M)$

Yukawa'54, Ikenaga'77 $x = \frac{2}{\pi}\arctg(\frac{1}{z}) = kL$, $k(\theta) = \frac{2}{L}\arctg(\frac{1}{z})$, $z(\theta)$ из-за $H \to H_i(\theta)$ internal field

$\omega^2 = \omega_H(\omega_H+\omega_M F) \to F = \frac{\omega^2-\omega_H^2}{\omega_M\omega_\perp} = \frac{1}{1+z_1^2(\theta)}$, $z_1^2 = \frac{1}{F}-1 = \frac{(\omega_\perp^2-\omega^2)}{(\omega^2-\omega_H^2)} = p$.

Все $H_n(\omega)$ имеют минимум $H_1 = H_n(\theta)$ и H_c – $H_c = H$ – cut-off by $\frac{\omega^2}{\gamma^2} = B_i H_i|_{\theta=0} = \omega_\perp^2(\theta=0)\frac{1}{\gamma^2}$

H_1 из $\frac{\omega}{\gamma} = H_i(\theta=0)$, $\omega = \omega_H(\theta=0)$. $H_c \le H_n(\omega) \le H_1$.

J.A. Eshbach JAP 34, 1298 (1963): при $\frac{\omega}{\gamma} < (BH)^{1/2}$ – oscil. SW, $H_2 > H_c$,

при $\frac{\omega}{\gamma} > (BH)^{1/2}$ – загрж. SW



$\mu(\theta) = A_i(\zeta)$, $\zeta = \left(\frac{H_d'(\theta_i)}{D}\right)^{1/3}(\theta-\theta_i)$ – Airy function at $\theta \approx \theta_i$

при $H < H_c$, MSW могут быть, но они отражаются to the edge $\theta > \theta_i$.

Если учитывать dyn. dipolar field $\neq 0$ $\left[\frac{\omega}{\omega_M} - h - h_d(\theta) + l_e^2\nabla^2\right]J^n(\theta) = 0$ – Schrödinger eq.

с потенциалом из-за $h_d(\theta) \sim H_d(\theta)$ – finite size.

но Eshbach wrote $\left[-\frac{2A}{M_s}\nabla^2 + \frac{\omega_\perp(\theta)}{\gamma} - \frac{\omega}{\gamma}\right]J^n(\theta) = 0$, $\omega_\perp^2(\theta) = B_i(\theta)H_i(\theta)\gamma^2$

Отсюда z_0 при $\lambda \ll R$, $kR \gg 1$, cylindrical SW можно "approximate as plane waves".

Хотя $J_n(kR) \approx \left(\frac{2}{\pi kR}\right)^{1/2}\cos\left(kR - \frac{n\pi}{2} - \frac{\pi}{4}\right)$ при $kR \gg 1$ (это неверно in both center $R \neq 0$) как plane waves $\sim \cos(kR - \varphi_0)$ не похожи.

Ikenaga'1977: cylindrical waves (disks), но все равно Damon'65 $y\,tg(\frac{1}{2}ykL) = 1$, $\theta = \gamma/R$,

$y = i\sqrt{f_M(\omega)}$, $\mu = \frac{BH-\omega^2}{H^2-\omega^2}$, $H \to H_i$, $\omega \to \frac{\omega}{\gamma}$. $\alpha_{n-1,N} = \frac{2R}{L}\int_0^{\theta_1} d\theta\, \eta^{-1}(\theta,H)\left[\arctg\frac{1}{y(\theta,H)}\right]$, $R\int_0^{\theta_1} d\theta\, k(\theta,H) = \alpha_{n-1,N}$

$k = \frac{2}{L}\frac{1}{y}\arctg\frac{1}{y}$, $y^2(\theta,H) = -\mu = \frac{BH-\omega^2}{\omega^2-H^2}$ $(H \to H_i(\theta))$ так же как в Yukawa-Abe'1974

Bound. cond. $J_{n-1}(kR) = 0$, $k = \alpha_{n-1,N}/R$ Эффективная $\int_0^{\theta_1} k(\theta,H) = \alpha_{n-1,N}$, при больш. $kR \gg 1$

$\alpha_{n-1,N} \approx \pi\left[N + \frac{1}{2}|n-1| - \frac{1}{4}\right]$, это дает $H_{res}(N)$, $N = 1,2,3,\dots$ точнее θ_1 (turning point) от 1.

т.е. нахождение $\theta_1 = \theta_1(H)$ из $\omega/\gamma = H_i(\theta_1)$. $\alpha_{n-1,N} = \frac{2R}{L}\int_0^{\theta_1(H)} d\theta\,\frac{1}{y}\arctg\frac{1}{y}$, $y = y(\theta,H)$.

$n=1$, $\alpha_{0,N} = \pi_N$, $\pi_N(H) = \frac{2}{\pi}\int_0^{\theta_1(H)} d\theta\,\frac{1}{y}\arctg\frac{1}{y}$ – можно посчитать с $y = y(\theta,H)$. Или

$\frac{1}{y^2} = \frac{H_i^2-\omega^2/\gamma^2}{\omega^2/\gamma^2 - H_i B_i} = \frac{\omega^2/\gamma^2 - H_i^2}{H_i B_i - \omega^2/\gamma^2} = \frac{\omega^2-\omega_H^2}{\omega_\perp^2-\omega^2} \ge 0$ $\frac{1}{y^2} \sim (\omega/\gamma - H_i) \to 0$ при $\theta \to \theta_1$.

→ M. Sparks, Bell. St. Com. 8, 431 (1980): MSW in infinite circular disk

magnetost. potential $\psi(\theta,\varphi,z) = J_n(k\theta)e^{in\varphi}\cos(k_z z - \frac{\nu}{2}\pi)e^{i\omega t} \to \omega^2 = \omega_H[\omega_H + 1\,\omega_d]$, $\nu = 0,1$

$\omega_d = \frac{\omega_M}{2}\frac{k^2}{k^2+k_z^2}$, $k_z^2 = -f(\omega)k^2$. $tg\left[\frac{1}{2}k_z L - \frac{1}{2}\nu\pi\right] = k/k_z$. $H_i \to H_i + D(k_\theta^2+k_z^2)$ exchange

$2\omega_d = \omega_M\frac{1}{1+(k_z/k)^2}$, $x = k_z/k$, $\nu = 0$ $x\,tg\left[\frac{1}{2}xkL\right] = 1$ дает $x_1(kL)$ и $2\omega_d = \frac{\omega_M}{1+x_1^2(kL)}$ – Fuji, Men kot

т.е. где дисп. $2\omega_d = \frac{\omega_M}{1+\chi_1^2(kL)}$, χ - root $x\,\mathrm{tg}\left[\frac{1}{2}x\,kL\right]=1$ у Sparks'то также но как у Damon'1965 где oo-film + $\vec{k}$-plane wave. Можно $k_y \to (k_x^2+k_y^2)=k$.

$m_\varphi, m_\rho \sim J_l(k\rho)\, e^{il\varphi} \cos(k_z z - \frac{1}{2}y\pi)$, $l=n+1$, within $kR \gg 1$ limit. $m_\rho|_R = 0$, $J_l(kR)=0$

Radially sym. modes, $l=0$. Using m_ρ, m_φ as trial function in a var. calculation of ω for finite R $\Rightarrow$ $2\omega_d = 4\pi M_s \gamma f(kL)$, $f(x)=1-\frac{1}{x}(1-e^{-x})$ как у Kalinikos-Slavin'1986 ($l=0$, no pinning on $z=\pm L/2$)

Для нормир. Slavin's eq. $\omega_2 = \omega_H(\omega_H+\omega_M f(kL))$ так хорошо работает для $H_{res}(\omega)$. Но сам интеграл из LL eq. и $f(kz\dots)$, то надо сдвиг. $m_\rho(\rho,z)$, $m_\varphi(\rho,z) \sim J_0(k\rho)\cos(k_z z)$,

$k_z \neq 0 \Rightarrow \langle \vec{m}\rangle_z = \frac{1}{L}\int_{-L/2}^{L/2} dz\, \vec{m}(\rho,z)$ будет зависеть от k_z.

$\frac{1}{L}\int_{-L/2}^{L/2} dz\, \vec{m}(\rho,z) = \vec{m}(\rho)\frac{1}{L}\int dz \cos k_z z = \vec{m}(\rho)\frac{1}{k_z L}\sin k_z z\Big|_{-L/2}^{L/2} = \vec{m}(\rho)\frac{2\sin(k_z L/2)}{k_z L}$, $k_z = \sqrt{-\mu}\,k \neq 0$, т.к. $k \neq 0$.

При strong pinning $k \to k_n = \pi n/R$. У Kalin.-Slavin $f(kL)$ получается при $k_z = 0$.

Вывод: вернулся к выводу eq. для $m_\rho = f(\rho)$ $\vec{m}_0$ от 10/I, 10/III, 26/VI 2002 и главное $f(dz(\dots)$ также с учетом $m_\rho(\rho,z)$, $m_\varphi(\rho,z) \sim J_0(k\rho)\cos(k_z z)$.

Note: 11/I 2003 в надежде на $H_{res}(\omega)$ Nicolet, $\frac{\omega}{\gamma} = H + H_d^{nn} + 2\pi M_s f(\beta k_n)$ за счет $H_k = 1.5$ kOe и малого $M_s = 460$ G (Ni). При малом M_s lin-$\omega(H)$ и $\omega(H)$-Slavin сильно отличаются.

УД 10/I 2002 г. $\vec{m} = \vec{m}_0 + \vec{\mu}(\rho, z, \varphi, t)$. Ищем $\mu_\pm(\rho,z,t) = e(\rho,z,t)\,e^{\pm i\varphi}$, $\Theta = \Theta_0 + e\theta$, $\Phi = \Phi_0 + \psi$

$-\frac{1}{\gamma}\dot{\vec{\mu}} = \vec{m}_0 \times (\vec{H}_m^0 + \vec{H}) + \vec{m}_0 \times \vec{h}_{ex} + \vec{\mu} \times \vec{H}_m^0 + \vec{m}_0 \times \vec{h}_m + \vec{\mu} \times \vec{H}$

$\vec{H}_m = \vec{H}_m^0 + \vec{h}_m$, $\vec{H}_{ex} = \vec{h}_{ex}$, $\vec{H} = \vec{H}_0 + \vec{h}_w$. Считаем $\vec{m}_0 \parallel Oz$, $\vec{m}_0 = M_s\hat{z}$. $\vec{\mu}(t)$ - SW.

$\langle\dots\rangle_z$ все УД. $\langle \vec{H}_m^0\rangle_z = \hat{z}\,G_{zz}\,m_0$. $\langle \vec{h}_{ex}\rangle_z = \frac{1}{L}\int dz \left[\alpha\nabla^2\vec{\mu}\right]$, $\vec{\mu} = (\mu_x, \mu_y) = (\mu_s, \mu_\varphi)$

$(\nabla^2\vec{\mu})_\varphi = \nabla^2\mu_\varphi - \frac{1}{\rho^2}\mu_\varphi + \frac{2}{\rho^2}\frac{\partial\mu_\rho}{\partial\varphi}$, $(\nabla^2\vec{\mu})_\rho = \nabla^2\mu_\rho - \frac{1}{\rho^2}\mu_\rho - \frac{2}{\rho^2}\frac{\partial\mu_\varphi}{\partial\varphi}$. $\nabla^2\mu = \nabla_\rho^2\mu + \frac{1}{\rho^2}\frac{\partial^2\mu}{\partial\varphi^2} + \frac{\partial^2\mu}{\partial z^2}$

$\nabla_\rho^2\mu = \frac{\partial^2\mu}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\mu}{\partial\rho}$. $\langle(\nabla^2\vec{\mu})_\varphi\rangle = \langle\nabla^2\mu_\varphi\rangle - \frac{\langle\mu_\varphi\rangle}{\rho^2} + \frac{2}{\rho^2}\frac{\partial}{\partial\varphi}\langle\mu_\rho\rangle$, $\langle(\nabla^2\vec{\mu})_\rho\rangle = \langle\nabla^2\mu_\rho\rangle - \frac{\langle\mu_\rho\rangle}{\rho^2} - \frac{2}{\rho^2}\frac{\partial}{\partial\varphi}\langle\mu_\varphi\rangle$

$\langle\nabla^2\mu_d\rangle \to \left\langle\frac{\partial^2}{\partial z^2}\mu_d\right\rangle = \frac{1}{L}\int dz \frac{\partial^2\mu_d}{\partial z^2} = \frac{1}{L}\frac{\partial\mu_d}{\partial z}\Big|_{-L/2}^{L/2} = 0$, т.к. $\frac{\partial\vec{\mu}}{\partial z} = 0$ на $z = \pm L/2$ (?)

$\mu_d(z)$ - even, $\partial\mu_d/\partial z$ - odd $f(z)$. Но $\partial\mu_{x,y}/\partial z = 0$, $\partial\mu_z/\partial z$ может $\neq 0$, но $\mu_z = 0$.

$\langle\nabla^2\vec{\mu}\rangle_z = \nabla^2\langle\vec{\mu}\rangle_z$, $\langle\vec{m}_0 \times \vec{h}_{ex}\rangle_z = \alpha\langle\vec{m}_0 \times \nabla^2\vec{\mu}\rangle = \alpha\vec{m}_0 \times \langle\nabla^2\vec{\mu}\rangle = \alpha\vec{m}_0 \times \nabla^2\langle\vec{\mu}\rangle_z$

$\langle\vec{m}_0 \times \vec{h}_m\rangle = \vec{m}_0 \times \langle\vec{h}_m\rangle$. $\langle\vec{\mu} \times \vec{H}_m^0\rangle$ не нулевой, $\vec{H}_m^0 = (H_m^{0\rho}, 0, H_m^{0z})$

$\vec{\mu} \times \vec{H}_m^0 = \begin{vmatrix} \hat{\rho} & \hat{\varphi} & \hat{z} \\ \mu_\rho & \mu_\varphi & 0 \\ H_\rho & 0 & H_z \end{vmatrix} = \hat{\rho}\,\mu_\varphi H_m^z - \hat{z}\,\mu_\varphi H_m^\rho - \hat{\varphi}\,\mu_\rho H_m^z$, $\vec{H}_m^0 = \vec{H}_m^0(\rho,z)$.

$-\frac{1}{\gamma}\langle\dot{\vec{\mu}}\rangle = \alpha\vec{m}_0 \times \nabla^2\langle\vec{\mu}\rangle + \vec{m}_0 \times \langle\vec{h}_m\rangle + \langle\vec{\mu}\rangle \times \vec{H} + \langle\vec{\mu} \times \vec{H}_0^m\rangle_z$

μ_ρ, μ_φ - even $f(z)$, $H_m^\rho(z)$ - odd. $\langle\mu_\varphi(z) H_m^\rho(z)\rangle_z = 0$

$\langle\vec{h}_m\rangle = \frac{1}{L}\int dz \int dz' \int d^2\vec{\rho}'\, \vec{G}(\vec{r}\,\vec{r}')\vec{\mu}(\rho',z')$ где $\vec{G}(\vec{r},\vec{r}')$ надо вычисл. $\langle\vec{\mu}\rangle \to \vec{\mu}$

$-\frac{1}{\gamma}\dot{\vec{\mu}} = \alpha\vec{m}_0 \times \nabla^2\vec{\mu} + \vec{\mu} \times \vec{H} + \vec{m}_0 \times \langle\vec{h}_m\rangle_z + \langle\vec{\mu} \times \vec{H}_m^0\rangle_z$ - УД усреднение.

$\langle\vec{\mu}(\rho,z)\rangle_z = \vec{\mu}(\rho)\frac{2\sin(k_z L/2)}{k_z L}$ при $\mu_d(\rho,z) \sim \cos k_z z$. См. УД (4), (6) Sparks'1970

→ M. Sparks, Phys. Rev. B 1, 3831 (1970). $\mathrm{arctg}\frac{\pi}{x} \to \frac{\pi}{x}\left[1 - \frac{1}{x}(1-e^{-x})\right]$ $x = kL$

Для $\vec{m}_0 \perp$ film plane $\frac{\omega^2}{\gamma^2} = \left[\langle H_i\rangle + Dk^2\right]\left[\langle H_i\rangle + Dk^2 + 2\omega_d\right]$, $\mathrm{arctg}\frac{\pi}{x} \to \frac{\pi}{x}f(x)$.

и $\omega^2 = \langle\omega_H\rangle\left[\langle\omega_H\rangle + \omega_M f(kL)\right]$, $\frac{1}{\gamma}\langle\omega_H\rangle = \langle H_i\rangle + Dk^2$. $\omega_d = \frac{1}{2}\omega_M f(kL)$ получается

и для cylindrical dot из общей формулы для ω_d при $k_z L \ll \pi$, $kR \gg 1$.

$\omega_d = -\frac{1}{2}\gamma M_s \langle\partial^+ G \partial^-\rangle$, $\langle\partial^+ G \partial^-\rangle = \frac{1}{N}\int d^3\vec{r}\, m_+^*(\vec{r}) \int d^3\vec{r}'\, \partial^+ \frac{1}{|\vec{r}-\vec{r}'|}\partial_1^- m_+(\vec{r}')$

$N = \int d^3\vec{r}\, |m_+|^2$, $\partial^\pm = \frac{\partial}{\partial x} \pm i\frac{\partial}{\partial y}$, $\partial_1^- = \frac{\partial}{\partial x'} - i\frac{\partial}{\partial y'}$, $m_\pm = m_x \pm i m_y$.

$\psi(\rho,\varphi,z) = J_n(k\rho)\cos(k_z z - \frac{1}{2}y\pi)e^{-in\varphi}$. Если брать trial function $m_+ = \partial^+\psi_z^-$ то

для ω_d получим $\omega_d = \frac{1}{2}\omega_M \frac{k^2}{k^2+k_z^2}$, $\mathrm{tg}\left(\frac{1}{2}k_z L - \frac{1}{2}y\pi\right) = k/k_z$ eq. Damon'65 для plane waves

Сделаем хорошее приближение $\omega_d = \frac{1}{2}\omega_M f(kL)$ при $kR \gg 1$. $\omega_d = 2\pi M_s^2 \sin^2\theta_k$,

θ_k угол $\hat{k}\,\hat{z}$ с $\vec{H}_0$, $\sin^2\theta_k = k_z^2/(k_z^2+k_\rho^2)$ → Damon'65 eq.

$\omega^2/\gamma^2 = H(H+2\omega_d)$, в circular-precession approximation $\omega_{ff} = H + \omega_d$, $H = H_i + Dk^2$, See

H. Benson, D.L. Mills, Ph. Rev. 188, 849 (1969).

Escaneado con CamScanner

(29/IV) Sparks' PRB' 1970 из уравнения $m_+ = \chi_- \partial^+ \psi$. Magnetost. dyn. field $\vec{h} = \nabla\psi$

$m_+ = \chi_- \partial^+ \psi = \chi_- h_+$, $\vec{h} = \nabla G \nabla \cdot \vec{m}$, $Gf(\vec{r}) = \int d^3\vec{r}' \frac{1}{|\vec{r}-\vec{r}'|} f(\vec{r}')$

$m_+(\vec{r}) = \chi_- \partial^+ \int d^3\vec{r}' \frac{\nabla \cdot \vec{m}(\vec{r}')}{|\vec{r}-\vec{r}'|} = \chi_- \int d^3\vec{r}' \, G_{+d}(\vec{r}\vec{r}') m_d(\vec{r}')$, χ_- - const

т.е. $m_+(\vec{r}) \sim J_e(kg) e^{-ilq} \cos(k_z z - \frac{1}{2}\gamma\pi)$ есть СФ интегрального ур.

с ядром $G_{+d}(\vec{r}\vec{r}')$. // Ищем $\mu(g)$ с ядром $g(g,g') = \int dx \, x f(\beta x) J_0(xg) J_0(xg')$

$\mu(g) \sim J_0(xg)$ имеет при $R \to \infty$. $\lambda \mu(g) = \int_0^R g(gg') \mu(g') dg'$, $g(gg') = \int dk \, k f(kL) J_0(kg) \times J_0(kg')$

$\lambda \mu(g) = \frac{1}{2} \int_0^R dg' g' \int dk k f(kL) J_0(kg) J_0(kg') \mu(g')$. Ищем $\mu(g) = J_0(xg)$,

$\lambda J_0(xg) = \frac{1}{2} \int_0^R dg' g' \int dk k f J_0(kg) J_0(kg') J_0(xg') = \frac{1}{2} \int dk k f(kL) J_0(kg) \int_0^R dg' g' J_0(kg') J_0(xg') =$

$= |R\to\infty| = \frac{1}{2} \int dk k f(kL) J_0(kg) \frac{1}{k} \delta(k-x) = \frac{1}{2} f(xL) J_0(xg)$, т.е. $\mu(g) = J_0(xg)$ есть

СФ при $R \to \infty$ с СЗ $\lambda = \frac{1}{2} f(xL)$. Если R - finite, то одна $J_0(xg)$ не есть СФ.

Можно искать $\mu(g) = \sum_\nu a_\nu J_0(x_\nu g)$ как 24/IIIT, но полагая $a_\nu = a_n \delta_{n\nu}$, все ν кроме

одного хотя и одно ν должны удовлетворять (MathCad), $a_\nu \approx 0{,}9$, for normalized $\mu(g)$.

$m_+(g,\varphi,z) \underset{(R\to\infty)}{=} \chi_- k e^{-il\varphi} J_l(kg) \cos(k_z z - \frac{1}{2}\gamma\pi)$ дает $\omega_d = \frac{1}{2}\omega_M \frac{k^2}{k^2+k_z^2}$, $tg(\frac{1}{2}k_z L - \frac{\gamma}{2}\pi) = \frac{1}{k_z}$ -

- equation Damon '65. Если еще и $kR \gg 1$, то $\omega_d = \frac{1}{2}\omega_M f(kL)$.

Я покажу, что $f_{Dam}(x) = \frac{1}{1+x_1^2(kL)}$ хорошо приближается $\beta f(x)$ при $x = \beta x_n = kL$

не очень большом, при $x = \beta x_n \le 8$, ошибка $< 3{,}7\%$.

Условия применимости $\omega_d = \frac{1}{2}\omega_M f(kL)$, $kR = x_n \gg 1$ и $\beta x_n \le 5$ (ошибка $< 3\%$)

$x_{min} = x_1$ (strong pinning) $= 2{,}40$, $x_n \gg 1$ хорошо выполняется с $x_2 = 5{,}52$, $x_3 = 8{,}65$, ... ($n=2$)

Ni dots $\beta = 0{,}14$, n up to 8, $(\beta x_n)_{max} = 0{,}14 \cdot 24{,}35 = 3{,}40$ ($\Delta f/f = 2{,}3\%$)

FeNi dots $\beta = 0{,}1$, $n_{max} = 8$, $(\beta x_n)_{max} = 0{,}1 \cdot 24{,}35 = 2{,}43$

CoFe dots $\beta = 0{,}04$, $n_{max} = 8$ ($R = 1000$ nm), $\beta x_8 = 0{,}04 \cdot 24{,}35 = 0{,}97$

CoFe dots $\beta = 0{,}4$, $n_{max} = 2$ ($R = 100$ nm), $\beta x_2 = 0{,}4 \cdot 5{,}52 = 2{,}21$

CoFe dots $\beta = 0{,}2$, $n_{max} = 7$ ($R = 200$ nm), $\beta x_7 = 0{,}2 \cdot 21{,}21 = 4{,}24$ ($\Delta f/f = 2{,}8\%$)

CoFe dots $\beta = 0{,}133$, $n_{max} = 10$ ($R = 300$ nm), $\beta x_{10} = 0{,}133 \cdot 30{,}64 = 4{,}08$ ($\Delta f/f = 2{,}7\%$)

CoFe dots $\beta = 0{,}267$, $n_{max} = 4$ ($R = 150$ nm), $\beta x_4 = 0{,}267 \cdot 11{,}79 = 3{,}15$ ($\Delta f/f = 2{,}1\%$)

CoFe dots $\beta = 0{,}1$, $n_{max} = 10$ ($R = 100$ nm), $\beta x_{10} = 0{,}1 \cdot 30{,}64 = 3{,}06$

Для всех Ni, FeNi, CoFe dots $\Delta f/f \le 3\%$, ошибка в $\frac{\omega_d}{\gamma} = H\left(H + \frac{1}{f}\omega_M f(kL)\right)$ еще меньше

3%, т.к. $\omega_M f(kL)$ под $\sqrt{(\dots)}$. Хуже всего $\left(\frac{\Delta f}{f}_{max}\right)$ для $R = 200, 300$ nm CoFe dots.

Надо еще учесть finite k_z in H_{ex}. $\chi = k_z/k$. $k_z = \chi_1(kL) \cdot k$, $k = k_s$.

$H_{ex} = \frac{2A}{M_s}(k_s^2 + k_z^2) = \frac{2A}{M_s}(1+\chi_1^2)k^2$, $\chi_1 = \chi_1(kL)$ first root $x \cdot tg(\frac{1}{2}xkL) = 1$.

$1 + \chi_1^2(kL) = \frac{\omega_M}{2\omega_d} \approx \frac{\omega_M}{\omega_M f(kL)} = \frac{1}{f(kL)}$, $kL = \beta x_n$, $k = x_n/R$. $H_{ex} = \frac{2A}{M_s}\left[1 + \frac{1}{f(kL)}\right]k^2 = \frac{2A}{M_s R^2}\left[1+f^{-1}\right]x_n^2$

$H_{ex}(n) = \frac{2A}{M_s}\frac{1}{R^2}\left[1 + \frac{1}{f(\beta x_n)}\right]x_n^2$. $(\beta x_n)_{min} = 0{,}04 \cdot 2{,}40 = 0{,}96$, $f(\beta x_{min}) = 0{,}357$

$\left[1 + \frac{1}{f_{min}}\right] = 3{,}8$ (!) Нет, $[\dots] = \frac{1}{f}$

finite pinning d. $x_n(d)$ надает с d // $x_n(d)$ при $\beta = const$, $x_n(d) \gg 1$ выполняется

все хуже и хуже и полагая $\omega_d = \frac{1}{2}\omega_M f(kL)$. Лучше использовать $f_D(x) = \frac{1}{1+\chi_1^2(kL)}$,

перестроить CoFe dots using $f_D(kL)$ и modified $H_{ex}(n)$, use finite d.

Note: Sparks' то берет $m_+ = \chi_- \partial^+ \psi$ и $m_- = \chi_+ \partial^- \psi$ и надо $\chi_+ \neq \chi_-$, $|m_+| \neq |m_-|$ to get

non-circular precession и $\omega^2 = \omega_H(\omega_H + 2\omega_d)$. $\omega = \omega_H + \omega_d$ - for circular precession.

Откуда $\chi_+ \neq \chi_-$, если ант. ур. Ландау-Лифшица? Причина non-circular precession?

Записать ант. ур. для $\mu = \mu_+(\theta, z)$ perp. magnetized circular dot, не усредняя по "z"

$H_{neg}(n) = G(x_n) - H_d^{nn} - H_{ex}(x_n)$, $G(x) = -2\pi M_s f + \sqrt{\left(\frac{\omega}{\gamma}\right)^2 + (2\pi M_s f)^2}$, $f = f(kL) \to f_D(kL)$

$H_d^{nn} = 4\pi M_s [-1 + H_{n,i}]$, i numbers dots β_i. $H_{ex}(x_n) \simeq 4\pi M_s \left(\frac{L^2}{R^2}\right)\left[1 + f^{-1}(kL)\right]x_n^2$. $x_n \to x_{n,i}$

Escaneado con CamScanner

File Radial. SW-cyl-dot3.mcd" Здесь всё то же, что res(n) - Damon-Sparks через немного иначе res(n) - Slavin. Проверил для R=1000 nm CoFe dot. $l_e = 6{,}7$ nm (R=500.1 nm) - наложил, $M_s = 1240$ G.

Из linear SW dispersion eq. $\omega = \omega_H + \omega_d$ и non-circular precession eqs. $\omega^2 = \omega_H(\omega_H + 2\omega_d)$ Damon-Sparks с $\omega_d = \frac{1}{2}\omega_M f_D(kL)$ или Sparks - Slavin $\omega_d = \frac{1}{2}\omega_M f(kL)$ интерполирует точки поворота $\omega/\gamma = H_i(\beta)$. Точки поворота $\beta_0 \to 1$ при $\beta \ll 1$, но для $\beta > 0.1$, $\beta_0 < 0{,}9$ могут быть важными.

(30/IV) Meetings с Ricardo (1D SW manuscript Intro Refs и сравн. с эксп.) и Ai-Ping - переслала мне new MS по Skyrmions (!?) in nanotubes. Agustina написала PRX - Neel Sk MS и Resp. letter to check/change. Перепишу conf.

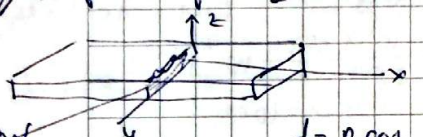
→ Для описания radial „drum" modes in CoFe dots можно/нужно:
• 1) описать pinning $d(\beta)$, т.к. β_0 up to 0.4; • 2) заменить $f(kL)$ на $f_D(kL)$ Damon; • 3) учесть finite k_z in Hex.

(1/V) Работа над Letter to Editor и Replies to Referees „Sk-Py dots" PRX-Berganza.

(2/V) Работа над MS text PRX Berganza. послал files to Agustina, Miriam, Oksana. Skyrme-обсуждение с Глебом „Drum" SW modes new calcs. Дополнил „Drum modes CoFe" MS by new eqs. в духе Damon-Sparks, послал Word и .tex files to Глеб и Сергей! P. Gruszecki прислал Simul. „SW scattering at film edge".

(3/V) Trip to Heraklion, HMM 2019: Bus DSS → Barajas 17/V – plane Bar. → Heraklion 18/V – Hotel Bar. 17/18/V
– Hotel Heraklion (18-23/V), доклад HMM – plane Her. – Aprunum 23/V – Hotel Aprunum 23/24/V –
– plane Aprunum – Barajas 24/V – Bus Bar. → DSS 24/V. • подгот. Sk-talk HMM-2019.

SW sim. Параметры для SW simulation by Ruben: $\mathcal{H} = -\frac{1}{2}\sum_{ij} J_{ij}\vec{S}_i\vec{S}_j - K_z\sum_j (S_j^z)^2 + K_x\sum_j (S_j^x)^2$



x - direction of SW propagation. Infinite film? Thickness?

$J = 5.77\cdot10^{-21}$ J, $K_x = 1.28\cdot10^{-22}$ J, $K_z = 4.01\cdot10^{-24}$ J

$L = 0{,}001$, $\mu = 2.5\,\mu_B$, $a = 0.4$ nm, $\gamma = 2.1\cdot10^5 \frac{m}{As}$, Simple cubic lattice.

Demag. fields are neglected (потому K_y доминирует, результат ≈ O_y намагничен?).

Unit cell volume $V_0 = a^3 = (0.4\cdot nm)^3 = (0.4\cdot10^{-7})^3 cm^3 = (4)^3\cdot10^{-24} cm^3 = 64\cdot10^{-24} cm^3$

$M_s = \mu/V_0 = 2.5\frac{\mu_B}{a^3} = 2.5\cdot\frac{9.274\cdot10^{-21}\,erg/G}{64\cdot10^{-24}\,cm^3} = 0.362\cdot10^3 G = 362$ G $(=362$ kA/m).

$A = \frac{zJ}{a} = |z=6$ simple cubic$| = 6\frac{5.77\cdot10^{-21}J}{4\cdot10^{-8}cm} = 8.66\cdot10^{-21}10^{8}\frac{erg}{cm} = 8.66\cdot10^{-6}\frac{erg}{cm}$

Вводимо выше $A = \frac{z}{2}\frac{J}{a}$ как следует из $E_{ex} = \sum_{ij} J_{ij}\cos k r_{ij}$ $|r_{ij}=a, J_{ij}=J|$

$A = 4.28\cdot10^{-6}$ erg/cm. A - undefinite, may be $A = \frac{z}{4}\frac{J}{a}$.

$\lambda = \frac{K_x}{K_z} = \frac{1.28\cdot10^{-22}J}{4.01\cdot10^{-24}J} = \frac{1.28}{4.01}10^2 = 31.9$ – очень гигант value.

Units of velocity $v_0 = \frac{\Delta_0}{t_0} = \frac{2\gamma}{M_s}\sqrt{AK_z} = 2\frac{2.1\cdot10^5\frac{m}{A\cdot s}}{362\,kA/m}\sqrt{AK_z}$.

$K_x = \frac{K_x}{V_0} = \frac{1.28\cdot10^{-22}J}{64\cdot10^{-24}cm^3} = \frac{1.28}{64}10^2\frac{J}{cm^3} = \frac{1.28}{64}10^9\frac{erg}{cm^3} = 2.0\cdot10^7\frac{erg}{cm^3}$ – гигант.

$K_z = \frac{1}{\lambda}K_x = 0{,}062\,J\cdot10^7\,erg/cm^3 = 6.27\cdot10^5\,erg/cm^3$, $H_a = \frac{2K_z}{M_s} = 2\frac{6.27\cdot10^5\frac{erg}{cm^3}}{362G} = 3.464$ kOe

$v_0 = \frac{2\cdot2.1}{362}\frac{m}{A\cdot s}\frac{m}{kA}\sqrt{AK_z}$. Лучше $\gamma/2\pi = 3.0\frac{GHz}{kOe}$, $v_0 = 2\frac{3.0\,GHz/kOe\cdot2\pi}{0.362\,kOe}\sqrt{4.28\cdot10^{-6}\frac{erg}{cm}\,6.27\cdot10^5\frac{erg}{cc}} =$

$= \frac{6.0}{0.362}\frac{GHz}{10^6 Oe^2}\cdot2\pi\frac{erg}{cm^2}\sqrt{4.28\cdot0.627} = \frac{6.0}{0.362}\frac{10^9}{10^6}\frac{1}{s}cm\sqrt{4.28\cdot0.627}\cdot2\pi = 170.6\,10^3\frac{cm}{s} = 1706\frac{m}{s}$ (!)

$v_- = v_0\left[\sqrt{1+\lambda} - 1\right] = 1706\frac{m}{s}\left[\sqrt{1+31.9} - 1\right] = 1.706\cdot\frac{km}{s}\cdot4.736 = 8.08$ km/s

$\Delta_0 = \sqrt{A/K_z} = \sqrt{4.28\cdot10^{-6}\frac{erg}{cm}/0.627\cdot10^6\frac{erg}{cm^3}} = \sqrt{\frac{4.28}{0.627}}\cdot10^{-6}cm = 2.61\cdot10^{-6}cm = 26.1$ nm



Из Fig-Ruben $\Delta_{SW}(v)$, $\Delta_0 = 12.0$ nm и $v_m = 21.4$ km/s + Neel SW.

$\Delta_{SW}(v) = \Delta_0\sqrt{1 - v^2/v_m^2}$. Если взять A из Δ_0-Ruben = 12 nm, то

$A = \Delta_0^2 K_z = (12)^2\,10^{-14}cm^2\,6.27\cdot10^5\frac{erg}{cm^3} = 902.9\cdot10^{-9}\,erg/cm = 0903\cdot10^{-6}\,erg/cm$

Escaneado con CamScanner

Уменьшение A от $4{,}28 \frac{erg}{cm} \cdot 10^{-6}$ до $0{,}903 \cdot 10^{-6} \frac{erg}{cm}$ снизит искажение скорости $v_0 = \frac{2\delta}{M_s}\sqrt{AK_z}$ в $\sqrt{\frac{4{,}28}{0{,}903}} = 2{,}177$ раз. Новое $v_0 = \frac{1}{2{,}177} \cdot 1706 \frac{m}{s} = 784 \frac{m}{s}$ и новое $v = \frac{8{,}08}{2{,}177} \frac{km}{s} = 3{,}71 \frac{km}{s}$.

Откуда же у Ruben $v_m = v_- = 21{,}4$ km/s (?), если Δ_0 совпадают?

Согласно Schlömann'и $\Delta(v)$, $\Delta(v \to v_-) = \sqrt{\frac{1}{1+\lambda}} \cdot \frac{\Delta(v)}{\Delta_0} = \frac{1}{(1+\lambda)^{3/4}} = 0{,}418$ при $\lambda = 31{,}9$.

Отношение $\Delta(v)$-Schlömann с $\frac{\Delta(v_-)}{\Delta_0} = 0{,}418$ не может описать $\Delta(v)$-Ruben, где $\frac{\Delta(v_-)}{\Delta_0} \to 0$ (<0,1).

// Может Ruben simulates DW in film or stripe и/или в маг. поле? Δ_0

Нужно DW profile $\theta(x,t)$. Это $\Phi(z) = \text{const}$ или $\Phi(z)$ - variable перемен?

E-mail's to Ruben, Burbea, Ricardo, Oleg T. и Ricardo. Reminder Gleb - «drum» modes.

Ricardo прислал план to change 1D-DW MS Introduction/Refs - послал ему comments.

(4/V) Revision 1D DW dynamics manuscript. В «Eroski» только начал.

(5/V) Загрузка EEC-Synergy 350-Skyrm. project - Runin-Bankowski (на рец.)

Пришла на рец. Zhao - Appl. Sci. - Skyrmion switching gate. Ещё. Design.

(6/V) Переписка с Exc. о re-subm. PRX - Skyrm. и Normal-COST. Собр. «Cheese bank» - проверить.

Написание 1D. DW manuscript (Leaf file). Ещё.

(7/V) Ruben прислал new parameters 1D DW Simulations: Ruben DW simul.

1) hexag. simple cubic lattice $a = 3{,}328 \cdot 10^{-10}$ m (было 3/V $a = 0{,}4$ nm), $V_0 = a^3$

2) anisotropy constant $K = (4.396 + 512) \cdot k_B / V_0$ [J/m^3], $a = 3{,}328 \cdot 10^{-8}$ cm

3) mag. moment $\mu = 4\mu_B$ of lattice site (было $\mu = 2.5\mu_B$)

4) $M_s = \mu / V_0$ [A/m]; 5) exch. stiffness $A = (115K \cdot k_B)/a$ [J/m]

$A = 115\,K \cdot 1{,}3806 \cdot 10^{-23}$ J/K $/ (3{,}328 \cdot 10^{-10}$ m$) = \frac{1{,}15 \cdot 1{,}3806}{3{,}328} 10^{-11} \left(\frac{J}{m}\right) = 4{,}77$ pJ/m

$A = 4{,}77$ pJ/m $= 4{,}77\ 10^{-12}\ 10^{7}$ erg/10^{2}cm $= 4{,}77 \cdot 10^{-7}$ erg/cm $= 0{,}477 \cdot 10^{-6}$ erg/cm

$K = (4.396 + 512)\,K \cdot 1{,}3806 \cdot 10^{-23} \frac{J}{K} \frac{1}{(3{,}328)^3 10^{-30}\ m^3} = \frac{2.096 \cdot 10^3 \cdot 1{,}3806}{(3{,}328)^3} 10^{-23}\ 10^{30} \frac{J}{m^3} =$

$= 7{,}85 \cdot 10^{-2}\ 10^{10}$ J/m^3 $= 7{,}85 \cdot 10^{8}$ J/m^3 (grant value). 1 J/m$^3 = 10$ erg/cm^3

$M_s = \frac{4\mu_B}{V_0} = 4 \cdot \frac{9.274 \cdot 10^{-21}\ erg/G}{(3{,}328)^3 10^{-30}\ m^3} = \frac{4 \cdot 9.274}{(3{,}328)^3} 10^{-21} \cdot 10^{24}\ G = 1{,}006 \cdot 10^3\ G$

$v_{max} = v_- = v_0(\sqrt{1+\lambda} - 1) \approx v_0\sqrt{\lambda} = \frac{2\delta}{M_s}\sqrt{AK_z}\sqrt{\frac{K_x}{K_z}} = \frac{2\delta}{M_s}\sqrt{AK_x}$, $K_x = K$ (?) Ruben

$v_{max} = 2\,\frac{2.802 \cdot (MHz/Oe) \cdot 2\pi}{1{,}006 \cdot 10^3\ G} \sqrt{0{,}477 \cdot 10^{-6} \frac{erg}{cm} \cdot 7.85 \cdot 10^{9} \frac{erg}{cm^3}} = 4\pi \frac{2.802}{1.006} 10^{-3} \times$

$\times \frac{MHz}{G \cdot Oe} \cdot \frac{erg}{cm^2} \sqrt{4{,}77 \cdot 7{,}85} \cdot 10 = 4\pi \cdot \frac{2.802}{1.006} 10^{-2}\ cm \cdot 10^{6} \frac{1}{s} \sqrt{4{,}77 \cdot 7{,}85} =$

$= 214{,}7 \cdot 10^4 \frac{cm}{s} = 214{,}7 \cdot 10^2 \frac{km}{s} = 21{,}47 \frac{km}{s}$, т.е. $v_{max} = 21{,}47 \left(\frac{km}{s}\right)$!

→ Note: $A = 0{,}477 \cdot 10^{-6} \frac{erg}{cm}$ (намного больше) $M_s = 1006$ G (Ok), $K = 7{,}85 \cdot 10^{8} \frac{J}{m^3} = 7.85 \cdot 10^{9} \frac{erg}{cm^3}$

— много неизвестно. Обычно $K \sim 10^7$ erg/cm^3 (даже для $FePt$)

Т.е. можно получить v_{max} (DW) $= 21{,}47$ km/s для очень grant нереалистич. K. //Ruben это признал.

// Вернулся из Киева. Из 3 300 € привезли ~250 € back. На билет 450 € домой

30% за 2017 и имплантат и 35 000 аванс за квартиру, взял и гонорар (~1 200 €), всего = 1650 €.

Л. чаю - 100 €, чаю - 100 €, Эдику - 100 €, Л. Дусе - 100 €.

Купил «Alsa» без тикета Тбилиси → Батуми на 17/V 9:00 из Тбилиси.

(8/V) С/п. «Lidl», «Carrefour» Olaberria. Закупки на Sol Skymag 2019. Тест/Сергей написали о трудностях fitting FMR drum modes. Compute by new equations:

A (La) Симеон нашёл и расст с yttr nagement bot radius R. Начал писать Аршин 23/V

(9/V) Reminder COST-Vosmar. Сон. Ikerbasque Workshop July 2. Купил тоже, Avaria', Аршин.

Drum modes. Надо привести ОР $\mu_z(\rho,z) = J_0(k_\rho \rho)\cos(k_z z)$ когда в теории Doman-Sparks k_z конечно. Сейчас $H_d(\rho,z)$ и усреднить $H_d(\rho,z)$ на $\mu_n(\rho,z) = J_0(2\pi\rho)\cos k_z z$, с правильной нормировкой $\mu_n(\rho,z)$. Требуем $\int d\rho\,\rho \int dz\,\mu_n^2(\rho,z)\frac{1}{L} = 1$

$k_z z = (k_z L)\,z/L$. $\int_{-L/2}^{L/2} dz \cos^2 k_z z = (k_z z = x) = k_z^{-1} \int dx \cos^2 x^{N_n} =$

$= \frac{1}{k_z}\frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right]_{-k_zL/2}^{k_zL/2} = \frac{1}{k_z}\left[\frac{k_zL}{2} + \frac{1}{2}\sin k_zL\right] = \frac{L}{2} + \frac{1}{2}\frac{\sin k_zL}{k_z} = \frac{L}{2}\left[1 + \frac{\sin k_zL}{k_zL}\right]$

k_zL ограничено от 0 до π или k_z от 0 до ∞.

При $\vec{M}_0(\vec{r}) = (0,0,M_{0z})$ и $M_{0z} = M_s = \text{const}$ – доп. заполнение по направлению.

$H_d^0(\vec{r}) = (H_{\rho}^0, 0, H_z^0)$, см. 1/I 2019. $H_z^0(\rho,\varphi,z) = M_s\int d^3\vec{r}'\, G_{zz}(\vec{r},\vec{r}')$

$\int d\varphi' G_{zz}(\vec{r},\vec{r}') = -2\pi\frac{\partial}{\partial z}\frac{\partial}{\partial z'}\int dk e^{-k|z-z'|}J_0(k\rho)J_0(k\rho')$ не зависит от φ.

$H_z^0(\rho,z) = -2\pi M_s\frac{\partial}{\partial z}\frac{\partial}{\partial z'}\int dk e^{-k|z-z'|}J_0(k\rho)J_0(k\rho')$ – уравнение

$\int d\rho\rho\int dz\,\mu_n^2(\rho,z) = \int d\rho\rho\int_{-\beta/2}^{\beta/2}dz\, J_0^2(x\rho)\cos^2\beta x_z z = \int_0^1 d\rho\rho J_0^2(x\rho)\int_{-\beta/2}^{\beta/2}dz\cos^2\beta x_z z =$

$= \frac{1}{2}\left[J_0^2(x) + J_1^2(x)\right]\frac{1}{2}\left[1 + \frac{\sin\beta x_z}{\beta x_z}\right]$ где ρ в ед. R, z в ед. L. $\frac{x}{R} = k_\rho$, $k_z = \frac{x_z}{R}$.

$dJ_0(x) = -xJ_1(x)$ – решение при $\rho = R$. $\int d\rho\rho\int dz\,\mu_n^2(\rho,z) = \frac{1}{2}\left(1 + \frac{d^2}{dx^2}\right)J_0^2(x)\frac{1}{2}\left[1 + \frac{\sin\beta x_z}{\beta x_z}\right] = N_n$

$N_n(x,x_z) = \frac{1}{4}\left(1 + \frac{d^2}{dx^2}\right)J_0^2(x)\left[1 + \frac{\sin\beta x_z}{\beta x_z}\right]$ и с $\mu_n(\rho,z) = \frac{1}{\sqrt{N_n}}J_0(x_n\rho)\cos\beta x_z z$

$k_z z = (k_z L)\left(\frac{z}{L}\right) = x_z\frac{L}{R}\left(\frac{z}{L}\right) \to \beta x_z z$. $z \in [-\frac{1}{2}, \frac{1}{2}]$.

$\frac{\partial^2}{\partial x^2}e^{-k|x|} = k^2e^{-k|x|} - C\delta(x)$ – ? дельта-функция появляется.

Нужно $x \neq 0$ $\frac{\partial^2}{\partial x^2}e^{\pm kx} = k^2e^{\pm kx}$. $\int_{-\infty}^{\infty}dx\frac{\partial^2}{\partial x^2}e^{-k|x|} = k^2\int_{-\infty}^{\infty}dx\, e^{-k|x|} - C\int_{-\infty}^{\infty}dx\,\delta(x)$

$H_z^0(\rho,\varphi,z) = M_s\int d\rho'\rho'\int d\varphi'\int dz' G_{zz}(\vec{r},\vec{r}') = M_s\int d\rho'\rho'\int dz'(-2\pi)\frac{\partial}{\partial z}\frac{\partial}{\partial z'}\int dk e^{-k|z-z'|}J_0(k\rho)J_0(k\rho')$

$\int_{-L/2}^{L/2}dz'\frac{\partial}{\partial z'}e^{-k|z-z'|} = e^{-k|z-z'|}\Big|_{-L/2}^{L/2} = e^{-k\left(\frac{L}{2}-z\right)} - e^{-k\left(z+\frac{L}{2}\right)} =$

$= e^{-kL/2}\left(e^{kz} - e^{-kz}\right) = 2e^{-kL/2}\operatorname{sh}(kz)$.

$H_z^0(\rho,z) = (-2\pi M_s)\frac{\partial}{\partial z}\int d\rho'\rho'\int dk\, 2e^{-kL/2}\operatorname{sh}(kz)J_0(k\rho)J_0(k\rho') = (-4\pi M_s)\int d\rho'\rho' J_0(k\rho)J_0(k\rho')\times$

$\times\int dk e^{-kL/2}k\operatorname{ch}(kz)$, $\int_0^R d\rho'\rho' J_0(k\rho') = \frac{1}{k^2}\int dx\, x J_0(x) = \frac{1}{k^2}xJ_1(x)\Big|_0^{kR} = \frac{R}{k}J_1(kR)$

$H_z^0(\rho,z) = (-4\pi M_s)\int dk k e^{-kL/2}\operatorname{ch}(kz)J_0(k\rho)\frac{R}{k}J_1(kR)$, $R \to 1$ (ρ в ед. R), $L \to \beta$.

$H_z^0(\rho,z) = (-4\pi M_s)\int dk e^{-\beta k/2}\operatorname{ch}(\beta kz)J_0(k\rho)J_1(k)$.

$\langle H_z^0(\rho,z)\rangle_z = \frac{1}{L}\int_{-L/2}^{L/2}dz\, H_z^0(\rho,z)$, $\int_{-\beta/2}^{\beta/2}dz\operatorname{ch}(\beta kz) = \frac{1}{\beta k}\operatorname{sh}(\beta kz)\Big|_{-1/2}^{1/2} = \frac{2}{\beta k}\operatorname{sh}(\beta k/2)$

$\langle H_z^0(\rho,z)\rangle_z = (-4\pi M_s)\int dk\frac{2}{\beta k}\operatorname{sh}(\beta k/2)e^{-\beta k/2}J_0(k\rho)J_1(k) = \left(\frac{-4\pi M_s}{\beta}\right)\int\frac{dk}{k}(1 - e^{-\beta k})J_0(k\rho)J_1(k)$

Сделано, совп. с 1/I 2019 г.

$H_z^0(\rho,z) = (-4\pi M_s)\int_0^\infty dx\, e^{-\beta x/2}\operatorname{ch}(\beta xz)J_0(x\rho)J_1(x)$.

$(H_z^0)_{nn} = \int d\rho\rho\int dz\,\mu_n^2(\rho,z)H_z^0(\rho,z) = \frac{1}{N_n}\int d\rho\rho\int dz\, H_z^0(\rho,z)J_0^2(x_n\rho)\cos^2(\beta x_z z)$

$(H_z^0)_{nn} = (-4\pi M_s)\frac{1}{N_n}\int d\rho\rho\int dz\int_0^\infty dx e^{-\beta x/2}\operatorname{ch}(\beta xz)J_0(x\rho)J_1(x)J_0^2(x_n\rho)\cos^2(\beta x_z z)$

10/V Ориг. налог на наличку 12.30 €. Meeting с Julian (рек. SolSkyMag и A.Fert)

$(H_z^0)_{nn}$ – матрица элементов static dipolar field cylindrical dot на $\mu_n(\rho,z) = J_0(x_n\rho)\cos k_z z$:

$\int_{-1/2}^{1/2}dz\operatorname{ch}(\beta xz)\cos^2(\beta x_z z) = \int dz\operatorname{ch}(\beta xz)\frac{1}{2}\left[1 + \cos(2\beta x_z z)\right] = \frac{1}{2}\int dz\operatorname{ch}(\beta xz) +$

$+\frac{1}{2}\int dz\operatorname{ch}(\beta xz)\cos(2\beta x_z z) = \frac{1}{2}\frac{1}{\beta x}\operatorname{sh}(\beta xz)\Big|_{-1/2}^{1/2} + \frac{1}{2}\Big[2\beta x_z\sin(2\beta x_z z)\operatorname{ch}(\beta xz) +$

$+\beta x\cos(2\beta x_z z)\operatorname{sh}(\beta xz)\Big]\frac{1}{\beta^2x^2 + 4\beta^2x_z^2}\Big|_{-1/2}^{1/2} = \frac{1}{\beta x}\operatorname{sh}(\beta x/2) + \frac{1}{\beta^2(x^2+4x_z^2)}\times$

$\times\left[2\beta x_z\sin(\beta x_z)\operatorname{ch}(\beta x/2) + \beta x\cos(\beta x_z)\operatorname{sh}(\beta x/2)\right]$, $x_z/R = k_\rho$, $k_z = \frac{x_z}{R}$, $k_\rho = k$.

$(H_z^0)_{nn} = (-4\pi M_s)\frac{1}{N_n}\int d\rho\rho\int dx e^{-\beta x/2}J_0(x\rho)J_1(x)J_0^2(x_n\rho)\left[\frac{1}{\beta x}\operatorname{sh}(\beta x/2) + \frac{1}{\beta^2(x^2+4x_z^2)}\times\right.$

$\left.\times\left\{2\beta x_z\sin(\beta x_z)\operatorname{ch}(\beta x/2) + \beta x\cos(\beta x_z)\operatorname{sh}(\beta x/2)\right\}\right]$, $x = \frac{k_z}{k}$ и $x\operatorname{tg}\left(\frac{1}{2}xkL\right) = 1 \to x_1(kL)$

$x = \frac{k_z}{k} = \frac{x_z}{x}$. $\frac{x_z}{x}\operatorname{tg}\left(\frac{1}{2}k_zL\right) = 1$, $x_z\operatorname{tg}\left(\frac{1}{2}\beta x_z\right) = x$ – Damon'65 eq.

$\int_{-1/2}^{1/2}dz\operatorname{ch}(\beta xz)\cos^2(\beta x_z z) = \frac{1}{\beta x}\operatorname{sh}(\beta x/2) + \frac{1}{\beta}\frac{1}{x^2+4x_z^2}\left[2x_z\sin(\beta x_z)\operatorname{ch}(\beta x/2) + x\cos(\beta x_z)\operatorname{sh}(\beta x/2)\right]$

Escaneado con CamScanner

$\sin\gamma = \frac{2u}{1+u^2}$, $\cos\gamma = \frac{1-u^2}{1+u^2}$, $u = \operatorname{tg}\frac{\gamma}{2}$, $\sin(\beta x_2) = \frac{2\operatorname{tg}(\beta x_2/2)}{1+\operatorname{tg}^2(\beta x_2/2)}$, $\operatorname{tg}(\beta x_2/2) = \frac{x}{x_2}$

$\sin(\beta x_2) = \frac{2(x/x_2)}{1+(x/x_2)^2} = \frac{2xx_2}{x^2+x_2^2}$, $f_D(x) = \frac{1}{1+z_1^2(x)}$, z_1 - first root $z\operatorname{tg}(\frac{1}{2}zx)=1$

$\sin(\beta x_2) = \frac{2(x_2/x)}{1+(x_2/x)^2} = 2\frac{x_2}{x} f_D(x)$. $z = k_2/k = \frac{x_2}{x}$, $x = kL$ здесь, $x = \beta x$.

$\cos(\beta x_2) = \frac{1-\operatorname{tg}^2(\beta x_2/2)}{1+\operatorname{tg}^2(\beta x_2/2)} = \frac{1-(x/x_2)^2}{1+(x/x_2)^2} = \frac{x_2^2-x^2}{x^2+x_2^2}$ (-1)

$[\dots] = [2x_2\sin(\beta x_2)\operatorname{ch}(\beta x/2) + x\cos(\beta x_2)\operatorname{sh}(\beta x/2)] = 2x_2\frac{2xx_2}{x^2+x_2^2}\operatorname{ch}(\beta x/2) + x\frac{(x_2^2-x^2)}{x^2+x_2^2}\operatorname{sh}(\beta x/2) =$ (-1)

$= \frac{1}{x^2+x_2^2}\left[4xx_2^2\operatorname{ch}(\beta x/2) + x(x_2^2-x^2)\operatorname{sh}(\beta x/2)\right]$ (+1)

$\frac{1}{4\pi M_s}(H_z^0)_{nn} = -\frac{1}{N_n}\int_0^{\pi/2} d\varphi \int_0^\infty dx\, I_0(x\varrho) J_1(x) J_0^2(x_n\varrho) e^{-\beta x/2}\left\{\frac{1}{\beta x}\operatorname{sh}(\beta x/2) + \frac{1}{\beta}\frac{1}{x^2+4x_2^2}[\dots]\right\}$

$N_n = \frac{1}{4}\left(1+\frac{d^2}{dx_2^2}\right)J_0^2(x)\left[1+\frac{\sin\beta x_2}{\beta x_2}\right]$, $x\to x_n$. $\sin(\beta x_2) = \frac{2x_2}{x} f_D(\beta x)$

$\sin(\beta x_2)/\beta x_2 = \frac{2}{\beta x} f_D(\beta x)$.

$e^{-\beta x/2}\{\dots\} = \frac{1}{\beta x}e^{-\beta x/2}\operatorname{sh}(\beta x/2) + \frac{1}{\beta}\frac{1}{(x^2+4x_2^2)}\frac{1}{x^2+x_2^2}\left[4xx_2^2 e^{-\beta x/2}\operatorname{ch}\frac{\beta x}{2} + x(x_2^2-x^2)e^{-\beta x/2}\operatorname{sh}\frac{\beta x}{2}\right] =$ (-1)

$= \frac{1}{2\beta x}(1-e^{-\beta x}) + \frac{1}{2\beta}\frac{1}{(x^2+4x_2^2)(x^2+x_2^2)}\left[4xx_2^2(1+e^{-\beta x}) + x(x_2^2-x^2)(1-e^{-\beta x})\right]$ (-1)

$\lim_{x_2\to 0} e^{-\beta x/2}\{\dots\}_{x_2=0} = \frac{1}{2\beta x}(1-e^{-\beta x}) + \frac{1}{2\beta}\frac{1}{x^2}\left[-x(1-e^{-\beta x})\right] = \frac{1}{\beta x}(1-e^{-\beta x})$

и $\frac{1}{4\pi M_s}(H_{zz}^0)_{nn} = -\frac{1}{N_n}\int_0^{\pi/2} d\varphi \int_0^\infty dx\, J_0(x\varrho)J_1(x)J_0^2(x_n\varrho)\frac{1}{\beta x}(1-e^{-\beta x})$ - вернемся чуть позже $x_2\to 0$.

$\frac{1}{\beta x}(1-e^{-\beta x}) = 1 - f(\beta x)$, $f(t) = 1 - \frac{1}{t}(1-e^{-t})$

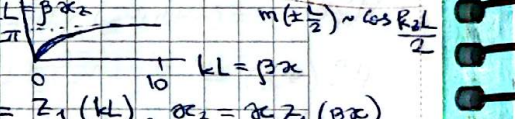
$N_n = \frac{1}{4}\left(1+\frac{d^2}{dx_n^2}\right)J_0^2(x_n)\left[1+2\frac{f(\beta x_n)}{\beta x_n}\right]$ окончательно через x_n. $f_D(x)$ - Дамон-Шарп.

$-\frac{1}{4\pi M_s}(H_{zz}^0)_{nn} = \frac{1}{2\beta}\frac{1}{N_n}\int_0^{\pi/2} d\varphi \int_0^\infty dx\, J_0(x\varrho)J_1(x)J_0^2(x_n\varrho)\left[\frac{1}{x}(1-e^{-\beta x}) + \frac{4xx_2^2(1+e^{-\beta x}) + x(x^2-x_2^2)(1-e^{-\beta x})}{(x^2+4x_2^2)(x^2+x_2^2)}\right]$

$(k_2)_{max} = \pi/L$ или $\beta x_2 \gg 1$. $k_2 L = (\beta x_2)_{max} = \pi$

т.е. $x_2 \le \frac{\pi}{\beta}$, тогда как $\varrho \le x \le \infty$, $0 \le \varrho \le \infty$.

$m(\frac{x}{2}) \sim \cos\frac{k_2 L}{2}$



Одна Сота дает $0.1 \le (\beta x_n) \le 4.24$ согласно з/к. $\frac{x_2}{x} = z_1(kL)$, $x_2 = x z_1(\beta x)$,

$z_1(x)$ 1й root of eq. $z\operatorname{tg}(\frac{1}{2}zx) = 1$, $x = \beta x$. Для $x \ll 1$, $\frac{1}{2}z_1^2 x = 1$, $z_1(x) = \sqrt{\frac{2}{x}}$

и $x_2 = x z_1(\beta x) = x\sqrt{\frac{2}{\beta x}} = \sqrt{\frac{2x}{\beta}}$, $\frac{x_2}{x} = \sqrt{\frac{2}{x}} \gg 1$ при $x\to 0$, но $\beta x_2 \le \pi$ при $x = \beta x \gg 1$

$x_2 = x z_1(\beta x) \to k_2 L = x z_1(x)$, $x = k_2 L = \beta x_n$. $x_2 \sim x$ при $\beta x_n \sim 1$.

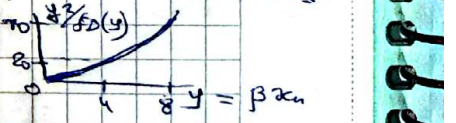
сравнимы

$I_0(x_n, x) = \int_0^{\pi/2} d\varphi\, J_0(x\varrho)J_0^2(x_n\varrho)$

$-\frac{1}{4\pi M_s}(H_z^0)_{nn} = \frac{1}{2\beta}\frac{1}{N_n}\int_0^\infty dx\, J_1(x) I_0(x_n, x)\left[\frac{1}{x}(1-e^{-\beta x}) + \frac{4xx_2^2(1+e^{-\beta x}) + x(x^2-x_2^2)(1-e^{-\beta x})}{(x^2+4x_2^2)(x_n^2+x_2^2)}\right]$ $x\to x_n$

$H_{ex}(u) = 4\pi M_s\left(\frac{l_e}{R}\right)^2 x_n^2 \frac{1}{f_D(\beta x_n)} = 4\pi M_s\left(\frac{l_e}{L}\right)^2\frac{(\beta x_n)^2}{f_D(\beta x_n)}$, т.к. $L = 40$ нм = const

$H_{ex}(u)$ растет почти квадратично с ростом βx_n.



$H_{m(u)}^{dyn} = 4\pi M_s f_D(\beta x_n)$, $\frac{H_{ex}(u)}{H_{m(u)}^{dyn}} = \left(\frac{l_e}{L}\right)^2\left[\frac{\beta x_n}{f_D(\beta x_n)}\right]^2$

при $x = \beta x \le 1$, $f_D(x) = f(x) \approx \frac{1}{2}x$, $\frac{x}{f_D(x)} \approx \frac{x}{\frac{1}{2}x} = 2$ и $\frac{H_{ex}(u)}{H_m(u)} = 4\left(\frac{l_e}{L}\right)^2$

но $L = 40$ нм, $l_e \simeq 5$ нм. Для малых βx_n, $\frac{H_{ex}(u)}{H_m(u)} \simeq 4\left(\frac{5}{40}\right)^2 = 4\frac{1}{8^2} = \frac{4}{64} = \frac{1}{16} \approx 6\%$

Одна Сота дает $(\beta x)_{max} = 4.2$ что $R = 200$ нм. $f_D(u) \approx 1$.

т.е. что $(\beta x_n)_{max} \approx 4$ $\frac{H_{ex}(u)}{H_m(u)} = \left(\frac{l_e}{L}\right)^2(\beta x_n)^2 = \left(\frac{5}{40}\right)^2(4)^2 = \frac{1}{8^2}\cdot 16 \simeq \frac{1}{4}$, магнитостатика доминирует

Для характерных (βx_n) $\frac{H_{ex}(u)}{H_m(u)} \approx 10\%$, но он вычисляется магнитостатическим подходом Дамон-Шарпа.

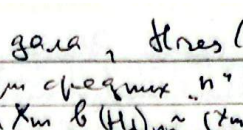
Варьируем M_s и l_e. $l_e(R)$ растет с $R\downarrow$. $l_e = \sqrt{A/2\pi M_s^2}$, $A = 2\pi M_s^2 l_e^2$

но $M_s(R)$ падает на $\sim 30\%$, и $l_e(R)$ растет $\sim$ в 3 раза, $A(R)$ растет с $R\downarrow$. $M_s(R)$ падает с $R\downarrow$

Переходим к Шарпа-аппроксимации $f_D(x) \to f(x) = 1 - (1-e^{-x})/x$, но уже в d-reduced, $k_2(kL)$ и H_{zmn}^0 с верхней нормировкой на $\mu_n(\varrho) = J_0(x_n\varrho)\cos k_2 z$.

Escaneado con CamScanner

(11/V) Замена $f_0(x) \to f(x)$ нищо не дава, $H_{res}(\eta)$ vadat zamenо силното $H_{res}^{exp}(\eta)$ или средните „n".



give R = 1000, 700, 500 nm

Вариянти: •1) reduced pinning $d(\beta)$ — $k_z=0$ (Slavin), $f(k)$ fitting ok; $k_z \neq 0$ (maxon fitting) give R близки

•2) Strong pinning $J_0(x_m)=0$ $k_z=0$ (Slavin) fitting is OK $f(k)$ give R големи; $f_D(x)$ Hillebrands, $f(x)$-Sparks

$k_z \neq 0$ - fitting OK for R

но какъв е критерия (turning points at $g \approx R$?) Strong pinning дава по-добро fitting $H_{res}^{exp}(\eta)$, чем „reduced pinning $d(\beta)$. Но защо magnetostatics dominates in dyn. fields дава голяма $k_z \neq 0$ [two examples of pure magnetostatic approach]. Вкл. Strong pinning $d \to \infty$, с $\bar{m}(R)=0$, хубаво, т.к. при $g \approx R$ есть turning points и $\bar{m}(g)$ имат свой $\sim J_0(\alpha g)$ зависимост осцилиращо по $e^{-\alpha g}$ при $g > g_0$.

(12/V) Написване 1D DW manuscript, нощна гледа/сравнено MathCad file с new $H_{res}(\eta)$ (micromagnetic) $M(g,z) \sim J_0(\alpha g)\cos(k_z z)$.

(13/V) Sol Skymag factures, переписка с A. Kirilyuk - COST reimbursement.

Сделано до обяда в Heraklion: •1) print Hotel Madrid, tickets Hotel Athenes, Hotel Heraklion •2) поправ. Skyrm. Int. talk ICNM'19-Heraklion, •3) review Appl. Sci. Skyrmion paper Zhao •4) confirm Hotel reserv. Heraklion; •5) Започнато 1D DW paper.

DW mass

Сим. 19/V 2014 г. $\frac{L}{\gamma^2} \cong 2.8 \cdot 10^{-20}$ g for $L = 100$ nm при $\frac{\gamma}{2\pi} = 3.0 \frac{MHz}{Oe}$.

$\gamma_{\gamma^2} = 2.8 \cdot 10^{-22} \frac{g}{nm}$, $\frac{1}{\gamma^2 \Delta} \cong 2.8 \cdot 10^{-22} \frac{g}{nm} \frac{1}{10nm} = 2.8 \cdot 10^{-23} \frac{g}{nm^2}$.

Döring mass uniaxial PM $m_{DW} = \frac{1}{2\pi\gamma^2\Delta} \cong \frac{2.8 \cdot 10^{-23}}{2\pi} g/nm^2 = 4.46 \cdot 10^{-24} \frac{g}{nm^2}$

Две stripe $L = 10$ nm, $w = 50$ nm $S = wL = 500\ nm^2$ $M_{DW} = m_{DW} S = 4.46 \cdot 10^{-24} \frac{g}{nm^2} \times$

$\times 5 \cdot 10^2 = 2.23 \cdot 10^{-21}$ g. // Saitoh-Tatara Nature '2004 Две Py stripes

$L = 45$ nm, $w = 70$ nm измерими $M_{DW} = 6.55 \cdot 10^{-20}$ g.

$S(\text{Saitoh}) = \left(\frac{70}{50}\right)\left(\frac{45}{10}\right) S = 6.3\,S$. Очаквам $M_{DW}(\text{Dör}) = 2.23 \cdot 6.3 \cdot 10^{-21}$ g $= 1.40 \cdot 10^{-20}$ g. Т.е. $M_{DW}(\text{Saitoh})$ измерено в ~ 4 пъти повече от unaxial Döring mass.

Запис „Atlantis" hotel. Neel-Py Skyrm/Bergamo → to Nanoscale (PBX не взема). Answ. DW-exp. refs. by Ricardo → comments. Нощна 1D-DW draft to Oksana.

(14/V) Сергей/глеб изчисляват разлики между $H_{res}(\eta)$ Cofe data по eq. APL'2004 и new eq. by Sparks за strong pinning. Обядно на Skype. Обядно Conf. с Julian.

(15/V) Написване 1D-DW paper. Тел. разговор с Rhodri: в романса Вероятно в 1D-DW paper P. Roy as co-author вместо с Ho Eq. $\Delta(V)$ give ultralarge K_u. С Л. в „Eroski" Urbil" и „Carrefour" Diartzun. Изпит does give HMM-trip.

(16/V) Нощна Solskymag flyers. С Л. в „Carrefour" Olaberria и „Lidl". Переписка с Oksana и Ricardo. JEMS изпраща мой Skyrmion talk as poster (!?), а Zukovic Sk-dyn. as oral.

(17-24/V) Conf. trip to HMM'2019, May 19-22, Heraklion, Greece. Irura → Madrid → Athenes → Heraklion → Athenes → Madrid → Irura. Изгубеният от January са сега CONSTANTO „450€ to Magnetism.eu" ecли ли не от немец 450€ год. fee [ESM school in Brno, Ghech Republic, Sept. 2019] for Ricardo. S.K. Kim изпраща нощна Vortex/Sk review give Ker. J. Magnetics за попълв 1500 $. 18/V bunga mosty Londres

(25/V) Отказка за Sparks непосредствен conf. Supplies. Бъдещ отдих дома. Отп. 25% clena. E-mail sketch of Cofe dot parameters to describe $H_{res}(\eta)$.

(26/V) Днешно: •1) 10T give P. to K и visit Tolosa 29/V; •2) appl. prolonga MEC-project; •3) 1D DW paper; •4) review Filipanova APL до 26/V; •5) Reminder Глеб: Mrs Mavs, Verba Dob •6) tickets Uppsala-Poznan и hotel Poznan, conf. fee Uppsala; •7) review 3D-Sk ERC project до 3/VI

→ Нощна new Skyrmion papers: 1) P. Fischer - Hopfions 2) C. Panagopoulos, PRA'19 (Cofe) в Skyrmions

3) Yu Li, Bloch-point switch PS (MH_z) 4) M. Sisodia - Sk-ST oscillator; 5) G. Yu Sk-track MC

6) J. Leliaert WBR (DW I); 7) M. Garcet - corrings.

1) P. Fischer - hopfions mereno ne nameu. Namer // Y. Liu, R.K. Lake, Jiadong Zang PRB98, 174433 (2018) u „Binding a hopfion in chiral magnetic nanodisk" Mag. dot sandwiched between two films with PMA. Dot L=70 nm, R=100 nm

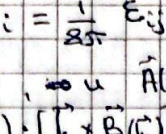
Q_H - Hopf invariant - topol. index of homotopy group $\pi_3(S^2)$, t.e. $S_3 \to S_2$ map PRB98

$Q_H = -\int d^3\vec{r}\, \vec{B}\cdot\vec{A}$, $B_i = \frac{1}{8\pi}\varepsilon_{ijk}\vec{m}\cdot(\partial_j\vec{m}\times\partial_k\vec{m})$ - emergent mag. field,

$\mathrm{Tot}\vec{A} = \vec{B}$. Ecuu $\vec{k}\cdot\vec{A}=0$, uu $\vec{A}(\vec{r}) = \frac{1}{N}\sum_{\vec{k}}\vec{A}(\vec{k})e^{i2\pi\vec{k}\cdot\vec{r}}$ $\vec{B}(\vec{r}) = \frac{1}{N}\sum_{\vec{k}}B(\vec{k})e^{i2\pi\vec{k}\cdot\vec{r}}$, то

$Q_H = i\frac{1}{N}\sum_{\vec{k}}\frac{1}{2\pi k^2}\vec{B}(-\vec{k})\cdot[\vec{k}\times\vec{B}(\vec{k})]$ - topol. charge in $\vec{k}$-space.

В gore monopole-antimonopole pair (MAP) state energy < E(hopfion), t.e. hopfion is metastable



$L_D = 4\pi A/D \sim 100$ nm - нохване на Bloch-MPD

$\Delta E_{H\to MAP} \approx 0.1\cdot10^{-17}$ J

$\Delta E_{H\to MAP}/k_BT \approx 240$, t.e. hopfion is created is thermostable. $\frac{\Delta E}{k_BT} = \frac{0.1\cdot10^{-17}\,J}{1.4\cdot10^{-23}\,J/K\cdot3\cdot10^2\,K} = \frac{0.1}{1.38\cdot3}10^4 = 2.4\cdot10^2 = 240$

$Q_H(S_3\to S_2)$ совпада с Hopf-index Kundu-Rybakov '82 и Komineas '96 (см. 31/V-1/VI 2018)

Eq. for $Q_H = \frac{1}{(4\pi)^2}\int d^3\vec{r}\int d^3\vec{r}'\,\varepsilon_{ijk}\gamma_i(\vec{r})\frac{R_j}{R^3}\gamma_k(\vec{r}')$, $\vec{R}=\vec{r}-\vec{r}'$, $\gamma_i = -\frac{1}{2}\varepsilon_{ijk}\vec{m}(\partial_j\vec{m}\times\partial_k\vec{m})$ (Komineas '96)

нохоне на eq. для вихровата magnetostatic energy

$E_m = -\frac{1}{2}\int d^3\vec{r}\,d^3\vec{r}'\,G_{\alpha\beta}(\vec{r}\vec{r}')M_\alpha(\vec{r})M_\beta(\vec{r}') = \frac{1}{2}\int d^3\vec{r}\,d^3\vec{r}'\,\frac{1}{R^3}\left[\vec{M}(\vec{r})\cdot\vec{M}(\vec{r}') - \frac{3}{R^2}(\vec{R}\vec{M})(\vec{R}\vec{M}')\right]$

$G_{\alpha\beta}(\vec{r}\vec{r}') = -\frac{\partial}{\partial x_\alpha\partial x_\beta}\frac{1}{|\vec{r}-\vec{r}'|} = -\frac{1}{R^3}\left[\delta_{\alpha\beta} - 3\frac{R_\alpha R_\beta}{R^2}\right]$, $\vec{R}=\vec{r}-\vec{r}'$, см. 6/V 2018, где вместо $\vec{M}(\vec{r})$ използвахме gyrocoupling density (emergent mag. field) $\vec{\gamma}(\vec{r})$ или $\vec{B}(\vec{r})$.

$Q_H = \frac{1}{(4\pi)^2}\int d^3\vec{r}\,d^3\vec{r}'\,\frac{1}{R^3}\vec{\gamma}(\vec{r})\cdot[\vec{R}\times\vec{\gamma}(\vec{r}')]$, $E_m = \frac{1}{2}\int d^3\vec{r}\,d^3\vec{r}'\,\frac{1}{R^3}\left[\vec{M}(\vec{r})\vec{M}(\vec{r}') - \frac{3}{R^2}(\vec{M}\cdot\vec{R})(\vec{M}'\cdot\vec{R})\right]$

$Q_H \neq 0$ ecuu ecuu $\Phi(\vec{r})$ - singular, $\vec{m}(\Theta,\Phi)$. Например $\Phi(x,y) = q\arctg(\frac{y}{x})$ for 2D Soliton

Бъдещо, $\vec{X}$ - coordinate, где $\Phi(\vec{r}-\vec{X})$ is singular ecuu Hopfion/Soliton center.

Arxiv papers by M. Garst o dyn. Skyrmion Strings.

(27/V) Обучение SolStkyMag'19 с Julian. Обучение тук-там с Ricardo. Skype с Бунгаев - нрабеното TOL MathCad и „Drum" modes сигнал Skype с Ингъл - бъкт. Sw porter to SolStkyMag. Ануи. Радан, booking room #4 Faculty Hotel, Aug. 31 - Sept. 11 (Maciej).

(28/V) Вечеря с Library representatives: залог Вечеря Friday 21 - June, $11^{00}-20^{00}$. Термин auditorio g^{oo} June 24. Poster holders на-нагло-упорити. Reminder to A. Arrott. Нощен. docs для Tax Return 2018 (Л. salary - 1500€).

(29/V) Visit Tax office Tolosa: беснуг 4.170€ ненужно, защо за 2017 ще за ужасно. Напреса payments. Нощ.: NIE tarjetas originals, 10-T или Л., нрамчика Irura - 4 час. Maciej - нощ. Faculty Hotel. нрегледвам

(30/V) Visit to Bar - Igeldo и tour Bureau San-Sebastian. Igeldo ~ 60 мест и не хвали обещаваното такса Conf. (6€ ядно и питно). Docs - Anorroga MINECO project - Запрос Оксана. Програма и conf. fee Sol-StkyMag 2019.

(31/V) С Л. в Lidl "Olaberria и Corregidor" Olaberria - Закупка coffee, chocolates, aqua. E-mails to Jungwirth (2 key invited) и Hernando (нормат talk).

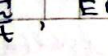
(1/VI) С Л. и ЮО. на нашия „Ordarrela", обед в „Ham" Eroski и Urbil - Закупка.

(2/VI) Update, formating Sol-StkyMag'19 conf. програма, нефиксиран Conf.

(3/VI) нощен. Travel report HMM'19, Heraklion, Greece. Но следен HMM'19!

• 1) P. Ho - C. Panagopoulos, PRA 11, 024064 (2019) Geometrically tailored Skyrmions in ML + нарушения at H=0. Skyrmions in ML Ir/Co/Pt/Al dots. Exch. и dipol. c = Rs/R ≈ 0.3-0.8 и силно нарушен с ролята D/A. в горах.

• 3) Yu Li - C. Panagopoulos, „Bloch point-mediated switching in mag. Skyrmions" такова HMM - abstract. Paper/preprint не намер. PS (THz) switching. Emergent electric fields $E(t) \sim \frac{\partial\vec{m}}{\partial t}$, $E(\omega)$ peak ~ mV-voltage. Simulations MuMax3.



Послан запрос до C. Panagopoulos - нрамчика.

Escaneado con CamScanner

(4/VI) New convocatorias AEI и prorroga MEC - project (референция с Окс.) С Л. Erofeev. Urbil. Update conf. program. Travel reports Greece и Donostia (Окс.).

(5/VI) SolSkyMag'2019 program и payment list. Бъмъ.

(6/VI) С Л. и Ю. в San Seb. - изпращам в Киев: Л. кане (12 kg) и Л. D (5 kg). В „Банка" - transfer Bakharev 440 €. Organ travel reports и Conf. fee invoice - Uppsala to Antxoni. С Л. в Olaberria „Merkal" и „Lidl". Помощ Л.

(7/VI) Уговаряване кореспонденция с посетители. Разбор paper Zivieri PRB'2019 - Sk-Sk fluctuations - все още към Sk radius R_S (fluctuations (Окс. email). Уговаряне с Ricardo о его CPM-talk (AFM-DW?) и SolSkyMag'19 poster. Бъмъ.

(8/VI) Среща с Ант. Цачев на плаж. Song' NJP - Sk-dynamics. Analyzing grant notifications by Emilia Moreira. Добавих Ricardo „1D DW" to posters.

(9/VI) С Л. и Ю. на Ondarreta playa. „Nam". Бъмъ.

(10/VI) Уговаряне с Окс. о Zivieri PRB'19 и Hotel San-Seb. Julian canceled Ana-Lluga trip. Draft Solicitud Prorroga FIS 2016.

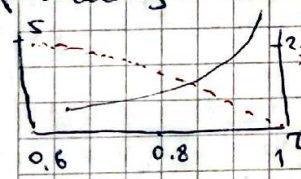
(11/VI) Уюр. Seminar talk DIAC by Ricardo „Supermagnonic DW motions AF", обсъждане, тънкости разбор по DW dynamics in AF и biaxials FM. Послан draft Prorroga FIS 2016 Окс.ане: new directions - Sk-Hall effect и 3D Skyrmions.

(12-15/VI) Уюр.среща с Л. на плажа в Madrid (Tres Cantos). Закупка в IKEA дюли SolSkyMag'2019. Carrefour - Olaberria. Offer reduced fee/hotel to B. Ivanov. Papa Martinez Perez/Zaragoza изпратени draft на paper по AE-vortex Co-nanodots.

(16/VI) HMM'2019 Skyrmion papers:

•4) N. Sisodia, S. Komineas, P.K. Muduli, PRB'2019. „Skyrmion gyrooscillator" Coupling of Sk-rotation motion and breathing mode." Simulations, Sk in dot $L = 5$ nm, $2R = 150$ nm. Co_2FeAl. Rotation frequency $f_G \approx 300$ MHz. $m_z(x,y,t)$ dynamic is not radially symmetric no com.e $\vec{X} = (X, Y)$ Sk-core position. $\langle m_z \rangle$ is not conserved due to DMI, $\langle m_z \rangle(t)$ oscillates с частоти f_G f_G comes because of f-breathing of free skyrmion. $\Phi = \varphi + \Phi_0$, $\Phi_0(t) \sim \sin \omega_G t$. $f_{BR} \approx 2$ GHz >> $f_G \approx 300$ MHz без STT.

•6) J. Leliaert - J. Mulkers. J. Phys. D 2018. „Coupling Sk-velocity to breathing mode in periodically notched nanotracks". $f_{BR} \approx 4.5 \div 7.8$ GHz как $F(K, M_S)$



$R_S(D) \to \infty$, $f_{BR}(D) \to 0$ when $D \to D_C = \frac{4}{\pi}\sqrt{AK}$

$\delta = \sqrt{A/K}$ - universal plot, see Gus, Materials'2018. Gyu $R_S(D)$,

$f_0 = \frac{\gamma}{\pi}\frac{K}{M_S} = \left(\frac{\gamma}{2\pi}\right) H_a$.

•6') L. Sanchez-Tejerina AF-DW Ni_2O/Pt stripes - не намерих paper (нали?).

•7) M. Garst. Skyrmion strings.

•7a) V.P. Kravchuk M. Garst. Arxiv - Feb. 2019. Solitons of Skyrmion string. Simul. Fe Ge

$V_{ess}(z) = \frac{1}{2}\varkappa_1\left(\frac{\partial \vec{R}}{\partial z}\right)^2 - \frac{1}{4}\varkappa_2\left(\frac{\partial \vec{R}}{\partial z}\right)^4$, $\varkappa_1, \varkappa_2 > 0$ - exchange of the string, see Gus' Sci Rep 2015, где $\varkappa_1$

System is infinite along thickness z - coordinate. $R = X + iY$ - Soliton position in x-y plane. $\Psi = X + iY = \Psi(z - vt)\exp\left[i(z - vt) + i\frac{v^2}{2}t\right] e^{(z)} e^{i\omega t}$

$\Psi(\xi) = A(\xi)e^{i\Phi(\xi)}$, $A(\xi) \sim \frac{1}{Ch(\xi/\xi_0)}$ - soliton profile. $\Psi(\xi - vt) \to \Psi(\xi)$, $\Psi(\xi = \pm\infty) = 0$

Fit to simulations $\Psi_{sim}(z,t) = R(z - Vt)\, e^{i[\Phi(z - Vt) + \delta t]}$ - propagating Soliton

// B статията от Gus' Sci Rep. 2015 где linear excitations of vortex string at finite thickness Kravchuk - Garst дават nonlinear excitations Sk-string (solitons) at infinite thickness.

$v = V/V_0$, $V_0 \approx 1.3$ km/s. $v \sim 1.0$ in main text, но in Suppl. mat. $V \approx 1\cdot10^{-5}\frac{km}{s} \div 1\cdot10^{-4}\frac{km}{s}$ $= (10^{-1} - 10^{-2})$ m/s очень мало. $v \sim 1$ km/s или $v \sim 10^{-1}$ m/s Все же?

FeGe: $A = 8.8$ pJ/m $M_S = 380$ G, $D = 1.58$ mJ/m^2. // Theory + simulations.

•7b) S. Seki, M. Garst ... Y. Tokura. arXiv Feb. 2019. Experiment string propagation in Cu_2OSeO_3 (stripe-shape). CCW/Bz/CW - three spin modes measured (2-5 GHz)

(17/VI) Уюр.среща с Л. в „Carrefour" и „Al Campo" Сиантъри, уюрчев. и конференция.

(18/VI) Уюрчев. и конференция, удоряка дома, Emanb Julian, Ricardo. Orrer-Madrid trip.

Escaneado con CamScanner

19/VI 21/VI Перемичка SolSkyMag'2019. Закупка продуктов и напитков.
21/VI Завоз на офер. "Toyota"-ван и имущество в библиотеку. Раскладка conf. bags и badges.
22-23/VI Отдых дома, уборка дома. Бумаги.
24-28/VI Sol-SkyMag 2019 conf. in Carlos Santa María Library.
32 registered participants, вкл. 19 students. Arrott, Tretiakov по 250€, Fort, Chubykalo – 0€.
Собрано cash (расчет): 10,480€. Cash (факт.) на утро 29/VI: 10.570€
(570€ - ю., 1000€ - ю., 9,000€ - А). Разница за ужин (+90€) вложил ю. за „Catamaran"
Вложили cash: 1) J. Gonzalez - 2,500€; 2) J. Zagra - 500€; 3) Оскар - 350€;
4) Irene - 250€; 5) Маргарита - 250€, 6) Л. на печ., цветы - 200€
10.570 - 2.500 - 500 - 850 - 200 = 6.520€ (расчет и факт. остаток на 3/VII)
Beg receipts: Arrott (400) + Atkinson (70) + Golovchanskyiwise (150) + Hulle Brands (55) +
+ Karazei (440) + Levy (16) + Livinenko (150) + Ouose (10) + Zelent (150) = 1,441€
Comm. receipts: Golovchansky (480) + Kondou (480) + Manica (480) + Mochizuki (480) +
+ Goli (480) + Rozhansky (480) + Suess (480) + Tretiakov (250) + Yu Hao (630) = 4.240€
29/30/VII Отдых дома. La Perla ресторан с Л., Ю., Ж. Zunbar/Chizhik прислали инф. о rejection Grupos Consolidados Basque Government project (21 из 100 баллов)
1/VIII Meeting с Julian - результаты и перспективы Sol-SkyMag conf. E-mail to A. Fort.
2/VIII Ikerbasque meeting, Hornani (~260 Ikerbasque scientists, IKP. в основном)
3/VIII Л. и Ю. сдали J. Gonzalez 2500€, посылка в Киев 4 кг. (37 kg) = 37·1,2+10€ = 55€
„Nam" lunch с Л., Ю., Ж. „Eroski" Urbil с Л., Ю. J. Camarero и Julian Prorroga
drafts. Analysis Grupos Consol. project evaluation — Оценка списания из-за 4 из 7
scientists (Lourdes Valentina, Alexander, J. Mari) и малой текущей активности, все держится
Се на Gonzalez, Gushchenko, Zunbar, + мало PhD Students, postdocs, занимающихся science
4/VII Л. полетел в Киев, с собой 2,400€. SkyTmion review. Отдых, бумаги.
5/VIII Сроно: 1) articlerets Uppsala - Poznan; 2) prorroga MEC - Sk project;
3) romance ocean CONSTANTO"; 4) TD - SW paper; 5) drum" sw nodes paper.
6) travel reports Dss Oksana и Киев.
6-7/VII Ретью Referee report и ноутс. Appeal на „Grupos Consolidados" Upps.
Gobierno Vasco proposal renewal (81 балл из 100, not supported) Поиск баз. неудобно
8/VII С Ю. в „Bankia", Зенкова не ответ и отменили встр. для „CONSTANTO" с Ю.
в „Lide", пастелерия „Diarteun". Ноутс. Travel reports Окс. и Киев. D.P. Ayuntamiento.
10/VII Отдал travel reports Antxoni. С Ю. и Julian G. в „Lugarritz" бар,
навещали „Tabotabo". Бумаги. Перемичка с Зугарой SK-Talk, Uppsala.
11/VII С Ю. и J. Zagra в ресторане Bertolosa, отдых, бумаги.
12/VIII Подготовил changes/comments к Solicitud prorroga SKYTRON FIS2016
project и послал to J. Camarero, Validares, Perna, Окс. Peer SkyTmion paper
bck Bergamo et al. rejected by ACS Nano (2 negative reports из 3 referees).
Звонил в Тех Августин из „Seguridad Social", asked консультацию, по поводу Ноутса.
13/VIII С Ю. и Julian по „Mercadona" Urbil и „El Perlon". Бумаги.
14/VIII Update Ikerbasque web-site (Sol-SkyMag 2019 etc.)
15/VIII Работа над Prorroga MEC SkyTmion project. С Ю. и Ж. в Окс. ноутс.
16/VIII Л. вернулся из Киева, с собой back 100€ из 2,400€ (1400€ в один,
400€ моей мамы, 200€ Л. мама, ~300€ мамины, прочее).
17/VIII С Л., Ю., Ж. на обед в „Nan" и потерял Diarteun" downtown. (Л. в „Mercadona"
„Eroski" Urbil. Hamcau Ricardo и Nacio)
18/VIII Закрыли Antxoni „Estado de cuentas" MEC project. Отдых дома.
19/VIII С Л. и Ю. на вилле Undarreta. J. Camarero и M. Validares прислали appl. MEC-prorroga.
20-21/VIII Расчет IVA „Constanto". Возврат ун. Л. и К.
22/VIII Расчет с Любовью „Constanto" за 2018.
23-25/VIII Подготовил app. Prorroga MEC-SkyTmion project. Новая file на подпись Camarero
Перемичка prorroga с Окс. 25/VII утер Juan Jose del Val.

Escaneado con CamScanner

(26/VII) J. Camarero приехал с Sigued prorroga Appl. Сделал Subm. to MEC-Website и уехал назначил APL/Exam vice-rector. С Л. в „Carrefour" Alaberria. Бумаги.

(27/VII) Получили APL-manuscript Mark et al. Vortex Hn. Submitted report.

(28/VII) План: 1) Jorbenado Ricardo. 2) 1D DW paper; 3) drum" modes paper; 4) review Vortex-Skyrm gas Kim; 5) arrange trip Uppsala-Poznan (talks, new sk-paper 3D dynamics).

(29/VII) Послали file Ricardo DW dyn. in AFM-coupled FM layers + comments eq.

(30/VII) С Л. в „Banka": deposit 5,760€ conf. fee, обещала cancel serv. fee. С Л. обед в „Nam" Ondaneta и „Eroski" Urbil. Напечатали 3 ч. Л., 10.

(31/VII) 3D-Skyrmion dynamics. 3D-Thiele eq.

Можно ли из LLG подставляя $\vec{r} \to \vec{r} - \vec{X}(t)$ $\vec{X} = (X, Y, Z)$ вывести generalized Thiele eq. for $\vec{X}(t)$.

$-\frac{1}{\gamma}\dot{\vec{m}} = \vec{M} \times \vec{H}$, $\frac{M_s}{\gamma}\dot{\vec{m}} = \vec{m} \times \frac{\partial W}{\partial \vec{m}}$, $\frac{1}{\gamma}(\dot{\vec{m}})_\alpha = \varepsilon_{\alpha\beta\gamma} m_\beta \frac{\partial W}{\partial m_\gamma}$

$\dot{m}_\alpha = \frac{d}{dt} m_\alpha(\vec{r} - \vec{X}) = \frac{\partial m_\alpha}{\partial r_\beta} \frac{\partial (\vec{r} - \vec{X})_\beta}{\partial t} = -\frac{\partial m_\alpha}{\partial r_\beta} \dot{X}_\beta = \gamma \cdot \varepsilon_{\alpha\beta\gamma} m_\beta \frac{\partial W}{\partial m_\gamma}$

$g_{\alpha\beta} = \vec{m} \cdot \left(\frac{\partial \vec{m}}{\partial x_\alpha} \times \frac{\partial \vec{m}}{\partial x_\beta}\right) = \vec{m} \cdot (\partial_\alpha \vec{m} \times \partial_\beta \vec{m}) = m_j \cdot (\partial_\alpha \vec{m} \times \partial_\beta \vec{m})_j =$

$= m_j \varepsilon_{jkl} (\partial_\alpha \vec{m})_k (\partial_\beta \vec{m})_l$. $(\vec{G} \times \dot{\vec{X}}) = \varepsilon_{\beta\chi\gamma} G_\beta \dot{X}_\gamma$. $G_\gamma = \varepsilon_{\gamma\alpha\beta} g_{\alpha\beta} \cdot (-\frac{1}{2})$

LL eq.: $\dot{m}_i = \gamma \varepsilon_{ijk} m_j \frac{\partial W}{\partial m_k}$, (ikj) give $\vec{m}$-components, $\alpha\beta$ for $\vec{X}$.

$\dot{m}_i = -\frac{\partial m_i}{\partial x_\beta} \dot{X}_\beta = \gamma \varepsilon_{ijk} m_j \frac{\partial W}{\partial m_k}$. $g_{\alpha\beta} = m_j \varepsilon_{jkl} \frac{\partial m_k}{\partial x_\alpha} \frac{\partial m_l}{\partial x_\beta}$

$-\frac{\partial m_l}{\partial x_\alpha} \frac{\partial m_i}{\partial x_\beta} \dot{X}_\beta = \gamma \varepsilon_{ijk} m_j \frac{\partial W}{\partial m_k} \frac{\partial m_l}{\partial x_\alpha}$. умн. на $m_n \varepsilon_{nli}$

$-m_n \varepsilon_{nli} \frac{\partial m_l}{\partial x_\alpha} \frac{\partial m_i}{\partial x_\beta} \dot{X}_\beta = \gamma m_n \varepsilon_{nli} \varepsilon_{ijk} m_j \frac{\partial W}{\partial m_k} \frac{\partial m_l}{\partial x_\alpha}$

$-g_{\alpha\beta} \dot{X}_\beta = \gamma m_n m_j \varepsilon_{nli} \varepsilon_{ijk} \frac{\partial m_l}{\partial x_\alpha} \frac{\partial W}{\partial m_k}$, $\frac{\partial W}{\partial m_k} = \frac{\partial W}{\partial x_\gamma} \frac{\partial x_\gamma}{\partial m_k}$ — new

$\varepsilon_{nli}\varepsilon_{ijk} = \left|\text{сумма по } i\right| = \varepsilon_{inl}\varepsilon_{ijk} = \delta_{nj}\delta_{lk} - \delta_{nk}\delta_{lj}$

$m_n m_j [\delta_{nj}\delta_{lk} - \delta_{nk}\delta_{lj}] \frac{\partial m_l}{\partial x_\alpha} \frac{\partial W}{\partial m_k} = m_j m_j \frac{\partial m_k}{\partial x_\alpha} \frac{\partial W}{\partial m_k} - m_k m_j \times$

$\times \frac{\partial m_j}{\partial x_\alpha} \frac{\partial W}{\partial m_k} = \frac{\partial m_k}{\partial x_\alpha} \frac{\partial W}{\partial m_k} - m_k \frac{\partial W}{\partial m_k} m_j \frac{\partial m_j}{\partial x_\alpha} = \left| m_j \frac{\partial m_j}{\partial x_\alpha} = 0 \right| = \frac{\partial W}{\partial x_\alpha} = -\frac{\partial W}{\partial X_\alpha}$

$-g_{\alpha\beta}\dot{X}_\beta = \gamma(-1)\frac{\partial W}{\partial X_\alpha}$, $g_{\alpha\beta}\dot{X}_\beta = \gamma \frac{\partial W}{\partial X_\alpha}$. $g_{\alpha\beta} = \varepsilon_{ijk} m_i \frac{\partial m_j}{\partial x_\alpha} \frac{\partial m_k}{\partial x_\beta}$

Т.е. для 3D $\vec{X}(t)$ уравнение Thiele сохранило 2D-вид, но вместо $\vec{G} = \{G_\alpha\}$ с $G_z \neq 0$, как общее $g_{\alpha\beta}$ — gyrocoupling density и $x_\alpha = x, y, z$. (gyrovector)

$\int dV$ дает $G_{\alpha\beta}\dot{X}_\beta = \gamma \frac{\partial}{\partial X_\alpha} \int dV w = \gamma \frac{\partial W}{\partial X_\alpha}$, $W = \int dV w$, w — energy density.

$\frac{M_s}{\gamma} G_{\alpha\beta}\dot{X}_\beta = \frac{\partial W}{\partial X_\alpha}$, $G_{\alpha\beta} = \int dV g_{\alpha\beta} = \int dV \varepsilon_{ijk} m_i \frac{\partial m_j}{\partial x_\alpha} \frac{\partial m_k}{\partial x_\beta} = \int dV \vec{m} \cdot \left[\frac{\partial \vec{m}}{\partial x_\alpha} \times \frac{\partial \vec{m}}{\partial x_\beta}\right]$

$\frac{M_s}{\gamma}$ можно (как обычно) внести в определение $G_{\alpha\beta}$): $G_{\alpha\beta}\dot{X}_\beta = \frac{\partial W}{\partial X_\alpha}$

$G_{\alpha\beta} = -G_{\beta\alpha}$ — antisym. tensor 3D. $\alpha, \beta, \gamma = x, y, z$.

$G_{\alpha\beta} = \begin{pmatrix} 0 & -G_3 & G_2 \\ G_3 & 0 & -G_1 \\ -G_2 & G_1 & 0 \end{pmatrix}$ соответствует вектору $\vec{G} = (G_1, G_2, G_3)$, $G_1 = G_{yz}$, $G_2 = G_{xz}$, $G_3 = G_{yx} = G_z$

Для 2D case (x, y) $G_{zy} = 0$, $G_{xz} = 0$ и $\vec{G} = (0, 0, G_z)$ — обычный gyrovector

Для 3D case (x, y, z), $\vec{m}(x, y, z, t)$, gyrovector $\vec{G} = (G_x, G_y, G_z)$ имеет 3 компоненты

Пусть Spherical particle, с $\vec{X}_{eq} = 0$, $W(X) = \frac{\varkappa}{2}\vec{X}^2 = \frac{\varkappa}{2}(X^2 + Y^2 + Z^2)$

$G_{\alpha\beta}\dot{X}_\beta = \begin{pmatrix} 0 & -G_3 & G_2 \\ G_3 & 0 & -G_1 \\ -G_2 & G_1 & 0 \end{pmatrix} \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} -G_3\dot{Y} + G_2\dot{Z}, & G_3\dot{X} - G_1\dot{Z}, & -G_2\dot{X} + G_1\dot{Y} \end{pmatrix} =$

$= \vec{G} \times \dot{\vec{X}}$ и eq. Thiele $\vec{G} \times \dot{\vec{X}} = \frac{\partial}{\partial \vec{X}} \frac{\varkappa}{2}\vec{X}^2 = \varkappa \vec{X}$, $\vec{G} \times \dot{\vec{X}} = \varkappa \vec{X}$.

$\vec{G} \times \vec{G} \times \dot{\vec{X}} = \vec{G}(\vec{G} \cdot \dot{\vec{X}}) - \dot{\vec{X}}(\vec{G} \cdot \vec{G}) = \vec{G}(\vec{G} \cdot \dot{\vec{X}}) - G^2 \dot{\vec{X}} = \varkappa \vec{G} \times \vec{X}$ — не упрощает.

Надо найти обратную матрицу $(G_{\alpha\beta})^{-1}$. $\hat{G}\dot{\vec{X}} = \varkappa \vec{X}$, $\vec{X} = \varkappa \hat{G}^{-1}\dot{\vec{X}}$

Escaneado con CamScanner

$\vec{X}(t) = \vec{X}_0 e^{\lambda t}$, $\dot{\vec{X}} = \lambda \vec{X}_0 e^{\lambda t} = \lambda \vec{X}$, $\lambda \vec{X} = \varkappa \hat{G}^{-1} \vec{X}$, сл. хр.

Чтобы $\det(\varkappa \hat{G}^{-1} - \lambda) = 0$ $\left[\varkappa \hat{G}^{-1} - \lambda\right] \vec{X} = 0$. $\hat{G} = \begin{pmatrix} 0 & -G_z & G_y \\ G_z & 0 & -G_x \\ -G_y & G_x & 0 \end{pmatrix}$

$\det \hat{G} = -G_x G_y G_z + G_x G_y G_z = 0$, $\hat{G}$ – антисимметр.

Если $\dot{\vec{X}} = \vec{\omega} \times \vec{X}$, $\vec{G} \times \vec{X} = \vec{G} \times (\vec{\omega} \times \vec{X}) = \vec{\omega}(\vec{G} \cdot \vec{X}) - \vec{X}(\vec{\omega} \cdot \vec{G}) = \varkappa \vec{X}$,

Это возм. только, если $\vec{\omega} \parallel \vec{X}$, но тогда $\dot{\vec{X}} = a \vec{X} \times \vec{X} = 0$, $\dot{\vec{X}} = 0$, $\omega = 0$.

$\vec{G} \times \vec{X} = \varkappa \vec{X}$ для spherical particle с $W(\vec{X}) = \frac{\varkappa}{2} \vec{X}^2$.

$\vec{X} = X_0 (\sin\theta \cos\Phi, \sin\theta \sin\Phi, \cos\theta)$ $\begin{pmatrix} 0 & -G_z & G_y \\ G_z & 0 & -G_x \\ -G_y & G_x & 0 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \varkappa \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$\vec{G}(\vec{G} \cdot \dot{\vec{X}}) - G^2 \dot{\vec{X}} = \varkappa \vec{G} \times \vec{X}$, $G_i (G_j \dot{X}_j) - G^2 \dot{X}_i$.

$\vec{A} \times \vec{G} \times \dot{\vec{X}} = \vec{G}(\vec{A} \dot{\vec{X}}) - \dot{\vec{X}}(\vec{A} \cdot \vec{G}) = \varkappa \vec{A} \times \vec{X}$, $\cos(\vec{A} \cdot \dot{\vec{X}}) = 0$, $\vec{A} \perp \dot{\vec{X}}$, тогда

$\dot{\vec{X}} = \frac{1}{(\vec{A} \cdot \vec{G})} \varkappa \vec{A} \times \vec{X}$, но $\vec{A} = \vec{A}(t)$. $\vec{\omega} = -\frac{\varkappa}{(\vec{A} \cdot \vec{G})} \vec{A}$ и $\dot{\vec{X}} = \vec{\omega} \times \vec{X}$ опять.

$\vec{G} = \text{const}$ (at small $|\vec{X}|$) $\Rightarrow$ движение $\vec{X}$ в плоскости $\perp \vec{G}$, $(\vec{G} \cdot \vec{X}) = 0$.

$\vec{G} \times \dot{\vec{X}} = \vec{G} \times (\vec{\omega} \times \vec{X}) = \vec{\omega}(\vec{G} \cdot \vec{X}) - \vec{X}(\vec{\omega} \cdot \vec{G}) = \varkappa \vec{X}$, $-\vec{X}(\vec{\omega} \cdot \vec{G}) = \varkappa \vec{X}$ и

$-(\vec{\omega} \cdot \vec{G}) = \varkappa$, $-\frac{\varkappa \vec{A}}{(\vec{A} \cdot \vec{G})} (-1) \vec{G} = \varkappa$ – тождество.

$\vec{G} = (G_1, G_2, G_3)$ – для sphere все направления эквивалентны,

$1 \rightleftarrows 2$, $2 \rightleftarrows 3$, $1 \rightleftarrows 3$. $\vec{G} = G(1,1,1)$. $G_i = G$ – ось ?

$\vec{m}(\vec{r}, t) = \vec{m}_0(\vec{r}) + \delta \vec{m}(\vec{X})$. какое $\vec{m}_0(\vec{r})$ взять чтобы найти $\vec{G}$?

и $W(\vec{X}) = \frac{\varkappa}{2}(X^2 + Y^2 + Z^2)$.

В плоскости $X'Y' \perp \vec{G}$, $X'(t), Y'(t) \to Z'(t) = X'(t) + iY'(t) = Z_0 e^{i\omega t}$, с $\omega = -\frac{\varkappa}{|G|}$.

$\vec{\omega} = -\frac{\varkappa \vec{G}}{|G|^2}$.

Вывод: 3D $\vec{X}(t) = (X(t), Y(t), Z(t))$ в сферич. частице движется в плоскости $\perp \vec{G}$ с частотой $\omega_G = \varkappa / |G|$. $W(\vec{X}) = \frac{\varkappa}{2} |\vec{X}|^2$.

Что это за солитон $\vec{m}_0(\vec{r})$. Сообщен ли он? нелегко. См. 11/VIII.

(1-6/VIII) Отдых дома, были, фильмы. С Л. и Ю. на Онтарио и Ohio-beaches.

О 3D solitons, gyrovector и topol. vector potential, Hopf invariant – см. Komineas, Phys. D '1996. Gyrovector $\gamma_i = -\frac{1}{2} \varepsilon_{ijk} \vec{m} \cdot (\partial_j \vec{m} \times \partial_k \vec{m}) = -\frac{1}{2} \varepsilon_{ijk} g_{jk}$

$g_{jk} = \vec{m} \cdot (\partial_j \vec{m} \times \partial_k \vec{m})$, $\nabla \cdot \vec{\gamma} = 0$, $\vec{\gamma} = \nabla \Pi \times \nabla \Phi = \text{rot} \vec{a}$, Hopf ind. $N = \frac{1}{4\pi} \int dV (\vec{a} \cdot \vec{\gamma})$

See E.A. Kuznetsov, A.V. Mikhailov, Ph. lett. 77A, 37 (1980) „On the topological meaning of canonical Clebsch variables" ($\Phi, \cos\Theta = \Pi$) для $\vec{m}(\Theta, \Phi)$ case

L.D. Faddeev lett. Math. Phys. 1, 289 (1976), „Some comments on many-dimen. solitons"

Topol. Current $J_\mu = \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{abc} \partial_\nu u^a \partial_\rho u^b \partial_\sigma u^c$, $\vec{u} = (u_1, u_2, u_3)$ – isovector fields ($\vec{m}$)

$\mu = 0,1,2,3$; $a = 1,2,3$. $J_0 = \text{div} \vec{P}$, $P_i = \varepsilon_{ikl} \varepsilon^{abc} u^a \partial_k u^b \partial_l u^c$

$\vec{u} \to \vec{m}$, $P_i = \varepsilon_{ikl} \vec{m} (\partial_k \vec{m} \times \partial_l \vec{m}) = -2 \gamma_i$ – gyrovector с точн. до множителя.

Topol. charge $Q = \int dV J_0 \neq 0$, только если $\text{div} \vec{\gamma} \neq 0$, т.е. $\vec{\gamma} = \nabla \Pi \times \nabla \Phi$ сингулярна.

$J_\mu = \varepsilon_{\mu\nu\rho\sigma} \partial_\nu \vec{m} \cdot (\partial_\rho \vec{m} \times \partial_\sigma \vec{m})$ и $J_0 = -2 \text{div} \vec{\gamma}$ – связь $\vec{\gamma}$ и topol. current.

$\vec{\gamma} = \text{rot} \vec{a}$, $\vec{a} = \Pi \nabla \Phi + \nabla F$, $\Delta F = -\text{div}(\Pi \nabla \Phi)$. eq. for potential F.

$\vec{\gamma} = \text{rot} \vec{a} = \text{rot}[\Pi \nabla \Phi + \nabla F] = \text{rot}(\Pi \nabla \Phi) + \text{rot}(\nabla F) = \Pi \cdot \text{rot}(\nabla \Phi) + \nabla \Pi \times \nabla \Phi = \nabla \Pi \times \nabla \Phi$

$\vec{\gamma} = \nabla \Pi \times \nabla \Phi$, $\Pi = \cos\Theta$, $\vec{m}(\Theta, \Phi)$. $N = \frac{1}{4\pi} \int dV (\vec{a} \cdot \vec{\gamma}) = \frac{1}{4\pi} \int dV (\vec{a} \cdot \text{rot} \vec{a})$ – Hopf inv.

$N = \frac{1}{4\pi} \int dV \, \Pi \nabla \Phi \cdot \vec{\gamma} = \frac{1}{4\pi} \int dV \, \Pi \nabla \Phi \cdot \text{rot}(\Pi \nabla \Phi) = \frac{1}{4\pi} \int dV \, \Pi \nabla \Phi \cdot (\nabla \Pi \times \nabla \Phi)$

Hopf index через Θ, Φ: $N = \frac{1}{4\pi} \int dV \, \Pi \nabla \Phi \cdot (\nabla \Pi \times \nabla \Phi)$ – Faddeev 1976.

(7-10/VIII) Отдых дома, были, ничего.

(11/VIII) Определение Hopf. index $N = Q_H$ в N-Faddeev 76 дает Kundu-Rybakov '82 [3½ стр 208]

$Q_H = \frac{m}{4\pi} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \, (1 - w) \left[\frac{\partial w}{\partial r} \frac{\partial v}{\partial \theta} - \frac{\partial w}{\partial \theta} \frac{\partial v}{\partial r}\right] = \pm n m$, n-integer. $\vec{r} = \vec{r}(r, \theta, \varphi)$ сферич. (Радиус-вектор)

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$m_z = \cos\theta = w(r,\theta)$, $\Phi = \operatorname{arctg}\frac{m_y}{m_x} = m\varphi + \nu(r,\theta)$, $\vec{m}(\theta,\Phi)$, $\vec{m}^2 = 1$ — ansatz Kundu

т.к. в 2D-case $\Phi = q\varphi + \Phi_0$, будем это обобщать на 3D-case $\vec{m}(x,y,z)$ Soliton

$\Phi_0 \to \nu(r,\theta)$, helicity сохраняется переменной

Комінеаз: $Q_H = \frac{1}{4\pi}\int dV\, \vec{a}\cdot\vec{\gamma}$, $\vec{\gamma} = \operatorname{rot}\vec{a}$ - gyrovector, $\vec{a}$ - vector potential

Flux of $\vec{\gamma}$: $q = \frac{1}{4\pi}\int_S d\vec{S}\cdot\vec{\gamma}$ для 2D-case $\vec{\gamma} = (0,0,\gamma_z)$ дает 2D topol. charge $q = \pm 1$.

1/VII 2018 я получил $q = \frac{1}{4\pi}\int d\vec{S}\cdot\vec{\gamma} = 1$ для hedgehog $\vec{m}(\vec{r}) = \frac{\vec{r}}{r}$, а Hopf index Q_H of hedgehog не получал. Далее $Q_H(\text{hedgehog}) = 0$ в ноб.

Note (Kundu etc) $Q_H = \pm m\cdot n$, n - integer, m из $\Phi = m\varphi + \nu(r,\theta)$ - vorticity $m = q$ - 2D topol. charge.

3D Soliton with $Q_H = 1$ (ansatz de Vega '78), $m_z = \cos\theta(\eta)$, $m_x + i m_y = \sin\theta\, e^{i(\beta-\varphi)}$ (η)

(η,β,φ) - toroidal coordinates, $\theta(0) = \pi$, $\theta(\infty) = 0$ - b.c. Для $\theta(\eta)$ можно записать функционал

$\vec{B}(\text{Kundu}) = -2\,\nabla w\times\nabla\Phi = -2\,\nabla(\cos\theta)\times\nabla\Phi = -2\,\vec{\sigma}\times\vec{\Omega} = -2\vec{\gamma}$ (Комінеаз)

Hedgehog: $\vec{m}(\vec{r}) = \frac{\vec{r}}{r}$, $m_j = \frac{r_j}{|\vec{r}|}$, $\Phi = \operatorname{arctg}\frac{m_y}{m_x} = \operatorname{arctg}\frac{y}{x} = \varphi$, $\nu = 0$ (Kundu)

$\vec{B} = \frac{-2m}{r^2\sin\theta}\left[\hat{r}\frac{\partial w}{\partial\theta} - \hat{\theta}\, r\frac{\partial w}{\partial r}\right] - \frac{2}{r}\hat{\varphi}\left[\frac{\partial w}{\partial r}\frac{\partial\nu}{\partial\theta} - \frac{\partial w}{\partial\theta}\frac{\partial\nu}{\partial r}\right] = \left|\begin{smallmatrix}\nu=0\\ m=1\end{smallmatrix}\right| =$

$= -\frac{1}{r^2\sin\theta}\left[\hat{r}\frac{\partial w}{\partial\theta} - \hat{\theta}\, r\frac{\partial w}{\partial r}\right]$, $w = \cos\theta = m_z = \frac{z}{r} = \cos\theta$, $\frac{\partial w}{\partial\theta} = -\sin\theta$

$\vec{r}(r,\theta,\varphi)$ - сферич. координаты, $\frac{\partial w}{\partial r} = \frac{\partial}{\partial r}m_z = \frac{\partial}{\partial r}\cos\theta = 0$.

$\vec{B} = -\frac{q}{r^2\sin\theta}\left[\hat{r}(-\sin\theta) - \hat{\theta}\, r\frac{\partial w}{\partial r}_{=0}\right] = 2\frac{\hat{r}}{r^2}$, $\int_{\text{upper semisphere}} d\vec{S}\cdot\vec{B} = 2\int r^2\sin\theta\, d\theta\, d\varphi \times$

$\times\frac{1}{r^2} = 2\int d\theta\sin\theta\, d\varphi = -2\pi\int d(\cos\theta) = -2\pi\cdot 2\cos\theta\Big|_0^{\pi/2} = -4\pi\left[\cos\frac{\pi}{2} - \cos 0\right] = 4\pi$

$\frac{1}{4\pi}\int_{\text{upper}} d\vec{S}\cdot\vec{B} = 1$. $\frac{1}{4\pi}\oint_{\text{sphere}} d\vec{S}\cdot\vec{\gamma} = \frac{1}{4\pi}\int_{\text{upper}} d\vec{S}\left(-\frac{1}{2}\vec{B}\right) + \frac{1}{4\pi}\int_{\text{down}} d\vec{S}\left(-\frac{1}{2}\vec{B}\right) = -\frac{1}{2} - \frac{1}{2} = -1$

$\vec{\gamma}(\vec{r}) = -\frac{1}{2}\vec{B}(\vec{r}) = -\frac{\hat{r}}{r^2}$. $g_{\gamma\delta} = -\frac{1}{2}\varepsilon_{\gamma\alpha\beta}g_{\alpha\beta}$, $G_\gamma = \int dV\, g_{\gamma\gamma}$ - гиперкомпл.

Если gyrovector density $\vec{\gamma}(\vec{r}) = c\frac{\hat{r}}{r^2}$, то $\vec{G} = \int dV\, c\frac{\vec{r}}{r^2}$, $G_i = c\int_{\text{sphere}} dV\frac{x_i}{r^2}$

$x_i \Rightarrow \vec{r}(x,y,z)$. Для сферы $G_z = c\int dr\, d\theta\, d\varphi\, r^2\sin\theta\frac{\cos\theta}{r^2} =$

$= c\int d\varphi\int d\theta\sin\theta\cos\theta = 2\pi c\int d(\sin\theta)\sin\theta = 2\pi c\frac{1}{2}\sin^2\theta\Big|_0^{\pi} = 0$

$G_{x,y} = c\int_{\text{sphere}} dr\, d\theta\, d\varphi\, r^2\sin\theta\frac{\sin\theta\sin\varphi}{r^2}(\cos\varphi) \sim \int d\varphi\begin{pmatrix}\sin\varphi\\ \cos\varphi\end{pmatrix} = 0 \Rightarrow \vec{G} = 0$

Eq. thiele (31/VII) $\vec{G}\times\dot{\vec{X}} = \alpha\dot{\vec{X}}$ при $\vec{G} = 0$ дает $\dot{\vec{X}}(t) = 0$ - гироторная

динамика hedgehog в sphere не существует (!)

Надо либо заменить sphere → cylinder, либо считать hedgehog гироскопическим динамика via Hopf-index Q_H.

3D Soliton

(12/VIII) Не удается Q (Faddeev) $= \int dV J_0 = \int dV\,\operatorname{div}\vec{\beta} = -2\int dV\,\operatorname{div}\vec{\gamma} = -2\oint d\vec{S}\cdot\vec{\gamma}$

и q из $\int dV\vec{\gamma} = \vec{G}$. Для 2D $\vec{m}(x,y)$, $\gamma_x = \gamma_y = 0$, $G_z = \int dV\gamma_z = L\int_S dx\,dy\,\gamma_z =$

$= 4\pi L q$, $q = \frac{1}{4\pi}\int dx\,dy\,\gamma_z$, $\gamma_z = g_{xy} = \vec{m}\cdot(\partial_x\vec{m}\times\partial_y\vec{m})$ - 2D top. charge

Т.е., не удается $\oint d\vec{S}\cdot\vec{\gamma}$ к $\int dx\,dy\,\gamma_z = \int dS_z\gamma_z$.

Комінеаз'96 удает $Q = \frac{1}{4\pi}\oint d\vec{S}\cdot\vec{\gamma}$ и $q = \frac{1}{4\pi}\int dx\,dy\,\gamma_z$. $Q = q$ только

если отсутствует flux $\int d\vec{S}\cdot\vec{\gamma}$ через поверхность S - незамкнутую.

$\vec{G} = \int dV\vec{\gamma}$ - charge of $\vec{\gamma}(\vec{r})$, а не $\int dV\,\operatorname{div}\vec{\gamma} = \oint d\vec{S}\cdot\vec{\gamma}$. И только в 2D-

case $\vec{m}(x,y)$, $\vec{G} = \int dV\vec{\gamma}(x,y) = L\int dx\,dy\,\gamma_z\hat{z}$ и $\int dV\,\operatorname{div}\vec{\gamma} = \oint dS_z\gamma_z$ похожи.

$\gamma_{z\neq 0}$, $\gamma_{x,y} = 0$

$\int dV\,\operatorname{div}\vec{\gamma} = \oint dS_z\gamma_z = \oint dx\,dy\,\vec{n}\cdot\vec{\gamma} = \int dx\,dy\,\gamma_z\Big|_{z=+\frac{L}{2}} - \int dx\,dy\,\gamma_z\Big|_{z=-\frac{L}{2}} = 0$,

т.к. $\vec{\gamma} \neq f(z)$. Hopf charge $Q_H = (1/4\pi)\int dV\vec{a}\cdot\vec{\gamma}$, $\vec{\gamma} = \operatorname{rot}\vec{a}$ - это gyrovector.

Escaneado con CamScanner

• Kundu '82 gave $\vec{B} = \frac{2}{m_z}(\nabla m_x \times \nabla m_y)$ - topol. field и $Q_H = \frac{-m}{8\pi}\int dr_z \int d\theta (1-m_z) B_\varphi$

$B_\varphi = B_\varphi(r,\theta)$, $\vec{n}(r,\theta,\varphi)$ инвариантен. Вихрь дает $Q_H \neq 0$.

$B_\varphi = -\frac{2}{r}\left[\frac{\partial w}{\partial r}\frac{\partial v}{\partial \theta} - \frac{\partial w}{\partial \theta}\frac{\partial v}{\partial r}\right]$, $w = \cos\theta$, $\Phi = m\varphi + S(r,\theta)$, $\vec{m}(\theta,\Phi)$

$B_\varphi(r,\theta) = -\frac{2}{r}\left[\frac{\partial w}{\partial r}\frac{\partial \Phi}{\partial \theta} - \frac{\partial w}{\partial \theta}\frac{\partial \Phi}{\partial r}\right]$. Komineas '06 $\vec{\gamma} = \nabla\Pi \times \nabla\Phi$, $\Pi = \cos\theta$

$\gamma_\varphi = (\nabla\Pi \times \nabla\Phi)_\varphi = (\nabla\Pi)_r(\nabla\Phi)_\theta - (\nabla\Phi)_r(\nabla\Pi)_\theta$, $\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\varphi}$

Ответ (Kundu) $\Pi, \Phi \neq f(\varphi)$. $\gamma_\varphi = \frac{\partial \Pi}{\partial r}\frac{1}{r}\frac{\partial \Phi}{\partial \theta} - \frac{\partial \Phi}{\partial r}\frac{1}{r}\frac{\partial \Pi}{\partial \theta}$, $\Pi = w$ (Kundu)

Выгодно B_φ (Kundu) $= -2\gamma_\varphi$. $\vec{B} = -2(\nabla w \times \nabla\Phi) = -2\vec{\gamma}$.

$B_z = \frac{2}{m_z}(\nabla m_x \times \nabla m_y)_z = \frac{2}{m_z}\left[(\nabla m_x)_x (\nabla m_y)_y - (\nabla m_x)_y (\nabla m_y)_x\right] =$

$= \frac{2}{m_z}\left[\frac{\partial m_x}{\partial x}\frac{\partial m_y}{\partial y} - \frac{\partial m_x}{\partial y}\frac{\partial m_y}{\partial x}\right]$ // $\vec{m}\cdot\left[\frac{\partial \vec{m}}{\partial x} \times \frac{\partial \vec{m}}{\partial y}\right] = m_x\left(\frac{\partial m_y}{\partial x}\frac{\partial m_z}{\partial y} - \frac{\partial m_z}{\partial x}\frac{\partial m_y}{\partial y}\right) +$

$+ m_y\left(\frac{\partial m_z}{\partial x}\frac{\partial m_x}{\partial y} - \frac{\partial m_x}{\partial x}\frac{\partial m_z}{\partial y}\right) + m_z\left(\frac{\partial m_x}{\partial x}\frac{\partial m_y}{\partial y} - \frac{\partial m_y}{\partial x}\frac{\partial m_x}{\partial y}\right)$ — нелепо.

Сравним $\vec{B} = -2(\nabla w \times \nabla\Phi)$ и $\vec{B} = \frac{2}{m_z}(\nabla m_x \times \nabla m_y)$, $m_x = \sin\theta\cos\Phi$

$\nabla m_x \times \nabla m_y = \nabla(\sin\theta\cos\Phi) \times \nabla(\sin\theta\sin\Phi) = [\sin\theta\nabla(\cos\Phi) + \cos\Phi\nabla(\sin\theta)] \times$

$\times [\sin\theta\nabla(\sin\Phi) + \sin\Phi\nabla(\sin\theta)] = \sin\theta\sin\Phi\nabla(\cos\Phi)\nabla(\sin\theta) + \cos\Phi\sin\theta \times$

$\times \nabla(\sin\theta) \times \nabla(\sin\Phi) = \sin\theta\sin\Phi\sin\Phi(-1)\cos\theta\nabla\Phi\times\nabla\theta + \cos\Phi\sin\theta\cos\theta\cos\Phi \times$

$\times \nabla\theta\times\nabla\Phi = \sin\theta\cos\theta(\nabla\theta\times\nabla\Phi)[\sin^2\Phi + \cos^2\Phi] = \sin\theta\cos\theta\nabla\theta\times\nabla\Phi$.

$\vec{B} = \frac{2}{m_z}(\nabla m_x \times \nabla m_y) = 2\sin\theta\nabla\theta\times\nabla\Phi = -2\nabla(\cos\theta)\times\nabla\Phi = -2\nabla w\times\nabla\Phi$ — верно.

$\vec{\gamma} = -\frac{1}{2}\frac{\vec{B}}{m_z} = \nabla w\times\nabla\Phi = \nabla\Pi\times\nabla\Phi = -\sin\theta\nabla\theta\times\nabla\Phi$

Удобно и геометрично нечто. $\vec{B} = \frac{2}{m_z}(\nabla m_x \times \nabla m_y)$, $\vec{\gamma} = -\frac{1}{m_z}(\nabla m_x \times \nabla m_y)$

$\vec{B} \sim \vec{\gamma}$ — emergent magnetic field related to $\vec{m}(\vec{r})$.

$\vec{\gamma} = \mathrm{rot}\,\vec{a}$, $\vec{a} = \frac{1}{4\pi}\vec{A}^e$, $A^e_\mu = (A^e_0, \vec{A}^e)$ emergent vector potential Gus'EPL'16.

Очевидно что hedgehog $\vec{m}(\vec{r}) = \frac{\vec{r}}{|r|}$ и $\vec{\gamma} = \frac{\hat{r}}{r^2}$ дает $\vec{G} = \int dV\vec{\gamma} = 0$

дает нулевой суммарный заряд. Будет ли $\vec{G} \neq 0$ для Soliton deVega'78?

• deVega'78 $\vec{j}_h = \vec{\gamma} = \frac{1}{4\pi c^2}\frac{(ch\eta - c\beta)^2}{sh\eta}\left[\hat{e}_\beta + sh\eta\,\hat{e}_\varphi\right]\sin\theta\cdot\theta'(\eta)$ в toroidal coordinates

$\vec{r}(\eta,\beta,\varphi)$, $m_z = \cos\theta$, $m_x + im_y = \sin\theta e^{i(\beta-\varphi)}$, $\theta(\eta)$. $\vec{\gamma}$ Gauss. $\vec{m}(\vec{r}) \Rightarrow Q_H = 1$.

$x + iy = c\frac{sh\eta}{ch\eta - \cos\beta}e^{i\varphi}$, Wiki $x+iy = a\frac{sh\tau}{ch\tau - \cos\sigma}e^{i\varphi}$, $(\eta,\beta,\varphi) = (\tau,\sigma,\varphi)$

$c = a$, $\eta = \tau$, $\beta = \sigma$, $\varphi = \varphi$.

$dV = c^3\frac{sh\eta}{(ch\eta - \cos\beta)^3}d\beta d\eta d\varphi$, $\vec{G} = \int dV\vec{\gamma} = \frac{c}{4\pi}\int d\eta d\beta d\varphi \frac{\sin\theta\theta'(\eta)}{(ch\eta - \cos\beta)}\left[\hat{e}_\beta + sh\eta\,\hat{e}_\varphi\right]$

Ответа будет $\sin\theta(\eta)$, $\theta(\eta)$? $\varphi \in [0, 2\pi]$, $\beta \in [-\pi, \pi]$, $\eta \geq 0$. Или $\beta \in [0, 2\pi]$.

$\theta(\eta=0) = \pi$, $\theta(\eta=\infty) = 0$ т.е. $\int_{-\pi}^{\pi}\frac{d\beta}{a - \cos\beta} = \left[a = ch\eta \geq 1\right] = \frac{2}{\sqrt{a^2-1}}\mathrm{arctg}\frac{(a+1)}{\sqrt{a^2-1}}\mathrm{tg}\frac{\beta}{2}\Big|_{-\pi}^{\pi} =$

$= \frac{2}{\sqrt{a^2-1}}\left[\mathrm{arctg}(+\infty) - \mathrm{arctg}(-\infty)\right] = \frac{2\pi}{\sqrt{a^2-1}} = \frac{2\pi}{sh\eta}$.

$\vec{G} = \frac{1}{2}c\int d\eta d\beta \frac{\sin\theta\theta'(\eta)}{(ch\eta - \cos\beta)}[\dots] = \pi c\int_0^\infty \frac{d\eta}{sh\eta}\sin\theta\theta'(\eta)\left[\hat{e}_\beta + sh\eta\,\hat{e}_\varphi\right]$

Если $\theta(\eta)$ без аномалий $\vec{G} = \int dV\vec{\gamma} = 0$ конечно.

Т.е., Soliton deVega'78 с $Q_H = 1$ имеет $\vec{G} \neq 0$ и должен иметь гироскопическую динамику.

deVega'78 дает частный выбор $\theta(\eta)$: $\cos\theta(\eta) = 1 - 2\frac{1}{ch^2\eta}$, $\cos\theta(\eta=0) = -1$

$\vec{G} = \pi c\int_0^\infty \frac{d\eta}{sh\eta}\sin\theta\theta'(\eta)\left[\hat{e}_\beta + sh\eta\,\hat{e}_\varphi\right]$, $-\sin\theta\theta'(\eta) = (-2)\frac{d}{d\eta}\frac{1}{ch^2\eta}$, $\cos\theta(\eta=\infty) = 1$

$\times(-2)\frac{1}{ch^2\eta}sh\eta = \frac{4}{ch^3\eta}sh\eta$ и $\vec{G} = \pi c(-4)\int_0^\infty \frac{d\eta}{ch^3\eta}\left[\hat{e}_\beta + sh\eta\,\hat{e}_\varphi\right] = G_\beta\hat{e}_\beta + G_\varphi\hat{e}_\varphi$

$G_\varphi = -4\pi c\int_0^\infty \frac{d\eta}{ch^3\eta}sh\eta = -4\pi c\int\frac{d(ch\eta)}{ch^3\eta} = -4\pi c\left(-\frac{1}{2}\right)\frac{1}{ch^2\eta}\Big|_0^\infty = 2\pi c[0-1] = -2\pi c$

$G_\beta = -4\pi c\int_0^\infty \frac{d\eta}{ch^3\eta} = -4\pi c\left|\frac{1}{2}\frac{sh\eta}{ch^2\eta} + \frac{1}{2}\mathrm{arctg}(sh\eta)\right|_0^\infty = -4\pi c\cdot\frac{\pi}{4} = -\pi^2 c$

$\vec{G} = -\pi^2 c\,\hat{e}_\beta - 2\pi c\,\hat{e}_\varphi = -\pi c[\pi\hat{e}_\beta + 2\hat{e}_\varphi]$ - gyrovector of deVega '78 soliton

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c - parameter of toroidal coordinates, $x+iy = c\frac{\mathrm{sh}\,\eta\, e^{i\varphi}}{(\mathrm{ch}\,\eta - \cos\beta)}$, $z = c\frac{\sin\beta}{(\mathrm{ch}\,\eta - \cos\beta)}$

$\eta = \infty$ corresp. to the closed curve (filament) $\to x^2+y^2 = c^2,\ z=0$.

η = const curves = circles of the filament.

Kundu-Rybakov' 1982 gauge in cylindrical coordinates (ρ, φ, z) — 3D topol. charge

Hopf-index $Q_H = \frac{m}{4\pi}\int_0^\infty d\rho \int_{-L/2}^{L/2} dz\,(1-w)\left[\frac{\partial w}{\partial \rho}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial z}\frac{\partial v}{\partial \rho}\right]$. $m_z = \cos\theta = w(r,\theta)$

$\vec{r} = (r,\theta,\varphi)$ - spherical or $\vec{r} = (\rho, z, \varphi)$ cylindrical coordinates. $\Phi = \arctg\frac{m_y}{m_x} = m\varphi + v(r,\theta)$

$m_z = \cos\theta = w(\rho,z)$, $\Phi(\vec{r}) = m\varphi + v(\rho,z)$, $v_{end\,p.} = \pm n\frac{\pi}{2} \to Q_H = \pm nm$. $\vec{m}(\theta,\Phi)$

$m_x + i m_y = \sin\theta\, e^{i\Phi}$

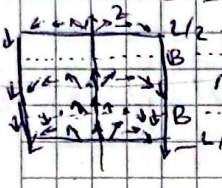
Можно нумеровать $w(\rho,z)$ и $v(\rho,z)$ in cyl. tot, чтобы $Q_H = \pm 1$ ($n=1, m=1$)

Kundu gaer $w = w(\rho)$, $v = kz$, $Q_H = \frac{kL}{4\pi}\int_0^\infty d\rho\,(1-w)\frac{\partial w}{\partial \rho} = \frac{kL}{2\pi}$ если $\cos\theta(0) = -1$, $\cos\theta(\infty) = +1$ — Skyrmion b.c. in infinite film.

T.e. ansatz $m_z(\rho)$, $\Phi = \varphi + kz$ даёт $Q_H \neq 0$, $Q_H = \frac{kL}{2\pi}$, L - size of dot $\parallel Oz$.

Выбрав $k = 2\pi/L$ получим soliton с $Q_H = 1$ (!). $m_z(\rho)$, $\Phi = \varphi + \frac{2\pi}{L}z$.

$\Phi_0(z) = \frac{2\pi}{L}z$, $\Phi_0(0) = 0$, $\Phi_0(\pm\frac{L}{2}) = \pm\pi$ — в middle plane, и на поверх. dot $z = \pm\frac{L}{2}$. Будут Neel-Skyrmions. Bloch - Skyrmions $z = \pm\frac{L}{4}$.



Будет ли решение $m_z(\rho)$, $\Phi(\varphi,z) = \varphi + kz$ метастабильно? Нужно ли для стабилизации DMI?

Slonczewski equation (без DMI и W_m) $\left[K + A(\nabla\Phi)^2\right]\sin 2\theta - 2A\nabla^2\theta = 0$

$-2A(\nabla^2\Phi)\sin^2\theta - 2A\sin 2\theta\,(\nabla\Phi\cdot\nabla\theta) = 0$

$\nabla\Phi = \hat{\rho}\frac{\partial\Phi}{\partial\rho} + \hat{\varphi}\frac{\partial\Phi}{\rho\partial\varphi} + \hat{z}\frac{\partial\Phi}{\partial z} = \frac{\hat{\varphi}}{\rho} + k\hat{z}$, $\theta(\rho) \to \nabla\theta = \hat{\rho}\theta'(\rho)$, $\nabla\Phi\cdot\nabla\theta = 0$

$\nabla^2\Phi = 0$ $\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\varphi^2} + \frac{\partial^2\Phi}{\partial z^2} = 0$ удовл. $\left[K + A\left(\frac{1}{\rho^2} + k^2\right)\right]\sin 2\theta = +2A\nabla^2\theta$

$\nabla^2\theta(\rho) = \theta''_{\rho\rho} + \frac{1}{\rho}\theta'_\rho$, $\nabla^2\theta = \theta''_{\rho\rho} + \frac{1}{\rho}\theta'_\rho = \frac{1}{2}\sin 2\theta\left[\frac{1}{\Delta^2} + k^2 + \frac{1}{\rho^2}\right]$, $\Delta^2 = A/K$ —

также ур-е для $\theta(\rho)$ имеет смысл для W_m и DMI. Уже $k\neq 0$ это $\frac{1}{\Delta^2} \to \frac{1}{\Delta^2} + k^2$.

$W[\theta,\Phi] = \int dV\left[A\left\{(\nabla\theta)^2 + \sin^2\theta(\nabla\Phi)^2\right\} + K\sin^2\theta\right] = A\int dV\left[(\nabla\theta)^2 + \sin^2\theta\left(\frac{1}{\Delta^2} + (\nabla\Phi)^2\right)\right] =$

$= A\int dV\left[(\nabla\theta)^2 + \left(\frac{1}{\Delta^2} + k^2 + \frac{1}{\rho^2}\right)\sin^2\theta\right] = W(k=0) + Ak^2\int dV\sin^2\theta = W(k=0) + W_{ex}(k)$

Extra energy из-за $k\neq 0$ $W_{ex} = Ak^2\int dV\sin^2\theta = A\left(\frac{2\pi}{L}\right)^2\int dV\sin^2\theta$ будет мала при $L \gg \Delta$, thick dots/films. Но при $L \gg \Delta$, т.е. $W_m(L)$ будет вклад из-за top/bottom surface Neel Skyrmions при $z = \pm L/2$. $L \gg \Delta$ ограничен bulk DMI или ≈ 0 interface $z = +L/2$ или/и $z = -L/2$ DMI. Bulk DMI stabilizes Bloch-Sk ($\simeq 2$ и 5 layer Skyrmions). Interface DMI это не может стабилизировать 3 Neel-Sk.

Проверим численно или аналитически, будет ansatz $\theta(\rho)$, $m_z(\rho)$, $\Phi = \varphi + kz$ новое coorbs. metastable state $\vec{m}(\vec{r})$ in a circular dot.

$\vec{\gamma} = \nabla\Pi\times\nabla\Phi = -\sin\theta\,\nabla\theta\times\nabla\Phi = -\sin\theta\,\hat{\rho}\,\theta'_\rho\times\left(\frac{1}{\rho}\hat{\varphi} + k\hat{z}\right) = -\sin\theta\theta'_\rho\left[\frac{1}{\rho}\hat{\rho}\times\hat{\varphi} + k\hat{\rho}\times\hat{z} = -\sin\theta\theta'_\rho\left(\frac{1}{\rho}\hat{z} - k\hat{\varphi}\right]\right.$, $\vec{G} = \int dV\,\vec{\gamma} = -\int d\rho\,\rho\int d\varphi\,dz\,\sin\theta\theta'_\rho\left[\frac{1}{\rho}\hat{z} - k\hat{\varphi}\right] =$

$= -2\pi L\int d\rho\,\rho\,\sin\theta\theta'_\rho\left[\frac{1}{\rho}\hat{z} - k\hat{\varphi}\right]$, $G_z = -2\pi L\int d\rho\,\rho\sin\theta\theta'_\rho = 2\pi L\int d(\cos\theta) = 2\pi L\cos\theta\Big|_0^\infty \neq 0$

$G_\varphi = 2\pi Lk\int d\rho\,\rho\sin\theta\theta'_\rho = (-)$ неважно, т.к. $G_z \neq 0$ и $\vec{G} \neq 0$.

Итак есть two 3D solitons с $Q_H = 1$, Belavin'78 и Kundu'82, имеющие $\vec{G} \neq 0$. Soliton Kundu - нужен, но надо доказать его метастабильность. Soliton Belavin - сложнее, но это „a finite energy solution" будущее метастабильно.

• J. Whitehead. Proc. Nat. Acad. Sci. USA 33, 117 (1947)

S^2: $m_x^2 + m_y^2 + m_z^2 = 1$, S_3: $|z|^2 + |y|^2 = 1$ (z, y - complex coordinates for $S_3 \equiv R_4$).

Mapping $f: S_3 \to S_2$ given by $m_x + i m_y = 2\rho\, z^n y^m$, $m_z = \rho\left(|z|^{2n} - |y|^{2m}\right)$

$\rho = 1/(|z|^{2n} + |y|^{2m})$. Hopf invariant for $S_3 \to S_2$ mapping $Q_H = \pm nm$.

$z = z_0 e^{in\theta}$, $y = y_0 e^{im\theta}$ - fibres.

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• I.L. Bogolubsky. Ph. Lett A 126 (511, 1988). 3D solitons in lattice model. High-order exchange. $S_x = 2(N_1N_4 + N_2N_3)$, $S_y = 2(N_1N_3 - N_2N_4)$, $S_z = N_3^2 + N_4^2 - N_1^2 - N_2^2$, $N_i = \frac{2x_i}{1+r^2}$, $i=1,2,3$, $N_4 = \frac{1-r^2}{1+r^2}$, $r^2 = \sum_i x_i^2$. Такое распределение $\vec{S}(\vec{r})$ в виде gauge Hopf index $Q_H = 1$. $\vec{S}(\vec{r})$ похоже на $\vec{m}(\vec{r})$ Whitehead'a при $n=1$, $N=1$.

• P. Sutcliffe. PRL 118, 247203 (2017) Skyrmion knots in frustrated magnets.

Magn. field with $Q_H = 1$ (Hopf charge) is given by (...) and has the topology of circular skyrmion string, with radius λ, lying in plane $z=0$ and twisted once (twist = π-rotation?)

$\vec{m}(\vec{r})$: $m_x = \frac{4}{R^2}\left[2\lambda^2 xz - \lambda y(\lambda^2 - r^2)\right]$, $m_y = \frac{4}{R^2}\left[2\lambda^2 yz + \lambda x(\lambda^2 - r^2)\right]$, $\vec{m}^2 = 1$

$m_z(\vec{r}) = \frac{4}{R^2}\left[2\lambda^2 z^2 - \lambda^2 r^2 + \frac{1}{4}(\lambda^2 - r^2)^2\right]$, $R^2 = (\lambda^2 + r^2)^2$, λ - string radius.

можно найти $\vec{J} = -\frac{1}{m_z}(\nabla m_x \times \nabla m_y)$ и gyrovector $\vec{G} = \int dV \vec{J}^2$.

• D.A. Nicole. Phys. G: Nucl. Phys. 4, 1363 (1978) Solitons with $Q_H \neq 0$.

Знач. обозначения работ deVega 1978. $\vec{m}(\theta,\varphi) \to w = \operatorname{tg}\frac{\theta}{2} e^{i\varphi}$

map $w = \frac{N_1 + iN_3}{N_1 + iN_2}$, $N_i = \frac{2x_i}{1+R^2}$, $R^2 = \sum_i x_i^2$, $N_4 = \frac{1-R^2}{1+R^2}$

$m_x = N_1^2 + N_2^2 - N_3^2 - N_4^2$, $m_y = 2(N_1N_4 + N_2N_3)$, $m_z = 2(N_1N_3 - N_2N_4)$, $\vec{h} = \vec{J}$.

$\vec{h}$ Gyrofield: $h_i = \frac{1}{8\pi}\varepsilon_{ijk}\varepsilon_{abc} n_a \partial_j n_b \partial_k n_c = \frac{1}{4\pi}\varepsilon_{ijk}\sin\theta\,\partial_j\theta\,\partial_k\varphi =$

$= \frac{1}{2\pi i}\varepsilon_{ijk}\frac{1}{[1+|w|^2]^2}\partial_j w\,\partial_k w$, $w = \operatorname{tg}\frac{\theta}{2}e^{i\varphi}$, Hopf-index $H = -\int dV \vec{a}\cdot\vec{h}$, $\vec{h} = \operatorname{rot}\vec{a}$

$h_i = \frac{1}{8\pi}\varepsilon_{ijk}\vec{m}\cdot(\partial_j\vec{m}\times\partial_k\vec{m})$ или $\vec{h} = \frac{1}{8\pi}\varepsilon_{abc} m_a(\nabla m_b \times \nabla m_c)$

$w = \operatorname{tg}\frac{\theta}{2}e^{i\varphi} = \frac{1-R^2+2ix_3}{2(x_1+ix_2)} = \frac{(1+ix_3)^2}{2(x_1+ix_2)} - \frac{1}{2}(x_1 - ix_2)$, $\vec{r} = (x_1,x_2,x_3) = (x,y,z)$, $H=1$.

$\vec{r}\cdot\vec{e}_{z=0} \to \vec{m}(\vec{r})$.

$\operatorname{tg}\frac{\theta}{2}e^{i\varphi} = \frac{(1+iz)^2}{2(x+iy)} - \frac{1}{2}(x - iy)$. $B(\rho,\varphi,z)$: $\operatorname{tg}\frac{\theta}{2}e^{i\varphi} = \frac{1}{2\rho}e^{-i\varphi}(1+iz)^2 - \rho e^{-i\varphi}\frac{1}{2}$

$\operatorname{tg}\frac{\theta}{2}e^{i\varphi} = \frac{1}{2\rho}e^{-i\varphi}\left[(1+iz)^2 - \rho^2\right]$ // Видно что $w(x,y,z)$ не имеет ни нулей, ни полюсов, в отличие от 2D case, где $w(z) \sim (z - z_0)^2$. string radius $\lambda = 1$.

$\frac{1}{2}m_z = N_1N_3 - N_2N_4 = \frac{4}{(1+R^2)^2}x_1x_3 - \frac{2x_2}{(1+R^2)}\frac{(1-R^2)}{(1+R^2)} = \frac{2}{(1+R^2)^2}\left[2xz - y(1-R^2)\right] \to \frac{1}{2}m_x$ (Sutcl.)

$\left(\frac{1}{2}\right)m_x = N_1^2 + N_2^2 - N_3^2 - N_4^2 = \left(\frac{2x_1}{1+R^2}\right)^2 + \left(\frac{2x_2}{1+R^2}\right)^2 - \left(\frac{2x_3}{1+R^2}\right)^2 - \frac{(1-R^2)^2}{(1+R^2)^2} =$

$= \frac{4}{(1+R^2)^2}\left[x_1^2 + x_2^2 - x_3^2 - \frac{1}{4}(1-R^2)^2\right] = \frac{4}{(1+R^2)^2}\left[R^2 - 2x_3^2 - \frac{1}{4}(1-R^2)^2\right] \to -m_z$ (Sutcl.)

Nicole использ. $n_1 = \cos\Theta$, $n_2 + in_3 = \sin\Theta e^{i\Phi}$, надо $1\to3$, $2\to1$, $3\to2$ для сравнения. ($z=3$)

$w = \operatorname{tg}\frac{\theta}{2}e^{i\varphi} = \frac{1-R^2+2ix_2}{2(x_3+ix_1)} = \frac{(1+ix_2)^2}{2(x_3+ix_1)} - \frac{1}{2}(x_3 - ix_1) = \frac{(1+iy)^2}{2(z+ix)} - \frac{1}{2}(z - ix)$

$w = \frac{N_4 + iN_2}{N_3 + iN_1}$ или $(x_1,x_2,x_3) = (x,y,z)$. Sutcliffe'17 говорит Sk-string radius λ и использует индексы Nicole'78 и $\vec{m}(\vec{r})$ без больших скобок на Nicole etc.

Sutcliffe'17: $m_x + im_y = \sin\Theta e^{i\Phi} = \frac{4}{R^4}\left[2\lambda^2 z(x+iy) + i(x+iy)\lambda(\lambda^2 - r^2)\right]$, $R^2 = \lambda^2 + r^2$.

$m_x + im_y = \sin\Theta e^{i\Phi} = \frac{4}{(\lambda^2+r^2)^2}\left[2\lambda^2 z + i\lambda(\lambda^2 - r^2)\right](x+iy)$, $m_z = \frac{4}{(\lambda^2+r^2)^2}\left[2\lambda^2 z^2 - \lambda^2 r^2 + \frac{1}{4}(\lambda^2 - r^2)^2\right]$ -

дальше дано. 3D topol. soliton with Hopf index $Q_H = 1$ и string radius λ.

по плоскости $z=0$, $r^2 = \rho^2$ $m_z(\rho) = \frac{1}{(\lambda^2+\rho^2)^2}\left[(\lambda^2 - \rho^2)^2 - 4\lambda^2\rho^2\right]$. Sk-radius ρ_c найдем из $m_z(\rho) = 0$. $\rho_c = \lambda(\sqrt{2}\pm1) \sim \lambda$. λ может служить оценкой 2D Sk-radius, $\rho_c \approx \lambda$.

$m_z(\rho, z=0)$ имеет два нуля при $\rho = [0,\infty]$, $\rho_{c\pm} = \lambda(\sqrt{2}\pm1) \sim \lambda$

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Рассм. осциллирующие Sk-ring radius $\lambda(t)$? тогда λ есть conjugate variable.



- Sk-ring. Flux is quantized along Sk-string и 1 ring, $\parallel z$.

// Nicole'78 (и видимо Dolega'78) Bogolubsky'88 Sutcliffe'17 - все ввели одинаковый 3D soliton с $Q_H = 1$, $\vec{m}(\vec{r})$ в виде поверхности Sk-string (torus) с замкнутыми концами.

13-19/VIII Отдых дома. Вышла работа над Vortex/Sk review paper J. Magnetics. Отправила. APL Mat., Navas et al. was published. Обменялся оттисками. 50% АПС (1100 $).

20/VIII Meeting с Ricardo: подписали PhD program on нейроморфным EMag. School, (Brno, Sept. 2019) attendance. подготовил JEMS's poster, booked hotel Stockholm

21-23/VIII Printed JEMS poster, hotel, airtickets - reservations. Отдых, Отправка. Нам. Вышла. Закупка продуктов.

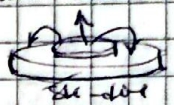
24-30/VIII Trip to Uppsala, Sweden, JEMS'2019. Oral talk by Zukhra on Sk-chain dynamics, my poster "Sk-stability". Restaurants с Л. Ужин. Экскурсия.

31/VIII Переезд с Л. в Poznan. Univ. residencia. С Л. в "tesco".

1/IX Отдых. С Л. в лес и озера возле Univ. A.M. Art. Skyrmion/Sk

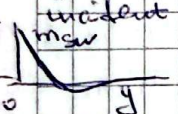
2/IX Беседа с Mateusz. Три Skyrmion projects: 1) Sk-gyrotropic motion in antidot array under current pulses (submitted); 2) в мягкие additional dot or ring на поверх. Sk-dot (RRL) на Skyrm stability due to stray fields; 3) Skyrmion dots on surface

3/IX

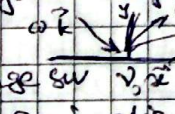


of soft magn. film — Skyrmion imprints non-uniform $\vec{m}(\vec{r})$ into film и Sk-gyrotropic or breathing oscillations generate SW in film/stripe - надо рассчитать Sk-image in soft film и eng oscillations.

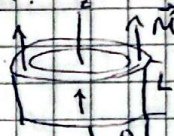
Беседа с Pawel: SW inelastic scattering by the film edge - мне рассчитать eigenmode profile of the edge mode и incident SW - edge SW interaction в духе RRL.



edge SW profile simul, $\vec{M}_0 = M_0 \hat{y}$ in the main approximation.



Ring stray field



$$\vec{H}_m(\vec{r}) = \int d^3\vec{r}'\, \hat{G}(\vec{r}\vec{r}')\,\vec{M}(\vec{r}'), \quad \vec{M}(\vec{r}) = (0,0,M_s) \text{ и } M_s = \text{const для ring}$$

$$H_m^\alpha(\vec{r}) = \int d^3\vec{r}'\, G_{\alpha z}(\vec{r}\vec{r}')\,M_s, \quad H_m^\varphi = 0 \text{ в силу симметрии.}$$

$$H_m^\rho(\vec{r}) = M_s \int d^3\vec{r}'\, G_{\rho z}(\vec{r}\vec{r}'), \quad H_m^z(\vec{r}) = M_s \int d^3\vec{r}'\, G_{zz}(\vec{r}\vec{r}')$$

$$G_{\rho z}(\vec{r}\vec{r}') = -\frac{\partial}{\partial\rho}\frac{\partial}{\partial z'}\frac{1}{|\vec{r}-\vec{r}'|}, \quad G_{zz}(\vec{r}\vec{r}') = -\frac{\partial^2}{\partial z\partial z'}\frac{1}{|\vec{r}-\vec{r}'|}$$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \int dk\, e^{-k|z-z'|} J_0(k|\vec{\rho}-\vec{\rho}'|) = \int dk\, e^{-k\Delta z}\sum_\mu e^{i\mu(\varphi-\varphi')} J_\mu(k\rho) J_\mu(k\rho')$$

$$\int_0^{2\pi} d\varphi'\, J_0(k|\vec{\rho}-\vec{\rho}'|) = \int d\varphi' \sum_\mu e^{i\mu(\varphi-\varphi')} J_\mu J_\mu(\cdot) = 2\pi J_0(k\rho)J_0(k\rho')$$

$$H_\rho(\vec{r}) = H_\rho(\rho,z) = M_s \int d\varphi' \int d\rho'\rho' \int dz' \left(-\frac{\partial}{\partial\rho}\frac{\partial}{\partial z}\right) \int dk\, e^{-k|z-z'|} J_0(k|\vec{\rho}-\vec{\rho}'|) =$$

$$= -2\pi M_s \frac{\partial}{\partial\rho}\frac{\partial}{\partial z'} \int d\rho'\rho' \int dz' \int dk\, e^{-k|z-z'|} J_0(k\rho) J_0(k\rho')$$



$$H_z(\rho,z) = -2\pi M_s \frac{\partial}{\partial z} \int d\rho'\rho' \int dz' \frac{\partial}{\partial z'} \int dk\, e^{-k|z-z'|} J_0(k\rho) J_0(k\rho')$$

$$\int_0^L dz' \frac{\partial}{\partial z'} e^{-k|z-z'|} = e^{-k|z-z'|}\Big|_{z'=0}^{z'=L} = e^{-k|z-L|} - e^{-k|z|} = \left\{\begin{array}{l} |z-L| = -(z-L),\ z<0 \\ |z| = -z,\ z<0 \end{array}\right\} =$$

$$= e^{k(z-L)} - e^{kz} = e^{kz}\left(e^{-kL}-1\right), \quad z<0.$$

$$H_\rho(\rho,z) = -2\pi M_s \frac{\partial}{\partial\rho} \int d\rho'\rho' \int dk\, J_0(k\rho) J_0(k\rho') e^{kz}\left(e^{-kL}-1\right), \quad \frac{\partial}{\partial\rho} J_0(k\rho) = -k J_1(k\rho)$$

$$H_\rho(\rho,z) = -2\pi M_s \int_{r_1}^{r_2} d\rho'\rho' \int dk\, k\, J_1(k\rho) J_0(k\rho') e^{kz}\left(1-e^{-kL}\right)$$

$$\int_{r_1}^{r_2} d\rho'\rho' J_0(k\rho') = \frac{1}{k^2} k\rho' J_1(k\rho')\Big|_{r_1}^{r_2} = \frac{1}{k}\left[r_2 J_1(kr_2) - r_1 J_1(kr_1)\right]$$

$$H_\rho(\rho,z) = -2\pi M_s \int_0^\infty dk\, J_1(k\rho)\left[r_2 J_1(kr_2) - r_1 J_1(kr_1)\right] e^{kz}\left(1-e^{-kL}\right)$$

$$H_z(\rho,z) = -2\pi M_s \frac{\partial}{\partial z}\int d\rho'\rho' e^{kz}\left(e^{-kL}-1\right) J_0(k\rho) J_0(k\rho') dk = -2\pi M_s \int_0^\infty dk\, e^{kz}\left(e^{-kL}-1\right) J_0(k\rho)\left[r_2 J_1(kr_2) - r_1 J_1(kr_1)\right]$$

$H_\rho(\rho,z) = -2\pi M_s \int_0^\infty dk\, J_1(k\rho)\left[\zeta J_1(\zeta k) + J_1(k)\right] e^{k\beta z}\left(1-e^{-k\beta}\right)$, $\zeta = \frac{r_2}{r_1}$, $\beta = \frac{L}{r_1}$, $z<0$

$H_z(\rho,z) = -2\pi M_s \int_0^\infty dk\, e^{k\beta z}\left(e^{-k\beta}-1\right) J_0(k\rho)\left[\zeta J_1(\zeta k) - J_1(k)\right]$, ρ in units of r_1, z in units of L

r_1 - inner ring radius, r_2 - outer ring radius, L - ring thickness, $z<0$

— $H_\rho(\rho,z)$ и $H_z(\rho,z)$ изразени чрез $J_1(\rho,z) = \int_0^\infty dk\, J_1(k\rho) J_1(k\zeta) e^{-k\beta|z|}$ и $J_0(\rho,z) = \int_0^\infty dk\, J_0(k\rho) J_1(k\zeta) e^{-k\beta|z|}$.

Сам. J.T. Conway 'IEEE TMI 2001': $I_{\lambda\mu\nu}(r_1,r_2,z) = \int_0^\infty ds\, s^\lambda J_\mu(sr_1) J_\nu(sr_2) e^{-s|z|}$

$I_{000}(r_1,r_2,z) = \frac{k}{\pi\sqrt{r_1 r_2}} K(k)$, I_{010} - сложно, $I_{011}(r_1,r_2,z) = \frac{1}{3\pi k\sqrt{r_1 r_2}}\left[(2-k^2)K(k) - 2E(k)\right]$

$k^2 = \frac{4r_1 r_2}{(r_1+r_2)^2+z^2}$

$\frac{1}{2\pi M_s} H_\rho(\rho,z) = \int dk\, J_1(k\rho)\left[\zeta J_1(\zeta k) - J_1(k)\right] e^{-k\beta|z|}\left(e^{-k\beta}-1\right) =$

$= \zeta \int_0^\infty dk\, J_1(k\rho) J_1(\zeta k)\left[e^{-k\beta z} - e^{k\beta(z-1)}\right] - \int_0^\infty dk\, J_1(k\rho) J_1(k)\left[e^{k\beta z} - e^{k\beta(z-1)}\right]$

$\int_0^\infty dk\, J_1(k\rho) J_1(k\zeta) e^{-k|z|} = \frac{1}{\pi k\sqrt{\rho\zeta}}\left[(2-k^2)K(k) - 2E(k)\right]$, $k^2 = \frac{4\rho\zeta}{(\rho+\zeta)^2+z^2}$

$-|z| = \beta z$ or $|z| = \beta(z-1)(-1)$.

Poznan

(4-7/IX) Работа с Matensz над броненото ring stray field на SK-stability — в circular dot. 6/IX пристигане, посрещане с Maciej. groups. Matensz дни. имена обсъдено SK-project 2.300 PLN as per diem 31/VIII - 11/IX.

(9/IX) Беседа с Pawel: имена - edge mode profile and SW-edge SW scattering. 2) Бъдещи Becker SW in Neel DW calc. ing Alieu PRB '2011 дали сравнени с simulations.

Бъдещето с Matensz (MCMD) Neel SK energy in ring field as $F(R_s)$.

(10/IX) пристигане с Л. в Stockholm. Изпращане в Stockholm downtown.

(11/IX) Път Stockholm - Arlanda airport и flight to Bilbao. Пристигане 10 P.M.

(12/IX) Reports on Stockholm/Uppsala и Poznan trips. Отдих цяла.

Matensz изпраща MathCad files of SK-ring расчети.

(13/IX) Беседа с Julian (реч бъдещи за Sept., нови applications SolSkyMag to library, Ruben (AF project, обсъдени ему заключения Sh-Gordon paper) и Karin (engineer Office 2nd Ai-Ping, дъщеря. Напомняне ще ще дадена, дъщеря ей свой laptop "HP"). В „Nami" и Enoshima lobbil с Л. и Ю.

(14/IX) Отпет Uppsala - Poznan. Пристигане с Л. в „Carrefour Market".

(15/IX) Ранно пристигане с Ruben и Ricardo по 1D DW sine Gordon MS. Bockenhoff Overleaf file. Осмисляне 1D-DW $\Phi(3)$ расчет 24/IV. 3/7/V изработване на параметрични зависимости. $\Delta(\kappa)$ Ruberia (K is given). 1D - DW sine Gordon

Днес 11/II пристигане Боровец. Чернова сделки:

- 1) Movilidad" uPV/EHU appl. in Nov. (travel agent, имена, пощата);
- 2) "Dynamics SW modes paper - finish writing + submit (Porto trip in Oct.)
- 3) Review Vortex/Skyrmion SK-Kim до 31 Oct. 2019 (написване SK-part и подобряване фиг.)
- 4) AF - DW dynamics с Ruben-Ricardo (сделки Beccaga to MS)
- 5) 1D-DW motion / sine Gordon (заключения и submit in Sept.) - Subm. PRB 29/I/19.
- 6) Изпълнение „Chase" bank account. 8/II - 3 пътувания $ to „Bankia"
- 7) изработване и заключения Skyrm pair/chain MS с Зуфрин, Qe
- 8) Расчет ring/SK-dot и Skyrm-dot oscillation generation of SW in mag. film (Kelly)
- 9) Non-elastic SW scattering on film-edge (Pawel) и DW string oscillations

(16/IX) Изпращане SolSkyMag/Logo Library appl. у Julian и Ruben осведомена в Library. Беседа с Ricardo (имена ежем. 450€ год. заключения 1D DW manuscript) - нови работи JEMS'2019, нови projects on Skyrmions и SW-scattering с Poznan. Изпращане 1D DW paper give Phys. Rev. B с Л. и Ю. в Zeltwitz, Германия в C. Argiñano restaurant (15/IX).

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(17/IX) Sec. 5 of 1D DW manuscript Δ ⊓ v и v ⌐ c H . Это только для AF or weak-FM.

Напомню ну $v = \frac{\gamma H}{d}\Delta$, $\Delta(v) = \Delta_0\sqrt{1 - v^2/c^2}$ через нашу ссылку

$v(H) = \frac{c\mu H}{\sqrt{c^2 + \mu^2 H^2}}$, $v(H) = c$ saturates и $v(H) \propto \mu H$ at small H как в FM. $\mu = \frac{\gamma H}{\Delta}$ здесь. c – some limiting GW velocity.

Baryakhtar et al. topical. Magnetic Solitons '1994 : AF – weak FM

$\frac{1}{M_0^2} W(\vec{m}, \vec{l}) = \frac{1}{2}\lambda(\nabla\vec{l})^2 + \frac{1}{2}\delta\vec{m}^2 + W_a(\vec{l}) + W_D - \vec{H}\cdot\vec{m}$ для AF или weak FM

DMI $W_D = \vec{d}\cdot(\vec{m}\times\vec{l})$, $d = |\vec{d}|$. $c = \frac{1}{2}\gamma M_0\sqrt{d\delta}$, $[d] = Oe\,cm^2$.

Для weak FM, $\vec{m} \sim \frac{d}{\delta}\vec{l}$, $\mu = \frac{2\gamma d\Delta_0}{\lambda\delta}$, λ – damping parameter.

Для обычного FM : подвижность $\mu(F) = \frac{\gamma\Delta_0}{d}$. $\mu(AF) = \frac{2d}{\delta}\mu(F) << \mu(F)$, $d/\delta << 1$,

хотя $\mu(AF)$ и $\mu(F)$ могут быть сравнимы, т.к. Δ_0 и $\lambda(d)$ – разные.

Теорию $v(H)$ weak FM см. A.K. Zvezdin, JETP Lett. 29, 513 (1979).

Новые comments on Sec. 5 1D DW paper to Ruben, Ricardo.

L.D. Faddeev et al. 2007 : sine Gordon $\varphi_{tt} - \varphi_{xx} + \frac{m^2}{\beta}\sin\varphi = 0$ имеет реш.

$\varphi(x,t) = \frac{4Q}{\beta}\arctg\exp\left[\frac{m}{\sqrt{1-v^2}}(x - vt - x_0)\right]$, Q – topol. charge, $Q = \pm 1$ for solitons (antisol.)

$v = \frac{1 - x_0^2}{1 + x_0^2}$, v и $x_0 = x(t=0)$ связаны (!). У меня $\beta = 2$, $m = \frac{1}{\Delta_0}$ – mass.

(18/IX) Обсудили Ricardo недостатки Sec. 5 1D DW paper и дали ему новые "twisted magnons" Nat. Com. 2019. Один Uppsala-Roman referee report + to Rubrom. Обсуд. Ricardo $v(H)$ для weak FM.

(19/IX) Пришел на ред. RRL – Sisodia. MICINN – projects all (by 17/X). переписка и тел. разговор с Оксаной. Договорились о prefix. подать co-ordinated project on nanowires, one PI – Madrid, и 2 PI – Donostia. Написание Kin Vortex/Skyrm. review. Я, посылки Ruben : 13 kg N.D. + 11 kg None

(20/IX) Julian Conv/Lecb. в моем nanowire" – proposal as 50% scientist. Julian на пенсию в October 2022 (70 лет), с тем чтобы получать гранты конференции членов кес. С N.и 10. на моем Ondarreta и "Nan" pitches.

(21/IX) 1D – DW paper. Ruben прислал parameters для опис. $\Delta(v)$ Sec. 5, см. 7/X. Ruben sent me two different sets of parameters to plot $\Delta(v)$:

Set #1 (2/X) : $K_x = 1.28\cdot10^{-22}$ J, $K_z = 4.01\cdot10^{-24}$ J etc. и set #2 (7 May) : $K = 7.85\cdot10^8 \frac{J}{m^3}$ и $A = \frac{J}{m} = 4.77\frac{pJ}{m}$

set #2 дал $v_{max} = 21.47$ km/s, тогда как на Fig. 4 of MS, $v_{max} = 4.98$ km/s,

т.е. видимо Ruben used set #1 to plot Fig. 4, $\Delta(v)$ simulated.

M. Coey, Magnetism 2009 : $A = \frac{JS^2 z}{a}$, S – local spin, a – lattice period,

z is the number of atoms per unit cell (z = 1 simple cubic, z = 2 b.c.c. z = 4 f.c.c.)

$\mathcal{H} = -2\sum_{i>j} J_{ij}\vec{S}_i\cdot\vec{S}_j$, J_{ij} – exch. integrals. $\mathcal{H} = -\sum_{ij} J_{ij}\vec{S}_i\vec{S}_j$

Ruben (7/X) $J = 5.77\cdot10^{-21}$ J (simple cubic lat.). Atom. moment $\mu = 2.5\,\mu_B$

$\mu = g\mu_B S = 2.5\,\mu_B$, $gS = 2.5$, $g \approx 2$. $S_{eff} = \frac{2.5}{2} = \frac{5}{4}$ (!?)

A (Ruben) $= \frac{5.77\cdot10^{-21}\,J}{0.4\cdot10^{-9}\,m}\left(\frac{5}{4}\right)^2\cdot 1 = 22.54\ pJ/m \approx 22.5\ pJ/m$

$v_{max} = \frac{2\gamma}{M_s}\sqrt{AK_z}\left[\sqrt{1+\lambda} - 1\right] = \left|\lambda = K_x/K_z \approx 32 >> 1\right| \approx \frac{2\gamma}{M_s}\sqrt{AK_z}\sqrt{\lambda} \approx \frac{2\gamma}{M_s}\sqrt{AK_x}$

$v_0 = \frac{2\gamma}{M_s}\sqrt{AK_z}$ (см. 3/X) $= 2.2\pi\frac{3.0\ GHz/kOe}{0.362\ kOe}\sqrt{2.25\frac{erg}{cm}10^{-6}\ 6.27\cdot10^5\frac{erg}{cm^3}} = 12\pi\frac{1.0\ GHz}{0.362\cdot10^6\frac{erg}{cm^2}}\sqrt{2.25\cdot0.627} =$

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$= \frac{12\pi}{0.362} \cdot \frac{10^9 Hz}{10^6 (Oe)^2} \cdot \frac{erg}{cm^3} \cdot cm \sqrt{2,25 \cdot 0,627} = 104,14 \cdot 10^3 \frac{cm}{s} \sqrt{2,25 \cdot 0,627} = 123,7 \cdot 10^3 \frac{cm}{s} = 1237 \frac{m}{s}$

$v_0 = 1237$ m/s. $\left[\sqrt{1+\lambda} - 1\right] = \sqrt{1+31,9} - 1 = 4,736 \Rightarrow v_{max} = v_0 \left[\sqrt{1+\lambda} - 1\right] =$

$= 1237 \frac{m}{s} \cdot 4,736 = 5858 \frac{m}{s} = 5,858$ km/s — Ruben declared $v_m = 4,98 \frac{km}{s}$.

Можно будет $(\gamma/2\pi) = 2.8 \frac{GHz}{kOe}$, а не $3,0 \frac{GHz}{kOe} \rightarrow v_{max} = 5,858 \cdot \frac{2.8}{3.0} = 5,467 \frac{km}{s}$

У Рубена $\Delta_0 = \sqrt{\frac{A}{K_z}} = 11,6$ nm $\Rightarrow A = \Delta_0^2 (K_z)^2 = [11,6 \cdot 10^{-7} cm]^2 \left[6,27 \cdot 10^5 \frac{erg}{cm^3}\right] =$

$= 1,346 \cdot 10^{-12} cm^2 \cdot 6,27 \cdot 10^5 \frac{erg}{cm^3} = 8,437 \cdot 10^{-7}$ erg/cm

Такое число $A = 0,8437 \cdot 10^{-6}$ erg/cm дает число $v_0 = 1237 \frac{m}{s} \sqrt{\frac{0,844}{2,25}} = 758 \frac{m}{s}$

$v_m = 758 \frac{m}{s} \cdot 4,736 = 3,588 \frac{km}{s}$ (Ruben declared $v_m = 4,98$ m/s).

Ruben $|K| = 115 K \cdot k_B = 115 \cdot K \cdot 1,381 \cdot 10^{-23} \frac{J}{K} = 158,8 \cdot 10^{-23}$ J — manuscript.

Ruben (2[V]) $K_x = 1,28 \cdot 10^{-22}$ J и J $= 5,77 \cdot 10^{-21}$ J. Откуда дано $|K| = 15,88 \cdot 10^{-22}$ J в тексте 1D - DW manuscript?

С п.ч. в Сибирь, зар. номер. В „Eroski" Urbel. Editing Sec. 5 1D DW paper.

22/IV Написал и submitted Zeff. report on Siroda - Sk to RRL. 1D DW $d \neq 0$ $H \neq 0$

$v(H) = \frac{\mu H}{\sqrt{1+(\mu H/c)^2}}$ Сом. 17/IV is valid for weak FM (Zvezdin'79), μ - DMI.

Zvezdin - ЭЕТР Lett. 1979 показал что $\varphi(x,t) = 2 \operatorname{arctg} e^{\frac{(x-vt)}{\Delta(v)}}$ $\vec{l}(\theta, \varphi)$, $\theta = const$ решение соответствует для $d \neq 0$, $H \neq 0$ weak FM, если намагниченность подчиняется $v(H)$.

Для weak FM при $d=0$, $H=0$ это sine-Gordon for $\varphi(x,t)$. При $d \neq 0$, $H \neq 0$ sine-Gordon нет, а DW решение $\operatorname{tg} \varphi/2 = \exp(\pm \frac{(x-vt)}{\Delta})$ сохраняется.

Верно ли это для FM с biaxial anisotropy и $\lambda = K_x/K_z >> 1$?

Schlyer-Walker 1974, Eq. (3) при $\varphi(x,t) = const$, $\theta(x,t)$ и $d \neq 0$, $H \neq 0$ из энергии

$$E = \int dx \left[-HM_z + 2\pi M_x^2 - \frac{K}{M_0^2} M_z^2 + \frac{A}{M_0^2} (\nabla \vec{M})^2 \right], \quad K = K_z, \quad 2\pi M_0^2 = K_x$$

$\dot{\theta} = 4\pi\gamma M_0 \sin\theta \sin\varphi \cos\varphi$, $d\dot{\theta} = 4\pi\gamma M_0 \sin\theta \cos\theta \cos^2\varphi + \frac{2K\gamma}{M_0} \sin\theta \cos\theta +$

$+ \gamma H \sin\theta - \frac{2\gamma A}{M_0} \frac{\partial^2 \theta}{\partial x^2}$. $\Delta_0 = \sqrt{\frac{A}{K_z}}$, $t_0 = \frac{1}{\gamma H_a} = \frac{1}{\gamma} \frac{M_s}{2K_z}$ units of length and time.

$\frac{d\theta}{dt} = \frac{1}{t_0} \frac{d\theta}{d(t/t_0)} = \frac{1}{t_0} \frac{d\theta}{d\tau} = \frac{2K_z\gamma}{M_s} \frac{d\theta}{d\tau} \rightarrow \frac{2K_z\gamma}{M_s} \frac{d\theta}{d\tau} = 4\pi\gamma M_s \sin\theta \sin\varphi \cos\varphi$

$\frac{4\pi\gamma M_s \cdot M_s}{2K_z\gamma} = \frac{2\pi M_s^2}{K_z} = \frac{K_x}{K_z} = \lambda$. $\frac{d\theta}{d\tau} = \lambda \sin\theta \sin\varphi \cos\varphi$

$- \frac{2\gamma A}{M_s} \theta''_{xx} + \left[4\pi\gamma M_s \cos^2\varphi + \frac{2K\gamma}{M_s}\right] \sin\theta\cos\theta = d\dot{\theta} - \gamma H \sin\theta$

$\frac{\partial^2}{\partial x^2} = \Delta_0^{-2} \frac{\partial^2}{\partial (x/\Delta_0)^2}$, $\frac{2\gamma A}{M_s} \Delta_0^{-2} = \frac{2\gamma A}{M_s} \frac{K_z}{A} = \frac{2\gamma K}{M_s}$, $\frac{4\pi\gamma M_s \cdot M_s}{2K\gamma} = \lambda$, $H_a = 2K/M_s$.

$- \frac{2AK}{M_s} \theta''_{xx} + \frac{2KK}{M_s} \left[1 + \lambda \cos^2\varphi\right] \sin\theta\cos\theta = d \frac{2K\gamma}{M_s} \dot{\theta}_\tau - \gamma H \sin\theta$, $\frac{\gamma H \cdot M_s}{2K\gamma} = \frac{H}{H_a}$

$\begin{cases} -\theta''_{xx} + (1 + \lambda\cos^2\varphi) \sin\theta\cos\theta = d\dot{\theta} - h\sin\theta \\ \dot{\theta} = \lambda \sin\theta \sin\varphi\cos\varphi \end{cases}$, $h = H_z/H_a$.

- equation for $\theta(x,t)$ при $d \neq 0$, $H \neq 0$.

$-\theta''_{xx} + (1 + \lambda\cos^2\varphi) \sin\theta\cos\theta = d\lambda \sin\theta \sin\varphi\cos\varphi - h\sin\theta = \sin\theta (d\lambda \sin\varphi\cos\varphi - h)$

Минимизируем выбором $\varphi(H)$ так. Чтобы $d\lambda \sin\varphi\cos\varphi = h$. $1 + \lambda\cos^2\varphi = \frac{1}{\Delta^2}$

тогда $\theta''_{xx} = \frac{1}{\Delta^2} \sin\theta\cos\theta$, $\operatorname{tg}\frac{\theta}{2} = \exp\left(-\frac{(x-vt)}{\Delta}\right)$, $\theta = 2\operatorname{arctg} e^{-x/\Delta}$

$\theta_x = 2 \frac{1}{1+e^{-2x/\Delta}} \left(-\frac{1}{\Delta}\right) e^{-x/\Delta} = -\frac{1}{\Delta} \frac{1}{ch\, x/\Delta} = -\frac{1}{\Delta} \sin\theta$, $\sin\theta = \frac{1}{ch\, x/\Delta}$, $\cos\theta = th\, x/\Delta$

$\theta/\sin\theta = \left(-\frac{1}{\Delta}\right) \frac{1}{\sin\theta} \theta_x(v) = \frac{v}{\Delta}$, $\frac{v}{\Delta} = \lambda \sin\varphi\cos\varphi \rightarrow \frac{dv}{\Delta} = h$

$\frac{v}{\Delta(v)} = \frac{h}{d}$. Если $\Delta(v) = \sqrt{1 - v^2/c^2}$, то $v(h) = \frac{h}{d} \frac{1}{\sqrt{1 + (h/dc)^2}}$

Abs, units $v(h) = 2\gamma/M_s \sqrt{AK_z} \frac{h}{d} \frac{1}{\sqrt{\ldots}}$



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$\frac{2\gamma\sqrt{AK_z}}{M_s}\frac{HM_s}{2K_z} = \gamma\sqrt{\frac{A}{K_z}}H = \gamma\Delta_0 H$, $v(H) = \frac{\gamma\Delta_0 H}{\alpha}\frac{1}{\sqrt{1+(h/\alpha\epsilon)^2}}$, $h = \frac{H}{H_d}$.

$\frac{h}{c} = \frac{HM_s}{2K_z}\frac{v_0}{v_-} = \frac{HM_s}{2K_z}\frac{1}{v_-}\frac{2\gamma}{M_s}\sqrt{AK_z} = \frac{\gamma H}{v_-}\sqrt{AK_z} = \frac{\gamma H\Delta_0}{v_-}$.

$K^2(v) = \frac{1}{2}(v^2-\lambda) - 1 + \frac{1}{2}\sqrt{(v^2-v_-^2)(v^2-v_+^2)}$. Если $\lambda >> 1$, то $v_- = v_+ = c = \sqrt{\lambda}$

$K^2(v) = \frac{1}{2}(v^2-c^2) - 1 + \frac{1}{2}(v^2-c^2) \approx (v^2-c^2) = -\varkappa^2(v)$.

$\varkappa^2 = (c^2-v^2) = c^2(1-v^2/c^2)$ $\varkappa(v) = c\sqrt{1-v^2/c^2}$, $c = \sqrt{\lambda}$, $v_0 = \frac{2\gamma}{M_s}\sqrt{AK_z}$

Но $\varkappa(v) \to \max$ при $v \to v_- = c$. Надо что-то не так.

Generalized Walker eq $\frac{v}{\Delta(v)} = \frac{h}{\alpha}$ учитывает DW width $\Delta(v) = \frac{1}{\varkappa(v)}$

Standard Walker '74 Solution $v(H) = \frac{\gamma\Delta_0 H}{\alpha}\frac{1}{\sqrt{1+\frac{\lambda}{2}(1-\sqrt{1-h^2})}}$, $h = \frac{H}{H_w}$, $H_w = 2\pi\alpha M_s$

$\frac{v}{\Delta(v)} = \frac{h}{\alpha}$ is special solution for any λ.

$\Delta(v)$ надо взять из $\varkappa(v) = 1/\Delta(v)$.

Ruben уточнил parameters of DW simulations, set #3.

теперь это chain granularity $6\cdot10^4 a$, $a = 0{,}3328$ nm lattice period.

μ of atom, $= 4\,\mu_B$; exch. integral $J = 115\,K\cdot k_B$, $M_s = J/V_0$, $V_0 = a^3$.

Exch. stiffness $A = J/a$ (уточнить, где z и S?)

$k_x = 115\,K\cdot k_B$ (x along spin chain), $k_z = 1{,}302\cdot10^{-24}$ J.

The set #3 parameters, кроме K_x, K_z, близки к Set #2 (sent by Ruben 7/V)

$M_s = \frac{4\mu_B}{V_0} = 1{,}006$ G. $A(\text{Ruben}) = \frac{J}{a} = 4.77$ pJ/m $= 0{,}477\cdot10^{-6}$ erg/cm

$k_x = 115\,K\cdot k_B = J = 115\,K\cdot 1{,}3806\cdot10^{-23}\frac{J}{K} = 158{,}8\cdot10^{-23}$ J. $k_x = K_x V_0$, $k_z = K_z V_0$

$\lambda = \frac{K_x}{K_z} = \frac{k_x}{k_z} = \frac{158{,}8\cdot10^{-23}}{1{,}302\cdot10^{-24}}\frac{J}{J} = 121{,}94\cdot10 = 1219{,}4 >>> 1$.

$K_x = k_x/V_0 = 158{,}8\cdot10^{-23}\,J/(3{,}328)^3 10^{-30}\,m^3 = 4{,}308\cdot10^7\frac{J}{m^3} = 4{,}308\cdot10^8\frac{erg}{cm^3}$

$K_z = 1{,}302\cdot10^{-24}\,J/(3{,}328)^3 10^{-30}\,m^3 = 0{,}0353\cdot10^6\frac{J}{m^3} = 0{,}353\cdot10^6\frac{erg}{cm^3}$

$v_- = \frac{2\gamma}{M_s}\sqrt{AK_z}\left[\sqrt{1+\lambda}-1\right] \approx \frac{2\gamma}{M_s}\sqrt{AK_z}\sqrt{\frac{K_x}{K_z}} = \frac{2\gamma}{M_s}\sqrt{AK_x}$ (m/s)

$v_- = \frac{2\cdot2{,}8\,MHz/Oe\cdot2\pi}{1{,}006\cdot10^3\,Oe}\sqrt{0{,}477\cdot10^{-6}\frac{erg}{cm}\,4{,}308\cdot10^8\frac{erg}{cm^3}} = \frac{5{,}6\cdot2\pi10^6 Hz\cdot10\frac{erg}{cm^2}\sqrt{0{,}477\cdot4{,}308}}{1{,}006\cdot10^3(Oe)^2} =$

$= \left|\frac{erg}{cm^2\cdot Oe^2} = cm\right| = \frac{2\pi\cdot5{,}6\cdot10^7}{1{,}006\cdot10^3}\sqrt{0{,}477\cdot4{,}308}\left(\frac{cm}{s}\right) = 50{,}14\cdot10^4\frac{cm}{s} = 5{,}014\left(\frac{km}{s}\right)$

$\sqrt{\lambda} = \sqrt{1219{,}4} = 34{,}920$, $\sqrt{1+\lambda}-1 = 33{,}934$

Aharoni: Intro Theory Ferromagnetism $E_{ex} = JS^2\sum\sum\left[(\vec{S}_i\cdot\vec{\nabla})\vec{m}\right]^2$. $\vec{S}_i$ – ближ. соседи узла i: $\vec{S}_i = a(\pm1,\pm1,\pm1)$ for simple cubic lattice, J – ex. integral ближ. соседей $H_{ex} = -J\sum\vec{S}_i\vec{S}_j$ $\to A = \frac{JS^2}{a}z$, $z = 1,2,4$ – S.c., bcc., f.cc.

Если $\mu = g\mu_B S = 4\mu_B$, то $S(\text{ion}) = 2$.

Alternative: A из $\Delta_0 = \sqrt{\frac{A}{K_z}}$, $A = \Delta_0^2 K_z = (11{,}6)^2 10^{-18} m^2\cdot3{,}53\cdot10^4\,J/m^3 =$

$= 4750\cdot10^{-11}\,J/m = 4{,}75$ pJ/m, это хорошо сов. с A(Ruben) $= J/a = 4{,}77$ pJ/m

Refined $v_- = 5{,}014\frac{km}{s}\frac{(\sqrt{1+\lambda}-1)}{\sqrt{\lambda}} = 5{,}014\cdot\frac{33{,}934}{34{,}920}\frac{km}{s} = 4{,}872\left(\frac{km}{s}\right)$

(23/IX) Revised 1D-DW manuscript (Sec. 5) и новая ссылка to co-authors

(24/IX) Нагрев. New res. tasks on dyl. nanowires – MCIU-proposal – sent to Oksana

(25/IX) Написание 1D-DW paper. переписка с Ricardo

(26/IX) Л. с Ю. в downtown, отправили 2 посылки (14kg + D. (Л., мама) и 13 kg (мама, муж. комбайн 140€) – 48 €. Сорвали с крыш. задний bumper – La Concha

Escaneado con CamScanner

(27/IX) С Л. и Ю. – Lecaka, Navarra и „Carrefour" Oiartzun. Обед в City Walk, Lintzerin.

(28/IX) Changes to Res Tasks – Nanowire proposal. Fizz к Usr/Sk. Review Gate S.K. Kim. Charilaou et al. Em. el. fields in nanowires. PRL 121, 097202 (2018): Emergent el. field upon ghanneling BP in nanowire $E_m \sim 0.5$ MV/m – giant.

(30/IX) Орган Антхони на ОНИ. Invoice 1100 $ – APL Mat. Участие в 1st day Conf. „Passion for knowledge". Домашка с Ю. и Ж. Задача. Обед в „Laporta" с Л., Ю., Ж. и Ирина в party вечер с Ю., Ж.

(1/X) Entered to GAUR и wagmeau отчет Ricardo. Лит. обзор wires и nanocarbon wire – proposal. E-mail to Verba с PRB-helicity MS.

(2/X) Поездка в Киев (с сестрой 1,500 €). Разбор нац. Nanowire proposal. Новая Орс. inv. capabilities / sw cylinder / Methodologia и Revised Memoria. (Sec. 2).

(3/X) Revising Introduction к 1D DW paper. Title of Nanowire proposal.

(4/X) Публикация UCIU об отчете по „Skyrmion" project Nov. 14, 2019, in Madrid.

→ S.K. Kim, Y. Tserkovnyak, APL 111, 032401 (2017). Fast vortex oscillations in ferrimagnetic disk near angular momentum compensation point.

Gyrofreq. ω_G – AFM

Ур-е для ω_G in ferromagnet (при $S_1 \approx S_2$) $\gg \omega_G$ (ferromagnet), ~0.1 GHz. (~10 GHz) compens. point

$\omega_G(FM) = \mathcal{H}/G$, $\omega_G(AFM) = |G \to 0| = \pm\omega_{AFM}$, $\omega_{AFM} = \sqrt{\frac{\varkappa}{M_v}}$, где M_v is the vortex mass. Оценка $\omega_{AFM}/2\pi = 30$ GHz (!) для $Co_{1-x}Tb_x$.

$M_v = 2\pi\rho L \ln(R/R_c)$, ρ (mass density) $= \frac{\hbar^2}{2Jz a^3}$, $J = 35$ meV, $a = 0.4$ nm, Z-G.

$\rho \approx \frac{\hbar^2}{12 \cdot J a^3} = \frac{(1.05 \cdot 10^{-34}\ J\cdot s)^2}{12\,(0.4\ nm)^3\ 35\ meV} = \frac{(1.05)^2 \cdot 10^{-68}\ J^2 s^2}{12 \cdot (4\cdot 10^{-10}\ m)^3\ 35 \cdot 10^{-3} \cdot 10^{-19} \cdot 1.6\ J} =$

$= \frac{1.1 \cdot 10^{-68}\ J^2 s^2}{12 \cdot 64 \cdot 10^{-30}\ m^3\ 35 \cdot 1.6 \cdot 10^{-22}\ J} = \frac{1.1}{12 \cdot 64 \cdot 56} \frac{10^{-68}}{10^{-52}} \frac{J s^2}{m^3} = 2.3 \cdot 10^{-5} \cdot 10^{-16} \frac{J s^2}{m^3}$

$\rho \approx 2.3 \cdot 10^{-21} \frac{J s^2}{m^3} = |J = kg \cdot m^2/s^2| = 2.3 \cdot 10^{-21}\ kg/m$.

$M_v = 2\pi\rho \ln\left(\frac{R}{R_c}\right) \cdot L = \left|\begin{matrix} L = 10\ nm \\ R/R_c \approx 10 \end{matrix}\right| = 6 \cdot 10\ nm\ 2.3 \cdot 2.3 \cdot 10^{-21} \frac{kg}{m} = 6 \cdot (2.3)^2 \cdot 10^{-29}\ kg$

$M_v = 32 \cdot 10^{-29}\ kg = 3.2 \cdot 10^{-28}\ kg = 3.2 \cdot 10^{-25}\ g$ (много занижена)

Нормальная $M_v \sim 10^{-19}$ g (dynamic Gus. Siekep. 2015), $M_v(DW) \sim 10^{-21}$ g.

T.e. Kim-Tserkovnyak (APL'17) занижают M_v по крайней мере в 10^4 раз, что влияет к завышению $\omega_{AFM} = \omega_G$ Частота $\sim \sqrt{\frac{\varkappa}{M_v}}$, в $\sqrt{10^4} = 100$ раз (!) Нормальная $\omega_{AFM}/2\pi \approx 30$ GHz/100 ≈ 0.3 GHz такого не наблюдали как в FM.

Eq. $\rho = \frac{\hbar^2}{2Jz a^3}$ где mass density будет из formal magnon mass $M_m = \frac{\hbar^2}{Ja^2}$.

$M_v = 2\pi L \ln(R/R_c)\,\rho = 2\pi L \ln\frac{R}{R_c} \cdot \frac{\hbar^2}{2Jz a^3} = \left|Jz \to J\right| = 2\pi \frac{L}{a} \ln\left(\frac{R}{R_c}\right) \cdot M_m$

Занижение M_v (vortex mass) at least in 10^4 times – полное непонимание → формальное решение Задачи о gyrotropic frequency in AFM (comp. ferri magnet)

Оценка $\omega_G(AFM)$ может быть верна только если M_v – static AFM mass. Dynamic $M_v(AFM)$ значительно больше (!?). А предполагаемая dyn. $M_v(FM) \approx M_v(AFM)$

Сом. Ivanov Low Temp. Phys. 2019 Energy density of AFM

$W = \frac{1}{2} E_{ex} \vec{m}^2 + \frac{1}{2} A (\nabla \vec{l})^2 + \omega_a(\vec{l}) - \vec{M}\vec{H} - \frac{1}{2}\vec{M}\vec{H}_m$ AFM $S_1 = S_2 = S$, $\omega_{ex} = \frac{E_{ex}}{2\hbar S}$

Lagrangian AFM $L = T - W$, $T = \frac{\hbar S}{a\omega_{ex}} \int d^3\vec{r} \left(\frac{\partial \vec{l}}{\partial t}\right)^2$, $W = \int d^3\vec{r} \left[\frac{1}{2} A (\nabla \vec{l})^2 + \dots\right]$

$\vec{l}(\theta, \varphi) \to \left(\frac{\partial \vec{l}}{\partial x}\right)^2 = (\dot{\theta})^2 + \sin^2\theta (\dot{\varphi})^2$. Mass density $T = \frac{1}{2}\int d^3\vec{r}\, \rho_M (\dot{\vec{l}})^2$, $\rho_M = \frac{2\hbar S}{\omega_{ex}}$

$\rho_M = \frac{4\hbar^2 S^2}{E_{ex}}$, $E_{ex} \sim J$ (AFM intersublattice exch. integral), SW velocity $c^2 = A\omega_{ex}/\hbar^2 S$.

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Ivanov, PHT 1995: $W = M_0^2 \int dV \left[\frac{1}{2} \delta \vec{m}^2 + \frac{1}{2} \alpha (\nabla \vec{l})^2 + w_a(\vec{l}) + \dots \right]$

$M_0 = g\mu_B S/v_0$, v_0 – unit cell volume. $\frac{1}{2} M_0^2 \delta = \frac{1}{2} E_{ex}$, $\frac{1}{2} \alpha M_0^2 = \frac{1}{2} A$.

$c = \frac{1}{2} \gamma M_0 \sqrt{\alpha \delta}$. $\delta = \frac{16 J a A_0}{(g\mu_B)^2} = \frac{16 J a}{(g\mu_B)^2} \frac{v_0}{2a} = \frac{8 J v_0}{(g\mu_B)^2}$

$E_{ex} = M_0^2 \delta = (g\mu_B S)^2 \frac{8 J v_0}{(g\mu_B)^2} = 8 J v_0 S^2$, J из $\mathcal{H}_{ex} = J \sum_l \vec{S}_l \vec{S}_{l+1}$

$\mathcal{S}_M = \frac{4\hbar^2 S^2}{E_{ex}} = \frac{4\hbar^2 S^2}{8 J v_0 S^2} = \frac{1}{2} \frac{\hbar^2}{J v_0}$, $v_0 = a^3$ для simple cubic lattice.

$[\mathcal{S}_M] = kg/m$. Kim-Tserkovnyak 2017 дают $\mathcal{S}_M = \frac{\hbar^2}{2 J z a^3}$, z – число ближ. соседей ($z=6$)

Т.е. при $J \to (Jz)$, $v_0 = a^3$ eq. Ivanov '2019, Ivanov '95 и Kim-Tserkovnyak '17 совпадают. // Т.е. Tserkovnyak взял формально верную static AF effective mass density. Но M_V (AF) может значительно вырасти из-за dynamics (interaction with SW), static – это никто не учитал.

Переслет $T = \frac{1}{2} \int dV \mathcal{S}_M (\dot{\vec{l}})^2$ в $T = \frac{1}{2} M_V (\dot{\vec{X}})^2$ в eq. Thiele надо проверить. (для AFM)

// Published Ivanov-Galkina-Zaspel PRA 2019 с той же идеей. Там же ω_G eq. vortex mass M_V, которая мала для AFM и $\omega_{G/RT} = 10-20$ GHz.

(5-14/I) Работа над Memoria "Nanowires" proposal и заполнение UPV/EHU Tasks application in Sede electronica. 14/I new appl. was submitted.

(15/I) Л. вернулась из Киева, с собой back (1330 испрат.) 120€ из 1,500€

(16/I – 17/I) Дополн. документы для MCIU „Nanowires" proposal. (Маня 300€)

(18/I) С Julan Gonzalez в пр. резервации для trip в Porto. Л. посещала Л.Д. – 16€ Киев

(19/I – 31/I) Trip с Лорой в Porto (общежитие Зам. 500€ сэкон.)

19/I – изъяли, о проблемах SKYTRON project до 29/XII 2020. переписка с Оксаной (Skyrton project office), Verba (PRB), Ricardo (DW paper), Zelent (review JMMM), S.K. Kim (review paper JOM). Отпуск. решения на Fassbender-ACS Appl. Mat. Vortex ω_G in Cr-irradiated $\mathcal{S}_M$ data. Беседа с Виницем (SW eq. by Sparks с кого можно научиться fitting по A, M_S – решили оставить fitting by eq. Slavin из Kakazei' APL '2004 и chap. manuscript to ACS Nano) и R. Verba (on mean simulations $\mathcal{H}_{res}$ (H)) пояснения fitted M_S, A), финансов. progress по написанию „Dzyan modes" paper из-за дождя. exp. part by Dobrovolskiy и неделимся тем, что мы сделали. С Л. в ресторане на ocean и Duero river. < Л. „Lidl" и „Carrefour" – 31/I.

Ricardo submitted DW dynamics MS to PRB (29/I) – BK13956 Gushanko

(1/II) Progress. и submitted reviewer's port Ramasubramanian-Fassbender 'vortex' ACS Nano.

(2/II) Invitation letters SolSkyMag '2020 to JM. Kosterlitz и 10 inv. speakers на подготовка Skyrton project areas (slides). С Л., Ю. в „Carrefour" оплатили.

(3/II) Ск. мне разо от 15/IX где должно стоять „Chase" bank accounts. R. Verba submitted Helicity MS to PRB (1/II). // Подал Abstract на Trends in Magnetism 2020, 7-11 sept., Cefalu, Italy до 28/II 2020 и JEMS '2020, Lisbon, July 27-31, до 1/III 2020.

(4/II) Skyrmion review. С J. Gonzalez обед рес. „Lukas" (A. Talbot).

Использ. грант „SKYTRON" project (informe intermedio) на Sept. 20, 2018:

Total 30.959,49€ = 20.692,87€ (personal) + 979,57€ (fung.) + 9.287,05€ (visjes/dietas inscr.)

На 25/VIII 2019 (prorroga request) – Total 43.191,72€ (без генеральных).

Расходы в Skyrton project FIS2016 20/IX – 31/XII 2018 (Estado de cuentas 14/XII 2018)

Viajes y dietas: 219,53€ + 898,27€ + 593,19€ = 1710,99€

Расходы Skyrton 1/I – 25/VIII 2019 (viajes/d): 5353,06€ + 35,00€ (poster) = 5388,06€

25/07

Расходы "Skyron" 10/IX 2018 – 25/VII 19: 1710,99 + 5.388,06 = 7.099,05 €

Total expenses расчетное на 25/07/2019 = total (20/09/2018) + расходы (24/09/18 – 25/07/19) = 30.959,49 + 7.099,05 = 38.058,54 €

Хочешь ли в progress report (25/07/2019) я написал 43.191,72 € (?).

Сумма расходов 43.191,72 € = 70.000,00 € – 26.808,28 € (Estado 18/07/2019) формально верна. Но я не могу найти сумму расходов 43.191,72 € суммируя свои расходы personal, fungibles, viajes Jan.2017 – July 2019. // Вероятно upv/ehu добавили свои, скрытые, расходы etc. (~5 k€) to my saldo disponible calculation на 18/07/2019. И не MCIU не дописало ~ 5 k€?

(5/IX) Анализ сумм расходов "Skyron" FIS2016 project.
Моя записка 4/IX была в том что расходы "Informe intermedio" есть на 30/06/2018, а не на 20/09/2018 как я считал.
На 04/11/2019: Fungible – 1014,57 €, Otros gastos (publ.) – 1016,82 €
Viajes/dietas – 20.204,56 €

(5-7/XI) Skyron (slides) report. Переписка с J. Camarero, Оксана, Cristina (UAM/EHU). Order travel Porto.

(8/XI) E-mail Slavin + 6 invited speakers letters. Emus. SolSkyMag. invit.

(9/XI) "Al Campo", Arbi", Carregar с п.,10: 9 invited speakers confirmed.

(11/XI) Беседа с Julian (conf.) и Ricardo (current work on AF dyn.). Сдача Антрон' order flip-Porto. Переписка с Окс. (Klavi) и Elena Prats (MCIU).

(12/XI) Подпис. бумага SKYRON" (14/X MCIU). Переписка SolSkyMag. п. поломка Kiwi 1.D. 12€

(13-16/XI) поездка в Madrid с п. Доклад "Skyron" project 14/XI в MCIU by Julio (вопрос комиссии – ref joint papers ура-ников subprojects?). Беседа с Agustina Окс., Miriam – Ay-Need Skyrmions-manuscript; (?) MFM – более сложное, чем вначале и предполагалось (flower, sk, vortex, SD states). Беседа с P. Pema и X (IMDEA) о St. stability in FM dots. Оксана – предложили добавить analytic part к их simulations.

(17/XI) Invitations speakers SolSkyMag 2020.

(18/XI) Написание Vortex/Skyrm. review для S.K. Kim JOM.

(19/XI) Meeting с Ruben и Ricardo: AF-DW dynamics. Ruben предложил написать short paper on cell size influence on DW width simulations и inertia effects DW dynamics на начальной стадии ~10 ps, H_{ac} (t) of DW motion $\frac{H_{ac}}{0}$ t, t_2 t. до ~ 40 ps, transient DW dynamics involving A-F(FM) DW mass.

(20/XI) → Shen-Tretiakov APL'2019: переписка расчеты/траектории AF и FM-Skyrmions. и ранние на рез. Moon – 1D model Skyrm. motion / Adv. Theory Simul. '2019

→ Wang-Bradaes 'PRL' 2019 "Current-driven dynamics of magnetic Hopfions" – прочитать, изучить, проверить Reg's.

(21-30/XI) Написание Vortex/Skyrmion review – JOM. Переписка SolSkyMag 2020. Переписка с Uroz, Maciej – visit Poznan – Feb. 2020. Отель Madrid и Las Vegas-Окс. 26/XI N. Reyren sem. talk., Skyrm!

(1/XII) Купил авиабилеты BIO → Palma (PMI) Ukeling" и PMI → Berlin "Ryanair" Booked hotel Palma de Mallorca 3/4/XII и 16/17/XII 2020.

(2/XII) Лит. обзор "Magnetic Skyrmions". Review rep. Moon "1D descript. Skyrmions"

(3/XII) Updating Google Scholar и Chess.com sites (photo attached).

(3-4/XII) Написание new paper Vortex/Skyrmion для JOM.

(5/XII) DIPC Seminar A. Hernando "Spin Hall effect": с п. "Lidl". Пос. Kiwi 30 € N.D 9kg, Пенка 7 kg

(6-7/XII) Написание Vortex/Sk review JOM. С п.,10. J. downtown Viko y pintxos.

(8/XII) Закончил Vortex/Sk JOM review и послал его S.K. Kim. Ожидаю ответ.

Contacted Gleb по сист. "Domain modes" MS – no response.

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(9/XII) Вернулся к 3D - solitons (Aug. 2019) и сделал что-то для 3D Skyrmions in thick (opt) multilayers.

• 1) E.G. Tveten, PRL 110, 127208 (2013) – eq. Thiele для AF. Нет gyroforce, но есть small mass term, $M_v \sim 1/J$ – в рамках 1-mode, $J \to \infty$.

• 2) S. Selzer – U. Nowak, PRL 117, 107201 (2016): Inertia-free thermally Driven DW motion in AF. M_{DW} (AF) simulations и наноленовые speculations.

Отсутствует в AF нет tilting DW (moving) from easy x2 plane $\Rightarrow M_{DW}=0$
но Fig. 3 допускает small tilt $\eta_y \ll 1$, small $M_{DW} \sim 1/J$. J – AF exch. integral

Soliton mass $M_v \sim 1/J$ должна быть в 2D-case $\vec{m}(x,y,t)$. Противоречие между Tveten PRL'2013 (не есть 1D-DW AF mass) и Selzer PRL'2016 (нет 1D-DW AF mass)

(10/XII) • 3) L. Shen et al. APL'2019 „ST nano-oscillators based on AF skyrmions"
F-Skyrmion STO: current J is large ($J > J_{c2}$) – no gyrotropic Sk motion, skyrmion is expelled from the dot.
F-Sk oscillator
Steady orbit radius $u_0(J)$ и gyrofrequency $\omega_G(u_0)$ in non-linear regime можно найти u-Φ методом Guo NRL'2014, $u=|s|$, $s=|s|e^{i\Phi}$ отсюда $\dot{u}=0$, $u=u_0(J)$
$\omega_G(u) = \frac{1}{u G(u)} \frac{\partial W}{\partial u}$, damping $d(u)$, ST-term $\gamma(u)$ – спектр. F-Skyrmion motion. ST-force $\vec{F}_{ST}$ будет для radial vortex reference layer
$\omega_G(u)$, $G(u)$, $d(u)$, $\gamma(u)$ – зависят от moving Sk-ansatz $\vec{m}(\vec{s},t) = \vec{m}[\vec{s}, X(t)]$.
simplest – BP ansatz $f(z,s) = \frac{i}{2}(z-s)$. Тогда u/u $J \to 0$, $u_0 \to 0$ дает конечное $\omega_G(u_0 \to 0) = \omega_G^0$ из Guo IEEE Magn. Lett. '17.
Для Sk-ansatz $\vec{m}[\vec{s}, \vec{X}(t)]$ можно брать rigid DeBonte ansatz $\operatorname{tg}\frac{\Theta}{2} = \frac{R_s}{s} e^{-\frac{(s-R_s)}{\Delta}}$, где $s \to |\vec{s}-\vec{s}|$ для moving Sk $\vec{s}(t) = \vec{X}(t)/R$.

(11/XII) AF-Skyrmion: AF-Sk oscillator frequency определяется AF-Sk mass $M_v(AF)$, т.к. $\vec{G}=0$ и damping
$M_v(AF) \approx 8.4 \cdot 10^{-24}$ g очень мала (parameters by Shen et al.), что дает $f_G(AF) = 10-20$ GHz
частоты. Sk-radius AF-Sk $R_s = \frac{\Delta}{\sqrt{2}(1-D/D_c)}$ в разных направлениях, т.к. $\vec{G}=0$
eq. для F-Sk и только u/u $D \to D_c - 0$.

•• 4) F. García-Sanchez et al. New J. Ph. 2016, Ferromagnetic Sk-STO, показано что F-Sk $\omega_G(2\pi)$ мала ≤ 100 MHz, orbit radius всегда $R_0 \approx R$ (Sk-trajectory близка к dot edge), $J_{c1}=0$, но $J_{c2} \neq 0$. Используется vortex $\vec{m}(\vec{r})$ of ref. layer с $\Phi = \varphi + \Phi_0$, где helicity $\Phi_0 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ – обычный и radial vortices.

• 5) S. Zhang et al. New J. Phys. 17, 023061 (2015) „Current-induced FM-Skyrmion oscillator"
Вторая, first paper по F-Sk STO. Simulations. Возможно as García-Sanchez – Bloch Skyrmion и $\vec{m}_{ref} \| \hat{z}$ – uniform (!). f_G up to 1.4 GHz, $\Delta f \sim 1$ MHz.

• 6) W. Jiang et al. IEEE Magn. Lett. 9, 3102905 (2018) – F-Skyrmion oscillator – simul. Neel-Sk dots. Только ref. layer $\vec{p}(r)$ – radial vortex и large $\alpha_{damp} = 0.3$ берут
и Sk-gyration by SP current J с $f_G = 30-120$ MHz. Orbit radius $R_0 \approx R$ как у García-Sanchez'16.

(12/XII) Посылка с книгой 44€: книга (8,5 kg), т. Ане (8,5 kg), Наталье (7,5 kg).
Comida Navidad с Ириной в „Nam". Гриб сообщил о new exp. Advance на thick $L = 80-350$ nm Py dots ($R = 250$ nm). Посчитал $\omega_G(n,L)$ и посчитал гриду $n = 1,2,3,4$
$\omega(n,L)$ имеет min as function of L or $\beta = L/R$.

(13/XII) Skyrmion spin-torque oscillator. Sk – STNO
Сделано Guo NRL'2014 $\dot{\Phi} - \omega_G(u) = d(u)\frac{\dot{u}}{u}$, $\dot{u} = [\gamma(u) - d(u)\dot{\Phi}]$ и система УД для Soliton core position $s = u e^{i\Phi}$, $u = |s|$, $s = (X+iY)/R$. $u(t)$ – soliton orbit radius.
Steady motion $\dot{u}=0 \to \gamma(u_0) = d(u_0)\dot{\Phi} = d(u_0)\omega_G(u_0) \to$ корень $u_0(J)$, $\omega_G(u) = \frac{1}{R^2 u G(u)} \frac{\partial W(u)}{\partial u}$
u/u $u = u_0(J)$ дает STO frequency $\omega_0(J)$ и J_{c1}, J_{c2} – crit. currents.

Escaneado con CamScanner

Soliton orbit radius & gyration frequency are determined by $W(u)$, $G(u)$ damping $D(u)$ & spin-torque $\gamma(u) = a(u)/G(u)$ — non-linear functions of $u = |s|$. Gus' NRL'14 исследовали вихрь - STO. // Одно скирмион STO, без функций $W(u)$, $G(u)$, $D(u)$ и $\gamma(u)$ будет грубым + non-diag. damping + reference layer $\vec{p}(\vec{s})$ polarization $\vec{p}$ не обязательно $\vec{p} = p_z \hat{z}$ с $p_z = \text{const}(\vec{s})$.

Garcia-Sanchez'16 и Shen'19 рассмотр. $\vec{p}(\vec{s}) = (\cos\Phi, \sin\Phi, 0)$, $\Phi = \varphi + \Phi_0$ с разными Soliton helicity $\Phi_0 = 0, \pi$ (radial vortex), $\Phi_0 = \pm\pi/2$ (vortex), где уравновешивает вихрь (vortex) core для reference layer.

Skyrmion ST oscillator

Идея: брать simple Skyrmion ansatz с нормами $u_0(r)$, $\omega_0(r)$ для разных ref layer polarization $\vec{p}(\vec{s})$ — для ГМ Skyrmion STO.

Сначала Gus' JPhCS'2011: $F^{\alpha}_{ST} = -\frac{\partial}{\partial x_\alpha}\int dV w_s$, $\frac{\partial w_s}{\partial \vec{m}} = a\,\vec{m}\times\vec{p}$, $a = M_s \oplus J$.

$F^{\alpha}_{ST} = -\int dV \frac{\partial w_s}{\partial x_\alpha} = -\int dV \frac{\partial w_s}{\partial \vec{m}}\cdot\frac{\partial \vec{m}}{\partial x_\alpha} = -\int dV\, a[\vec{m}\times\vec{p}]\cdot\frac{\partial \vec{m}}{\partial x_\alpha} = a\int dV\, \vec{p}\cdot\left[\vec{m}\times\frac{\partial \vec{m}}{\partial x_\alpha}\right]$

$F^{\alpha}_{ST} = aL\int d^2\vec{s}\;\vec{p}(\vec{s})\cdot\left[\vec{m}\times\frac{\partial \vec{m}}{\partial x_\alpha}\right]$, $\alpha = x, y$ — Spin-torque Slonczewski force.

Ранее Gus'2010, 11, 14 использовали $\vec{p} = p_z\hat{z}$ с $p_z = \text{const} = P$.

$\left(\vec{m}\times\frac{\partial \vec{m}}{\partial s}\right)_z = \sin^2\theta\frac{\partial\Phi}{\partial s}$, $f(z) = |f|e^{i\Phi}$, $w = \operatorname{tg}\frac{\theta}{2}e^{i\varphi}$, $\vec{m} = \vec{m}(\theta,\Phi)$.

Для skyrmion $f(z) = w(z)$, т.к. на вихрь - area rule $w = f/|f|$.

26/XII 2013г.: $\left(\vec{m}\times\frac{\partial \vec{m}}{\partial s}\right)_z = \frac{2i}{(1+w\bar{w})^2}\left(w\frac{\partial \bar{w}}{\partial s} - \bar{w}\frac{\partial w}{\partial s}\right) = \frac{2i}{(1+f\bar{f})^2}\left[f\frac{\partial \bar{f}}{\partial s} - \bar{f}\frac{\partial f}{\partial s}\right]$.

Future BP soliton model for skyrmion, $f(z) = \frac{1}{c}e^{i\Phi_0}(z-s)$ — rigid BP skyrmion

$\bar{f} = \frac{1}{c}e^{-i\Phi_0}(\bar{z}-\bar{s})$, $\frac{\partial \bar{f}}{\partial s} = 0$, $\left[f\frac{\partial \bar{f}}{\partial s} - \bar{f}\frac{\partial f}{\partial s}\right] = -\bar{f}\frac{\partial}{\partial s}\left[\frac{1}{c}e^{i\Phi_0}(z-s)\right] =$

$= -\frac{1}{c}e^{-i\Phi_0}(\bar{z}-\bar{s})\frac{1}{c}e^{i\Phi_0}(-1) = \frac{1}{c^2}(\bar{z}-\bar{s})$, $\left(\vec{m}\times\frac{\partial \vec{m}}{\partial s}\right)_z = \frac{2i}{(1+f\bar{f})^2}\frac{(\bar{z}-\bar{s})}{c^2}$

Имм $\left(\vec{m}\times\frac{\partial \vec{m}}{\partial s}\right)_z = \sin^2\theta\frac{\partial\Phi}{\partial s}$, $\Phi = \operatorname{Im}(\ln f)$, $\frac{\partial\Phi}{\partial s} \neq \operatorname{Im}\frac{1}{f}\frac{\partial f}{\partial s} = \frac{1}{|f|^2}\operatorname{Im}\left(\bar{f}\frac{\partial f}{\partial s}\right)$ неверно, надо $\frac{\partial\Phi}{\partial s} = \frac{\partial}{\partial s}\operatorname{Im}(\ln f)$

$\left(\vec{m}\times\frac{\partial \vec{m}}{\partial s}\right)_z = \frac{4|f|^2}{(1+|f|^2)^2}\frac{1}{|f|^2}\operatorname{Im}\left(\bar{f}\frac{\partial f}{\partial s}\right)$ — верно.

$\frac{\partial}{\partial s} = \frac{1}{2}\left(\frac{\partial}{\partial s_x} - i\frac{\partial}{\partial s_y}\right)$ и $\frac{\partial}{\partial s_x}$, $\frac{\partial}{\partial s_y}$ по отдельности. $s = s_x + is_y$, $\bar{s} = s_x - is_y$

$\left(\vec{m}\times\frac{\partial \vec{m}}{\partial s_x}\right)_z = \frac{2i}{(1+f\bar{f})^2}\left[f\frac{\partial \bar{f}}{\partial s_x} - \bar{f}\frac{\partial f}{\partial s_x}\right] = (\cdot)[\cdots]$, $f = \frac{1}{c}e^{i\Phi_0}(z-s)$, $\bar{f} = \frac{1}{c}e^{-i\Phi_0}(\bar{z}-\bar{s})$

$\frac{\partial \bar{f}}{\partial s_x} = \frac{1}{c}e^{-i\Phi_0}(-1)\frac{\partial \bar{s}}{\partial s_x} = -\frac{1}{c}e^{-i\Phi_0}$, $\frac{\partial f}{\partial s_x} = \frac{1}{c}e^{i\Phi_0}(+1)\frac{\partial s}{\partial s_x} = \frac{1}{c}e^{i\Phi_0}(-1)$.

$\left(\vec{m}\times\frac{\partial \vec{m}}{\partial s_x}\right)_z = \frac{2i}{(1+f\bar{f})^2}\left[\frac{1}{c}e^{i\Phi_0}(z-s)\frac{(-1)}{c}e^{-i\Phi_0} - \frac{1}{c}e^{-i\Phi_0}(\bar{z}-\bar{s})\frac{(-1)}{c}e^{i\Phi_0}\right] =$

$= (\cdot)\frac{1}{c^2}\left[-(z-s)+(\bar{z}-\bar{s})\right] = \frac{1}{c^2}\frac{2i}{(1+f\bar{f})^2}\left[-2iy + 2i s_y\right] = \frac{4}{c^2}\frac{1}{(1+f\bar{f})^2}(y-s_y)$

$\left(\vec{m}\times\frac{\partial \vec{m}}{\partial s}\right)_z = \frac{1}{2}\left(\vec{m}\times\frac{\partial \vec{m}}{\partial s_x}\right)_z - \frac{i}{2}\left(\vec{m}\times\frac{\partial \vec{m}}{\partial s_y}\right)_z = \frac{2i}{c^2}\frac{1}{(1+f\bar{f})^2}(\bar{z}-\bar{s}) =$

$= \frac{2i}{c^2}\frac{1}{(\cdot)^2}\left[(x-s_x) + i(y-s_y)\right] = \frac{2}{c^2}\frac{1}{(\cdot)^2}\left[+(y-s_y) + i(x-s_x)\right]$

$\left(\vec{m}\times\frac{\partial \vec{m}}{\partial s_x}\right)_z = \frac{4}{c^2}\frac{1}{(1+f\bar{f})^2}(y-s_y)$, $\left(\vec{m}\times\frac{\partial \vec{m}}{\partial s_y}\right)_z = -\frac{4}{c^2}\frac{1}{(1+f\bar{f})^2}(x-s_x)$

$B(s,\bar{s}) = \int_{\text{dot}} d^2\vec{s}\;\frac{(\bar{z}-\bar{s})}{(1+f\bar{f})^2} = \left|f\bar{f} = \frac{1}{c^2}|z-s|^2\right| = \int_{\text{dot}} dz\,d\bar{z}\,(\bar{z}-\bar{s})\,F(|z-s|)$. $s \to se^{i\varphi_s}$, $\bar{s}\to\bar{s}e^{-i\varphi_s}$

$B(s,\bar{s}) = B(\bar{s},s) = \int dz\,d\bar{z}\,(\bar{z}-s)F(|z-s|) = \int dz\int d\bar{z}\,(\bar{z}-s)F(|z-\bar{s}|) = \left|\begin{smallmatrix}z\to\bar{z}\\ \bar{z}\to z\end{smallmatrix}\right| =$

$= \int d\bar{z}\,dz\,(z-s)F(|z-\bar{s}|) = \int dz\,d\bar{z}\,(z-s)F(|z-s|)$, $B(s,\bar{s}) = \bar{s}\Lambda(|s|)$, $\Lambda(0)\neq 0$.

$B(|s|\bar{s}) = s\Lambda(|s|)$. $\bar{B}(s,\bar{s}) = B(\bar{s},s)$, Тогда $B(s,\bar{s}) = E(\bar{s})\Lambda(|s|)$,

$\bar{B}(s,\bar{s}) = \bar{E}(\bar{s})\Lambda = E(s)\Lambda(|s|)$ — но можно показать что $E(s) = \bar{s}$.

$(\bar{z}-\bar{s}) \sim \bar{f}$. $\frac{\bar{f}}{(1+f\bar{f})^2} = -\frac{\partial}{\partial s}\frac{1}{(1+f\bar{f})} \sim \frac{\partial}{\partial z}\frac{1}{(1+f\bar{f})} = \frac{\partial}{\partial z}\ln(1+f\bar{f}) \sim \frac{1}{f}\frac{\partial}{\partial f}\ln(1+f\bar{f})$

$c^2(1+f\bar{f}) = c^2 + |z-s|^2 = c^2 + s^2 + s^2 - 2rs\cos(\varphi-\varphi_s)$

Escaneado con CamScanner

$F_{ST}^{-} \sim \int d^2\vec{s}\, \frac{(\vec{z}-\vec{s})}{dot\,(1+|f|^2)^2} = B(s\bar{s}) = \bar{s}\Lambda(|s|)$, $F_{ST}^{x} \sim \int ds\,s\,d\varphi \frac{(s\sin\varphi - s\sin\varphi_s)}{dot\,[c^2+s^2+s^2-2ss\cos(\varphi-\varphi_s)]^2} \sim$

$\sim \int d\varphi' \frac{[s\sin(\varphi'+\varphi_s)-s\sin\varphi_s]}{[c^2+s^2+s^2-2ss\cos\varphi']^2} = \int_0^{2\pi} d\varphi' \frac{[s\cos\varphi'-s]\sin\varphi_s}{[c^2+s^2+s^2-2ss\cos\varphi']^2} = \sin\varphi_s \times$

$\times \int_0^{2\pi} d\varphi \frac{(s\cos\varphi - s)}{[c^2+s^2+s^2-2ss\cos\varphi]^2} = \left| \frac{\partial}{\partial s}(c^2+s^2+s^2-2ss\cos\varphi) = 2s-2s\cos\varphi \right| =$

$= \sin\varphi_s \left(+\frac{\partial}{\partial s}\right)\frac{1}{2}\int_0^{2\pi} \frac{d\varphi}{[s^2+c^2+s^2-2ss\cos\varphi]}$; $\int_0^{2\pi} \frac{d\varphi}{(1\pm a\cos\varphi)} = \frac{2\pi}{\sqrt{1-a^2}}$, $a = \frac{2s}{c^2+s^2+s^2}$

$\int_0^{2\pi} \frac{d\varphi}{[c^2+s^2+s^2-2ss\cos\varphi]} = \frac{1}{(c^2+s^2+s^2)}\int_0^{2\pi} \frac{d\varphi}{1-a\cos\varphi} = \frac{1}{(c^2+s^2+s^2)} \frac{2\pi}{\sqrt{1-\frac{4s^2s^2}{(c^2+s^2+s^2)^2}}} =$

$= \frac{2\pi}{\sqrt{(c^2+s^2+s^2)^2-4s^2s^2}}$. $F_{ST}^{x} \sim \int_0^R ds\,s\sin\varphi_s \frac{1}{2}\frac{\partial}{\partial s}\frac{2\pi}{\sqrt{(c^2+s^2+s^2)^2-4s^2s^2}} = \left| \frac{\partial}{\partial s} = 2s\frac{\partial}{\partial(s^2)},\ s^2=t \right| =$

$= s\cdot\sin\varphi_s \frac{\partial}{\partial t}\cdot 2\pi \int_0^R ds\,s \frac{1}{\sqrt{(c^2+s^2+t)^2-4s^2t}} = \left| s^2=x,\ 2sds=dx \right| = \pi s\sin\varphi_s \frac{\partial}{\partial t}\int_0^{R^2+1} dx \frac{1}{\sqrt{(c^2+x+t)^2-4xt}}$

$(c^2+x+t)^2-4xt = c^4+2c^2(x+t)+(x+t)^2-4xt = x^2+x(2c^2-2t)+c^4+2c^2t+t^2$

$F_{ST}^{x} \sim \pi s\sin\varphi_s \frac{\partial}{\partial t}\int_0^1 dx \frac{1}{\sqrt{x^2+2(c^2-t)x+(c^2+t)^2}} = s\sin\varphi_s\,\psi(t)$ – вычисление аналитически

$t = s^2$

т.к. дано $\vec{P} = P_z\hat{z}$, $P_z = const$ $\vec{F}_{ST}(\vec{s})$ вычислена аналитически для rigid BP model.



пусть $s \le 1-c$, иначе s_k - edge $m_z = 0$ касается dot edge → исчезнет rigid s_k-profile.

s_k-energy [Gus'IEJmagJett'12] $W = A(\nabla m_\alpha)^2 + W_0 - Km_z^2 + W_m$

Thin dot $W_m = 2\pi M_s^2 m_z^2$ и $K \to K - 2\pi M_s^2 = 2\pi M_s^2(Q-1) = K_{eff}$. Граница $Q \approx 1$ для BP – s_k

Для BP ansatz дает DMI вместо $A \to A' = A - \frac{1}{2}|D|R_c$

s_k-energy density $W = A'(\nabla m_\alpha)^2 - Km_z^2$, $W_{ex} = \frac{8A'}{(1+|f|^2)^2}\left|\frac{\partial f}{\partial z}\right|^2 \frac{1}{R^2} =$

$= \frac{8A'}{R^2}\frac{1}{(1+|f|^2)^2}\frac{1}{c^2} = \frac{8A'}{R^2c^2}\frac{1}{(1+|f|^2)^2}$, $c = R_c/R$ – core radius.

$m_z^2 = \cos^2\theta = 1-\sin^2\theta$, $\sin^2\theta = \frac{4|f|^2}{(1+|f|^2)^2}$

$W(s) = \int dV\, W$, $W = W_{ex}+W_a$, $W(s) = L\int d^2\vec{s}\left[\frac{8A'}{c^2R^2}\frac{1}{(1+|f|^2)^2} + K + K\frac{4|f|^2}{(1+|f|^2)^2}\right]$

чтобы нечего $K = 0$.

$W(s) = \frac{8A'L}{c^2R^2}\int d^2\vec{s}\,\frac{1}{(1+|f|^2)^2}$ Надо $\frac{\partial W(s)}{\partial s}$, $s = |s| = u$ здесь.

$c^2+c^2|f|^2 = c^2+|z-s|^2 = c^2+s^2+s^2-2ss\cos(\varphi-\varphi_s)$.

$\frac{1}{(1+|f|^2)^2} = \left[\frac{1}{c^2+s^2+s^2-2ss\cos\varphi}\right]^2 = -\frac{1}{2c}\frac{\partial}{\partial c}\frac{1}{[c^2+s^2+s^2-2ss\cos\varphi]}$

$W(s) = (\cdot)\int ds\,s\int d\varphi \frac{1}{(1+|f|^2)^2} = (\cdot)\frac{(-1)}{2c}\frac{\partial}{\partial c}\int ds\,s\int_0^{2\pi} d\varphi \frac{1}{[c^2+s^2+s^2-2ss\cos\varphi]} = (\cdot)\frac{(-1)}{2c}\frac{\partial}{\partial c}\int ds\,s \times$

$\times \frac{2\pi}{\sqrt{(c^2+s^2+s^2)^2-4s^2s^2}} = (\cdot)\frac{(-1)}{2c}\frac{\partial}{\partial c}\pi\int_0^1 dx \frac{1}{\sqrt{(c^2+x+s^2)^2-4xs^2}}$ – считается аналитически

Гировектор $|G| = \frac{M_s L}{\gamma}\int d^2\vec{s}\,\frac{4}{(1+|f|^2)}\left|\frac{\partial f}{\partial z}\right|^2 = \frac{M_s L}{\gamma}\cdot\frac{4}{c^2}\int d^2\vec{s}\,\frac{1}{(1+|f|^2)^2} = \frac{M_s L\cdot 4}{\gamma c^2}\frac{c^2R^2}{8A'L}W_{ex}$, $W_{ex} \to W$

$|G| = \frac{1}{2}\frac{M_s}{\gamma}\cdot\frac{R^2}{A'}W(s)$.

Damping $D_{\alpha\beta} = -\alpha_G\frac{M_s}{\gamma}L\int d^2\vec{s}\left(\frac{\partial\vec{m}}{\partial s_\alpha}\cdot\frac{\partial\vec{m}}{\partial s_\beta}\right) = -\alpha_G\frac{M_s L}{\gamma}\int d^2\vec{s}\left[\frac{\partial\theta}{\partial s_\alpha}\frac{\partial\theta}{\partial s_\beta} + \sin^2\theta\frac{\partial\varphi}{\partial s_\alpha}\frac{\partial\varphi}{\partial s_\beta}\right]$

Для rigid model $\frac{\partial F}{\partial s_\alpha} = -\frac{\partial F}{\partial x_d}$, т.к. $x_\alpha \to (x_\alpha - s_\alpha)$. $D_{xx}+D_{yy} \sim \left[\frac{\partial\theta}{\partial s_x}\frac{\partial\theta}{\partial s_x} + \frac{\partial\theta}{\partial s_y}\frac{\partial\theta}{\partial s_y} + \right.$

$\left. + \sin^2\theta\left(\frac{\partial\varphi}{\partial s_x}\right)^2 + \sin^2\theta\left(\frac{\partial\varphi}{\partial s_y}\right)^2\right] = \left[\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2 + \sin^2\theta\left(\frac{\partial\varphi}{\partial x}\right)^2 + \sin^2\theta\left(\frac{\partial\varphi}{\partial y}\right)^2\right] =$

$= \left[(\nabla\theta)^2 + \sin^2\theta(\nabla\varphi)^2\right] = (\nabla m_\alpha)^2 \sim W_{ex}$. $D_{xx}+D_{yy} = -\alpha_G\frac{M_s L}{\gamma}\int d^2\vec{s}\left[(\nabla m_\alpha)^2\right] = -\alpha_G\frac{M_s L}{\gamma A'}W_{ex}$

$W_{ex} = \int A'(\nabla m_\alpha)^2 d^2\vec{s}$. $D_{xx} = D_{yy} = -\alpha_G\frac{M_s L}{2\gamma A'}W_{ex}(s)$, $W_{ex} \to W$.

$w = \operatorname{tg}\frac{\theta}{2}e^{i\varphi}$, $|f| = |w| = \operatorname{tg}\frac{\theta}{2}$, $\theta = 2\operatorname{arctg}|f|$, $\frac{\partial\theta}{\partial x_\alpha} = 2\frac{1}{1+|f|^2}\frac{\partial|f|}{\partial x_\alpha}$, $\frac{\partial|f|}{\partial x_\alpha} = \frac{\partial}{\partial x_\alpha}\sqrt{f\bar{f}} = \sqrt{\bar{f}}\frac{\partial\sqrt{f}}{\partial x_\alpha} +$

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$$\partial_x\sqrt{|f|^2}=\partial_x\sqrt{f\bar f}=\frac{1}{2\sqrt{f\bar f}}\left(\partial_x f\cdot\bar f+f\cdot\partial_x\bar f\right)=\frac{\bar f}{2\sqrt{f\bar f}}\partial_x f+\frac{f}{2\sqrt{f\bar f}}\partial_x\bar f=\frac{2}{|f|}\left[\bar f\frac{\partial f}{\partial x}+f\frac{\partial\bar f}{\partial x}\right]$$

$$\frac{\partial}{\partial x}\frac{4}{(1+f\bar f)^2}=\frac{\partial}{\partial x}\frac{4}{(1+|f|^2)^2}=\frac{16}{|f|^2(1+|f|^2)^2}\left[\bar f\frac{\partial f}{\partial x}+f\frac{\partial\bar f}{\partial x}\right]$$

$$\frac{\partial}{\partial x}\frac{2f}{|f|}=\frac{2}{|f|}\left[\frac{\partial f}{\partial x}+f\frac{\partial}{\partial x}\frac{f}{|f|}\right]$$

Через $x_0\to\frac{s}{2}$

$$\frac{\partial}{\partial s}\frac{2f}{|f|}=\frac{2}{|f|}\left[\frac{\partial f}{\partial s}+f\frac{\partial}{\partial s}\frac{f}{|f|}\right]=\frac{2f}{|f|}\cdot\frac{\partial}{\partial s}\frac{|f|}{f}=\frac{2f}{|f|}\cdot\left[\frac{\partial|f|}{\partial s}\frac{f}{f^2}-\frac{1}{f}\frac{\partial f}{\partial s}\right]=\frac{2}{|f|}\frac{\partial|f|}{\partial s}-\frac{2}{f}\frac{\partial f}{\partial s}$$

$$\frac{\partial}{\partial\bar s}\frac{2f}{|f|}=\frac{2}{|f|}\left[f\frac{\partial}{\partial\bar s}\frac{f}{|f|}+\frac{\partial f}{\partial\bar s}\right]=\frac{2f}{|f|}\cdot\frac{\partial}{\partial\bar s}\frac{|f|}{f}=\frac{2f}{|f|}\cdot\left[\frac{\partial|f|}{\partial\bar s}\frac{f}{f^2}-\frac{1}{f}\frac{\partial f}{\partial\bar s}\right]=\frac{2}{|f|}\frac{\partial|f|}{\partial\bar s}-\frac{2}{f}\frac{\partial f}{\partial\bar s}$$

$$\frac{\partial\vec m}{\partial s_x}\cdot\frac{\partial\vec m}{\partial s_y}\cdot\frac{\partial}{\partial s}=\frac{1}{2}\left(\frac{\partial}{\partial s_x}-i\frac{\partial}{\partial s_y}\right),\quad\frac{\partial}{\partial\bar s}=\frac{1}{2}\left(\frac{\partial}{\partial s_x}+i\frac{\partial}{\partial s_y}\right),\quad\frac{\partial}{\partial s_x}=\frac{\partial}{\partial s}+\frac{\partial}{\partial\bar s},\quad\frac{\partial}{\partial s_y}=\left[\frac{\partial}{\partial s}-\frac{\partial}{\partial\bar s}\right]\frac{1}{i}$$

$$\left(\frac{\partial\vec m}{\partial s}+\frac{\partial\vec m}{\partial\bar s}\right)\cdot\frac{1}{i}\left(\frac{\partial\vec m}{\partial s}-\frac{\partial\vec m}{\partial\bar s}\right)=\frac{1}{i}\left[\left(\frac{\partial\vec m}{\partial s}\right)^2-\left(\frac{\partial\vec m}{\partial\bar s}\right)^2\right]=-2\frac{1}{i}\left[\left(\frac{\partial\vec m}{\partial s}\right)^2-\left(\frac{\partial\vec m}{\partial\bar s}\right)^2\right]=$$

$$=-2\,\mathrm{Im}\left(\frac{\partial\vec m}{\partial s}\right)^2\cdot\quad\frac{\partial\vec m}{\partial\bar s}-?\quad m_x+im_y=\frac{2f}{1+f\bar f},\quad m_z=\frac{1-f\bar f}{1+f\bar f}.$$

$$\vec m=m_-\hat e_-+m_+\hat e_++m_z\hat z,\quad\hat e_\pm=\frac{1}{\sqrt2}(\hat x\mp i\hat y)$$

$$\vec m=m_-\frac{1}{\sqrt2}(\hat x+i\hat y)+m_+\frac{1}{\sqrt2}(\hat x-i\hat y)+\hat z m_z,\quad m_x=\frac{1}{\sqrt2}(m_-+m_+),\quad m_y=\frac{i}{\sqrt2}(m_--m_+)$$

$$m_x+im_y=\frac{1}{\sqrt2}(m_-+m_+)-\frac{1}{\sqrt2}(m_--m_+)=\sqrt2\,m_+\cdot\quad m_x-im_y=\sqrt2\,m_-\cdot\quad 2m_+m_-+m_z^2=1$$

$$(\vec m_1\cdot\vec m_2)=m_{+1}m_{-2}+m_{-1}m_{+2}+m_{z1}m_{z2}.\quad\vec m=(m_+,m_-,m_z)$$

$$m_+=\frac{1}{\sqrt2}(m_x+im_y)=\frac{1}{\sqrt2}\frac{2f}{1+f\bar f},\quad m_-=\frac{1}{\sqrt2}(m_x-im_y)=\bar m_+=\frac{1}{\sqrt2}\frac{2\bar f}{1+f\bar f}.\quad\frac{\partial\vec m}{\partial s}-?$$

$$\frac{\partial m_+}{\partial s}=\sqrt2\frac{\partial}{\partial s}\frac{f}{1+f\bar f}=\sqrt2\frac{\partial f/\partial s\,(1+f\bar f)-f\,\partial_s(f\bar f)}{(1+f\bar f)^2}=\sqrt2\frac{\partial f/\partial s}{(1+f\bar f)^2},\quad\text{если }\frac{\partial\bar f}{\partial s}=0$$

$$\frac{\partial m_-}{\partial s}=\sqrt2\frac{\partial}{\partial s}\frac{\bar f}{1+f\bar f}=\sqrt2\,\bar f\frac{(-1)}{(1+f\bar f)^2}\frac{\partial}{\partial s}(f\bar f)=-\sqrt2\frac{\bar f^2}{(1+f\bar f)^2}\frac{\partial f}{\partial s}$$

$$\frac{\partial m_z}{\partial s}=\frac{\partial}{\partial s}\frac{(1-f\bar f)}{(1+f\bar f)}=\frac{1}{(1+f\bar f)^2}\left[\frac{\partial}{\partial s}(1-f\bar f)(1+f\bar f)-(1-f\bar f)\frac{\partial}{\partial s}(1+f\bar f)\right]=$$

$$=\frac{1}{(1+f\bar f)^2}\left[-\bar f\frac{\partial f}{\partial s}(1+f\bar f)-(1-f\bar f)\bar f\frac{\partial f}{\partial s}\right]=\frac{1}{(1+f\bar f)^2}(-2)\bar f\frac{\partial f}{\partial s}$$

$$\left(\frac{\partial\vec m}{\partial s}\right)^2=2\frac{\partial m_+}{\partial s}\frac{\partial m_-}{\partial s}+\left(\frac{\partial m_z}{\partial s}\right)^2=2\sqrt2\frac{\partial f/\partial s}{(1+f\bar f)^2}(-\sqrt2)\bar f^2\frac{\partial f/\partial s}{(1+f\bar f)^2}+\frac{4}{(1+f\bar f)^4}\bar f^2\left(\frac{\partial f}{\partial s}\right)^2\equiv0\ (!)$$

– для жёсткой BP – skyrmion model $f(z)=\frac{1}{c}e^{i\varphi_0}(z-s)$, где $\frac{\partial\bar f}{\partial z}=0$, $\frac{\partial f}{\partial\bar s}=0$.

$\frac{\partial\vec m}{\partial s_x}\cdot\frac{\partial\vec m}{\partial s_y}=-2\,\mathrm{Im}\left(\frac{\partial\vec m}{\partial s}\right)^2=0$, $D_{xy}=0$ – это только для BP-soliton с $\frac{\partial\bar f}{\partial s}=0$.

Для TVA (two-vortex model), $f(z)=i\frac{c}{c}\left[z-\frac{(s+\bar sz^2)}{1+s\bar s}\right]$, $\frac{\partial f}{\partial s}\neq0$, $D_{xy}\neq0$.

Но мы видели, что в общ. дин. skyrmion case $\frac{\partial f}{\partial s}=0$, $\frac{\partial\bar s}{\partial s}=0$?? Это уже только в приближении слабых BP skyrmion, жёсткий BP.

Moving skyrmion ansatz – нарушается, $f=f(z,s,\bar s)$ и вообще $D_{xy}\neq0$.

Все подр. в YD Gus/NRL'14 $i\dot s+\omega_G(u)s=-d(u)s+if(u)s$ изменения и поправки внутри жёсткий BP skyrmion model/ansatz.

$$J(c,s)=\int_0^1\frac{dx}{\sqrt{(c^2+s^2+x)^2-4xs^2}}\qquad SQ(c,s,x)=x^2+2(c^2-s^2)x+(c^2+s^2)^2$$

$$\mathrm{Det}(c,s)=b^2-4ac=4(c^2-s^2)^2-4(c^2+s^2)^2=-16c^2s^2<0\ \text{– нет действ. корней}$$

$$J(c,s)=\int_0^1\frac{1}{\sqrt{ax^2+bx+c}}=\left|a>0,\ b^2<4ac\right|=\mathrm{Arsh}\frac{2x+2(c^2-s^2)}{\sqrt{16c^2s^2}}\Big|_0^1=$$

$$=\mathrm{Arsh}\left[\frac{x+(c^2-s^2)}{2cs}\right]=\mathrm{Arsh}\left[\frac{1+c^2-s^2}{2cs}\right]-\mathrm{Arsh}\left[\frac{c^2-s^2}{2cs}\right],\quad\mathrm{Arsh}(x)=\ln\left[x+\sqrt{x^2+1}\right]$$

$c=R/R$ skyrmion core radius, $s=|\vec s|=u=|\vec x|/R$ sk. displacement from dot center.

Skyrmion дрейф by rigid BP ansatz $f(z)=\frac{1}{c}e^{i\varphi_0}(z-s)$. // $\vec P=P_z\hat z$ дрейф

ST-force $F_{ST}^a(s)=M_s\partial JL\,P_z\int d^2\vec s\left(\vec m\times\frac{\partial\vec m}{\partial s_a}\right)_z$, $\int d^2\vec s\left(\vec m\times\frac{\partial\vec m}{\partial s}\right)_z=|s=s_x+is_y|=$

$$=\int d^2\vec s\,\frac{2i}{c^2}\frac{(\bar z-\bar s)}{(1+f\bar f)^2}\cdot\quad F_{ST}^x=M_s\partial JLP_z\int d^2\vec s\left(\vec m\times\frac{\partial\vec m}{\partial s_x}\right)_z=M_s\partial JLP_z\frac{4}{c^2}\int d^2\vec s\,\frac{(y-s_y)}{(1+f\bar f)^2}=$$

$$=(\dots)\cdot\pi s_y\frac{\partial}{\partial(s^2)}J(c,s)\ \text{ч}.\quad\hat z\times\vec s=\begin{vmatrix}\hat x&\hat y&\hat z\\0&0&1\\s_x&s_y&0\end{vmatrix}=\hat y s_x-\hat x s_y.\quad F_{ST}^y=(\dots)(-1)\pi s_x\frac{\partial}{\partial(s^2)}J(c,s)$$

$$\vec F_{ST}=-\pi(\dots)(\hat z\times\vec s)\frac{\partial}{\partial(s^2)}J(c,s)=a(s)(\hat z\times\vec s),\quad a(s)=-\pi(\dots)\frac{\partial}{\partial(s^2)}J(c,s)\ \text{ч}.\ \text{см 23/XII}$$

Escaneado con CamScanner

Steady SK-core orbit $\chi(u) = d(u)\,\omega_G(u)$, $\frac{a(u)}{|G(u)|} = -\frac{D(u)}{|G(u)|}\frac{1}{R^2 u|G(u)|}\frac{\partial W(u)}{\partial u}$

(M/m) $a(u) = -\frac{D(u)}{R^2(u)\cdot|G(u)|}\frac{\partial W(u)}{\partial u} = -\frac{1}{R^2}\frac{D(u)}{u|G(u)|}\frac{\partial W(u)}{\partial u}$

$J(c,s) = \mathrm{Arsh}\left[\frac{1+c^2-s^2}{2cs}\right] - \mathrm{Arsh}\left[\frac{c^2-s^2}{2cs}\right]$, $\frac{c^2-s^2}{2cs} + \sqrt{\frac{(c^2-s^2)^2}{4c^2s^2}+1} =$

$= \frac{1}{2cs}\left[c^2-s^2+\sqrt{(c^2-s^2)^2+4c^2s^2}\right] = \frac{1}{2cs}\left[c^2-s^2+\sqrt{(c^2+s^2)^2}\right] = \frac{c}{s}$

$\frac{1+c^2-s^2}{2cs} + \sqrt{\frac{(1+c^2-s^2)^2}{4c^2s^2}+1} = \frac{1}{2cs}\left[1+c^2-s^2+\sqrt{(1+c^2-s^2)^2+4c^2s^2}\right] =$

$= \frac{1}{2cs}\left[1+c^2-s^2+\sqrt{1+2(c^2-s^2)+(c^2+s^2)^2}\right]$

$J(c,s) = \ln\left\{\frac{1}{2c^2}\left[1+c^2-s^2+\sqrt{1+2(c^2-s^2)+(c^2+s^2)^2}\right]\right\}$, $J(c,0) = \ln\left(1+\frac{1}{c^2}\right)$

Для $\vec{F}_{ST}$ надо $\frac{\partial}{\partial t}J(c,t)\Big|_{t=s^2} = \frac{\partial}{\partial t}\ln\frac{1}{2c^2}\left[1+c^2-t+\sqrt{1+2(c^2+t)+(c^2+t)^2}\right] =$

$= \frac{1}{1+c^2-t+\sqrt{(\cdots)}}\left[-1+\frac{1}{2\sqrt{(\cdots)}}\left(-2+2(c^2+t)\right)\right] = \left[\frac{c^2+t-1}{\sqrt{(\cdots)}}-1\right]\frac{1}{1+c^2-t+\sqrt{(\cdots)}}$

$a(s) = -\pi\left(\frac{4}{c^2}M_s\beta J L P_z\right)\frac{\partial}{\partial(s^2)}J(c,s)$, $W_{ex}(s) = \frac{8A'L}{c^2R^2}(-\pi)\frac{\partial}{\partial(c^2)}J(c,s)$

$G(s) = \frac{1}{2}\frac{M_s R^6}{\gamma A'}W_{ex}(s)$, $D = D_{xx} = D_{yy}$, $D(s) = -d_G\frac{M_s L^6}{2\gamma A'}W_{ex}(s)$

Eq. gas orbit radius of Skyrmion, $u = |s| = s$: $a(s) = -\frac{1}{R^2}\frac{D(s)}{sG(s)}\frac{\partial W(s)}{\partial s}$, $W = W_{ex}$ local

$a(s) = -\frac{1}{R^2}\frac{D(s)}{G(s)}\frac{1}{s}\frac{\partial W(s)}{\partial s}$, $\frac{D(s)}{G(s)} = -\frac{d_G M_s L}{(2\gamma A')}W_{ex}(s)\frac{(2\gamma A')}{M_s R^2}\frac{1}{W_{ex}(s)}$ ≠ мех. см. 22.01

$= -d_G\frac{L}{R^2}$. $a(s) = -\frac{1}{R^2}\frac{(-d_G)L}{R^2}\frac{1}{s}\frac{\partial W_{ex}(s)}{\partial s}$, $-\pi(\cdot)\frac{\partial}{\partial s^2}J(c,s) = \frac{d_G L}{2R^4}\frac{\partial}{\partial(s^2)}\times$

$+(-\pi)\frac{8A'L}{c^2R^2}\frac{\partial}{\partial(c^2)}J(c,s)$, $(\cdot)\frac{\partial}{\partial(s^2)}J(c,s) = \frac{2d_G L}{R^4}\frac{8A'L}{c^2R^2}\frac{\partial}{\partial(s^2)}\frac{\partial}{\partial(c^2)}J(c,s)$

$(\cdot) = \frac{4}{c^2}M_s\beta J L P_z\cdot\frac{\partial}{\partial(s^2)}J(c,s) = J_1(c,s)$, $\frac{4}{c^2}M_s\beta J L P_z J_1(c,s) = \frac{16 d_G A' L^2}{c^2 R^6}\frac{\partial}{\partial(c^2)}J_1(c,s)$

$\to M_s\beta J P_z = \frac{4 d_G A' L}{R^6}\frac{1}{J_1(c,s)}\frac{\partial}{\partial(c^2)}J_1(c,s)$ - eq. for $s_0(J)$ - orbit radius.

$[F_{ST}] = [M_s\beta J]\left[\frac{V}{R}\right] = [M_s\beta J]\cdot cm^2 = \frac{erg}{cm}$, $[M_s\beta J] = erg/cm^3$.

$\left[\frac{A'L}{R^6}\right] = \frac{erg}{cm}\frac{1}{cm^3}$, надо $R^6 \to R^3$, $P_z = 1$. $M_s\beta J = \frac{4 d_G A' L}{R^3}\frac{1}{J_1(c,s)}\frac{\partial}{\partial(c^2)}J_1(c,s)$

$J_0 = \frac{4 d_G A' L}{M_s\beta R^3}$, $\frac{1}{J_1(c,s)}\frac{\partial}{\partial(c^2)}J_1(c,s) = J/J_0$ ← eq. for $s_0(J)$ - P_z

$J_1(c,s) = \frac{\partial}{\partial(s^2)}J(c,s) = \frac{1}{\sqrt{1+2(c^2-s^2)+(c^2+s^2)^2}}\frac{\left(c^2+s^2-1-\sqrt{1+2(c^2-s^2)+(c^2+s^2)^2}\right)}{\left(1+c^2-s^2+\sqrt{1+2(c^2-s^2)+(c^2+s^2)^2}\right)}$

Если $c\ll 1$, $s\ll 1$, $\sqrt{1+2(c^2-s^2)+(c^2+s^2)^2} \approx 1+c^2-s^2$, $J_1(c,s) \approx \frac{1}{1+c^2-s^2}\frac{(c^2+s^2-1+1-c^2+s^2)}{(1+c^2-s^2+1+c^2-s^2)} =$

$\approx \frac{1}{(1+c^2-s^2)}\frac{2(s^2-1)}{2(1+c^2-s^2)} = -\frac{(1-s^2)}{1+2(c^2-s^2)} = -(1-s^2)(1-2c^2+2s^2) = -1+2c^2-s^2$

$J_1(c,s) \approx -1+2c^2-s^2$, $\frac{\partial}{\partial(c^2)}J_1 = +2$, $\frac{2}{-1+2c^2-s^2} = J/J_0$, $\frac{1}{1-2c^2+s^2} = 2c^2-s^2+1$

$-2(1+2c^2-s^2) = J/J_0$, $2s^2 = \frac{J}{J_0}+2(1+2c^2)$, $s^2(J) = \frac{J}{2J_0}+1+2c^2$,

$s^2(J)$ растет с $J\uparrow - OK$, но $s^2(0) = 1+2c^2$ - finite - is wrong.

Moreover, $s^2 > 1$ не имеет смысла. $J=0$

Разложение по $c^2\ll 1$, $s^2\ll 1$ не проходит.

$\frac{\partial}{\partial(c^2)}\ln J_1(c,s) = J/J_0$.

$\ln J_1 = \left(\frac{J}{J_0}\right)c^2 + C_1$, $J_1(c,s) = E\cdot e^{Jc^2/J_0}$, $\frac{\partial}{\partial(s^2)}J(c,s) = Ee^{Jc^2/J_0}$

$s^2(J)$ очень сильно растет с J, но можно посчитать $J(s^2)$.

При $s\to 0$, $J_1(c,0) = \frac{1}{(1+c^2)}\frac{(-2)}{2(1+c^2)} = -\frac{1}{(1+c^2)^2} = Ee^{Jc^2/J_0}$ - невозможно (J, т.к. $E = e^{C_1} \geq 0$.

Если $J_1(c,0)\to -J_1(c,0)$, $\frac{1}{(c^2+1)^2} = Ee^{Jc^2/J_0}$ - дает finite current $J = J_{c1}$.

$W_{ex}(s) = -8\pi\frac{A'L}{c^2R^2}\frac{\partial}{\partial(c^2)}J(c,s)$, $\frac{\partial J(c,s)}{\partial c^2} < 0$, $W_{ex}(s)$ растет с $s\uparrow$, если $A' = A - \frac{1}{2}|D|R_c < 0$

$|D| > D_c = 2A/R_c$

Escaneado con CamScanner

Math Cad „ Skyrm - ST-oscillator, mcd " чистая

S(J) расчет

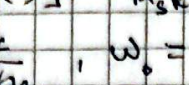


и расч J_{cr}, J_{cr} (exploding the Core from dot)

$$\omega_G(s) = \frac{1}{R^2}\frac{1}{SG(s)}\frac{\partial W(s)}{\partial s} = \frac{2}{R^2}\frac{1}{G(s)}\frac{\partial}{\partial(s^2)}W_{ex}(s) = \frac{2}{R^2}\frac{2\gamma A'}{M_s R^2}\frac{1}{W_{ex}}\frac{\partial}{\partial(s^2)}W_{ex}(s) =$$

$$= \frac{4\gamma A'}{M_s R^4}\frac{1}{\partial J/\partial(c^2)}\frac{\partial}{\partial(s^2)}\left[\frac{\partial J}{\partial(c^2)}\right] = \frac{4\gamma A'}{M_s R^4}\frac{1}{\partial J/\partial(c^2)}\frac{\partial}{\partial(c^2)}J_1(c,s)\cdot\frac{\partial J}{\partial(s^2)} = J_1(c,s)$$

$$\omega_G(s) = \omega_0\cdot\frac{\partial J_1(c,s)/\partial c}{\partial J(c,s)/\partial c},\quad \omega_0 = \frac{4\gamma A'}{M_s R^4}$$ - Maitland gas





Вывод: для reference layer polarization $\vec{P} = P_z\hat{z}$, $P_z = const$, можно записать зависимость $S_0(J)$ и $\omega_G(J)$ даже при $Q=1$ и неучете ее field of current.

$f(z) = e^{i\Phi_0}(z-s)/c$ - BP rigid model, задание модели от skyrm phase Φ_0 нет.

Sk - ST oscillator $\vec{P}(z)$

(15/XII) Более общий.

(16/XII) Надо найти и исследовать. $\vec{F}_{ST}^{d} = a\int dV\,\vec{P}(\vec{r})\left[\vec{m}\times\frac{\partial\vec{m}}{\partial x_d}\right]$ дает более общую зависимость

например для vortex $\vec{P}(r) = (\cos\Phi, \sin\Phi, 0)$, $\Phi = \varphi + \Phi_0$, Φ_0 - variable, как у Garcia-Sanchez 2016.

$$m_x + im_y = \frac{2f(z)}{1+f\bar{f}},\quad m_z = \frac{1-f\bar{f}}{1+f\bar{f}}.$$

$$\frac{\partial m_+}{\partial s_d} = \frac{\partial}{\partial s_d}\frac{2f}{1+f\bar{f}} = \frac{2}{(1+f\bar{f})^2}\left[\frac{\partial f}{\partial s_d}(1+f\bar{f}) - f\frac{\partial}{\partial s_d}f\bar{f}\right] = \left|\frac{\partial\bar{f}}{\partial s_d}\neq 0 \text{ для BP model}\right|$$

$$= \frac{2}{(\cdot)^2}\left[\frac{\partial f}{\partial s_d}(1+f\bar{f}) - f\bar{f}\frac{\partial f}{\partial s_d}\right] = \frac{2}{(1+f\bar{f})^2}\frac{\partial f}{\partial s_d},\quad \frac{\partial f}{\partial s_d} = \frac{\partial}{\partial s_d}\frac{e^{i\Phi_0}}{c}(z-s) =$$

$$= \frac{1}{c}e^{i\Phi_0}(-1)\frac{\partial}{\partial s_d}(s_x + is_y),\quad \frac{\partial f}{\partial s_x} = -\frac{1}{c}e^{i\Phi_0},\quad \frac{\partial f}{\partial s_y} = -i\frac{e^{i\Phi_0}}{c}\qquad s = s_x + is_y$$

$$\frac{\partial f}{\partial s} = \frac{1}{2}\left(\frac{\partial}{\partial s_x} - i\frac{\partial}{\partial s_y}\right) = \frac{1}{2}\left(-\frac{1}{c}e^{i\Phi_0} - i(-i)\frac{e^{i\Phi_0}}{c}\right) = -\frac{1}{c}e^{i\Phi_0}\cdot\frac{\partial}{\partial s}(m_x + im_y) =$$

$$= \frac{2}{(1+f\bar{f})^2}\left(-\frac{1}{c}\right)e^{i\Phi_0} = -\frac{2}{c}\frac{e^{i\Phi_0}}{(1+f\bar{f})^2}.\qquad \vec{m} = m_-\hat{e}_- + m_+\hat{e}_+ + m_z\hat{e}_z$$

$$\hat{e}_\pm = \frac{1}{\sqrt{2}}(\hat{x}\mp i\hat{y}),\quad \hat{e}_+\cdot\hat{e}_+ = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}(\hat{x}-i\hat{y})(\hat{x}-i\hat{y}) = \frac{1}{2}(1-1) = 0,\quad \hat{e}_-\cdot\hat{e}_- = 0$$

$$\hat{e}_+\cdot\hat{e}_- = \frac{1}{2}(\hat{x}-i\hat{y})(\hat{x}+i\hat{y}) = \frac{1}{2}(1+1) = 1.\quad \hat{e}_\pm\cdot\hat{z} = 0.\quad \hat{e}_+\times\hat{e}_- = i\hat{e}_z,\quad \hat{e}_\pm\times\hat{z} = \mp\hat{e}_\pm\cdot i$$

$$\vec{m}_1\times\vec{m}_2 = (m_{1-}\hat{e}_- + m_{1+}\hat{e}_+ + m_{1z}\hat{z})\times(m_{2-}\hat{e}_- + m_{2+}\hat{e}_+ + m_{2z}\hat{z}) = m_{1-}m_{2+}(-i)\hat{z} +$$

$$+ m_{1-}m_{2z}i\hat{e}_- + m_{1+}m_{2-}i\hat{z} + m_{1+}m_{2z}(-i)\hat{e}_+ + m_{1z}m_{2-}(-i)\hat{e}_- + m_{1z}m_{2+}i\hat{e}_+ =$$

$$= i\hat{e}_-(m_{1-}m_{2z} - m_{1z}m_{2-}) + i\hat{e}_+(m_{1z}m_{2+} - m_{1+}m_{2z}) + i\hat{z}(m_{1+}m_{2-} - m_{1-}m_{2+})$$

Вычисления (1.1) Spherical coordinate system:

Covariant spherical coordinates $x_{\pm1} = \mp\frac{1}{\sqrt{2}}(x\pm iy)$, $x_0 = z$, $\mu = \pm1, 0$

Contravariant sph. coordinates $x^{\pm1} = -\frac{1}{\mp}\frac{1}{\sqrt{2}}(x\mp iy)$, $x_0 = z$, $x^\mu = (-1)^\mu x_\mu$

Covar. spherical Basis vectors $\hat{e}_{\pm1} = \mp\frac{1}{\sqrt{2}}(\hat{x}\pm i\hat{y})$, $\hat{e}_0 = \hat{z}$ $\qquad x^\mu = (x_\mu)^*$

Contravar. sph. basis vectors $\hat{e}^{\pm1} = \mp(\frac{\hat{x}\mp i\hat{y}}{\sqrt{2}})$, $\hat{e}_0 = \hat{z}$ (а обычно $\hat{e}_\pm = \frac{1}{\sqrt{2}}(\hat{x}\mp i\hat{y})$,

т.е. contravar. basis вектора $\hat{e}_+ = -\hat{e}^{+1}$, $\hat{e}_- = \hat{e}^{-1}$. $\hat{e}^\mu = (-1)^\mu\hat{e}_{-\mu}$, $\hat{e}^\mu = (\hat{e}_\mu)^*$

$\hat{e}^\mu\hat{e}_\nu = e_\mu^* e_\nu = \delta_{\mu\nu}$, $e_\mu\times e_\nu = -i\varepsilon_{\mu\nu\lambda}e_\lambda$, $e^\mu\times e^\nu = i\varepsilon_{\mu\nu\lambda}e^\lambda$

$$\vec{A} = \sum_{\mu=0,\pm1} A^\mu\hat{e}_\mu = \sum_\mu A_\mu\hat{e}^\mu,\quad A_{+1} = -A^{-1} = -\frac{1}{\sqrt{2}}(A_x + iA_y),\quad A_{-1} = -A^{+1} = \frac{1}{\sqrt{2}}(A_x - iA_y)$$

$$\vec{A}\cdot\vec{B} = \sum_\mu A_\mu B^\mu = \sum_\mu A^\mu B_\mu.\qquad \vec{A}\times\vec{B} = \sum_\mu[\vec{A}\times\vec{B}]_\mu e^\mu = \sum_\mu[\vec{A}\times\vec{B}]^\mu e_\mu$$

$\frac{\partial\vec{m}}{\partial s_d} = \vec{B}$, $\vec{m} = \vec{A}$. $\vec{P}\cdot[\vec{A}\times\vec{B}] = \sum P_\mu[\vec{A}\times\vec{B}]^\mu = P_{+1}i(A_0B_{-1} - A_{-1}B_0) + P_0 i(A_{-1}B_{+1} - A_{+1}B_{-1}) +$

$+ P_{-1}i(A_{+1}B_0 - A_0B_{+1})$, $P_{\pm1} = \mp\frac{1}{\sqrt{2}}(P_x \pm iP_y)$. $A_{\pm1} = \mp\frac{1}{\sqrt{2}}(m_x \pm im_y)$,

$A_{+1} = -\frac{1}{\sqrt{2}}\frac{2f}{(1+f\bar{f})}$, $A_{-1} = \frac{1}{\sqrt{2}}\frac{2\bar{f}}{1+f\bar{f}}$, $A_0 = m_z$, $B_{\pm1} = \mp\frac{1}{\sqrt{2}}\left(\frac{\partial m_x}{\partial s_d} \pm i\frac{\partial m_y}{\partial s_d}\right) \Rightarrow$

$B_{+1} = -\frac{1}{\sqrt{2}}\frac{\partial}{\partial s_d}\frac{2f}{(1+f\bar{f})}$, $B_{-1} = \frac{1}{\sqrt{2}}\frac{\partial}{\partial s_d}\frac{2\bar{f}}{(1+f\bar{f})}$, $B_0 = B_z = \frac{\partial m_z}{\partial s_d}$

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$$\vec{P}\cdot[\vec{A}\times\vec{B}] = iP_{+1}\left(m_z\sqrt{2}\frac{\partial}{\partial z}\frac{\bar{\zeta}}{(1+\zeta\bar{\zeta})} - \frac{\partial m_z}{\partial z}\sqrt{2}\frac{\bar{\zeta}}{(1+\zeta\bar{\zeta})}\right) + iP_z\left\{\sqrt{2}\frac{\bar{\zeta}}{(1+\zeta\bar{\zeta})}(-\sqrt{2})\frac{\partial}{\partial z}\frac{\zeta}{(1+\zeta\bar{\zeta})} - \right.$$

$$\left. - (-\sqrt{2})\frac{\zeta}{1+\zeta\bar{\zeta}}\sqrt{2}\frac{\partial}{\partial z}\frac{\bar{\zeta}}{1+\zeta\bar{\zeta}}\right\} + iP_{-1}\left(-\sqrt{2}\frac{\zeta}{(1+\zeta\bar{\zeta})}\frac{\partial m_z}{\partial z} - m_z(-\sqrt{2})\frac{\partial}{\partial z}\frac{\zeta}{(1+\zeta\bar{\zeta})}\right) =$$

$$= iP_{+1}\{+1\} + iP_z\{z\} + iP_{-1}\{-1\}$$

$\frac{\partial}{\partial z}\to\frac{\partial}{\partial s}$, $\zeta_{sk}\sim\frac{\partial}{\partial s}$

$$\frac{1}{\sqrt{2}}\{+1\} = m_z\frac{\partial}{\partial s}\frac{\bar{\zeta}}{1+\zeta\bar{\zeta}} - \frac{\partial m_z}{\partial s}\frac{\bar{\zeta}}{1+\zeta\bar{\zeta}} = \left(\frac{1-\zeta\bar{\zeta}}{1+\zeta\bar{\zeta}}\right)\frac{(-1)\bar{\zeta}\cdot\zeta_{/s}}{(1+\zeta\bar{\zeta})^2} - \frac{\partial}{\partial s}\frac{(1-\zeta\bar{\zeta})}{(1+\zeta\bar{\zeta})}\frac{\bar{\zeta}}{1+\zeta\bar{\zeta}} =$$

$$= -\bar{\zeta}^2\frac{\partial\zeta}{\partial s}\frac{(1-\zeta\bar{\zeta})}{(1+\zeta\bar{\zeta})^3} - \frac{\bar{\zeta}}{(1+\zeta\bar{\zeta})}\cdot\frac{-\bar{\zeta}\zeta_{/s}(1+\zeta\bar{\zeta}) - (1-\zeta\bar{\zeta})\bar{\zeta}\zeta_{/s}}{(1+\zeta\bar{\zeta})^2} =$$

$$= \bar{\zeta}^2\frac{\zeta_{/s}}{(1+\zeta\bar{\zeta})^3}\left[-(1-\zeta\bar{\zeta}) + (1+\zeta\bar{\zeta}) + (1-\zeta\bar{\zeta})\right] = \bar{\zeta}^2\frac{\partial\zeta}{\partial s}\frac{1}{(1+\zeta\bar{\zeta})^2}$$

$\frac{\partial\bar{\zeta}}{\partial z}=0$ для BP Solitons

$$\frac{1}{\sqrt{2}}\{-1\} = -\frac{\zeta}{(1+\zeta\bar{\zeta})}\frac{\partial}{\partial s}\frac{(1-\zeta\bar{\zeta})}{(1+\zeta\bar{\zeta})} + \frac{(1-\zeta\bar{\zeta})}{(1+\zeta\bar{\zeta})}\frac{\partial}{\partial s}\frac{\zeta}{(1+\zeta\bar{\zeta})} = -\frac{\zeta}{(1+\zeta\bar{\zeta})}\frac{-\bar{\zeta}\zeta_{/s}(1+\zeta\bar{\zeta}) - (1-\zeta\bar{\zeta})\bar{\zeta}\zeta_{/s}}{(1+\zeta\bar{\zeta})^2} +$$

$$+ \frac{(1-\zeta\bar{\zeta})}{(1+\zeta\bar{\zeta})}\frac{\zeta_{/s}(1+\zeta\bar{\zeta}) - \zeta\bar{\zeta}\zeta_{/s}}{(1+\zeta\bar{\zeta})^2} = \frac{\zeta\bar{\zeta}\zeta_{/s}}{(1+\zeta\bar{\zeta})^3}\left[(1+\zeta\bar{\zeta}) + (1-\zeta\bar{\zeta})\right] + \frac{(1-\zeta\bar{\zeta})}{(1+\zeta\bar{\zeta})^3}\frac{\partial\zeta}{\partial s} =$$

$$= \frac{\partial\zeta}{\partial s}\frac{1}{(1+\zeta\bar{\zeta})^3}\left[\zeta\bar{\zeta}\cdot 2 + 1 - \zeta\bar{\zeta}\right] = \frac{\partial\zeta}{\partial s}\frac{1}{(1+\zeta\bar{\zeta})^2}$$

$$\left(\frac{1}{\sqrt{2}}\right)^2\{z\} = -\frac{\bar{\zeta}}{(1+\zeta\bar{\zeta})}\frac{\partial}{\partial s}\frac{\zeta}{(1+\zeta\bar{\zeta})} + \frac{\zeta}{(1+\zeta\bar{\zeta})}\frac{\partial}{\partial s}\frac{\bar{\zeta}}{1+\zeta\bar{\zeta}} = -\frac{\bar{\zeta}}{(1+\zeta\bar{\zeta})}\frac{\partial\zeta}{\partial s}\frac{(1+\zeta\bar{\zeta}) - \zeta\bar{\zeta}\zeta_{/s}}{(1+\zeta\bar{\zeta})^2} +$$

$$+ \frac{\zeta}{(1+\zeta\bar{\zeta})}\frac{-\bar{\zeta}\zeta_{/s}(1+\zeta\bar{\zeta}) - \zeta\bar{\zeta}\zeta_{/s}}{(1+\zeta\bar{\zeta})^2} = -\frac{\bar{\zeta}}{(1+\zeta\bar{\zeta})^3}\frac{\partial\zeta}{\partial s} - \frac{\zeta\bar{\zeta}^2\zeta_{/s}}{(1+\zeta\bar{\zeta})^3} = -\frac{\bar{\zeta}}{(\cdot)^3}\frac{\partial\zeta}{\partial s}\left[1+\zeta\bar{\zeta}\right] =$$

$$= -\bar{\zeta}\frac{1}{(1+\zeta\bar{\zeta})^2}\frac{\partial\zeta}{\partial s}.$$

Bruno Dec. 2013 $\left(\vec{m}\times\frac{\partial\vec{m}}{\partial s}\right)_z = \frac{2i}{(1+w\bar{w})^2}\left[w\frac{\partial\bar{w}}{\partial s} - \bar{w}\frac{\partial w}{\partial s}\right]$

$w\to\zeta$ для Skyrm.

$$\vec{P}\cdot\left[\vec{m}\times\frac{\partial\vec{m}}{\partial s}\right] = iP_{+1}\sqrt{2}\frac{\bar{\zeta}^2}{(1+\zeta\bar{\zeta})^2}\frac{\partial\zeta}{\partial s} + iP_{-1}\sqrt{2}\frac{1}{(1+\zeta\bar{\zeta})^2}\frac{\partial\zeta}{\partial s} + iP_z\frac{(-2)\bar{\zeta}}{(1+\zeta\bar{\zeta})^2}\frac{\partial\zeta}{\partial s} =$$

$$= i\frac{1}{(1+\zeta\bar{\zeta})^2}\frac{\partial\zeta}{\partial s}\left[\sqrt{2}P_{+1}\bar{\zeta}^2 + \sqrt{2}P_{-1} - 2P_z\zeta\right], \quad \zeta(z) = \frac{1}{c}e^{i\varphi_0}(z-s)$$

$\vec{P}$-vortex или Skyrmion. $P_x+iP_y = \frac{2w}{1+w\bar{w}}$, $P_z = \frac{1-w\bar{w}}{1+w\bar{w}}\cdot p$ - reference layer

$$P_{+1} = -\frac{1}{\sqrt{2}}(P_x+iP_y) = -\frac{1}{\sqrt{2}}\frac{2w}{1+w\bar{w}}, \quad P_{-1} = \frac{1}{\sqrt{2}}(P_x-iP_y) = \frac{1}{\sqrt{2}}\frac{2\bar{w}}{(1+w\bar{w})}, \quad P_0 = P_z$$

BP-Sk

$$\vec{P}\cdot\left[\vec{m}\times\frac{\partial\vec{m}}{\partial s}\right] = \frac{i}{(1+\zeta\bar{\zeta})^2}\frac{\partial\zeta}{\partial s}\left[\sqrt{2}\bar{\zeta}^2\left(\frac{1}{\sqrt{2}}\right)\frac{2w}{(1+w\bar{w})} + \sqrt{2}\frac{1}{\sqrt{2}}\frac{2\bar{w}}{(1+w\bar{w})} - 2\zeta\frac{(1-w\bar{w})p}{(1+w\bar{w})}\right] =$$

$$= \frac{i}{(1+\zeta\bar{\zeta})^2}\frac{\partial\zeta}{\partial s}\frac{2}{(1+w\bar{w})}\left[-w\bar{\zeta}^2 + \bar{w} - 2\bar{\zeta}p(1-w\bar{w})\right], \quad p\text{ - polarit reference}$$

Нужен $w=\zeta_p$ для Skyrmion ref. layer или $w=\begin{cases}\zeta_p(z)\text{ - v. core}\\ \zeta_p/|\zeta|\text{ - one core}\end{cases}$ - vortex/Skyrmion

$\zeta_p(z) = \frac{\lambda}{z}e^{i\chi}\cdot z$, $z = R_p/R$ - ref. layer, $\varphi_p = \varphi+\chi$ - фаза Soliton ref. layer

Если ref. layer vortex без core (Garcia-Sanchez '16, Shen '2019),

$w(z) = \zeta_p(z)/|\zeta_p| = e^{i\chi}\frac{z}{|z|} = e^{i\varphi_p} = e^{i(\varphi+\chi)}$, $P_x+iP_y = w$, $P_x = \cos\varphi_p$

Для ref. vortex без core $w = e^{i(\varphi+\chi)}$, $P_y = \sin\varphi_p$

$$= -e^{i(\varphi+\chi)}\frac{1}{c^2}e^{-2i\varphi_0}(\bar{z}-\bar{s})^2 + e^{-i(\varphi+\chi)}, \quad -w\bar{\zeta}^2 + \bar{w} - 2\bar{\zeta}p(1-w\bar{w}) =$$

если $s=0$, $w\cdot w\bar{\zeta}^2 = e^{-i(\varphi+\chi)} - e^{i(\varphi+\chi)}\frac{1}{c^2}e^{-2i\varphi_0}\rho^2 e^{-2i\varphi} = e^{-i(\varphi+\chi)} -$, $w\bar{w}=1$ ($P_z=0$).

$$-\frac{\rho^2}{c^2}e^{-i\varphi}e^{i\chi}e^{-2i\varphi_0} = e^{-i(\varphi+\chi)}\left[1 - \frac{\rho^2}{c^2}e^{2i(\chi-\varphi_0)}\right] - \text{coupling } \vec{p}\cdot\vec{m}$$

max при $\chi=\varphi_0$?

$$\to F_{sT} = a\int dV\,\vec{P}(\vec{r})\cdot\left[\vec{m}\times\frac{\partial\vec{m}}{\partial s}\right] = a\int dV\frac{2i}{(1+\zeta\bar{\zeta})^2(1+w\bar{w})}\frac{\partial\zeta}{\partial s}[\dots] = \left|\partial\zeta/\partial s = -\frac{1}{c}e^{i\varphi_0}\right| =$$

$$= a\left(-\frac{2i}{c}\right)e^{i\varphi_0}\int dV\frac{1}{(1+\zeta\bar{\zeta})^2(1+w\bar{w})}\left[\bar{w} - w\bar{\zeta}^2 - 2\bar{\zeta}p(1-w\bar{w})\right]$$ для BP-Sk $\zeta(z) = \frac{1}{c}e^{i\varphi_0}(z-s)$

F_{sT} для случая $w(z) = e^{i\varphi_p}$, $w\bar{w}=1$ - ref. vortex без core, $a = M_s\,\delta J$ Zhang

$$F_{sT}(\text{ref. vortex без core}) = a\left(-\frac{i}{c}\right)e^{i\varphi_0}\int dV\frac{1}{(1+\zeta\bar{\zeta})^2}\left[e^{-i\varphi_p} - e^{i\varphi_p}\bar{\zeta}^2\right], \quad \varphi_p = \varphi+\chi.$$

Escaneado con CamScanner

$e^{i\Phi_0}\left[\bar{w} - w\bar{f}^2 - 2\bar{f}p(1-w\bar{w})\right]$ = $|\bar{w}\sim \bar{f}_p \sim e^{-i\chi},\ \bar{f}\sim e^{-i\Phi_0}|$ – зависит от $\Phi_0-\chi=\psi$

Note: $\vec{F}_{ST}$ для ref. layer skyrmion/vortex с фазой χ, зависит от $\psi=\Phi_0-\chi$, где Φ_0 – фаза free layer skyrmion $f(z)=\frac{1}{a}e^{i\Phi_0}(z-s)$

т.к. Φ_0, χ input кроме $\vec{F}_{ST}$ не появляются (ни в $W(s)$, ни в $G(s)$, ни в $D(s)$), то и $S_0(J)$, $\omega_G(J)$ будут зависеть лишь от разности фаз $(\Phi_0-\chi)$ of Solitons in free (Φ_0) и reference (χ)-layers.

$\vec{F}_{ST}$ (free v. core) $= -\frac{i}{a}aL\,e^{i\Phi_0}\int \frac{d^2\vec{s}}{(1+\bar{f}f)^2}\left[e^{-i(\varphi+\chi)} - e^{i(\varphi+\chi)}\frac{1}{c^2}e^{-2i\Phi_0}(\bar{z}-\bar{s})^2\right] =$

$= -\frac{i}{a}aL\int \frac{d^2\vec{s}}{(1+\bar{f}f)^2}\left[e^{-i\varphi}e^{i\psi} - e^{i\varphi}e^{-i\psi}\frac{1}{c^2}(\bar{z}-\bar{s})^2\right]$, $\vec{F}_{ST} \sim \frac{\partial}{\partial s} = \frac{1}{2}\frac{\partial}{\partial s_x} - \frac{i}{2}\frac{\partial}{\partial s_y} \sim \frac{1}{2}\vec{F}^x - \frac{i}{2}\vec{F}^y$

$e^{i\varphi}(\bar{z}-\bar{s})^2 = e^{i\varphi}(\bar{z}^2 - 2\bar{z}\bar{s} + \bar{s}^2) = e^{i\varphi}(s^2e^{-2i\varphi} - 2\rho e^{-i\varphi}se^{i\varphi_s} + s^2e^{-2i\varphi_s})$

(17/XII) Вчитывала корректуры Vortex-Sk review JOM и list of changes. Positive ref. reports on SW-PRB manuscript, переписка с Дис., Ricardo.

(18/XII) $1+\bar{f}\bar{f} = 1+\frac{1}{c^2}|z-s|^2 = 1+\frac{1}{c^2}\left[\rho^2+s^2-2\rho s\cos(\varphi-\varphi_s)\right]$, $s=|\vec{s}|$ – const.

$I(\varphi_s,\psi) = \int_0^{2\pi} d\varphi\, \frac{G(\varphi-\varphi_s)}{G[\cos(\varphi-\varphi_s)]}\left[e^{-i\varphi}e^{i\psi} - e^{i\varphi}e^{-i\psi}\frac{1}{c^2}\left(\rho^2e^{-2i\varphi} - 2\rho s e^{-i\varphi}e^{-i\varphi_s} + s^2e^{-2i\varphi_s}\right)\right] =$

$= \int d\varphi\, G\cdot\left[e^{-i\varphi}e^{i\psi} - \frac{1}{c^2}e^{-i\psi}\left(\rho^2e^{-i\varphi} - 2\rho s e^{-i\varphi_s} + s^2e^{i\varphi}e^{-2i\varphi_s}\right)\right] = \left|\varphi-\varphi_s=\varphi'\right| =$

$= \int_{-\varphi_s}^{2\pi-\varphi_s} d\varphi'\, G(\cos\varphi')\left[e^{-i\varphi'}e^{i(\psi-\varphi_s)} - \frac{1}{c^2}e^{i\psi}\left(\rho^2e^{-i\varphi'}e^{-i\varphi_s} - 2\rho s e^{-i\varphi_s} + s^2e^{i\varphi'}e^{-i\varphi_s}\right)\right] =$ $\quad \varphi=\varphi'+\varphi_s$

$= \int_0^{2\pi} d\varphi\, G(\cos\varphi)\left[e^{-i\varphi}e^{i(\psi-\varphi_s)} - \frac{1}{c^2}e^{-i\psi}e^{-i\varphi_s}\left(\rho^2e^{-i\varphi} - 2\rho s + s^2e^{i\varphi}\right)\right] =$

$= e^{-i\varphi_s}\int_0^{2\pi(\pi)} d\varphi\, G(\cos\varphi)\left[e^{-i\varphi}e^{i\psi} - \frac{1}{c^2}e^{i\psi}\left(\rho^2e^{-i\varphi} - 2\rho s + s^2e^{i\varphi}\right)\right]$, $\psi = \Phi_0 - \chi$

$\Phi_P = \varphi+\chi$ – polarizer phase

$I(\varphi_s) = e^{-i\varphi_s}\int_{(-\pi)}^{\pi} d\varphi\, G(\cos\varphi)\left[e^{i\psi}\cos\varphi - \frac{1}{c^2}e^{-i\psi}\left(\rho^2\cos\varphi - 2\rho s + s^2\cos\varphi\right)\right]$

$I(\varphi_s,\psi) = e^{-i\varphi_s}\int d\varphi\, G(\cos\varphi)\left[\left\{e^{i\psi} - \frac{1}{c^2}e^{-i\psi}\left(\rho^2+s^2\right)\right\}\cos\varphi + 2\frac{\rho s}{c^2}e^{-i\psi}\right]$

$G(\cos\varphi) = \frac{1}{(1+\bar{f}\bar{f})^2} = \frac{c^4}{(c^2+\rho^2+s^2-2\rho s\cos\varphi)^2}$. Будет $I_0 = \int_0^{2\pi} d\varphi\, G(\varphi)$ и $I_1 = \int_0^{2\pi} d\varphi\, G(\cos\varphi)\cdot\cos\varphi$

При $s\to 0$, $I(\varphi_s,\psi)\to 0$. Т.е. $I(\varphi_s,\psi)\sim s$ или $\sim s^2$.

$I_0 = \int d\varphi\, G(\cos\varphi) = c^4\int d\varphi \frac{1}{(c^2+\rho^2+s^2-2\rho s\cos\varphi)^2} = \frac{c^4}{(c^2+\rho^2+s^2)^2}\int_0^{2\pi} d\varphi \frac{1}{(1-a\cos\varphi)^2} =$

$= \left|\frac{2\rho s}{c^2+\rho^2+s^2} = a\right| = \frac{c^4}{(c^2+\rho^2+s^2)^2}\frac{2\pi}{(1-a^2)^{3/2}}$, $1-a^2 = 1 - \frac{4\rho^2s^2}{(c^2+\rho^2+s^2)^2} = \frac{1}{(c^2+\rho^2+s^2)^2}\times$

$\times\left[(c^2+\rho^2+s^2)^2 - 4\rho^2s^2\right] = \frac{1}{(\dots)^2}\left[c^4 + (\rho^2+s^2)^2 + 2c^2(\rho^2+s^2) - 4\rho^2s^2\right] = \frac{1}{(\dots)^2}\left[c^4 + 2c^2(\rho^2+s^2) + (\rho^2-s^2)^2\right]$

$I(\varphi_s,\psi) = e^{-i\varphi_s}\left\{e^{i\psi} - \frac{1}{c^2}e^{-i\psi}(\rho^2+s^2)\right\}I_1 + e^{-i\varphi_s}\frac{2}{c^2}\rho s e^{-i\psi}I_0$. $I_0 = I_0(s^2)$.

$I_1 = \int_0^{2\pi} d\varphi\, G(\cos\varphi)\cos\varphi = c^4\int d\varphi \frac{\cos\varphi}{(c^2+\rho^2+s^2-2\rho s\cos\varphi)^2} = \frac{c^4}{(c^2+\rho^2+s^2)^2}\int_0^{2\pi} d\varphi \frac{\cos\varphi}{(1-a\cos\varphi)^2}$

$I(a) = \int_0^{2\pi} d\varphi \frac{1}{(1-a\cos\varphi)}$, $\frac{dI}{da} = \int d\varphi(-1)\frac{1}{(1-a\cos\varphi)^2}(-\cos\varphi) = \int d\varphi \frac{\cos\varphi}{(1-a\cos\varphi)^2}$

$I(a) = \frac{2\pi}{\sqrt{1-a^2}}$, $\frac{dI}{da} = 2\pi\left(-\frac{1}{2}\right)(1-a^2)^{-3/2}(-2a) = 2\pi\frac{a}{(1-a^2)^{3/2}}$, $I_1 = \frac{c^4}{(\dots)^2}\left(\frac{dI}{da}\right)$

$I(\varphi_s,\psi) = e^{-i\varphi_s}\left\{e^{i\psi} - \frac{1}{c^2}e^{-i\psi}(\rho^2+s^2)\right\}\frac{c^4}{(c^2+\rho^2+s^2)^2}\cdot 2\pi\frac{a}{(1-a^2)^{3/2}}$, $\left|a = \frac{2\rho s}{(c^2+\rho^2+s^2)} < 1\right|$

$+ e^{-i\varphi_s}\frac{2}{c^2}\rho s e^{-i\psi}\frac{c^4}{(c^2+\rho^2+s^2)^2}\frac{2\pi}{(1-a^2)^{3/2}} = \frac{2\pi e^{-i\varphi_s}}{(1-a^2)^{3/2}}\frac{c^4}{(c^2+\rho^2+s^2)^2}\left[a\left\{\dots\right\} + \frac{2}{c^2}\rho s e^{-i\psi}\right]$

$I(\varphi_s,\psi) = 4\pi\rho s e^{-i\varphi_s}\frac{1}{(1-a^2)^{3/2}}\frac{c^4}{(c^2+\rho^2+s^2)^2}\left[\frac{1}{(c^2+\rho^2+s^2)}\left\{e^{i\psi} - \frac{1}{c^2}e^{-i\psi}(\rho^2+s^2)\right\} + \frac{1}{c^2}e^{-i\psi}\right]$

$I(\varphi_s,\psi) = 8\pi\rho s e^{-i\varphi_s}\frac{\cos\psi}{(1-a^2)^{3/2}}\cdot\frac{c^4}{(c^2+\rho^2+s^2)^3}$, Вот так да! $\left[\dots\right] = \frac{1}{(c^2+\rho^2+s^2)}\cdot 2\cos\psi$

Escaneado con CamScanner

$$F_{ST}^- = -\frac{i}{c}(M_s \partial J L)\, e^{i\Phi_0} \int d^2\vec{s}\, \frac{[\dots]}{(1+\vec{s}\vec{s})^2} = -\frac{i}{c}(M_s\partial JL)\int ds\, s\, d\varphi\, \frac{\left[e^{-i\varphi}e^{i\psi} - e^{i\varphi}e^{-i\psi}\frac{1}{c^2}(\vec{z}-\vec{s})^2\right]}{(1+\vec{s}\vec{s})^2} =$$

$$= -\frac{i}{c}(\dots)\int ds\, s\, d\varphi\, G(\cos\varphi)\left[e^{-i\varphi}e^{i\psi} - \frac{1}{c^2}e^{-i\psi}(\dots)\right]e^{-i\varphi_s} = -\frac{i}{c}(\dots)\int ds\, s\, I(\varphi_s\psi) =$$

$$= -\frac{i}{c}(\dots)\int ds\, s\cdot 8\pi s\, e^{-i\varphi_s}\frac{\cos\psi}{(1-a^2)^{3/2}}\frac{c^4}{(c^2+s^2+s^2)^3} = -ic^3(\dots)\,8\pi s\, e^{-i\varphi_s}\cos\psi\int\frac{ds\,s^2}{(1-a^2)^{3/2}}\frac{1}{(c^2+s^2+s^2)^3}$$

$$F_{ST}^- = -i\vec{s}\,B(s,c,\psi),\quad B(s,c,\psi) = c^3(M_s\partial JL)\,8\pi\cos\psi\int ds\,\frac{s^2}{(1-a^2)^{3/2}}\frac{1}{(c^2+s^2+s^2)^3}\ \text{-reel.}$$

$$\vec{F}_{ST} = (M_s\partial JL)\int d^2\vec{s}\; P(\vec{z})\cdot\left[\hat{m}\times\frac{\partial\hat{m}}{\partial s}\right],\quad \frac{\partial}{\partial s} = \frac{1}{2}\left(\frac{\partial}{\partial s_x} - i\frac{\partial}{\partial s_y}\right) \Rightarrow F_{ST}^- = \frac{1}{2}F_{ST}^x - \frac{i}{2}F_{ST}^y$$

$$F_{ST}^- = -i\vec{s}B = \frac{1}{2}F_{ST}^x - \frac{i}{2}F_{ST}^y,\quad F_{ST}^x = 2Re(-i\vec{s}B) = 2BRe(-i\vec{s}) = 2BRe[-i(s_x - is_y)] = -2Bs_y$$

$$F_{ST}^y = -2Im(-i\vec{s}B) = -2BIm[-i(s_x - is_y)] = 2s_xB,\quad (\hat{z}\times\vec{s}) = -\hat{x}s_y + \hat{y}s_x$$

$$\vec{F}_{ST} = \hat{x}F_{ST}^x + \hat{y}F_{ST}^y = \hat{x}(-2Bs_y) + \hat{y}(2s_xB) = 2B(-\hat{x}s_y + \hat{y}s_x) = (\hat{z}\times\vec{s})B$$ — ST force

Сохранился баг GuskRL'14 $\vec{F}_{ST}(\vec{s}) = (\hat{z}\times\vec{s})a(s)$, $s=u\Rightarrow|s|$ как дано $\vec{P} = P_z\hat{z}$, $P_z = const$

Vortex ref. layer $\vec{F}_{ST} = (\hat{z}\times\vec{s})\,B(s,c,\psi)$. Немонотонность $\vec{F}_{ST}(s)$ играет в $B(s) = B(s^2)$.

$$B(s,c,\psi) = 8\pi c^3\cos\psi\,(M_s\partial JL)^{KR}\int_0^1 ds\,\frac{s^2}{(1-\tilde{a}^2)^{3/2}}\frac{1}{(c^2+s^2+s^2)^3},\quad \tilde{a} = \frac{2ps}{(c^2+s^2+s^2)}.$$

Reference layer – vortex $\vec{m}_p = (\cos\Phi_p, \sin\Phi_p, 0)$ без core, $\psi = \Phi_0 - \chi$, $\Phi_p = \varphi + \chi$

ψ – разность фаз sky vision (free layer) и vortex (reference layer $\vec{P}(\vec{r})$).

Note: $B(\psi)\sim\cos\psi$, при $\psi = \Phi_0 - \chi = \pm\pi/2$, $\vec{F}_{ST}(\psi) = 0$, ST не эффективен.

Due steady motion $\dot{z}\times\vec{s} = \frac{d\vec{s}}{dt}$, $\Rightarrow \vec{F}_{ST}\sim\frac{d\vec{s}}{dt}$ (по кругу) $\vec{F}$ (.) и компенсация drive force damping force $-D\dot{\vec{s}}$ –

– должен быть first critical current J_{c1} to excite gyromode.

у Garcia-Sanchez'16 simulated $w_G(\psi=-\pi/2) = 0$, т.е. $\vec{F}_{ST}$ не действует (!) при $\psi = \pm\pi/2$.

19/XII Получил Estado de Cuentas MCIU и Skaioteck projects (19,900 и 3,100 €)

Beседа с Julian: хотят приехать visiting to CAF, Ordizia. Обсудил написать to Haldan, invite to SciSky Mag 2020. Beседа с Ricardo: 1) Кафедра FM-Sk oscillator project 2) рассм. писать ответ to Reviewers DW-APB paper с п. и т.д. link". П. 2 нос. книгу 25€ : перев и п. D.

20/XII Second proof JOM V/Sk-review: publish as is.

Сначала GuskRL'14 и 14/XII (P_z-case) steady Sk-core orbit radius минус eq. $\dot{y}(u) = d(u)\,\omega_G(u)$, $y(u) = \frac{a(u)}{G(u)}$, $\vec{F}_{ST}(\vec{s}) = (\hat{z}\times\vec{s})\,a(u)$, $u=|s|$

Для Vortex ref. layer $\vec{P} = \vec{P}_v(\vec{r})$ 12/XII показано, что $\vec{F}_{ST}(\vec{s}) = (\hat{z}\times\vec{s})\,B(s)$,

т.е. $a(s)$ - P_z case $\rightarrow$ $B(s)$ - P_v case.

$$y(s) = \frac{B(s)}{G(s)},\quad \frac{B(s)}{G(s)} = -\frac{D(s)}{G(s)}\omega_G(s),\quad \omega_G(s) = \frac{1}{R^2}\frac{1}{sG(s)}\frac{\partial W(s)}{\partial s};\quad d(s) = -D(s)/G(s)$$

$$B(s) = -D(s)\frac{1}{R^2 sG(s)}\frac{\partial W}{\partial s},\quad D(s)/G(s) \text{ тест, см. 12/XII} = d_GL/R^2 \text{ для rigid Sk-model и } W = W_{ex}$$

$$B(s) = \frac{d_GL}{R^2}\frac{1}{R^2 s}\frac{\partial W_{ex}}{\partial s} = \frac{2d_GL}{R^4}\frac{\partial W_{ex}}{\partial(s^2)},\quad W_{ex}(s) = -8\pi\frac{A'L}{c^2R^2}\frac{\partial}{\partial(c^2)}J(c,s)$$

$$J(c,s) = \int_0^1 dx\,\frac{1}{\sqrt{(c^2+s^2+x)^2 - 4xs^2}}$$ – исследовать 14/XII. $W_{ex}(s)$ можно найти явно.

$$W_{ex}(s) = \frac{8A'L}{c^2R^2}\int d^2\vec{s}\,\frac{1}{(1+\vec{s}\vec{s})^2},\quad \frac{1}{(1+\vec{s}\vec{s})^2} = G(\cos\varphi),\quad I_0(s) = \int_0^{2\pi}d\varphi\,G(\cos\varphi) = \int_0^{2\pi}d\varphi\,\frac{1}{(1+\vec{s}\vec{s})^2} =$$

$$= \frac{c^4}{(c^2+s^2+s^2)^2}\frac{2\pi}{(1-\tilde{a}^2)^{3/2}},\quad \tilde{a} = \frac{2ps}{(c^2+s^2+s^2)}.\quad W_{ex}(s) = \frac{8A'L}{c^2}\int_0^1 ds\,s\int_0^{2\pi}d\varphi\,\frac{1}{(1+\vec{s}\vec{s})^2} =$$

$$= \frac{8A'L}{c^2}\int_0^1 ds\,s\,I_0(s,s),\quad I_0(s,s) = \frac{c^4}{(c^2+s^2+s^2)^2}\frac{2\pi}{(1-\tilde{a}^2)^{3/2}}.$$

$$\rightarrow B(s) = 8\pi c^3\cos\psi\,(M_s\partial JL)\int_0^1 ds\,\frac{s^2}{(1-\tilde{a}^2)^{3/2}}\frac{c^0}{(c^2+s^2+s^2)^3} = \frac{2d_GL}{R^4}\frac{\partial}{\partial(s^2)}\frac{8A'L}{c^2}2\pi c^4 \times$$

$$\times\int_0^1 ds\,s\,\frac{1}{(c^2+s^2+s^2)^2}\frac{1}{(1-\tilde{a}^2)^{3/2}}$$ – eq. for Sk-orbit radius $s_0(J)$ – для Vortex ref. layer

Escaneado con CamScanner

$$F_{ST}^{\lambda} = M_s \delta J L \int d^2\vec{\rho}\, \vec{P}\cdot\left(\vec{m}\times\frac{\partial\vec{m}}{\partial x_\lambda}\right) = M_s\delta J L R \int_0^1 ds \int_0^{2\pi} d\varphi\, \vec{P}\cdot\left(\vec{m}\times\frac{\partial\vec{m}}{\partial\varphi}\right)$$

$[F_{ST}] = erg/cm = [M_s \delta J L R]$ — erg/cm, в definition $\vec{F}_{ST} = aL\int d^2\vec{\rho}\,\vec{P}\cdot\left[\vec{m}\times\frac{\partial\vec{m}}{\partial z}\right]$

Итого $a = M_s\delta J R$, а не $a = M_s\delta J$ как 18/XI

$8\pi c^3 \cos\psi\,(M_s\delta J R L)\int_0^1 ds(\dots) = 32\pi c^4 \frac{d_G A' L^2}{R^4 c^2}\frac{\partial}{\partial(s^2)}\int_0^1 ds\, s(\dots)\cdot R^2$ ($\int_0^1 ds(\dots)$ — c)

Параметрическая размерность для R^n, т.к. L играет здесь

$(M_s\delta J)\cos\psi\int_0^1 ds(\dots) = \frac{4}{c}\frac{L}{R^3} d_G A' \frac{\partial}{\partial(s^2)}\int_0^1 ds\, s(\dots)$, $J_1 = \frac{4}{c}\frac{d_G A' L}{R^3 M_s\delta}$ Vortex $\vec{P}$

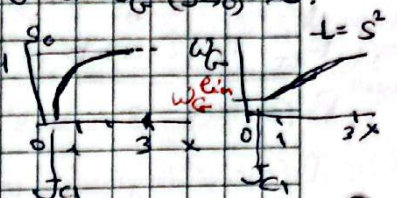
Eq. for $s_0(J)$

$$\frac{J}{J_1}\cos\psi\int_0^1 ds\,\frac{s^2}{(1-\tilde{a}^2)^{3/2}}\frac{1}{(c^2+s^2+s^2)^3} = \frac{\partial}{\partial(s^2)}\int_0^1 ds\, s\,\frac{1}{(1-\tilde{a}^2)^{3/2}}\frac{1}{(c^2+s^2+s^2)^2}$$

$\tilde{a} = 2Rs/(c^2+s^2+s^2)$. Eq. для $s_0^2(J)$. Вес в решении $S(J\to J_{c1})\to 0$?

$\omega_G(s) = \frac{2}{R^2}\frac{1}{G(s)}\frac{\partial}{\partial(s^2)}W(s)$. При $J\cos\psi\to 0$, $J\to 0$, $\frac{\partial W}{\partial(s^2)}\to 0$ и $\omega_G(J\to 0)\to 0$.

$x = \frac{J}{J_1}\cos\psi$, строим $x(s^2)$. „Skyrm_ST_oscillator.nb"



Снова выходит что есть J_{c1}, $\omega_G \propto$ линейная $f(\phi)$.

$\omega_G(J\to 0)$ не $\to 0$ как у Garcia-Sanchez'16, ω_G имеет смысл при $J \gtrsim J_{c1}$ вопреки комменту.

Нормировка current J: 1) P_z, $J_0 = \frac{4 d_G A' L}{M_s\delta R^3}$; 2) Vortex $\vec{P}$, $J_1 = \frac{4}{c}\frac{d_G A' L}{R^3 M_s\delta}$ — отличие в числ. $\frac{1}{c}$, ожидаемо.

Нормировка ω_G: $\omega_G = \frac{1}{R^2}\frac{1}{sG(s)}\frac{\partial W}{\partial s} = \frac{2}{R^2 G(s)}\frac{\partial}{\partial(s^2)}W_{ex}(s) = \frac{2}{R^2}\left[\frac{M_s}{2\gamma A'}W_{ex}\right]^{-1}\frac{\partial W_{ex}}{\partial(s^2)} =$

$= \frac{2}{R^2}\frac{2\gamma A'}{M_s}\frac{1}{W_{ex}}\frac{\partial W_{ex}}{\partial s^2} = \frac{4\gamma A'}{M_s R^2}\frac{1}{W_{ex}}\frac{\partial W_{ex}}{\partial s^2}$, $\frac{1}{W_{ex}}\frac{\partial W_{ex}}{\partial(s^2)} = \frac{1}{\frac{\partial}{\partial c}J(c,s)}\frac{\partial}{\partial(s^2)}\frac{\partial J(c,s)}{\partial c} = \frac{\partial y_1(c,s)/\partial c}{\partial J(c,s)/\partial c}$

$\omega_G = \omega_0\cdot\frac{\partial y_1/\partial c}{\partial J/\partial c}$, $\omega_0 = \frac{4\gamma A'}{M_s R^2}$, $A' = A - \frac{1}{2}|D|R_c$ <0 for stable ske.

Gus' IEEE Magn. Lett. 2007, $\omega_G^{lin}(\phi=1) = \omega_M\frac{c^2}{(1+c^2)^2}\frac{1}{\pi}\frac{1}{\pi c^2}\left[\frac{1}{2}\pi c(|D|-1)\right] =$

$= \omega_M\frac{c^2}{(1+c^2)^2}\frac{1}{\pi}\frac{L^2}{R^2}\left[\frac{1}{2}\frac{R}{L_E}c|D|\frac{L_E}{A}-1\right] = \omega_M\frac{c^2}{(1+c^2)^2}\frac{1}{\pi}\frac{2A}{M_s^2}\frac{1}{R^2}\left[\frac{1}{2}R_c\frac{|D|}{A}-1\right] =$

$= -\omega_M\frac{c^2}{(1+c^2)^2}\frac{2}{\pi}\frac{A'}{R^2 M_s^2}$, $\omega_G^{lin} = -\omega_M\frac{2}{\pi}\frac{c^2}{(1+c^2)^2}\frac{A'}{R^2 M_s^2}$, $\omega_G^{lin}(c,s) = -\frac{4\gamma A'}{M_s R^2}(-1)\frac{\partial y_1/\partial c}{\partial J/\partial c} =$

$= -\frac{\omega_M}{\pi}\frac{A'}{M_s^2 R^2}(-1)\frac{\partial y_1/\partial c}{\partial J/\partial c} \Rightarrow -\omega_M\frac{A'}{R^2 M_s^2} = \frac{\pi}{2}\frac{1}{c^2}(1+c^2)^2\omega_G^{lin}$

$\omega_G(c,s) = \frac{1}{\pi}\cdot\frac{\pi}{2}\frac{1}{c^2}(1+c^2)^2\omega_G^{lin}(1) = \frac{1}{2}\frac{1}{c^2}(1+c^2)^2\omega_G^{lin}(-1)\frac{\partial y_1/\partial c}{\partial J/\partial c} = \omega_0(-1)\frac{\partial y_1/\partial c}{\partial J/\partial c}$

Scale $\omega_0 = \frac{1}{2c^2}(1+c^2)^2\omega_G^{lin}$. При $c=\frac{1}{2}$, $\frac{1}{2c^2}(1+c^2)^2 = \frac{1}{2\cdot 1/4}\left(1+\frac{1}{4}\right)^2 = 2\cdot\frac{25}{16} = \frac{25}{8} \approx 3{,}13$

Т.к. ω_G^{lin} для typical set parameters ≈ 1 GHz, то $\omega_0 = \frac{1}{2c^2}(1+c^2)^2\omega_G^{lin} = (3\div 10)\omega_G^{lin}$, может $\to 10$ GHz при $c=0{,}2$.

$\omega_G(c, s\to 0) = \omega_G^{lin}(c)$, т.е. при $J\to J_{c1}$, $s_0(J_{c1}) = 0$, $\omega_G(c, J_{c1}) = \omega_G^{lin}(c)(-1)$

$\omega_G(J_{c2}) \approx 3\omega_G(J_{c1}) \approx 3\omega_G^{lin}$ может достичь 3 GHz.

P_z $J_0 = \frac{4 d_G |A'| L}{M_s\delta R^3} = \frac{1}{\delta}\frac{d_G L}{M_s\delta R^3}\left(\frac{\gamma}{M_s R^2}|A'|\right)M_s R^2 = \frac{d_G L}{M_s^2\delta R}\frac{\omega_0}{\gamma}$, $\omega_0 = \frac{4\gamma}{M_s R^2}|A'|$, $J_{c1} \approx 1{,}6 J_0$ при $c=0.5$.

Vortex $\vec{P}$ $J_1 = \frac{4^2}{c}J_0 = \frac{4^2}{c}\frac{L}{R}\frac{d_G}{\delta}\frac{\omega_0}{\gamma}$. $x_{c1}\approx 0{,}4$, $J_{c1}\cos\psi \approx 0{,}4 J_1$.

Sk-velocity (max) $\approx \omega_G(J_{c2})\cdot R = 3\,GHz\cdot 100\,nm\cdot 2\pi \approx 18\cdot 10^9\frac{1}{s}\cdot 10^{-7}m \approx 1800\,m/s$ (!)

и это при $v_{sk}\approx 2000$ m/s со Skyrmion number не противоречит?

$x = \frac{J}{J_1}\cos\psi$. Если $J = const$ и постоянно $\omega_G(\psi)$, то $\omega_G(\psi)$ повторит $\omega_G(J)$, $\omega_G(x)$ на участке $x\in[0, b]$. $J/J_1 = b$

$F_{ST}(vortex) \sim c^3$ следует из вида $\vec{P}\cdot(\vec{m}\times\frac{\partial\vec{m}}{\partial z}) \sim \frac{\partial}{\partial z}[\vec{m}-\vec{w}f^2 - 2f\vec{P}(1-\vec{w}\vec{m})]$

$\partial/\partial z \sim \frac{1}{c}$, $\vec{w}\sim 1/c^0$, $f^2\sim Y_0^2$, $\vec{P}\sim Y_c$, $F_{ST}(vortex)\sim\frac{1}{c}\left[\frac{1}{c^0}-\frac{1}{c^2}-0\right]\sim$ нечет. $f(c)$.

что вихрь без core

Escaneado con CamScanner

(21/XII) Нормировка current $J_0(P_2) = \frac{4 d_G |A'| L}{M_S \beta R^3}$, $J_1(\text{vortex } \vec{P}) = \frac{1}{c} \frac{d_G |A'| L}{M_S \beta R^3} = \frac{1}{c} J_0(P_2)$

(22/XII) Eq. for $S_0(J)$ P_2 – 2g. layer и vortex $\vec{P}$ 2g. layer цилиндры к бугру, где $\vec{F}_{ST}$ часто разлагаются вдоль. Y_c в J_1 из-за бугра $\vec{F}_{ST}(\text{vortex } \vec{P})$.

Для P_2: $a(s) = -D(s) \frac{1}{R^2 s G(s)} \frac{\partial W}{\partial s} = -D(s) \omega_G(s)$

Для vortex $\vec{P}$: $B(s) = -D(s) \omega_G(s)$. Бугр ST-terms $a(s)$ и $B(s)$ разлагаются.

$D(s) = D_{xx} = D_{yy} = -d_G \frac{M_S L}{\gamma} \int d^2\vec{\rho} \left(\frac{\partial \vec{m}}{\partial x_\alpha} \cdot \frac{\partial \vec{m}}{\partial x_\beta} \right)_{\alpha=\beta=x}$. $D_{xx} + D_{yy} = -d_G \frac{M_S L}{\gamma} \times$

$\times \int d^2\vec{\rho} \left[\frac{\partial \vec{m}}{\partial x} \frac{\partial \vec{m}}{\partial x} + \frac{\partial \vec{m}}{\partial y} \frac{\partial \vec{m}}{\partial y} \right] = \left| \vec{m} = \vec{m}(\vec{\rho} - \vec{X}) \right| = -d_G \frac{M_S L}{\gamma} \int d^2\vec{\rho} \left[\left(\frac{\partial \vec{m}}{\partial x} \right)^2 + \left(\frac{\partial \vec{m}}{\partial y} \right)^2 \right] =$

$= -d_G \frac{M_S L}{\gamma} \int d^2\vec{s} (\nabla \vec{m})^2$, $W_{ex} = A' L \int d^2\vec{s} (\nabla \vec{m})^2$, $D_{xx} + D_{yy} = 2D = -d_G \frac{M_S}{\gamma A'} W_{ex}$, см. 13/XII

$D(s) = -d_G \gamma^{-1} M_S W_{ex}(s)$. $a(s) = d_G \frac{M_S}{2\gamma A'} W_{ex} \omega_G(s)$, $\vec{F}_{ST} = (\hat{z} \times \hat{s}) a(s)$

$[a] = \left[\frac{\omega}{\gamma} M_S \right] [2A'] \left[\frac{W_{ex}}{A'} \right] = \frac{\text{erg}}{\text{cm}^3} \frac{\text{erg}}{\text{erg/cm}} = \text{erg/cm}^2$. Надо $[a] = [F_{ST}] = \text{erg/cm}$

Уточ. $a(s) = d_G \frac{M_S R}{2\gamma A'} W_{ex} \omega_G(s)$, $\omega_G = \frac{1}{R^2 s G(s)} \frac{\partial W}{\partial s} = \frac{2}{R^2} \frac{1}{G(s)} \frac{\partial W}{\partial (s^2)}$, $W = W_{ex} + \dots$

Нам: $G(s) = \frac{M_S L}{\gamma} \int d^2\vec{\rho} \frac{4}{(1+\delta f)^2} \left| \frac{\partial f}{\partial z} \right|^2$, $W_{ex} = \int dV A' (\nabla \vec{m})^2 = A' L \int d^2\vec{\rho} (\nabla \vec{m})^2 = A' L \int d^2\vec{\rho} \frac{8}{(1+\delta f)^2} \left| \frac{\partial f}{\partial z} \right|^2$

$G(s) = \frac{M_S L}{\gamma} 4 \int d^2\vec{\rho} \frac{1}{(1+\delta f)^2} \left| \frac{\partial f}{\partial z} \right|^2 = 4 \frac{M_S L}{\gamma} \frac{\frac{1}{2} W_{ex}}{A' L} = \frac{1}{2} \frac{M_S W_{ex}}{\gamma A'} = \frac{1}{2} \frac{M_S}{\gamma A'} W_{ex}$

$a(s) = d_G \frac{M_S R}{2\gamma A'} W_{ex} \frac{2}{R^2} \frac{1}{G(s)} \frac{\partial W}{\partial (s^2)} = d_G \frac{M_S R}{(2\gamma A')} W_{ex} \frac{2}{R^2} \frac{(2\gamma A')}{M_S W_{ex}} \frac{\partial W}{\partial (s^2)} = \frac{2 d_G}{R} \frac{\partial W}{\partial (s^2)}$

Или $a(s) = -\frac{D(s)}{G(s)} \frac{1}{R^2 s} \frac{\partial W}{\partial s} \cdot R$, $\frac{-D(s)}{G(s)} = d_G \frac{M_S}{2\gamma A'} W_{ex}(s) \frac{2\gamma A'}{M_S W_{ex}} = d_G$ (!)

$a(s) = d_G \frac{1}{sR} \frac{\partial W}{\partial s} = \frac{2 d_G}{R} \frac{\partial W}{\partial (s^2)}$. $\frac{D(s)}{G(s)} = -d_G$ для $\delta(z)$ any.

$\frac{D(s)}{G(s)} = -\frac{d_G L}{R^2}$ 14/XII и 20/XII посчитано неверно. Надо $\frac{D(s)}{G(s)} = -d_G$ for any $\delta(z)$.

Для P_2: $a(s) = \frac{2 d_G}{R} \frac{\partial W}{\partial (s^2)}$, ... для vortex $\vec{P}$: $B(s) = \frac{2 d_G}{R} \frac{\partial W}{\partial (s^2)}$,

где $\vec{F}_{ST} = (\hat{z} \times \hat{s}) a(s)$ ... $\vec{F}_{ST} = (\hat{z} \times \hat{s}) B(s)$. Это eqs. for $S_0^2(J)$

$B(s) = 8\pi c^3 \cos\psi (M_S \beta J R L) \int_0^1 d\tilde{s} \frac{\tilde{s}^2}{(1-\tilde{a}^2)^{3/2}} \frac{1}{(c^2 + \tilde{s}^2 + \tilde{s}^2)^3} = \left(\frac{2 d_G}{R} \right) \frac{\partial W_{ex}(s)}{\partial (s^2)}$, если $W(s) = W_{ex}(s)$

$W_{ex}(s) = 8 A' L \int d^2\vec{\rho} \frac{1}{(1+\delta f)^2} \left| \frac{\partial f}{\partial z} \right|^2 = \frac{8}{c^2} A' L \cdot \int d^2\vec{\rho} \frac{1}{(1+\delta f)^2} \cdot \frac{1}{c^2} = \frac{8}{c^2} A' L \int d\varphi\, s\, ds \frac{2\pi c^4}{(1-\tilde{a}^2)^{3/2} (c^2 + \tilde{s}^2 + \tilde{s}^2)^2}$

$\delta(z) = \frac{1}{a} e^{i q_0 (z-s)}$

$8\pi c^3 \cos\psi (M_S \beta J R L) \int \frac{ds\, s^2}{(\dots)} = 8 A' L \cdot 2\pi c^2 \frac{\partial}{\partial (s^2)} \int d\varphi\, s\, ds (\dots) \left(\frac{2 d_G}{R} \right) = 32 A' L \pi c^2 \frac{d_G}{R} \frac{\partial}{\partial (s^2)} \int d\varphi\, s\, ds (\dots)$

→ $J_1 = 32 A' L \pi c^2 \frac{d_G}{R} \frac{1}{8\pi c^3 M_S \beta R L} = \frac{4}{c} \frac{A' d_G}{R^2 M_S \beta}$, (ранее 20/XII было $J_1 = \frac{4}{c} \frac{d_G A'}{M_S \beta} \frac{L}{R^3}$)

Для vortex $\vec{P}$ $J_1 = \frac{4}{c} \frac{d_G A'}{M_S \beta R^2}$. Ур-е на $S_0(J)$ такое же как 20/XII $\frac{J}{J_1} \cos\psi(\dots) = 1$.

Для P_2: $a(s) = -\pi \left(\frac{4}{c^2} M_S \beta J L \right) \frac{\partial}{\partial (s^2)} J(c,s)$. $W_{ex}(s) = \left(\frac{8}{c^2} A' L \right) \int d^2\vec{\rho} \frac{1}{(1+\delta f)^2} =$

$= (\cdot)(-1) \frac{\partial}{\partial (c^2)} \pi J(c,s)$, $-\pi \left(\frac{4}{c^2} M_S \beta J L R \right) \frac{\partial}{\partial (s^2)} J(c,s) = \frac{2 d_G}{R} \frac{\partial}{\partial (s^2)} \left(\frac{8}{c^2} A' L \right) (+\pi) \frac{\partial J(c,s)}{\partial (c^2)}$

$\frac{\partial}{\partial (s^2)} J(c,s) = J_1(c,s)$, $\frac{4}{c^2} M_S \beta J L R J_1(c,s) = \frac{8}{c^2} A' L \frac{2 d_G}{R} \frac{\partial}{\partial (c^2)} J_1(c,s)$

$J_1(c,s) = J_0 \frac{\partial}{\partial (c^2)} J_1(c,s)$, $J_0 = \frac{8}{c^2} A' L \frac{2 d_G}{R} \frac{1 \cdot a^2}{4 M_S \beta L R} = 4 \frac{d_G A'}{M_S \beta R^2}$. $\frac{J}{J_0} = \frac{\partial}{\partial (c^2)} \ln J_1(c,s)$

Corrected $J_0 = 4 d_G A' / (M_S \beta R^2)$. 14/XII было $J_0 = 4 \frac{d_G A'}{M_S \beta} \left(\frac{L}{R^3} \right)$, лишний множитель $\left(\frac{L}{R} \right)$.

Согласно 20/XII теперь $J_0 = \frac{d_G}{\Phi} \frac{\omega_0}{\gamma}$, $J_1 = \frac{1}{c} \frac{d_G}{\Phi} \frac{\omega_0}{\gamma}$, где $\omega_0 = \frac{4\gamma}{M_S R^2} |A'|$, $\omega_0 = \frac{1}{2c^2} (1+c^2)^2 \omega_G^{\text{lin}}$ –

– имеет частоту, $\omega_0 \sim \omega_G^{\text{lin}} \sim 1\,\text{GHz}$

$W_{ex}(s) = \left(\frac{8}{c^2} A' L \right) \int d^2\vec{\rho} \frac{1}{(1+\delta f)^2}$ в P_2 – между и $\vec{P}$ vortex между считается по-разному.

Escaneado con CamScanner

$$\frac{\partial}{\partial(s^2)}\int_0^{2\pi}\frac{1}{(1+\delta\xi)^2} = \frac{\partial}{\partial(s^2)}\int_0^1 d\rho\,\rho\int_0^{2\pi}d\varphi\,\frac{1}{\left[1+c^2-(\rho^2+s^2-2\rho s\cos(\varphi-\varphi_s))\right]^2} = \left(|z-s|^2=\rho^2+\dots\right) =$$

$$= c^4\frac{\partial}{\partial(s^2)}\int d\rho\,\rho\int d\varphi\,\frac{1}{\left[c^2+\rho^2+s^2-2\rho s\cos\varphi\right]^2}, \quad J_\varphi = \int_0^{2\pi}d\varphi\,\frac{1}{\left[c^2+\rho^2+s^2-2\rho s\cos\varphi\right]^2}$$

1) $J_\varphi = -\frac{\partial}{\partial(c^2)}\int d\varphi\,\frac{1}{\left[c^2+\rho^2+s^2-2\rho s\cos\varphi\right]} = -\frac{\partial}{\partial(c^2)}\frac{1}{(c^2+\rho^2+s^2)}\int_0^{2\pi}d\varphi\,\frac{1}{(1-a\cos\varphi)} = -\frac{\partial}{\partial(c^2)}\frac{2\pi}{(c^2+\rho^2+s^2)\sqrt{1-a^2}} =$

$= -\frac{\partial}{\partial(c^2)}\frac{2\pi}{\sqrt{(c^2+\rho^2+s^2)^2-4\rho^2s^2}}$, $a=\frac{2\rho s}{c^2+\rho^2+s^2}$. инт. в P_2 - между.

2) $J_\varphi = \int d\varphi\,\frac{1}{(1-a\cos\varphi)^2}\,\frac{1}{(c^2+\rho^2+s^2)^2} = \frac{1}{(c^2+\rho^2+s^2)^2}\frac{2\pi}{(1-a^2)^{3/2}}$ - инт. в $\vec P$ вне между

$\frac{\partial}{\partial(c^2)}\frac{1}{\sqrt{(\dots)}} = -\frac{1}{2}\frac{1}{\left[(c^2+\rho^2+s^2)^2-4\rho^2s^2\right]^{3/2}}\cdot 2(c^2+\rho^2+s^2) = -\frac{1}{(c^2+\rho^2+s^2)^2}\left[\frac{1}{\sqrt{1+a^2}}\right]^3$

$\frac{\partial}{\partial s}J_\varphi = \frac{\partial}{\partial s}\int d\varphi\,\frac{1}{\left[c^2+\rho^2+s^2-2\rho s\cos\varphi\right]^2} = -2\int d\varphi\,\frac{2(s-\rho\cos\varphi)}{[\dots]^3} = -4\int d\varphi\,\frac{(s-\rho\cos\varphi)}{\left[c^2+\rho^2+s^2-2\rho s\cos\varphi\right]^3} =$

$= -\frac{4}{(c^2+\rho^2+s^2)^3}\int_0^{2\pi}d\varphi\,\frac{(s-\rho\cos\varphi)}{(1-a\cos\varphi)^3}$ - берется аналитически. Тип $\int d\rho\,\rho(\dots)$ часто встречается в задаче.

Для $P_z \Rightarrow$ $a(s) = -\frac{4\pi}{c^2}(M_s\,\delta\,J\,L\,R)\frac{\partial}{\partial(s^2)}J(c,s)$, $J(c,s) = \int_0^1 d\rho\,\frac{1}{\sqrt{(c^2+s^2+\rho^2)^2-4\rho^2s^2}}$.

Для вект. $\vec P$ $\Rightarrow$ $B(s) = 8\pi c^3\cos\psi\,(M_s\,\delta\,J\,L\,R)\int_0^1 d\rho\,\frac{\rho^2}{(1-\tilde a^2)^{3/2}}\frac{1}{(c^2+\rho^2+s^2)^3}$

Зав. больше $a(s)$ или $B(s)$ для каких c? ($J=\text{const}$)

$\frac{B(s)}{a(s)} = -2c^3\cos\psi\cdot\frac{\int_0^1 d\rho\,\frac{\rho^2}{(1-\tilde a^2)^{3/2}}\frac{1}{(c^2+\rho^2+s^2)^3}}{\frac{\partial}{\partial(s^2)}J(c,s)}$, $\tilde a = \frac{2\rho s}{(c^2+\rho^2+s^2)}$, $\frac{\partial}{\partial(s^2)}J = \frac{1}{2s}\frac{\partial J}{\partial s}$

Ratio $B(s)/a(s) = 0.1-0.2$ for $c=0.4-0.6$ при $J=\text{const}$

$J_1(\text{vortex }\vec P) = \frac{1}{c}J_0(P_z)$, $J_1 > J_0$, т.е. J_{c1} в два раза больше J для вихря $-\vec P$. the same M_s, δ, L, R.

Больше при других равных усл.

Вопрос: Зачем вихрь $\vec P$ осциллятор нужен, если P_z - это экспериментально?

Mathead расс при $c=0.5$: для P_z $J_{c1}=1.6J_0$, $J_{c2}=3.1J_0$, $J_1=2J_0$

$J_{c1}(P_z) = 1.6J_0$ для вих $\vec P$ $\frac{J_{c1}\cos\psi}{J_1}=0.4$, $\frac{J_{c2}\cos\psi}{J_1}\approx 3$, $J_{c2}(\text{вихр}) = 6J_0$.

$J_{c1}(\text{вихр}) = 0.4\cdot 2J_0 = 0.8J_0$, $\cos\psi=1$, т.е. $J_{c1}(\text{вихр}) < J_{c1}(P_z)$ in abs. units.

$\frac{J}{J_0} = -\frac{1}{J_1(c,s)}\frac{\partial}{\partial(c^2)}J_1(c,s) = -\frac{\partial}{\partial(c^2)}\ln J_1(c,s)$, при $s\to 0$, $J\to J_{c1}$

$J(c,s) = \ln\left[1+c^2-s^2+\sqrt{1+2(c^2-s^2)+(c^2+s^2)^2}\right] - \ln(2c^2) = |s\to 0| = \ln[1+c^2-s^2+$

$+\sqrt{1+2c^2-2s^2+c^4+2c^2s^2}\,] - \ln(2c^2)$, $\sqrt{1+2c^2+c^4+2(c^2-1)s^2} = (1+c^2)\sqrt{1+\frac{2(c^2-1)s^2}{(c^2+1)^2}} =$

$= 1+c^2+\frac{(c^2-1)}{(c^2+1)}s^2$, $1+c^2-s^2+\sqrt{\dots} = 1+c^2-s^2+1+c^2+\frac{(c^2-1)}{(c^2+1)}s^2 =$

$= 2(1+c^2)+\frac{s^2}{c^2+1}\left(c^2-1-(c^2+1)\right) = 2(1+c^2)-\frac{2}{(1+c^2)}s^2$.

$J(c,s) = \ln\left[2(1+c^2)-\frac{2}{(1+c^2)}s^2\right] - \ln(2c^2) = J(c,0)+\ln\left(1-\frac{s^2}{(1+c^2)^2}\right) \approx -\frac{s^2}{(1+c^2)^2} + J(c,0)$

$J_1(c,s) = \frac{\partial}{\partial(s^2)}J(c,s) = -\frac{1}{(1+c^2)^2}$, $\frac{J}{J_0} = -\frac{\partial}{\partial(c^2)}\ln\frac{1}{(1+c^2)^2} = \frac{\partial}{\partial(c^2)}2\ln(1+c^2) =$

$= 2\frac{1}{1+c^2}$, т.е. $\frac{J_{c1}}{J_0} = \frac{2}{1+c^2}$, $\frac{J_{c1}}{J_0} = 1.6$ при $c=0.5$. Ok.

$J_{c1}(P_z) = \frac{2}{(1+c^2)}J_0$ - получен аналитически. теперь вихрь $\vec P$.

$J_{\vec P}(c,\rho) = \int_0^1 d\rho\,\rho^2\frac{1}{(1-\tilde a^2)^{3/2}}\frac{1}{(c^2+\rho^2+s^2)^3} = \int d\rho\,\rho^2(1+\tilde a^2)^{3/2}\frac{1}{(\dots)^3}$, $J_{\vec P}(c,s=0) =$

$= \int_0^1 d\rho\,\rho^2\frac{1}{(c^2+\rho^2)^3} = |\rho^2=x,\ 2\rho\,d\rho = dx| = \int dx\frac{1}{(c^2+x)^3} = -c^2\int dx\frac{1}{(c^2+x)^3} = \left[-\frac{x}{4(c^2+x)^2}+\right.$

$\left.+\frac{x}{8c^2(x+c^2)}+\frac{1}{8c^3}\operatorname{arctg}\frac{x}{c^2}\right]_0^1 = -\frac{1}{4(c^2+1)^2}+\frac{1}{8c^2(c^2+1)}+\frac{1}{8c^3}\operatorname{arctg}\frac{1}{c} = \frac{(1-c^2)}{8c^2(1+c^2)^2}+\frac{1}{8c^3}\operatorname{arctg}\frac{1}{c}$

$W_{P_z}(s) = \frac{2}{c^2}A'L\int d\rho\,\frac{\rho^2}{(1+\tilde a^2)^{3/2}} = -8\pi\frac{\partial}{\partial(c^2)}J(c,s)\frac{1}{c^2}A'L$ ✓

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$$\frac{\partial}{\partial(s^2)} W_{ex}(s) = -\frac{8\pi}{c^2} A'L \frac{\partial}{\partial(s^2)} \frac{\partial}{\partial(c^2)} J(c,s) \overset{c^4}{=} -\frac{8\pi}{c^2} A'L \frac{\partial}{\partial(c^2)} \frac{\partial}{\partial(s^2)} \left[J(c,0) - \frac{s^2}{(1+c^2)^2} \right] = |s\to 0| =$$

$$= +\frac{8\pi}{c^2} A'L \frac{\partial}{\partial(c^2)} \frac{1}{(1+c^2)^2} = -\frac{8\pi}{c^2} A'L \frac{+2}{(1+c^2)^3} = -\frac{16\pi}{c^2(1+c^2)^3} A'L \cdot \overset{c^4}{}$$

Eq. for J_{c1}, $S\to 0$: $8\pi c^3 \cos\psi \,(M_s \delta J R L)\, J_B(c,s=0) = \frac{2dG}{R} \left.\frac{\partial W(s^2)}{\partial s^2}\right|_{s=0}$

$8\pi c^3 \cos\psi\,(M_s \delta J R L) J_B = \frac{2dG}{R}(-16\pi)\frac{A'L}{c^2(1+c^2)^3}$, $J_{c1}(\text{отн.})\cdot\cos\psi = \frac{32\pi dG|A'|L}{R\cdot c^2(1+c^2)^3} \times$

$\times \frac{1}{8\pi c^3 (M_s \delta R L) J_B(c,0)}$, $J_1 = \frac{4}{c}\frac{dG|A'|}{M_s \delta R^2}$. Sk-STNO

$\cos\psi J_{c1} = J_1 \frac{1}{c^4(1+c^2)^3 J_B(c,0)}$, $c^4(1+c^2)^3 J_B(c,0) = \frac{1}{8}c^2(1-c^4) + \frac{1}{8}c(1+c^2)^3 \operatorname{arctg}\frac{1}{c} = F(c)$

$J_{c1}\cos\psi = J_1 / F(c)$. $F(0{,}5) = 0{,}164$, $J_{c1}\cos\psi = J_1 \cdot 6{,}08$ (!?), Mathcad дает $\frac{J_1}{J_1} \cong 0{,}4$.

Какая ошибка в $J_{c1}(\omega_{ex}\vec{P})$.

(23/XII) Проверим $a(s)$. $\vec{F}_{ST} = (\hat{z}\times\vec{s})\,a(s)$. $F^x_{ST} = (M_s \delta J L R) P_z \int d^2\vec{\rho}\left(\vec{m}\times\frac{\partial\vec{m}}{\partial s_x}\right)_z$

$\left(\vec{m}\times\frac{\partial\vec{m}}{\partial s_x}\right)_z = \frac{4}{c^2}\frac{(y-s_y)}{(1+\rho\bar{\rho})^2}$, $F^x_{ST} = (M_s \delta J L R)\int d^2\vec{\rho}\,\frac{4}{c^2}\frac{(\rho\sin\varphi - s\sin\varphi_s)}{(1+\rho\bar{\rho})^2} =$

$= (\dots)\frac{4}{c^2}\int d^2\vec{\rho}\,\frac{(\rho\sin\varphi - s\sin\varphi_s)}{\left[1+\frac{1}{c^2}(\rho^2+s^2-2s\rho\cos(\varphi-\varphi_s)\right]^2} = (\dots)4c^2\int d^2\vec{\rho}\,\frac{(\rho\sin\varphi - s\sin\varphi_s)}{\left[c^2+\rho^2+s^2-2s\rho\cos(\varphi-\varphi_s)\right]^2}$

$\int_0^{2\pi} d\varphi \frac{(\rho\sin\varphi - s\sin\varphi_s)}{\left[c^2+\rho^2+s^2-2s\rho\cos(\varphi-\varphi_s)\right]^2} = |\varphi' = \varphi-\varphi_s| = \int_{-\varphi_s}^{2\pi-\varphi_s} d\varphi' \frac{(\rho\sin(\varphi'+\varphi_s) - s\sin\varphi_s)}{\left[c^2+\rho^2+s^2-2s\rho\cos\varphi'\right]^2} =$

$= \int_0^{2\pi} d\varphi'(\dots) = \int_{-\pi}^{\pi} d\varphi'(\dots) = |\varphi'\to\varphi| = \int_{-\pi}^{\pi} d\varphi \frac{[\rho\sin\varphi\cos\varphi_s + \rho\cos\varphi\sin\varphi_s - s\sin\varphi_s]}{[c^2+\rho^2+s^2-2s\rho\cos\varphi]^2} =$

$= \int d\varphi \frac{(\rho\cos\varphi - s)\sin\varphi_s}{[c^2+\rho^2+s^2-2s\rho\cos\varphi]^2} = \sin\varphi_s \frac{\partial}{\partial s}\int d\varphi \frac{1}{[c^2+\rho^2+s^2-2s\rho\cos\varphi]}(-1)(-\tfrac{1}{2}) =$

$= s\sin\varphi_s \frac{\partial}{\partial(s^2)}\int d\varphi \frac{1}{[c^2+\rho^2+s^2-2s\rho\cos\varphi]} = s\sin\varphi_s \frac{\partial}{\partial(s^2)}\frac{1}{(c^2+\rho^2+s^2)}\int d\varphi \frac{1}{1-a\cos\varphi} =$

$= s\sin\varphi_s \frac{\partial}{\partial(s^2)}\frac{1}{(\dots)}\frac{2\pi}{\sqrt{1-\frac{(2s\rho)^2}{(\dots)^2}}} = 2\pi s\sin\varphi_s \frac{\partial}{\partial(s^2)}\frac{1}{\sqrt{(c^2+\rho^2+s^2)^2 - 4s^2\rho^2}}$

$F^x_{ST} = (M_s \delta J L R)\,4c^2 \int d\rho\rho \int d\varphi \frac{(\rho\sin\varphi - s\sin\varphi_s)}{[c^2+\rho^2+s^2-2\rho s\cos(\varphi-\varphi_s)]^2} = (\dots)4c^2\int d\rho\rho\cdot 2\pi s\sin\varphi_s \frac{\partial}{\partial(s^2)}\frac{1}{\sqrt{(\dots)}} =$

$= (M_s \delta J L R)\,8\pi c^2 s\sin\varphi_s \frac{\partial}{\partial(s^2)}\int d\rho\rho \frac{1}{\sqrt{(c^2+\rho^2+s^2)^2 - 4s^2\rho^2}} = |\rho^2 = x,\ d\rho\cdot 2\rho = dx| =$

$= (M_s \delta J L R)\,4\pi c^2 s\sin\varphi_s \frac{\partial}{\partial(s^2)} J(c,s) = (M_s \delta J L R)\,4\pi c^2 (s_y)\frac{\partial}{\partial(s^2)} J(c,s)$

$\hat{z}\times\vec{s} = -\hat{x}s_y + \hat{y}s_x$, $\left(\vec{m}\times\frac{\partial\vec{m}}{\partial s_y}\right)_z = -\frac{4}{c^2}\frac{1}{(1+\rho\bar{\rho})^2}(x-s_x)$. $x = \rho\cos\varphi \longrightarrow$

$\to \rho\cos(\varphi'+\varphi_s) = \rho\cos\varphi'\cos\varphi_s - \rho\sin\varphi'\sin\varphi_s \to \rho\cos\varphi\cos\varphi_s$, $(x-s_x)\to(\rho\cos\varphi - s)\cos\varphi$

$F^y_{ST} = (M_s \delta J L R)\,4\pi c^2 (-s_x)\frac{\partial}{\partial(s^2)} J(c,s)$, $\vec{F}_{ST} = (\hat{z}\times\vec{s})\,a(s)$,

$a(s) = -4\pi c^2 (M_s \delta J L R)\frac{\partial}{\partial(s^2)} J(c,s)$, так как 13/XII получили и 14/XII, 22/XII

использовали $a(s) = -\frac{4\pi}{c^2}(M_s L R \delta J)\frac{\partial}{\partial(s^2)} J(c,s)$. Отличие в фор. множителе c^4

из-за $\int d^2\vec{\rho}\,\frac{(\cdot)}{(1+\vec{\rho}\bar{\rho})^2} = c^4\int d^2\vec{\rho}\,\frac{(\cdot)}{[c^2+\rho^2+s^2-2s\rho\cos(\varphi-\varphi_s)]^2}$.

Теперь $\frac{B(s)}{a(s)} = -2c\cos\psi \frac{1}{\frac{\partial}{\partial(s^2)}J(c,s)}\int_0^1 d\rho \frac{\rho^2}{(1-\rho^2)^{3/2}}\frac{1}{(c^2+\rho^2+s^2)^3}$ и J_0 другое.

$B(s)/a(s) = (1\div 2)$ дает $c = 0{.}5$ (напомним $c = 0{.}1 - 0{,}2$ от 22/XII).

$W_{ex} = \int dV\, w_{ex} = \int dV \frac{8A}{(1+f\bar{f})^2}\left|\frac{\partial f}{\partial z}\right|^2 = \frac{8AL}{c^2}\int d^2\vec{\rho}\,\frac{1}{(1+f\bar{f})^2} = |\rho \text{ in units of } R| =$

$= \frac{8AL}{c^2}\int d^2\vec{\rho}\,\frac{1}{\left[1+\frac{1}{c^2}|\vec{\rho}-\vec{s}|^2\right]^2} = \frac{8AL}{c^2}c^4\int_0^1 d\rho\rho\int_0^{2\pi} d\varphi \frac{1}{[c^2+\rho^2+s^2-2\rho s\cos(\varphi-\varphi_s)]^2} = 8AL c^2\int_0^1 d\rho\rho\, J_\varphi$

$J_\varphi = \int_0^{2\pi} d\varphi \frac{1}{[c^2+\rho^2+s^2-2\rho s\cos\varphi]^2} = -\frac{\partial}{\partial(c^2)}\frac{2\pi}{\sqrt{(c^2+\rho^2+s^2)^2-4\rho^2 s^2}}$, см. 22/XII.

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$W_{ex} = 8A'Lc^2 \int_0^1 ds\, s\,(-2\pi)\frac{\partial}{\partial(c^2)} \frac{1}{\sqrt{(c^2+s^2+s^2)^2 - 4s^2s^2}} = -16\pi A'Lc^2 \frac{\partial}{\partial(c^4)} J(c,s)$ для P_2 case.

Ур-е на $g_0^2(J)$ P_2 case $a(s) = \frac{2dc}{R}\frac{\partial}{\partial(s^2)} W$, $W = W_{ex}$ поиск. $\frac{1}{2}w_3 J^2 \cdot g\,dg = \frac{1}{2}dx$, $x = g^2$.

$-4\pi c^2 (M_s \delta J L R)\frac{\partial}{\partial(s^2)} J(c,s) = -16\pi A'Lc^2 \left(\frac{2dc}{R}\right)\frac{\partial}{\partial(s^2)}\frac{\partial}{\partial(c^2)} J(c,s)$

$\frac{\partial}{\partial(s^2)} J(c,s) = J_1(c,s)$. $-4\pi c^2 (M_s \delta J L R) J_1(c,s) = -8\pi A'Lc^2 \frac{2dc}{R}\frac{\partial}{\partial(c^2)} J_1(c,s)$

$J/J_0 = -\frac{1}{J_1(c,s)}\frac{\partial}{\partial(c^2)} J_1(c,s)$, $J_0 = \frac{16\pi dc\,|A'|L c^2}{R\cdot 4\pi c^2 M_s \delta L R} = 4\frac{dc\,|A'|}{M_s \delta R^2}$

J_0 сократил свое значение (ком 22/XII) из-за компенсации дополнительных c^4-множителей, появившихся в $a(s)$ и $W_{ex}(s)$.

При вычислении $\vec{P}$-vortex case c^4 в $B(s)$ и $W_{ex}(s)$ не теряем, $s_0^2(J)$ сократ. был.

$\vec{P}$-vortex: $B(s) = \frac{2dc}{R}\frac{\partial W}{\partial(s^2)}$ eq. for $s_0^2(J)$. Для расчета J_{c1} $W_{ex}(s)$ вычисляем как в P_2-case, описанном выше.

$W_{ex}(s) = 8\pi |A'| Lc^2 \frac{\partial}{\partial(c^2)} J(c,s)$.

$B(s) = 8\pi c^3 (M_s \delta J L R) J_B(c,s)\cos\psi$, $J_B(c,s) = \int_0^1 \frac{ds\, s^2}{(1-s^2)^{3/2}} \frac{1}{(c^2+s^2+s^2)^3}$

$\cos\psi\, 8\pi c^3 (M_s \delta L J R) J_B(c,s) = \frac{2dc}{R}\frac{\partial}{\partial(s^2)} 8\pi |A'| Lc^2 \frac{\partial}{\partial(c^2)} J(c,s)$

Введем $J_1 = \frac{J}{J_0}\frac{dc\,|A'|}{M_s \delta R^2}$, $8\pi c^3 \cos\psi\, M_s \delta J L R J_B = \frac{16\pi}{R} Lc^2 \cdot \frac{c}{4} M_s \delta R^2 J_1 \frac{\partial}{\partial(s^2)}\frac{\partial}{\partial(c^2)} J(c,s)$

$J\cos\psi \cdot J_B(c,s) = \frac{1}{2} J_1 \frac{\partial}{\partial(s^2)}\frac{\partial}{\partial(c^2)} J(c,s)$, $x = \frac{J\cos\psi}{J_1}$

$x \cdot J_B(c,s) = \frac{1}{2}\frac{\partial}{\partial(s^2)}\frac{\partial}{\partial(c^2)} J(c,s)$ – eq. для $s_0^2(J)$ $\vec{P}$-vortex, где $W_{ex}(s)$ считается как в P_2 case.

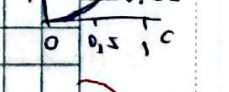
При $J = J_{c1}$, $x = x_1$, $s_0(J_{c1}) = 0$. Надо разложить $J(c,s)$ до s^2.

$J(c,s)\big|_{s\to 0} = J(c,0) - \frac{s^2}{(1+c^2)^2}$, $\frac{\partial J}{\partial(s^2)}\big|_{s\to 0} = -\frac{1}{(1+c^2)^2}$, $\frac{\partial}{\partial(s^2)}\frac{\partial J}{\partial(c^2)} = \frac{\partial}{\partial(c^2)}\frac{(-1)}{(1+c^2)^2} =$

$= \frac{2}{(1+c^2)^3}$. $x_1 J_B(c,0) = \frac{1}{(1+c^2)^3}$.

$J_B(c,s=0) = \frac{(1-c^2)}{8c^2(1+c^2)^2} + \frac{1}{8c^3}\operatorname{arctg}\frac{1}{c}$, $\frac{1}{x_1} = \frac{1}{8c^2}(1-c^4) + \frac{1}{8}\frac{(1+c^2)^3}{c^3}\operatorname{arctg}\frac{1}{c} = F_1(c)$

$F_1(0,s) = 2{,}63$, $x_1 = \frac{1}{F_1(0,s)} = 0{,}38$. $J_{c1}\cos\psi = J_1 \frac{1}{F_1(c)}$, $J_{c1}(c)\cos\psi = \frac{J_1}{F_1(c)}$, $\frac{J_{c1}}{J_0}$

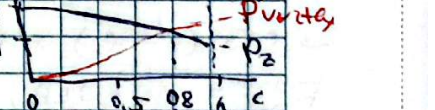


$J_{c1}(c)$ of $\vec{P}$-vortex case max for $c \ll 1$, но быстро падает с c.

Ошибка 22/XII при расчете $J_{c1}(\vec{P}_{vort})$ была в том, что в $W_{ex}(P_2)$ пропущен c^4. !

Для P_2 case $J_{c1} = \frac{2}{1+c^2} J_0$, для vortex $\vec{P}$ $J_{c1}\cos\psi = \frac{J_1}{F_1(c)}$ и $J_1 = \frac{1}{c} J_0$.

$\frac{J_{c1}(P_2)}{J_0} = \frac{2}{(1+c^2)}$ и $\frac{J_{c1}(\text{vort}\,\vec{P})}{J_0}\cdot\cos\psi = \frac{1}{cF_1(c)}$



При $c < 0{,}8$ $J_{c1}(\vec{P}_{vort}) < J_{c1}(P_2)$, $c = R_c/R$.

$F_1(c) = |c\to 0| = \frac{1}{8}\frac{1}{c^2} + \frac{1}{8c^3}\left(\frac{\pi}{2} - c\right) \approx \frac{\pi}{16}\frac{1}{c^3}$, $\frac{J_{c1}(c)}{J_0} \sim c^2$ при $c \ll 1$ для vortex $\vec{P}$.

$J_{c1}/J_1 \sim c^3 \ll 1$, for review Garcia-Sanchez 16 claimed $J_{c1}(\text{vort}) = 0$.

24/XII Написание MS „Skyrmion spin-torque oscillator" for pss-RRL и Adv. Th. Sim.

25/XII $W_{ex}(s)$, $W_{an}(s)$ посчитаны иначе Vc,s в рамках „rigid BP Soliton model" 7/V 2016 между расчета такой не как 14/XII, но общее одно $\frac{\partial}{\partial(c^2)} J(c,s)$, $c^2 = x$

$W_{ex}(s) = 8\pi AL\left[1 - \frac{c^2}{\sqrt{(1+c^2+s^2)^2 - 4s^2}}\,\frac{1+c^2+s^2+\sqrt{(\dots)}}{1+c^2-s^2+\sqrt{(\dots)}}\right]$, $\sqrt{(\dots)} = \sqrt{(1+c^2+s^2)^2 - 4s^2}$

$W_{ex}(s) = 8\pi AL\left[1 - \frac{c^2}{Q(c,s)}\,\frac{1+c^2+s^2+Q(c,s)}{1+c^2-s^2+Q(c,s)}\right]$, $Q(c,s) = \sqrt{(1+c^2+s^2)^2 - 4s^2}$.

$W_{an}(s) = 4c^2 KV\left[\ln\left\{\frac{1}{2c^2}\left(1+c^2-s^2+Q(c,s)\right)\right\} + \frac{c^2}{Q(c,s)}\cdot\frac{1+c^2+s^2+Q(c,s)}{1+c^2-s^2+Q(c,s)} - 1\right]$, $W_{an} = -K\int dV \cos^2\theta + KV$

$W_{an} = K\int dV \sin^2\theta$, $V = \pi R^2 L$ – dot volume. // См. paper Skyrmion mass Aug.-Sept. 2016 (!)

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Уже $W_m = 2\pi M_s^2 \int dW \cos^2\theta$ дало тип dot gaca $K \to K = K - 2\pi M_s^2 = 2\pi M_s^2 (Q-1)$

Steady Sk.-orbit radius из eq. $a(s) = \frac{2d_0}{R}\frac{\partial}{\partial s^2} W$, $W = W_{ex} + W_z + W_m + W_a + W_J$

ЗМІ-WD берет к перенормировке $A \to A' = A - \frac{1}{2}|D|R_c^2$, W_m уже берет $K \to K' = K - 2\pi M_s^2$.

Осталось найти W_J - Zeeman energy of $\vec{m}$ in Oe-field of CPP current.

Oe-field

$W_J = -M_s \int dV\, \vec{m}(\vec{r},\vec{x}) \cdot \vec{H}_J$, $\vec{H}_J = (0, H_J^\varphi, 0) = H_J \hat{\varphi}$ is centered at $\rho = 0$.

$H_J = H_J(\rho)$.

$W_J = -M_s L \int d\rho\rho \int d\varphi\, m_\varphi(\rho,\varphi) H_J(\rho)$, $m_\varphi = -m_x \sin\varphi + m_y \cos\varphi = \mathrm{Im}\left[e^{-i\varphi}(m_x + i m_y)\right]$

$m_\varphi = \sin\Theta \sin(\Phi - \varphi)$, $m_\rho = \sin\Theta\cos(\Phi-\varphi)$, $m_x + i m_y = \frac{2f(z)}{(1+ff^*)}$ for skyrmion.

$m_\varphi = \mathrm{Im}\left[\frac{2}{1+f\bar{f}} e^{-i\varphi} f\right] = \frac{2}{(1+f\bar{f})} \mathrm{Im}\left[e^{-i\varphi} \frac{1}{c} e^{i\Phi_0}(z - s)\right]$.

Ибо $\sin^2\Theta = 1 - \cos^2\Theta = 1 - \left(\frac{1-f\bar{f}}{1+f\bar{f}}\right)^2 = \frac{1}{(1+f\bar{f})^2}\left[(1+f\bar{f})^2 - (1-f\bar{f})^2\right] = \frac{4f\bar{f}}{(1+f\bar{f})^2}$

$f = |f| e^{i\Phi}$, $e^{i\Phi} = \frac{f}{|f|} = \frac{f}{\sqrt{f\bar{f}}} = \sqrt{f/\bar{f}}$, $i\Phi = \frac{1}{2}\ln\frac{f}{\bar{f}} = i\Phi(s)$

$m_\varphi = \frac{2}{c}\frac{1}{(1+f\bar{f})} \mathrm{Im}\left[e^{i\Phi_0}(z e^{-i\varphi} - s e^{-i\varphi})\right] = \left|z = \rho e^{i\varphi},\ s = u e^{i\varphi_s},\ u = |s|\right| =$

$= (\dots) \mathrm{Im}\left[\rho e^{i\Phi_0} - e^{i\Phi_0} u e^{i(\varphi_s - \varphi)}\right] = \frac{2}{c(1+f\bar{f})}\left[\rho\sin\Phi_0 - |s|\sin(\Phi_0 + \varphi_s - \varphi)\right]$

$\int_0^{2\pi} d\varphi\, m_\varphi = \frac{2}{c}\int_0^{2\pi} d\varphi \frac{\left[\rho\sin\Phi_0 - s\cdot\sin(\Phi_0 + \varphi_s - \varphi)\right]}{(1+f\bar{f})}$, $f\bar{f} = \frac{1}{c^2}\left(\rho^2 + s^2 - 2\rho s\cos(\varphi - \varphi_s)\right)$

$\varphi' = \varphi - \varphi_s$, $\sin(\Phi_0 - \varphi') = \sin\Phi_0\cos\varphi' - \cos\Phi_0\sin\varphi'$, $\int_0^{2\pi(\pi)} d\varphi' \frac{\sin\varphi'}{F(\cos\varphi')} = 0$

$\int_0^{2\pi} d\varphi\, m_\varphi = \frac{2}{c}\sin\Phi_0 \int_0^{2\pi} d\varphi \frac{(\rho - s\cos\varphi)}{1 + \frac{1}{c^2}(\rho^2 + s^2 - 2\rho s\cos\varphi)} = 2c\sin\Phi_0 \int_0^{2\pi} d\varphi \frac{(\rho - s\cos\varphi)}{c^2 + \rho^2 + s^2 - 2\rho s\cos\varphi} =$

$\neq 2c\sin\Phi_0 \frac{1}{2}\frac{\partial}{\partial s}\int_0^{2\pi} d\varphi \ln\left(c^2 + \rho^2 + s^2 - 2\rho s\cos\varphi\right) \simeq c\sin\Phi_0 \frac{\partial}{\partial s}\int_0^{2\pi} d\varphi \ln\left[a \pm b\cos\varphi\right] = c\sin\Phi_0 \frac{\partial}{\partial s} \times$

$\times 2\pi\left[\ln\left(a + \sqrt{a^2 - b^2}\right) - \ln 2\right]$, $a^2 - b^2 = (c^2 + \rho^2 + s^2)^2 - 4\rho^2 s^2$, $a = c^2 + \rho^2 + s^2$, $b = 2\rho s$

$\to \int_0^{2\pi} d\varphi\, m_\varphi = 2\pi c\sin\Phi_0 \frac{\partial}{\partial s}\ln\left(a + \sqrt{a^2 - b^2}\right) = 2\pi c\sin\Phi_0 \frac{1}{a + \sqrt{a^2 - b^2}}\left[\frac{\partial a}{\partial s} + \frac{1}{2\sqrt{a^2 - b^2}}\left(2a\frac{\partial a}{\partial s} - 2b\frac{\partial b}{\partial s}\right)\right] =$

$= \frac{2\pi c\sin\Phi_0}{a + \sqrt{a^2 - b^2}}\left[2s + \frac{1}{\sqrt{a^2 - b^2}}\left\{a\cdot 2s - 2\rho s\cdot 2\rho\right\}\right] = \frac{2\pi c\cdot 2s\sin\Phi_0}{a + \sqrt{a^2 - b^2}}\left[1 + \frac{(\rho^2 - 2\rho^2)}{\sqrt{a^2 - b^2}}\right] = 0$ при $s = 0$?

Нет. надо $\to \int_0^{2\pi} d\varphi\, m_\varphi = 2c\sin\Phi_0 \frac{1}{2}\frac{\partial}{\partial s}\int_0^{2\pi} d\varphi \ln\left(c^2 + \rho^2 + s^2 - 2\rho s\cos\varphi\right) = c\sin\Phi_0 \frac{\partial}{\partial s} \times$

$\times 2\pi\left[\ln\left(a + \sqrt{a^2 - b^2}\right) - \ln 2\right] = \frac{2\pi c\sin\Phi_0}{a + \sqrt{a^2 - b^2}}\left[\frac{\partial a}{\partial s} + \frac{1}{\sqrt{a^2 - b^2}}\left(a\frac{\partial a}{\partial s} - b\frac{\partial b}{\partial s}\right)\right] =$

$= \frac{2\pi c\sin\Phi_0}{a + \sqrt{a^2 - b^2}}\left[2s + \frac{1}{\sqrt{a^2 - b^2}}\left(a\cdot 2s - 2\rho s\cdot 2s\right)\right] = \frac{4\pi c\rho\sin\Phi_0}{a + \sqrt{a^2 - b^2}}\left[1 + \frac{a - 2s^2}{\sqrt{a^2 - b^2}}\right]$ в reg. R

Интеграл по ρ забывает от $H_J(\rho)$. // Видно, что $W_J \neq 0$ только если $\Phi_0 = \pm\frac{\pi}{2}$, Bloch-Sk.

Ур-е Maxwell eq. $\mathrm{rot}\vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}$ при $\vec{E} = 0$ $\mathrm{rot}\vec{B} = \mu_0 \vec{J}$, $\vec{J}$ - current density

$\int_S d^2\vec{\rho}\, \mathrm{rot}\vec{B} = \mu_0 \int_S d^2\vec{\rho}\, \vec{J} = \mu_0 I$, $\int_S d\vec{S}\,\mathrm{rot}\vec{B} = \oint_C d\vec{l}\cdot\vec{B}$, $\vec{B} = \mu_0\vec{H}$ ($\vec{S} \parallel \vec{J}$)



Для контура на расст. ρ, C_ρ: $\oint_{C_\rho} d\vec{l}\cdot\vec{B} = \oint \rho d\varphi \hat{\varphi}\cdot\vec{B} = \rho\oint d\varphi B_\varphi =$

$= \mu_0 \int_{S_\rho} d^2\vec{\rho}\, \vec{J}\cdot\hat{z} = \mu_0 J_z \int_{S_\rho} d^2\vec{\rho} = \mu_0 J_z \cdot \pi\rho^2$, $2\pi\rho B_\varphi(\rho) = \mu_0 J \pi\rho^2$, $B_\varphi(\rho) = \frac{1}{2}\rho\mu_0 J$, $[J] = \frac{A}{m^2}$

или $H_\varphi(\rho) = \frac{1}{2}\rho J$. $J = \mathrm{const}$, SI units. $J = I/(\pi R^2)$, R - dot radius. $[H_\varphi] = A/m$.

$W_J(s) = -M_s L \int d\rho\rho\, H_J(\rho) \int d\varphi\, m_\varphi(\vec{\rho},\vec{s}) = \left|H_J = k H_\varphi,\ k - \text{коэфф. пересчета } A/m \text{ в Oe}\right| =$

$= -M_s L k \int_0^R d\rho\rho \left(\frac{1}{2}\rho J\right)\cdot\left[\frac{4\pi c\rho\sin\Phi_0}{a + \sqrt{a^2 - b^2}}\left[1 + \frac{a - 2s^2}{\sqrt{a^2 - b^2}}\right]\right]$ в reg. R $= -M_s L k R^3 J \int_0^1 d\rho\rho^2 \left[\frac{4\pi c\rho\sin\Phi_0}{a + \sqrt{a^2 - b^2}}[\dots]\right]$

$W_J(s) = -2\pi c\sin\Phi_0 k L R^3 M_s J \int_0^1 d\rho \frac{\rho^3}{a + \sqrt{a^2 - b^2}}\left[1 + \frac{a - 2s^2}{\sqrt{a^2 - b^2}}\right]$, $a^2 - b^2 = (c^2 + \rho^2 + s^2)^2 - 4\rho^2 s^2$

Oe-field term

$W_J(s) = -2c\sin\Phi_0 V k R J M_s \cdot J_c(s)$, $J_c(s) = \int_0^1 d\rho \frac{\rho^3}{a + \sqrt{a^2 - b^2}}\left[1 + \frac{a - 2s^2}{\sqrt{a^2 - b^2}}\right]$, $V = \pi R^2 L$, $c = \frac{R_c}{R}$

$kRJ = h_J$ - пр. поле вихря, $[R] = m$, $[J] = A/m^2$, $[h_J] = k[RJ] = k[A/m] = Oe$.

Для V сферы $R \sim L$ в CGS, $[R] = [L] = cm$. $W_J(s) = -2c\sin\Phi_0 (h_J M_s V)\cdot J_c(s)$.

Escaneado con CamScanner

$J = 1\ MA/cm^2 = 1\cdot10^6 A\ \frac{1}{10^{-4}m^2} = 1\cdot10^{10}\frac{A}{m^2}$ и $R = 100$ nm Coerc. field $\to RJ = 10^2\cdot10^{-9}m\cdot10^{10}\frac{A}{m^2} = 10^3\frac{A}{m}$.

$1\ Oe = \frac{1}{4\pi}10^3 A/m$, т.е. плотность тока $J = 1\ MA/cm^2$ и $R = 100$ nm Coerc. поле $kRJ = h_J = 4\pi\ Oe \approx 12\ Oe$

Critical ST current $J_0 = 4\frac{dG|A'|}{M_S\delta R^2}$, $\delta = \frac{hy}{2|e|L\mu_0 M_S}$ в SI.

Com. 19/XII 2013

При $M_S = 800$ G, $y = 0.2$ (RM), $L = 5$ nm, $\delta = 0.131\cdot10^{-7}$ m.

$\delta(L) = \frac{5\ nm}{L(nm)}\cdot0.131\cdot10^{-7}$ m при $L = 1$ nm $\delta(1nm) = 0.655\cdot10^{-7}$ m.

Typical $\delta J_0 = 4\cdot\frac{0.1\cdot10^{-6}\ erg/cm}{10^3\ G\cdot(100\ nm)^2} = 4\cdot\frac{0.1\cdot10^{-6}\ erg}{cm\cdot10^3 G\cdot10^{-10}cm^2} = 0.4\cdot10\ (Oe) \approx 4\ Oe$

т.е. искал ST-current J_0, coerc. ST mag. field $\delta J_0 \approx 4$ Oe, а $J = 1\ \frac{MA}{cm^2}$ Coerc. max. field $h_J \approx 12$ Oe. Gonzalez-Sanchez: δ max J up to 60 GA/m^2 $= 60\cdot10^9 A\ \frac{1}{10^4 cm^2} = 60\cdot10^5\ A/cm^2 = 6\ MA/cm^2$, т.е. $h_J \approx 6\cdot12\ Oe = 72\ Oe$

Zeeman energy $W_J(s) = -2\pi\sin\Phi_0(h_J M_S V)J_c(s)$. Какая величина $J_c(s)$ для typical c,s? Coerc. соответствует $H_0 = \delta J_0 = 4$ Oe to $J_0(SI) = \frac{1}{4\pi}10^3 A/m \approx 300\frac{A}{m}$ или при $\delta \approx 1\cdot10^{-7}$ m

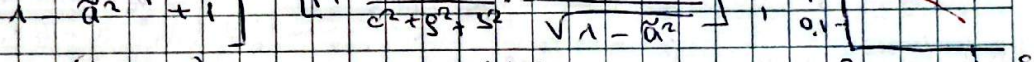
$J_0 = \frac{1}{\delta}H_0 = 3\cdot10^2\frac{A}{m}\ \frac{1}{10^{-7}m} = 3\cdot10^2\cdot10^7\frac{A}{m^2} = 3\ \frac{GA}{m^2} \approx 0.3\ \frac{MA}{cm^2}$

J_0 (Skyrmion) $\ll J_{c1}$ (vortex) ≈ 3.5 MA/cm² из Gus'NRL'14.

$J_{c1}(P_z\text{-Skyrm}) \approx 1.6 J_0$, $J_{c1}(\vec{P}\text{vort/st}) \approx 0.8 J_0$ при $c = 0.5$.

$$J_c(s) = \int_0^1 d\varphi\ \frac{s^3}{c^2+s^2+s^2+\sqrt{(c^2+s^2+s^2)^2-4s^2s^2}}\left[1+\frac{c^2+s^2-s^2}{\sqrt{(c^2+s^2+s^2)^2-4s^2s^2}}\right] =$$

$$= \int_0^1 d\varphi\ \frac{s^3}{(c^2+s^2+s^2)\left[\sqrt{1-\tilde{a}^2}+1\right]}\left[1+\frac{c^2+s^2-s^2}{c^2+s^2+s^2}\cdot\frac{1}{\sqrt{1-\tilde{a}^2}}\right],$$



$W_J(c=0.5, s) \approx -(h_J M_S V)\cdot0.2$ или $w_J = \frac{1}{V}W_J \approx h_J M_S\cdot0.2 = 0.2\cdot12\ Oe\cdot10^3 G = 2.4\cdot10^3\ \frac{erg}{cm^3}$

Тогда как $W_{ex} \sim \frac{|A'|}{R^2} = \frac{10^{-6}erg/cm}{(100\ nm)^2} = \frac{10^{-6}erg}{cm\cdot10^{10}cm} \approx 10^4\frac{erg}{cm^3}$ (при $J = 1\ \frac{MA}{cm^2}$)

Вывод: для Bloch-Skyrmion W_J сравнима с W_{ex} даже при $J = 1\ \frac{MA}{cm^2}$ relatively compact $J_c(s)$ убывает с $s\uparrow$, $W_J(s)$ растет с $s\uparrow$ и ω_G тоже растет с $s\uparrow$ (как для vortex-sto)

$W_{an}(s) = K\int dV\sin^2\theta = 4KV F_a(c,s)$, $F_a(c,s) = c^2\left[\ln\left(\frac{1+c^2-s^2+Q(c,s)}{2c^2}\right) + \frac{c^2}{Q}\frac{1+c^2+s^2+Q}{1+c^2-s^2+Q} - 1\right]$

$F_a(c,s) > 0$, но растет с $s\uparrow$, т.к. при $K = 2\pi M_S^2(Q-1) > 0$, $W_{an}(s)$ растет с $s\uparrow$, а при $Q < 1$, $W_{an}(s)$ растет с $s\uparrow$, это подтверждает вид ω_G: $\omega_G^{lin} \sim (1-Q)$. $W_{an} \sim (1-Q)s^2$.

Note: $F_{ST}(s) \sim s\alpha(s)$ линейна (для P_z case) как для vortex-STO, $F_{ST}(s) \sim sB(s)$ $\vec{P}$ vortex – выпукла до $s = 0.8$, но линейна с $s \geq 0.3$

Итог $W_{an}(s)$, $W_J(s)$ используют eq. $\alpha(s) = \frac{2dG}{R}\frac{\partial W}{\partial(s^2)}$ для $s_0^2(J)$ и для $\omega_G(s)|_{s=s_0(J)}$ orbit radius

26-27/XII Написание Skyrmion – STNO manuscript. Sk – STNO

28/XII Вычисление spin-torque functions $\alpha(s)$, $B(s)$ для P_z и vortex $\vec{P}$ при $s \to 0$.

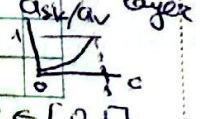
$$\alpha(s) = -4\pi c^2(M_S\delta JLR)\frac{\partial}{\partial(s^2)}J(c,s) = -4\pi c^2(M_S\delta JLR)J_1(c,s)$$

$$J_1(c,s\to0) = \frac{1}{(1+c^2)}\frac{(c^2+1-(1+c^2))}{(c^2+1+(1+c^2))} = -\frac{1}{(1+c^2)^2} < 0$$

$$B(s,c,\psi) = 8\pi c^3(M_S\delta JRL)\cos\psi\int_0^1 d\varphi\ \frac{s^3}{(1-\tilde{a}^2)^{3/2}}\frac{1}{(c^2+s^2+s^2)^3},\quad J_B(s\to0) = \int_0^1 d\varphi\ s^2\times$$

$$\times\frac{1}{(c^2+s^2)^3} = \frac{1}{8c^2}\left[\frac{1+c^2}{(1+c^2)^2} + \frac{1}{c}\operatorname{arctg}\frac{1}{c}\right]\cdot\ \alpha(s\to0) = \frac{4\pi c^2}{(1+c^2)^2}(M_S\delta JLR) - \text{Skyrmion free layer}$$

$$B(s\to0) = \pi c\left[\frac{1-c^2}{(1+c^2)^2} + \frac{1}{c}\operatorname{arctg}\frac{1}{c}\right]\cos\psi\,(M_S\delta JRL),\quad B(s) \sim \cos\psi.$$



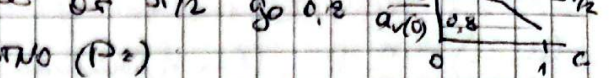
Note: для vortex STNO [Gus NRL'14] с P_z, $\alpha_v(s\to0) = \pi RLM_S\delta J$.

$\alpha_{sk}(0)/\alpha_{vort}(0) = \frac{4c^2}{(1+c^2)^2} = F_s(c)$ растет от 0 до 1 при $c \in [0,1]$.

Escaneado con CamScanner

т.е. ST-force core vortex STNO (P_2) > ST-force skyrmion STNO (P_2) особенно для малых sk-radius $c = R_0/R$. // $B(s,0)/a_{vort}(0) = \left[\frac{c(1-c^2)}{(1+c^2)^2} + \operatorname{arctg}\frac{1}{c}\right]\cos\psi$.

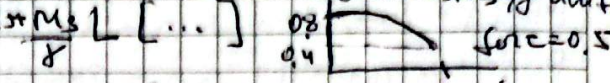
При $\cos\psi = 1$, $B(0)/a_v(0) = F_2(c)$ падает от $\pi/2$ до 0,2



т.е. sk-STNO ST-force > ST-force vortex STNO (P_2)

$W_{ex} = 8\pi|A'|L c^2 \frac{\partial}{\partial(c^2)} J(c,s)$ и exact $W_{ex} = 8\pi AL[\dots]$ от 25 $\frac{pA}{m}$

$|G| = \frac{M_s}{2\gamma A} W_{ex} = \frac{M_s}{2\gamma A} 8\pi AL[\dots] = 4\pi\frac{M_s}{\gamma} L[\dots]$



$|G|(s)$ падает с ростом $u = |s|$, что подтверждается [Garcia-San. 16] [Shen'19]

$Q^2(c,s) = (1+c^2+s^2)^2 - 4s^2 = (1+c^2)^2 + 2s^2(1+c^2) + s^4 - 4s^2 = (c^2+s^2)^2 + 2(c^2+s^2) + 1 - 4s^2 = 1 + 2(c^2-s^2) + (c^2+s^2)^2 \Rightarrow$ $Y_1(c,s) = \frac{1}{Q(c,s)} \frac{[c^2+s^2-1-Q(c,s)]}{[1+c^2-s^2+Q(c,s)]} \Rightarrow$

$Y_1(c,s) = \frac{(-1)}{Q} \frac{[Q+1-s^2-c^2]}{[Q+1-s^2+c^2]}$. $W_{ex} = 8\pi AL\left[1 - \frac{c^2}{Q}\frac{Q+1+c^2+s^2}{Q+1+c^2-s^2}\right]$. Reg. $Y_B(c,s) = -Y_1(c,s)$

$Y_B(c,s) = \int_0^1 d\tilde{s}\, \frac{s^2}{(1-\tilde{s}^2)^{3/2}} \frac{1}{(c^2+s^2+\tilde{s}^2)^3} = \left|\tilde{s} = \frac{2s}{(c^2+s^2+\tilde{s}^2)}\right| = \int_0^1 d\tilde{s}\, \frac{s^2}{\left[(c^2+s^2+\tilde{s}^2)^2 - 4s^2\tilde{s}^2\right]^{3/2}}$

$Y_B(c,s)$ можно взять, но будет очень неудобно.

Насколько велики $W_{an}(s)$ и $W_J(s)$ по сравн. с $W_{ex}(s)$? $K \to 2\pi M_s^2(Q-1)$

$\frac{W_{an}(s)}{W_{ex}(s)} = \frac{KV}{8\pi AL}\frac{[an]}{[ex]} = \frac{2\pi M_s^2(Q-1)V}{8\pi AL}\frac{[an]}{[ex]}$, $\frac{2\pi M_s^2(Q-1)\pi R^2 L}{8\pi AL} = \frac{R^2}{l_{ex}^2}\frac{1}{8}(Q-1)$

$l_{ex}^2 = \frac{A}{2\pi M_s^2} \sim 5$ nm typical. $\frac{W_{an}(s)}{W_{ex}(s)} = \frac{1}{8}\left(\frac{R}{l_{ex}}\right)^2 (Q-1)\frac{4c^2[\dots]}{[1-\frac{c^2}{Q}(\dots)]}$, $\frac{[an]}{[ex]} \approx 1$ $\forall s$ и при $c=0,5$ и падает с ↓ Q.

Уровень $l_{ex} = \sqrt{\frac{1\cdot10^{-6}\,erg}{cm\cdot 2\pi\cdot(10^3 G)^2}} = 10^{-6}\frac{1}{\sqrt{2\pi}}\,cm \approx 4{,}0\,nm$, $\frac{W_{an}(s)}{W_{ex}(s)} \approx \frac{1}{8}(Q-1)\left(\frac{R}{l_{ex}}\right)^2 \sim 10(Q-1)$ при $R = 50$ nm

При $Q=1$ $W_{an}=0$, но при $Q=0.9$, $\frac{W_{an}(s)}{W_{ex}(s)} \approx 1$ оценочно.

$J = 1\,MA/cm^2 = 10\,GA/m^2$ создает $h_J \approx 12\,Oe$. $\frac{W_J(s)}{W_{ex}(s)} = \frac{h_J M_s V}{8\pi AL}\frac{[an]}{[ex]}$, $\frac{[an]}{[ex]} = 0{,}2 \div 0{,}4$ для $c=0,5$

$\frac{h_J M_s V}{8\pi AL} = \frac{h_J M_s \pi R^2 L}{8\pi AL} = \frac{R^2\cdot 2\pi M_s^2}{8A}\frac{h_J}{2\pi M_s} = \frac{1}{2}\left(\frac{R}{l_{ex}}\right)^2\left(\frac{h_J}{2\pi M_s}\right)$, $\frac{h_J}{2\pi M_s} \approx \frac{12\cdot Oe}{6\cdot10^3 G} \approx 2\cdot10^{-3}$ для $J = 10\frac{GA}{m^2}$

$\frac{W_J(s)}{W_{ex}(s)} \approx \frac{1}{2}\left(\frac{R}{l_{ex}}\right)^2\left(\frac{h_J}{2\pi M_s}\right) \sim \frac{1}{4}10^{-3}\left(\frac{R}{l_{ex}}\right)^2$. При $R/l_{ex} = 10$, $\frac{W_J(s)}{W_{ex}(s)} \approx 10^{-1}$ для $J = 10\div40\frac{GA}{m^2}$

Но с ростом R важность W_{an} и W_J растет как $\sim R^2$.

ST-force зависит от $M_s \mathcal{E} L = M_s L \frac{\hbar\gamma}{2e|L|M_s} = \frac{\hbar}{2|e|}\gamma$, flux quantum (SI) $= \frac{h}{2e} = 2{,}068\cdot10^{-15}$ Wb

$1\,Wb = T\cdot m^2 = \frac{kg\cdot m^2}{A\cdot s^2}$, $M_s \mathcal{E} L = \frac{\gamma}{2\pi}\Phi_0 = \frac{\gamma}{2\pi}2{,}068\cdot10^{-15}\,Wb$, $Wb = \frac{Joule}{A}$

F_{ST} scales $\sim M_s \mathcal{E} L J R = \frac{\gamma}{2\pi}2{,}068\cdot10^{-15} Wb\cdot J R$, $M_s \mathcal{E} L J = \frac{\gamma}{2\pi}2{,}068\cdot10^{-15}\frac{Wb}{m}\cdot J$

При $J = 1\,MA/cm^2 = 10\,GA/m^2$, $M_s \mathcal{E} L J = \frac{\gamma}{2\pi}2{,}068\cdot10^{-15}\frac{Joule}{A}\cdot10\frac{GA}{m^2} = \frac{\gamma}{2\pi}2{,}068\cdot10^{-5}\frac{Joule}{m^2}$

$Joule/m^2 = 10^7 erg/10^4 cm^2 = 10^3 erg/cm^2$.

Токи $J_0 = 4dc\frac{|A'|}{M_s \mathcal{E} R^2}$ создают $M_s \mathcal{E} L J_0 = M_s \mathcal{E} L\, 4dc\frac{|A'|}{M_s \mathcal{E} R^2} = 4dc L\frac{A'}{R^2} = \frac{2}{\pi}dc M_s^2 L\left(\frac{l_{ex}}{R}\right)^2$,

если взять $l_{ex}^2 = |A'|/(2\pi M_s^2)$. Токи $J = 1\,MA/cm^2$ создают $(M_s \mathcal{E} L J) = \frac{\gamma}{2\pi}2{,}068\cdot10^{-2}\frac{erg}{cm^2}$.

$\gamma = 0 \div 1$ - spin polarization of CPP current. При $R = 100$ nm $\frac{1}{R}(M_s L \mathcal{E} J) = \frac{\gamma}{2\pi}\cdot2{,}07\cdot10^3\frac{erg}{cm^3}$ - малая energy density, но она сравн. с $dc W_{ex}$ - малой.

Note: у Garcia-Sanchez'16 $R_c \approx 1$ nm, когда 1-SD state, не удивительно, что $W(s)$ слабо зависит от s.

$W_{an} \sim (Q-1)$ можно приближенно считать $Q = 1$, а $W_J \sim J$ и минимум ищется просто

(29) XII Orbit sk radius α $a(s) = \frac{2d_G}{R}\frac{\partial W}{\partial(s^2)}$ | $a_p(s) = 4\pi c^2 (M_s 2L) JR\, I_p(c,s) - P_z$

$a_v(s) = 8\pi c^3 (M_s 2L)\cos\psi \cdot JR\, I_v(c,s)$ — vort $\vec{P}$

$I_p(c,s) = \frac{1}{Q(c,s)}\frac{Q+1-s^2-c^2}{Q+1-s^2+c^2}$, $Q^2(c,s) = (1+c^2+s^2)^2 - 4s^2$, $I_v(c,s) = \int_0^1 d\varphi \frac{\varphi^2}{[(\varphi^2+c^2+s^2)^2 - 4\varphi^2 s^2]^{3/2}}$

$(M_s 2L) = \frac{\gamma}{2\pi}\Phi_0$, flux quantum $\Phi_0 = 2{,}068\cdot 10^{-15}$ Wb. $\Phi_0 \to \Phi_q$. γ – spin polarizat.

Другие $\frac{1}{R}a(s) = \frac{2d_G}{R^2}\frac{\partial W}{\partial(s^2)}$, $\frac{1}{R}a_p(s) = 4\pi c^2\left(\frac{\gamma\Phi_q}{2\pi}\right) J\, I_p(c,s)$, $\frac{a_v(s)}{R} = 8\pi c^3\cos\psi\left(\frac{\gamma\Phi_q}{2\pi}\right) J\, I_v(c,s)$

$[\Phi_q J] = $ Joule/m² или erg/cm².

P_z: $2c^2(\gamma\Phi_q) J\, I_p(c,s) = \frac{2d_G}{R^2}\frac{\partial W}{\partial(s^2)}$. $\vec{P}_{vort}$: $4c^3\cos\psi\,(\gamma\Phi_q) J\, I_v(c,s) = \frac{2d_G}{R^2}\frac{\partial W}{\partial(s^2)}$

Нам уже можно W_{ex} 23/III обмен: $P_z \to J/J_0 = -\frac{1}{J_1(c,s)}\cdot\frac{\partial J_1(c,s)}{\partial(c^2)}$, $\vec{P}_v \to \varkappa J_B(c,s) = \frac{1}{2}\frac{\partial}{\partial(c^2)} J_1(c,s)$

$P_z \to -\frac{J}{J_0} J_1(c,s) = \frac{\partial J_1(c,s)}{\partial(c^2)}$, $\vec{P}_v \to 2\frac{J}{J_1}\cos\psi\, J_B(c,s) = \frac{\partial J_1(c,s)}{\partial(c^2)}$. $\varkappa = \frac{J}{J_1}\cos\psi$. $J_B(c,s) = I_v(c,s)$

$J_0 = 4\frac{d_G|A'|}{M_s 2R^2}$, $J_1 = \frac{4}{c}\frac{d_G|A'|}{M_s 2R^2}$. $W = W_{ex} + W_{an} + W_J$ (γM_I in W_{ex}, W_m in W_{an}).

Опр. $b_p = 2c^2\gamma\Phi_q I_p(c,s)$, $b_v = 4c^3\cos\psi\,\gamma\Phi_q I_v(c,s)$, тогда $b(s)J = \frac{2d_G}{R^2}\frac{\partial W}{\partial(s^2)}$, $b = b_p, R$

$W = W' + W_J$, т.е. вводим W_J. $W_J(s) = -2c\sin\Phi_0 h_J M_s V J_c(s)$,

$h_J = kRJ$, k – коэф. перехода (A/m) to Oe, $k = \frac{4\pi}{10^3}$ Oe/(A/m), $(\alpha = \frac{10^3}{4\pi}$ (A/m)

$W_J(s) = -2c\sin\Phi_0 kRM_s V J_c(s)\cdot J = -b_J(s)J$, $b_J(s) = 2c\sin\Phi_0 kRM_s V J_c(s)$

$b(s)J = \frac{2d_G}{R^2}\frac{\partial}{\partial(s^2)}[W' + W_J]$, $b(s)J = \frac{2d_G}{R^2}[W' - b_J J] = \frac{2d_G}{R^2}\frac{\partial W'}{\partial(s^2)} - \frac{2d_G}{R^2}J\frac{\partial b_J}{\partial(s^2)}$

Уже W_J берем к $\left[b(s) + \frac{2d_G}{R^2}\frac{\partial b_J}{\partial(s^2)}\right]J = \frac{2d_G}{R^2}\frac{\partial W'}{\partial(s^2)}$. Пусть $W' = W_{ex}$

$W_{ex} = 8\pi|A'|c^2\frac{\partial}{\partial(c^2)}J(c,s)$, $A' < 0$. $\frac{\partial W_{ex}}{\partial(s^2)} = (-)\frac{\partial}{\partial(s^2)}\frac{\partial}{\partial(c^2)}J(c,s) = 8\pi|A'|c^2\frac{\partial}{\partial(c^2)}J_1(c,s)$

$\left[b(s) + \frac{2d_G}{R^2}\frac{\partial b_J}{\partial(s^2)}\right]J = \frac{2d_G}{R^2}8\pi|A'|Lc^2\frac{\partial J_1}{\partial(c^2)}$, $\frac{2d_G}{R^2}\frac{8\pi|A'|L}{J_0} = \frac{16\pi d_G|A'|L}{R^2\cdot 4d_G|A'|} =$

$= 4\pi(M_s 2L) = 4\pi\frac{\gamma}{2\pi}\Phi_q = 2\gamma\Phi_q$. $\left[b(s) + \frac{2d_G}{R^2}\frac{\partial b_J}{\partial(s^2)}\right]\frac{J}{J_0} = 2\gamma\Phi_q c^2\frac{\partial J_1}{\partial(c^2)}(c,s)$

$0 < I_p(c,s) = -J_1(c,s)$. Пусть P_z и $b_J = 0$

$2c^2\gamma\Phi_q(-1)J_1(c,s)\frac{J}{J_0} = 2\gamma\Phi_q c^2\frac{\partial J_1}{\partial(c^2)}$, $\frac{J}{J_0} = -\frac{1}{J_1}\frac{\partial J_1}{\partial(c^2)}$ верно

Т.е. уже W_J сработает и заменит $b(s) \to \left[b(s) + \frac{2d_G}{R^2}\frac{\partial b_J}{\partial(s^2)}\right]$.

$T(s) = \frac{1}{b(s)}\frac{2d_G}{R^2}\frac{\partial b_J}{\partial(s^2)}$, $T_p(s) = \frac{1}{2c^2\gamma\Phi_q I_p(c,s)}\frac{2d_G}{R^2}\frac{\partial}{\partial(s^2)}J_c(s)\cdot 2c\sin\Phi_0 kRM_s V =$

$= \frac{2d_G\sin\Phi_0 kM_s V}{c\cdot\gamma\Phi_q R}(-) = 2\pi d_G\sin\Phi_0\frac{kM_s RL}{c\gamma\Phi_q}\frac{1}{I_p(c,s)}\frac{\partial}{\partial(s^2)}J_c(s)$

$k\frac{1}{Wb} = k\frac{1}{\text{Joule}} = k\frac{(A/m)}{\text{Joule}}\cdot m = \frac{4\pi}{10^3}\frac{1\,Oe}{\text{Joule}}\cdot m = 1\,Oe\,\frac{10^{-2}cm}{10^{-7}erg} = 1\,Oe\cdot 10^{-5}\frac{cm}{erg} =$

$= 1\,Oe\cdot 10^{-5}\frac{cm}{(Oe\cdot G)cm^3} = 1\cdot 10^{-5}\frac{4\pi\cdot 10^{-3}}{G\cdot cm^2}\cdot\frac{kM_s RL}{\Phi_q} = \frac{k}{2{,}068\cdot 10^{-15}Wb}\cdot M_s RL =$

$= \frac{1}{2{,}068}10^{15}\cdot 10^{-5}\frac{1}{G\cdot cm^2}M_s RL = \frac{10^{+10}}{2{,}068}\frac{M_s RL}{G\cdot cm^2}\frac{4\pi}{10^3}$ берем $M_s = 10^3$ G, $R = 100\,nm = 10^{-5}cm$, $L = 1\,nm = 10^{-7}cm$

$\frac{kM_s RL}{\Phi_q} = \frac{10^{10}}{2{,}068}\frac{10^3 G\cdot 10^{-5}\cdot 10^{-7}cm^2}{G\cdot cm^2}\frac{4\pi}{10^3} = \frac{10\cdot 4\pi}{2{,}068\cdot 10^3} = 4{,}84\cdot\frac{4\pi}{10^3}$ ($R = 100\,nm$, $L = 1\,nm$) $= 6{,}08\cdot 10^{-2}$

$T_p(s) = 2\pi d_G\sin\Phi_0\frac{1}{c\gamma}\left(\frac{kM_s RL}{\Phi_q}\right)\frac{1}{I_p(c,s)}\frac{\partial}{\partial(s^2)}J_c(s)$, $J_c(s) = \int_0^1 d\varphi\frac{\varphi^3}{a+\sqrt{a^2-b^2}}\left[1 + \frac{a-2s^2}{\sqrt{a^2-b^2}}\right]$

$d_G = 0{,}1$, $\gamma = 0{,}5$, $\Phi_0 = \pi/2$ $T_p(s) = \frac{2\pi}{5}\left(\frac{kM_s RL}{\Phi_q}\right)\frac{1}{cI_p(c,s)}\frac{\partial}{\partial(s^2)}J_c(s)$, $a = c^2+s^2+s^2$, $b = 2s$

Mathcad даст $\frac{1}{c}\frac{1}{I_p}\frac{\partial}{\partial(s^2)}J_c(s) \approx 0{,}5 \div 1{,}5$ для типичных c,s и $T_p(s) \approx 1\cdot 5\cdot(0{,}5 \div 1{,}5) = (3 \div 8)\frac{4\pi}{10^3}$

Т.е. для P_z Oe-field term в 3-8 раз больше, чем ST-term ($R = 100\,nm$) при $\sin\Phi_0 = 1$.

Escaneado con CamScanner

Может $\frac{k}{W_B}$ - невярно? Да. Получаме W_J в SI-units като ST-force.

$E_J \to W_J = -\mu_0 \int dV \vec{M}\cdot\vec{H}$, $\vec{H} = (0, H_\varphi, 0)$, $H_\varphi = \frac{1}{2} gJ$, $[H_\varphi] = A/m$

$W_J = -\mu_0 M_s \int dV\, m_\varphi H_\varphi = -\mu_0 M_s L \int_0^R ds\, s\,(\tfrac{1}{2}gJ) \int d\varphi\, m_\varphi = -\mu_0 M_s L \tfrac{1}{2} J \int_0^R ds\, s^2 \times$

$\times \left[\frac{\pi R c \sin\Phi_0}{a+\sqrt{a^2-b^2}}\left(1+\frac{a-2s^2}{\sqrt{a^2-b^2}}\right)\right] = -2\pi c \mu_0 M_s J \sin\Phi_0 L R^3 \int_0^1 dg \frac{g^3}{a+\sqrt{a^2-b^2}}\left[1+\frac{a^2-2s^2}{\sqrt{a^2-b^2}}\right] =$

$= -2\pi c \sin\Phi_0 \mu_0 M_s J L R^3 J_c(s)$. $\mu_0 M_s H_J = \mu_0 M_s J R$, $[\mu_0 H^2] = [\mu_0]\frac{A^2}{m^2} =$

$= \frac{H}{m}\frac{A^2}{m^2} = \text{Joule}/m^3$. $W_J(s) = -b_J(s) J$, $b_J(s) = 2\pi c \sin\Phi_0 \mu_0 M_s L R^3 J_c(s)$ (SI)

$\tau(s) = \frac{1}{b(s)} \frac{2d_G}{R^2} \frac{\partial b_J}{\partial(s^2)}$. $\tau_p(s) = \frac{1}{2c^2 \gamma\Phi_q I_p(c,s)} \frac{2d_G}{R^2} \frac{\partial}{\partial(s^2)} J_c(s) \cdot 2\pi c \sin\Phi_0 \mu_0 M_s L R^3 =$

$= 2\pi d_G \sin\Phi_0 \frac{\mu_0 M_s L R}{\gamma\Phi_q} \frac{1}{c I_p(c,s)} \frac{\partial}{\partial(s^2)} J_c(s)$, $\left(\frac{\mu_0 M_s R L}{\Phi_q}\right) = 4\pi\cdot 10^{-7}\frac{H}{m}\frac{M_s R L}{2{,}068\cdot 10^{-15} Wb} =$

$= [Wb = H\cdot A] = 4\pi\cdot 10^{-7}\frac{H}{m}\frac{1}{2{,}07\cdot 10^{-15}\cdot H\cdot A} M_s R L = 10^{-7}\frac{1}{m\cdot A}\frac{1}{2{,}07\cdot 10^{-15}}(4\pi M_s R L)$

$4\pi M_s = 10^3$ kA/m сравнен $M_s = 10^3$ G. $\left(\frac{\mu_0 M_s R L}{\Phi_q}\right) = \frac{1}{m\cdot A}\frac{4\pi\cdot 10^{-7}}{2{,}07\cdot 10^{-15}} M_s R L = \left[\begin{matrix}R=100\,nm\\ L=1\,nm\end{matrix}\right] =$

$= \frac{1}{m\cdot A}\frac{4\pi}{2{,}07} 10^8 \cdot 10^6 \frac{A}{m} 10^{-7} m\cdot 10^{-9} m = \frac{4\pi}{2{,}07} 10^{-2} = \frac{10}{2{,}07}\frac{4\pi}{10^3}$. В k-метода

смятане на множител $4\pi\cdot 10^{-3}$. $\left(\frac{k M_s R L}{\Phi_q}\right)_{CGS} = \left(\frac{\mu_0 M_s R L}{\Phi_q}\right)_{SI} = \frac{10}{2{,}07}\cdot\frac{4\pi}{10^3} = 6{,}08\cdot 10^{-2}$

Получаме $\tau_p(s) = (3\div 8)\cdot\frac{4\pi}{10^3} = (0{,}036 \div 0{,}096) \simeq (0{,}04 \div 0{,}1)$ при $R = 100$ nm.

$\tau_v(s) = \frac{1}{b_v}\frac{2d_G}{R^2}\frac{\partial b_J}{\partial(s^2)} = \frac{1}{4c^3\cos\psi\,\gamma\Phi_q}\frac{1}{I_v}\frac{2d_G}{R^2}\frac{\partial}{\partial(s^2)} 2\pi c\sin\Phi_0 \mu_0 M_s R^3 L J_c(s) = \left|\begin{matrix}\psi=0\\ \Phi_0 = 3\pi/2\end{matrix}\right| =$

$= \frac{\pi}{c^2}\frac{d_G}{\gamma}\left(\frac{\mu_0 M_s R L}{\Phi_q}\right)\frac{1}{I_v(c,s)}\frac{\partial}{\partial(s^2)} J_c(s) = \frac{\pi d_G}{\gamma}(6{,}08\cdot 10^{-2})\frac{1}{c^2 I_v(c,s)}\frac{\partial}{\partial(s^2)} J_c(s)$

$\tau_v(s) = \frac{\pi}{0.5}\cdot 0{,}1\,(6{,}08\cdot 10^{-2})(1\div 2) = (3{,}8 - 7{,}6)\cdot 10^{-2} = (0{,}04 \div 0{,}08)$ поправка на (к J-term ST)

Oe-field $\lesssim 8\%$ при $\vec{J} \times \vec{P}$. Note: $\frac{\partial b_J}{\partial(s^2)} < 0$, $W_J(s) = -J b_J(s)$, $\frac{\partial W_J}{\partial s^2} = -J\frac{\partial b_J}{\partial s^2} > 0$,

т.е. увеличаването s причинява и ръст на $W_J(s)$ при $\sin\Phi_0 \neq 0$ (Bloch-Sk) и при $J>0$, $\sin\Phi_0 > 0$.

Oe-field ($\uparrow J$) H_φ, $\vec{H}_J = (0, H_\varphi, 0)$, $H_\varphi = \frac{1}{2} gJ$, CCW при $J>0$.

$\left[b(s) + \frac{2d_G}{R^2}\frac{\partial b_J}{\partial(s^2)}\right]\frac{J}{J_0} = 2\gamma\Phi_q c^2 \frac{\partial}{\partial(c^2)} J_1(c,s)$, $b(s) = b_p(s)$ или $b = b_v(s)$,

$W = W_{ex} + W_J$

$J_0 = 4\frac{d_G |A'|}{M_s \vartheta R^2}$, $M_s \vartheta L = \frac{h\gamma}{2|e|} = \frac{\gamma}{2\pi}\Phi_q$, $\Phi_q = \frac{h}{2|e|}\frac{2\pi}{2|e|} = \frac{h}{2|e|}$.

P_z: $b_p(s) = 2c^2\gamma\Phi_q I_p(c,s)$ vrtP: $b_v(s) = 4c^3\cos\psi\,\gamma\Phi_q I_v(c,s)$ — разделяме $b(s)$ на $2\gamma\Phi_q$

$\left[b(s) + \frac{2d_G}{\gamma\Phi_q R^2}\frac{\partial b_J}{\partial(s^2)}\right]\frac{J}{J_0} = c^2\frac{\partial}{\partial(c^2)} J_1(c,s)$. разделяме на c^2.

$\left[b(s) + \frac{d_G}{\gamma\Phi_q R^2 c^2}\frac{\partial b_J}{\partial(s^2)}\right]\frac{J}{J_0} = \frac{\partial}{\partial(c^2)} J_1(c,s)$, $I_p(c,s) = -J_1(c,s)$,

$b_p(s) = I_p(c,s)$, $b_v(s) = 2c\cos\psi \cdot I_v(c,s)$

$b_J(s) = 2c\sin\Phi_0 \mu_0 M_s V R J_c(s) = B_J J_c(s)$, $\frac{B_J}{\gamma\Phi_q R^2 c^2} \frac{2c\sin\Phi_0 \mu_0 M_s (\pi R^2 L) R}{\gamma\Phi_q R^2 c^2} =$

$= \frac{2\pi}{c}\sin\Phi_0 \left(\frac{\mu_0 M_s R L}{\gamma\Phi_q}\right)$. $\left[b(s) + \frac{2\pi d_G \sin\Phi_0}{c\gamma}\left(\frac{\mu_0 M_s R L}{\Phi_q}\right)\frac{\partial}{\partial(s^2)} J_c(s)\right]\frac{J}{J_0} = \frac{\partial}{\partial(c^2)} J_1(c,s)$

$S = 2\pi R L$

$\Phi_s = \mu_0 M_s S$-flux

$\left[b(s) + \frac{d_G}{\gamma}\sin\Phi_0 \left(\Phi_s/\Phi_q\right)\frac{1}{c}\frac{\partial}{\partial(s^2)} J_c(s)\right]\frac{J}{J_0} = \frac{\partial}{\partial(c^2)} J_1(c,s)$ eq. $s(J)$.

$S = f(J/J_0)$. $J_0 = \frac{4 d_G |A'|}{M_s \vartheta R^2} = \frac{1}{M_s \vartheta L}\frac{4 d_G |A'| L}{R^2} = \frac{2\pi}{\gamma\Phi_q}\cdot\frac{4 d_G |A'| L}{R^2}$. $[J_0] = \frac{1}{Wb}\frac{\text{Joule}}{m^2} = \frac{A}{m^2}$

$1 = \frac{\Phi_s}{\mu_0 M_s 2\pi R L}$, $J_0 = \frac{2\pi d_G}{\gamma\Phi_q}\frac{|A'| L}{R^2}$, $A' = A - \frac{1}{2}|D|Rc$. $J_0 \sim L/R^2$.

Escaneado con CamScanner

(30/XII) Vons Ex. Sk review is published in J. Magnolias. Как $c_{eq}(s)$ зависит от s?

$W_{ex}(c,s) = 2\pi L A'(c)\left[1 - \frac{c^2}{Q}\frac{(c)}{(c)}\right] = 8\pi L A'(c) I_{ex}(c,s)$, $A' = A - \frac{1}{2}|D|Rc < 0$

$\frac{1}{2\pi AL} W_{ex}(c,s) = \left(1 - \frac{DR}{2A}c\right) I_{ex}(c,s)$, $D = |D| > 0$. $\lambda = \frac{DR}{2A}$, $\lambda = \frac{1\,mJ/m^2 \cdot 100nm}{2\cdot 10 pJ/m} =$ typical

$= \frac{10^{-3} J \cdot 10^{-7} m \cdot m}{m^2 \cdot 2\cdot 10^{-12} J \cdot 10} = 5^-$, $\frac{1}{2\pi AL} W_{ex}(c,s) = (1 - \lambda c) I_{ex}(c,s)$, $\lambda c > 1$

$W_{ex}(c,s)$ имеет min при $c_{eq} \approx 1 + \frac{1}{\lambda}$ и $s=0$, с ростом s, $c_{eq}(s)$ сильно растет. Но $c_{eq} \gtrsim 1$ – это quasi 1-SD state, not skyrmion (это при $K=0$).

$W(c,s) = W_{ex}(c,s) + W_{an}(c,s) = 2\pi AL(1-\lambda c) I_{ex}(c,s) + KV I_K(c,s)$

$\frac{1}{2\pi AL} W(c,s) = (1-\lambda c) I_{ex}(c,s) + \frac{1}{2}(Q-1)\left(\frac{R}{l_{ex}}\right)^2 I_K(c,s)$, $I_K(c,s) = 4c^2[\ln(\ldots) + \ldots]$

$\frac{1}{2\pi AL} W(c,s) = (1-\lambda c) I_{ex}(c,s) + \lambda I_K(c,s)$, $\lambda = \frac{1}{2}(Q-1)\left(\frac{R}{l_{ex}}\right)^2$, $l_{ex}^2 = A/(2\pi M_s^2)$

при $\lambda \sim (Q-1) < 0$, $Q < 1$, $W(c,s)$ имеет min при $c_{eq} = 0{,}6$–$0{,}7$, $c_{eq}(s)$ сильно растет с s.

Сам Aranda IMMM'18 $\frac{1}{4\pi M_s^2 V} E_{sk}(c) = \frac{4}{\sqrt{2}} \frac{1}{1+c^2}\left(1 - \frac{1}{2}|d|c^2\right) + 2(Q+1)c^2\left[\ln\left(1+\frac{1}{c^2}\right) - \frac{1}{1+c^2}\right] + 1/2$

within BP-model at $s=0$. $E_{sk}(c)$ при $Q>1$ дает metastable sk с $c_{eq} \sim 0{,}1$, а при $Q<1$ – stable sk с $c_{eq} \approx 0{,}7$ (large radius sk). BP-ansatz лучше работает при $c \approx 0{,}1$ (small radius skyrmions).

J_{c2} → Second critical current J_{c2} (s=1, skyrmion expelling from dot)

$Q(c,s) = \sqrt{(1+c^2+s^2)^2 - 4s^2}$, $Q(c,s=1) = \sqrt{(2+c^2)^2 - 4} = \sqrt{c^4+4c^2} = c\sqrt{c^2+4} = Q(c,1)$

$I_p(c,s) = \frac{1}{Q(c,s)} \frac{Q+1-s^2-c^2}{Q+1-s^2+c^2}$, $I_p(c,s=1) = \frac{1}{Q(c,1)} \frac{Q(c,1)-c^2}{Q(c,1)+c^2}$

$\frac{Q-c^2}{Q+c^2} = \frac{(Q-c^2)^2}{(Q+c^2)(Q-c^2)} = \frac{(Q-c^2)^2}{(Q^2-c^4)} = \frac{(Q-c^2)^2}{c^2(c^2+4)-c^4} = \frac{1}{4c^2}(Q-c^2)^2 =$

$= \frac{1}{4c^2}[Q^2 + c^4 - 2Qc^2] = \frac{1}{4c^2}[c^2(c^2+4) + c^4 - 2c^2 Q] = \frac{1}{4c^2}[2c^2(c^2+2) - 2c^2 Q]$

$I_p(c,s=1) = \frac{1}{Q}\frac{Q-c^2}{Q+c^2} = \frac{1}{Q}\frac{1}{4c^2}[2c^2(c^2+2) - 2c^2 Q] = \frac{1}{2}\frac{(c^2+2)}{Q(c,1)} - \frac{1}{2} = \frac{1}{2}\left[\frac{c^2+2}{Q} - 1\right]$

P1: $I_p(c,1)\frac{J_{c2}}{J_0} = -\frac{\partial}{\partial(c^2)} I_p(c,1)$, $\frac{\partial}{\partial c} I_p(c,1) = \frac{1}{2}\frac{1}{Q^2}\left[2cQ - (c^2+2)\frac{\partial Q}{\partial c}\right]$

$\frac{J_{c2}}{J_0} = -\frac{1}{I_p(c,1)}\frac{1}{2c}\frac{\partial I_p}{\partial c} = -\frac{2}{\left[\frac{c^2+2}{Q} - 1\right]}\frac{1}{2c}\frac{1}{2}\frac{1}{Q^2}\left[2cQ - (c^2+2)\frac{\partial Q}{\partial c}\right]$

$\frac{\partial Q}{\partial c} = \frac{\partial}{\partial c}\left[c\sqrt{c^2+4}\right] = \sqrt{c^2+4} + c\frac{1\cdot 2c}{2\sqrt{c^2+4}} = \sqrt{(\cdot)} + \frac{c^2}{\sqrt{c^2+4}} = \frac{1}{\sqrt{c^2+4}}\left[c^2+4+c^2\right] = 2\frac{(c^2+2)}{\sqrt{c^2+4}}$

$J_{c2}/J_0 = -\frac{1}{2}\frac{1}{[c^2+2-Q]}\frac{1}{cQ}\left[2cQ - (c^2+2)2\cdot\frac{(c^2+2)}{\sqrt{c^2+4}}\right] = -\frac{1}{2cQ}\frac{2}{[\ldots]}\frac{1}{\sqrt{c^2+4}}\left[c^2(c^2+4) - (c^2+2)^2\right] =$

$= -\frac{1}{cQ}\frac{1}{[c^2+2-Q]}\frac{1}{\sqrt{c^2+4}}(-4) = \frac{4}{c^2(c^2+4)}\frac{1}{[c^2+2-c\sqrt{c^2+4}]} = \frac{1}{c^2(c^2+4)}\left[c^2+2+c\sqrt{c^2+4}\right]$

P2 $\frac{J_{c2}}{J_0}(c) = \frac{1}{c^2(c^2+4)}\left[c^2+2+c\sqrt{c^2+4}\right]$, $J_{c2}(c) \sim 1/c^2$ велик при $c \to 0$

$Q(c=0,s) = (1-s^2)$ и $Q(c\to 0, s=1) \to 0$. $I_p(c,s) \sim \frac{1}{Q(c\to 0,1)} \to \infty$, следовательно ST-term $\sim J\, I_{sp}(c,s)$, $I_{sp}(c,1) \to 0$, $J_{c2} \to \infty$. Т.е. при $c \to 0$, $c\to 0, s=1$ система не зубчатая и $F_{ST} \sim J$.

Что с $\vec{P}$: $\frac{J}{J_0} 2c\cos\psi\, I_V(c,1) = -\frac{1}{2c}\frac{\partial}{\partial c} I_p(c,1)$, $I_V(c,1) = \int_0^1 ds \frac{s^2}{\left[(s^2+c^2+1)^2 - 4s^2\right]^{3/2}}$

$\frac{J_{c2}}{J_0}\cos\psi = \frac{1}{2c\, I_V(c,1)}(-)\frac{\partial}{\partial(c^2)} I_p(c,1)$, $I_V(c\to 0,1) \sim 1/c^2$ (нужно), $J_{c2}(c) \sim 1/c^2$ при $c \to 0$.

Note: $J_{c2}^V(c)/J_{c2}^P(c) \approx 2\ \forall c$ (точнее 1,9–2,4).

(31/XII) Сильно. review paper JOM и magnetic vortices and skyrmions" С.Л. Бычков-Гончаров "Hidden Oberberria. Предложение от R. es Energy USA to review project by E. Chudnovsky, Topology"

Аннотация Donovick Google Scholar 9325 cit. / h=51 на 31/XII 2019г.

(1/I) 2020 Написание SK-STNO manuscript. С Л., Ю., Ј. в бар "La Concha", финиш с Л. и А., Бостън. Hotel Paris Apr'20.

(2/I) Non-linear gyrofrequency $\omega_G(s) = \frac{2}{R^2}\frac{1}{G(s)}\frac{\partial W(s)}{\partial(s^2)}$, $W_{ex}\cdot\frac{M_s}{2\gamma A} = G(s)$

$\omega_G(s) = \frac{4\gamma|A'|}{R^2 M_s}\frac{1}{W_{ex}(s)}\frac{\partial W}{\partial(s^2)}$. Если $W = W_{ex}$, т.е. W_{in} и W_J пренебрежимо (Qx1)

то $\omega_G(s) = -\omega_0 \frac{\partial J_1(cs)/\partial c}{\partial J(cs)/\partial c}$, $\omega_0 = \frac{4\gamma|A'|}{M_s R^2}$, $A' = A - \frac{1}{2}|z|Rc < 0$

$\omega_G^{lin}(s\to 0, Q=1) = +\frac{2}{\pi}\omega_M \frac{c^2}{(1+c^2)^2}\frac{|A'|}{R^2 M_s^2} = \frac{2c^2}{(1+c^2)^2}\frac{4\gamma|A'|}{R^2 M_s} = \frac{2c^2}{(1+c^2)^2}\omega_0$, Ск. 20/III

т.е. frequency scale $\omega_0 = \frac{1}{2c^2}(1+c^2)^2\omega_G^{lin}$, ω_0 и ω_G^{lin} нрм $c \simeq 0.5$.

$\omega_0 = \frac{4\gamma|A'|}{M_s R^2} = 4\gamma\cdot 2\pi M_s \frac{|A'|}{2\pi M_s^2 R^2} = 2\omega_M\left(\frac{l_{ex}}{R}\right)^2$, $l_{ex}^2 = |A'|/(2\pi M_s^2)$, $\omega_M = 4\pi\gamma M_s$

Дано $|A'| \simeq 10$ pJ/m, $M_s = 1000$ G, $l_{ex} = \sqrt{\frac{10^{-6}\,erg}{cm\cdot 2\pi\, 10^6 G^2}} = \frac{1}{\sqrt{2\pi}}10^{-6}cm = 0.4\cdot 10^{-6}cm =$

$\omega_M/2\pi = 4\pi\left(\frac{\gamma}{2\pi}\right)M_s \simeq 4\pi\cdot 3\frac{GHz}{kOe}\cdot 10^3 G \simeq 37.7$ GHz $= 4.0$ nm

Дано $R = 100$ nm, $\omega_0/2\pi = 2\frac{\omega_M}{2\pi}\left(\frac{4.0\,nm}{100\,nm}\right)^2 = 2\cdot 37.7\cdot 16\cdot 10^{-4}$ GHz $\simeq 120$ MHz (120.6)

Дано $R = 50$nm, $\frac{\omega_0}{2\pi} \simeq 480$ MHz. Нрм $\gamma/2\pi = 3.0$ GHz/kOe.

Числено в MathCad $\omega_G(s)/\omega_0$ изменя от 0.32 до 1.205 (P_z) и (vort $\vec{P}$), т.е. в 3.77 раза (нрм $c = 0.5$).

Current scale: $x = \frac{J}{J_1}\cos\psi$, $J_1(vort\vec{P}) = \frac{1}{2}J_0$, $x = c\frac{J}{J_0}\cos\psi \to x = c\frac{J}{J_0}$, $\cos\psi = 1$

$c = 0.5$, $x = 0.5\frac{J}{J_0}$

(3-4/I) Нумерация Figs. 1-3 в SK-STNO manuscript. Написание и оформяне текста. С Л. и Ю. в Carrefour и "Lidl". Обаждане

(5/I) Аналу invited SolSkyMag 2020, референция с candidates (Mantafis, Grundler, Blügel, Fangohr, Klaüi, Davelder). Update inv. list — 22 confirmed.

(6/I) Написание отказ Referees of 1D-DW ARB paper.

• H.J. Mikeska J. Phys. C 11, L29 (1978) дава Sin-Gordon за $\Phi(z,t)$, $\hat{z}\perp$ ПЛН,

$\mathcal{H} = -J\sum_n \vec{S}_n\cdot\vec{S}_{n+1} + A\sum_n (S_n^z)^2 - \mu B_x\sum_n S_n^x$, $A>0$ ПЛН xy, B_x и ПЛН $\approx$ ОЛН

$\frac{\partial^2\Phi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = m^2\sin\Phi$, $\Theta = \frac{1}{2AS}\dot{\Phi}$, mass $m \sim \sqrt{B_x/J}$. без доказателства

Anyway это дава kink solution $\Phi(z,t) = 4\arctg e^{\pm\gamma m z'}$, $\cos\Phi = 1 - \frac{2}{ch^2\gamma m z'}$,

$\gamma = 1/\sqrt{1-u^2/c^2}$, $z' = z - ut - z_0$, а не дава DW.

Ск. comments к Magyari 1984, Mikeska 1981 и Mikeska 1983. 18/IV 2019

• Mikeska JAP 1981: $\vec{S}_n = S(\cos\theta_n\cos\Phi_n, \cos\theta_n\sin\Phi_n, \sin\theta_n)$, $\hat{z}\perp$ ПЛН xy, $\theta \to \pi/2 - \theta$

$$\left.\begin{aligned}\dot\theta &= JS\Phi_{zz} - \mu B_x\sin\Phi\\ \dot\Phi &= 2AS\theta - JS\theta_{zz} + \mu B_x\cos\Phi\end{aligned}\right\}$$

Ако $\theta_{zz} = 0$ и $\mu B_x \ll 2AS$, то $\theta = \frac{1}{2AS}\dot\Phi$ и получаваме Sin-Gordon как Mikeska 78

т.е. $\theta \ll 1$ линеаризация, θ - отклон. от ПЛН xy $\Phi_{zz} - \frac{1}{c^2}\ddot\Phi = m^2\sin\Phi$, где

$c^2 = 2S^2JS$ и $m^2 = \mu B_x/JS \ll 1$. Long wave limit $(qa)\ll 1$, a-период на реш.

Тогава $\gamma m = \frac{m}{\sqrt{1-u^2/c^2}} \ll 1$, т.е. дори $u\to c$, $u^2 \ll c^2$ $\dot\Phi = 2A\theta$ само при $u^2 \ll c^2$.

$m^2 = \frac{\mu B}{JS} = \left(\frac{\mu B}{AS}\right)\left(\frac{A}{J}\right) \ll\ll 1$, т.к. $\frac{\mu B}{AS} \ll 1$ и $\frac{A}{J} \ll 1$ дава RM.

При $\lambda = \frac{AS}{\mu B_x} \gg 1$, $m^2 \to 0$ и нея SG-equation, а просто $\Phi_{zz} - \frac{1}{c^2}\ddot\Phi = 0$

R.h.s. sine-Gordon by Mikeska at $\lambda \gg 1$ ($\lambda = \frac{K_z}{K_t}$ у нея) изглежда неудобно

r.h.s. sine-Gordon by Sklyanin. У нея при $\lambda \gg 1$, $\xi(\bar z) = \text{const}$ $\Theta_{xx} - \frac{1}{c^2}\ddot\Theta = \frac{1}{2}\sin 2\Theta$

2.h.s не работи. SG сухранява свой вид

Преминаваме и $\bar t = t + ut$, $\Theta_{zt} = 0 \to \Theta_{\bar t\bar t} = 0$, $\Theta_{\bar z} = \text{const}$, $\dot\Theta = \Theta_{\bar z}\frac{\partial\bar z}{\partial t} = -u\Theta_{\bar z}$

$= \text{const}$. Но у Mikeska $\dot\theta = \dot\Phi/2AS$ и $\theta = \text{const} \Rightarrow \dot\Phi = \text{const}$ (не SG)

Escaneado con CamScanner

// Есть ли multiplication of $\mathcal{L}_{kin} = -\Phi\dot{\theta}\sin\theta$ или $\mathcal{L}_{kin} = -\dot{\Phi}\cos\theta$? и/или $\Phi_x = \Phi_z = 0$ Walker-type of solutions?

Eqs.(1) of 1D-DW paper при $\Phi_x = 0$ $\left\{ \begin{array}{l} \dot{\theta} = \lambda\cos\Phi\sin\Phi\sin\theta \\ \dot{\Phi}\sin\theta = [1+\lambda\cos^2\Phi]\cos\theta\sin\theta - \theta_{xx} \end{array} \right\}$

если $\Phi = \pi/2 - \varepsilon$, $\cos\Phi\sin\Phi = \varepsilon$ и $\dot{\theta} = \lambda\varepsilon\sin\theta$, тогда $\sin\theta = \dot{\theta}/\lambda\varepsilon$ и

$\mathcal{L}_{kin} = -\Phi\dot{\theta}\sin\theta = -(\frac{\pi}{2} - \varepsilon)\dot{\theta}\sin\theta = \varepsilon\dot{\theta}\sin\theta = \varepsilon\dot{\Phi}\frac{\dot{\theta}}{\lambda\varepsilon} = \frac{1}{\lambda}(\dot{\theta})^2$, $\mathcal{L}_{kin} = \frac{1}{\lambda}(\dot{\theta})^2$.

$\Phi_z = 0 \to \dot{\Phi} = 0$ и $\theta_{xx} = [1+\lambda\cos^2\Phi]\sin\theta\cos\theta$, $[1+\lambda\cos^2\Phi] = 1/\Delta^2(y)$

$\theta_{xx} = \frac{1}{\Delta^2}\sin\theta\cos\theta$, $\theta_{\xi\xi} = \frac{1}{2\Delta^2}\sin 2\theta$ - SG eq., если $y(\xi) = \xi/\Delta$ is linear.

Eq.(5) при $\Phi_z = 0$, $v y_\xi = \lambda\cos\Phi\sin\Phi$, $\eta_\xi = \frac{\lambda}{v}\sin\Phi\cos\Phi$, $\eta(\xi) = \xi\left(\frac{\lambda}{v}\sin\Phi\cos\Phi\right) + C$

$y_{\xi\xi} + [1+\lambda\cos^2\Phi - (y_\xi)^2]\sin y = 0$, $y_{\xi\xi} + [\frac{1}{\Delta^2} - (y_\xi)^2]\sin y = 0$ имеет solitonное решение

решение $y_\xi = \frac{1}{\Delta}$, $y(\xi) = \xi/\Delta$, если $\frac{1}{\Delta} = \frac{\lambda}{v}\sin\Phi\cos\Phi$, $\frac{\lambda}{v}\frac{\sin\Phi\cos\Phi}{\sqrt{1+\lambda\cos^2\Phi}} = 1$.

(7-8/I) Написание Sk-STNO paper. Sk-STNO

(9/I) Meeting с Julian: A.text в пределах 27/I; Задача не надо ехать 4/II в CAF - только Word; Julian Jankowski support Solglykmag у CAF и Tour Bureau SD. Make Contact City Hall отн. Welcome Party 22/VI и Conf. Dinner Londres 24/VI. Meeting с Ricardo: Он предложил доклад (плохо обоснованный) по вычислению DW position $q(t)$ in response to SOT current $J(t)$ within F1/F2 layers model с AF coupling energy среднем (параметры M_{s2} Au).

(10/I) Очистил domain name и Solglykmag.es" to Hostalia Bonnaw. Updated Ikerb.site CV+...

Sk-STNO paper в RRL-Word template. Edit the paper.

(11/I) Editing Sk-STNO paper, improved. Fig. Sk-image in bot" для RRL-contents. с A. в „Lide", Carrefour „ Olaberria

(12/I) Оценил J_{c2} с параметрами Shen'APL'2019. Верно ли $J = 20\frac{MA}{cm^2} > J_{c2}$?

Shen'19: $L = 1$ nm, $A = 15$ pJ/m, $K = 0.6$ MJ/m^3, $M_s = 580$ kA/m, $D = 3$ mJ/m^2, $\eta = 0.4$, $\alpha_G = 0.01 \div 0.2$ [Sampaio'2013], $R = 60$ nm. Sk-radius R_c не указан.

$A' = A - DR_c\frac{1}{2} = 15\frac{pJ}{m} - \frac{1}{2}3\cdot10^{-3}\frac{J}{m^2}R_c = 15\frac{pJ}{m} - 1.5\cdot10^{-3}\frac{J}{m^2}\cdot 60\,nm\cdot c =$

$= (15 - 90\cdot c)\,pJ/m = |\text{мусть } c = 0.5| = -30\,pJ/m$. Берем $\alpha_G = 0.1$.

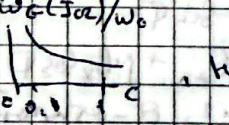
$J_0 = \frac{8\pi\alpha_G L|A'|}{\eta\Phi_2 R^2} = \frac{8\pi\cdot 0.1\cdot 1nm\cdot 30\,pJ/m}{0.4\cdot 2.07\cdot10^{-15}\frac{J}{A}\cdot(60nm)^2} = \frac{8\pi\cdot0.1\cdot10^{-9}\cdot30\cdot10^{-12}J}{0.4\cdot2.07\cdot10^{-15}\frac{J}{A}\cdot36\cdot10^{-16}m^2} =$

$= \frac{8\pi\cdot0.1\cdot3\cdot10^{-20}J\cdot A}{0.4\cdot2.07\cdot10^{-30}\,3.6\,m^2J} = \frac{8\pi\cdot0.1\cdot3}{0.4\cdot2.07\cdot3.6}10^{10}\frac{A}{m^2} = 2.53\cdot10^{10}\frac{A}{m^2} = 2.53\frac{MA}{cm^2} = J_0$

Имеем $J_{c2}/J_0 = 6.12$, т.е. $J_{c2} = 6.12\cdot J_0 = 6.12\cdot 2.53\frac{MA}{cm^2} = 15.49\frac{MA}{cm^2}$.

Т.е. Shen APL'19 выбрал $J = 20\,MA/cm^2 > J_{c2} = 15.5\,MA/cm^2$ ($\alpha_G = 0.1$, $c = 0.5$). $\eta = 0.4$

$J(Shen) = 20\,MA/cm^2 = 200\,GA/m^2$

$\frac{\omega_G(J_{c2})}{\omega_0}$ as $F(c)$ достигает 5.24 при $c = 0.1$ 

, но в act. temperature не все так просто.

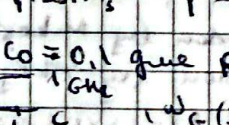
$\omega_G(J_{c2}) = F(c)\cdot\omega_0(c)$, $\omega_0(c) = \omega_0'\left|\frac{R}{2l_0}c - 1\right|$,

$l_D = A/D$. $\omega_0' = 2\omega_M\left(\frac{l_e}{R}\right)^2$. $l_e^2 = A/2\pi M_s^2$. Расчет $\omega_G(J_{c2})$ as function of "c"

at $R = const$. $\omega_G(J_{c2}) = F(c)\,2\omega_M\left(\frac{l_e}{R}\right)^2\left[\frac{R}{2l_0}c - 1\right]$, $\frac{2l_0}{R} = \frac{2A}{DR} = c_0$. $c_0 = 0.1$ $R = 100$ nm

Пусть $R = 100$ nm, $\omega_M/2\pi = 37.7\,GHz$, $l_e = \sqrt{\frac{A}{2\pi M_s^2}} = \sqrt{\frac{1.5\cdot10^{-6}erg/cm}{2\pi\cdot(10^3)^2G^2}} = \sqrt{\frac{1.5}{2\pi}}\cdot10\,nm = 4.9$ nm

$\frac{1}{2\pi}\omega_G(J_{c2}) = 2\cdot37.7\,GHz\left(\frac{4.9nm}{100nm}\right)^2\left[\frac{c}{c_0} - 1\right]\cdot F(c)$

$\omega_G(J_{c2})/2\pi = \left(\frac{c}{0.1} - 1\right)F(c)\cdot0.181\,GHz$. 

, $\omega_G(J_{c2})$ достигает $\approx 1\,GHz$ при $c \approx 1$.

$\forall c, R$, $\omega_G^{J_{c2}}(c,R) = F(c)\,2\omega_M\left(\frac{l_e}{R}\right)^2\left[\frac{2R}{2A}c - 1\right]$, при $A = 15\cdot10^{-6}\frac{erg}{cm}$ $D = 3\,mJ/m^2$, $M_s = 1000$ G

Max $\omega_G(J_{c2})$ при large $c = 0.8$-0.9 и small $R \approx 25$ nm // $2A/D = 10$ nm, $l_e' = 4.9$ nm

$\forall c=0.9$ Max $\frac{\omega_G^{J_{c2}}}{2\pi}(c,R) = 2,62$ GHz достигается $\forall R = 22$ nm.

Current scale $J_0(c,R) = \frac{8\pi d_G L}{\gamma \Phi_2} \frac{1}{R^2} |A'|$, $\omega_0 = 2\omega_M \frac{l_e^2}{R^2}\left[\frac{DR}{2A}c - 1\right]$

$\omega_0 = 2\omega_M \frac{l_{ex}^2}{R^2}$, $l_{ex}^2 = \frac{|A'|}{2\pi M_s^2} = \frac{A}{2\pi M_s^2}\left[\frac{DR}{2A}c - 1\right] = l_e^2\left[\frac{DR}{2A}c - 1\right]$

$\omega_0 = 2\omega_M \frac{|A'|}{2\pi M_s^2}\frac{1}{R^2} = 2\cdot 4\pi\gamma M_s \frac{|A'|}{2\pi M_s^2} R^{-2} = \frac{4\gamma}{M_s}|A'|\frac{1}{R^2}$

$\frac{|A'|}{R^2} = \frac{M_s}{4\gamma}\omega_0$ и $J_0 = \frac{8\pi d_G L}{\gamma \Phi_2}\frac{M_s}{4\gamma}\omega_0 = \frac{2\pi d_G L M_s}{\gamma\gamma\Phi_2}\omega_0$, $l_e^2 = \frac{A}{2\pi M_s^2}$

$J_0(c,R) = \frac{8\pi d_G L}{\gamma\Phi_2}\frac{A}{R^2}\left[\frac{DR}{2A}c - 1\right] = 16\pi^2 M_s^2 \frac{d_G L}{\gamma\gamma\Phi_2}\left(\frac{l_e}{R}\right)^2\left[\frac{DR}{2A}c - 1\right]$,

$16\pi^2 M_s^2 \frac{d_G L}{\gamma\Phi_2} = 16\pi^2 (10^3 G)^2 \frac{0.1\cdot 1\,nm}{0.5\cdot 2,07\cdot 10^{-15}\frac{J}{A}} = 16\pi^2 10^6 \frac{erg}{cm^3}\frac{0.1\cdot 1\,nm\cdot A}{1,034\cdot 10^{-15} J} =$

$= 16\pi^2\cdot 10^6 \frac{10^{-7}J}{10^{-6}m^3}\frac{0.1\cdot 10^{-9}m\cdot A}{1,034\cdot 10^{-15}J} = \frac{16\pi^2}{1,034}\cdot 10^{10}\frac{A}{m^2} = 152,72\cdot 10^{10}\frac{A}{m^2}$

$J_0(c,R) = 15,27\cdot 10^{11}\frac{A}{m^2}\left(\frac{l_e}{R}\right)^2\left[\frac{DR}{2A}c - 1\right]$, $D/2A = (10\,nm)^{-1}$, $l_e = 4,9\,nm$

$J_0(c,R) = 15,27\cdot 10^{11}\frac{A}{m^2}\left(\frac{4,9\,nm}{R}\right)^2\left[\frac{R(nm)}{10\,nm}c - 1\right]$ Для cont. $\vec{P}$

$J_{c1}^{\circlearrowleft}/J_0 = F_1(c)$, $J_{c2}^{\circlearrowleft}(c)/J_0 = F_2(c)$ построим численно as function of „c". (Sk-radius)

$J_{c1}^{\circlearrowleft}(c,R) = F_1(c)\cdot J_0(c,R)$ и $J_{c2}^{\circlearrowleft}(c,R) = F_2(c)\cdot J_0(c,R)$ — in abs. units A/m^2.

Из-за множителя $J_0(c)$ и $\left(\frac{c}{c_0}-1\right)$ $J_{c2}^{\circlearrowleft}(c)$ не расет с „c" $\forall R = 50$ nm а надает $J_0(c)$ не меньше чем $J_{c1}^{\circlearrowleft}(c)$ $\forall R$. или слабо расет с „c" $\forall R = 100$ nm.

Из-за множителя $J_0(R) \sim \frac{1}{R^2}\left(\frac{DR}{2A}c - 1\right)$, $J_{c1}^{\circlearrowleft}(R)$, $J_{c2}^{\circlearrowleft}(R)$ достигают max при small $R \approx 25 \div 35$ nm и large $c = 0.8 \div 0.9$ — typ. примерно тогда же когда $\omega_G^{J_{c2}}(R)$ имеет max

Минимизировать $J_{c1}^{\circlearrowleft}(c,R) \equiv$ брать малые „c" и/или ~~large~~ R — small.

Максимизировать $J_{c2}^{\circlearrowleft}(c,R) \equiv$ брать большие „c" и/или small R.

Условия max $J_{c2}^{\circlearrowleft}(c,R)$ и max $\omega_G^{J_{c2}}(c,R)$ — совпадают, large „c" и small R.

Для P_2 - polarizer уже $J_0(c,R)$ приводит к abs. $J_{c1}^2(c,R)$ качественно похожим на $J_{c2}^{\circlearrowleft}(c,R)$ и $J_{c1}^2 > J_{c1}^{\circlearrowleft}$ при $c \le 0.8$, и $J_{c2}^2 < J_{c2}^{\circlearrowleft}$ $\forall c,R$, abs. units.

Уточнение: max $J_{c2}^{\circlearrowleft}(c,R)$ при small R достигается для large „c", а при large R - при small „c". Причем $J_{c2}^{\circlearrowleft}$(small c, large R) < $J_{c2}^{\circlearrowleft}$(large c, small R), а также $\omega_G^{J_{c2}}(c,R)$ достигает max при large „c", small R.

Max $J_{c1}^{\circlearrowleft}(c,R)$, $J_{c2}^{\circlearrowleft}(c,R)$ достигаются at small R and large „c"(J_{c1}) ~~small „c"~~ (J_{c2}).

Min $J_{c1}^{\circlearrowleft}(c,R)$ при small R, small „c".

Если подбирать c,R из max $\omega_G^{J_{c2}}(c,R)$ т.е. abs. $J_{c1}(c,R)$ будет не очень мал, но $J_{c2}^{\circlearrowleft}(c,R)$ будет велик. [large c, small R] Например $J_{c2}^{\circlearrowleft}(0.9, 22\,nm) = 2,073\cdot 10^9\frac{GA}{m^2}$ is giant.

Max current J у Garcia-Sanchez '16 is 55 GA/m^2 (!)

Deadline Abstracts • SkyMag, 6-9 Apr. 2020, Paris is Feb. 10, 2020 / TMAG-2020, Italy Sept. 7-11, Abs. до Feb. 28/02
• JEMS, 27-30/VII, 2020, Lisbon, is March 14, 2020

Ratio $\omega_G(J_{c2})/\omega_G(J_{c1})$? $\omega_G(J_{c1}) = \omega_G^{lin} = \frac{2c^2}{(1+c^2)^2}\omega_0$, $\omega_0(c,R) = 2\omega_M\left(\frac{l_e}{R}\right)^2\left[\frac{DR}{2A}c - 1\right]$

$\omega_G(J_{c2}) = F(c)\,\omega_0 \rightarrow \frac{\omega_G(J_{c2})}{\omega_G(J_{c1})} = F'(c)\frac{(1+c^2)^2}{2c^2}$ Зависит только от $c = R_c/R$.

$\frac{\omega_G(J_{c2})}{\omega_G(J_{c1})}$ падает с ростом „c" от 267,35 при $c=0.1$ до 1,445 при $c=0.9$, 1,672 $c=0.8$ (Данные из Vortex STNO)

Editing Sk-STNO paper для RRL и changing Figs.

Escaneado con CamScanner

13/I · Subm. Skyrmion - STNO manuscript to pss-RRL. Subm. abstract to Skymag'2020, Paris.

14/I Meeting с Julian. Presentation by Ricardo $Mn_2(Pd)Au$ в индонезийском two-FM layers coupled by AF-exchange. параметры of layered Mn_2Au (4-FM слоя, no 2-слое Ванюше). Rigid 1D Neel DW ansatz for $\vec{l}(\cos\theta, \sin\theta\cos\varphi, \sin\theta\sin\varphi)$ $\theta = \pi/2$, $\cos\varphi = \text{th}\left(\frac{x-q(t)}{\Delta(t)}\right)$. Eq. for $q(t)$ - DW position as function of current $J(t) \to H_{SOT}(t)$ field pulse $\ddot{q} + \alpha\dot{q} = H_{SOT}(t)\Delta(t)$, $\ddot{q}$ отвечает за инерционный AF-mass term $m_{DW} \sim \frac{1}{J_{AF}}$. Simulated velocity $\dot{q}(t)$ $V(t)$ 40 km/s, 10 ps, t; moving boundary effect if only simulated $\Delta(t)$ numerical Бо Еммануил. $\dot{q}(t)$ хорошо согл. с simulations by Ruben или standard AF-DW mass $\sim \gamma/J_{AF}$ и $\Delta(t) = \sqrt{1 - v^2(t)/v_m^2}$, где $v_m \approx 43$ km/s из других simulations by Ruben / Pierre.

Obsaan to Supfard Ricardo's trips to JEMS'20 и lost-school Porto (Aug. 2020).

15/I Replies to Referees and editing 1D-DW PRB, см. начало 6/I 2020.

1991 – A.J. Mikeska, M. Steiner, Adv. Phys. 40, N3, 191 year

$$\mathcal{H} = -J\sum_n \vec{S}_n \vec{S}_{n+1} + D\sum_n (S_n^z)^2 - C\sum_n (S_n^x)^2$$, OZ нормальна к DW plane xy

z in units a, chain nepmog, time t in units $1/(JS)$

$d = \frac{2D}{J}$, $c = \frac{2C}{J}$. $\vec{S} = S(\underset{x}{\sin\theta\cos\Phi}, \underset{y}{\sin\theta\sin\Phi}, \underset{z}{\cos\theta})$

Решение $\vec{S}/S = \frac{1}{\text{ch}\mu s}(\pm\text{th}\mu s, \cos\Phi_0, \sin\Phi_0)$, $\mu = \sqrt{d}\sqrt{1 - \frac{v^2}{d}} \overset{(c=0)}{=} \sqrt{d}\sin\Phi_0$, $s = z - ut$

Φ_0, μ, u удовлетв. (при $c \neq 0$) $\begin{cases} \mu = \sqrt{c + d\sin^2\Phi_0} \\ \sin\Phi_0\cos\Phi_0 = \mu\frac{u}{d} \end{cases}$ имп реш. ($\Phi_0 = 0$) $\begin{cases} 1 + \lambda\cos^2\Phi_0 = \frac{1}{\Delta^2} \\ \frac{v}{\lambda\Delta} = \cos\Phi_0\sin\Phi_0 \end{cases}$, $\mu \equiv \frac{\sqrt{c}}{\Delta(v)}$

Видно $\mu = \frac{\sqrt{c}}{\Delta}$, $\frac{u\sqrt{c}}{d} = \frac{v}{\lambda}$, $\lambda = \frac{K_2}{K_y} = \frac{d}{c} \gg 1$. У меня x in units $\Delta_0 = \sqrt{A/K_2}$, v - velocity in units $\Delta_0/t_0 = 2\gamma\sqrt{AK_2}/M_s$, t in $t_0 = 1/\gamma H_a$, $H_a = 2K_2/M_s$.

и $\sin\Phi_0 \to \cos\Phi_0$ (мое), $\Phi_0^M = \pi/2 - \Phi_0$.
Mikeska.

Eq. (2.20)-Mikeska'91: $E(u) = 2S^2\sqrt{JD}\left[1 - \frac{u^2}{d} \pm \sqrt{\left(1 - \frac{u^2}{d}\right)^2 + \frac{uc}{d}\left(1 + \frac{c}{d}\right)}\right]^{1/2}$?
(for DW energy)

мое $E_{DW}(v) = 4K_2\Delta_0 \cdot \varkappa(v)$, $K = k + i\varkappa$, $K^2(v) = \frac{1}{2}(v^2 - \lambda) - 1 \pm \frac{1}{2}\sqrt{(v^2 - v_-^2)(v^2 - v_+^2)}$, $v_\pm = \sqrt{1+\lambda} \pm 1$

При $v < v_-$, $K(v) = i\varkappa(v)$. $\frac{c}{d} = \frac{1}{\lambda} \ll 1$. $v_+ v_- = (\sqrt{1+\lambda}+1)(\sqrt{1+\lambda}-1) = \lambda$.

$\frac{u\sqrt{c}}{d} = \frac{1}{\sqrt{c}}\frac{uc}{d} = \frac{1}{\lambda}\frac{u}{\sqrt{c}} = \frac{v}{\lambda}$, т.е. $\frac{u}{\sqrt{c}} = v$ (u - velocity Mikeska).

При $u \to 0$ $E_{DW}^{Mik}(u \to 0) = 2S^2\sqrt{JD}\left[1 \pm 2\sqrt{\frac{4}{\lambda}\left(1 + \frac{1}{\lambda}\right)} + 1\right]^{1/2} = 2S^2\sqrt{JD}\left[1 \pm \sqrt{1 + \frac{4}{\lambda}\left(1 + \frac{1}{\lambda}\right)}\right]^{1/2} =$

$= (\cdot)\left[1 \pm \sqrt{\left(1 + \frac{2}{\lambda}\right)^2}\right]^{1/2} = (\cdot)\left[1 \pm \left(1 + \frac{2}{\lambda}\right)\right]^{1/2} = (\cdot)\left\{\sqrt{2 + \frac{2}{\lambda}}\right\}$ – неверно.

Мое $K^2(v) \underset{v \to 0}{=} -\frac{\lambda}{2} - 1 \pm \frac{1}{2}\sqrt{v^2 v_+^2} = -\frac{\lambda}{2} - 1 \pm \frac{1}{2}\lambda = \begin{cases} -1 \\ -1 - \lambda \end{cases} = -\varkappa^2(v \to 0)$

$\varkappa^2(0) = 1$ для + ветви и $E_{DW}(v \to 0) = 4K_2\Delta_0 \cdot \varkappa(v \to 0) = 4K_2\Delta_0$ – верно.

M.M. Bogdan, A.S. Kovalev, Z. Phys. B 71, 341 (1988). Exact SW на фоне moving 1D DW in biaxial FM, DW типа $\Phi(z) = \omega z$. Это обобщение Walker'1971. Взято за (!)

A. How et. al. Soliton theory for realistic DW dynamics. PRB 40, 4808 (1989).

$\frac{1}{\gamma}\dot{\theta} = H_{tot}^{\Phi}$, $-\frac{1}{\gamma}\sin\theta\dot{\Phi} = H_{tot}^{\theta}$, $\underbrace{x - ct}_{z}$, $\hat{z}$ along DW propagation как у Mikeska.

$y = \frac{\pi}{2} - \theta \ll 1$ из-за $W_m = 2\pi M_s^2\cos^2\theta$, xy - is easy plane. $\vec{m} = (\underset{z}{\sin\theta\cos\Phi}, \underset{y}{\sin\theta\sin\Phi}, \underset{x}{\cos\theta})$

$\vec{H} \| \hat{x}, \partial/\partial H$. $H_{tot}^{\theta} = H_0\cos\Phi y + 4\pi M_s y + \frac{2K}{M_s}y\cos^2\Phi - \frac{2A}{M_s}\left[\frac{\partial^2 y}{\partial z^2} + y(\Phi_{z0}^x)^2\right]$

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$\sin\theta = \sin\left(\frac{\pi}{2} - y\right) = \cos y \approx 1 - O(y^2)$, $-\frac{1}{\gamma}\frac{d\Phi}{dt} = 4\pi M_s y + \frac{2K}{M_s}\cos^2\Phi\, y - \frac{2A}{M_s}\left[\frac{\partial^2 y}{\partial z^2} + y\left(\frac{\partial\Phi}{\partial z}\right)^2\right]$

How - normalization z in $\delta = \sqrt{A/k}$ units, time in $\tau = t/\omega_0$ units, $\omega_0 = \gamma\sqrt{8\pi K}$

$\frac{d}{dt} \to \frac{1}{\tau}\frac{d}{d(t/\tau)} = \omega_0 \frac{d}{d(\omega_0 t)} = \gamma\sqrt{8\pi K}\frac{d}{dt}$, $\frac{\partial^2}{\partial z^2} = \frac{1}{\delta^2}\frac{\partial^2 y}{\partial(z/\delta)\partial(z/\delta)} = \frac{1}{\delta^2}\frac{\partial^2 y}{\partial z^2}$

$-\frac{1}{\gamma}\sqrt{8\pi K}\dot\Phi = 4\pi M_s y + \frac{2K}{M_s}\cos^2\Phi\, y - \frac{2A}{M_s}\frac{1}{\delta^2}\left[\frac{\partial^2 y}{\partial z^2} + y\left(\frac{\partial\Phi}{\partial z}\right)^2\right]$

$\frac{2A}{M_s}\frac{1}{\delta^2} = \frac{2A}{M_s}\frac{K}{A} = \frac{2K}{M_s}$, $-\sqrt{8\pi K}\dot\Phi = 4\pi M_s y + \frac{2K}{M_s}\cos^2\Phi\, y - \frac{2K}{M_s}\left[y_{zz} + y(\Phi_z)^2\right]$

Def. $\nu = \frac{\sqrt{4\pi K}}{4\pi M_s}$ и assume $\nu \ll 1$. $\nu^2 = \frac{4\pi K}{(4\pi)^2 M_s^2}$, $\frac{2K}{M_s}\frac{1}{4\pi M_s} = \frac{2}{4\pi}\frac{K}{M_s^2} = 2\nu^2$

$-\sqrt{2}\nu\dot\Phi = y + 2\nu^2\left[y\cos^2\Phi - y_{zz} - y(\Phi_z)^2\right]$. $\nu = \sqrt{\frac{K}{4\pi M_s^2}} = \sqrt{\frac{1}{2\lambda}}$,

λ (use) $= \frac{K_z}{K_x} \to \frac{K_z}{K_z} = \frac{K}{2\pi M_s^2}$, $\nu^2 = \frac{1}{2\lambda} \ll 1$, при $\lambda \gg 1$.

Сильное $\nu^2 \ll 1$, тогда вообще можно пренебречь $\nu^2[\dots]$ членом, тогда $\dot y = -\sqrt{2}\nu\dot\Phi$ и $\ddot y = -\sqrt{2}\nu\ddot\Phi$ (нет)

$\frac{1}{\gamma}\dot\theta = -\frac{1}{\gamma}\dot y = -\frac{1}{\gamma}\left(-\sqrt{2}\nu\dot\Phi\right) = \frac{\sqrt{2}}{\gamma}\nu\ddot\Phi = H^\Phi_{tot}(\Phi)$ и дает sine-Gordon для $\Phi(z,t)$.

Но $\frac{1}{\gamma}\dot\theta$ в dim.less variables, $\frac{d}{dt} \to \gamma\sqrt{8\pi K}\frac{d}{dt}$, $\frac{1}{\gamma}\dot\theta = \sqrt{8\pi K}\frac{d\theta}{dt} = -\sqrt{\pi 8K}\frac{dy}{dt}$

$-\sqrt{8\pi K}\frac{dy}{dt} = -\sqrt{8\pi K}\left(-\sqrt{2}\nu\frac{d^2\Phi}{dt^2}\right) = \sqrt{2}\sqrt{8\pi K}\ddot\Phi\nu = H^\Phi_{tot}$.

$H^\Phi_{tot} = -\frac{2K}{M_s}\sin\Phi\cos\Phi + \frac{2A}{M_s}\frac{\partial^2\Phi}{\partial z^2}$, $\sqrt{2}\sqrt{8\pi K}\nu\ddot\Phi = -\frac{2K}{M_s}\sin\Phi\cos\Phi + \frac{2A}{M_s}\frac{K}{A}\Phi_{zz}$

$\sqrt{2}\sqrt{8\pi K}\ddot\Phi\nu = \frac{2K}{M_s}\left[\Phi_{zz} - \sin\Phi\cos\Phi\right]$, $\sqrt{2}\sqrt{8\pi K}\nu = \sqrt{2}\sqrt{8\pi K}\sqrt{\frac{4\pi K}{4\pi M_s}} =$

$= \sqrt{2}\sqrt{2}\cdot\frac{4\pi K}{4\pi M_s} = \sqrt{2}\left(\frac{2K}{M_s}\right) = \frac{2K}{M_s}$, $\ddot\Phi = \Phi_{zz} - \sin\Phi\cos\Phi$, $\Phi_{zz} - \ddot\Phi = \frac{1}{2}\sin 2\Phi$

Def. $\psi = 2\Phi$, $\psi_{zz} - \psi_{tt} = \sin\psi$ Sine-Gordon, Eq. (2.18) How 1989.

Я использ. normalization time $t_0 = \frac{1}{\gamma H_a}$, $\omega_0 = \gamma H_a$, $H_a = \frac{2K_z}{M_s}$

$\frac{d^2\Phi}{dt^2} \Rightarrow \tau^2\frac{d^2\Phi}{dt^2} = |t\text{-abs. units}| = \frac{\tau^2}{t_0^2}\frac{d^2\Phi}{d(t/t_0)^2} = \frac{\tau^2}{t_0^2}\frac{d^2\Phi}{dt^2}$, где t in t_0 units.

$\tau^2/t_0^2 = \frac{1}{\gamma^2 8\pi K}\gamma^2\left(\frac{2K}{M_s}\right)^2 = \frac{1}{8\pi K}\frac{4K^2}{M_s^2} = \frac{K}{2\pi M_s^2} = 2\left(\frac{K}{4\pi M_s^2}\right) = 2\nu^2 = \frac{1}{\lambda}$.

Т.е. в наших переменных $\Phi_{zz} - \frac{1}{\lambda}\Phi_{tt} = \frac{1}{2}\sin 2\Phi$ что верно учит. $\Phi \to \theta$ и $\lambda = c^2$.

→ Вопросы y_{zz}, Φ_z не надо забывать, они учтены в членах $\sim\lambda$ и сами исчезают при $\nu \ll 1$, $\lambda \gg 1$.

$-\sqrt{8\pi K}\frac{dy}{dt} = \frac{2K}{M_s}\left[\Phi_{zz} - \sin\Phi\cos\Phi\right]$, $\sqrt{4\pi K} = 4\pi M_s\cdot\nu$, $\frac{2K}{M_s}\frac{1}{4\pi M_s} = 2\nu^2$.

(1) $-\sqrt{2}\nu\frac{dy}{dt} = 2\nu^2\left[\Phi_{zz} - \sin\Phi\cos\Phi\right]$ и $-\sqrt{2}\nu\dot\Phi = y + 2\nu^2\left[y\cos^2\Phi - y_{zz} - y(\Phi_z)^2\right]$.

Если ν^2-terms in (2) пренебречь, $y = -\sqrt{2}\nu\dot\Phi$, то (1): $2\nu^2\ddot\Phi = 2\nu^2\left[\Phi_{zz} - \sin\Phi\cos\Phi\right]$ (2)

дает солитонное $\theta = 0$, если ν^2 terms пренебречь и забыть, и нет Sine-Gordon. Т.е. method How 1989 противоречит (1).

Если method How 1989 ($y = \frac{\pi}{2} - \theta \ll 1$, или $\Phi = \frac{\pi}{2} - \varepsilon$, $\varepsilon \ll 1$ в УД (z,t) и $\nu \to 0$ (x,t) и $\lambda \gg 1$) разбивает, то он должен сравнить для идеи (x,t) систем Eq. (2).

Eq.(2) $\begin{cases}\dot\theta\sin\theta = \left[\lambda\cos\Phi\sin\Phi + \Phi_{xx}\right]\sin^2\theta + \theta_x\Phi_x\sin 2\theta\\ \dot\Phi\sin\theta = \left[1 + \lambda\cos^2\Phi + (\Phi_x)^2\right]\cos\theta\sin\theta - \theta_{xx}\end{cases}$, $\Phi = \frac{\pi}{2} - \varepsilon$, $\varepsilon \ll 1$, $\sin\Phi = \cos\varepsilon = 1 - O(\varepsilon^2)$, $\cos\Phi = \sin\varepsilon \approx \varepsilon$, но $\varepsilon(x,t)$.

$\dot\theta = \left[\lambda\varepsilon - \varepsilon_{xx}\right]\sin\theta - \varepsilon_x\theta_x 2\cos\theta$ и $-\dot\varepsilon\sin\theta = \left[1 + \lambda\varepsilon^2 + (\varepsilon_x)^2\right]\cos\theta\sin\theta - \theta_{xx}$

$\lambda \gg 1$ тогда: $\dot\theta = \lambda\varepsilon\sin\theta$ и $-\dot\varepsilon\sin\theta = \cos\theta\sin\theta - \theta_{xx}$ если $\lambda\varepsilon^2 \ll 1$,

$\varepsilon = \frac{1}{\lambda}\frac{\dot\theta}{\sin\theta}$, $\lambda\dot\varepsilon = \frac{\ddot\theta}{\sin\theta} + \dot\theta\frac{(-1)\cos\theta}{\sin^2\theta}\dot\theta = \frac{\ddot\theta}{\sin\theta} - (\dot\theta)^2\frac{\cos\theta}{\sin^2\theta}$. $-\sin\theta\dot\varepsilon = -\sin\theta\times$

$\times\frac{1}{\lambda}\left[\frac{1}{\sin\theta}\ddot\theta - (\dot\theta)^2\frac{\cos\theta}{\sin^2\theta}\right] = -\frac{1}{\lambda}\ddot\theta + \frac{1}{\lambda}(\dot\theta)^2\operatorname{ctg}\theta = \cos\theta\sin\theta - \theta_{xx}$, нет Sine-Gordon из-за $\frac{1}{\lambda}(\dot\theta)^2\operatorname{ctg}\theta$-term. Если оставить член ε^2 in $[\dots]$, то

$-\sin\theta\dot\varepsilon = -\frac{1}{\lambda}\ddot\theta + \frac{1}{\lambda}(\dot\theta)^2\operatorname{ctg}\theta = \left[1 + \lambda\cdot\frac{1}{\lambda^2}\frac{(\dot\theta)^2}{\sin^2\theta}\right]\cos\theta\sin\theta - \theta_{xx}$ и $-\frac{1}{\lambda}\ddot\theta = \sin\theta\cos\theta - \theta_{xx}$

- exact SG-eq. $\theta_{xx} - \frac{1}{c^2}\ddot\theta = \frac{1}{2}\sin\theta\cos\theta$, $c^2 = \lambda$, для $\theta(x,t)$.

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$\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \sin\theta\cos\theta$ имеет реш. $\cos\theta = th\left(\frac{x-vt}{\Delta}\right)$, $\sin\theta = \frac{1}{ch(z/\Delta)}$, $\xi = x - vt$.

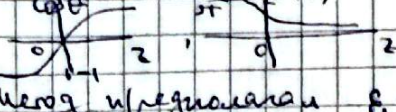
$z = \xi/\Delta$, $\Delta = \sqrt{1-v^2/c^2}$

$\frac{d\theta}{dt} = \frac{d}{dt}\theta\left(\frac{\xi}{\Delta}\right) = \frac{d}{dz}\theta(z)\frac{dz}{dt} = \frac{1}{\Delta}\theta'(z) = -\frac{v}{\Delta}\theta'(z)$, $\frac{d^2\theta}{dt^2} = -\frac{v}{\Delta}\frac{d}{dt}\theta'(z) = -\frac{v}{\Delta}\frac{d}{dz}\theta'(z)\frac{dz}{dt} =$

$= -\frac{v}{\Delta}\left(-\frac{v}{\Delta}\right)\theta''(z) = \frac{v^2}{\Delta^2}\theta''(z)$. $\theta_{xx} = \frac{1}{\Delta^2}\theta''(z)$, $\frac{1}{\Delta^2}\theta''(z) - \frac{1}{c^2}\frac{v^2}{\Delta^2}\theta''(z) = \sin\theta\cos\theta$

$\frac{(1-v^2/c^2)}{\Delta^2}\theta''(z) = \sin\theta\cos\theta$ или $\theta''(z) = \sin\theta\cos\theta$ при $\Delta^2 = (1-v^2/c^2)$.

$\sin\theta = \frac{1}{ch z}$, $\cos\theta\,\theta' = -\frac{1}{ch^2 z}sh z = -\frac{1}{ch z}th z = -\sin\theta\cos\theta$, $\theta' = -\sin\theta$



$\theta' = -\sin\theta < 0$, т.е. $\frac{\theta'}{\sin\theta} = (-1)$ или $\frac{\theta'}{\sin\theta} = (+1)$.

Метод нуклеации (Kaw, 1989) $\xi = \frac{1}{\lambda}\frac{\dot\theta}{\sin\theta}$ и скорость $\dot\xi$. $\dot\xi = \frac{1}{\lambda}\frac{1}{\sin\theta}\left(-\frac{v}{\Delta}\right)\theta'(z) = -\frac{v}{\lambda\Delta}\frac{\theta'(z)}{\sin\theta} =$

$= \frac{v}{\lambda\Delta}$, т.к. $\theta'(z)/\sin\theta = -1$ дает реш. SG для $\theta(x,t)$. $\xi(x,t) = \frac{v}{\lambda\Delta} = const$,

но если $\xi(x,t) = const$, $\dot\xi = 0$, $\xi_x = 0$ и $0 = [1+\lambda\xi^2]\sin\theta\cos\theta - \theta_{xx}$

$1 + \lambda\xi^2 = 1 + \lambda\frac{v^2}{\lambda^2\Delta^2} = \frac{1}{\Delta^2}\left[\Delta^2 + \frac{v^2}{\lambda}\right] = \frac{1}{\Delta^2}\left[1 - \frac{v^2}{c^2} + \frac{v^2}{\lambda}\right] = 1/\Delta^2$ и $\theta_{xx} = \frac{1}{\Delta^2}\sin\theta\cos\theta$,

т.е. SG вида $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \sin\theta\cos\theta$ неинтегрируемо никуда.

// Method Kaw'89 ($\xi << 1$, $\xi(x,t)$, $\lambda >> 1$) дает SG для $\theta(x,t)$, откуда следует $\xi(x,t) = const$, а при $\xi(x,t) = const$ методом Kaw'89 SG для $\theta(x,t)$ никуда не интегрируется.

→ т.е. Method Kaw'89 противоречив и не дает SG для $\theta(x,t)$.

Применен travelling ansatz $\xi = x - vt$ и assumption $\Phi_3 = 0$ для SD (η, Φ), $tg\theta/2 = e^{-\eta}$ сферу дает SG, точнее $\theta_{33} = \frac{1}{\Delta^2}\sin\theta\cos\theta$, эквивал. SG Eq(6)

$\theta_{xx} - \frac{1}{v^2}\ddot\theta = \frac{1}{2\Delta_0^2}\sin 2\theta$. Если $\Phi(\xi) = \Phi_0 = \pi/2 - \varepsilon$ и $\varepsilon << 1$, $\lambda >> 1$, откуда SG $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \frac{1}{2}\sin\theta\cos\theta$ $c^2 = \lambda$.

Вывод: Method Kaw'1989 не дает SG, а метод $\Phi_3 = 0$ дает SG и в частности SG при $\varepsilon << 1, \lambda >> 1$.

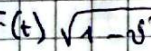
Kosevich'1990: $v_{DW} = v_g$, ω (lab. frame) $= \tilde\omega$ (mov. frame) $+ v_g k$, при $\tilde\omega = 0$ $\omega = v_g k$, $\frac{\omega}{k} = v_{ph}$, $v_{ph} = v_g = v_{DW}$. For DW ω - imaginary и k - imaginary, $v = \frac{\omega}{k}$ real.

Schlömann'71: v_{DW} (tail) $= v = v_{ph}$ (SW)

(16/1) Написание Replies to Referees Comments 1D-DW PRB manuscript.

(17/1) Meeting с Ricardo: 1) обсуждение Replies Ref. 1D-PRB и работ Wieser PRB'10, Wieser' PRB'10, Hill' PRL'2018. Slides Ricardo по AF-DW dynamics

$\dot q(t) = v_{DW}(t)$ as function $H_{ext}(t)$. Получил решение 1) $m\ddot q + \alpha\dot q = F(t)\sqrt{1-(\dot q)^2/v_m^2}$

$m\dot v + \alpha v = F(t)\sqrt{1 - v^2/v_m^2}$, где $F(t)$  - field/current pulse shape, $F(t) \sim t$.

2) записал и решить уравнения для $\dot q$ и $\Delta(t)$ в духе Thiaville'2005, т.е. dynamics (ps-scale) of DW width $\Delta(t) \to$ уже решить описание $v(t)$ as func. $H_{ext}(t)$

$\dot v\frac{1}{\sqrt{1-v^2/c^2}}$? $\frac{d}{dt}\sqrt{1-v^2/c^2} = \frac{1}{2\sqrt{1-v^2/c^2}}(-2)v/c^2\,\dot v = -\frac{v\dot v}{c^2}\frac{1}{\sqrt{1-v^2/c^2}}$

$u = \sqrt{1-v^2/c^2}$, $m\frac{\dot v}{\sqrt{1-v^2/c^2}} + \frac{\alpha v}{\sqrt{1-v^2/c^2}} = F(t)$, $v = c\sqrt{1-u^2}$.

$\frac{\dot v}{\sqrt{1-v^2/c^2}} = -\frac{c^2}{v}\frac{du}{dt} = -\frac{c}{\sqrt{1-u^2}}\dot u$, $\frac{v}{\sqrt{1-v^2/c^2}} = \frac{c\sqrt{1-u^2}}{u} \to -\frac{mc}{\sqrt{1-u^2}}\dot u + \alpha\frac{c}{u}\sqrt{1-u^2} = F(t)$

т.е. получаем в виде $F(t)$: $\dot v = -\beta v + F(t)\sqrt{1-v^2/c^2}$, если steady solution $\dot v = 0$ при $F(t) = const$

А другое решение при $F(t) = const$, $\dot v = f(v)$, $\frac{dv}{f(v)} = dt$, $\int\frac{dv}{f(v)} = t - t_0$,

$I = \int_{v_0}^{v}\frac{dv}{f(v)} = \int\frac{dv}{F\sqrt{1-v^2/c^2} - \beta v} = \frac{1}{F}\int\frac{dv}{\sqrt{1-v^2/c^2} - \beta' v} = \left|v = c\sin z\right| = \frac{1}{F}\int\frac{c\cos z\cos z}{\cos z - \beta' c\sin z} =$

$= \frac{c}{F}\int\frac{dz}{1-\beta' c\,tg z}$ - gamma Spanier. $\int\frac{dx}{a+b\,tg x} = \frac{xa}{a^2+b^2} + \frac{b}{a^2+b^2}\ln(a\cos x + b\sin x)$

$I = \frac{c}{F}\left[\frac{z}{1+(\beta' c)^2} - \frac{\beta' c}{1+(\beta' c)^2}\ln(\cos z - \beta' c\sin z)\right]_{v_0}^{v} = (t - t_0)$, где $v/c = \sin z$.

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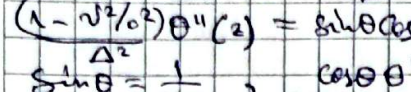
$\Theta_{xx} - \frac{1}{c^2}\Theta_{tt} = \sin\Theta\cos\Theta$ имеет реш. $\cos\Theta = \text{th}\left(\frac{x-vt}{\Delta}\right)$, $\sin\Theta = \frac{1}{\text{ch}(\xi/\Delta)}$, $\xi = x - vt$, $\Delta = \sqrt{1-v^2/c^2}$

$\frac{d\Theta}{dt} = \frac{d}{dt}\Theta\left(\frac{\xi}{\Delta}\right) = \frac{d}{d\xi}\Theta(\xi)\frac{d\xi}{dt} = \frac{1}{\Delta}\Theta'(\xi) = -\frac{v}{\Delta}\Theta'(\xi)$, $z = \xi/\Delta$, $\frac{d^2\Theta}{dt^2} = -\frac{v}{\Delta}\frac{d}{dt}\Theta'(z) = -\frac{v}{\Delta}\frac{d}{dz}\Theta'(z)\frac{dz}{dt} =$

$= -\frac{v}{\Delta}\left(-\frac{v}{\Delta}\right)\Theta''(z) = \frac{v^2}{\Delta^2}\Theta''(z)$. $\Theta_{xx} = \frac{1}{\Delta^2}\Theta''(z)$, $\frac{1}{\Delta^2}\Theta''(z) - \frac{1}{c^2}\frac{v^2}{\Delta^2}\Theta''(z) = \sin\Theta\cos\Theta$

$\frac{(1-v^2/c^2)}{\Delta^2}\Theta''(z) = \sin\Theta\cos\Theta$ или $\Theta''(z) = \sin\Theta\cos\Theta$ при $\frac{1}{\Delta^2} = (1-v^2/c^2)$.

$\sin\Theta = \frac{1}{\text{ch}z}$, $\cos\Theta\,\Theta' = -\frac{1}{\text{ch}^2 z}\text{sh}z = -\frac{1}{\text{ch}z}\text{th}z = -\sin\Theta\cos\Theta$, $\Theta' = -\sin\Theta$



$\Theta' = -\sin\Theta < 0$, т.е. $\frac{\Theta'}{\sin\Theta} = (-1)$ или $\frac{\Theta'}{\sin\Theta} = (+1)$.

Метод инвариантов $\varepsilon = \frac{1}{\lambda}\frac{\dot\Theta}{\sin\Theta}$ и оператор $\dot\varepsilon$. $\dot\varepsilon = \frac{1}{\lambda}\frac{1}{\sin\Theta}\left(-\frac{v}{\Delta}\right)\Theta'(z) = -\frac{v}{\lambda\Delta}\frac{\Theta'(z)}{\sin\Theta} =$
(Kaw 1989)

$= \frac{v}{\lambda\Delta}$, т.к. $\Theta'(z)/\sin\Theta = -1$ дает реш. SG для $\Theta(x,t)$. $\varepsilon(x,t) = \frac{v}{\lambda\Delta} = \text{Const}$,

но если $\varepsilon(x,t) = \text{const}$, $\dot\varepsilon = 0$, $\varepsilon_x = 0$ и $0 = [1+\lambda\varepsilon^2]\sin\Theta\cos\Theta - \Theta_{xx}$

$1 + \lambda\varepsilon^2 = 1 + \lambda\frac{v^2}{\lambda^2\Delta^2} = \frac{1}{\Delta^2}\left[\Delta^2 + \frac{v^2}{\lambda}\right] = \frac{1}{\Delta^2}\left[1 - \frac{v^2}{c^2} + \frac{v^2}{\lambda}\right] = 1/\Delta^2$ и $\Theta_{xx} = \frac{1}{\Delta^2}\sin\Theta\cos\Theta$,

т.е. SG вида $\Theta_{xx} - \frac{1}{c^2}\Theta_{tt} = \sin\Theta\cos\Theta$ нелинейно неинвариантно.

// Method Kaw'89 ($\varepsilon \ll 1$, $\varepsilon(x,t)$, $\lambda \gg 1$) дает SG для $\Theta(x,t)$, откуда очевидно $\varepsilon(x,t) = \text{const}$, а при $\varepsilon(x,t) = \text{const}$ методом Kaw'89 SG для $\Theta(x,t)$ нельзя получить.

→ т.е. Method Kaw'89 несовместим и не дает SG для $\Theta(x,t)$.

Применен travelling ansatz $\xi = x - vt$ и assumption $\Phi_\xi = 0$ для YD (η, Φ), $\text{tg}\Theta/2 = e^{-\eta}$ сразу дает SG, причем $\Theta_{\xi\xi} = \frac{1}{\Delta^2}\sin\Theta\cos\Theta$, эквивал. SG Eq(6)

$\Theta_{xx} - \frac{1}{v_e^2}\ddot\Theta = \frac{1}{2\Delta_e^2}\sin 2\Theta$. Если $\Phi(\xi) = \Phi_0 = \pi/2 - \varepsilon$ и $\varepsilon \ll 1$, $\lambda \gg 1$, откуда SG $\Theta_{xx} - \frac{1}{c^2}\Theta_{tt} = \frac{1}{2}\sin\Theta\cos\Theta$ $c^2 = \lambda$.

Выводы: Method Kaw '1989 не дает SG, а метод $\Phi_\xi = 0$ дает SG и в пределе SG при $\varepsilon \ll 1$, $\lambda \gg 1$.

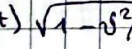
Kaseichi'1990: $v_{DW} = v_g$, ω (lab. frame) $= \tilde\omega$ (mov. frame) $+ v_g k$, при $\tilde\omega = 0$ $\omega = v_g k$, $\frac{\omega}{k} = v_{ph}$, $v_{ph} = v_g = v_{DW}$. For DW ω - imaginary и k - imaginary, $v = \frac{\omega}{k}$ real.

Schlömann'71: v_{DW} (tail) $= v = v_{ph}$ (SW)

(16/I) Написание Replies to Referees comments 1D-DW PRB manuscript.

(17/I) Meeting с Ricardo: 1) обсуждение Replies Ref. 1D-PRB и работ Wieser's PRB'ов, Wieser' PRB'10, Hill' PRL'2018. Slides Ricardo по AF-DW dynamics

$\ddot q(t) = \dot v_{DW}(t)$ as function $H_{ext}(t)$. Получили решение $m\ddot q + \alpha\dot q = F(t)\sqrt{1-(\dot q)^2/v_m^2}$

$m\dot v + \alpha v = F(t)\sqrt{1-v^2/v_m^2}$, где $F(t)$  - field/current pulse shape, $F(t) \sim t$.

2) записал и решить уравнения для $\dot q$ и $\Delta(t)$ в духе Thiaville'2005, чтоб. dynamics (ps-scale) of DW width $\Delta(t)$ → получить аналитическое $v(t)$ as func. $H_{ext}(t)$

$\dot v \frac{1}{\sqrt{1-v^2/c^2}}$? $\frac{d}{dt}\sqrt{1-v^2/c^2} = \frac{1}{2\sqrt{1-v^2/c^2}}(-2)v/c^2\,\dot v = -\frac{v\dot v}{c^2}\frac{1}{\sqrt{1-v^2/c^2}}$

$u = \sqrt{1-v^2/c^2}$, $\to m\frac{\dot v}{\sqrt{1-v^2/c^2}} + \frac{\alpha v}{\sqrt{1-v^2/c^2}} = F(t)$, $v = c\sqrt{1-u^2}$.

$\frac{\dot v}{\sqrt{1-v^2/c^2}} = -\frac{c^2}{v}\frac{du}{dt} = -\frac{c}{\sqrt{1-u^2}}\dot u$, $\frac{v}{\sqrt{1-v^2/c^2}} = \frac{c\sqrt{1-u^2}}{u} \to -\frac{mc}{\sqrt{1-u^2}}\dot u + \alpha\frac{c}{u}\sqrt{1-u^2} = F(t)$

в случаем в α и $F(t)$: $\dot v = -\alpha v + F(t)\sqrt{1-v^2/c^2}$, если steady solution $\dot v = 0$ при $F(t) = \text{const}$

А другое решение при $F(t) = \text{Const}$, $\dot v = f(v)$, $\frac{dv}{f(v)} = dt$, $\int\frac{dv}{f(v)} = t - t_0$

$I = \int_{v_0}^{v}\frac{dv}{f(v)} = \int\frac{dv}{F\sqrt{1-v^2/c^2} - \alpha v} = \frac{1}{F}\int\frac{dv}{\sqrt{1-v^2/c^2} - \alpha' v} = \left|v = c\sin z\right| = \frac{1}{F}\int\frac{c\cos z\,dz}{\cos z - \alpha' c\sin z} =$ $\alpha' = \alpha/F$.

$= \frac{c}{F}\int\frac{dz}{1 - \alpha' c\,\text{tg}z}$ - громоздкое выражение. $\int\frac{dx}{a+b\,\text{tg}x} = \frac{xa}{a^2+b^2} + \frac{b}{a^2+b^2}\ln(a\cos x + b\sin x)$

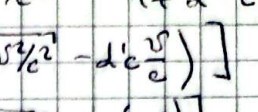
$I = \frac{c}{F}\left[\frac{z}{1+(\alpha' c)^2} - \frac{\alpha' c}{1+(\alpha' c)^2}\ln(\cos z - \alpha' c\sin z)\right]_{v_0}^{v} = (t - t_0)$, где $v/c = \sin z$.

Escaneado con CamScanner

Due steady state $\dot v = \text{const}$, $t\to\infty$, $\ddot v = 0$ и $\alpha v = F\sqrt{1-v^2/c^2}$, $\alpha' = \alpha/F$.

$(\alpha' v)^2 = 1 - v^2/c^2$, $\left(\alpha'^2 + \frac{1}{c^2}\right)v^2 = 1$, $v_s^2 = \frac{1}{\alpha'^2 + 1/c^2} = \frac{c^2}{1+\alpha'^2 c^2}$

$\frac{F}{c}t(v) + \text{const} = \frac{1}{\alpha'^2c^2+1}\left[\arcsin\frac{v}{c} - \alpha' c \ln\left(\sqrt{1-v^2/c^2} - \alpha' c\frac{v}{c}\right)\right]$



$t(v) = \frac{c}{F}\frac{1}{1+\frac{\alpha^2c^2}{F^2}}\left[\arcsin\frac{v}{c} - \frac{\alpha c}{F}\ln\left(\sqrt{1-v^2/c^2} - \frac{\alpha}{F}v\right)\right] + \text{const}$

$t(v) = \frac{\alpha}{F^2+\alpha^2c^2}\left[F\arcsin\frac{v}{c} - \alpha c\ln\left(\sqrt{1-v^2/c^2} - \frac{\alpha}{F}v\right)\right]$, отсюда $v(t)$ в неявном виде и v в ед. c, $\nu = \frac{\alpha c}{F}$, $t(v) = \frac{1}{\alpha}\frac{\nu}{1+\nu^2}\left[\arcsin v - v\ln\left(\sqrt{1-v^2} - \nu v\right)\right]_{v_0}^{v} + t_0$

$t(v_s)\to\infty$, v_s - steady state velocity $v_s = \frac{c}{\sqrt{1+\nu^2}}$. при $\nu \ll 1$, $v_s \approx c$.

при $t=t_0$, $v(t_0) = v_0$ - граничное условие

Сн. 2/v.2

Исходное ур-ие $\frac{\nu}{\alpha}\dot v = -\nu v + \sqrt{1-v^2}$ или $\frac{1}{\alpha}\dot v = -v + \frac{1}{\nu}\sqrt{1-v^2}$, $\nu = \frac{\alpha c}{F}$.

$\alpha t = \tau$ - slow time scale $\frac{dv}{d\tau} = -v + \frac{1}{\nu}\sqrt{1-v^2}$, при $H_{ext}(t) = \text{const}$.

(18/I) Improvements 1D DW-PRB paper. С Л. "Lidl", "Carrefour" Olaberria.

(19/I) С Л., Ю., Т. на Ондарреta и "Nam"! // Вернулись к 6/I - nullification L_{kin}.

В Micromagnetics для action $S\to\min$, надо standard lagrangian $L = L_{kin} - E \to -L$

Lagrangian density $\mathcal{L}$ (energy density in units $2K_2$, $[K_2] = \text{erg/cm}^3$) см. 2/v.19г.

$\mathcal{L} = \mathcal{L}_{kin} - e(\theta,\varphi)$, $\mathcal{L}_{kin} = -\cos\theta\dot\varphi = v\cos\theta\varphi_\xi = v\varphi\sin\theta\theta_\xi$, $\xi = x - vt$,

t в единицах t_0, $t_0 = 1/\gamma H_a$, $H_a = \frac{2K_2}{M_s}$, x,ξ в единицах $\Delta_0 = \sqrt{A/K_2}$.

Action $S = \int dt\int dx\,\mathcal{L}$, под travel wave ansatz $S = L = \int d\xi\,\mathcal{L}$

Energy density (in units $2K_2$) $e(\theta,\varphi) = \frac{1}{2}\left[(\theta_\xi)^2 + \sin^2\theta(\varphi_\xi)^2 + \sin^2\theta(1+\lambda\cos^2\varphi)\right]$

$L_{kin} = \int d\xi\,\mathcal{L}_{kin} = \int d\xi\, v\cos\theta\varphi_\xi = v\int d\xi\left[\sin\theta\cdot\varphi\theta_\xi + \frac{\partial}{\partial\xi}(\cos\theta\varphi)\right] = v\int d\xi\,\sin\theta\varphi\theta_\xi +$

$+ v\cos\theta(\xi)\varphi(\xi)\Big|_{-\infty}^{\infty} = \int d\xi\left[v\sin\theta\varphi\theta_\xi\right] + \text{const}$, $\cos\theta(\pm\infty) = \pm1$ или ∓1, $\varphi(\pm\infty) = 0$ условие

$\mathcal{L}_{kin}^{(\theta,\varphi)} = v\varphi\sin\theta\theta_\xi$, Walker $\varphi(\xi) = \varphi_0$, $\mathcal{L}_{kin}^{W} = v\varphi_0\sin\theta\theta_\xi$, $L_{kin}^{W} = \int d\xi\,\mathcal{L}_{kin}^{W} =$

$= v\varphi_0\int d\xi\,\sin\theta\theta_\xi = -v\varphi_0\int d\xi\frac{\partial}{\partial\xi}(\cos\theta) = -v\varphi_0\cos\theta(\xi)\Big|_{-\infty}^{+\infty} = -2v\varphi_0$, при

$\cos\theta(\pm\infty) = \pm1$, $\frac{\partial\theta}{\partial\xi} < 0$



Нужно $\mathcal{L}_{kin}(\theta,\varphi) = v\varphi\sin\theta\theta_\xi$, т.к. подбором $\varphi(\xi) = \text{const}$ не удовлетворяется,

$\tg\frac{\theta}{2} = e^{-y}$, $\sin\theta = \frac{1}{\ch y}$, $\theta(+\infty) = 0$, $\cos\theta = \th y$, $\cos\theta\,\theta_\xi = -\frac{1}{\ch^2 y}\sh y\cdot y_\xi = -\sin\theta\cos\theta\, y_\xi$

$\theta_\xi = -\sin\theta\, y_\xi$ и $\mathcal{L}_{kin}(y,\varphi) = v\varphi\frac{1}{\ch y}(-1)\frac{1}{\ch y}y_\xi = -v\varphi\frac{y_\xi}{\ch^2 y}$

В 1D-DW PRB paper использовано: $\mathcal{L}(\theta,\varphi) = e(\theta,\varphi) - \mathcal{L}_{kin}$ to search $\min\int d\xi\,\mathcal{L}(\theta,\varphi)$.

Пусть $\varphi(\xi) = \varphi_0$ (Walker), $\mathcal{L}(\theta,\varphi_0) = e(\theta,\varphi_0) - v\varphi_0\sin\theta\theta_\xi = \frac{1}{2}\left[\theta_\xi^2 + \sin^2\theta(1+\lambda\cos^2\varphi_0)\right] - v\varphi_0\sin\theta\theta_\xi$

Сом. Eq.(5) MS $1+\lambda\cos^2\varphi_0 = \frac{1}{\Delta^2}$, $\mathcal{L}(\theta,\varphi_0) = \frac{1}{2}\left[\theta_\xi^2 + \frac{1}{\Delta^2}\sin^2\theta\right] - v\varphi_0\sin\theta\theta_\xi = \mathcal{L}\left(\theta,\frac{\partial\theta}{\partial\xi}\right)$

$\frac{\partial}{\partial\xi}\frac{\partial\mathcal{L}}{\partial\theta_\xi} = \frac{\partial}{\partial\xi}\left[\theta_\xi - v\varphi_0\sin\theta\right] = \theta_{\xi\xi} - v\varphi_0\cos\theta\theta_\xi$, $\frac{\partial\mathcal{L}}{\partial\theta} = \frac{1}{2\Delta^2}2\sin\theta\cos\theta\Big|_{(\xi)} - v\varphi_0\theta_\xi\cos\theta$

$\theta_{\xi\xi} - v\varphi_0\cos\theta\theta_\xi = \frac{1}{\Delta^2}\sin\theta\cos\theta - v\varphi_0\cos\theta\theta_\xi$, φ_0 - знач. не зав-ит от ξ (Walker) в ур-ии Эйлера.

$\theta_{\xi\xi} = \frac{1}{2\Delta^2}\sin 2\theta$, $\tg\frac{\theta}{2} = e^{-\xi/\Delta}$, $\theta(\xi) = 2\arctg e^{-\xi/\Delta}$, $\theta_\xi = 2\frac{1}{1+e^{-2\xi/\Delta}}(-1)e^{-\xi/\Delta}\frac{1}{\Delta} =$

$= -\frac{1}{\Delta}\frac{1}{\ch(\xi/\Delta)}$, $\theta_{\xi\xi} = -\frac{1}{\Delta}\frac{\partial}{\partial\xi}\frac{1}{\ch(\xi/\Delta)} = -\frac{1}{\Delta}\frac{(-1)}{\ch^2(\xi/\Delta)}\frac{1}{\Delta}\sh(\xi/\Delta) = \frac{1}{\Delta^2}\frac{1}{\ch(\xi/\Delta)}\th(\xi/\Delta) = \frac{1}{\Delta^2}\sin\theta\cos\theta$

Т.е. если $y = \xi/\Delta$, то $\theta_{\xi\xi} = \frac{1}{2\Delta^2}\sin 2\theta$. $y = \xi/\Delta$ при $\varphi(\xi) = \varphi_0 = \text{const}$, $\frac{1}{\Delta^2} = 1+\lambda\cos^2\varphi_0$

$\theta(\xi) = \theta(x,t) = \theta(x-vt)$, $\theta_{xx} = \theta_{\xi\xi}$, $\theta_{tt} = v^2\theta_{\xi\xi}$, $\theta_t = \theta_\xi\cdot(-v)$, $\theta_{xx} - \frac{1}{v^2}\theta_{tt} =$

$= \left(1 - v^2/v_-^2\right)\theta_{\xi\xi} = \frac{(1-v^2/v_-^2)}{2\Delta^2}\sin 2\theta$, $\Delta_e^2 = \frac{\Delta^2}{(1-v^2/v_-^2)}$ даст Eq.(6) из MS.

Escaneado con CamScanner

T.e. Eq(6)-ms $\theta_{xx} - \frac{1}{v^2}\theta_{tt} = \frac{1}{2\Delta_e^2}\sin 2\theta$ получается из $\mathcal{L}(\theta,\Phi_0) = \frac{1}{2}(\theta_z)^2 + \frac{1}{2\Delta^2}\sin^2\theta - v\Phi_0\sin\theta\theta_z$ — вариац. подход без проблем. Также мы знаем Eq.(6) надо изменить.

$\mathcal{I}_{kin}(\theta,\Phi_0) = v\Phi_0\sin\theta\theta_z$ — solver. $L^w_{kin} = \int dz\, \mathcal{I}_{kin}(\theta,\Phi_0) = -2v\Phi_0$.

$L = \int dz\, \mathcal{L}(\theta,\Phi) = \int dz\, e(\theta,\Phi) - \int dz\, L_{kin} = \int dz\, e(\theta,\Phi) - v\int dz\, \Phi\sin\theta\theta_z = L[\theta,\Phi]$

$\forall \Phi(z)=\Phi_0$, $L[\theta,\Phi_0] = \int dz\, e(\theta,\Phi_0) - \int dz\, \mathcal{I}_{kin}(\theta,\Phi_0) = \frac{1}{2}\int dz\,[\theta_z^2 + \frac{1}{\Delta^2}\sin^2\theta] + 2v\Phi_0$

Cm. Lang-Bishop (70/IV/82). minimize $I(\theta,\Phi) = \int dx\, \mathcal{H}(x) + I_{kin}(v)$, $I_{kin}(v) = -Pv$

$P = S\int dx\,(1-\cos\theta)\frac{\partial\Phi}{\partial x}$, $I_{kin}(v) = -\int dx\, vp$, $P = \int dx\, p$, $-vp = -vS(1-\cos\theta)\frac{\partial\Phi}{\partial x}$ →

→ $vS\cos\theta\frac{\partial\Phi}{\partial x} = vS\cos\theta\Phi_z = i_{kin}(v)$. Мой $\mathcal{I}_{kin} = v\cos\theta\Phi_z$ или $\mathcal{I}_{kin} = v\Phi_z\sin\theta\theta_z$.

В моих единицах $S(\text{lang}\cdot 2\pi) = 1$. $I_{kin}(v) = -Pv = L_{kin} = -2v\Phi_0$, т.е.

$\forall z\in\Phi(z)=\Phi_0$, momentum $P = 2\Phi_0$.

Eq.(2) $\forall \Phi_z=0$, $\Phi_x=0$, $\dot\Phi=0$ (travel.wave ansatz) : $\left\{\begin{array}{l}\dot\theta\sin\theta = \lambda\cos\Phi_0\sin\Phi_0\sin^2\theta \\ \theta_{xx} = (1+\lambda\cos^2\Phi_0)\cos\theta\sin\theta\end{array}\right\}$ $\Phi(z)=\Phi_0$

$\left\{\begin{array}{l}\dot\theta = \lambda\sin\Phi_0\cos\Phi_0\sin\theta \\ \theta_{xx} = \frac{1}{\Delta^2}\sin\theta\cos\theta\end{array}\right\}$, $\lambda\sin\Phi_0\cos\Phi_0 = \frac{v}{\Delta}$, $\dot\theta = \frac{v}{\Delta}\sin\theta$, $\mathcal{I}_{kin}(\theta,\Phi_0) = -\Phi_0\dot\theta\sin\theta$

$\sin\theta = \frac{\Delta}{v}\dot\theta$, $\mathcal{I}_{kin} = -\Phi_0\dot\theta\sin\theta = -\Phi_0\dot\theta\frac{\Delta}{v}\dot\theta = -\frac{\Phi_0\Delta}{v}(\dot\theta)^2$ или $\mathcal{I}_{kin} = -\Phi_0\frac{v}{\Delta}\sin^2\theta$

$\mathcal{L}(\theta,\Phi_0) = \frac{1}{2}(\theta_x)^2 + \frac{1}{2\Delta^2}\sin^2\theta + \Phi_0\frac{\Delta}{v}(\dot\theta)^2$ на основе УД для $\theta(x,t)$.

Какой $\mathcal{L}(\theta)$ берет к $\theta_{xx} - \frac{1}{v^2}\ddot\theta = \frac{1}{2\Delta_e^2}\sin 2\theta$.

$\mathcal{L}(\theta) = \frac{1}{2}(\theta_x)^2 + \frac{1}{2\Delta_0^2}\sin^2\theta - a(\dot\theta)^2 = \mathcal{L}(\theta(x,t))$, $\frac{\partial\mathcal{L}}{\partial\theta} = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot\theta} + \frac{\partial}{\partial x}\frac{\partial\mathcal{L}}{\partial(\theta_x)}$

$\frac{1}{\Delta_e^2}\sin\theta\cos\theta = \frac{d}{dt}(-2a\dot\theta) + \frac{\partial}{\partial x}(\theta_x) = -2a\ddot\theta + \theta_{xx}$, $2a = \frac{1}{v^2}$

$\mathcal{L}(\theta) = \frac{1}{2}(\theta_x)^2 + \frac{1}{2\Delta_e^2}\sin^2\theta - \frac{1}{2}\frac{1}{v^2}(\dot\theta)^2$ и формула Л. $\forall\Phi(z)=\Phi_0$ $\mathcal{L}(\theta,\Phi_0) = \frac{1}{2}(\theta_x)^2 + \frac{1}{2\Delta^2}\sin^2\theta + \Phi_0\frac{\Delta}{v}(\dot\theta)^2$

$\frac{1}{\Delta_e^2} = \frac{1}{\Delta^2}(1 - v^2/v^2)$. $\frac{1}{2\Delta_e^2}\sin^2\theta - \frac{1}{2v^2}(\dot\theta)^2 = \frac{1}{2\Delta^2}\left(1-\frac{v^2}{v^2}\right)\sin^2\theta - \frac{1}{2v^2}(\dot\theta)^2 =$

$= \frac{1}{2\Delta^2}\sin^2\theta - \frac{1}{2\Delta^2}\frac{v^2}{v^2}\sin^2\theta - \frac{1}{2v^2}(\dot\theta)^2 = \frac{1}{2\Delta^2}\sin^2\theta - \frac{1}{2\Delta^2}\frac{1}{v^2}\Delta^2(\dot\theta)^2 - \frac{1}{2v^2}(\dot\theta)^2 =$

$= \frac{1}{2\Delta^2}\sin^2\theta - \frac{1}{v^2}(\dot\theta)^2 \Rightarrow \mathcal{L}(\theta) = \frac{1}{2}(\theta_x)^2 + \frac{1}{2\Delta^2}\sin^2\theta - \frac{1}{v^2}(\dot\theta)^2$, ganami Sc-Eq.(6)

Точно $\mathcal{L}(\theta,\Phi_0) = \frac{1}{2}(\theta_x)^2 + \frac{1}{2\Delta^2}\sin^2\theta + \Phi_0\frac{\Delta}{v}(\dot\theta)^2$ — разумеется остался $(\dot\theta)^2$ член.

Particular case $\lambda\gg 1$, $\Phi_0 = \pi/2 - \varepsilon$, $\sin\Phi_0\cos\Phi_0 = \cos\varepsilon\sin\varepsilon \approx \varepsilon$, $\dot\theta = \lambda\varepsilon\sin\theta$

$\mathcal{I}_{kin}(\theta,\varepsilon) = -\Phi_0\dot\theta\sin\theta = -(\pi/2-\varepsilon)\dot\theta\sin\theta = \varepsilon\sin\theta\dot\theta - \frac{\pi}{2}\sin\theta\dot\theta = \frac{1}{\lambda}(\dot\theta)^2 - \frac{\pi}{2}\sin\theta\dot\theta$

$e(\theta,\varepsilon) = \frac{1}{2}[(\theta_x)^2 + \sin^2\theta(1+\lambda\varepsilon^2)]$, $\lambda\varepsilon = \frac{v}{\Delta}$, $1+\lambda\varepsilon^2 = \left(\frac{\lambda\varepsilon}{v}\right)^2 = \frac{1}{\Delta'^2}$, но $\Delta = \Delta' = \sqrt{1-v^2/c^2}$ Зрев

$\mathcal{L}_{(\theta,\varepsilon)} = e(\theta,\varepsilon) - \mathcal{I}_{kin}(\theta,\varepsilon) = \frac{1}{2}(\theta_x)^2 + \frac{1}{2\Delta'^2}\sin^2\theta - \frac{1}{\lambda}(\dot\theta)^2$. Ferstl int. $(\theta_x)^2 = \frac{1}{\Delta'^2}\sin^2\theta$, и/м 1972.

$\mathcal{L}(\theta,\varepsilon) = (\theta_x)^2 - \frac{1}{\lambda}(\dot\theta)^2$ (!?). // В 1D-DW manuscript $\mathcal{L}(\theta\dot\theta) = \theta_x^2 + \sin^2\theta - \dot\theta^2/c^2$.

24/IV я остановил работу 1D DW dynamics, так и не вычислив $\mathcal{L}(\theta,\varepsilon)$ вид.

Возможно я брал $\mathcal{L}(\theta,\varepsilon) = (\theta_x)^2 + \sin^2\theta - \frac{1}{c^2}(\dot\theta)^2$ методом $m_x = \varepsilon\sin\psi \ll 1$, $c^2 = \lambda$.

(20/I) Ревизия 1D DW paper и ответов Referees. Письмо to Ricardo y Ruben. Публикация D. Berkov на Solid State Mag 2020. 1 abstract на 20/I.

(21/I) Переписка Ricardo/Ruben о 1D-DW paper. Разбор Wieser'PRB'2010.

Wieser'2010 plotted DW velocity in units $v_0 = \gamma a J/\mu_s$, $v_0 = \frac{J}{\hbar}a$

если $\mu_s = g\mu_B S$, $S=1$, $D \ll J \approx 10^{-20}$ Joule, $a = 0{,}3$ nm, $v_0 \approx 300$ km/s

Velocity $v_{DW}/v_0 \approx \frac{b}{2}\frac{\Delta}{a}$, $\Delta = \sqrt{Ja^2/2D_e}$, $b = B_z/B_0$, $B_0 = J/\mu_s = B_{ex}$.

$v_{DW}(B_z)$ имеет max, Walker breakdown и/м $D_h/J < 1$. $v_{DW}(B_z)$ saturates at $\frac{D_h}{J} \gg 1$.

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(21/I) План работы (см. 11/II, 15/IX 2019):

1) AF-DW dynamics с Ricardo, Ruben, SK-transient Subm. Nat. Com.
2) ~~Re-submit~~ 1D-DW manuscript to PRB
3) $\Phi_0(9)$ - Sk static, $BS^2 \to N$ Sk transform. Breath mode. Φ_0 - Verba PRB
4) Sk-pair, 1D chain, PSS - Zukhra Q_t^m
5) Subm. "Drum" modes MS, Takaaki, Dobrovolskiy - Subm. Nano Lett. 30/III
6) "Movilidad" UPV/EHU travel grant (deadline Mar.)
7) Paper "ring/Sk-dot (Zelent)
8) Non-elastic SW scattering film edge (Pawel) и DW-string oscillat.
9) Laguna Sk-STNO, subm. to RRL 13/I
10) Вчера в revised MS „Neel-Sk in soft dots" - Berganza Subm. Nanoscale
11) Conf. SkyMag'2020, 6-9 Apr., Paris - Abs., ~~hotel~~, ~~fee~~, tickets - go Feb. 10, 2020 Conf. cancelled
12) TMAG-2020, Sept. 7-11, Palermo, Italy - Abs., hotel, fee, tickets - go Feb. 28, 2020
13) JEMS'2020, July 27-30, Lisbon - Abs., hotel, fee, tickets - go March 14, 2020 Change → Dec. 2020
14) Rev. APL '2020 Hung/Sk STNO
15) CONSTANTO IVA ~~XII~~кв. 2019 и Н. 3/год. тах 2019
16) Нодг. и Беседа с А. Fert, ~27-31/I. о бодж. собш. работе.
17) Нодг. trip в Poznan 3-17/II, Bus tickets Berlin ⇄ Poznan
18) Recom. letter A. Thiaville for D. Givord "award". sent to Cros 17/I
19) 1D DW dynamics, solution $\varepsilon(\xi)$, $\lambda \gg 1$
20) Nanotube edge modes, Парсад exp.
21) Нодеза декл. IVA I кв. 2020 go 25/IV, Замме. Tax office на May 2020

(22/I) Meeting с Julian (Conf., visit A. Fert) и Ricardo (Replies to 1D DW Referees, changes to MS, new paper by Komineas, AF-Sk mass). Paper IVA CONSTANTO.

(23/I) Проверим нонсенс sine-Gordon behavior FM by Enz, Mikeska, Dodd нодеги $\Phi = \pi/2 - \varepsilon$, $\varepsilon \ll 1$, нолевое $\varepsilon(x,t)$ и $\varepsilon(x,t) = \text{Const}$, $\varepsilon_x = \frac{\partial \varepsilon}{\partial x} = 0$ и $\neq 0$ Пусть $\varepsilon(x,t) \ll 1$. $\cos\Phi \sin\Phi = \sin\varepsilon\cos\varepsilon = \varepsilon + O(\varepsilon^3)$, $\cos^2\Phi = \sin^2\varepsilon \simeq \varepsilon^2 + O(\varepsilon^4)$

Eq. (2) of 1D-DW MS $\left\{ \begin{array}{l} \dot\Theta \sin\Theta = [\lambda\varepsilon - \varepsilon_{xx}]\sin^2\Theta - \varepsilon_x\Theta_x \sin 2\Theta \\ -\dot\varepsilon \sin\Theta = [1 + \lambda\varepsilon^2 + (\varepsilon_x)^2]\cos\Theta\sin\Theta - \Theta_{xx} \end{array} \right\}$ с точностью до ε^2.

Dodd'1982 даже в начальных eq. где $\dot\Theta, \dot\Phi$ игнорируется член с $(\Phi_x)^2$.

$\dot\Theta = [\lambda\varepsilon - \varepsilon_{xx}]\sin\Theta - \varepsilon_x\Theta_x \cdot 2\cos\Theta$, тогда получаем $\dot\Theta = \lambda\varepsilon\sin\Theta$ надо $\varepsilon_x = 0$.

Если $\varepsilon_x \neq 0$ тогда, то $\ddot\Theta = \lambda\dot\varepsilon\sin\Theta + \lambda\varepsilon\cos\Theta\dot\Theta - \frac{d}{dt}(\varepsilon_{xx}\sin\Theta) - \frac{d}{dt}(\varepsilon_x\Theta_x 2\cos\Theta) =$

$= -\lambda\left[(1+\lambda\varepsilon^2 + (\varepsilon_x)^2)\cos\Theta\sin\Theta - \Theta_{xx}\right] + \lambda\varepsilon\cos\Theta\left[(\lambda\varepsilon - \varepsilon_{xx})\sin\Theta - 2\varepsilon_x\Theta_x\cos\Theta\right] -$

$- \frac{d}{dt}\left[\varepsilon_{xx}\sin\Theta + 2\varepsilon_x\Theta_x\cos\Theta\right]$, $\Theta_{xx} - \frac{1}{\lambda}\Theta_{tt} = (1+\lambda\varepsilon^2 + \varepsilon_x^2)\cos\Theta\sin\Theta - \varepsilon\cos\Theta\left[(\lambda\varepsilon - \varepsilon_{xx})\sin\Theta -\right.$

$\left. - 2\varepsilon_x\Theta_x\cos\Theta\right] + \frac{1}{\lambda}\frac{d}{dt}(\varepsilon_{xx}\sin\Theta + 2\varepsilon_x\Theta_x\cos\Theta) = (1+\varepsilon_x^2)\sin\Theta\cos\Theta + \varepsilon\cos\Theta\left(\varepsilon_{xx}\sin\Theta + 2\frac{\varepsilon_x\Theta_x}{\cos\Theta}\right) +$

$+ \frac{1}{\lambda}\frac{d}{dt}(\varepsilon_{xx}\sin\Theta + 2\varepsilon_x\Theta_x\cos\Theta)$. Если $\varepsilon_x = 0$, то $\Theta_{xx} - \frac{1}{c^2}\Theta_{tt} = \sin\Theta\cos\Theta$ - точное SG.

→ $\Theta_{xx} - \frac{1}{\lambda}\Theta_{tt} = (1+\varepsilon_x^2)\sin\Theta\cos\Theta + \varepsilon\cos\Theta(\varepsilon_{xx}\sin\Theta + 2\varepsilon_x\Theta_x\cos\Theta) + \frac{1}{\lambda}\frac{d}{dt}(\varepsilon_{xx}\sin\Theta + 2\varepsilon_x\Theta_x\cos\Theta)$

Вблизи yD с точн. до ε^2. Если $\varepsilon_x^2 \ll 1$, $\varepsilon\varepsilon_{xx} \ll 1$, $\varepsilon\varepsilon_x \ll 1$ слагаемые и более высокие то $\Theta_{xx} - \frac{1}{\lambda}\Theta_{tt} = \frac{1}{\lambda}\frac{d}{dt}(\varepsilon_{xx}\sin\Theta + 2\varepsilon_x\Theta_x\cos\Theta) + \sin\Theta\cos\Theta$ и надо $\frac{1}{\lambda}\frac{d}{dt}(\ldots)$ тоже $\to 0$.

$\Theta_{xx} - \frac{1}{\lambda}\Theta_{tt} = \sin\Theta\cos\Theta$ - SG.

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SG $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \sin\theta\cos\theta$ имеет реш. $\cos\theta = th\left(\frac{\xi}{\Delta}\right)$, $\sin\theta = \frac{1}{ch(\xi/\Delta)}$, $\xi = x - vt$, $\Delta = \sqrt{1 - v^2/c^2}$

$\theta_{xx} = \frac{1}{\Delta^2}\theta''(z) = \frac{1}{\Delta^2}\sin\theta\cos\theta$, $z = \xi/\Delta$, $(\cdot)' = \frac{d}{dz}(\cdot)$. Вернемся к ε для θ, ε

$-\dot\varepsilon\sin\theta = [1 + \lambda\varepsilon^2 + \varepsilon_x^2]\sin\theta\cos\theta - \sin\theta\cos\theta\frac{1}{\Delta^2}$, $\dot\varepsilon = \left[\frac{1}{\Delta^2} - 1 - \lambda\varepsilon^2 - \varepsilon_x^2\right]\cos\theta$

$\dot\theta = -\frac{v}{\Delta}\theta'(z) = \frac{v}{\Delta}\sin\theta$, $\theta_x = \frac{d\theta}{dx} = \theta'_z\frac{dz}{dx} = \frac{1}{\Delta}\theta'(z) = -\frac{1}{\Delta}\sin\theta$

$\frac{v}{\Delta}\sin 2\theta = (\lambda\varepsilon - \varepsilon_{xx})\sin^2\theta - \varepsilon_x\sin 2\theta\left(+\frac{1}{\Delta}\right)\sin\theta$. $z = \xi/\Delta = (x - vt)/\Delta$.

$\frac{v}{\Delta} = \lambda\varepsilon - \varepsilon_{xx} + \frac{1}{\Delta}\varepsilon_x 2\cos\theta$. Если снова $\varepsilon_x = 0$, то $\varepsilon = \frac{v}{\lambda\Delta}$ и $\dot\varepsilon = 0$,

т.е. $\varepsilon_x = 0$, $\dot\varepsilon = 0$, $\varepsilon = v/\lambda\Delta$ совместимо с $\varepsilon(\xi)$ и $\frac{d\varepsilon}{d\xi} = 0$, $\frac{d\varphi}{d\xi} = 0$. traveling wave $+\varphi_\xi = 0$

Если $\theta(x,t) = \theta(\xi)$ как следует из SG, то и $\varepsilon(x,t) = \varepsilon(\xi)$ или $\varepsilon = \varepsilon(z)$.

$\dot\varepsilon = \frac{d\varepsilon(z)}{dt} = \varepsilon'(z)\frac{dz}{dt} = -\frac{v}{\Delta}\varepsilon'(z)$, $\varepsilon_x = \varepsilon'(z)\frac{dz}{dx} = \frac{1}{\Delta}\varepsilon'(z)$.

$-\frac{v}{\Delta}\varepsilon' = \left[\frac{1}{\Delta^2} - 1 - \lambda\varepsilon^2 - \frac{1}{\Delta^2}(\varepsilon')^2\right]\cos\theta$, $\frac{v}{\Delta} = \lambda\varepsilon - \frac{1}{\Delta^2}\varepsilon'' + 2\cos\theta\frac{1}{\Delta^2}\varepsilon'$

Совместимы ли эти два eqs. for $\varepsilon(z)$? При $\varepsilon(z) = const$, $\varepsilon' = 0$ — да.

$\varepsilon_{xx} - \frac{2}{\Delta}\cos\theta\,\varepsilon_x - \lambda\varepsilon = -\frac{v}{\Delta}$, $\cos\theta = th(\xi/\Delta)$. Видно $\varepsilon(z) \sim v$.

Если $\Delta \to 0$, $\frac{1}{\Delta} \to \infty$ (1) $(\varepsilon')^2 = 1$, (2) $-\varepsilon'' + 2\cos\theta\,\varepsilon' = 0$, — несовместимы.
$v \to c$, $\varepsilon'' = 0$

$\varepsilon'(z)$ – odd function $z \to \varepsilon(z)$ – even function z и $\varepsilon(z) \sim v$.

Интеграл движ. $\theta_x^2 + \Phi_x^2\sin^2\theta = \sin^2\theta(1 + \lambda\cos^2\Phi)$, $\cos\left(\frac{\pi}{2} - \varepsilon\right) = \sin\varepsilon = \tilde\varepsilon$ $+O(\varepsilon^3)$

$\theta_x^2 + \varepsilon_x^2\sin^2\theta = \sin^2\theta(1 + \lambda\varepsilon^2)$, $\theta_x^2 = \frac{1}{\Delta^2}\sin^2\theta$

$\frac{1}{\Delta^2} + \varepsilon_x^2 = 1 + \lambda\varepsilon^2$, $\varepsilon_x^2 = 1 + \lambda\varepsilon^2 - \frac{1}{\Delta^2}$, $\dot\varepsilon = -\frac{v}{\Delta}\varepsilon' = \left[\frac{1}{\Delta^2} - 1 - \lambda\varepsilon^2 - \left(1 + \lambda\varepsilon^2 - \frac{1}{\Delta^2}\right)\right] \times$

$-\frac{v}{\Delta}\varepsilon' = 2\cos\theta\left[\frac{1}{\Delta^2} - 1 - \lambda\varepsilon^2\right]$. Но $\varepsilon_x^2 = 1 + \lambda\varepsilon^2 - \frac{1}{\Delta^2}$ можно решить только $\times\cos\theta$

$\frac{1}{\Delta^2} = x^2$, $\frac{d\varepsilon}{dx} = \sqrt{1 - x^2 + \lambda\varepsilon^2}$, $1 - x^2 = a^2(\varepsilon)$, $1 - \frac{1}{\Delta^2} = \frac{1}{\Delta^2}(\Delta^2 - 1) = \frac{1}{\Delta^2}\left[\left(1 - \frac{v^2}{c^2}\right) - 1\right] < 0$

$\frac{d\varepsilon}{\sqrt{\lambda\varepsilon^2 - a^2}} = dx$, $\frac{1}{\sqrt\lambda}\int\frac{d\varepsilon}{\sqrt{\varepsilon^2 - b^2}} = x + c$. $b^2 = \frac{1}{\lambda}a^2 = \frac{1}{\lambda}(x^2 - 1) = \frac{1}{\lambda}\left(\frac{1}{\Delta^2} - 1\right) > 0$

$\ln|\varepsilon + \sqrt{\varepsilon^2 - b^2}| = Arch|\varepsilon| = \sqrt\lambda x$. $\varepsilon(x) = ch(\sqrt\lambda x)$? $y = \varepsilon/b$

$\frac{1}{\sqrt\lambda b}\int\frac{dy}{\sqrt{y^2 - 1}} = x$, $arcch\,y = \sqrt\lambda x$, $\varepsilon(x) = b\,ch(\sqrt\lambda x)$, $x > b = \frac{v}{\lambda\Delta}$.

$b^2 = \frac{1}{\lambda}\left(\frac{1}{\Delta^2} - 1\right) = \frac{1}{\lambda\Delta^2}(1 - \Delta^2) = \frac{1}{\lambda\Delta^2}\left[1 - \left(1 - \frac{v^2}{c^2}\right)\right] = \frac{1}{\lambda\Delta^2}\frac{v^2}{c^2} = \frac{v^2}{\lambda^2\Delta^2}$

$\frac{d\varepsilon}{dy} = \frac{d\varepsilon}{d\xi} \to \varepsilon(\xi) = b\,ch(\sqrt\lambda\xi)$ $b = \frac{v}{\lambda\Delta} = \varepsilon_0$, $\varepsilon(\xi) = \frac{v}{\lambda\Delta}ch(\sqrt\lambda\xi)$ — nonsense,

$\varepsilon(\xi)$ – расходится при $\xi \to \pm\infty$.

Вывод: SG $\theta_{xx} - \frac{1}{c^2}\theta_{tt} = \sin\theta\cos\theta$ имеет Ens/Dodd инвариантная variable $\varepsilon(x,t)$ ищем и преобразуем ε_x^2, $\varepsilon\varepsilon_x$, $\varepsilon\varepsilon_{xx}$, $\frac{1}{\lambda}\varepsilon_{xx}$, $\frac{1}{\lambda}\varepsilon_x$ значения приводят к senseless $\varepsilon(\xi) = \varepsilon(x,t)$, которое $\to\infty$ при $|\xi| \to \infty$. $\varepsilon \ll 1$.

Остается только $\varepsilon(x,t) = const(x)$. Eq.(1) и eqs. Ens, Dodd при $\varepsilon(x,t) = const(x)$

→ тогда $\frac{d\varepsilon}{dx} = 0$, $\dot\theta = \lambda\dot\varepsilon\sin\theta$ $-\dot\varepsilon\sin\theta = (1 + \lambda\varepsilon^2)\sin\theta\cos\theta - \theta_{xx}$, откуда неизвестная $\varepsilon(t)$ через $\ddot\theta = \lambda\ddot\varepsilon\sin\theta + \lambda\dot\varepsilon\cos\theta\dot\theta$ получим SG $\theta_{xx} - \frac{1}{\lambda}\theta_{tt} = \sin\theta\cos\theta$, предполагая $\dot\varepsilon \neq 0$.

1) $\dot\theta = \frac{v}{\Delta}\sin\theta$ в ans SG, $\varepsilon = \frac{1}{\lambda}\frac{\dot\theta}{\sin\theta} = \frac{\lambda v}{\Delta} = const$ и $\dot\varepsilon = 0$, $\dot\varepsilon = 0$ не подходит для решения SG

2) $\frac{d\varepsilon}{dx} = \varepsilon_x = 0$ и $\dot\varepsilon \neq 0$ несовместимо с travel. wave ansatz $\theta(\xi)$, $\varepsilon(\xi)$ или $\varphi(\xi)$

$\varepsilon_x = \varepsilon_\xi$, $\dot\varepsilon = \varepsilon_\xi\dot\xi = -v\varepsilon_\xi$ и нельзя $\varepsilon_x = 0$ и $\dot\varepsilon \neq 0$, $\dot\varepsilon = -v\varepsilon_x$.

→ надо отказ $\varepsilon(\xi)$ с использованием traveling wave ansatz $\vec m(x - vt)$.

Получили responses do Referees и 1D-2W manuscript и sent comments Ricardo.

(24/I) Запросили Gyfbiotti info о CMD-GEFES 2020, Madrid, Aug. 31 – sept. 4.

(25/I) 1D-2W manuscript was resubmitted to PRB. C n. «Carrefour» Olaberria.

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(27/I) Завършил и sent to V. Gros nomination A. Thuaille на „D. Gibert" prize. Бълг.

(28/I) Обсъжд. с Ruben и Ricardo AF-DW dyn. (inertial) in response to $H_{sat}(t)$ и Ricardo project with Unai (Berlin) - Spin Peltier effect due to DW pair collision in AF Mn_2Au ($\Delta T(r)$ near DW). Публикува A. Fert. Domag me A. Fert (Ricardo/Kon) 1D DW dynamics, mapping LG to Sine Gordon.

(29/I) Подгот. slides за „Skyrm-STNO" presentation give A. Fert/SkyMag.

(30/I) Presentation „Sk-STNO" talk give Ruben и Ricardo (A. Fert не знае). Запис A. Fert о before 31/I. Review и correct Ref. responses to Ba-Felicity ARB.

(31/I) Presentation „Sk-STNO" give A. Fert, J. Gonzalez и Ricardo. Обед в „Nam" с Л. в „Carrefour" и „Lidl" Alberria.

(4/II) Отдаване дрехи, бельо, якове + Fabor с Ант. П. носилки: 14kg (Л.Д.), 6kg (Ана) 31€

(2/II) Принт. и взеване board passes BIO → Palma, Palma → Berlin. Peer. APL MS Liang et al. „Sk-STNO-synking neurons" - reject.

(3/II) Пристигане BIO → Palma - Hotel „HM" Gran Fiesta. Poznan

(4/II) Пристигане Palma → Berlin (STR). Flixbus to Poznan. Изход в спирка „Avenida", taxi, настаняване в HAM Residencia.

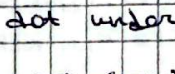
(5/II) Meet. с Y. Klos, M. Zelent, P. Gruszecki. Обсъжд. static Skyrmion energy simulation с Zelent. Zelent взима 1.300 PLN на ден разходи. С Л. и Ана в „Carrefour" Вечеря.

(6/II) Л. цели ранг. Изпит, защита, сбогуване. Обсъжд SW-edge scattering с Pawel и Sk-calcs с Nedelcu. Сем. семинар 11/II и бъдещ Zaragoza 3/III.

(7/II) Обсъжд. Sk-energy, R_s calc. с Zelent. DeBonis ansatz дава малки R_s по сравн. с MuMax и слабо зависи от DMI. Бълг.

(8/II) Отдаване в Residencia, бълг. Обсъжд. с Игор Рера paper on photon-vortex interaction и PRB'18 - Kusminskiy. У Рера $\vec{M}.\vec{H}$ interaction - bilinear, у Kusminskiy $\vec{P}.\vec{E}$ interaction - cubic. $\vec{H}, \vec{E}$ - mer., en. поле сбер. Белтъм. $\Delta\vec{P} = \alpha\vec{E}$, $\alpha = i\hat{f}\vec{M} \rightarrow$ Hint ~ $\vec{M}.\vec{E}.\vec{E}$.

(10-13/II) Ревю artificial Sk Network и SW edge scattering-Pawel. Domag сем „Skyrmion STNO". Резюме.

(14/II) Dipolar field from ISD dot under it 

, $z<0$, см. 3/IV Zang $\tau_1 \to 0$, $R_2 \to R_2$

$$H_\rho(\rho,z) = -2\pi M_s R_2 \int_0^\infty dk\, J_1(k\rho) J_1(kR_2) e^{kz}(1-e^{-kL})$$
$$H_z(\rho,z) = -2\pi M_s R_2 \int_0^\infty dk\, J_0(k\rho) J_1(kR_2) e^{kz}(e^{-kL}-1)$$

ρ in units of R_2, z in units of L, $k\rho = (kR_2)\frac{\rho}{R_2} \to k\rho$, $kz = (kR_2)\frac{z}{L}\frac{L}{R_2} = kz\beta$
k in units $1/R_2$, $kL = kR_2\frac{L}{R_2} = \beta k$

$$\begin{cases} H_\rho(\rho,z) = -2\pi M_s \int_0^\infty dk\, J_1(k\rho) J_1(k) \exp(kz\beta)(1-e^{-\beta k}) \\ H_z(\beta,z) = -2\pi M_s \int_0^\infty dk\, J_0(k\rho) J_1(k) \exp(kz\beta)(e^{-k\beta}-1) \end{cases}$$ see J.T. Conway 2001

(15/II) Обеди с Л. „Ресо". Отбери Referees „Sk-STNO" RRL. Resubmitted RRL.

(16/II) Домаи. Poznan → Berlin (Schönefield) → Palma → Bilbao. Hora hotel Palma.

(11/II) Сроч. e-mails.

(12/II) Уточнен 2 invited speakers: Berkov и Kirilyuk. E-mail to Caleb Уточнен PRB „1D DW dyn." и PRB „Art. Skyrmions" и „Dimm modes" Ruben бъдещи уточнени Arxiv: „AF DW dynamics" — уточнен/Comment.

→ Спомо: 1) Л. отчет IWA 2016-18; 2) Abstracts - Palermo до feb. 28; 3) SW-int. Pawel. 4) Haldane invitation; 5) Visit to Pepe; 6) отчет Poznan.

Връзки с Гълъб: уикенд 28/II - 3/III ко-ние (лич. дорожу ≤200€); уикенд на SolSkyMag (Conf. fee - cash, лич. дорожу ≤200€ и Hotel); мин с Лорен в Porto и 13-26/VIII до JEMS за отчет уточн. Гълъб; канд. на бел. as invited to OST-school Magnonics' 2020-Porto, Dimm modes уточнен 20XX.

(19/II) Обсъжд. с Julian SolSkyMag (уточнени Haldan, задат Abstracts to decide о Welcome Party/Conf. event) и Unai - Skyrm. Fellow (UPV/EHU иска да 49% на STNO).

Abs. SW-DW scat. и 1D DW dyn. → JEMS, Sk-STNO → Palermo (?).

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20/II IWA Gustambo" - Excel file 2016 2017, 2018. Reference c Ricardo, Ruben, Pepa.

Review draft paper AF=DW dyn. by Ruben - Overleaf.

21/II Nogues - u nocias to Interbasque e-mail o noggerence Unai Atxirika - Res. Fellow application (>60%), send to Andrea, "Invitation D. Haldane nogr. Abstract TMAG 2020 "Sk-SOMO" - neuronal problem registration.

22/II Review/update MS Ruben 1D AF DW dynamics. 1D DW AF

2018 4/II Nonlocal interaction energy due to exchange of two Neel 1D DW

$\theta=\frac{\pi}{2}$, $\varphi=\varphi(x)$, $\varphi_1(x)=2q_1\arctg\left(e^{\frac{x-X_1}{\Delta}}\right)$, $\varphi_2(x)=2q_2\arctg\left(e^{\frac{x-X_2}{\Delta}}\right)$

$q=\frac{1}{\pi}\int dx\,\frac{\partial\varphi}{\partial x}=\frac{1}{\pi}\left[\varphi(+\infty)-\varphi(-\infty)\right]=\frac{1}{\pi}\left[\pm\frac{\pi}{2}-\left(\mp\frac{\pi}{2}\right)\right]=\pm 1$.

$E_{ex}=\int dx\,A\left[(\nabla\theta)^2+\sin^2\theta(\nabla\varphi)^2\right]=\left|\theta=\pi/2,\ \varphi(x)\right|=A\int_{-\infty}^{\infty}dx\,(\nabla\varphi)^2$, $(\nabla\varphi)^2=\left(\frac{\partial\varphi}{\partial x}\right)^2$

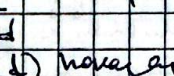
$\varphi(x)=\varphi_1(x)+\varphi_2(x)$, $\frac{\partial\varphi}{\partial x}=\frac{\partial\varphi_1}{\partial x}+\frac{\partial\varphi_2}{\partial x}=\frac{q_1}{\Delta}\frac{1}{\mathrm{ch}\xi_1}+\frac{q_2}{\Delta}\frac{1}{\mathrm{ch}\xi_2}$, $\xi_i=\left(\frac{x-X_i}{\Delta}\right)$

$\left(\frac{\partial\varphi}{\partial x}\right)^2=\frac{1}{\Delta^2}\left[\frac{q_1}{\mathrm{ch}\xi_1}+\frac{q_2}{\mathrm{ch}\xi_2}\right]^2=\frac{1}{\Delta^2}\left[\frac{1}{\mathrm{ch}^2\xi_1}+\frac{1}{\mathrm{ch}^2\xi_2}+\frac{2q_1q_2}{\mathrm{ch}\xi_1\,\mathrm{ch}\xi_2}\right]$

$E_{ex}^{int}=\frac{A}{\Delta^2}\int dx\,\frac{2q_1q_2}{\mathrm{ch}\xi_1\,\mathrm{ch}\xi_2}=2q_1q_2\frac{A}{\Delta^2}\int dx\,\frac{1}{\mathrm{ch}\xi_1\,\mathrm{ch}\xi_2}=2q_1q_2\frac{A}{\Delta}I(d)$, $d=\left(\frac{X_1-X_2}{\Delta}\right)$

$I(d)=2\Delta\frac{d}{\mathrm{sh}d}\Rightarrow E_{ex}^{int}=4q_1q_2\frac{A}{\Delta}\frac{d}{\mathrm{sh}d}$, $d=\left(\frac{X_1-X_2}{\Delta}\right)$ reduced DW$_1$–DW$_2$ distance.

When $q_1q_2=+1$, $E_{ex}^{int}(d)=\frac{4A}{\Delta}\frac{d}{\mathrm{sh}d}$ repulsive; when $q_1q_2=-1$ attract



na Fig. 3a - Ruben $q_1q_2=-1$, nonetheless $E_{ex}^{int}(d)$ nova ana repulsive, a ne attractive?

$E=A(\nabla\vec{m})^2-K\sin^2\Phi+E_m$. E_m (1D DW) $=2\pi M_s^2(m_y)^2$ in bulk.

Due thin film $\frac{E_m}{M_s^2}=N_{zz}m_z^2+N_{xx}m_x^2+N_{yy}m_y^2=N_{yy}m_y^2$

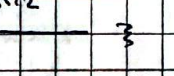
$N_{zz}\times 4\pi\to 0$, $N_{xx}\to 0$, $N_{yy}\sim\frac{L}{w}\ll 1$. $\frac{E_m}{M_s^2}=N_{yy}m_y^2\Rightarrow$

$m_y=\sin\Theta\sin\Phi=\sin\Phi$.

$E_m=N_{yy}M_s^2\sin^2\Phi\ll E_a=-K\sin^2\Phi$, $\forall w$. $N_{yy}\ll 1$. E_m influenced perturb nova. Ea ruler tenko E_a^{int}?

$\sin\Phi=\sin(\Phi_1+\Phi_2)=\sin\Phi_1\cos\Phi_2+\sin\Phi_2\cos\Phi_1$

$\Phi(x)=\arctg\left(\frac{x}{\Delta}\right)$, $\frac{\pi}{4}$, 0 – za nekemo, gorsmo





U Ruben, Fig. 3a $\frac{\partial\Phi}{\partial x}<0$ bcenga - 360 360° Neel DW, $\Phi(\xi)=\frac{\pi}{2}-4\arctg(e^{\xi})$

Hago $q=+1$, $\Phi(\xi)=2\arctg(e^{\xi})-\pi/2$

$q=-1$, $\Phi(\xi)=\frac{\pi}{2}-2\arctg(e^{\xi})$

$\Phi(\xi)=q\arctg(e^{\xi})$

ova cuncarne 180° - DW.

Ovo cuncarne E_{ex}^{int} - DW

$\Phi(\xi)=q\left[2\arctg e^{\xi}-\pi/2\right]$, 90° - Neel DW - ner.

2. 4/II 2018 e ovnca Exch. interaction

DW pair $\Phi=\Phi_1+\Phi_2=2q_1\arctg(e^{\xi_1})+2q_2\arctg e^{\xi_2}-\frac{\pi}{2}(q_1+q_2)=\Phi(\xi)$

$\frac{\partial\Phi}{\partial x}=2q_1\frac{\partial}{\partial x}\arctg e^{\xi_1}+2q_2\frac{\partial}{\partial x}\arctg e^{\xi_2}=2q_1\frac{1}{1+e^{2\xi_1}}e^{\xi_1}\frac{1}{\Delta}+2q_2\frac{1}{e^{2\xi_2}+1}$

$=\frac{q_1}{\Delta}\frac{1}{\mathrm{ch}\xi_1}+\frac{q_2}{\Delta}\frac{1}{\mathrm{ch}\xi_2}$. $\Phi(\xi)=2q\arctg(e^{\xi})$, $\Phi\in[0,\pi]$ (± 1)

Razlika E_{ex}^{int} 4/II 2018, Beyer, no vmesto da mu nagovorim $\Phi(\xi)=q\left[2\arctg e^{\xi}-\pi/2\right]$

$\sin(\Phi_1+\Phi_2)=\sin\left[2q_1a_1+2q_2a_2-\frac{\pi}{2}(q_1+q_2)\right]=(-1)^{q_1+q_2}\sin(2q_1a_1+2q_2a_2)$

$a_i=\arctg(e^{\xi_i})$, $(-1)^{q_1+q_2}=-q_1q_2$, $\sin(\Phi_1+\Phi_2)=-q_1q_2\sin(2q_1a_1+2q_2a_2)$

$\sin^2\Phi=\sin^2(2q_1a_1+2q_2a_2)$, $\sin^2\Phi=\sin^2\Phi_1\cos^2\Phi_2+\sin^2\Phi_2\cos^2\Phi_1+2\sin\Phi_1\sin\Phi_2\cos\Phi_1\cos\Phi_2$

$\Phi_i\to 2q_ia_i$ zgrec. $\sin\Phi=\sin(2\arctg e^{\xi_i})=q\sin(2\arctg e^{\xi_i})=q\frac{1}{\mathrm{ch}\xi_i}$,

$\cos\Phi=(-q)^0\cos(2\arctg e^{\xi_i})=\cos(2\arctg e^{\xi_i})=-q\,\mathrm{th}\xi_i$. $\sin(2q_ia_i)=q_i\frac{1}{\mathrm{ch}\xi_i}$

$\xi_i=(x-x_i)/\Delta$ $\cos(2q_ia_i)=-q_i\mathrm{th}\xi_i$ - ner $\cos(2a_i)=-\mathrm{th}\xi_i$

Escaneado con CamScanner

$$\sin^2\Phi = \frac{1}{ch^2\xi_1}\,th^2\xi_2 + \frac{1}{ch^2\xi_2}\,th^2\xi_1 + 2\,\frac{1}{ch\xi_1\,ch\xi_2}\,th\xi_1\,th\xi_2 = \left(\frac{th\xi_2}{ch\xi_1} + \frac{th\xi_1}{ch\xi_2}\right)^2$$

Введем $d = (X_1 - X_2)/\Delta$. $\xi_1 = \frac{x'}{\Delta}$, $\xi_2 = \frac{x'}{\Delta} + d$, $x' = x - X_1$, $\xi_1 - \xi_2 = d\,(-1)$.

$$\sin^2\Phi = \frac{1}{ch^2\xi_1}\,\frac{1}{ch^2\xi_2}\,(sh\xi_1 + sh\xi_2)^2 .$$ Note: $\Phi(\xi) = q\left[2\,\mathrm{arctg}\,e^{\xi} - \frac{\pi}{2}\right] = q\cdot gd(\xi)$

$gd(\xi) = 2\,\mathrm{arctg}\,e^{\xi} - \pi/2$ – гудермановская. $\sin\Phi = \sin[q\,gd(\xi)] = q\sin[gd(\xi)] =$

$= q\,th\xi$, $\cos\Phi = \cos(q\,gd(\xi)) = \cos[gd(\xi)] = \frac{1}{ch\xi}$

$\Phi_i(\xi_i) = q_i\cdot gd(\xi_i) \to \sin\Phi_i = q_i\,th\xi_i$, $\cos\Phi_i = 1/ch\xi_i$

$\Phi(x) = \Phi_1 + \Phi_2$, $\sin\Phi(1) = \sin\Phi_1\cos\Phi_2 + \cos\Phi_1\sin\Phi_2 = q_1\,th\xi_1\,\frac{1}{ch\xi_2} +$

$+ \frac{1}{ch\xi_2}\,q_2\,th\xi_2$, $\sin^2\Phi = \frac{th^2\xi_1}{ch^2\xi_2} + \frac{th^2\xi_2}{ch^2\xi_1} + 2q_1q_2\,\frac{th\xi_1\,th\xi_2}{ch\xi_1\,ch\xi_2}$ – ответ.

$$\sin^2\Phi = \frac{1}{ch^2\xi_1\,ch^2\xi_2}\,(q_1\,sh\xi_1 + q_2\,sh\xi_2)^2$$

1D DW interaction

$q_1q_2=+1$, $sh\xi_1 + sh\xi_2 = 2sh\frac{1}{2}(\xi_1+\xi_2)\,ch\frac{1}{2}(\xi_1-\xi_2) = 2sh\frac{1}{2}\left(\frac{2x'}{\Delta}+d\right)ch\frac{d}{2}$

$q_1q_2=-1$, $sh\xi_1 - sh\xi_2 = 2sh\frac{1}{2}(\xi_1-\xi_2)\,ch\frac{1}{2}(\xi_1+\xi_2) = 2sh\frac{d}{2}\cdot ch\left(\frac{x'}{\Delta}+\frac{d}{2}\right)(-1)$

$2ch^2\xi = ch2\xi + 1$. $W_a = \int dx\,E_a = -K\int_{-\infty}^{\infty} dx\,\sin^2\Phi$ – наименьшая работа.

$2ch^2\xi_2 = 1 + ch2\left(\frac{x'}{\Delta}+d\right) = 1 + ch\frac{2x'}{\Delta}\,ch2d + sh\frac{2x'}{\Delta}\,sh2d$

$$E_a^{int} = -K\int dx\,\sin^2\Phi = -K\int dx'\,\frac{1}{ch^2\xi_1}\,\frac{1}{\frac{1}{2}\left[1 + ch\frac{2x'}{\Delta}\,ch2d + sh\frac{2x'}{\Delta}\,sh2d\right]}\,4sh^2\left(\frac{x'}{\Delta}+\frac{d}{2}\right)ch^2\frac{d}{2} =$$

$$= -8K\Delta\int_{-\infty}^{\infty}\frac{dx}{ch^2x}\,\frac{sh^2(x+d/2)\,ch^2(d/2)}{[1 + ch2x\,ch2d + sh2x\,sh2d]}$$ при $q_1q_2=+1$. $\Delta = \sqrt{A/K}$.

$$E_{int}(d) = E_{ext}^{int} + E_a^{int} = \frac{4A}{\Delta}\,\frac{d}{shd} - 8K\Delta\int_{-\infty}^{\infty}\frac{dx}{ch^2x}\,\frac{sh^2(x+\frac{d}{2})\,ch^2(d/2)}{[1 + ch2x\,ch2d + sh2x\,sh2d]}$$
$q_1q_2=+1$

$2sh^2\xi = ch2\xi - 1$, $J(d) = \int_{-\infty}^{\infty}\frac{dx}{ch^2x}\,\frac{[ch(2x+d)-1]}{[ch2(x+d)+1]}$, посл. тр. – Рунге.

$$\frac{E_{int}(d)}{4\sqrt{AK}} = \frac{d}{shd} - sh^2\frac{d}{2}\,J(d)$$ См. "1D_DW_interaction.mcd"

$E_{int}(d)$ быстро падает с ростом d при $q_1q_2=+1$ – отталкивание DWs.

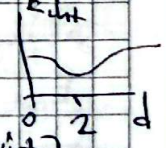
Если $q_1q_2=-1$, то $sh(x+\frac{d}{2}) \to ch(x+\frac{d}{2})$ в числителе $J(d)$, $2ch^2\xi = ch2\xi + 1$.

и $chd/2 \to shd/2$.

$$\frac{1}{4\sqrt{AK}}\,E_{int}(d) = -\frac{d}{shd} - sh^2\frac{d}{2}\int_{-\infty}^{\infty}\frac{dx}{ch^2x}\,\frac{[ch(2x+d)+1]}{[ch2(x+d)+1]}$$
$q_1q_2=-1$

Интересно, что для $q_1q_2=-1$, $E_{int}(d)$ имеет min при $d \approx 2$ (!),

и это без всякого SOT mag. field, которое сжимает DW pair (attractive int.)



(23/II) Учтем E_{int} из-за SOT mag. field $\parallel Oy$ (Fig. 3a Ruben).

$W_z = -\vec{H}\vec{M} = -H_y M_s \sin\Phi$, $\sin\Phi = \sin(\Phi_1+\Phi_2) = q_1\,\frac{th\xi_1}{ch\xi_2} + q_2\,\frac{th\xi_2}{ch\xi_1}$

$\Phi_i(\xi_i) = q_i\,gd(\xi_i)$, $\sin\Phi_i = \sin\left[q_i\left(2\,\mathrm{arctg}\,e^{\xi_i} - \frac{\pi}{2}\right)\right] = \sin\left(2q_i\,\mathrm{arctg}\,e^{\xi_i}\right)\cos\left(q_i\frac{\pi}{2}\right) -$

$- \cos\left(2q_i\,\mathrm{arctg}\,e^{\xi_i}\right)\sin\left(q_i\frac{\pi}{2}\right) = -q_i\cos\left(2\,\mathrm{arctg}\,e^{\xi_i}\right)$. ($\sin\left(q_i\frac{\pi}{2}\right) = 0$)

$\Theta = 2\,\mathrm{arctg}\,e^{\xi}$, $\Theta'_\xi > 0$, $0 \to \pi$, $\cos\Theta(\xi)$ падает с $\xi\uparrow$, $\cos\Theta = -th\xi$.

$\sin\Phi_i = -q_i\cos\left(2\,\mathrm{arctg}\,e^{\xi_i}\right) = -q_i(-th\xi_i) = q_i\,th\xi_i$

$\cos\Phi_i = \cos\left[q_i\left(2a_i - \frac{\pi}{2}\right)\right] = \cos(2q_ia_i)\cos\left(q_i\frac{\pi}{2}\right) + \sin(2a_iq_i)\sin\left(q_i\frac{\pi}{2}\right) =$

$= q_i\sin(2a_i)\,q_i = q_i^2\sin\left(2\,\mathrm{arctg}\,e^{\xi_i}\right) = 1/ch\xi_i$, $\cos\Phi_i = 1/ch\xi_i$, т.е. $\cos\Phi_i, \sin\Phi_i$

знаки как 22/II $gd(\xi)$-method и $E_{an}^{int}(d)$ – верная.

Escaneado con CamScanner

$E_z = \int dx W_z = -H_y M_s \int_{-\infty}^{\infty} dx \sin\Phi$ $\int dx \sin\Phi = \int dx \left[q_1 \frac{th\xi_1}{ch\xi_1} + q_2 \frac{th\xi_2}{ch\xi_2} \right]$

Диполи в ед. Δ, $d = X_1 - X_2$, $\xi_1 = x'$, $\xi_2 = x' + d$, $x' = x - X_1$, $\xi_1 - \xi_2 = -d$

$\xi_1 = x - X_1$, $\xi_2 = x - X_2$ в ед. Δ.

$\int dx' \sin\Phi = \int \left[q_1 \frac{th x'}{ch(x'+d)} + q_2 \frac{th(x'+d)}{ch x'} \right] dx = \int \frac{dx}{ch\xi_1 ch\xi_2} \left(q_1 sh\xi_1 + q_2 sh\xi_2 \right)$

а) $q_1 q_2 = +1$, $\int dx \sin\Phi = q_1 \int \frac{dx}{ch\xi_1 ch\xi_2} (sh\xi_1 + q_1 q_2 sh\xi_2) = q_1 \int \frac{dx}{ch\xi_1 ch\xi_2} (sh\xi_1 + sh\xi_2) =$

$= q_1 \int \frac{dx'}{ch\xi_1 ch\xi_2} 2ch\frac{d}{2} sh\left(x' + \frac{d}{2}\right) = 2q_1 ch\frac{d}{2} \int \frac{dx}{ch x \cdot ch(x+d)} sh\left(x + \frac{d}{2}\right) = 2q_1 ch\frac{d}{2} J_a(d)$

$J_a(d) = \int_{-\infty}^{\infty} \frac{dx}{ch x} \frac{sh(x + \frac{d}{2})}{ch(x+d)}$ — интеграл

$\int dx \sin\Phi = q_1 \int \left[\frac{th x}{ch(x+d)} + \frac{th(x+d)}{ch x} \right] dx$

б) $q_1 q_2 = -1$,

$\int_{-\infty}^{\infty} dx \frac{th\xi_1}{ch\xi_2} = \int dx \frac{th x}{ch(x+d)} = |x+d = x'| = \int_{-\infty}^{\infty} dx' \frac{th(x'-d)}{ch x'} = \int \frac{dx}{ch x} th(x-d) =$

$= \int \frac{dx}{ch x} \frac{(th x - th d)}{1 + th x \cdot th d} = \int dx\, th x \frac{1}{(ch x - sh x \cdot th d)} - th d \int dx \frac{1}{(ch x - sh x \cdot th d)}$

а) $q_1 q_2 = +1$, $\int dx \sin\Phi = 2q_1 ch\frac{d}{2} \cdot J_a(d)$, $J_a(d) = \int_{-\infty}^{\infty} \frac{dx}{ch x} \frac{sh(x + d/2)}{ch(x+d)}$

б) $q_1 q_2 = -1$, $\int dx \sin\Phi = q_1 \int \frac{dx}{ch\xi_1 ch\xi_2} (sh\xi_1 - sh\xi_2) = q_1 (-2) sh\frac{d}{2} \int \frac{dx}{ch x} \frac{ch(x + \frac{d}{2})}{ch(x+d)} =$

$= -2q_1 \cdot sh\frac{d}{2} J_b(d)$, $J_b(d) = \int_{-\infty}^{\infty} \frac{dx}{ch x} \frac{ch(x + d/2)}{ch(x+d)}$

а) $q_1 q_2 = +1$, $E_z = -H_y M_s \Delta \cdot 2q_1 ch\frac{d}{2} J_a(d)$; б) $q_1 q_2 = -1$, $\frac{E_z}{H_y M_s \Delta} = 2q_1 sh\frac{d}{2} J_b(d)$

Общая форма $q_1 sh\xi_1 + q_2 sh\xi_2 = sh(q_1\xi_1) + sh(q_2\xi_2) = 2sh\frac{1}{2}(q_1\xi_1 + q_2\xi_2) \cdot ch\frac{1}{2}(q_1\xi_1 - q_2\xi_2)$

$= 2q_1 sh\frac{1}{2}(\xi_1 + q\xi_2) ch\frac{1}{2}(\xi_1 - q\xi_2)$, $q = q_1 q_2 = \pm 1$.

$\xi_1 + q\xi_2 = x' + q(x'+d) = (1+q)x' + qd$, $\xi_1 - q\xi_2 = (1-q)x' - qd$

$\int dx' \sin\Phi = \int dx' \frac{1}{ch x' \cdot ch(x'+d)} 2q_1 sh\left[\frac{(1+q)x' + qd}{2}\right] ch\frac{1}{2}\left[(1-q)x' - qd\right]$

$E_z = -H_y M_s \Delta \cdot 2q_1 \int \frac{dx}{ch x} \frac{sh\frac{1}{2}[(1+q)x + qd]\, ch\frac{1}{2}[(1-q)x - qd]}{ch(x+d)}$ — забыли ли q_1 и $q = q_1 q_2$

Для анизотропии $E_a = -K_\Delta \int dx \sin^2\Phi$, $\sin^2\Phi = \frac{1}{ch^2 x ch^2(x+d)} sh^2\frac{1}{2}[(1+q)x + qd] ch^2\frac{1}{2}[(1-q)x - qd]$

для забывания знака $q = q_1 q_2$.

$\Phi = \Phi_1 + \Phi_2 = q_1 \left[2arctg e^{\xi_1} - \frac{\pi}{2} \right] + q_2 \left[2arctg e^{\xi_2} - \frac{\pi}{2} \right] \to \left(\frac{\partial\Phi}{\partial x}\right)^2 = \left(\frac{q_1}{ch\xi_1} + \frac{q_2}{ch\xi_2} \right)^2 =$

$= \frac{1}{ch^2\xi_1} + \frac{1}{ch^2\xi_2} + 2q \frac{1}{ch\xi_1 ch\xi_2}$, $E_{ex} = \frac{A}{\Delta} \int dx \left(\frac{\partial\Phi}{\partial x}\right)^2 = \frac{A}{\Delta} \int dx \left[\frac{1}{ch^2\xi_1} + \frac{1}{ch^2\xi_2} + 2q \frac{1}{ch\xi_1 ch\xi_2} \right]$

$E_{ex}^{int} = 2q \frac{A}{\Delta} \int \frac{dx}{ch\xi_1 ch\xi_2} = 2q \frac{A}{\Delta} \int_{-\infty}^{\infty} \frac{dx}{ch x \cdot ch(x+d)} = 2q \frac{A}{\Delta} I(d)$, $I(d) = \int_{-\infty}^{\infty} \frac{dx}{ch x \cdot ch(x+d)} = \frac{2d}{sh d}$

забыли ли $q = q_1 q_2$ раньше.

$E_{int} = E_{ex}^{int}(q) + E_a(q) + E_z(q_1, q)$, $E_{int} = E_{int}(d)$ — зависит от расстояния между E_z. $d = \left(\frac{X_1 - X_2}{\Delta}\right)$.

$\frac{E_z(d)}{H_y M_s \Delta} = -2q_1 \int_{-\infty}^{\infty} \frac{dx}{ch x} \frac{sh\frac{1}{2}[(1+q)x + qd]\, ch\frac{1}{2}[(1-q)x - qd]}{ch(x+d)} = I_z(d, q_1, q)$

$E_a(d) = \frac{A}{\Delta} \left[\int \frac{dx}{ch^2 x} + \int \frac{dx}{ch^2(x+d)} + 2q \int \frac{dx}{ch x \cdot ch(x+d)} \right] = Const(d) + 2q \frac{A}{\Delta} I(d) = Const + 2q \frac{A}{\Delta} \frac{d}{sh d}$

$I_z(d, q_1, +1) = -2q_1 ch\frac{d}{2} \int_{-\infty}^{\infty} \frac{dx}{ch x} \frac{sh(x + d/2)}{ch(x+d)}$ — Мешает не знаем

$F(d) = \int_{-\infty}^{\infty} \frac{dx}{ch x} \frac{sh(x + d/2)}{ch(x+d)}$, $F(-d) = \int_{-\infty}^{\infty} \frac{dx}{ch x} \frac{sh(x - d/2)}{ch(x-d)} = \left| \begin{matrix} x' = -x \\ x = -x' \end{matrix} \right| = -\int_{-\infty}^{\infty} \frac{dx}{ch x} \frac{sh(-x - d/2)}{ch(-x-d)}$

$= -\int_{-\infty}^{\infty} \frac{dx}{ch x} \frac{sh(x + d/2)}{ch(x+d)} = -F(d)$, $F(d)$ — нечет. $F(0) = 0$. $E_z(d)$ — нечетная

Полные вопросы / comments on 1D topol. charges и DW interaction

Ricardo, Pierre. Proof of 1D-DW paper PRB.

Escaneado con CamScanner

(24/II) Беседа с Julian о SolSkyMag'2020. Conf. перемяска, letters to Unai (Ikerbasque Fellowship) и Ricardo - proofs.

(25/II - 28/II) Подготовка Excel tables по IVA и обозначения нормы Base IVA - разход не совп. с Разходи Form 200 по канону за прибыль.

(27/II) Мнения Глеб. Работа с ним над „Drum modes" MS given Nenolett. (от ухудшили MS-text от 2/2019).

(29/II) Мой и П. дело ромб. дома (Глеб подарил 2×50€).

(1/III) работа над „Drum modes" MS. Отдых дома. Agustina sent revised Py dot paper.

(2/III) Вопреки с Julian. Обед в Помпеи с Глеб. Принтам Sk-SFNO RRL.

(3/III) перемяска с invited SolSkyMag. Включая, поправили и послали to Agustina et al. „Skyrmion Py dot" MS. Начали разделять ено на 2 MS - exp./simul. и theory/sim.

(4/III) перемяска SolSkyMag. носили Глеб revised „Drum modes" MS to complete (by Dobrovolsk.

(5/III) Отдел travel - Poznan, перемяска SolSkyMag-2020.

(6/III) Отдел Poznan и отдел Vilp - Kakazei. С П. в „Eroski" и Urbil. Вычитка proof Sk-oscillator RRL. Conf. program.

(7/III) Носили proof Sk-oscil." to RRL. С П. в „Lidl", „Carrefour - Olaberria.

(8/III) составл. програм SolSkyMag. Фильм и Зарега" дома.

(9/III) Reminders to invited Speakers (все ответили, кроме Nikitov).

(9-10/III) Составление program SolSkyMag'2020 и перемяска conf.

(11/III) Вычитка 2nd proof 1D-Dw PRB, ответ company-presenter, Получ. invited. revised theory sec. и text „Drum modes" и носили Глеб, Олекс., Abs. JEMS'20.

(12/III) Отделили Hotel Paris (SkyMag, Пр. 6-9). Сделано: •1) договорил. и publish SolSkyMag program; •2) определились с Restoran, Welcome SolSkyMag; •3) abs. JEMS •4) Сдать отчеты Poznan и San Seb. /Глеб; •5) ответ Налог. инспекции; •6) Visit to Pepa. •IJUAM - отчет.

Берганза прислала the last version „Skyrmions in Py-dots" дала Atmoscale. Работа над SolSkyMag-2020 program". Глеб просил о 2-х posters Timofeev.

(13/III) Оксана закончила R_S, Δ и $m_z(s)$, $m_s(s)$ Neel-Sk in thick Py-dots для 2nd paper by Berganza et al. (theory + Simul.). Berganza-Py dots MFM

Расчеты Neel-Sk metastable states с $\beta = \frac{L}{R} = (0.1 \div 0.9)$ и $\frac{R}{l_{ex}} = 3.0 - 1500$ cylinder. dot в целом 22-23/I 2018 (!). β - R MPD и больше l_{ex}. Затем в Berganza-MS брали BP-model of Neel-Sk, т.е. $m_z(s) = \frac{c^2 - s^2}{c^2 + s^2}$, $m_s(s) = \frac{2cs}{c^2+s^2}$, R_c - Sk-radius.

$c = R_c/R$. $E[\vec{m}(s)] = E(c, \beta, R) = E_{ex}(c, R) + E_m(c, \beta)$ - dot energy.

$E(c, \beta, R) \to$ min дает Sk-radius equilibrium $c_0(\beta, R)$ и equil. $E_0(\beta, R) = E(c_0, \beta, R)$ MathCad „Skyrmion-stability-dot.mcd". Брали для Py dots $Q = 0$ ($K_u = 0$), $DMI = 0$.

Подготовили ответ Оксане и другим co-authors о способе вычисл. MPD - Berganza - для Neel-Skyrmions. Updated conf. Program и вывесили на website.

(14/III) Просмотрен „Drum modes" draft и носили comments to Dobrovol.

Оксана просит Neel/Bloch Sk-profiles для Berganza dot sizes 60×30 nm Видимо $2R = 60$ nm - dot diameter (в Berganza-PRX subm. May 2019 было $2R = 70$ nm), а dot thickness is $L = 30$ nm. B/N-Sk Berganza MS

$$e(r_s) = \frac{1}{4\pi M_S^2 V} E(r_s) = \left(\frac{l_{ex}}{R}\right)^2 \int_0^1 ds\, s \left[(\theta_s')^2 + \frac{1}{s^2}\sin^2\theta\right] + e_m(\beta)_{r_s}, \quad \beta = L/R \text{ cyl. dot.}$$

$$e_m(c, \beta) = \frac{1}{\beta}\int_0^\infty dk\,(1 - e^{-\beta k})\, I_2^2(k) + \int_0^\infty dk\, k f(\beta k)\, I_S^2(k), \quad \begin{cases} I_2(k) = \int_0^1 ds\, s\, J_0(ks)\, m_z(s) \\ I_0(k) = \int_0^1 ds\, s\, J_1(ks)\, m_s(s) \end{cases}$$

$c = r_s = R_s/R$ - reduced Sk-radius. $l_{ex}^2 = A/(2\pi M_S^2)$.

$e(c, \beta, \tau) = e_{ex}(c, \tau) + e_m(c, \beta)$, $\tau = R/l_{ex}$. $(L, R) \to (\beta, \tau)$ dot sizes. $e(c, L, R)$

BP ansatz $\operatorname{tg}\frac{1}{2}\theta(s) = \frac{s}{c}$ (или s/r_s). $m_z(s) = \cos\theta(s)$ $m_\rho = \sin\theta \sin\Phi_0$, B-Sk $\Phi_0 = \frac{\pi}{2}$

$\sin\theta(s) = \frac{2cs}{c^2+s^2}$ - BP ansatz profile of B/N-Sk. Neel-Sk $m_s = \sin\theta(s)\cos\Phi_0$, $\Phi_0 = 0$

Берем $L = 30$ nm ($R = 30$ nm $\to \beta = 1$. $A(Py) = 1.3 \cdot 10^{-6}$ erg/cm, $M_S = 800$ G $l_{ex} = 5.7$ nm $\tau = R/l_{ex} = 30\text{ nm}/5.7\text{ nm} = 5.3$. Согласно MPD Jan 2018 Neel-Sk не есть

Escaneado con CamScanner

metastable zone $\beta = 1,0$ vs. Вземам $\beta = 0,85$, $z = 5.3$ Neel-Sk under

min $E(c)$ при $c_0 = 0,32$. $e(c_0) = 39,656 \cdot 10^{-2}$, Minimum $E(c)$ по-ниско от Bloch-Sk under min $E(c)$ при $c_0 = 0,75$ и $e(c_0) = 11,367 \cdot 10^{-2}$ голяма при $\beta = 1,0$. Ако Bloch-Sk $c_0(\beta)$ зависимост само при $\beta = 0.8 - 1.0$.

Sk-number $Q = \frac{1}{2}[m_z(0) - m_z(R)]$, $m_z(s) = \frac{c^2 - s^2}{c^2 + s^2}$, $m_z(0) = 1$, $m_z(1) = \frac{c^2-1}{c^2+1}$

$2Q = 1 - \frac{(c^2-1)}{(c^2+1)} = \frac{1}{(1+c^2)}\left[(c^2+1) - (c^2-1)\right] = \frac{2}{1+c^2}$, $Q(c) = \frac{1}{(1+c^2)}$

$Q_N(0,32) = 0,91$; $Q_B(0,75) = 0,64$. $R_s^B = 0,75 \cdot R = 23\text{nm} < R$

Local explanations to Oksana + gp. Co-authors

(15/III) Orgot gona. Бълг. Силвия contacted inv. Solid State Mag условно.

(16/III) Subm. SW-SW scat. to JEMS. Приемовия SW-invited abstract. Local horarios 2019 + подкрепа form to Cristina, UAV/EHU.

(17/III) Ирина на реф. Tonelo-APL. Еще нагр. реф. Kim-JAP. Бълг.

Bergamza submitted by-dot skyrmion MS to NanoScale (1st exp. part).

(18/III) Чистене review Back et al. „The 2020 Skyrmionics Roadmap" (arXiv).

Cu_2OSeO_3 damping $\alpha = 10^{-4}$ (magnetic insulator) [Stasinopoulos APL 111, 032408 (2017)]

(19/III) Напомняне с Beek (EPS) и Oksana. Ревюм нагарват на EPS-CMD Prize 2020.

(20/III) Подготовка application на EPS-CMD Prize 2020.

(24/III) Подготовка appl. на EPS-CMD Prize. Рев. MS Park-Kim SW-SW-JAP (major)

(22/III) Рев. MS Tonelo et al. Pl/Co/Ta nanodome Sk. Refer to other journal.

(23/III) Възстанових official cash payments to CONSTANTO conf. fee 2016-2017-2018 + local 10% to prepare invoices на рамково с 10% ДДС. Напомняне с Oksc. Julian, Papa. Започна Dzyaloshinskii о статия „Drum modes" MS

(24/III) Offered to Bergamza conv. calc. Neel-Sk energy и $m_{z,s}(s)$ profiles in Py + dots (see 14/III) + Flowchart conv. 450 € goes за advertisements CONSTANTO

(25/III) Напом. + local Oksana revised EPS Prize nomination.

(26/III) Oksana Subm. my nomination to EPS / Vander Beek + напоследък още 2 започна support. letters.

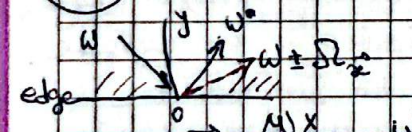
(26/III – 30/III) Започни 3+ support. letters, от които 2 вече не отвърнаха, 2 откликнаха (Hillebrands, Thiaville) + 30 одобрения + Ug 30 confirmations, 28 local supporting letters, бл.р. Fert и Arrott.

(31/III) С Л. Бълг. Soc. Soc за March. 30/III „Drum modes" напомняне: Бълг. gona. Thanks to Arrott + Stoll for EPS prize support. lett. Nanolett.

(1/IV) Comments к simulations Bergamza Sk, Vortex, SD-states Py dots.

(2/IV) JEMS'20 in Lisbon отложена на 1-11/VII 20г. edge SW scattering

(3/IV) Inelastic SW scattering by film edge (Pawel) in Word file 10/IV 2020



Edge SW – scatt. δk_x wavevector $\vec{x} \parallel Ox$ (edge)

Bulk plane SW – $(\omega, \vec{k}, \vec{k})$ initial, $(\omega, \vec{k}, \vec{K})$ scattered state or final state.

$\omega_{\vec{k}}^{\vec{k}} = \omega_{\vec{K}}^{\vec{k}} \pm \delta k_{\vec{x}}$, $\vec{k} = \vec{K} \pm \vec{x}$.

$\vec{m}(\theta, \Phi)$ $\theta = \theta_v + \psi$ $\Phi = \Phi_v + \varphi$ (θ, ψ) - SW. $\theta_v = \theta_0 + y$, $\Phi_v = \Phi_0 + z$ –

– moving soliton (SW) background в друга сис. EPL 2016

90°-Neel DW near the film edge ($y = 0$) - plane.

Edge SW = eSW, bulk plane SW = pSW.

Principal point is eSW-pSW interaction resulting in pSW inelastic scat

Exch. energy $W_{ex}(\vec{m}_s) = A(\nabla_\alpha \vec{m}_s)^2$ дава $W_{int} = -A_{\vec{j}} \vec{A}^e$, $\vec{j} = i$

pSW current, $\Psi = e^{i\varphi} + i\sin\theta_s \psi$ – pSW wave function.

$W_{int}(e_x) = -A\vec{j} \cdot (\nabla z + y \nabla \Phi_0)$. Static DW $\theta_0 = \frac{\pi}{2}$, $\Phi_0 = \Phi_0(y)$

– 90° Neel DW с ширина Δ. $\Phi_0(y)$ може и не монотонно –

Полученото $W_{int}(e_x)$ есть pSW-eSW interaction ур-ие – $J_{kin} = \frac{M_s}{\gamma}(1 - \cos\theta)\dot{\Phi}$

Изчисляване само въз основа на сравн. с $W_{int}(e_x)$?

Escaneado con CamScanner

$\frac{\gamma}{M_s}\mathcal{L}_{kin} = (1-\cos\theta)\dot\Phi = -\cos\theta\dot\Phi = -\cos(\theta_0+y+ee)(\dot\xi+\dot\psi) = -(\dot\xi+\dot\psi)\times$
$\times[\cos\theta_0\cos(y+ee) - \sin\theta_0\sin(y+ee)] = -(\dot\xi+\dot\psi)\cos\theta_0\left[1-\frac{1}{2}(y+ee)^2\right] +$
$+(\dot\xi+\dot\psi)\sin\theta_0(y+ee)$ (?) В Gus' EPL'2016 Lagrange equation

$(\gamma/M_s)\mathcal{L}_{int} = \sin\theta_0(\dot\xi ee - \psi\dot y) + \cos\theta_0 y(\dot\xi ee - \dot y\psi)$

$\mathcal{L}_{int} = \sin\theta_0(\dot\Phi_0 ee - \psi\dot\Theta_0)$ с точн. до членов linear on SW (ee,ψ)

$\mathcal{L}_{int} = \sin(\theta_0+y)(\dot\xi ee - \psi\dot y) = (\dot\xi ee - \psi\dot y)[\sin\theta_0 + y\cos\theta_0]$ опис.

(aB) и (aab) interaction, a - DW, b - SW. А надо (abb) interaction - quadratic по SW (ee,ψ) как в $W_{ex}(int)$ // т.е., в Dadoun PRL'19 а used неполное $\mathcal{L}_{int}(kin)$. Надо $\mathcal{L}_{kin} = -(\dot\xi+\dot\psi)\cos\theta_0\left[1-\frac{1}{2}(y+ee)^2\right] + (\dot\xi+\dot\psi)\sin\theta_0\times$
по симметризованное. $\times(y+ee)$

$\mathcal{L}_{kin} = -\cos\theta\dot\Phi = -\frac{d}{dt}(\cos\theta\Phi) - \sin\theta\Phi\dot\theta = -\sin\theta\Phi\dot\theta$.

$\mathcal{L}_{kin} = -\sin(\theta_0+y+ee)(\Phi_0+\xi+\psi)(\dot y+\dot{ee}) = -(\Phi_0+\xi+\psi)(\dot y+\dot{ee})\times$
$\times[\sin\theta_0\cos(y+ee) + \cos\theta_0(y+ee)]$. $\mathcal{L}_{kin}(1) = -\cos\theta\dot\Phi$, $\mathcal{L}_{kin}(2) = -\Phi\sin\theta$

Чисто $\theta_0 = \frac{\pi}{2}$, $\vec m_0(\theta_0\Phi_0)$ is in film plane → Neel DW, $\cos\theta_0 = 0$.

$\mathcal{L}_{kin}(1) = \sin\theta_0(\dot\xi+\dot\psi)(y+ee)$, $\mathcal{L}_{kin}(2) = -(\Phi_0+\xi+\psi)(\dot y+\dot{ee})\sin\theta_0\cos(y+ee)$

$\mathcal{L}_{kin}(1)$ не содержит cubic (abb) terms. $\mathcal{L}_{kin}(2) = -(\Phi_0+\xi+\psi)(\dot y+\dot{ee})\times$
$\times\left[1-\frac{1}{2}(y+ee)^2\right] = (\text{bilinear}) + \frac{1}{2}(\Phi_0+\xi+\psi)(\dot y+\dot{ee})(y+ee)^2 \Rightarrow \frac{1}{2}\Phi_0(\dot y+\dot{ee})(y+ee)^2$
$= \frac{1}{6}\Phi_0\frac{d}{dt}(y+ee)^3$ записывается как полная производная по $d/dt(..)$.

Вывод: $\mathcal{L}_{kin}$ не содержит cubic (abb) interaction terms - Все (a-b) или pSW - eSW interaction из-за $W_{int}(ex)$.

(4/XI) Давай обсудим. Что demag-field $H_d(y)$ он учтен в/м $\vec M_0(\vec r) = M_s\hat y$ (!) constant $\vec M(\vec r)\parallel oy$. Для DW near $y=0$ plane, the gauge field approach to account DW - SW interaction неприменим (pSW - eSW interaction). Надо искать DW-SW interaction исходя из non-linear LLG.

Ground state: $\vec M_0(\vec r) = -M_s\hat y$, static dip-field $H_d(y)$ опис. Semiinfinite film

Linear SW: bulk (not localized) SW + edge-localized SW

Dyn. magnetization $\vec m(\vec r,t) = \sum a_\nu(t)\vec m_\nu(\vec r)$, $\vec m_\nu(\vec r)$ mode profiles

a_ν - SW amplitudes, ν - mode number. $a_\nu(t) = a_\nu(0)e^{i\omega_\nu t}$.

Если подставить $\vec m(\vec r,t)$ в LLG, то в linear по a_ν - approximation получим $\hat\Gamma m_\nu(\vec r) = \omega_\nu^2 m_\nu(\vec r)$ уравнение на eigenmodes $m_\nu(\vec r)$, eigenfreq. ω но если учесть в LLG члены $\sim a_\nu^2$, то они описывают слабое взаимодействие между eigenmodes (3-magnon scattering).

Dip. $H_d^\alpha(\vec r) = \int d^3\vec r'\left[G_{dx}M_x(\vec r') + G_{dy}M_y(\vec r') + G_{dz}M_z(\vec r')\right]$

У нас $M_y^0 \neq 0$, $M_{x,y}^0 = 0$, $H_d^\alpha(\vec r) = \int d3\vec r'\, G_{dy}(\vec r\vec r')M_y(\vec r')$

$M_y(\vec r') = -M_s = const$, $H_d^\alpha(\vec r) = -M_s\int d3\vec r'\, G_{dy}(\vec r\vec r')$, $\alpha = x,y,z$

$H_d^z(r) = 0$ из-за $\int dz(..) = 0$, $H_d^x(\vec r) = (-M_s)\int dx'dy'dz'\, G_{xy}(\vec r\vec r')$

$G_{d\beta}(\vec r\vec r') = G_{d\beta}(\vec r - \vec r')$. После усред. по толщине z (L-thickness) -

$H_d^x(x,y) = (-M_s)\int dx'dy' G_{xy}(x-x', y-y') = (-M_s)\int dx'dy'\frac{1}{(2\pi)^2}\int d^2\vec k e^{i\vec k(\vec r-\vec r')}(iT)\frac{k_xk_y}{k^2}$

$\int_{-\infty}^{\infty} dx' e^{-(k_xx')} = 2\pi\delta(k_x)$ при ∞-film $\parallel ox$ и $\int d^2\vec k\,\delta(k_x)k_y(..) = 0$.

$H_d^x(x,y) = 0$. Остается $H_d^y(x,y)$ - не зависит от x для ∞-film $\parallel ox \to H_d^y(y)$

Использован Gus' JMMM' 2011 Green function in rect. elements."

Escaneado con CamScanner

$\frac{\gamma}{M_s}\mathcal{L}_{kin} = (1-\cos\theta)\dot\Phi = -\cos\theta\dot\Phi = -\cos(\theta_0+y+\varepsilon\varepsilon)(\dot\xi+\dot\psi) = -(\dot\xi+\dot\psi)\times$
$\times[\cos\theta_0\cos(y+\varepsilon\varepsilon) - \sin\theta_0\sin(y+\varepsilon\varepsilon)] = -(\dot\xi+\dot\psi)\cos\theta_0\left[1-\frac{1}{2}(y+\varepsilon\varepsilon)^2\right] +$
$+(\dot\xi+\dot\psi)\sin\theta_0(y+\varepsilon\varepsilon)$ (?) B Gus' EPL'2016 Lorentz eq.

$(\gamma/M_s)\mathcal{L}_{int} = \sin\theta_0(\dot\xi\varepsilon\varepsilon - \psi\dot y) + \cos\theta_0 y(\dot\xi\varepsilon\varepsilon - \dot y\psi)$

$\mathcal{L}_{int} = \sin\theta_0(\dot\Phi_v\varepsilon\varepsilon - \psi\dot\Theta_v)$ < term. go zuenob linear on SW $(\varepsilon\varepsilon,\psi)$

$\mathcal{L}_{int} = \sin(\theta_0+y)(\dot\xi\varepsilon\varepsilon - \psi\dot y) = (\dot\xi\varepsilon\varepsilon - \psi\dot y)[\sin\theta_0 + y\cos\theta_0]$ anuc.

(aB) и (aab) interaction, a - DW, b - SW. Л надо (abb) interaction - quadratic по SW $(\varepsilon\varepsilon,\psi)$ как в $W_{ex}(int)$ // т.е., в Dadoen RPL'19 я искал неправ. $\mathcal{L}_{int}(kin)$. Надо $\mathcal{L}_{kin} = -(\dot\xi+\dot\psi)\cos\theta_0\left[1-\frac{1}{2}(y+\varepsilon\varepsilon)^2\right] + (\dot\xi+\dot\psi)\sin\theta_0\times$
по самой линеаризованной. $\times(y+\varepsilon\varepsilon)$,

$\mathcal{L}_{kin} = -\cos\theta\dot\Phi = -\frac{d}{dt}(\cos\theta\Phi) - \sin\theta\Phi\dot\theta = -\sin\theta\Phi\dot\theta$.

$\mathcal{L}_{kin} = -\sin(\theta_0+y+\varepsilon\varepsilon)(\Phi_0+\xi+\psi)(\dot y+\dot\varepsilon\varepsilon) = -(\Phi_0+\xi+\psi)(\dot y+\dot\varepsilon\varepsilon)\times$
$\times[\sin\theta_0\cos(y+\varepsilon\varepsilon) + \cos\theta_0(y+\varepsilon\varepsilon)]$. $\mathcal{L}_{kin}(1) = -\cos\theta\dot\Phi$, $\mathcal{L}_{kin}(2) = -\Phi\sin\theta\dot\theta$

Пусть $\theta_0 = \frac{\pi}{2}$, $\vec m_0(\Phi_0)$ is in film plane → Neel DW, $\cos\theta_0 = 0$.

$\mathcal{L}_{kin}(1) = \sin\theta_0(\dot\xi+\dot\psi)(y+\varepsilon\varepsilon)$, $\mathcal{L}_{kin}(2) = -(\Phi_0+\xi+\psi)(\dot y+\dot\varepsilon\varepsilon)\sin\theta_0\cos(y+\varepsilon\varepsilon)$

$\mathcal{L}_{kin}(1)$ не содержит cubic (abb)-terms. $\mathcal{L}_{kin}(2) = -(\Phi_0+\xi+\psi)(\dot y+\dot\varepsilon\varepsilon)\times$
$\times\left[1-\frac{1}{2}(y+\varepsilon\varepsilon)^2\right] = (\text{bilinear}) + \frac{1}{2}(\Phi_0+\xi+\psi)(\dot y+\dot\varepsilon\varepsilon)(y+\varepsilon\varepsilon)^2 \Rightarrow \frac{1}{2}\Phi_0(\dot y+\dot\varepsilon\varepsilon)(y+\varepsilon\varepsilon)^2 =$
$= \frac{1}{6}\Phi_0\frac{d}{dt}(y+\varepsilon\varepsilon)^3$ запишется как полная производная по $d/dt(\dots)$.

→ **Вывод**: $\mathcal{L}_{kin}$ не содержит cubic (aBB) interaction terms - Все (a-B) или pSW - eSW interaction из-за $W_{ex}(ex)$.

(4/XI) Panel above, two demag-field $H_d(y)$ on curve w/m $\vec M_0(\vec r) = M_s\hat y$ (!) constant $\vec M(\vec r) \parallel Oy$. Near DW near y=0 plane, the gauge field approach to account DW-SW interaction не применим (pSW-eSW interaction). Надо искать DW-SW interaction исходя из non-linear LLG.

Ground state: $\vec M_0(\vec r) = -M_s\hat y$, static dip-field $H_d(y)$ anuc. Semiinfinite film

Linear SW: bulk (not localized) SW + edge-localized SW

Dyn. magnetization $\vec m(\vec r,t) = \sum a_\nu(t)\vec m_\nu(\vec r)$, $\vec m_\nu(\vec r)$ mode profiles, a_ν - SW amplitudes, ν - mode number, $a_\nu(t) = a_\nu(0)e^{i\omega_\nu t}$.

Если подставить $\vec m(\vec r,t)$ в LLG, то в linear по a_ν - approximation получим $\hat\Gamma m_\nu(\vec r) = \omega_\nu^2 m_\nu(\vec r)$ уравнение на eigenmodes $m_\nu(\vec r)$, eigenfreq. ω_ν.
Но если учесть в LLG члены с a_ν^2, то они описуют слабое взаимодействие между eigenmodes (3-magnon scattering).

Dip. $H_d^\alpha(\vec r) = \int d^3\vec r'\left[G_{dx}M_x(\vec r') + G_{dy}M_y(\vec r') + G_{dz}M_z(\vec r')\right]$

У нас $M_y^0 \neq 0$, $M_{x,y}^0 = 0$, $H_d^\alpha(\vec r) = \int d^3\vec r' G_{dy}(\vec r\vec r')M_y(\vec r')$

$M_y(\vec r') = -M_s = \text{const}$, $H_d^\alpha(\vec r) = -M_s\int d^3\vec r' G_{dy}(\vec r\vec r')$, $\alpha = x,y,z$

$H_d^z(r) = 0$ из-за $\int dz(\dots) = 0$, $H_d^x(\vec r) = (-M_s)\int dx'dy'dz' G_{xy}(\vec r\vec r')$

$G_{d\beta}(\vec r\vec r') = G_{d\beta}(\vec r - \vec r')$. После усред. по толщине z (L-thickness) →

$H_d^x(x,y) = (-M_s)\int dx'dy' G_{xy}(x-x', y-y') = (-M_s)\int dx'dy'\frac{1}{(2\pi)^2}\int d^2k\, e^{-i\vec k(\vec r-\vec r')}\frac{k_xk_y}{(\vec k\vec r)}\frac{f(kL)}{k^2}$

$\int_{-\infty}^{\infty} dx' e^{-ik_x x'} = 2\pi\delta(k_x)$ т.к. ∞-film $\parallel Ox$ → $\int d^2\vec k\,\delta(k_x)k_y(\dots) = 0$.

$H_d^x(x,y) = 0$. Остается $H_d^y(x,y)$ - не зависит от x для ∞-film $\parallel Ox$ → $H_d^y(y) \neq 0$

Использовали Gus' JMMM'2011 Green function in rect. elements."

$\vec{M}_0$ Py - semiinfinite film, $K_u = 0$ ecc. torsore Exchange + dip. energy

LLG in (θ,φ) representation com J.C. Slonczewski, JMMM 12 108 (1979)

$$\begin{cases} \frac{M_s}{\gamma}\dot{\varphi}\sin\theta = \frac{\delta W}{\delta\theta} = A(\nabla\varphi)^2\sin 2\theta - 2A\nabla^2\theta - \vec{H}\cdot\frac{\partial\vec{m}}{\partial\theta} \\ -\frac{M_s}{\gamma}\dot{\theta}\sin\theta = \frac{\delta W}{\delta\varphi} = -2A\nabla^2\varphi\sin^2\theta - 2A(\nabla\varphi\cdot\nabla\theta)\sin 2\theta - \vec{H}\cdot\frac{\partial\vec{m}}{\partial\varphi} \end{cases}$$

$\vec{H}$ is magnetostatic field. $\vec{m} = \vec{m}(\theta,\varphi)$. $\vec{H}(\vec{r}) = M_s\int d^3r'\,\hat{G}(\vec{r}\vec{r}')\,\vec{m}(\vec{r}')$.

SW excitations: $\theta = \theta_0 + \varepsilon\theta$, $\varphi = \varphi_0 + \varepsilon\psi$, we keep (y,z) of φ_0, i.e. $\theta_0 = \pi/2$. $\vec{M}_0$ in film plane. For SW propagation $\varphi_0 = -\pi/2$, $\vec{M}_0 = -M_s\hat{y}$

$\sin\theta = \sin(\theta_0+\varepsilon\theta) = \sin\theta_0\cos\varepsilon\theta + \cos\theta_0\sin\varepsilon\theta = \sin\theta_0(1-\frac{1}{2}\varepsilon\theta^2)$

$\sin 2\theta = \sin(2\theta_0 + 2\varepsilon\theta) = \sin 2\theta_0\cos 2\varepsilon\theta + \cos 2\theta_0\sin 2\varepsilon\theta = -2\varepsilon\theta + O(\varepsilon^3)$

$$\begin{cases} \frac{M_s}{\gamma}\dot{\psi}\sin\theta_0 = A(\nabla\psi)^2(-1\varepsilon\theta) - 2A\nabla^2\varepsilon\theta + \vec{H}\frac{\partial\vec{m}}{\partial\theta} \\ -\frac{M_s}{\gamma}\dot{\varepsilon\theta}\sin\theta_0 = -2A\nabla^2\psi\sin^2\theta_0(1-\varepsilon\theta^2) - 2A(\nabla\psi\cdot\nabla\varepsilon\theta)(-1\varepsilon\theta) - \vec{H}\frac{\partial\vec{m}}{\partial\varphi} \end{cases}$$

edge-SW scattering

C term go $\varepsilon\theta^2, \varepsilon\theta\psi, \psi^2$: $\frac{M_s}{\gamma}\dot{\psi} = -2A\nabla^2\varepsilon\theta - \vec{H}\cdot\frac{\partial\vec{m}}{\partial\theta}$; $-\frac{M_s}{\gamma}\dot{\varepsilon\theta} = -2A\nabla^2\psi - \vec{H}\cdot\frac{\partial\vec{m}}{\partial\varphi}$

Bugno: non eliminare nei $(\varepsilon\theta,\psi)$ terms ove due magnetostatics (!).

$\vec{m} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) = \hat{x}\sin\theta\cos\varphi + \hat{y}\sin\theta\sin\varphi + \hat{z}\cos\theta$

$\frac{\partial\vec{m}}{\partial\theta} = \hat{x}\cos\theta\cos\varphi + \hat{y}\cos\theta\sin\varphi - \hat{z}\sin\theta$; $\frac{\partial\vec{m}}{\partial\varphi} = -\hat{x}\sin\theta\sin\varphi + \hat{y}\sin\theta\cos\varphi$

$\frac{\partial\vec{m}}{\partial\theta} = \hat{x}\cos(\theta_0+\varepsilon\theta)\cos(\varphi_0+\psi) + \hat{y}\cos(\theta_0+\varepsilon\theta)\sin(\varphi_0+\psi) - \hat{z}\sin(\theta_0+\varepsilon\theta) =$

$= \hat{x}[\cos\theta_0\cos\varepsilon\theta - \sin\theta_0\sin\varepsilon\theta][\cos\varphi_0\cos\psi - \sin\varphi_0\sin\psi] + \hat{y}[\cos\theta_0\cos\varepsilon\theta - \sin\theta_0\sin\varepsilon\theta][\sin\varphi_0\cos\psi + \cos\varphi_0\sin\psi] - \hat{z}[\sin\theta_0\cos\varepsilon\theta + \cos\theta_0\sin\varepsilon\theta] = \hat{x}[-\varepsilon\theta][\psi] + \hat{y}[-\varepsilon\theta][-\cos\psi] - \hat{z}\cos\varepsilon\theta =$

$= \hat{x}(-\varepsilon\theta\psi) + \hat{y}\varepsilon\theta - \hat{z}(1-\frac{1}{2}\varepsilon\theta^2)$; $\frac{\partial\vec{m}}{\partial\varphi} = -\hat{x}\sin(\theta_0+\varepsilon\theta)\sin(\varphi_0+\psi) + \hat{y}\sin(\theta_0+\varepsilon\theta)\cos(\varphi_0+\psi) =$

$= -\hat{x}[\sin\theta_0\cos\varepsilon\theta + \cos\theta_0\sin\varepsilon\theta][\sin\varphi_0\cos\psi + \cos\varphi_0\sin\psi] + \hat{y}[\sin\theta_0\cos\varepsilon\theta + \cos\theta_0\sin\varepsilon\theta][\cos\varphi_0\cos\psi - \sin\varphi_0\sin\psi] =$

$= -\hat{x}[\cos\varepsilon\theta][-\cos\psi] + \hat{y}[\cos\varepsilon\theta][\psi] = \hat{x}(1-\frac{\varepsilon\theta^2}{2})(1-\frac{1}{2}\psi^2) + \hat{y}\psi$.

$\frac{\partial\vec{m}}{\partial\theta} = -\hat{x}\varepsilon\theta\psi + \hat{y}\varepsilon\theta + \hat{z}(-1+\frac{1}{2}\varepsilon\theta^2)$; $\frac{\partial\vec{m}}{\partial\varphi} = \hat{x}(1-\frac{1}{2}\varepsilon\theta^2-\frac{1}{2}\psi^2) + \hat{y}\psi$.

Dip. $\vec{H} = \vec{H}_0 + \vec{h}$, $\vec{H}_0 = H_y^0\hat{y} + \vec{h}$. $\vec{h}$ cooperative terms $\sim \varepsilon\theta, \psi, \varepsilon\theta^2, \psi^2, \varepsilon\theta\psi$ — dyn. dip. field

$\vec{H}\cdot\frac{\partial\vec{m}}{\partial\theta} = (\vec{H}_0+\vec{h})\cdot[-\hat{x}\varepsilon\theta\psi + \hat{y}\varepsilon\theta + \hat{z}(-1+\frac{1}{2}\varepsilon\theta^2)] = H_y^0\varepsilon\theta + h_y\varepsilon\theta - h_z$

$\vec{H}\cdot\frac{\partial\vec{m}}{\partial\varphi} = (\vec{H}_0+\vec{h})\cdot[\hat{x}(1-\frac{1}{2}\varepsilon\theta^2-\frac{1}{2}\psi^2) + \hat{y}\psi] = H_y^0\psi + h_x + h_y\psi$.

$$\begin{cases} \frac{M_s}{\gamma}\dot{\psi} = -2A\nabla^2\varepsilon\theta - M_sH_y^0\varepsilon\theta - M_sh_y\varepsilon\theta + M_sh_z \\ -\frac{M_s}{\gamma}\dot{\varepsilon\theta} = -2A\nabla^2\psi - M_sH_y^0\psi - M_sh_x - M_sh_y\psi \end{cases}$$

bugno: non eliminare h_x, h_z e non h_y onde avere SW-SW interaction

$h_x(\vec{r}) = \int dx'dy'dz'\left[G_{xx}(\vec{r}\vec{r}')m_x(\vec{r}') + G_{xy}m_y + G_{xz}m_z\right]M_s$.

N.B. $\int dz\int dz'\,G_{\alpha\beta}(zz') = G_{\alpha\beta}(\vec{\rho}\vec{\rho}') \neq 0$ remains even $\alpha\neq z$ and $\beta\neq z$

$h_x(\vec{\rho}) = \int dx'dy'\left[G_{xx}(\vec{\rho}\vec{\rho}')m_x(\vec{\rho}') + G_{xy}(\vec{\rho}\vec{\rho}')m_y(\vec{\rho}')\right]M_s$

$h_y(\vec{\rho}) = M_s\int dx'dy'\left[G_{yx}(\vec{\rho}\vec{\rho}')m_x(\vec{\rho}') + G_{yy}(\vec{\rho}\vec{\rho}')m_y(\vec{\rho}')\right]$

$h_z(\vec{\rho}) = M_s\int dx'dy'\left[G_{zz}(\vec{\rho}\vec{\rho}')m_z(\vec{\rho}')\right]$

$m_x = \sin\theta\cos\varphi = \sin(\theta_0+\varepsilon\theta)\cos(\varphi_0+\psi) = [\sin\theta_0\cos\varepsilon\theta + \cos\theta_0\sin\varepsilon\theta][\cos\varphi_0\cos\psi - \sin\varphi_0\sin\psi] = [\cos\varepsilon\theta][\psi] = \psi$, $m_z = \cos(\theta_0+\varepsilon\theta) = \cos\theta_0\cos\varepsilon\theta - \sin\theta_0\sin\varepsilon\theta$

$m_y = \sin\theta\sin\varphi = \sin(\theta_0+\varepsilon\theta)\sin(\varphi_0+\psi) = [\sin\theta_0\cos\varepsilon\theta + \cos\theta_0\sin\varepsilon\theta][\sin\varphi_0\cos\psi + \cos\varphi_0\sin\psi] = [\cos\varepsilon\theta][-\cos\psi] = -(1-\frac{1}{2}\varepsilon\theta^2)(1-\frac{1}{2}\psi^2) = -1 + \frac{1}{2}\varepsilon\theta^2 + \frac{1}{2}\psi^2$, $m_y = \frac{1}{2}\varepsilon\theta^2$

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Дуж. $m_x = +\psi$, $m_y = \frac{1}{2}\varepsilon\varepsilon^2 + \frac{1}{2}\psi^2$, $m_z = -\varepsilon\varepsilon$, $m_y \sim \varepsilon\varepsilon^2, \psi^2$, т.к. $\hat{y}$ осн $\vec{M}_0$ диз.

$h_x(\vec{s}) = M_s \int dx'dy' \left[G_{xx}\psi + G_{xy}\frac{1}{2}(\varepsilon\varepsilon^2+\psi^2)\right]$, $h_z(\vec{s}) = M_s \int dx'dy' G_{zz}\,\varepsilon\varepsilon(-1)$

$h_y(\vec{s}) = M_s \int dx'dy' \left[G_{yx}\psi + G_{yy}\frac{1}{2}(\varepsilon\varepsilon^2+\psi^2)\right] \approx M_s \int dx'dy' G_{yx}\psi$

$$\begin{cases} \frac{M_s}{\gamma}\dot{\psi} = -2A\nabla^2\varepsilon\varepsilon - M_s H_y^0 \varepsilon\varepsilon - M_s^2 \varepsilon\varepsilon \int dx'dy' G_{yx}\psi + M_s^2(-1)\int dx'dy' G_{zz}\,\varepsilon\varepsilon \\ -\frac{M_s}{\gamma}\dot{\varepsilon\varepsilon} = -2A\nabla^2\psi - M_s H_y^0\psi - M_s^2 \int dx'dy' \left[G_{xx}\psi + G_{xy}\frac{1}{2}(\varepsilon\varepsilon^2+\psi^2)\right] - M_s^2\psi\int dx'dy' G_{yx}\psi \end{cases}$$

$$\begin{cases} \frac{1}{\gamma}\dot{\psi} = -\frac{2A}{M_s}\nabla^2\varepsilon\varepsilon - H_y^0\varepsilon\varepsilon - M_s\int dx'dy' G_{zz}\,\varepsilon\varepsilon - M_s\varepsilon\varepsilon\int dx'dy' G_{yx}\psi \\ -\frac{M_s}{\gamma}\dot{\varepsilon\varepsilon} = -\frac{2A}{M_s}\nabla^2\psi - H_y^0\psi - M_s\int dx'dy' G_{xx}\psi - \frac{1}{2}M_s\int dx'dy' G_{xy}(\varepsilon\varepsilon^2+\psi^2) - M_s\psi\int dx'dy' G_{yx}\psi \end{cases}$$

– SW–SW interaction terms (квадратичные, cond. cubic terms in $\mathcal{L}$)

→ Линейные SW: $\begin{cases} \frac{1}{\gamma}\dot{\psi} = -\frac{2A}{M_s}\nabla^2\varepsilon\varepsilon - H_y^0\varepsilon\varepsilon - M_s\int dx'dy' G_{zz}\,\varepsilon\varepsilon \\ -\frac{1}{\gamma}\dot{\varepsilon\varepsilon} = -\frac{2A}{M_s}\nabla^2\psi - H_y^0\psi - M_s\int dx'dy' G_{xx}\psi \end{cases}$ $\Psi = \varepsilon\varepsilon + i\sin\Theta_0\psi - \ldots$

$m_x = \psi$, $m_z = -\varepsilon\varepsilon$ $\Psi = \varepsilon\varepsilon + i\psi$

$G_{zz}(\vec{k}) = -4\pi[1 - f(kL)]$, $G_{xx}(\vec{k}) = -4\pi\frac{k_x^2}{k^2}f(kL)$, $G_{\alpha\beta}(\vec{s}\vec{s}') = \frac{1}{(2\pi)^2}\int d^2\vec{k}\, G_{\alpha\beta}(\vec{k})e^{i\vec{k}\Delta\vec{s}}$

$\frac{1}{\gamma}\dot{\psi} = -\left[\frac{2A}{M_s}\nabla^2 + H_y^0 + M_s\int dx'dy' G_{zz}^{\,x}\right]\dot{\varepsilon\varepsilon} = -[\ldots](-\gamma)\left[\frac{2A}{M_s}\nabla^2 + H_y^0 + M_s\int dx'dy' G_{xx}^{\,x}\right]\psi(-1)$

$-\frac{1}{\gamma^2}\ddot{\psi} = \left[\frac{2A}{M_s}\nabla^2 + H_y^0 + M_s\int dx'dy' G_{zz}^{\,x}\right]\left[\frac{2A}{M_s}\nabla^2 + H_y^0 + M_s\int dx'dy' G_{xx}^{\,x}\right]\psi$

$\psi(\vec{s},t) = \psi(\vec{s})e^{i\omega t}$ даст $\frac{\omega^2}{\gamma^2}\psi = \left[\frac{2A}{M_s}\nabla^2 + H_y^0 + M_s\int dx'dy' G_{zz}^{\,x}\right]\left[\frac{2A}{M_s}\nabla^2 + H_y^0 + M_s\int dx'dy' G_{xx}^{\,x}\right]\psi$

$G_{zz} \neq G_{xx}$, усреднение диполей и LLG не сводится к уравнению для $\Psi = \varepsilon\varepsilon + i\psi$

При $kL \ll 1$, thin film $G_{zz}(\vec{k}) \approx -4\pi$, $G_{xx}(\vec{k}) \ll 1$, $G_{zz}(\vec{s}\vec{s}') = \frac{-4\pi}{(2\pi)^2}\int d^2\vec{k}\, e^{i\vec{k}(\vec{s}-\vec{s}')} = -4\pi\delta(\vec{s}-\vec{s}')$.

$\mathrm{Tr}\left[G_{\alpha\beta}(\vec{s}\vec{s}')\right] = -4\pi\delta(\vec{s}-\vec{s}')$.

$\frac{\omega^2}{\gamma^2}\psi = \left[\frac{2A}{M_s}\nabla^2 + H_y^0 - 4\pi M_s\right]\left[\frac{2A}{M_s}\nabla^2 + H_y^0\right]\psi(\vec{s})$, $H_y^0 = H_y^0(y)$.



Две edge modes $H_y^0(y)$ с max при $y \approx 0$ близко.

Две bulk modes (not localized) $H_y^0 \approx 0$ и $\nabla^2 \to -k^2$

$\frac{\omega^2(k)}{\gamma^2} = \left(\frac{2A}{M_s}k^2 + 4\pi M_s\right)\frac{2A}{M_s}k^2$, $\psi(\vec{s}) \approx e^{i\vec{k}\vec{s}}$. Добавим ext. field $\vec{H} \parallel \vec{M}_0$

Lin. SW $k \to 0$ $\frac{1}{\gamma^2}\omega^2(k) = \left[H + 4\pi M_s + \frac{2A}{M_s}k^2\right]\left[H + \frac{2A}{M_s}k^2\right]$ reduced to Kittel eq.

Conacno Slonczewski '79 variation $\delta\vec{m}$ ведет к variation of dipolar energy density $\delta w_m = -\vec{H}M_s\delta\vec{m}$. А если под $\delta\vec{m}$ понимать SW, а под δw_m – dip. energy of excited SW? Тогда w_{int}^m будучи δw_m.

$\vec{H}\cdot\delta\vec{m} = \vec{H}\cdot\vec{m}_{SW} = (\vec{H}_0 + \vec{h})\cdot\vec{m} = H_0^y m_y + \vec{h}\cdot\vec{m}$, $H_0^y m_y = H_0^y\frac{1}{2}(\varepsilon\varepsilon^2+\psi^2)$ –

– сокращ. $H_y^0\varepsilon\varepsilon$, $H_y^0\psi$ членам в уравнении и не содержит SW–SW interaction.

$\vec{h}\cdot\vec{m} = m_x h_x + m_y h_y + m_z h_z = \psi M_s\int d^2\vec{s}'\left[G_{xx}\psi + \frac{1}{2}G_{xy}(\varepsilon\varepsilon^2+\psi^2)\right] + \frac{1}{2}(\varepsilon\varepsilon^2+\psi^2)\times$

$\times M_s\int d^2\vec{s}'\left[G_{yx}\psi + G_{yy}\frac{1}{2}(\varepsilon\varepsilon^2+\psi^2)\right] - \varepsilon\varepsilon(-1)M_s\int d^2\vec{s}'\, G_{zz}\,\varepsilon\varepsilon$

Cubic part $w_{int}^m = -M_s\vec{H}\delta\vec{m} = -M_s\vec{h}\cdot\vec{m} = -M_s^2\left[\frac{1}{2}\psi\int d^2\vec{s}'\, G_{xy}(\varepsilon\varepsilon^2+\psi^2) + \frac{1}{2}(\varepsilon\varepsilon^2+\psi^2)\times\right.$

$\left.\times\int d^2\vec{s}'\, G_{yx}\psi\right]$

$W_{int}^m = -\frac{M_s^2}{2}\left[\psi\int d^2\vec{s}'\, G_{xy}(\varepsilon\varepsilon^2+\psi^2) + (\varepsilon\varepsilon^2+\psi^2)\int d^2\vec{s}'\, G_{yx}\psi\right]$, $G_{xy}(\vec{s}\vec{s}') = G_{yx}(\vec{s}\vec{s}')$.

SW $\vec{m}$: $m_x = \psi$, $m_y = \frac{1}{2}(\varepsilon\varepsilon^2+\psi^2)$, $m_z = -\varepsilon\varepsilon$. $\Theta = \Theta_0 + \varepsilon\varepsilon$, $\Phi = \Phi_0 + \psi$.

Полное $m_y = -1 + \frac{1}{2}(\varepsilon\varepsilon^2+\psi^2)$. $\vec{m}^2 = m_x^2 + m_y^2 + m_z^2 = \psi^2 + \varepsilon\varepsilon^2 + \left[-1 + \frac{1}{2}(\varepsilon\varepsilon^2+\psi^2)\right]^2 =$

$= \psi^2 + \varepsilon\varepsilon^2 + 1 - (\varepsilon\varepsilon^2+\psi^2) + \frac{1}{4}(\varepsilon\varepsilon^2+\psi^2)^2 = 1 + O(\varepsilon\varepsilon^4, \psi^4)$.

Note $W_{int}^m = -M_s^2\left[m_x\int d^2\vec{s}'\, G_{xy}m_y + m_y\int d^2\vec{s}'\, G_{yx}m_x\right]$. Ор $\varepsilon\varepsilon, \psi$ надо перевести к a_k^+, a_k или a_k', a_k SW operators (или SW creation/ann.).

Due circular symmetry $\Psi = m_x + im_z = \psi - i\varepsilon\varepsilon = -i(\varepsilon\varepsilon + i\psi) = \sum_{\vec{k}} a_{\vec{k}}\, m_{\vec{k}}(\vec{r})e^{i\omega_k t}$ даёт SW переход.

Escaneado con CamScanner

Gurevich-Melkov Ch.10: $\vec{m}(\vec{r},t) = \sum_{\vec{k}} \vec{m}_{\vec{k}}(t) e^{-i\vec{k}\vec{r}}$ (Fourier Series)

$a_{\vec{k}} \sim (m_{kx} + i m_{ky})$ give circular symmetry.

$a_{\vec{k}} = \frac{1}{M_s}[m_{kx} + i m_{ky}]$, $a^*_{-\vec{k}} = [m_{kx} - i m_{ky}]/M_s$

(I/II) SW amplitudes $a_{\vec{k}}$ (Bose-operators in quantum approach) blagodarya

bcega verno $a_{\vec{k}} = m_{kx} - i m_{ky}$, $a^*_{-\vec{k}} = m_{kx} - i m_{ky}$. No yD give $a_{\vec{k}}$, $a^*_{-\vec{k}}$ are coupled due to magnetostatics when $\sin\theta_{\vec{k}}$ ($\theta_{\vec{k}} = \angle(\vec{k}, \vec{M}_0)$) $\neq 0$. Unlike dyn. magnetostatics when $\vec{k} \parallel \vec{M}_0$ has linear circular symmetry in energy (yD) we get uncoupled b terms $a_{\vec{k}}$. Hago bbogurb when pazoame Bogoliubova:

$a_{\vec{k}} \to c_{\vec{k}}$, $a_{\vec{k}} = u_{\vec{k}} c_{\vec{k}} + v_{\vec{k}} c^*_{-\vec{k}}$, $a^*_{-\vec{k}} = v^*_{\vec{k}} c_{\vec{k}} + u^*_{\vec{k}} c^*_{-\vec{k}}$

$u_{\vec{k}} = \frac{1}{\sqrt{2}}\sqrt{\frac{A_{\vec{k}}}{\omega_{\vec{k}}} + 1}$, $v_{\vec{k}} = -\frac{1}{\sqrt{2}}\sqrt{\frac{A_{\vec{k}}}{\omega_{\vec{k}}} - 1}\, e^{2i\varphi_{\vec{k}}}$, $\varphi_{\vec{k}} = \arctg\left(\frac{k_y}{k_x}\right)$, $\omega^2_{\vec{k}} = A^2_{\vec{k}} - |B_{\vec{k}}|^2$

$B_{\vec{k}} = \frac{1}{2}\omega_M \sin^2\theta_{\vec{k}}\, e^{i2\varphi_{\vec{k}}}$, $\vec{k}(\theta_{\vec{k}}, \varphi_{\vec{k}})$ b cferene koord. c oz $\|\vec{M}_0$.

CP of magnon operators (White, Ch.8) $\hat{c}_k$, $\hat{c}_k|c_k\rangle = c_k|c_k\rangle$ bgg $\hat{c}_k$

"coherent" magnon state $|c_k\rangle = e^{-\frac{1}{2}|c_k|^2} \sum_{n_k=0}^{\infty} \frac{1}{\sqrt{n_k!}} (\hat{c}_k)^{n_k} |n_k\rangle$

$\hat{n}_i = S - S^z_i$ spin deviation operator c

cof. znacheniem $n_i = S - M_s = S - m$, $\hat{S}_z|m\rangle = m|m\rangle$, $|n_i\rangle = \frac{1}{\sqrt{N}} \sum_{\vec{k}} e^{i\vec{k}\vec{r}_i} |n_{\vec{k}}\rangle$

$c_{\vec{k}} = |c_k| e^{i\beta_{\vec{k}}}$ - SW amplitude, $|c_k|^2 = \langle c_k|\hat{n}_k|c_k\rangle$.

$\langle c_k|S^x_i(t)|c_k\rangle = S_{\vec{k}} \cos(\vec{k}\vec{r}_i - \omega_{\vec{k}} + \beta_{\vec{k}})$, $\langle c_k|S^y_i(t)|c_k\rangle = \varepsilon_{\vec{k}} S_{\vec{k}} \sin(\vec{k}\vec{r}_i - \omega_{\vec{k}} t + \beta_{\vec{k}})$

$S_{\vec{k}} = \sqrt{2S}(u_k - v_k)|c_k|$, ellipticity $\varepsilon_k = \frac{u_k + v_k}{u_k - v_k}$. When $B_{\vec{k}} = 0$, $\omega_k = A_k$, $\varepsilon_{\vec{k}} = 1$ - circular oscillations. $\varepsilon_k \neq 1$ bg-za magnetostatics $B_{\vec{k}} \neq 0$ (dyn. magnetostatics).

Propepa when ellipticity of precession is sample shape, give when $N_x = N_y$

$N_x \neq N_y$ ($N_z = N_x$ y Pamel) - static magnetostatics.

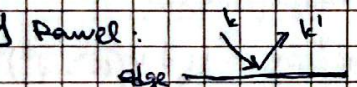
White, Ch.8, § 8.2.c Nonlinear processes gaer 3-magnon dip-interaction [Schlömann'1961]

$\mathcal{H}^{(3)}_{dip} = \sum_{\vec{k}\vec{k}', k\neq 0} \left[g_k \hat{c}_k \hat{c}_{k'} \hat{c}^+_{k+k'} + g^*_k \hat{c}^+_k \hat{c}^+_{k'} \hat{c}_{k+k'} \right]$

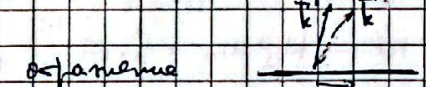
confluence Splitting

$g_{\vec{k}} = -\frac{4\pi}{V}\sqrt{2NS}\, g^2\mu_B^2 \cos\theta_{\vec{k}} \sin\theta_{\vec{k}} e^{i\varphi_{\vec{k}}}$ gue bulk, plane SW ($\vec{k}_\perp, \vec{k}$).

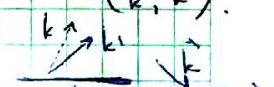
y Pamel:



edge

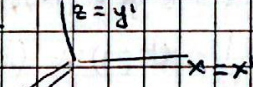


splitting



Hyeer $\vec{k}$ - reflected magnon, $\vec{k}$ - surface (eSW) magnon

$g_{\vec{k}} \sim \cos\theta_k \sin\theta_k \sim k_z k_{x,y}/k^2$



$g_{\vec{k}}(\text{Pamel}) \sim k_z k_{x,y'} = k_y k_{x,z}$

$\hat{y} \parallel \vec{M}_0$, xyz

$g_{\vec{k}}(\text{Pamel}) \sim k_y k_z$,

т.е. reflected pSW gamma umeet $\vec{k}$ b nnockocth nnenku xOy c $k_y \neq 0$, $k_x \neq 0$!)

Tornee $g_{\vec{k}} \sim \cos\theta_k \sin\theta_k e^{i\varphi_k} = \cos\theta_k \sin\theta_k (\cos\varphi_k + i\sin\varphi_k) = k_z(k_x + ik_y)$

$g_{\vec{k}}(\text{Pamel}) \sim k_y(k_x + ik_z)$, $|g_{\vec{k}}(\text{Pamel})|^2 \sim k_y^2(k_x^2 + k_z^2) \sim k_y^2 k_x^2$, т.к. $k_z = 0$.

D. Stancil - SW. 2009 (see Schlömann's Tech. reports '1959 + V.S. Livov turbul. 1994)

Chapt. 9.

LLG in the form $\dot{M}_+ = 2i\gamma M_z \frac{\delta W}{\delta M_-}$, $\dot{M}_- = -2i\gamma M_z \frac{\delta W}{\delta M_+}$, $M_\pm = M_x \pm iM_y$, W - energy density

Замена $M_+ = a\sqrt{(a^*a)}\sqrt{2\gamma M_s}$, $M_- = a^* f(a^*a)\sqrt{2\gamma M_s}$ даст yD $\dot{a} = i\frac{\delta W}{\delta a^*}$, $\dot{a}^* = -i\frac{\delta W}{\delta a}$

где $f(a^*a) = \sqrt{1 - \frac{\gamma}{2M_s} a^*a}$, $M_z = M_s - \gamma a^*a$. Это classical Holst-Primakov.

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$\dot a = i\frac{\delta W}{\delta a^*}$, $\dot a^* = -i\frac{\delta W}{\delta a}$ - canonical form. $W(a,a^*) = \mathcal{H}(a,a^*)$ - polinom.

$a_{\vec k}(t) = \frac{1}{V}\int d^3\vec r\, a(\vec r,t) e^{-i\vec k\vec r}$, $a(\vec r,t) = \sum_{\vec k} a_{\vec k}(t) e^{i\vec k\vec r}$

$\dot a_{\vec k} = i\frac{\partial \mathcal{H}}{\partial a_{\vec k}^*}$, $\dot a_{\vec k}^* = -i\frac{\partial \mathcal{H}}{\partial a_{\vec k}}$. the Hamiltonia eqs. of motion.

Note: Gur.-Melkov сразу вводят $a_k = (m_{kx} + i m_{ky})$, $a_{-k}^* = (m_{kx} - i m_{ky})$ без всякого Holst.-Primakov transformation.

$\mathcal{H}^{(2)} = \sum_k \left[A_k a_k^* a_k + \frac{1}{2} B_k a_k^* a_{-k}^* + \frac{1}{2} B_k^* a_k a_{-k} \right] \to \mathcal{H}^{(2)} = \sum_k \omega_k b_k^* b_k$ где

Замена $b_k = u_k a_k + v_k a_{-k}^*$, $b_k^* = u_k^* a_k^* + v_k^* a_{-k}$ и $\omega_k^2 = A_k^2 - |B_k|^2$

V.S. L'vov. Wave turbulence 1994. Ch. 3.

Станем вводя canonical variables следуя Ch.3 L'vov. Lvov рассматр. a, a^* как удовлетв. to $\dot a = i\frac{\partial \mathcal{H}}{\partial a^*}$, $\dot a^* = -i\frac{\partial \mathcal{H}}{\partial a}$ canonical Hamiltonian eqs.

У меня (Zagara Pawel) $\Psi(\vec r,t) = m_x + i m_z = -i(\varphi + i\psi) = \sum_{\vec k} a_{\vec k}(t)\mu_{\vec k}(\vec r)$, где $\vec k$ - нумерует СФ, $\mu_k(\vec r)$ profile of $\vec k$-eigenmode, a_k - is the $\vec k$-mode amplitude.

Melkov берет $\mu_k(\vec r) = e^{i\vec k\vec r}$, тогда $\sum a_k e^{ikr} = a$, $m_{x'} + i m_{y'} = a$, ось $\parallel \vec M_0$

Найдем W in terms of $\delta W(\varphi)(\psi)$ и диагонализуем ее.

Лин. δW $\begin{cases} \frac{1}{\gamma}\dot\psi = \frac{\delta W}{\delta\theta} = -\frac{2A}{M_s}\nabla^2\varphi - H_y^0\varphi - M_s\int d^2\vec\rho'\, G_{zz}\varphi \\ -\frac{1}{\gamma}\dot\varphi = \frac{\delta W}{\delta\Phi} = -\frac{2A}{M_s}\nabla^2\psi - H_y^0\psi - M_s\int d^2\vec\rho'\, G_{xx}\psi \end{cases}$

$W = \int dV\left[A\left((\nabla\theta)^2 + (\nabla\Phi)^2\sin^2\theta\right) - \frac{1}{2}\vec H_m\cdot\vec M\right]$. $\theta = \theta_0 + \varphi$, $\Phi = \Phi_0 + \psi$, $\vec m = \vec m_{sw}$

$(\nabla\theta)^2 + (\nabla\Phi)^2\sin^2\theta = (\nabla\varphi)^2 + (\nabla\psi)^2\sin^2\theta_0\left(1 - \frac{1}{2}\varphi^2\right)^2 = (\nabla\varphi)^2 + (\nabla\psi)^2 + O(\psi^2\varphi^2)$

$\vec H\cdot\vec M = (\vec H_0 + \vec h)(\vec m_0 + \vec m)M_s = M_s\vec H_0\vec m_0 + M_s\vec h\cdot\vec m_0 + M_s\vec H_0\vec m + M_s\vec h\cdot\vec m = |\vec m_0 = -\hat y| =$

$= -M_s h_y + M_s H_y^0 m_y + M_s\vec h\cdot\vec m$; $h_y = M_s\int d^2\vec\rho'\left[G_{yx}\psi + G_{yy}\frac{1}{2}(\varphi^2+\psi^2)\right]$

Далее $\delta W_m = -M_s\vec H\delta\vec m = -M_s\vec H\cdot\vec m_{sw}$, $\vec H\cdot\vec m_{sw} = H_y^0 m_y + \vec h\cdot\vec m = H_y^0 m_y +$

$+ \psi M_s\int d^2\vec\rho'\, G_{xx}\psi + \varphi M_s\int d^2\vec\rho'\, G_{zz}\varphi$ + (cubic terms). $\delta W = \delta W_{ex} + \delta W_m$

Quadratic $W(sw) = \delta W = \left[(\nabla\varphi)^2 + (\nabla\psi)^2\right] - M_s H_y^0\frac{1}{2}(\varphi^2+\psi^2) - M_s^2\psi\int d^2\vec\rho'\, G_{xx}\psi -$

$- M_s^2\varphi\int d^2\vec\rho'\, G_{zz}\varphi$. $W(sw)$ - не имеет канонич. формы по (φ,ψ).

$W(sw) = A\left[(\nabla\varphi)^2 + (\nabla\psi)^2\right] - \frac{1}{2}M_s H_y^0(\varphi^2+\psi^2) - M_s^2\psi\int d^2\vec\rho'\, G_{xx}\psi - M_s^2\varphi\int d^2\vec\rho'\, G_{zz}\varphi$.

$\varphi + i\psi = e^{i\pi/2}(m_x + i m_z) = e^{i\pi/2}\sum_{\vec k} a_{\vec k}(t)\mu_{\vec k}(\vec\rho) = a(\vec\rho,t) = \varphi(\vec\rho,t) + i\psi(\vec\rho,t)$

$a(\vec\rho,t)$ canonic variable, но кроме нее в квадратичном приближении.

// Согласно Melkov a - canonic, а согласно L'vov надо classic H.-Primakov для кубического, и - сделать всего одно линейное по (φ,ψ) - δW variables.

$W(sw) = L\int d^2\vec\rho\, w(sw)$, где $G_{\alpha\beta}(r,z)$ удерживаем по z. $H_y^0 = f(y)$. $W(sw) = \mathcal{H}$.

$\frac{1}{L}W(\delta W) = \int d^2\vec\rho\left\{A\left[(\nabla\varphi)^2 + (\nabla\psi)^2\right] - \frac{1}{2}M_s H_y^0(\varphi^2+\psi^2)\right\} - M_s^2\int d^2\rho d^2\rho'\, G_{xx}\psi(\rho)\psi(\rho') -$

$\Psi = \varphi + i\psi = \sum_{\vec k} a_{\vec k}\mu_{\vec k}(\vec\rho) = a(\vec\rho,t)$ - подставим. $- M_s^2\int d^2\rho d^2\rho'\, G_{zz}\varphi(\rho)\varphi(\rho')$

$|\nabla\Psi|^2 = \nabla\Psi\cdot\nabla\Psi^* = |\nabla(\varphi + i\psi)|^2 = |\nabla\varphi + i\nabla\psi|^2 = (\nabla\varphi)^2 + (\nabla\psi)^2$, $\varphi^2 + \psi^2 = |\Psi|^2$

$\nabla\Psi\cdot\nabla\Psi^* = \nabla\sum_k a_k\mu_k\cdot\nabla\sum_{k'} a_{k'}^*\mu_{k'}^* = \sum_{kk'} a_k a_{k'}^*\nabla\mu_k\cdot\nabla\mu_{k'}^*$

$\varphi^2 + \psi^2 = \Psi\cdot\Psi^* = \sum_k a_k\mu_k\cdot\sum_{k'} a_{k'}^*\mu_{k'}^* = \sum_{kk'} a_k a_{k'}^*\mu_k\mu_{k'}^*$.

$\frac{1}{L}W_{sw} = \int d^2\vec\rho\left\{A\sum_{kk'} a_k a_{k'}^*\nabla\mu_k\nabla\mu_{k'}^* - \frac{1}{2}M_s H_y^0\sum_{kk'} a_k a_{k'}^*\mu_k\mu_{k'}^*\right\}$. Видимо $\int d^2\vec\rho\,\mu_k\mu_{k'}^* = \delta_{kk'}$ косяк

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$k = (0, \vec{k})$ 0 - edge localized SW $\vec{k}$ - plane SW.

In each term $a_0 a_{\vec{k}}^*$, $a_k a_0^*$ - direct processes $\omega_{\vec{k}} \to \omega_k$, $\omega_{\vec{k}} - \omega_k$ - перерезание.

$$W_{ex} = L \int d^2\vec{\rho}\, A \left\{ a_0 a_0^* \nabla\mu_0 \cdot \nabla\mu_0^* + \sum_{\vec{k}\vec{k}'} a_{\vec{k}} a_{\vec{k}'}^* \nabla\mu_{\vec{k}} \cdot \nabla\mu_{\vec{k}'}^* \right\}$$

Мусса pSW - точные plane waves $\mu_{\vec{k}}(\vec{\rho}) = e^{i\vec{k}\vec{\rho}}$, $\nabla\mu_{\vec{k}} \cdot \nabla\mu_{\vec{k}'}^* = (i\vec{k})(-i\vec{k}')\mu_{\vec{k}}\mu_{\vec{k}'}^*$

$$W_{ex} = L \int d^2\vec{\rho}\, A \left\{ a_0 a_0^* \nabla\mu_0 \cdot \nabla\mu_0^* + \sum_{\vec{k}\vec{k}'} a_{\vec{k}} a_{\vec{k}'}^* (\vec{k}\cdot\vec{k}') \mu_{\vec{k}}\mu_{\vec{k}'}^* \right\}, \quad \int d^2\vec{\rho}\, \mu_{\vec{k}}\mu_{\vec{k}'}^* =$$

$= \int d^2\vec{\rho}\, e^{i\vec{k}\vec{\rho}} e^{-i\vec{k}'\vec{\rho}} = \int d^2\vec{\rho}\, e^{i(\vec{k}-\vec{k}')\vec{\rho}} = \delta_{\vec{k}\vec{k}'} S$, S - film square

Возьмем нормированные $\mu_{\vec{k}}(\vec{\rho})$, $\int dV \mu_{\vec{k}}\mu_{\vec{k}'}^* = \delta_{\vec{k}\vec{k}'} V$

$$W_{ex} = AL \int d^2\vec{\rho}\, |\nabla\mu_0|^2 a_0^* a_0 + A \sum_{\vec{k}} k^2 a_{\vec{k}}^* a_{\vec{k}}$$ Пусть $L \int d^2\vec{\rho}\, |\nabla\mu_0|^2 = k_0^2 V$

$$W_{ex} = AV \sum_k k^2 a_k^* a_k$$ - ex. Hamiltonian is diagonal. $k = (k_0, \vec{k})$

$$\frac{1}{L} W_{loc}^m = -\frac{1}{2} M_s \int d^2\vec{\rho} \sum_{kk'} H_y^0(y) \mu_k \mu_{k'}^* a_k a_{k'}^* = \left| \int d^2\vec{\rho}\, H_y^0 \mu_k \mu_{k'}^* = \right| \text{ignoring dif.sat } \omega_{\vec{k}} \rightleftarrows \omega_k \left| = \right.$$

$$= \int d^2\vec{\rho}\, H_y^0 \mu_0 \mu_0^* + \int d^2\vec{\rho}\, H_y^0 \mu_k \mu_{k'}^* \Big| = -\frac{1}{2} M_s \sum_{\substack{k\vec{k}' \\ \vec{k}\neq\vec{k}'}} \int d^2\vec{\rho}\, H_y^0 \mu_{\vec{k}} \mu_{\vec{k}'}^* a_{\vec{k}} a_{\vec{k}'}^* -$$

$$-\frac{1}{2} M_s \int d^2\vec{\rho}\, H_y^0 |\mu_0|^2 a_0 a_0^*$$ - нет $\vec{k} \neq \vec{k}'$ описывает $\omega_{\vec{k}} \to \omega_{\vec{k}'}$ two magnon processes на краю пленки edge $H_y^0(y)$.

$$W_{loc} = W_{ex} + W_m = AV \sum_k k^2 a_k^* a_k - \frac{1}{2} M_s L \sum_k \int d^2\vec{\rho}\, H_y^0 \mu_k \mu_k^* a_k a_k^* - \frac{M_s}{2} \sum_{\vec{k}\vec{k}'} \int d^2\vec{\rho}\, H_y^0 \mu_{\vec{k}} \mu_{\vec{k}'}^* a_{\vec{k}} a_{\vec{k}'}^*$$

$$W_{loc} = AV \sum_k k^2 a_k^* a_k + \frac{1}{2} M_s L \int d^2\vec{\rho} \sum_k H_y^0 \mu_k \mu_k^* a_k a_k^* - \frac{1}{2} M_s L \int d^2\vec{\rho} \sum_{\vec{k}\neq\vec{k}'} H_y^0 \mu_{\vec{k}} \mu_{\vec{k}'}^* a_{\vec{k}} a_{\vec{k}'}^*$$

$$W_{loc} = \sum_k \Omega_k a_k^* a_k - \sum_{\vec{k}\neq\vec{k}'} \Lambda_{\vec{k}\vec{k}'} a_{\vec{k}} a_{\vec{k}'}^*, \quad \Omega_{\vec{k}} = AVk^2 - \frac{1}{2} M_s L \int d^2\vec{\rho}\, H_y^0 |\mu_k|^2$$

$$\Lambda_{\vec{k}\vec{k}'} = -\frac{1}{2} M_s L \int d^2\vec{\rho}\, H_y^0 \mu_{\vec{k}} \mu_{\vec{k}'}^*$$

$$\frac{1}{M_s^2 L} W_{nonloc} = -\int d^2\vec{\rho}\, d^2\vec{\rho}' \left[G_{xx}(\vec{\rho}) \psi(\vec{\rho}) \psi(\vec{\rho}') + G_{zz}(\vec{\rho}) e(\vec{\rho}) e(\vec{\rho}') \right]$$ - какие биграммы зависят a_k, a_k^*?

$$G_{\beta\beta}(\vec{\rho}, \vec{\rho}') = \frac{1}{(2\pi)^2} \int d^2\vec{k}\, G_{\beta\beta}(\vec{k}) e^{i\vec{k}(\vec{\rho}-\vec{\rho}')}, \quad G_{xx}(\vec{k}) = -4\pi \frac{k_x^2}{k^2} f(kL), \quad G_{zz}(\vec{k}) = +4\pi [1 - f(kL)]$$

$W(sw) = W_{loc} + W_{nonloc}$ - квадратичная, не учтены эффекты размагничивания.

(6/w) Вернемся к V.S. L'vov, Wave turbulence, 1994, Ch. 3. $W(\vec{m}) \to W(a, a^*)$

LG: $\dot{M}_+ = 2i\gamma M_z \frac{\delta W}{\delta M_-}$, $\dot{M}_- = -2i\gamma M_z \frac{\delta W}{\delta M_+}$, W - energy density

Подстановка $M_+ = \sqrt{2\gamma M_s}\, a f(x)$, $M_- = a^* f(x) \sqrt{2\gamma M_s}$, $f(x) = \sqrt{1 - \frac{x}{2M_s}}$, $x = a^* a$

$\lambda = \sqrt{2\gamma M_s}$. $\dot{M}_+ = \lambda \dot{a} f(x) + \lambda a f'(x) \frac{d}{dt}(a^* a) = \lambda \dot{a} f(x) + \lambda a f'(x) (\dot{a}^* a + a^* \dot{a})$

use $\dot{a} = i \frac{\delta W}{\delta a^*}$, $\dot{a}^* = -i \frac{\delta W}{\delta a}$ (можно наоборот). $\frac{\delta W}{\delta M_-} = \frac{\delta W}{\delta a} \frac{\partial a}{\partial M_-} + \frac{\delta W}{\delta a^*} \frac{\partial a^*}{\partial M_-}$

$M_- = \lambda a^* f(x)$, $1 = \lambda \frac{\partial a^*}{\partial M_-} + \lambda a^* f'(x) \left[\frac{\partial a^*}{\partial M_-} a + a^* \frac{\partial a}{\partial M_-} \right]$

$f'(x) = \frac{1}{2f} \left(-\frac{\gamma}{2M_s} \right)$, $2f = \lambda 2f \frac{\partial a^*}{\partial M_-} + \lambda a^* \left(-\frac{\gamma}{2M_s} \right) \left[\frac{\partial a^*}{\partial M_-} a + a^* \frac{\partial a}{\partial M_-} \right]$

$2f = \frac{\partial a^*}{\partial M_-} \cdot \lambda \cdot \left[2f - \frac{\gamma}{2M_s} x \right] - \frac{\gamma}{2M_s} \lambda (a^*)^2 \frac{\partial a}{\partial M_-}$ - много.

$\dot{M}_+ = \lambda f(x) i \frac{\delta W}{\delta a^*} + \lambda a f'(x) \left[-i \frac{\delta W}{\delta a} a + a^* i \frac{\delta W}{\delta a^*} \right] = \lambda i \frac{\delta W}{\delta a^*} \left[f + x f' \right] -$

$-i \lambda f' a^2 \frac{\delta W}{\delta a}$. $\frac{\partial M_+}{\partial a} = \lambda f + \lambda a f'(x) a^*$, $\frac{\partial M_-}{\partial a} = \lambda a^* f'(x) a^*$

$\frac{\partial M_+}{\partial a^*} = \lambda a f'(x) a$, $\frac{\partial M_-}{\partial a^*} = \lambda f + \lambda a^* f'(x) a$. $\dot{M}_+ = i\lambda (x f' + f) \frac{\delta W}{\delta a^*} - i \lambda f' a^2 \frac{\delta W}{\delta a}$

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$$\dot M_+ = i\lambda(f+xf')\frac{\delta W}{\delta a^*} - i\lambda f' a^2 \frac{\delta W}{\delta a} = i\lambda(f+xf')\left[\frac{\delta W}{\delta M_+}\frac{\partial M_+}{\partial a^*} + \frac{\delta W}{\delta M_-}\frac{\delta M_-}{\partial a^*}\right] -$$

$$- i\lambda f' a^2\left[\frac{\delta W}{\delta M_+}\frac{\partial M_+}{\partial a} + \frac{\delta W}{\delta M_-}\frac{\partial M_-}{\partial a}\right] = i\lambda(f+xf')\left[\frac{\delta W}{\delta M_+}\lambda a^2 f' + \frac{\delta W}{\delta M_-}\lambda(f+xf')\right] -$$

$$- i\lambda f' a^2\left[\frac{\delta W}{\delta M_+}\lambda(f+xf') + \frac{\delta W}{\delta M_-}\lambda f'(a^*)^2\right] = i\lambda^2\frac{\delta W}{\delta M_-}\left[(f+xf')^2 - f'^2 x^2\right]$$

$$\dot M_+ = i\lambda^2\frac{\delta W}{\delta M_-}\left[f^2 + 2xff'\right], \quad f + 2xf' = f + 2x\frac{1}{2f}\left(-\frac{\gamma}{2M_s}\right)$$

$$f^2 + 2xff' = f^2 + x\left(-\frac{\gamma}{2M_s}\right) = 1 - \frac{\gamma}{2M_s}x - \frac{\gamma}{2M_s}x = 1 - \frac{\gamma}{M_s}x, \quad x = a^*a.$$

$M_z = M_s - \gamma x$, $\lambda^2 = 2\gamma M_s$. $\lambda^2[f^2 + 2xff'] = 2\gamma M_s\left(1 - \frac{\gamma}{M_s}x\right) = 2\gamma(M_s - \gamma x) =$

$= 2\gamma M_z$

$\dot M_+ = i2\gamma M_z\frac{\delta W}{\delta M_-}$ indeed. Т.е. если $\dot a = -i\frac{\delta W}{\delta a^*}$, $\dot a^* = -i\frac{\delta W}{\delta a}$ и $M_+ = \lambda a f(x)$,

то можно воспользоваться LLG.

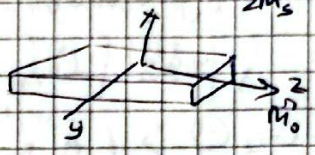
→ Вывод: LLG $\dot M_+ = 2i\gamma M_z\frac{\delta W}{\delta M_-}$, $\dot M_- = -2i\gamma M_z\frac{\delta W}{\delta M_+}$ и $\left(\dot a = i\frac{\delta W}{\delta a^*}, \dot a^* = -i\frac{\delta W}{\delta a}\right)$ canon. YD.

Эквивалентны, если $M_+ = \lambda a f(x)$, $M_- = \lambda a^* f(x)$, $\lambda = \sqrt{2\gamma M_s}$, $f(x) = \sqrt{1 - \frac{\gamma}{2M_s}x}$

$(x = a^*a)$

H. Bertram, V. Safonov, J. Jh. IEEE TM 38, 2514 (2002).

$W = \frac{1}{2M_s}\left[H_x M_x^2 + H_y M_y^2\right]$, $\vec H = -\frac{\delta W}{\delta \vec M}$



$M_x = 2^{-1}\lambda(a + a^*)$; $M_y = \frac{1}{i}\lambda(a^* - a)$, $\lambda = \sqrt{2\gamma M_s}$.

$M_+ = M_x + iM_y = \lambda a$, $M_- = M_x - iM_y = \lambda a^*$ → $\mathcal{H} = A a^* a + \frac{1}{2}B(a^{*2} + a^2)$,

$A = \frac{1}{2}\gamma(H_x + H_y)$, $B = \frac{1}{2}\gamma(H_x - H_y)$. $\vec H$ совпадает при $\vec M = \vec M_0 = M_s\hat z$.

Это основа теории спиновых волн обычного метода (Liv. Stanich) при $f(x) \approx 1$, $\frac{a^*a\gamma}{2M_s} \ll 1$.

E. Schlömann. Ph. Rev. 121, 1312 (1961).

Quantum Hol.-Prim. $M_x + iM_y = \lambda a^+\left[1 - \frac{g\mu_B}{2M_s}a^+a\right]^{1/2}$, $M_x - iM_y = \lambda\left[\dots\right]^{1/2}a$

$M_z = M_0 - g\mu_B a^+a$, $\lambda = \sqrt{2g\mu_B M_s}$, $[a(\tau), a^+(\tau')] = \delta(\tau - \tau')$

$a(\vec r) = \frac{1}{\sqrt V}\sum_{\vec k} a_{\vec k} e^{i\vec k\vec r}$, $[a_{\vec k}, a^+_{\vec k'}] = \delta_{\vec k - \vec k'}$. $\mathcal{H}^{(3)} = \sum_{kk'k''}\left(\Phi_{kk'k''}a_k a_{k'} a^+_{-k''} + \text{c.c.}\right)$

– 3-magnon scattering. $\Phi_{kk'k''} = C\sin 2\theta_k e^{-i\varphi_{k_x}}\times\delta_{k+k'+k''}$, $C = -\pi g\mu_B\left(2g\mu_B\frac{M_s}{V}\right)^{1/2}$

Именно эту Scat. amplitude $\Phi_{kk'k''}$ использовал White, Ch. 8 as $g_{\vec k}$.

В элементах операторов $c_k = u_k a_k + v_k a^+_{-k}$, $\Phi_{kk'k''}$ выражение сложное,

и вычислил u_k, v_k как у L'vov 1994.

E. Schlömann Phys. Rev. 116, 828 (1959). Canonic variables c_k и YD, classical

H.-prim. и decay of uniform mode $c_0 c_k^* c_{-k}^*$ и g-зависимость $\mathcal{H}^{(3)}$ и exch. $\mathcal{H}^{(4)}$ outer.

H. Suhl. J. Phys. Chem. Solids 1, 209 (1957) The theory of FMR at high signal powers.

Bulk sample, plane waves $\vec H_d(\vec r) = -4\pi\sum_{\vec k}\frac{\vec k(\vec k\cdot\vec M_k)}{k^2}e^{i\vec k\vec r}$, $|\vec k| > k_{min}$

$\vec M(\vec r, t) = \sum_{\vec k}\vec M_{\vec k}(t)e^{i\vec k\vec r}$, $\vec M_k(t) = \frac{1}{V}\int dV\,\vec M(\vec r, t)e^{-i\vec k\vec r}$, $\vec m = \vec M/M_s$.

Def. $a_{\vec k} = m_{k_x} + im_{k_y}$, $a^*_{-k} = m_{k_x} - im_{k_y}$, $m_{kx} = \frac{1}{2}(a_k + a^*_{-k})$; $m_{ky} = \frac{1}{2i}(a_k - a^*_{-k})$

$\vec m(\vec r, t) = \sum_{\vec k}\vec m_{\vec k}(t)e^{i\vec k\vec r}$. $m_z = 1 - \frac{1}{2}m_+ m_-$, $m_z = 1 - \frac{1}{2}\sum_{kk'}a_k a^*_{k'-k}e^{ikr}$

$a_k = m^+_k$, $a^*_{-k} = m^-_k$. Нахождение $m_+(a)$, $m_z(a)$ в LLG нелинейные non-linear eqs. для a_k, a^*_{-k} (как у Mattis, Ch. 10) без всякого H.-prim. canonic a_k и canonic YD (!), т.е. для метода расчета non-lin SW

(1) Suhl – 1957 a_k, a^*_{-k}, LLG и (2) Schlömann – L'vov (1959) – canonic YD, H.P.

Если $m_z(a_k, a^*_{-k})$ подставить в $W(\vec{m})$, то получим the energy $W(\vec{m})$ density decomposition по a_k, a^*_{-k}, $W(a)$ содерж. уд a_k, a^*_{-k} из LLG. т.е., $W(a)$ и уд a_k, a^*_{-k} можно получить независимо, не прибегая к canonic eqs. $\dot{a}_k = i\frac{\delta W}{\delta a^*_k}$ как в merge Schlömann–Liou. Tan gener $\frac{\pi}{2}$.

$m_+ = (m_x + i m_z) = e^{-i\pi/2}(ee + i\psi) = \sum_k a_k \mu_k(\vec{s})$, $\vec{k} \to k\,(k_0, \vec{k})$ edge sw pbc

$m_- = (m_x - i m_y) = e^{i\pi/2}(ee - i\psi) \rightleftarrows \sum_k a^*_k \mu^*_k(s)$.

Группа $e^{\pm i\pi/2}$ в a, a^*, $ee + i\psi = \sum_k a_k \mu_k$, $ee - i\psi = \sum_k a^*_k \mu^*_k$.

$ee(\vec{s}_i) + i\psi(\vec{s}_i) = \sum_k a_k(t)\mu_k(\vec{s})$. $ee = \mathrm{Re}\sum_k a_k\mu_k = \frac{1}{2}\sum_k (a_k\mu_k + a^*_k\mu^*_k)$

$\psi = \mathrm{Im}\sum_k a_k\mu_k = \frac{1}{2i}\sum_k (a_k\mu_k - a^*_k\mu^*_k)$

$m_+ = \sum_k m^+_k e^{ikr} = \sum_k a_k e^{ikr}$, $m_- = \sum_k m^-_k e^{-ikr} = \sum_k a^*_{-k} e^{ikr}$

$(m_-)^* = \sum_k a_k e^{-ikr} = \sum_k a_k e^{ikr} = m_+$, что оправдывает введение a_k и a^*_{-k}.

тогда $\frac{1}{M_s^2 L} W_{nonloc} = -\int d^2s\, d^2s' \left[G_{xx}\psi(\vec{s})\psi(\vec{s}') + G_{zz}\, ee(\vec{s})\, ee(\vec{s}') \right] =$

$= -\int d^2s\, d^2s' \Big[G_{xx}\frac{1}{2}\sum_k (a_k\mu_k(\vec{s}) + a^*_k\mu^*_k(\vec{s}))\sum_{k'}(a_{k'}\mu_{k'}(\vec{s}') + a^*_{k'}\mu^*_{k'}(\vec{s}'))\frac{1}{2} +$

$+ G_{zz}\frac{(-1)}{4}\sum_k (a_k\mu_k(\vec{s}) - a^*_k\mu^*_k(\vec{s}))\sum_{k'}(a_{k'}\mu_{k'}(\vec{s}') - a^*_{k'}\mu^*_{k'}(\vec{s}')) \Big] =$

$= -\frac{1}{4}\int d^2s\, d^2s' \Big[G_{xx}\sum_{kk'}\{ a_k\mu_k(\vec{s})\, a_{k'}\mu_{k'}(\vec{s}') + a^*_k a_{k'}\mu^*_k(\vec{s})\mu_{k'}(\vec{s}') + a_k a^*_{k'}\mu_k(\vec{s})\mu^*_{k'}(\vec{s}') +$

$+ a^*_k a^*_{k'}\mu^*_k(\vec{s})\mu^*_{k'}(\vec{s}')\} - G_{zz}\sum_{kk'}\{ a_k a_{k'}\mu_k(\vec{s})\mu_{k'}(\vec{s}') - a^*_k a_{k'}\mu^*_k(\vec{s})\mu_{k'}(\vec{s}') -$

$- a_k a^*_{k'}\mu_k(\vec{s})\mu^*_{k'}(\vec{s}') + a^*_k a^*_{k'}\mu^*_k(\vec{s})\mu^*_{k'}(\vec{s}')\} \Big]$

$\frac{1}{M_s^2 L} W_{nonloc} = -\frac{1}{4}\int d^2s\, d^2s' \Big[(G_{xx} - G_{zz})\, a_k a_{k'}\mu_k(\vec{s})\mu_{k'}(\vec{s}') + (G_{xx} + G_{zz})\, a^*_k a_{k'}\mu^*_k(\vec{s})\mu_{k'}(\vec{s}') +$

$+ (G_{xx} + G_{zz})\, a_k a^*_{k'}\mu_k(\vec{s})\mu^*_{k'}(\vec{s}') + (G_{xx} - G_{zz})\, a^*_k a^*_{k'}\mu^*_k(\vec{s})\mu^*_{k'}(\vec{s}') \Big] \sum_{kk'}(\cdot)$. $G_{\alpha\beta}(\vec{s}\vec{s}') = G_{\beta\alpha}(\vec{s}'\vec{s})$

$\frac{1}{M_s^2 L} W_{nonloc} = \sum_{kk'}\left[B_{kk'}\, a_k a_{k'} + B^*_{kk'}\, a^*_k a^*_{k'} + 2C_{kk'}\, a^*_k a_{k'} \right]$

$B_{kk'} = -\frac{1}{4}\int d^2s\, d^2s' (G_{xx} - G_{zz})\mu_k(\vec{s})\mu_{k'}(\vec{s}')$; $C_{kk'} = -\frac{1}{4}\int d^2s\, d^2s' (G_{xx} + G_{zz})\mu^*_k(\vec{s})\mu_{k'}(\vec{s}')$

Вкл. $M_s^2 L$ в коэффициенты $B_{kk'}$, $C_{kk'}$.

$W(a) = \sum_k \Omega_k a^*_k a_k - \sum_{k\neq k'} A_{kk'} a_k a^*_{k'} + \sum_{kk'}\left[B_{kk'}\, a_k a_{k'} + B^*_{kk'}\, a^*_k a^*_{k'} + 2C_{kk'} \right]$

– в квадратичном приближении. $\sum_{kk'}(\cdot)$ сокращается из-за real dyn. interac

$W^m_{int} = -\frac{1}{2}M_s^2\left[\psi\int d^2s'\, G_{xy}(ee^2 + \psi^2) + (ee^2 - \psi^2)\int d^2s'\, G_{yx}\psi \right]$ – energy dens

$\Rightarrow$

$W^m_{int} = -\frac{1}{2}M_s^2\Big[\frac{1}{2i}\sum_k (a_k\mu_k(\vec{s}) - a^*_k\mu^*_k(\vec{s}))\int d^2s'\, G_{xy}\sum_{k'k''} a_{k'}a^*_{k''}\mu_{k'}(\vec{s}')\mu^*_{k''}(\vec{s}') +$

$+ \sum_{kk'} a_k a^*_{k'}\mu_k(\vec{s})\mu^*_{k'}(\vec{s})\int d^2s'\, G_{yx}\frac{1}{2i}\sum_{k''}\left[a_{k''}\mu_{k''}(\vec{s}') - a^*_{k''}\mu^*_{k''}(\vec{s}') \right]$, $W^m_{int} = \int dV w^m_{int}$, $dV = L\int d^2s$

$W^m_{int} = -\frac{1}{4i}M_s^2 L\sum_{kk'k''}\int d^2s\int d^2s'\, G_{xy}\Big[(a_k\mu_k(\vec{s}) - a^*_k\mu^*_k(\vec{s}))\, a_{k'}a^*_{k''}\mu_{k'}(\vec{s}')\mu^*_{k''}(\vec{s}') +$

$+ a_k a^*_{k'}\mu_k(\vec{s})\mu^*_{k'}(\vec{s})\left(a_{k''}\mu_{k''}(\vec{s}') - a^*_{k''}\mu^*_{k''}(\vec{s}') \right)\Big]$, $k \rightleftarrows k'$, $s \rightleftarrows s'$ во втором члене

* Нет недиаг. квазичастичных quadratic $W(a)$, т.к. canonic уд не связаны, ω_k – из LLG

Edge SW Scattering (Raval)

$$W_{int}^{m} = -\frac{1}{4i} M_s^2 L \sum_{kk'k''} \int d^2\vec{s}\, d^2\vec{s}'\, G_{xy} \left(a_k \mu_k(s) - a_k^* \mu_k^*(s)\right)\left[a_{k'} a_{k''}^* \mu_{k'}(s') \mu_{k''}^*(s') + a_{k''} a_{k'}^* \mu_{k''}(s') \mu_{k'}^*(s')\right]$$

$$\int d^2\vec{s}\, d^2\vec{s}'\, G_{xy} \mu_k(s) \mu_{k'}(s') \mu_{k''}^*(s') = \mathcal{D}_{kk'k''}$$

$$E_{kk'k''} = G_y \int d^2\vec{s}\, d^2\vec{s}'\, \mu_k^*(s) \mu_{k'}(s') \mu_{k''}^*(s'), \quad F_{kk'k''} = \int d^2\vec{s}\, d^2\vec{s}'\, G_{xy} \mu_k(s) \mu_{k''}(s') \mu_{k'}^*(s')$$

$$G_{kk'k''} = \int d^2\vec{s} \int d^2\vec{s}'\, \mu_k^*(s) \mu_{k''}(s') \mu_{k'}^*(s') G_{xy} = \mathcal{D}^*_{k'k''k}, \quad E_{kk'k''} = F^*_{kk'k''}$$

$$W_{int}^{m} = -\frac{1}{4i} M_s^2 L \sum_{kk'k''} \left[a_k a_{k'} a_{k''}^* \mathcal{D}_{kk'k''} - a_k^* a_{k'}^* a_{k''} \mathcal{D}^*_{kk'k''} - a_k^* a_{k'} a_{k''}^* E_{kk'k''} + a_k a_{k'}^* a_{k''} F_{kk'k''}\right]$$

→ вклады $\frac{1}{2} M_s^2 L$ в коэф. $\mathcal{D}, E, F, G$

$$W_{int}^{m} = -\frac{1}{2i} \sum_{kk'k''} \left[a_k a_{k'} a_{k''}^* \mathcal{D}_{kk'k''} - \mathcal{D}^*_{kk'k''} a_k^* a_{k'}^* a_{k''} + F_{kk'k''} a_k a_{k'}^* a_{k''} - F^*_{kk'k''} a_k^* a_{k'} a_{k''}^*\right]$$

$k'' \to k$, $k'' \to k'$, $k' \to k$

$$W_{int}^{m} = -\frac{1}{2i} \sum_{kk'k''} \left[\mathcal{D}_{kk'k''} a_k a_{k'} a_{k''}^* - \mathcal{D}^*_{k''k'k} a_k a_{k''}^* a_{k'}^* + (F_{k''kk'}) F_{kk''k'} a_k a_{k'} a_{k''}^* - F^*_{k'kk''} a_k a_{k'}^* a_{k''}^*\right]$$

$$W_{int}^{m} = -\frac{1}{2i} \sum_{kk'k''} \left[\left(\mathcal{D}_{kk'k''} + F_{kk''k'}\right) a_k a_{k'} a_{k''}^* - \left(\mathcal{D}^*_{k''k'k} + F^*_{k'kk''}\right) a_k a_{k'}^* a_{k''}^*\right]$$

$$P_{kk'k''} = \mathcal{D}_{kk'k''} + F_{kk''k'} = 2\mathcal{D}_{kk'k''}$$

$$\mathcal{D}^*_{k'k'k} = \int d^2\vec{s}\, d^2\vec{s}'\, G_{xy} \mu_{k''}(s) \mu_{k'}(s') \mu^*_k(s'), \quad Q^*_{kk'k''} = \mathcal{D}^*_{k''k'k} + F^*_{k'kk''}$$

$$F_{k'kk''} = \int d^2\vec{s} \int d^2\vec{s}'\, G_{xy} \mu_{k'}(s) \mu_{k''}(s') \mu^*_k(s')$$

$$W_{int}^{m} = -\frac{1}{2i} \sum_{kk'k''} \left[2\mathcal{D}_{kk'k''} a_k a_{k'} a_{k''}^* - Q^*_{kk'k''} a_k a_{k'}^* a_{k''}^*\right]$$

$\vec{k} + \vec{k}' = \vec{k}''$  confluence  splitting $\vec{k} = \vec{k}' + \vec{k}''$

$$\mathcal{D}_{kk'k''} = \int d^2\vec{s}\, d^2\vec{s}'\, G_{xy} \mu_k(s) \mu_{k'}(s') \mu^*_{k''}(s')$$ — случаи confluence $k, k'' \approx$ pSW, $k' -$ eSW

Мусор $\mu_k(\vec{s}) = e^{i\vec{k}\vec{s}}$, $\mu^*_{k''}(\vec{s}) = e^{-i\vec{k}''\vec{s}}$

$$\mathcal{D}_{kk'k''} = \int d^2\vec{s} \int d^2\vec{s}'\, G_{xy}(\vec{s}\vec{s}') e^{i\vec{k}\vec{s}} e^{-i\vec{k}''\vec{s}'} \mu_{k'}(s')$$ Edge SW $\mu_{k'} = \mu_0(y) e^{i\varkappa x}$ (s)

$$\mathcal{D}_{kk'k''} = \int d^2\vec{s} \int d^2\vec{s}' \frac{1}{(2\pi)^2} \int d^2\vec{p}\, e^{i\vec{p}(\vec{s}-\vec{s}')} (-4\pi) \frac{p_x p_y}{p^2} f(pL) e^{i\vec{k}\vec{s}} e^{-i\vec{k}''\vec{s}'} \mu_0(y') e^{i\varkappa x'}$$

$$\int d^2\vec{s}\, e^{i\vec{p}\vec{s}} e^{i\vec{k}\vec{s}} = \int_0^\infty dx\, e^{i(p_x+k_x)x} \int_0^\infty dy\, e^{i(p_y+k_y)y} = 2\pi\delta(p_x+k_x) \int_0^\infty dy\, e^{i(p_y+k_y)y}$$

Semi-infinite

$$\int_0^\infty dx\, e^{ikx} = \int_{-\infty}^{\infty} dx \left[\cos kx + i \sin kx\right] = \frac{1}{2}\int_{-\infty}^{\infty} dx\, e^{ikx} + i\int_0^\infty dx \sin kx \approx \pi\delta(k)$$

$$\int d^2\vec{s}\, e^{i\vec{p}\vec{s}} e^{i\vec{k}\vec{s}} \approx 2\pi^2 \delta(p_x+k_x)\delta(p_y+k_y)$$

$$\mathcal{D}_{kk'k''} = \frac{(-4\pi)}{(2\pi)^2} 2\pi^2 \int d^2\vec{s}' \int d^2\vec{p}\, e^{-i\vec{p}\vec{s}'} \frac{p_x p_y}{p^2} f(pL) e^{-i\vec{k}''\vec{s}'} \mu_0(y') e^{i\varkappa x'} \delta(p_x+k_x)\delta(p_y+k_y) =$$

$$= -2\pi \int d^2\vec{s}'\, e^{i\vec{k}\vec{s}' + k_x k_y} f(kL) e^{-i\vec{k}''\vec{s}'} \mu_0(y') e^{i\varkappa x'} = -2\pi \frac{k_x k_y}{k^2} f(kL) \int_{-\infty}^{\infty} dx' \int_0^\infty dy'\, e^{x}$$

$$\times e^{ik_x x'} e^{ik_y y'} e^{-i(k''_x x' + k''_y y')} \mu_0(y') e^{i\varkappa x'}, \quad \int_{-\infty}^{\infty} dx' e^{i(k_x - k''_x + \varkappa)x'} = 2\pi\delta(k_x + \varkappa - k''_x)$$

$$\mathcal{D}_{kk'k''} = \mathcal{D}_{k\varkappa k''} = -(2\pi)^2 \frac{k_x k_y}{k^2} f(kL) \delta(k_x + \varkappa - k''_x) \int_0^\infty dy' e^{i(k_y - k''_y)y'} \mu_0(y')$$

$$\mathcal{D}_{k\varkappa k''} = -(2\pi)^2 \frac{k_x k_y}{k^2} f(kL) \int_0^\infty dy' e^{i(k_y - k''_y)y'} \mu_0(y')$$

мусор $\mu_0(y) = e^{-y/\Delta}$  $k'' \to k'$

$$\int_0^\infty dy\, e^{i(k - \frac{1}{\Delta})y} = \int dy \left[\cos ky + i \sin ky\right] e^{-y/\Delta} =$$

$$= i\frac{k}{\gamma_\Delta^2 + k^2} + \frac{\gamma_\Delta}{\gamma_\Delta^2 + k^2} = \Delta \frac{1 + ik\Delta}{(1 + k^2\Delta^2)}, \quad k = k_y - k'_y, \quad k_x + \varkappa = k'_x$$

$$\mathcal{D}_{k\varkappa k'} = -(2\pi)^2 \frac{k_x k_y}{k^2} f(kL) \Delta \frac{1 + i(k_y - k'_y)\Delta}{\left[1 + \Delta^2 (k_y - k'_y)^2\right]}$$

— не забыли ли $\varkappa, k'_x$ при выполнении $k_x + \varkappa = k'_x$

Escaneado con CamScanner

$$D_{\vec{k}\vec{k}'} = -2\pi^2 \cdot \sin 2\varphi_{\vec{k}} \, f(kL)\Delta \, \frac{1 + i(k_y - k_y')\Delta}{\left[1 + (k_y - k_y')^2\Delta^2\right]} \sim \sin 2\varphi_k$$

$\omega_{\vec{k}} + \omega_{\vec{x}} = \omega_{\vec{k}'}$, $\vec{k} + \vec{x} = \vec{k}'$ Вот как для амплитуда $\vec{k} \to \vec{k}'$ "confluence" process

$\sim \sin 2\varphi_{\vec{x}}$, max при $\varphi_k = \frac{\pi}{4}$. $k_y = k_y'$ — сохр. импульса

$\operatorname{tg}\varphi_k = \frac{k_y}{k_x}$, $\operatorname{tg}\varphi_{k'} = \frac{k_y'}{k_x'} = k_y/k_x$

$D_{\vec{k}\vec{k}'} = -2\pi^2 \sin 2\varphi_k \, f(kL)\Delta$ — на перпендикулярах $\varphi_k(\vec{s})$, $\varphi_{k'}(\vec{s}')$.

Оценка аннигиляции и confluence', нормальное edge sw при отражении от gelu edge.

(7/IV) Обзор Google-sch. papers Suhl, Schlömann, Ivon, Zakharov. Мультиплет Wint и нормальное решение линеаризованной задачи.

$$W_{int}^m = -\frac{1}{2} M_s^2 L \int d^2\vec{s}\, d^2\vec{s}' \, G_{xy}\left[\psi(\vec{s})\left(\varepsilon\varepsilon^2(\vec{s}') + \psi^2(\vec{s}')\right) + \left(\varepsilon\varepsilon^2(\vec{s}) + \psi^2(\vec{s})\right)\psi(\vec{s}')\right]$$

$G_{xy} = G_{yx}$ и $\vec{s} \rightleftarrows \vec{s}'$

$$W_{int}^m = -M_s^2 L \int d^2\vec{s}\, d^2\vec{s}' \, G_{xy}(\vec{s}\vec{s}')\left[\psi^2(\vec{s}) + \varepsilon\varepsilon^2(\vec{s})\right]\psi(\vec{s}')$$

$$\varepsilon\varepsilon + i\psi = \sum_k a_k(t)\,\varphi_k(\vec{s}'), \quad \psi(\vec{s}, t) = \operatorname{Im}\sum_k a_k \varphi_k(\vec{s})$$

$$W_{int}^m = -M_s^2 L \int d^2\vec{s}\, d^2\vec{s}' \, G_{xy} \sum_{kk'} a_k a_{k'}^* \varphi_k \varphi_{k'}^*(\vec{s}) \operatorname{Im}\sum_{k''} a_{k''}\varphi_{k''}(\vec{s}')$$

$$W_{int}^m = -M_s^2 L \operatorname{Im} \int d^2\vec{s}\, d^2\vec{s}' \sum_{kk'k''} a_k a_{k'}^* a_{k''} \varphi_k(\vec{s}) \varphi_{k'}^*(\vec{s}) \varphi_{k''}(\vec{s}')$$

$$\operatorname{Im}\sum_{k''} a_{k''}\varphi_{k''} = \frac{1}{2i}\left[\sum_{k''} a_{k''}\varphi_{k''} - \sum_{k''} a_{k''}^*\varphi_{k''}^*\right]$$

$$W_{int}^m = -M_s^2 L \frac{1}{2i} \int d^2\vec{s}\, d^2\vec{s}' \, G_{xy} \sum_{kk'k''} a_k a_{k'}^* \varphi_k(\vec{s}) \varphi_{k'}^*(\vec{s}) \left[a_{k''}\varphi_{k''}(\vec{s}') - a_{k''}^*\varphi_{k''}^*(\vec{s}')\right]$$

Def. $D_{kk'k''} = \int d^2\vec{s}\, d^2\vec{s}' \, G_{xy} \varphi_k(\vec{s}) \varphi_{k'}^*(\vec{s}) \varphi_{k''}(\vec{s}')$

$$W_{int}^m = -\frac{1}{2i} M_s^2 L \sum_{kk'k''}\left[D_{kk'k''} a_k a_{k'}^* a_{k''} - D_{kk'k''}^* a_k a_{k'}^* a_{k''}^*\right]$$

$k' \rightleftarrows k''$

$$W_{int}^m = -\frac{1}{2i} M_s^2 L \sum_{kk'k''}\left[D_{kk'k'} a_k a_{k'} a_{k''}^* - D_{kk'k''}^* a_k a_{k'}^* a_{k''}\right]$$

Amplitude $\left(\vec{k} \rightleftarrows \vec{k}', \vec{k}''\right)$ confluence $= D_{kk''k'}$, ampl. $\left(\vec{k} \rightleftarrows \vec{k}', \vec{k}\right)$ Splitting $= D_{kk'k''}^*$

Вид $D_{kk'k''} = \int d^2\vec{s} \int d^2\vec{s}' \, G_{xy}(\vec{s}\vec{s}') \varphi_k(\vec{s}) \varphi_{k'}^*(\vec{s}') \varphi_{k''}(\vec{s}')$ задан

sw-профиль $\varphi_k(\vec{s})$, $k = (k_0, \vec{k})$ — решение линейной задачи

$$\begin{cases} \frac{1}{\gamma}\dot{\psi} = -\frac{2A}{M_s}\nabla^2\varepsilon\varepsilon - H_y^0\varepsilon\varepsilon - M_s\int d^2\vec{s}'\, G_{zz}\,\varepsilon\varepsilon(\vec{s}') \\ -\frac{1}{\gamma}\varepsilon\varepsilon = -\frac{2A}{M_s}\nabla^2\psi - H_y^0\psi - M_s\int d^2\vec{s}'\, G_{xx}\,\psi(\vec{s}') \end{cases}$$

$\varepsilon\varepsilon(t) = \varepsilon\varepsilon\, e^{i\omega t}$, $\psi(t) = \psi(\vec{s})\, e^{i\omega t + i\varphi_0}$

$$\begin{cases} i\frac{\omega}{\gamma}\psi(\vec{s}) = -\frac{2A}{M_s}\nabla^2\varepsilon\varepsilon - H_y^0\varepsilon\varepsilon - M_s\int d^2\vec{s}'\, G_{zz}\,\varepsilon\varepsilon(\vec{s}') \\ -i\frac{\omega}{\gamma}\varepsilon\varepsilon(\vec{s}) = -\frac{2A}{M_s}\nabla\psi - H_y^0\psi - M_s\int d^2\vec{s}'\, G_{xx}\,\psi(\vec{s}') \end{cases}$$

translation $\parallel Ox$, $\varepsilon\varepsilon(\vec{s}) = \varepsilon\varepsilon(y)e^{iqx}$, $\psi(\vec{s}) = \psi(y)e^{iqx}$

$$\begin{cases} i\frac{\omega}{\gamma}\psi(y) = -\frac{2A}{M_s}\left[-q^2 + \frac{\partial^2}{\partial y^2}\right]\varepsilon\varepsilon(y) - H_y^0\varepsilon\varepsilon - M_s e^{-iqx}\int d^2\vec{s}'\, G_{zz}\,\varepsilon\varepsilon(y')\, e^{iqx'} \\ -i\frac{\omega}{\gamma}\varepsilon\varepsilon(y) = -\frac{2A}{M_s}\left[-q^2 + \frac{\partial^2}{\partial y^2}\right]\psi(y) - H_y^0\psi - M_s e^{-iqx}\int d^2\vec{s}'\, G_{xx}\,\psi(y')\, e^{iqx'} \end{cases}$$

$$\int_{-\infty}^{\infty} dx' \, G_{zz}(\vec{s}\vec{s}')\, e^{iqx'} = \int dx' e^{iqx} \frac{1}{(2\pi)^2}\int d^2\vec{k}\, e^{i\vec{k}(\vec{s}-\vec{s}')}(-4\pi)(1 - f(kL)) = \frac{-4\pi}{(2\pi)^2}\int dx' e^{iqx'} \times$$

$$\times \int d^2\vec{k}\, e^{ik_x(x-x')} e^{ik_y(y-y')}(1-f) = -\frac{1}{\pi}\int dx' e^{i(q-k_x)x'} \int dk_x e^{ik_x x} e^{ik_y(y-y')}(1-f) =$$

$$= -\frac{1}{\pi}\int d^2\vec{k}\, e^{ik_x x} e^{ik_y(y-y')}(1-f)\, 2\pi\delta(q-k_x) = -2e^{iqx}\int dk_y e^{ik_y(y-y')}(1-f)\Big|_{k_x = q}$$

$$\int_{-\infty}^{\infty} dx' G_{xx}(\vec{s}\vec{s}') e^{iqx'} = \int dx' e^{iqx'} \frac{1}{(2\pi)^2} \int d^2\vec{k} e^{i\vec{k}(\vec{s}-\vec{s}')} (-4\pi) \frac{k_x^2}{k^2} f(kL) =$$

$$= -\frac{1}{\pi} \int d^2\vec{k} \underbrace{\int dx' e^{i(q-k_x)x'}}_{=2\pi\delta(q-k_x)} e^{ik_x x} e^{ik_y(y-y')} \frac{k_x^2}{k^2} (\cdot f) = \left| \int_{-\infty}^{\infty} dx' e^{i(q-k_x)x'} = 2\pi\delta(q-k_x) \right| =$$

$$= -2 e^{iqx} \int dk_y e^{ik_y(y-y')} \frac{q^2}{k^2} (\cdot f)_{q=k_x}$$

$$e^{-iqx} \int d^2\vec{s}' G_{xx} \varphi(y') e^{iqx'} = -2 \int dy' \varphi(y') \int dk_y e^{ik_y(y-y')} \frac{q^2}{k^2} (\cdot f) =$$

$$= \int_0^{\infty} dy' g_{xx}(yy') \varphi(y'), \quad g_{xx}(yy') = -2 \int_{-\infty}^{\infty} dk_y e^{ik_y(y-y')} \frac{q^2}{q^2+k_y^2} \left[f\left(\sqrt{q^2+k_y^2}L\right) \right]$$

$$e^{-iqx} \int d^2\vec{s}' G_{zz} \psi(y') e^{iqx'} = -2 \int dy' \psi(y') \int dk_y e^{ik_y(y-y')} \left[1 - f\left(\sqrt{q^2+k_y^2}L\right)\right] =$$

$$= \int_0^{\infty} dy' g_{zz}(yy') \psi(y'), \quad g_{zz}(yy') = -2 \int_{-\infty}^{\infty} dk_y e^{ik_y(y-y')} \left[1 - f\left(\sqrt{q^2+k_y^2}L\right)\right]$$

Lin SW

$$\begin{cases} i\frac{\omega}{\gamma}\psi(y) = \frac{2A}{M_s}\left[q^2 - \frac{\partial^2}{\partial y^2}\right]\varphi(y) - H_y^0(y)\varphi(y) - M_s \int_0^{\infty} dy' g_{zz}(yy')\varphi(y') \\ i\frac{\omega}{\gamma}\varphi(y) = \frac{2A}{M_s}\left[q^2 - \frac{\partial^2}{\partial y^2}\right]\psi(y) - H_y^0(y)\psi(y) - M_s \int_0^{\infty} dy' g_{xx}(yy')\psi(y') \end{cases}$$

Semi-inf. stem $0 \le y < \infty$, $\omega \to -\omega$

SW: $\varphi(\vec{s},t) = \varphi(y) e^{i(qx-\omega t)}$, $\psi(\vec{s},t) = \psi(y) e^{i(qx-\omega t)}$ – все свелось к SW-profiles along y axis $\perp$ film edge. $qL \approx 1$. Грубо $qL \ll 1$.

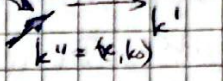
Эту систему решим 4/IV when $L \to 0$, когда $\varphi(y), \psi(y) \sim e^{ik_y y}$ для pSW

Звоним маме в конец по Skype.

(8/IV) Максимально упростим scatter. amplitude $D_{kk'k''}$.

$$D_{kk'k''} = \int d^2\vec{s} \int d^2\vec{s}' G_{xy} \mu_k(\vec{s}) \mu_{k'}^*(\vec{s}') \mu_{k''}(\vec{s}')$$

$k, k' \to \vec{k}, \vec{k}'$ – pSW, k'' – eSW



Все $\mu_k(\vec{s}) = \mu_k(y) e^{ik_x x}$ – в силу transl. inv. вдоль film edge ∂_x.

$$D_{kk'k''} = \int dx\, dy \int dx'\, dy' \frac{1}{(2\pi)^2} \int d^2\vec{p}\, e^{i\vec{p}(\vec{s}-\vec{s}')} (-4\pi) \frac{p_x p_y}{p^2} f(pL) \mu_k(y) e^{ik_x x} \mu_{k'}^*(y') e^{-ik'_x x'} \mu_{k''}(y') e^{ik''_x x'} =$$

$$= \frac{(-4\pi)}{(2\pi)^2} \int dx\, dy \int dx'\, dy' \int dp_x\, dp_y \frac{p_x p_y}{p^2} f(pL) \times$$

$$\times e^{ip_x(x-x')} e^{ip_y(y-y')} e^{ik_x x} e^{-ik'_x x'} e^{ik''_x x'} \mu_k(y) \mu_{k'}^*(y') \mu_{k''}(y')$$

$$\int_{-\infty}^{\infty} dx\, e^{ip_x x} e^{ik_x x} e^{-ik'_x x} = \int dx\, e^{i(p_x+k_x-k'_x)x} = 2\pi\delta(p_x + k_x - k'_x)$$

$$\int_{-\infty}^{\infty} dx'\, e^{-ip_x x'} e^{ik''_x x'} = \int dx' e^{i(k''_x - p_x)x'} = 2\pi\delta(p_x - k''_x)$$

$$D_{kk'k''} = \frac{(-4\pi)}{(2\pi)^2} (2\pi)^2 \int_0^{\infty} dy\, dy' \int_{-\infty}^{\infty} dp_x\, dp_y \frac{p_x p_y}{p^2} f(pL) e^{ip_y(y-y')} \delta(p_x + k_x - k'_x) \delta(p_x - k''_x) \times$$

$$\int_{-\infty}^{\infty} dp_x F(p_x) \delta(p_x + k_x - k'_x) \delta(p_x - k''_x) = F(k''_x) \delta(k''_x + k_x - k'_x), \quad p_x = k''_x$$

$$\times \mu_k(y) \mu_{k'}^*(y) \mu_{k''}(y')$$

$$D_{kk'k''} = (-4\pi) \int dy\, dy' \int dp_y \frac{p_x p_y}{p^2} f(pL)\Big|_{p_x = k''_x} e^{ip_y(y-y')} \mu_k(y) \mu_{k'}^*(y) \mu_{k''}(y') \times$$

$$\times \delta(k_x + k''_x - k'_x)$$

$$g_{xy}(yy', p_x) = -2 \int_{-\infty}^{\infty} dp_y \frac{p_x p_y}{p^2} f(pL) e^{ip_y(y-y')}, \quad p_x = k''_x = x \text{ – exogenic w.vector } \vec{x}$$

$$D_{kk'k''} = 2\pi \int_0^{\infty} dy\, dy' g_{xy}(yy', p_x) \mu_k(y) \mu_{k'}^*(y) \mu_{k''}(y') \delta(k_x + k''_x - k'_x), \quad p_x = k''_x$$

В lin. SW задаем $g_{xx}(yy', q) = -2 \int_{-\infty}^{\infty} dk_y e^{ik_y(y-y')} \frac{q^2}{q^2+k_y^2} f\left(\sqrt{q^2+k_y^2}L\right)$

$$g_{zz}(yy', q) = -2 \int_{-\infty}^{\infty} dk_y e^{ik_y(y-y')} \left[1 - f\left(\sqrt{q^2+k_y^2}L\right)\right], \quad \mu_{k''}(y) = \mu_x(y) e^{ixx}$$

$$D_{kk'x} = 2\pi \int_0^{\infty} dy\, dy' g_{xy}(yy', x) \mu_k(y) \mu_{k'}^*(y') \mu_{x k_0}(y'), \quad g_{xy}(yy', x) = -2 \int_{-\infty}^{\infty} dp_y \frac{x p_y}{p^2} f(pL) e^{ip_y \Delta y}$$

$$D_{kk'x} = 2\pi \int_0^{\infty} dy\, dy' g_{xy}(yy', x) \mu_k(y) \mu_{k'}^*(y') \mu_x(y') \delta(k_x + x - k'_x), \quad k'' = (x, k_0) = (k''_x, k''_y)$$

$p = \sqrt{x^2+p_y^2}$

$\delta(k_x + x - k'_x)$ можно убрать если в $\sum_{kk'k''}(\dots)$ подразумевать $k''_x = x = k'_x - k_x$.

Escaneado con CamScanner

Note: $t_{i,k}$, $g_{xy}(yy';x) \sim x$, so $D_{kk'x} \sim x$. Pure pSW scattering acts to (eSW)

Если использовать для pSW $\mu_k(\vec\rho) = \frac{1}{\sqrt{S}} e^{i\vec k\vec\rho}$, $\int d^2\rho\, \mu_k \mu^*_{k'} = \frac{1}{S}\int d^2\rho\, e^{i\vec k(\vec\rho-\vec\rho')} =$

$= \frac{1}{S}\int dx\, e^{ik(x-x')}\Big|_{-\infty}^{\infty} = \frac{1}{S}\int dx\, e^{i(k-k')x} = \frac{1}{S}\, S\,\delta_{kk'} = \delta_{kk'}$, то

$D_{k'k''x} = \frac{2\pi}{S}\int_0^\infty dy\,dy'\, g_{xy}(yy';x)\, e^{ik_y y} e^{-ik'_y y'} \mu_x(y'')$, $\quad p^2 = x^2 + p_y^2$

$\int_0^\infty dy\, e^{i(k_y - k'_y)y} g_{xy}(yy';x) = \int_0^\infty dy\, e^{i(k_y-k'_y)y}(-2)\int_{-\infty}^{\infty} d\rho_y \frac{x\rho_y}{\rho^2} f(\rho L) e^{i\rho_y(y-y')} =$

$= \Big|\int_0^\infty dy\, e^{i(k_y - k'_y + \rho_y)y} \overset{?}{=} \pi\,\delta(k_y - k'_y + \rho_y)\Big| = (-2)\pi\,\frac{x\rho_y}{\rho^2} f(\rho L)\, e^{-i\rho_y y}\Big|_{\rho_y = k'_y - k_y}$

$D_{k'k''x} = \frac{2\pi}{S}\int dy'\, \mu_x(y')\,(-2\pi)\frac{x\rho_y}{\rho^2} f(\rho L)\, e^{-i\rho_y y'} = -\frac{(2\pi)^2}{S}\frac{x\rho_y}{\rho^2} f(\rho L)\int_0^\infty dy\, \mu_x(y) e^{-i\rho_y y}$

$D_{k'k''x} = -\frac{(2\pi)^2}{S}\frac{x\rho_y}{\rho^2} f(\rho L)\int_0^\infty dy\, e^{-i\rho_y y}\mu_x(y)$, $\quad \rho_y = k_y' - k_y$, $\rho^2 = x^2 + \rho_y^2$.

Вид $D_{kk'k''}$ — более удобен, т.к. $D_{kxk''}$ был зависим от $\vec k'$ — волнового вектора падающей волны.

$\Rightarrow D_{kk''k'} a_k a_{k'} a^*_{k''}$ $\qquad D_{kk'k'} = \int dx\, dy \int d^2\rho'\, G_{xy}\, \mu_k(\vec\rho)\, \mu^*_{k''}(\vec\rho)\, \mu_{k'}(\vec\rho')$

Применяем Винт-усреднение от $\pm i/\bar\nu$. $G_{xy}(\vec\rho\vec\rho') = \frac{1}{(2\pi)^2}\int d^2\vec k\, G_{xy}(\vec k) e^{i\vec k(\vec\rho-\vec\rho')}$

$G_{xy}(\vec k) = -4\pi\frac{k_x k_y}{k^2} f(kL)$, $G_{yx}(\vec\rho\vec\rho') = G_{xy}(\vec\rho\vec\rho')$. $G_{xy}(\vec\rho',\vec\rho) = \frac{1}{(2\pi)^2}\int d^2\vec k\, \times$

$\times G_{xy}(\vec k)\, e^{i\vec k(\vec\rho'-\vec\rho)} = \frac{1}{(2\pi)^2}\int d^2\vec k\, G_{xy}(\vec k)\, e^{-i\vec k(\vec\rho-\vec\rho')} = G^*_{xy}(\vec\rho\vec\rho')$

$G_{xy}(\vec\rho',\vec\rho) = G^*_{xy}(\vec\rho\vec\rho') = G_{xy}(\vec\rho\vec\rho')$, т.к. $G_{xy}(\vec\rho\vec\rho')$ вещ.

Теперь $D_{kk'k''} \sim k_x k_y$ Винт $\pm i/\bar\nu$ надо в $\int dx\,dy\,(\dots)$ одну $\mu_k(\vec\rho)$ ср-но от $\vec\rho$.

Т.е. $\pm i/\bar\nu$ надо брать $W^m_{int} = -M_s^2 L \int d^2\vec\rho\, d^2\vec\rho'\, G_{xy}\, \psi(\vec\rho)\,[\psi^2(\vec\rho') + \varphi^2(\vec\rho')]$.

$\psi(\vec\rho)[\psi^2(\vec\rho') + \varphi^2(\vec\rho')] = \mathrm{Im}\sum_k a_k \mu_k(\vec\rho)\sum_{k'k''} a_{k'} a^*_{k''}\mu_{k'}(\vec\rho')\mu^*_{k''}(\vec\rho') =$

$= \frac{1}{2i}\Big[\sum_k a_k\mu_k(\vec\rho) - \sum_k a^*_k\mu^*_k(\vec\rho)\Big]\sum_{k'k''} a_{k'}a^*_{k''}\mu_{k'}(\vec\rho')\mu^*_{k''}(\vec\rho') =$

$= \frac{1}{2i}\sum_{kk'k''}\Big[a_k a_{k'} a^*_{k''}\mu_k(\vec\rho)\mu_{k'}(\vec\rho')\mu^*_{k''}(\vec\rho') - a^*_k a_{k'} a^*_{k''}\mu^*_k(\vec\rho)\mu_{k'}(\vec\rho')\mu^*_{k''}(\vec\rho')\Big] =$

$= \frac{1}{2i}\sum_{kk'k''}\Big[a_k a_{k'} a^*_{k''}\mu_k(\vec\rho)\mu_{k'}(\vec\rho')\mu^*_{k''}(\vec\rho') - a^*_{k''} a^*_{k'} a_k \mu^*_k(\vec\rho)\mu_{k'}(\vec\rho')\mu_{k''}(\vec\rho')\Big] =$ $\quad k'' \gtrless k'$

$= \frac{1}{2i}[\dots]$, $\quad W^m_{int} = -M_s^2 L\int d^2\vec\rho\, d^2\vec\rho'\, G_{xy}\,\frac{1}{2i}\sum_{kk'k''}[\dots]$

$W^m_{int} = -\frac{1}{2i}M_s^2 L\sum_{kk'k''}\Big[D_{kk'k''}\, a_k a_{k'} a^*_{k''} - D^*_{kk'k''}\, a^*_k a^*_{k'} a_{k''}\Big]$

$D_{kk'k''} = \int d^2\vec\rho\int d^2\vec\rho'\, G_{xy}(\vec\rho\vec\rho')\,\mu_k(\vec\rho)\,\mu_{k'}(\vec\rho')\,\mu^*_{k''}(\vec\rho')$ теперь

$\mu_k(\vec\rho) = \mu_k(y) e^{ik_x x}$, $\quad G_{xy}(\vec\rho\vec\rho') = \frac{1}{(2\pi)^2}(-4\pi)\int d^2\vec p\, e^{i\vec p(\vec\rho-\vec\rho')}\dots$

$\int dx\int dx'\, e^{ip_x(x-x')} e^{ip_y(y-y')} e^{ik_x x} e^{ik'_x x'} e^{-ik''_x x''} = \int_{-\infty}^{\infty} dx\, e^{i(p_x+k_x)x}\int_{-\infty}^{\infty} dx'\dots$

$= (2\pi)^2\,\delta(p_x + k_x)\,\delta(k'_x - p_x - k''_x)$

$D_{kk'k''} = \int_0^\infty dy\,dy'\,\frac{(-4\pi)}{(2\pi)^2}\int d^2\vec p\, e^{ip_y(y-y')}\frac{p_x p_y}{p^2} f(\rho L)\,\mu_k(y)\,\mu_{k'}(y')\,\mu^*_{k''}(\dots)$

$+ (2\pi)^2\,\delta(p_x + k_x)\,\delta(k'_x - p_x - k''_x)$, $\quad p_x = -k_x$ — вошедшая в вектор $\vec k$ (а не $\vec k'$)

$D_{kk'k''} = -4\pi\int dy\,dy'\int dp_y\, e^{ip_y(y-y')}\frac{(-k_x)p_y}{p^2} f(\rho L)\,\mu_k(y)\,\mu_{k'}(y')\,\mu^*_{k''}(y')\,\delta(\dots)$

$D_{kk'k''} = 2\pi\int dy\,dy'\, g_{xy}(yy';k_x)\,\mu_k(y)\,\mu_{k'}(y')\,\mu^*_{k''}(y')\,\delta(k_x + k'_x - k''_x)$ — амплитуда рассеяния

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$$g_{xy}(yy', -k_x) = -2\int_0^\infty dp_y \frac{(-k_x)p_y}{k_x^2+p_y^2} f(pL) e^{ip_y(y-y')}, \quad p^2 = k_x^2 + p_y^2.$$

Дело не в определении $D_{k'xk}$ — исследовал одну $\mu_k(y)$ или две. Надо выбирать представление, где все вычисл. через initial SW

wave vector k

Удобно $k \to \vec{k}$, $k' \to \vec{k}'$, $x \to \vec{x}$ как 6/iv.

Amplitude SW edge-scattering

$$D_{k'xk} = 2\pi \int_0^\infty dy\, dy'\, g_{xy}(yy', -k_x)\, \mu_{k'}^*(y)\, \mu_x(y')\, \mu_k^*(y')\, \delta(k_x + x - k_x')$$

теперь представление $\mu_{k'}(y) = \frac{1}{\sqrt{S}} e^{ik_y y}$, $\mu_k(y) = \frac{1}{\sqrt{S}} e^{-ik_y y}$

$$D_{k'xk} = \frac{2\pi}{S} \int dy\, dy'\, g_{xy}(yy', -k_x)\, e^{ik_y y} \mu_x(y') e^{-ik_y' y'} \delta(\cdot)$$

$$\int_0^\infty dy\, g_{xy}(yy', -k_x) e^{ik_y y} = -2\int_0^\infty dy e^{ik_y y} \int dp_y \frac{(-k_x)p_y}{p^2} f(pL) e^{ip_y(y-y')} =$$

$$= -2\int dp_y \frac{(-k_x)p_y}{p^2} f(pL) e^{-ip_y y'} \int_0^\infty dy\, e^{i(k_y+p_y)y} = -2\int dp_y \frac{(-k_x)p_y}{p^2} f(pL) e^{-ip_y y'} \pi\delta(p_y + k_y) =$$

$\to 2\pi$

$$= -2\frac{k_x k_y}{k^2} f(kL)\pi e^{ik_y y'}; \quad D_{k'xk} = -\frac{(2\pi)^2}{S}\int_0^\infty dy' \mu_x(y') e^{-ik_y' y'} \delta(\cdot) \frac{k_x k_y}{k^2} f(kL) e^{ik_y y'}$$

$$D_{k'xk} = -2\frac{(2\pi)^2}{S}\delta(k_x + x - k_x')\frac{k_x k_y}{k^2} f(kL) \int_0^\infty dy\, \mu_x(y) e^{i(k_y - k_y')y},$$

y-component of $\vec{k}$ pSW сохраняется только приближенно (!).

Вид edge SW $\mu_x(y)$ неизвестен. Из Simul. Pawel $\mu_x(y)$ предполагаем

Тогда $\mu_x(y) \sim e^{-y/\Delta}$ localized near film edge $y=0$ plane.

$N\int_0^\infty dy\, \mu_x^2(y) = 1$ нормировка. $N\int_0^\infty dy e^{-\frac{2}{\Delta}y} = N\int dy\, e^{xy} = N\frac{1}{x} e^{xy}\Big|_{-\infty}^{0} =$

$= N(-\frac{2}{\Delta})^{-1} e^{-\frac{2}{\Delta}y}\Big|_0^\infty = N(-\frac{\Delta}{2})\left[0 - 1\right] = \frac{1}{2}N\Delta = 1, \quad N = \frac{2}{\Delta}$

$\mu_x(y) = \frac{1}{\sqrt{N}} e^{-y/\Delta}$, $N = \Delta/2$. $\int_0^\infty dy\, e^{-y/\Delta} e^{i(k_y - k_y')y} = \Delta \frac{1 + i(k_y - k_y')\Delta}{1 + (k_y - k_y')^2\Delta^2}$

$$D_{k'xk} = -2\frac{(2\pi)^2}{S}\frac{k_x k_y}{k^2} f(kL)\sqrt{\frac{2}{\Delta}}\Delta \frac{1 + i(k_y - k_y')\Delta}{\left[1 + (k_y - k_y')^2\Delta^2\right]} \delta(k_x + x - k_x'), \quad \frac{k_x k_y}{k^2} = \frac{1}{2}\sin 2\varphi_k$$

(amplitude $\vec{k} \to \vec{k}'$, SW confluence (absorption) с edge SW (x) в пл. pSW - plane waves и $\mu_x = \frac{1}{\sqrt{N}} e^{-y/\Delta}$ exp. localized at film edge ($y=0$) plane.

$F(x) = \frac{1+ix}{1+x^2}$, $|F(x)|^2 = \frac{1}{1+x^2}$, $F(k\Delta)$ — uncer max при $k = 0$, ширина $\sim \frac{1}{\Delta}$

расчет $D_{k'xk}$ см 6/iv усложн при неуверенности.

Надо выполнять $\omega_k + \omega_x = \omega_{k'}$ и $k_x + x = k_x'$, $k_y \approx k_y'$ одновременно.

$D_{k'xk}$ может не зависеть от x (eSW), но может зависеть из-за преобразования $\omega_k + \omega_x = \omega_{k'}$. Надо знать ω_k, ω_x.

(9/iv) Т.к. $D_{k'xk}$ удобнее через $\mu_x(y) = \frac{1}{\sqrt{S}} e^{ik_y y}$ для pSW, то используем это и далее ≈ решение ДЗ от 7/iv для linear SW

$$\int_0^\infty dy'\, g_{zz}(yy')\, e_0(y') = \int dy' g_{zz}(yy') e_0 e^{ik_y y'} = te_0 \int dy' e^{ik_y y'} (-2)\int dp_y e^{ip_y(y-y')}(1-f) =$$

$$= -2te_0 \int dp_y e^{ip_y y}(1-f)\int_0^\infty dy' e^{i(k_y - p_y)y'} = -2te_0 \int dp_y e^{ip_y y}(1-f)\pi\delta(k_y - p_y) =$$

$$= -2\pi t e_0 e^{ik_y y}\left[1 - f\left(\sqrt{q^2 + k_y^2}L\right)\right]; \quad \int dy' g_{zz}(y') e_0(y') \approx -2\pi t e_0 e^{ik_y y}\left[1 - f(W)\right]$$

$$\int_0^\infty dy' g_{xx}(yy')\psi(y') = \int_0^\infty dy' g_{xx}(yy')\psi_0 e^{ik_y y'} = \psi_0 \int dy' e^{ik_y y'}(-2)\int dp_y e^{ip_y(y-y')}\frac{q^2}{(q^2+p_y^2)} =$$

$$= -2\psi_0 \int dp_y e^{ip_y y}\frac{q^2}{q^2+p_y^2} f \int_0^\infty dy' e^{i(k_y - p_y)y'} = -2\psi_0 \int dp_y e^{ip_y y}\frac{q^2}{(q^2+p_y^2)} f\, \pi\delta(p_y - k_y)$$

$$= -2\pi\psi_0 e^{ik_y y}\frac{q^2}{(q^2+k_y^2)} f\left(\sqrt{q^2+k_y^2}L\right); \quad \int dy' g_{xx}(yy')\psi(y') \approx -2\pi\psi_0 e^{ik_y y}\frac{q^2}{k^2} f(kL)$$

$\frac{\partial^2}{\partial y^2} e_0(y) = -k_y^2 e_0$, $\frac{\partial^2}{\partial y^2}\psi(y) = -k_y^2\psi$, $q = k_x$. ДЗ 7/iv уточнить

$$\begin{cases} -\frac{i\omega}{\gamma}\psi_0 e^{ik_y y} = \frac{2A}{M_s^2}k^2 \ell_0 e^{ik_y y} - H_y^0 \ell_0 e^{ik_y y} + 2\pi M_s \ell_0 e^{ik_y y}[1-f(kL)] \\ \frac{i\omega}{\gamma}\ell_0 e^{ik_y y} = \frac{2A}{M_s}k^2\psi_0 e^{ik_y y} - H_y^0\psi_0 e^{ik_y y} + 2\pi M_s \psi_0 e^{ik_y y}\frac{k_y^2}{k^2}f(kL) \end{cases}$$

$$-\frac{i\omega}{\gamma}\psi_0 = \left(\frac{2A}{M_s}k^2 - H_y^0 + 2\pi M_s(1-f)\right)\ell_0 \;;\quad \frac{i\omega}{\gamma}\ell_0 = \left(\frac{2A}{M_s}k^2 - H_y^0 + 2\pi M_s\frac{k_y^2}{k^2}f\right)\psi_0$$

$$\frac{\omega^2}{\gamma^2} = \left[\frac{2A}{M_s}k^2 + H - H_y^0 + 2\pi M_s(1-f)\right]\left[\frac{2A}{M_s}k^2 + H - H_y^0 + 2\pi M_s\frac{k_y^2}{k^2}f\right],$$

$f = f(kL)$, $k^2 = k_x^2 + k_y^2$, $H = -H_y > 0$, $H_y^0 > 0$ $\sqrt{-H_y^0(y)}$ y $\vec{M}_0 = -M_s\hat{y}$. $\vec{H} = +H_y\hat{y}$ $H_y < 0$.

$H_y^0(y)$, k_y имеет $H_y^0 \to \langle k_y|H_y^0|k_y\rangle = \frac{1}{L_y}\int_0^\infty dy\, e^{-ik_y y} H_y^0(y) e^{+ik_y y} = \frac{1}{L_y}\int_0^\infty dy\, H_y^0(y) = \langle H_y^0\rangle_y$

$\int_0^\infty dy\, e^{ik_y y} = \pi\delta(k_y)$ - исчезающе малая. Standard $\int_{-\infty}^{\infty} dy\, e^{ik_y} = 2\pi\delta(k)$.

$\int_0^\infty dx\, e^{ikx}e^{-\varepsilon x}$, $\int_{-\infty}^{\infty} dx\, e^{ikx-\varepsilon|x|} = \int_{-\infty}^{\infty} dx\cos kx\, e^{-\varepsilon|x|}$ $\quad 2\int_0^\infty dx\cos kx\, e^{-\varepsilon x} = 2\frac{\varepsilon}{\varepsilon^2+k^2}$

$\lim_{\varepsilon\to 0}\int_{-\infty}^{\infty} dx\, e^{ikx-\varepsilon|x|} = \lim_{\varepsilon\to 0} 2\frac{\varepsilon}{\varepsilon^2+k^2} = 2\pi\lim_{\varepsilon\to 0}\frac{1}{\pi}\frac{\varepsilon}{k^2+\varepsilon^2} = 2\pi\delta(k)$

$\int_0^\infty dx\, e^{ikx-\varepsilon x} = \int_0^\infty dx\cos kx\, e^{-\varepsilon x} + i\int_0^\infty dx\sin kx\, e^{-\varepsilon x} = \frac{\varepsilon}{k^2+\varepsilon^2} + i\frac{k}{k^2+\varepsilon^2}$

$\lim_{\varepsilon\to 0}\int_0^\infty dx\, e^{ikx-\varepsilon x} = \lim_{\varepsilon\to 0}\frac{\varepsilon}{\varepsilon^2+k^2} + \lim_{\varepsilon\to 0} i\frac{k}{k^2+\varepsilon^2} = \pi\delta(k) + i/k$

$\delta(k) = \int_{-\infty}^{\infty} e^{ikx}\frac{dx}{2\pi} = \int_{-\infty}^{\infty}\frac{dx}{2\pi}\cos kx = \frac{1}{\pi}\int_0^\infty dx\cos kx$, поэтому e^{ikx}, $\psi(x)$, $\cos kx$ чтение $\sin kx$

10/IV Переведем Western Union online можно в Киев 300 €.
Ответ Вас: в Shyfrman MPD сфл. доп. т.к. MPD in relative β, $\frac{R_t}{l_{ex}}$ units one gradep constant, Гиперчувствительность A, l_{ex} gaer гиперчувствительность of absolute critical $R_c = \gamma_c \cdot l_{ex}$. Намагничивание edge SW нанокомпозит (Ramel).

11/IV Перпендикулярная и ориентированность pSW.
Мысль Semi-inf. film есть stripe with width w (потом $\frac{w}{L}\to\infty$).

$\int_0^w dy\, e^{ik_y y} = \frac{1}{ik}e^{iky}\Big|_0^w = \frac{1}{ik}\left(e^{ikw}-1\right) = \frac{1}{ik}e^{ikw/2}\left(e^{ikw/2}-e^{-ikw/2}\right) = \frac{2}{k}e^{ikw/2}\sin\frac{kw}{2}$

$\delta(x) = \lim_{\varepsilon\to 0}\frac{1}{\pi}\frac{\varepsilon}{(\varepsilon^2+x^2)}$, $\delta_N(x) = \frac{1}{2\pi}\frac{\sin(N+\frac{1}{2})x}{\sin\frac{x}{2}}$, $\delta(x) = \lim_{N\to\infty}\frac{1}{2\pi}\frac{1}{N}\frac{\sin^2(\frac{Nx}{2})}{\sin^2\frac{x}{2}}$

$\lim_{N\to\infty} x\to 0$, $\delta(x) = \frac{1}{\pi}\frac{\sin Nx}{x}$ $\to$ $\int_{-w/2}^{w/2} dy\, e^{iky} = 2e^{ikw/2}\cdot\pi\delta(k) = 2\pi\delta(k)$.

$\lim_{w\to\infty}\int_{-w/2}^{w/2} e^{iky} = \frac{1}{ik}e^{iky}\Big|_{-w/2}^{w/2} = \frac{e^0}{k}\frac{1}{2i}\left(e^{ikw/2}-e^{-ikw/2}\right) = \frac{2}{k}\sin\frac{kw}{2} = 2\pi\delta(k)$.

w/s $y=0$ в центре stripe, $-\frac{w}{2} y \le \frac{w}{2}$. Аргумент $\delta(k) = \lim_{n\to\infty}\frac{1}{\pi}\frac{\sin nx}{x}$

9/IV $\int_0^\infty dy'\, e^{i(k_y-p_y)y'} = 2\pi\delta(k_y-p_y)$ и pSW dispersion relation

pSW $\frac{1}{\gamma^2}\omega_k^2 = \left[\frac{2A}{M_s}k^2 + H - H_y^0 + 4\pi M_s(1-f)\right]\left[\frac{2A}{M_s}k^2 + H - H_y^0 + 4\pi M_s\frac{k_x^2}{k^2}f\right]$, f

$\exists\, \vec{M}_0 = -M_s\hat{y}$, $\vec{H} = H_y\hat{y}$, $H_y < 0$, $H = -H_y$, $H_y^0 > 0$, $H_y^0(y) \to \langle H_y^0\rangle_y$

$\langle H_y^0\rangle = 4\pi M_s N_{yy}$. Eq. (24) Gus Jmmm 2011, $\langle H_m^d\rangle = M_s G_{dd}(0)$, $N_{dd} = -G_{dd}$

Saturated rect. dot $N_{yy} = -\frac{4p}{\pi}\ln\left(\frac{p}{2}\right)$, $p = \frac{t}{w} \ll 1$, $w = \frac{t}{w} \sim 1$.

В ранней amplitude 8/IV Dietrich надо $\pi \to 2\pi$.

Eq. Kalinikos-Slavin $\omega_n^2 = \left[\omega_H + l\omega_M k^2\right]\left[\omega_H + l\omega_M k^2 + \omega_M F_{nn}\right]$, $P_{00}(kL) = f(kL)$

$F_{nn}(\theta=\pi/2) = P + \left[1 - P(\cos^2\varphi + 1) + \frac{\omega_M P(1-P)}{\omega_H}\sin^2\varphi\right]$, $\omega_n^2 = \omega_H(\omega_H + \omega_M F_{nn})$

$\omega_n^2 = \omega_H^2 + \omega_M\left\{P\omega_H(1+P\cos^2\varphi) + \omega_M(1-P)P\sin^2\varphi\right\}$, $\frac{k_x^2}{k^2} = \cos^2\varphi_k = \sin^2\varphi$, $\varphi = \hat{n}_0\hat{k}$

$\omega_n^2 = \omega_H^2 + \omega_M\omega_H(1-P\cos^2\varphi) + \omega_M^2 P(1-P)\sin^2\varphi = [\omega_H + \omega_M(1-f)][\omega_H + \omega_M f\sin^2\varphi]$

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Т.е., eq. ω_n^2 Kalinikos-Slavin'86 совпадает с eq.(24) Gus'JMMM'11 при $\theta=\pi/2$, $\varphi=\frac{\pi}{2}-\varphi_k$

Нашли Word file "Edge SW Scattering" и послали Ирине, Maciej, Pawel.

Обзор 3-magnon scattering. Есть papers by C. Patton и R. Camley.

Schultheiss, Verba et al. PRL '2019 "whispering gallery magnons in mag. vortex"

3-magnon scattering of radial SW vortex state cyl. dot to azim. SW

$(n_0, 0) \to (n_1, m) + (n_2, -m)$ $a(\rho,t) = \sum_{n,m} a_{n,m}(t)\, g_{nm}(\rho) e^{im\varphi}$

n - radial, m - azim. index. Хотя $g_{nm}(\rho)$ не ортогональны, но правила отбора дают $m_1 = -m_2$, $n_1 \neq n_2$ для $ff^{(3)}$ - 3-magnon dipole interaction.

m" up to 14, n up to 5 — SW azimuthal localized near dot edge и похожи на "whispering gallery" modes. $\omega_{n_0 0} = \omega_{n_1 m} + \omega_{n_2 -m}$

(12/IV) Рассм. FMR exp. и simulations end"-localized modes in long cylindrical nanowires. Будем nanowires ($R=60$ nm, $L \geq 1\mu$m) are in vortex state. Вопросы Маркаду о FMR/Simulations. Все modes are azimuthal ($m=\pm 1$)?

(13/IV) Анализ slides и movies by Farkhad "end" modes. Будем это high-order vortex gyrotropic modes с $k_z \neq 0$. Exch. $\omega_{ex} \sim 1/L^2$, modes are не dipolar.

(14/IV) Анализ exp./simulations by Farkhad of the "nanowire" vortex-end modes. Ирина, JAP - Kim.

(15/IV) E-mail to Fähnle - gyrofield, VC reversal.

Vortex end-modes wires

Ding'Sci. Rep.'14, Gus'Sci. Rep.'15 — High order V. gyromodes in thick dots.

Nanowire length $L \approx 1000$ nm → V. mass may be important. Но Ив. Бычко. repeat ω_ν, $a_\nu(\rho)$ azimuthal ($m=\pm 1$) eigenfrequencies и mode profiles — корректно неизбежно. Без V. mass нельзя $\vec{s}(z) = \vec{X}(z)/R$ v.c. position.

Thiele eq. $\lambda \vec{s}(z) = \left[-\gamma \frac{\partial^2}{\partial z^2} + U\right] \vec{s}(z)$, $U\vec{s}(z) = \int_0^1 dz' \varkappa(zz') \vec{s}(z') - \mu \vec{s}(z)$

Magnetostatic kernel

Две long cylindrical wires $\varkappa(zz') = 8\pi M_s^2 \beta \int_0^\infty dk\, e^{-\beta k|z-z'|} I^2(k)$, $I(k) = \int_0^1 d\rho\rho\, J_1(k\rho)$

$\beta = L/R \gg 1$ ($R=60$ nm, $L \geq 1\mu$m). ρ в ед. R, z in units L.

Можно ли дать U более простое при $\beta \to \infty$?

$K(z) = \int_0^\infty dk\, e^{-\beta k z} I^2(k)$, $z \geq 0$. $\varkappa(zz') = 8\pi M_s^2 \beta K(|z-z'|)$

$K(x) = \int_0^\infty dk\, e^{-xk} I^2(k)$, $x = \beta z$. Но $x \gg 1$ всюду при $\beta \to \infty$ кроме $z=0$.

$\int dz' \varkappa(zz') S(z') \approx |\beta \gg 1| = S(z) \int dz' \varkappa(z'z')$, т.к. $\varkappa(zz')$ имеет max при $z=z'$.

$K(z \to 0) = \int_0^\infty dk\, I^2(k) = \int_0^\infty dk \int_0^1 d\rho\rho\, J_1(k\rho) \int_0^1 d\rho'\rho'\, J_1(k\rho') = \int_0^1 d\rho\rho\, d\rho'\rho' \times$

$\times \int_0^\infty dk\, J_1(k\rho) J_1(k\rho')$. $K(z) = \int_0^\infty dk\, e^{-\beta k z} \int d\rho\rho\, J_1(k\rho) \int d\rho'\rho' J_1(k\rho') = \int d\rho\rho \int d\rho'\rho' \times$

$\times \int_0^\infty dk\, e^{-\beta k z} J_1(k\rho) J_1(k\rho')$ // Cм. J.T. Conway IEEE Tr. Magn. 37, 2977 (2001)

$K(\beta) = \int_0^\infty dk\, e^{-\beta k z} I^2(k) = |kz = t, z\,dk = dt| = \frac{1}{z} \int_0^\infty dt\, e^{-\beta t} I^2(t/z)$

$I(k) = I(t/k) = \int_0^1 d\rho\rho\, J_1(\rho t/z)$, $K(\beta \gg 1) = \frac{1}{\beta} \frac{1}{z} I^2(0) = \frac{0}{\beta z}$ (?).

$K(\beta) = \int_0^\infty dt\, e^{-\beta t} q(t)$; $q(t) = \frac{1}{z} I^2(t/z)$, $I(t/z) = \int_0^1 d\rho\rho\, J_1(\rho t/z)$

При $t \to 0$ $I(t/z) = \int_0^1 d\rho\rho\, c\left(\rho \frac{t}{z}\right) = c\frac{t}{z} \int_0^1 d\rho\rho^2 = \frac{c}{3}\frac{t}{z}$, $q(t) \underset{t\to 0}{\approx} \frac{1}{z}\left(\frac{c}{3}\right)^2 \frac{t^2}{z^2} = at^2$

$K(\beta) = \int_0^\infty e^{-\beta t} q(t) = \left|\frac{\lambda - \mu}{\mu = 1, \lambda = 3}\right| \approx \Gamma(3)\frac{a}{\beta^3} = \Gamma(3)\left(\frac{c}{3}\right)^2 \frac{1}{z^3} \cdot \frac{1}{\beta^3}$ при $\beta \to \infty$.

$K(z) = \int_0^\infty dk\, e^{-\beta k z} I^2(k) \approx \Gamma(3)\left(\frac{c}{3}\right)^2 \frac{1}{(\beta z)^3}$ при $\beta \to \infty$. Но $z \to |z-z'|$ и

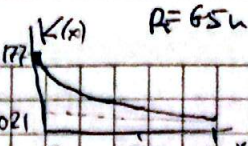
$\varkappa(zz') \sim \frac{1}{\beta|z-z'|^3}$, $S(z)$ из $\int_0^1 dz' \frac{1}{\beta|z-z'|^3} S(z')$ дает $\int dz'(\ldots)$ с особенностью при $z=z'$

$R = 65\,nm$, $L = 1\,\mu m$

$K(x) = \int_0^\infty dk\, e^{-xk} I^2(k)$ Mathcad дал

$x = \beta|z-z'|$, $I(k) = \int_0^1 ds\, J_1(ks)$.

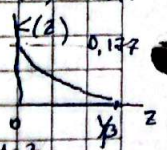
$K(0) = 0.177$ конечно.



$K(x)$ быстро падает с $x = \beta|z-z'|$

Разложим $K(x) \sim \frac{1}{x^3}$ при $x \gg 1$.

Magn. kernel $\chi(z,z') = 8\pi M_s^2 \beta\, K\left[\beta|z-z'|\right]$.



$K(z)$ существенно при $z \le 1/\beta$, $|z-z'| \le 1/\beta$ при $\beta \to \infty$

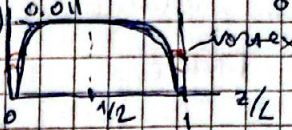
$$\int_0^1 dz'\, K(\beta|z-z'|)\, S(z') = \left|\begin{matrix} -(z-z') = \zeta \\ z' - z = \zeta \end{matrix}\right| = \int_{-z}^{1-z} d\zeta\, K(\beta|\zeta|)\, S(\zeta+z) \approx S(z) \int_{-z}^{1-z} d\zeta\, K(\beta|\zeta|)$$

$$\int_{-z}^{1-z} d\zeta\, K(\beta|\zeta|) = \int_{-z}^{0} d\zeta\, K(\beta|\zeta|) + \int_0^{1-z} d\zeta\, K(\beta|\zeta|) = \int_0^z d\zeta\, K(\beta|\zeta|) + \int_0^{1-z} d\zeta\, K(\beta|\zeta|)$$

$$\int_0^1 dz'\, K(\beta|z-z'|)\, S(z') \approx \left[\int_0^z d\zeta\, K(\beta|\zeta|) + \int_0^{1-z} d\zeta\, K(\beta|\zeta|)\right] S(z) \approx \int_0^1 dz'\, K(\beta|z-z'|)\, S(z) =$$

$\beta \to \infty$

$= F(z) S(z)$, $F(z) = \int_0^1 dz'\, K(\beta|z'-z|)$



узкая GM локализация

$$\lambda \vec{S}(z) = -\gamma \frac{\partial^2 \vec{S}}{\partial z^2} - \mu \vec{S}(z) + \int_0^1 dz'\, \vec{\chi}(z,z')\, \vec{S}(z')$$

$$\int_0^1 \chi\, \vec{S}(z')\, dz' = 8\pi M_s^2 \beta \int_0^1 dz'\, K(\beta|z-z'|)\, \vec{S}(z') \equiv c F(z) \vec{S}(z) \text{ при } \beta \gg 1, \quad c = 8\pi M_s^2 \beta.$$

$$+\gamma \frac{\partial^2 \vec{S}}{\partial z^2} = -(\lambda+\mu)\vec{S}(z) + F(z)\vec{S}(z), \quad \gamma \frac{\partial^2 \vec{S}}{\partial z^2} + \left[\lambda + \mu - 8\pi M_s^2 \beta F(z)\right]\vec{S}(z) = 0$$

$\lambda = \frac{\omega}{\gamma} g = \frac{\omega}{\gamma} \cdot \frac{2\pi M_s}{\gamma} = \frac{2M_s}{\gamma}\omega$, γ и μ – exch. coefficients, see Dirg's Rep'ty.

т.е. задача при $\beta \gg 1$ сводится к 1D Schrödinger equation с потенциалом $F(z)$. $F(z)$ имеет резкие min near $z \approx 0$, $z \approx L$ – nanowire ends, где vortex GM и будет локализован.

В библиотеке: изучен the potential $F(z)$ и зависимость by $\cos(qz)$, где q eq. By Mathieu с res./here определяется vortex localized at rectan. element edges

$F(z) = \int_0^1 dz'\, K(\beta|z-z'|)$ можно упростить при $z \approx 0$ и/или $z \approx 1$.

$$F(z) = \int_0^z dz'\, K\left(\beta(z-z')\right) + \int_z^1 dz'\, K\left(\beta(z'-z)\right) = \int_0^z dz' \int_0^\infty dk\, e^{-k\beta(z-z')} I^2(k) +$$

$$+ \int_z^1 dz' \int_0^\infty dk\, e^{-k\beta(z'-z)} I^2(k) = \int_0^\infty dk\, I^2(k) \left[\int_0^z dz'\, e^{-\beta k(z-z')} + \int_z^1 dz'\, e^{-\beta k(z'-z)}\right] =$$

$$= \int dk\, I^2 \left[e^{-\beta k z} \int_0^z dz'\, e^{\beta k z'} + e^{\beta k z} \int_z^1 dz'\, e^{-\beta k z'}\right] = \int dk\, I^2 \left[e^{-\beta k z} \frac{1}{\beta k} e^{\beta k z'}\Big|_0^z + \right.$$

$$\left. + e^{\beta k z} \frac{(-1)}{\beta k} e^{-\beta k z'}\Big|_z^1\right] = \int \frac{dk}{\beta k} I^2 \left[e^{-\beta k z}\left(e^{\beta k z} - 1\right) - e^{\beta k z}\left(e^{-\beta k} - e^{-\beta k z}\right)\right]$$

$$= \frac{1}{\beta} \int \frac{dk}{k} I^2(k) \left[1 - e^{-\beta k z} - e^{\beta k z} e^{-\beta k} + 1\right] = \frac{2}{\beta} \int_0^\infty \frac{dk}{k} I^2(k) \left[1 - e^{-\beta k/2} \operatorname{ch} \beta k\left(z - \tfrac{1}{2}\right)\right]$$

$$F(z) = \frac{2}{\beta} \int_0^\infty \frac{dk}{k} I^2(k) \left[1 - e^{-\beta k/2} \operatorname{ch} \beta k\left(z - \tfrac{1}{2}\right)\right],$$ ферман. сим. $z = \frac{1}{2}$ см. 25/III

$$\int_0^\infty \frac{dk}{k} I^2(k) = \int_0^1 ds\, s \int_0^1 ds'\, s' \int_0^\infty \frac{dk}{k} J_1(ks) J_1(ks') = \int_0^1 ds\, s \int_0^1 ds'\, s' \int_0^\infty \frac{dx}{x} J_1(x)$$

$$\int_0^\infty \frac{dx}{x} J_{\nu+2n+1}(x) J_{\nu+2n+1}(x) = \frac{\delta_{mn}}{\left[4n+2\nu+2\right]}, \quad \nu > -1, \text{ Гр.-Р.} \quad \int_0^\infty \frac{dx}{x} J_{\cdot}$$

Значение $F(z) = \frac{a_0}{2} + a_1 \cos q\left(z - \frac{1}{2}\right) + \ldots = \sum a_n \cos q_n\left(z - \right.$

при $z' \in \left[-\frac{1}{2}, \frac{1}{2}\right]$, где $q = \frac{1}{(2L)}$, надо $\cos \frac{\pi n z'}{L} = \cos 2\pi n z' =$

$a_n = \left|L = 1/2\right| = 2 \int_{-1/2}^{1/2} dz'\, F(z') \cos\left(2\pi n z'\right)$, $z' = z - 1/2$.

Преобразование Фурье eq. Матье $F(z') = \frac{1}{2} a_0 + a_1 \cos 2\pi z'$ и т.д., a_2, q

$$\int_{-1/2}^{1/2} \operatorname{Cosh}(\beta k z) \cos q_n z\, dz = \frac{1}{\beta} \left[\operatorname{Sh} \beta k z \cdot \cos q_n z\right]_{-1/2}^{1/2} \frac{(\beta k)}{(\beta k)^2 + q_n^2} + \frac{q_n}{(\beta k)^2 + q_n^2} \operatorname{ch} \beta k z \cdot \sin q_n z$$

$$= 2 \frac{\beta k}{(\beta k)^2 + q_n^2} \operatorname{Sh}(\beta k/2) \cos q_n/2 = 2(-1)^n \frac{\beta k}{(\beta k)^2 + q_n^2} \operatorname{Sh} \beta k/2$$

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$F(z) = C_1 + C_2^0(-1)\frac{2}{\beta}\int\frac{dk}{k} I^2 e^{-\beta k/2} ch\beta kz$, $F(z) = \sum_n a_n \cos q_n z$, $q_n = 2\pi n$.

$a_n = 2\int_{-1/2}^{1/2} dz\, F(z)\cos q_n z$, $a_0 = 2\int_{-1/2}^{1/2} dz F(z) = 2\int_{-1/2}^{1/2} dz\left[C_1 - \frac{2}{\beta}\int\frac{dk}{k} I^2 e^{-\frac{\beta k}{2}} ch\beta kz\right] =$

$= 2C_1 - \frac{4}{\beta}\int\frac{dk}{k} I^2 e^{-\beta k/2}\int_{-1/2}^{1/2} dz\, ch(\beta kz) = 2C_1 - \frac{4}{\beta}\int\frac{dk}{k} I^2 e^{-\beta k/2}\frac{1}{\beta k} sh\beta kz\Big|_{-1/2}^{1/2} =$

$= \frac{4}{\beta}\int\frac{dk}{k} I^2 - \frac{8}{\beta^2}\int\frac{dk}{k^2} I^2 e^{-\beta k/2} sh(\beta k/2) = a_0$

$a_n (n\neq 0) = 2\int_{-1/2}^{1/2} dz \cos q_n z\left[C_1 - \frac{2}{\beta}\int\frac{dk}{k} I^2 e^{-\beta k/2} ch\beta kz\right] = -\frac{4}{\beta}\int\frac{dk}{k} I^2 e^{-\beta k/2} \times$

$\times\int_{-1/2}^{1/2} dz \cos q_n z \cdot ch\beta kz = -\frac{4}{\beta} 2(-1)^n \int\frac{dk}{k} I^2 e^{-\beta k/2}\frac{\beta k}{(\beta k)^2 + q_n^2} sh\beta k/2$

$a_0 = \frac{4}{\beta}\int_0^\infty \frac{dk}{k} I^2(k) f(\beta k)$, $a_n = -\frac{8}{\beta}(-1)^n \beta\int dk \frac{1}{(\beta k)^2 + q_n^2} I^2(k) e^{-\beta k/2} sh\beta k/2$,

$a_n = (-1)^{n+1}\cdot 4\cdot\int_0^\infty dk \frac{I^2(k)}{(\beta k)^2 + q_n^2}(1 - e^{-\beta k})$, $q_n = 2\pi n$. $F(z) = \frac{a_0}{2} + \sum_{n=1}^{N} a_n \cos q_n z$, $n = 1,2,\dots$

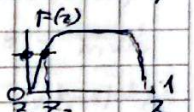
К сожалению для $\beta = 1000\,nm/65\,nm = 15.385$, a_n с ростом "n" убывают медленно и n=0,1 членов недостаточно. При $N = 25\text{-}30$ $F(z)$ отлично воспроизводится, See "Vortex_end_modes_wires.mcd".

Vortex-end modes – wires

(16/IV) Надо представить the potential $F(z)$ в виде $F(z) = a + b\cos qz$ с 3 parameters a, b, q, чтобы заменить Maxwell eq. method as Gub. JPCM'2014

Alternative – использовать Бор-Зоммерфельд eq. $\int_{z_1}^{z_2}\frac{p(z)}{\omega} dz = n + \gamma$,

где ω_n - discret eigenfrequencies. Здесь т-точки поворота $z_1(\omega)$, $z_2(\omega)$ из условия $p(z) \sim \sqrt{\omega - F(z)} = 0$



Note: $\chi(zz')$ и $F(z)$ расчитаны для cylindrical dot/wire в пренебрежении V. core R_c. Для wires $\frac{L}{R} >> 1$, V. core radius $R_0 \sim R$, $R_c(z)$ и пренебрегать им нельзя. Для long wire/thick dot с $R_c(z)$ надо пересчитать. Недавно появилась и важность vortex mass $M_v(L)$ для ферр. wires (Cu, M_v - Oct.-Dec. 2014)

• March 2014: $S(z,\omega) = U(z\omega) e^{i\Phi(z\omega)}$, thick V. dot, 3D для $U(z)$ и $\Phi(z)$, но будем $|S(z)|$ и фазу $\Phi(z)$

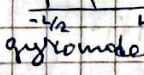
• July 2014: thick V. dot, $S(z) = S_x(z) + i S_y(z) = \sum_{n=-\infty}^{\infty} C_n e^{iq_n z}$, $C_n = \frac{1}{L}\int_{-\frac{L}{2}}^{L/2} dz\, S(z) e^{-iq_n z}$, $q_n = \frac{2\pi n}{L}$

$\lambda = \omega\frac{g}{\mu}$ (μ сокращаем в λ) $\lambda S(z) = -\gamma S'' + \int dz' \chi(zz') S(z') \to \lambda C_n = \sum_{n'} \Lambda_{nn'} C_{n'}$

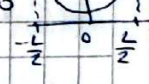
$\Lambda_{nn'} = \gamma q_n^2 \delta_{nn'} + K_{nn'}$, $K_{nn'} = \frac{1}{L}\int dz\int dz' \chi(zz') e^{-iq_n z} e^{iq_{n'} z'}$ – нашел аналитически

Решаем $\hat{\Lambda}\vec{c} = \lambda\vec{c}$ eq. на Fourier-coefficients $\vec{c} = (c_1, c_2, c_3, \dots)$, c_n даст eigenfunctions $S_n(z)$ и eigenfrequencies ω_n. $\omega_0(L)$ оказалась гораздо выше ω_0^{exp}

Это заложено в Nov.-Dec. 2014 версии vortex mass $M_v(L)$.

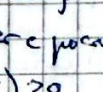
Для thick dot 

($L \leq 100\,nm$) – считаем MathCad. Для wires, $\frac{L}{R} >> 1$, но $\beta < 1$:



– end vortex функции. Дело в ядре $\chi(zz')$ – kernel.

Local magnetostatics: $J(z) = \int dz' \chi(zz') = 2\int_0^\infty \frac{dk}{k} I^2(k)\left[1 - e^{-\beta k/2} ch\beta kz\right]$, $z \in [-\frac{1}{2}, \frac{1}{2}]$



оставляя при $z - \frac{1}{2} \to z$, $\frac{1}{\beta} J(z) = F(z)$, $J(0) - J(\pm 1/2) > 0$ и растет с ростом β

$J(\pm\frac{1}{2}) = 2\int\frac{dk}{k} I^2(k)\left[1 - e^{-\beta k/2}\frac{1}{2}(e^{\beta k/2} + e^{-\beta k/2})\right] = \int_0^\infty \frac{dk}{k} I^2(k)(1 - e^{-\beta k}) > 0$.

Центры $S_0(z)$ и $S_n(z)$ localization at the faces (wire ends) растет βf.

Еще 30/III Mz. использовал $J(z) \approx J(0)\cos pz$ → Mathieu eq. $\frac{d^2S}{dz^2} + [a - 2q\cos 2z]S(z) = 0$

$z = pz/2$, $a = \left(\frac{2}{p}\right)^2\frac{\lambda}{\gamma}$, $q = \frac{1}{2}\left(\frac{2}{p}\right)^2\frac{J(0)}{\gamma}$ → Центр a, $\lambda_n = \frac{\omega_n}{\omega_M}$ с разумными a_n.

→ Вывод: взяв $J(z) = \beta F(z) = J(0)\cos pz$ и подобрав "p" можно заменить аналитич. метод Maxwell. Можно аналитич. Бор-Зоммерфельд или численно фурье-разл. $S(z) = \sum_n C_n e^{iq_n z}$. Надо знать $\chi(zz')$ с учетом $R_c(z)$

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Aug. 2014: $M_z(z) = M(0)(1-\beta^2 z^2)$, $\beta^2 = -\frac{1}{2}\frac{M''(0)}{M(0)} > 0$ maximum $\psi s'' + [\lambda - J_0 + J_0\beta^2 z^2]S(z) = 0$

$y'' + (\frac{1}{4}x^2 - a)y(x) = 0$ eq. parabolic cylinder, harmonic oscillator problem.

Oct. 2014: $x(zz')$ evaluated by two-vortex ansatz - TVA $\vec{m}[\vec{s}, \vec{X}(z,t)]$ hyperbolas

Vortex core $c = R_c/R \to 0$. Do gave $I(k) = \int_0^1 dg\, g\, J_1(kg)$ true core.

$\int_0^1 dg(\dots) = \int_0^c dg(\dots) + \int_c^1 dg(\dots)$. Dance Sci. Rep. 14,15 δ for $\int_c^1 dg(\dots)$ (out core) $\to$ see V.Cr.

Hago vrecho $R_c(z)$, $c(z) = \frac{R_c(z)}{R}$ give nanowires, t.k. $R_c \gtrsim R$ - R_c hence hyperbolas.

$W_m^{vol}(TVA) = W_m^{vol}(\text{in core}) + W_m(\text{out core}, c\neq 0)$ - volume magn. energy TVA-ansatz.

Dve finite R_c, c nago ewe gobavimo face magn. energy W_m^{face} ug za $m_z \neq 0$.

Prve core $W_m^{vol}(\text{out core}) = \frac{1}{2}V\int dz\, dz'\, \chi(zz')\, \vec{S}(z)\vec{S}(z')$, $\chi(zz') = 2\pi M_s^2 \beta \int_0^\infty dk\, e^{-k\beta|z-z'|} I(k,z) I(k,z')$

$I(k,z) = \int_{c(z)}^1 dg\, g J_1(kg)$, $c(z) = \frac{R_c(z)}{R}$ $I(k) = \int_0^1 dg\, g J_1(kg)$.

$W_m^{vol}(\text{in core}) = \frac{1}{2}\int_{core} dV dV' \frac{g(\vec{r})g(\vec{r}')}{|\vec{r}-\vec{r}'|}$, $g(\vec{r}) = -M_s \operatorname{div}\vec{m}(\vec{r})$ - volume charge

$W_m^{vol}(\text{in-core}) = \pi^2 M_s^2 \int dz \int dz' \int dk\, e^{-k|z-z'|}\left[4c(z)c(z')\vec{S}(z)\cdot\vec{S}(z')\Lambda(k,z)\Lambda(k,z') + \right.$

$\left. + \Lambda'(k,z)\Lambda'(k,z')\left(\frac{\partial\vec{S}}{\partial z}\cdot\frac{\partial\vec{S}}{\partial z'}\right)\right]$

$\Lambda(k,z) = \int_0^{c(z)} dg\, g F(g) J_1(kg)$

$\Lambda'(k,z) = \int_0^{c(z)} dg\, g F(g)(1+g^2) J_1(kg)$, $F(g) = \frac{4g}{c^2}\frac{1}{[1+g^2/c^2]^2}$, $c(z) = \frac{R_c(z)}{R}$ Conv. 7/XII 2014.

Чтобы перейти к безразмерным, взяв $[W_m] = erg$. z' in units L, g in units R

$\operatorname{div}\vec{m} = \frac{8g}{c}\frac{(s_x\sin\varphi - s_y\cos\varphi)}{(1+g^2/c^2)^2}\frac{1}{R} + \frac{4}{c^2}\frac{(1+g^2)}{[1+g^2/c^2]^2}\left(\vec{\beta}\cdot\frac{\partial\vec{S}}{\partial z}\right)\frac{1}{L} = \frac{1}{R}\left[\frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y} + \frac{1}{\beta}\frac{\partial m_z}{\partial z}\right]$

$e^{-k|z-z'|} \to |kg = (kR)\frac{g}{R} = (kR)g$, $kz = (kR)\frac{1}{R}Lz = \beta kz$, k in units $1/R$, z in units $|=$

$= e^{-\beta k|z-z'|}$, $\left(\frac{\partial\vec{S}}{\partial z}\cdot\frac{\partial\vec{S}}{\partial z'}\right) \to \frac{1}{\beta^2}\left(\frac{\partial\vec{S}}{\partial z}\cdot\frac{\partial\vec{S}}{\partial z'}\right)$ & $\operatorname{div}\vec{m}'$.

$W_m^{vol}(\text{in-core}) = \pi^2 M_s^2 R^4 L^2 \int dz \int dz' \frac{1}{\beta}\int dk\, e^{-\beta k|z-z'|}\frac{1}{R^2}\left[4cc'\vec{S}\vec{S}(1)\Lambda\Lambda(1) + \frac{1}{\beta^2}\Lambda'\Lambda'(1)\frac{\partial\vec{S}}{\partial z}\cdot\frac{\partial\vec{S}}{\partial z'}\right]$

$\pi^2 R^4 L^2 \frac{1}{R^3} = \pi^2 R L^2 = \pi R^2 L \frac{L}{R}\pi = \pi\beta V$, $V = \pi R^2 L$, $\beta = L/R$.

$W_m^{vol}(\text{in-core}) = \pi M_s^2 V\beta \int dz \int dz' \int dk\, e^{-\beta k|z-z'|}\left[4c(z)c(z')\Lambda(k,z)\Lambda(k,z')\vec{S}(z)\vec{S}(z') + \right.$

$\left. + \frac{1}{\beta^2}\Lambda'(kz)\Lambda(kz')\left(\frac{\partial\vec{S}}{\partial z}\cdot\frac{\partial\vec{S}}{\partial z'}\right)\right]$

Magn. in-core TVA energy, полный

Зависимость $c(z)$ все усложняет, но обе $I(k,z)$, $\Lambda(k,z)$, $\Lambda'(kz)$ вычисляются.

$W_m^{face}(c,s) = M_s^2 R^3 \int_0^\infty dk\left[1 - e^{-\beta k}\right]\sum_{\mu=-\infty}^{\infty}|K_\mu(k)|^2$, $K_\mu(k) = \int d^2\vec{\rho}\, J_\mu(k\rho)e^{i\mu\varphi}$

$c = R_c/R$ здесь на торцах цил. dot/wire, $c = c(z=0,L)$ core $z=0,L$ и $c = c(z=$

W_m^{face} посчитана 17/III 2014 г. $m_z = \frac{1-|\zeta|^2}{1+|\zeta|^2} = \frac{2}{1+|\zeta|^2} - 1$, $\zeta($

$|\zeta(z)|^2 = d\left[1 - 2\varepsilon\zeta\cos\varphi + \varepsilon^2\zeta^2\right]$, $d = \frac{\zeta^2}{c^2 z_0^2}$, $z_0 = \frac{(1+s^2)}{(1-s^2)}$, $\varepsilon = \frac{s}{(1-s^2)} = s +$

$\frac{1}{1+|\zeta|^2} = \frac{1}{1+d}\left[1 + \frac{2\varepsilon\zeta d}{1+d}\cos\varphi - \frac{d}{1+d}\varepsilon^2\zeta^2 + \frac{4\varepsilon^2\zeta^2 d^2}{(1+d)^2}\cos 2\varphi + O(\varepsilon^3)\right]$

При $\int_{core} d^2\vec{\rho}(\dots)$ надо учитывать один вклад с интегрированием при

$W_m = W_m^{vol}(\text{in core}) + W_m^{vol}(\text{out core}) + W_m^{face}$. Усложнение $c(z) = \frac{R_c}{R}$

Dve face energy $\vec{m}(\vec{r},t) = \vec{m}[\vec{s}, \vec{S}(z,t)]$ взято при $z=0,L$ или $z=\pm 1/2$ и не зависит от $\vec{S}(z)$. Будем W_m^{face} брать free energy в Bound Conditions $z=1$

При $c(z) = c = const$

$W_m^{vol}(\text{in-core}) = \pi M_s^2 V\beta \int dz\, dz' e^{-\beta k|z-z'|}\left[4c^2\Lambda^2(k)\vec{S}(z)\vec{S}(z') + \frac{1}{\beta^2}\Lambda'^2(k)\left(\frac{\partial\vec{S}}{\partial z}\cdot\frac{\partial\vec{S}}{\partial z'}\right)\right]dk$

Escaneado con CamScanner

(17/IV) Ruben сделал submission AF-DW to Nature Phys. не нравится. со знаком $\frac{\partial \vec{S}}{\partial z}\cdot\frac{\partial \vec{S}}{\partial z'}$ в W_m^{vol} имеет вид vertical exchange. Насколько он близок к long wires по сравн. с $W_{ex}=\frac{1}{2}gV\int dz\left(\frac{\partial \vec{S}}{\partial z}\right)^2$, где $g=M_s^2\left(\frac{l_e}{L}\right)^2\left[\ln\frac{1}{c}+\frac{5}{4}\right]$, $l_e=\sqrt{2A/M_s^2}$? $W_{ex}(c)$ – wires

$W_m^{vol}=\pi M_s^2\frac{V}{\beta}\int dz\,dz'\,e^{-\beta k|z-z'|}\Lambda'^2(k)\left(\frac{\partial \vec{S}}{\partial z}\cdot\frac{\partial \vec{S}}{\partial z'}\right)^2=\pi M_s^2\frac{V}{\beta}\int dz\,dz' K_v(zz')\times\left(\frac{\partial \vec{S}}{\partial z}\cdot\frac{\partial \vec{S}}{\partial z'}\right)$

W_m^{vol} где $\frac{\partial \vec{S}}{\partial z}$ term. $K_v(zz')=\int dk\,e^{-\beta k|z-z'|}\Lambda'^2(k)$ имеет при $\beta\gg1$ sharp peak at $z=z'$. $\int dz' K_v(zz')\frac{\partial \vec{S}}{\partial z'}(z')=\frac{\partial \vec{S}}{\partial z}(z)\int dz' K_v(zz')$

$W_m^{vol}=\pi M_s^2\frac{V}{\beta}\int dz\,F_v(z)\left(\frac{\partial \vec{S}}{\partial z}\right)^2$ in local approx. $F_v(z)=\int dz' K_v(zz')$

$F_v(z)\int dz'(\dots)$ близко как $F_v(z)\int dz\,(\dots)$ от 16/IV. $I(k)\to\Lambda'(k)$.

$F_v(z)=\frac{2}{\beta}\int\frac{dk}{k}\Lambda'^2(k)\left[1-e^{-\beta k/2}\operatorname{ch}\beta k\left(z-\frac{1}{2}\right)\right]$, $\Lambda'(k)=\int_0^c ds\,s\,F(s)(1+s^2)J_1(ks)$



$F_v(z)$ max при $z=1/2$ и min at $z=0,1$. Оценим max $F_v(z)$.

$W_m^{vol}=\pi M_s^2\frac{V}{\beta}\int dz\,F_v(z)\left(\frac{\partial \vec{S}}{\partial z}\right)^2$ сравн. с $W_{ex}=\frac{1}{2}gV\int dz\left(\frac{\partial \vec{S}}{\partial z}\right)^2$

$\pi M_s^2\frac{1}{\beta}F_v(z)$ сравн. с $\frac{1}{2}g$. Max $\frac{\pi M_s^2}{\beta}F(z)=\frac{\pi M_s^2}{\beta}\frac{2}{\beta}\int\frac{dk}{k}\Lambda'^2(k)\left[1-e^{-\beta k/2}\right]$

$\frac{2\pi M_s^2}{\beta^2}\int\frac{dk}{k}\Lambda'^2(k)\left(1-e^{-\beta k/2}\right)$ сравн. с $\frac{1}{2}g=\frac{1}{2}M_s^2\left(\frac{l_e}{L}\right)^2\left(\ln\frac{1}{c}+\frac{5}{4}\right)=\frac{M_s^2}{2}\frac{1}{\beta^2}\left(\frac{l_e}{R}\right)^2\left(\ln\frac{1}{c}+\frac{5}{4}\right)$

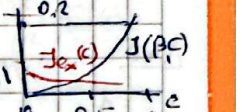
$l_e^2/L^2=\frac{1}{\beta^2}(l_e/R)^2$. $\Lambda'(k,c)=\int_0^c ds\,s\,F(s)(1+s^2)J_1(ks)$

$J(\beta,c)=\int_0^\infty\frac{dk}{k}\Lambda'^2(k,c)\left(1-e^{-\beta k/2}\right)$ сравн. с $\frac{1}{4\pi}\left(\frac{l_e}{R}\right)^2\left(\ln\frac{1}{c}+\frac{5}{4}\right)=J_{ex}(c)$

typical $l_e=20$ nm, $c=0.5$, $R=65$ nm $\frac{1}{4\pi}\left(\frac{20}{65}\right)^2\left(\ln 2+\frac{5}{4}\right)=0.015$.

При $c>0.7$ $J(\beta,c)\gg J_{ex}(c)$, но при $c=0.3\div0.6$ $J(\beta,c)\approx J_{ex}(c)$ сравнимы

$J(\beta,c)$ не зависит от β при $\beta>10$.



Note: $W_{ex}=\frac{1}{2}gV\int dz\left(\frac{\partial \vec{S}}{\partial z}\right)^2$ также посчитана для v. core, при $c\to0$

$W_{ex}=A(\nabla\vec{m})^2$, $(\nabla\vec{m})^2=(\nabla m_x)^2+(\nabla m_y)^2+(\nabla m_z)^2=\left(\frac{\partial\vec{m}}{\partial z}\right)^2+\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)^2$

ФКА: $f(\zeta)=-i\frac{c}{c}\frac{(\zeta-s)(\bar{s}\zeta-1)}{(1+s\bar{s})}$, $\zeta=x+iy\to\zeta$. In-core $w=f(\zeta)$, $|f|\le1$, out-core $w=\frac{2f}{1+|f|^2}\to\frac{f}{|f|}$

$m_x+im_y=m_+=\frac{2w}{1+w\bar{w}}$, $m_z=\frac{1-w\bar{w}}{1+w\bar{w}}$.

$s=s_x+is_y=s(z)$. $m_x=\operatorname{Re}\frac{2w}{1+w\bar{w}}=\frac{2}{1+w\bar{w}}\operatorname{Re}w$, $m_y=\frac{2}{1+w\bar{w}}\operatorname{Im}w$

$\frac{\partial w}{\partial z}=\left|\text{in-core, } w=f(\zeta)\right|=\frac{\partial}{\partial z}\frac{(-i)}{c}\frac{1}{(1+s\bar{s})}(\zeta-s)(\bar{s}\zeta-1)=\frac{(-i)}{c}\left[(\zeta-s)(\bar{s}\zeta-1)\times\right.$

$\left.\times\frac{(-1)}{(1+s\bar{s})^2}\frac{\partial}{\partial z}(s\bar{s})+\frac{1}{(1+s\bar{s})}(\bar{s}\zeta-1)(-1)\frac{\partial s}{\partial z}+\frac{(\zeta-s)}{(1+s\bar{s})}\zeta\frac{\partial\bar{s}}{\partial z}\right]=$ (с form. go s^2)$=$

$=-\frac{i}{c}\left[+\zeta\frac{\partial}{\partial z}(s\bar{s})-(\bar{s}\zeta-1)\frac{\partial s}{\partial z}+(\zeta-s)\zeta\frac{\partial\bar{s}}{\partial z}\right]=-\frac{i}{c}\left[\zeta\frac{\partial s}{\partial z}\bar{s}+\zeta s\frac{\partial\bar{s}}{\partial z}-(\bar{s}\zeta-1)\frac{\partial s}{\partial z}+\right.$

$\left.+(\zeta^2-s\zeta)\frac{\partial\bar{s}}{\partial z}\right]=-\frac{i}{c}\left[\frac{\partial s}{\partial z}+\zeta^2\frac{\partial\bar{s}}{\partial z}\right]$, $\frac{\partial w}{\partial z}=-\frac{i}{c}\left(\frac{\partial s}{\partial z}+\zeta^2\frac{\partial\bar{s}}{\partial z}\right)$ – in-core

$\frac{\partial m_x}{\partial z}=\frac{\partial}{\partial z}\frac{2}{1+w\bar{w}}\operatorname{Re}w=\frac{2(-1)}{(1+w\bar{w})^2}\frac{\partial}{\partial z}(w\bar{w})\operatorname{Re}w+\frac{2}{1+w\bar{w}}\operatorname{Re}\frac{\partial w}{\partial z}$

$\frac{\partial m_+}{\partial z}=\frac{\partial}{\partial z}\frac{2w}{1+w\bar{w}}=\frac{(-2)}{(1+w\bar{w})^2}\frac{\partial}{\partial z}(w\bar{w})\cdot w+\frac{2w^0}{1+w\bar{w}}\frac{\partial w}{\partial z}=\frac{2w^0}{(1+w\bar{w})^2}\times$

$\times\left[-\frac{\partial}{\partial z}(w\bar{w})\cdot w+(1+w\bar{w})\frac{\partial w}{\partial z}\right]=\frac{2}{(1+w\bar{w})^2}\left[-w\left(\frac{\partial w}{\partial z}\bar{w}+w\frac{\partial\bar{w}}{\partial z}\right)+(1+w\bar{w})\frac{\partial w}{\partial z}\right]=$

$=\frac{2}{(1+w\bar{w})^2}\left[\frac{\partial w}{\partial z}-w^2\frac{\partial\bar{w}}{\partial z}\right]$, $\frac{\partial\bar{w}}{\partial z}=\frac{i}{c}\left(\frac{\partial s}{\partial z}+\bar{\zeta}^2\frac{\partial\bar{s}}{\partial z}\right)$ – in core.

$w(\zeta)=f(\zeta)=-\frac{i}{c}\frac{1}{(1+s\bar{s})}(\zeta-s)(\bar{s}\zeta-1)$ – in core.

In-core $w\bar{w}=\frac{1}{c^2}\frac{1}{(1+s\bar{s})^2}|\zeta-s|^2|\bar{s}\zeta-1|^2=|w|^2=|f|^2$.

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$\frac{\partial m_+}{\partial z} = \frac{2}{(1+w\bar{w})^2}\left[\frac{\partial w}{\partial z} - w^2\frac{\partial \bar{w}}{\partial z}\right]$, In-core $\frac{\partial m_+}{\partial z} = \frac{2}{(1+w\bar{w})^2}\left[\frac{i}{c}\left(\frac{\partial s}{\partial z} + \zeta^2\frac{\partial \bar{s}}{\partial z}\right) - w^2\frac{i}{c}\left(\frac{\partial \bar{s}}{\partial z} + \bar{\zeta}^2\frac{\partial s}{\partial z}\right)\right]$. $\frac{\partial m_x}{\partial z} = \mathrm{Re}\frac{\partial m_+}{\partial z}$, $\frac{\partial m_y}{\partial z} = \mathrm{Im}\frac{\partial m_+}{\partial z}$

$\left(\frac{\partial m_x}{\partial z}\right)^2 = \left(\mathrm{Re}\frac{\partial m_+}{\partial z}\right)^2$. Видно, что с точностью до s^2, надобно

In core $w = \zeta(\zeta)$, $w(s=0) = \zeta(\zeta, s=0) = \frac{i}{c}\zeta$. $z\bar{z} = |(x+iy)|^2 = x^2+y^2 = s^2 = \zeta\bar{\zeta}^2$ In-core $w(s=0)$, $\bar{w}(\bar{s}=0)$

In-core $\frac{\partial m_+}{\partial z} = \frac{2}{\left[1+s^2/c^2\right]^2}\left[-\frac{i}{c}\left(\frac{\partial s}{\partial z} + \zeta^2\frac{\partial \bar{s}}{\partial z}\right) - \left(+\frac{\zeta^2}{c^2}\right)\frac{i}{c}\left(\frac{\partial \bar{s}}{\partial z} + \bar{\zeta}^2\frac{\partial s}{\partial z}\right)\right] =$

$= \left(-\frac{i}{c}\right)\frac{2}{(1+s^2/c^2)^2}\left[\frac{\partial s}{\partial z} + \zeta^2\frac{\partial \bar{s}}{\partial z} - \frac{\zeta^2}{c^2}\left(\frac{\partial \bar{s}}{\partial z} + \bar{\zeta}^2\frac{\partial s}{\partial z}\right)\right] =$

$= -\frac{i}{c}\frac{2}{(1+s^2/c^2)^2}\left[\frac{\partial s}{\partial z}\left(1 - s^4/c^2\right) + \left(1-\frac{1}{c^2}\right)\zeta^2\frac{\partial \bar{s}}{\partial z}\right]$?- справимся. Проверим.

$W_{ex} = A\int d^2\zeta\, dz\, (\nabla\vec{m})^2 = A\int d^2\zeta\, dz\, (\nabla_\zeta\vec{m})^2 + A\int d^2\zeta\, dz\left(\frac{\partial \vec{m}}{\partial z}\right)^2$ → даст $\vec{s}^2(z)$ → даст $\left(\frac{\partial \vec{s}}{\partial z}\right)^2$

$\int d^2\zeta\left(\nabla_\zeta\vec{m}\right)^2$, где $s(z)$, можно посчитать по 1-му приближению теории возмущений $w(\zeta)$, $\zeta = x+iy$.

(18/iv) Метод Ньютона: $f(z) = \frac{1}{c}\left[iz + \frac{1}{2}(a-\bar{a}z^2)\right]$ даст $\frac{E_{ex}}{M_s^2 V} = \left(\frac{L_e}{R}\right)^2\left[2 - \ln\frac{2c}{1+\sqrt{1-a^2}}\right]$

$W_{ex}(\nabla_\zeta\vec{m}) = M_s^2 S\left(\frac{L_e}{R}\right)^2\int dz\left[2 - \ln\frac{2c}{1+\sqrt{1-a^2}}\right]$, $V = SL \to \int dz$, $a \to |a|$.

$f(\zeta) = -\frac{i}{c}\frac{1}{1+s\bar{s}}(\zeta - s)(\bar{s}\zeta - 1)$ и $f(\zeta) = \frac{1}{c}\left[i\zeta + \frac{1}{2}(a-\bar{a}\zeta^2)\right]$ → сверка ans.

$(\zeta - s)(\bar{s}\zeta - 1) = \bar{s}\zeta^2 - \zeta - s\bar{s}\zeta + s = -\zeta(1+s\bar{s}) + s + \bar{s}\zeta^2$

$f(\zeta) = -\frac{i}{c}\frac{1}{1+s\bar{s}}\left[-\zeta(1+s\bar{s}) + s + \bar{s}\zeta^2\right] = \frac{1}{c}\left[i\zeta - i\frac{s+\bar{s}\zeta^2}{1+s\bar{s}}\right] = \frac{1}{c}\left[i\zeta + \frac{1}{2}(a-\bar{a}\zeta^2)\right]$

$\frac{a}{2} = -i\frac{s}{1+s\bar{s}}$, $-\frac{\bar{a}}{2} = -i\frac{\bar{s}}{1+s\bar{s}}$, $a(s) = 2i\frac{s}{(1+s\bar{s})}$, $s = s(z)$.

Note: при $|\zeta|<1$, $|s|<1$ путь отображения $W(a)$ будет $a \to s$. $f(\zeta, a)$ меняется по a.

$\frac{1}{1+\sqrt{1-a^2}} = \frac{1}{1+(1-\frac{1}{2}a^2)} = \frac{1}{2}\frac{1}{1-\frac{1}{4}a^2} = \frac{1}{2}\left(1+\frac{1}{4}a^2\right) + O(a^4)$. $a \to |a|$ будет

$W_{ex}(\nabla_\zeta\vec{m}) = M_s^2\pi R^2\left(\frac{L_e}{R}\right)^2\int dz\left[2 - \ln\left\{2c\,\frac{1}{2}\left(1+\frac{1}{4}|a|^2\right)\right\}\right] = (\ldots)\int dz\left[1 - \ln c - \ln\left(1+\frac{1}{4}|a|^2\right)\right]$

$W_{ex}(\nabla_\zeta\vec{m}) = \pi R^2 M_s^2\left(\frac{L_e}{R}\right)^2\int dz\left[2 - \ln c - \frac{1}{4}|a|^2\right]$, $|a|^2 = a\bar{a} = 4\frac{|s|^2}{(1+s\bar{s})^2} \approx 4|s|^2 + O(s^4)$

$W_{ex}(\nabla_\zeta\vec{m}) = \pi R^2 M_s^2\left(\frac{L_e}{R}\right)^2\int dz\left[2 - \ln c - |s|^2\right] = W_{ex}(s=0) - \pi R^2 M_s^2\left(\frac{L_e}{R}\right)^2\int dz\,\vec{s}^2(z)$

$W_{ex}(\nabla_\zeta\vec{m}) = W_{ex}(s=0) - M_s^2 V\left(\frac{L_e}{R}\right)^2\int dz\,\vec{s}^2(z) = (s=0) - \frac{\mu}{2}V\int dz\,\vec{s}^2(z)$, $\mu = 2M_s^2(L_e/R)^2$ –

– можно также μ называть в Ding, Sai, Rep. 2014. $\left(W_{ex}(\zeta_\zeta\vec{m}) = -\frac{1}{2}\mu V\int dz\,\vec{s}^2(z)\right)$

Нашли $\int d^2\zeta\, dz\left(\frac{\partial \vec{m}}{\partial z}\right)^2$, т.е. $\left(\frac{\partial \vec{s}}{\partial z}\right)^2$ → нам в $W_{ex}[s(z)]$.

$w(\zeta\zeta) = \left\{\begin{matrix} f(\zeta), & |\zeta|\le 1, \text{ core} \\ s(\zeta)/|\zeta|, & |\zeta|>1 \end{matrix}\right\}$, $f(\zeta) = \frac{1}{c}\left[i\zeta + \frac{1}{2}(a-\bar{a}\zeta^2)\right]$, $\zeta = x+iy$, $m_+ = \frac{2w}{1+w\bar{w}}$, $m_2 = \frac{1-w\bar{w}}{1+w\bar{w}}$

Core: $\frac{\partial w}{\partial z} = \frac{\partial}{\partial z}f(\zeta) = \frac{\partial}{\partial z}\frac{1}{c}\left[i\zeta + \frac{1}{2}(a-\bar{a}\zeta^2)\right] = \frac{1}{2c}\left[\frac{\partial a}{\partial z} - \frac{\partial \bar{a}}{\partial z}\zeta^2\right]$. Вне core – сложнее.

$\frac{1}{2}\frac{\partial m_+}{\partial z} = \frac{\partial}{\partial z}\frac{w}{(1+w\bar{w})} = -\frac{w}{(1+w\bar{w})^2}\frac{\partial}{\partial z}(w\bar{w}) + \frac{1}{(1+w\bar{w})}\frac{\partial w}{\partial z} = \frac{1}{(1+w\bar{w})^2}\left[\frac{\partial w}{\partial z}(1+w\bar{w}) - w\left(\frac{\partial w}{\partial z}\bar{w} + w\frac{\partial \bar{w}}{\partial z}\right)\right] = \frac{1}{(1+w\bar{w})^2}\left[\frac{\partial w}{\partial z} - w^2\frac{\partial \bar{w}}{\partial z}\right]$.

$\left(\frac{\partial m_+}{\partial z}\right)^2 = \left(\frac{\partial m_x}{\partial z} + i\frac{\partial m_y}{\partial z}\right)^2 = \left(\frac{\partial m_x}{\partial z}\right)^2 - \left(\frac{\partial m_y}{\partial z}\right)^2 + 2i\frac{\partial m_x}{\partial z}\frac{\partial m_y}{\partial z}$ – много.

$\left|\frac{\partial m_+}{\partial z}\right|^2 = \left(\frac{\partial m_x}{\partial z}\right)^2 + \left(\frac{\partial m_y}{\partial z}\right)^2$. $W_{ex} = \left(\frac{\partial \vec{m}}{\partial z}\right)^2 = \left|\frac{\partial m_+}{\partial z}\right|^2 + \left(\frac{\partial m_z}{\partial z}\right)^2$

$\frac{\partial m_z}{\partial z} = \frac{\partial}{\partial z}\left(\frac{1-w\bar{w}}{1+w\bar{w}}\right) = \frac{\partial}{\partial z}\frac{1-m}{1+m} = \left|m = w\bar{w}\right| = \frac{\partial}{\partial z}f(m) = \frac{\partial f}{\partial m}\frac{\partial m}{\partial z} = \frac{\partial}{\partial m}\frac{(1-m)}{(1+m)}\frac{\partial m}{\partial z} =$

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$= \frac{(-1)(1+w)-(1-w)\cdot 1}{(1+w)^2}\, w_z' = \frac{(-2)}{(1+w\bar w)^2}\frac{\partial}{\partial z}(w\bar w) = \frac{(-2)}{(1+w\bar w)^2}\left[\frac{\partial w}{\partial z}\bar w + w\frac{\partial \bar w}{\partial z}\right]$

$\left(\frac{\partial \vec m}{\partial z}\right)^2 = \left|\frac{\partial m_+}{\partial z}\right|^2 + \left(\frac{\partial m_z}{\partial z}\right)^2 = \left|\frac{2}{(1+w\bar w)^2}\left(\frac{\partial w}{\partial z} - w^2\frac{\partial \bar w}{\partial z}\right)\right|^2 + \frac{4}{(1+w\bar w)^4}\left[\frac{\partial w}{\partial z}\bar w + w\frac{\partial \bar w}{\partial z}\right]^2 =$

$= \frac{4}{(1+w\bar w)^4}\left[\left(\frac{\partial w}{\partial z} - w^2\frac{\partial \bar w}{\partial z}\right)\left(\frac{\partial \bar w}{\partial \bar z} - \bar w^2\frac{\partial w}{\partial \bar z}\right) + \left(\frac{\partial w}{\partial z}\bar w + w\frac{\partial \bar w}{\partial z}\right)^2\right] =$

$= \frac{4}{(\cdot\cdot)^4}\left[\frac{\partial w}{\partial z}\frac{\partial \bar w}{\partial \bar z} - \bar w^2\left(\frac{\partial w}{\partial z}\right)^2 - w^2\left(\frac{\partial \bar w}{\partial z}\right)^2 + w^2\bar w^2\frac{\partial w}{\partial z}\frac{\partial \bar w}{\partial \bar z} + \left(\frac{\partial w}{\partial z}\right)^2\bar w^2 + w^2\left(\frac{\partial \bar w}{\partial z}\right)^2 + 2w\bar w\frac{\partial w}{\partial z}\frac{\partial \bar w}{\partial z}\right] =$

$= \frac{4}{(\cdot\cdot\cdot)^4}\frac{\partial w}{\partial z}\frac{\partial \bar w}{\partial \bar z}\left(1 + w^2\bar w^2 + 2w\bar w\right) = \frac{4}{(1+w\bar w)^2}\frac{\partial w}{\partial z}\frac{\partial \bar w}{\partial \bar z} \Rightarrow \boxed{\left(\frac{\partial \vec m}{\partial z}\right)^2 = \frac{4}{(1+w\bar w)^2}\left|\frac{\partial w}{\partial z}\right|^2}$

In core: $\left|\frac{\partial w}{\partial z}\right|^2 = \frac{\partial w}{\partial z}\frac{\partial \bar w}{\partial \bar z} = \frac{1}{2c}\left[\frac{\partial a}{\partial z} - \bar z^2\frac{\partial \bar a}{\partial z}\right]\frac{1}{2c}\left[\frac{\partial \bar a}{\partial \bar z} - z^2\frac{\partial a}{\partial \bar z}\right] = \frac{1}{4c^2}\left[\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial \bar z} - \bar z^2\left(\frac{\partial a}{\partial z}\right)^2 - z^2\left(\frac{\partial \bar a}{\partial \bar z}\right)^2 + z^2\bar z^2\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial \bar z}\right]$.

$\left|\frac{\partial w}{\partial z}\right|^2 \sim a^2\cdot\frac{4}{(1+w\bar w)^2}\Big|_{a=0}$ – теорем.

In core $w\bar w = |f|^2_{a=0} = |z|^2/c^2 = \frac{s^2}{c^2}$. Out core $w\bar w = \frac{f}{|f|}\frac{\bar f}{|f|} = 1$.

$a = -2i\frac{s}{1+3s} \approx -2is + O(s^3)$. $\frac{\partial a}{\partial z} \to -2i\frac{\partial s}{\partial z}$, $\frac{\partial \bar a}{\partial \bar z} \to +2i\frac{\partial \bar s}{\partial \bar z}$.

$\left|\frac{\partial w}{\partial z}\right|^2 = \frac{1}{4c^2}\left[4\cdot\frac{\partial s}{\partial z}\frac{\partial \bar s}{\partial \bar z} - \bar z^2 4(-1)\left(\frac{\partial s}{\partial z}\right)^2 - z^2 4(-1)\left(\frac{\partial \bar s}{\partial \bar z}\right)^2 + z^2\bar z^2 4\frac{\partial s}{\partial z}\frac{\partial \bar s}{\partial \bar z}\right] =$

$= \frac{1}{c^2}\left[\left(1 + z^2\bar z^2\right)\frac{\partial s}{\partial z}\frac{\partial \bar s}{\partial \bar z} + \bar z^2\left(\frac{\partial s}{\partial z}\right)^2 + z^2\left(\frac{\partial \bar s}{\partial \bar z}\right)^2\right]$, $z\bar z = |z|^2 = s^2$, $z = se^{i\varphi}$

In core $\left(\frac{\partial \vec m}{\partial z}\right)^2 = \frac{4}{(1+s^2/c^2)^2}\frac{1}{c^2}\left[\left(1+s^4\right)\frac{\partial s}{\partial z}\frac{\partial \bar s}{\partial \bar z} + s^2e^{-i2\varphi}\left(\frac{\partial s}{\partial z}\right)^2 + s^2e^{i2\varphi}\left(\frac{\partial \bar s}{\partial \bar z}\right)^2\right]$

$W_{ex}\left(\frac{\partial \vec m}{\partial z}, \text{in core}\right) = A\int d\sigma\vec{\ }\int dz\left(\frac{\partial \vec m}{\partial z}\right)^2 = \left|\int d\sigma\vec{\ } = \int ds\, s\int_0^{2\pi}d\varphi,\ \int d\varphi\, e^{in\varphi} = 2\pi\delta_{n0}\right| =$

$= A\int ds\, s\, 2\pi\int dz\,\frac{4}{c^2}\frac{\left(1+s^4\right)}{(1+s^2/c^2)^2}\frac{\partial s}{\partial z}\frac{\partial \bar s}{\partial \bar z} = 2\pi A R^2 L\int_0^c ds\, s\int_0^1 dz\,\frac{4}{c^2}\frac{(1+s^4)}{(1+s^2/c^2)^2}\frac{1}{L^2}\frac{\partial s}{\partial z}\frac{\partial \bar s}{\partial \bar z}$

$2\pi A R^2 L\frac{1}{L^2} = (\pi R^2 L)\,2A\,\frac{1}{L^2} = V\cdot\frac{2A}{M_s^2}\frac{1}{L^2}M_s^2 = V\left(\frac{Le}{L}\right)^2 M_s^2$, $Le^2 = 2A/M_s^2$.

$W_{ex}(\text{in core}) = M_s^2 V\left(\frac{Le}{L}\right)^2\int_0^c ds\, s\int_0^1 dz\,\frac{4}{c^2}\frac{(1+s^4)}{(1+s^2/c^2)^2}\frac{\partial s}{\partial z}\frac{\partial \bar s}{\partial \bar z}$, $J(c) = \int_0^c ds\, s\,\frac{4}{c^2}\frac{(1+s^4)}{(1+s^2/c^2)^2}$

$W_{ex}(\text{in core}) = M_s^2 V\left(\frac{Le}{L}\right)^2 J(c)\int dz\left(\frac{\partial \vec s}{\partial z}\right)^2$, $\frac{\partial s}{\partial z}\frac{\partial \bar s}{\partial \bar z} = \left|\frac{\partial s}{\partial z}\right| = \left(\frac{\partial \vec s}{\partial z}\right)^2$

$J(c) = \frac{4}{c^2}\int_0^c ds\, s\,\frac{(1+s^4)}{(1+s^2/c^2)^2} = \left|s^2/c^2 = x\right| = 2\int_0^1 dx\,\frac{(1+c^4x^2)}{(1+x)^2} = 2\int_0^1\frac{dx}{(1+x)^2} + 2c^4\int_0^1 dx\,\frac{x^2}{(1+x)^2} =$

$= 2\frac{(-1)}{(1+x)}\Big|_0^1 + c^4\cdot 2\left[1+x - 2\ln(1+x) - \frac{1}{1+x}\right]\Big|_0^1 = (-2)\left[\frac{1}{2} - 1\right] + 2c^4\left[2 - 2\ln 2 - \frac{1}{2}\right] =$

$= 1 + 2c^4\left(\frac{3}{2} - 2\ln 2\right) = 1 + c^4(3 - 4\ln 2)$.

Out core: $\frac{\partial w}{\partial z} = \frac{\partial}{\partial z}\frac{f}{|f|} = \frac{\partial}{\partial z}\frac{f}{\sqrt{f\bar f}} = \frac{\partial}{\partial z}\sqrt{\frac{f}{\bar f}} = \frac{1}{2\sqrt{\frac{f}{\bar f}}}\frac{\partial}{\partial z}\frac{f}{\bar f} = 2\sqrt{\frac{\bar f}{f}}\frac{f'\bar f - f\bar f'}{(\bar f)^2} =$

$= \frac{1}{2}\frac{1}{\sqrt{f\bar f}}\frac{1}{\bar f}\left(f'\bar f - f\bar f'\right)$, $f' = \frac{\partial}{\partial z}f = \frac{1}{2c}\left(\frac{\partial a}{\partial z} - z^2\frac{\partial \bar a}{\partial \bar z}\right) \sim a$

$\left|\frac{\partial w}{\partial z}\right|^2 = \frac{1}{4}\frac{1}{f\bar f}\left|\frac{1}{\bar f}\left(f'\bar f - f\bar f'\right)\right|^2 = \left|\text{т.к. } f \sim a,\ \bar f \text{ и } a,\ \text{то } f, \bar f \text{ имеют } a^2,\ \text{поэтому } f(z) = cz/c \text{ имеет } a = 0\right| =$

$= \frac{1}{4}\frac{1}{(z\bar z/c^2)}\left|\frac{1}{(-i\bar z/c)}\left(\frac{\partial f}{\partial z}\cdot(-i\bar z/c) - i\frac{z}{c}\frac{\partial \bar f}{\partial \bar z}\right)\right|^2 = \frac{1}{4}\frac{c^2}{s^2}\left|\frac{\partial f}{\partial z} + \frac{\partial \bar f}{\partial \bar z}\right|^2$

$\frac{\partial f}{\partial z} + \frac{\partial \bar f}{\partial \bar z} = \frac{1}{2c}\left[\frac{\partial a}{\partial z} - z^2\frac{\partial \bar a}{\partial z} + \frac{\partial \bar a}{\partial \bar z} - \bar z^2\frac{\partial a}{\partial \bar z}\right] = \frac{1}{2c}\left[\left(1-\bar z^2\right)\frac{\partial a}{\partial z} + \left(1-z^2\right)\frac{\partial \bar a}{\partial \bar z}\right]$

$\left|\frac{\partial w}{\partial z}\right|^2 = \frac{1}{4}\frac{c^2}{s^2}\frac{1}{4c^2}\left[\left(1-\bar z^2\right)\frac{\partial a}{\partial z} + \left(1-z^2\right)\frac{\partial \bar a}{\partial \bar z}\right]\left[\left(1-z^2\right)\frac{\partial \bar a}{\partial \bar z} + \left(1-\bar z^2\right)\frac{\partial a}{\partial z}\right] =$

$= \frac{1}{16s^2}\left[z\left(1-z^2\right)\left(1-\bar z^2\right)\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial \bar z} + \left(1-\bar z^2\right)^2\left(\frac{\partial a}{\partial z}\right)^2 + \left(1-z^2\right)^2\left(\frac{\partial \bar a}{\partial \bar z}\right)^2\right]$. Out core $w\bar w = 1$.

$\left(\frac{\partial \vec m}{\partial z}\right)^2 = 4\left|\frac{\partial w}{\partial z}\right|^2$. $z = se^{i\varphi}$, при $\int_0^{2\pi}d\varphi$ все члены $\sim z^2, \bar z^2$ зануляются

$\left|\frac{\partial w}{\partial z}\right|^2 = \frac{1}{16s^2}\left[2\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial \bar z}(\cdot) + \left(\frac{\partial a}{\partial z}\right)^2 + \left(\frac{\partial \bar a}{\partial \bar z}\right)^2\right]$, $a = -2is + O(s^3)$, $(\cdot) = z^{1+}\bar z^{1+} = s^4 + 1$

Escaneado con CamScanner

Out core $\left|\frac{\partial w}{\partial z}\right|^2 = \frac{1}{4\rho^2}\left[2(1+\rho^4)\frac{\partial s}{\partial z}\frac{\partial \bar{s}}{\partial z} - \left(\frac{\partial s}{\partial z}\right)^2 - \left(\frac{\partial \bar{s}}{\partial z}\right)^2\right]$ — нужно вернуться. [где ошибка?]

$\frac{\partial w}{\partial z} = \frac{\partial}{\partial z}\frac{f}{|f|} = \frac{\partial}{\partial z}\frac{f}{\sqrt{f\bar{f}}} = \frac{\partial}{\partial z}\left(\frac{f}{\bar{f}}\right) = \frac{1}{2}\sqrt{\frac{\bar{f}}{f}}\frac{\partial}{\partial z}\frac{f}{\bar{f}} = \frac{1}{2}\sqrt{\frac{\bar{f}}{f}}\frac{(f'\bar{f} - f\bar{f}')}{(\bar{f})^2} =$

$= \frac{1}{2}\frac{1}{\sqrt{f\bar{f}}}\frac{1}{\bar{f}}\left[f'\bar{f} - f\bar{f}'\right]$, $f(\zeta,z) = f(\zeta,a) = \frac{1}{a}\left[i\zeta + \frac{1}{2}(a - \bar{a}\zeta^2)\right]$

$f' = \frac{\partial f}{\partial z} = \frac{1}{2c}\left[\frac{\partial a}{\partial z} - \zeta^2\frac{\partial \bar{a}}{\partial z}\right]$, $\bar{f}' = \frac{\partial}{\partial z}\frac{1}{a}\left[-i\zeta + \frac{1}{2}(\bar{a} - a\zeta^2)\right] = \frac{1}{2c}\left[\frac{\partial \bar{a}}{\partial z} - \zeta^2\frac{\partial a}{\partial z}\right]$

$f'\bar{f} - f\bar{f}' = \frac{1}{2c}\left(\frac{\partial a}{\partial z} - \zeta^2\frac{\partial \bar{a}}{\partial z}\right)\left(-\frac{i}{a}\zeta\right) - \frac{i}{c}\zeta\frac{1}{2c}\left(\frac{\partial \bar{a}}{\partial z} - \zeta^2\frac{\partial a}{\partial z}\right) =$

$= -\frac{i}{2c^2}\zeta\left[\frac{\partial a}{\partial z} - \zeta^2\frac{\partial \bar{a}}{\partial z} + \frac{\partial \bar{a}}{\partial z} - \zeta^2\frac{\partial a}{\partial z}\right] = -\frac{i}{2c^2}\zeta\left[(1-\zeta^2)\frac{\partial a}{\partial z} + (1-\zeta^2)\frac{\partial \bar{a}}{\partial z}\right]$

$\frac{\partial w}{\partial z} = \frac{1}{2}\frac{1}{\sqrt{f\bar{f}}}\frac{1}{\left(-\frac{i}{c}\zeta\right)}\left(-\frac{i}{2c^2}\zeta\right)[\cdots] = \frac{1}{2}\frac{1}{\sqrt{\zeta^2/c^2}}\frac{1}{2c}\left[(1-\zeta^2)\frac{\partial a}{\partial z} + (1-\zeta^2)\frac{\partial \bar{a}}{\partial z}\right] =$

$= \frac{1}{4\rho}\left[(1-\zeta^2)\frac{\partial a}{\partial z} + (1-\zeta^2)\frac{\partial \bar{a}}{\partial z}\right] = \frac{\partial w}{\partial z}$, $\zeta = \rho e^{i\varphi}$

Out core, $w_2 = 0$, $w\bar{w} = 1$ и $\left(\frac{\partial w}{\partial z}\right)^2 = \left|\frac{\partial w}{\partial z}\right|^2 = \frac{1}{4}\left[\frac{\partial w}{\partial z}\frac{\partial \bar{w}}{\partial z} - w^2\left(\frac{\partial \bar{w}}{\partial z}\right)^2 - \bar{w}^2\left(\frac{\partial w}{\partial z}\right)^2 + (w\bar{w})^2\frac{\partial w}{\partial z}\frac{\partial \bar{w}}{\partial z}\right]$

$\left(\frac{\partial w}{\partial z}\right)^2 = \frac{1}{4}\left[2\frac{\partial w}{\partial z}\frac{\partial \bar{w}}{\partial z} - w^2\left(\frac{\partial \bar{w}}{\partial z}\right)^2 - \bar{w}^2\left(\frac{\partial w}{\partial z}\right)^2\right]$, $w(a=0) = \frac{f(0)}{|f(0)|} = e^{i\varphi}$

$\left(\frac{\partial w}{\partial z}\right)^2 = \frac{1}{4}\left[2\frac{\partial w}{\partial z}\frac{\partial \bar{w}}{\partial z} + e^{2i\varphi}\left(\frac{\partial \bar{w}}{\partial z}\right)^2 + e^{-2i\varphi}\left(\frac{\partial w}{\partial z}\right)^2\right]$.

$e^{-2i\varphi}\left(\frac{\partial w}{\partial z}\right)^2 = \frac{1}{16\rho^2}e^{-2i\varphi}\left[(1-\rho^2e^{-2i\varphi})\frac{\partial a}{\partial z} + (1-\rho^2e^{2i\varphi})\frac{\partial \bar{a}}{\partial z}\right]^2 =$ [опустим $\rho\rho$ несущ.]

$= \frac{1}{16\rho^2}e^{-2i\varphi}\left[2(1-\rho^2e^{-i2\varphi})(1-\rho^2e^{2i\varphi})\frac{\partial a}{\partial z}\frac{\partial \bar{a}}{\partial z} + (1+\rho^4e^{4i\varphi} - 2\rho^2e^{2i\varphi})\left(\frac{\partial \bar{a}}{\partial z}\right)^2\right] =$

$= \frac{1}{16\rho^2}e^{-2i\varphi}\left[-2\rho^2e^{2i\varphi}\frac{\partial a}{\partial z}\frac{\partial \bar{a}}{\partial z} - 2\rho^2\left(\frac{\partial \bar{a}}{\partial z}\right)^2e^{2i\varphi}\right]$, оставляем только члены с $\int d\varphi\, (\cdots) \neq 0$.

$\left(\frac{\partial w}{\partial z}\right)^2 = \frac{1}{2}\left|\frac{\partial w}{\partial z}\right|^2 + \frac{1}{4}\left[\frac{1}{16\rho^2}\left[-2\rho^2\frac{\partial a}{\partial z}\frac{\partial \bar{a}}{\partial z} - 2\rho^2\left(\frac{\partial a}{\partial z}\right)^2\right] + \frac{1}{16\rho^2}\left[-2\rho^2\frac{\partial a}{\partial z}\frac{\partial \bar{a}}{\partial z} - 2\rho^2\left(\frac{\partial a}{\partial z}\right)^2\right]\right] =$

$= \frac{1}{2}\left|\frac{\partial w}{\partial z}\right|^2 - \frac{1}{32}\left[2\frac{\partial a}{\partial z}\frac{\partial \bar{a}}{\partial z} + \left(\frac{\partial \bar{a}}{\partial z}\right)^2 + \left(\frac{\partial a}{\partial z}\right)^2\right]$ — ? ρ^2 где потерян?

Далее $\left|\frac{\partial w}{\partial z}\right|^2 = \frac{1}{16\rho^2}\left[2\frac{\partial a}{\partial z}\frac{\partial \bar{a}}{\partial z}(1+\rho^4) + \left(\frac{\partial a}{\partial z}\right)^2 + \left(\frac{\partial \bar{a}}{\partial z}\right)^2\right]$.

Если вообще исключить начальные члены $\left(\frac{\partial a}{\partial z}\right)^2$, $\left(\frac{\partial \bar{a}}{\partial z}\right)^2$, что будет? Интересно ли Ding'ам? Rep: 2014?

$\left(\frac{\partial \vec{w}}{\partial z}\right)^2 = \frac{4}{(1+w\bar{w})^2}\left|\frac{\partial w}{\partial z}\right|^2 = \left|\frac{\partial w}{\partial z}\right|^2 = \frac{1}{8}\frac{(1+\rho^4)}{\rho^2}\frac{\partial a}{\partial z}\frac{\partial \bar{a}}{\partial z}$, $a = -2i\rho + O(\rho^3)$

$\left(\frac{\partial \vec{w}}{\partial z}\right)^2 = \frac{1}{2}\frac{(1+\rho^4)}{\rho^2}\frac{\partial \vec{s}}{\partial z}\frac{\partial \vec{s}}{\partial z}$. $W_{ex}\left(\frac{\partial \vec{w}}{\partial z}, out\right) = A\int d^2\vec{s}\, dz\left(\frac{\partial \vec{w}}{\partial z}\right)^2 =$

$= A\int d\rho\rho\int d\varphi\int dz\,\frac{1}{2}\frac{(1+\rho^4)}{\rho^2}\frac{\partial \vec{s}}{\partial z}\frac{\partial \vec{s}}{\partial z} = 2\pi A R^2\int_c d\rho\rho\int dz\,\frac{1}{2}\frac{(1+\rho^4)}{\rho^2}\frac{\partial \vec{s}}{\partial z}\frac{\partial \vec{s}}{\partial z} =$

$= 2\pi A R^2 L\,\frac{1}{L^2}\frac{1}{2}\int_c^1 d\rho\rho\,\frac{(1+\rho^4)}{\rho^2}\int dz\left(\frac{\partial \vec{s}}{\partial z}\right)^2 = \frac{1}{2}M_3^2 V\left(\frac{l_e}{L}\right)^2 J_1(c)\int dz\left(\frac{\partial \vec{s}}{\partial z}\right)^2$

$J_1(c) = \int_c^1 d\rho\rho\,\frac{(1+\rho^4)}{\rho^2} = \int_c^1 d\rho\left(\frac{1}{\rho} + \rho^3\right) = \left[\ln\rho + \frac{1}{4}\rho^4\right]_c^1 = \ln\frac{1}{c} + \frac{1}{4}(1-c^4)$

Ding'ам $W_{ex} = \frac{1}{2}V\gamma\int dz\left(\frac{\partial \vec{s}}{\partial z}\right)^2$. Сейчас $W_{ex}\left(\frac{\partial \vec{w}}{\partial z}, out\right) = \frac{1}{2}V\gamma'\int dz\left(\frac{\partial \vec{s}}{\partial z}\right)^2$,

$\gamma = M_3^2(l_e/L)^2\left[\ln\frac{1}{c} + \frac{5}{4}\right]$ $\gamma' = M_3^2\left(\frac{l_e}{L}\right)^2 J_1(c) = M_3^2(l_e/L)^2\left[\ln\frac{1}{c} + \frac{1}{4}(1-c^4)\right]$

$W_{ex}(in\,core) = M_3^2V\left(\frac{l_e}{L}\right)^2 J(c)\int dz\left(\frac{\partial \vec{s}}{\partial z}\right)^2 = \frac{1}{2}V\gamma''\int dz\left(\frac{\partial \vec{s}}{\partial z}\right)^2$, $\gamma'' = 2M_3^2\left(\frac{l_e}{L}\right)^2 J(c)$

$J(c) = 1 + c^4(3 - 4\ln 2)$. Now $\gamma = \gamma''(core) + \gamma'(out) = M_3^2\left(\frac{l_e}{L}\right)^2\left[\ln\frac{1}{c} + \frac{1}{4}(1-c^4) + 2 + 2c^4(3-4\ln 2)\right] = (\cdot)\left[\ln\gamma_c + \frac{9}{4} - \frac{1}{4}c^4 + 2c^4(3-4\ln 2)\right]$, не сокращается

$\gamma(Ding) = (\cdot)\left[\ln\frac{1}{c} + 5/4\right]$ далее при $c\to 0$.

Escaneado con CamScanner

(19/IV) Фархад прислал new end modes" wires videos и уточнил параметры.
Экспериментальные (Vazquez): $Fe_{28}Co_{63}Cu_{10}$ wires, $R = 60$ nm, $L = 25$–30 μm
Simulations: $R = 60$ nm, $L = 1$ μm, $M_s = 1600$ G, $A = 2{,}5\cdot10^{-6}$ erg/cm, $\alpha = 0{,}01$
Ошибка 18/IV в W_{ex}(core). Надо $W_{ex}(\text{core}) = M_s^2 V \left(\frac{l_e}{L}\right)^2 J(c) \int dz \left(\frac{\partial \vec{s}}{\partial z}\right)^2$, где $J(c) = 1 + c^4(3 - 4\ln 2)$. Note: $J(c\to 0) = 1$ — конечно.

Out core: $\frac{\partial w}{\partial z} = \frac{1}{2}\frac{1}{\sqrt{f\bar f}}\frac{1}{f}\left[f'\bar f - f\bar f'\right]$ $\quad f(\zeta) = \frac{1}{c}\left[i\zeta + \frac{1}{2}(a - \bar a\zeta^2)\right]$

$\bar f(\zeta) = \frac{1}{c}\left[-i\bar\zeta + \frac{1}{2}(\bar a - a\bar\zeta^2)\right]$

$\frac{\partial f}{\partial z} = \frac{1}{2c}\left[\frac{\partial a}{\partial z} - \zeta^2\frac{\partial \bar a}{\partial z}\right]$, $f(a\to 0) = \frac{1}{c}i\zeta$, $\bar f(a\to 0) = -\frac{1}{c}i\bar\zeta$ — здесь 18/IV ошибка.

$\frac{\partial w}{\partial z} = \frac{1}{2}\frac{1}{\sqrt{\zeta\bar\zeta/c^2}}\frac{1}{\left(-\frac{1}{c}i\bar\zeta\right)}\left[f'\left(-\frac{1}{c}i\bar\zeta\right) - \frac{1}{c}i\zeta f'\right] = \frac{1}{2}\frac{c}{f}\left[f' + \frac{\zeta}{\bar\zeta}\bar f'\right]$, $f' = \frac{\partial f}{\partial z}$

$\left|\frac{\partial w}{\partial z}\right|^2 = \frac{c^2}{4f^2}\left(f' + \frac{\zeta}{\bar\zeta}\bar f'\right)\left(\bar f' + \frac{\bar\zeta}{\zeta}f'\right) = \frac{c^2}{4f^2}\left[f'\bar f' + \frac{\zeta}{\bar\zeta}(\bar f')^2 + \frac{\bar\zeta}{\zeta}(f')^2 + f'\bar f'\right]$

$\frac{\zeta}{\bar\zeta}(f')^2 = e^{-2i\varphi}\frac{1}{4c^2}\left[\left(\frac{\partial a}{\partial z}\right)^2 + \zeta^4\left(\frac{\partial \bar a}{\partial z}\right)^2 - 2\zeta^2\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial z}\right]$

$\left|\frac{\partial w}{\partial z}\right|^2 = \frac{c^2}{4f^2}\left[2\frac{1}{4c^2}\left(\frac{\partial a}{\partial z} - \zeta^2\frac{\partial \bar a}{\partial z}\right)\left(\frac{\partial \bar a}{\partial z} - \bar\zeta^2\frac{\partial a}{\partial z}\right) + \frac{e^{-2i\varphi}}{4c^2}\left[\left(\frac{\partial a}{\partial z}\right)^2 + \zeta^4\left(\frac{\partial \bar a}{\partial z}\right)^2 - 2\zeta^2\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial z}\right] + \right.$

$\left. + \frac{e^{2i\varphi}}{4c^2}\left[\left(\frac{\partial \bar a}{\partial z}\right)^2 + \bar\zeta^4\left(\frac{\partial a}{\partial z}\right)^2 - 2\bar\zeta^2\frac{\partial \bar a}{\partial z}\frac{\partial a}{\partial z}\right]\right]$, $\int_0^{2\pi} d\varphi(\dots)$ оставляет только $e^{\pm i\varphi}$-члены. $\zeta = \rho e^{i\varphi}$.

$\left|\frac{\partial w}{\partial z}\right|^2 = \frac{c^2}{4f^2}\left[\frac{1}{2c^2}\left(1 + \zeta^2\bar\zeta^2\right)\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial z} - 2\zeta^2\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial z}\frac{e^{-2i\varphi}}{4c^2} - 2\bar\zeta^2\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial z}\frac{e^{2i\varphi}}{4c^2}\right]$

$\zeta^2 e^{-2i\varphi} = \rho^2 e^{2i\varphi}e^{-2i\varphi} = \rho^2$, $\bar\zeta^2 e^{2i\varphi} = \rho^2 e^{-2i\varphi}e^{2i\varphi} = \rho^2$

$\left|\frac{\partial w}{\partial z}\right|^2 = \frac{c^2}{4f^2}\left[\frac{1}{2c^2}(1+\rho^4)\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial z} - \frac{2}{4c^2}\rho^2\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial z} - \frac{2}{4c^2}\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial z}\right]$

$\left|\frac{\partial w}{\partial z}\right|^2 = \frac{c^2}{4f^2}\frac{1}{2c^2}\left[(1+\rho^4) - 2\rho^2\right]\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial z} = \frac{1}{8}\frac{1}{f^2}(1-\rho^2)^2\frac{\partial a}{\partial z}\frac{\partial \bar a}{\partial z}$, $a = -2i\zeta + O(\zeta^3)$

$\left|\frac{\partial w}{\partial z}\right|^2 = \frac{1}{2f^2}(1-\rho^2)^2\frac{\partial \zeta}{\partial z}\frac{\partial \bar\zeta}{\partial z}$. $\left(\frac{\partial \vec m}{\partial z}\right)^2 = \frac{4}{(1+w\bar w)^2}\left|\frac{\partial w}{\partial z}\right|^2 = \left|w\bar w = 1 \text{ out core}\right| = \left|\frac{\partial w}{\partial z}\right|^2$

$W_{ex}(\text{out core}) = A\int d^2\vec\rho\int dz\left(\frac{\partial \vec m}{\partial z}\right)^2 = A\cdot 2\pi\int d\rho\rho\int dz\left|\frac{\partial w}{\partial z}\right|^2 = 2\pi A R^2 L\int d\rho\rho\, dz\frac{1}{L^2}\left|\frac{\partial w}{\partial z}\right|^2$

$2\pi A R^2 L\frac{1}{L^2} = 2AVL^2 = \frac{2A}{M_s^2}M_s^2 V\frac{1}{L^2} = M_s^2 V\left(\frac{l_e}{L}\right)^2$, ρ in R-units, z in L-units.

$W_{ex}(\text{out}) = M_s^2 V\left(\frac{l_e}{L}\right)^2\int_a^1 d\rho\rho\int dz\frac{1}{2f^2}(1-\rho^2)^2\frac{\partial s}{\partial z}\frac{\partial \bar s}{\partial z} = M_s^2 V\left(\frac{l_e}{L}\right)^2 J_1(c)\int dz\frac{\partial \vec s}{\partial z}\frac{\partial \vec s}{\partial z}$

$J_1(c) = \int_a^1 d\rho\rho\frac{1}{2\rho^2}(1-\rho^2)^2 = \frac{1}{2}\int_a^1 d\rho\frac{1}{\rho}\left(1 - 2\rho^2 + \rho^4\right) = \frac{1}{2}\left[\ln\rho - 2\frac{\rho^2}{2} + \frac{1}{4}\rho^4\right]_a^1 =$

$= \frac{1}{2}\left[\ln 1 - \ln c - 1 + c^2 + \frac{1}{4} - \frac{1}{4}c^4\right] = \frac{1}{2}\left(\ln\frac{1}{c} - \frac{3}{4} + c^2 - \frac{1}{4}c^4\right)$

$W_{ex}(\text{out}) = M_s^2 V\left(\frac{l_e}{L}\right)^2 J_1(c)\int dz\left(\frac{\partial \vec s}{\partial z}\right)^2 = \frac{1}{2}\eta_0 V\int dz\left(\frac{\partial \vec s}{\partial z}\right)^2$, $\eta_0 = 2M_s^2(l_e/L)^2 J_1(c)$

$\eta_0 = M_s^2\left(\frac{l_e}{L}\right)^2\left(\ln\frac{1}{c} - \frac{3}{4} + c^2 - \frac{1}{4}c^4\right)$

$W_{ex}(\text{core}) = M_s^2 V\left(\frac{l_e}{L}\right)^2 J(c)\int dz\left(\frac{\partial \vec s}{\partial z}\right)^2 = \frac{1}{2}\eta_{in} V\int dz\left(\frac{\partial \vec s}{\partial z}\right)^2$, $\eta_{in} = 2M_s^2(l_e/L)^2 J(c)$

$\eta_{in} = M_s^2\left(\frac{l_e}{L}\right)^2 2\left[1 + c^4(3 - 4\ln 2)\right]$, $W_{ex}\left(\frac{\partial \vec m}{\partial z}\right) = W_{ex}(\text{core}) + W_{ex}(\text{out}) = \frac{1}{2}\eta V\int dz\left(\frac{\partial \vec s}{\partial z}\right)^2$

$\eta = \eta_{in} + \eta_0 = M_s^2\left(\frac{l_e}{L}\right)^2\left[2 + 2c^4(3 - 4\ln 2) + \ln\frac{1}{c} - \frac{3}{4} + c^2 - \frac{1}{4}c^4\right]$

$\eta = M_s^2\left(\frac{l_e}{L}\right)^2\left[\ln\frac{1}{c} + \frac{5}{4} + c^2 + \left(\frac{23}{4} - 8\ln 2\right)c^4\right]$, $\eta(\text{Ding'14}) = M_s^2\left(\frac{l_e}{L}\right)^2\left[\ln\frac{1}{c} + \frac{5}{4}\right]$

т.е. я уточнил свой расчет у для Ding'14. При $c \ll 1$ $\eta = \eta_{\text{Ding}}$.
(finite core radius c)

$W_{ex}(\nabla\vec m) = -\frac{1}{2}\mu V\int dz\, \vec s^2(z)$, $\mu = 2M_s^2\left(\frac{l_e}{L}\right)^2 = \mu(\text{Ding'14})$ — при учете finite $c = \frac{R_c}{R}$ не учитывается.

$23/4 - 2\ln 2 = 0{,}205$. Учет finite c^4 уменьшил $W_{ex}(\partial\vec m/\partial z)$.

Escaneado con CamScanner

(20) (IV) т.к. finite v. core radius $c = R_c/R$ уже и в W_m и в W_{ex}, то можно найти две eigenfrequencies / eigenvalues of vortex gyromodes

Numerics (Simulations by Farkhad): $R = 60$ nm, $L = 1$ nm, $M_s = 1600$ G, $A = 2{,}5 \cdot 10^{-6}$ erg/cm

$$l_e = \sqrt{2A/M_s^2} = \sqrt{\frac{2.25 \cdot 10^{-6}\ \text{erg/cm}}{(1.6)^2 \cdot 10^6\ \text{G}^2}} = \sqrt{\frac{5}{(1.6)^2}} \cdot 10\ \text{nm} = 13{,}98\ \text{nm} \approx 14{,}0\ \text{nm}$$

$\beta = L/R = 16.667$. Если $c = c(z)$, то $c(z)$ надо оставить внутри $\int dz(\dots)$.

Чуть-что $c(z) = c = \text{const}$. Находим $\omega_n(c)$, $m_n(z, c) = S_n(z, c) \cdot e^{i\varphi}$.

Сначала всё в TVA – two-vortex ansatz с $\vec{m}_s(\text{dyn}) = 0$.

$$W_m = W_m^{vol} = W_m^{vol}(\text{in core}) + W_m^{vol}(\text{out}) = 4\pi M_s^2 \beta V \int dz' dz \int e^{-k|z-z'|\beta} I(k,z) I(k,z')\, dk\, \vec{S}(z)\vec{S}(z')$$

$$+ \pi M_s^2 \beta V \int dz \int dz' \int dk\, e^{-\beta k|z-z'|} \left[4c^2 \mathcal{N}^2(k)\, \vec{S}(z)\vec{S}(z') + \frac{1}{\beta^2} \mathcal{N}'^2(k) \left(\frac{\partial \vec{S}}{\partial z} \cdot \frac{\partial \vec{S}}{\partial z'} \right) \right]$$

$$W_m = 4\pi M_s^2 V \beta \int dz \int dz' \int_0^\infty dk\, e^{-k|z-z'|} \left[I(k,z) I(k,z') + c^2 \mathcal{N}^2(k) \right] \vec{S}(z)\vec{S}(z') +$$

$$+ \pi M_s^2 V \frac{1}{\beta} \int dz \int dz' \int dk\, e^{-\beta k|z-z'|} \mathcal{N}'^2(k) \left(\frac{\partial \vec{S}}{\partial z} \cdot \frac{\partial \vec{S}}{\partial z'} \right), \quad c(z) = \text{const}, \quad I(k,z) = I(k) = c \int_0^1 d\rho\, \rho\, J_1(k\rho)$$

$$W_m = 4\pi M_s^2 V \beta \int dz dz' \int dk\, e^{-k\beta|z-z'|} \left[I^2(k) + c^2 \mathcal{N}^2(k) \right] \vec{S}(z)\vec{S}(z') +$$

$$+ \pi M_s^2 V \frac{1}{\beta} \int dz dz' \int dk\, e^{-\beta k|z-z'|} \mathcal{N}'^2(k) \left(\frac{\partial \vec{S}}{\partial z} \cdot \frac{\partial \vec{S}}{\partial z'} \right), \quad I(k) = \int_c^1 d\rho\, \rho\, J_1(k\rho)$$

$$\mathcal{N}(k) = \int_0^c d\rho\, \rho\, F(\rho) J_1(k\rho), \quad \mathcal{N}'(k) = \int_0^c d\rho\, \rho\, F(\rho)(1+\rho^2) J_1(k\rho), \quad F(\rho) = \frac{4\rho}{c^2} \frac{1}{(1+\rho^2/c^2)^2}, \quad F(c) = \frac{1}{c}$$

$$W_m = \frac{1}{2} V \int dz dz'\, \varkappa(zz')\, \vec{S}(z)\vec{S}(z') + \pi M_s^2 \frac{V}{\beta} \int dz dz' \int dk\, e^{-\beta k|z-z'|} \mathcal{N}'^2(k) \left(\frac{\partial \vec{S}}{\partial z} \cdot \frac{\partial \vec{S}}{\partial z'} \right)$$

New $\varkappa(z,z') = 8\pi M_s^2 \beta \int dk\, e^{-\beta k|z-z'|} \left[I^2(k) + c^2 \mathcal{N}^2(k) \right]$ – но справа с Ding'14 уже $\varkappa(zz') = 8\pi M_s^2 \beta \int dk\, e^{-\beta k|z-z'|} I^2(k)$, $I^2(k) \to I^2(k) + c^2 \mathcal{N}^2(k)$ – и конечно finite core

Def. $\delta(zz') = 2\pi M_s^2 \frac{1}{\beta} \int_0^\infty dk\, e^{-k|z+z'|\beta} \cdot \mathcal{N}'^2(k)$

$$W_m = \frac{1}{2} V \int dz dz'\, \varkappa(zz')\, \vec{S}(z)\vec{S}(z') + \frac{1}{2} V \int dz dz'\, \delta(zz') \left(\frac{\partial \vec{S}}{\partial z} \cdot \frac{\partial \vec{S}}{\partial z'} \right)$$

Note: у нас finite v. core и нужно к перенормировке $\varkappa(zz')$ и появляется $\delta(zz')$ – term, non-local exchange. U P(zz') term 25/IV

$$W_{ex} = W_{ex}(\partial_z \vec{m}) + W_{ex}(\partial_z \vec{m}) = -\frac{1}{2} \mu V \int dz\, \vec{S}^2(z) + \frac{1}{2} \gamma V \int dz \left(\frac{\partial \vec{S}}{\partial z} \right)^2$$

$$\mu = \mu(\text{Ding'14}) = 2M_s^2 \left(\frac{l_e}{R} \right)^2, \quad \gamma = M_s^2 \left(\frac{l_e}{L} \right)^2 \left[\ln \frac{1}{c} + \frac{5}{4} + c^2 + \left(\frac{13}{4} - 8\ln 2 \right) c^4 \right]$$

$$W = W_m + W_{ex} = \frac{V}{\pi} \int dz\, w, \quad \frac{W}{\pi} [\vec{S}(z), \vec{S}'(z)] = -\frac{1}{2} \mu \vec{S}^2 + \frac{1}{2} \gamma \left(\frac{\partial \vec{S}}{\partial z} \right)^2 + \frac{1}{2} \int dz' \varkappa(zz') \vec{S}(z) \cdot \vec{S}(z') +$$

$$W = L \int dz\, E = R^2 L \int dz\, w = \frac{1}{\pi} V \int dz\, w \text{ – conv. Ding'14.} \quad + \frac{1}{2} \int dz' \delta(zz') \left(\frac{\partial \vec{S}}{\partial z} \cdot \frac{\partial \vec{S}}{\partial z'} \right)$$

$$\vec{g} \times \vec{S} = -\frac{\partial w}{\partial \vec{S}} + \frac{\partial}{\partial z} \frac{\partial w}{\partial (\partial \vec{S}/\partial z)}; \quad \vec{g} \times \vec{S} = \vec{g} \times \vec{\omega} \times \vec{S} = \vec{\omega}(\vec{g} \cdot \vec{S}) - \vec{S}(\vec{g} \cdot \vec{\omega}) = \vec{S} g_z \omega(-1)$$

Thiele eq. $G_{\alpha\beta} \dot{X}_\beta = -\partial_\alpha W \left(-\frac{\delta W}{\partial X_\alpha} \right)$, $\vec{G} = G\hat{z}$, $G = G_z = G_{xy}$ def. Gus MRL'2014.

$g_z = g_{xy} = g$, $-\omega g \vec{S} = -\frac{\partial w}{\partial \vec{S}} = -\frac{\partial}{\partial \vec{S}} \left[-\frac{1}{2} \mu \vec{S}^2 \right] = \mu \vec{S}$, $\frac{\omega g}{\pi} \vec{S} = \mu \vec{S}$. $\lambda = \frac{\omega g}{\pi}$.

$$-\lambda \vec{S} = \frac{1}{\pi} \left(-\frac{\delta W}{\delta \vec{S}} \right) = \frac{1}{\pi} \left[-\frac{\partial w}{\partial \vec{S}} + \frac{\partial}{\partial z} \frac{\partial w}{\partial (\partial \vec{S}/\partial z)} \right] = \left[-\frac{\partial}{\partial \vec{S}} + \frac{\partial}{\partial z} \frac{\partial}{\partial (\partial \vec{S}/\partial z)} \right] \times$$

$$\times \left[-\frac{1}{2} \mu \vec{S}^2 + \frac{1}{2} \gamma \left(\frac{\partial \vec{S}}{\partial z} \right)^2 + \frac{1}{2} \int dz' \varkappa(zz') \vec{S}(z)\vec{S}(z') + \frac{1}{2} \int dz' \delta(zz') \left(\frac{\partial \vec{S}}{\partial z} \cdot \frac{\partial \vec{S}}{\partial z'} \right) \right]$$

$$-\lambda \vec{S} = \mu \vec{S} + \gamma \frac{\partial^2 \vec{S}}{\partial z^2} - \int dz' \varkappa(zz') \vec{S}(z') + \frac{\partial}{\partial z} \frac{\partial}{\partial (\partial \vec{S}/\partial z)} \frac{1}{2} \int dz' \delta(zz') \left(\frac{\partial \vec{S}}{\partial z} \cdot \frac{\partial \vec{S}}{\partial z'} \right)$$

$$\lambda \vec{S} = -\mu \vec{S} - \gamma \frac{\partial^2 \vec{S}}{\partial z^2} + \int dz' \varkappa(zz') \vec{S}(z') - \frac{\partial}{\partial z} \frac{\partial}{\partial (\partial \vec{S}/\partial z)} \frac{1}{2} \int dz' \delta(zz') \left(\frac{\partial \vec{S}}{\partial z} \cdot \frac{\partial \vec{S}}{\partial z'} \right)$$

$$\lambda \vec{S} = -\mu \vec{S} - \gamma \frac{\partial^2 \vec{S}}{\partial z^2} + \int dz' \varkappa(zz') \vec{S}(z') - \frac{\partial}{\partial z} \int dz' \delta(zz') \frac{\partial \vec{S}(z')}{\partial z'}, \quad \frac{\partial \delta(zz')}{\partial z} = \delta'(zz')$$

$$(\lambda + \mu) \vec{S}(z) = -\gamma \frac{\partial^2 \vec{S}}{\partial z^2} + \int dz' \varkappa(zz') \vec{S}(z') - \int dz' \delta'(zz') \frac{\partial \vec{S}(z')}{\partial z'}$$

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$\frac{\partial}{\partial z}\int dz' B(zz')\frac{\partial \vec S(z')}{\partial z'} \approx \frac{\partial}{\partial z}\left[\frac{\partial \vec S(z)}{\partial z}\int dz' B(zz')\right] = \int dz' B(zz')\frac{\partial^2 \vec S}{\partial z^2} +$
$+\frac{\partial \vec S}{\partial z}\int dz' B'(zz')$

Лок. local approxim. $\frac{\partial}{\partial z}\frac{\partial}{\partial(\partial \vec S/\partial z)}\cdot\frac{1}{2}\int dz' B(zz')\left(\frac{\partial \vec S}{\partial z}\cdot\frac{\partial \vec S}{\partial z'}\right) =$

$= \frac{\partial}{\partial z}\frac{\partial}{\partial(\partial \vec S/\partial z)}\left[\frac{1}{2}\left(\frac{\partial \vec S}{\partial z}\right)^2\int dz' B(zz')\right] = \frac{\partial}{\partial z}\left[\frac{\partial \vec S}{\partial z}\cdot\int dz' B(zz')\right] =$

$= \frac{\partial^2 \vec S}{\partial z^2}\cdot\int dz' B(zz') + \frac{\partial \vec S}{\partial z}\cdot\int B'(zz')dz', \quad B'(zz') = \frac{\partial}{\partial z}B(zz').$

Функционал $W[\vec S(z), \vec S'(z)] = W[\vec S(z), \frac{\partial \vec S}{\partial z}]$ содержит non-local term $\frac{1}{2}\int dz' B(zz')\left(\frac{\partial \vec S}{\partial z}\cdot\frac{\partial \vec S}{\partial z'}\right)$. Как его вариировать?

Также в local approx. $\frac{1}{2}\nu(z)\left(\frac{\partial \vec S}{\partial z}\right)^2$, $\nu(z) = \int dz' B(zz')$ как вариировать?

$B(zz') = 2\pi M_s^2\frac{1}{\beta}\int dk\, e^{-\beta k|z-z'|}\mathcal{V}'^2(k)$

1D case $\vec S = S \to y$, $z \to x$, $F[x, y, y'] = \frac{1}{2}\int dx' B(xx')y'(x)y'(x')$

Local approx. $F[x, y, y'] = \frac{1}{2}\nu(x)(y')^2$, $\nu(x) = \int dx' B(xx')$.

$I = \int dx\, F(x, y, y')$. Buldygin. Var. calculus $\to$ $\frac{\partial F}{\partial y} - \frac{d}{dx}\frac{\partial F}{\partial y'} = 0$

$\frac{d}{dx}(\cdot)$ – полная производная, что даст

$\frac{\partial^2 F}{\partial y'^2}y'' + \frac{\partial^2 F}{\partial y \partial y'}y' + \frac{\partial^2 F}{\partial x \partial y'} - \frac{\partial F}{\partial y} = 0$, dif. eq. Нужно найти $y(x)$.

$F(x, y, y') = -\frac{1}{2}\mu y^2 + \frac{1}{2}\gamma(y')^2 + \frac{1}{2}\nu(x)(y')^2$

$\frac{\partial F}{\partial y'} = \gamma y' + \nu y'$, $\frac{\partial^2 F}{\partial x \partial y'} = \nu' y'$, $\frac{\partial^2 F}{\partial y \partial y'} = 0$, $\frac{\partial^2 F}{\partial y'^2} = \gamma + \nu(x)$, $\frac{\partial F}{\partial y} = -\mu y$

$[\gamma + \nu(x)]y'' + \nu' y' + \mu y = 0$, что и нужно

$\frac{\partial F}{\partial y} - \frac{d}{dx}\frac{\partial F}{\partial y'} = 0$, $\frac{d}{dx}\frac{\partial F}{\partial y'} = \frac{d}{dx}[\nu(x)y' + \gamma y'] = \nu' y' + \nu y'' + \gamma y''$

Local: $W = W_0 + \frac{1}{2}\nu(z)\left(\frac{\partial \vec S}{\partial z}\right)^2$, $\frac{d}{dz}\frac{\partial}{\partial \vec S'}\frac{1}{2}\nu(\vec S')^2 = \frac{d}{dz}[\nu \vec S'] = \nu'\vec S' + \nu \vec S''$

$(\lambda+\mu)\vec S(z) = -\gamma \vec S'' + \int dz' \varkappa(zz')\vec S(z') + \nu'\vec S' + \nu \vec S''$, $\nu = \nu(z) = \int dz' B(zz')$

$W = W_0 + \frac{1}{2}\int dz' B(zz')\left(\frac{\partial \vec S}{\partial z}\cdot\frac{\partial \vec S}{\partial z'}\right) = \frac{1}{2}\int dz' B(zz')\vec S'(z)\cdot\vec S'(z')$, $\vec S' = \partial \vec S/\partial z$.

$\frac{d}{dz}\frac{\partial}{\partial \vec S'(z)}[\cdot] = \frac{d}{dz}\int dz' B(zz')\vec S'(z') = \int dz' \frac{dB(zz')}{dz}\vec S'(z') = \int dz' B(zz')\vec S'(z')$

$\int dz' \frac{dB(zz')}{dz}\vec S'(z') = -\int dz' \frac{dB(zz')}{dz'}\vec S'(z') = -\int dB(zz')\cdot\vec S'(z') = \left|\begin{array}{l} dv = -dB, \ v = -B \\ \vec S' = \vec u, \ \vec S'' = \vec u' \end{array}\right|$

$= -B\vec S'\Big|_0^1 + \int B\vec S''dz'$, если $\vec S'(z=0,1) = 0$, то $\int dz' \frac{dB(zz')}{dz}\vec S'(z') = \int dz' B(zz')\vec S''$

$W_m = -\int dV \frac{1}{2}\vec H\vec M$, $\delta W_m = -\frac{1}{2}\int dV[\delta\vec H\cdot\vec M + \vec M\cdot\delta\vec H] = -\int W \vec M\delta\vec H$

$W_m = -\frac{1}{2}\vec H\cdot\vec M$, $\vec H[\vec M]$, $\delta W_m = -\frac{1}{2}[\delta\vec H\cdot\vec M + \vec H\cdot\delta\vec M] = -\vec H\cdot\delta\vec M$, $\vec H\delta\vec M = \vec M\cdot\delta\vec H$

$W_3 = \frac{1}{2}\vec h(z)\vec S(z)$, $\vec h(z) = \int dz' \varkappa(zz')\vec S(z')$, $\delta W_3 = \frac{1}{2}[\vec S\cdot\delta\vec h + \vec h\cdot\delta\vec S] = \vec h\delta\vec S$

$W_2 = \frac{1}{2}\int dz' B(zz')\vec S'(z)\vec S'(z')$, $\vec h(z) = \int dz' B(zz')\vec S'(z')$, $W_2 = \frac{1}{2}\vec h_{(z)}\cdot\vec S'(z)$

$\delta W_2 = \frac{1}{2}\delta(\vec h\vec S') = \frac{1}{2}[\vec S'\cdot\delta\vec h + \vec h\cdot\delta\vec S']$, $\vec S'\delta\vec h = \vec S'_{(z)}\cdot\int dz' B\,\delta\vec S'(z')$

$\vec h\cdot\delta\vec S' = \delta\vec S'(z)\int dz' B\vec S'(z')$, $\lim\int dz(\ldots)$, $\vec S'\delta\vec h = \vec h\cdot\delta\vec S'$, т.к. $z \rightleftarrows z'$, $B(zz') = B(z'z)$

$\delta W_2 = \vec h\cdot\delta\vec S'$, $\frac{\delta W_2}{\delta\vec S'} = \vec h = \int dz' B(zz')\vec S'(z')$

Local: $\frac{d}{dz}\frac{\partial}{\partial \vec S'}\frac{1}{2}\nu(\vec S')^2 = \nu'\vec S' + \nu\vec S''$, Nonlocal: $\frac{d}{dz}\frac{\partial}{\partial \vec S'}\frac{1}{2}\int dz' B\vec S'(z)\vec S'(z') =$

Nonlocal: $\frac{\partial}{\partial \vec S'}\frac{d}{dz}\frac{1}{2}\int dz' B\vec S'(z)\vec S'(z') = \int dz' B(zz')\vec S''(z') \to \vec S''(z)\int dz' B(zz') = \nu(z)\vec S''(z)$

$= \frac{d}{dz}\int dz' B(zz')\vec S'(z')$

Разница в чем $\nu'\vec S'$ (между local и nonlocal) исчезает, т.к. $\nu' = \frac{d\nu}{dz} = \frac{d}{dz}\int dz' B(zz') = \frac{1}{dz}(\text{const}) = 0$, если $B(zz') \to \delta(z-z')$ в local approximation

Это надо учесть в $\int dz' \frac{dB(zz')}{dz} \vec{S}'(z') = \int dz' B(zz') \vec{S}''(z')$. $\int_{-\infty}^{\infty} \delta'(x)\varphi(x)dx = -\int_{-\infty}^{\infty} \delta(x)\varphi'(x)dx$

$B(zz') \to \delta(z-z')$. Свёртка $\delta' * f = \delta * f' = f'$ (известно).

Local approx. $(\lambda+\mu)\vec{S}(z) = \left[-\gamma + \nu(z)\right]\frac{\partial^2 \vec{S}}{\partial z^2} + \vec{S}(z)\int dz' \chi(zz')$, $\nu(z) = \int dz' B(zz')$

Удаление 2-го члена не позволяет сделать local approximation to yet.

Если $B(zz') \to \delta(z-z')$ на неограниченном, то $\int dz' \frac{dB(zz')}{dz}\vec{S}'(z') = \int dz' B(zz')\vec{S}''(z')$ неограничивается интегрированием по частям.

Удаление $\frac{\partial \vec{S}}{\partial z}$ в 1D означало бы удаление lin. k_z-term (in $\omega(k_z)$) – non-reciprocity, что возможно для vortex-state наночастиц, $\vec{k} \uparrow\downarrow \vec{p}$, p – vortex core polarization, $\vec{p} = \pm\hat{z}$.

17/IV. $B(zz') = 2\pi M_S^2 \frac{1}{\beta} K_v(zz')$, $K_v(zz') = \int_0^\infty dk\, e^{-k|z-z'|\beta}\, \Lambda^{-2}(k)$, $F_v(z) = \int_0^1 dz' K_v(zz')$

$\nu(z) = \int_0^1 dz' B(zz') = 2\pi M_S^2 \frac{1}{\beta} F_v(z)$, $F_v(z) = \frac{2}{\beta}\int_0^\infty \frac{dk}{k}\Lambda^{-2}(k)\left[1 - e^{-\beta k/2}\operatorname{ch}\beta k z\right]$

Max $F_v(z)$ at $z=0$ и min $F_v(z)$ at $z = \pm\frac{1}{2}$. $\min F_v(z) = \frac{1}{\beta}\int \frac{dk}{k}\Lambda^{-2}(k)\left(1 - e^{-\beta k}\right) \geq 0$

$\nu(z) = \frac{2\pi M_S^2}{\beta}\left[K_v(z) > 0\right]$.

$-\gamma$ и $\nu(z)$ конкурируют. При $c \gtrsim 0.5$, $\nu - \gamma > 0$ и $\gamma D_S \frac{\omega}{\gamma} = \chi^2 \frac{\partial^2 S}{\partial z^2}$

Запишем решение $S(z) \sim e^{ik_z z}$, $S'' = -k_z^2 S$ for some ω.

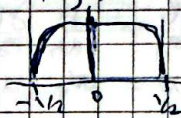
Т.е. $B(zz')$ как и $\chi(zz')$ dipolar terms способствуют localization of eigenmodes at the wire ends, $z = \pm 1/2$. MathCad $F_v(z)$ считает очень быстро при $|z| \lesssim 0.1$ или.

$K_v(x) = \int_0^\infty dk\, e^{-xk}\Lambda^{-2}(k)$, $x = \beta|z-z'| \geq 0$, $x \in [0, \beta]$, $K_v(x)$ падает с $x\uparrow$ быстро

$e^{-\beta k/2}\operatorname{ch}\beta k z = \frac{1}{2}e^{-\beta k/2}\left(e^{\beta k z} + e^{-\beta k z}\right) = \frac{1}{2}\left(e^{\beta k(z-1/2)} + e^{-\beta k(z+1/2)}\right)$ term

$K(x) = \int dk\, \omega^{-xk} I^2(k)$ как $\chi(zz')$

$B(x) \sim K_v(x)$ остаётся очень мало при $x=0$ примерно 1, или по $|z-z'|$ примерно $1/\beta$.



Vortex G'M

24/IV 2014. Метод функции Грина для решения интегро-дифф ур. для $S(z,t)$.

$S(z) = \sum_{n=-\infty}^{\infty} c_n e^{iq_n z}$, $c_n = \frac{1}{L}\int_{-L/2}^{L/2} dz\, S(z) e^{-iq_n z}$, $L \to \infty$, $q_n = 2\pi n$.

$K_{nn'} = \int dz\, dz'\, \chi(zz') e^{iq_n z} e^{iq_{n'} z'}$, $\chi(zz') = \beta\int dk\, e^{-k|z-z'|} I^2(k)$ in units $2\pi M_S^2$.

$I_{nn'} = \int_{-1/2}^{1/2} dz\, dz'\, e^{-\beta k|z-z'|} e^{-iq_n z} e^{iq_{n'} z'} = 2\left[\frac{\beta k\,\delta_{nn'}}{q_n^2 + \beta^2 k^2} + (-1)^{n+n'}\frac{(\beta^2 k^2 - q_n q_{n'})}{(\beta^2 k^2 + q_n^2)(\beta^2 k^2 + q_{n'}^2)}\left(e^{-\beta k} - 1\right)\right]$

Можно (nov. 2014) $\int ne(\beta k) = \int dz\, dz'\, e^{-\beta k|z-z'|}\cos q_n z \cdot \cos q_{n'} z'$ если расчет только самое $\vec{S}(z)$

К расчету 24/VII добавить $B(zz')$ член. $\frac{\partial^2}{\partial z^2} S(z') = \sum_n \frac{\partial^2}{\partial z^2} c_n e^{iq_n z'} = \sum_n (-q_n^2) c_n e^{iq_n z'}$

$\frac{1}{L}\int_{-1/2}^{1/2} e^{i(q_n - q_{n'})z} dz = \delta_{nn'}$. $(\lambda+\mu) \to \lambda$. $\lambda\sum_n c_n e^{iq_n z} = \gamma\sum_n q_n^2 c_n e^{iq_n z} + \int dz' \chi(zz')\sum_n c_n e^{iq_n z'} +$

$+ \int dz' B(zz')\sum_n (-q_n^2) c_n e^{iq_n z'}$

Умн. на $e^{-iq_{n'}z}$ и $\int_{-1/2}^{1/2} dz(\ldots)$: $\lambda\sum_n c_n \int dz\, e^{iq_n z} e^{-iq_{n'} z} = \gamma\sum_n q_n^2 c_n \int dz\, e^{iq_n z} e^{-iq_{n'} z} +$

$+ \int dz\, dz'\, \chi(zz')\sum_n c_n e^{iq_n z'} e^{-iq_{n'} z} + \int dz\, dz'\, B(zz')\sum_n (-q_n^2) c_n e^{iq_n z'} e^{-iq_{n'} z}$

$\lambda c_{n'} = \gamma q_{n'}^2 c_{n'} + \sum_n K_{n'n} c_n + \sum_n \left\{ + q_n^2 \int dz\, dz'\, B(zz') e^{iq_n z'} e^{-iq_{n'} z}\right\} c_n$

$L_{n'n} = \int dz\, dz'\, B(zz') e^{-iq_{n'} z} e^{iq_n z'} \Rightarrow \lambda c_{n'} = \gamma q_{n'}^2 c_{n'} + \sum_n K_{n'n} c_n + \sum_n (+q_n^2 L_{n'n}) c_n$

$\lambda c_n = \gamma q_n^2 c_n + \sum_{n'}\left\{K_{nn'} + q_{n'}^2 L_{nn'}\right\} c_{n'}$, $\Lambda_{nn'} = \gamma q_n^2 \delta_{nn'} + K_{nn'} + q_{n'}^2 L_{nn'}$

$\lambda c_n = \sum_{n'} \Lambda_{nn'} c_{n'}$, $\Lambda\vec{c} = \lambda\vec{c}$, $L_{nn'} = \int dz\, dz'\, B(zz') e^{-iq_n z} e^{iq_{n'} z'}$

Что сравнение с экспер. Si-Rap. и расчет $\chi(zz') \to K_{nn'}$ new и добавить $L_{nn'}$ из $B(zz')$

Escaneado con CamScanner

21/IV $K_{nn'} = 8\pi M_s^2 \beta \int dz\,dz' e^{-iq_n z} e^{iq_{n'} z'} \int dk e^{-\beta k|z-z'|} \left[I^2(k) + c^2\Lambda^2(k)\right] =$

$= 8\pi M_s^2 \beta \int dk \left[I^2(k) + c^2\Lambda^2(k)\right] J_{nn'}(k)$

$L_{nn'} = \int dz\,dz' \delta(z-z') e^{-iq_n z} e^{iq_{n'} z'} = \frac{2\pi M_s^2}{\beta} \int dz\,dz' e^{-iq_n z} e^{iq_{n'} z'} \int dk e^{-\beta k|z-z'|} \Lambda^2(k)$

$L_{nn'} = \frac{2\pi M_s^2}{\beta} \int dk \Lambda^2(k) J_{nn'}(k)$, $J_{nn'}(k) = \int dz\,dz' e^{-\beta k|z-z'|} e^{-iq_n z} e^{iq_{n'} z'}$

$(\lambda+\mu) C_n = \sum_{nn'} C_{n'} \left[\gamma q_n^2 \delta_{nn'} + K_{nn'} - q_n^2 L_{nn'}\right]$, $q_n = 2\pi n$.

$\lambda = \frac{\omega g}{\pi} = \frac{\omega}{\omega_M} 8\pi M_s^2$, $\mu = 2M_s^2 \left(\frac{l_e}{R}\right)^2$; $\gamma = M_s^2 \left(\frac{l_e}{L}\right)^2 \left[\ln\frac{1}{c} + \frac{5}{4} + c^2 + c^4(\ldots)\right]$

$\frac{\mu}{8\pi M_s^2} = \frac{1}{8\pi M_s^2} 2M_s^2 (l_e/R)^2 = \frac{1}{4\pi}\left(\frac{l_e}{R}\right)^2$, $\frac{\gamma}{8\pi M_s^2} = \frac{1}{8\pi}\left(\frac{l_e}{L}\right)^2 \left[\ln\frac{1}{c} + \frac{5}{4} + \ldots\right]$

$\frac{1}{8\pi M_s^2} K_{nn'} = \beta \int dk \left[I^2(k) + c^2\Lambda^2(k)\right] J_{nn'}(k)$, $\frac{L_{nn'}}{8\pi M_s^2} = \frac{1}{4\beta} \int dk \Lambda^2(k) J_{nn'}(k)$

$\frac{\lambda+\mu}{8\pi M_s^2} = \lambda' = \frac{\omega}{\omega_M} + \frac{1}{4\pi}\left(\frac{l_e}{R}\right)^2$, $\lambda' C_n = \sum_{nn'} \Lambda_{nn'} C_{n'}$, $\Lambda_{nn'} \to \frac{\Lambda_{nn'}}{8\pi M_s^2}$

$\Lambda_{nn'} = \frac{1}{8\pi}\left(\frac{l_e}{L}\right)^2 \left[\ln\frac{1}{c} + \frac{5}{4} + \ldots\right] q_n^2 \delta_{nn'} + \beta \int dk \left[I^2(k) + c^2\Lambda^2(k)\right] J_{nn'}(k) -$

$- q_{n'}^2 \frac{1}{4\beta} \int dk \Lambda^2(k) J_{nn'}(k)$

$\frac{1}{4\pi}\left(\frac{l_e}{R}\right)^2 = \frac{1}{4\pi}\left(\frac{14{,}0\ nm}{60\ nm}\right)^2 = 4{,}33 \cdot 10^{-3}$ (мало по сравн. с ω/ω_M, но сравн. с ω_c/ω_M.

$\omega_M/2\pi = (\gamma/2\pi)\, 4\pi M_s = 3{,}0 \frac{GHz}{kOe} \cdot 4\pi \cdot 1{,}6\ kG\ (FeCo) = 60{,}32\ GHz$.

УД что C_n in dimensionless units, $\lambda' = \frac{\omega}{\omega_M} + \frac{1}{4\pi}\left(\frac{l_e}{R}\right)^2$. λ'_n → спектр задачи ω_n.

$\omega_n = \omega_M \left(\lambda'_n - \frac{1}{4\pi}\frac{l_e^2}{R^2}\right)$. Ищем λ'_n и S_n – СЗ и СВ of the matrix $\Lambda_{nn'}$.

$J_{nn'} = J_{n'n} \Rightarrow K_{nn'} = K_{n'n}$, $L_{nn'} = L_{n'n}$. Матрица $\Lambda_{nn'}$ – симметричная, но вещ.

Тогда $q_n^2 L_{nn'}$ тоже – $\Lambda_{nn'}$ должна симметричной.

$S(z) = S_x(z) + iS_y(z) = \sum_n C_n e^{iq_n z}$, $S^*(z) = S_x(z) - iS_y^*(z) = \sum_n C_n^* e^{-iq_n z}$

$\Lambda_{nn'}$ – вещ. $\Rightarrow$ соб/eigenvectors C_n are real, $S_x(z) = \sum_n C_n \cos q_n z$, $S_y(z) = \sum_n C_n \sin(q_n z)$

// Сим. 25/VII/2014. Eq. Thiele для $\vec{S}(zt)$ дает $-i\dot{g}S(zt) = +\gamma S''(zt) + \int dz' \chi(zz') S(z't)$

(момент) здесь $S(zt) = S_x(zt) + iS_y(zt)$. Fourier trans. $S(zt) = S(z\omega) e^{i\omega t}$

дает $\lambda S(z\omega) = -\gamma S'' + \int dz' \chi(zz') S(z'\omega)$, $\lambda = \omega g/\pi$, eq. для $S(z\omega) = S_x(z\omega) + iS_y(z\omega)$

$\vec{g} \times \vec{s} = -\frac{\delta w}{\delta \vec{s}}$ в Sci. Rep. 14 брали $\dot{\vec{S}} = \vec{\omega} \times \vec{S}$. $\left(\pm - \omega g \vec{S}\right)$, $\lambda \vec{S} = \frac{\delta W}{\delta \vec{S}}$, $\lambda = \frac{g\omega}{\pi}$.

Дано $\dot{\vec{S}} = \vec{\omega} \times \vec{S}$, $\vec{g} \times \vec{s} = -g\omega \vec{S}$. $\dot{S}(zt) = i\omega S(zt)$, $S = S_x + iS_y$.

$\vec{\omega} \times \vec{S} = \omega\left[\hat{y} S_x - \hat{x} S_y\right]$, $\vec{\omega} = \omega \hat{z}$. $\dot{S}_x = \omega S_y$, $\dot{S}_y = \omega S_x$

$\dot{S}_x + i\dot{S}_y = -\omega S_y + i\omega S_x = i\omega(S_x + iS_y) = i\omega S = \dot{S}$. $\dot{\vec{S}} = \vec{\omega} \times \vec{S}$ совпал.

$\dot{S} = i\omega S$ дает $S_x + iS_y = S$. УД что $S = S_x + iS_y$ или $\vec{S} = S_x \hat{x} + S_y \hat{y}$.

Нашли симметр в $\Lambda_{nn'}$, надо + $q_n^2 L_{nn'}$ тоже. Тогда $q_n^2 L_{nn'}$ делает $\Lambda_{nn'}$ – несим.

$S(z) = \sum_n C_n e^{iq_n z}$, здесь S это S_x или S_y, $\vec{S}(S_x, S_y)$. $n = -N, -N+1, \ldots N$.

$\Lambda_{nn'}$ – real, все λ_n, C_n – real. $S_n(z) = \sum_n C_n \cos q_n z + i\sum_n C_n \sin q_n z$.

Противоречие, если $S = S_x, S_y$, то $S(z)$ должно быть real. Но некоторые $S_n(z)$ are imaginary. $S(z) = \sum_n C_n e^{iq_n z} = S_n^*(z) = \sum_n C_n^* e^{-iq_n z} =$

$= \sum_n C_{-n}^* e^{iq_n z}$, real $S(z) \Rightarrow C_n = C_{-n}^*$. Но если C_n – real, $C_n = C_{-n}$, $C_n^* = C_n$

$S(z) = \sum_n C_n \cos q_n z + i\sum_n C_n \sin q_n z = \left| C_n \sin(2\pi n z) \leftrightarrow n C_n = (-n) C_{-n} = (-n) C_n, \sum_n(\ldots) = 0 \right| =$

$= \sum_n C_n \cos q_n z$ – $S(z)$ – real. MathCad (Vortex_TM_long-wires.mcd) дает

либо $C_n = C_{-n}$, либо $C_n = -C_{-n}$, что real и mirror $S(z)$, respectively.

$S(z) = C_0 + \sum_{n<0} C_n e^{iq_n z} + \sum_{n>0} C_n e^{iq_n z} = C_0 + \sum_{n>0} C_{-n} e^{-iq_n z} + \sum_n C_n e^{iq_n z} =$

$= C_0 + \sum_{n>0}\left[C_{-n}(\cos q_n z - i\sin q_n z) + C_n(\cos q_n z + i\sin q_n z)\right] = C_0 + \sum_{n>0}(C_n + C_{-n}) \cos q_n z +$

$+ i\sum_{n>0}(C_n - C_{-n}) \sin q_n z$. $\vec{S}(z) = \begin{cases} C_0 + 2\sum_{n>0} C_n \cos q_n z, & C_n = C_{-n} \\ 2i\sum_{n>0} C_n \sin q_n z, & C_n = -C_{-n} \end{cases}$ – real, СВ. $S(z) = S_{x,y}(z)$.

– mirror СВ.

22/iv СФ метода Фурье распадается на рег. $S_n(z)$ и конт. $S_n(\xi)$.

Lowest mode ω_0 – рег. $S_0(z)$  сильно локализована на wire ends $z=\pm l/2$.

К сожалению $\omega_0, \omega_1, \ldots$ сильно близки ($\omega_0/2\pi \approx 4$ ГГц) и дают $\omega_n/2\pi$ сильно неплотно по сравн. с Single Particle $\omega_0/2\pi$ ($\lambda=1$ см) ≈ 0.3 ГГц.

Источники ошибок Фурье метода:

1) Игнорируем все coefficients в $\Lambda_{nn'}$ и λ, λ', μ
2) разложение по $e^{iq_n z}$ плохо для описания localized end modes, ψ_0 надо брать $N \gg 1$ & $\varphi(z) = \sum_{n=1}^{N} c_n e^{iq_n z}$, а недиагональные $\Lambda_{nn'}$ сильно падают с ростом n, n', $n \neq n'$ компоненты Λ – диагональ $n=n'$
3) vortex mass M_v (interaction с azim. SW) больше для wires $L=1000$ нм, чем $L=200$ нм, качественно изменяется 1D $\psi(z,t)$.

Display $\Lambda_{nn'}$ для $N=20$ показал, что все $\Lambda_{nn'} \sim 10^{-3}$, почти не зависят от n,n' т.е. (2) верно, плоские волны $e^{iq_n z}$ не адекватны для low-ω_n end modes. $\Lambda_{nn} \sim 1 - 0.1$ — разложение по $e^{iq_n z}$ сработало в Sci. Rep. 2014, 15 $\omega_S = \frac{\gamma}{2\pi}$ exch. dominating ($\sim \frac{1}{L^2}$) при $L=50-100$ нм. Расчет M_v – очень сложен, т.к. $\vec{m}_{SW}(z,\varphi)$ неоднородны.

$$(\lambda+\mu)\vec{S}(z) = -\gamma^2 \frac{\partial^2 \vec{S}}{\partial z^2} + \int dz' \chi(zz') \vec{S}(z') - \int dz' \beta(zz') \frac{\partial^2 \vec{S}(z')}{\partial z'^2}$$ можно сделать к обычному 1D – уж только при $\chi(zz') \to \delta(z-z')$, $\beta(zz') \to \delta(z-z')$.

Разложение по ф. Мэтье? Eq. Mathieu $y'' + (a - 2q\cos 2x) y(x) = 0$ $\to$ $ce_n(x,q)$, $se_n(x,q)$

$\int_0^{2\pi} dx\, ce_n \cdot ce_m = \int_0^{2\pi} dx\, se_n\, se_m = \pi\delta_{nm}$, $\int_0^{2\pi} dx\, ce_n\, se_m = 0$.

The Mathieu functions $ce_n(x,q)$, $se_n(x,q)$ form a complete set, i.e. any periodic (π, 2π-period) $f(x)$ can be expanded as series in $ce_n(x,q)$ and $se_n(x,q)$

Def $u_n(z) = ce_n(z,q) + i\, se_n(z,q)$. Найти $\int_{-l/2}^{l/2} dz \int dz'\, e^{-\beta k|z-z'|} u_n(z) u_{n'}^*(z')$ подробно

$J_{nn'}(k) = \int dz \int dz'\, e^{-\beta k|z-z'|} e^{-iq_n z} e^{iq_{n'} z'}$ где $u_n(z) = e^{iq_n z}$.

В MathCad нет встроенных Mathieu functions, $\int dz \int dz' f(zz') u_n(z) u^*(z')$ – численно найти.

Т.к. wires are long $\beta = \frac{L}{R} \gg 1$, можно будет искать $\beta \to \infty$. Если $15/\pi$ исходно

$K(z) = \int_0^\infty dk\, e^{-\beta k z} I^2(k)$ если $F(\beta z)$, $z \to |z-z'|$ и помнить $\beta|z-z'| \gg 1$, т.к. если $z=z'$ max

$K(x) = \int_0^\infty dk\, e^{-xk} I^2(k)$, $x = \beta|z-z'|$ посчитать можно, они имеют острый при $x=0$, $z=z'$, $\beta \gg 1$.

Даже если local approx. $(\lambda+\mu)\vec{S}(z) = +\gamma^2 \vec{S}''(z) + \vec{S}(z)\int dz' \chi(zz') - S''(z)\int dz' \beta(zz')$

$\lambda' \vec{S}(z) = -\left[\gamma + \int dz' \beta(zz')\right] \vec{S}''(z) + \int dz' \chi(zz') \cdot \vec{S}(z)$ – не сводится к $\lambda\vec{S}$.

Можно $\beta(zz') = 2\pi M_S^2 \frac{1}{\beta} \int_0^\infty dk\, e^{-\beta k|z-z'|} \Lambda'^2(k)$, $\nu(z) = \int dz' \beta(zz')$

$\nu(z) = 2\pi M_S^2 \frac{1}{\beta} \cdot \frac{2}{\beta} \int_0^\infty \frac{dk}{k} \Lambda'^2(k) \left[1 - e^{-\beta k/2} \mathrm{ch}\, \beta k z\right]$, $\lambda\vec{S} = -(\gamma + \nu(z))\vec{S}''(z) + \vec{S}(z)$

$\max \nu(z) = \nu(0) = 4\pi M_S^2 \frac{1}{\beta^2} \int_0^\infty \frac{dk}{k} \Lambda'^2(k)(1 - e^{-\beta k/2})$, $\Lambda'(k) = \int_0^c dq\, q F(q)(1+q^2) J_1(kq)$

$F(\beta) = \frac{4g}{c^2} \frac{1}{(1+g/c)^2} = \frac{4}{c} \frac{\tau}{(1+\tau^2)^2}$, $\tau = g/c$. $\Lambda'(k) = c^2 \int_0^1 d\tau \tau \frac{4}{c} \frac{\tau}{(1+\tau^2)^2}(1+\tau^2\xi^2) J_1(k\tau c)$

$k\tau c = u$, $\nu(0) = 4\pi M_S^2 \frac{1}{\beta^2} \int_0^\infty \frac{du}{u} (1 - e^{-\beta u/2c}) \Lambda'^2(u)$, $\Lambda'(u) = 4c \int_0^1 d\tau \tau^2 \frac{(1+\tau^2\xi^2)}{(1+\tau^2)^2} J_1(\tau u)$

$g/\beta c \to 1$. $\int_0^\infty du \frac{1}{u} e^{-xu} \Lambda'^2(u)$, $\Lambda'(u) \underset{u\to 0}{\sim} u$, $\int_0^\infty \frac{du}{u} u^2 e^{-xu} = \int_0^\infty du\, u e^{-xu} = \left|ux = y\right|$

Пример: $I(x) = \int_0^\infty dt\, e^{-xt} q(t)$, если $q(0)=0$, $q(t) \underset{x\to\infty}{\sim} t$, то $\int_0^\infty dt\, e^{-xt} q(t) \sim \frac{1}{x^2}$. $c = \frac{1}{x^2}$

$\nu(0) = 4\pi M_S^2 \frac{1}{\beta^2} \left[\int_0^\infty \frac{du}{u} \Lambda'^2(u) - \int_0^\infty \frac{du}{u} e^{-xu} \Lambda'^2(u)\right] = (*) \left[\int_0^\infty \frac{du}{u} \Lambda'^2 - \frac{c}{x^2}\right]$, $x = \beta/2c \gg 1$

При $\beta \to \infty$, $\nu(0) = 4\pi M_S^2 \int_0^\infty \frac{du}{u} \Lambda'^2(u,c) \cdot \frac{1}{\beta^2}$ сравнить с $\gamma(c) = M_S^2 \left(\frac{he}{R}\right)\left[\ln\frac{1}{c} + \frac{5}{4}\right] \frac{1}{\beta^2}$

Итак при $\beta \to \infty$ $\nu(0)$ и $\gamma \sim 1/\beta^2 \to 0$ обе, при $c > 0.3$ ν-term $> \gamma(c)$, при $c > 0.6$, $\nu(0) \gg \gamma(c)$.

Escaneado con CamScanner

→ Чтобы $\beta \to \infty$ получался $\nu(z)$ и ν выбрасывать одновременно.

$\lambda + \mu = \lambda' = 8\pi M_s^2 \left(\frac{\omega}{\omega_M} + \frac{1}{4\pi} \frac{Le^2}{R^2} \right)$ от β не зависит. Осталась только $\int dz' \varkappa(zz')$

$\lambda' \vec{S}(z) = \int dz' \varkappa(zz') \vec{S}(z')$, $\varkappa(zz') = 8\pi M_s^2 \beta \int dk e^{-\beta k |z-z'|} [I^2(k) + c^2 \Lambda^2(k)]$

Ясно, что $\vec{S}(z) = S_0 = const$ не может быть решением.

Все же оценка для $\vec{S}(z) = const$. $\lambda' \int dz S^2(z) = \int dz dz' \varkappa(zz') \vec{S}(z) \vec{S}(z')$

$\lambda' = \int dz dz' \varkappa(zz')$. $J_{00}(k) = 2 \left[\frac{1}{\beta k} + \frac{1}{(\beta k)^2} (e^{-\beta k} - 1) \right] = \frac{2}{\beta k} f(\beta k)$

$\frac{1}{8\pi M_s^2} K_{00} = \beta \int dk [I^2(k) + c^2 \Lambda^2(k)] J_{00}(k) = 2 \int \frac{dk}{k} f(\beta k) [I^2(k) + c^2 \Lambda^2(k)]$

$\frac{\omega_0}{\omega_M} = -\frac{1}{4\pi} \left(\frac{Le}{R} \right)^2 + 2 \int \frac{dk}{k} f(\beta k) [I^2(k) + c^2 \Lambda^2(k)] = \omega_0(\beta, c) / \omega_M$.

Для $\omega_0(\beta, c) = 6 - 8$ GHz для $c = 0,3 - 0,7$ и $\beta \approx 17$, очень высокая.

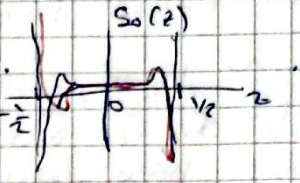
$\lambda' \vec{S}(z) = \int dz' \varkappa(zz') \vec{S}(z')$, $\varkappa(zz') = \beta \int dk e^{-\beta k |z-z'|} [I^2(k) + c^2 \Lambda^2(k)]$

$\varkappa(z,z') \to \varkappa(z_i, z_j) = \varkappa_{ij}$, $z_j = \frac{j}{N}$, $j = 0, 1, \dots N$. С увелич. N, ω_0 падает.

$S_0(z)$ локализована на $z = \pm 1/2$ или около них.

Видимо localized "end" modes всего двух тех. имеется.

Остальные — полученные SW с $q_z \neq \frac{\pi n}{L}$.



$S_0(z) = \frac{1}{2} \delta(z - 1/2) \pm \frac{1}{2} \delta(z + 1/2)$ - first iteration

$\lambda' S(z) = \frac{1}{2} \int dz' \varkappa(zz') S(z') = \frac{1}{2} \int dz' [\varkappa(zz')] [\delta(z' \mp 1/2) \pm \delta(z' + 1/2)] =$

$= \frac{1}{2} \varkappa(z, \frac{1}{2}) \pm \frac{1}{2} \varkappa(z, -\frac{1}{2}) = \frac{1}{2} \beta \int dk [I^2(k) + c^2 \Lambda^2(k)] \left[e^{+\beta k (z - \frac{1}{2})} \pm e^{-\beta k (z + \frac{1}{2})} \right] =$

$= \frac{1}{2} \beta \int dk F(k) e^{-\beta k/2} \left(e^{\beta k z} \pm e^{-\beta k z} \right) = \beta \int dk F(\beta) e^{-\beta k/2} \begin{pmatrix} ch \beta k z \\ sh \beta k z \end{pmatrix}$

Second iteration $S_0(z) = \frac{\beta}{\lambda'} \int dk [I^2(k) + c^2 \Lambda^2(k)] e^{-\beta k/2} \begin{pmatrix} ch \beta k z \\ sh \beta k z \end{pmatrix}$

Пусть $\varphi_n(z)$ известны. Спектр $\lambda'_n = \frac{1}{\int dz S_n^2(z)} \int dz dz' \varkappa(zz') S_n(z) S_n(z')$

Хотя $S_0(z)$-2nd iter. похожи на $S_0(z)$? решения инт. eq., λ'_0, $\frac{\omega_0}{2\pi} = 3,3$ GHz (или $\beta = 1000/60$, $c = 0,5$) слишком велико.

Revised and 8th Glob ФНТ - thick dot MS

23/IV Верна ли $\omega_0(\beta)$ при $\beta \ll 1$, $S_0(z) = const$?

$\forall \beta$: $\frac{\omega_0(\beta)}{\omega_M} = 2 \int \frac{dk}{k} f(\beta k) [I^2(k) + c^2 \Lambda^2(k)] - \frac{1}{4\pi} \left(\frac{Le}{R} \right)^2$

$c\Lambda(k) = c \int_c^1 ds\, s F(s) J_1(ks) = 4 \int_c^1 ds\, s \frac{1}{(1 + s^2/c^2)^2} J_1(ks)$, $I(k) = \int_c^1 ds\, J_1(ks)$

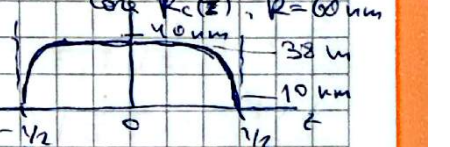
Если пренебречь in-core, $c \to 0$, то $\frac{\omega_0(\beta)}{\omega_M} = 2 \int \frac{dk}{k} f(\beta k) I^2(k) - \frac{1}{4\pi} \left(\frac{Le}{R} \right)^2$

Gus'JAP'02 (IVA): $\omega_0 = \frac{1}{2} \gamma M_s \frac{4}{9} [4\pi F_V(\beta) - \left(\frac{R_0}{R} \right)^2 \frac{1}{2}] = \omega_M \left[2 F_V(\beta) - \frac{1}{4\pi} \left(\frac{R_0}{R} \right)^2 \right]$

$F_V(\beta) = \int \frac{dt}{t} \left[\int_0^1 ds\, J_1(st) \right]^2 f(\beta t)$. $\omega_0(\beta)$ at $S_0(z) = const$ в полосе здесь

$\omega_0 \to$ Gus' JAP' 2002, $R_0 = Le$, for $\forall \beta$.

Может дело в зависимости $R_c(z)$ вблизи $z = \pm 1/2$



$R_c(0) = 38$ nm, $c = \frac{38}{60} = 0,63$, $R_c(\pm 1/2) = 10$ nm, $c = 0,167$

$c\Lambda(k) = 4 \int_0^c ds\, s \frac{1}{(1 + s^2/c^2)^2} J_1(ks) = |s/c = r| = 4c^2 \int_0^1 dr \frac{r^2}{(1 + r^2)^2} J_1(kcr)$.

$\lim_{c \to 0} c\Lambda(k) = 4c^2 \int_0^1 dr \frac{r^2}{(1 + r^2)^2} J_1(kcr) \approx 2c^3 k \int_0^1 dr \frac{r^3}{(1 + r^2)^2} = 0$, если $\int_0^\infty dk(\omega)$ сходится.

Проверим in-core Wm calc. от 7/IX 2014.

In-core, $w(\xi\bar{\xi}) = f(\xi) = \frac{i}{c} \left[\xi - \frac{(s + \bar{s}\xi^2)}{1 + s\bar{s}} \right]$, $\frac{\partial m_z}{\partial z} = \frac{\partial}{\partial z} \frac{(1 - f\bar{f})}{(1 + f\bar{f})} = \frac{-1(1 + f\bar{f}) - (1 - f\bar{f})}{(1 + f\bar{f})^2} \times$

$\times \frac{\partial}{\partial z} (f\bar{f}) = \frac{(-2) \partial_z (f\bar{f})}{(1 + f\bar{f})^2}$. $f(\xi) \approx \frac{i}{c} [\xi - s - \bar{s}\xi^2] + O(s^3)$, $f\bar{f} = \frac{i}{c} (\xi - s - \bar{s}\xi^2) (-\frac{i}{c}) (\bar{\xi} - \bar{s} - s\bar{\xi}^2)$

$= \frac{1}{c^2} [\xi\bar{\xi} - \bar{s}\xi - s\xi^2\bar{\xi} - s\bar{\xi} - \bar{s}\xi^2\bar{\xi}] = \frac{1}{c^2} [\xi\bar{\xi} - \bar{s}\xi(1 + \xi\bar{\xi}) - s\bar{\xi}(1 + \xi\bar{\xi})]$

$\frac{\partial}{\partial z} (f\bar{f}) = \frac{\partial}{\partial z} \frac{1}{c^2} [\xi\bar{\xi} - (1 + \xi\bar{\xi})(\bar{s}\xi + s\bar{\xi})] = -\frac{1}{c^2} (1 + \xi\bar{\xi}) \left(\bar{\xi} \frac{\partial s}{\partial z} \xi + \frac{\partial \bar{s}}{\partial z} \bar{\xi} \right)$

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$$\frac{\partial m_+}{\partial z} = \frac{(-2)}{(1+f\bar f)^2}\frac{\partial}{\partial z}(f\bar f) = \frac{(-2)}{(1+f\bar f)^2}\frac{(-1)}{c^2}(1+\zeta\bar\zeta)\left(\frac{\partial\bar s}{\partial z}\zeta + \frac{\partial s}{\partial z}\bar\zeta\right) = \frac{4}{c^2}\frac{(1+\zeta^2)}{(1+\zeta^2/c^2)^2}\left(\vec\zeta\cdot\frac{\partial\vec s}{\partial z}\right)$$

$$\bar\zeta\frac{\partial s}{\partial z} + \zeta\frac{\partial\bar s}{\partial z} = \frac{\partial}{\partial z}(\bar\zeta s + \zeta\bar s) = \frac{\partial}{\partial z}2\,\mathrm{Re}(\bar\zeta s) = \frac{\partial}{\partial z}2(\vec\zeta\cdot\vec s)$$

$$\frac{\partial m_+}{\partial x} + \frac{\partial m_y}{\partial y} = 2\,\mathrm{Re}\frac{\partial m_+}{\partial\zeta}, \quad 2\frac{\partial}{\partial\zeta} = \frac{\partial}{\partial x} - i\frac{\partial}{\partial y}, \quad 2\frac{\partial}{\partial\zeta}m_+ = \left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)(m_x + i m_y)$$

$$\frac{\partial m_+}{\partial\zeta} = \frac{\partial}{\partial\zeta}\frac{2f}{1+f\bar f} = 2f\frac{(-1)}{(1+f\bar f)^2}\bar f\frac{\partial f}{\partial\zeta} + \frac{2}{(1+f\bar f)}\frac{\partial f}{\partial\zeta} = 2\frac{\partial f}{\partial\zeta}\frac{1}{(1+f\bar f)^2}\left[-f\bar f + (1+f\bar f)\right] =$$

$$= \frac{2}{(1+f\bar f)^2}\frac{\partial f}{\partial\zeta}, \quad \frac{\partial s}{\partial\zeta} = \frac{\partial}{\partial\zeta}\frac{i}{c}\left[\zeta - s - \bar s\zeta^2\right] = \frac{i}{c}\left[1 - 2s\zeta\right]$$

$$\frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y} = 2\,\mathrm{Re}\frac{\partial m_+}{\partial\zeta} = 2\,\mathrm{Re}\frac{2}{(1+f\bar f)^2}\frac{i}{c}(1-2\bar s\zeta) = \frac{8}{(1+f\bar f)^2}\mathrm{Re}(\bar s\zeta i)\frac{1}{c}$$

$$\mathrm{Re}(i\bar s\zeta) = \mathrm{Re}\,i(s_x - is_y)(x+iy) = \mathrm{Re}\,i(s_x x - is_y x + s_x iy + s_y y) = s_y x - s_x y$$

$$\frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y} = -\frac{8}{c}\frac{1}{(1+f\bar f)^2}(s_y x - s_x y) = \frac{8s}{c}\frac{1}{(1+s^2/c^2)^2}(s_x\sin\varphi - s_y\cos\varphi)$$

$$\underline{\mathrm{div}\,\vec m\ (\text{in core})} = \frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y} + \frac{\partial m_z}{\partial z} = \frac{8s}{c}\frac{1}{(1+s^2/c^2)^2}(s_x\sin\varphi - s_y\cos\varphi) + \frac{4}{c^2}\frac{(1+s^2)}{(1+s^2/c^2)^2}\left(\vec s\cdot\frac{\partial\vec s}{\partial z}\right)$$

В точках $\mathrm{div}\vec m$ (core) от $t/\chi 1+2$.

$$\mathrm{div}\,\vec m\ (\text{in core}) = F(s)\left[2c(s_x\sin\varphi - s_y\cos\varphi) + (1+s^2)\left(\frac{\partial s_x}{\partial z}\cos\varphi + \frac{\partial s_y}{\partial z}\sin\varphi\right)\right], \quad F(s) = \frac{4s}{c^2}\frac{1}{(1+s^2/c^2)^2}$$

// χ-term gives $\omega_0/2\pi \simeq 9$ GHz when $s_0(z) = \mathrm{const}$ and 4,3 GHz when $s_0(z) \simeq \delta(z \pm \nu_2)$, $c = 70\,\mathrm{nm}/60\,\mathrm{nm} = 0{,}167$ (Physical Review Letters)

We have very nonuniform, non-parabolic $W_m^{face}(s)$ and $W_m^{vol} \sim \left(\frac{\partial\vec s}{\partial z}\right)$.

Необычный эффект дает $\chi/2\pi = 3{,}0$ GHz/kOe $\to$ $\delta/2\pi = 2{,}8$ GHz/kOe (Sinai).

$$\frac{\omega_M}{2\pi} = \left(\chi/2\pi\right)\cdot 4\pi\cdot 1{,}6\cdot 10^3\ \mathrm{kOe} = 56{,}3\ \mathrm{GHz}.$$

// Ester Berganza присылала slides Simul. $\vec M(\vec\kappa)$ in cyl., dots и semi-shperes. Запомн. ЕИА генер. CONSTANTO, 1 vb-nono.

(24/IV) E-mail to Berganza. Работа в сарае. 5 часов.

(25/IV) Face ($z = \pm\nu_2$) magnetostatic energy W_m^f, см. 16/IV. W_m^{face} magneto-statics

$$W_m^f = M_s^2 R^3 \int_0^\infty dk\,(1 - e^{-\beta k})\sum_{\mu=-\infty}^{\infty}|K_\mu(k)|^2, \quad K_\mu(k) = \int d^2\vec\rho\ J_\mu(k\rho)e^{i\mu\varphi}m_z(\vec\rho)$$

17/III 2014 и далее, Очень $K_\mu(k)$ через $f(\zeta,s) = \frac{i}{c}\left[\zeta - \frac{s+\bar s\zeta^2}{1+s\bar s}\right]$ core.

Ygodnee $f(\zeta,a) = \frac{i}{c}\left[i\zeta + \frac{1}{2}(a - \bar a\zeta^2)\right]$ и через $a(s) = -2i\frac{s}{(1+s\bar s)}$

In core $w(z\bar z) = f(z)$. $w_z(\vec s, a) = \frac{1-|f|^2}{1+|f|^2}$, у core border $|f(\zeta,a)| = 1$

$$|f|^2 = \frac{1}{c^2}\left[i\zeta + \frac{a}{2} - \frac{\bar a}{2}\zeta^2\right]\left[-i\bar\zeta + \frac{\bar a}{2} - \frac{a}{2}\bar\zeta^2\right] = \frac{1}{c^2}\left[\zeta\bar\zeta + \frac{i}{2}(\zeta\bar a - a\bar\zeta) + i\zeta\left(-\frac{a}{2}\right)\bar\zeta^2 + \right.$$

$$\left. + \frac{1}{4}a\bar a - \frac{1}{4}a^2\bar\zeta^2 + \frac{i}{2}\bar a\zeta^2\bar\zeta - \frac{1}{4}\bar a^2\zeta^2 + \frac{1}{4}\zeta^2\bar\zeta^2 a\bar a\right] = \frac{1}{c^2}\left[\zeta\bar\zeta + \frac{i}{2}(\zeta\bar a - a\bar\zeta) + \right.$$

$$\left. + \frac{i}{2}\zeta\bar\zeta(\bar a\zeta - \bar\zeta a) + \frac{1}{4}a\bar a - \frac{1}{4}(a^2\bar\zeta^2 + \bar a^2\zeta^2) + \frac{1}{4}\zeta^2\bar\zeta^2 a\bar a\right]$$

$$|f(\zeta,a)|^2 = \frac{1}{c^2}\left[\zeta\bar\zeta + \frac{i}{2}(\zeta\bar a - a\bar\zeta)(1+\zeta\bar\zeta) + \frac{1}{4}a\bar a(1+\zeta^2\bar\zeta^2) - \frac{1}{4}(a^2\bar\zeta^2 + \bar a^2\zeta^2)\right]$$

Ищем W_m^f с точностью до $\sim a^2$, с уравнением единицы у core border $|f(\zeta,a)|^2 = 1$.

$$s(\zeta,s) = \frac{i}{c}\left[\zeta - s - \bar s\zeta^2\right] + O(s^3), \quad \zeta - s = \zeta', \quad \zeta^2 = (\zeta'+s)^2 = \zeta'^2 + 2\zeta's + s^2$$

$$s(\zeta',s) = \frac{i}{c}\left[\zeta' - \bar s\zeta'^2 - 2s\bar s\zeta'\right]$$ и далее коэффициенты с точностью до $\zeta = s$.

$$|f(\zeta',s)|^2 = \frac{i}{c}\left(\zeta' - \bar s\zeta'^2 - 2s\bar s\zeta'\right)\left(\frac{-i}{c}\right)\left[\bar\zeta' - s\bar\zeta'^2 - 2s\bar s\bar\zeta'\right] = \frac{1}{c^2}\left[\zeta'\bar\zeta' - s\zeta'\bar\zeta'^2 - 2s\bar s\zeta'\bar\zeta' - \right.$$

$$\left. - \bar s\zeta'^2\bar\zeta' + s\bar s\zeta'^2\bar\zeta'^2 - 2s\bar s\zeta'\bar\zeta'^2\right] = \frac{1}{c^2}\left[\zeta'\bar\zeta' - \zeta'\bar\zeta'(s\bar\zeta' + \bar s\zeta') - 4s\bar s\zeta'\bar\zeta' + s\bar s\zeta'^2\bar\zeta'^2\right]$$

$$|f(\zeta',s)|^2 = \frac{1}{c^2}\left[1 - (s\bar\zeta' + \bar s\zeta') - 4s\bar s + s\bar s\zeta'\bar\zeta'\right]\zeta'\bar\zeta', \quad |f(\zeta',s)|^2 = 1$$ далее core border

$$1 = \frac{\rho'^2}{c^2}\left[1 - (s\bar\zeta' + \bar s\zeta') + s\bar s\rho'^2 - 4s\bar s\right], \quad s\bar\zeta' + \bar s\zeta' = 2\,\mathrm{Re}\,s\bar\zeta' = 2(\vec s\cdot\vec\rho'), \quad \vec\rho'(x';y')$$

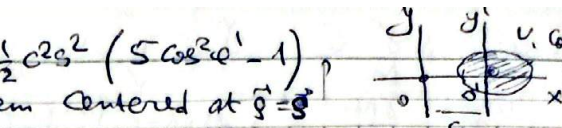
$$s^2(\varphi)\left[\rho'^2 - 2\vec s\cdot\vec\rho' + s\bar s(\rho'^2 - 4)\right] = c^2, \quad \rho'^2\left[1 - 2\vec s\cdot\vec\rho' + s\bar s(\rho'^2 - 4)\right] = c^2, \quad \rho'\Big|_{s=0} = c.$$

Черно (15/IV 2014) $\frac{\rho'(\varphi')}{c} = 1 + c\cos(\varphi' - \varphi_s) + 2s^2 + \frac{1}{2}c^2s^2\left(5\cos^2(\varphi' - \varphi_s) - 1\right)$

$(\vec s\cdot\vec\rho') = s\rho'\cos(\varphi' - \varphi_s)$, $\vec s = |s|$ задано. Удобно $\varphi_s = 0$, т.е. $\vec s$ вдоль Ox.

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V. Core border : $\frac{S'(\varphi')}{(\text{Norm. } S^2)} = 1 + \varepsilon_3 \cos\varphi' + \frac{1}{2}\varepsilon^2 + \frac{1}{2}\varepsilon^2 S^2 \left(5\cos^2\varphi' - 1\right)$, $\vec{\rho} = \vec{s} - \vec{s}_c$, shifted coord. system centered at $\vec{\rho} = \vec{s}$



$W_m = M_s^2 R^3 I_k(s)$, $I_k(s) = \int dk (1 - e^{-\beta k}) \sum_\mu |K_\mu(k,s)|^2$, см. 19/I 2017г.

Вклад в gyrofrequency ω_f? $S = S(z)$, но для face m. energy, $S = S(z = \pm 1/2)$ не есть $S(z)$

тех. gyromodes $S(z=1/2) = S(z=-1/2)$, интег. modes $S(z=+1/2) = -S(z=-1/2)$.

$W_m^f(s)$, $I_k(s)$ зависят от s как s^2, $W_m^f(s) = W_m^f(0) + \frac{1}{2}\varkappa' s^2$.

$\omega_s = \frac{1}{R^2 G s}\frac{\partial W_m}{\partial s} = \frac{2}{R^2 G}\frac{\partial W_m}{\partial (s^2)} = \frac{2}{R^2 G}\frac{\varkappa'}{2} = \frac{\varkappa'}{R^2 G}$, $W_m^f(s) = W_m^f(0) + \frac{1}{2} R^2 G \omega_s s^2$

$\omega_f = \frac{\omega_M}{2\pi^2}\frac{1}{\beta}\frac{1}{s}\frac{\partial I_k}{\partial s}$, т.к. $\sum_\mu |K_\mu(k,s)|^2 = P_0(k) + \frac{1}{2}P_2(k) s^2$

$\omega_f = \frac{\omega_M}{2\pi^2}\frac{1}{\beta}\frac{1}{s}\frac{\partial}{\partial s}\int dk (1 - e^{-\beta k})\left[P_0 + \frac{1}{2}P_2 s^2\right] = \frac{\omega_M}{2\pi^2}\frac{1}{\beta}\int_0^\infty dk (1 - e^{-\beta k}) P_2(k)$

т/к $\beta \gg 1$, long wire, $\omega_f(\beta) \sim \frac{1}{\beta}(1 - e^{-\beta k}) \to 0$ если $G = 2\pi M_s L$ (?)

$\vec{G} = \frac{M_s}{\gamma}\int dV \sin\theta \underset{(\nabla\theta \times \nabla\varphi)}{=} \frac{M_s}{\gamma}\int dz \int d^2\rho \sin\theta (\nabla\theta \times \nabla\varphi)$. ЗДЗ $\vec{m} = \vec{m}(\vec{r}, t) = \vec{m}[\vec{\rho}, \vec{S}(z,t)]$

$\vec{m} = \vec{m}(\theta, \varphi)$. ЗД topol. charge и $\vec{G}$ - definition см. 31/VII - 12/VIII 2019г.

Проверим W_m^f. $W_m^f = \frac{1}{2} M_s^2 \int dS \int dS' \frac{m_z(\vec{\rho}z) m_z(\vec{\rho}'z')}{|\vec{r} - \vec{r}'|} = \frac{1}{2} M_s^2 \int dz_\rho dz_{\rho'} m_z m_z(\cdot) \times$

$\times \int dk e^{-k|z-z'|} \sum_\mu J_\mu(\rho) J_\mu(\rho') e^{i\mu(\varphi - \varphi')}$.



$m_z(\vec{\rho}z) = \vec{m}\cdot\vec{n} = \begin{cases} m_z, z=L \\ -m_z, z=0 \end{cases}$, $\vec{n} = \begin{cases} \hat{z}, z=L \\ -\hat{z}, z=0 \end{cases}$, $(\vec{m}\cdot\vec{n})(\vec{m}\cdot\vec{n}') = m_z m_z' - m_z m_z' - m_z m_z' + (-m_z)(-m_z') =$

$= 2m_z m_z' - 2m_z m_z'$, $(z=L, z'=L)$ $(z=0, z'=L)$ $(z=L, z'=0)$ $(z=0, z'=0)$

$(\Delta z = 0)$ $(\Delta z = \pm L)$

$W_m^f = \frac{1}{2} M_s^2 \int dz_\rho dz_{\rho'} \int dk \sum_\mu J_\mu J_\mu(\cdot) e^{i\mu(\varphi - \varphi')} 2\left[m_z m_z' e^0 - m_z m_z' e^{-k|\Delta z|}\right] =$

$= \frac{1}{2} M_s^2 \int dz_\rho dz_{\rho'} \int dk (1 - e^{-kL}) \sum_\mu J_\mu J_\mu(\cdot) e^{i\mu(\varphi - \varphi')} m_z(\vec{\rho}) m_z(\vec{\rho}') =$

$= M_s^2 R^3 \int dk (1 - e^{-\beta k}) \sum_\mu |K_\mu(k)|^2$, $K_\mu(k) = \int dz_\rho \vec{J}_\mu(k\rho) e^{i\mu\varphi} m_z(\vec{\rho})$.

$m_z(L) m_z'(L) + m_z(0) m_z'(0) - m_z(0) m_z'(L) - m_z(2) m_z'(0)$, $m_z = m_z(\vec{\rho}, z)$, $m_z' = m_z(\vec{\rho}', z')$

Динг'14: $g_{d\beta} = \frac{M_s}{\gamma}\int dz_\rho \vec{m}(\partial_\alpha \vec{m} \times \partial_\beta \vec{m})$, $g_{d\beta}\dot{X}_\beta = \frac{\delta E}{\delta X_\alpha}$, $E = \int dz_\rho \vec{w}$, $\vec{w}$ - energy density $\frac{erg}{cm^3}$

$W(\vec{\rho}, z, s)$, $E(z,s) = \int dz_\rho W(\vec{\rho}, z, s) = R^2 W(s, z)$. $W = L\int dz E = R^2 L \int dz w$.

$W = V \int dz \frac{1}{2\pi} W(s)$. $\vec{g} \times \vec{s} = -\frac{\partial w}{\partial s} + \frac{\partial}{\partial z}\left(\frac{\partial w}{\partial s_z'}\right)$, z in units of L, $g = 2\pi M_s/\gamma$

$W_m^f = M_s^2 R^3 \int dk (1 - e^{-\beta k}) \sum_\mu |K_\mu(k)|^2$, $K_\mu(k,s) = \int dz_\rho \vec{J}_\mu(k\rho) e^{i\mu\varphi} m_z(\vec{\rho}, s)$

$W = W(s, s_z')$ и надо вариировать вблизи $s = 0$, $\frac{\partial w}{\partial s} = -\frac{\partial w}{\partial s} + \frac{\partial}{\partial z}\left(\frac{\partial w}{\partial s_z'}\right)$.

Плотность W по z. $W = \frac{1}{2\pi} V \int dz \bar{w}$.

W_m^f не содержит $\int dz (\cdots)$. $m_z(\vec{\rho}, s)$, здесь $s = s(z = \pm 1/2)$

Поэтому $W_m^f = \frac{1}{2\pi} V \int dz w_m^f$ и $\frac{W_m^f}{2\pi} = \frac{1}{2}\varkappa_f s^2$, то $\vec{s}(z) = \varkappa_f \vec{s}(z) + (\text{остальные члены})$

$\lambda = g w / \pi$, то вклад в gyrofrequency $\omega_f = \frac{\pi \varkappa_f}{g} = \frac{\pi}{2\pi M_s/\gamma}\varkappa_f = \frac{1}{2}\frac{\gamma}{M_s}\varkappa_f$

Face - $W_m^f = \frac{1}{2\pi} V \int dz w_m^f = \frac{1}{2\pi} V w_m^f$, $w_m^f = \frac{2\pi}{V} W_m^f$ (s) $= \frac{2\pi}{V}\left[W_m^f(0) + \frac{1}{2}\varkappa' s^2\right] = w_m^f(0) + \frac{1}{2}\pi\varkappa_f s^2$

$\frac{1}{2}\varkappa'\frac{\pi}{V} = \frac{1}{2}\pi\varkappa_f$, $\varkappa_f = \frac{\varkappa'}{V}$. $\omega_f = \frac{\varkappa'}{R^2 G} = \frac{\varkappa_f V (\pi R^2 L)^{-1}}{R^2 2\pi M_s L} = \frac{1}{2}\frac{\gamma}{M_s}\varkappa_f$.

т.е. если $\varkappa' = \varkappa_f V$ и $\varkappa_f$ не зависит от "2" - всё ok.

$W_m^f = M_s^2 R^3 I_k(s)$, $I_k(s) = \int dk (1 - e^{-\beta k}) \sum_\mu |K_\mu|^2$, $K_\mu = K_\mu(k,s)$ и не зависит от β

$\sum_\mu |K_\mu(k,s)|^2 = P_0(k) + \frac{1}{2}P_2(k) s^2$, $W_m^f = M_s^2 R^3 \int dk (1 - e^{-\beta k})\left[P_0(k) + \frac{1}{2}P_2(k) s^2\right] =$

$= W_m^f(0) + \frac{1}{2} M_s^2 R^3 \int dk (1 - e^{-\beta k}) P_2(k) s^2 = W_m^f(0) + \frac{1}{2}\varkappa' s^2$, $\varkappa' = M_s^2 R^3 \int dk (1 - e^{-\beta k}) P_2(k)$

Escaneado con CamScanner

$W_s = \frac{1}{2}\frac{\gamma}{M_s}\chi_s = \frac{1}{2}\frac{\gamma}{M_s}\frac{1}{V}x' = \frac{1}{2}\frac{\gamma}{M_s}\frac{1}{\pi R^2 L} M_s^2 R^3 \int dk (1-e^{-\beta k}) P_2(k)$

$W_s = \frac{\gamma M_s}{2\pi}\frac{1}{\beta}\int_0^\infty dk (1-e^{-\beta k}) P_2(k) \to 0$ при $\beta \to \infty$ и $W_m^s = \frac{1}{\pi} V W_m^s$ } на единицу.

Вклад W_s — это даже макроскопических. $W_m^s = \frac{1}{2}\chi_s S^2$

Все в рамках assumption $g_{\alpha\beta} = \frac{M_s}{\gamma}\int dz' \vec{m}(\partial_z \vec{m} \times \partial_\beta \vec{m})$, как в Ding'14,

и $g_{\alpha\beta}\dot{X}_\beta = -\frac{\delta E}{\delta X_\alpha}$, E energy per unit dot thickness (thickness energy density)

$E(S)$, $\vec{S} = \vec{X}(z)/R = \vec{S}(z)$. $W = L\int_z E = L\int dz\, E$, $E = E[S(z)] = E(z)$

Вернемся исходному (z is fixed) eq. Thiele $g_{\alpha\beta}\dot{S}_\beta = -\frac{\delta W}{\delta S_\alpha}$, $W = E/R^2$ для 3D texture $\vec{m}(\vec{r},t) = \vec{m}[\vec{r}, \vec{X}(z,t)]$ еще надо доказать (!).

31/III 2019 я доказал, что для 3D texture $\vec{m}(\vec{r},t)$ и rigid change $\vec{r} \to \vec{r} - \vec{X}(t)$

имеем полную $g_{\alpha\beta}\dot{X}_\beta = \frac{\partial W}{\partial X_\alpha}$ и $G_{\alpha\beta}\dot{X}_\beta = \frac{\partial W}{\partial X_\alpha}$, $W = \int dV \bar{w}$, где $\alpha = x,y,z$

$g_{\alpha\beta} = \vec{m}\cdot\left(\frac{\partial \vec{m}}{\partial x_\alpha} \times \frac{\partial \vec{m}}{\partial x_\beta}\right)$, $G_{\alpha\beta} = \int dV g_{\alpha\beta}$, $g_{\alpha\beta}$ — gyrocoupling density, $g_{\alpha\beta}(x,y,z)$.

$G_{\alpha\beta}$ — antisym. constant($\vec{r}$) tensor, every coordinate gyrovector $\vec{G} = (G_1, G_2, G_3)$

2D Thiele $G_{\alpha\beta}\dot{X}_\beta = \frac{\partial W}{\partial X_\alpha}$ сохраняет свой вид $G_1 = G_{zy}$, $G_2 = G_{xz}$, $G_3 = G_{yx}$.

в 3D case, где $\vec{X} = (X,Y,Z)$ — 3D и gyrovector $\vec{G} = (G_x, G_y, G_z)$ имеет 3 components,

Две thick dots/wires $\vec{m}(\vec{r},t) = \vec{m}[\vec{S}, \vec{X}(z,t)]$ как rigid v. $\vec{m}(\vec{r},t) = \vec{m}[\vec{S} - \vec{X}(z,t)]$

Впервые в 2013, потом Ding'2014 я доказал исходное 3D $g_{\alpha\beta}\dot{X}_\beta = -\delta E/\delta X_\alpha$.

(Инвариант).

Т.к. W_m^s дает $\omega_s(\beta) \to 0$ при $\beta \gg 1$, необходимо ввести фактор $W_m^{vol} \sim \left(\frac{\partial \vec{S}}{\partial z}\right)$.

Сент. 7/X 2014 при $C \neq \delta(z)$: $W_m^{vol} = -\mu\pi^2 \int dz dz' \int dk\, e^{-k|z-z'|} \int_0^\infty d\beta g F(\beta) J_1(k\beta) \int_0^\infty d\beta' g F(\beta') J_1(k\beta') M_s^2 \times$

$\times (1+\beta^2) c \left(\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right)_z = -4\pi^2 c M_s^2 \int dz dz' \int dk\, \Lambda(k)\Lambda'(k) e^{-k|z-z'|} \left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right]_z$

$W_m^{vol}\left(\frac{\partial \vec{S}}{\partial z}\right) = -\int dz dz' P(zz') \left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right]_z$, $P(zz') = 4\pi^2 M_s^2 c \int dk\, e^{-k|z-z'|} \Lambda(k)\Lambda'(k)$

Units: goodbye units. $R^4 (\beta \to \beta/R)$, $L^2 (dz dz' \to \frac{1}{L^2} dz dz')$, $\frac{1}{R}(k \to kR)$ и $\frac{1}{R^2}\frac{1}{\beta}$ из $-\beta^2$

$\operatorname{div}\vec{m} = \frac{1}{R}\left(\frac{\partial m_x}{\partial \beta} + \frac{\partial m_y}{\partial \beta} + \frac{1}{\beta}\frac{\partial m_z}{\partial z}\right)$. $R \cdot L^2 \cdot \frac{1}{R}\frac{1}{R^2\beta} = R\frac{L^2}{L/R} = R^2 L = \frac{1}{\pi} V$, см. 16/IV

$P(zz') = 4\pi M_s^2 V c \int dk\, e^{-k|z-z'|\beta} \Lambda(k)\Lambda'(k)$. Standard $W_m^{vol} = -\frac{1}{2} V \int dz dz' P(zz') \left(\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right)$

$W_m^{vol}\left(\frac{\partial \vec{S}}{\partial z}\right) = -\frac{1}{2} V \int dz dz' P(zz') \left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right]_z$, $\left(P(zz') = 8\pi M_s^2 c \int dk\, e^{-\beta k|z-z'|} \Lambda(k)\Lambda'(k)\right)$

$\hat{z}\cdot\left[\frac{\partial \vec{S}}{\partial z}(z) \times \vec{S}(z')\right] = \frac{\partial S_y}{\partial z} S_x(z') - \frac{\partial S_x}{\partial z} S_y(z') = \frac{1}{2i}\left(\frac{\partial \bar{S}}{\partial z} S(z') - \frac{\partial S}{\partial z}\bar{S}(z')\right)$

Вывод 26/IV: $\frac{1}{\pi}\bar{W}\left[S(z), S'(z)\right] = -\frac{1}{2}\mu\vec{S}^2 + \frac{1}{2}\mu\left(\frac{\partial \vec{S}}{\partial z}\right)^2 + \frac{1}{2}\int dz' \chi(zz')\vec{S}(z)\cdot\vec{S}(z') + \frac{1}{2}\int dz' \delta(zz')\left(\frac{\partial \vec{S}}{\partial z}\cdot\frac{\partial \vec{S}}{\partial z'}\right)$

$W = \frac{V}{\pi}\int_z \bar{W}$, $\frac{1}{\pi}\bar{W}_m^{vol}\left(\frac{\partial \vec{S}}{\partial z}\right) = -\frac{1}{2}\int dz' P(zz')\left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right]_z$

Сент. $\frac{1}{\pi} W_m^{vol}(\partial\vec{S}/\partial z) = -\frac{1}{4}\int dz' P(zz')\left[\frac{\partial \vec{S}(z)}{\partial z} \times \vec{S}(z')\right]\cdot\hat{z} - \frac{1}{4}\int dz' P(zz')\left[\frac{\partial \vec{S}(z')}{\partial z'} \times \vec{S}(z)\right]\cdot\hat{z}$

$\left[\vec{S}'(z) \times \vec{S}(z')\right]\cdot\vec{k} = \vec{S}'(z)\cdot\left[\vec{S}(z') \times \hat{z}\right]$, $\frac{\partial}{\partial z}\frac{\partial}{\partial \vec{S}'}\hat{z}\cdot\left[\vec{S}'(z) \times \vec{S}(z')\right] = \frac{\partial}{\partial z}\left[\vec{S}(z') \times \hat{z}\right]$

$\left[\vec{S}'(z') + \vec{S}(z)\right]\cdot\hat{z} = \vec{S}(z)\cdot\left[\hat{z} \times \vec{S}'(z')\right]$, $\frac{\partial}{\partial \vec{S}}\left[\vec{S}'(z') \times \vec{S}(z)\right]\cdot\hat{z} = \hat{z} \times \vec{S}'(z')$

$\frac{1}{\pi} W_m^{vol}$, $\frac{1}{\pi}\frac{\delta \bar{W}_m}{\delta \vec{S}} = \frac{1}{\pi}\frac{\partial w_m}{\partial \vec{S}} - \frac{1}{\pi}\frac{\partial}{\partial z}\frac{\partial w_m}{\partial(\partial \vec{S})} = \frac{\partial}{\partial \vec{S}}\left[-\frac{1}{4}\int dz' P(zz')\vec{S}'(z)\cdot\left[\vec{S}(z') \times \hat{z}\right] -\right.$

$\left. -\frac{1}{4}\int dz' P(zz')\vec{S}(z)\cdot\left[\hat{z} \times \vec{S}'(z')\right]\right] - \frac{\partial}{\partial z}\frac{\partial}{\partial(\partial \vec{S})}[\ldots] = -\frac{1}{4}\int dz' P(zz')\left[\hat{z} \times \vec{S}'(z')\right] +$

$+ \frac{\partial}{\partial z}\frac{1}{4}\int dz' P(zz')\left[\vec{S}(z') \times \hat{z}\right] = -\frac{1}{4}\int dz' P(zz')\left[\hat{z} \times \frac{\partial \vec{S}}{\partial z'}\right] + \frac{1}{4}\int dz' \frac{\partial P(zz')}{\partial z'}\left[\vec{S}(z') \times \hat{z}\right]$

Escaneado con CamScanner

$\vec{g}\times\vec{S} = -\frac{\delta W}{\delta \vec{S}}$, $\vec{g}\times\vec{S} = \vec{g}\times\vec{\omega}\times\vec{S} = -g\omega\vec{S}$, $g\omega\vec{S} = \frac{\delta W}{\delta \vec{S}}$, $\lambda\vec{S} = \frac{1}{\mu}\frac{\delta W}{\delta \vec{S}}$, $\lambda = \frac{g\omega}{\mu}$

Thiele eq. $\lambda\vec{S} = \frac{1}{\mu}\frac{\delta W}{\delta \vec{S}} = -\frac{1}{4}\int dz' P(zz')\left[\hat{z}\times\frac{\partial \vec{S}}{\partial z'}\right] + \frac{1}{4}\int dz' \frac{\partial P(zz')}{\partial z}\left[\vec{S}(z')\times\hat{z}\right] + (\dots)$ old terms

$\int dz' \frac{\partial P(zz')}{\partial z}\vec{S}(z') = \left|\dots\right|$. given $P(zz') = P(z-z')$, $\frac{\partial P}{\partial z} = -\frac{\partial P}{\partial z'}$,

$\int dz' \frac{\partial P}{\partial z}\vec{S}(z') = (-1)\int dz' \frac{\partial P}{\partial z'}\vec{S}(z') = \left|\begin{matrix}\vec{S}(z') = u \\ dz'\frac{\partial P}{\partial z'} = dv\end{matrix}\right| = (-1)\left[\int u\,dv\right] =$

$= (-1)\left[uv - \int v\,du\right] = (-1)\left[\vec{S}(z')P(zz')\Big|_{-}^{+} - \int P(zz')\vec{S}'(z')\,dz'\right]$

$\frac{1}{\mu}\frac{\delta W_m}{\delta \vec{S}} = \frac{1}{4}\left[\int dz' P(zz')\vec{S}'(z')\times\hat{z} + \frac{1}{4}\int dz' \frac{\partial P}{\partial z}\vec{S}(z')\times\hat{z}\right] = \frac{1}{4}\left[\int dz' P(zz')\vec{S}'(z') + \right.$

$\left. + \int dz' \frac{\partial P}{\partial z}\vec{S}(z')\right]\times\hat{z}$, $\int dz' P(zz')\vec{S}'(z') + \int dz' \frac{\partial P}{\partial z}\vec{S}(z') = \int dz' P\vec{S}'(z') -$

$- \vec{S}(z')P(zz')\Big|_{-}^{+} + \int dz' P\vec{S}'(z') = 2\int dz' P(zz')\vec{S}'(z') - \vec{S}(z')P(zz')\Big|_{-}^{+}$

Другие члены. $\int dz' P(zz')\vec{S}'(z') = \int dz' \frac{\partial \vec{S}}{\partial z'}P(zz') = \left|\begin{matrix}dz'\frac{\partial \vec{S}}{\partial z'} = dv \\ u = P\end{matrix}\right| =$

$= \int u\,dv = uv - \int v\,du = \vec{S}(z')P(zz')\Big|_{-}^{+} - \int \vec{S}(z')\frac{\partial P}{\partial z'}dz'$

$\vec{A} = \int dz' P\frac{\partial \vec{S}}{\partial z'} + \int dz' \frac{\partial P}{\partial z}\vec{S}(z') = \vec{S}(z')P(zz')\Big|_{-}^{+} - \int dz' \frac{\partial P}{\partial z'}\vec{S}(z') + \int dz' \frac{\partial P}{\partial z}\vec{S}(z') =$

$= 2\int dz' \frac{\partial P}{\partial z}\vec{S}(z') + \vec{S}(z')P(zz')\Big|_{-}^{+}$, $\vec{S}(z')P(zz')\Big|_{z'=-1/2}^{z'=+1/2} = \vec{S}(1/2)P(z,\tfrac{1}{2}) -$

$- \vec{S}(-\tfrac{1}{2})P(z,-\tfrac{1}{2}) = \left|\begin{matrix}\text{нужно, чтобы} \\ \vec{S}(z)\ \text{even}\end{matrix}\right| = \vec{S}(\tfrac{1}{2})\left[P(z,\tfrac{1}{2}) - P(z,-\tfrac{1}{2})\right] = \vec{S}(\tfrac{1}{2})\,8\pi M_s^2 c \times$

$\times\left[\int dk\, e^{-\beta k|z-1/2|}\Lambda\Lambda' - \int dk\, e^{-\beta k|z+1/2|}\Lambda\Lambda'\right] = \vec{S}(1/2)\,8\pi M_s^2 c\int dk\,\Lambda(k)\Lambda'(k)\times$

$\times\left[e^{-\beta k(1/2-z)} - e^{-\beta k(z+1/2)}\right] = (\cdot)\int dk\,\Lambda\Lambda' e^{-\beta k/2}\,2\sinh(\beta k z) = \vec{S}(\tfrac{1}{2})16\pi M_s^2 c\int dk\,\Lambda\Lambda' e^{-\beta k/2}\times$

$\times\sinh\beta k z$

B.c. нужно $\vec{S}(z')P(zz')\Big|_{-1/2}^{+1/2} \overset{?}{\to} 0$, хотя и при $\beta \gg 1$, вообще говоря, не очевидно.

$\frac{1}{\mu}\frac{\delta W_m}{\delta \vec{S}} = \frac{1}{4}\left[\int dz' P\vec{S}'(z') + \int dz' \frac{\partial P}{\partial z}\vec{S}(z')\right]\times\hat{z} = \frac{1}{2}\int dz' \frac{\partial P}{\partial z}\vec{S}(z')\times\hat{z}$

Thiele

$$\lambda\vec{S} = \frac{1}{2}\int dz' \frac{\partial P}{\partial z}\vec{S}(z')\times\hat{z} - \mu\vec{S} - \gamma\frac{\partial^2\vec{S}}{\partial z^2} + \int dz' \alpha(zz')\vec{S}'(z') - \int dz' \frac{\partial \beta}{\partial z}\frac{\partial \vec{S}(z')}{\partial z'}$$

Thiele ур-е для $\vec{S}(z)$ через $\vec{S}(z')$

$\int dz' \frac{\partial \beta(zz')}{\partial z}\frac{\partial \vec{S}(z')}{\partial z'} = \left|\text{инт. по частям}\right| = \int dz' \beta(zz')\frac{\partial \vec{S}}{\partial z'}(z')$, $\vec{S}\times\hat{z} = S_y\hat{x} - S_x\hat{y}$

При $\beta \to \infty$, exchange-like γ-, и β-члены можно выбросить, т.к. они $\sim \frac{1}{\beta^2}$. для ферромагнетика

$$\lambda'\vec{S}(z) = \int dz' \alpha(zz')\vec{S}(z') + \frac{1}{2}\int dz' \frac{\partial P}{\partial z}(zz')\vec{S}(z')\times\hat{z}, \quad \lambda' = \lambda + \mu = 8\pi M_s^2\left(\frac{\omega}{\omega_M} + \right.$$

$$\begin{cases}\lambda' S_x = \int dz' \alpha S_x(z') + \frac{1}{2}\int dz' P' S_y(z') \\ \lambda' S_y = \int dz' \alpha S_y(z') + \frac{1}{2}\int dz' P'(-S_x)\end{cases} \xrightarrow[\text{appr.}]{\text{local}} \begin{cases}\lambda' S_x = S_x\int dz'\alpha + \frac{1}{2}S_y\int dz' P' \\ \lambda' S_y = S_y\int dz'\alpha - \frac{1}{2}S_x\int dz' P'\end{cases}$$

$S = S_x + iS_y$, $\lambda' S = S\int dz'\alpha + \frac{1}{2}\int dz' P'(S_y - iS_x) = S\int dz'\alpha + \frac{1}{2}e^{-i\pi/2}\int dz' P' S(z)$

$\lambda' S(z) = \left[\int dz'\alpha(zz') + \frac{1}{2}e^{-i\pi/2}\int dz' P'_z(zz')\right]S(z)$, $e^{-i\pi/2} = -i$.

$|\lambda'|^2|S(z)|^2 = \left[\left(\int dz'\alpha\right)^2 + \frac{1}{4}\left(\int dz' P'_z\right)^2\right]|S(z)|^2$, и что? Local appr. плохо, см. $\alpha z/\nu$.

$\lambda' S(z) = \int dz'\left[\alpha(zz') - \frac{1}{2}i P'_z(zz')\right]S(z') = \int dz' A(zz')S(z')$

$A(zz') = \alpha(zz') - \frac{1}{2}i P'_z(zz')$, $A_{mn} = A(z_m, z'_n) = \alpha(z_m, z'_n) - \frac{1}{2}i P'_z(z_m, z'_n)$

$A_{nm} = \alpha(z_n, z'_m) - \frac{1}{2}i P'_z(z_n, z'_m)$, $P'_z = \frac{\partial}{\partial z}P(zz') = -\frac{\partial}{\partial z'}P(zz')$

$A_{nm} = \alpha(z_m, z_n) - \frac{1}{2}i\frac{\partial P}{\partial z_n}(z_n, z_m) = \alpha_{mn} - \frac{1}{2}i\left(-\frac{\partial P_{nm}}{\partial z_m}\right) = \alpha_{mn} + \frac{1}{2}i\frac{\partial P_{mn}}{\partial z_m}$

$A_{mn} = \alpha_{mn} - \frac{1}{2}i\frac{\partial P_{mn}}{\partial z_m}$, $A_{nm} = \alpha_{mn} + \frac{1}{2}i\frac{\partial P_{mn}}{\partial z_m} = A^*_{mn}$ — матрица A_{mn} эрмитова, спектр λ', ω/ω_M — real (!), $\frac{\partial P_{mn}}{\partial z_m} = \frac{\partial}{\partial z_m}P(z_m, z_n)$

Можно $A(zz') = \alpha(zz') - \frac{i}{2}\frac{\partial P(zz')}{\partial z}$, $A(z'z) = \alpha(z'z) - \frac{i}{2}\frac{\partial P}{\partial z}(z'z)$, $\frac{\partial P}{\partial z'} = -\frac{\partial P}{\partial z}$,

$A(z'z) = \alpha(zz') - \frac{i}{2}(-1)\frac{\partial P(zz')}{\partial z} = \alpha(zz') + \frac{1}{2}\frac{\partial P(zz')}{\partial z} = A^*(zz')$.

Ядро свойства kernel $A(zz')$ $\overset{\text{ур-е}}{\Rightarrow}$ $\alpha(zz') = \alpha(z'z)$, $P(zz') = P(z'z)$. $\alpha, P \sim f(|z-z'|)$

Escaneado con CamScanner

$\frac{\partial P(zz')}{\partial z} = 8\pi M_s^2 c \int dk\, e^{-\beta k|z-z'|} \Lambda(k)\Lambda'(k)$, $\frac{\partial}{\partial z} e^{-\beta k|z-z'|} = \begin{cases} z>z' \\ z<z' \end{cases}$

$z>z'$, $\frac{\partial}{\partial z} e^{-\beta k|z-z'|} = \frac{\partial}{\partial z}(-) \cdot \frac{\partial}{\partial z} e^{-\beta k|z-z'|} = e^{-\beta k|z-z'|}(-\beta k)\frac{\partial}{\partial z}|z-z'|$

$z>z'$, $\frac{\partial}{\partial z}|z-z'| = \frac{\partial}{\partial z}(z-z') = 1$; $z<z'$, $\frac{\partial}{\partial z}|z-z'| = \frac{\partial}{\partial z}(z'-z) = -1$

Def. $\theta(z-z') = \begin{cases} 1, z>z' \\ -1, z<z' \end{cases} = \mathrm{sign}(z-z')$, $\theta(0)=0$ условно, $\theta(x)$ – ноги.

$\frac{\partial P(zz')}{\partial z} = 8\pi M_s^2 c \int dk \Lambda\Lambda' \frac{\partial}{\partial z} e^{-\beta k|z-z'|} = 8\pi M_s^2 c (-\beta)\theta(z-z') \times$ + $\int dk\, k e^{-\beta k|z-z'|} \Lambda(k)\Lambda'(k)$

$\frac{\partial P(zz')}{\partial z} = -8\pi M_s^2 c \beta \theta(z-z') \int_0^\infty k e^{-\beta k|z-z'|} \Lambda(k)\Lambda'(k)$

$A(zz') = \alpha(zz') - \frac{i}{2} P'_z(zz') = 8\pi M_s^2 \beta \int dk\, e^{-\beta k|z-z'|}\left[I^2(k) + c^2\Lambda^2(k)\right] +$
$+ i 8\pi M_s^2 c\beta\theta(z-z') \int dk\, k e^{-\beta k|z-z'|} \Lambda(k)\Lambda'(k)$

In units $(8\pi M_s^2)$: $A(zz') = \beta \int dk\, e^{-\beta k|z-z'|}\left[I^2(k) + c^2\Lambda^2(k)\right] + \frac{i}{2} c\beta\theta(z-z') \int dk\, k e^{-\beta k|z-z'|}\Lambda(k)\Lambda'(k)$

Инт. уравнение $\lambda' S(z) = \int dz' A(zz') S(z')$. Далее $\lambda' \vec{S}(z) = \int dz' \alpha(zz') \vec{S}(z')$ (линейная)

Урав $\omega_n^{vol}(\vec{S}/\partial z)$ – term нормальных ω_0 – frequency (!!!).

При $N=50$ $\omega_0/2\pi$ нормировка ≈ 1.1 GHz ядро 0.6 GHz при $\beta = \frac{1000}{60}$, $c=0.5$, $\omega_n/2\pi = 56.3$ GHz.

Но спектр ω_n имеет зависимость от дин. матрицы

Вне N, с ростом N все частоты падают.

Сфокусированные цил. вихри (vortex state) при $\beta \gg 1$:

1) гибкие localized GM, even and odd ω_0 около $z=0$ plane

2) non-localized GM, $\vec{S}_n(z) \sim \cos q_n z, \sin q_n z$ с ω_n падающими с ростом q_n, как в BVMSW, где $q_n \parallel Oz \parallel \vec{M}_0$ (vortex core)

ω_n — bulk SW – quasi-continuous $\omega_n(q_z)$

GM localized doublet $= \omega_0$

n – mode number

при q_z^n, $\vec{q} \parallel Oz$

Это δ_{ij} - и δ-terms

Not. Сам. не видел "Drum modes" нигде на репозиторий. Куда ? Appl. Phys. Rev.? Commun. Physics?

Comments to Dobrovolskiy et al.

(27/IV) Работа над PHT-paper, SW/GM in thick vortex dots" после ген. version после описание GM-spectrum/wires и compare to Farkhad (также singul. $S_n(z)$)

(28/IV) Описание для сборника чем нашли singul. profiles $S_n(z)$ of vortex GM/wires. Либо после AP-DW sym." Мб to Nat. Commun. Вернулся напомнил редакцию в Nanoscale на "Py-dome shape sk-dots" мб (major revision) с переписью с Орл. и Eisen" что Nanoscale paper и new Py-dot $\vec{M}(\vec{r})$ singul. paper.

(29/IV) Вычисл $\omega_0(\beta)$ и ω_n long wires (GM-vortex-state wires accounting V. Core) с учетом back manipulations с учет vortex mass M_v, see Gris Sci. Rep. '2015

Vortex mass

$\frac{g}{\gamma}\omega \vec{S}(z) = -\lambda \frac{\partial^2 \vec{S}}{\partial z^2} - \mu \vec{S}(z) + \int dz' \left[\alpha(zz') - \omega^2 M_v(zz')\right]\vec{S}(z')$, $M_v(z,z') = \frac{\beta}{\pi^2} \sum \frac{1}{\nu_{nj}^2}$

$M_v(z,z')$ is nonlocal и выражается через $a_\nu(g,z)$, ω_ν – eigenmodes/eigen of azimuthal $m=\pm 1$ SW modes, $\vec{e}(\vec{g},z,t) = a_\nu(g,z)\cos(m\varphi - \omega t)$, $\Theta = \Theta_\nu + \Theta_{ee}$ $\Phi = \Phi_{ee}$ (SW)

В Gris' Sci. Rep' 15 интегр. ур-е с $\alpha(zz')$, $M_\nu(zz')$ не решали,

учтем non-local terms in diagonal approximation по exchange с $\cos q_n z$

$\lambda \to \gamma$ в YD Sci. Rep. 15, $\gamma = M_s^2 (ka/L)^2 \left[\ln \frac{1}{kc} + 5/4 + \dots\right]$, см. (19/IV).

Eigenfn. с Φ $u_n(z) = \sqrt{\frac{2}{1+\delta_{n0}}} \cos q_n z$, $q_n = n\pi/(n\pi/L)$. $M_\nu^{nn'} = \int dz dz' M_\nu(zz') u_n(z) u_{n'}^*(z')$

$M_\nu^{nn} = M_\nu^n$. Renormalized GM freq. $\omega_n' = \frac{g}{2M_\nu^n}\left[\sqrt{1 + 4M_\nu^n \omega_n/g} - 1\right]$, $g = 2\pi \frac{M_s}{\gamma}$,

ω_n – GM frequency at $M_\nu = 0$.

Azim. SW $a_\nu(g,z)$, ω_ν для long cyl. wires найти точно. В Sci. Rep. 14,15 учитывали только $a_\nu(g)$ разложив по $J_1(kg)$. Для wires, z-coordinate нужна. Функции $\cos q_n z$ подставили, будет азим. SW могут быть "end-localized".

Ключ $a_\nu(g,z)$, функции в конечном V. Core radius $c = R_c/R$, даже $c(z)$ с $c = 0.17 - 0.63$

Вопрос, как надо учитывать если $a_\nu(g,z)$ очень сложно. В Sci. Rep. – брали $c \to 0$, ignoring V. Core

Escaneado con CamScanner

// $R = 60$ nm, $L = 1\,\mu$m, $R_c = 10 \div 38$ nm, azimuth. SW смешано переплетено с GM. Разделение eigenmodes на GM (localized near v. core) и azim. SW (localized free core) и инициирование v. core — плохое приближение для wires с $c = 0.17 \div 0.63$. M_V для GM можно учесть как fitting parameter, renormalizing $\omega_n(M_V = 0)$. Сравн. $M_V^n = M_V^0$ для main low frequency GM localized на wire ends и $M_V^n = M_V^P$ для non-localized GM, норм plane-waves $\cos qz$, $\sin qz$.

Аналогично: не вводим Thiele eq. для v. core center $\vec{S}(z,t) = \frac{\vec{X}(z,t)}{R}$ а работаем LLG eq. для $\theta(\vec{r},t) = a(s,z)\cos(m\varphi - \omega t)$, $\psi(\vec{r},t) = b(s,z)\sin(m\varphi - \omega t)$, где θ, ψ малые SW, $\Theta = \Theta_0 + \theta$, $\Phi = \Phi_0 + \psi$, $\vec{m}(\Theta, \Phi)$, получим систему связанных 2-х интегро-дифф. (или интегральных без exch.) уравнений для $a(s,z)$, $b(s,z)$. Все DF components $G_{\alpha\beta}(\vec{r},\vec{r}') \neq 0$ — высокая степень связности. Нет малого параметра. ERTE appl. для теории

So/IV. Причина квантования частот wire?

Simulations $\omega_n/2\pi$ = 0.3 1.5 2.4 3.0 GHz, $L = 1\,\mu$m

Experiment $\omega_n/2\pi$ = 0.33 1.5 2.5 3.3 GHz, L-? ($L = 20\,\mu$m?)

$\omega = \omega(\vec{k})$, discrete $\vec{k} = \vec{k}_m \to$ discrete $\omega_n = \omega(\vec{k}_n)$.

$\vec{k} = (k_s, k_\varphi, k_z)$. $k_s \approx$ const, no radial zeros of dyn $\vec{m}(s,\varphi,z)$ for cooper. azimuthal index $m = \pm 1$ при возбуждении by in-plane xy variable mw field, $k_\varphi = 1/R = $ const. $k_z^n \approx \frac{n\pi}{L}$ для plane waves along z, но для plane waves (not localized at wire ends $z = \pm L/2$), $\Delta\omega_n \sim \omega_M \left(\frac{l_e}{L}\right)^2$, $l_e = 14$ nm, $L = 1\,\mu$m, $\frac{\Delta\omega_n}{2\pi} = 56{,}3\ \text{GHz}\left(\frac{14\,\text{nm}}{1000\,\text{nm}}\right)^2 = 56{,}3\ \text{GHz} \cdot 2\cdot 10^{-4} \approx 0{,}011$ GHz — очень мало по сравн. с simulations / exp., где $\Delta\omega_n/2\pi \approx 1$ GHz.

→ Вывод: квантование ω_n возможно только за счет $\mathrm{Im}(k_z)$, т.е. из-за localized $\vec{S}(z,t)$, или азим. SW at wire ends. Это подтверждается слабой зависимостью ω_n от L.



Но если ω_n-quantized из-за wells at wire ends, spin modes are strongly localized, то членами $\frac{1}{\beta^2}\frac{\partial^2 \vec{S}}{\partial z^2}$ и перекрестного члена, $\frac{\partial \vec{S}}{\partial z}, \frac{\partial \vec{S}}{\partial z^2}$ можно $\sim \beta, \beta^2$

(*) $(\lambda+\mu)\vec{S}(z) = -\gamma\frac{\partial^2 \vec{S}}{\partial z^2} + \int dz' \chi(zz')\vec{S}(z') - \int dz' \frac{\partial \chi(zz')}{\partial z}\frac{\partial \vec{S}(z')}{\partial z'} + \frac{1}{2}\int dz' \frac{\partial P(zz')}{\partial z}\vec{S}(z')\times\hat{z}$

$\frac{1}{\pi}\overline{W}[\vec{S}(z),\vec{S}'(z)] = -\frac{1}{2}\mu\vec{S}^2 + \frac{1}{2}\gamma\left(\frac{\partial \vec{S}}{\partial z}\right)^2 + \frac{1}{2}\int dz' \chi(zz')\vec{S}(z')\cdot\vec{S}(z) + \frac{1}{2}\int dz' \beta(zz')\left(\frac{\partial \vec{S}}{\partial z}\cdot\frac{\partial \vec{S}}{\partial z'}\right) - \frac{1}{2}\int dz' P(zz')\left[\frac{\partial \vec{S}(z)}{\partial z}\times\vec{S}(z')\right]\cdot\hat{z}$

μ и γ - exchange $\|\hat{s}$ и $\|\hat{z}$, $\chi(zz')$, $\beta(zz')$, $P(zz')$ - volume magnetostatics accounting finite v. core $c = c(z)$.

$\chi(zz')$ из-за $\left(\frac{\partial m_x}{\partial x}, \frac{\partial m_y}{\partial y}\right)$, $\beta(zz')$ из-за $\left(\frac{\partial m_z}{\partial z}\right)$, $P(zz')$ cross-term $\left(\frac{\partial \vec{m}}{\partial s}\right)$ и $\frac{\partial m_z}{\partial z}$ в exp.

$\mathrm{div}\vec{m} = \frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y} + \frac{\partial m_z}{\partial z}$. $\beta(zz')$ и $P(zz') \neq 0$ только из-за $c \neq 0$, где $m_z \neq 0$, v. core

$W = \frac{1}{\pi}V\int dz\,\overline{w}$. Energy density $\overline{w}$ и Thiele eq. для $\vec{S}(z,\omega) = \vec{S}(z)$ дают самое для vortex-ground state of cylindrical wire, length L, radius R.

Далее 25/IV применяем $\beta(zz')$ и γ-члены, что позволяет сделать GD и интегральную eq. для $\vec{S}(z) \to S(z)$, решить их численно и найти doublet of localized modes at $\omega_0(\beta)$ - the lowest eigenfrequency. Но все наши ω_n - are localized at wire ends, а не только ω_0-mode!

Т.к. $\frac{\partial \beta(zz')}{\partial z} = -\frac{\partial \beta(zz')}{\partial z'}$, то в β-член можно перейти и $\frac{\partial^2 \beta}{\partial z^2}\vec{S}(z')$ избавиться от $\frac{\partial \vec{S}}{\partial z}/\frac{\partial \vec{S}}{\partial z'}$

Переход к интегр. eq. для $\vec{S}(z)$ меняет только γ-член.

Local approx., $\beta \to \infty$: $z' \to z$ в $\vec{S}(z')$: $\frac{1}{\pi}\overline{W} = -\frac{1}{2}\mu\vec{S}^2 + \frac{1}{2}\gamma\left(\frac{\partial \vec{S}}{\partial z}\right)^2 + \frac{1}{2}\int dz' \chi(zz')\vec{S}^2(z) + \frac{1}{2}\int dz' \beta(zz')\left(\frac{\partial \vec{S}}{\partial z}\right)^2 - \frac{1}{2}\int dz' P(zz')\left[\frac{\partial \vec{S}}{\partial z}\times\vec{S}(z)\right]_z$

Def. $\tilde{\chi}(z) = \int dz' \chi(zz')$, $\tilde{\beta}(z) = \int dz' \beta(zz')$, $\tilde{P}(z) = \int dz' P(zz')$

1/2

$$\frac{1}{\pi}\bar{W} = -\frac{1}{2}\mu\vec{S}^2 + \frac{1}{2}\gamma\left(\frac{\partial\vec{S}}{\partial z}\right)^2 + \frac{1}{2}\vec{\varkappa}(z)\vec{S}^2(z) + \frac{1}{2}\nu(z)\left(\frac{\partial\vec{S}}{\partial z}\right)^2 - \frac{1}{2}\pi(z)\left[\frac{\partial\vec{S}}{\partial z}\times\vec{S}(z)\right]\cdot\hat{z}$$

$$\frac{1}{\pi}\frac{\delta W}{\delta\vec{S}} = \frac{1}{\pi}\frac{\partial W}{\partial\vec{S}} - \frac{1}{\pi}\frac{\partial}{\partial z}\frac{\partial W}{\partial(\vec{S}')} = -\mu\vec{S} + \vec{\varkappa}(z)\vec{S} - \gamma\frac{\partial^2\vec{S}}{\partial z^2} + \frac{\delta}{\delta\vec{S}}[\ell] + \frac{\delta}{\delta\vec{S}}[P]$$

$$\frac{\delta}{\delta\vec{S}}[\ell] = \left(\frac{\partial}{\partial\vec{S}} - \frac{\partial}{\partial z}\frac{\partial}{\partial(\vec{S}')}\right)\left[\frac{1}{2}\nu(z)(\vec{S}')^2\right] = -\frac{\partial}{\partial z}\frac{\partial}{\partial(\vec{S}')}\left[\frac{1}{2}\nu(z)(\vec{S}')^2\right] = -\frac{\partial}{\partial z}\left[\nu(z)\vec{S}'\right] = -\nu'\vec{S}' - \nu\vec{S}''(z)$$

$$\frac{\delta}{\delta\vec{S}}[P] = \left(\frac{\partial}{\partial\vec{S}} - \frac{\partial}{\partial z}\frac{\partial}{\partial(\vec{S}')}\right)\left[-\frac{1}{2}\pi(z)\left[\vec{S}'(z)\times\vec{S}\right]\cdot\hat{z}\right] = -\frac{1}{2}\pi(z)\frac{\partial}{\partial\vec{S}}\left[\vec{S}'\times\vec{S}\right]\cdot\hat{z} +$$

$$+\frac{\partial}{\partial z}\frac{\partial}{\partial(\vec{S}')}\frac{1}{2}\pi(z)\left[\vec{S}'(z)\times\vec{S}\right]\cdot\hat{z}. \qquad \hat{z}\cdot\left[\vec{S}'\times\vec{S}\right] = \vec{S}'\cdot(\vec{S}\times\hat{z}) = \vec{S}'\cdot(\hat{z}\times\vec{S}')$$

$$\frac{\delta}{\delta\vec{S}}[P] = -\frac{1}{2}\pi(z)\frac{\partial}{\partial\vec{S}}\left[\vec{S}\cdot(\hat{z}\times\vec{S}')\right] + \frac{\partial}{\partial z}\frac{\partial}{\partial(\vec{S}')}\left[\frac{1}{2}\pi(z)\vec{S}'\cdot(\vec{S}\times\hat{z})\right] = -\frac{1}{2}\pi(z)(\hat{z}\times\vec{S}') +$$

$$+\frac{\partial}{\partial z}\left[\frac{1}{2}\pi(z)(\vec{S}\times\hat{z})\right] = -\frac{1}{2}\pi(\hat{z}\times\vec{S}') + \frac{1}{2}\pi'(z)(\vec{S}\times\hat{z}) + \frac{1}{2}\pi(\vec{S}'\times\hat{z}) = \pi(\vec{S}'\times\hat{z}) + \frac{1}{2}\pi'(\vec{S}\times\hat{z})$$

$$\lambda\vec{S} = \frac{1}{\pi}\frac{\delta W}{\delta\vec{S}} = \left[-\mu + \vec{\varkappa}(z)\right]\vec{S}(z) - \gamma\vec{S}''(z) - \nu'\vec{S}' - \nu\vec{S}''(z) + \pi(\vec{S}'\times\hat{z}) + \frac{1}{2}\pi'(\vec{S}\times\hat{z})$$

$$(\lambda+\mu)\vec{S}(z) = \vec{\varkappa}(z)\vec{S}(z) - \gamma\vec{S}''(z) - \nu'(z)\vec{S}'(z) - \nu(z)\vec{S}''(z) + \pi(\vec{S}'\times\hat{z}) + \frac{1}{2}\pi(z)(\vec{S}\times\hat{z})$$

Local approx.

$$(\lambda+\mu)\vec{S}(z) = \vec{\varkappa}(z)\vec{S}(z) + \frac{1}{2}\pi'(z)(\vec{S}\times\hat{z}) - \nu'(z)\vec{S}'(z) + \pi(\vec{S}'\times\hat{z}) - \gamma\vec{S}''(z) - \nu(z)\vec{S}''(z)$$

Ке сводится к 1D-ому $\lambda'\vec{S}(z) = \left[-\gamma\frac{\partial^2}{\partial z^2} + U(z)\right]\vec{S}(z)$, $\lambda = g\omega/\pi$.

Даже при $\pi(z)=0$, $\nu(z)=0$, $(\lambda+\mu)\vec{S} = \vec{\varkappa}(z)\vec{S} + \pi(z)(\vec{S}'\times\hat{z}) - [\gamma+\nu(z)]\vec{S}''$ -

не сводится к ур-ю $\gamma\to\gamma+\nu(z)$ - variable и $(\vec{S}'\times\hat{z})$ - term. $\lambda' = \lambda+\mu$.

ур-ие получается лишь при $\pi(z)=0$, $\nu(z)=0$, что нереалистично.

Вывод: интегро-дифф. ур-е $(*)$ для $\vec{S}(z)$ не решить, local approx. тоже не дает (дифф. ур. с variable coeffs.), но остается решить Фурье, в соотв. переходим от $\vec{S}$ к $S = S_x + iS_y$, где P-term сов. $2S/\sqrt{\pi}\ \varkappa(zz') \to \varkappa(zz') - \frac{i}{2}\frac{\partial P(zz')}{\partial z}$

Метод Фурье. 2-term в 1D ур-ии. в виде $-\frac{\partial z(zz')}{\partial z}\frac{\partial S(z')}{\partial z'}$

$$z(zz') = 2\pi M_S^2\frac{1}{\beta}\int_0^\infty dk\, e^{-\beta k|z-z'|}\Lambda^2(k). \qquad P(zz') = 8\pi M_S^2 c\int dk\, e^{-\beta k|z-z'|}\Lambda(k)\Lambda'(k)$$

$$\frac{\partial z(zz')}{\partial z} = -2\pi M_S^2\,\Theta(z-z')\int dk\,k\,e^{-\beta k|z-z'|}\Lambda^2, \qquad \frac{\partial P(zz')}{\partial z} = -8\pi M_S^2 c\beta\,\Theta(z-z')\int dk\,k\,e^{(\cdot)}\Lambda\Lambda'$$

$$\frac{\partial}{\partial z}e^{-\beta k|z-z'|} = -\beta k e^{-\beta k|z-z'|}\frac{\partial|z-z'|}{\partial z} = -\beta k\,\Theta(z-z')e^{-\beta k|z-z'|}, \quad \Theta(z) = \begin{cases}1, z>0\\ 0, z<0\end{cases}$$

$$S(z) = \sum_n C_n e^{iq_n z}. \quad \lambda'\sum_n C_n e^{iq_n z} = \gamma\sum_n q_n^2 C_n e^{iq_n z} + \int dz'\,\varkappa(zz')\sum_n C_n e^{iq_n z'} -$$

$$-\int dz'\frac{\partial z(zz')}{\partial z}\sum_n C_n iq_n e^{iq_n z'}. \quad \text{Умн. на } e^{-iq_{n'}z} \text{ и } \int_{-1/2}^{1/2}(\dots). \quad \int_{-1/2}^{1/2}dz\,e^{i(q_n-q_{n'})z} = \delta_{nn'}.$$

$$\lambda' C_{n'} = \gamma q_{n'}^2 C_{n'} + \int dz\,dz'\,\varkappa(zz')\sum_n C_n e^{iq_n z'}e^{-iq_{n'}z} - \int dz\,dz'\frac{\partial z(zz')}{\partial z}\sum_n C_n iq_n e^{iq_n z'}e^{-iq_{n'}z}$$

$\varkappa \to \varkappa - \frac{i}{2}\frac{\partial P}{\partial z} = A(zz')$. $A_{n'n} = \int dz\,dz'\,A(zz')e^{iq_n z'}e^{-iq_{n'}z}$, $S_{n'n} = \int dz\,dz'\frac{\partial z(zz')}{\partial z}e^{iq_n z'}e^{-iq_{n'}z}$

$$\lambda' C_{n'} = \gamma q_{n'}^2 C_{n'} + \sum_n A_{n'n}C_n - \sum_n S_{n'n}iq_n C_n, \quad \lambda' C_n = \gamma q_n^2 C_n + \sum_{n'} A_{nn'}C_{n'} - \sum_{n'} S_{nn'}iq_{n'}C_{n'}$$

$$\lambda' C_n = \sum_{n'}\left[\gamma q_n^2\delta_{nn'} + A_{nn'} - iq_{n'}S_{nn'}\right]C_{n'}, \quad \left(\lambda' C_n = \sum_{n'}\Lambda_{nn'}C_{n'}\right)$$

$\Lambda_{nn'} = \gamma q_n^2\delta_{nn'} + A_{nn'} - iq_{n'}S_{nn'}$. $A_{nn'} = K_{nn'} - \frac{i}{2}\left(\frac{\partial P}{\partial z}\right)_{nn'}$, $K_{nn'} = \int dz\,dz'\,\varkappa(zz')\times$

$\times e^{-iq_n z}e^{iq_{n'}z'}$

$S_{nn'} = \int dz\,dz'\frac{\partial z}{\partial z}(zz')e^{-iq_n z}e^{iq_{n'}z'}$. $S_{nn'} = \int dz\,dz'e^{-iq_n z}e^{iq_{n'}z'}(-2\pi M_S^2)\Theta(z-z')\int dk\,k\,e^{-\beta k|z-z'|}\Lambda^2$

$\left(\frac{\partial P}{\partial z}\right)_{nn'} = \int dz\,dz'e^{-iq_n z}e^{iq_{n'}z'}(-8\pi M_S^2 c\beta)\Theta(z-z')\int dk\,k\,e^{-k\beta|z-z'|}\Lambda\Lambda'$

Канон. $\int dz\,dz'\,e^{-iq_n z}e^{iq_{n'}z'}\Theta(z-z')e^{-k\beta|z-z'|} = \tilde{J}_{nn'}(k)$.

$\tilde{J}_{n'n} = \int dz\,dz'\,e^{-iq_{n'}z}e^{iq_n z'}\Theta(z-z')e^{-\beta k|z-z'|} = \int dz\,dz'e^{-iq_{n'}z'}e^{iq_n z}(-1)\Theta(z-z')e^{-\beta k|\cdot|} =$

$\tilde{J}_{n'n} = -\tilde{J}^*_{nn'}$. $A_{nn'} = K_{nn'} - \frac{i}{2}\left(\frac{\partial P}{\partial z}\right)_{nn'}$, $\left(\frac{\partial P}{\partial z}\right)_{nn'} = (-8\pi M_S^2 c\beta)\int dk\,k\,\Lambda\Lambda'\tilde{J}_{nn'}(k) = -\tilde{J}^*_{nn'}$ $\Rightarrow$

$A_{nn'} = K_{nn'} + i4\pi M_S^2 c\beta\int dk\,k\,\Lambda\Lambda'\tilde{J}_{nn'}(k)$, $S_{nn'} = (-2\pi M_S^2)\int dk\,k\,\Lambda^2(k)\tilde{J}_{nn'}(k)$

Escaneado con CamScanner

In units $(8\pi M_s^2)$: $\Lambda_{nn'} = \frac{1}{8\pi}\left(\frac{l_0}{L}\right)^2\left[\ln\frac{1}{c}+\frac{5}{4}+\dots\right]q_n^2\delta_{nn'} + K_{nn'} - \frac{i}{2}\left(\frac{\partial P}{\partial z}\right)_{nn'} - iq_{n'}S_{nn'}$

$K_{nn'} = \beta\int dk\left[I^2(k)+c^2\Lambda^2(k)\right]J_{nn'}(k)$, $\left(\frac{\partial P}{\partial z}\right)_{nn'} = -c\beta\int dk\,k\,\Lambda\Lambda'\bar{J}_{nn'}(k)$

$S_{nn'} = -\frac{1}{4}\int dk\,k\,\Lambda'^2(k)\bar{J}_{nn'}(k)$, $A_{nn'} = K_{nn'} + \frac{i}{2}c\beta\int dk\,k\,\Lambda\Lambda'\bar{J}_{nn'}(k)$

$M_{nn'} = i\bar{J}_{nn'}$, $M^*_{nn'} = -i\bar{J}^*_{nn'} = -i(-\bar{J}_{n'n}) = i\bar{J}_{n'n} = M_{n'n}$, т.е. матрица $i\bar{J}_{nn'}$ – эрмитова. $iS_{nn'} \sim i\bar{J}_{nn'}$ – эрмитова.

$\Lambda_{nn'} = \gamma q_n^2\delta_{nn'} + K_{nn'} + \frac{i}{2}c\beta\int dk\,k\,\Lambda\Lambda'\bar{J}_{nn'}(k) - iq_{n'}\left(+\frac{1}{4}\right)\int dk\,k\,\Lambda'^2\bar{J}_{nn'}(k)$

$\Lambda_{nn'} = \gamma q_n^2\delta_{nn'} + \frac{i}{2}\int dk\,\Lambda'(k)k\left[c\beta\Lambda(k)+\frac{1}{2}q_{n'}\Lambda\right]\bar{J}_{nn'}(k) + K_{nn'}$. Ок, но $q_{n'}$-term матрица эрмитова. Ок из 3-го члена.

$\int dz'\frac{\partial\beta(zz')}{\partial z}\frac{\partial S(z')}{\partial z'} = \left|z\to z'\right| = \int dz\frac{\partial\beta(z'z)}{\partial z'}\frac{\partial S(z)}{\partial z} = -\int dz\frac{\partial\beta(zz')}{\partial z}\frac{\partial S(z)}{\partial z}$

$S(z) = \sum_n c_n e^{iq_n z}$, $S(z') = \sum_n c_n e^{iq_n z'}$, $\frac{\partial S(z')}{\partial z'} = \sum_n c_n iq_n e^{iq_n z'} \longrightarrow$

$\to \frac{1}{2}\sum_n c_n iq_n e^{iq_n z'} + \frac{1}{2}\sum_{n'} c_{n'} iq_{n'} e^{iq_{n'}z'} = \frac{1}{2}\sum_n c_n iq_n e^{iq_n z'} + \frac{1}{2}\sum_m c_m iq_m e^{iq_m z'}$

$\int dz'\beta'_z(zz')\frac{1}{2}\left[\sum_n c_n iq_n e^{iq_n z'} + \sum_m c_m iq_m e^{iq_m z'}\right]e^{-iq_{n'}z} = \frac{1}{2}\int dz\,dz'\sum_n c_n q_n\beta' e^{iq_n z'}e^{-iq_{n'}z} +$

$+\frac{1}{2}\int dz\,dz'\sum_m c_m iq_m\beta' e^{iq_m z'}e^{-iq_{n'}z} = \frac{1}{2}\sum_n c_n iq_n S_{n'n} + \frac{1}{2}\sum_m c_m iq_m S_{n'm} =$

$= \frac{1}{2}\sum_{n'} c_{n'} iq_{n'} S_{nn'} + \frac{1}{2}\sum_{n'} c_{n'} iq_{n'} S_{nn'} = \left(\frac{1}{2}\right)\sum_{n'} iq_{n'} S_{nn'} c_{n'}$.

$\delta\Lambda_{nn'} = iq_{n'}S_{nn'}$, $\delta\Lambda_{n'n} = iq_n S_{n'n} = iq_n(-1)S^*_{nn'}$, $\delta\Lambda^*_{nn'} = -iq_{n'}S^*_{nn'}$

$\delta\Lambda_{nn'} \neq \delta\Lambda^*_{nn'}$, т.к. $q_n \neq q_{n'}$. Нужно всего $q_{n'} \to \frac{1}{2}(q_n+q_{n'})$ – но это эвристика!

$\int dz'\frac{\partial\beta}{\partial z}S'(z') = (-)\int dz'\frac{\partial\beta}{\partial z'}S'(z') = \left|dz'\frac{\partial\beta}{\partial z'} = dv,\ u = S' - \text{нет.},\ dz'\frac{\partial S}{\partial z'} = dv,\ u = \frac{\partial\beta}{\partial z},\ du = \frac{\partial}{\partial z'}u\,dz'\right| =$

$= \frac{\partial\beta}{\partial z}S(z')\Big|_{-}^{+} - \int S(z')\frac{\partial u}{\partial z'}dz' = \frac{\partial\beta(zz')}{\partial z}S(z')\Big|_{-1/2}^{1/2} - \int dz'\frac{\partial^2\beta}{\partial z\partial z'}S(z')$

$\frac{\partial\beta}{\partial z}S(z')\Big|_{-1/2}^{1/2} = \beta'\left(z,\frac{1}{2}\right)S(1/2) - \beta'\left(z,-\frac{1}{2}\right)S(-1/2) \neq 0$, т.к. $S(\pm 1/2)\neq 0$ в общем случае.

Без повернутых членов $\int dz'\frac{\partial\beta(zz')}{\partial z}\frac{\partial S(z')}{\partial z'} = -\int dz'\frac{\partial^2\beta(zz')}{\partial z\partial z'}S(z')$

$\Lambda_{nn'} = \gamma q_n^2\delta_{nn'} + K_{nn'} + \frac{i}{2}\int dk\,k\,\Lambda'(k)\left[c\beta\Lambda(k)+\frac{1}{4}(q_n+q_{n'})\Lambda'(k)\right]\bar{J}_{nn'}(k)$ – уравнение.

$\bar{J}_{nn'}(k) = \int dz\,dz'\,\theta(z-z')e^{-\beta k|z-z'|}e^{-iq_n z}e^{iq_{n'}z'}$, $q_n = 2\pi n$. $\theta(x) = \text{sign}(x)$.

$\theta(z)e^{-x|z|} = \begin{cases} -e^{xz}, & z<0 \\ e^{-xz}, & z>0 \\ 0, & z=0 \end{cases}$, $\bar{J}_{nn'} = \int dz\int dz'\,e^{-\beta k(z-z')}e^{-iq_n z}e^{iq_{n'}z'} +$

$+\int dz\,dz'(-1)e^{\beta k(z-z')}e^{-iq_n z}e^{iq_{n'}z'} = \int dz\,e^{+(-\beta k+iq_n)z}\cdot\int_{z>z'}dz'\,e^{+(\beta k+iq_{n'})z'} -$

$-\int dz\,e^{(\beta k-iq_n)z}\int_{z<z'}dz'\,e^{+(-\beta k+iq_{n'})z'} = \frac{(-1)}{(\beta k+iq_{n'})}e^{-(\beta k+iq_{n'})z}\Big|_{-1/2}^{1/2}\cdot\frac{1}{(x+iq_{n'})}e^{(x+iq_{n'})z'}\Big|_{-1/2}^{1/2} -$

$-\frac{1}{(x-iq_n)}e^{(x-iq_n)z}\frac{e^{z'(-x+iq_{n'})}}{(-x+iq_{n'})}\Big|_{-1/2}^{1/2} = \frac{(-1)}{(x+iq_n)(x+iq_{n'})}\left[e^{-(\beta k+iq_{n'})\frac{1}{2}} - e^{+(\beta k+iq_{n'})\frac{1}{2}}\right]\times$

$\times\left\{e^{(x+iq_{n'})\frac{1}{2}} - e^{-(x+iq_{n'})\frac{1}{2}}\right\}\Big] + \frac{1}{(x-iq_n)(x-iq_{n'})}\left\{e^{(x-iq_n)\frac{1}{2}} - e^{-(x-iq_n)\frac{1}{2}}\right\}\left\{e^{(-x+iq_{n'})\frac{1}{2}} - e^{-(-x+iq_{n'})\frac{1}{2}}\right\} =$

$= \left|e^{\pm iq_n/2} = e^{\pm i\pi n} = (-1)^n = \cos(\pi n)\right| = \frac{(-1)}{(x+iq_n)(x+iq_{n'})}(-1)^n\left\{e^{-\beta k/2} - e^{\beta k/2}\right\}(-1)^{n'}\left\{e^{x/2} - e^{-x/2}\right\} +$

$+\frac{1}{(x-iq_n)(x-iq_{n'})}(-1)^{n+n'}\left\{e^{x/2} - e^{-x/2}\right\}\left\{e^{-x/2} - e^{x/2}\right\} = \frac{(-1)(-1)^{n+n'}}{(x+iq_n)(x+iq_{n'})}(-4)\sinh^2\frac{x}{2} +$

$+\frac{(-1)^{n+n'}}{(x-iq_n)(x+iq_{n'})}(-4)\sinh^2\frac{x}{2} = 4(-1)^{n+n'}\sinh^2\frac{x}{2}\left[\frac{1}{(x+iq_n)(x+iq_{n'})} - \frac{1}{(x-iq_n)(x-iq_{n'})}\right]$

$\frac{1}{(\cdot)} - \frac{1}{(\cdot)^*} = 2i\cdot\text{Im}\frac{1}{(x+iq_n)(x+iq_{n'})} = 2i\,\text{Im}\cdot\frac{(x-iq_n)(x-iq_{n'})}{(x^2+q_n^2)(x^2+q_{n'}^2)} =$

$= \frac{2i}{(\cdot)(\cdot)}\text{Im}\left[x^2 - q_n q_{n'} - iq_n x - iq_{n'}x\right] = \frac{2i}{(x^2+q_n^2)(x^2+q_{n'}^2)}(-1)\times(q_n+q_{n'})$

Escaneado con CamScanner

$$\bar{J}_{nn'} = 4(-1)^{n+n'} \sh^2\frac{\chi}{2}\,(-2i\chi)\frac{(q_n+q_{n'})}{(\chi^2+q_n^2)(\chi^2+q_{n'}^2)} = -8i(-1)^{n+n'}\sh^2\frac{\chi}{2}\,\frac{\chi(q_n+q_{n'})}{(\chi^2+q_n^2)(\chi^2+q_{n'}^2)}$$

Если в $\bar{J}_{nn'}$ раскрыть замену $\theta(z-z')$ на 1, то $\bar{J}_{nn'} \to J_{nn'}$ от 20/IV

$$\frac{1}{(\cdot)} + \frac{1}{(\cdot)^*} = 2\,\mathrm{Re}\,\frac{1}{(\chi+iq_n)(\chi+iq_{n'})} = 2\cdot\frac{(\chi^2 - q_n q_{n'})}{(\chi^2+q_n^2)(\chi^2+q_{n'}^2)}$$

$$J_{nn'} = 4(-1)^{n+n'}\sh^2\frac{\chi}{2}\cdot 2\,\mathrm{Re}\,\frac{1}{(\chi+iq_n)(\chi+iq_{n'})} = 8(-1)^{n+n'}\sh^2\frac{\chi}{2}\,\frac{(\chi^2-q_n q_{n'})}{(\chi^2+q_n^2)(\chi^2+q_{n'}^2)},\ \chi=\beta k$$

Вот так да! Сильно отлич. от $\bar{J}_{nn'}$ 20/IV. Где $\delta_{nn'}$ член?

Видимо здесь $\bar{J}_{nn'}$, $J_{nn'}$ вычислены неверно // Надо $\int dz \int dz' f(zz') =$

$$= \int_{-1/2}^{1/2} dz \left[\int_{-1/2}^{z} dz' f(zz') + \int_{z}^{1/2} dz' f(zz')\right]$$ ($z'<z$ / $z'>z$) как 24/VIII 2014г.

$R_{nn'} = iA_{nn'}\bar{J}_{nn'}$, $A_{nn'} = A_{n'n}$ (real). $R_{n'n} = iA_{n'n}\bar{J}_{n'n} = iA_{nn'}(-1)\bar{J}^*_{nn'}$, $R^*_{nn'} = -iA_{nn'}\bar{J}^*_{nn'}$

$R^*_{nn'} = R_{n'n} \to R_{nn'}$ - Эрмитова.

$$\bar{J}_{nn'} = \int_{-1/2}^{1/2} dz \int_{1/2}^{1/2} dz'\,\theta(z-z')\,e^{-\chi|z-z'|}e^{-iq_n z}e^{iq_{n'}z'} \Rightarrow \int_{-1/2}^{1/2} dz' \theta(z-z')e^{-\chi|z-z'|}e^{-iq_n z}e^{iq_{n'}z'} =$$

$$= \left|z-z'=\zeta\right| = \int_{z'}^{z} d\zeta\,(+1)\,e^{-\chi(z-z')}e^{-iq_n z}e^{iq_{n'}z'} + \int_{z}^{1/2} d\zeta\,(-1)e^{\chi(z-z')}e^{-iq_n z}e^{iq_{n'}z'} =$$

$$= e^{-\chi z}e^{-iq_n z}\int_{-1/2}^{z} dz'\,e^{(\chi+iq_{n'})z'} - e^{\chi z}e^{-iq_n z}\int_{z}^{1/2} dz'\,e^{(-\chi+iq_{n'})z'} = e^{-iq_n z}\left[e^{-\chi z}\times\right.$$

$$\times\frac{1}{(\chi+iq_{n'})}e^{(\chi+iq_{n'})z'}\Big|_{-1/2}^{z} - e^{\chi z}\frac{1}{(-\chi+iq_{n'})}e^{(-\chi+iq_{n'})z'}\Big|_{z}^{1/2}\Big] = e^{-iq_n z}\Big[e^{-\chi z}\frac{1}{(\chi+iq_{n'})}\times$$

$$\times\left\{e^{(\chi+iq_{n'})z} - e^{-(\chi+iq_{n'})\frac{1}{2}}\right\} - e^{\chi z}\frac{1}{(-\chi+iq_{n'})}\left\{e^{(-\chi+iq_{n'})\frac{1}{2}} - e^{(-\chi+iq_{n'})z}\right\}\Big] =$$

$$= e^{-iq_n z}\left[\frac{1}{(\chi+iq_{n'})}\left\{e^{iq_{n'}z} - e^{-(\chi+iq_{n'})\frac{1}{2}}e^{-\chi z}\right\} - \frac{1}{(-\chi+iq_{n'})}\left\{e^{\chi z}e^{(-\chi+iq_{n'})\frac{1}{2}} - e^{iq_{n'}z}\right\}\right] =$$

$$= e^{-iq_n z}\left[e^{iq_{n'}z}\left(\frac{1}{\chi+iq_{n'}} + \frac{1}{(-\chi+iq_{n'})}\right) - \frac{1}{(\chi+iq_{n'})}e^{-(\chi+iq_{n'})\frac{1}{2}}e^{-\chi z} - \frac{1}{(-\chi+iq_{n'})}e^{\chi z}e^{(-\chi+iq_{n'})\frac{1}{2}}\right]$$

$$\frac{1}{\chi+iq_{n'}} + \frac{1}{(-\chi+iq_{n'})} = \frac{\chi - iq_{n'} + (-\chi - iq_{n'})}{\chi^2+q_{n'}^2} = -2i\frac{q_{n'}}{\chi^2+q_{n'}^2}, \quad \int dz\, e^{i(q_n-q_{n'})z} = \delta_{nn'}.$$

$$\bar{J}_{nn'} = \int dz\, e^{-iq_n z}\left[e^{iq_{n'}z}(-2i)\frac{q_{n'}}{\chi^2+q_{n'}^2} - \frac{1}{(\chi+iq_{n'})}e^{-(\chi+iq_{n'})\frac{1}{2}}e^{-\chi z} - \frac{1}{(-\chi+iq_{n'})}e^{(-\chi+iq_{n'})\frac{1}{2}}e^{\chi z}\right] =$$

$$= (-2i)\frac{q_{n'}}{\chi^2+q_{n'}^2}\delta_{nn'} - \frac{1}{(\chi+iq_{n'})}e^{-(\chi+iq_{n'})\frac{1}{2}}\int dz\, e^{-(\chi+iq_n)z} - \frac{1}{(-\chi+iq_{n'})}e^{(-\chi+iq_{n'})\frac{1}{2}}\int dz\, e^{(\chi-iq_n)z} =$$

$$= (-2i)\frac{q_{n'}}{\chi^2+q_{n'}^2}\delta_{nn'} - \frac{1}{(\chi+iq_{n'})}e^{-\frac{\chi}{2}}(-1)^{n'}\frac{(-1)}{(\chi+iq_n)}e^{-(\chi+iq_n)z}\Big|_{-1/2}^{1/2} - \frac{1}{(-\chi+iq_{n'})}e^{-\frac{\chi}{2}}(-1)^{n'}\frac{1}{(\chi-iq_n)}e^{(\chi-iq_n)z}\Big|_{-1/2}^{1/2} =$$

$$= (-2i)\frac{q_n\delta_{nn'}}{\chi^2+q_n^2} + \frac{1}{(\chi+iq_{n'})(\chi+iq_n)}e^{-\chi/2}(-1)^{n'}\left[e^{-(\chi+iq_n)\frac{1}{2}} - e^{(-\chi+iq_n)(-\frac{1}{2})}\right] +$$

$$+ \frac{(-1)^{n'}}{(\chi-iq_n)(\chi+iq_{n'})}e^{-\chi/2}\left[e^{(\chi-iq_n)\frac{1}{2}} - e^{(\chi-iq_n)(-\frac{1}{2})}\right] = -2i\frac{q_n\delta_{nn'}}{\chi^2+q_n^2} + \frac{e^{-\chi/2}(-1)^{n'+n}}{(\chi+iq_n)(\chi+iq_{n'})}\left[e^{-\frac{\chi}{2}} - e^{\frac{\chi}{2}}\right] +$$

$$+ \frac{(-1)^{n+n'}}{(\chi-iq_n)(\chi-iq_{n'})}e^{-\chi/2}\left[e^{\chi/2} - e^{-\chi/2}\right] = -2i\frac{q_n\delta_{nn'}}{\chi^2+q_n^2} + (-1)^{n+n'}(1-e^{-\chi})\left[\frac{1}{(\chi-iq_n)(\chi-iq_{n'})} + \frac{1}{(\chi+iq_n)}\times\right.$$

$$\left.\times\frac{1}{(\chi+iq_{n'})}\right]$$

$$\frac{1}{(\cdot)} - \frac{1}{(\cdot)^*} = 2i\,\mathrm{Im}\,\frac{1}{(\chi-iq_n)(\chi-iq_{n'})} = 2i\,\mathrm{Im}\,\frac{(\chi+iq_n)(\chi+iq_{n'})}{(\chi^2+q_n^2)(\chi^2+q_{n'}^2)} = \frac{2i}{(\cdot)(\cdot)}\chi(q_n+q_{n'})$$

$$\bar{J}_{nn'} = -2i\frac{q_n\delta_{nn'}}{(\chi^2+q_n^2)} + (-1)^{n+n'}(1-e^{-\chi})\frac{2i\chi(q_n+q_{n'})}{(\chi^2+q_n^2)(\chi^2+q_{n'}^2)},\quad \chi=\beta k,\ \bar{J}_{nn'} = \bar{J}_{nn'}(k)$$

В $A_{nn'}$ входит $i\bar{J}_{nn'} = \frac{2q_n\delta_{nn'}}{(\chi^2+q_n^2)} - 2(1-e^{-\chi})(-1)^{n+n'}\frac{\chi(q_n+q_{n'})}{(\chi^2+q_n^2)(\chi^2+q_{n'}^2)}$, $\chi=\beta k$.

$$J_{nn'} = \frac{2\chi\delta_{nn'}}{(\chi^2+q_n^2)} - 2(1-e^{-\chi})(-1)^{n+n'}\frac{(\chi^2-q_n q_{n'})}{(\chi^2+q_n^2)(\chi^2+q_{n'}^2)}$$

Вставил $\bar{J}_{nn'}$ в MathCad „Vortex_TM_long_wires.mcd". Считает быстро.

ω_0 теперь singlet. $\omega_0/2\pi \approx 4.2$ GHz. $\Delta\omega_n$ увеличилось по сравн. с ним. пр. расчетом. $S(z)$ сильно локализована при $z=\pm 1/2$.

Проверить баг 4D. Надо $\vec{S} \to S = S_x + iS_y$ в ин. форме. $\overline{W}[S, S']$.

Escaneado con CamScanner

(2/IV) Лекция в саду, поносили дрова. E-mails to Wrachtrup и Iranmehr (postdoc candid.)

$\vec{S}^2 = S\bar{S} = (S_x + iS_y)(S_x - iS_y) = S_x^2 + S_y^2$. $(\vec{S}')^2 = S'\bar{S}'$. Проверка s-eq

$\vec{S}(z)\vec{S}(z') = S(z)\bar{S}(z') = [S_x(z) + iS_y(z)][S_x(z') - iS_y(z')] = S_x(z)S_x(z') + S_y(z)S_y(z') +$
$+ i[S_y(z)S_x(z') - S_y(z')S_x(z)] = S_x(z)S_x(z') + S_y(z)S_y(z')$ если $z \rightleftarrows z'$ сим. одна

б: $\vec{S}'\vec{S}' = \vec{S}'(z)\vec{S}'(z') = S'(z)\bar{S}'(z')$ если одна сим. отн. $z \rightleftarrows z'$.

P: $\bar{S}'(z)S(z') - S'(z)\bar{S}(z') = [S_x'(z) - iS_y'(z)][S_x(z') + iS_y(z')] - [S_x'(z) + iS_y'(z)][S_x(z') - iS_y(z')]$

$= -2iS_y'(z)S_x(z') + 2iS_x'(z)S_y(z') = 2i[S_x'(z)S_y(z') - S_y'(z)S_x(z')] =$

$= 2i\cdot\hat{z}\cdot\left[\frac{\partial\vec{S}(z')}{\partial z} \times \vec{S}(z')\right]$.

$\frac{1}{\pi}W[S,S'] = -\frac{1}{2}\mu S\bar{S} + \frac{\gamma}{2}S'\bar{S}' + \frac{1}{2}\int dz'\varkappa(zz')S(z)\bar{S}(z') + \frac{1}{2}\int dz'\vartheta(zz')S'(z)\bar{S}'(z) - \frac{1}{2}\int dz'P\left[\frac{\bar{S}'(z)S(z') - S'(z)\bar{S}(z')}{2i}\right]$

Eq. Thiele $\vec{g}\times\dot{\vec{S}} = -\frac{\delta W}{\delta\vec{S}}$, $\frac{\delta W}{\delta\vec{S}} = \frac{\partial W}{\partial S} - \frac{\partial}{\partial z}\frac{\partial W}{\partial(S')}$. $\left\{\begin{matrix} g\dot{S}_y = -\frac{\delta W}{\delta S_x} \\ -g\dot{S}_x = -\frac{\delta W}{\delta S_y}\end{matrix}\right\}$. $g\dot{S}_y - ig\dot{S}_x = -ig(\dot{S}_x + i\dot{S}_y) = -ig\dot{S}$

$-ig\dot{S} = (-1)\left[\frac{\delta W}{\delta S_x} + i\frac{\delta W}{\delta S_y}\right]$, $\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$, $\frac{\partial}{\partial\bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)$

$\frac{\delta W}{\delta S_x} + i\frac{\delta W}{\delta S_y} = 2\frac{\delta W}{\delta\bar{S}}$, $ig\dot{S} = 2\frac{\delta W}{\delta\bar{S}}$, $S(zt) = S(z)\exp(i\omega t)$, $\dot{S} = i\omega S$. $\dot{\bar{S}} = -i\omega\bar{S}$

$ig\dot{S} = ig(i\omega)S = -g\omega S$, $g\omega S = -2\frac{\delta W}{\delta\bar{S}}$. $-ig\dot{\bar{S}} = -ig(-i\omega)\bar{S} = 2\frac{\delta W}{\delta S}$

$\vec{g}\times\dot{\vec{S}} = \vec{g}\times(\vec{\omega}\times\vec{S}) = \omega g_z(-1)\vec{S}$, $g_z > 0$ (Sci. Rep. 14), $-\omega g_z\vec{S} = -\frac{\delta W}{\delta\vec{S}}$, $\lambda\vec{S} = \frac{1}{\pi}\frac{\delta W}{\delta\vec{S}}$

$g_z = g > 0$. $\vec{g} = g_z\hat{z}$. Изменим знак g. $g\omega S = 2\frac{\delta W}{\delta\bar{S}}$, $-ig\dot{S} = 2\frac{\delta W}{\delta\bar{S}}$, $ig\dot{\bar{S}} = 2\frac{\delta W}{\delta S}$

В Ring Sci. Rep '14 я ввел $\lambda = g\omega/\pi$, $\lambda S = \frac{2}{\pi}\frac{\delta W}{\delta\bar{S}}$ и $\lambda\bar{S} = \frac{2}{\pi}\frac{\delta W}{\delta S}$.

$\lambda S = 2\frac{\delta}{\delta\bar{S}}\left(\frac{W}{\pi}\right) = 2\left[\frac{\partial}{\partial\bar{S}} - \frac{\partial}{\partial z}\frac{\partial}{\partial(\bar{S}')}\right]\left(\frac{W}{\pi}\right) = 2\left[\frac{\partial}{\partial\bar{S}} - \frac{\partial}{\partial z}\frac{\partial}{\partial(\bar{S}')}\right]\left[-\frac{1}{2}\mu S\bar{S} + \frac{\gamma}{2}S'\bar{S}' + \right.$

$\left. + \frac{1}{2}\int dz'\varkappa S(z)\bar{S}(z') + \frac{1}{2}\int dz'\vartheta(z)S'(z)\bar{S}'(z') - \frac{1}{4i}\int dz'P\{\bar{S}'(z)S(z') - S'(z)\bar{S}(z')\}\right] =$

$= -\mu S - 2\frac{\partial}{\partial z}\frac{\partial}{\partial(\bar{S}')}\frac{1}{2}\gamma S'\bar{S}' + 2\frac{\partial}{\partial\bar{S}}\frac{1}{2}\int dz'\varkappa S(z')\bar{S}(z) - 2\frac{\partial}{\partial z}\frac{\partial}{\partial(\bar{S}')}\frac{1}{2}\int dz'\vartheta(z)S'(z')\bar{S}'(z') +$

$+ 2\left[\frac{\partial}{\partial\bar{S}} - \frac{\partial}{\partial z}\frac{\partial}{\partial(\bar{S}')}\right]\left(-\frac{1}{4i}\right)\int dz'P\{\bar{S}'(z)S(z') - S'(z)\bar{S}(z')\}$

$\lambda S = -\mu S - \gamma S'' + \int dz'\varkappa(zz')S(z') - \frac{\partial}{\partial z}\int dz'\vartheta(zz')S'(z') + 2\frac{\delta}{\delta\bar{S}}[\text{P-term}]$

$W_\varkappa \sim \int dz\,dz'\varkappa(zz')S(z)\bar{S}(z') = \int dz\,dz'\varkappa(zz')S(z)\bar{S}(z) = \int dz\left[\int dz'\varkappa(zz')\bar{S}(z')\right]$

W_ϑ - аналогично $W_\varkappa$.

$W_P \sim \int dz\,dz'\frac{i}{2}\left[\frac{\partial\vec{S}(z)}{\partial z} \times \vec{S}(z')\right]P(zz') = \int dz\,dz'P(zz')\hat{z}\cdot\left[\frac{\partial\vec{S}(z')}{\partial z'} \times \vec{S}(z)\right]$ ($W_\varkappa$)

Под знаком $\int dz\,dz'P(zz')$ $\bar{S}'(z)S(z') - S'(z)\bar{S}(z')$ и $\bar{S}'(z')S(z) - S'(z')\bar{S}(z)$ эквивалентны.

$2\left[\frac{\partial}{\partial\bar{S}} - \frac{\partial}{\partial z}\frac{\partial}{\partial(\bar{S}')}\right]\left(-\frac{1}{4i}\right)\int dz'P\left\{\begin{matrix}\bar{S}'(z)S(z') - S'(z)\bar{S}(z') \\ S'(z')S(z) - S'(z')\bar{S}(z)\end{matrix}\right\} =$

$= \left(\frac{-1}{2i}\right)\left\{\begin{matrix} -\partial/\partial z\int dz'P\,S(z') \\ -\int dz'P\,S'(z')\end{matrix}\right\}$, $\frac{\delta W_P}{\delta\bar{S}}$ будут эквивалентны, если $\int dz'\frac{\partial P}{\partial z}S(z') = \int dz'P\frac{\partial S}{\partial z'}$

Выберем $\int dz'\frac{\partial P}{\partial z}(zz')S(z')$ (ген. см. 25/IV)

$2\frac{\delta}{\delta\bar{S}}[\text{P-term}] = \frac{1}{2i}\int dz'\frac{\partial P}{\partial z}S(z') = -\frac{1}{2}\int dz'\frac{\partial P(zz')}{\partial z}S(z')$, что совп. с расчетом 25/IV

$\lambda S = -\mu S - \gamma S'' + \int dz'\left[\varkappa(zz') - i\frac{\partial P(zz')}{\partial z}\right]S(z') - \int dz'\frac{\partial\vartheta(zz')}{\partial z}S'(z')$ (*) SS предсказ. 25/IV

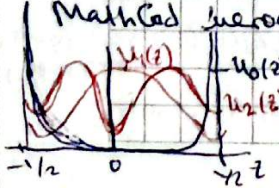
$\lambda = g\omega/\pi$

Это уд совпадает с уд в $\vec{S}$-представлении от 30/IV, где $\vec{S}\to S$, $\vec{S}(z)\times\hat{z}\to -iS(z)$

уд (*) использовал в методе Фурье 1/V, где $q_{n'} \to \frac{1}{2}(q_n + q_{n'})$ эвристика, чтоб сделать диаг. матрицу $\lambda_{nn'}$. Эрмитовой.

MathCad методом Фурье посчитал $\frac{\omega_0}{2\pi} = 4{,}147$ GHz, $\frac{\omega_1}{2\pi} = 5{,}407$, $\frac{\omega_2}{2\pi} = 5{,}537$, $\frac{\omega_3}{2\pi} = 5{,}776$ GHz (N=100) mat2z*

$S_n(z) = u_n e^{i\varphi_n(z)}$, $u_n(z) = |S_n(z)|$



Note: $S_0(z)$ localized near wire ends $z = \pm 1/2$, $S_1(z) \approx \cos(\pi z)$, $S_1(\pm\frac{1}{2}) \neq 0$

Из одной из $S_1(z), S_2(z), S_3(z)$ не $=0$, кроме $S_0(z)$, что ≈ 0 или $z = 0$, при $z = \pm 1/2$, или $S_n(\pm 1/2) \neq 0$, или $S_n'(\pm 1/2) \neq 0$

Escaneado con CamScanner

Какая vortex mass нужна, чтоб из $\omega_0/2\pi = 4{,}15$ GHz сделать 0,3 GHz simulated.

$\omega_n' = \frac{g}{2M_v'}\left[\sqrt{1+4M_v'\omega_n/g} - 1\right]$, $g = \frac{2\pi M_s}{\gamma}$. Пусть mass density $M_v = \frac{m}{\gamma^2}$ (на eg. диполе),

$\frac{g}{M_v} = \frac{2\pi M_s}{\gamma \cdot \frac{m}{\gamma^2}} = \frac{2\pi\gamma M_s}{m}$, $\omega_n \frac{M_v}{g} = \omega_n \frac{m}{2\pi\gamma M_s} = \frac{\omega_n}{\omega_M}\frac{2m}{2}$, ω_n - freq at $M_v = 0$.

$\omega_n' = \frac{\omega_M}{4m}\left[\sqrt{1+8m\frac{\omega_n}{\omega_M}} - 1\right] = \omega_n'(m)$, чтоб из $\frac{\omega_0}{2\pi} = 4{,}15$ сделать $\frac{\omega_0}{2\pi} = 0{,}3$ GHz

надо $m = 1200$ (!), т.е. vortex mass $M_v/L = M_v = \frac{1200}{\gamma^2}$ - super giant value.

В Sci. Rep '15 M_v достигала $10/\gamma^2$ при $L = 100$ nm. Сейчас $L = 1000$ nm.

3/V // Решаем спектр. задачу $\hat{O}s(z) = \lambda s(z)$, $\lambda = g\omega/\pi = \frac{2\pi M_s}{\gamma}\frac{\omega}{\pi} = \frac{\omega}{\omega_M}(8\pi M_s^2)$

$\lambda = \frac{\omega}{\omega_M}$, если оператор $\hat{O}s(z)$ in units $8\pi M_s^2$. Vortex mass

$\hat{O}s(z) = -\mu s - \eta s'' + \int dz'\left[\chi(zz') - \frac{1}{2}\frac{\partial P(zz')}{\partial z}\right]s(z') - \int dz' \frac{\partial \chi(zz')}{\partial z}\frac{\partial s(z')}{\partial z}$

Если eq. Thiele $\vec{g}\times\vec{s} = -\frac{\delta W}{\delta \vec{s}}$, $\lambda\vec{s} = \frac{1}{\pi}\frac{\delta W}{\delta \vec{s}}$, $\lambda s = \frac{2}{\pi}\frac{\delta W}{\delta s}$, $s = s_x + i s_y$, то $\vec{s}(s_x, s_y)$, то

$\lambda = \omega/\omega_M$ и спектр $\hat{O}s_n(z) = \lambda_n s_n(z)$, даст eigenfreq. $\omega_n = \omega_M \lambda_n$.

Если eq. Thiele $M_v\ddot{\vec{s}} + \vec{g}\times\dot{\vec{s}} = -\frac{\delta W}{\delta \vec{s}}$, включает v. mass (пусть M_v is local и $M_v = \text{const}$)

$\vec{s}(zt) = \vec{s}(z)e^{i\omega t} \to -M_v\omega^2\vec{s} - i\omega g\vec{s} = -\frac{\delta W}{\delta \vec{s}}$, $\left[M_v\omega^2 + i\omega g\right]\vec{s} = \frac{\delta W}{\delta \vec{s}}$

Запишем в виде $\frac{1}{\pi}\frac{\delta W}{\delta \vec{s}} = \hat{O}\vec{s} = \lambda\vec{s}$, $\lambda(\omega) = \frac{1}{\pi}\left[M_v\omega^2 + i\omega g\right]$

Находим спектр λ_n, но при $\omega^2 M_v \neq 0$ и $\lambda_n \neq \omega_n g/\pi = \omega_n/\omega_M$.

$\frac{1}{\pi}M_v\omega^2 = \frac{1}{\pi}M_v\frac{\omega^2}{(4\pi)^2\gamma^2 M_s^2}(4\pi)^2\gamma^2 M_s^2 = \left(\frac{\omega}{\omega_M}\right)^2 16\pi\gamma^2 M_s^2 M_v =$

$= \left(\frac{\omega}{\omega_M}\right)^2 2\gamma^2 M_v (8\pi M_s^2) \to \left(\frac{\omega}{\omega_M}\right)^2 2m$ in units $(8\pi M_s^2)$, $M_v = \frac{m}{\gamma^2}$.

$\lambda(\omega) = 2m\left(\frac{\omega}{\omega_M}\right)^2 + \left(\frac{\omega}{\omega_M}\right)$. $2mx^2 + x - \lambda = 0$, $x = \omega/\omega_M$. Нам $x(\lambda) > 0$.

$x_{1,2} = \frac{1}{4m}\left[-1 \pm \sqrt{1 - 4\cdot 2m(-\lambda)}\right]$, $x(\lambda) = \frac{1}{4m}\left[\sqrt{1+8m\lambda} - 1\right]$

Находим спектр λ_n из $\hat{O}\vec{s} = \lambda\vec{s}$, но eigenfreq. $x_n = \frac{\omega_n}{\omega_M} = \frac{1}{4m}\left[\sqrt{1+8m\lambda_n} - 1\right]$

и только при $M_v = 0$, $m = 0$, $x_n = \frac{\omega_n}{\omega_M} = \lambda_n$. $\hat{O}\vec{s}_n(z) = \lambda_n\vec{s}_n(z)$

Если $m >> 1$, то $x_n = \frac{\omega_n}{\omega_M} \approx \sqrt{\frac{\lambda_n}{2m}} \to 0$ as $\frac{1}{\sqrt{m}}$.

Спектр $\omega_n = \omega_M x_n(\lambda_n)$ верен настолько, насколько верно $M_v\ddot{\vec{s}} + \vec{g}\times\dot{\vec{s}} = -\frac{\delta W}{\delta \vec{s}}$ eq. Thiele. Т.е. M_v is local и $M_v(z) = M_v' = \text{const}$. Если $M_v = \text{const}$ то матрица

элементов $(M_v)_{nn'} = \int dz\, M_v(zz')u_n(z)u_{n'}^*(z') = \int dz\, M_v(z)\delta(z-z')u_n(z)u_{n'}^*(z')dz' =$

$= \int dz\, M_v(z)\underbrace{u_n(z)u_{n'}^*(z)}|u_n(z)|^2 = M_v\int dz\,|u_n(z)|^2\delta_{nn'} = M_v\delta_{nn'}$ - diagonal и от n'' не зависит.

Ортонорм. сф: $\int dz\, u_n(z)u_{n'}^*(z) = \delta_{nn'}$

// Из Sci. Rep. '15 следует что $M_v(zz') = M_v\delta(z-z')$ - грубое приближение.

$d_{ss}(s,s';z,z') = \frac{\partial^2}{\partial s\partial s'}\int dk\, e^{-\beta k|z-z'|}J_1(ks)J_1(ks')$, $M_v(z,z') = \frac{\beta}{4\gamma^2}\sum_\nu \frac{1}{\omega_\nu^2}\gamma_\nu(z)N_\nu(z')$

$N_\nu(z') = \int ds_s ds_z \int dr_z\, m_0(r_z)\, d_{ss}(s,z;r_z)\, a_\nu(s,z)$. Если $d_{zz}(\tau,\gamma') = \lambda\delta(\tau-\gamma')$,

$\tau = (r_z', z')$, $\Gamma(\eta,\eta') = \frac{1}{4}\int d\tau\, d_{ss}(\eta,\tau)d_{zz}(\tau-\gamma') = \frac{\lambda}{4}d_{ss}(\eta,\eta')$, $\eta = (s,z)$.

$\int d\gamma'\,\Gamma(\eta,\eta')\,a(\gamma') = \frac{\lambda}{4}\int d\gamma'\, d_{ss}(\eta,\gamma')a(\gamma') = \omega^2 a(\eta)$

$\int ds_s dz\, d_{ss}(s,z;r_z)a_\nu(s,z) = \omega_\nu^2 a_\nu(r,z)\frac{4}{\lambda}$. $N_\nu(z) = \int dr_z m_0(r_z)\frac{4}{\lambda}\omega_\nu^2 a_\nu(r,z)$

$M_\nu(z,z') = \frac{\beta}{4\gamma^2}\sum_\nu \omega_\nu^{-2}\gamma_\nu(z)\frac{4}{\lambda}\omega_\nu^2\int dr_z m_0(r_z)a_\nu(r,z) = \frac{\beta}{\lambda\gamma^2}\sum_\nu \gamma_\nu(z)\int dr_z m_0(r)a_\nu(r,z)$

$M_\nu(z,z') = \frac{\beta}{\lambda\gamma^2}\sum_\nu \int ds_s\, m_0(s)a_\nu(s,z)\int dr_z\, m_0(r)a_\nu(r,z)$. Если $\sum_\nu a_\nu(s,z)a_\nu(r,z) \sim \delta(s-r)\delta(z-z')$,

то $M_{(z,z')} = \frac{\beta}{\lambda\gamma^2}\int ds_s\, m_0(s)\int dr_z\, m_0(r)\sum_\nu a_\nu(s,z)a_\nu(r,z') = \frac{\beta}{\lambda\gamma^2}\int ds_s\, m_0(s)\int dr_z m_0(r)\times$

$\times\delta(s-r)\frac{1}{s}\delta(z-z') = \frac{\beta M}{\lambda\gamma^2}\delta(z-z')\int ds_s\, m_0^2(s) = M_v\delta(z-z')$.

Escaneado con CamScanner

$M_\nu(z,z') = M_\nu \delta(z-z')$ возможно ли $d_{zz}(\rho z, \rho' z') = \lambda \frac{1}{\rho} \delta(\rho-\rho')\delta(z-z')$

$d_{zz}(y,y') = \lambda \delta(y-y')$ в ядре $\Gamma(\tau, y')$, что эффективно СФ $a_\nu(\rho, z)$ azim. SW.

$d_{zz}(y,y') = \frac{\partial^2}{\partial z \partial z'} \int dk e^{-\beta k|z-z'|} J_1(k\rho) J_1(k\rho')$, $y = (\rho, z)$. Возможно ли $d_{zz}(y,y') = \lambda \frac{1}{\rho}\delta(\rho-\rho')\delta(z-z')$? Если да, то в каком пределе? $\beta \to 0$? $\beta \to \infty$?

Верно ли $\sum_\nu a_\nu(\rho,z) a_\nu(\rho' z') = \mu \frac{1}{\rho}\delta(\rho-\rho')\delta(z-z')$? Видимо да. Условие полноты системы СФ.

Note: $a_\nu(\rho,z)$ из Gus'Sci. Rep.'15 число magnetostatic azim. SW ($m = \pm 1$), что $\approx$ верно если $(R_c/R)^2 \ll 1$, large R-dots. $a_\nu(\rho,z)$ нормированы без учета finite V. core radius $c = R_c/R$, что разумно если $c \ll 1$, large R-dots.

→ Вывод: если $M_\nu(z,z') = M_\nu \delta(z-z')$, то с СФ $\vec{S}_n(z)$ из $\hat{O}\vec{S}_n(z) = \lambda_n \vec{S}_n(z)$ верно. Но λ_n надо перенормировать в соб. частоты как $\frac{\omega_n}{\omega_M} = \frac{1}{4m}\left[\sqrt{1+8m\lambda_n} - 1\right]$, где $m = \gamma^2 M_\nu$, M_ν - vortex mass per unit dot thickness L.

// Berganza прислала Referee's replies и revised Ag-dot "Nanoscale" paper.

Работа над Replies to Referees of Berganza "Nanoscale" MS.

(4/V) Работа над peer-review of Berganza MS, послал обе files to Oksana. Рец. на APL Mat. MS by Ma et al "Vortex core switch." и Deac-ACS-AMI MS "Vortex GM in ion-implanted Py disks." (second review). MS-Berganza

(5/V) CMD-GEFES Conf. in Madrid Aug. 31 - Sept. 4, 2020 перенесена на 2022. Оксана прислала $\vec{H}_m(\vec{r})$ вокруг Py-dome-shaped dot. Крипта $\vec{M}_m(\vec{r})$.

Замечания Оксане: 1) magnetostatic field of dome-shaped dot не может быть такими как на fig. Nanoscale из-за небольших B_n напряженности in dot basal plane. $\vec{H}_m(\vec{r})$ simulated Oxana. Σ-home dipole, хотя $\langle \vec{M} \rangle_V \neq 0$

2) обсудили 2D-topol. charge $Q(z)$ не инвариантна, только $\int dz Q(z)$ из-за дискретности component G_z, $\vec{G} = (G_x, G_y, G_z)$ для 3D $\vec{M}(\vec{r})$ как у нас. 3D topol. charge мы не считали. Neel-sk ≈ hedgehog из-за подобности поверхности сферы $\vec{m}^2 = 1$ для Neel-sk и 3D hedgehog

3) Curvature effects для Py-dome shaped dots - это имп. Curvature induced DMI может быть только для thin 2D shells (caps) соч. Gaididei' APL 2014

4) Оксе. не знает на каких wires Farkhad делал FMR, не знает имена и wire $L = 1\mu m$, $R = 60$ nm, $M_s = 1600$ G была in vortex state с $R_c(z)$.

5) обещала найти из simulations $H_m^z(z)$ near basal dot plane $z=0$ и прислать мне.

(6/V) Сделан inv. talk nomination S. Urazhdin на MMM'2020.

E-mail to Ricardo о AF-DW dyn. Puzzle rush. Отдых.

(7/V) Открыла A. Abrosia список papers '2019 (они небыли вкл. ранее) Dabrowski сделан re-submit. „Drum modes" MS to Small. Ricardo

(8/V) Puzzle rush. Фильмы, отдых дома. Работа в саду. AF-DW

(9/V) перенесена Sol-SkyMag. Чтение Ricardo „AF-DW dynamics" file, Sec. 1.

(10/V) Ivanov LTP'1995 об. что нач. поле и DMI нарушают Lorentz-inv. DMI = 0, а нач. поле учтено как $\vec{H}_{so}$ (Sot field): $\vec{H}_{so}$ для $\vec{m}_1$, $-\vec{H}_{so}$ для $\vec{m}_2$ - подтвержден AFM. преобразование mag. anisotropy и damping:

$\dot{\vec{m}}_1 = \omega_e(\vec{m}_1 \times \vec{m}_2) - \gamma(\vec{m}_1 \times \vec{H})$; $\dot{\vec{m}}_2 = -\omega_e(\vec{m}_1 \times \vec{m}_2) + \gamma(\vec{m}_2 \times \vec{H})$, $\vec{H} = \vec{H}_{so}$ here

$\dot{\vec{\ell}} = 2\omega_e(\vec{\ell} \times \vec{m}) - \gamma(\vec{m} \times \vec{H})$, $\dot{\vec{m}} = -\gamma(\vec{\ell} \times \vec{H})$, $\vec{m} = \frac{1}{2}(\vec{m}_1 + \vec{m}_2)$, $\vec{\ell} = \frac{1}{2}(\vec{m}_1 - \vec{m}_2)$

ω_e ~ interlayer exch. AF integral, определение ω_e для Mn_2Au проверить

$\vec{\ell} \times \dot{\vec{\ell}} = 2\omega_e \vec{\ell} \times (\vec{\ell} \times \vec{m}) - \gamma \vec{\ell} \times (\vec{m} \times \vec{H}) \approx 2\omega_e [\vec{\ell}(\vec{\ell} \cdot \vec{m}) - \vec{m}(\vec{\ell} \cdot \vec{\ell})] = -2\omega_e \vec{m}$

$\vec{m} + \vec{\ell} = \vec{m}_1$, $\vec{m} - \vec{\ell} = \vec{m}_2$, $\vec{m}_1 \times \vec{m}_2 = (\vec{m} + \vec{\ell})(\vec{m} - \vec{\ell}) = \vec{\ell} \times \vec{m} - \vec{m} \times \vec{\ell} = 2\vec{\ell} \times \vec{m}$

$\vec{m}_1 - \vec{m}_2 = 2\vec{\ell} = 2\omega_e(\vec{m}_1 \times \vec{m}_2) - \gamma(\vec{m}_1 + \vec{m}_2) \times \vec{H} = 2\omega_e \cdot 2(\vec{\ell} \times \vec{m}) - 2\gamma \vec{m} \times \vec{H}$

$\dot{\vec{\ell}} = 2\omega_e(\vec{\ell} \times \vec{m}) - \gamma(\vec{m} \times \vec{H})$, $\dot{\vec{m}}_1 + \dot{\vec{m}}_2 = 2\dot{\vec{m}} = -\gamma(\vec{m}_1 \times \vec{H}) + \gamma(\vec{m}_2 \times \vec{H}) = -2\gamma(\vec{\ell} \times \vec{H})$

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$\vec{m} \approx -\frac{1}{2\omega_e} \vec{l} \times \dot{\vec{l}}$ следует из Eq.(6)(7)-Ricardo. Ricardo имеет $\vec{m} \simeq \frac{1}{2\omega_e} \vec{l} \times \dot{\vec{l}}$, Eq.(9)

Однако точное из него (4): $\vec{m} \simeq \frac{1}{2\omega_e} \vec{l} \times \dot{\vec{l}}$, $\vec{m} = \frac{\hbar}{4Js} \vec{l} \times \dot{\vec{l}} - \frac{a}{4} \nabla \vec{l}$. $\nabla \vec{l}$ term leads to $\mathcal{L} = -\frac{1}{2} \hbar s \vec{l} \cdot (\dot{\vec{l}} \times \nabla \vec{l})$ вклад в лагранжиан, который не дает вклада в классическое уравнение ур-я. Использ. $\omega_e \sim J$ (AFM exch. integral).

В Mn_2Au у-е FM плоскости с integral $J_3 > 0$ in-plane: (A, A', B, B')

$\omega_e = \frac{2\gamma}{M_s} |J|$, $J = 4J_1 + J_2$ - откуда $\vec{l}$ голосов? Mn_2Au - See Roy PRB16

A' и B' слои вокруг от общих точек A, B слои с AF-coupling $J_2 < 0$, куда J_1 делось? J_1 входит в ω_e, но не входит в AB-model given by Eq.(24). Почему в Eq.(24) $E = \ldots - |J_2| (\vec{m}_1 \cdot \vec{m}_2)$? - FM exchange между A и B-planes?

→ Т.е. 3 models used: ① $E_{ex} = -\sum J_{ij} \vec{m}_i \vec{m}_j$ (Mn_2Au), 4 FM layers A, A', B, B' in unit cell with FM exch. coupling (J_3) within A, A', B, B' planes and AF coupling A'B, B'A (J_1) и AF coupling AB (J_2); ② 2 FM-layers ($J_3 > 0$), A и B, with AF coupling given by combination $J = 4J_1 + J_2 < 0$; ③ 2 FM-layers (exch. stiffness $A_3 \sim J_3$), AF-coupled by $J_2 < 0$, Eq.(24)-Ricardo.

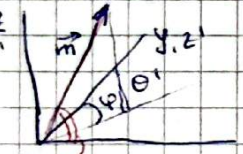
Нужно: как перейти от model ① -> to ②? Эквивалентны ли models ② и ③? Какое отношение model ③-Ricardo имеет к Mn_2Au? (кроме общих $\vec{l}$ ssr)

E-mail с этими вопросами to Ruben и Ricardo.

11/12 Model ③: Ricardo - вводит $\vec{m}_A = \vec{m} = (\sin\varphi \cos\varepsilon, \cos\varphi, \sin\varphi \sin\varepsilon)$ делая/пользуя 2 угла - φ - is azimuthal angle (около оси Oy) in basal plane xOy, ε is a small out-plane angle". Standard $\vec{m}(\theta, \varphi) = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$, θ около от polar axis Oz, φ azimuthal in-plane xOy angle.

Запись $\vec{m} = (\sin\varphi\cos\varepsilon, \cos\varphi, \sin\varphi\sin\varepsilon)$ означает $\vec{m} = (\sin\varphi\cos\varepsilon, \sin\varphi\sin\varepsilon, \cos\varphi) =$

$= (\sin\theta'\cos\varphi', \sin\theta'\sin\varphi', \cos\theta') = \vec{m}(\theta'\varphi')$, $\varphi = \theta'$, $\varepsilon = \varphi'$ в $x'y'z'$, $\begin{pmatrix} x = x' \\ y = z \\ z = y \end{pmatrix}$



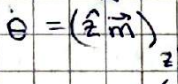
$\varphi' = \varepsilon$ угол в плоскости $zx = y'x'$ относительно от оси $x = x'$.

$(\vec{m}_1 \cdot \vec{m}_2) = \cos\theta_1' \cos\theta_2' + \cos(\varphi_1' - \varphi_2') \sin\theta_1' \sin\theta_2' = \cos\varphi_1 \cos\varphi_2 +$

$+ \cos(\varepsilon_1 - \varepsilon_2) \sin\varphi_1 \sin\varphi_2$, что совп. с Eq.(26) Ricardo.

Тогда ε - угол не есть угол отклонения $\vec{m}$ от плоскости xOy - basal plane.

В xyz, $\vec{m}(\theta, \varphi)$ 

. Нельзя $\varepsilon = \pi/2 - \theta$, т.к. углы в разных плоскостях $\varepsilon = \varphi'$ угол в плоскости $zx = x'y'$, $\theta = (\hat{z}\vec{m})$ 

$\cos\varepsilon = (\vec{m})_{x'y'} = m_x = m_{x'}$

$(\vec{m})_{xy} = \sin\theta$, $(\vec{m})_{x'y'} = \sin\theta'$, $m_{x'} = (\vec{m})_{x'y'} \cos\varepsilon = \sin\theta' \cos\varepsilon = \sin\varphi \cos\varepsilon$

Отклонение от xy-plane это $m_z = \cos\theta = m_{y'} = \sin\varphi \sin\varepsilon$ аналог. by ε и φ angles.

$\cos\theta = \sin\varphi \sin\varepsilon$, можно считать, что $m_z = \cos\theta = \sin\varphi \sin\varepsilon$ small ур-я $\varepsilon \ll 1$.

Угол отклонения $\vec{m}$ от плоскости xOy есть $\beta = \pi/2 - \theta$, $\cos\theta = \sin\varphi \sin\varepsilon$

$\sin\beta = \sin(\pi/2 - \theta) = \cos\theta = \sin\varphi \sin\varepsilon$; $\beta \ll 1$.

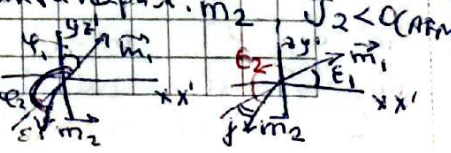
Easy plane anisotropy (Mn_2Au) $K_{21} = 1{,}303 \cdot 10^{-22}$ J, $|J_2| = k_B \cdot 532 K = 1{,}38 \cdot 10^{-23} \frac{J}{K} \times$

$K_{21}/|J_2| = 1{,}303/73{,}42 = 0{,}0177$ или $\simeq 1{,}8\%$ $\times 532 K = 7342 \cdot 10^{-22}$ J

AFM exchange предпочитает в плоскости β ($\varphi_1 - \varphi_2 = \pi$) надо вводить одновременно $E_{ex} = -J_2 (\vec{m}_1 \cdot \vec{m}_2)$. Т.к. $E_{ex} \gg E_a, E_{exch}$, то $\vec{m}_1 \simeq$ анти-парал. $\vec{m}_2$, $J_2 < 0$ (AFM)

$\vec{m}_1(\theta_1', \varphi_1') = \vec{m}_1(\varphi_1, \varepsilon_1)$, $\vec{m}_2(\theta_2', \varphi_2') = \vec{m}_2(\varphi_2, \varepsilon_2)$

$\varphi_2 = \pi + \varphi_1 + \delta$; $\varepsilon_2 = \varepsilon_1 + \pi + \gamma$, $\varepsilon, \gamma \ll 1$



Escaneado con CamScanner

(12/V) Взех на реф. Gao "SW pulse-induced DW inertia" Ann. Physik

$(\vec{m}_1\cdot\vec{m}_2)=(\vec{m}_A\cdot\vec{m}_B)=\cos\varphi_1\cos(\pi+\varphi_1+\delta)+\sin\varphi_1\sin(\pi+\varphi_1+\delta)\cos[\pi+\gamma]=$

$=\cos\varphi_1\left[\cos\pi\cos(\varphi_1+\delta)\right]+\sin\varphi_1\left[\sin\pi\cos(\cdot)+\cos\pi\sin(\varphi_1+\delta)\right](-1)\cos\gamma=$

$=-\cos\varphi_1\cos(\varphi_1+\delta)+\sin\varphi_1\sin(\varphi_1+\delta)\cos\gamma=(\vec{m}_1\cdot\vec{m}_2)$

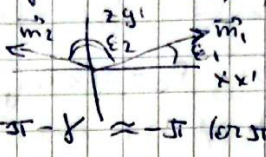
Ricardo, Eq. (27): let assume $\varphi_1(x)\cong\frac{\pi}{2}$, $\varphi_2(x)\cong-\frac{\pi}{2}$, близо near DW center, где $\vec{m}_1,\vec{m}_2(x)\parallel Ox$, тогда $(\vec{m}_1\cdot\vec{m}_2)=\cos\gamma=-\cos(\varepsilon_1-\varepsilon_2)$ и

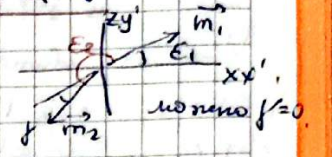
$E_{ex}=-J_2(\vec{m}_1\cdot\vec{m}_2)=J_2\cos(\varepsilon_1-\varepsilon_2)=-J_2\cos\gamma$, $\overset{\gamma=0}{\Rightarrow}$ $J_2=-|J_2|<0$, $E_{ex}=|J_2|\cos\gamma$?

Видимо номера $\varphi_2=\theta_2'=-\pi/2$, first polar angle $\varphi_2=\theta_2'\in[0,\pi]$, уравнение $\varphi_2=\pi+\varphi_1+\delta$ не выполняется. $\varphi=\theta'$ polar angle в $x'y'z'$.

Надо в центре DW $\varphi_1=\varphi_2=\pi/2$, $E_{ex}=-J_2(\vec{m}_1\cdot\vec{m}_2)=-J_2\cos(\varepsilon_1-\varepsilon_2)=$

$=|J_2|\cos(\varepsilon_1-\varepsilon_2)=-|J_2|\cos\gamma$, $\gamma=0$ дает min E_{ex}, $\varepsilon_2=\pi+\varepsilon_1+\gamma$.

Ricardo (at $\gamma=0$) дает $\varepsilon_2=-\varepsilon_1-\pi$ 

, а надо 

, когато $\gamma=0$.

$\varepsilon_1-\varepsilon_2=\varepsilon_1-[-\varepsilon_1-\pi]=\pi+2\varepsilon_1$,

а надо $\varepsilon_1-\varepsilon_2=\varepsilon_1-[\pi+\varepsilon_1+\gamma]=-\pi-\gamma\approx-\pi$ (or π)

Ricardo $\cos(\varepsilon_1-\varepsilon_2)=\cos(\pi+2\varepsilon_1)=-\cos2\varepsilon_1$, а надо $\cos(\varepsilon_1-\varepsilon_2)=\cos(\pi+\gamma)=-\cos\gamma$

Large $|J_2|$ предсказва $\gamma\to0$. Предсказване Ricardo $E_{ex}=-|J_2|\cos2\varepsilon_1\to-|J_2|$ $\varepsilon_1\to0$ физ. несмислено (Phys. nonsense)

$(E_{ex}=-|J_2|\cos\gamma)$

Вече Ricardo предсказване $\varepsilon_1\to0$ ид-ма $|J_2|$ large означава easy-plane xy anisotropy. Изотропно exchange $E_{ex}=-J_2(\vec{m}_1\cdot\vec{m}_2)$ не может да бъде отговорно за any anisotropy.

На мейл to Ricardo и Ruben с уведомяване на Omn.Soc с $\varphi_2=-\pi/2$, $\varepsilon_2=-\varepsilon_1-\pi$, тъй като не е вярно $E_{ex}=-|J_2|\cos2\varepsilon_1$

(13/V) Получих от Ricardo и Ruben отговор. Те много помнят spherical angles. И смятат, че $E_{ex}(\vec{m}_1\cdot\vec{m}_2)$ може да бъде anisotropic (ид-ма невярно уред. средно. ъгъл $\vec{m}_1,\vec{m}_2$). Бъдещ. финансир. Clave Soc.Sec.

(14/V) Получих отказ от Dept. Trabajo y Justicia, тъй като ERTE-appl. (поли основание Soc. Sec. на порц) на пара-автономна получила status „Silencio administrativo", тъй като отказ означава положително решение по умолчание. Подадох appl. в Soc. Sec. и приложих Certificado de Silencio Administrativo.

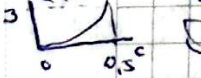
(15/V) Dipolar and quadrupolar moments of Py dome-shaped dot / Bergamaschi. Com. papers with Zukhra on C-M in coupled Sk-arrays and cylindr. dot in Neel/Bloch Sk-state (параметър Φ_0)

Q_2^0 – Sk – dot

dip. moment $P_z=\langle M_z\rangle_V=\rho M_S V\left[2c^2\ln\left(1+\frac{1}{c^2}\right)-1\right]$, quadrupolar moment

$Q_2^0=2M_SVc\cos\Phi_0\left[1-c^2\ln\left(1+1/c^2\right)\right]$, $F(c)=c^2\ln(1+1/c^2)$

Ratio $\frac{Q_2^0}{P_z}=\frac{2M_SVc\cos\Phi_0[1-F(c)]}{M_SV[2F(c)-1]}=2c\cos\Phi_0\frac{[1-F(c)]}{[2F(c)-1]}=\cos\Phi_0\,Q(c)$, $\cos\Phi_0\neq0$ for Neel-Sk

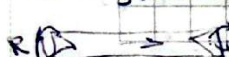


$Q(c)=2c\frac{[1-F(c)]}{[2F(c)-1]}$, $c=R_c/R$ - reduced skyrmion radius.

Перемина с Exc. по новата $H_m(\vec{r})$ of dome-shaped dot и Q_2^0.

Нараства много MS по vortex-GM in thick cyl. dots (wires) с $L\sim R$, $L>R$

(16/V) Vortex (L,R) Дълъг cylindrical dot (wire) с рос-ом thickness (length) L at R fixed vortex state is ground state at small L, intermediate L. At $L>L_c$ vortex is unstable (metastable), ground state $(L>L_{SD})$ is wire-end localized vortex



, при дългомерни рогове L, end-localized vortex states $\perp$ SD, slower state at the wire ends $z=\pm L/2$.

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Найдем $L_c(R)$ при котором vortex minimum нарождается. Предполагаем $R_c \ll 2$, $c \neq f(z)$, $c = R_c/R$, имеем "static core" и "Bessel" shape

$W_{ex}(\vec{s},\vec{m}) = M_s^2 \beta \left(\frac{L_e}{R}\right)^2 \int dz \left[2 - \ell n c(z)\right] \to W_{ex}(\vec{s},\vec{m}) = M_s^2 \pi R^2 \left(\frac{L_e}{R}\right)^2 L (2 - \ell n c)$

$W_m^{face}(c,s) = M_s^2 R^3 \int_0^\infty dk (1 - e^{-\beta k}) \sum_\mu |K_\mu(k)|^2$, $K_\mu(k) = \int d^2\vec{s}\, J_\mu(ks) e^{i\mu\varphi} m_z(\vec{s},c,s)$

For static vortex $s=0$, $m_z(\vec{s},c) = m_z(s,c)$, $K_\mu(k) = \int ds\, s\, J_\mu \int_0^{2\pi} d\varphi e^{i\mu\varphi} m_z(s,c) =$

$= 2\pi \delta_{\mu 0} \int ds\, s\, J_0(ks)\, m_z(s,c) = \delta_{\mu 0} K_0(k)$.

$W_m^{face}(c) = M_s^2 R^3 \int dk (1 - e^{-\beta k}) \sum_\mu 4\pi^2 \delta_{\mu 0}^2 \left[\int ds\, (\ldots)\right]^2 = 4\pi^2 M_s^2 R^3 \int dk (1 - e^{-\beta k}) \left[\int ds\, s\, J_0(ks) m_z(s,c)\right]^2$

$W(c) = W_{ex}(c) + W_m^{face}(c)$, $\frac{1}{M_s^2 V} W(c) = \frac{4\pi}{\beta} \int_0^\infty dk (1 - e^{-\beta k}) I_0^2(k,c) + \left(\frac{L_e}{R}\right)^2 (2 - \ell n c)$, $V = \pi R^2 L$

$I_0(k,c) = \int_0^c ds\, s\, J_0(ks) m_z(s,c)$, $m_z(s,c) = \frac{c^2 - s^2}{c^2 + s^2}$. $\min W(c)$ по c: $c = \frac{R_c}{R}$, $c_0 = c_0\left(\beta, \frac{R}{L_e}\right)$, $\frac{R}{L_e} = \frac{1}{\Gamma}$.

Где $c(z)$ нужно для $\frac{\partial c}{\partial z}$ - ген. вклада в exchange и $W_m^{vol}\left(\frac{\partial m_z}{\partial z}\right)$ energy.

$W(c) = \frac{4\pi}{\beta} \int dk (1 - e^{-\beta k}) I_0^2(k,c) + \frac{1}{\Gamma^2}(2 - \ell n c)$, $\frac{\partial W}{\partial c} = \frac{2\pi}{\beta} \int dk (1 - e^{-\beta k}) I_0(k,c) \frac{\partial I_0(k,c)}{\partial c} - \frac{1}{\Gamma^2} \frac{1}{c}$

Use'ду solution при $\left(\frac{L}{L_e}\right) \gg 1$, $R_c = 0{,}68 L_e \left(\frac{L}{L_e}\right)^{1/3}$, $c_0 = 0{,}68 \frac{1}{\Gamma}\left(\frac{L}{L_e}\right)^{1/3} = 0{,}68\, \beta^{1/3}/\Gamma^{2/3}$

$\min c_0(\beta,\Gamma) = 0{,}68 \frac{1}{\Gamma}(\beta/\Gamma)^{1/3}$ at $\beta/\Gamma \gg 1$ - use'ду. $L/L_e \gg 1$, $\beta/\Gamma \gg 1$

$\frac{\partial I_0(k,c)}{\partial c} = \frac{\partial}{\partial c} \int_0^c ds\, s\, J_0(ks) m_z(s,c)$, $\frac{\partial m_z}{\partial c} = \frac{\partial}{\partial c}\left[\frac{c^2 - s^2}{c^2 + s^2}\right] = \frac{2c(c^2+s^2) - (c^2-s^2)2c}{(c^2+s^2)^2} = 4\frac{s^2 c}{(c^2+s^2)^2}$

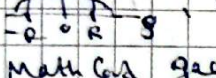
$I_1(k,c) = \frac{\partial I_0}{\partial c}(k,c) = \int_0^c ds\, s\, J_0(ks) \frac{4s^2 c}{(c^2+s^2)^2}$, $\frac{\partial W}{\partial \Gamma} = \frac{2\pi}{\beta} \int dk (1 - e^{\beta k}) I_0(k,c) I_1(k,c) - \frac{1}{\Gamma^2}\frac{1}{c}$.

Matlab "Vortex_thick_dot_Stability.mcd" дает, что use'ду $c_0(\beta)$ и $\beta^{1/3}$ не точно.

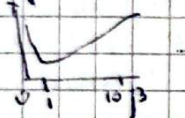
Remark $\left(\frac{2\pi}{\beta} \int dk (1 - e^{-\beta k}) I_0(k,c) I_1(k,c) = \frac{1}{\Gamma^2 c}\right)$ есть "c" $(\beta\Gamma \gg 1)$ $\Rightarrow c_0(\beta,\Gamma)$ equilibrium.

$W_{cc}''[c = c_0(\beta,\Gamma)] = 0$ дает $\beta_c(\Gamma)$. Еще option: $\min W(c)$ может нарождаться и при $W_{cc}'' = 0$, а $c_0(\beta,\Gamma)$ достигает 1. При $c \geq 1$ (остается $\vec{m}(s)$ уже не вихрь),

а энергия к LSD



, see Gus' JAP '2004. $g_{zz}(ss') = -\frac{4\pi}{\beta} \int dk (1 - e^{-\beta k}) J_0(ks) I_0(ks')$



Matlab дает странное, при $\beta < 1$ не может быть $c_0 \approx 0{,}7$.

Use'ду: $\frac{W_m^{us}}{M_s^2 V} = 4\pi \frac{R_c^3}{R^2 L}\left[g(0) - g\left(\frac{L}{R_c}\right)\right]$, $g(t) = \frac{1}{2\pi} \int_0^1 dx \int_0^1 dx' \int_0^{2\pi} d\varphi \times$

$\times \frac{xx' z(x) z(x')}{\sqrt{x^2 + x'^2 - 2xx'\cos\varphi + t^2}}$, $z(x) = \frac{1 - x^2}{1 + x^2}$, $x = s/R_c$, $b = R_c$ - vortex radius.

$\frac{1}{\sqrt{(\vec{x} - \vec{x}')^2 + t^2}} = \int dk\, J_0(k|\vec{x} - \vec{x}'|) e^{-kt} = \int dk e^{-kt} \sum_\mu e^{i\mu\varphi} J_\mu(kx) J_\mu(kx')$

$\frac{1}{M_s^2 V} W_m^{us} = 4\pi \frac{R_c^3}{R^2 L}\left[g(0) - g\left(\frac{L}{R_c}\right)\right]$. $g(t) = \frac{1}{2\pi} \int_0^1 dx\, x \int_0^1 dx' x' \int_0^{2\pi} d\varphi\, z(x) z(x') \int dk e^{-kt} \times$

$\times \sum_\mu e^{i\mu\varphi} J_\mu(kx) J_\mu(kx') = \int_0^1 dx\, x \int_0^1 dx' x' z(x) z(x') \int dk\, e^{-kt} J_0(kx) J_0(kx')$

$\frac{1}{M_s^2 V} W_m^{us} = 4\pi \frac{R_c^3}{R^2 L} \int_0^1 dx\, x \int_0^1 dx' x' z(x) z(x') \int dk \left[1 - e^{-kL/R_c}\right] J_0(kx) J_0(kx') =$

$= \left| x = s/R_c,\ \Gamma = \frac{s}{R},\ x = \frac{1}{\Gamma R} = \frac{\Gamma}{c} \right| = 4\pi \frac{R_c^3}{R^2 L} \frac{1}{c^4} \int_0^c d\Gamma\Gamma \int_0^c d\Gamma' z\left(\frac{\Gamma}{c}\right) z\left(\frac{\Gamma'}{c}\right) \int dk \left(1 - e^{-kL/R_c}\right) \times$

$\times J_0\left(k\frac{\Gamma}{c}\right) J_0\left(k\frac{\Gamma'}{c}\right) = \left| k/c = p,\ \frac{kL}{R_c} = pc\frac{L}{R_c} = p\beta \right| = 4\pi \frac{R_c^3}{R^2 L} \frac{1}{c^4} c \int_0^c d\Gamma\Gamma \int_0^c d\Gamma'\Gamma' z\left(\frac{\Gamma}{c}\right) z\left(\frac{\Gamma'}{c}\right) \times$

$\times \int dp (1 - e^{-\beta p}) J_0(p\Gamma) J_0(p\Gamma') = \frac{4\pi}{\beta} \int_0^c d\Gamma\Gamma \int_0^c d\Gamma'\Gamma' m_z(\Gamma,c) m_z(\Gamma',c) \int dk (1 - e^{-\beta k}) J_0(k\Gamma) J_0(k\Gamma') =$

$= \frac{4\pi}{\beta} \int dk (1 - e^{-\beta k}) I_0^2(k,c)$, $I_0(k,c) = \int_0^c d\Gamma\Gamma\, m_z(\Gamma,c) J_0(k\Gamma)$ сбой, с моей W_m^{face}

У меня была ошибка в Matlab: брал $\int_0^1 g(\ldots)$, а надо $\int_0^c dg(\ldots)$.

$\frac{\partial^2 W}{\partial c^2} = \frac{\partial}{\partial c}\left[\frac{2\pi}{\beta} \int dk (1 - e^{-\beta k}) I_0(k,c) I_1(k,c) - \frac{1}{\Gamma^2}\frac{1}{c^2}\right] = \frac{2\pi}{\beta} \int dk (1 - e^{\beta k}) \left[I_1^2(k,c) + I_0(k,c) I_2(k,c)\right] + \frac{1}{\Gamma^2}\frac{1}{c^3}$

$I_2(k,c) = \frac{\partial}{\partial c} I_1(k,c) = \frac{\partial}{\partial c} \int_0^c ds\, s\, J_0(ks) \frac{4s^2 c}{(c^2+s^2)^2} = c\, J_0(kc) \frac{4c^3}{(2c^2)^2} + \int_0^c ds\, s\, J_0(ks) 4s^2 \frac{\partial}{\partial c} \frac{c}{(c^2+s^2)^2}$

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$I_2(k,c) = J_0(kc) + 4\int_0^c d\rho\rho^3 J_0(k\rho)\frac{[c^2-\rho^2)^2 - 4c^4]}{(c^2+\rho^2)^4}$, $\lim I_1(kc) = \frac{2}{\pi c}\int_0^c d\rho\rho J_0 m_z(\rho,c)$ ur. bpx. npegena не нашо

т.к. $\int_0^c \rho J_0(k\rho) m_z(\rho,c)\Big|_{\rho=c} = 0$

$\frac{\partial^2 W}{\partial c^2} = \frac{2\pi l}{\beta}\int_0^\infty dk\,(1-e^{-\beta k})\left[I_1^2(k,c) + I_0(k,c) I_2(k,c)\right] + \frac{1}{\pi^2}\frac{1}{c^2} = W''_{cc}(c,\beta,z)$

Анализ Mathcad показал, что $W''_{cc}(c,\beta,z) > 0$ $\forall$ разумных β, z. т.е. $c_0(\beta,z)$ - vortex min всегда есть? Надо $W''_{cc}[c_0(\beta,z),\beta,z]$ sign. Usov $c_0(\beta,z) = 0{,}684\frac{1}{\pi}\left(\frac{L}{Le}\right)^{\frac{1}{3}} = 0{,}684\frac{1}{\pi}(\beta z)^{\frac{1}{3}}$ очень хорошее прибл. при $\frac{L}{Le} = \beta z \gg 1$ Mcad дал корр. не 0,68 (mon'94), а 0,685. $W''_{cc}[c_0(\beta,z),\beta,z] > 0$ $\forall \beta, z$ при $\beta \geq 0.2$, $z \geq 2$

При переходе Vortex → ||SD (small L) W''_{cc} goes to -0, а $z_{crit} > 0$. В Gus'JMMM'02 Vortex stability из sign $\frac{\partial^2 W}{\partial s^2}\Big|_{s=0}$.

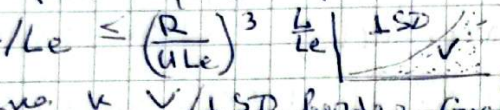
1) Vortex → ||SD translational mode (TM) instability, $\frac{\partial^2 W}{\partial s^2}\Big|_{s=0} = 0$, $\omega_{TM} = 0$
2) Vortex → ⊥SD - breathing mode instability $\frac{\partial^2 W}{\partial c^2}\Big|_{c_0} = 0$ (large β) (small R, L)

17/V $c_0(\beta,z) = u\frac{1}{\pi}(\beta z)^{\frac{1}{3}}$, $u = 0{,}685$. Border of Vortex existence из $c_0(\beta,z) = 1$, $R_c = R$, $c_0 > 1$ = ⊥SD. $\frac{u}{\pi}(\beta z)^{\frac{1}{3}} \leq 1$, $u\left(\frac{L}{Le}\right)^{\frac{1}{3}} \leq \frac{R}{Le}$

Usov solution

V. core radius

$L/Le \leq \left(\frac{R}{uLe}\right)^3$ — граница V/⊥SD border Gus/Met JMMM'02 — equilibrium $E_V(L,R) = E_{\perp SD}(\beta)$



Если $L_e = 14{,}0$ nm, $R = 60$ nm (typical wires) $L_c/L_e = \left(\frac{R}{uL_e}\right)^3$, $L_c = 14\text{ nm}\cdot\left(\frac{60}{0{,}685\cdot 14}\right)^3 = 3{,}43\cdot 10^3$ nm

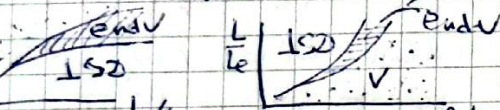
Vortex is stable in nanowire at $L < L_c = 3{,}43$ μm

Wires: Vortex stability

Это approximation $R_c \neq f(z)$, $c \neq f(z)$.

→ $L < L_c(R)$, $L_c(R) = L_e\left(\frac{R}{uL_e}\right)^3$ - условие single vortex stability in cyl. dot/wire with length L, radius R и $L_e = \sqrt{2A/M_s^2}$.

Но реально (при L lower) V→⊥SD будет V→end-V, затем end-V→⊥SD



end-V state не на Metlov JMMM'02 MPD и нельзя как найти $L_c(R)$ для V→end-V transition

pC-Vortex

Нами были MS Vortex GM in thick dots/wires. Это C,P-Vortex.

Energy density $\bar{\bar{w}}[\vec{S}(z),\vec{S}'(z')]$ это/и содержит x-, z- и P-terms coming from W_m^{vol}, т.е. из $\operatorname{div}\vec{m} = \frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y} + \frac{\partial m_z}{\partial z}$, $\frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y} = 2\,\mathrm{Re}\frac{\partial m_+}{\partial \zeta}$ ~ $m_+ = \frac{2w}{1+ww}$

$w(\zeta,\bar{\zeta}) = \begin{cases} f(\zeta), & |f|\leq 1 \\ f(\bar{\zeta})/|f|, & |f|\geq 1 \end{cases}$, $f(\zeta) = -\frac{i}{c}\mathbb{C}\cdot\frac{(z-s)(\bar{s}'z-1)}{(1+s\bar{s})}$ т.е. $\frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y} \sim \mathbb{C}$, v.chirality

$m_z = p\frac{(1-w\bar{w})}{(1+w\bar{w})}$ in v. core, $m_z \sim p$.

x-term ~ $\vec{S}(z)\vec{S}'(z')$ содерж. $\left(\frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y}\right)^2$, in-core и out-core, $\sim \mathbb{C}^2$, от $\mathbb{C}$ не зависит

z-term ~ $\frac{\partial\vec{S}}{\partial z}\cdot\frac{\partial\vec{S}(z')}{\partial z'}$ содерж. $\frac{\partial m_z}{\partial z}\frac{\partial m_z(z')}{\partial z'} \sim p^2$, от p не зависит

P-term ~ $\frac{\partial\vec{S}}{\partial z}\times\vec{S}(z')$ содерж. $\left(\frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y}\right)\cdot\frac{\partial m_z}{\partial z} \sim p\mathbb{C}$, т.е. в P-term in energy $\bar{\bar{w}}$ и $\gamma_D\vec{S}(z)$ надо включить $p\mathbb{C}$ - product. $P(zz') \to p\mathbb{C}P(zz')$

$\frac{1}{\pi}\bar{\bar{w}}[\vec{S}(z),\vec{S}'(z)] = (\mu,\eta,x,z\text{-terms}) - \frac{1}{2}p\mathbb{C}\int dz' P(zz')\left[\frac{\partial\vec{S}(z)}{\partial z}\times\vec{S}(z')\right]\cdot\hat{z}$

P-term (determined by v. core шнур) дает non-local DMI (!!!), задающую - vortex chirality. Dynamic non-local DMI для v. core position $\vec{S}(z)$.

$\left[\frac{\partial\vec{S}}{\partial z}\times\vec{S}(z')\right]\cdot\hat{z} = \frac{\partial S_x}{\partial z}S_y(z') - \frac{\partial S_y}{\partial z}S_x(z')$. // W_m^{vol} из-за z^2 даст finite vortex core $c \neq 0$ и/или даже и non-local dynamic DMI in thick dots / long wires, если $\vec{S} \neq \vec{S}'(z)$.

P-term и/или даже и energy в GM frequency $\Delta W \sim p\mathbb{C}$, freq. splitting $p\mathbb{C} = \pm 1$.

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$\lambda \vec{S} = (\mu, y, z, x\text{-terms}) + \frac{1}{2}\int dz' \frac{\partial P}{\partial z}(zz')\vec{S}(z') \times \hat{z}\, pC$ где $\lambda = pgw/\pi$.

$\frac{\partial P(zz')}{\partial z} = -8\pi M_s^2 cp\,\Theta(z-z')\int dk\, k\, e^{-\beta k|z-z'|} J(k)J'(k)$

Кроме очевидного $\frac{1}{\pi}\vec{W}_p = -\frac{1}{2}pC\int dz' P(zz')\left[\frac{\partial \vec{S}}{\partial z}\times\vec{S}(z')\right]\cdot\hat{z} \approx -\frac{1}{2}pC \times$

$\times\left[\frac{\partial \vec{S}}{\partial z}\times\vec{S}(z)\right]\cdot\hat{z}\int dz' P(zz') \approx -\frac{1}{2}pC\, k_z \vec{S}^2(z)\int dz' P(zz') \to -\frac{1}{2}pC\, k_z \vec{S}^2 \int dz dz' P(zz')$

$\frac{1}{\pi}\vec{W}_p = \frac{1}{\pi}\frac{1}{2}\varkappa_p \vec{S}^2$, $\varkappa_p = \pi p C k_z \int dz dz' P(zz') \Rightarrow \lambda\vec{S} = \frac{1}{\pi}\frac{\Sigma\vec{W}}{\Sigma\vec{S}}$, $\vec{W}_p = \frac{1}{2}\varkappa_p\vec{S}^2$

$g\omega\vec{S} = \frac{\Sigma\vec{W}}{\Sigma\vec{S}} = \frac{\partial\vec{W}}{\partial\vec{S}} \Rightarrow g\Delta\omega_p\vec{S} = \frac{\partial}{\partial\vec{S}}\vec{W}_p = \varkappa_p\vec{S}$, $\Delta\omega_p = \frac{1}{g}\varkappa_p$. Найти $\varkappa_p$. $g = \frac{2\pi M_s}{\gamma}$

$\Delta\omega_p = \frac{\gamma}{2\pi M_s} 8\pi M_s^2 c \int dz dz' pC\, e^{-\beta k|z-z'|} J(k)J'(k)dk \cdot \pi p C k_z$, $\int dz dz' e^{-\beta k|z-z'|} = \frac{2}{k}f(\beta k)$ $\Rightarrow$

$\Delta\omega_p = \gamma 4\pi M_s c \cdot \pi p C k_z \int dk \frac{2}{k} f(\beta k) J(k)J'(k) = \omega_M \cdot 2cpC k_z \int_0^\infty \frac{dk}{k} f(\beta k) J(k)J'(k)$

$J(k) = \int_0^c dg\, g\, F(g) J_1(kg)$, $J'(k) = \int_0^c dg\, g\, F(g)(1+g^2) J_1(kg)$, $F(g) = \frac{4g}{c^2}\frac{1}{[1+g^2/c^2]^2}$

Splitting $pC=+1$ и $pC=-1$ $\Delta\omega_p^{\pm} = \omega_M \cdot 4c k_z \int_0^\infty \frac{dk}{k} f(\beta k) J(k)J'(k)$

k_z дает non-localized GM modes $k_z \neq \pi n$. Для localized ent-GM $k_z \to \varkappa_z \sim \frac{1}{\Delta z}$

Будучи $k_z \sim 10$, $\varkappa_z \sim 100$, т.к. $\Delta z \cong 0{,}01$ для long wires. $\Delta z \sim 10$nm

и/или $L = 1000$ nm

$\Delta\omega_p = \omega_M k_z I_p(\beta, c)$, $I_p(\beta,c) = 4c\int_0^\infty \frac{dk}{k} f(\beta k) J(k,c) J'(k,c)$.

$I_p(\beta,c)$ растет с ростом β $\sqrt{\beta}$. $I_p(\beta, c=0{,}5) = (0{,}01 - 0{,}06)$ n/m $\beta \in [0.1, 5]$

$\Delta\omega_p^{\pm} \simeq \omega_M \cdot 10 \cdot (1 \div 6)\cdot 10^{-2} \overset{5}{\approx} (0{,}1 - 0{,}6)\,\omega_M$, т.к. $\frac{\omega_M}{2\pi} \sim 30$ GHz (Py),

→ Splitting $\Delta\omega_p^{\pm}/2\pi \approx 3 - 18$ GHz для ν, где $c = 0{,}5$ и рост в гигант (!) и расте- с ст.

Послал e-mail Рахмату с предложением найти ω_n для $p=+1$ и $pC=+1$, $pC=-1$.

(18/V) Обзор simulations Pawel „Edge SW scattering". Brief comment.

Переписка с Oec. и Elder по Nanoscale - Sk dot paper. Начертил sim. $\vec{H}_m(\vec{r})$.

(19/V 25/V) Сдвиг. Работа к сагу. Фишка. Бинт

(21/V) Величина frequency pC-splitting cylindr. wires $\Delta\omega_p^{\pm}$ от 17/V

Самого забыл. Begin $\int dz dz' P(zz')\left[\frac{\partial\vec{S}}{\partial z}\times\vec{S}(z)\right]\cdot\hat{z} = \left|\text{local appr.}\right| =$

$= \int dz dz' P(zz')\cdot\left[\frac{\partial\vec{S}}{\partial z}\times\vec{S}(z)\right]\cdot\hat{z} = \int dz dz' P\, k_z^{(z)}\left[\vec{S}(z)\times\vec{S}(z)\right]\cdot\hat{z} = 0$

Потому $\frac{\partial\vec{S}(z)}{\partial z}\times\vec{S}(z') \approx k_z\vec{S}^2(z)$. Тождество мало.

Формально SS $\psi\Phi(21/V) \Rightarrow \lambda S = (\mu, y, z\text{-terms}) + \int dz'\left[\varkappa(zz') - \frac{i}{2}\frac{\partial P(zz')}{\partial z}\right] s(z')$

$pC \to -pC$, $P \to -P$ будет и $\lambda S = (\mu, y, z) + \int dz'\left[\varkappa + \frac{i}{2}\frac{\partial P}{\partial z}(zz')\right] s(z')$ Проще

не менее: спектр λ_n но в dyn. matrix будет недоступ $\Lambda_{nn'}$ (1/V) Следует

P-term $\Rightarrow \Lambda_{nn'} = (\ldots) + \frac{i}{2}cp\int dk\, k J(k)J'(k)\bar{J}_{nn'}(k)$, $\bar{J}_{nn'}$ pure imaginary,

$i\bar{J}_{nn'}$ is real, и $\Lambda_{nn'}$ is real. $pC \to -pC$, $P \to -P$ меняет величину $\Lambda_{nn'} \Rightarrow$

меняет спектр ω_n (или λ_n).

$\frac{1}{\pi}\vec{W}_p(z) = -\frac{1}{2}pC\int dz' P(zz')\left[\frac{\partial\vec{S}(z)}{\partial z}\times\vec{S}(z')\right]\cdot\hat{z}$. $W = L\int_z E = R^2 L\int dz w = V\int dz \frac{1}{\pi} w$

$W_p = V\left(-\frac{1}{2}pC\right)\int dz dz'\left[\frac{\partial\vec{S}(z)}{\partial z}\times\vec{S}(z')\right]\cdot\hat{z}\, P(zz')$, $P(zz') = 8\pi M_s^2 c\int dk\, e^{\beta k|z-z'|} J(k)J'(k)$

$\left[\frac{\partial\vec{S}(z)}{\partial z}\times\vec{S}(z')\right]\cdot\hat{z} = \frac{1}{2}\left[\frac{\partial\vec{S}(z)}{\partial z}s(z') - \frac{\partial s}{\partial z}\vec{S}(z')\right]$ в ss-представлении, см. 21/V

Если $s(z) = u(z)e^{i\varphi(z)}$ $u=|s|$, $\neq 0$ $\frac{\partial\vec{S}}{\partial z}s(z') - \frac{\partial s}{\partial z}\vec{S}(z) = \left[u'e^{i\varphi} + u(i\varphi')\right]u(z')e^{i\varphi(z')} -$

$\vec{S}(z) = u(z)e^{-i\varphi(z)}$

$-\left[u'e^{i\varphi} + ui\varphi' e^{i\varphi}\right]u(z')e^{i\varphi(z')} = \left[u' - ui\varphi'\right]u(z)e^{i(\varphi(z)-\varphi(z'))} - \left[u' + iu\varphi'\right]e^{i(\varphi(z)-\varphi(z'))}u(z')$

но нет упрощения.

Вывод: Письмо подтвердил, что смена знака pC ведет к иным eigenfrequencies

Набор $\omega_n/2\pi = (0{,}3;\ 1{,}5;\ 2{,}4,\ 3{,}0)$ GHz сменился на $\omega_n/2\pi = (0{,}5,\ 1{,}8,\ 3{,}0,\ 4{,}3)$ GHz

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(22/V) получих e-mail от Polish Sci. Center защо application на POLS grant, 100-200 kE, на 1-2 года. Maciej съгласихме дано Polish host inst. Проблема в том, что PI-project should be employed „full-time" in Poland. по правилата Ikerbasque „if ≤ 12 month stay outside Basque Country is linked to fellowship/appointment with any kind of payment, the Ikerbasque salary is reduced to 80% of the current amount."
Кроме того, Ikerbasque authorization term finishs with 3-year assessment period, т.е. in June 2021. переписка с Maciej. Бълг.

(23/V) Реферат. MS Gao. An. der Phys. DW mass - SW. Бълг.

(24/V) Cita Tax Office - Tolosa на 23/VI, 11^00. От там надо письмо-autorization.

(25/V) Написах и пратих reject-отказ на Gao - An. der Phys. Working hours дне Julain по „Skyrmion" project in 2019, замолих и пратих и на Cristina.

(26/V) Анализ условий POLS-project (Poland). переписка с Maciej. Входяща IWA „Constanto" - 2016-2018.

(27/V) Editing CV-Tartakovskaya. Изпратих Elana и „3D magn. textures" to Maciej as grant PI и теря. Skype с Tartakovska она согласилась на 1-2 года зимъ и работа в Poznan по POLS-project, if accepted.

(28/V) Начало писане POLS Horizon project, подбор Refs (писане пона).

(29/V) Събрах Refs, написах Introduction и Short description/plan.

(30/V) Написване POLS Introduction, переписка с Ленов.

(31/V) Отдих. Бълг. Res. Plan POLS.

(1/VI) Обсъждане плана работа по Vortex modes long wires с Фархад. Написване POLS proposal - переписка с Ленов.

(2/VI) Анализ Micr. Simulations Gyrovector in dome-shape Py dots. Кратка slides. Изчислих gyrovector $G_z = f(L,R)$ for dome-shape Py dot. Если знать $z(L,R) \sim L/R$ (L,R), то $\omega_G \sim 1/[G_z \chi] = f(L,R)$ - skyrmion gyrotropic frequency. Звънна Оксана

(3-9/VI) Написване POLS proposal „3D magnetization textures". переписка с Тартаковская и Krawczyk. Обсъждане GM, SW wires с Фархад.

(10/VI) Анализ papers-Submission activity и conf. (план 21/I). TMAG 2020, Sept. 7-11, Palermo перенесена на Sept. 6-10, 2021 - отменен hotel. JEMS'2020, July 27-30, Lisbon перенесена на Dec. 7-11, 2020, Lisbon → reserve hotel. отменен

(11/VI) Написване POLS proposal, подбор Figs. Рецензия SA дне Julian.

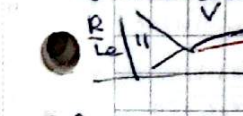
(12/VI) Изпратих MCIII-wire proposal: начало June 1, 2020, 60 kE costes Skype с Глеб: обсъждане Appeal to Small и „Drum modes", мд directes. Изпратих revised file to Gleb, Alexander (надо сделать appeal, а не ответы to Referees). Skype с Игорь: изпратих to visit Donostia ≤ 1 мес. обсъждане MS SW-edge с Pawel. Proof ФНТ „GM, SW thick dots" Получих 5 нов. CMD-letts. Год 2020-EPS Prize изпратих to Jörg Wrachtrup, Germany (Google H=92, 37.700 citations).

(13/VI) Запълних Excel-отчет Skyrm, грант FP3-2019 и sent to Cristina. ИФТ/ЕНИ (не хватает 4-х trip reports.

(14/VI) Edited POLS „3D textures" proposal и писах to Maciej, Ленов.

(15/VI) Отдых дома. Бълг. Maciej submitted POLS proposal. GM wires

(16/VI) Написване MS на thick dots/wires - GM. 16-17/IX отделно V → 1SD transition на $C_0(\beta,R) \approx 1$, $C_0 = R_c/R$ equilibrium.



Ко спасено Metlov JMMM'02 $R_{eq}(L) > R_c(L)$ и $L_c(R)$ дне V → 1SD нужна оценка на $R_{eq}(L)$. Тя дава минимум $L_c(R)$, тъй: $L/L_e = (R/uL_e)^3$, $u = 0,685$ от 17/IX usov's $C_0(\beta,z) = u\frac{1}{z}(\beta z)^{1/3}$, $\beta = L/R$, $z = R/L_e$ отдих хорошее идея Найден $W(c)$ изп. $c = C_0(\beta z)$. $W_v(c,\beta,z) = W_{1SD}(\beta)$ дава $R_{eq}(L)$.

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$\frac{1}{M_s^2 V} W_v(k) = \frac{4\pi}{p} \int_0^\infty dk \,(1-e^{-pL}) I_0^2(k) + \left(\frac{k_z}{k}\right)^2 (2-\ell_{ne})$, $I_0(k) = \int_0^c ds\, s\, J_0(ks)\, m_z(s) = I_0(k_z)$

Это универсальное для $S(z)$ и всех тонких слоев $z = 0, L$ с зарядами.

Для 1 SD $W_{ex}=0$, $W_m^1 = \frac{4\pi}{M_s^2 V} \int dk\,(1-e^{-pL}) I_0^1(k)$, $I_0^1(k) = \int_0^1 ds\, s\, J_0(ks)\, m_z(s) = ?$

$I_0^1(k) = \frac{1}{k^2}\int_0^k dx\, x J_0(x) = \frac{1}{k^2} x J_1(x)\Big|_0^{kr} = \frac{1}{k^2} k J_1(k) = \frac{1}{k} J_1(k)$, $r = R/\ell_e$.

$W_m^1(p) = \int_0^\infty \frac{4\pi}{p} dk\,(1-e^{-pL}) \frac{1}{k^2} J_1^2(k)$. $W_v[c_0(p,z), p, z] = W_m^1(p)$

$c_0(p,z) = \frac{u}{v}(p^2\ell_e^2)^{1/4} = \frac{u}{v}(L/\ell_e)^{1/2}$. $\leftrightarrow$ даёт $p_c(z)$ и $\frac{L_{cr}}{R} = f(\frac{z}{R})$

$L_{cr}/\ell_e = \frac{R}{\ell_e} f(R/\ell_e)$. Найти $f(\ell_e)$. $L_{cr}^2 = 2A/M_s^2$. Vortex thick dot stability

L_{cr}: из equil. V-1 SD идет резкое $\frac{L_{cr}}{\ell_e} = \left(\frac{R}{\ell_e}\right)^3$ из $c_0(p,z) = 1$ incl

при $\ell_e = 14$ nm, $R = 60$ nm, $L_{cr} = R f(\frac{R}{\ell_e}) = 2.358 \cdot 10^3$ nm $= 2.36$ μm (17/IV) Было $L_c = 3.43$ μm

Итог $\frac{L_{cr}(R)}{L_c(R)}$ близко насчитано с ростом $R/\ell_e > 3$ при $\approx 0.70 = \ln 2 = 0.693$.

Это $\frac{R}{\ell_e} \geq 1.1$ такое было точное bifurcation point Metlov'02 MPD.

Это для универсальное $m_z = f(z)$, $R_0(z) = f(z)$ uniform vortex along length L.

Сент. 19/IX $W_{ex} = \frac{1}{2} J V \int dz \left(\frac{\partial S}{\partial z}\right)^2 + \frac{1}{2} \mu V \int dz\, S^2(z)$, $\mu = 2M_s^2 (\ell_e/R)^2$,

$y = M_s^2 (\ell_e/L)^2 \left[\ln\frac{1}{\delta} + \frac{5}{4} + a^2 + \left(\frac{23}{4} - 8\ln 2\right) a^4\right]$

(17/IV) Написание GM - thick dots/wires MS. Other modes "second rev. small

Note: решение такой магнитостатич. problem - int. equation дает существ. low-f localized at the wire end modes, even и odd по отн. к $z=0$ wire middle plane. То решение интегр.-дифф. eq. $\hat{F}\vec{S}(z) = \lambda \vec{S}(z)$ некоторым фурье $S(z) = \sum_n c_n e^{iq_n z}$ дает только одну end-localized моду $u_0(z)$, $S_n(z) = u_n(z) e^{i\Phi_n(z)}$ $u_n(z) = |S_n(z)|$, $S = S_x + iS_y$,

Почему? Из-за z^2 в $\hat{F}$? Как сделать $u_n(z)$? А сделать $\Phi_0(z)$ будет ω_0-функцией?

(18/VI) Написание вортекс GM wires" MS. Берганза сделал "Pydots" MS to Nano scale

(19/VI) Берганза "Pydots" Nanoscale was accepted. Л. пос. Л. Д + Нат. + Перед - 60 €

Перешли рук в Денвер после 13/III - Л., 10.

GM - thick dots

Eq. Ландау $\dot{S} + \frac{\delta W}{\delta S} = 0$, $S = \vec{S}(x,t)$. Если обычно $S(x,t) = S_x(x,t) + iS_y(x,t)$, $\vec{S}(S_x, S_y)$, то

$i\dot{S}(x,t)g = 2\frac{\delta W}{\delta \bar{S}}$ $\frac{\partial W}{\partial S} = \frac{1}{2}\left(\frac{\partial W}{\partial S_x} + i\frac{\partial W}{\partial S_y}\right) \to \frac{\delta W}{\delta S} = \frac{1}{2}\left(\frac{\delta W}{\delta S_x} + i\frac{\delta W}{\delta S_y}\right)$ (умножен.

$S(x,t) = S(x) e^{i\omega t}$, $\dot{S}(x,t) = i\omega S(x) e^{i\omega t} = i\omega S(x,t)$, $ig\dot{S}(x,t) = ig\, i\omega S(x,t) = -\omega g S(x,t)$

$-\omega g S(x,t) = 2\frac{\delta W}{\delta \bar{S}}$, $g \to -g$, $\omega g S(x,t) = 2\frac{\delta W}{\delta \bar{S}}$, $\frac{\omega g}{J\pi} = \lambda$, $\lambda S(x,t) = \frac{2}{J\pi}\frac{\delta W}{\delta \bar{S}}$

$\frac{1}{\pi}\frac{\delta W}{\delta \bar{S}}$ см. 9/X. $\lambda S(x,t) = -\mu S - \gamma S^3 + \int dx' \left[\chi(xx') - \frac{1}{2}\frac{\partial^2 \beta(xx')}{\partial x^2}\right] S(x't) - \int dx' \frac{\partial \beta(xx')}{\partial x} S'(x't)$, $S' = \frac{\partial S}{\partial x}$

Если $S(x,t) = S(x) e^{i\omega t}$, то $e^{i\omega t}$ сокращается и будет $S(x)$ также, $S'(x) = \frac{\partial S}{\partial x}$

тем или другим $S(x,t)$, $S(x,t) = S_x(x,t) + iS_y(x,t) = S(x) e^{i\omega t}$, Is $S(x)$ real? Если $p=0$,

то $S(x)$ is real, т.к. $\lambda, \mu, \gamma, \chi, \beta$ are real. $S(x)$-real $\to$ $\begin{cases} S_x(x,t) = S(x)\cos\omega t \\ S_y(x,t) = S(x)\sin\omega t \end{cases}$

Если $S(x) = S_1 + iS_2$ - complex, то $S_x(x,t) = \mathrm{Re}\left[(S_1 + iS_2) e^{i\omega t}\right] =$

$= \mathrm{Re}\left[(S_1 + iS_2)(\cos\omega t + i\sin\omega t)\right] = S_1\cos\omega t - S_2\sin\omega t$, $S_y(x,t) = S_2\cos\omega t + S_1\sin\omega t$

$S_x^2 + S_y^2 = S_1^2\cos^2\omega t + S_2^2\sin^2\omega t - 2S_1S_2\sin\omega t\cos\omega t + S_2^2\cos^2\omega t + S_1^2\sin^2\omega t +$

$+ 2S_1S_2\cos\omega t\sin\omega t = S_1^2 + S_2^2 = |S(x,t)|^2 = |S(x) e^{i\omega t}|^2 = |S(x)|^2$

$S(x) = \sum_{n=-\infty}^{\infty} c_n e^{iq_n x}$, если $c_n = c_{-n}$, $S(x) = c_0 + 2\sum c_n \cos q_n x$ - чет. если $c_n = -c_{-n}$

$S(x) = \sum_{n<0} c_n e^{iq_n x} + \sum_{n\geq 0} c_n e^{iq_n x}$, $|q_n = 2\pi n| = 2i\sum_{n>0} c_n \sin q_n x$ - нечет.

$S_x(x,t) = \sqrt{S_1^2+S_2^2}\left[\cos\omega t\cos\alpha + \sin\omega t\sin\alpha\right] = \sqrt{S_1^2+S_2^2}\cos(\omega t + \alpha)$, $\alpha = \arccos\frac{S_1}{\sqrt{S_1^2+S_2^2}}$

$S_y(x,t) = \sqrt{\dots}\left[\sin\omega t\cos\alpha + \sin\alpha\cos\omega t\right] = \sqrt{S_1^2+S_2^2}\sin(\omega t + \alpha)$, $\sin\alpha = \frac{S_2}{\sqrt{S_1^2+S_2^2}}$

$S(x) = S_1(x) + iS_2(x)$, $\vec{S}(x,t)$ описана эллипсом, но в уравн. фазы, если $\frac{S_2}{\sqrt{S_1^2+S_2^2}}$

MathCad для $p=0$, тоже либо $c_n = c_{-n}$, либо $c_n = -c_{-n}$) т.е. все $S_n(z)$ моды либо чет., либо нечет. по отн. к $z=0$ plane. $z \in [-\ell_z, \ell_z]$

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Более точно не $S(z,t) = S(z)e^{i\omega t}$, а $S(z,t) = \int d\omega\, S(z,\omega) e^{i\omega t}$.

$-ig\dot{S}(z,t) = 2\frac{\delta W}{\delta S} = \pi\left[-\mu S - \gamma S'' + \int dz' \chi(zz') S(z',\omega) + \dots\right]$, $-\frac{ig}{\pi}\frac{d}{dt}\int d\omega\, S(z\omega)e^{i\omega t} =$

$= -\mu\int d\omega\, S(z\omega)e^{i\omega t} - \gamma\frac{\partial^2}{\partial z^2}\int d\omega\, S(z\omega)e^{i\omega t} + \int dz'\chi(zz')\int d\omega\, S(z'\omega)e^{i\omega t} + \dots$

Умножаем $\int d\omega\, e^{i\omega t}$: $\frac{1}{\pi}g\omega S(z\omega) = -\mu S(z\omega) - \gamma\frac{\partial^2}{\partial z^2}S(z\omega) + \int dz'\chi(zz')S(z'\omega) + \dots$

так что eq. $\lambda S = -\mu S - \gamma S'' + \int dz'\chi(zz')S(z')$ написано именно для $S(z,\omega)$, а не для $S(z,t)$. ω – фурье-компонента (спектр), тогда

$\lambda S(z) = -\mu S - \gamma S''(z) + \int dz'\left[\chi(zz') - \frac{i}{2}\frac{\partial\rho}{\partial z}(zz')\right]S(z') - \int dz'\frac{\partial\delta(zz')}{\partial z}S'(z')$ для $S(z,\omega)$

$\lambda = \omega g/\pi$, для каждого S λ_n, или ω_n, $S(z,\omega) = S_n(z)\delta(\omega-\omega_n)$ и

$S_n(z,t) = \int d\omega\, S(z,\omega)e^{i\omega t} = \int d\omega\, S_n(z)\delta(\omega-\omega_n)e^{i\omega t} = S_n(z)e^{i\omega_n t}$.

$S_n(z,t) = S_{nx}(zt) + iS_{ny}(zt)$. $S_n(z)$ are real for $P=0$, т.к. УД вещественное для $P=0$. При $P=0$ можно $S_n(z)\to S_n(z)e^{i\Phi_n}$, если $\Phi_n(z) = \text{const}$. Или иначе, при $P=0$ УД для $S_x(z)$, $S_y(z)$ – одинаковы → можно УД для S. $P\neq 0$ перемешивает УД для S_x и S_y / поэтому при $P\neq 0$ УД для $S(z) = S_x(z) + iS_y(z)$ и уже, ведь $P\neq 0$ не перемешивает УД для S_n и $\bar{S}_n$, а УД для S только одно, хотя и комплексное. При $P\neq 0$ для eq. для S_x, S_y.

$P\neq 0 \to S_n(z)$ is complex, $S_n(z) = U_n(z)e^{i\Phi_n(z)}$, $U_n(z) = |S_n(z)|$.

$S_n(z,t) = S_n(z)e^{i\omega_n t} = U_n(z)e^{i[\omega_n t + \Phi_n(z)]} = S_{nx}(zt) + iS_{ny}(zt)$

• $S_{nx}(zt) = U_n(z)\cos[\omega_n t + \Phi_n(z)]$, • $S_{ny}(zt) = U_n(z)\sin[\omega_n t + \Phi_n(z)]$ – circular v. core motion with variable phase $\Phi_n(z)$ при $P\neq 0$. $\Phi_n(z)$ вообще на $S_n(zt)$ modes intensity [Ding, APL '2014].

Вопрос: ли $\vec{S} = \vec{\omega} + \vec{S}$? $\dot{\vec{S}} = \omega\hat{z}\times\vec{S} = \omega\begin{vmatrix} x & y & z \\ 0 & 0 & 1 \\ S_x & S_y & 0 \end{vmatrix} = \omega(\hat{y}S_x - \hat{x}S_y)$

$\dot{S}_x = -\omega S_y$, $\dot{S}_y = \omega S_x$, $\dot{S}_x = \frac{d}{dt}u\cos(\omega t+\Phi) = -u\sin(\omega t+\Phi)\omega = -\omega S_y$ – вращается, т.к. Vorbit is circular for any value of z.

→ Выводы: 1) частота $\Phi_n(z) \neq \text{const}$ если $P\neq 0$; $S_n(zt) = U_n(z)e^{i[\omega_n t + \Phi_n(z)]}$

2) при $P=0$ $S_n(z)$ либо вещ. (pure real), либо нечет. (pure im.), а при $P\neq 0$ $S_n(z)$ ни вещ., ни нечет. (имеет неограничен.), $\mathrm{Re}\,S_n(z)\neq 0$, $\mathrm{Im}\,S_n(z)\neq 0$

3) при $P\neq 0$ $\mathrm{Re}\,S_n(z)$ – вещ., $\mathrm{Im}\,S_n(z)$ – нечет., где Re и $\mathrm{Im}\neq 0$.

$S_n(z) = U_n(z)e^{i\Phi_n(z)}$, $\mathrm{Re}\,S_n(z) = U_n(z)\cos\Phi_n(z)$ – вещ., $\mathrm{Im}\,S_n(z) = U_n(z)\sin\Phi_n(z)$ – нечет.

$U_n(z) = |S_n(z)|$ всегда вещ., $\Rightarrow \Phi_n(z)$ – нечет. функция z, $z\in[-\frac{1}{2}, \frac{1}{2}]$.

$S_n(-z) = S_n^*(z)$ свойство СФ при $P\neq 0$ и $P=0$.

$\frac{1}{\pi}\bar{W}_P[S,\bar{S}] = -\frac{1}{4i}\int_{-1/2}^{1/2}dz'\,P(zz')\left[\frac{\partial\bar{S}(z)}{\partial z}S(z') - \frac{\partial S(z)}{\partial z}\bar{S}(z')\right]$, удобно, т.к. сами $S(-z) = \bar{S}(z)$ (см. выше)

$\frac{1}{\pi}\bar{W}_P(-z) = -\frac{1}{4i}\int_{-1/2}^{1/2}dz'\,P(-z,z')\left[\frac{\partial\bar{S}(z)}{\partial z}S(z') + \frac{\partial S(z)}{\partial z}\bar{S}(z')\right] = \frac{1}{4i}\int_{-1/2}^{1/2}dz'\,P(zz')\left[-\frac{\partial\bar{S}(-z)}{\partial z}S(-z') + \right.$

$\left. + \frac{\partial S(-z)}{\partial z}\bar{S}(-z')\right] = \frac{1}{4i}\int dz'\,P(zz')\left[-\frac{\partial}{\partial z}S(z)\bar{S}(z') + \frac{\partial}{\partial z}\bar{S}(z)S(z')\right] = \frac{1}{\pi}\bar{W}[z]$ – чтд. и проще, чем интеграл $\int_{-1/2}^{1/2}(\dots)$

Т.е. СФ $S_n(zt) = \int S_n(z\omega)e^{i\omega t}d\omega$, $S_n(z\omega) = S_n(z)$ удобно. $S_n(-z) = \bar{S}_n(z)$.

$S(-z) = S_1(-z) + iS_2(-z) = (S_1(z) + iS_2(z))^* = S_1(z) - iS_2(z)$, $S_1(-z) = S_1(z)$, $S_2(-z) = -S_2(z)$

$\Phi_n(z)$ нечет., от z численно неограниченно в какая-то ΦИТ '2020 для $n=0,1$ vortex GM. Bug/Gan. $U(z)$, $\Phi(z)$ неизвестны еще 24-25/VII 2014 г.

20/VII Бадмаев с А. и Ю.

$P\neq 0$ magnetostatic term gaer $\Phi(z)$, $S(z) = U(z)e^{i\Phi(z)}$. В Sci. Rep '2014 нарисовали in core area, т.е. $c\to 0$. Как P зависит от c при $c = \frac{R_c}{R}\to 0$?

$F(s) = \frac{4s}{c^2}\frac{1}{(1+s^4/c^2)^2}$, $0\le s\le c$. $\Lambda(k) = \int_0^c ds\,sF(s)J_1(ks)$, $s/c = x$, $x\le 1$, $F(s) = \frac{4}{e}\times\frac{1}{(1+x^2)^2}$

$\Lambda(k) = c^2\int_0^1 dx\,x\frac{4}{e}\frac{x^2}{(1+x^2)^2}J_1(kcx) = 4c\int_0^1 dx\,\frac{x^2}{(1+x^2)^2}J_1(kcx)$, $\Lambda'(k) = \Lambda(k)$ с $c\to 0$

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$P(zz') \simeq 8\pi M_s^2 c\rho C \int dk e^{-\beta k|z-z'|} \Lambda^2(k) = 8\pi M_s^2 \rho C \int_0^\infty dx e^{-\frac{\beta}{c}x|z-z'|} \Lambda^2(x)$, $x=kc$

$\Lambda(x) = 4a \int_0^1 dx \frac{x^2}{(1+x^2)^2} J_1(\pi x)$, $P(zz') = 8\pi M_s^2 \rho C \cdot 16 c^2 \int_0^\infty dx e^{-\frac{\beta}{c}x|z-z'|} \left[\int_0^1 \frac{x^2}{(1+x^2)^2} J_1(\pi x)\right]^2$

Далее при $z=z'$ $\int_0^\infty dx\, J_1(\pi x) J_1(\pi x')$ [6.512 Гр.-Р.] конечен.

$P(zz') \sim c^2 f(c) \sim c^2$ или c^n, $n>2$. $P(zz') \to 0$ при $c \to 0$.

Но exch. $y(c) \sim \ln 1/c$ расходится при $c \to 0$.

// $\Phi_n(z)$ с $\frac{\partial \Phi_n}{\partial z} \neq 0$ появляется при учете v. corr (finite c) и растет с ростом L

$\Phi_n(z)$ ведет к уменьшению FMR n-mode intensity.

(21/VII - 22/VI) Численные расчеты $S_n(z) = \Phi_n(z) e^{i\Phi_n(z)}$ для $R = 100$ nm, PMA, L - variable. High-freq. doublets при малых L.

① Decreasing $\Phi_n(z)$ of the lowest GM появляется при больших L. Бинд.

(23/VI) Вызов Tax office, Volosa. Вернут 3 500 €. Уменьшилась ипотека. E-mail to Ливон от AF-DW paper. С п.и Ю. на Ondarreta.

(24/VI) Отдых дома. Бинд.

Gyrofield

(25/VI) Отменить/переразервировать Hotel Gijon до June 30.

Оп. налог на (2011) машину. Сделать вторник MCIU-wires project.

Zeme Referee's comments on Ivan's AF DW dyn. + MS - Mat. Com.

Gyrofield: Неверное определение $\mathcal{L}_{kin}$ и gyrofield in Supp. Mat. #1.

Gus' PRL'2008 $\mathcal{L}_{kin} = \frac{M_s}{\gamma} \frac{(\vec{n}\cdot[\dot{\vec{n}}\times\vec{n}])}{1+\vec{n}\cdot\vec{n}}$ - kin. Lagrangian density

Gyrofield $\vec{h} = \frac{1}{M_s}\frac{\partial \mathcal{L}_{kin}}{\partial \vec{n}}$. Номер надо $\frac{\delta \mathcal{L}_k}{\delta \vec{n}} = \frac{\partial \mathcal{L}_k}{\partial \vec{n}} - \frac{d}{dt}\frac{\partial \mathcal{L}_k}{\partial(\dot{\vec{n}})}$?

$\mathcal{L}_{kin} \to W_z = -\vec{m}\cdot\vec{h}$, $\mathcal{L} = \mathcal{L}_{kin} - W$, $\delta\mathcal{L} = \delta\mathcal{L}_k + \ldots = \vec{h}\,\delta\vec{m} M_s = \frac{\delta \mathcal{L}_k}{\delta \vec{m}}\delta\vec{m} + \ldots$

$\mathcal{L}_{kin} = M_s \vec{m}\cdot\vec{h} \to \frac{\delta \mathcal{L}_k}{\delta \vec{m}} = M_s \vec{h}$, $\vec{h} = \frac{1}{M_s}\frac{\delta \mathcal{L}_k}{\delta \vec{m}}$. $\frac{M_s}{\gamma} = J$.

$\mathcal{L}_k(\vec{m}) = J \frac{\vec{n}\cdot(\dot{\vec{m}}\times\vec{m})}{1+\vec{n}\cdot\vec{m}}$. $\vec{a}\cdot(\vec{b}\times\dot{\vec{c}}) = \dot{\vec{m}}\cdot(\vec{n}\times\vec{m}) = \vec{m}\cdot(\dot{\vec{n}}\times\vec{m})$

$\mathcal{L}_k = J \frac{(\dot{\vec{m}}\times\vec{n})\cdot\vec{m}}{1+\vec{n}\cdot\vec{m}}$, $\frac{\delta \mathcal{L}_k}{\delta \vec{m}}$? Нужна $\frac{\partial \mathcal{L}_k}{\partial \vec{m}} = \frac{\partial}{\partial \vec{m}} J \frac{(\dot{\vec{m}}\times\vec{n})\cdot\vec{m}}{1+\vec{n}\cdot\vec{m}} =$

$= J \frac{1}{(1+\vec{n}\cdot\vec{m})^2}\left[(\dot{\vec{m}}\times\vec{n})(1+\vec{n}\cdot\vec{m}) - (\dot{\vec{m}}\times\vec{n})\cdot\vec{m}\,\vec{n}\right] = \frac{J}{(1+\vec{n}\cdot\vec{m})^2}\left[\vec{a}(1+\vec{n}\cdot\vec{m}) - (\vec{a}\cdot\vec{m})\vec{n}\right]$

$\vec{a} = \dot{\vec{m}}\times\vec{n}$. $\vec{a}(\vec{n}\cdot\vec{m}) - \vec{n}(\vec{a}\cdot\vec{m}) = \vec{m}\times(\vec{a}\times\vec{n}) = \vec{m}\times\{(\dot{\vec{m}}\times\vec{n})\times\vec{n}\} =$

$= \vec{m}\times\{\underset{a}{\vec{n}}\times(\underset{b}{\dot{\vec{m}}}\times\underset{c}{\vec{n}})\}(-1) = -\vec{m}\times\{\dot{\vec{m}}(\vec{n}\cdot\vec{n}) - \vec{n}(\vec{n}\cdot\dot{\vec{m}})\} = -\vec{m}\times\{\dot{\vec{m}} - \vec{n}(\vec{n}\cdot\dot{\vec{m}})\}$

$\frac{\partial \mathcal{L}_k}{\partial \vec{m}} = J\frac{1}{(1+\vec{n}\cdot\vec{m})^2}\left[\dot{\vec{m}}\times\vec{n} - \vec{m}\times\dot{\vec{m}} + \vec{m}\times\vec{n}(\vec{n}\cdot\dot{\vec{m}})\right]$

В Gus' PRL'08 $\mathcal{L}_{kin} = J\frac{(\dot{\vec{m}}\times\vec{m})_z}{(p+m_z)}$, где $(\vec{h})_z = -\frac{1}{\gamma}\frac{(\dot{\vec{m}}\times\vec{m})_z}{(p+m_z)^2}$, $\begin{cases}\vec{n}=\hat{z}, z<0, p=+1\\ \vec{n}=-\hat{z}, z>0, p=-1\end{cases}$

$\frac{\partial \mathcal{L}_k}{\partial \dot{\vec{m}}} = \frac{\partial}{\partial \dot{\vec{m}}} J\frac{\dot{\vec{m}}\cdot(\vec{n}\times\vec{m})}{(1+\vec{m}\cdot\vec{n})} = J\frac{(\vec{n}\times\vec{m})}{(1+\vec{n}\cdot\vec{m})}$, $\frac{d}{dt}\frac{\partial \mathcal{L}_k}{\partial \dot{\vec{m}}} = \frac{d}{dt}J\frac{(\vec{n}\times\vec{m})}{(1+\vec{n}\cdot\vec{m})} =$

$= J\frac{1}{(1+\vec{n}\cdot\vec{m})^2}\left[(\dot{\vec{n}}\times\vec{m})(1+\vec{n}\cdot\vec{m}) - (\vec{n}\times\vec{m})(\dot{\vec{n}}\cdot\vec{m})\right]$

$\frac{\delta \mathcal{L}_k}{\delta \vec{m}} = \frac{\partial \mathcal{L}_k}{\partial \vec{m}} - \frac{d}{dt}\frac{\partial \mathcal{L}_k}{\partial(\dot{\vec{m}})} = \frac{J}{(1+\vec{n}\cdot\vec{m})^2}\left[\dot{\vec{m}}\times\vec{n} - \vec{m}\times\dot{\vec{m}} + \vec{m}\times\vec{n}(\dot{\vec{n}}\cdot\vec{m}) - (\vec{n}\times\dot{\vec{m}})(1+\vec{n}\cdot\vec{m}) + (\vec{n}\times\vec{m})(\dot{\vec{n}}\cdot\vec{m})\right] =$

$= \frac{J}{(1+\vec{n}\cdot\vec{m})^2}\left[-\vec{m}\times\dot{\vec{m}} + 2\dot{\vec{m}}\times\vec{n} - (\vec{n}\times\dot{\vec{m}})(\vec{n}\cdot\vec{m})\right]$

→ $\frac{\delta \mathcal{L}_k}{\delta \vec{m}} = \frac{J}{(1+\vec{n}\cdot\vec{m})^2}\left[-\vec{m}\times\dot{\vec{m}} + (\dot{\vec{m}}\times\vec{n})\{2+\vec{n}\cdot\vec{m}\}\right]$. Вот уже! Или $\frac{\delta \mathcal{L}_k}{\delta \vec{m}}$, или $\frac{\partial \mathcal{L}_k}{\partial \dot{\vec{m}}}$

не совпадает с gyrofield из Gus' PRL'08 ($h_z \sim (\dot{\vec{m}}\times\vec{m})_z$) Нет, все ок - h_z см 27/VI

Gus' PRL'16: $\mathcal{L}_{kin} = \vec{C}(\vec{m})\cdot D_0\vec{m}$, $\vec{C}(\vec{m}) = J_c\frac{(\vec{m}\times\vec{m}')}{(1+\vec{m}\cdot\vec{n})}$, $D_0\vec{m} = (\partial_0 - \hat{A}_0)\vec{m} = \partial_0\vec{m} - \vec{m}\times\vec{A}_0$

$\partial_0\vec{m} = \frac{\partial\vec{m}}{\partial x_0} = \frac{1}{c}\frac{\partial\vec{m}}{\partial t}$, $x_0 = ct$. Gyrofield $\vec{H}_g(\vec{m}) = -\frac{c}{\gamma}\vec{A}_0(\vec{m})$. $\vec{A}_0$ определен Eq. (2)

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Eje $\alpha=\varphi$, $\beta=\theta$, $\gamma=\pi-\varphi$. $\vec{m}(\theta,\varphi)$, $A^x_\mu = \sin\gamma\partial_\mu\beta - \sin\beta\cos\gamma\partial_\mu\alpha =$
$= \sin\varphi\partial_\mu\theta + \sin\theta\cos\theta\partial_\mu\varphi$, $A^y_\mu = -\cos\varphi\partial_\mu\theta + \sin\theta\sin\varphi\partial_\mu\varphi$,
$A^z_\mu = -\partial_\mu\varphi + \cos\theta\partial_\mu\varphi = (\cos\theta-1)\partial_\mu\varphi$, $\partial_\mu = \frac{\partial}{\partial x_\mu}$, $\frac{\partial}{\partial x_0} = \frac{1}{c}\frac{\partial}{\partial t}$

Gyrofield $\vec{H}_g = -\frac{c}{\gamma}\vec{A}_0$, $H_g^x = -\frac{1}{\gamma}\left[\sin\varphi\dot\theta + \sin\theta\cos\varphi\dot\varphi\right]$, $H_g^y = -\frac{1}{\gamma}\times$
$\times\left[-\cos\varphi\dot\theta + \sin\theta\sin\varphi\dot\varphi\right]$, $H_g^z = -\frac{1}{\gamma}\left[\cos\theta-1\right]\dot\varphi$

Gyrofield ug Gus'PRL'08, $h_z = -\frac{1}{\gamma}\frac{(\vec m\times\dot{\vec m})_z}{(p+m_z)^2}$, $p=\pm1$.

$(\vec m\times\dot{\vec m})_z = m_x\dot m_y - m_y\dot m_x = \sin\theta\cos\varphi\frac{d}{dt}(\sin\theta\sin\varphi) - \sin\theta\sin\varphi\frac{d}{dt}(\sin\theta\cos\varphi)$
$= \sin\varphi\cos\varphi(\cos\theta\dot\theta\sin\varphi + \sin\theta\cos\varphi\dot\varphi) - \sin\theta\sin\varphi(\cos\theta\dot\theta\cos\varphi - \sin\theta\sin\varphi\dot\varphi) =$
$= \sin^2\theta\cos^2\varphi\dot\varphi + \sin^2\theta\sin^2\varphi\dot\varphi = \sin^2\theta\dot\varphi$

$p+m_z = p+\cos\theta = \cos\theta+1$, $(\vec m\times\dot{\vec m})_z = \sin^2\theta\dot\varphi = (1-\cos^2\theta)\dot\varphi$

$h_z = -\frac{1}{\gamma}\frac{(1-\cos^2\theta)}{(\cos\theta+1)^2}\dot\varphi$, como en $H_g^z = -\frac{1}{\gamma}(\cos\theta-1)\dot\varphi$ (EPL'16).
(PRL'08)

$\frac{1-\cos^2\theta}{(\cos\theta+1)^2} = \frac{(1-\cos\theta)(1+\cos\theta)}{(1+\cos\theta)^2} = \begin{cases}\frac{1-\cos\theta}{1+\cos\theta}, & p=+1\\ (1+\cos\theta)/(1-\cos\theta), & p=-1\end{cases} = \frac{1-p\cos\theta}{1+p\cos\theta}$, $p=\pm1$.

Gyrofields ug Gus'PRL'08 u Gus'EPL'16 ne cobinagaiot. Paguice onpegenaem?
B Gus'EPL'16 $\vec H_g$ defined as m. field from soliton $\vec m(\theta,\varphi)$ acting on SW $\vec m_s$

$\mathcal{L}_{kin} = \frac{M_s}{\gamma}\frac{\vec n\cdot(\vec m\times\dot{\vec m})}{1+\vec n\vec m}$, $(\vec m\times\dot{\vec m})_z = \sin^2\theta\dot\varphi$, $\vec n\cdot\vec m = \cos\theta$ если $\vec n=\hat z$.

$\mathcal{L}_{kin} = \frac{M_s}{\gamma}\frac{(\vec m\times\dot{\vec m})_z}{1+m_z} = \frac{M_s}{\gamma}\frac{\sin^2\theta\dot\varphi}{1+\cos\theta} = \frac{M_s}{\gamma}\frac{(1-\cos^2\theta)}{1+\cos\theta}\dot\varphi = \frac{M_s}{\gamma}(1-\cos\theta)\dot\varphi$, $\vec n=\hat z$.
$\vec m(\theta,\varphi)$, верно.

(26/VI) Перепрезервирано Hotel Gijon на 17–23/VII с правен отказ до 1/VII. Бъдещ

(27/VI) Чрез Gus'PRL'08, $\mathcal{L}_{kin}$ записан в виде $\mathcal{L}_{kin} = M_s\vec m\cdot\vec h$, $\vec h$ - gyrofield
$\vec h = \frac{1}{M_s}\frac{\delta\mathcal{L}_{kin}}{\delta\vec m}$, $\vec h = \vec h(\vec m,\dot{\vec m})$, $\dot{\vec m}\times\vec n = \dot{\vec m}\times\hat z = \begin{vmatrix}\hat x & \hat y & \hat z\\ \dot m_x & \dot m_y & \dot m_z\\ 0 & 0 & 1\end{vmatrix} = \hat x\dot m_y - \hat y\dot m_x$, $\vec n=\hat z$.
Если брать само h_z и $\vec h = \frac{1}{M_s}\frac{\partial\mathcal{L}_k}{\partial\dot{\vec m}}$ или $\vec h = \frac{1}{M_s}\frac{\delta\mathcal{L}_k}{\delta\dot{\vec m}}$, то $h_z = -\frac{1}{\gamma}\frac{(\vec m\times\dot{\vec m})_z}{(1+\vec n\cdot\vec m)^2}$
$\vec m\times\vec n = \hat x m_y - \hat y m_x$
ковариантен, но равно $\partial/\partial\dot{\vec m}$ или вариация $\delta/\delta\dot{\vec m}$. h_z из Gus'PRL'08, верно.
Нужно найти $(\vec H)_z = P$, $(P+m_z)^2 = (1+pm_z)^2 = (1+p\cos\theta)^2 = (1+\vec n\cdot\vec m)^2$ очевидно, что $\vec H$
с $p=\pm1$ у. спин поляризация трябва $\partial/\partial\dot{\vec m}$ или $\delta/\delta\dot{\vec m}$ в лагранж. $\vec H$?

$\mathcal{L}_k = M_s\vec m\cdot\vec h$, если $\delta/\delta\dot{\vec m}$, то $\vec H_k = M_s\vec m\cdot\frac{1}{M_s}\frac{\delta\mathcal{L}_k}{\delta\dot{\vec m}} = \vec m\cdot\frac{\delta\mathcal{L}_k}{\delta\dot{\vec m}} = \frac{J\vec m\cdot}{(1+\vec n\cdot\vec m)^2}\left[-\vec m\times\dot{\vec m} + \right.$
$\left. + (\vec m\times\vec n)(2+\vec n\cdot\vec m)\right] = \frac{J}{(1+\vec n\cdot\vec m)^2}\vec m\cdot(\dot{\vec m}\times\vec n)(2+\vec n\cdot\vec m) = J\frac{(2+\vec n\cdot\vec m)}{(1+\vec n\cdot\vec m)^2}\vec n\cdot(\vec m\times\dot{\vec m})$ будем $= \frac{J}{(1+\vec n\cdot\vec m)}\vec n\cdot(\vec m\times\dot{\vec m})$,
если "z" $\to 1$. Если $\partial/\partial\dot{\vec m}$, $\vec H_k = \vec m\cdot\frac{\delta\mathcal{L}_k}{\delta\dot{\vec m}} = \vec m\cdot\frac{J}{(1+\vec n\vec m)^2}\left[-\vec m\times\dot{\vec m} + \dot{\vec m}\times\vec n + (\vec m\times\vec n)(\vec n\cdot\vec m)\right]$
$= \frac{J}{(1+\vec n\cdot\vec m)^2}\vec m\cdot(\vec m\times\vec n)$ - защото не вървят.
$\vec m\cdot(\vec m\times\vec n) = \vec m\cdot(\vec m\times\hat z) = \vec m\cdot\left[\hat x m_y - \hat y m_x\right] = m_x\dot m_y - m_y\dot m_x = (\vec m\times\dot{\vec m})_z = \sin^2\theta\dot\varphi$
Нещо друго $\frac{\delta\mathcal{L}_k}{\delta\dot{\vec m}} = \frac{J}{(1+\vec n\vec m)^2}\left[-\vec m\times\dot{\vec m} + (\vec m\times\vec n)(1+\vec n\cdot\vec m)\right]$, всъщност $\vec m\cdot\frac{\delta\mathcal{L}_k}{\delta\dot{\vec m}} = \frac{J}{(1+\vec n\vec m)}\vec n\cdot(\vec m\times\dot{\vec m}) = \mathcal{L}_k$
И также $\delta\mathcal{L}_k/\delta\dot{\vec m}$ не совпадает с $\frac{\delta\mathcal{L}_k}{\delta\dot{\vec m}} = \frac{\partial\mathcal{L}_k}{\partial\dot{\vec m}} - \frac{d}{dt}\frac{\partial\mathcal{L}_k}{\partial(\dot{\vec m})}$, остават $\vec h$-комп.
нищо $h_x, h_y \perp \vec n$ ($\vec n=\hat z$) и са $\vec h$-компоненти $\perp\vec n$.

Хамил $\mathcal{L}_k = \frac{M_s}{\gamma}\frac{\vec n\cdot(\vec m\times\dot{\vec m})}{(1+\vec n\vec m)}$ записан в виде $\mathcal{L}_k = M_s\vec m\cdot\vec h$. Это нужно, чтобы
$\vec h = \frac{1}{\gamma}\frac{1}{(1+\vec n\vec m)^2}\left[-\vec m\times\dot{\vec m} + (\vec m\times\vec n)(1+\vec n\cdot\vec m)\right]$ ду-поле, но нужно $\vec H = \frac{1}{M_s}\frac{\delta\mathcal{L}_k}{\delta\dot{\vec m}}$
или $\delta\mathcal{L}_k/\delta\dot{\vec m} = \frac{\partial\mathcal{L}_k}{\partial\dot{\vec m}} - \frac{d}{dt}\frac{\partial\mathcal{L}_k}{\partial(\dot{\vec m})}$, вариационна производна. Хамил $h_z = \frac{1}{M_s}\cdot\frac{\delta\mathcal{L}_k}{\delta\dot m_z}$
Нямам $\vec h = M_s\vec H_k/\delta\dot{\vec m}$ да $\vec h$-компоненти $\perp\vec n$. Тези стъпки

Escaneado con CamScanner

$$\vec{h} = (\vec{h}\cdot\vec{n})\vec{n} + \vec{h}(\perp\vec{n}) = -\frac{1}{\gamma}\frac{(\vec{m}\times\dot{\vec{m}})\cdot\vec{n}}{(1+\vec{n}\cdot\vec{m})^2}\vec{n} + \frac{1}{\gamma}\frac{(\vec{m}\times\dot{\vec{m}})}{(1+\vec{n}\cdot\vec{m})}$$

$$\vec{m} = m_n\vec{n} + \vec{m}(\perp\vec{n}),\quad \vec{n}\times\vec{m} = \vec{n}\times[m_n\vec{n}+\vec{m}_\perp] = \vec{n}\times\vec{m}_\perp,\quad \dot{\vec{m}}\cdot(\vec{n}\times\vec{m}) = \dot{\vec{m}}\cdot(\vec{n}\times\vec{m}_\perp) =$$
$$= \vec{n}\cdot(\vec{m}_\perp\times\dot{\vec{m}}),\quad \vec{m}\times\dot{\vec{m}} = (m_n\vec{n}+\vec{m}_\perp)\times(\dot{m}_n\vec{n}+\dot{\vec{m}}_\perp) = m_n\vec{n}\times\dot{\vec{m}}_\perp + \vec{m}_\perp\times(\dot{m}_n\vec{n}) + \vec{m}_\perp\times\dot{\vec{m}}_\perp$$
$$\vec{m}_\perp\times\dot{\vec{m}} = \vec{m}_\perp\times(\dot{m}_n\vec{n}),\quad \vec{n}\cdot(\vec{m}\times\dot{\vec{m}}) = \vec{m}(\vec{n}\times\vec{m}) = \vec{n}\cdot(\vec{m}_\perp\times\dot{\vec{m}}) = \vec{n}\cdot[\vec{m}_\perp\times\dot{m}_n\vec{n} + \vec{m}_\perp\times\dot{\vec{m}}_\perp]$$
$$\vec{n}\cdot(\vec{m}\times\dot{\vec{m}}) = \vec{n}\cdot(\vec{m}_\perp\times\dot{\vec{m}}_\perp)$$

$$\vec{h}_{in} = \frac{M_S}{\gamma}\frac{\vec{n}\cdot(\vec{m}\times\dot{\vec{m}})}{(1+\vec{n}\cdot\vec{m})} = \frac{M_S}{\gamma}\frac{\vec{n}\cdot(\vec{m}_\perp\times\dot{\vec{m}}_\perp)}{(1+m_n)},\quad \text{para } \vec{n}=\hat{z}\ \text{ges.}\quad (\vec{m}_\perp\times\dot{\vec{m}}_\perp) = m_x\dot{m}_y - m_y\dot{m}_x$$

$$h_n = \frac{1}{M_S}\frac{\delta\mathcal{L}_K}{\delta m_n} = \frac{1}{\gamma}\left[\frac{\partial}{\partial m_n} - \frac{d}{dt}\frac{\partial}{\partial\dot{m}_n}\right]\frac{\vec{n}\cdot(\vec{m}_\perp\times\dot{\vec{m}}_\perp)}{(1+m_n)} = \frac{1}{\gamma}\vec{n}(\vec{m}_\perp\times\dot{\vec{m}}_\perp)\frac{\partial}{\partial m_n}\frac{1}{(1+m_n)} =$$
$$= -\frac{1}{\gamma}\frac{1}{(1+m_n)^2}\vec{n}\cdot(\vec{m}_\perp\times\dot{\vec{m}}_\perp).\quad \vec{h}_\perp\ \text{mismo tu vas } \vec{h}_\perp = \frac{1}{M_S}\frac{\delta\mathcal{L}_K}{\delta\vec{m}_\perp}\ ?$$

$$\frac{\delta\mathcal{L}_K}{\delta\vec{m}_\perp} = \left[\frac{\partial}{\partial\vec{m}_\perp} - \frac{d}{dt}\frac{\partial}{\partial\dot{\vec{m}}_\perp}\right]\frac{M_S}{\gamma}\frac{\vec{n}\cdot(\vec{m}_\perp\times\dot{\vec{m}}_\perp)}{(1+m_n)} = \frac{M_S}{\gamma}\frac{1}{(1+m_n)}\left[\frac{\partial}{\partial\vec{m}_\perp} - \frac{d}{dt}\frac{\partial}{\partial\dot{\vec{m}}_\perp}\right]\vec{n}\cdot(\vec{m}_\perp\times\dot{\vec{m}}_\perp)$$

Def $\vec{m}_\perp = \vec{\mu}$, $\left[\frac{\partial}{\partial\vec{\mu}} - \frac{d}{dt}\frac{\partial}{\partial\dot{\vec{\mu}}}\right]\vec{n}\cdot(\vec{\mu}\times\dot{\vec{\mu}}) = \frac{\partial}{\partial\vec{\mu}}(\dot{\vec{\mu}}\times\vec{n})\cdot\vec{\mu} - \frac{d}{dt}\frac{\partial}{\partial\dot{\vec{\mu}}}(\vec{n}\times\vec{\mu})\cdot\vec{\mu} =$

$= (\dot{\vec{\mu}}\times\vec{n}) - \frac{d}{dt}(\vec{n}\times\vec{\mu}) = \dot{\vec{\mu}}\times\vec{n} - \vec{n}\times\dot{\vec{\mu}} = 2\dot{\vec{\mu}}\times\vec{n}$. $\frac{\delta\mathcal{L}_K}{\delta\vec{\mu}} = \frac{M_S}{\gamma}\frac{1}{(1+m_n)}\cdot 2\dot{\vec{\mu}}\times\vec{n}$,

sorge como $\vec{h}_\perp = \frac{1}{\gamma}\frac{(\vec{m}\times\dot{\vec{m}})}{(1+m_n)}$. $\vec{h}_\perp = \frac{1}{M_S}\frac{\delta\mathcal{L}_K}{\delta\vec{m}}$ no funciona. $\vec{h}_\perp = \frac{1}{2}\frac{1}{M_S}\frac{\delta\mathcal{L}_K}{\delta\vec{m}}$

Hemican Word file "Gyrofield" is noticed end to Ruben y Ricardo to re-write Suppl. Note #1 of "AF DW dynamics" manuscript.

22/VII Hemican other 2nd Referee o Gyrofield / Kinetic field u noticed Edited GW-wires Abstract Farkhad gue Virtual MMM' Ruben/Ric 2020

29/VII C.N. B "Carrefour" Olaberria. Cocabur u noticed BUPV/EHU budgets y MCIU-wires project (60k€ costes directos). 7,000 € computers. E-mail to Ruben AF DW dyn" paper, gyrofield estimations

30/VII Oizop u approved Ricardo AF abstracts to Virtual MMM'20 i CMD'20 Conference

1/VIII E-mail to Ruben AF DW dyn. MS. Skype c. Fiad, Proof AIP thick dots" E-mail to Igor o Pawel "GW edge scattering" MS

2/VIII E-mail to Antxoni. Proof "GM thick dots" AIP. Proof. Bilbao. 3D charge BP-Exc. e-mail

3/VIII Ruben re-submit "AF DW dyn." MS to Commun. Phys. Paracer GM wires. Roc. & Lenell : N.D. 11.5 kg, N. Irene 6.5 kg, Unne 4.0 kg — 48€

4/VIII C.N. Carrefour, Olaberria. Hamiltonian Vortex GM paper // Nam, Quadratic

5/VIII Gue vortex pC-product closure $\times P(zz') \to pC$, $\lambda = \frac{1}{\pi}g\omega \to \lambda = p\frac{1}{\pi}g\omega$. Meaning Onsager $p=+1$ is fixed + number of vortex chirality

1D gue $S(z\omega) = S_I + iS_R$: $\lambda S(z) = -\left[\nu\frac{\partial^2}{\partial z^2} - \mu\right]S(z) + \int dz'\left[x(zz') - \frac{i}{2}P'_z(zz')\right]S(z') -$

$$P'_z(zz') = \frac{\partial P(zz')}{\partial z},\quad B'_z = \frac{\partial B(zz')}{\partial z}$$

$$- \int dz' B'_z(zz')\frac{\partial S(z')}{\partial z'}$$

$(\cdots)$ complex conjugate 1D : $\lambda\bar{S}(z) = -[\cdots]\bar{S}(z) + \int dz'\left[x(zz') + \frac{i}{2}P'_z(zz')\right]\bar{S}(z') -$

Given $\bar{S}(z) = S(-z)$ gue cases. neg.

$$- \int dz' B'_z(zz')\frac{\partial\bar{S}(z')}{\partial z'}$$

$$\lambda S(-z) = -\left[\frac{\partial^2}{\partial z^2} - \mu\right]S(-z) + \int dz'\left[x(zz') + \frac{i}{2}P'_z(zz')\right]S(-z') - \int dz' B'_z(zz')\frac{\partial S(-z')}{\partial z'}$$

$-z\to z$: $\lambda S(z) = -[\cdots]S(z) + \int dz'\left[x(-z,z') + \frac{i}{2}\frac{\partial P(-z,z')}{\partial(-z)}\right]S(-z') - \int dz'\frac{\partial B(-z,z')}{\partial(-z)}(-1)\frac{\partial S(-z')}{\partial(-z')}(-1)$

$$\int_{-1/2}^{1/2} dz'\, x(-z,z')S(-z') = \int_{-1/2}^{1/2} dz'\, x(-z,z')S(-z') = \left|-z'=\xi\right| = \int_{-1/2}^{1/2} d\xi\, x(-z,-\xi)S(\xi)(-1)$$

Comparing definitions $x(zz') = x(z',z)$, $B(zz') = B(z',z)$, $P(zz') = P(z',z)$

le x, u B, u P acts functions of $|z-z'|$.

$$\int_{-1/2}^{1/2} dz'\, x(-z,z')S(-z') = (-1)\int_{1/2}^{-1/2} d\xi\, x(z,\xi)S(\xi) = \int_{-1/2}^{1/2} d\xi\, x(z,\xi)S(\xi)$$

$$\int_{-1/2}^{1/2} dz'\frac{\partial P(-z,z')}{\partial(-z)}S(-z') = \left|-z'=\xi\right| = -\int_{1/2}^{-1/2} d\xi\frac{\partial P(-z,-\xi)}{\partial(-z)}S(\xi) = \int_{-1/2}^{1/2} d\xi\frac{\partial P(z,\xi)}{\partial(-z)}S(\xi)(-1) =$$

Escaneado con CamScanner

$$= +\int_{-1/2}^{1/2} dz\, \frac{\partial P(z,z')}{\partial z} s(z) = -\int_{-1/2}^{1/2} dz\, \frac{\partial P(z,z')}{\partial z} s(z)$$

$$\int_{-1/2}^{1/2} dz' \frac{\partial \mathcal{Z}(-z,z')}{\partial(-z)} \frac{\partial s(z')}{\partial z'} = \left|z' = -\xi\right| = -\int_{1/2}^{-1/2} d\xi \frac{\partial \mathcal{Z}(-z,-\xi)}{\partial(-z)} \frac{\partial s(+\xi)}{\partial(-\xi)} = -\int_{-1/2}^{1/2} d\xi \frac{\partial \mathcal{Z}(z,\xi)}{\partial(-z)} \frac{\partial s(+\xi)}{\partial(+\xi)}$$

$$= -\int_{1/2}^{-1/2} d\xi \frac{\partial \mathcal{Z}(z,\xi)}{\partial z} \frac{\partial s(\xi)}{\partial \xi} = \int_{-1/2}^{1/2} d\xi \frac{\partial \mathcal{Z}(z,\xi)}{\partial z} \frac{\partial s(\xi)}{\partial \xi}$$

$$** \quad \lambda s(z) = -[\dots] s(z) + \int dz' \chi(zz') s(z') + \frac{i}{2}(-1) \int_{-1/2}^{1/2} d\xi \frac{\partial P}{\partial z}(z,\xi) s(\xi) - \int d\xi \frac{\partial \mathcal{Z}(z,\xi)}{\partial z} \frac{\partial s(\xi)}{\partial \xi}$$

$\lim z' \to \xi$, eq. $(*)$ coincides с eq. $(**)$.

Вывод: если $\bar{s}(z) = s(-z)$, то eq. для $s(z)$ и $\bar{s}(z)$ совпадают.

$$* \quad \lambda s(z) = -\left[\gamma \frac{\partial^2}{\partial z^2} - \mu\right] s(z) + \int dz' \left[\chi(zz') - \frac{i}{2} \frac{\partial P(zz')}{\partial z}\right] s(z') - \int dz' \frac{\partial \mathcal{Z}(zz')}{\partial z} \frac{\partial s(z')}{\partial z'}$$

А если $P \to -P$? $\quad \lambda s(z) = [\gamma, \mu, \chi, s] + \frac{i}{2} \int dz' \frac{\partial P(zz')}{\partial z} s(z') \quad (***)$

$\lambda \bar{s}(z) = [\gamma \mu \chi s] - \frac{i}{2} \int dz' \frac{\partial P(zz')}{\partial z'}$ (terms) $\bar{s}(z')$ даст eq. для $s(z)$ с $P \to -P$,

если $\bar{s}(z) = s(-z)$.

$+P$: $\lambda s(z) = [\gamma \mu \chi s] - \frac{i}{2} \int dz' \frac{\partial P}{\partial z}(zz') s(z')$, $\quad -P$: $\lambda \bar{s}(z) = [\dots] - \frac{i}{2} \int dz' \frac{\partial P(zz')}{\partial z} \bar{s}(z')$

$\to$ Все (λ_i, Φ) для $+P$ и $-P$ эквивалентны если $s(z) = \bar{s}(z)$, т.е. $s(z)$ — real, $\Phi(z) = \text{const}$, только $\mathrm{Re}\, s(z) \neq 0$, $\mathrm{Im}\, s(z) = u(z) \sin\Phi(z) = 0$, $\Phi(z) = 0, \pm\pi$.

Но при $c = Re/R \neq 0$, L большое $\mathrm{Im}\, s_n(z) \neq 0$ если $|P| \neq 0$ и спектр λ_n зависит существенно от $\mathrm{sign}(P) = \mathrm{sign}(PC)$.

$$-\frac{i}{2}\left(\frac{\partial P}{\partial z}\right)_{nn'} = \frac{i}{2} cp \int dk\, k \mathcal{J}(k) \Lambda'(k) \bar{J}_{nn'}(k) = cp \int dk\, k \mathcal{J} \left[\frac{q_n \delta_{nn'}}{(\chi^2 + q_n^2)} + (-1)^{n+n'} \frac{\chi (q_n + q_{n'})(e^{-\chi} - 1)}{(\chi^2 + q_n^2)(\chi^2 + q_{n'}^2)}\right]$$

$\chi = \beta k$, $q_n = 2\pi n$ при $s(z) = \sum_n c_n e^{i q_n z}$ — Фурье разложение. $\quad n = -N, -N+1, 0, \dots N$.

$$-\frac{i}{2}\left(\frac{\partial P}{\partial z}\right)_{nn'} \sim \mathrm{sign}(P) \left[\frac{q_n \delta_{nn'}}{\chi^2 + q_n^2} + (-1)^{n+n'} \frac{\chi (q_n + q_{n'})(e^{-\chi} - 1)}{(\chi^2 + q_n^2)(\chi^2 + q_{n'}^2)}\right]$$

$$(-1)^{n+n'} (q_n + q_{n'}) \sim (-1)^{n+n'} (n + n')$$

$$\mathrm{diag}\left(\frac{\partial P}{\partial z}\right)_{nn'} \sim \left(\frac{q_n}{\chi^2 + q_n^2} + \frac{2\chi q_n (e^{-\chi} - 1)}{(\chi^2 + q_n^2)^2}\right] \mathrm{sign}(P) \sim q_n \mathrm{sign}(P) \sim n\, \mathrm{sign}(P)$$

$$\text{non-diag}\left(\frac{\partial P}{\partial z}\right)_{nn'} \sim \mathrm{sign}(P) \left[(-1)^{n+n'} \frac{\chi (q_n + q_{n'})}{(\chi^2 + q_n^2)(\chi^2 + q_{n'}^2)} (e^{-\chi} - 1)\right] \sim \mathrm{sign}(P) (-1)^{n+n'} (n + n')$$

$$\frac{1}{V} W_m^P [\bar{s}(z)] = \frac{1}{2} \int dz dz' (-1) P(zz') \left[\frac{\partial \bar{s}}{\partial z} s(z') - \frac{\partial s}{\partial z} \bar{s}(z')\right] = \frac{1}{4i} (-1) \int dz dz' P(zz') \left[\frac{\partial \bar{s}(z)}{\partial z} s(z') - \frac{\partial s}{\partial z} \bar{s}(z')\right] =$$

$$= -\frac{1}{4i} \int dz dz' P(zz') \left[\frac{\partial s(-z)}{\partial z} s(z') - \frac{\partial s}{\partial z} s(-z')\right] = (-1) \int dz dz' P(zz') \frac{\partial s(-z)}{\partial z} s(z') - (-1) \int dz dz' P(zz') \frac{\partial s}{\partial z} s(-z') =$$

$$\frac{\partial s}{\partial z}(z) s(-z') = \left|\begin{matrix} -z = \xi \\ -z' = \xi' \end{matrix}\right| = (-1)(-1) \int_{1/2}^{-1/2} d\xi \int_{1/2}^{-1/2} d\xi' P(-\xi, -\xi') \frac{\partial s(\xi)}{\partial \xi} (-1) s(\xi') - (-1) \int_{1/2}^{-1/2} d\xi \int_{1/2}^{-1/2} d\xi' (-1) P(z, -\xi') \frac{\partial s(z)}{\partial z} s(\xi')$$

$$= (-1) \int dz dz' P(\xi, -z') \frac{\partial s(\xi)}{\partial \xi} s(z') - (-1) \int dz dz' P(zz') \frac{\partial s(z)}{\partial z} s(-z') = \left|(-1) = -1/4i\right| =$$

$$= (-) \int_{-1/2}^{1/2} d\xi \int_{-1/2}^{1/2} d\xi' P(\xi, \xi') \frac{\partial s(\xi)}{\partial \xi} s(-\xi') - (-1) \int_{-1/2}^{1/2} dz dz' P(zz') \frac{\partial s(z)}{\partial z} s(-z') = (-)(-2) \int dz dz' P(zz') \frac{\partial s(z)}{\partial z} s(-z')$$

Сим. zlk: $\quad \frac{\partial \bar{s}}{\partial z} s(z') - \frac{\partial s}{\partial z} \bar{s}(z') = u'(z) u(z') \left(\frac{1}{2i}\right) \sin\left[\Phi(z') - \Phi(z)\right] - u(z) i \Phi'(z) u(z') 2\cos\left[\Phi(z') - \Phi(z)\right]$

$$= 2i\, u(z') \left[u'(z) \sin\left[\Phi(z') - \Phi(z)\right] - u(z) \Phi'(z) \cos\left[\Phi(z') - \Phi(z)\right]\right] \equiv 0, \text{ если } \Phi(z) = \text{const}$$

$$\frac{1}{V} W_m^P [\bar{s}(z)] = -\frac{1}{2} \int dz dz' P(zz') \left[u'(z) \sin\left[\Phi(z') - \Phi(z)\right] - u(z) \Phi'(z) \cos\left[\Phi(z') - \Phi(z)\right]\right] u(z'),$$

$W_m^P \neq 0$ только если $\Phi(z) \neq \text{const}$ прямое зеркало $\frac{\partial \bar{s}}{\partial z} s(z') - \frac{\partial s}{\partial z} \bar{s}(z')$.

Escaneado con CamScanner

pc

YD для $S(z)$, $+P$ такое как для $\bar{S}(z)$ $w/m -P \Rightarrow w/m P \neq 0$

Каждому λ_n - СЗ $\lambda_n S_n(z) = [\gamma, \mu, \varkappa, \delta]_n S_n(z) - \frac{i}{2}\int dz' \frac{\partial P(zz')}{\partial z} S_n(z')$

Соответствует пара СФ $S_n(z)$ и $\bar{S}_n(z)$ // Каждое СЗ λ_n 2-х кратно

Вырождение. $\frac{1}{J} W_m P = +\frac{1}{4i}\int dz\, dz' P(zz') \left[\frac{\partial S(z)}{\partial z} \bar{S}(z') - P \frac{\partial \bar{S}(z)}{\partial z} S(z') \right]$

Точнее λ_n не вырождение, когда $+P$ и $-P$ - это разные $N_0(P)$ config.

Если $w/m +P$ СФ есть λ_n, СФ $S_n(z)$, то $w/m -P$ СЗ λ_n а СФ - $\bar{S}_n(z)$.

w/m смена $\text{sign}(P)$ с $+P$ на $-P$, λ_n сохраняется, а СФ $S_n(z)$ превращ. в $\bar{S}_n(z)$.

Спектр $\lambda_n(P)$ не зависит от $\text{sign}(P) = \text{sign}(pC)$, но $+P$ соотв. СФ $S_n(z)$, а $-P$ соотв. $\bar{S}_n(z)$.

Это видно из матричных $\left(\frac{\partial P}{\partial z}\right)_{nn'} \sim \text{sign}(P)(n+n')$, $w/m P \to -P$ и $n \to -n$

$\left(\frac{\partial P}{\partial z}\right)_{nn'}$ сохраняет свой вид. $S(z) = \sum_n C_n e^{iq_n z}$, $n \to -n$, $S(z) \to \bar{S}(z) = \sum_{n'} C_{n'} e^{-iq_{n'} z}$, $n' \to -n'$

т.к. C_n - коэф. are real, $\lambda_0 = \frac{1}{2\pi} P \omega_0 / \pi$ вырожден. P, но не pC.

$S_\nu(z) = \sum_n C_n^{(\nu)} e^{iq_n z}$ $w/m +P$, $\bar{S}_\nu(z) = \sum_n C_n^{(\nu)} e^{-iq_n z}$ $w/m -P$.

$S_\nu^+(z) = U_\nu(z) e^{i\Phi_\nu(z)}$ $w/m +P$, $\bar{S}_\nu(z) = U_\nu(z) e^{-i\Phi_\nu(z)}$ $w/m -P$, это $S_\nu(z,\omega)$

$S_\nu^+(z,t) = S_\nu^+(z\omega_\nu) e^{i\omega_\nu t} = U_\nu(z) e^{i[\Phi_\nu(z) + \omega_\nu t]} = S_{\nu x}^+(z,t) + i S_{\nu y}^+(z,t)$

$S_{\nu x}^+(z,t) = U_\nu(z) \cos[\omega_\nu t + \Phi_\nu(z)]$, $S_{\nu y}^+(z,t) = U_\nu(z) \sin[\omega_\nu t + \Phi_\nu(z)]$ для $+P$.

Для $-P$ СФ $\bar{S}_\nu(z) = U_\nu(z) e^{i\Phi_\nu(z)}$, $\bar{S}_\nu(z) = S_\nu(z)$, СФ $\bar{S}_\nu(z,t) = \bar{S}_\nu(z) e^{i\omega_\nu t} = U_\nu(z) e^{-i\Phi_\nu(z)} e^{i\omega_\nu t} =$

$= \bar{S}_{\nu x}(z,t) + i \bar{S}_{\nu y}(z,t)$

$\bar{S}_{\nu x}(z,t) = U_\nu(z) \cos[\omega_\nu t - \Phi_\nu(z)]$, и $\bar{S}_{\nu y}(z,t) = U_\nu(z) \sin[\omega_\nu t - \Phi_\nu(z)]$

Если P (v. polarization) is fixed, $P=+1$, то $\text{sign}(\omega_\nu) > 0$ is fixed

Разные sign ω приводят к разным СФ $S_\nu^+(z,t)$ СФ разлагаются by $\text{sign}[\Phi_\nu(z)]$, $\left\{ \begin{matrix} S_\nu^+(z,t) \text{ w/m } +P \\ \bar{S}_\nu(z,t) \text{ w/m } -P \end{matrix} \right\}$, это

→ Т.е. $\text{sign}(\omega)$ оказывается на v. core dynamics $\vec{S}(z,t)$, хотя frequency splitting ω_n (or λ_n) нет. Важно!

(6/VII) Письмо V. Prida, Oviedo - visit? Были с N. Carregosa, Claberria.

w/m небольших L - singlet ω_0 и doublet ω_1, ω_2. Из-за чего doublet?

Далее набор дублетов - все моды, кроме ω_0 - дублеты ($N=30$).

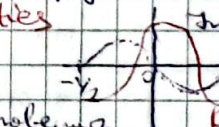
Вычисление по сериям x-, P-, z-terms сохраняет дублеты.

// Для СФ дублета ω_1, ω_2 $S_1(z)$ и $S_2(z)$ - СФ разлагаются фазами $\Phi_2(z) \equiv -\Phi_1(z)$

и соотношением $\text{Re}S(z)$ и $\text{Im}S(z)$. GM mode intensities

В Ding Sc. Rep. 14, APL'14 (видно Fig. 2 $|m_x(z,\omega_n)|^2$ - plots

$m_x(z) \sim S_{1y}(z) = \text{Im}S_1(z)$ имеет $= y$ (=0) w/m $z=0$. проблемы



(7/VII) Начинаю писать обзор 3rd generation AF. DW dyn. paper и послал to Ruben. Skype. Тема: 1) модуль мн у знака у Ding это in figs 2; 2) contact Aurelio.

Oviedo сем. можно visit ~ 20-21 July.

Simulated GM thick dot mode intensities (recall) $I_n^s(z) \sim |\vec{m}_n(z,\omega_n)|^2$ или $\sim |m_n^x(z,\omega_n)|^2$

Ca-g 11/VIII 2014. Integral GM mode intensity $I_m \sim \frac{1}{\int dz |S_n(z\omega_n)|^2} \left| \int dz\, S_n(z\omega_n) \right|^2$, где $S_n(z,\omega_n)$ - n-th eigenmode. Или 14/VIII 2014 ?

integral mode intensities $I_n = I_n^x \sim \frac{1}{d\omega_n} \left| \int dz\, S_n^x(z,\omega_n) \right|^2$ // Local $f(\vec{r})$ и интегр. $\sum dV(m)$ intensities

Local $I_n(\vec{r})$ simulated Fig. 2 Step'14. APL'14. Integral intensities

$I_n = \int dV I_n(\vec{r})$ simulated and measured By Brandt Band FMR.

(8/VII) Начинаю вычислять GM modes profiles - MathCad: $S_1(z)$, $S_2(z)$ и $\vec{S}(z)$.

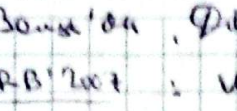
Brandt PRB'2004: спектры $f''_{xx}(\omega)$: Py - dot, $R = 20$ nm, $L = 26 - 60$ nm, т.е. $m_x(z,\omega)$ plotted

Ding'14 также plotted $m_x(z,\omega_n)$ для $n=0,1,2$. И Brandt'04 и Ding'14 дали plotted

Escaneado con CamScanner

для $n=1$ GM max $m_x(\vec{r})$ near top face $z=+1/2$. У меня $\mathrm{Im}\, S_1(\omega)$ имеет max при $z = \pm 1/4$. $\mathrm{Im}\, S_1(\omega) = S_{1y}(z) \sim \langle m_x(\vec{r}) \rangle_0 = \overline{m_x}(z)$. Аналогично Simulations Bonetti, Pirro'08, Puszkarski'14 и наши new GM calculations.

● Van Hertel PRB'2007: vortex type oscillations in rectangles



с медленным расположением ν, core $\vec{X}$ at $z=\pm\frac{1}{2}$, cyclic trajectories не учитывались

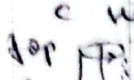
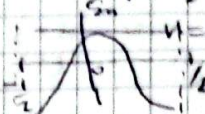


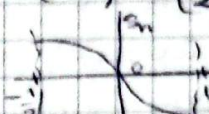
Схема для $n=0,1,2$ GM, n-y знак $\vec{X}_n(z)$ вдоль z, Δ - phase difference $\vec{X}_n(z,t)$ at $z=+1/2$ (top), $z=-\frac{1}{2}$ (bottom). Zhang $\omega_n(k)$ нет, интересно Δ_n нет, только δm_z snapshots только $n=0$ GM имеет FMR-интерес $\sim |\int dV \vec{m}_n(\vec{r}, \omega_n)|^2$. Это-то сумно не нам в моем MathCad c_n - расчет(!).

Метод Фурье (Zhang 2014), с которым вычисляли $z_0/\bar{z}$ и осциллировали расчет частот $\Delta_{nn'}$ фундаментально неправильно. $S(z) = \sum_n c_n e^{iq_n z}$, где $c_n = \frac{1}{L}\int_{-1/2}^{1/2} dz\, S(z) e^{-iq_n z}$, $L \to 1$, $q_n = 2\pi n$, предполагается периодичность $\lim L \to 0$, когда $W_m \to 0$ и остается только W_{ex}: $S_n(z) \sim \cos(2\pi n z)$, $\sin(2\pi n z)$.

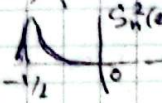
Надо exch. eigenfunctions $S_n(z) \sim \cos\pi n\left(z+\frac{1}{2}\right) = \cos(\pi n z)\cos\left(\frac{\pi n}{2}\right) - \sin(\pi n z)\sin\frac{\pi n}{2}$

$\sim \begin{cases} \cos(\pi n z), & n - \text{odd even}, 0, 2, \ldots \\ \sin(\pi n z), & n - \text{odd}, \ n = 1, 3, \ldots \end{cases}$





как в расчетах



правильное dynamics $I_n(z) \sim |S_n(z)|^2$ для $n=1$ мода

$S(z) = \sum_n c_n e^{iq_n z}$, $q_n = 2\pi n$ - предполагает periodic $S(z)$ на $z \in [-\frac{1}{2}, \frac{1}{2}]$ с периодом $L=1$. Это физически не обосновано, в общем случае. Хотя для lowest mode $S_0(z)$ может работать.

Отсюда, что $S_n(z)$ имеет период dot thickness (wire length) L, а неточное решение $S_n(-1/2) \neq S_n(1/2)$, ведь $z = \pm 1/2$ эквивалентны для $S(z)$ с периодом $L=1$ на $z \in [-1/2, 1/2]$ интервале. Это сильное, физически не обоснованное, ограничение на bug ФФ $S_n(z)$.

$e_n(z) = e^{iq_n z}$, $\int dz\, e_n(z) e_n^*(z') = \int_{-1/2}^{1/2} dz\, e^{iq_n z} e^{-iq_{n'} z} = \int dz\, e^{i(q_n - q_{n'})z} = |q_n - q_{n'} \neq 0|$

$= \int \cos q_n z\, dz + i\int dz \sin q_n z = \frac{1}{q_n}\sin q_n z \Big|_{-1/2}^{1/2} + i\left(-\frac{1}{q_n}\right)\cos q_n z \Big|_{-1/2}^{1/2} = |q_n = 2\pi n| =$

$= \frac{1}{(q_n - q_{n'})}\sin(q_n - q_{n'})z \Big|_{-1/2}^{1/2} - \frac{i}{(q_n - q_{n'})}\cos(q_n - q_{n'})z \Big|_{-1/2}^{1/2} = \frac{2}{(q_n - q_{n'})}\left[\sin(q_n - q_{n'})\frac{1}{2}\right]$

При $q_n \neq q_{n'}$ $\sin\frac{1}{2}(q_n - q_{n'}) = \sin\pi(n - n') = 0$, $\frac{2}{x}\sin\frac{x}{2} \to 1$ при $x = q_n - q_{n'} \to 0$

$\int_{-1/2}^{1/2} dz\, e_n(z) e_{n'}^*(z) = \delta_{nn'}$, $e_n(z) = e^{iq_n z}$, $q_n = 2\pi n$.

Exch. ФФ $S_n(z) = \cos q_n\left(z + \frac{1}{2}\right)$, удовл. $\frac{\partial S_n}{\partial z}\Big|_{z=\pm 1/2} = 0$, откуда $q_n = \pi n$, не подходит

$\varphi_n(z) = \cos q_n z$, $z \in [0, 1]$. $\frac{\partial \varphi_n}{\partial z} = -q_n \sin q_n z \Big|_0^1$, $\frac{\partial \varphi_n}{\partial z}\Big|_0 = 0$, $\frac{\partial \varphi_n}{\partial z}\Big|_1 = -q_n \sin q_n = 0$, $q_n = \pi n$. Не можем требовать $\frac{\partial S_n}{\partial z}\Big|_{z=0,1} = 0$ - exch. b.c., как есть $W_{ex}(L)$.

① Для $L \to \infty$ подходит для всех интеграл Фурье $S(z) = \int dq\, C(q) e^{iqz}$, это для continuous spectrum $\lambda(q) = \gamma q^2 + F(q)$ $F(q) \xrightarrow{L\to\infty} 8\pi^2 W_m$.

Long wires $L \to q_n$, $\lambda(q) \to \lambda_n = \lambda(q_n)$ можно приближенно вычислять ● разрешенные q_n, например $q_n = n\pi/L$ из boundary conditions $z = 0, L$. Но это нашу физическую сущность localized modes at $z = 0, L$ для которых $q_n = \frac{\pi(n+1)}{L}$ нет.

② Finite L, даже $L \leq 1c$. Решения по eigenfunctions $\cos q_n z$, $\sin q_n z$ удовл. возбужденному $\pm\frac{\partial \varphi_n}{\partial z} + d\varphi_n\Big|_{z=\pm L} = 0$ found. boundaries. Нет ли решения $\lambda = \lambda_n$ из W_m effect? (25/IV)

Escaneado con CamScanner

Нужно чтобы $\frac{\partial \varphi_n(z)}{\partial z}\Big|_{z=\pm 1/2} = 0$. Это из-за $P \neq 0$ $\varphi_n(z)$ are complex.

Иначе $\varphi_n(z) = \cos \pi n (z + 1/2)$, $\frac{\partial \varphi_n}{\partial z}\Big|_{z=\pm 1/2} = -(\pi n)\sin(\pi n)(z+\frac{1}{2}) = 0$, $\sin \pi n = 0$.

Иначе $\varphi_n(z) = \cos q_n (z + 1/2)$ или с $z \in [0,1]$ $\varphi_n = \cos q_n z$ как в Ding'14, с $q_n = \pi n$.

Если $\varphi_n(z) = u_n(z) e^{i\Phi_n(z)}$, то $\frac{\partial \varphi_n}{\partial z} = u_n' e^{i\Phi_n} + u_n i\Phi_n' e^{i\Phi_n} = \varphi_n\left[\frac{u_n'}{u_n} + i\Phi_n'\right] = 0$ (2)

дает $\frac{u_n'}{u_n} + i\Phi_n' = 0$ или $u_n(z = \pm 1/2) = 0$. $\frac{\partial \varphi_{nx}(z)}{\partial z} = 0$, $\frac{\partial \varphi_{ny}}{\partial z} = 0$ at $z = \pm 1/2$.

→ Задача подобрать полный/ортогон. набор СФ $\varphi_n(z)$, удовлетворяющими $\frac{\partial \varphi_n}{\partial z} + d\varphi_n = 0\Big|_{z=\pm 1/2}$, тогда $S(z) = \sum_n C_n \varphi_n(z)$ и $\int \varphi_{nn'}$ на $\varphi_n, \varphi_{n'}(z)$.

Иначе $\varphi_n(z) = e^{iq_n z}$, $q_n = 2\pi n$, т.к. период $z = 1$ и в.с. не выполняются.

$\varphi_n(x) = \sin nx \longrightarrow \int_0^{2\pi} dx\, \varphi_n(x)\varphi_m(x) = \delta_{mn}\pi$ - ортогональность, где $\cos nx$

$\varphi_n(x) = \cos nx$ $\int d\varphi_m \varphi_n = 2\pi\, \delta_{m=n=0}$

Наборы СФ $\sin nx$, $\cos nx$ как решение Sturm-Liouville $y'' + n^2 y = 0$ - полные системы СФ. $2\pi \to 1$, $\varphi_n(x) = \sin(2\pi n x)$, $\cos(2\pi n x)$.

Иначе $\varphi_n(x) = \cos\left(\frac{n\pi x}{\ell}\right)$, $\varphi_n(x) = \sin\left(\frac{n\pi x}{\ell}\right)$ удобно. $\int_0^\ell dx\, \varphi_m(x)\varphi_n(x) = \frac{\ell}{2}\delta_{mn}$ $m,n = 1,2,3$

Для $\cos(\cdot)$ надо вкл. $n=0$ тогда $\varphi_0(x) = 1$, $\int \varphi_n^2(x) = \ell$. $n=0$

Разложение по $\varphi_n(z) = \cos(n\pi z)$, период $\cos(\pi n z)$ $T = (1/2)^{-1} = 2$, $\ell = 1$, $\frac{\partial \varphi_n}{\partial z}\Big|_{z=0,1} = 0$ делал в Ding' Sci. Rep. '14.

Если $\frac{\partial \varphi_n}{\partial z} + d\varphi_n = 0\Big|_{0,1}$, то СФ $\varphi_n(z) = \frac{(-1)^n}{q_n}\cos(q_n z) + \frac{1}{d}\sin q_n z$, $q_n = n\pi$.

$\frac{\partial \varphi_n}{\partial z} + d\varphi_n\Big|_{z=0,1} = \pm \sin q_n z + \frac{1}{d}\left[q_n \cos q_n z\right] \pm \frac{d}{q_n}\cos q_n z + \sin q_n z = 0$, т.к. $\sin q_n z\Big|_{0,1} = 0$

$\varphi_n(z) = \frac{1}{d}\sin q_n z - \frac{1}{q_n}\cos q_n z$ удобно. в.с. $\frac{\partial \varphi}{\partial z} + d\varphi\Big|_{z=0,1} = 0$, но $\varphi_n(z)$ - не real, а из-за $P \neq 0$ $S(z) = \sum C_n \varphi_n(z)$ should be complex. C_n - complex?

9/VII Иначе можно $\varphi_n(z) = \cos(q_n z)$, $q_n = \pi n$, $\int_0^1 dz\, \varphi_n(z)\varphi_{n'}(z) = \left(\frac{1+\delta_{n0}}{2}\right)\delta_{nn'}$.

16/VII 2014г. $f_{nn'}(x) = \int_0^1 dz\, dz'\, e^{-x|z-z'|}\varphi_n(z)\varphi_{n'}(z') = \left[\frac{4}{(1+\delta_{n0})(\delta_{n'0}+1)}\right]^{1/2} \frac{x}{x^2+q_n^2} \times$

$\varphi_n(z) = \cos q_n z$, $q_n = \pi n$. $\times\left[(1+\delta_{n0})\delta_{nn'} - \frac{x}{(x^2+q_{n'}^2)}\left\{1 - e^{-x}(-1)^n\right\}\left\{1 + (-1)^{n+n'}\right\}\right]$, $N_n = \frac{1}{2}(1+\delta_{n0})$ $x = \beta k$.

$\delta_{nn'}(x)$ похожа на $J_{nn'}(x) = \int_{-1/2}^{1/2} dz\, dz'\, e^{-x|z-z'|} e^{-iq_n z} e^{iq_{n'}z'}$ от 20/IV

Найти $\delta_{nn'}(x) = \int_0^1 dz\, dz'\, \theta(z-z')\, e^{-x|z-z'|}\varphi_n(z)\varphi_{n'}(z')$, $\varphi_n(z) = \cos q_n z \frac{1}{\sqrt{N_n}}$ для $P_{nn'}$, $S_{nn'}$.

$q_n = \pi n$.

$\int dx\, e^{ax}\cos bx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right]$

$f_{nn'}(x) = \frac{1}{\sqrt{N_n N_{n'}}}\int_0^1 dz \int_0^1 dz'\, \theta(z-z')\cos q_n z \cos q_{n'} z' e^{-x|z-z'|} = \frac{1}{\sqrt{(\cdot)}}\int_0^1 dz \left[\int_0^z dz' (+1) \times\right.$

$\left. \times \cos q_n z \cos q_{n'} z' e^{-x(z-z')} + \int_z^1 dz' (-1)\cos q_n z \cdot \cos q_{n'} z' e^{-x(z'-z)}\right] = \frac{1}{\sqrt{(\cdot)}}\int_0^1 dz \times$

$\times\left[e^{-xz}\cos q_n z \int_0^z dz' e^{xz'}\cos q_{n'} z' + (-1)\cos q_n z\, e^{xz}\int_z^1 dz' e^{-xz'}\cos q_{n'} z'\right] =$

$= \frac{1}{\sqrt{(\cdot)}}\int_0^1 dz \cos q_n z \left[e^{-xz}\frac{e^{xz}}{q_{n'}^2+x^2}\left\{x\cos q_{n'} z' + q_{n'}\sin q_{n'} z'\right\}\Big|_0^z - e^{xz}\frac{e^{-xz'}}{x^2+q_{n'}^2}\left\{-x\cos q_{n'} z' + q_{n'}\sin q_{n'} z'\right\}\Big|_z^1\right]$

$= \frac{1}{\sqrt{(\cdot)}}\int_0^1 dz \frac{\cos q_n z}{x^2+q_{n'}^2}\left[e^{-xz}\left\{e^{xz}\left(x\cos q_{n'} z + q_{n'}\sin q_{n'} z\right) - x\right\} - e^{xz}\left\{e^{-x}\left(-x\cos q_{n'} + q_{n'}\sin q_{n'}\right) - \right.\right.$

$\left.\left. - e^{-xz}\left(-x\cos q_{n'} z + q_{n'}\sin q_{n'} z\right)\right\}\right]$

$\cos q_{n'} = \cos(\pi n') = (-1)^{n'}$, $\sin q_{n'} = \sin(\pi n') = 0$

$\delta_{nn'}(x) = \frac{1}{\sqrt{N_n N_{n'}}}\frac{1}{x^2+q_{n'}^2}\int_0^1 dz \cos q_n z\left[x\cos q_{n'} z + q_{n'}\sin q_{n'} z - xe^{-xz}e^{xz}e^{-x}(-x)(-1)^{n'} + \right.$

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$+ \left(-x\cos q_{n'}z + q_{n'}\sin q_{n'}z\right)\Big]$, $\int_0^1 dz\cos q_n z\cos q_{n'}z = N_n\delta_{nn'}$, $\int_0^1 dz\, q_n q_{n'} = \delta_{nn'}$

$$\int_0^1 dz\cos q_n z\cdot\sin q_{n'}z = \left|n\neq n'\right| = -\frac{1}{2}\cdot\frac{\cos(q_{n'}-q_n)z}{(q_{n'}-q_n)} - \frac{1}{2}\frac{\cos(q_n+q_{n'})z}{(q_n+q_{n'})}\Big|_0^1 =$$

$$= -\frac{1}{2}\frac{\cos(q_{n'}-q_n)}{q_{n'}-q_n} - \frac{1}{2}\frac{\cos(q_n+q_{n'})}{q_{n'}+q_n} + \frac{1}{2}\frac{1}{(q_{n'}-q_n)} + \frac{1}{2}\frac{1}{(q_n+q_{n'})} =$$

$$= -\frac{1}{2}\frac{(-1)^{n'-n}}{(q_{n'}-q_n)} - \frac{1}{2}\frac{(-1)^{n+n'}}{(q_n+q_{n'})} + \frac{1}{2}\frac{1}{(q_{n'}-q_n)} + \frac{1}{2}\frac{1}{(q_n+q_{n'})} =$$

$$= \frac{1}{2}\frac{1}{(q_n+q_{n'})}\left[1-(-1)^{n+n'}\right] + \frac{1}{2}\frac{1}{(q_{n'}-q_n)}\left[1-(-1)^{n'-n}\right] = \frac{1}{2}\left[1-(-1)^{n+n'}\right]\times$$

$$\times\left[\frac{1}{q_n+q_{n'}} + \frac{1}{q_{n'}-q_n}\right] = \frac{1}{2}\left[1-(-1)^{n+n'}\right]\frac{2q_{n'}}{q_{n'}^2-q_n^2} = \left[1-(-1)^{n+n'}\right]\frac{q_{n'}}{q_{n'}^2-q_n^2}$$

$$\bar{f}_{nn'}(x) = \frac{1}{\sqrt{N_n N_{n'}}}\frac{1}{x^2+q_{n'}^2}\Big[xN_n\delta_{nn'} + \left(1-(-1)^{n+n'}\right)\frac{q_{n'}^2}{q_n\neq q_{n'}\; q_{n'}^2-q_n^2} - \left\{e^{-xz}\right\}_{(z=1)}+$$

$$+(-x)N_n\delta_{nn'} + q_{n'}\int_0^1 dz\cos q_n z\sin q_{n'}z\Big] = \frac{1}{\sqrt{N_n N_{n'}}}\frac{1}{x^2+q_{n'}^2}\Big[2\left(1-(-1)^{n+n'}\right)\frac{q_{n'}^2}{q_{n'}^2-q_n^2} +$$

$$+\int_0^1 dz\cos q_n z\cdot x\left\{-e^{-xz}+(-1)^{n'}e^{-x}e^{xz}\right\}\Big]\,,\quad n\neq n'\text{ - неравенство.}$$

$$\int_0^1 dz\cos q_n z\cdot\sin q_n z = \left|q_n z = x\right| = \frac{1}{q_n}\int_0^{q_n} dx\cos x\sin x = \frac{1}{2}\frac{1}{q_n}\sin^2 x\Big|_0^{q_n} = \frac{1}{2q_n}\sin^2 q_n = 0$$

$$\bar{f}_{nn'}(x) = \frac{1}{\sqrt{N_n N_{n'}}}\int_0^1 dz\int_0^1 dz'\,\theta(z-z')e^{-x|z-z'|}\cos q_n z\cos q_{n'}z'$$

$$\int_0^1 dz'(\dots) = \int_0^z dz'(+1)e^{-x(z-z')}\cos q_{n'}z' + \int_z^1 dz'(-1)e^{-x(z'-z)}\cos q_{n'}z' =$$

$$= e^{-xz}\int_0^z dz' e^{xz'}\cos q_{n'}z' - e^{xz}\int_z^1 dz' e^{-xz'}\cos q_{n'}z' = e^{-xz}\left[x\cos q_{n'}z' + q_{n'}\sin q_{n'}z'\right]\frac{e^{xz'}}{x^2+q_{n'}^2}\Big|_0^z -$$

$$- e^{xz}\frac{e^{-xz'}}{x^2+q_{n'}^2}\left[-x\cos q_{n'}z' + q_{n'}\sin q_{n'}z'\right]_z^1 = \frac{e^{-xz}}{x^2+q_{n'}^2}\Big[e^{xz}\left(x\cos q_{n'}z + q_{n'}\sin q_{n'}z\right) -$$

$$- x\Big] - \frac{e^{xz}}{x^2+q_{n'}^2}\left[e^{-x}\left(-x\cos q_{n'} + q_{n'}\sin q_{n'}\right) - e^{-xz}\left(-x\cos q_{n'}z + q_{n'}\sin q_{n'}z\right)\right] =$$

$$= \frac{1}{x^2+q_{n'}^2}\left[x\cos q_{n'}z + q_{n'}\sin q_{n'}z - xe^{-xz} - e^{xz}e^{-x}(-x)(-1)^{n'} + (-x)\cos q_{n'}z + q_{n'}\sin q_{n'}z\right] =$$

$$= \frac{1}{x^2+q_{n'}^2}\left[2q_{n'}\sin q_{n'}z - xe^{-xz} + xe^{-x}e^{xz}(-1)^{n'}\right]$$

$$\bar{f}_{nn'}(x) = \frac{1}{\sqrt{N_n N_{n'}}}\int dz\cos q_n z\int_0^1 dz'(\dots) = \frac{1}{\sqrt{N_n N_{n'}}}\frac{1}{x^2+q_{n'}^2}\int_0^1 dz\cos q_n z\left[2q_{n'}\sin q_{n'}z - xe^{-xz} + xe^{-x}e^{xz}(-1)^{n'}\right]$$

$$\int_0^1 dz\cos q_n z\sin q_{n'}z = \left[1-(-1)^{n+n'}\right]\frac{q_{n'}}{(q_{n'}^2-q_n^2)}\,,\quad n\neq n'\text{, иначе }(dz(\dots)=0.$$

$$\int_0^1 dz\, e^{-xz}\cos q_n z = \frac{e^{-xz}}{x^2+q_n^2}\left[-x\cos q_n z + q_n\sin q_n z\right]_0^1 = \frac{1}{x^2+q_n^2}\left[e^{-x}(-x)(-1)^n - (-x)\right] =$$

$$= \frac{x}{x^2+q_n^2}\left[1-e^{-x}(-1)^n\right]\,,\quad \int_0^1 dz\cos q_n z\, e^{xz} = -\frac{x}{x^2+q_n^2}\left[1-e^{x}(-1)^n\right]$$

$$\bar{f}_{nn'}(x) = \frac{1}{\sqrt{N_n N_{n'}}}\frac{1}{x^2+q_{n'}^2}\Big[\left\{1-(-1)^{n+n'}\right\}\frac{2q_{n'}^2}{(q_{n'}^2+q_n^2)} - x\cdot\frac{x}{(x^2+q_n^2)}\left\{1-e^{-x}(-1)^n\right\} +$$

$$+ xe^{-x}(-1)^{n'}\frac{(-x)}{(x^2+q_n^2)}\left\{1-e^{x}(-1)^n\right\}\Big]\,,\quad -\frac{x^2}{(x^2+q_n^2)}\left\{1-e^{-x}(-1)^n\right\} - \frac{x^2}{(x^2+q_n^2)}e^{-x}(-1)^{n'}\ (*)$$

$$(*)\ \left\{1-e^{x}(-1)^n\right\} = -\frac{x^2}{(x^2+q_n^2)}\left[1-e^{-x}(-1)^n + e^{-x}(-1)^{n'}\left\{1-e^{x}(-1)^n\right\}\right] =$$

$$= -\frac{x^2}{(x^2+q_n)^2}\left[1-e^{-x}(-1)^n + e^{-x}(-1)^{n'} - (-1)^{n+n'}\right] = -\frac{x^2}{(x^2+q_n^2)}\left\{1-(-1)^{n+n'}\right\}\left[1-e^{-x}(-1)^n\right]$$

$$\bar{f}_{nn'}(x) = \frac{1}{\sqrt{N_n N_{n'}}}\frac{1}{(x^2+q_{n'}^2)}\left\{1-(-1)^{n+n'}\right\}\Big[\frac{2q_{n'}^2}{(q_{n'}^2-q_n^2)} - \frac{x^2}{(x^2+q_n^2)}\left\{1-e^{-x}(-1)^n\right\}\Big]$$

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$f_{nn'}(x) = \int dz dz' \varphi_n(z)\varphi_{n'}(z') e^{-x|z-z'|}$, $\varphi_n(z) = \frac{1}{\sqrt{N_n}}\cos q_n z$, $q_n = \pi n$, $N_n = \frac{1}{2}(1+\delta_{n0})$

$$f_{nn'}(x) = \frac{1}{\sqrt{N_n N_{n'}}}\left[(1+\delta_{n0})\delta_{nn'} - \frac{x}{(x^2+q_{n'}^2)}\left\{1+(-1)^{n+n'}\right\}\left\{1-e^{-x}(-1)^n\right\}\right]\frac{x}{(x^2+q_n^2)}$$

$f_{n'n} = \int dz dz' \varphi_{n'}(z)\varphi_n(z') e^{-x|z-z'|} = |z \rightleftarrows z'| = \int dz dz' \varphi_{n'}(z)\varphi_n(z) e^{-x|z-z'|} = f_{nn'}$ — сим.

$$\delta f_{nn'} = \frac{1}{\sqrt{N_n N_{n'}}} \frac{(-x^2)}{(x^2+q_n^2)(x^2+q_{n'}^2)}\left\{1+(-1)^{n+n'}\right\}\left\{1-e^{-x}(-1)^n\right\}$$

$$\delta f_{n'n} = \frac{1}{\sqrt{N_n N_{n'}}} \frac{(-x^2)}{(x^2+q_n^2)(x^2+q_{n'}^2)}\left\{1+(-1)^{n+n'}\right\}\left\{1-e^{-x}(-1)^{n'}\right\} = \delta f_{nn'}$$? только если $(-1)^n = (-1)^{n'}$

$(-1)^{n+n'} = 1$, $(-1)^{n+n'} = 1$, $n, n' = 0, 1, 2, \dots$, $n \cup n'$ — одной четности. тоже и всё.

$\bar{f}_{n'n} = \int dz dz' \Theta(z-z') e^{-x|z+z'|}\varphi_{n'}(z)\varphi_n(z') = |z \rightleftarrows z'| = -\int dz dz' \Theta(z-z') e^{-x|z-z'|}\varphi_{n'}(z)\varphi_n(z) =$

$= -\bar{f}_{nn'}$ — anti-sym

$$\bar{f}_{nn'}(x) = \frac{1}{\sqrt{N_n N_{n'}}}\left\{1-(-1)^{n+n'}\right\}\frac{1}{(x^2+q_{n'}^2)}\left[1+\left(\frac{q_{n'}^2+q_n^2}{q_{n'}^2-q_n^2}\right) - \frac{x^2}{(x^2+q_n^2)}\left\{1-e^{-x}(-1)^n\right\}\right]$$

$$\bar{f}_{n'n} = \frac{1}{\sqrt{(\cdot)}}\{\text{сим.}\}\frac{1}{(x^2+q_n^2)}\left[1+\frac{(q_n^2+q_{n'}^2)}{(q_n^2-q_{n'}^2)} - \frac{x^2}{(x^2+q_{n'}^2)}\left\{1-e^{-x}(-1)^{n'}\right\}\right] \neq -\bar{f}_{nn'}$$?

$\bar{f}_{nn'} \neq 0$ только если $(-1)^{n+n'} \neq 1$, $n \cup n'$ — разной четности. $(-1)^n = -(-1)^{n'}$

$I_{nn'} = \int_0^1 dz \cos q_n z \cdot \sin q_{n'} z = \int_0^1 dz \cos(\pi n z)\sin(\pi n' z) = |\pi z = x| = \frac{1}{\pi}\int_0^\pi dx \cos nx \cdot \sin n' x =$

$= \frac{1}{\pi}\cdot\frac{2n'}{(n')^2-n^2}$ если $n+n'$ — нечет., иначе $=0$. (n, n' разной четности) $I_{nn'} = \left[1-(-1)^{n+n'}\right]\frac{q_{n'}}{q_{n'}^2-q_n^2} = \frac{2q_{n'}}{(q_{n'}^2-q_n^2)}$ (верно)

$$\bar{f}_{nn'}(x) = \frac{1}{\sqrt{N_n N_{n'}}}\int_0^1 dz \int_0^1 dz' \Theta(z-z') e^{-x|z-z'|}\cos q_n z \cdot \cos q_{n'} z' = |z-z' = \xi| =$$

$$= \frac{1}{\sqrt{(\cdot)}}\int_0^1 dz \int_0^{z-1} d\xi\,(-1)\,\Theta(\xi) e^{-x|\xi|}\cos q_n z \cdot \cos q_{n'}(z-\xi)$$

$z-1 \le 0$

$\cos q_n z \cdot \cos q_{n'}(z-\xi) = \frac{1}{2}\cos\left[q_n z + q_{n'}(z-\xi)\right] + \frac{1}{2}\cos\left[q_n z - q_{n'}(z-\xi)\right]$, $z \in [0,1]$

$\int_z^{z-1} d\xi\,\Theta(\xi)F(\xi) = \int_z^0 d\xi\,\Theta(\xi)F(\xi) + \int_0^{z-1} d\xi\,\Theta(\xi)F(\xi) = -\int_0^z d\xi\,\Theta(\xi)F(\xi) + \int_0^{z-1} d\xi\,\Theta(\xi)F =$

$= -\int_0^z d\xi F(\xi) + \int_0^{z-1} d\xi\,(-1)F(\xi)$, $F(\xi) = e^{-x|\xi|}\cos q_{n'}(z-\xi) = e^{-x|\xi|}\,\mathrm{Re}\,e^{i q_{n'}(z-\xi)}$

$\xi > 0$, $\xi < 0$

$\int_z^{z-1} d\xi\,\Theta(\xi)F(\xi) = -\int_0^z d\xi\, e^{-x\xi}\,\mathrm{Re}\,e^{i q_{n'}(z-\xi)} - \int_0^{z-1} d\xi\, e^{x\xi}\,\mathrm{Re}\,e^{i q_{n'}(z-\xi)}$

$F(\xi) = e^{-x|\xi|}\left[\cos q_{n'} z \cos q_{n'}\xi + \sin q_{n'} z \sin q_{n'}\xi\right] = e^{-x|\xi|}\left[\cos q_{n'} z\,\mathrm{Re}\,e^{i q_{n'}\xi} + \right.$

$\left. + \sin q_{n'} z\,\mathrm{Im}\,e^{i q_{n'}\xi}\right]$. $\int d\xi e^{x\xi} e^{i q_{n'}\xi} = \int d\xi\, e^{(x+i q_{n'})\xi} = \frac{1}{x+i q_{n'}} e^{(x+i q_n)\xi}$

$a = \frac{1}{2}(a+\bar{a}) + \frac{1}{2}(a-\bar{a}) = \frac{1}{2}\left[a'+ia''+a'-ia''\right] + \frac{1}{2}\left[a'+ia''-a'+ia''\right] = a'+ia''$

$\mathrm{Re}(a) = \frac{1}{2}(a+\bar{a})$, $\mathrm{Im}(a) = \frac{1}{2i}(a-\bar{a})$

$\int_z^{z-1} d\xi\,\Theta(\xi)F(\xi) = -\int_0^z d\xi\, e^{-x\xi}\left[\cos q_{n'} z\,\mathrm{Re}\,e^{i q_{n'}\xi} + \sin q_{n'} z\cdot\mathrm{Im}\,e^{i q_{n'}\xi}\right] -$

$- \int_0^{z-1} d\xi\, e^{x\xi}\left[\cos q_{n'} z\,\mathrm{Re}\,e^{i q_{n'}\xi} + \sin q_{n'} z\cdot\mathrm{Im}\,e^{i q_{n'}\xi}\right] = \left[-\cos q z\,\mathrm{Re}\frac{1}{-x+iq}e^{(-x+iq)\xi} - \right.$

$\left. - \sin q z\,\mathrm{Im}\frac{1}{-x+iq}e^{(-x+iq)\xi}\right]_0^z - \left[\cos q z\,\mathrm{Re}\frac{1}{x+iq}e^{(x+iq)\xi} + \sin q z\,\mathrm{Im}\frac{1}{x+iq}e^{(x+iq)\xi}\right]_0^{z-1} =$

$= -\cos q z\,\mathrm{Re}\frac{1}{-x+iq}e^{(-x+iq)z} + \mathrm{Re}\frac{\cos q z}{-x+iq} - \sin q z\,\mathrm{Im}\frac{1}{-x+iq}e^{(-x+iq)z} +$

$+ \sin q z\,\mathrm{Im}\frac{1}{-x+iq} - \cos q z\,\mathrm{Re}\frac{1}{x+iq}e^{(x+iq)(z-1)} + \cos q z\,\mathrm{Re}\frac{1}{x+iq} -$

$- \sin q z\,\mathrm{Im}\frac{1}{x+iq}e^{(x+iq)(z-1)} + \sin q z\,\mathrm{Im}\frac{1}{x+iq}$ — сокращаем чтобы $\int_0^z dz' (\dots)$ остался.

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На new eigenfunctions $\varphi_n(z) = \frac{1}{\sqrt{N_n}} \cos q_n z$, $q_n = n\pi$, $N_n = \frac{1}{2}(1+\delta_{n0})$

$K_{nn'} = \int dz\,dz'\, \chi(zz')\varphi_n(z)\varphi_{n'}(z') = 8\pi M_s^2 \beta \int dz\,dz' \varphi_n(z)\varphi_{n'}(z') \int dk e^{-\beta k|z-z'|} \times$

$\times [I^2(k) + c^2 \Lambda^2(k)] =$

$= 8\pi M_s^2 \beta \int dk [I^2(k) + c^2\Lambda^2(k)] f_{nn'}(\beta k)$, $x = \beta k$. $J_{nn'} \to f_{nn'}$.

• $\lambda s(z) = \left[-\gamma \frac{\partial^2}{\partial z^2} - \mu\right] s(z) + \int dz' \chi(zz') s(z') - \frac{i}{2}\int dz' \frac{\partial P(zz')}{\partial z} s(z') - \int dz' \frac{\partial \Xi(zz')}{\partial z} \frac{\partial s}{\partial z'}$

$s(z) = \sum_n c_n \varphi_n(z)$. Если $\frac{\partial s(z)}{\partial z}\Big|_{z=0,1} = 0$ (2с/IV), то $\int dz' \frac{\partial \Xi(zz')}{\partial z} \frac{\partial s(z')}{\partial z'} = \int dz' \Xi(zz') \frac{\partial^2 s}{\partial z'^2}$

• $\lambda s(z) = \left[-\gamma \frac{\partial^2}{\partial z^2} - \mu\right] s(z) + \int dz' \chi(zz') s(z') - \frac{i}{2}\int dz' \frac{\partial P(zz')}{\partial z} s(z') - \int dz' \Xi(zz') \frac{\partial^2 s(z')}{\partial z'^2}$

$\frac{\partial \varphi_n}{\partial z} = -\frac{1}{\sqrt{N_n}} q_n \sin q_n z$, $\frac{\partial^2 \varphi_n}{\partial z^2} = -\frac{1}{\sqrt{N_n}} q_n^2 \cos q_n z = -q_n^2 \varphi_n(z)$

$\lambda \sum_n c_n \varphi_n(z) = \left[-\gamma \frac{\partial^2}{\partial z^2} - \mu\right] \sum_n c_n \varphi_n(z) + \int dz' \chi(zz') \sum_n c_n \varphi_n(z') - \frac{i}{2}\int dz' \frac{\partial P}{\partial z}(zz') \sum_n c_n \varphi_n(z') -$

$- \int dz' \Xi(zz') \frac{\partial^2}{\partial z'^2} \sum_n c_n \varphi_n(z')$

$\lambda \sum_n c_n \varphi_n(z) = \sum_n \left[\gamma q_n^2 - \mu\right] c_n \varphi_n(z) + \int dz' \chi(zz') \sum_n c_n \varphi_n(z') - \frac{i}{2}\int dz' \frac{\partial P}{\partial z} \sum_n c_n \varphi_n(z') +$

$+ \int dz' \Xi(zz') \sum_n c_n q_n^2 \varphi_n(z')$

$\times \varphi_{n'}(z)$ и $\int dz$, $\int dz\, \varphi_n(z)\varphi_{n'}(z) = \delta_{nn'}$

$\lambda c_{n'} = \left[\gamma q_{n'}^2 - \mu\right] c_{n'} + \int dz\,dz' \chi(zz') \sum_n c_n \varphi_n(z')\varphi_{n'}(z) - \frac{i}{2}\int dz\,dz' \frac{\partial P}{\partial z} \sum_n c_n \varphi_n(z')\varphi_{n'}(z) +$

$+ \int dz\,dz' \Xi(zz') \sum_n c_n q_n^2 \varphi_n(z')\varphi_{n'}(z)$, $n \rightleftarrows n'$

$\lambda c_n = \left[\gamma q_n^2 - \mu\right] c_n + \sum_{n'} c_{n'} \int dz\,dz' \chi(zz') \varphi_{n'}(z')\varphi_n(z) - \frac{i}{2}\sum_{n'} c_{n'} \int dz\,dz' \frac{\partial P}{\partial z} \varphi_{n'}(z')\varphi_n(z) +$

$+ \sum_{n'} c_{n'} \int dz\,dz' \Xi(zz') \varphi_{n'}(z')\varphi_n(z) q_{n'}^2$, $L_{nn'} = \int dz\,dz' \Xi(zz') \varphi_n(z)\varphi_{n'}(z)$

$\lambda c_n = \left[\gamma q_n^2 - \mu\right] c_n + \sum_{n'} c_{n'} \left[K_{nn'} - \frac{i}{2}\left(\frac{\partial P}{\partial z}\right)_{nn'} + q_{n'}^2 L_{nn'}\right]$, тогда $\lambda c_n = \sum_{n'} J_{nn'} c_{n'}$

$J_{nn'} = (\gamma q_n^2 - \mu)\delta_{nn'} + K_{nn'} - \frac{i}{2}\left(\frac{\partial P}{\partial z}\right)_{nn'} + q_{n'}^2 L_{nn'}$, $J_{nn'} \neq J_{n'n}$ из-за 2 член. (доказательство)

$\left[-i\left(\frac{\partial P}{\partial z}\right)_{nn'}\right]^* = i\left(\frac{\partial P}{\partial z}\right)_{nn'} = -i\left(\frac{\partial P}{\partial z}\right)_{n'n}$ & сами $\bar{f}_{nn'} = -\bar{f}_{n'n}$ - антисим.

Because в УД сами $\frac{\partial s}{\partial z}$, сам. 2с/IV: $\lambda s(z) = (-\dots) - \int dz' \frac{\partial \Xi(zz')}{\partial z} \frac{\partial s(z')}{\partial z'}$

Но $\int dz' \frac{\partial \Xi}{\partial z} \frac{\partial s(z')}{\partial z'} = \int dz' \Xi(zz') \frac{\partial^2 s(z')}{\partial z'^2}$ если $\frac{\partial s}{\partial z}\Big|_{z=0,1} = 0$, тогда Вронск. $\varphi_n(z) \sim \cos q_n z$.

Сам. 1/IV $\int dz' \frac{\partial \Xi}{\partial z} \frac{\partial s(z')}{\partial z'} = \frac{\partial \Xi}{\partial z}(z, \tfrac{1}{2}) s(\tfrac{1}{2}) - \frac{\partial \Xi}{\partial z}(z, -\tfrac{1}{2}) s(-\tfrac{1}{2}) - \int dz' \frac{\partial^2 \Xi(zz')}{\partial z\,\partial z'} s(z')$

$\Xi(zz') = 2\pi M_s^2 \frac{1}{\beta} \int dk\, e^{-\beta k|z-z'|} \Lambda^2(k)$, $\frac{\partial}{\partial z} e^{-x|z-z'|} = e^{-x|z-z'|}(-x)\frac{\partial |z-z'|}{\partial z} =$

$= -x\theta(z-z') e^{-x|z-z'|}$ $\theta(z-z') = \mathrm{sign}(z-z')$. $\frac{\partial \Xi}{\partial z}(zz') = (\cdot)\int dk \Lambda^2(k) \frac{\partial}{\partial z} e^{-\beta k|z-z'|} =$

$= (\cdot)(-\beta)\theta(z-z')\int dk k e^{-\beta k|z-z'|} \Lambda^2(k)$, $(\cdot) = 2\pi M_s^2/\beta$

поверхностный член → $\frac{\partial \Xi}{\partial z}(z, \frac{1}{2}) s(\tfrac{1}{2}) - \frac{\partial \Xi}{\partial z}(z, -\tfrac{1}{2}) s(-\tfrac{1}{2}) = (\cdot)(-\beta)(-1)\int dk k e^{-\beta k|z-\frac{1}{2}|} \Lambda^2(k) s(\tfrac{1}{2}) -$

$- (\cdot)(-\beta)(+1)\int dk k e^{-\beta k|z+1/2|} \Lambda^2(k) s(-\tfrac{1}{2}) = (\cdot)\beta \int dk k e^{-\beta k(\frac{1}{2}-z)} \Lambda^2 s(\tfrac{1}{2}) +$

$+ \beta(\cdot)\int dk k e^{-\beta k(z+\frac{1}{2})} \Lambda^2 s(-\tfrac{1}{2}) = \beta(\cdot)\int dk k e^{-\beta k/2} \Lambda^2(k)\left[e^{\beta k z} s(\tfrac{1}{2}) + e^{-\beta k z} s(-\tfrac{1}{2})\right]$

$\frac{\partial^2 \Xi}{\partial z\,\partial z'} = \frac{\partial}{\partial z}\frac{\partial \Xi}{\partial z'}$, $\frac{\partial^2}{\partial z\,\partial z'} e^{-x|z-z'|} = \frac{\partial}{\partial z}\left[-x\theta(z-z') e^{-x|z-z'|}\right] = -x\frac{\partial \theta}{\partial z} e^{-x|z-z'|} -$

$- x\theta \frac{\partial}{\partial z} e^{-x|z-z'|} = x\frac{\partial \theta(z)}{\partial z} e^{-x|z-z'|} - x\theta(z-z') x\theta(z-z') e^{-x|z-z'|}$

Верно ли $\frac{\partial \theta(z)}{\partial z} = \delta(z)$? Если да, то $\frac{\partial^2 e^{-x|z-z'|}}{\partial z\,\partial z'} = x\delta(z-z') - x^2 e^{-x|z-z'|}$

$1 = \int_{-\infty}^{\infty} \delta(z)\,dz = \frac{1}{2}\int_{-\infty}^{\infty} \frac{\partial \theta}{\partial z} dz = \frac{1}{2}\theta(+\infty) - \frac{1}{2}\theta(-\infty) = \frac{1}{2} - \left(-\frac{1}{2}\right) = \frac{2}{2}$, $\frac{\partial \theta}{\partial z} = 2\delta(z)$

$\frac{\partial^2}{\partial z\,\partial z'} e^{-x|z-z'|} = 2x\delta(z-z') - x^2 e^{-x|z-z'|}$, тогда дает surface term $s(\pm\frac{1}{2})$?

Escaneado con CamScanner

10/VII Два пути сделать $\Lambda_{nn'}$ эрмитовой:

1) Заменить $q_{n'}^2 \to \frac{1}{2}(q_n^2 + q_{n'}^2)$ сохранив $\varphi_n(z) = \frac{1}{\sqrt{\nu_n}} \cos q_n z$, $q_n = n\pi$

2) Записать ν-term в виде $\frac{\partial^2 S}{\partial z \partial z'}$ $S(z')$ без $\frac{\partial S}{\partial z}$, $\frac{\partial^2 S}{\partial z'^2}$ и применить surface ($\sigma_1 \pm \nu_2$) terms.

11/VII Нужно позвонить Грегу в conditions при $z = \pm\frac{1}{2}$ и заменить набор для разложения $\cos q_n z \to \sin q_n z$ на $\cos q_n z$. Тогда при этом будет учтена Wetting

$$\frac{\partial \varphi_n}{\partial z} + d\varphi_n = 0, \quad \varphi_n(z) = \frac{1}{d}\cos q_n z + q_n^{-1}\sin q_n z, \quad -q_n \sin q_n z + \cos q_n z +$$

$$+ d\left(\frac{1}{d}\cos q_n + \frac{1}{q_n}\sin q_n z\right) = -\left(\frac{q_n}{d} + \frac{d}{q_n}\right)\sin q_n z = 0 \to q_n = n\pi$$

Повторяем замену (1) $q_{n'}^2 \to \frac{1}{2}(q_n^2 + q_{n'}^2)$.

$$\left(\frac{\partial P}{\partial z}\right)_{nn'} = \int dz dz' \varphi_n(z)\varphi_{n'}(z') \frac{\partial P}{\partial z} = \int dz dz' \varphi_n(z)\varphi_{n'}(z')(-2\pi M_s^2 c\beta)\Theta(z-z')\int_0^\infty dk k e^{-\beta k|z-z'|}\Lambda\Lambda$$

$$\left(\frac{\partial P}{\partial z}\right)_{nn'} = -2\pi M_s^2 c\beta \int_0^\infty dk k \Lambda(k)\Lambda'(k)\bar{f}_{nn'}(k), \quad \bar{f}_{nn'}(k) = \int dz dz' e^{-\beta k(z-z')}\Theta(z-z')\varphi_n(z)\varphi_{n'}(z)$$

$\Lambda_{nn'} \to \Lambda_{nn'}/(2\pi M_s^2)$. $\Lambda_{nn'} = (\gamma q_n^2 - \zeta^4)\delta_{nn'} + K_{nn'} - \frac{i}{2}\left(\frac{\partial P}{\partial z}\right)_{nn'} + \frac{1}{2}(q_n^2 + q_{n'}^2)L_{nn'}$

- $K_{nn'} = \beta\int_0^\infty dk\left[I^2(k) + c^2\Lambda^2(k)\right]\bar{f}_{nn'}(\beta k)$, $\left(\frac{\partial P}{\partial z}\right)_{nn'} = -c\beta\int dk k \Lambda(k)\Lambda'(k)\bar{f}_{nn'}(k)$

$L_{nn'} = \int dz dz' z(zz')\varphi_n(z)\varphi_{n'}(z') = 2\pi M_s^2 \frac{1}{\beta}\int dz dz' \varphi_n\varphi_{n'}\int dk e^{-\beta k|z-z'|}\Lambda'^2(k)$

- $L_{nn'} = \frac{1}{4\beta}\int dk \Lambda'^2(k) f_{nn'}(k)$, $f_{nn'}(k) = \int dz dz' \varphi_n(z)\varphi_{n'}(z') e^{-\beta k|z-z'|}$

Осталось $f_{nn'}(k) = \int dz dz' \Theta(z-z') e^{-\beta k|z-z'|}\varphi_n(z)\varphi_{n'}(z')$, $q_n = n\pi$.

File "vortex-TM-long-wires-1.med". Бывш. Отель Hotels Orjan Santiago.

14/VII Бывш. C N. в „Corfour" Market. C N. и W. Франция в Villabona.

12/VII MathCad не считает $\left(\frac{\partial P}{\partial z}\right)_{nn'}$. Найти $f_{nn'}$ аналитически

$$\frac{\partial P}{\partial z} = c\frac{\partial}{\partial z}\int dk e^{\beta k|z-z'|}\Lambda(k)\Lambda'(k), \quad \left(\frac{\partial P}{\partial z}\right)_{nn'} = \int_0^1 dz dz' \frac{\partial P}{\partial z}\varphi_n(z)\varphi_{n'}(z'), \quad \varphi_n(z) = \frac{\cos q_n z}{\sqrt{\nu_n}}, \quad q_n = n\pi$$

$$\int_0^1 dz \frac{\partial P}{\partial z}\cos q_n z = \left| dz\frac{\partial P}{\partial z} = dv, \; u = \cos q_n z, \; du = -q_n \sin q_n z, \; v = P(z,z') \right| = \cos q_n z\, P(z,z')\Big|_{z=0}^{1} -$$

$$- \int dz\, P(z,z')(-q_n)\sin q_n z = \cos q_n P(1,z') - P(0,z') + q_n\int dz\, P(z,z')\sin q_n z$$

$P(z,z') = c\int dk\, e^{-\chi|z-z'|}\Lambda(k)\Lambda'(k)$, $c\int dk \Lambda(k)\Lambda(k')$ можно снять, осталось $e^{-\chi|z-z'|}$

$$\int_0^{z'} dz \cos q_n z\, e^{-\chi|z-z'|} = (-1)^n e^{-\chi(z'-z)} - e^{-\chi z'} + q_n\int_0^{z'} dz\, e^{-\chi|z-z'|}\sin q_n z$$

$$\int_0^{z'} dz\, e^{-\chi|z-z'|}\sin q_n z = \int_0^{z'} dz\, e^{-\chi(z'-z)}\sin q_n z + \int_{z'}^{z} dz\, e^{-\chi(z-z')}\sin q_n z = e^{-\chi z'}\int_0^{z'} dz\, e^{\chi z}\sin q_n z +$$

$$+ e^{\chi z'}\int_{z'}^{z} dz\, e^{-\chi z}\sin q_n z = \left| \int dx e^{ax}\sin bx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)\right| =$$

$$= e^{-\chi z'}\left[\frac{e^{\chi z}}{\chi^2+q_n^2}(\chi\sin q_n z - q_n\cos q_n z)\right]_0^{z'} + e^{\chi z'}\left[\frac{e^{-\chi z}}{\chi^2+q_n^2}(-\chi\sin q_n z - q_n\cos q_n z)\right]_{z'}^{1} =$$

$$= \frac{1}{\chi^2+q_n^2}\left[e^{-\chi z'}\left\{e^{\chi z'}(\chi\sin q_n z' - q_n\cos q_n z') - (-q_n)\right\} + e^{\chi z'}\left\{e^{-\chi}(-\chi\sin q_n - q_n\cos q_n) - \right.\right.$$

$$\left.\left. - e^{-\chi z'}(-\chi\sin q_n z' - q_n\cos q_n z')\right\}\right] = \frac{1}{\chi^2+q_n^2}\left[\chi\sin q_n z' - q_n\cos q_n z' + e^{-\chi z'}q_n(-1)^n + \right.$$

$$\left. + e^{\chi z'}e^{-\chi}(-q_n)(-1)^n - (-\chi\sin q_n z' - q_n\cos q_n z')\right] = \frac{1}{\chi^2+q_n^2}\left[2\chi\sin q_n z' + q_n(-1)^n e^{-\chi z'} - q_n(-1)^n e^{-\chi}e^{\chi z'}\right]$$

$$\int_0^1 dz\left[\int_0^{z'} dz \frac{\partial}{\partial z}e^{-\chi|z-z'|}\right]\cos q_n z = (-1)^n e^{-\chi z'}e^{\chi z'} - e^{-\chi z'} + \frac{q_n}{\chi^2+q_n^2}\left[2\chi\sin q_n z' + q_n(-1)^n e^{-\chi z'} - q_n(-1)^n e^{-\chi}e^{\chi z'}\right]$$

$$\int_0^1 dz'\left[\int_0^{z'} dz \frac{\partial}{\partial z}e^{-\chi|z-z'|}\right]\cos q_n z\cos q_{n'} z' = \int_0^1 dz'\cos q_{n'} z'\left[(-1)^n e^{-\chi}e^{\chi z'} - e^{-\chi z'} + \frac{q_n}{\chi^2+q_n^2}\left\{2\chi\sin q_n z' + q_n(-1)^n e^{-\chi z'} - \right.\right.$$

$$\left.\left. - q_n(-1)^n e^{-\chi}e^{\chi z'}\right\}\right] =$$

$$= \int dz'\cos q_{n'} z'\left[(-1)^n e^{-\chi}e^{\chi z'} - e^{-\chi z'}\right] + \frac{q_n}{\chi^2+q_n^2}\int dz'\cos q_{n'} z'\left\{2\chi\sin q_n z' + q_n e^{-\chi z'} - q_n(-1)^n e^{-\chi}e^{\chi z'}\right\} =$$

$$= \frac{2\chi q_n}{\chi^2+q_n^2}\int dz'\cos q_{n'} z'\sin q_n z' + \int_0^1 dz'\cos q_{n'} z'\left[(-1)^n e^{-\chi}e^{\chi z'} - e^{-\chi z'}\right]\left[1 - \frac{q_n^2}{\chi^2+q_n^2}\right]$$

(q_n)

Escaneado con CamScanner

$$\int_0^1 dz' \cos q_{n'} z' \sin q_{n'} z' = \left| \begin{matrix} q_{n'} z' = x \\ dz' = \frac{1}{q_{n'}} dx \end{matrix} \right| = \frac{1}{q_{n'}} \int_0^{\pi n'} dx \cos x \sin x = \frac{1}{2q_{n'}} \int_0^{\pi n'} dx \sin 2x =$$

$$= \frac{1}{2q_{n'}} (-1) \frac{1}{2} \cos 2x \Big|_0^{\pi n'} = -\frac{1}{4q_{n'}} \cos 2x \Big|_0^{\pi n'} = -\frac{1}{4q_{n'}} \left[\cos 2\pi n' - 1 \right] = 0 \quad \forall n \neq 0$$

$\forall n' = 0 \quad \int_0^1 dz' \sin q_{n'} z = 0 \quad \forall n \to \int_0^1 dz' \cos q_n z' \cdot \sin q_{n'} z' = \int_0^1 dx \sin q_n x \cos q_{n'} x =$

$$= \left| \text{T. Poma } 2.555 \right| = -\frac{1}{2} \frac{1}{(q_n + q_{n'})} \cos(q_n + q_{n'}) x \Big|_0^1 - \frac{1}{2} \frac{1}{(q_n - q_{n'})} \cos(q_n - q_{n'}) x \Big|_0^1 =$$

$$= -\frac{1}{2} \frac{1}{(q_n + q_{n'})} \left[\cos(q_n + q_{n'}) - 1 \right] - \frac{1}{2(q_n - q_{n'})} \left[\cos(q_n - q_{n'}) - 1 \right], \quad \cos(q_n + q_{n'}) = (-1)^{n+n'}$$

$$\cos(q_n - q_{n'}) = (-1)^{n+n'}$$

$$\int_0^1 dx \sin q_n x \cos q_{n'} x = -\frac{1}{2} \frac{1}{(q_n + q_{n'})} \left((-1)^{n+n'} - 1 \right) - \frac{1}{2(q_n - q_{n'})} \left((-1)^{n+n'} - 1 \right) =$$

$$= -\frac{1}{2} \left[(-1)^{n+n'} - 1 \right] \left[\frac{1}{q_n + q_{n'}} + \frac{1}{q_n - q_{n'}} \right] = -\frac{1}{2} \left[(-1)^{n+n'} - 1 \right] \frac{2q_n}{q_n^2 - q_{n'}^2} = -\left[(-1)^{n+n'} - 1 \right] \frac{q_n}{(q_n^2 - q_{n'}^2)}$$

$$\int_0^1 dz' \int_0^1 dz \left[\frac{\partial}{\partial z} e^{-x|z-z'|} \right] \cos q_n z \cdot \cos q_{n'} z' = \frac{2x q_n}{x^2 + q_n^2} \left[1 - (-1)^{n+n'} \right] \frac{q_n}{(q_n^2 - q_{n'}^2)} + \frac{x^2}{x^2 + q_n^2} \cdot$$

$$\cdot \int_0^1 dz' \cos q_{n'} z' \left[(-1)^n e^{-x} e^{xz'} - e^{-xz'} \right] \Rightarrow$$

$$= (\cdot) + \frac{x^2}{x^2 + q_n^2} \left[\int_0^1 dz' \cos q_{n'} z' e^{xz'} \cdot (-1)^n e^{-x} - \int dz' \cos q_{n'} z' e^{-xz'} \right] = (\cdot) + \frac{x^2}{x^2 + q_n^2} \left[(-1)^n e^{-x} \cdot \right.$$

$$\cdot \left\{ \frac{e^{xz'}}{x^2 + q_{n'}^2} \left(x \cos q_{n'} z' + q_{n'} \sin q_{n'} z' \right) \right\} - \left\{ \frac{e^{-xz'}}{x^2 + q_{n'}^2} \left(-x \cos q_{n'} z' + q_{n'} \sin q_{n'} z' \right) \right\} \Big]_0^1 =$$

$$= (\cdot) + \frac{x^2}{x^2 + q_n^2} \left[(-1)^n e^{-x} \left\{ \frac{e^x}{x^2 + q_{n'}^2} \left(x (-1)^{n'} \right) - \frac{1}{x^2 + q_{n'}^2} x \right\} - \left\{ \frac{e^{-x}}{x^2 + q_{n'}^2} \left(-x (-1)^{n'} \right) - \frac{1}{x^2 + q_{n'}^2} (-x) \right\} \right]$$

$$= (\cdot) + \frac{x^2}{(x^2 + q_n^2)(x^2 + q_{n'}^2)} \left[(-1)^{n+n'} x - (-1)^n e^{-x} x + e^{-x} x (-1)^{n'} - x \right] = (\cdot) + \frac{x^2 \cdot x}{(x^2 + q_n^2)(x^2 + q_{n'}^2)} \cdot$$

$$\cdot \left[(-1)^{n+n'} - 1 - (-1)^n e^{-x} + e^{-x} (-1)^{n'} \right] = (\cdot) + \frac{x^3}{(x^2 + q_n^2)(x^2 + q_{n'}^2)} \left[(-1)^{n+n'} - 1 \right] \left[e^{-x} (-1)^n + 1 \right]$$

$$\int_0^1 dz\, dz' \left[\frac{\partial}{\partial z} e^{-x|z-z'|} \right] \cos q_n z \cdot \cos q_{n'} z' = \frac{2x}{x^2 + q_n^2} \left[1 - (-1)^{n+n'} \right] \frac{q_n^2}{(q_n^2 - q_{n'}^2)} + \frac{x^3 \left[(-1)^{n+n'} - 1 \right]}{(x^2 + q_n^2)(x^2 + q_{n'}^2)} \left[e^{-x} (-1)^n + 1 \right] =$$

$$= \left[1 - (-1)^{n+n'} \right] \left[\frac{2x}{x^2 + q_n^2} \frac{q_n^2}{q_n^2 - q_{n'}^2} - \frac{x^3}{(x^2 + q_n^2)(x^2 + q_{n'}^2)} \left\{ e^{-x} (-1)^n + 1 \right\} \right] = F_{nn'}(x)$$

$$\frac{2x q_n^2}{(q_n^2 - q_{n'}^2)} - \frac{x^2}{(x^2 + q_{n'}^2)} = \frac{1}{(x^2 + q_{n'}^2)(q_n^2 - q_{n'}^2)} \left[2q_n^2 (x^2 + q_{n'}^2) - x^2 (q_n^2 - q_{n'}^2) \right] = \frac{q_n^2 x^2 + 2 q_n^2 q_{n'}^2 + x^2 q_{n'}^2}{(x^2 + q_{n'}^2)(q_n^2 - q_{n'}^2)}$$

$$\int_0^1 dz \int_0^1 dz' \left[\frac{\partial}{\partial z} e^{-x|z-z'|} \right] \cos q_n z \cdot \cos q_{n'} z' = \left[1 - (-1)^{n+n'} \right] \times \left[\frac{1}{(x^2 + q_n^2)(x^2 + q_{n'}^2)} \cdot \frac{(q_n^2 x^2 + q_{n'}^2 x^2 + 2 q_n^2 q_{n'}^2)}{(q_n^2 - q_{n'}^2)} - \right.$$

$$\left. - \frac{x^2 e^{-x} (-1)^n}{(x^2 + q_n^2)(x^2 + q_{n'}^2)} \right] = \left[1 - (-1)^{n+n'} \right] \times \frac{1}{(x^2 + q_n^2)(x^2 + q_{n'}^2)} \left[\frac{q_n^2 x^2 + q_{n'}^2 x^2 + 2 q_n^2 q_{n'}^2}{(q_n^2 - q_{n'}^2)} - x^2 e^{-x} (-1)^n \right]$$

$\forall n \to n', n' \to n, \quad (-1)^n \to (-1)^{n'} = (-1)^{n+n'} (-1)^n = \left| (-1)^{n+n'} = -1 \right| = -(-1)^n$ - annulus

$$\left(\frac{\partial P}{\partial z} \right)_{nn'} = \int dz\, dz' \varphi_n(z) \varphi_{n'}(z') \frac{\partial P}{\partial z} = \frac{1}{\sqrt{N_n N_{n'}}} \int dz\, dz' \cos q_n z \cdot \cos q_{n'} z' \cdot c \int dk \Lambda(k) \Lambda'(k) \left[\frac{\partial}{\partial z} e^{-x|z-z'|} \right]$$

$$= \frac{1}{\sqrt{N_n} \sqrt{N_{n'}}} c \int dk \Lambda(k) \Lambda'(k) \int dz\, dz' \cos q_n z \cdot \cos q_{n'} z' \left[\frac{\partial}{\partial z} e^{-x|z-z'|} \right], \quad c \text{ giusto coupon},$$

$$\left(\frac{\partial P}{\partial z} \right)_{nn'} = -c\beta \int dk\, k \Lambda(k) \Lambda'(k) \bar{f}_{nn'}(k), \quad \bar{f}_{nn'}(k) = -\frac{1}{\beta k} \frac{1}{\sqrt{N_n N_{n'}}} \int dz\, dz' \cos q_n z \cos q_{n'} z' \left[\frac{\partial}{\partial z} e^{-x|z-z'|} \right]$$

Se calcolare con $\frac{\partial}{\partial z} e^{-x|z-z'|} = -\beta k \, \mathrm{sign}(z - z') e^{-x|z-z'|}$, $x = \beta k$

$$\left(\frac{\partial P}{\partial z} \right)_{nn'} = \frac{1}{\sqrt{N_n N_{n'}}} c \int dk \Lambda(k) \Lambda'(k) \left[1 - (-1)^{n+n'} \right] \frac{x}{(x^2 + q_n^2)(x^2 + q_{n'}^2)} \left[\frac{x^2 (q_n^2 + q_{n'}^2) + 2 q_n^2 q_{n'}^2}{(q_n^2 - q_{n'}^2)} - x^2 e^{-x} (-1)^n \right]$$

$$\bar{f}_{nn'}(k) = \int dz\, dz' \, \theta(z - z') e^{-\beta k |z - z'|} \varphi_n(z) \varphi_{n'}(z') = -\frac{1}{\beta k} \int dz\, dz' \varphi_n(z) \varphi_{n'}(z') \left[\frac{\partial}{\partial z} e^{-x|z-z'|} \right]$$

$\bar{f}_{nn'}(x) = -\frac{1}{\sqrt{N_n N_{n'}}} \frac{1}{x} F_{nn'}(x)$ - MathCad ти сказано когда $\partial \neq 0$. Интеграл MathCad находит, но нужно в разложении $S(z) = \sum_{n=0,1,2,\ldots} C_n \cos(\pi n z)$, где коэффициенты C_n комплексные из-за $p \neq 0$

$\varphi_n(z) = \frac{1}{\sqrt{N_n}} \cos(\pi n z), \quad z \in [0, 1]$ или $\varphi_n(z) = \frac{1}{\sqrt{N_n}} \cos \pi n (z + 1/2)$ - при $z \in [-1/2, 1/2]$

$S(z) = \sum_n C_n \cos \pi n (z + 1/2)$, При $L = 500$ nm спрямление (Φ, no odd, no even, $\partial S / \partial z = 0$ не бывает.

Escaneado con CamScanner

13/VII План сред. работ: а) Замм. Начал на машинку; б) когда Хелет вернет Антуан; в) Замечания декларации CONSTANTO IVA-ПКВ и публик. 2019 г) проверить AF-Dw dyn. Ricardo; д) проверить GM thick dots/wires; е) проверить SW edge scattering panel; ж) отпуск в Порто; з) new comp. кум. Зам. план работ 21/I-papers.

Mn_2Au

Запросил Ricardo overleaf file of new DW-AF dyn. paper. Блиц.

14/VII С. Ю. в Анкете. С П. и Ю. по имени Дева, Огурых, Блиц. Recr. IVA-ПКВ.

15/VII Ю. оплатила Visa Taza. Receipt приложим CONSTANTO за 2019.

16/VII Работа над Overleaf file "Inertial DW AF Mn_2Au dynamics" Ricardo Приложение со средст. скоростей координат и с Lagrangian Eq. (2), где Sent 14/VII

J_3 input в $A_3(\vec{n}_1\vec{n}_2)^2$ term, а J_2 в $\frac{1}{|J_2|}(\dot{\vec{l}})^2$-term, $-|J_2|\vec{l}_1\cdot\vec{l}_2$ term expand?

Запрос Ruben'y to explain Eq. (2). Сколько неэквивалентных Mn-lattice sites и сколько магнитных неэквивалентных подрешеток в Mn_2Au? 2 Mn узла и 4-подрешетки или один Mn-узел и 2-маг. подрешетки?

Comment: PRL'16, 200 in Mn_2Au the two magnetic sublattices form inversion partners" ($\vec{L}=\vec{m}_1-\vec{m}_2$ here).

17/VII Ruben в подписку strange Lagrangian Eq. (2) и послал pdf file "Inertial DW motion..." Запомнить, что послал 4/I 2020 и когда будет. Ricardo.

Ruben использовал 2d. Неудоб. using of spherical coordinates.

Ruben использ. Ref. E.G. Tveten, Phys. Rev. B 93, 104408 (2016) "Intrinsic magnetization of AF textures"

P.E. Roy PRB 2016: "... the (Mn_2Au) system consist of two AF-coupled Sublattices carrying total moments $\vec{m}_A$ и $\vec{m}_B$. Уд. $\begin{cases}\dot{\vec{m}}_A = \omega_e[\vec{m}_A\times\vec{m}_B]+(..)\\ \dot{\vec{m}}_B = \omega_e[\vec{m}_A\times\vec{m}_B]+(...)\end{cases}$, где $\omega_e = \frac{2\delta}{\mu_0 M_{Mn}}|J|$, $J = 4J_1+J_2$, $J_2<0$ означает обращение 4-х spin sublattices к 2-spin sublattices, $\vec{m}_A$ и $\vec{m}_B$. (?)

A.B. Shick et al. PRB 81, 212409 (2010): "... (Mn_2Au) calculations are performed considering $MoSi_2$ body-centered tetragonal structure with exp lattice parameters $a = 6.291$ a.u., $c = 16.142$ a.u.. "The tetragonal unit cell with two formula units (Mn_2Au)..."

Видно, что есть 2 non-equiv. sites for Mn-atoms: а) в центре тетрагонов (2 Mn atoms in tetragon centers); б) на ребрах tetragon. unit cells (8 atoms/4 = 4/4 + 4/4 = 2 Mn atoms).

Shick утверждает, что все $\vec{\mu}_{Mn}$ одинаковы и $|\vec{\mu}_{Mn}| = 3.2\,\mu_B$. $|K_2|$ total / not local

В centers of tetragons $\pm\vec{\mu}_{Mn}$, на ребрах tetragon. cell $\pm\vec{\mu}_{Mn}$.

2 и 3rd (A)(B), 5 и 6th (A)(B) Mn-planes (Fig. 1 Ricardo) are not equivalent

2th, 6th planes (A/B) имеют $J_3\left(\frac{A}{B}\right)$, а 3-rd, 5th planes $\left(\frac{B}{A}\right)$, имеют $J_3\left(\frac{B}{A}\right)$. Т.е. для FM exchange J_3 надо вводить $J_3\left(\frac{A}{B}\right)$ и $J_3\left(\frac{B}{A}\right)$, тогда J_3 (body Mn) и J_3 (apex, Mn), (apex) – ребро = edge

S. Khmelevskyi et al. APL 93, 162503 (2008). Вклад J_1, J_2, J_3 для Mn_2Au и предсказал high $T_N \sim 1600$ K. Вклад J_3 (body Mn) и самый сильный. 2,6th planes между Mn(body) – Mn(body), т.к. 2nd Mn(body) за пределами unit cell. Но J_3(Mn-body) может $\neq 0$. Кроме того, Блиц. сделал для Mn(body) есть 4 Mn(edge) с расст. $\frac{\sqrt{3}}{2}a$, J_1, а расстояние Mn(body) – Mn(body) = a. Для J_2 $z_{Mn-Mn} = \frac{c}{3} \approx 9\cdot 0{,}85$, т.е. 9,1a J_3 $z_{Mn-Mn} = a$. Mn(edge) не эквивал. Mn(body). Для Mn(body) есть только J_1, для Mn(edge) есть J_2 и J_3 в приближении Блиц. Сделал.

Mn_2Au не сводится к standard 2-spin sublattice AF. В Mn_2Au есть два типа Mn-sites, B(body)-Mn и e(edge Mn) и 4-spin sublattices $\pm\vec{\mu}_B$, $\pm\vec{\mu}_e$, хотя $|\vec{\mu}_B| = |\vec{\mu}_e|$, b-sites и e-sites of Mn не эквивалентны ни в кристаллическом так и в магнитных полях. Mn(B) имеет только J_1, Mn(e) имеет J_2 и J_3 integrals.

Escaneado con CamScanner

Надо ввести намагниченности $\vec{m}_{dj}$ $d = b, e$ – тип Mn узла (body, edge) и $j = 1,2$ номер Mn-слоя если d is fixed. Mn_2Au

$\vec{m}_{b1}$ (layer 2), $\vec{m}_{b2}$ (layer 6), $\vec{m}_{e1}$ (layer 3), $\vec{m}_{e2}$ (layer 5) номера layers как на Fig. 1 Ricardo's manuscript. Из $\vec{m}_{dj}$ составить лин. комбинации интересующие и преобразование группы симметрии Mn_2Au

$\vec{m}_{b1} = \vec{m}_1$, $\vec{m}_{b2} = \vec{m}_2$, $\vec{m}_{e1} = \vec{m}_3$, $\vec{m}_{e2} = \vec{m}_4$. Static $\vec{m}_2 = -\vec{m}_1$, $\vec{m}_3 = -\vec{m}_4$

① $\vec{m}_1 + \vec{m}_2 + \vec{m}_3 + \vec{m}_4 = 0$ (static) – FM vector

② $\vec{m}_1 + \vec{m}_2 - \vec{m}_3 - \vec{m}_4 = 0$ (static), ③ $\vec{m}_1 - \vec{m}_2 - \vec{m}_3 + \vec{m}_4 = 2\vec{m}_1 - 2\vec{m}_3$ (static)

④ $\vec{m}_1 - \vec{m}_2 + \vec{m}_3 - \vec{m}_4 = 2\vec{m}_1 + 2\vec{m}_3$ (static). Для Mn_2Au (static) $\vec{m}_1 = \vec{\mu}$, $\vec{m}_3 = -\vec{\mu}$

поэтому static ① = 0, ② = 0 ∀ AF layered, ③ = $2(\vec{m}_1 - \vec{m}_3) = 4\vec{\mu}$, ④ = 0

Combination $\vec{l} = \vec{m}_1 - \vec{m}_2 - \vec{m}_3 + \vec{m}_4$, $\vec{l}$ (static) $= 4\vec{\mu}$ to. для Mn_2Au

видно, что комбинация интерпретируется для Mn_2Au.

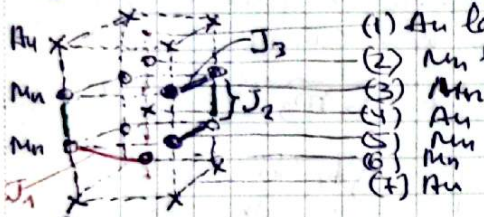
$\vec{l}(Mn_2Au) = \vec{m}_{b1} - \vec{m}_{b2} - \vec{m}_{e1} + \vec{m}_{e2}$, $\vec{l}(Mn_2Au) = \vec{m}_A(2) - \vec{m}_B(6) - \vec{m}_B(3) + \vec{m}_A(5)$

в обозначениях Ricardo.

Ricardo/Ruben вводят $\vec{l}_1 = \vec{m}_A(5) - \vec{m}_B(6)$ и $\vec{l}_2 = \vec{m}_B(3) - \vec{m}_A(2)$, $\vec{l} = \vec{l}_1 - \vec{l}_2$ (sum) комбинации двух векторов $\vec{m}_{dj}$ для определения $\vec{l}$, а не всех 4-х $\vec{m}_{dj}$ как я

(18/VII) AF-vector $\vec{l}$ должен естественно вводиться из ур-я для $\vec{m}_i$ of Mn_2Au unit cell.

$$E = -\sum_{ij} J_{ij}\vec{m}_i\cdot\vec{m}_j + K_z\sum_i(\vec{m}_i\cdot\hat{z})^2 - K_y\sum_i(\vec{m}_i\cdot\hat{y})^2 - M_s\sum_i\vec{m}_i\cdot\vec{H}$$ – energy density (per unit cell V · volume) для Mn_2Au. $K_z = -K_{z\perp} > 0$, $K_y = K_{z\parallel} > 0$, i.j – сум. по ближ. соседям, $K_{z\perp}$, $K_{z\parallel}$ анизотропии, K_z велико, а K_y то надо для Neel DW in xOy easy plane formed by K_z. Mn_2Au



(1), Mn layer are (2), (3), (5), (6)

J_1 – exch. вдоль диагонали (2) и (3), (6) и (5) Mn atoms

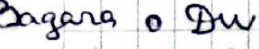
J_2 – exch. вдоль Oz (3) и (4) Mn atoms

J_3 – exch. of Mn atoms in (3) and (5) planes

J_1, J_2, J_3 – схема Khanelevskyi 'APL'08, приближение ближ. соседей

$J_1 < 0$, $J_2 < 0$ (оба $|J_1|$, $|J_2|$ – велики), $J_3 > 0$.

Mn atoms are in Body (b) positions in (2), (6) – layers и в edge (e) positions in (3), (5) layers.

Задача о DW dynamics in  double layer film with AF coupling J_1, easy xOy-plane и hard Ox axis (Neel DW) решалась в А.К. Звездин, JETP 80 (1995) 734 (1995) т/к $J_1 \gg k_z = 2\pi M_s^2$, $K_x = K$ решение для in-plane azimuthal $\varphi(x,t)$ angle есть sin-Gordon $\frac{1}{c^2}\ddot{\varphi} - \nabla^2\varphi + \frac{1}{2\Delta^2}\sin 2\varphi = 0$, $\Delta = \sqrt{\frac{A}{K}}$, $c = \frac{\gamma}{M_s}\sqrt{2AJ_1}$, $\theta_{1,2} = \pi/2$.

$\frac{\pi}{2} + \varphi(x,t) = \pm 2\arctan\left[\exp\left(\pm\frac{x - vt}{\Delta\sqrt{1 - v^2/c^2}}\right)\right]$, A – exch. stiffness of FM layers,

у нас всего $K_x = K$ – hard axis Ox anisotropy.

$K_y > 0$ easy Oy axis, задача как Neel DW

$\frac{\Delta^2}{c^2}\ddot{\varphi} - \Delta^2(\partial_x^2\varphi)^2 + \frac{1}{2}\sin 2\varphi = 0$, $\frac{\Delta^2}{c^2} = \frac{A}{K}\frac{M_s^2}{\gamma^2 2AJ_1} = \frac{M_s^2}{2\gamma^2 K J_1}$

$\frac{M_s^2}{2\gamma^2 J_1}\ddot{\varphi} - A(\partial_x^2\varphi) + \frac{1}{2}K\sin 2\varphi = 0$, $\mathcal{L} = \frac{M_s^2}{2\gamma^2 J_1}(\dot{\varphi})^2 - A(\partial_x\varphi)^2 - K\sin^2\varphi$

$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = \frac{\delta\mathcal{L}}{\delta\varphi} - \frac{\partial\mathcal{L}}{\partial\varphi} - \frac{\partial}{\partial x}\frac{\partial\mathcal{L}}{\partial(\partial_x\varphi)} = -K\sin 2\varphi + 2(\partial_x^2\varphi)A = \frac{M_s^2}{\gamma^2 J_1}\ddot{\varphi}$; $m_{DW} \sim \frac{1}{J_1}$,

решение $\varphi(x,t)$ и $\mathcal{L}$ – Звездин для double-layer film такое же как у Ricardo (May 4, 2020), где $J_1 \to |J_2|$, $m_{DW} \sim 1/|J_2|$, $A \sim J_3$, J_1 исчезло.

Вопрос: можно ли в 4-layer AF Mn_2Au свести к 2 layer model Звездин, где только interlayer (3)(5) coupling J_2 is present? Как быть с J_1 и спинами (2), (6), которые через J_1 взаимодействуют со спинами (3), (5)?

Escaneado con CamScanner

Mn_2Au Hagoo расм. equations of motion $\vec{m}_{B_1}=\vec{m}_1$ (2), $\vec{m}_{B_2}=\vec{m}_2$ (6), $\vec{m}_{e_1}=\vec{m}_3$ (3), $\vec{m}_{e_2}=\vec{m}_4$ (5)



Gayer magnetizations. Ищем решения (DW) неоднородные вдоль Ox. Вдоль Oz, Oy система однородна, т.е. периодична с „c" (Oz), и „a" (Oy)

Варианты: 1) chain ∥Ox, состоящая из unit cells; вдоль Oy, Oz нет повторов
2) thin film xOy, film thickness = c, периодичность вдоль Oy. $\vec{m}(x,t)$
3) Bulk AF $\vec{m}(x,t)$, периодичность вдоль z (c) и вдоль y (a).

Gomonay PRL'16 расм. „bulk" case, Roy PRB'16 расм. ≈ „bulk" 150×150×5 unit cells
$a=0.333$ nm, $c=0.854$ nm. „Chain" шириной 0,333 nm экс. не сделаешь.
Расм. случай of ultrathin Mn_2Au film, $t=c=0.854$ nm (2 cases).

$E_{ex} = -\sum_{i,j} J_{ij}\vec{m}_i\cdot\vec{m}_j = -J_1\vec{m}_1\cdot 4\vec{m}_3 - J_3\sum_j \vec{m}_j^{\parallel x}\cdot\vec{m}_3 - J_3\vec{m}_3\cdot\vec{m}_3^{\parallel y} - J_2\vec{m}_3\cdot\vec{m}_4 -$
per unit cell (2) (3) (3) (3) (3) (6) (5)
$-J_3\vec{m}_4\cdot\vec{m}_4^{\parallel x} - J_3\vec{m}_4\cdot\vec{m}_4^{\parallel y} - J_1\vec{m}_2\cdot 4\vec{m}_4 = -4J_1\vec{m}_1\cdot\vec{m}_3 - 4J_1\vec{m}_2\cdot\vec{m}_4 -$
(5) (5) (6) (5)
$- J_2\vec{m}_3\cdot\vec{m}_4 - J_3\vec{m}_3\cdot\vec{m}_3^{\parallel y} - J_3\vec{m}_4\cdot\vec{m}_4^{\parallel y} - J_3\vec{m}_3\cdot\vec{m}_3^{\parallel x} - J_3\vec{m}_4\cdot\vec{m}_4^{\parallel x}$

$\vec{m}\cdot\vec{m}^{\parallel x} = \vec{m}(0)\vec{m}(\Delta\hat{x}) = \vec{m}(0)\cdot\left[\vec{m}(0) + \frac{\partial\vec{m}}{\partial x}a(\pm) + \frac{1}{2}a^2\frac{\partial^2\vec{m}}{\partial x^2}\right] = \vec{m}(0)\times$
$\times\left[\vec{m}(0) + \frac{1}{2}a^2\frac{\partial^2\vec{m}}{\partial x^2}\right]$, знак $\pm a\frac{\partial\vec{m}}{\partial x}$ сокращается при $\sum_i$ вдоль Ox.

$E_{ex} = -4J_1\vec{m}_1\cdot\vec{m}_3 - 4J_1\vec{m}_2\cdot\vec{m}_4 - J_2\vec{m}_3\cdot\vec{m}_4 - J_3\vec{m}_3\vec{m}_3^{\parallel y} - J_3\vec{m}_4\cdot\vec{m}_4^{\parallel y} -$
$- J_3\vec{m}_3\left[\vec{m}_3 + \frac{1}{2}a^2\frac{\partial^2\vec{m}_3}{\partial x^2}\right]^{\parallel x} - J_3\vec{m}_4\left[\vec{m}_4 + \frac{1}{2}a^2\frac{\partial^2\vec{m}_4}{\partial x^2}\right]^{\parallel x}$

Учредительный 2оо в соседних узлах (3) и (5) - planes $\vec{m}_3(i)\vec{m}_3(j) = \vec{m}_3^2 = \mu^2$,
$\vec{m}_4(i)\vec{m}_4(j) = \vec{m}_4^2 = \mu^2$ внутри unit cell, т.е. J_3 достаточно сильное,
чтобы не допускать неоднородных $\vec{m}_3$, $\vec{m}_4$ внутри unit cell.
μ = μи единиц согласно Shick 2010, Khmelevskyi '08 для всех Mn-sites.
Члены типа $J_3\mu^2$ можно опустить в E_{ex} как Const.

$E_{ex} = -4J_1\vec{m}_1\cdot\vec{m}_3 - 4J_1\vec{m}_2\cdot\vec{m}_4 - J_2\vec{m}_3\cdot\vec{m}_4 - J_3\frac{1}{2}a^2\vec{m}_3\cdot\frac{\partial^2\vec{m}_3}{\partial x^2} - \frac{J_3}{2}a^2\vec{m}_4\cdot\frac{\partial^2\vec{m}_4}{\partial x^2}$

$\vec{m}\frac{\partial^2\vec{m}}{\partial x^2}$: $\int_a^b dx\,\vec{m}\cdot\frac{\partial^2\vec{m}}{\partial x^2} = \int d\left(\frac{\partial\vec{m}}{\partial x}\right)\cdot\vec{m} = \vec{m}\cdot\frac{\partial\vec{m}}{\partial x}\Big|_a^b - \int dx\,\frac{\partial\vec{m}}{\partial x}\cdot\frac{\partial\vec{m}}{\partial x}$

Если $\frac{\partial\vec{m}}{\partial x}\Big|_{a,b} = 0$ exch. boundary conditions, то $\vec{m}\cdot\frac{\partial^2\vec{m}}{\partial x^2} = -\left(\frac{\partial\vec{m}}{\partial x}\right)^2$

$E_{ex} = -4J_1\vec{m}_1\cdot\vec{m}_3 - 4J_1\vec{m}_2\cdot\vec{m}_4 - J_2\vec{m}_3\cdot\vec{m}_4 + A\left(\frac{\partial\vec{m}_3}{\partial x}\right)^2 + A\left(\frac{\partial\vec{m}_4}{\partial x}\right)^2$, $A=\frac{1}{2}a^2J_3>0$.

Все $\vec{m}_n = \vec{m}_n(x)$, где $x=\text{const}$ в пределах unit cell.

$E_{an} + E_H = K_z\sum_i(\vec{m}_i\cdot\hat{z})^2 - K_y\sum_i(\vec{m}_i\cdot\hat{y})^2 - M_S\sum_i\vec{m}_i\cdot\vec{H}_i$ $\quad -\dot{\vec{m}}_i\frac{1}{\gamma} = \vec{m}_i\times\vec{H}_{eff}^i$

$\frac{\delta E}{\delta\vec{m}} = \frac{\partial E}{\partial\vec{m}} - \frac{\partial}{\partial x}\frac{\partial E}{\partial(\partial\vec{m}/\partial x)}$ $\quad \vec{H}_{eff}^i = -\frac{\delta E}{\delta\vec{m}_i}\frac{1}{M_S}$

E_{an} может иметь K_z^i, K_y^i - разные величины на разных Mn-sites i. Кристаллический Эффект.

$M_S\vec{H}_{eff}^{(1)} = -\frac{\delta E}{\delta\vec{m}_1} = 4J_1\vec{m}_3 - 2K_z m_1^z\hat{z} + 2K_y m_1^y\hat{y} + \vec{H}_1 M_S$

$M_S\vec{H}_{eff}^{(2)} = -\frac{\delta E}{\delta\vec{m}_2} = 4J_1\vec{m}_4 - 2K_z m_2^z\hat{z} + 2K_y m_2^y\hat{y} + \vec{H}_2 M_S$

$M_S H_{eff}^{(3)} = -\frac{\delta E}{\delta\vec{m}_3} = 4J_1\vec{m}_1 + J_2\vec{m}_4 - 2A\frac{\partial^2\vec{m}_3}{\partial x^2} - 2K_z m_3^z\hat{z} + 2K_y m_3^y\hat{y} + \vec{H}_3 M_S$

$M_S\vec{H}_{eff}^{(4)} = -\frac{\delta E}{\delta\vec{m}_4} = 4J_1\vec{m}_2 + J_2\vec{m}_3 - 2A\frac{\partial^2\vec{m}_4}{\partial x^2} - 2K_z m_4^z\hat{z} + 2K_y m_4^y\hat{y} + \vec{H}_4 M_S$

$-\frac{1}{\gamma}\dot{\vec{m}}_1 = \frac{\vec{m}_1}{M_S}\times\left[4J_1\vec{m}_3 - 2K_z m_1^z\hat{z} + 2K_y m_1^y\hat{y} + \vec{H}_1 M_S\right]$, $-\frac{1}{\gamma}\dot{\vec{m}}_2 = \vec{m}_2\times\left[4J_1\vec{m}_4 - 2K_z m_2^z\hat{z} + 2K_y m_2^y\hat{y} + \vec{H}_2 M_S\right]\frac{1}{M_S}$

$-\frac{1}{\gamma}\dot{\vec{m}}_3 = \frac{1}{M_S}\vec{m}_3\times\left[4J_1\vec{m}_1 + J_2\vec{m}_4 - 2A\frac{\partial^2\vec{m}_3}{\partial x^2} - 2K_z m_3^z\hat{z} + 2K_y m_3^y\hat{y} + \vec{H}_3 M_S\right]$

$\frac{1}{\gamma}\dot{\vec{m}}_4 = \frac{1}{M_S}\vec{m}_4\times\left[4J_1\vec{m}_2 + J_2\vec{m}_3 - 2A\frac{\partial^2\vec{m}_4}{\partial x^2} - 2K_z m_4^z\hat{z} + 2K_y m_4^y\hat{y} + \vec{H}_4 M_S\right]$

Escaneado con CamScanner

$\vec{H} = \vec{H}^{ex}$ здесь $\vec{H}_1 = \vec{H}_4 = +\vec{H}$, $\vec{H}_2 = \vec{H}_3 = -\vec{H}$. Уравнения движения для $\vec{m}_1$ и $\vec{m}_2$. $\rho = \frac{M_s}{\gamma}$

$-\rho \dot{\vec{m}}_1 = 4J_1 \vec{m}_1 \times \vec{m}_3$; $\rho \dot{\vec{m}}_2 = 4J_1 \vec{m}_2 \times \vec{m}_4$; $-\rho \dot{\vec{m}}_3 = \vec{m}_3 \times \left[4J_1 \vec{m}_1 + J_2 \vec{m}_4 - 2A \frac{\partial^2 \vec{m}_3}{\partial x^2}\right]$

$-\rho \dot{\vec{m}}_4 = \vec{m}_4 \times \left[4J_1 \vec{m}_2 + J_2 \vec{m}_3 - 2A \frac{\partial^2 \vec{m}_4}{\partial x^2}\right]$

$-\rho \dot{\vec{m}}_1 - \rho \dot{\vec{m}}_3 = \vec{m}_3 \times \left[J_2 \vec{m}_4 - 2A \frac{\partial^2 \vec{m}_3}{\partial x^2}\right]$, $-\rho \dot{\vec{m}}_2 - \rho \dot{\vec{m}}_4 = \vec{m}_4 \left[J_2 \vec{m}_3 - 2A \frac{\partial^2 \vec{m}_4}{\partial x^2}\right]$

$-\rho (\dot{\vec{m}}_1 + \dot{\vec{m}}_3 + \dot{\vec{m}}_2 + \dot{\vec{m}}_4) = -2A \vec{m}_3 \times \frac{\partial^2 \vec{m}_3}{\partial x^2} - 2A \vec{m}_4 \times \frac{\partial^2 \vec{m}_4}{\partial x^2}$. Проверим $\vec{l} = \vec{m}_1 - \vec{m}_2 - \vec{m}_3 + \vec{m}_4$

$-\rho \dot{\vec{l}} = -\rho [\dot{\vec{m}}_1 - \dot{\vec{m}}_2 - \dot{\vec{m}}_3 + \dot{\vec{m}}_4] = 4J_1 \vec{m}_1 \times \vec{m}_3 + \vec{m}_4 \times \left[4J_1 \vec{m}_2 + J_2 \vec{m}_3 - 2A \frac{\partial^2 \vec{m}_4}{\partial x^2}\right] -$

$-4J_1 \vec{m}_2 \times \vec{m}_4 - \vec{m}_3 \times \left[4J_1 \vec{m}_1 + J_2 \vec{m}_4 - 2A \frac{\partial^2 \vec{m}_3}{\partial x^2}\right] = 8J_1 \vec{m}_1 \times \vec{m}_3 - 8J_1 \vec{m}_2 \times \vec{m}_4 -$

$-2J_2 \vec{m}_3 \times \vec{m}_4 - 2A \vec{m}_4 \times \frac{\partial^2 \vec{m}_4}{\partial x^2} + 2A \vec{m}_3 \times \frac{\partial^2 \vec{m}_3}{\partial x^2}$. // Видно что $\frac{\partial^2 \vec{m}_i}{\partial x^2} = 0$, $\dot{\vec{m}} = 0$.

// Если считать $\vec{m}_1, \vec{m}_2$ – slave variables из-за J_1, $\vec{m}_1 \uparrow\uparrow \vec{m}_3$, $\vec{m}_2 \uparrow\uparrow \vec{m}_4$, то

$-\rho \dot{\vec{m}}_3 = \vec{m}_3 \times \left[J_2 \vec{m}_4 - 2A \frac{\partial^2 \vec{m}_3}{\partial x^2}\right]$, $-\rho \dot{\vec{m}}_4 = \left[J_2 \vec{m}_3 - 2A \frac{\partial^2 \vec{m}_4}{\partial x^2}\right]$ – eqs. for $\vec{m}_3, \vec{m}_4$

Или $\begin{cases} -\rho \dot{\vec{m}}_3 = \vec{m}_3 \times \left[J_2 \vec{m}_4 - 2A \frac{\partial^2 \vec{m}_3}{\partial x^2} - 2K_z m_3^z \hat{z} + 2K_y m_3^y \hat{y} - \vec{H} M_s\right] \\ -\rho \dot{\vec{m}}_4 = \vec{m}_4 \times \left[J_2 \vec{m}_3 - 2A \frac{\partial^2 \vec{m}_4}{\partial x^2} - 2K_z m_4^z \hat{z} + 2K_y m_4^y \hat{y} + \vec{H} M_s\right] \end{cases}$ условие $\vec{m}_1 = -\vec{m}_3$, $\vec{m}_2 = -\vec{m}_4$

Сравнение. $E^* = -J_2 \vec{m}_3 \cdot \vec{m}_4 + A \left(\frac{\partial \vec{m}_3}{\partial x}\right)^2 + A \left(\frac{\partial \vec{m}_4}{\partial x}\right)^2 + 2K_z \left[(m_3^z)^2 + (m_4^z)^2\right] -$

$-2K_y \left[(m_3^y)^2 + (m_4^y)^2\right] + 2M_s (\vec{m}_3 - \vec{m}_4) \cdot \vec{H}$ – effective 2-layer (3)//(5) model с easy plane (xy) анизотропии $2K_z$, easy axis anisotropy $\parallel Oy = 2K_y$, stiffness A, interlayer exch. integral J_2 и specific Zeeman energy (отсюда Zvezdin'99)

Возможно надо $A = \frac{1}{2} a^2 J_3 \to a^2 J_3$ // E^* сводится к Zvezdin'99 model

$\Delta = \sqrt{\frac{A}{K}}$ – Zvezdin'99, где $E = \sum_i \left[2\pi M_s^2 \cos^2 \theta_i + \frac{1}{2} K m_{yi}^2 + \frac{A}{2} \left[(\nabla \theta)^2 + \sin^2 \theta_i (\nabla \varphi_i)^2\right]\right] + \frac{1}{2} J_1 \vec{m}_1 \cdot \vec{m}_2$

• Zvezdin'99: $E = \sum_i \left[\left(2\pi M_s^2 - \frac{1}{2}K\right) m_{zi}^2 - \frac{1}{2} K m_{xi}^2 + \frac{A}{2} \left\{(\nabla \theta_i)^2 + \sin^2 \theta_i (\nabla \varphi_i)^2\right\}\right] + \frac{1}{2} J_1 \vec{m}_1 \cdot \vec{m}_2$

Наш $E^* = A^* \left(\frac{\partial \vec{m}_3}{\partial x}\right)^2 + A^* \left(\frac{\partial \vec{m}_4}{\partial x}\right)^2 + 2K_z \left[(m_3^z)^2 + (m_4^z)^2\right] - 2K_y \left[(m_3^y)^2 + (m_4^y)^2\right] - J_2 \vec{m}_3 \cdot \vec{m}_4$

модели совпадают, если $\frac{A}{2} = A^*$ (мое), $x \rightleftarrows y$, $2K_z = 2\pi M_s^2 - \frac{1}{2}K > 0$, $2K_y = \frac{1}{2}K$, $J_1 = -2J_2 > 0$

$\Delta = \sqrt{\frac{A}{K}} = \sqrt{\frac{2A^*}{4K_y}} = \sqrt{\frac{A^*}{2K_y}} = \sqrt{\frac{a^2 J_3}{2K_y}} = a\sqrt{\frac{J_3}{2K_y}}$, $\frac{J_3}{K_y} = \frac{115\, k_B \cdot K}{V \cdot 7 \cdot (1{,}8548 \cdot 10^{-25}\, J)\frac{1}{V}} =$

$= \frac{115 K \cdot 1{,}381 \cdot 10^{-23} J/K}{7 \cdot 1{,}8548 \cdot 10^{-25} J} = \frac{158{,}815}{12{,}9836} \cdot 10^2 = 12{,}232 \cdot 10^2$ $\to \Delta = 0{,}333\, nm \sqrt{\frac{12{,}232}{2} \cdot 10} = 8{,}235\, nm$

$M_s = \frac{J_{mn}}{V} = \frac{4\mu_B}{a^2 c} = \frac{4 \cdot 9{,}274 \cdot 10^{-21}\, erg/G}{(0{,}333)^2 \cdot 10^{-14}\, cm^2 \cdot 0{,}854 \cdot 10^{-7}\, cm} = \frac{4 \cdot 9{,}274}{(0{,}333)^2 \cdot 0{,}854} G = 391{,}7\, G$

Max Dw velocity $c = \frac{\gamma}{M_s} \sqrt{2AJ_1} = \frac{\gamma}{M_s} \sqrt{2 \cdot 2A^* \cdot 2|J_2|} = \frac{2\gamma}{M_s} \sqrt{2A^* |J_2|} = \frac{2\gamma}{M_s} a \sqrt{2 J_3 |J_2|}$, $J_i \to \frac{J_i}{V}$

$c = \frac{2 \cdot 2\pi \cdot 0{,}333\, nm}{0{,}392\, kOe} 2{,}80 \frac{GHz}{kOe} \sqrt{2 \frac{115 K \cdot k_B \cdot 532 K \cdot k_B}{V^2}} = (-)\, 1K \cdot k_B \sqrt{230 \cdot 532}$

$V = (0{,}333)^2 \cdot 0{,}854\, nm^3$, $c = \frac{4\pi}{0{,}392\, kOe} 2{,}80 \frac{GHz}{kOe} \frac{1}{0{,}333 \cdot 0{,}854\, nm^3} 1{,}381 \cdot 10^{-23} J \sqrt{230 \cdot 532} =$

$= \frac{4\pi}{0{,}392 \cdot \frac{erg}{cm^3} \cdot 10^6} \cdot 2{,}80\, GHz \frac{1}{0{,}333 \cdot 0{,}854 \cdot 10^{-14} cm^2} 1{,}381 \cdot 10^{-23} J \cdot 349{,}8 = \frac{4\pi \cdot 2{,}80 \cdot 1{,}381 \cdot 349{,}8}{10^6 \cdot 0{,}392 \cdot 10^{-7} J \cdot 0{,}333 \cdot 0{,}854} \frac{J}{cm}$

$\times \frac{10^9\, Hz}{10^{14}} \cdot 10^{-23} = \frac{4\pi \cdot 2{,}80 \cdot 1{,}381 \cdot 349{,}8}{0{,}392 \cdot 0{,}333 \cdot 0{,}854} \cdot \frac{10^7}{10^6} \frac{cm}{s} = 152{,}47 \cdot \frac{10^3 \cdot 10^7}{10^6} \frac{cm}{s} = 152{,}47 \cdot 10^5 \frac{km}{s}$ (?)

c (corrected) $= 152{,}47 \cdot 10^{-1}\, km/s = 15{,}25\, km/s$

M_s: 4 muons per unit cell, 2 $c + \vec{\mu}_{Mn}$, 2 $c - \vec{\mu}_{Mn}$. Поэтому $M_s = \frac{2\mu_{Mn}}{V} = 783{,}4\, G$

Но согласно Shick' 2010 $\mu_{Mn} = 3{,}2 \mu_B$ и тогда $M_s = \frac{2\mu_{Mn}}{V} = 626{,}7\, G$

$c = \frac{2\gamma}{M_s} a \sqrt{2 J_3 |J_2|} \sim 1/M_s$ тогда меньше 15,25 km/s.

Escaneado con CamScanner

Ricardo (overleaf) gives $v_{max} = c = \frac{2\gamma}{\mu_0 M_s} \sqrt{S^2 |J_2| A_3} = \frac{2\gamma}{\mu_0 M_s} S \sqrt{|J_2| J_3 ac}$, $S = \frac{5}{2}$

или $v_m = \frac{2\gamma}{M_s} S a \sqrt{\frac{c}{a}} \sqrt{J_3 |J_2|}$ чаще сам Звездин'99: $c = \frac{2\gamma a}{M_s} \sqrt{2 J_3 |J_2|}$

Ratio $\frac{v_m}{c} = S \sqrt{\frac{c}{a}} \frac{1}{\sqrt{2}} = \frac{5}{2} \sqrt{\frac{c}{2a}} = \frac{5}{2} \sqrt{\frac{0,854}{2 \cdot 0,333}} = 2,83$.

$\frac{v_m}{c} = \frac{43,15 \text{ (Ricardo)}}{15,25 \text{ (exp)}} = 2,83$, т.е. Ricardo used $M_s = 392$ G и 1 atom Mn / cell

с $\mu_{Mn} = 4\ \mu_B$

Мое определение вектора AF $(\vec{m}, \vec{l})$ совпад. с Ivanov 1993 $L = \sum_j \vec{M}_j - \sum_j \vec{M}_j$

A.F. Andreev et al. УФН 1980

$L = \frac{1}{2} \int dV \left[\beta \dot{\vec{l}}^2 - q_{ik} \frac{\partial \vec{l}}{\partial x_i} \frac{\partial \vec{l}}{\partial x_k} \right] \rightarrow \vec{m} = \gamma \beta [\vec{l} \times \dot{\vec{l}}]$

V.G. Bar'yakhtar et al. УФН 146 417 (1985)

$\vec{m} = \frac{\vec{M}_1 + \vec{M}_2}{2M_0}$, $\vec{l} = \frac{\vec{M}_1 - \vec{M}_2}{2M_0}$, $W = M_0^2 \left[\frac{1}{2} \delta \vec{m}^2 + \frac{1}{2} a_{ik} \frac{\partial \vec{l}}{\partial x_i} \frac{\partial \vec{l}}{\partial x_k} + W_a + W_{DM} + W_H \right]$, $W = \int dV w$.

Max DW velocity $c = \frac{1}{2} \gamma M_0 \sqrt{a \delta}$, $\vec{m} = \frac{2}{\gamma \delta M_0} [\dot{\vec{l}} \times \vec{l}]$, $\vec{m}^2 + \vec{l}^2 = 1$

Ivanov' LTP'1995: $w = M_0^2 \left[\frac{1}{2} \delta \vec{m}^2 + \frac{1}{2} a (\nabla \vec{l})^2 + \dots \right]$ $c = \frac{1}{2} \gamma M_0 \sqrt{a \delta}$

Grekova, LTP'2018: $W_{ex} = \frac{1}{2 M_s} H_{ex} \vec{M}^2$, $\vec{L} = \vec{M}_1 - \vec{M}_2$, $\vec{M} = \vec{M}_1 + \vec{M}_2$, $|\vec{M}_1| = |\vec{M}_2| = M_0$, $M_s = 2M_0$, $\vec{l} = \frac{1}{2}(\vec{M}_1 + \vec{M}_2)$, $\vec{m} = \frac{\vec{M}_1 + \vec{M}_2}{2M_0}$

δ-model $L = \int dV \left[\frac{M_0}{\gamma^2 H_{ex}} (\dot{\vec{l}})^2 - \frac{2M_0}{\gamma H_{ex}} \vec{H}_{eff} \cdot (\vec{l} \times \dot{\vec{l}}) - \frac{A}{2} (\nabla \vec{l})^2 - \dots \right]$

$c = \gamma \sqrt{\frac{A H_{ex}}{2M_0}}$, $W_{ex} = \frac{1}{4\mu_0} H_{ex} \vec{M}^2 = \frac{1}{4\mu_0} H_{ex} (\vec{M}_1 + \vec{M}_2)^2 = M_0 \mu H_{ex} \vec{m}^2$

Obvious definition $c = \gamma \sqrt{\frac{1}{2} a M_0^2 \frac{1}{2} \delta} = \frac{1}{2} \gamma M_0 \sqrt{a \delta}$, $W_{ex} = \frac{1}{2} \delta \vec{m}^2 M_0^2$, откуда $H_{ex} = \frac{1}{2} \delta M_0$, $A = a M_0^2$

Тоже совп. с Bar.'1985, Ivanov'95

$W_{ex} = \frac{1}{2} M_0^2 \delta \vec{m}^2 = \frac{1}{2} \delta M_0^2 \frac{1}{4} \left(\frac{\vec{M}_1}{M_0} + \frac{\vec{M}_2}{M_0} \right)^2 = \frac{1}{8} \delta M_0^2 (\vec{m}_1 + \vec{m}_2)^2 = \frac{1}{4} \delta M_0^2 \vec{m}_1 \cdot \vec{m}_2 + \text{const}$

W_{ex} (Ricardo) $= |J_2| \vec{m}_1 \cdot \vec{m}_2$, $|J_2| = \frac{1}{4} \delta M_0^2$, $|\vec{M}_i| = M_0$, W_{ex} (Zvezdin) $= \frac{1}{2} J_1 \vec{m}_1 \cdot \vec{m}_2$

A-B

J_1 (Zvezdin) $= 2|J_2| = \frac{1}{2} \delta M_0^2$, $W_{ex}(2v) = \frac{1}{2} A (\nabla \vec{m})^2$, $W_{ex} = \frac{1}{2} a M_0^2 (\nabla \vec{l})^2 = \frac{A}{2} (\nabla \vec{l})^2$, $A = a M_0^2$

Zvezdin'99 $c = \frac{\gamma}{M_0} \sqrt{2 A J_1} = \frac{\gamma}{M_0} \sqrt{2 a M_0^2 \frac{1}{2} \delta M_0^2} = \gamma M_0 \sqrt{a \delta}$, Вопрос да (?)

→ c (Zvezdin'99) в 2-раза больше „c" defined by Bar'85, Ivanov'95, Gal'2018

Kosevich'1990: $w_{ex} = \frac{1}{2} A \vec{M}^2 + \frac{1}{8} a \left(\frac{\partial \vec{L}}{\partial x_i} \right)^2$, $\vec{L} = \vec{M}_1 - \vec{M}_2$, $\vec{M}_1 + \vec{M}_2 = \vec{M}$, $|\vec{M}_i| = M_0$

c (Kos) $= \gamma M_0 \sqrt{A a}$, W_{ex} (Kos) $= \frac{1}{2} A \vec{m}^2 \cdot 4 M_0^2 + \frac{1}{8} a (\nabla \vec{l})^2 \cdot 4 M_0^2$, W_{ex} (Bar) $= M_0^2 \left[\frac{1}{2} \delta \vec{m}^2 + \frac{1}{2} a (\nabla \vec{l})^2 \right]$

A (Kos) $= \frac{1}{4} \delta$, a (Kos) $= a$, c (Kos) $= \gamma M_0 \sqrt{\frac{1}{4} \delta a} = \frac{1}{2} \gamma M_0 \sqrt{a \delta}$ – как в Bar'1985.

Norem c (Zvezdin) $= \frac{\gamma}{M_0} \sqrt{2 A J_1}$, совп. с „c" defined by Bar, Kosevich, Ivanov?

Это из-за double-layer film?

A.K. Zvezdin Pis ZETF 23 (10) 605 (1979): $W = \frac{a}{2} \vec{m}^2 + A (\nabla \vec{l})^2 + \dots$

$c = 2\gamma \frac{1}{M_0} \sqrt{\frac{a}{2} A}$, a (2v) $= \delta M_0^2$, A (2v) $= \frac{1}{2} a M_0^2$, c (2v) $= \frac{2\gamma}{M_0} \sqrt{\delta M_0^2 \frac{1}{2} a M_0^2} = \gamma M_0 \sqrt{2 a \delta}$,

Тоже совп. с Zvezdin'99 и Bar.'85. Heq. „c" не рассмотрен

A.K. Zvezdin et al. LTP 36, 226 (2010): $W_{ex} = \frac{a}{2} \vec{m}^2 + A (\nabla \vec{l})^2$, $c = \gamma \sqrt{\frac{2A}{\chi_\perp}}$, $\chi_\perp = \frac{M^2}{a}$

$c = \gamma \sqrt{2A \frac{a}{M^2}} = \frac{\gamma}{M} \sqrt{2Aa} = \frac{\gamma}{M_0} \sqrt{2 \delta M_0^2 \frac{1}{2} a M_0^2} = \gamma M_0 \sqrt{a \delta}$ – также как в Zvezdin'99, но совп. с Zvezdin'79 + Bar.'85.

Комплекс Zvezdin, скорее всего „c" def. и поэтому его „c" совп. с Bar'85

Большой Титов „Antiferromagnets" у Зубарева

(20/VII) Зухра написала Turov's Book (djvu), Глеб написал - PDF.
Коротичка с Ricardo, Ruben o meeting в Donostia. Блин.

Eq. (9.10) Туров: $E_{ex} = J_0 \vec{M}^2 + J_1 \vec{L}_a^2 + J_2 \vec{L}_b^2 + J_3 \vec{L}_c^2$ для
4-х подрешеток. AF. $\vec{L}_a = \vec{M}_1 - \vec{M}_2 - \vec{M}_3 + \vec{M}_4$, $\vec{L}_b = \vec{M}_1 - \vec{M}_2 + \vec{M}_3 - \vec{M}_4$
$\vec{L}_c = \vec{M}_1 + \vec{M}_2 - \vec{M}_3 - \vec{M}_4$, $\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \vec{M}_4$

Ф.11. AF dyn., Туров следует И.В. Барьяхтар, Б.А. Иванов, ФНТ 5 (N1) 154 (1979)
и Б.А. Иванов, Лк конференция ФНТ 1975, 1, 355. (1983, Sov. Sci. Rev. A. Phys. 6, 183)
Двухподрешеточная AF, равномодульное приближение $|\vec{M}_1| = |\vec{M}_2| = M_0$
$\vec{L}^2 + \vec{M}^2 = 4M_0^2$. $\mathcal{L} = \frac{1}{2} E^{-1} M_0^2 \left[\frac{1}{a^2} (\dot{\vec{l}})^2 - (\nabla \vec{l})^2 \right]$ + (zeem, anisotropy, DMI)

$\vec{m}^2 + \vec{l}^2 = 1$, $\vec{m} \cdot \vec{l} = 0$, исходная energy density $W = \frac{1}{2} E \vec{M}^2 + \frac{1}{2} E^{-1} a^2 (\nabla \vec{L})^2 + (an, \chi, DMI)$

$\dot{\vec{l}} = -2\gamma M_0 E \vec{m} \times \vec{l}$, откуда, $\vec{m} = \frac{1}{2M_0 E} \frac{1}{\gamma} (\vec{l} \times \dot{\vec{l}})$, $\dot{\vec{m}} = -2\gamma M_0 E a^2 (\vec{l} \times \nabla^2 \vec{l})$

$\vec{l} \times \left[\ddot{\vec{l}} - c^2 \nabla^2 \vec{l} \right] = (an, \chi, DMI)$, где $c = \gamma a \sqrt{2 H_E H_E'}$ - limit. velocity

$H_E = E M_0$, $H_E' = 2 E' M_0$, $c = 2\gamma M_0 a \sqrt{E E'}$, $\vec{m} = \vec{M}/2M_0$, $\vec{l} = \vec{L}/2M_0$

Bar'85: $W_{ex} = M_0^2 \left[\frac{1}{2} \delta \vec{m}^2 + \frac{1}{2} \alpha (\nabla \vec{l})^2 \right]$, $W_{ex}(\text{Тур}) = \frac{1}{2} E M_0^2 \cdot 4 \vec{m}^2 +$
$+ \frac{1}{2} E' a^2 (2M_0)^2 (\nabla \vec{l})^2$,
откуда $\frac{1}{2} \delta M_0^2 = 2 E M_0^2$, $E = \frac{1}{4} \delta$, $M_0^2 \frac{1}{2} \alpha = 2 E' a^2 M_0^2$, $E' a^2 = \frac{1}{4} \alpha$

$c = 2\gamma M_0 \sqrt{E E' a^2} = 2\gamma M_0 \sqrt{\frac{1}{4}\delta \cdot \frac{1}{4}\alpha} = \frac{1}{2} \gamma M_0 \sqrt{\alpha \delta}$, точно как в Bar'85 Turov'95

• Барьяхтар-Иванов ФНТ'1979: вывод 2-model 2-х подрешеточного AF

$c = \gamma M_0 \sqrt{2\tilde{E}(\alpha - J')} = |\alpha' = 0, \beta, \beta' = 0| = \gamma M_0 \sqrt{2\delta\alpha}$, но возможно α, δ другие

$W_{ex} = \int dV \left[\delta \vec{M}_1 \cdot \vec{M}_2 + \frac{\alpha}{2} \left\{ \left(\frac{\partial \vec{M}_1}{\partial x_i} \right)^2 + \left(\frac{\partial \vec{M}_2}{\partial x_i} \right)^2 \right\} \right]$, $\vec{M}_1 \cdot \vec{M}_2 = \frac{1}{4} (\vec{M}_1 + \vec{M}_2)^2 - (\vec{M}_1 - \vec{M}_2)^2 \frac{1}{4} =$
$= \frac{1}{4} \vec{M}^2 - \frac{1}{4} \vec{L}^2 = \frac{1}{4} 4 M_0^2 \vec{m}^2 - \frac{1}{4} \cdot 4 M_0^2 \vec{l}^2 = M_0^2 (\vec{m}^2 - \vec{l}^2)$, $\vec{m}^2 + \vec{l}^2 = 1$

$\vec{M}_1 \cdot \vec{M}_2 = M_0^2 \left[\vec{m}^2 - (1 - \vec{m}^2) \right] = 2 M_0^2 \vec{m}^2 + const$. $(\partial_x \vec{M}_1)^2 = (\dots)$.

$\vec{l} = \frac{1}{2M_0} (\vec{M}_1 - \vec{M}_2)$, $\vec{m} = \frac{1}{2M_0} (\vec{M}_1 + \vec{M}_2)$, $\vec{l} + \vec{m} = \frac{\vec{M}_1}{M_0}$, $\vec{m} - \vec{l} = \frac{\vec{M}_2}{M_0}$

$(\partial_x \vec{M}_1)^2 + (\partial_x \vec{M}_2)^2 = M_0^2 \left[\partial_x (\vec{l} + \vec{m}) \right]^2 + M_0^2 \left[\partial_x (\vec{m} - \vec{l}) \right]^2 = |\text{сначала только } \vec{l}| =$
$= M_0^2 (\partial_x \vec{l})^2 + M_0^2 (\partial_x \vec{l})^2 = 2 M_0^2 \left(\frac{\partial \vec{l}}{\partial x_i} \right)^2$

$W_{ex}(Bar'79) = \delta \vec{M}_1 \vec{M}_2 + \frac{\alpha}{2} \left[(\partial_x \vec{M}_1)^2 + (\partial_x \vec{M}_2)^2 \right] = 2 M_0^2 \vec{m}^2 \delta + \frac{\alpha}{2} 2 M_0^2 \left(\frac{\partial \vec{l}}{\partial x_i} \right)^2$

сравн. с $W_{ex}(Bar'85) = \frac{1}{2} \delta M_0^2 \vec{m}^2 + \frac{1}{2} M_0^2 \alpha (\nabla \vec{l})^2$, $2\delta(79) = \frac{1}{2}\delta$, $\alpha(79) = \frac{1}{2}\alpha$

$c(Bar'79) = \gamma M_0 \sqrt{2 \delta(79) \alpha(79)} = \gamma M_0 \sqrt{2 \cdot \frac{1}{4}\delta \cdot \frac{1}{2}\alpha} = \frac{1}{2} \gamma M_0 \sqrt{\alpha\delta}$ - то же
как в Bar' 85, Ivanov 95, Kos' 1990 и Turov 2001 (!!).

// Bar-Ivanov ФНТ'79 получена рец. 14/VII 1978, published in July 1979
Zvezdin'79, JETP Lett. Subm. 12/II 1979, published 20/V 1979. T.e. Zvezdin
subm. 2-model paper позже Bar-Ivanov, но published раньше.
+ Zvezdin ответил на мой вопрос о „c" и написал JETP Lett.'79 cor. Zvezdin JETP'95

(21/VII) Zvezdin' JETP Lett'79 corr.: $W_{ex} = \frac{a}{2} \vec{m}^2 + A (\nabla \vec{l})^2$, $\vec{m} = \frac{\vec{M}_1 + \vec{M}_2}{2M}$, $\vec{l} = \frac{\vec{M}_1 - \vec{M}_2}{2M}$ Zv-JMMM'2020

$c^2 = 4\gamma^2 \frac{A H_E}{M}$, $a/M = 2H_E$, $c^2 = 4\gamma^2 \frac{A}{M} \frac{a}{2M}$, $c = \frac{2\gamma}{M} \sqrt{\frac{1}{2} A a}$.

$W_{ex}(Bar) = \frac{1}{2} \delta M_0^2 \vec{m}^2 + \frac{1}{2} \alpha M_0^2 (\nabla \vec{l})^2$, $a = \delta M_0^2$, $A = \frac{1}{2} \cdot 2 M_0^2$, тогда $M_0 = M$,
$c(Zvezdin) = \frac{2\gamma}{M_0} \sqrt{\frac{1}{2} \frac{1}{2} \alpha M_0^2 \delta M_0^2} = \gamma M_0 \sqrt{\alpha\delta}$, такое же как Zvezdin'99, 2019,
но в 2 раза больше „c" Bar'85, Kosevich'90, Ivanov'95.

Escaneado con CamScanner

(22/VII) Organ Antwort' travel-Report-Poznan » Glob Dynamika за Feb 2020

Meeting с Ricardo: обсуждение AF/DW theorial dyn. Mn_2Au manuscript. Ricardo got 2 new bulk Gordon losses" неупруги в y-polar axis (in plane easy axis Mn_2Au) experim. cuccine координат, по нашите в z-polar ? cuccine (?). Strange - проверка z - tetragonal axis Mn_2Au 2 weeks 1993 Gne double layer FM coupled film used 02 layers и нещ 2nd Gne-Gordon. Ricardo gne new Refs to Mn_2Au: Nature.com 14 Mar 2015 (Barthem, Gweed) и V. Barthem, D. Givord, Nat. Commun. 4, 2892 (2013).

Barthem'13, experiment: $\mu_{mm} = 4,0 \mu_B$ at 2K, high $T_N = 1500-1600K$, $M_S = 780$ G (Mn sublatt) и ground "easy plane" anisotropy ≈ 25 K/Mn_2Au f.u. with $K_2 = 7,3 \cdot 10^6$ J/m^3

$H_A = \frac{2K_2}{M_S} = 2 \cdot \frac{7,3 \cdot 10^4 \text{ erg/cm}^3}{0,18 \cdot 10^3 \text{ G}} = \frac{2 \cdot 7,3}{0,18} 10^4 \text{ Oe} = 18,1 \cdot 10^4 \text{ Oe} = 187 \text{ kOe}$ (!)

(23/VII) Проверка DW equations (sine-Gordon) за analogia Ricardo (от знак Kzll неизмерен). Нагрели шибани „CONSTANTO" gne gen. sinus.

(24/VII) Gne double layer FM (films (AF coupled) with two sublattice AF zamcon eqs. of motion $\vec{m}_1, \vec{m}_2$, нерешен к $\vec{m} = \vec{m}_1 + \vec{m}_2$, $\vec{l} = \vec{m}_1 - \vec{m}_2$, изкл. $\vec{m}$ и нерешен к $\vec{l}$-equation of 3-model. В числ $W_2 = -\vec{m} \cdot \vec{H}$, bigger energy $W_2 = -\vec{l} \cdot \vec{H}$ coorder. to SOT fields in definition Gomonay 'ARI16 нарушена ос z чрез $\vec{H}$ в YD (kos. Bar. Gurev Ivanov, a в Tveten' PRL 2013 = си есе) $\vec{l} = \vec{l}(\theta, \varphi)$, gne φ - azimuth. angle използва Bass Neel DW ansatz

$\varphi(x,t) = 2 \arctg\left[e^{-\frac{(x-vt)}{\Delta(v)}}\right]$ Neel DW in basis xy-plane, Oy - easy axis $\perp K_{2\perp}$.

Torque $\varphi(x,t) = 2 \arctg\left[e^{\frac{x-vt}{\Delta}}\right] - \frac{\pi}{2}$, $\varphi(x,t,q) = 2 \arctg\left[e^{\frac{x-q}{\Delta}}\right] - \pi/2$, $q(t)$ - DW center position at motion $\parallel Ox$. $\theta(x,t)$ изкл. от YD. Можем $\theta(x,t) \approx \pi/2$. Осъществ Thiele eq. for $q(t)$, и чрез добавим H_{SOT} и damping.

Mn_2Au

• M. Arana, SM Rezende APL 111, 192409 (2017). Magnons in Mn_2Au-BLS

Три magnon peaks $\omega_1/2\pi = 121$ GHz, $\omega_2/2\pi = 3,24$ THz, $\omega_3/2\pi = 4,2$ THz $k \approx 0$.

$\omega_1 = \gamma \sqrt{2H_{a1} H_e}$, $\omega_2 = \gamma \sqrt{2H_{a2} H_e}$, $H_e = 2S z J / g\mu_B$, $H_{ai} = 2S D_i / g\mu_B$

$\mathcal{H} = \sum_{ij} 2J_{ij} \vec{S}_i \vec{S}_j + \sum_i D_1 (S_i^z)^2 + \sum_i \frac{D_2}{S^2} (S_i^x)^2 (S_i^y)^2$, $D_1, D_2 > 0$

D_1 - easy xy-plane ⊥ tetragon. axis [001], $H_{a1} \approx 53$ kOe (Barthem'13 numerno $H_A = \frac{2K_2}{M_S} = 187$ kOe)

ОЛН ⟨110⟩ direction, xy∥ОЛН, D_2 cooperative $H_{a2} \approx 70$ Oe.

J - AF intersublattice exchange (S не указана, z=NN броя), $H_e \approx 1300$ T.

Есть calculated $\omega_1(k)$, $\omega_2(k)$ с числ. J, D_1, D_2 parameters

$\omega_1(k)$ почти линейна, $v_{sw} = \frac{\omega_1}{k} = \frac{5,5 \text{ THz} \cdot 2\pi}{0,1 \cdot \frac{\pi}{a}} = \frac{55 \cdot 10^{12}}{s^{-1}} 0,333 \text{ nm} \cdot 2\pi =$

$= \frac{5,5 \cdot 0,333}{0,1\pi} 10^{13} \cdot 10^{-9} \frac{m}{s} = 0,583 \cdot 10^4 \text{ m/s} \cdot 2\pi = 3663 \text{ km/s}$, $v_{gr} = \frac{d\omega}{dk} = \frac{\omega}{k}$ gne linear $\omega(k)$. v_{gr} (max gne ω_1-branch) $= 5,83$ km/s $\cdot 2\pi = 36,63$ km/s

→ Note! 14/VII в answer с нов стор. J_3, J_2 с $M_S = 392$ G как $c = 15,25$ km/s corrected

если взем $M_S = 780$ G [Barthem'13], то $c = \frac{392}{780} \cdot 15,25 \frac{km}{s} = 7,66$ km/s

M_S = 2 mom /V, т.к. 2 Mn atoms coexchange ↑↑ moments/cell with volume/c.

Final parameter BLS Arana'19 $v_{gr} = 5,83$ km/s и теор. $c = 7,66$ km/s gabarito Бъдещи. Dz ↓ guarantee.

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• V. Barthem, ... D. Givord, JMMM 406, 289 (2016) Экс. найдено 250 Mn2Au имеет cycloid in-plane "easy axes" ⟨110⟩, вдоль диагоналей unit cell.

(25/VII) Согласно Gomonay PRL'16, SOT - вклад в energy density есть $W_H = -\vec{L}\cdot\vec{H}_{SOT}$, $\vec{L}=\vec{M}_1-\vec{M}_2$, $\vec{H}_{SOT}=\vec{H}(t)$, time-dependent, in general.

Вывод 3-модель, мех. SW velocity и SW motion $\theta(x,t)$ - Gomonay '16 См. 29-30/IX 2017 г. (!). • Следуем ch. 9, Kosevich Phys. Rep. 1990

AF Volkov 137

$$W_{AF} = \frac{1}{2}A\vec{M}^2 - \vec{L}\vec{H} + \frac{1}{8}\alpha\left(\frac{\partial\vec{L}}{\partial x_i}\right)^2 + W_a(\vec{L}),\quad \vec{L}=\vec{M}_1-\vec{M}_2,\ \vec{M}=\vec{M}_1+\vec{M}_2$$

$$\begin{cases} -\frac{1}{\gamma}\dot{\vec{M}} = \vec{M}\times\vec{H}_M + \vec{L}\times\vec{H}_L \\ -\frac{1}{\gamma}\dot{\vec{L}} = \vec{L}\times\vec{H}_M + \vec{M}\times\vec{H}_L \end{cases},\quad \vec{H}_M = -\frac{\delta W}{\delta\vec{M}} = -\frac{\partial W}{\partial\vec{M}} + \frac{\partial}{\partial x_i}\frac{\partial W}{\partial(\partial\vec{M}/\partial x_i)} = -\frac{\partial}{\partial\vec{M}}\frac{1}{2}A\vec{M}^2 = -A\vec{M}$$

$$\vec{H}_L = -\frac{\partial}{\partial\vec{L}}\left[-\vec{L}\vec{H} + W_a(\vec{L})\right] + \frac{\partial}{\partial x_i}\frac{\partial}{\partial(\partial\vec{L}/\partial x_i)}\left[\frac{1}{8}\alpha\left(\frac{\partial\vec{L}}{\partial x_i}\right)^2\right] = \vec{H} - \frac{\partial W_a}{\partial\vec{L}} + \frac{\alpha}{4}\frac{\partial^2\vec{L}}{\partial x_i^2}$$

$$-\frac{1}{\gamma}\dot{\vec{M}} = \vec{M}\times[-A\vec{M}] + \vec{L}\times\left[\vec{H}+\vec{H}_a+\frac{\alpha}{4}\frac{\partial^2\vec{L}}{\partial x_i^2}\right] = \vec{L}\times\left[\vec{H}+\vec{H}_a+\frac{\alpha}{4}\nabla^2\vec{L}\right],\quad \vec{H}_a = -\frac{\partial W_a}{\partial\vec{L}}$$

$$-\frac{1}{\gamma}\dot{\vec{L}} = \vec{L}\times[-A\vec{M}] + \vec{M}\times\left[\vec{H}+\vec{H}_a+\frac{\alpha}{4}\nabla^2\vec{L}\right] \approx -A\vec{L}\times\vec{M} + \vec{M}\times\vec{H} \approx -A\vec{L}\times\dot{\vec{M}}$$

$\frac{1}{\gamma}\dot{\vec{L}} = A\vec{L}\times\vec{M}$, имеем $\vec{M} = a\vec{L}\times\dot{\vec{L}}$, $\frac{1}{\gamma}\ddot{\vec{L}} = A\dot{\vec{L}}\times a(\vec{L}\times\dot{\vec{L}}) = aA\,\dot{\vec{L}}\times(\vec{L}\times\dot{\vec{L}}) =$

$= aA\left[\ddot{\vec{L}}(\vec{L}\cdot\dot{\vec{L}}) - \dot{\vec{L}}(\dot{\vec{L}}\cdot\vec{L})\right] = aA(-1)\dot{L}^2L^2$, $\frac{1}{\gamma} = -aAL^2$, $a = -\frac{1}{\gamma AL^2}$

$\vec{M} = a\vec{L}\times\dot{\vec{L}} = -\frac{1}{\gamma AL^2}\vec{L}\times\dot{\vec{L}} = \frac{1}{\gamma A}\vec{\ell}\times\dot{\vec{\ell}}$ нормир. в $-\frac{1}{\gamma}\dot{\vec{M}} = (1..)$

$\dot{\vec{M}} = \frac{d}{dt}\frac{1}{\gamma A}\vec{\ell}\times\dot{\vec{\ell}} = \frac{1}{\gamma A}\frac{d}{dt}(\vec{\ell}\times\dot{\vec{\ell}}) = \frac{1}{\gamma A}\dot{\vec{\ell}}\times\dot{\vec{\ell}}$, $-\frac{1}{\gamma}\frac{1}{\gamma A}\ddot{\vec{\ell}}\times\vec{\ell} = \vec{L}\times\left[\vec{H}+\vec{H}_a+\frac{\alpha}{4}\nabla^2\vec{L}\right]$

$\vec{\ell} = \vec{L}/2M_0$, $L^2 = 4M_0^2$, $\frac{\partial}{\partial\vec{L}} = (2M_0)^{-1}\frac{\partial}{\partial\vec{\ell}}$, $\vec{H}_a = -\frac{\partial W_a}{\partial\vec{L}} = -\frac{1}{2M_0}\frac{\partial W_a}{\partial\vec{\ell}}$

$$-\frac{1}{\gamma^2A}\ddot{\vec{\ell}}\times\vec{\ell} = 2M_0\vec{\ell}\times\left[\vec{H} - \frac{1}{2M_0}\frac{\partial W_a}{\partial\vec{\ell}} + \frac{\alpha}{4}2M_0\nabla^2\vec{\ell}\right] = \vec{\ell}\times\left[2M_0\vec{H} - \frac{\partial W_a}{\partial\vec{\ell}} + \alpha M_0^2\nabla^2\vec{\ell}\right]$$

$$\vec{\ell}\times\left[\alpha M_0^2\nabla^2\vec{\ell} - \frac{1}{\gamma^2A}\ddot{\vec{\ell}} + 2M_0\vec{H} - \frac{\partial W_a}{\partial\vec{\ell}}\right] = 0,\quad \text{def. } c^2 = \gamma^2A\alpha M_0^2,\ c = \gamma M_0\sqrt{\alpha A}$$

$$\vec{\ell}\times\left[\nabla^2\vec{\ell} - \frac{1}{c^2}\ddot{\vec{\ell}} + \frac{2}{\alpha M_0}\vec{H} - \frac{1}{\alpha M_0^2}\frac{\partial W_a}{\partial\vec{\ell}}\right] = 0$$

уравнение совпадает с уравнением 30/IX 2017, $\vec{H} = \vec{H}_{SOT}$, $\vec{H}$ - term acyr. eqn.

Damping: для FM dissip. function $R = \frac{1}{2}\eta(\dot{\vec{M}})^2$, $\eta = \frac{\alpha_G}{\gamma M_s}$, $\vec{H}_{eff} = \vec{H} - \eta\dot{\vec{M}}$

$R = \frac{1}{2}\alpha_G\frac{M_s}{\gamma}(\dot{\vec{m}})^2$, $\vec{M} = M_s\vec{m}$. Для AF, $M_s \to 2M_0$, $\vec{M}\to\vec{L}$, $R_{AF} = \frac{1}{4}\frac{\alpha_G}{\gamma M_0}(\dot{\vec{L}})^2$ — как в Gomonay PRB 2010.

Gomonay LTP 40, 17 (2014) дает $R_{AF} = \frac{1}{2}G_1(\dot{\vec{M}})^2 + \frac{1}{2}G_2(\dot{\vec{L}})^2$ согласуется на Shiino PRL 2016 и уравнение SW-A с учетом только G_2 K. Holst et al. PRL 106, 107206 (2011)

и дает $\alpha_G = G_2/|\vec{L}| = \frac{1}{2M_0}G_2$, $R_{AF}(\dot{\vec{L}}) = \frac{1}{2}\alpha_G\cdot 2M_0(\dot{\vec{\ell}})^2 = \alpha_G M_0(\dot{\vec{\ell}})^2$

$R_{AF} = \frac{G_1}{2}(\dot{\vec{M}})^2 + \frac{1}{2}G_2(\dot{\vec{L}})^2$ означает $\vec{H}_M \to \vec{H}_M - \frac{G_1}{\gamma}\dot{\vec{M}}$, $\vec{H}_L \to \vec{H}_L - \frac{G_2}{\gamma}\dot{\vec{L}}$

$$-\frac{1}{\gamma}\dot{\vec{M}} = \vec{M}\times\left[-A\vec{M} - \frac{G_1}{\gamma}\dot{\vec{M}}\right] + \vec{L}\times\left[\vec{H}+\vec{H}_a+\frac{1}{4}\alpha\frac{\partial^2\vec{L}}{\partial x_i^2} - \frac{G_2}{\gamma}\dot{\vec{L}}\right] \cong \|\vec{M}\| \ll \|\vec{L}\| =$$

$$= \vec{L}\times\left[\vec{H}+\vec{H}_a+\frac{1}{4}\alpha\frac{\partial^2\vec{L}}{\partial x_i^2} - \frac{G_2}{\gamma}\dot{\vec{L}}\right],\quad -\frac{1}{\gamma}\dot{\vec{L}} = \vec{L}\times\left[-A\vec{M} - \frac{G_1}{\gamma}\dot{\vec{M}}\right] + \vec{M}\times\left[\vec{H}+\vec{H}_a+\frac{\alpha}{4}\nabla^2\vec{L} - \frac{G_2}{\gamma}\dot{\vec{L}}\right]$$

$$\approx -\vec{L}\times\left[A\vec{M} + \frac{G_1}{\gamma}\dot{\vec{M}}\right] \approx -A\vec{L}\times\vec{M},\quad \vec{M} = \frac{1}{\gamma A}\dot{\vec{\ell}}\times\vec{\ell}\ \text{сохраняется.}$$

$$-\frac{1}{\gamma}\dot{\vec{M}} = -\frac{1}{\gamma^2A}\ddot{\vec{\ell}}\times\vec{\ell} = \vec{L}\times\left[\vec{H}+\vec{H}_a+\frac{1}{4}\alpha\frac{\partial^2\vec{L}}{\partial x_i^2} - \frac{G_2}{\gamma}\dot{\vec{L}}\right],\ \text{т.е. } \vec{H}\to\vec{H} - \frac{G_2}{\gamma}\dot{\vec{L}}$$

$\frac{2}{\alpha M_0}\vec{H} \to \frac{2}{\alpha M_0}\left(\vec{H} - \frac{G_2}{\gamma}\frac{1}{|\vec{L}|}\dot{\vec{\ell}}\right) = \frac{2}{\alpha M_0}\vec{H} - \frac{2G_2}{\gamma\alpha M_0}\frac{1}{2M_0}\dot{\vec{\ell}}$, $\frac{2}{\alpha M_0}\vec{H}\to\frac{2}{\alpha M_0}\vec{H} - \frac{G_2}{\gamma\alpha M_0^2}\dot{\vec{\ell}}$

уравнение для $\vec{\ell}$: $\vec{\ell}\times\left[\nabla^2\vec{\ell} - \frac{1}{c^2}\ddot{\vec{\ell}} + \frac{2}{\alpha M_0}\vec{H} - \frac{1}{\alpha M_0^2}\frac{\partial W_a}{\partial\vec{\ell}} - \frac{G_2}{\gamma\alpha M_0^2}\dot{\vec{\ell}}\right] = 0$, $c^2 = \gamma^2M_0^2\alpha A$

Gomonay PRB 2010: $R_{AF} = \frac{1}{4}\frac{\alpha_G}{\gamma M_0}(\dot{\vec{L}})^2$, откуда $G_2 = \frac{1}{2}\frac{\alpha_G}{\gamma M_0}$, $|\vec{M}| = M_0$

$\vec{H}_r = -\frac{G_2}{\gamma}\dot{\vec{L}} = -\frac{G_2}{\gamma}2M_0\dot{\vec{\ell}} = -(\alpha_G/\gamma^2)\dot{\vec{\ell}}$, минимая γ M_0 определением,

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$R_{AF} = \frac{1}{2} G_2 (\dot{\vec{l}})^2$. Relax field $\vec{H}_2 = -\frac{\partial R_{AF}}{\partial(\dot{\vec{l}})} = -G_2 \dot{\vec{l}}$, $G_2 = \frac{1}{2}\frac{\lambda G}{\gamma M_0}$

$\vec{H}_2 = -G_2 \dot{\vec{l}} = -\frac{1}{2}\frac{\lambda G}{\gamma M_0} 2M_0 \dot{\vec{l}} = -\frac{\lambda G}{\gamma}\dot{\vec{l}}$ — вторник 30/IX 2017 г.

исчезновение $\vec{H}_L = -\nu \dot{\vec{l}}$, $\vec{H}_2 = -\nu \dot{\vec{l}}$, $\nu = G_2 = \frac{\lambda G}{2\gamma M_0}$, λG — damping (Gilbert) и LLG M_2.

$\frac{2}{\lambda M_0} c^2 = \frac{2}{\lambda M_0}\gamma^2 M_0^2 \lambda A = 2\gamma^2 M_0 A$, $\frac{1}{\lambda M_0^2} c^2 = \frac{1}{\lambda M_0^2}\gamma^2 M_0^2 \lambda A = \gamma^2 A$

$\vec{l} \times \left[c^2 \nabla^2 \vec{l} - \ddot{\vec{l}} + 2\gamma^2 M_0 A \vec{H} - \gamma^2 A \frac{\partial w_a}{\partial \vec{l}} - \frac{\lambda G \gamma A}{2M_0} \dot{\vec{l}} \right] = 0$, $\frac{G_2 c^2}{\gamma^2 M_0^2} = \frac{\lambda G}{2\gamma M_0}\gamma^2 A$

Damping term нелинейно. $\vec{H}_2 = -G_2 \dot{\vec{l}}$, $\frac{2}{\lambda M_0}\vec{H} \to \frac{2}{\lambda M_0}(\vec{H} - G_2 \dot{\vec{l}}) =$

$= \frac{2}{\lambda M_0}\vec{H} - \frac{2}{\lambda M_0} G_2 2M_0 \dot{\vec{l}} = \frac{2}{\lambda M_0}\vec{H} - \frac{2}{M_0 \alpha} 2M_0 \frac{1}{2}\frac{\lambda G}{\gamma M_0} = (\cdot) - \frac{\lambda G}{\gamma}\frac{2}{\lambda M_0}\dot{\vec{l}}$

$\frac{2}{\lambda M_0}\vec{H} \to \frac{2}{\lambda M_0}\left(\vec{H} - \frac{\lambda G}{\gamma}\dot{\vec{l}}\right)$, $c^2 \frac{2}{\lambda M_0} = 2\gamma^2 A M_0 \frac{\lambda G}{\gamma}\frac{2}{\lambda M_0} c^2 = \frac{\lambda G}{\gamma} 2\gamma^2 A = 2\lambda G \gamma A M_0$

$\vec{l} \times \left[c^2 \nabla^2 \vec{l} - \ddot{\vec{l}} + 2\gamma^2 M_0 A \vec{H} - \gamma^2 A \frac{\partial w_a}{\partial \vec{l}} - 2\lambda G \gamma A M_0 \dot{\vec{l}} \right] = 0$ — уравнение Ландау-Лифшица-Гильберта-Слончевского

Для 2-х подрешеточного АГ с $\vec{H} = \vec{H}_{SOT}$ field и damping (Слончевский с 30/IX 2017)

Зверлин '79: $W_{ex} = \frac{a}{2}(\vec{m})^2 + \ldots$, $H_E = \frac{1}{2}\frac{a}{M_0}$, $\omega_E = 2\gamma H_E = \gamma \frac{a}{M_0}$

типа 2оа: $W_{ex} = \frac{1}{2} E \vec{M}^2 + \ldots$, $H_E = E M_0$, Кошкин: $W_{ex} = \frac{1}{2} A \vec{M}^2 + \ldots$, $A = E$

Мусса $H_e = 2E M_0 = 2AM_0$, $\omega_e = \gamma H_e$.

(*) $\vec{l} \times \left[c^2 \nabla^2 \vec{l} - \ddot{\vec{l}} + \gamma \omega_e \vec{H} - \gamma^2 A \frac{\partial w_a}{\partial \vec{l}} - \lambda G \omega_e \dot{\vec{l}} \right] = 0$, $\vec{l} = \vec{l}(\theta, \Phi)$

Лагранжиан (Кошкин) $\mathcal{L} = \frac{1}{2}\frac{\lambda M_0^2}{c^2}\left[(\dot{\theta})^2 + \sin^2\theta(\dot{\Phi})^2\right] - \frac{1}{2}\lambda M_0^2\left[(\nabla\theta)^2 + \sin^2\theta(\nabla\Phi)^2\right] - W_a - W_H$

$W_a = x_z (l_z)^2 - x_y l_y^2$, $W_H \overset{SOT}{=} -\vec{l}\cdot\vec{H} = -2M_0 \vec{l}\cdot\vec{H}$, для Mn_2Au case $x_z > 0$, $x_y > 0$

$\vec{H}_{SOT} \sim \hat{z} \times \vec{j}$, $\vec{j} = j\hat{x}$, $\vec{H}_{SOT} = H\hat{y} \parallel O_y$. $l_y H_y = H \sin\theta \sin\Phi$. $\vec{l}\cdot\vec{H} = l_y H_y$

$W_a = x_z \cos^2\theta - x_y \sin^2\theta \sin^2\Phi$. Damping term $R_{AF} = \frac{1}{2} G_2 (\dot{\vec{l}})^2 = \frac{\lambda G}{\gamma} M_0 (\dot{\vec{l}})^2$

$\frac{\delta \mathcal{L}}{\delta \xi} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\xi}} = -\frac{\partial R}{\partial \dot{\xi}}$, $\mathcal{L} = \mathcal{L}_{kin} - W$, $\mathcal{L}_{kin} = \frac{1}{2}\frac{1}{\gamma^2 A}\left[(\dot{\theta})^2 + \sin^2\theta(\dot{\Phi})^2\right]$, $\xi = \theta, \Phi$.

$R_{AF} = \frac{\lambda G}{\gamma} M_0 (\dot{\vec{l}})^2 = \frac{\lambda G}{\gamma} M_0 \left[(\dot{\theta})^2 + \sin^2\theta(\dot{\Phi})^2\right]$. $\dot{\vec{l}} = \frac{d}{dt}(\sin\theta\cos\Phi\,\hat{x} + \sin\theta\sin\Phi\,\hat{y} + \cos\theta\,\hat{z})$

$\frac{\partial R}{\partial \dot{\theta}} = \frac{\lambda G}{\gamma} M_0 2\dot{\theta}$, $\frac{\partial R}{\partial \dot{\Phi}} = \frac{\lambda G}{\gamma} M_0 \sin^2\theta \cdot 2\dot{\Phi}$, $R_f = \frac{1}{2}\frac{\lambda G}{\gamma}(\dot{\vec{m}})^2 \frac{M_S^2}{M_S^2} = \frac{1}{2}\frac{\lambda G}{\gamma} M_S (\dot{\vec{m}})^2$

т.е. relax terms дают АГ, такие же как для FM $\vec{m}(\theta,\Phi)$, $\vec{l}(\theta,\Phi)$, $\frac{M_0 M_S}{|M_1| = |M_2| = M_0} \to 2M_0$.

$\frac{\partial w_a}{\partial \theta} = \frac{\partial}{\partial \theta}\left[-x_z \sin^2\theta - x_y \sin^2\theta \sin^2\Phi\right] = -\frac{\partial}{\partial\theta}\sin^2\theta\left[x_z + x_y \sin^2\Phi\right] =$

$= -2\sin\theta\cos\theta\,(x_z + x_y \sin^2\Phi)$, $\frac{\partial w_a}{\partial \Phi} = -x_y \sin^2\theta \frac{\partial}{\partial\Phi}\sin^2\Phi = -x_y \sin^2\theta \sin 2\Phi$.

$\frac{\partial w_H}{\partial \theta} = \frac{\partial}{\partial\theta}\left[-2M_0 \vec{l}\cdot\vec{H}\right] = -2M_0 H \frac{\partial}{\partial\theta} l_y = -2M_0 H \frac{\partial}{\partial\theta}\sin\theta\sin\phi = -2M_0 H \cos\theta \sin\Phi$

$\frac{\partial w_H}{\partial \Phi} = -2M_0 H \frac{\partial}{\partial\Phi}\sin\theta\sin\Phi = -2M_0 H \sin\theta\cos\Phi$, см 30/IX 2017

(**) $\begin{cases} \ddot{\theta} - c^2\nabla^2\theta - \left[(\dot{\Phi})^2 - c^2(\nabla\Phi)^2\right]\sin\theta\cos\theta + \gamma^2 A\left[-\sin 2\theta(x_z + x_y\sin^2\Phi) + 2M_0 H(\cos\theta\sin\Phi\right] = -\frac{\lambda G M_0}{\gamma}\dot{\theta} \times \gamma^2 A \\ \frac{\partial}{\partial t}\left[\sin^2\theta\dot{\Phi}\right] - c^2\,\mathrm{div}(\sin^2\theta\nabla\Phi) + \gamma^2 A\left[-x_y \sin^2\theta\sin 2\Phi - 2M_0 H\sin\theta\cos\Phi\right] = -\frac{\lambda G}{\gamma} 2M_0 \sin^2\theta\dot{\Phi}\gamma^2 A \end{cases}$

Возможны ПАГ'ки (плоские ДВ $\theta(x,t)$, $\Phi(x,t)$), где Mn_2Au с easy plane xy перпендикулярно $\nabla \to \frac{\partial}{\partial x}\hat{x}$, ДВ вдоль оси x. Пусть $\theta = \pi/2$ (т.к. x_z очень большой), уравнение Φ будет

$\ddot{\Phi} - c^2\partial_x^2\Phi + \gamma^2 A\left[-x_y \sin 2\Phi - 2M_0 H\cos\Phi\right] = -\frac{\lambda G}{\gamma} 2M_0 \gamma^2 A \dot{\Phi}$

$-x_y \sin 2\Phi - 2M_0 H_y \cos\Phi = 0$, мало $\cos\Phi = 0$, $\Phi = \pm\frac{\pi}{2}$, мало $\sin\Phi = \pm 1$, $H_y = \pm\frac{x_y}{M_0}$, $H_y \parallel x$, $H_y < x_y/M_0$.

$\ddot{\Phi} - c^2\Phi''_{xx} - \gamma^2 A x_y \sin 2\Phi - 2M_0 H\gamma^2 A\cos\Phi = -\frac{\lambda G}{\gamma} 2M_0 \gamma^2 A\dot{\Phi}$

Чтобы $H = 0$, $\lambda G = 0$ $\ddot{\Phi} - c^2\Phi''_{xx} - \gamma^2 A x_y \sin 2\Phi = 0$ — уравнение sine-Gordon с $\sin 2\Phi$-членом. Т.е. sine-Gordon с period $x_y < 0$, ОЛН $\parallel O_x$.

Но Zvezdin '79, Zvezdin '99 получили sine-Gordon для $\Phi(x,t)$ и/или $\theta = \pi/2$, чем отличаются?

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В Zvezdin'79 SG для $\Phi(x,t)$ получается при $\theta = \frac{\pi}{2}$. $W_a = \frac{1}{2}b_1 l_x^2 + \frac{1}{2}b_3 l_z^2$

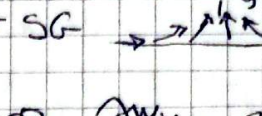
$\Phi(x,t) = 2\arctg e^{\frac{x-vt}{\Delta}}$, $\Delta^{-1} \sim w_A = \sqrt{\omega_d^2 - \omega_E\omega_1} = \left|\omega_d^{=0}_{DMI}\right| = \sqrt{-\omega_E\omega_1}$, $\omega_1 = \frac{\gamma b_1}{M_0}$

При $b_1 > 0$, ОЛН - Oy, $w_A > 0$ и Δ = imaginary, как sine-Gordon по x у меня. Если $b_1 < 0$, Δ - is real, есть SG, но ОЛН если Ox

Т.е. условия получения SG у меня и Zvezdin'79 согласуются, SG получается при DW-propagation direction (Ox) совпадает с ОЛН (ox)

Но у Zvezdin'99 (после $x \leftrightarrow y$) DW-direction есть Ox, а ОЛН $\parallel$ Oy — это за счет $\vec{M}_1(\theta,\varphi_1)$, $\vec{M}_2(\theta_2,\varphi_2)$ - 2D solution с $\theta_1 = \theta + \varepsilon$, $\theta_2 = \pi - \theta + \varepsilon$, $\varphi_1 = \varphi + \beta$, $\varphi_2 = \pi + \varphi - \beta$, $\varepsilon, \beta \ll 1$, $\theta = \pi/2 \to$ SG для $\Phi(x,t)$

Если у меня $\varkappa_y = -|\varkappa_y| < 0$, ОЛН - Ox, то будет SG → но Hy не действует на текущую DW. Надо H_x.



$W_H = -2M_0 \vec{l}\cdot\vec{H} = -2M_0 H_x l_x = -2M_0 H \sin\theta\cos\varphi$, $\frac{\partial W_H}{\partial\theta} = -2M_0 H\cos\theta\cos\varphi$, $\frac{\partial W_H}{\partial\varphi} = 2M_0 H\sin\theta\sin\varphi$

Ур-е для $\Phi(x,t)$: $\ddot{\Phi} - c^2\partial_x^2\Phi + \gamma^2 A\left[|\varkappa_y|\sin 2\Phi + 2M_0 H_x \sin\varphi\right] = -\left(\frac{d_G 2M_0}{\gamma}\right)\gamma^2 A\dot{\Phi}$.

При $H = 0, d_G = 0$, $\frac{1}{c^2}\ddot{\Phi} - \Phi''_{xx} + \frac{\gamma^2 A|\varkappa_y|}{c^2}\sin 2\Phi = 0$ - SG

$\frac{1}{c^2}\ddot{\Phi} - \Phi''_{xx} + \frac{1}{2\Delta^2}\sin 2\Phi = 0$, $\Phi(x,t) = 2\arctg\left[e^{\frac{x-vt}{\Delta(v)}}\right]$, $\Delta(v) = \Delta_0\sqrt{1 - v^2/c^2}$

$\frac{1}{2\Delta_0^2} = \frac{\gamma^2 A\varkappa_y}{c^2}$. $\Delta_0^2 = \frac{1}{2}\frac{c^2}{\gamma^2 A\varkappa_y} = \frac{1}{2}\frac{\gamma^2 M_0^2 2A}{\gamma^2 A\varkappa_y} = \frac{M_0^2 d}{2\varkappa_y}$, $W_{ex}(\vec{l}) = \frac{1}{8}d(\vec{\nabla}\vec{l})^2 = \frac{1}{2}dM_0^2(\vec{\nabla}\vec{l})^2 = A^*(\vec{\nabla}\vec{l})^2$

$\Delta_0^2 = A'/\varkappa_y$, где $A' = \frac{1}{2}dM_0^2$ ~ AF exchange stiffness.

$\Phi(x,t) = \Phi(\xi) = 2\arctg e^{\xi}$, $\Phi(-\infty) = 0$, $\Phi(+\infty) = \pi$, $\cos\Phi = \th\xi(-1)$, $\sin\Phi = \frac{1}{\ch\xi}$

Use $\Phi(x,t) = \Phi(x, q(t)) = 2\arctg e^{\left(\frac{x-q}{\Delta}\right)}$, $\Delta = \Delta_0\sqrt{1 - \dot{q}^2/c^2}$ или $\Delta(\dot{q}) = \text{const}$

$\frac{1}{c^2}\ddot{\Phi} - \Phi''_{xx} + \frac{1}{2\Delta_0^2}\sin 2\Phi + \frac{\gamma^2 A\cdot 2M_0 H_x}{c^2}\sin\Phi = -\frac{1}{c^2}\frac{2M_0 d_G}{\gamma}\gamma^2 A\dot{\Phi}$

$\frac{\gamma^2 A\cdot 2M_0 H_x}{c^2} = \frac{\gamma^2 A\cdot 2M_0 H_x}{\gamma^2 M_0^2 dA} = \frac{2H_x}{dM_0}$, $\frac{1}{c^2}\frac{2M_0 d_G}{\gamma}\gamma^2 A = \frac{1}{\gamma^2 M_0^2 dA}\frac{2M_0 d_G}{\gamma}\gamma^2 A = \frac{2d_G}{dM_0\gamma}$

$\frac{1}{c^2}\ddot{\Phi} - \Phi''_{xx} + \frac{1}{2\Delta_0^2}\sin 2\Phi + \frac{2H_x}{dM_0}\sin\Phi = -\frac{2d_G}{dM_0\gamma}\dot{\Phi}$ - без разложения SG.

$\Phi(x,t) = 2\arctg\left[e^{\frac{x-q}{\Delta}}\right]$, $\frac{d\varphi}{dt} = \frac{d\Phi}{dq}\dot{q} \ast \Phi(\xi) = 2\arctg e^{\xi}$, $\xi = \frac{x-q}{\Delta}$

$\frac{d\varphi}{dt} = \frac{d\varphi}{d\xi}\dot{\xi} = 2\frac{1}{e^{2\xi}+1}e^{\xi}\dot{\xi} = \frac{1}{\ch\xi}\dot{\xi} = \sin\Phi\dot{\xi}$, $\frac{\partial^2\Phi}{\partial t^2} = \frac{\partial}{\partial t}\dot{\Phi} = \frac{\partial}{\partial t}(\sin\Phi\dot{\xi}) =$

$= \cos\Phi\frac{d\Phi}{d\xi}(\dot{\xi})^2 + \sin\Phi\ddot{\xi} = \sin\Phi\cos\Phi(\dot{\xi})^2 + \sin\Phi\cdot\ddot{\xi}$, $\frac{d\Phi}{d\xi} = \sin\Phi$

$\frac{\partial\Phi}{\partial x} = \frac{d\Phi}{d\xi}\frac{\partial\xi}{\partial x} = \sin\Phi\cdot\xi'_x$, $\frac{\partial^2\Phi}{\partial x^2} = \frac{\partial}{\partial x}(\sin\Phi\xi'_x) = \cos\Phi\frac{d\Phi}{d\xi}(\xi'_x)^2 + \sin\Phi\,\xi''_{xx}$

$\frac{1}{c^2}\left[\sin\Phi\cos\Phi(\dot{\xi})^2 + \sin\Phi\ddot{\xi}\right] - \sin\Phi\cos\Phi(\xi'_x)^2 - \sin\Phi\,\xi''_{xx} + \frac{1}{\Delta_0^2}\sin\Phi\cos\Phi +$

$+ \frac{2H_x}{dM_0}\sin\Phi = -\frac{2d_G}{2M_0\gamma}\sin\Phi\dot{\xi}$, $\cos\Phi\left[\frac{1}{c^2}(\dot{\xi})^2 - (\xi'_x)^2 + \frac{1}{\Delta_0^2}\right] + \frac{1}{c^2}\ddot{\xi} - \xi''_{xx} + \frac{2H_x}{dM_0} = -\frac{2d_G}{\gamma dM_0}\dot{\xi}$

$\xi = \frac{x-q}{\Delta(\dot{q})}$, $\xi'_x = \frac{1}{\Delta}$, $\xi''_{xx} = 0$, $\dot{\xi} = \frac{d}{dt}\left(\frac{x-q}{\Delta}\right) = \frac{d}{dt}(x-q)\Delta - (x-q)\dot{\Delta}$

$\ddot{\xi} = -\frac{d}{dt}\frac{\dot{q}\Delta + (x-q)\dot{\Delta}}{\Delta^2} = -\frac{\frac{d}{dt}(\dot{q}\Delta)\Delta^2 + \frac{d}{dt}\left[(x-q)\dot{\Delta}\right]\Delta^2 - \left[\dot{q}\Delta + (x-q)\dot{\Delta}\right]2\Delta\dot{\Delta}}{\Delta^4} = -\frac{\dot{q}\Delta - (x-q)\dot{\Delta}}{\Delta^2} =$

$= -\frac{1}{\Delta^4}\left[(\ddot{q}\Delta + \dot{q}\dot{\Delta})\Delta^2 + (-\dot{q}\dot{\Delta} + (x-q)\ddot{\Delta})\Delta^2 - (\dot{q}\Delta + (x-q)\dot{\Delta})2\Delta\dot{\Delta}\right]$

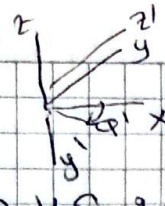
Через $\mathcal{L}(\theta = \pi/2, \varphi) = \frac{1}{2}\frac{dM_0^2}{c^2}(\dot{\Phi})^2 - \frac{1}{2}dM_0^2(\Phi'_x)^2 + (-1)|\varkappa_y|\sin^2\Phi + 2M_0 H_x\cos\Phi$

$\mathcal{L}(\Phi) = \frac{1}{2}\frac{dM_0^2}{c^2}\sin^2\Phi(\dot{\xi})^2 - \frac{1}{2}dM_0^2\sin^2\Phi(\xi'_x)^2 - |\varkappa_y|\sin^2\Phi + 2M_0 H_x\cos\Phi$

$\mathcal{L}(\Phi) = \frac{1}{2}\frac{dM_0^2}{c^2}\sin^2\Phi\left[\frac{\dot{q}\Delta + (x-q)\dot{\Delta}}{\Delta^2}\right]^2 - \frac{1}{2}dM_0^2\sin^2\Phi\frac{1}{\Delta^2} - \varkappa_y\sin^2\Phi + 2M_0 H_x\cos\Phi$

$\dot{\Delta} = \frac{\partial}{\partial t}\Delta_0\sqrt{1 - \dot{q}^2/c^2} = \Delta_0\frac{1}{2\sqrt{\ldots}}\frac{d}{dt}(\dot{q}^2/c^2) = \Delta_0\frac{1}{2\sqrt{\ldots}}\frac{1}{c^2}2\dot{q}\ddot{q} = \frac{\Delta_0}{c^2}\frac{\Delta_0}{\Delta}\dot{q}\ddot{q}$

Escaneado con CamScanner



В векторном УД (α) мишень $\frac{\partial w_a}{\partial \vec{\alpha}}$ зависит от системы координат.

$\vec{H} = H_y \hat{y} = H_{z'} \hat{z}'$ - easy field. $w_a = x_z (l_z^2) - x_y l_y^2$, $y \to z'$, $z \to -y'$

$w_a' = x_z l_{y'}^2 - x_y l_{z'}^2 = x_z \sin^2\theta' \sin^2\varphi' - x_y \cos^2\theta'$, $x_i > 0$

В УД для θ, φ, записав мишень $w_a \to w_a'$, $w_H \to w_H' = -2M_0 l_{z'} H_{z'} =$
при $xyz \to x'y'z'$ переходе и $\theta \to \theta'$, $\varphi \to \varphi'$. $= -2M_0 H \cos\theta'$,

$\frac{\partial w_a'}{\partial \theta'} = x_z 2\sin\theta'\cos\theta' \sin^2\varphi' + x_y 2\sin\theta'\cos\theta'$, $\frac{\partial w_a'}{\partial \varphi'} = x_z \sin^2\theta' 2\sin\varphi'\cos\varphi'$

$\frac{\partial w_H'}{\partial \theta'} = 2M_0 H \sin\theta'$, $\frac{\partial w_H'}{\partial \varphi'} = 0$. В УД $(\theta\varphi)$ $\frac{\partial w_a}{\partial \theta} \to \frac{\partial w_a'}{\partial \theta'}$, $\frac{\partial w_a}{\partial \varphi} \Rightarrow \frac{\partial w_a'}{\partial \varphi'}$

(**) $\begin{cases} (\theta,\varphi \to \theta',\varphi') + \gamma^2 A[x_z \sin 2\theta' \sin^2\varphi' + x_y \sin 2\theta' + 2M_0 H \sin\theta'] = \text{damp}(\theta,\varphi \to \theta'\varphi') \\ (\theta,\varphi \to \theta'\varphi') + \gamma^2 A[x_z \sin^2\theta' \sin 2\varphi' + 0] = \text{damping}(\theta,\varphi \to \theta'\varphi') \end{cases}$

Энергия в УД (**) берем $\varphi' = 0$ (strong easy plane $x'z'$) Neel DW - вращение $\vec{l}$ в $x'y'$ plane. = xy-plane, Oz' easy axis

$\varphi' = 0$ превращает 2-е уравнение (**) в тождество

1-е ур. при $\varphi' = 0$ $\ddot{\theta} - c^2\theta''_{xx} + \gamma^2 A[x_y \sin 2\theta + 2M_0 H_y \sin\theta] = -\frac{2\alpha M_0}{\gamma}\gamma^2 A \dot{\theta}$, $\theta' \to \theta$

УД для $\theta(x,t)$ - SG $\ddot{\theta} - c^2\theta''_{xx} + \gamma^2 A x_y \sin 2\theta = 0$ при $H = 0$, $\alpha = 0$, $x_y > 0$ ($\vec{\theta} \parallel H - \theta_x$)

т.е. в xyz наши мишень SG для $\Phi(x,t)$, при $\theta = \pi/2$, $x_y < 0$ ($\vec{\theta} \parallel H - \theta_y$). Не сход. с Zvezdin 1999 Comment

В $x'y'z'$ наши мишень SG для $\theta(x,t)$, при $\varphi' = 0$, $x_y > 0$ ($\vec{\theta} \parallel H - \theta_y$).

$x'y'z'$ с polar axis $Oz' \parallel \theta_y \parallel \vec{\theta} \parallel H$ in xy-plane предложили Ricardo.

26/VII

• H. Yang, PRB 100, 024407 (2019): DW propagation beyond relativistic limit AF Mn_2Au DW and SW

$w_{ex} = \frac{a}{2}\vec{m}^2 + \frac{A}{2}(\nabla\vec{n})^2$, $\vec{m} = \frac{1}{2S}(\vec{S}_a + \vec{S}_b)$, $\vec{n} = \frac{1}{2S}(\vec{S}_a - \vec{S}_b)$, $c = \gamma\sqrt{\frac{1}{2}AA}$

w_{ex} (Bar '85) $= M_0^2[\frac{1}{2}\delta\vec{m}^2 + \frac{1}{2}\alpha(\nabla\vec{l})^2]$, $a = \delta M_0^2$, $A = \alpha M_0^2$

c (Yang) $= \gamma\sqrt{\frac{1}{2}\delta M_0^2 \alpha M_0^2} = \gamma M_0 \frac{1}{\sqrt{2}}\sqrt{\alpha\delta}$, c (Bar) $= \frac{1}{2}\gamma M_0\sqrt{\alpha\delta}$, т.е.

c (Yang) $= \sqrt{2}\cdot c$ (Bar), в $\sqrt{2}$ раз больше c (Bar). В Mn_2A, $a \sim J_2$, $A \sim J_3$, а не все $\sim J$ как у Yang '19. $\vec{H}_{SF} \parallel Oz$, Oz is easy axis, т.е. расш.

случай Comonay PRL '2016, не имеющий отношение к Mn_2Au.

Формально c (Yang) $= \gamma\sqrt{\frac{1}{2}Aa} = \gamma\sqrt{\frac{1}{2}\delta JS^2 \cdot d^2 JS^2} = 2\gamma JS^2 d$, где

d - lattice period, $\vec{v}_{DW} \parallel Oz$. Утверждают, что $J(Mn_2Au) = 24$ meV

$\mathcal{H}_{ex}$ (Yang) $= J\sum \vec{S}_{ai}\cdot\vec{S}_{bj} = JS^2\sum_{ij\parallel Oz}\vec{m}_{ai}\cdot\vec{m}_{bj}$, $\mathcal{H}_{ex}(Mn_2Au) = J_2\sum_{ij\parallel Oz}\vec{m}_i\vec{m}_j$

$JS^2 = |J_2| = (532\,K)\cdot k_B = 532\,K \cdot 8{,}617\,\frac{eV}{K}\cdot 10^{-5} = 532\cdot 8{,}617\cdot 10^{-2}$ meV $= 45{,}84$ meV

как $J = 25$ meV может быть? при $S = 3/2$ будет $J = 20{,}4$ meV

c (Yang) $= 2\gamma JS^2 d = |JS^2 = |J_2|, d = a| = 2\gamma|J_2|c = 2\cdot 2\pi\frac{\gamma}{2\pi}|J_2|\,0{,}854$ nm

$|J_2| = 45{,}84$ meV $= 45{,}84\cdot 10^{-3}\cdot 1{,}60\cdot 10^{-19}$ J $= 45{,}84\cdot 1{,}60\cdot 10^{-22}\cdot 10^7$ erg $= 7{,}33\cdot 10^{-14}$ erg

c (Yang) $= 2\cdot 2\pi\cdot 2{,}8\,\frac{MHz}{Oe}\cdot 7{,}33\cdot 10^{-14}$ erg $\cdot 0{,}854\cdot 10^{-7}$ cm $\cdot \frac{erg}{Oe}\cdot cm = \frac{Oe^2}{cm^3}\frac{cm}{Oe} = \frac{Oe}{cm^4}$

Размерность c (Yang) $\frac{Oe}{cm^4}$ неверная. $a = \delta JS^2 = \delta M_0^2$,

$A = \alpha M_0^2 = \alpha J^2 S^2$, не может быть, т.к. δ - dimensionless, $[\alpha] = cm$.

Видимо надо $J \to J/V$, V - unit cell volume. Тогда limit velocity DW

c (Yang) $= 2\gamma|J_2|c$, $[\gamma\frac{J_2}{V}c] = \frac{1}{s}\frac{1}{Oe}\frac{erg}{cm^3}cm = \frac{1}{s}\frac{1}{Oe}\frac{Oe^2\cdot cm^3}{cm^3}cm = Oe\cdot\frac{cm}{s}$,

надо еще на M_s разделить. Cm. 18/VII, где $c = \frac{2\gamma}{M_s}\frac{a}{V}\sqrt{2J_3|J_2|}$

c (Yang) $= \frac{2\gamma}{M_s}\frac{|J_2|}{V}c = \frac{2\gamma}{M_s}\frac{|J_2|}{a^2}$, c (Yang) $= 4\pi\frac{2{,}8\,MHz/Oe}{0{,}78\cdot 10^3 Oe}\frac{7{,}33\cdot 10^{-14}\,erg}{(0{,}333)^2\,10^{-14}\,cm^2} =$

$= 4\pi\frac{2{,}8\cdot 10^6}{0{,}78\cdot 10^3}\frac{7{,}33}{(0{,}333)^2}\frac{cm}{s} = 2{,}98\cdot 10^3\cdot 10^3\frac{cm}{s} = 29{,}8$ km/s, а не 42km/s claimed.

Escaneado con CamScanner

Note. Two $\omega_n(k)$ simulated by Arana APL'2017 и Otxoa' Phys. Com. 2020 существенно разнятся при больших k и в величине $\omega(\pi/a)-\omega(0)$ Bandwidth

Расхождение почему? Хотя v_{gr} хорошо согласованы, 36,6 km/s (Arana), 43,2 (Otxoa) т.к. $\omega_n(k)$ близки при малых k, где $v_{gr}(k)$ имеет max. $c = \max v_{gr}(k)$.

→ При немного $x'y'z'$ ($Oz' \parallel$ by как у Ricardo), $\Phi' = 0$ (strong $xy = x'z'$ anisotropy) получим Sine-Gordon для $\Theta(x,t)$: $\ddot{\Theta} - c^2 \frac{\partial^2\Theta}{\partial x^2} + \gamma^2 A \varkappa_y \sin 2\Theta = 0$

$\frac{1}{c^2}\ddot{\Theta} - \Theta''_{xx} + \frac{1}{2\Delta_0^2}\sin 2\Theta = 0$, $\Delta_0^2 = (\alpha M_0^2/2\varkappa_y)$, $W_{ex}(\cos\vartheta) = \frac{1}{2}A\vec{m}^2 + \frac{1}{8}\alpha(\nabla\vec{L})^2$

$\vec{M}^2 = (\vec{M}_1 + \vec{M}_2)^2 = \vec{M}_1^2 + \vec{M}_2^2 + 2\vec{M}_1\vec{M}_2 = 4M_0^2 - \vec{m}^2$, $\vec{L} = 2M_0\vec{l}$, $\vec{l}^2 + \vec{m}^2 = 1$, $|\vec{M}_1| = M_0$

$W_{ex}(\cos\vartheta) = \frac{1}{2}(4M_0^2)A\vec{m}^2 + \frac{1}{8}(4M_0^2)\alpha(\nabla\vec{l})^2$

$\Theta(x,t) = 2\operatorname{arctg}\left[e^{\frac{x-vt}{\Delta(v)}}\right]$, $\Delta(v) = \Delta_0\sqrt{1 - v^2/c^2}$, $c = \gamma M_0\sqrt{\alpha A}$

Подставим $\Theta(x, q(t)) = 2\operatorname{arctg}\left[e^{\pm\frac{x-q}{\Delta}}\right]$ в полное ур. с $H \neq 0$ и $d_c \neq 0$.

$\ddot{\Theta} = \sin\Theta\cos\Theta(\dot{\xi})^2 + \sin\Theta\ddot{\xi}$, $\Theta''_{xx} = \sin\Theta\cos\Theta(\xi'_x)^2 + \sin\Theta(\xi''_{xx})$, $\frac{d\Theta}{d\xi} = \sin\Theta$

$\dot{\Theta} = \sin\Theta\dot{\xi}$, $\xi = \frac{(x-q)}{\Delta}$, $\frac{1}{c^2}\ddot{\Theta} - \Theta''_{xx} + \frac{1}{2\Delta_0^2}\sin\Theta\cos\Theta + \frac{2H}{\alpha M_0}\sin\Theta = -\frac{2d_c}{\alpha M_0\gamma}\dot{\Theta}$

$\dot{\xi} = \frac{d}{dt}\left(\frac{x-q}{\Delta}\right) = -\frac{1}{\Delta}\dot{q}$, $\ddot{\xi} = -\frac{1}{\Delta}\ddot{q}$, $\xi'_x = \frac{\partial}{\partial x}\left(\frac{x-q}{\Delta}\right) = \frac{1}{\Delta}$, $\xi''_{xx} = 0$, зависимость

$\Delta(\dot{q}) = \Delta_0\sqrt{1 - \dot{q}^2/c^2}$ пока игнорируем. $h = 2H/\alpha M_0$, $2d_c/(\alpha M_0\gamma) = \eta$

$\frac{1}{c^2}\left[\sin\Theta\cos\Theta\frac{\dot{q}^2}{\Delta^2} + \sin\Theta(-1)\frac{\ddot{q}}{\Delta}\right] - \left[\sin\Theta\cos\Theta\frac{1}{\Delta^2}\right] + \frac{1}{2\Delta_0^2}\sin\Theta\cos\Theta + h\sin\Theta = +\eta\sin\Theta\frac{\dot{q}}{\Delta}$

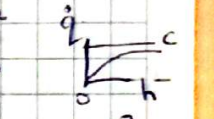
$\frac{1}{c^2}\left[\cos\Theta\frac{\dot{q}^2}{\Delta^2} - \frac{\ddot{q}}{\Delta}\right] - \cos\Theta\frac{1}{\Delta^2} + \frac{1}{2\Delta_0^2}\cos\Theta + h = \eta\frac{\dot{q}}{\Delta}$.

$-\frac{1}{c^2\Delta}\ddot{q} + \cos\Theta\left[\frac{\dot{q}^2}{c^2\Delta^2} - \frac{1}{\Delta^2} + \frac{1}{\Delta_0^2}\right] + h = \eta\frac{\dot{q}}{\Delta}$, пренебрегаем $\frac{\dot{q}^2}{c^2\Delta^2} - \frac{1}{\Delta^2} + \frac{1}{\Delta_0^2} = 0$

$\frac{1}{\Delta^2}(\dot{q}^2/c^2 - 1) = (-1)\frac{1}{\Delta_0^2}$, $\Delta^2 = \Delta_0^2(1 - \dot{q}^2/c^2)$, $\Delta(\dot{q}) = \Delta_0\sqrt{1 - \dot{q}^2/c^2}$.

$-\frac{1}{c^2\Delta}\ddot{q} - \eta\frac{\dot{q}}{\Delta} = -h$, $\frac{1}{c^2\Delta}\ddot{q} + \frac{\eta}{\Delta}\dot{q} = h$, $h = 2H\gamma/\alpha M_0$ – Sat. field.

Если $\ddot{q}$ пренебречь, то $\dot{q} = \frac{\Delta}{\eta}h$ – steady state motion, тогда $\dot{q}^2 = (h^2/\eta^2)\Delta^2 = \left(\frac{h}{\eta}\right)^2\Delta_0^2(1 - \dot{q}^2/c^2) = \beta(1 - \dot{q}^2/c^2)$, $\dot{q}^2(1 + \beta/c^2) = \beta$



$\dot{q}^2 = \frac{\beta}{1+\beta/c^2} = \frac{\beta c^2}{c^2+\beta}$, $\dot{q} = c\sqrt{\frac{\beta}{\beta + c^2}}$, $\beta = \frac{h^2}{\eta^2}\Delta_0^2$. при $\beta \gg c^2$ $\dot{q}(h)$ насыщ.

$m_{DW} = \frac{1}{c^2\Delta} = \frac{1}{c^2\Delta_0\sqrt{1 - \dot{q}^2/c^2}} = \frac{m^0_{DW}}{\sqrt{1 - \dot{q}^2/c^2}}$, $m^0_{DW} = \frac{1}{c^2\Delta_0} = \frac{1}{\gamma^2 M_0^2\alpha A}\sqrt{\frac{\alpha M_0^2}{2\varkappa_y}}$ –

DW mass density. Eq. (48) и Eq. (68) pdf-file Ricardo dated 4/2020.
Eq. (30)

$\ddot{q} + \frac{2d_c\gamma|J_2|}{M_s}\dot{q} = \frac{2\gamma^2\Delta|J_2|}{M_s}H_y^{sat}$ Сравн. с моим $\ddot{q} + \eta c^2\dot{q} = c^2\Delta h$

$\eta c^2 = \frac{2d_c\gamma^2 M_0^2\alpha A}{\alpha M_0\gamma} = 2d_c\gamma M_0 A = \frac{2d_c\gamma}{M_0}|J_2|$, сравн. при $|J_2| = AM_0^2$, V – unit cell volume. Надо $J_2 \to J_2/V$,

$c^2\Delta h = \gamma^2 M_0^2\alpha A\Delta\frac{2H}{\alpha M_0} = 2\gamma^2 M_0 AH\Delta = \frac{2\gamma^2\Delta}{M_0}|J_2|H$, $\frac{|J_2|}{V} = AM_0^2$.

Т.е. мои $\dot{q}$ eq. совпадает с eqs (48), (68), (30) Ricardo при $|J_2| = AM_0^2V$.

Ex. stiffness α is hidden in $\Delta_0^2 = \frac{\alpha M_0^2}{2\varkappa_y}$.

(27/VII) Meeting with Ruben и Ricardo concerning Mn_2Au – DW motion MS

I suggested: 1) Subm. to ARA; 2) Замечать Theory Sec. III to my $\vec{s}$-$\vec{q}$ (i.e. approach allowing avoid considering $\varepsilon(\Phi')$, just $\varepsilon = 0$, $\Phi' = 0$ strong easy plane anis.) 3) Описать в Sec. II что Mn_2Au – A-substrate AF, но мог учитываться только FM layers #3 и #5, considering that layers #2, #6 are paramagnetic 4) Найти теор. величину "c" = max v_{gr} из J_3, z, M_s (Ricardo. $J_2 \to J_2 + 4J_1$)

5) Travel exp. Ruben to HDV/EHU напрямую по "расходам"; 6). correct "K2H → 2K2H" stick.

Escaneado con CamScanner

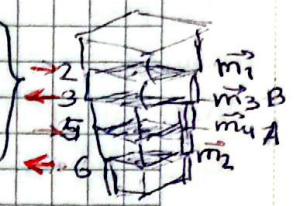
Вернемся к 4-Sublattice model 18/VII, $\vec{m}_1, \vec{m}_2, \vec{m}_3, \vec{m}_4$ вместо Mn_1 и Mn_2 — positions. $\vec{m}_1 = \vec{m}_{B1}$, $\vec{m}_4 = \vec{m}_{B2}$ Body Mn moments. Все исключения из dynamics assuming $\vec{m}_1 \uparrow\uparrow \vec{m}_3$, $\vec{m}_2 \downarrow\uparrow \vec{m}_4$. $\vec{L} = \vec{m}_1 - \vec{m}_2 - \vec{m}_3 + \vec{m}_4$ обладает свойством вектора AF $\vec{L}$, имеем static $\vec{L} = 4\vec{m}$, если $\vec{m}_1 = \vec{m}$, $\vec{m}_3 = -\vec{m}$. $\vec{m}_3 = \vec{m}_{e1}$, $\vec{m}_4 = \vec{m}_{e2}$ B – Body centered Mn atoms, e – edge centered Mn atoms

$$-\dot{\vec{m}}_1 + \dot{\vec{m}}_4 - \dot{\vec{m}}_2 - \dot{\vec{m}}_3] = 4J_1 \vec{m}_2 \times \vec{m}_3 + \vec{m}_4 \times [4J_1 \vec{m}_2 + J_2 \vec{m}_3 - 2A\partial_x^2 \vec{m}_4] -$$

$$-4J_1 \vec{m}_2 \times \vec{m}_1 - \vec{m}_3 \times [4J_1 \vec{m}_1 + J_2 \vec{m}_4 - 2A\partial_x^2 \vec{m}_3] = 8J_1 \vec{m}_1 \times \vec{m}_3 - 4J_1 \vec{m}_2 \times \vec{m}_4 -$$

$$-2J_2 \vec{m}_3 \times \vec{m}_4 + 4J_1 \vec{m}_1 \times \vec{m}_2 + 2A\partial_x^2 \vec{m}_4 \times \vec{m}_4 + 2A\vec{m}_3 \times \partial_x^2 \vec{m}_3 .$$ Если $\vec{l}$, то можем $\vec{l}$ домножить здесь крестом. Def: $\dot{\vec{m}} = \vec{m}_3/\delta$

$$\begin{cases} -\dot{\vec{m}}_1 = 4J_1 \vec{m}_1 \times \vec{m}_3 ; & -\dot{\vec{m}}_3 = \vec{m}_3 \times [4J_1 \vec{m}_1 + J_2 \vec{m}_1 - \partial_x^2 \vec{m}_3 \cdot 2A] \\ -\dot{\vec{m}}_2 = 4J_1 \vec{m}_2 \times \vec{m}_4 ; & -\dot{\vec{m}}_4 = \vec{m}_4 \times [4J_1 \vec{m}_2 + J_2 \vec{m}_3 - 2A\partial_x^2 \vec{m}_4] \end{cases}$$



Roy PRB'2016: $\dot{\vec{m}}_A = \omega_e [\vec{m}_A \times \vec{m}_B]$, $\dot{\vec{m}}_B = -\omega_e [\vec{m}_A \times \vec{m}_B]$

$\dot{\vec{m}}_4 = \omega_e [\vec{m}_4 \times \vec{m}_3]$, $\dot{\vec{m}}_3 = -\omega_e [\vec{m}_4 \times \vec{m}_3]$, $\omega_e = \frac{2\delta}{\mu_s} |4J_1 + J_2|$.

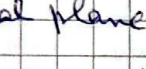
→ Если будем в коор $\dot{\vec{m}}_i$ 4D, $\vec{m}_1 = \vec{m}_4$, $\vec{m}_2 = \vec{m}_3$, то можно (для $\delta_3 \partial_x^2(\cdot)$)

$$-\dot{\vec{m}}_3 = \vec{m}_3 \times \vec{m}_4 (4J_1 + J_2), \quad -\dot{\vec{m}}_4 = \vec{m}_4 \times \vec{m}_3 (4J_1 + J_2)$$

$\vec{m}_1$ (2 слоя) $\parallel$ $\vec{m}_4$ (5 слоя) и $\vec{m}_2$ (6 слоя) $\parallel$ $\vec{m}_3$ (3 слоя), но зеро разы $\vec{m}_1 = \vec{m}_4$?? $\vec{m}_2 = \vec{m}_3$??
Это далее не доведено слои в Mn2 An unit cell. слои (2)-(5) (3)-(4) не рассматриваем. Можем только $\vec{m}_1 \parallel \vec{m}_4$, $\vec{m}_2 \parallel \vec{m}_3$ в static cofig.

$$-\dot{\vec{m}}_1 + \dot{\vec{m}}_4) = 4J_1 \vec{m}_1 \times \vec{m}_3 + \vec{m}_4 \times [4J_1 \vec{m}_2 + J_2 \vec{m}_3 - 2A\partial_x^2 \vec{m}_4]$$

$$-\dot{\vec{m}}_2 + \dot{\vec{m}}_3) = 4J_1 \vec{m}_2 \times \vec{m}_4 + \vec{m}_3 \times [4J_1 \vec{m}_1 + J_2 \vec{m}_4 - 2A\partial_x^2 \vec{m}_3] ?$$

In xy-Basal plane 

$E_{ex} = J_3 \sum_{ij} \vec{m}_i \vec{m}_j = 2J_3 \vec{m}_0 \vec{m}_1 + J_3' \vec{m}_0 \vec{m}_1 +$

$+ J_3' \vec{m}_0 \vec{m}_{x1} + J_3' \vec{m}_0 \vec{m}_{-1}$. O-Mn atoms – 4 in unit cell, но каждый из них на 1/4 принадлежит some selected unit cells. Но таких Mn-layers два per unit cell (назад × 2). $\vec{m}_1 = \vec{m}(0 + \hat{x}a) = \vec{m}(0) + \frac{\partial \vec{m}}{\partial x} a + \frac{1}{2} \frac{\partial^2 \vec{m}}{\partial x^2} a^2$, т.к. сосед. а→-а

Для одной Mn плоскости слоя (3) или (5):

$$E_{ex} = J_3' \vec{m}_0 (\vec{m}_1 + \vec{m}_{-1}) = J_3' \vec{m}_0 \left[\vec{m}(0) + \frac{\partial \vec{m}}{\partial x} a + \frac{1}{2} \frac{\partial^2 \vec{m}}{\partial x^2} a^2 + \vec{m}(0) - \frac{\partial \vec{m}}{\partial x} a + \frac{1}{2} \frac{\partial^2 \vec{m}}{\partial x^2} a^2 \right] =$$

$$= J_3' \left[2\vec{m}_0 + 2 \frac{\partial^2 \vec{m}}{\partial x^2} a^2 \right] \vec{m}_0 = 2J_3 \vec{m} \cdot \frac{\partial^2 \vec{m}}{\partial x^2} a^2 = -2J_3 a^2 \left(\frac{\partial \vec{m}}{\partial x} \right)^2$$

$$E_{ex} \text{ (на ед. } J_3) = -J_3 [\vec{m}_3 \cdot \vec{m}_3^{\,\parallel x}]_{(3 \text{ layer})} - J_3 [\vec{m}_4 \cdot \vec{m}_4^{\,\parallel x}]_{(5 \text{ layer})} = -\left[-2J_3 a^2 \left(\frac{\partial \vec{m}}{\partial x} \right)^2 - J_3 a^2 \left(\frac{\partial \vec{m}}{\partial x} \right)^2 \right] =$$

$$= 2J_3 a^2 \left(\frac{\partial \vec{m}}{\partial x} \right)^2 . \quad E_{AF} = \frac{1}{8} \alpha (\nabla \vec{L})^2 = \frac{1}{8} \alpha (2M_0)^2 (\nabla \vec{l})^2 = \frac{1}{2} \alpha M_0^2 (\nabla \vec{l})^2$$

$\vec{m}$ здесь это $\vec{m}_A = \vec{m}_4$ или $\vec{m}_B = \vec{m}_3$ намагниченности 5- или 3- подрешеток.

$\vec{l} = \vec{m}_A - \vec{m}_B$, $\vec{m} = \frac{1}{2}(\vec{m}_A + \vec{m}_B)$, $\vec{l}^2 + \vec{m}^2 = \frac{\vec{m}_A^2}{4} \cdot 2 + \frac{\vec{m}_B^2}{4} \cdot 2 = 1/2 + 1/2 = 1$, $\vec{m}_A^2 = \vec{m}_B^2 = 1$

$\vec{m}_A = \vec{l} + \vec{m}$, $\vec{m}_B = -(\vec{l} - \vec{m})$, $\left(\frac{\partial \vec{m}_A}{\partial x} \right)^2 = \left[\frac{\partial}{\partial x} (\vec{l} + \vec{m}) \right]^2 = \left[2 \frac{\partial \vec{l}}{\partial x} \frac{\partial \vec{m}}{\partial x} + \left(\frac{\partial \vec{l}}{\partial x} \right)^2 + \left(\frac{\partial \vec{m}}{\partial x} \right)^2 \right] =$

$= |$ если $\frac{\partial \vec{l}}{\partial x} \cdot \frac{\partial \vec{m}}{\partial x} = 0$ и производная единицы $\sim \vec{m}| = \left(\frac{\partial \vec{l}}{\partial x} \right)^2$

$$E_{ex} = 2J_3 a^2 \left(\frac{\partial \vec{m}_A}{\partial x} \right)^2 = 2J_3 a^2 \left(\frac{\partial \vec{l}}{\partial x} \right)^2 \text{ и } E_{AF} (\cos 90) = \frac{1}{2} \alpha M_0^2 \left(\frac{\partial \vec{l}}{\partial x} \right)^2$$

$\alpha = 4J_3 a^2 / M_0^2 V$, $c = \gamma M_0 \sqrt{\alpha A}$. $W_{AF} (\cos 90) = \frac{1}{2} A \vec{M}^2 = \frac{1}{2} (2M_0)^2 \vec{m}^2 A = 2M_0^2 A \vec{m}^2$.

$E_{ex} (J_2) = -J_2 \vec{m}_3 \vec{m}_4 = -J_2 \vec{m}_A \vec{m}_B$ per unit cell (4 слоя, AB × 4)

$E_{ex} (J_2) = -J_2 (\vec{m} + \vec{l})(\vec{m} - \vec{l}) |_{=0,1} = -J_2 (\vec{m}^2 - \vec{l}^2) = -J_2 [\vec{m}^2 - (1 - \vec{m}^2)] = -2J_2 \vec{m}^2$

$-J_2 = |J_2|$, $|J_2| = AM_0^2$ или $|J_2| = AM_0^2 V$, что и получили из совместного $\dot{q}$ eq. с $\dot{q}$ – Ricardo.

Escaneado con CamScanner

Коэффициенты 2-model, $d = \frac{4J_3 a^2}{V M_0^2}$, $A = \frac{|J_2|}{M_0^2 V}$.

$c = \gamma M_0 \sqrt{d \cdot A} = \gamma M_0 \sqrt{\frac{4J_3 a^2}{V M_0^2} \frac{|J_2|}{M_0^2 V}} = \gamma M_0 \frac{2a}{V M_0^2} \sqrt{J_3 |J_2|} = \frac{2a\gamma}{M_0 V} \sqrt{J_3 J_2}$

Эта c^* в $\sqrt{2}$ меньше c - Zvezdin'99. Zvezdin'99 в 2-раза завысил c^*, но (18/VII) (18/VII)
18/VII за занижена $d = J_3$ в 2-раза. $c = \frac{2a\gamma}{M_0 V} \sqrt{J_3 |J_2|}$ сов. определение c - Cos'90.

Что такое M_0? Модуль $|\vec{M}_i|$ двухподрешеточной модели A-B layers.

$M_0 = \frac{\mu_{Mn}}{V}$, т.к. A или B подрешетка имеет один Mn-atom per unit cell V.

$c = \frac{2a\gamma}{\mu_{Mn}} \sqrt{J_3 |J_2|}$, $c = \frac{4\pi \cdot 2{,}80 \frac{MHz}{Oe}}{4\pi \mu_B} \sqrt{\quad}$ или $\mu_{Mn} = 4\mu_B$. $\frac{2\gamma}{\mu_{Mn}} = \frac{2\gamma}{4\mu_B} = \frac{1}{\hbar}$

$\frac{\gamma}{\mu_B} = \frac{g\mu_B}{\hbar \mu_B} = \frac{g}{\hbar} \approx \frac{2}{\hbar}$, $c = \frac{a}{\hbar} \sqrt{J_3 |J_2|}$, $J_3 = 135\,K \cdot 1{,}381 \cdot 10^{-16} \frac{erg}{K} = 1{,}588 \cdot 10^{-14}$

$\hbar = 1{,}0546 \cdot 10^{-27}$ erg·s. $c = \frac{0{,}333 \cdot 10^{-7}\,cm}{1{,}0546 \cdot 10^{-27}\,erg\,s} \cdot 10^{-14} \sqrt{1{,}588 \cdot 7{,}347}\,erg = 10{,}79$ km/s, $|J_2| = 532\,K\, 1{,}381 \cdot 10^{-16} \frac{erg}{K} = 7{,}347 \cdot 10^{-14}$ erg

Но $c = 10{,}8$ km/s, мало по сравнению с $c = 43{,}2$ km/s simulated Ruben. см. c-(18/VII)

Может надо $J_2 \to (J_2 + 4J_1)$? Доп. множитель $\sqrt{(1 + 4\frac{J_1}{J_2})} = \sqrt{1 + 4 \cdot \frac{396}{532}} = 1{,}99$ - не сильно дело, будет $c = 21{,}6$ km/s. Вычисление $J_2 \to (J_2 + 4J_1)$ нет оправдания.

Arana et al. (APL'17) дает для кинетики SW bands Mn$_2$Au

$\omega_k^2/\gamma^2 = H_e(H_{a1} + H_{a2}) + H_{a1}H_{a2} - \gamma_k H_e(H_{a1} - H_{a2}) + H_e^2(1 - \gamma_k^2)$

где $H_e = 1300\,T = 13 \cdot 10^3$ kOe. $H_e = 2S_z J/g\mu_B$, видимо $J \sim J_2$.

$H_{a1} \approx 53$ kOe easy-plane anisotropy field, $H_{a2} = 0{,}072$ kOe in-plane 4-th order an. field

при $k \to 0$ $\omega_0/\gamma = \sqrt{2H_{a2}H_e}$, $\omega_0/2\pi \approx 1.2$ GHz. при $k/k_m = \frac{1}{\pi} ka \approx 0.1$ можно H_{a2} выбросить. $\omega_k^2/\gamma^2 = H_e H_{a1} - \gamma_k H_e H_{a1} + H_e^2(1 - \gamma_k^2)$

$\omega_k^2/\gamma^2 = H_e[H_{a1}(1 - \gamma_k) + H_e(1 - \gamma_k^2)]$, $\gamma_k = \cos\left(\frac{\pi}{2}\frac{k}{k_m}\right) = \cos\left(\frac{1}{2}ka\right)$.

пусть и $H_{a1} \ll H_e$, H_{a1} выбросим. $\omega_k/\gamma = H_e\sqrt{1 - \gamma_k^2} = H_e \sin\frac{1}{2}ka$

$1 - \cos^2\frac{1}{2}ka = \sin^2\frac{1}{2}ka$. $\omega_k \approx \gamma H_e \frac{1}{2}ka$, $v_{gr} = \frac{d\omega k}{dk} = \frac{1}{2}\gamma H_e a$.

при $H_e = 1{,}3 \cdot 10^4$ kOe и $a = 0{,}333$ nm, $v_{gr} = 0{,}5 \cdot 2\pi \cdot 2{,}8 \frac{GHz}{kOe} 1{,}3 \cdot 10^4\,kOe \cdot 0{,}333 \cdot 10^{-7}\,cm =$

$= \pi \cdot 2{,}8 \cdot 1{,}3 \cdot 0{,}333 \cdot 10^9 \cdot 10^4 \cdot 10^{-7} \frac{cm}{s} = 3{,}8 \cdot 10^6 \frac{cm}{s} = 38{,}1$ km/s (24/VII а оценили $v_g = 36{,}6 \frac{km}{s}$)

$v_{gr} \approx \frac{1}{2}\gamma H_e a$ определен by exch. field H_e.

$2\frac{S}{F}zJ = g\mu_B H_e = 2 \cdot 9{,}274 \cdot 10^{-21} \frac{erg}{Oe} \cdot 13 \cdot 10^4 \cdot 10^3\,Oe = 24{,}11 \cdot 10^{-14}$ erg. $|J_2| = 7{,}35 \cdot 10^{-14}$ erg.

Откуда у Arana giant H_e и больше $2S_z J \approx 3|J_2|$? $\frac{2S_z J}{|J_2|} = 3{,}28$

Если $(2S_z J)$-Arana заменить на $|J_2|$, то $v_{gr} = 38{,}1/3{,}28 = 11{,}62$ km/s, что близко к моему расчетному $c = 10{,}79$ km/s (!!!)

Arana'17 утверждает что $H_e = 1{,}3 \cdot 10^4$ kOe взято из $\chi_\perp$ measurements Barthem-2013. Barthem'2013, $\chi_\perp = 5 \cdot 10^{-4}$ at 300 K ($\perp \vec{L}$). Turov'2001, $\chi_\perp = \frac{1}{E}$, $H_e = EM_0$

$w_{ex} = \frac{1}{2}E\vec{M}^2 + \ldots$, $w_{ex}(\cos 90) = \frac{1}{2}A\vec{M}^2$, $E = A = \frac{|J_2|}{M_0^2 V}$, $\chi_\perp = \frac{1}{E} = \frac{M_0^2 V}{|J_2|}$

M_0 (Arana) = 780 G. $M_0^2 V = (0{,}78)^2 10^6\,G^2 (0{,}333\,nm)^2 (0{,}854\,nm) =$

$= (0{,}78)^2 10^6\,G^2 (0{,}333)^2 \cdot 0{,}854 \cdot 10^{-21}\,cm^3 = 57{,}6 \cdot 10^{-3} \cdot 10^6 \cdot 10^{-21} = 5{,}76 \cdot 10^{-17}$ erg

Если $\chi_\perp = 5 \cdot 10^{-4}$, то $|J_2| = \frac{M_0^2 V}{\chi_\perp} = \frac{5{,}76 \cdot 10^{-17}\,erg}{5 \cdot 10^{-4}} = 1{,}152 \cdot 10^{-13}$ erg - мало

Но Кнеллеровские дают $|J_2| = 7{,}347 \cdot 10^{-14}$ erg, в 1,568 раз меньшее значение.

$\frac{4J_1 + J_2}{J_2} = 4\frac{J_1}{J_2} + 1 = 3{,}98$. Видно $\frac{2z\frac{S}{F}J}{|4J_1 + J_2|} = 3{,}28/3{,}98 = 0{,}824 \approx 1$

Escaneado con CamScanner

Barthem'13 $\chi_\perp = \frac{M_S}{H_e}$ верно только, т.к. 

– работает не сама подрешетка, а strong easy plane anisotropy (?) at powder Mn_2Au.

$\chi(powder) = \frac{2}{3}\chi_\perp$, $\chi_\perp = \frac{3}{2}\chi = \frac{3}{2}\cdot 5\cdot 10^{-4}$, $|J_2| = \frac{M_0^2 V}{\chi_\perp} = \frac{(1)}{\frac{3}{2}\cdot 5\cdot 10^{-4}} = 7{,}68\cdot 10^{-14}$ erg,

это в хорошем согласии с $|J_2| = 7{,}35\cdot 10^{-14}$ erg by Kharchenko'08 (!)

Но $\chi = \frac{2}{3}\chi_\perp + \frac{1}{3}\chi_\parallel$ и $\chi = \frac{2}{3}\chi_\perp$ только если $\chi_\parallel \to 0$, at low temperatures.

Видно из Fig. 1 Barthem'13, что $\chi(400K) = \chi(100K)$ не зависит от T.

Откуда же брали $H_e = 1{,}3\cdot 10^4$ kOe у Arana'17 и Barthem'13 если $\chi = \frac{2}{3}\chi_\perp$ exp.

Где разница $|J_2|$?

$\chi_\perp = \frac{1}{W'}$, $W' = \frac{1}{\chi_\perp} = \frac{1}{\frac{3}{2}\chi} = \frac{1}{1{,}5\cdot 5\cdot 10^{-4}} = 1330$ – верно.

Но видимо либо Eq.(2) Barthem'13 $\frac{3}{2}NkT_N = WM_S^2 + W'M_S^2$ ошибочно, либо $\chi_\perp \neq 1/W'$. Зачем оно использовалось? Зачем $H_e = 1330 M_S = W'M_S$.

$H_e = 1330 \cdot 780\ G = 1{,}037\cdot 10^6\ G\ (Oe)$ – если use Eq.(2). $H_e = 1{,}037\cdot 10^3$ kOe.

Откуда взяли $H_e = 1{,}3\cdot 10^4$ kOe? Откуда $W'M_S^2 = N(4J_1 + J_2)k_B$?

$\frac{1}{\chi_\perp}M_S^2 = N(4J_1 + J_2)$, $\chi_\perp = \frac{M_0^2 V}{|J_2|}$ и $\chi_\perp = \frac{M_S^2}{N|4J_1 + J_2|}$ как совместить?

N – Mn atoms concentration. $N = \frac{4}{V}$, $\chi_\perp = \frac{M_S^2 V}{4|4J_1 + J_2|}$. Откуда $WM_S^2 = J_3$?

$N = \frac{4}{(0{,}533)^2\, 0{,}854\cdot 10^{-21}\ cm^3} = 42{,}2\cdot 10^{21}\ cm^{-3}$

$\frac{3}{2}k_B T_N N = N\cdot 4J_3 + N|4J_1 + J_2|$, $\frac{3}{2}T_N = 4J_3 + |4J_1 + J_2| = 4{,}115K + (4{,}396 + 5{,}32)$

$\frac{3}{2}T_N = 4(1{,}15 + 3{,}96)_K + 5{,}32\ K = 25{,}76\ K$, $T_N = \frac{2}{3}\cdot 25{,}76 = 17{,}1K$ (но не 14,35K как в статье)

Верная $\chi_\perp = \frac{M_S^2 V}{|J_2|}$, но никак не $\chi_\perp = \frac{M_S^2 V}{4|4J_1 + J_2|}$.

Надо не путать пароль с Eq.(2), а взять $|J_2| = \frac{M_0^2 V}{\chi_\perp} = \frac{M_0^2 V}{\frac{3}{2}\chi} = 7{,}68\cdot 10^{-14}$ erg

и $H_e = \frac{|J_2|}{g\mu_B} = \frac{7{,}68\cdot 10^{-14}\ erg}{2\cdot 9{,}2741\cdot 10^{-21}\ \frac{erg}{Oe}} = 0{,}414\cdot 10^4\ Oe = 4{,}14\cdot 10^3$ kOe, а не

$H_e = 1{,}3\cdot 10^4$ kOe. Тогда $v_{gr} = \frac{1}{2}\gamma H_e a = 38{,}1\ \frac{km}{s}\cdot \frac{0{,}414}{1{,}3} = 12{,}13\ \frac{km}{s}$, а не $38{,}1\ \frac{km}{s}$.

Если взять не $|J_2| = 3{,}6\cdot 10^{-14}$ erg из экс. $\chi_\perp$, а J_2 – Kharchenko'08, то $v_{gr} = 11{,}62\ km/s$

→ Вывод: $H_e = 1{,}3\cdot 10^4$ kOe и $v_{gr} \approx 38{,}1\ \frac{km}{s}$ Arana (2017) – Brance. Из $\chi_\perp$ measurements Barthem'13 следует $H_e = 4{,}14\cdot 10^3$ kOe и $v_{gr} = 12{,}13\ km/s$, ближе к моей теор. оценке $c = 10{,}79\ km/s$.

(28/VII) Arana'17: $v_{gr} = \frac{1}{2}\gamma H_e a = \frac{1}{2}\frac{g\mu_B}{\hbar}\frac{|J_2|}{g\mu_B}a = \frac{1}{2\hbar}a|J_2|$, у меня $c = \frac{a}{\hbar}\sqrt{J_3|J_2|}$

Они близки численно, если $\frac{1}{2}J_2 = \sqrt{J_3 J_2}$, $J_3 = \frac{1}{4}J_2$.

$\frac{J_3}{J_2} = \frac{115K}{532K} = 0{,}216$. Полное совпадение при $J_3/|J_2| = 0{,}25$.

v_{gr} – Arana и c – мое посчитаны в 2-подреш. модели, но Arana – 2-sublattice model (J_2 только) и моя 2-subl. model (J_2, J_3) – разные.

Совпадение v_{gr} – Arana и моей c'' – случайно. Но и v_{gr}(Arana) = 38,1 km/s никак не подтверждает верность $c = 43{,}3\ km/s$ – Ruben-sim.

При old $H_e = 1{,}3\cdot 10^4$ kOe было $K_{21} = \frac{1}{2}H_{a1}M_S = \frac{1}{2}\cdot 53\ kOe\cdot 0{,}78\cdot 10^3\ G = 20{,}67\cdot 10^6\ erg/cm^3$

Calc. Shick'2010, $K_{21} = 1{,}303\cdot 10^{-22}\ J/V$, $V = a^2 c = (0{,}333)^2\, 0{,}854\cdot nm^3 = 0{,}0947\cdot 10^{-24}\ cm^3$

$K_{21}(Shick) = \frac{1{,}303\cdot 10^{-15}\ erg}{0{,}947\cdot 10^{-22}\ cm^3} = 1{,}376\cdot 10^7\ \frac{erg}{cm^3}$, $K_{21}^{old}(Arana) = 2{,}067\cdot 10^7\ erg/cc$

$(\omega_2/\gamma)^2 = 2H_{a1}H_e$, $H_{a1} = \frac{\omega_2^2}{\gamma^2}\frac{1}{2H_e}$. Уменьшим H_e с $1{,}3\cdot 10^4$ kOe to $4{,}14\cdot 10^3$ kOe

увеличим H_{a1} с 53,0 kOe to $53\cdot \frac{13{,}0}{4{,}14} = 166{,}4$ kOe, что $\approx$ соответ. $K_{21} = 25K$ [Barth'13]

Escaneado con CamScanner

K_{21} (Shick) $= 1{,}9548 \cdot 10^{-22}$ J $= \frac{1}{V} \frac{1{,}8542 \cdot 10^{-22} \text{ J} \cdot k_B}{1{,}381 \cdot 10^{-23}} = \frac{1}{V} 13{,}4 \text{ K} \cdot k_B$, тогда как Bartheim'13 даёт $K_{21} \cong 25 \text{ K} \cdot k_B / V$ · V — unit cell volume.

AF DW velocity scale in Mn_2Au is $v_0 = \frac{a}{\hbar} \sqrt{J_3 |J_2|} = 10{,}79$ km/s.

Можно записать $c = \lambda v_0$, где λ зависит от числа ближ. соседей, определенных M_0 и J_2^{eff} при переходе к 2-Sublattice σ-model. При $\lambda = 4$ (Ricardo), получим $c = 43{,}16$ km/s сбл. to $c(\text{sin.}) = 43{,}3 \frac{\text{km}}{\text{s}}$

Переписка с Оксаной по новому symmetry of Mn_2Au lattice и exch. integrals необходимых для описания layered AF Mn_2Au. Спросим Ruben about exchange in (2), (6) Mn-body planes J_4. Orbes: $J_4 = J_3$

(29/VII) переписка с Оксаной и S Khmelevskyi об exch. integrals Mn_2Au. Написал Ricardo и Ruben об $v_0 = \frac{a}{\hbar} \sqrt{J_3 |J_2|}$ и $v_{max} = B \cdot v_0$ def., где B зависит от числа ближ. соседей $J_2 \to J_2^{eff}$ и от определения M_0 of Mn_2Au при сведении к 2-Sublat model

(30/VII) Написал Ruben о различии v_{max} ($J_1 J_2 J_3$) model и simulated и с ($J_1 J_2 J_3 J_4$)-model. Предложил to-do simulations putting $J_4 = 0$ и $J_4 = 0, J_1 = 0$

Это не нужно, т.к. $J_4 = J_3$ обосновал Khmelevskyi:

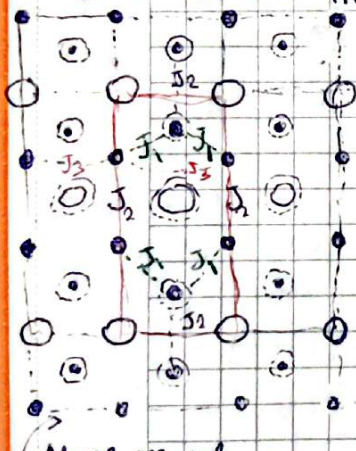
(31/VII) Сообщил из объяснений Khmelevskyi, что в Mn_2Au будет 4 Mn planes (1, 2, 3, 4). Причем [1], [3] — одинаковы, planes [2], [4] одинаковы. [1] переходит в [4], а [2]-plane переходит в [3] plane при инверсии $\vec{r} \to -\vec{r}$ отн. центра unit cell. На inversion на exch. integrals $J_1 J_2 J_3$ величины не влияет. Поэтому $J_4[1]$, $J_4[4] = J_3[2]$, $J_3[3]$, $J_4 = J_3$. Магнитная структура $\vec{m}^n_{Mn}$ directions на J_i уже не влияет. Написал о вопросе Simul. model $J_4 = J_3$ to Ruben.

(1/VIII) Explanations of crystal and magnetic symmetry of Mn_2Au to Oxc и Ruben. Вернемся к 4-sublattice model 18/VII, see also 27/VII. Mn_2Au — v_{max}

$$\frac{E_{ex}}{\text{unit cell}} = -4J_1 \vec{m}_1 \cdot \vec{m}_3 - 4J_1 \vec{m}_2 \cdot \vec{m}_4 - J_2 \vec{m}_3 \cdot \vec{m}_4 + A_3 \sum_i \left(\frac{\partial \vec{m}_i}{\partial x}\right)^2, \quad i = 1,2,3,4$$

т.к. сказал, что J_4 is $\vec{m}_1 = \vec{m}_{B1}$, $\vec{m}_2 = \vec{m}_{B2}$ - planes, ранее 18/VII $i = 3,4$, сейчас $J_4 = J_3$.

Исходя как 18/VII сказал $\vec{m}_1, \vec{m}_2$ — slave variables $\vec{m}_1 \downarrow\uparrow \vec{m}_3$, $\vec{m}_2 \downarrow\uparrow \vec{m}_4$, и исключить $\vec{m}_1, \vec{m}_2$ из ЛД.



● — Mn, ○ — Au, ◉, ○, — Mn и Au в глубине экрана, ∥ c axis

— unit cell

Число J_2 bonds: (4 на ребрах u.c. ●—○) $\times \frac{1}{4}$ +
(1) Mn planes
(2) + (2 bonds ◉—○) $\times \frac{1}{2} = 1 + 1 = 2$ bonds J_2/unit cell

Число J_1 - bonds: 4 для одной пары planes (2)-(3) +
(3)
(4) + 4 другие пары (5)-(6) planes ○—◉ $= 4 + 4 = 8$ J_1 bonds/unit cell

$$\frac{E_{ex}}{\text{u.c.}} = -4J_1 \vec{m}_1 \cdot \vec{m}_3 - 4J_1 \vec{m}_2 \cdot \vec{m}_4 - J_2 \vec{m}_3 \cdot \vec{m}_4 - J_2 \vec{m}_1 \cdot \vec{m}_2 + \frac{A}{3} \sum_i \left(\frac{\partial \vec{m}_i}{\partial x}\right)^2$$

Если взять в приближении Roy. APB'16: $\vec{m}_1 = \vec{m}_4$ (B1) (e2), $\vec{m}_2 = \vec{m}_3$ (B2) (e1)

Mn_2Au xz-plane ↳ то $\frac{E_{ex}}{\text{u.c.}} = -4J_1 \vec{m}_3 \cdot \vec{m}_4 \cdot 2 - 2J_2 \vec{m}_3 \cdot \vec{m}_4 + 2A_3 \left(\frac{\partial \vec{m}_3}{\partial x}\right)^2 + 2A_3 \left(\frac{\partial \vec{m}_4}{\partial x}\right)^2$

(1/2)

$$\frac{E_{ex}}{\text{u.c.}} = -2(4J_1 + J_2)\vec{m}_3 \cdot \vec{m}_4 + 2A_3 \left(\frac{\partial \vec{m}_3}{\partial x}\right)^2 + 2A_3 \left(\frac{\partial \vec{m}_4}{\partial x}\right)^2, \quad \vec{m}_3 = \vec{m}_B, \ \vec{m}_4 = \vec{m}_A \text{ Ricardo}$$

$$\frac{E_{ex}}{\text{u.c.}} = -2(4J_1 + J_2)\vec{m}_A \cdot \vec{m}_B + 2A_3 \left(\frac{\partial \vec{m}_A}{\partial x}\right)^2 + 2A_3 \left(\frac{\partial \vec{m}_B}{\partial x}\right)^2 \quad \text{Введем } \vec{l} = \frac{1}{2}(\vec{m}_A - \vec{m}_B)$$

$$\vec{m} = \frac{1}{2}(\vec{m}_A + \vec{m}_B)$$

Escaneado con CamScanner

$\vec{m}_A = (\vec{m}+\vec{l})$, $\vec{m}_B = \vec{m}-\vec{l}$, $\vec{m}^2+\vec{l}^2=1$. $\vec{m}_A\cdot\vec{m}_B = (\vec{m}+\vec{l})(\vec{m}-\vec{l}) = \vec{m}^2-\vec{l}^2 = 2\vec{m}^2-1 =$

$= 1-\vec{l}^2-\vec{l}^2 = -2\vec{l}^2+1$, $\left(\frac{\partial \vec{m}_A}{\partial x}\right)^2+\left(\frac{\partial \vec{m}_B}{\partial x}\right)^2 = \left[\frac{\partial}{\partial x}(\vec{m}+\vec{l})\right]^2+\left[\frac{\partial}{\partial x}(\vec{m}-\vec{l})\right]^2 =$

$= (\vec{m}')^2+(\vec{l}')^2+2\vec{m}'\cdot\vec{l}'+\vec{m}'^2+\vec{l}'^2-2\vec{m}'\cdot\vec{l}' = 2\vec{l}'^2+2\vec{m}'^2$

если $\vec{m}'^2 = \left(\frac{\partial \vec{m}}{\partial x}\right)^2 \ll \vec{l}'^2 = \left(\frac{\partial \vec{l}}{\partial x}\right)^2$, то $\left(\frac{\partial \vec{m}_A}{\partial x}\right)^2+\left(\frac{\partial \vec{m}_B}{\partial x}\right)^2 = 2\left(\frac{\partial \vec{l}}{\partial x}\right)^2$

$\frac{E_{ex}}{u.c.} = -2(4J_1+J_2)(+2)\vec{m}^2 + 2A_3\cdot 2\left(\frac{\partial \vec{l}}{\partial x}\right)^2 = 4|4J_1+J_2|\vec{m}^2 + 4A_3\left(\frac{\partial \vec{l}}{\partial x}\right)^2$

Сом. 25/VII W_{ex} (2-model) $= \frac{1}{2}A\vec{M}^2 + \frac{1}{8}\alpha\left(\frac{\partial \vec{L}}{\partial x_i}\right)^2$, $\vec{L} = 2M_0\vec{l}$, $\vec{M} = 2M_0\vec{m}$

$\vec{L} = \vec{M}_1 - \vec{M}_2$, $\vec{M} = \vec{M}_1+\vec{M}_2$

$W_{ex}(2) = \frac{1}{2}A\,4M_0^2\vec{m}^2 + \frac{1}{8}\alpha\cdot 4M_0^2\left(\frac{\partial \vec{l}}{\partial x}\right)^2 = 2AM_0^2\vec{m}^2 + \frac{1}{2}\alpha M_0^2\left(\frac{\partial \vec{l}}{\partial x}\right)^2$

Сравнивая $\frac{E_{ex}}{u.c.}$ и exch. density W_{ex}, получим $4|4J_1+J_2| = 2AM_0^2$, $4A_3 = \frac{1}{2}\alpha M_0^2$

$A = \frac{2}{M_0^2}|4J_1+J_2|$, $\alpha = \frac{8A_3}{M_0^2}$, $c = \gamma M_0\sqrt{\alpha A} = \gamma M_0\sqrt{\frac{8A_3}{M_0^2}\cdot\frac{2}{M_0^2}|4J_1+J_2|} =$

$= 4\frac{\gamma}{M_0}\sqrt{A_3|4J_1+J_2|}$. Совпад в MS-Ricardo $v_{max} = \frac{2\gamma}{M_0}\sqrt{|J_2^{eff}|A_3}$. Одинаковые ли A_3 у меня и Ricardo?

Ebert no 27/VII $A = \frac{|J_2^{eff}|}{M_0^2}$, $\alpha = \frac{4J_3a^2}{VM_0^2}$.

A_3 выразим через J_3. $\frac{E_{ex}}{u.c.} = A_3\sum_i\left(\frac{\partial \vec{m}_i}{\partial x}\right)^2$, $i = 1,2,3,4$ по всем Mn - layers в ячейке

Для одного Mn-layer $E_{ex} = -J_3\sum_j \vec{m}_j\cdot\vec{m}_{j+1} = -J_3\vec{m}_0\cdot(\vec{m}_1+\vec{m}_{-1}) = J_3a^2\left(\frac{\partial \vec{m}}{\partial x}\right)^2$ - 27/VII

С другой стороны для одного Mn-layer $\frac{E_{ex}}{u.c.} = A_3\left(\frac{\partial \vec{m}}{\partial x}\right)^2$, т.е. $A_3 = J_3a^2$.

$\alpha = \frac{8A_3}{M_0^2} = \frac{8}{M_0^2}J_3a^2$ (Ebert $\alpha = \frac{4J_3a^2}{VM_0^2}$ 27/VII). т.е. A,α (2-model) удвоились и $J_2 \to J_2^{eff}$ по сравн. с расчетом 27/VII.

$c = \frac{4\gamma}{M_0}\sqrt{A_3|4J_1+J_2|} = \frac{4\gamma}{M_0}\sqrt{J_3a^2|4J_1+J_2|} = \frac{4\gamma}{M_0}a\sqrt{\left(4\frac{J_1}{J_2}+1\right)}\sqrt{J_3|J_2|}$. $J_i \to J_i/V$.

Сом. 27/VII $M_0 = \frac{\mu_{Mn}}{V}$, т.к. A или B подрешетка имеет один Mn/u.c. $M_0 = |\vec{M}_A| = |\vec{M}_B|$

$\mu_{Mn} = 4\mu_B$, $\frac{\gamma}{M_0V} = \frac{g\mu_B}{\hbar}\frac{1}{\mu_{Mn}} = \frac{2\mu_B}{\hbar}\frac{1}{4\mu_B} = \frac{1}{2}\frac{1}{\hbar}$

$c = 4\frac{\gamma}{M_0V}a\left(4J_1/J_2+1\right)^{1/2}\sqrt{J_3|J_2|} = 4\frac{1}{2\hbar}a\left(4J_1/J_2+1\right)^{1/2}\sqrt{J_3|J_2|} = 2\left(\frac{4J_1}{J_2}+1\right)^{1/2}v_0$

$c = Bv_0$, $B = 2\left(\frac{4J_1}{J_2}+1\right)^{1/2} = 3.989$, $v_0 = \frac{a}{\hbar}\sqrt{J_3|J_2|} \approx 10.79$ km/s.

Mn2Ru2Sn $c = 2\left(\frac{4J_1}{J_2}+1\right)^{1/2}\frac{a}{\hbar}\sqrt{J_3|J_2|}$, v_{max} численно $c = 43.04\ \frac{km}{s}$ в отлич. согласии с Simulated $c = 43.3\ \frac{km}{s}$

Уточним $v_0 = \frac{a}{\hbar}\sqrt{J_3|J_2|} = a\frac{k_B}{\hbar}\sqrt{J_3J_2(K)} = 0,3328\cdot 10^{-7}\ cm\ \frac{1,381\cdot 10^{-16}\ erg/K}{1,0546\cdot 10^{-27}\ erg\cdot s}\sqrt{115.532} =$

$= 0,3328\ \frac{1,381}{1,0546}\cdot 10^6\sqrt{1,15\cdot 4,32}\ \frac{cm}{s} = 1,0779\cdot 10^6\ \frac{cm}{s} = 10,779\ \frac{km}{s}$

$c = 3,989\times 10,779\ km/s = 42,999\ km/s = 43,0\ km/s$ (calculated, 2-model).

(!) Удивительно, 2-model получилась из 4-sublattice model и $\vec{m}_{A1} = \vec{m}_{B2}$, $\vec{m}_{B2} = \vec{m}_{B1}$, равенства $\vec{m}_i$ [1] и [3], [2] и [4] Mn planes дает очень хорошее v_{max}, один в один simulations. И [1][3] и [2],[4] Mn-planes тоже хорошее приближение друг друга.

A_3 мне упр. $\frac{E_{ex}}{u.c.} = 2A_3\left[\left(\frac{\partial \vec{m}_A}{\partial x}\right)^2+\left(\frac{\partial \vec{m}_B}{\partial x}\right)^2\right] = 4A_3\left(\frac{\partial \vec{l}}{\partial x}\right)^2$, $A_3 = J_3a^2$

A_3 (Ricardo) $= A_3'$: $\frac{E_{ex}}{V} = A_3'\left(\frac{\partial \vec{m}}{\partial x}A\right)^2 = A_3'\left(\frac{\partial \vec{l}}{\partial x}\right)^2$, $\vec{l}_1 = \frac{1}{2}(\vec{m}_A-\vec{m}_B) = \vec{m}_A$ по

of one A(S) layer c) (3) или (5)-(6) layers

у меня E_{ex} of one Mn-layer $E_{ex}/u.c. = A_3\left(\frac{\partial \vec{m}}{\partial x}\right)^2$ и E_{ex} of 4-Mn layers

$E_{ex}/u.c. = 2A_3\left[\left(\frac{\partial \vec{m}_A}{\partial x}\right)^2+\left(\frac{\partial \vec{m}_B}{\partial x}\right)^2\right] = 4A_3\left(\frac{\partial \vec{l}}{\partial x}\right)^2$

Escaneado con CamScanner

Видно что для Mn layer $A_3 = A_3' = J_3 a^2$, то в 2-model Lagrangian входит $E_{ex}/u.c. = 4A_3 \left(\frac{\partial \vec{x}}{\partial x}\right)^2$ у меня и $\frac{E_{ex}}{u.c.} = A_3' \left(\frac{\partial \vec{x}_i}{\partial x}\right)^2$ т.е. $A_3' = 4A_3$. v_{max} (Ricardo) $= \frac{2\beta}{M_0} \sqrt{|J_2^{eff}| A_3'} = \frac{4\beta}{M_0} \sqrt{|J_2^{eff}| A_3}$ у Ricardo

$J_2^{eff} = 4J_1 + J_2$. $A_3' = 4 J_3 a^2$.

Число J_3 bonds along ∂x): $\left[\underset{(3)}{2} + \underset{(5)}{2}\right)\left(\frac{1}{4}\right)^0 + \left[\underset{(2)}{2} + \underset{(6)}{2}\right] \cdot \frac{1}{2} = 4 \cdot 2 = 6$

но A_3 входит иначе:

$E_{ex}/u.c. = A_3 \sum_i \left(\frac{\partial \vec{m}_i}{\partial x}\right)^2$, $i = 1,2,3,4$ и даже (2) и (5) - Mn слоев

$E_{ex}^{(j)}/V = A_3 \left(\frac{\partial \vec{m}_j}{\partial x}\right)^2 = -J_3 \left[\vec{m}_0 \cdot (\vec{m}_1 + \vec{m}_{-1})\right] \times \frac{1}{4} = +J_3 \left[4a^2 \left(\frac{\partial \vec{m}_j}{\partial x}\right)^2\right] \frac{1}{4} = J_3 a^2 \left(\frac{\partial \vec{m}_j}{\partial x}\right)^2$

даже каждого Mn из j-layer, V_4 с-ти атома на u.c.

$\vec{m}_0$ на Mn-layer берется и рядом, но вес каждого атома/per u.c. $= \frac{1}{4}$. (#3 и #5)

Response на BP 3D topol. charge Oksana y Jose Roldan sent me (y них $Q_H = 0$ для hedgehog и vortex like Bloch points). Проверить.

(2/VIII) Пример Julian Zapper Q-B+ Ruben/Ricardo о вычислении M_s Mn_2Au within 2-model. Robuttal AF DW Mn_2Au" to recubin. to Phys. Commun. Lett. "

(3/VIII) Editing robut Lett. Commun. Phys. и MS „Walker-like DW breakdown in layred antiferromagnets..." text - носчан to Ruben. Long e-mail from Ricardo on $(J_1 J_2 J_3)$ - 2 model of AF DW Mn_2Au dynamics — отвечен

(4/VIII) Проверка 4-х книг по solitons sent me by Ruben. Три из них - - M. Nakahara „Geometry, topology and physics" 2003, N. Manton, P. Sutcliffe „Topological Solitons" 2004, M. Danajski „Solitons, instantons and twistors" 2010 имеет вз PDF files через libgen. T. Vojang „Solitons in Field Theory" разрешил Ruben'y заказать через мой Syngrmion project.

Имеет и сканировал 2 книги „Micro-, nanowires" Ed. M. Vazquez, 2015 и 2020 „Magnetic" в общ с ch. by S. Jamet „DW and Bloch points" - BP classification.

(5/VIII) С А. в „Etxeski" Urbil, „Carrefour Malizet", Tolosa. перепился с Ruben. Рассм. вычисление A_3 - stiffness again.

$E_{ex} = -J_3^x \sum_j \vec{m}_j \cdot \vec{m}_{j+1} = -J_3^x \vec{m}_1 \cdot \vec{m}_2 - J_3^x \vec{m}_3 \vec{m}_4 = -2J_3^x \vec{m}_1 \cdot \vec{m}_2 = -2J_3^x \vec{m}_1 \cdot \vec{m}_2$



xy - basal plane of Mn-atoms + unit cell

xy это (3)(5) planes

$\vec{m}_2 = \vec{m}_1 + a\frac{\partial \vec{m}_1}{\partial x} + \frac{1}{2}a^2 \frac{\partial^2 m_1}{\partial x^2}$ → $E_{ex} = -2J_3 \vec{m}_1 \cdot \left[\vec{m}_1 + a\frac{\partial \vec{m}_1}{\partial x} + \frac{1}{2}a^2 \frac{\partial^2 \vec{m}_1}{\partial x^2}\right] =$

$= \frac{1}{2}\left[(-a) + (+a)\right] = -2J_3 \vec{m}_1 \left[\vec{m}_1 + \frac{1}{2}a^2 \frac{\partial^2 \vec{m}_1}{\partial x^2}\right] = -2J_3 \vec{m}_1 \cdot \frac{1}{2}a^2 \frac{\partial^2 \vec{m}_1}{\partial x^2} = -J_3 \vec{m} \cdot \frac{\partial^2 \vec{m}}{\partial x^2} a^2$

$E_{ex} = J_3 a^2 \left(\frac{\partial \vec{m}}{\partial x}\right)^2$, с другой стороны definition A_3 in i-layer $E_{ex} = A_3 \left(\frac{\partial \vec{m}_i}{\partial x}\right)^2$

т.е. $A_3 = J_3 a^2$. Если рассм. body Mn-planes



то для central Mn atom $E_{ex} = -J_3 \vec{m}_0 \vec{m}_{+1} - J_3 \vec{m}_0 \cdot \vec{m}_{-1} =$

$= -J_3 \vec{m}_0 \left[\vec{m}_{+1} + \vec{m}_{-1}\right] = -J_3 \vec{m}_0 \left[\vec{m}_0 + a\frac{\partial \vec{m}_0}{\partial x} + \frac{1}{2}a^2 m'' \; \vec{m}_0 + m_0 - a\frac{\partial \vec{m}_0}{\partial x} + \frac{1}{2}a^2 m''\right] =$

$= -J_3 \vec{m}_0 \left[2\vec{m}_0 + a^2 \frac{d^2 \vec{m}_0}{dx^2}\right] = -J_3 a^2 \frac{d^2 \vec{m}_0}{dx^2} \cdot \vec{m}_0 = J_3 a^2 \left(\frac{d\vec{m}}{dx}\right)^2$ и $A_3 = J_3 a^2$ снова

$\frac{E_{ex}}{u.c.} = 2A_3 \left[\left(\frac{\partial \vec{m}_A}{\partial x}\right)^2 + \left(\frac{\partial \vec{m}_B}{\partial x}\right)^2\right] = 4A_3 \left(\frac{\partial \vec{l}}{\partial x}\right)^2$, $\vec{l} = \frac{1}{2}(\vec{m}_A - \vec{m}_B)$.

Eq. (14) Ricardo May 4-like for A-Mn layer: $E_A = A_3 \left(\frac{\partial \vec{m}_A}{\partial x}\right)^2 - |J_2| \vec{m}_A \cdot \vec{m}_B$

Для пары A-B Mn-слоев $E = -2|J_2| \vec{m}_A \cdot \vec{m}_B + 2A_3 \left[\left(\frac{\partial \vec{m}_A}{\partial x}\right)^2 + \left(\frac{\partial \vec{m}_B}{\partial x}\right)^2\right]$, надо → to have AF A-B coupling. Тогда $E_{ex}/u.c. = 2|4J_1 + J_2| \vec{m}_A \cdot \vec{m}_B + 2A_3 [\ldots]$

Escaneado con CamScanner

Видно $2|J_2|$ сокращ. $2|4J_1+J_2|$, тогда $J_2 \to J_2^{eff} = 4J_1 + J_2$, а не $J_{2eff} = 2|4J_1+J_2|$ как в Ricardo et al. Aug. 3,

$\nu_{max}(Ricardo) = \frac{2\gamma}{M_0}\sqrt{|J_2^{eff}|A_3} = \frac{2\gamma}{M_0}\sqrt{(4J_1/J_2+1)|J_2|4A_3} = |A_3 = J_3 a^2| =$

$= \frac{2\gamma}{M_0}\sqrt{(4J_1/J_2+1)}\cdot 2a\sqrt{J_2 J_3} = \frac{4\gamma a}{M_0}\sqrt{\left(\frac{4J_1}{J_2}+1\right)}\frac{1}{\sqrt{2}}\sqrt{J_2 J_3} = \frac{4\gamma a}{M_0 V}\sqrt{\left(\frac{4J_1}{J_2}+1\right)}\sqrt{J_2 J_3}$.

$\frac{4\gamma}{M_0 V} = \frac{4\gamma}{\mu_B n} = \frac{4\gamma}{4\mu_B} = \frac{\gamma}{\mu_B} = \frac{g\mu_B}{\hbar}\frac{1}{\mu_B} = \frac{g}{\hbar} = \frac{2}{\hbar}$, $\nu_{max} = \frac{2a}{\hbar}\sqrt{\left(\frac{4J_1}{J_2}+1\right)}\sqrt{|J_2|J_3}$ —

Вопрос ν_{max}, нормировано $1/\sqrt{2}$

$\frac{F_{ex}}{V_{uc}} = 4|4J_1+J_2|\vec{m}^2 + 4A_3\left(\frac{\partial\vec{l}}{\partial x}\right)^2$, $\vec{l} = \frac{1}{2}(\vec{m}_A - \vec{m}_B)$, $\vec{m} = \frac{1}{2}(\vec{m}_A + \vec{m}_B)$,

A, B — 1-ая и 3-ая AFM слои. $A_3 = a^2 J_3$

В модели $W_{ex} = \frac{1}{2}A\vec{M}^2 + \frac{1}{2}\alpha\left(\frac{\partial\vec{L}}{\partial x_i}\right)^2 = 2AM_0^2\vec{m}^2 + \frac{1}{2}\alpha M_0^2\left(\frac{\partial\vec{l}}{\partial x_i}\right)^2$, откуда

(Kosevich'90) $A = \frac{2}{M_0^2}|4J_1+J_2|$ и $\alpha = \frac{8A_3}{M_0^2}$. $c = \gamma M_0\sqrt{\alpha A} = \frac{4\gamma a}{M_0}\sqrt{J_3|4J_1+J_2|}$

Сам $M_0 V = \mu_{mn} = 4\mu_B$, $J_i \to J_i/V$, V — unit cell volume, то $c = \frac{2a}{\hbar}\sqrt{\left(4\frac{J_1}{J_2}+1\right)}\sqrt{|J_2|J_3}$

Используя J_1, J_2, J_3 из Khmelevskyi '2008 получаем $\Theta = 43.0$ meV/f.u. для Mn_2Au.

Намагниченность $\vec{M} = \frac{1}{\delta A}\vec{l}\times\dot{\vec{l}}$ получаем из ЛЛ для $\vec{l}$ (25/VII):

(1) $\ddot{\vec{l}} \times [c^2\Delta\vec{l} - \vec{l}\times + 2\gamma^2 M_0 A\vec{H} - \gamma^2 A\frac{\partial w_a}{\partial\vec{l}} - 2\alpha\gamma A M_0\dot{\vec{l}}] = 0$, $\vec{H}$ — вн. field.

Лагранжиан $\mathcal{L} = \frac{1}{2}\frac{\alpha M_0^2}{c^2}(\dot{\vec{l}})^2 - \frac{1}{2}\alpha M_0^2(\nabla\vec{l})^2 - W_a - W_H = \mathcal{L}_{kin} - W$. Mn_2Au

$W_a = -\vec{L}\cdot\vec{H} = -2M_0\vec{l}\cdot\vec{H}$, $\vec{l} = \vec{l}(\theta,\varphi)$ даст систему ЛЛ для θ, φ.

Note: в ЛЛ для $\vec{l}$ нам $\Theta\Phi$ A (uniform AF exchange) входит явно, а α (non-uniform exchange) спрятан в $c^2 = \gamma^2 M_0^2 \alpha A$.

$\mathcal{L}_{kin} = \frac{1}{2}\frac{\alpha M_0^2}{c^2}(\dot{\vec{l}})^2 = \frac{1}{2}\frac{1}{\gamma^2 A}(\dot{\vec{l}})^2$. Используем систему координат $\vec{l}(\theta', \varphi')$ с Oz' in-plane of Mn_2Au unit cell $Oz' \| Oy$ и ОЛН-ey. Берем $\varphi' = 0$ — strong easy plane $x'z'$. $W_a' = x_z l_{y'}^2 - x_y l_{z'}^2$, $x_z, x_y > 0$, $x_z \gg x_y$.

Нормир. eq. для $\theta'(x,t)$. Доб. $\theta' = \theta$ (26/VII):

$\frac{1}{c^2}\ddot{\theta} - \frac{\partial^2\theta}{\partial x^2} + \frac{1}{\Delta_0^2}\sin\theta\cos\theta + \frac{2H}{\alpha M_0}\sin\theta = -\frac{2\alpha\epsilon}{\alpha M_0\gamma}\dot{\theta}$, $\Delta_0^2 = \frac{\alpha M_0^2}{2\pi_y}$, $H = H_y$ Stat

DW mass

Логарифм $\theta(x,t) = \theta[\xi, q(t)] = 2\arctg[\exp(\frac{x-q}{\Delta})]$ и ур-ие для $\theta(q,t)$ дает:

$\frac{1}{c^2\Delta}\ddot{q} + \frac{\gamma}{\Delta}\dot{q} = h$, $\Delta = \Delta_0\sqrt{1-\dot{q}^2/c^2}$, $\gamma = \frac{2\alpha\epsilon}{\alpha M_0\gamma}$, $h = \frac{2H}{\alpha M_0}$. $[\alpha] = cm^2$.

Standard Döring eq. (Malozemoff, p. 14): $m_w\ddot{x} + \beta\dot{x} + \xi x = M_0 H$. $[m_w] = \frac{g}{cm^2}$

$h = \frac{2H}{M_0 d} = M_0 H\left(\frac{2}{\alpha M_0^2}\right)$, $\frac{1}{2}\alpha M_0^2\frac{1}{c^2\Delta}\ddot{q} + \frac{\gamma}{\Delta}\cdot\frac{1}{2}\alpha M_0^2\dot{q} = M_0 H$

$m_{DW} = \frac{1}{2}\alpha M_0^2\frac{1}{c^2\Delta} = \frac{1}{2}\frac{\alpha M_0^2}{c^2\Delta_0}\frac{1}{\sqrt{1-\dot{q}^2/c^2}} = \frac{m^0_{DW}}{\sqrt{1-\dot{q}^2/c^2}}$.

$m^0_{DW} = \frac{1}{2}\frac{\alpha M_0^2}{c^2\Delta_0} = \frac{1}{2}\frac{\alpha M_0^2}{\gamma^2 M_0^2\alpha A\Delta_0} = \frac{1}{2}\frac{1}{\gamma^2 A\Delta_0}$; $[A] = 1$, $[m^0_{DW}] = g/cm^2$

$m^0_{DW} = \frac{1}{2}\frac{1}{\gamma^2 A\Delta_0}$, $A = \frac{2}{M_0^2 V}|4J_1+J_2|$, $\Delta_0 = \sqrt{\frac{\alpha M_0^2}{2\pi_y}}$.

$|J_2| = 7.347\cdot10^{-14}$ erg, $|J_1| = 396K\cdot1.381\cdot10^{-16}$ erg/K $= 5.469\cdot10^{-14}$ erg

$V = a^2 c = (0.3328)^2\cdot0.8539\cdot nm^3 = 9.457\cdot10^{-2}\cdot10^{-21}cm^3$ $M_0 = 392$ G

$A = \frac{2}{(392)^2 10^8 G^2\cdot 9.457\cdot10^{-23}cm^3}(4\cdot5.469 + 7.347)\cdot10^{-14}$ erg $= 40.219\cdot10^3$ (!)

$\gamma/2\pi = 28.025\cdot10^3$ MHz/T $\cdot (1\,Oe)^2 = erg/cm^3 = \frac{1}{cm^3}\cdot 1g\cdot\frac{cm^2}{s^2} = 1g\frac{1}{cm\cdot s^2}$

Escaneado con CamScanner

$\gamma^2 = (2\pi)^2 \left(28{,}025 \cdot 10^9\right)^2 \left(\frac{MHz}{T}\right)^2 = (28{,}025)^2 \cdot 10^6 \cdot 10^{12} \frac{1}{s^2} \frac{1}{(10^4 Oe)^2} = (28{,}025)^2 \times$

$\times (2\pi)^2 \cdot 10^{18} \frac{1}{s^2} \cdot 10^{-8} \frac{1}{(Oe)^2}$, $\frac{1}{s^2 \cdot (Oe)^2} = \frac{1}{s^2 erg/cm^3} = \frac{cm^3}{s^2} \frac{1 \cdot s^2}{g \cdot cm^2} =$

$= \frac{cm}{g}$. $\gamma^2 = (28{,}025 \cdot 2\pi)^2 \cdot 10^{10} \frac{cm}{g} = 31\,006{,}52 \cdot 10^{10} \frac{cm}{g} = 3{,}1007 \cdot 10^{14} \frac{cm}{g}$

$\frac{1}{\gamma^2} = \frac{1}{3{,}1007 \cdot 10^{14} \frac{cm}{g}} = 0{,}3225 \cdot 10^{-14} \frac{g}{cm}$ где $g = 2$, $\gamma = 8\,794\,B/\hbar$.

$W_0 = \alpha_x l_{x_i}^2 - \alpha_y l_{z_i}^2$, $W_a(\vec{m}) = K_{211} \sum_i (\vec{m}_i \cdot \hat{x})^2 - K_{211} \sum_i (\vec{m}_i \cdot \hat{y})^2$, $d_y = a_2$

W_a (in-plane) $= -K_{211} \sum_i (\vec{m}_i \cdot \hat{y})^2 = -2K_{211} \left[(\vec{m}_A \cdot \hat{y})^2 + (\vec{m}_B \cdot \hat{y})^2\right]$ BW width

$\vec{m}_A = \vec{m} + \vec{l}$, $\vec{m}_B = \vec{m} - \vec{l}$, $(m_A^y)^2 + (m_B^y)^2 = (m_y + l_y)^2 + (m_y - l_y)^2 =$

$= m_y^2 + 2m_y l_y + l_y^2 + m_y^2 - 2m_y l_y + l_y^2 = 2(m_y^2 + l_y^2) \approx 2l_y^2$

W_a (in xy plane) $= -2K_{211} \cdot 2l_y^2 = -4K_{211} \cdot l_y^2$, и $W_a' = -\alpha_y l_z^2 = -\alpha_y l_y^2$,

Q-ежега $\alpha_y = 4K_{211}$, $\Delta_0 = \sqrt{\frac{\chi M_0^2}{2\alpha_y}} = \sqrt{\frac{M_0^2}{2\alpha_y} \frac{8}{M_0^2 a^2} \frac{J_3}{V}} = 2a \sqrt{\frac{J_3}{\alpha_y V}}$

$\Delta_0 = a \sqrt{\frac{J_3}{K_{211} V}}$, $\frac{J_3}{K_{211} V} = \frac{1{,}588 \cdot 10^{-14} erg}{K_{211} \cdot V}$, $K_{211} = 7 \cdot K_{411} = 7 \cdot 1{,}8548 \cdot 10^{-25} \frac{J}{V}$

Roy PRB'16 считает $K_{411}/Mn\text{-ion} = 1{,}8548 \cdot 10^{-25}$ J на основе $H_{44} = 100$ Oe в экс. Barthem '2013. Roy'16 берет $K_{21} = +1{,}303 \cdot 10^{-22}$ J с ссылкой на Shick'2010 хотя Shick'2010 дает $K_{21} = +2{,}44 \frac{meV}{f.u.}$, $\uparrow Mn$ не на один Mn-ion.

$\frac{K_{21}}{f.u.} = +2{,}44\ meV = 2{,}44 \cdot 1{,}6022 \cdot 10^{-19} J \cdot 10^{-3} = 3{,}9094 \cdot 10^{-22} J$, $1\ f.u.\ Mn_2Au = 2\ Mn\text{-ions}$

$K_{21}/Mn\text{-ion} = 1{,}955 \cdot 10^{-22} J$ (см. Shick, а Roy PRB'16 берет $\frac{K_{21}}{Mn\text{-ion}} = 1{,}303 \cdot 10^{-22} J$.

Shick'2010 дает $K_{411}/f.u. = 0{,}01\ meV = 1{,}602 \cdot 10^{-24} J$, $\frac{K_{411}}{1\,Mn} = 0{,}801 \cdot 10^{-24} J$,

Roy'2016 сильно занижает $K_{411}/Mn\text{-ion} = 1{,}8548 \cdot 10^{-25} J$ с ссылкой на Barthem'2013

// по смыслу все K_2, K_4 энергии на один Mn-ion, single-ion anisotropy.

Если $\frac{K_{411}}{1\,Mn} = 1{,}8548 \cdot 10^{-25} J$ - Roy'16 это Ok, Ricardo in MS дает занижение $\rightarrow$

где V - unit cell volume, 4-Mn ions. $K_{411} = 1{,}8548 \cdot 10^{-25} \frac{J}{V}$,

Возьмем пока $K_{411} = 1{,}8548 \cdot 10^{-25} J$ и $K_{211} = 7 \cdot K_{411}$, тогда

в Δ_0 будет $K_{211} V = 7 \cdot K_{411} \cdot V = 7 \cdot 1{,}8548 \cdot 10^{-25} J = 1{,}8548 \cdot 10^{-18} erg \cdot 7$ 1/7

$\Delta_0 = a \sqrt{\frac{1{,}588 \cdot 10^{-14} erg}{7 \cdot 1{,}8548 \cdot 10^{-18} erg}} = 10^2 a \sqrt{\frac{1{,}588}{1{,}8548 \cdot 7}} = 0{,}925 \cdot 10^2 a = 0{,}925 \cdot 0{,}3328\, 10^2\ nm = \sqrt{28{,}5}\ nm =$

$= 19{,}77\ nm$.

В Ricardo MS Δ_0 (Simul.) $= 19{,}78$ nm. $\frac{28{,}5}{19{,}78} = 1{,}44$, $\sqrt{2} = 1{,}41$, разница $\times \sqrt{2}$ раз - ?

$W_a = -2K_{211} \sum_i (\vec{m}_i \cdot \hat{y})^2$, $W_a = \frac{1}{V} W_a = -\frac{K_{211}}{V} \sum_i (\vec{m}_i \cdot \hat{y})^2 = -4 \frac{K_{211}}{V} l_y^2$,

поэтому $\alpha_y = 4K_{211}/V$, где K_{211} - single-ion anisotropy (per Mn-ion) in Joules

$\Delta_0 = a \sqrt{\frac{J_3}{K_{211}}}$. E_a (Shick'2010) $= \ldots + K_{211}^* \sin^2\theta \cos 2\varphi = 2K_{211}^* \sin^2\theta \cos^2\varphi + \ldots =$

$= -2K_{211}^* m_y^2 = -K_{211} m_y^2$, $K_{211}^* = 7 \cdot K_{411}$ у Shick

$K_{211} = 2K_{211}^* = 14 \cdot K_{411}$ - берем сами $\frac{K_{211}}{K_{411}}$ как у Shick'2010, но K_{411}

берем не из Shick'10, а $K_{411}/Mn\text{-ion}$ $1{,}8548 \cdot 10^{-25} J$ берем изначально

Roy'2016 из Barthem'2013, $K_{211} = 14 \cdot K_{411}$. Тогда Δ_0 будет еще меньше

Нельзя, какую K_{211} и из каких соображений брать?

пусть пока $\Delta_0 = \Delta_0$(sim.) $= 19{,}78$ nm $m_{sw}^0 = \frac{1}{2} \delta^2 A \Delta_0 = \frac{1}{2} 1{,}022 \cdot 10^4 \cdot 19{,}78 \cdot 10^{-7} \cdot 0{,}3225 \cdot 10^{-14} g/cm =$

$= \frac{0{,}3225}{2 \cdot 4{,}022 \cdot 19{,}78} \cdot 10^{-11} g/cm^2 = 2{,}027 \cdot 10^{-3} \cdot 10^{-11} g/cm^2 = 2{,}03 \cdot 10^{-14} g/cm^2$

Escaneado con CamScanner

m^0_{DW} (Mn_2Au) $= 2.03 \cdot 10^{-14} \frac{g}{cm^2}$. Melkov 1996 gave experiment $m^0_{DW} = 0.5 \cdot 10^{-10} \frac{g}{cm^2}$.

Ricardo (Isle May 4, 2020) gave: $m^0_{DW} = \frac{(\mu_0 M_S)^2}{2\gamma^2 |J_2| \Delta_0}$, $m^0_{DW} = 2.6 \cdot 10^{-13} \frac{kg}{m^2} = 2.6 \cdot 10^{-14} \frac{g}{cm^2}$.

$m^0_{DW} = \frac{1}{2} \frac{1}{\gamma^2 A \Delta_0} = \frac{1}{2\gamma^2 \Delta_0} \frac{1}{\frac{2}{M_0^2 V} |4J_1 + J_2|} = \frac{1}{4} \frac{M_0^2 V}{\gamma^2 \Delta_0 |4J_1 + J_2|}$

m^0_{DW} (Ricardo) $= \frac{M_S^2}{2\gamma^2 \Delta_0 |J_2|}$, $|J_2| = |J_2| \frac{S^2}{V}$, $S = 5/2$, V - unit cell volume.

m^0_{DW} (Ricardo) $= \frac{M_S^2 V}{2\gamma^2 \Delta_0 |J_2| S^2} \cdot 2|4J_1 + J_2| = 2\left(4\frac{J_1}{J_2} + 1\right)|J_2| = 2 \cdot 3.977 |J_2| = 7.95 |J_2|$

$J_2 = 532K \cdot k_B$.

$S^2 |J_2| = 6.25 |J_2|$.

$\frac{m^0_{DW}(meas)}{m^0_{DW}(Ricardo)} = \frac{S^2 |J_2|}{2(4\frac{J_1}{J_2} + 1)|J_2|} = \frac{S^2}{2(4\frac{J_1}{J_2} + 1)} = \frac{6.25}{7.95} = 0.786$, ecuu

Означ. $M_0 = M_S = 4\mu_B / V$, $\Delta_0(meas) = \Delta_0(Ricardo)$, $\frac{m^0_{DW}(meas)}{m^0_D(Ricardo)} = \frac{2.03}{2.6} = 0.781$ близка

и $\frac{S^2}{2(4\frac{J_1}{J_2} + 1)} = 0.786$.

→ Borbog: в $q(t)$ equation Ricardo надо $m^0_{DW} \to 0.786\, m^0_{DW}$, если Δ_0 is the same.

Мое $\frac{1}{c^2 \Delta} \ddot{q} + \frac{\eta}{\Delta} \dot{q} = h$, $\ddot{q} + \eta c^2 \dot{q} = h c^2 \Delta$, у Ricardo Eq. (8) $\ddot{q} + \frac{2 d_G \gamma |J_2|}{M_0 M_S} \dot{q} = (...)$

ηc^2 и $h = M_0 H$ (χM_0^2), $\frac{1}{c^2 \Delta} \frac{1}{2} d M_0^2 \ddot{q} + \frac{\eta}{\Delta} \frac{1}{2} d M_0^2 \dot{q} = M_0 H$, $\frac{1}{2} d M_0^2 = m_{DW}$

$\frac{1}{2} d M_0^2 \frac{\eta}{\Delta} = \frac{1}{2\Delta} d M_0^2 \eta = \frac{1}{2\Delta} \frac{8 A_3}{M_0^2} M_0^2 \eta = \frac{1}{2\Delta} d M_0^2 \frac{2 d_G}{d M_0 \gamma} = \frac{d_G M_0}{\gamma \Delta}$

Мое $m_{DW} \ddot{q} + \frac{d_G M_0}{\gamma \Delta} \dot{q} = M_0 H$, Ricardo: $\ddot{q} + \frac{2 d_G \gamma |J_2|}{M_S} \dot{q} = \frac{2\gamma^2 \Delta |J_2|}{M_S} H = H M_S \times \left(\frac{2\gamma^2 \Delta |J_2|}{M_S^2}\right)$

$\frac{2 d_G \gamma |J_2|}{M_S} \cdot \frac{M_S^2}{2\gamma^2 \Delta |J_2|} = \frac{d_G M_S}{\gamma \Delta} \Rightarrow \frac{M_S^2}{2\gamma^2 \Delta |J_2|} \ddot{q} + \frac{d_G M_S}{\gamma \Delta} \dot{q} = M_S H$.

$m_{DW}(Ric) \ddot{q} + \frac{d_G M_S}{\gamma \Delta} \dot{q} = M_S H$ - eq. Ricardo, $m_{DW}(Ric) = \frac{M_S^2}{2\gamma^2 \Delta |J_2|}$,

$\Delta = \Delta_0 \sqrt{1 - \dot{q}^2/c^2}$, с мое и Ricardo можно было решить, и Δ_0 - решение

$m_{DW}(meas) = \frac{1}{4} \frac{M_0^2}{\gamma^2 \Delta |4J_1 + J_2|}$, $m_{DW}(Ric) = \frac{M_S^2}{2\gamma^2 \Delta |J_2|}$, J_3-3я J_3-член.

$J_i \to J_i c$ и меня $J_2 = J_2 S^2 / V$ - Ricardo

$\frac{m_{DW}(meas)}{m_{DW}(Ricardo)} = \frac{S^2}{2(4\frac{J_1}{J_2} + 1)} \frac{\Delta(Ric)}{\Delta(kos)}$, и можно если $\Delta(Ric) = \Delta(kos)$

и/или $\Delta_0(Ric) = \Delta(kos)$ и $c(Ric) = c(kos)$ - это

Отношение $= 0.786$, $m^0_{DW}(kos) / m^0_{DW}(Ric) = 0.786 \cdot \frac{\Delta_0(Ric)}{\Delta_0(kos)}$

$m_{DW} \ddot{q} + G \dot{q} = M_0 H$, $\frac{G(kos)}{G(Ric)} = \frac{\Delta(Ric)}{\Delta(kos)}$ - damping terms Ratio.

$G = \frac{d_G M_0}{\gamma \Delta}$.

⇒ Borbog: уд $m_{DW} \ddot{q} + G \dot{q} = M_0 H$, $\begin{cases} m_{DW}(Ric) \to m_{DW}(kos) = 0.786 \frac{\Delta(Ric)}{\Delta(kos)} m_{DW}(Ric) \\ G(Ric) \to G(kos) = \frac{\Delta(Ric)}{\Delta(kos)} \cdot G(Ric), \end{cases}$

Зависит от q - eq. Ricardo

если $\Delta(Ric) = \Delta(kos)$, то просто $m_{DW}(Ric) \to 0.786 \cdot m_{DW}(Ric)$. Это $M_0 = M_S$ (kos) (Ric)

Theor. $\Delta_0 = a \sqrt{\frac{J_3}{K_{2\perp}}}$, where $K_{2\perp}$ - in anisotropy OAH Hoy на один Mn-ion.

$K_{2\perp}$ - ? Теория $\Delta_0 = \Delta_0(exp) = 19.78$ nm, $K_{2\perp} = \frac{a^2}{\Delta_0^2} J_3 = \left(\frac{0.3328\,nm}{19.78\,nm}\right)^2 J_3 =$

$= 2.83 \cdot 10^{-4} J_3 = 2.83 \cdot 10^{-4} \cdot 1.588 \cdot 10^{-14}\, erg = 4.49 \cdot 10^{-18}\, erg = 4.49 \cdot 10^{-25}\, J / 1\,Mn$

$K_{2\perp}^{Sim} = \frac{a^2}{\Delta_0^2} J_3 = 4.49 \cdot 10^{-25}\, J / 1\,Mn$. Если в MS $K_{2\perp} = 7 K_{2\perp} = 12.98 \cdot 10^{-25}\, J / 1\,Mn$.

E-mails to Ruben y Ricardo: мои мой результат Mn_2Au DW dyn. paper, перепроверить работу коэффициенты m_{DW}, G in $q(t)$ equation исходя из 3-model, включая $K_{2\perp}$ in simul.? // Л. носочки Л.Д. 17€, 7kg в Киев.

8/VIII План сред. работ (см. 13/VIII) 1 от) Опц. начал. на 30 м до Sept. 14;
2) продвинуть Gu thick data/wires; 3) 32 layer charge HV + Osc. Base;
4) Sw edge scanning panel; 5) new co-p, + comp. details; 6) Beam Mon

Eq. for Dw center position $\frac{1}{c^2\gamma}\ddot{q} + \frac{\gamma}{\Delta}q = h$ Характерно m_{DW} бедный!

Сравним $\frac{1}{c^2}\ddot{q} + \gamma\dot{q}$. Если $h + h(t)$, то $\frac{1}{c^2\Delta}\ddot{q} + \frac{\gamma}{\Delta}\dot{q} = ht + \text{const}$, $h = \frac{2M}{\mu_0 M_0^2 A}$

Сравн. $\frac{1}{c^2}\ddot{q} + \gamma\dot{q}$. $\frac{1}{c^2}\frac{\ddot{q}}{\dot{q}} + \gamma = \frac{1}{c^2\gamma}\frac{\ddot{q}}{\dot{q}} + 1 = \frac{1}{c^2\gamma}(\dot{q}/q) + 1 = \frac{1}{2\gamma M_0 \alpha_0 A}(\dot{q}/q) + 1 = \frac{1}{2\alpha_G A}\frac{\omega_q}{\omega_M} + 1$, $\omega_q = \dot{q}/q$, $\omega_M = \gamma M_0 = 2\pi \cdot 3\,GHz$

$\omega_q \approx v \cdot \Delta_0^{-1} \approx 10\,\frac{km}{s} / (10\,nm) = 10\,\frac{1}{s}\frac{10^3 m}{10 \cdot 10^{-9}} = 10^{12}\,rad = (2\pi)\,10^{12}\,Hz$ $\quad 10,4 \cdot 10^3 G = 2\pi \cdot 1,2\,GHz$

$\frac{1}{c^2\gamma}\frac{\dot{q}}{q} \approx \frac{1}{2 \cdot 0,01 \cdot 4,02 \cdot 10^4}\left(\frac{2\pi \cdot 1,2\,GHz}{\frac{1}{2\pi}10^{12}\,Hz}\right)^{-1} = 2$ $\quad 1\,rad = \frac{2\pi}{2\pi}\,rad = 1\,Hz$

$= \frac{1}{8 \cdot 10^2}\frac{1}{2\pi}\frac{10^{12}\,Hz}{2\pi \cdot 1,2 \cdot 10^9\,Hz} = \frac{1}{(2\pi)^2}\frac{1}{2 \cdot 1,2}10 \approx 0,026$.

Т.е. $\frac{(m_{DW}\ddot{q})}{G\dot{q}} = \frac{1}{c^2\gamma}\frac{\ddot{q}}{\dot{q}} \approx \frac{1}{c^2\gamma}\frac{\dot{q}}{q} \propto 0,03$ дает ничего. $v_{DW} \sim 10\,\frac{km}{s}$

Вывод: эффект DW инерции (m_{DW} - anti(ferromagnet) огромен, но как DW не сравнимо с damping term/effect.

DW eq. $\ddot{q} + \gamma c^2\dot{q} = hc^2\Delta = \Delta_0 hc^2\sqrt{1 - v^2/c^2}$. $\dot{q} = v$, $\ddot{q} = \dot{v}$,

$\dot{v} = -\gamma c^2 v + \Delta_0 hc^2\sqrt{1 - v^2/c^2}$. Ср. с 17/I $\dot{v} = -\alpha v + F(t)\sqrt{1 - v^2/c^2}$,

$\alpha = \gamma c^2$, $F(t) = c^2 h(t)\Delta_0$. Если $F(t) = F_{const}$, то eq. для $v(t)$ можно решить $\int(\dots)$

$t(v) = \frac{c}{F^2 + \alpha^2 c^2}\left[F\arcsin\frac{v}{c} - \alpha c\ln\left(\sqrt{1 - v^2/c^2} - \frac{\alpha}{F}v\right)\right]_{v_0}^{v} + t_0$, $v(t_0) = v_0$.

$t(v) = \frac{1}{c}\frac{1}{(h^2\Delta_0^2 + \gamma^2 c^2)}\left[h\Delta_0\arcsin\left(\frac{v}{c}\right) - \gamma c^2\ln\left(\sqrt{1 - v^2/c^2} - \frac{\gamma}{h\Delta_0}v\right)\right]_{v_0}^{v} + t_0$. $[h] = \frac{1}{cm^2}$

$[\gamma] = \frac{s}{cm^2}$

Удобно как 17/I ввести dimensionless velocity $u = v/c$ и time $\tau = \Delta t$, $\nu = \frac{\alpha c}{F}$.

$t(u) = \frac{\nu}{(1 + \nu^2)}\left[\arcsin u - \nu\ln\left(\sqrt{1 - u^2} - \nu u\right)\right]_{u_0}^{u} + \tau_0$, $u(\tau_0) = u_0$ $\quad \nu = h_0/h$

$\tau = \Delta t = \gamma c^2 t$, $\nu = \frac{\alpha c}{F} = \frac{\gamma c^2 \cdot c}{c^2 h \Delta_0} = \frac{\gamma c}{h\Delta_0}$, $[\nu] = \frac{s}{cm^2}\frac{cm}{s}\frac{1}{cm^2 cm} = 1$ $\quad \nu = \frac{\alpha_G c}{\Delta_0 \gamma H}$

$\gamma c^2 = \frac{2\alpha_G}{\alpha M_0 \gamma}\gamma^2 M_0^2 \alpha A = 2\alpha_G \gamma M_0 A$, $[\gamma c^2] = \frac{1}{s}$. $\left(\frac{du}{d\tau} = -u + \frac{1}{\nu}\sqrt{1 - u^2}\right)$ $\quad H_0 = \frac{\alpha_G c}{\gamma\Delta_0}$

Steady velocity v (u_s) $\to \infty$, $\frac{du_s}{d\tau} = 0$, $u_s = \frac{1}{\sqrt{1 + \nu^2}}$, $\nu = \frac{\gamma c}{h\Delta_0}$, $u_s = 1$

$\ddot{q} + \frac{G}{m_{DW}}\dot{q} = \frac{1}{m_{DW}}M_0 H$, $\dot{v} = -\frac{G}{m_{DW}}v + \frac{1}{m_{DW}}M_0 H$, $\alpha = \frac{G}{m_{DW}}$, $F\sqrt{1 - \frac{v}{c}} = M_0 H$

$m_{DW} = m_{DW}^0/\sqrt{1 - v^2/c^2}$, $F = \frac{M_0 H}{m_{DW}^0}$. $\nu = \frac{\gamma c^3}{F} = \gamma c^2\frac{m_{DW}^0}{M_0 H} \sim \frac{m_{DW}^0}{H}$. $H_0 = \frac{\alpha_G c}{\gamma\Delta_0}$

$\nu = \frac{\gamma c}{h\Delta_0} = \frac{2\alpha_G c}{\alpha M_0 \gamma \Delta_0}\frac{dM_0}{2H} = \frac{\alpha_G c}{\Delta_0 \gamma H}$, $m_{DW}^0 = \frac{1}{2}\frac{1}{\gamma^2 A\Delta_0}$, $c^2 = \gamma^2 M_0^2 \alpha A$.

$\nu = \frac{H_0}{H}$

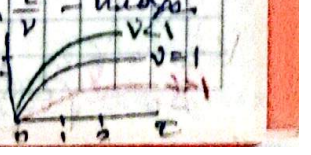
$\tau(u) \to \infty$ when $\ln(\dots) \to \infty$, $\sqrt{1 - u^2} \to \nu u$, $u \to u_s = \frac{1}{\sqrt{1 + \nu^2}}$ when $m_{DW} \to 0$

1) Small H, $\nu \gg 1$, $u(\tau) = u_0 e^{-\tau}$ - как при $u \to 0$. $\tau = -\frac{\nu^2}{1 + \nu^2}\ln(\dots) = -\ln(\dots)$

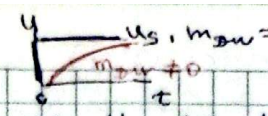
$\sqrt{1 - u^2} - \nu u = e^{-\tau}$. При $\tau \to \infty$ $u(\tau) \to u_s = \frac{1}{\sqrt{1 + \nu^2}}$

2) Large H, $\nu < 1$, $du/d\tau = \frac{1}{\nu}\sqrt{1 - u^2}$, $\tau(u) = \nu\arcsin u$, $u(\tau) = \sin\frac{\tau}{\nu}$ - медленно

$u(\tau) = \sin\frac{\tau}{\nu}$ подходит при $\frac{\tau}{\nu} < \pi/2$, т.е. $\tau < 1$.



→ $u(\tau)$ стремится к u_s при $\tau \to \infty$, но при $\nu > 1$ (small H) насыщение медленное, при $\nu <$ (large H) - быстрое.

Если $m_{DW}=0$, то $u(\tau)=u_s$ 

$u_s=\frac{1}{\sqrt{1+\nu^2}}$ → 4 н/ш $H\to\infty$

Inertia effect $m_{DW}\neq 0$, $u(t)$ достигает u_s не сразу с увелич. t (mass) $v(\tau)=\frac{c}{\sqrt{1+\nu^2}}$, $\nu=\nu(H)$

Шкала времени $\tau=dt=\gamma c^2 t=2d_G\gamma M_0 A t$, $\omega_e=\gamma\cdot 2AM_0$, $\tau=d_G\omega_e t$

$\omega_e/2\pi=\frac{\gamma}{2\pi}2AM_0=\left(\frac{\gamma}{2\pi}\right)2\cdot 4{,}02\cdot 10^4\cdot 0{,}392\cdot 10^3 G=2{,}8\cdot 8{,}04\cdot 0{,}392\cdot 10^7 G$ $\frac{MHz}{G}=$

$=8{,}82\cdot 10^7 MHz=8{,}82\cdot 10^{13} Hz=88{,}2$ THz – exch. frequency Mn_2Au.

$d_G\frac{\omega_e}{2\pi}=0{,}01\cdot 88{,}2$ THz $=0{,}882$ THz, $\tau=\frac{t}{t_0}$, $t_0=\frac{1}{d_G\omega_e}$

$t_0=\frac{1}{2\pi\cdot 0{,}882\cdot 10^{12}Hz}=0{,}180$ ps $(1 ps=10^{-12}s)$. $u(t)=u_s-\frac{\nu}{1+\nu^2}e^{-\tau\frac{(1+\nu^2)}{\nu^2}}$ $\tau\gg 1$

н/ш $t=1-10$ ps, $t/t_0\gg 1$, $\tau\gg 1$ и $u(\tau)\approx u_s$, эффект m_{DW} мал,

→ m_{DW} важна при $\tau\leq 1$. При $t\gg 1$ m_{DW} уже неважна, $u(\tau)=u_s$ – steady motion at given H. m_{DW} возникает с $\nu\uparrow$, $H\downarrow$

$\nu=\gamma c^3\frac{m^0_{DW}}{M_0 H}=\frac{2d_G}{dM_0\gamma}c\,\gamma^2 M_0^2 dA\cdot\frac{m^0_{DW}}{M_0 H}=\frac{d_G c}{\Delta_0\gamma H}$, $\frac{1}{\nu}=\frac{\gamma H\Delta_0}{d_G c}=\frac{H}{H_0}$ $\Delta_0^2=\frac{dM_0^2}{2\pi_y}$

Scale H is $H_0=\frac{d_G c}{\gamma\Delta_0}=\frac{d_G}{\gamma\Delta_0}\gamma M_0\sqrt{dA}=\frac{d_G M_0\sqrt{dA}}{M_0\sqrt{\frac{d}{2\pi_y}}}=d_G\sqrt{2\pi_y A}$

$H_0=d_G\sqrt{2\pi_y A}$, $\pi_y=\frac{4K_{211}}{V}$ K_{211} easy axis constant per Mn-ion. $W_a=\pi_z l_z^2-\pi_y l_y^2$

$A=\frac{2}{M_0^2 V}|4J_1+J_2|=4{,}02\cdot 10^4$ для Mn_2Au $V=9{,}457\cdot 10^{-23}$ cm^3.

Пусть $K_{211}=K_{211}^{sim}=4{,}49\cdot 10^{-25}\frac{J}{1Mn}=4{,}49\cdot 10^{-18}$ erg/1 Mn, тогда

$\pi_y=\frac{4K_{211}^{sim}}{V}=\frac{4}{9{,}457\cdot 10^{-23}cm^3}4{,}49\cdot 10^{-18}erg=\frac{4\cdot 4{,}49}{9{,}457}\cdot 10^5\frac{erg}{cm^3}=1{,}90\cdot 10^5\frac{erg}{cm^3}$

$H_0=d_G\sqrt{2\cdot\pi_y A}=0{,}01\sqrt{2\cdot 4{,}02\cdot 10^4\cdot 1{,}90\cdot 10^5}\cdot Oe=10^{-2}\cdot 10^4\sqrt{8{,}04\cdot 19{,}0}\,Oe=$

$=12{,}36\cdot 10^2 Oe=1{,}24$ kOe

$H_0=1{,}24$ kOe для Mn_2Au.

$\nu=\frac{H_0}{H}=\frac{1{,}24\ kOe}{(10-60)mT}=\frac{1{,}24\ kOe}{(100\div 600)Oe}\approx 2\div 12$. По Ricardo берем $d=10^{-3}$ (?)

Тогда $d_G\frac{\omega_e}{2\pi}=8{,}82$ THz $\cdot 10^{-2}=0{,}0882$ THz и $t_0=1{,}80$ ps и

$H_0=10^{-1}\,1{,}24$ kOe $=124$ Oe, тогда $\nu=\frac{124\,Oe}{(100-600)Oe}=0{,}2-1{,}2$

$d_G\downarrow$ → $t_0\uparrow$, падает typical $\tau=\frac{t}{t_0}$, сильнее эффект m_{DW}

$d_G\downarrow$ → $H_0\downarrow$; $\frac{H}{H_0}$ растет при том же H, $\nu\downarrow$, $u(\tau)$ быстрее насыщается, слабее эффект m_{DW}

Выводы: time scale DW motion $t_0\sim d_G A$, SOT field scale $H_0=d_G\sqrt{2\pi_y A}$, easy axis – by анизотропу определ. не только Δ_0, но и H_0. Anisotropy

9/VIII Ruben обвинил о K_{211}. Table 1 in Walker-like DW freak down.... Mn_2Au

$K_{211}=7\cdot K_{411}$, $K_{411}=1{,}855\cdot 10^{-25}$ J (на 1 Mn-ion надо помножать).

$\Delta_0=l_{ex}=\sqrt{A'/K'_{211}}$, $A'=156K\cdot k_B/a$ (table 1) $A'=\frac{156K\cdot 1{,}381\cdot 10^{-16}erg/K}{0{,}3328\cdot 10^{-7}cm}=$

$=\frac{156\cdot 1{,}381}{0{,}3328}10^{-9}\frac{erg}{cm}=647{,}3\cdot 10^{-9}erg/cm=6{,}47\cdot 10^{-7}erg/cm$. Надо K_{211} in $\frac{erg}{cm^3}$.

Найдем $K'_{211}=A'/l_{ex}^2=\frac{6{,}47\cdot 10^{-7}erg/cm}{(19{,}78)^2\cdot 10^{-14}cm^2}=1{,}654\cdot 10^{-2}\cdot 10^7\frac{erg}{cm^3}=1{,}654\cdot 10^5 erg/cm^3$

$(l_{ex}=19{,}78$ nm$)$

$\frac{K_{211}}{V}=7\cdot\frac{1{,}855\cdot 10^{-25}J}{9{,}457\cdot 10^{-23}cm^3}=1{,}373\cdot 10^{-2}J/cm^3=1{,}373\cdot 10^5 erg/cm^3$, т.е. $K'_{211}=\frac{K_{211}}{V}$ рассчитано. Сам 7/VIII $\Delta_0=a\sqrt{\frac{J_3}{K_{211}}}$, $[J_3]=$ erg or J. $[K_{211}]=$ erg or J.

$\Delta_0=\sqrt{\frac{a^2J_3}{K_{211}}}=\sqrt{a^2J_3/V\cdot\frac{1}{K_{211}/V}}$, т.е. $A'=\frac{a^2J_3}{V}=\frac{J_3}{c}=\frac{1{,}588\cdot 10^{-14}erg}{0{,}8559\ nm}=$

Escaneado con CamScanner

$= \frac{1.588}{0.8539} \frac{10^{-14}\ erg}{10^{-7}\ cm} = 1.86 \cdot 10^{-7} \frac{erg}{cm}$. $A'(Ruben) = 6.47 \cdot 10^{-7} \frac{erg}{cm}$

Откуда Ruben взял $A' = 156\ K \cdot ka/a$? Такое A' соответ. $K_{2\perp}' = 1.654 \cdot 10^5 \frac{erg}{cm^3}$

если $l_{ex} = \Delta_0 = 19.78$ nm. $A'(Ruben)$ из какой работы?

Roy'PRB'16 берет $\frac{K_{4\perp}}{1\ Mn} = 1.8548\ J \cdot 10^{-25}$ с ссылкой на Barthem'13

А откуда $K_{2\perp} = 7 K_{4\perp}$ взяла из Shick'2010 (1) (нигде дословно было $K_{2\perp} = 14 K_{4\perp}$ т.к. $2K_{2\perp}^2 = K_{2\perp}$, $K_{2\perp}^2 = 7 K_{4\perp}$)

Barthem'13 (p.6) дает in-plane anisotropy fields exp. $Mn_2Au = 0.01\ T = \mu_0 H_a$.

E_A (basal plane$_{xy}$) $= -\frac{1}{2} K_{4\perp} \sum_i \left[(m_i^x)^4 + (m_i^y)^4 \right]$, $K_{4\perp}$ is per 1 Mn-ion.

E_A (xy) (per u.c.) $= -\frac{1}{2} K_{4\perp} \cdot 4 \left[m_x^4 + m_y^4 \right]$, 4 - Mn-atoms per u.cell.

$m_x^4 + m_y^4 = (\sin\theta\cos\varphi)^4 + (\sin\theta\sin\varphi)^4 = \sin^4\theta \left[\cos^4\varphi + \sin^4\varphi \right] = \left[\text{Эвакит} \right] =$

$= \sin^4\theta \left[\frac{1}{8}(\cos 4\varphi + 4\cos 2\varphi + \frac{6}{2}) + \frac{1}{8}(\cos 4\varphi - 4\cos 2\varphi + \frac{6}{2}) \right] =$

$= \frac{1}{4}\sin^4\theta \cos 4\varphi + \frac{3}{4}\sin^4\theta$, $\frac{E_A(Roy)}{(per\ u.c.)} = -\frac{1}{2} K_{4\perp} 4 \left[\frac{1}{4}\sin^4\theta\cos 4\varphi + \frac{3}{4}\sin^4\theta \right] =$

$= -\frac{1}{2} K_{4\perp} \sin^4\theta \cos 4\varphi + (\dots)$, $\frac{E_A(Shick)}{(per\ f.u.)} = K_{4\perp} \sin^4\theta \cos 4\varphi$

(u.c.) = 2 (f.u.) = 4 Mn-atoms. Поэтому $K_{4\perp}(Roy) \overset{def.}{=} K_{4\perp}(Shick)$ с точн. до знака.

$\frac{K_{4\perp}(Shick)}{1\ Mn} = 0.801 \cdot 10^{-24}\ J$. $\frac{K_{4\perp}(Roy)}{1\ Mn} = 1.855 \cdot 10^{-25}\ J$ с ссылкой на Barthem'13.

Т.е. Roy'16 берет $K_{4\perp}$ в $\frac{8.01}{1.855} = 4.32$ меньше чем simul. $K_{4\perp}$ - Shick 2010.

Недавно как Roy'16 переоценил $H_a^{xy} = 100$ Oe в $K_{2\perp} = 1.855 \cdot 10^{-25}\ J/1\ Mn$.

$E_A = K_1 (\vec{m}\cdot\vec{n})^2$, $\frac{\partial E_A}{\partial \vec{m}} = 2K_1 (\vec{m}\cdot\vec{n})\cdot\vec{n} = H_{a_1} M_S (\vec{m}\cdot\vec{n})\vec{n}$, $H_{a_1} = \frac{2K_1}{M_S}$

$E_A^{xy} = +\frac{1}{2} K_4 \left[(\vec{m}\cdot\hat{x})^4 + (\vec{m}\cdot\hat{y})^4 \right]$, $\frac{\partial E_A^{xy}}{\partial \vec{m}} = 2K_4 \left[(\vec{m}\cdot\hat{x})^3 \hat{x} + (\vec{m}\cdot\hat{y})^3 \hat{y} \right] =$

$\left(= H_{a4} M_S [\dots] \right)$, $H_{a4} = \frac{2K_4}{M_S}$.

Roy'2016 $\omega_{4\perp} = \frac{2\beta K_{4\perp}}{M_S}$, т.е. $H_{a4} = \frac{2K_{4\perp}}{M_S}$.

Если взять из соотн. Roy'16 $K_{4\perp} = \frac{1}{2} H_{a4} M_S = \frac{1}{2} 10^2\ Oe \cdot 780\ G = 3.9 \cdot 10^4 \frac{erg}{cm^3}$

(Barthem) (Barthem'13

$K_{2\perp}' = 7 \cdot K_{4\perp}' = 2.73 \cdot 10^5\ erg/cm^3$, это значение не соотн. ни с $K_{2\perp}$ из l_{ex}, ни с $K_{2\perp}' = K_{2\perp}/V$ из Roy'16.

2 $\frac{K_{2\perp}(Roy'16)}{V} = 2 \cdot 1.373 \cdot 10^5 \frac{erg}{cm^3} = 2.746 \cdot 10^5 \frac{erg}{cm^3}$, т.е. Roy'16 берет 2 Mn per u.c. V,

внутри в рамках 3-model, $K_{2\perp}' = 2 \frac{K_{2\perp}(Roy)}{V}$. $K_{4\perp}' = 2 \frac{K_{4\perp}(Roy)}{V}$, где

$K_{4\perp}(Roy'16) = 1.855\ J \cdot 10^{-25}/(1\ Mn)$ as in Table 1 (Dm-common. Phys.)

$A'(mae) = \frac{a^2 J_3}{V} = 1.86 \cdot 10^{-7} \frac{erg}{cm}$, $\sqrt{\frac{A(mae)}{K_{2\perp}'(mae)}} = \sqrt{\frac{1.86 \cdot 10^{-7}\ erg/cm}{2.73 \cdot 10^5\ erg/cm^3}} = \sqrt{\frac{1.86}{2.73}} \cdot 10^{-6}\ cm =$

$= 0.825 \cdot 10^{-6}\ cm = 8.25$ nm, если $A' \to 4A'(mae)$, то $\sqrt{\frac{4A'}{K_{2\perp}'}} = 2 \cdot 8.25 = 16.50$ nm

близко к $l_{ex} = 19.78$ nm by Ruben // как можно было получить $K_{2\perp}$, $K_{4\perp}$ количественно? Вопросы для Δ_0 и H_0?

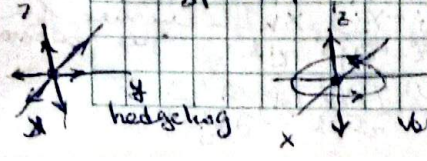
E-mail to Ruben о values $K_{2\perp}$, $K_{4\perp}$ in AF inertial Dw dyn" MS Mn_2Au.

BP - 3D charge

(10/VIII) **3D topol. charge** (Hopf index) of Bloch point (BP). File Fernandez-Roldan, July 27, 2020

Для вида BP, $\vec{m}(\vec{r}) = p\frac{\vec{r}}{|\vec{r}|}$ - hedgehog и $\vec{m}(\vec{r}) = \frac{1}{\rho}(-\nu y, \nu x, p z)$ -

vortex-type BP. $\nu = \pm 1$ - chirality (C), $p = \pm 1$ - polarity of BP, $\vec{r} = (x, y, z)$

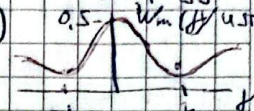


$-n/m_z = 0$, $\vec{m}(\vec{\rho}) = \frac{1}{\rho}(-\nu y, \nu x, 0)$ - vortex без „core"

$\rho = \sqrt{x^2 + y^2}$; See E. Feldkeller'1965 и W. Döring'1968.

• R. B. Elias, A. Verga, Eur. Ph. J. B 82, 159 (2011): $\vec{M}(r,\theta,\varphi) = M(r)\,\vec{m}_\gamma(\theta,\varphi)$
$\vec{r} = (r,\theta,\varphi)$ сферич. координаты, $|\vec{m}_\gamma|^2 = 1$, the ansatz by Döring '68
$\vec{m}_\gamma = \big(\cos(\varphi+\gamma)\sin\theta,\ \sin(\varphi+\gamma)\sin\theta,\ \cos\theta\big)$, т.е. $\Phi = \varphi+\gamma$, $\Theta = \theta$
$\gamma = 0$ соотв. to hedgehog BP (max W_m) $\gamma = \arccos(-\frac{11}{29})$ – spiral BP solution
в Döring '68, $\gamma_{skirm} = 113°$ из min $(W_x + W_m)$. Это все для $M(r)$ – function
$W_m = W_m(\gamma)$ максимум при $M_3(r) < M_3(0) = 0$

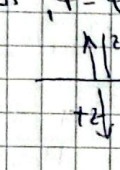
• O.V. Pylypovskyi et al. PRB 85, 224401 (2012). sphere
$\vec{r} = (r,\theta,\varphi)$. Многозначная ansatz $\Theta = p\theta + \frac{\pi}{2}(1-p)$, $\Phi = q\varphi + \gamma$, $p,q = \pm 1$
Bloch or Néel Feldkeller solution $\gamma_F = \arccos(-\frac{p}{2}) = \begin{cases} 120°, p=+1 \\ 60°, p=-1 \end{cases}$, Döring $\gamma_D = \arccos(-\frac{11}{29}) = 112°$
и свое для сферы finite radius R: $\gamma_0 = \arccos(-\frac{p}{4}) = \begin{cases} 105°, p=+1 \\ 75°, p=-1 \end{cases}$
$p=\pm1$ polarity, $q=\pm1$ vorticity надобно p,q чтобы $Q = \frac{1}{4\pi}\int d\theta d\Phi \sin\Theta = pq$ как для 2D vortex. $Q \sim \int d^2S\, \gamma_z$ более-менее?
если 3D $\vec{M}(\vec{r}) = M(r)\vec{m}_\gamma(\theta,\varphi)$ для BP. $M(r)$ не рассм. $\pm\gamma_0$ разделяет
ен. barrier $\approx \frac{1}{2}(W_m M_3^2 V)$. $\gamma = 0$ соотв. max $W_m(\gamma)$

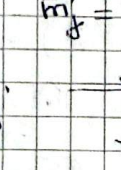


$\gamma_0 = 90°$ соотв. касательной к сфере $|\vec{r}|^2 = 1$



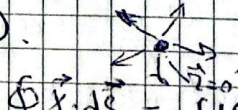
Fig. 25.2. BP [5, Jamet et al. Nanowires '2015] неточна, т.к. $\gamma \neq \pi/2$.
$Q=-1$ – antivortex in xy-plane – неприемлемо. Оставим $Q=+1$.
$p=+1$, $\Theta = \theta$, $\Phi = \varphi+\gamma$ $\vec{m}_\gamma = [\cos(\varphi+\gamma)\sin\theta,\ \sin(\varphi+\gamma)\sin\theta,\ \cos\theta]$ – hedgehog
$p=-1$, $\Theta = -\theta + \pi$, $\Phi = \varphi+\gamma$, $\vec{m}_\gamma = [\cos(\varphi+\gamma)\sin\theta,\ \sin(\varphi+\gamma)\sin\theta,\ -\cos\theta]$ – vortex-like





$(p=+1, \gamma=0)$ – hedgehog BP; $(p=-1, \gamma=\frac{\pi}{2})$ – vortex-like BP

Vorticity (emergent mag. field) $\vec{\gamma} = \nabla(\cos\Theta)\times\nabla\Phi = -\sin\Theta\,\nabla\Theta\times\nabla\Phi$ [Komineas '96]
$\nabla\Theta = p\nabla\theta$, $\nabla\Phi = q\nabla\varphi$ – не зависит от γ ($\Phi = q\varphi+\gamma$). $\vec{r} = (r,\theta,\varphi)$
$\nabla f(\vec{r}) = \nabla f(r,\theta,\varphi) = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\theta} + \frac{1}{r}\frac{1}{\sin\theta}\frac{\partial f}{\partial\varphi}\hat{\varphi}$
$\nabla\theta = \nabla f = \big|f(\vec{r}) = \theta\big| = \frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\theta} = \frac{1}{r}\hat{\theta}$, $\nabla\varphi = \frac{1}{r\sin\theta}\hat{\varphi}$
$\vec{\gamma} = -\sin\Theta\, p\nabla\theta\times q\nabla\varphi = -pq\sin\big[p\theta + \frac{\pi}{2}(1-p)\big]\frac{1}{r}\hat{\theta}\times\frac{1}{r\sin\theta}\hat{\varphi} =$
$= -pq\frac{1}{r^2}\hat{\theta}\times\hat{\varphi}$, $\hat{\theta}\times\hat{\varphi} = \cos\theta\cos^2\varphi\,\hat{x}\times\hat{y} + \cos\theta\sin^2\varphi\,\hat{x}\hat{y} + \ldots = \hat{r}$
$\vec{\gamma} = -pq\frac{\hat{r}}{r^2}$ не зависит от γ – (phase of BP).



Итак $\vec{\gamma}(\vec{r}) = \frac{\hat{r}}{r^2}$. $\oint_{сферы}\vec{\gamma}\cdot d\vec{S} = \int dV\,\mathrm{div}\,\vec{\gamma}$. $\oint_S \vec{\gamma}\cdot d\vec{S} = \int d\theta d\varphi\,\vec{\gamma}\cdot\hat{r}r^2\sin\theta$ ($d\theta d\varphi$)
$\oint_S \vec{\gamma}\cdot d\vec{S} = \int d\theta d\varphi\, r^2\sin\theta\,\hat{r}\cdot\vec{\gamma} = \int d\theta d\varphi\sin\theta\, r^2\frac{1}{r^2} = \int d\theta d\varphi\sin\theta = 4\pi$
$\int dV\,\mathrm{div}\,\vec{\gamma} = 4\pi$, как же $\mathrm{div}\,\vec{\gamma} = 0$ condition? $\mathrm{div}\,\vec{\gamma} = 4\pi\delta(\vec{r})$...
$\vec{m}^2 = 1$ означает $\mathrm{div}\,\vec{\gamma} = 0$. Имеющееся отклонение от $\vec{m}^2 = 1$ устранимо.
Komineas '96: Θ и Φ cannot be both differentiable for $\vec{m}(\vec{r})$ with Hopf index $N \neq 0$.
$\nabla\Theta \sim \frac{1}{r}\hat{\theta}$, $\nabla\Phi \sim \frac{1}{r\sin\theta}\hat{\varphi}$, $\Theta(\vec{r})$ и $\Phi(\vec{r})$ разрывны. При $r=0$ и $\sin\theta = 0$

$\vec{\gamma}(\vec{r}) = \frac{\hat{r}}{r^2}$, $\gamma_i(\vec{r}) = \frac{x_i}{r^3}$. $N = \frac{1}{(4\pi)^2}\varepsilon_{ijk}\int dV dV' \frac{x_i}{r^3}\frac{(x_j - x'_j)}{|\vec{r}-\vec{r}'|^3}\frac{x'_k}{r'^3}$ – можно все сразу?
$\vec{r} - \vec{r}' = \vec{R}$, $\frac{R_j}{R^3} = \frac{x_j}{R^3} = -\frac{\partial}{\partial x_j}\frac{1}{R}$. $N = \frac{1}{(4\pi)^2}\varepsilon_{ijk}\int dV dV' \frac{x_i x'_k}{r^3 r'^3}(-1)\frac{\partial}{\partial x_j}\frac{1}{R}$
$\frac{1}{R} = \frac{1}{|\vec{r}-\vec{r}'|}$ разложить по $Y_{lm}(\theta,\varphi)$ и $\int d\theta d\varphi(\ldots)$.

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BP (11/VIII) Kundu '1982: $m_a^2=1$, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu=2\varepsilon^{abc}\partial_\mu m_a \partial_\nu m_b \cdot m_c$

$\vec B=\mathrm{rot}\vec A$, Hopf index $Q_H=-\frac{1}{(8\pi)^2}\int d^3x\,\vec A\cdot\vec B$, $\vec B=\frac{2}{m_3}\nabla m_1\times\nabla m_2$ - emergent mag. field

$Q_H=-2\frac{1}{(8\pi)^3}\int d^3x\,d^3x'\frac{1}{R^3}\vec B'\cdot(\vec R\times\vec B)$, $\vec R=\vec x-\vec x'$, $\vec B=-2\nabla w\times\nabla\gamma$, $\vec B'=\vec B(\vec x')$

$m_3=\cos\beta=w(r,\varphi)$, $\gamma=\arctg\left(\frac{m_2}{m_1}\right)=m\varphi+v(r,\varphi)$,

где $\vec r(r,\varphi,z)$ - сферич. координаты $\vec m=\vec m(\beta,\gamma)$, $\beta=\theta$, $\gamma=\Phi$ сферич. углы $\vec m$.

$m_2=\cos\theta=w(r,\varphi)$, $\Phi=m\varphi+v(r,\varphi)$

Сравн. с оценкой Elias'11 или Pylyavskyi'12: $\theta=p\varphi+\frac{\pi}{2}(1-p)$, $\Phi=\varphi+\gamma$. ($q=1$)

$m_2=p\cos\varphi=w(r,\varphi)$ и $\gamma=v(r,\varphi)$, $m=1$.

$Q_H=\frac{q}{4\pi}\int_0^\infty dr\int_0^\pi d\varphi(1-p\cos\varphi)\left[\frac{\partial w}{\partial r}\frac{\partial v}{\partial\varphi}-\frac{\partial w}{\partial\varphi}\frac{\partial v}{\partial r}\right]=\frac{q}{4\pi}\int_0^\infty dr\int_0^\pi d\varphi(1-w)\times$

$\times\left[0\cdot\frac{\partial v}{\partial\varphi}-\frac{\partial}{\partial\varphi}(p\cos\varphi)\frac{\partial\gamma}{\partial r}\right]=+\frac{pq}{4\pi}\int_0^\infty dr\int_0^\pi d\varphi(1-p\cos\varphi)\sin\varphi\frac{\partial\gamma}{\partial r}=$

$=\frac{pq}{4\pi}\int_0^\pi d\varphi\sin\varphi\int_0^\infty dr\frac{\partial\gamma}{\partial r}=pq\frac{1}{2\pi}\int_0^\infty dr\frac{\partial\gamma}{\partial r}$, если $\gamma\neq f(\varphi)$

Вектор $\vec q$, где BP с $\gamma=\gamma(r)$, $Q_H=pq\frac{1}{2\pi}[\gamma(\infty)-\gamma(0)]$ - Hopf index BP

$\vec B=-2\nabla(m_2)\times\nabla\Phi$, тогда как Komineas'96 опр. $\vec\gamma=\nabla(m_2)\times\nabla\Phi$, $\vec\gamma=-\frac{1}{2}\vec B$

$Q_H=-\frac{q}{(8\pi)^3}\int d^3\vec r\,d^3\vec r'\frac{1}{R^3}\vec\gamma\cdot(\vec R\times\vec\gamma)(-2)^2=-\frac{1}{(4\pi)^3}\int d^3\vec r\,d^3\vec r'\frac{1}{R^3}\vec\gamma\cdot(\vec R\times\vec\gamma)$

N (Komineas') $=\frac{1}{(4\pi)^2}\varepsilon_{ijk}\int dv\,dv'\frac{1}{R^3}\gamma_i R_j\gamma'_k=\frac{1}{(4\pi)^2}\int dv\,dv'\frac{1}{R^3}\vec\gamma\cdot(\vec R\times\vec\gamma')$

$\vec\gamma\cdot(\vec R\times\vec\gamma')=\vec\gamma'\cdot(\vec\gamma\times\vec R)=-\vec\gamma'\cdot(\vec R\times\vec\gamma)$, $Q_H=\frac{1}{4\pi}N$ (Komineas)

Если $\vec B=\mathrm{rot}\vec A$, то $\vec A(\vec r)=\frac{1}{4\pi}\int dv'\frac{\nabla'\times\vec B(\vec r')}{|\vec r-\vec r'|}$ - Biot-Savart law.

$Q_H(BP)=pq\frac{1}{2\pi}[\gamma(\infty)-\gamma(0)]\neq0$, если $\gamma=\gamma(r)$, $Q_H=\pm1$ или $\gamma(\infty)-\gamma(0)=\pm2\pi$

$\vec\gamma=q\frac{\hat r}{r^2}\to\oint d\vec S\cdot\vec\gamma=\int dv\,\mathrm{div}\vec\gamma=4\pi q$, $\mathrm{div}\vec\gamma=4\pi q\delta(\vec r)$



R.G. Elias et al PRB'2014 предлагает использовать $\oint d\vec S\cdot\vec B=\int dv\,\mathrm{div}\vec B\neq0$

Вектор $\vec B=q\frac{\hat r}{r^2}-4\pi q\hat z\delta(x)\delta(y)$, emergent field along z-axis, где $\vec A$ снимает через $\sin\theta=0$, $\vec A(\vec r)=-\frac{q}{r}\mathrm{ctg}\theta\hat\varphi$, но $\vec A\cdot\vec B=0$ и $Q_H=0$.

$\oint d\vec S\cdot\vec B=\oint dS\hat r\cdot\left[q\frac{\hat r}{r^2}-4\pi q\hat z\delta(x)\delta(y)\right]=q\oint dS\frac{1}{r^2}-\oint dS(+4\pi q)\delta(x)\delta(y)=$

$=q\oint d\theta d\varphi\sin\theta r^2\frac{1}{r^2}-4\pi q\oint dS\,\delta(x)\delta(y)=4\pi q-4\pi q=0$ и $\int dV\,\mathrm{div}\vec B=0$.

$\vec A(\vec r)=A_\varphi\hat\varphi$, $\vec B=\mathrm{rot}\vec A=\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\varphi\sin\theta)\hat r-\frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi)\hat\theta=\frac{\hat r}{r\sin\theta}\frac{\partial}{\partial\theta}\left(-\frac{q}{r}\cos\theta\right)-$

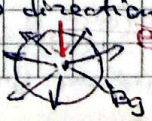
$A_\varphi=-\frac{q}{r}\mathrm{ctg}\theta$ $-\frac{1}{r}\frac{\partial}{\partial r}(-q\,\mathrm{ctg}\theta)\hat\theta=q\frac{\hat r}{r^2}$, $\vec B(\vec r)=q\frac{\hat r}{r^2}$, где все $\delta(x)\delta(y)$ зачем?

Elias et al. '2014 неверно применили перенормировку $\vec B$ из Y. Shnir. Magnetic monopoles. 2005.

$\vec B=g\frac{\vec r}{r^3}=\mathrm{rot}\vec A$ и $\mathrm{div}\vec B=4\pi g\delta(\vec r)$ и $\mathrm{div}\vec B=0$ одновременно. Где ошибка?

Dirac potential $\vec A(\vec r)=\frac{g}{r}\frac{(\vec r\times\vec n)}{(r-(\vec r\cdot\vec n))}$, где обычно берут $\vec n=\hat z$ Dirac string direction, $\vec A$ is singular along $\vec r-(\vec r\cdot\vec n)\,\vec n=0$ direction (полуось $\theta=\pi$, $z>0$)

$\vec B=\vec B_g+\vec B_{sing}=g\frac{\vec r}{r^3}-4\pi g\vec n\,\Theta(z)\delta(x)\delta(y)$



$\vec B_{sing}$, $\vec B_{sing}$ дает $\oint\vec B\cdot d\vec S=0$, $\mathrm{div}\vec B=0$

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$\vec{A} = \vec{A}_S = -\frac{q}{r}\frac{1+\cos\theta}{\sin\theta}\hat{\varphi}$ (singular at $\theta=0$, nonsingular $\forall r>0$), $\vec{A}_N = \frac{q}{r}\frac{1-\cos\theta}{\sin\theta}\hat{\varphi}$ (sing. $\theta=\pi$, $\hat{n}=-\hat{z}$, nonsingular $\forall r>0$), $\vec{A}^{Sch} = -\frac{q}{r}\frac{\cos\theta}{\sin\theta}\hat{\varphi} = \frac{1}{2}(\vec{A}_S + \vec{A}_N)$ (singular on the z-axis, no matter $\theta=0$ or $\theta=\pi$)

Ch 1. of Shiz book "Dirac monopoles"

$\vec{A} = A_\varphi\hat{\varphi}$ eq-za Dirac string $\parallel Oz$. $\vec{A}^{Sch}$ - Schwinger potential

[J. Schwinger, PR 144, 1087 (1966) PR 151, 1048 (1966)]

Nonsing Bose/ons $\vec{n}$ unare, на $\hat{z}$ - selected direction.

in spherical coordinate system

Вывод: $\operatorname{div}\vec{B}=0$, $\operatorname{div}\vec{j}=0$ нужно собственно $\vec{j} = q\frac{\hat{r}}{r^2}$, $\vec{B} = g\frac{\hat{r}}{r^2}$ monopole field, за счет B_{sing}, j_{sing} и Dirac string potential. Где обеспечено сферами $M_3(x)$, нужно $M_3(x) = M_3 = const$, $\vec{n}^2 = 1$.

BP - это monopole of emergent mag. field $\vec{B}$, и зарядом g

Это не ист. зарядов, а q - vorticity defined as $\Phi = q\varphi + \gamma$, q topol. charge.

$\vec{j}(\vec{r}) = q\frac{\hat{r}}{r^2} - 4\pi q\vec{n}\,\theta(z)\delta(x)\delta(y)$, $\operatorname{div}\vec{j}=0$, $\vec{A} = -\frac{q}{r}\frac{1+\cos\theta}{\sin\theta}\hat{\varphi}$ или $\vec{n}=\hat{z}$ Dirac ($\vec{A}=\vec{A}_S$) string.

E-mails to Oksana y Josa.

2D charge BP y Gyroelectr

(12/VIII) $\vec{r} = (r, \theta, \varphi)$, $\vec{m}(\theta, \Phi)$. BP ansatz $\Theta(\vec{r}) = p\theta + \frac{\pi}{2}(1-p)$, $\Phi(\vec{r}) = q\varphi + \gamma(r)$

Emergent mag. field $\vec{\gamma} = \vec{B} = -\sin\Theta\,\nabla\Theta\times\nabla\Phi$, $\sin\Theta = \sin\left[p\theta + \frac{\pi}{2}(1-p)\right] = \sin\theta$

$\nabla\Theta = \nabla\left(p\theta + \frac{\pi}{2}(1-p)\right) = p\nabla\theta = \frac{p}{r}\hat{\theta}$

$\nabla\Phi = \nabla(q\varphi + \gamma(r)) = q\nabla\varphi + \nabla\gamma(r) = \frac{q}{r\sin\theta}\hat{\varphi} + \frac{\partial\gamma}{\partial r}\hat{r}$.

$\vec{B} = -\sin\Theta\,\nabla\Theta\times\nabla\Phi = -\sin\theta\left[\frac{p}{r}\hat{\theta}\right]\times\left[\frac{q}{r\sin\theta}\hat{\varphi} + \frac{\partial\gamma}{\partial r}\hat{r}\right]$ $\hat{\theta}\times\hat{\varphi} = \hat{r}$

$\hat{\theta}\times\hat{\varphi} = \left[\cos\theta(\cos\varphi\hat{x} + \sin\varphi\hat{y}) - \sin\theta\hat{z}\right]\times\left[-\sin\varphi\hat{x} + \cos\varphi\hat{y}\right] = \cos\theta\cos^2\varphi\,\hat{x}\times\hat{y} +$

$+ \cos\theta\sin^2\varphi\,\hat{y}\times\hat{x}(-1) + \sin\theta\sin\varphi\,\hat{z}\times\hat{x} - \sin\theta\cos\varphi\,\hat{z}\times\hat{y} = \cos\theta\hat{z} + \sin\theta\sin\varphi\hat{y} +$

$\hat{\theta}\times\hat{r} = \left[\cos\theta(\cos\varphi\hat{x} + \sin\varphi\hat{y}) - \sin\theta\hat{z}\right]\times\left[\sin\theta\cos\varphi\hat{x} + \sin\theta\sin\varphi\hat{y} + \cos\theta\hat{z}\right] =$ $+ \sin\theta\cos\varphi\hat{x} = \hat{r}$

$= \cos\theta\cos\varphi\sin\theta\sin\varphi\,\hat{x}\times\hat{y} + \cos\theta\sin\varphi\sin\theta\cos\varphi\,\hat{y}\times\hat{x} + \cos^2\theta\cos\varphi\,\hat{x}\times\hat{z} +$ $-\hat{\theta}\times\hat{r} = \hat{\varphi}$

$+ \cos^2\theta\sin\varphi\,\hat{y}\times\hat{z} - \sin^2\theta\cos\varphi\,\hat{z}\times\hat{x} - \sin^2\theta\sin\varphi\,\hat{z}\times\hat{y} = \cos\varphi\hat{y} - \sin\varphi\hat{x} = \hat{\varphi}$

$\vec{B} = -\sin\theta\left[\frac{pq}{r^2\sin\theta}\hat{\theta}\times\hat{\varphi} + \frac{p}{r}\frac{\partial\gamma}{\partial r}\hat{\theta}\times\hat{r}\right] = -\sin\theta\left[\frac{pq}{r^2\sin\theta}\hat{r} + \frac{p}{r}\frac{\partial\gamma}{\partial r}\hat{\varphi}\right]$

$\vec{B} = -pq\frac{\hat{r}}{r^2} + \frac{p\sin\theta(-1)}{r}\frac{\partial\gamma}{\partial r}\hat{\varphi} = B_r\hat{r} + B_\varphi\hat{\varphi}$, B_φ не дает flux $\int\vec{B}$, т.к. $d\vec{S} = \hat{r}\,dS$

$\int_V dV\,B_r = \langle B_r\rangle_V = \frac{1}{V}\int dr\int_0^{2\pi}d\varphi\int_0^{\pi}d\theta\,r^2\sin\theta\left(-\frac{pq}{r^2}\right)$

$\langle B_r\hat{r}\rangle_V = \frac{1}{V}\int dr\int_0^{2\pi}d\varphi\int_0^{\pi}d\theta\,r^2\sin\theta\left(-\frac{pq}{r^2}\right)\left[\sin\theta\cos\varphi\hat{x} + \sin\theta\sin\varphi\hat{y} + \cos\theta\hat{z}\right] = 0$

т.к. $\int_0^{2\pi}d\varphi\sin\varphi = 0$, $\int_0^{2\pi}d\varphi\cos\varphi = 0$, $\int_0^{\pi}d\theta\sin\theta\cos\theta = 0$

$\langle B_\varphi\hat{\varphi}\rangle = \frac{1}{V}\int dr\int_0^{2\pi}d\varphi\int_0^{\pi}d\theta\,r^2\sin\theta\left(\frac{p}{r}\right)\sin\theta\frac{\partial\gamma}{\partial r}\left[-\sin\varphi\hat{x} + \cos\varphi\hat{y}\right] = 0$

т.о. $\gamma = \gamma(r)$ не спасает, $\int d\varphi(\ldots) = 0$, $\langle\vec{B}\rangle_V = \langle B_r\hat{r}\rangle + \langle B_\varphi\hat{\varphi}\rangle = 0$, и $\vec{G} = \langle\vec{B}\rangle_V = 0$, total gyroconductor $\vec{G} = 0$.

Можно ли найти vector-potential $\vec{A}$, $\vec{B} = \operatorname{rot}\vec{A}$? при $\gamma = \gamma(r)$?

$B_r = (\operatorname{rot}\vec{A})_r = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\varphi\sin\theta) - \frac{\partial A_\theta}{\partial\varphi}\right]$, $\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right] = 0$

$B_\varphi = \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right] = -\frac{p}{r}\sin\theta\frac{\partial\gamma}{\partial r}$. A_φ Вариант, т.к. $\vec{A}\cdot\vec{B} = A_\varphi B_\varphi + \ldots \neq 0$ - опреден. $\neq 0$ Hopf index.

Вывод: при $\gamma = const$ $\vec{A} = (0, 0, A_\varphi)$, $A_\varphi = -\frac{q}{r}\frac{1+\cos\theta}{\sin\theta}$, $\frac{\partial}{\partial r}(rA_\varphi) = 0$

Т.к. $B_\varphi \neq 0$, то хотя бы одна из $A_r, A_\theta \neq 0$. Пусть $A_\theta = 0 \to \frac{\partial A_r}{\partial\varphi} = p\sin\theta\,\gamma'$, $\gamma' = \frac{\partial\gamma}{\partial r}$

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$A_r = -p\cos\theta\,\gamma' + \text{const}(\theta)$, $\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\theta\sin\theta)\right] = B_r = -\frac{pq}{r^2}$

$\frac{\partial}{\partial\theta}(A_\theta\sin\theta) = -\frac{pq}{r}\sin\theta$, $A_\theta\sin\theta = \frac{pq}{r}\cos\theta + \text{const}'(\theta)$

$A_\theta = \frac{pq}{r}\frac{\cos\theta}{\sin\theta} + \frac{\text{const}'(\theta)}{\sin\theta}$, $\int dV A_\theta B_\theta = \int dr\int d\varphi\int d\theta\, r^2\sin\theta \times$

$\times\left[\frac{pq}{r}\frac{\cos\theta}{\sin\theta} + \frac{\text{const}'(\theta)}{\sin\theta}\right]\left(-\frac{p}{r}\right)\sin\theta\,\gamma' = \int dr\int d\varphi\int d\theta\, r^2\sin\theta\left(-\frac{p}{r}\right)\gamma' \times$

$\times\left[\frac{pq}{r}\cos\theta + \text{const}'(\theta)\right]$, $\int_0^\pi d\theta\sin\theta\cos\theta = 0$

$\int dV A_\theta B_\theta = \int dr\int d\varphi\int d\theta\, r^2\sin\theta\left(-\frac{p}{r}\right)\gamma' \cdot \text{const}'(\theta)$, $\text{const}'(\theta) = f_1(r,\varphi)$

$\int dV A_\theta B_\theta = (-p)\int dr\, r\int d\varphi\, f_1(r,\varphi)\frac{\partial\gamma}{\partial r}\int d\theta\sin\theta$, $\int_0^\pi d\theta\sin\theta = \cos\theta(-1)\Big|_0^\pi = 2$

$\int dV A_\theta B_\theta = -2p\int dr\, r\int d\varphi\, f_1(r,\varphi)\frac{\partial\gamma}{\partial r}$, $A_\theta = \frac{pq}{r}\frac{\cos\theta}{\sin\theta} + \frac{f_1(r,\varphi)}{\sin\theta}$

Калибровка $f_1(r,\varphi)$. Остальные eqs. $B_\theta = 0$, $\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} = \frac{\partial}{\partial r}(rA_\theta)$ и

и $B_\theta = (\text{rot}\vec{A})_\theta \to \frac{\partial A_r}{\partial\theta} = p\sin\theta\,\gamma' \to A_r = -p\cos\theta\gamma' + f(r,\varphi)$, $f(r\varphi) = \text{const}(\theta)$

$\frac{\partial A_r}{\partial\varphi} = \frac{\partial f(r,\varphi)}{\partial\varphi}$, $\frac{\partial}{\partial r}(rA_\theta) = \frac{\partial}{\partial r}\left[pq\cos\theta + \frac{r}{\sin\theta}f_1(r,\varphi)\right]$

$\frac{\partial f}{\partial\varphi} = \sin\theta\frac{\partial}{\partial r}(rA_\theta) = \sin\theta\frac{\partial}{\partial r}\left[\frac{r}{\sin\theta}f_1(r,\varphi)\right] = \frac{\partial}{\partial r}\left[rf_1(r,\varphi)\right]$

Т.е. $f(r,\varphi)$ и $f_1(r,\varphi)$ подобраны так $\to$ $\frac{\partial f(r,\varphi)}{\partial\varphi} = \frac{\partial}{\partial r}\left[rf_1(r,\varphi)\right]$.

Все это в предположении $A_\theta = 0$.

$B_\theta = 0 \to \frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} = \frac{\partial}{\partial r}(rA_\theta)$ - уже использовали.

$A_r = -p\cos\theta\,\gamma' + f(r,\varphi)$, $A_\theta = 0$, $A_\varphi = \frac{pq}{r}\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}f_1(r,\varphi)$

$H \sim \int dV A_\varphi B_\theta = -2p\int d\varphi\int dr\, r f_1(r,\varphi)\frac{\partial\gamma}{\partial r}$. $\int dr\, r f_1\gamma' = \int dr\,\gamma'(rf_1) =$

$(Q_H) = \left|\begin{matrix} dr\,\gamma' = dv,\ v = \gamma(r) \\ u = rf_1(r\varphi)\end{matrix}\right| = \int dv\cdot u = \gamma(r)rf_1(r,\varphi)\Big|_0^R - \int\gamma(r)\frac{\partial}{\partial r}(rf_1)\,dr =$

$= \gamma rf_1\Big|_0^R - \int\gamma(r)\frac{\partial f(r,\varphi)}{\partial\varphi}\,dr$. $\int dV A_\varphi B_\theta = -2p\int d\varphi\int dr\, r f_1\gamma' =$

$= -2p\int d\varphi\left[\gamma rf_1\Big|_0^R - \int dr\,\gamma(r)\frac{\partial f(r\varphi)}{\partial\varphi}\right] = -2p\int d\varphi\,\gamma(r)rf_1(r,\varphi)\Big|_0^R +$

$+ 2p\int d\varphi\int dr\,\gamma(r)\frac{\partial f(r,\varphi)}{\partial\varphi} = -2p\int d\varphi\,\gamma(r)rf_1(r,\varphi)\Big|_0^R + 2p\int dr\,\gamma(r)f(r,\varphi)\Big|_0^{2\pi}$

$f(r,\varphi)\Big|_0^{2\pi} = A_r(r,\varphi)\Big|_0^{2\pi} = 0$, т.к. $A_r(r2\pi) = A_r(0)$ наблюдается со на 2π на компоненте A_r.

$\int dV A_\varphi B_\theta = -2p\int d\varphi\, r\gamma(r)f_1(r,\varphi)\Big|_0^R$. Если $\gamma(r) = \text{const}$, $A_\varphi = -\frac{q}{r}\frac{1+\cos\theta}{\sin\theta}$,

сравн. с $A_\varphi = \frac{pq}{r}\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}f_1(r,\varphi)$, $p=-1$, $\frac{1}{\sin\theta}f_1(r,\varphi) = -\frac{q}{r}\frac{1}{\sin\theta}$,

$f_1(r,\varphi) = -q/r$. Если $f_1(r,\varphi)$ сохраняет свой вид и при $\gamma = \gamma(r)$, $\to$

$Q_H \sim \int dV A_\varphi B_\theta = -2p\int d\varphi\, r\gamma(r)\left(\frac{-q}{r}\right)\Big|_0^R = 2pq\int_0^{2\pi}d\varphi\,\gamma(r)\Big|_0^R = 4\pi pq\left[\gamma(R) - \gamma(0)\right]$

11/VIII при $q=1$ монополь по ед. Кундт'82 $Q_H = \frac{1}{2\pi}P\left[\gamma(R) - \gamma(0)\right]$, $R\to\infty$ -

- разные сферы где ощущ ВР. Гипотеза $f_1(r,\varphi) = -q/r$ - приходит.

$A_\varphi = \frac{pq}{r}\frac{\cos\theta}{\sin\theta} + \frac{pq}{r} = \frac{pq}{r}\frac{1+\cos\theta}{\sin\theta}$ ($p=-1$), $A_r = -p\cos\theta\frac{\partial f}{\partial r} + f(r,\varphi)$, $f(r,\varphi)$ неизвестна.

$B_\varphi = -\frac{p}{r}\sin\theta\frac{\partial\gamma}{\partial r}$ и даст $Q_H \neq 0$ при $\frac{\partial\gamma}{\partial r}\neq 0$

$Q_H \sim \int dV A_\varphi B_\theta = \int dV\left(\frac{-q}{r}\right)\frac{1+\cos\theta}{\sin\theta}\left(-\frac{p}{r}\right)\sin\theta\frac{\partial\gamma}{\partial r} = pq\int dV\frac{1}{r^2}(1+\cos\theta)\frac{\partial\gamma}{\partial r} =$

$= pq\int dr\,d\theta\,d\varphi\, r^2\sin\theta\frac{\sin\theta}{r^2}(1+\cos\theta)\frac{\partial\gamma}{\partial r} = 2\pi pq\int_R dr\frac{\partial\gamma}{\partial r}\int_0^\pi d\theta\sin\theta(1+\cos\theta) =$

$= 4\pi pq\left[\gamma(R) - \gamma(0)\right]$, т.е. $Q_H \neq 0$ из-за $\gamma(r)$, согласно Кундт 1982

Q_H будет целым если $\gamma(R) - \gamma(0) = 2\pi n$, $n = 0, 1, 2, \ldots$

Для $\gamma(r)$ записать ед. из $\frac{\delta W}{\delta\gamma} = 0$ и найти $\gamma(r)$, или хотя $\gamma(0)$, $\gamma(R)$.

Escaneado con CamScanner

Комин Оскана: Дона с Jose Cruenor дне dots-Bergama $N=4\cdot 6$, т.е. $Q_H = N/4\pi = 0.3-0.4$ и Q_H дадени от dot (semisphere, truncated sphere) size. Обещана и (не) нарастване на theoretical Bergama-dot paper когда ги будат готови. 2) опасения Оскана, что $\gamma(r)$ дает $Q_H \neq 0$ и есть новое – Q_H то для BP (ранее $Q_H \neq 0$ только для toroidal hopfions); 3) учет $M_S(r)$ и выбор к нов дет. $\vec{\gamma}(r)$ и Q_H за включения в Topol. charges theory, где $M_S(r) = \text{const}$ и $\vec{m}^2 = 1$. Обещан файл по (1) – $\gamma(r)$ BP.

(13/VIII) Gyro-coupling density (vorticity) $\vec{\gamma}$ другие определения чем в Papanicolaou 1993:
$(\vec{\gamma}) := \frac{1}{2}\varepsilon_{ijk}(\partial_j\vec{m}\times\partial_k\vec{m})\cdot\vec{m}$, а не как в Kaminski '96, отличие в знаке.
$\operatorname{div}(\vec{\gamma}) = 0$, total vorticity $\vec{\Gamma} = \int dV\vec{\gamma}$, $\vec{\Gamma}=0$ или infinite volume limit when $\vec{\gamma}$ decays faster than $\frac{1}{r^3}$, at large $\vec{r}$.
$\vec{\gamma} = -\nabla\Pi\times\nabla\Phi = \sin\Theta\nabla\Theta\times\nabla\Phi$. Здесь $\vec{\gamma} = \vec{B}$, $\vec{B} = \operatorname{rot}\vec{A}$ – emergent mag. field.
$\Theta = p\varphi + \frac{\pi}{2}(1-p)$, $\Phi = q\varphi + \gamma$, $p = \pm 1$, arbitrary integer q (удобно $q=+1$)

(1) Чисто нова $\gamma = \text{const}$. $\nabla\Theta = p\nabla\varphi$, $\nabla\Phi = \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(q\varphi+\gamma)\hat{\varphi} = \frac{q}{r\sin\theta}\hat{\varphi}$
$\sin\Theta = \sin\left[p\varphi + \frac{\pi}{2}(1-p)\right] = \sin(p\varphi)\cos\left(\frac{\pi}{2}(1-p)\right) = \sin\varphi$, $p=\pm1$ помно.
$\vec{B} = \sin\Theta\nabla\Theta\times\nabla\Phi = \sin\theta\, p\nabla\varphi\times\nabla\Phi = \sin\theta\, p\frac{1}{r}\frac{\partial}{\partial\theta}(\varphi)\hat{\theta}\times\frac{q}{r\sin\theta}\hat{\varphi} =$
$= \frac{pq}{r^2}\hat{\theta}\times\hat{\varphi} = \frac{pq}{r^2}\hat{r}$, где $\vec{r} = (r,\theta,\varphi)$ – сферич. координаты, $\hat{r} = \frac{\vec{r}}{|\vec{r}|}$.
$\vec{B} = \frac{pq}{r^2}\hat{r}$, обоз. $g = pq$, $\vec{B} = \frac{g}{r^2}\hat{r}$ – monopole field with charge g.
Dirac potential $\vec{A} = \frac{g}{r}\frac{(\vec{r}\times\vec{n})}{r-(\vec{r}\vec{n})}$. $\vec{n} = \hat{z}$, $\vec{r}\times\hat{z} = -r\hat{z}\times(\cos\theta\hat{r} - \sin\theta\hat{\theta}) =$
$= -r\sin\theta\,\hat{r}\times\hat{\theta} = -r\sin\theta\hat{\varphi}$, $\hat{r}\times\hat{\theta} = \hat{\varphi}$
$\vec{A} = \frac{g}{r}\frac{-r\sin\theta\hat{\varphi}}{r(1-\cos\theta)}$, $\frac{\sin\theta}{1-\cos\theta} = \frac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} = \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta} = \frac{1+\cos\theta}{\sin\theta}$
$\vec{A} = -\frac{g}{r}\frac{1+\cos\theta}{\sin\theta}\hat{\varphi}$. теперь чтоб $\operatorname{div}\vec{B}=0$ берем $\vec{B} = \vec{B}_g + \vec{B}_{sing}$
$\vec{B}_g = \frac{g}{r^2}\hat{r}$, $\vec{B}_{sing} = -4\pi g\hat{z}\Theta(z)\delta(x)\delta(y)$. Dirac-string вдоль полуоси $z>0$.
$\Theta(z) = 1$, $z>0$ иначе $\Theta=0$.
$Q_H \sim \int dV\vec{A}\cdot\vec{B} = \int dV\left(\frac{-g}{r}\right)\frac{1+\cos\theta}{\sin\theta}\hat{\varphi}\cdot\left[\frac{g}{r^2}\hat{r} - 4\pi g\hat{z}\Theta(z)\delta(x)\delta(y)\right] \equiv 0$ (!), т.к.
$\hat{\varphi}\cdot\hat{z} = (-\sin\varphi\hat{x}+\cos\varphi\hat{y})\cdot\hat{z} \equiv 0$, $\hat{\varphi}\cdot\hat{r} = -\sin\varphi\cdot\sin\theta\cos\varphi + \cos\varphi\sin\theta\sin\varphi \equiv 0$
→ т.е. $\forall\gamma(r) = \text{const}$ $Q_H(BP) \equiv 0$ даже учитывая $\vec{B}_{sing}$ (гарантия $\operatorname{div}\vec{B}=0$).
$\oint_S \vec{B}d\vec{S} = 0$, но $\int dS\hat{r}\cdot\vec{B}_g = \int dS\hat{r}\cdot\frac{g}{r^2}\hat{r} = g\int d\varphi d\theta\sin\theta = 4\pi g$.
$\operatorname{div}\vec{\gamma} = \partial_i\gamma_i = \frac{1}{2}\varepsilon_{ijk}\partial_i\left[\partial_j\vec{m}\times\partial_k\vec{m}\right]\cdot\vec{m} = \frac{1}{2}\varepsilon_{ijk}\left[\partial_i\partial_j\vec{m}\times\partial_k\vec{m}\right]\cdot\vec{m} +$
$+\frac{1}{2}\varepsilon_{ijk}\left[\partial_j\vec{m}\times\partial_i\partial_k\vec{m}\right]\cdot\vec{m} + \frac{1}{2}\varepsilon_{ijk}\left[\partial_j\vec{m}\times\partial_k\vec{m}\right]\cdot\partial_i\vec{m} = \frac{1}{2}\varepsilon_{ijk}\left[\partial_i\partial_j\vec{m}\times\partial_k\vec{m}\right]\cdot\vec{m} +$
$+\frac{1}{2}\varepsilon_{ijk}\left[\partial_i\partial_k\vec{m}\times\partial_j\vec{m}\right]\cdot\vec{m}(-1) + (3) = \frac{3}{2}\varepsilon_{ijk}\left[\partial_i\partial_j\vec{m}\times\partial_k\vec{m}\right]\cdot\vec{m} + \frac{1}{2}\varepsilon_{ijk}\left[\partial_j\vec{m}\times\partial_k\vec{m}\right]\partial_i\vec{m}$
$k\to j$, $j\to k$, $\varepsilon_{ikj} = -\varepsilon_{ijk}$ Не ясно.
$\operatorname{div}\vec{B} = \operatorname{div}\left[\sin\theta\nabla\theta\times\nabla\Phi\right] = -\operatorname{div}\left[\nabla(\cos\theta)\times\nabla\Phi\right] = (-1)\operatorname{rot}(\nabla(\cos\theta))\cdot\nabla\Phi +$
$+\nabla(\cos\theta)\cdot\operatorname{rot}(\nabla\Phi) = 0$, т.к. $\operatorname{rot}(\nabla\Phi)$, если $\Phi(\vec{r})$ дважды непрерывно дифференц.
$\vec{B} = \sin\Theta\nabla\Theta\times\nabla\Phi$ следует из $\vec{m} = (\sin\theta\cos\varphi, \sin\theta\sin\Phi, \cos\theta)$ с $\vec{m}^2 = 1$.

(2) $\gamma(r) \neq \text{const}$. $\nabla\Phi = \nabla(q\varphi) + \nabla\gamma = \frac{q}{r\sin\theta}\hat{\varphi} + \gamma'\hat{r}$, $\gamma' = \frac{\partial\gamma}{\partial r}$
$\vec{B} = \sin\theta\, p\frac{1}{r}\hat{\theta}\times\nabla\Phi = \sin\theta\frac{p}{r}\hat{\theta}\times\left[\frac{q}{r\sin\theta}\hat{\varphi} + \gamma'\hat{r}\right] = \frac{g}{r^2}\hat{r} + \sin\theta\frac{p}{r}\gamma'\hat{\theta}\times\hat{r} =$
$= \frac{g}{r^2}\hat{r} + \sin\theta\frac{p}{r}\gamma'\hat{\varphi} = B_r\hat{r} + B_\varphi\hat{\varphi}$, $B_r = g/r^2$, $B_\varphi = p\gamma'\sin\theta/r$ (-1). см. 28/VIII

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12/VIII Отмена $\int dV A_\varphi B_\varphi$, некорректна. Предполагаем A_φ сохраняет свой вид и $\gamma(r,\varphi)$, $A_\varphi = -\frac{g}{r}\frac{1+\cos\theta}{\sin\theta}$. Тогда $\int dV A_\varphi B_\varphi = \int dV \frac{P}{r}\gamma'\sin\theta \cdot \left(\frac{g}{r}\right)\frac{1+\cos\theta}{\sin\theta} = -Pg\int dr\int d\varphi\int d\theta\, r^2\sin\theta\,\frac{\gamma'}{r^2}(1+\cos\theta) = -2\pi Pg\int dr\gamma'\int d\theta\sin\theta\times(1+\cos\theta)$

$\int_0^\pi d\theta\sin\theta(\cos\theta+1) = \int_0^\pi d\theta\sin\theta = 2 \to \int dV A_\varphi B_\varphi = -4\pi Pg\int dr\gamma'$, $g = Pq$.

Но если взять $\int dV A_r B_r$, A_r из $B_\varphi = -\frac{1}{r}\frac{\partial A_r}{\partial\theta} = \frac{P}{r}\gamma'\sin\theta$

$\frac{\partial A_r}{\partial\theta} = -P\gamma'\sin\theta$, $A_r = +P\gamma'\cos\theta + f(r,\varphi)$. Это все считая $A_\theta = 0$.

$\int dV A_r B_r = \int dV\left[P\gamma'\cos\theta + f(r,\varphi)\right]\frac{g}{r^2} = g\int dr\int d\theta\int d\varphi\, r^2\sin\theta\,\frac{1}{r^2}\left[P\gamma'\cos\theta + f(r,\varphi)\right]$

$\int_0^\pi d\theta\sin\theta\cos\theta = 0$, $\int dV A_r B_r = g\int dr\int d\theta\sin\theta\int d\varphi f(r,\varphi) = 2g\int dr\int_0^{2\pi} d\varphi f(r,\varphi)$

$f(r,\varphi) = f(r,\varphi+2\pi)$ т.к. $A_r(\varphi) = A_r(\varphi+2\pi) \to \int_0^{2\pi} d\varphi f(r,\varphi) \equiv 0$ и $\int dV A_r B_r = 0$.

Kundu'82: $Q_H = -\frac{1}{(8\pi)^2}\int dV\vec{A}\cdot\vec{B}$, $\vec{B} = -2\nabla(\cos\theta)\times\nabla\Phi$. $\vec{B}_k = 2\vec{B}$ (моё)

$Q_H = -\frac{1}{(8\pi)^2}\int dV\vec{A}_k\cdot\vec{B}_k = -\frac{1}{(8\pi)^2}2\cdot 2\int dV\vec{A}\cdot\vec{B}$, $\vec{B} = \sin\theta\nabla\theta\times\nabla\Phi$.

$Q_H = -\frac{1}{(4\pi)^2}\int dV\vec{A}\cdot\vec{B}$ совп. с $M_H = -\frac{1}{(4\pi)^2}\int d^3\vec{r}\,\vec{A}\cdot\vec{B}_{em}$ Göbel PR Res. 2020.

$Q_H = -\frac{1}{(4\pi)^2}\int dV\vec{A}\cdot\vec{B} = -\frac{1}{(4\pi)^2}\int dV A_\varphi B_\varphi = \frac{-4\pi Pg}{-(4\pi)^2}\int dr\gamma' = \frac{Pg}{4\pi}\int dr\gamma'$

$Pg = P\cdot Pq = q$

$Q_H = \frac{q}{4\pi}\int dr\gamma'$ BP Но согласно Kundu'82, Eq. (9): $Q_H = \frac{q}{4\pi}\int dr\int_0^\pi d\theta\sin\theta\times$

$\times(1-P\cos\theta)\left[-\frac{\partial}{\partial\theta}(P\cos\theta)\frac{\partial\gamma}{\partial r}\right] = \frac{Pq}{4\pi}\int dr\int_0^\pi d\theta(1-P\cos\theta)\sin\theta\gamma' = \frac{Pq}{2\pi}\int dr\gamma'$ (?).

Если вычислить в $\vec{B}$ член $\vec{B}_{sing} = 4\pi g\,\theta(z)\delta(x)\delta(y)(-1)\hat{z} = B_s\hat{z} = B_s[\cos\theta\hat{r} - \sin\theta\hat{\theta}]$ см. 11/VIII.

$B_{sing}^r = B_s\cos\theta$, $B_s = -4\pi g\,\theta(z)\delta(x)\delta(y)$, $B_r = \frac{g}{r^2} + B_s\cos\theta$

$\int dV A_r B_r = \int dV\left[P\gamma'\cos\theta + f(r,\varphi)\right]\left[\frac{g}{r^2} + B_s\cos\theta\right] = \int dr\int d\theta\int d\varphi\, r^2\sin\theta\left[P\gamma'\cos\theta + f\right]\times$

$+\left[\frac{g}{r^2} + B_s\cos\theta\right] = \int dr\int d\theta\int d\varphi\left[P\gamma'\cos\theta + f\right]\left[g + B_s r^2\cos\theta\right]\sin\theta = \int dr\int d\theta\int d\varphi\sin\theta\times$

$\times P\gamma'\cos\theta\left[g + B_s r^2\cos\theta\right]$, $B_s r^2 = (-4\pi g)\,\theta(z)\,\delta(x)\delta(y)r^2 = -4\pi g\,\theta(z)z^2\delta(x)\delta(y) =$

$= -4\pi g\,\theta(z)\delta(x)\delta(y)\frac{\cos^2\theta}{r^2}r^2$, $\int d\theta\sin\theta\cos\theta\left[g + B_s'\cos^3\theta\right] =$

$= -\int d(\cos\theta)\left[g\cos\theta + B_s'\cos^4\theta\right] = -\frac{g}{2}\cos^2\theta\Big|_0^\pi + \frac{1}{5}B_s'\cos^5\theta\Big|_0^\pi$

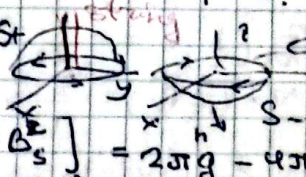
Лучше $\int d\theta\sin\theta\cos\theta\left[g + B_s r^2\cos\theta\right] = -\frac{g}{2}\cos^2\theta - \frac{1}{3}B_s r^2\cos^3\theta\Big|_0^\pi = \frac{2}{3}B_s r^2$

$\int dV A_r B_r = \int dr\int d\varphi P\gamma'\frac{2}{3}B_s r^2 = \frac{2}{3}2\pi P\int dr\gamma' B_s r^2 = \frac{4\pi}{3}P\int dr\gamma' r^2(-4\pi g)\theta(z)\delta(x)\delta(y)$

$\int dr\int d\theta\int d\varphi\sin\theta\cos\theta\left[g + (-4\pi g)\theta(z)\delta(x)\delta(y)r^2\cos\theta\right] = \int dr d\theta d\varphi\sin\theta\cos\theta(-4\pi g)\theta(z)$

$\times\delta(x)\delta(y)\left[\delta(x)\delta(y)\right]^0\cos\theta = \left|\delta(x)\delta(y)\text{ оставляем только }\theta=0\right| = 0$. $\int dr A_r B_r = 0$.

Теперь Gauss $\int d\vec{S}\cdot\nabla\times\vec{A} = \oint\vec{A}d\vec{l} = \int d\vec{S}\cdot\vec{B}$



$\int_{S+} d\vec{S}\cdot\vec{B} = \int_{S+} d\vec{S}\cdot\left[\vec{B}_g + \vec{B}_{sing}\right] = \int_{S+} dS\left[B_g^z + B_s^z\right] = 2\pi g - 4\pi g = -2\pi g$

$\int_{S-} = \int d\vec{S}\cdot\vec{B} = \int d\vec{S}\vec{B}_g = 2\pi g$ $\oint_S d\vec{S}\vec{B} = \int_{S+} d\vec{S}\cdot\vec{B} + \int_{S-} d\vec{S}\cdot\vec{B} = -2\pi g + 2\pi g = 0$

$\oint\vec{A}d\vec{l} = \int_0^{2\pi} r\,d\varphi A_\varphi = \int_0^{2\pi} d\varphi r\left(-\frac{g}{r^2}\right)\frac{1+\cos\theta}{\sin\theta} = \left|\theta=\pi/2\right| = -2g\int_0^{2\pi} d\varphi = -g\,2\pi = -2\pi g$

$\int_{S+} d\vec{S}\cdot\vec{A} = \oint\vec{A}d\vec{l}$. Напомним, что $A_\varphi = -\frac{g}{r}\frac{1+\cos\theta}{\sin\theta}$ for any A_r, A_θ.

$A_r = P\frac{\partial\gamma}{\partial r}\cos\theta + f(r,\theta)$ - любое $f(r)$ неважно, если A_θ такое.

$\int_{S+} d\vec{S}\cdot\vec{B} = \int_{S+} dS\,B_r = \oint_c\vec{A}d\vec{l} = \int_0^{2\pi} d\varphi r A_\varphi$, $B_\varphi \sim \gamma'$ и $A_r \sim \gamma'$ неважны.

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T. e. Vector potential component $A_\varphi = -\frac{q}{r}\frac{\cos\theta+1}{\sin\theta}$ one gauge-wise equiv $\gamma = \gamma(\varphi)$ и $\nabla\Phi = \frac{1}{r\sin\theta}\hat{\varphi} + \ldots = (\ldots) + \gamma'\hat{\varphi}$, так что B_z не зависит от γ.

• F. Wilczek, A. Zee PRL 51 (1983) 2250

Hopf invariant $H = -\frac{1}{2\pi}\int d^3x\, A_\mu J^\mu$, $J^\mu = \frac{1}{8\pi}\varepsilon^{\mu\nu\lambda}\varepsilon^{abc} n^a \partial_\nu n^b \partial_\lambda n^c$

$J^\mu = \frac{1}{8\pi}\varepsilon^{\mu\nu\lambda}\vec{n}\cdot(\partial_\nu\vec{n}\times\partial_\lambda\vec{n})$ - topol. current, $J^0 = \frac{1}{8\pi}\varepsilon^{ij}\varepsilon^{abc} n^a \partial_i n^b \partial_j n^c$

$J^0 = \frac{1}{8\pi}\varepsilon^{ij}\vec{n}\cdot(\partial_i\vec{n}\times\partial_j\vec{n})$, $Q = \int d^2x\, J^0$ - skyrmion number, $Q = 1$ (±1).

Но $\vec{B} = \vec{J}$, $B_i = \frac{1}{2}\varepsilon_{ijk}\vec{m}\cdot(\partial_j\vec{m}\times\partial_k\vec{m})$. Видимо $J^\mu = \frac{1}{4\pi}B_{\mu\nu}$ (?) ($\mu=0,1,2$ и $i=1,2$)

Т. е. Q завершено 2D (там $i,j=1,2$; $i,j=x,y$) и $Q = \int dx_1 dx_2\, J^0$.

Но для 3D - проблем $B_i = \frac{1}{2}\varepsilon_{ijk}\vec{m}(\partial_j\vec{m}\times\partial_k\vec{m})$, $i,j,k=1,2,3$; $i,j,k = x,y,z$

Т. е. совпадение ф-л Wilczek'83 надо еще доделать, см. 31/V 2018.

Kundu'82: $m_z = \cos\Theta = w(r,\varphi)$, $\Phi = \operatorname{arctg}\frac{m_y}{m_x} = q\varphi + v(r,\varphi)$, $v(r,\varphi) = \gamma(r)$ мое

$\vec{B} = -2\nabla(\cos\Theta)\times\nabla\Phi$, часть уравнения как Комнеевы'96, Фарнем.93

Kundu $\vec{B} = -\frac{2q}{r^2\sin\Theta}\left[\hat{r}\frac{\partial w}{\partial\varphi} - \hat{\varphi}r\frac{\partial w}{\partial r}\right] - 2\frac{1}{r}\hat{\varphi}\left[\frac{\partial w}{\partial r}\frac{\partial v}{\partial\varphi} - \frac{\partial w}{\partial\varphi}\frac{\partial v}{\partial r}\right]$ $\vec{J} = \vec{B} = -\nabla(\cos\Theta)\times\nabla\Phi$

Kundu $B_\varphi = -\frac{2}{r}\left[\frac{\partial w}{\partial r}\frac{\partial r}{\partial\varphi} - \frac{\partial w}{\partial\varphi}\frac{\partial r}{\partial r}\right]$, $Q_H = \frac{q}{4\pi}\int dr\int d\varphi\,(1-w)\left[\frac{\partial w}{\partial r}\frac{\partial v}{\partial\varphi} - \frac{\partial w}{\partial\varphi}\frac{\partial v}{\partial r}\right]$

$= \frac{q}{4\pi}\int dr\int d\varphi\,(1-w)\left(\frac{-r}{2}\right)B_\varphi = -\frac{q}{8\pi}\int dz\int_0^\pi d\varphi\,(1-w)\,B_\varphi(k)$, $\vec{B}(k) = 2\vec{B}$ (мое)

$Q_H = -\frac{q}{4\pi}\int dz\, r\int d\varphi\,(1-\cos\Theta)\,B_\varphi$ совпало Kundu'82.

$\cos\Theta = \rho\cos\varphi$, B_φ (мое) $= \rho\gamma'\sin\varphi/r$. $Q_H = -\frac{q}{4\pi}\int dz\int d\varphi\,(1-\rho\cos\varphi)\frac{\rho}{r}\gamma'\sin\varphi =$

$= -\frac{q\rho}{4\pi}\int dz\,\gamma'\int_0^\pi d\varphi\,\sin\varphi\,(1-\rho\cos\varphi) = -\frac{q\rho}{2\pi}\int dz\,\gamma'$, т.к. $\int_0^\pi d\varphi\sin\varphi = 2$.

$Q_H = -\frac{q\rho}{2\pi}\int dz\,\gamma'$, 11/VIII было $Q_H = \frac{\rho q}{2\pi}\int dz\,\gamma'$ - совпало Kundu'82.

$B_\varphi = \frac{1}{2}B_\varphi(k) = -\frac{1}{r}\left[\frac{\partial m_z}{\partial r}\frac{\partial\gamma}{\partial\varphi} - \frac{\partial m_z}{\partial\varphi}\frac{\partial\gamma}{\partial r}\right] = \frac{1}{r}\frac{\partial m_z}{\partial\varphi}\gamma' = \frac{\gamma'}{r}\frac{\partial}{\partial\varphi}(\rho\cos\varphi) =$

$= -\frac{\rho}{r}\sin\varphi\,\gamma'$ - то ф-ле Kundu'82. Т. е. из $\vec{B} = \sin\Theta\nabla\Theta\times\nabla\Phi$, $B_\varphi = \rho\gamma'\sin\varphi/r$.

Верно ли у Kundu $\vec{B} = \frac{2}{m_z}\nabla m_x\times\nabla m_y$ дающее Eq. (8)? Да.

$Q_H = -\frac{2}{(8\pi)^3}\int dV dV'\frac{\vec{B}'\cdot(\vec{R}\times\vec{B})}{R^3}$ (Kundu) $= \left|\vec{B}(k) = 2\vec{B}\right| = -\frac{8}{(8\pi)^3}\int dV dV'\vec{B}'\cdot(\vec{R}\times\vec{B})$ (мое) $\frac{1}{R^3}$

$\frac{8}{(8\pi)^3} = \frac{8}{8^3\pi^3} = \frac{1}{8\cdot\pi^3} = \frac{1}{4\pi}\frac{1}{(4\pi)^2}$, $Q_H = -\frac{1}{(4\pi)^2}\int dV dV'\vec{B}'\cdot(\vec{R}\times\vec{B})\frac{1}{R^3}$

$\int dV\,\vec{A}\cdot\vec{B} = \int dV\vec{B}\cdot\frac{1}{4\pi}\int dV'\frac{\nabla'\times\vec{B}(r')}{|r-r'|} = \frac{1}{4\pi}\int dV\int dV'\vec{B}\cdot\frac{\nabla'\times\vec{B}(r')}{|r-r'|}$ $\vec{R} = \vec{r}-\vec{r}'$

$\frac{1}{R}\nabla_R$? $\operatorname{rot}\left(\frac{1}{r}\vec{J}\right) \Rightarrow \nabla\times(\Psi\vec{A}) = \Psi\nabla\times\vec{A} + \nabla\Psi\times\vec{A}$: см. 20/VIII

$\operatorname{rot}\left(\frac{1}{r}\vec{B}\right) = \frac{1}{r}\operatorname{rot}\vec{B} + \nabla\left(\frac{1}{r}\right)\times\vec{B} = \frac{1}{r}\operatorname{rot}\vec{B} - \frac{\vec{r}}{r^3}\times\vec{B}$

$\nabla\left(\frac{1}{r}\right) = \frac{\partial}{\partial r}\left(\frac{1}{r}\right)\hat{r} = -\frac{1}{r^2}\hat{r} = -\frac{\vec{r}}{r^3}$. $\frac{1}{r}\operatorname{rot}\vec{B} = \operatorname{rot}\left(\frac{1}{r}\vec{B}\right) + \frac{\vec{r}}{r^3}\times\vec{B}$

$\frac{1}{r}\nabla\times\vec{B} = \nabla\times\left(\frac{1}{r}\vec{B}\right) + \frac{\vec{r}}{r^3}\times\vec{B}$, $\frac{\nabla'\times\vec{B}(r')}{|r'-r|} = \nabla'\times\left(\frac{1}{|r'-r|}\vec{B}(r')\right) + \frac{(\vec{r}'-\vec{r})}{|r'-r|^3}\times\vec{B}(\vec{r}')$ -

- если 1-ую функцию, то $\frac{\nabla'\times\vec{B}(r')}{R} = -\frac{\vec{R}}{R^3}\times\vec{B}(\vec{r}')$ - см. 21/VIII

$\int dV\vec{A}\cdot\vec{B} = \frac{1}{4\pi}\int dV dV'\vec{B}\cdot(-1)\frac{1}{R^3}\vec{R}\times\vec{B}(\vec{r}')$ (?), $\vec{B}\cdot[\vec{R}\times\vec{B}'] = \vec{B}'\cdot(\vec{R}\times\vec{B})(-1)$

$\int dV\vec{A}\vec{B} = +\frac{1}{4\pi}\int dV dV'\frac{1}{R^3}\vec{B}'\cdot(\vec{R}\times\vec{B})$, $Q_H = -\frac{1}{(4\pi)^2}\int dV\vec{A}\cdot\vec{B} = -\frac{1}{(4\pi)^3}\int dV dV'\frac{\vec{B}'\cdot(\vec{R}\times\vec{B})}{R^3}$

Eq (7) Q_H - kundu верно, если $\vec{B}\cdot\nabla'\times\left[\frac{1}{|r-r'|}\vec{B}(\vec{r}')\right]$ функция.

$\int dV\,\nabla\times\vec{A} = -\oint_S\vec{A}\times d\vec{S}$, $\int dV'\nabla'\times\left[\frac{1}{R}\vec{B}(\vec{r}')\right] = \int dV\nabla\times\left[\frac{1}{R}\vec{B}(\vec{r}')\right] =$

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$= \oint_{S=\partial V} \frac{1}{R}\vec{B}(\vec{r})\times d\vec{S} = 0$ если $\vec{B}(\vec{r})$ достаточно быстро падает с $|\vec{r}|=r$ росом.

Рапаниколаou'93 указал, что $\vec{B}(\vec{r}) = \gamma(\vec{r})$ должна падать быстрее, чем $\frac{1}{r^3}$.

$B(\vec{r}) \sim \frac{1}{r^{3+d}}$, $r\to\infty$ соответствует $\vec{m}(\vec{r}) \xrightarrow{r\to\infty} \vec{m}_0 = const$, т.е. localized soliton. $\vec{m}(\vec{r}) = \vec{m}_0 + \frac{c}{r^n}$, $\frac{\partial \vec{m}}{\partial r} \underset{r\to\infty}{\sim} \frac{1}{r^{n+1}}$ $n > 1/2$

$B(\vec{r}) \sim \left(\frac{\partial \vec{m}}{\partial r}\right)^2 \sim \frac{1}{r^{2n+2}}$, $2n+2 > 3$, $n > 1/2$, $\vec{m}(r) - \vec{m}_0 \sim \frac{1}{r^n}$.

Поэтому $Q_H = -\frac{1}{(4\pi)^2}\int dV \vec{A}\cdot\vec{B}$ [Eq.(3) Kundu] является общей а

($Q_H = -\frac{1}{(4\pi)^3}\int dV \int dV' \frac{1}{R^3}\vec{B}'\cdot(\vec{R}\times\vec{B})$ [Eq.(7) Kundu, Eq.(3.7) Komin.'96] является частным случаем, верным лишь для localized solitons с $\vec{B}(\vec{r}) \underset{r\to\infty}{\sim} \frac{1}{r^{3+d}}$

16 сент. 21/VIII у меня для ВР $\vec{B} \sim \frac{1}{r^2}$ ($B_r \sim 1/r^2$, $B_\theta \sim \gamma'/r$) и Eq.(7)-Kundu, Eq.(3.7)-Komin. не работают. Неудивительно, что мой Q_H-расчет для ВР-soliton не совпал с расчетом по Eq.(7),(9)-Kundu'82. ВР не есть localized

// Видимо $Q_H = -\frac{1}{(4\pi)^2}\int dV \vec{A}\cdot\vec{B}$ is integer, только если $\vec{m}(\vec{r}) \xrightarrow[r\to\infty]{Solit.} \vec{m}_0$ - для localized solitons (!). E(local.s.) - конечна.

// Energy of BP $\to\infty$ при soliton volume size $R\to\infty$, $E_{ex} \sim R$. (sphere radius)

Это неявно есть в Döring'68 и явно в Rybporvskyi'2012.

→ но с E(BP) нет проблем, подобно E(vortex) все ок in restricted geometry, dots, wires, etc.

→ Вывод: Q_H (localized soliton, ∞ sample) is integer, Q_H (BP, $\gamma(r)$) - любое, особенно in restricted geometry. Но для $Q_H(BP) \neq 0$ надо $\gamma = \gamma(r)$.

$Q_H(BP) = \frac{q}{4\pi}\int dr\gamma'$, $\gamma' = \frac{\partial\gamma}{\partial r}$. $\Phi = q\varphi + \gamma(r)$.

Если для localized soliton $\vec{B} \sim \frac{1}{r^{3+d}}$, $r\to\infty$, то $E(local.s.) \sim \int_0^R \frac{dr r^2}{r^{3+d}} \sim \frac{1}{R^d}$ - конечна при $R\to\infty$ (infinite sample).

14/VIII Написал Оксане и Jose, что Q_H (dots Bergamza) надо считать по Eq.(3.6)-Komin, а не по Eq.(3.7), которое неприемлемо для finite samples

Верно ли $\vec{B} = \frac{1}{m_z}\nabla m_x \times \nabla m_y$? $m_x = \sin\theta\cos\varphi$, $m_y = \sin\theta\sin\varphi$ Да

$\nabla m_x \times \nabla m_y = \nabla(\sin\theta\cos\varphi)\times\nabla(\sin\theta\sin\varphi) = [\cos\theta\cos\varphi\nabla\theta - \sin\theta\sin\varphi\nabla\varphi]\times$

$\times[\cos\theta\sin\varphi\nabla\theta + \sin\theta\cos\varphi\nabla\varphi] = [\cos\theta\sin\theta\cos^2\varphi + \sin\theta\cos\theta\sin^2\varphi](\nabla\theta\times\nabla\varphi) =$

$= \sin\theta\cos\theta(\nabla\theta\times\nabla\varphi)$, $\vec{B} = \frac{1}{\cos\theta}\sin\theta\cos\theta(\nabla\theta\times\nabla\varphi) = \sin\theta(\nabla\theta\times\nabla\varphi)$ - верно.

Ур-я Slonczewski'1979 ($\alpha\mu_0 K = 0$):

$$\begin{cases} \frac{\delta W}{\delta\theta} = A(\nabla\varphi)^2\sin 2\theta - 2A\nabla^2\theta - \vec{H}\frac{\partial\vec{M}}{\partial\theta} = 0 \\ \frac{\delta W}{\delta\varphi} = -2A(\nabla^2\varphi)\sin^2\theta - 2A(\nabla\varphi\cdot\nabla\theta)\sin 2\theta - \vec{H}\frac{\partial\vec{M}}{\partial\varphi} = 0 \end{cases}$$

ВР $\begin{cases}\theta = p\varepsilon + \frac{\pi}{2}(1-p) \\ \varphi = q\varphi + \gamma(r)\end{cases}$, $\sin\theta = \sin\left[p\varepsilon + \frac{\pi}{2}(1-p)\right] = \sin(p\varepsilon)\cos\left(\frac{\pi}{2}(1-p)\right) = \sin\varepsilon$

$\cos\theta = \cos\left[p\varepsilon + \frac{\pi}{2}(1-p)\right] = \cos(p\varepsilon)\cos\left(\frac{\pi}{2}(1-p)\right) = p\cos\varepsilon$, $\sin 2\theta = p\sin 2\varepsilon$

$\nabla\theta = p\nabla\varepsilon = p\frac{1}{r}\hat{\varepsilon}$, $\nabla\varphi = \frac{1}{r\sin\varepsilon}\hat{\varphi}\frac{\partial}{\partial\varphi}\Phi + \hat{r}\frac{\partial\Phi}{\partial r} = \frac{q}{r\sin\varepsilon}\hat{e} + \frac{\partial\gamma}{\partial r}\hat{r}$, $\vec{r} = (r,\varepsilon,\varphi)$

$\nabla^2\theta = \nabla\cdot\left(\frac{p}{r}\hat{\varepsilon}\right) = \mathrm{div}\left(\frac{p}{r}\hat{\varepsilon}\right) = \frac{1}{r\sin\varepsilon}\frac{\partial}{\partial\varepsilon}\left[\frac{p}{r}\sin\varepsilon\right] = \frac{p}{r^2}\mathrm{ctg}\,\varepsilon$

$\nabla^2\varphi = \mathrm{div}(\nabla\varphi) = \mathrm{div}\left[\gamma'\hat{r} + \frac{q}{r\sin\varepsilon}\hat{\varphi}\right] = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2\gamma'] + \frac{1}{r\sin\varepsilon}\frac{\partial}{\partial\varepsilon}\left[\frac{q}{r\sin\varepsilon}\right] =$

$\nabla\varphi\cdot\nabla\theta = \left[\gamma'\hat{r} + \frac{q}{r\sin\varepsilon}\hat{\varphi}\right]\cdot\left[\frac{p}{r}\hat{\varepsilon}\right] = 0$ $= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\gamma')$

$A\left(\frac{q^2}{r^2\sin^2\varepsilon} + \gamma'^2\right)p\sin 2\varepsilon - 2A\frac{p}{r^2}\mathrm{ctg}\,\varepsilon = \vec{H}\cdot\frac{\partial\vec{M}}{\partial\theta}$, $\theta = \theta(\vec{r})$, $\Phi = \Phi(\vec{r})$

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$-2A\frac{1}{r^2}\frac{\partial}{\partial\theta}(r^2\gamma')\sin^2\theta = \vec{H}\cdot\frac{\partial\vec{M}}{\partial\varphi}$ Eg W_m, $\frac{\partial}{\partial r}(r^2\gamma')=0$, $\gamma'(r)=\frac{C}{r^2}$

$A\left[\frac{q^2}{r^2\sin^2\theta}+\frac{C^2}{r^4}\right]p\sin^2\theta = 2A\frac{p}{r^2}\operatorname{ctg}\theta$, откуда $C=0$

$\frac{q^2}{r^2}\frac{2\sin\theta\cos\theta}{\sin^2\theta} = 2\frac{q^2}{r^2}\operatorname{ctg}\theta = \frac{2}{q}\operatorname{ctg}\theta$, $q=1$: $\gamma'(r)=0$, $\gamma(r)=\text{const}$

т.е. Eg magnetostatics W_m, $\gamma'(r)=\gamma_0=\text{const}$ и $q=1 \Rightarrow Q_M=0$ (BP).

Для среды W_m, $\vec{H}_m$ in spherical coordinates чтобы решить Green

$G_{\alpha\beta}(\vec{r},\vec{r}')$ с $\alpha,\beta = r,\theta,\varphi$ – magnetostatic Green functions. $G_{\alpha\beta} = -\frac{\partial}{\partial x_\alpha}\frac{\partial}{\partial x'_\beta}\frac{1}{|\vec{r}-\vec{r}'|}$

$\vec{m} = m_r\hat{r}+m_\theta\hat{\theta}+m_\varphi\hat{\varphi}$; $\vec{m} = m_x\hat{x}+m_y\hat{y}+m_z\hat{z} = \sin\theta\cos\Phi(\sin\theta\cos\varphi\cdot\hat{r}+\cos\theta\cos\varphi\hat{\theta}-\sin\varphi\hat{\varphi})+\sin\theta\sin\Phi(\sin\theta\sin\varphi\hat{r}+\cos\theta\sin\varphi\hat{\theta}+\cos\varphi\hat{\varphi})+\cos\theta(\cos\theta\hat{r}-\sin\theta\hat{\theta}) = \sin\theta\cos\Phi[\sin\theta\cos\theta\hat{r}+\cos\theta\cos\varphi\hat{\theta}-\sin\varphi\hat{\varphi}]+\sin\theta\sin\Phi[\sin\theta\sin\varphi\hat{r}+\cos\theta\sin\varphi\hat{\theta}+\cos\varphi\hat{\varphi}]+p\cos\theta[\cos\theta\hat{r}-\sin\theta\hat{\theta}]$

$\begin{cases} m_r = \sin^2\theta\cos(\Phi-\varphi)+p\cos^2\theta = \sin^2\theta\cos\gamma+p\cos^2\theta \\ m_\theta = \sin\theta\cos\theta\cos(\Phi-\varphi)-p\sin\theta\cos\theta = \sin\theta\cos\theta\cos\gamma-p\sin\theta\cos\theta \\ m_\varphi = \sin\theta\sin(\Phi-\varphi) = \sin\theta\sin\gamma \end{cases}$ BP magnetost.

т.к. $\Phi = q\varphi+\gamma$, $(q=1)$.

$\operatorname{div}\vec{m} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2m_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(m_\theta\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial m_\varphi}{\partial\varphi} =$

$= \frac{\sin^2\theta}{r^2}\frac{\partial}{\partial r}(r^2\cos\gamma)+\frac{1}{r^2}\frac{\partial}{\partial r}(p r^2\cos^2\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}[\sin^2\theta\cos\theta(\cos\gamma-p)]+$

$+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(\sin\theta\sin\gamma) = \frac{\sin^2\theta}{r^2}\frac{\partial}{\partial r}(r^2\cos\gamma)+\frac{p\cos^2\theta}{r^2}\cdot 2r+\frac{(\cos\gamma-p)}{r\sin\theta}\frac{\partial}{\partial\theta}[\sin^2\theta\cos\theta]$

$\frac{\partial}{\partial\theta}(\sin^2\theta\cos\theta) = 2\sin\theta\cos\theta\cdot\cos\theta+\sin^2\theta(-\sin\theta) = \sin\theta[2\cos^2\theta-\sin^2\theta] =$

$\Leftarrow = \sin\theta[3\cos^2\theta-1]$

$\operatorname{div}\vec{m} = \frac{\sin^2\theta}{r^2}\frac{\partial}{\partial r}(r^2\cos\gamma)+\frac{2p}{r}\cos^2\theta+\frac{1}{r}(\cos\gamma-p)(3\cos^2\theta-1)$, $\operatorname{div}\vec{m} = f(\cos\gamma)$

Сферы, $\vec{n}=\hat{r}$, $\operatorname{Div}\vec{m} = (\vec{m}\cdot\vec{n}) = m_r = \sin^2\theta\cos\gamma+p\cos^2\theta$ – surface charge.

Пусть есть volume m. charges $(R\to\infty)$ $W_m^{vol}(\gamma) = \frac{1}{2}\int dV dV'\frac{\rho(\vec{r})\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}$

$\rho(\vec{r}) = -\mu_0\operatorname{div}\vec{m}(\vec{r})$, $\frac{1}{|\vec{r}-\vec{r}'|} = 4\pi\sum_{lm}\frac{1}{(2l+1)}\frac{r_<^l}{r_>^{l+1}}Y^*_{lm}(\theta',\varphi')Y_{lm}(\theta,\varphi)$

$W_m^{vol}(\gamma) = \frac{1}{2}\mu_0^2\int dr\int d\theta\int d\varphi\, r^2\sin\theta\int dr'\int d\theta'\int d\varphi' r'^2\sin\theta'\frac{\operatorname{div}\vec{m}(\vec{r})\operatorname{div}\vec{m}(\vec{r}')}{|\vec{r}-\vec{r}'|} =$

$= \frac{1}{2}\mu_0^2\int dV\int dV'\operatorname{div}\vec{m}(\vec{r})\operatorname{div}\vec{m}(\vec{r}')4\pi\sum_{lm}\frac{1}{(2l+1)}\frac{r_<^l}{r_>^{l+1}}Y^*_{lm}(\theta',\varphi')Y_{lm}(\theta,\varphi)$

$q_{lm}(r') = \int d\theta\int d\varphi\sin\theta\, Y_{lm}(\theta,\varphi)\operatorname{div}\vec{m}(\vec{r}')$. $\operatorname{div}\vec{m}$ не зависит от φ.

$W_m = \frac{1}{2}\mu_0^2\int dr r^2\int dr' r'^2 4\pi\sum_{lm}\frac{1}{(2l+1)}\frac{r_<^l}{r_>^{l+1}}q_{lm}(r)q^*_{lm}(r')$

$\int d\theta Y_{lm}(\theta,\varphi) = \int d\varphi\sqrt{\frac{2l+1}{4\pi}}\sqrt{\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)e^{im\varphi} = 2\pi\sqrt{\frac{2l+1}{4\pi}}P_l^0(\cos\theta)\delta_{m0}$

$q_{lm}(r) = \int d\theta\sin\theta\operatorname{div}\vec{m}(r)\cdot 2\pi\sqrt{\frac{2l+1}{4\pi}}P_l(\cos\theta)\delta_{m0} = q_l\delta_{m0}$ (r)

$W_m = \frac{1}{2}\mu_0^2\int dr r^2\int dr' r'^2 4\pi\sum_l\frac{1}{(2l+1)}\frac{r_<^l}{r_>^{l+1}}q_l(r)q_l(r')$.

$q_l(r) = 2\pi\sqrt{\frac{4l+1}{4\pi}}\int d\theta\sin\theta P_l(\cos\theta)\operatorname{div}\vec{m}(\vec{r})$, умножим $2\pi\sqrt{\frac{2l+1}{4\pi}}$ на q_l.

$W_m = \frac{1}{2}\mu_0^2(2\pi)^2\int dr r^2\int dr' r'^2\sum_l\frac{r_<^l}{r_>^{l+1}}q_l(r)q_l(r')$, $q_l(r) = \int d\theta\sin\theta P_l(\cos\theta)\operatorname{div}\vec{m}(\vec{r})$

Для осевой симметрии (нет φ-зависимости) $\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{r_>}\sum_l\left(\frac{r_<}{r_>}\right)^l P_l(\cos\theta)$, $\theta = \hat{r}\hat{r}'$

$\operatorname{div}\vec{m}(\vec{r}) = \frac{2p}{r}-\sin^2\theta\sin\gamma\cdot\gamma'+\frac{1}{r}(\cos\gamma-p)(1+\cos^2\theta)$, $\theta\to\theta$ здесь.

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$P_2(x) = \frac{1}{2}(3x^2-1)$, $x = \cos\theta$, $P_0(x) = 1$. $\cos^2\theta = \frac{1}{3}(2P_2(x)+1)$, $\sin^2\theta = \frac{2}{3} - \frac{2}{3}P_2(x)$

$\operatorname{div}\vec{m}(\vec{r}) = \frac{2\rho}{r} - \sin\gamma\gamma' \frac{2}{3}[1-P_2(x)] + \frac{1}{r}(\cos\gamma - \rho)\left[1 + \frac{2}{3}P_2 + \frac{1}{3}\right]$

$\operatorname{div}\vec{m}(\vec{r}) = \left[\frac{2\rho}{r} - \frac{2}{3}\sin\gamma\gamma' + \frac{1}{r}(\cos\gamma-\rho)\frac{4}{3}\right]P_0(x) + \left[\frac{2}{3}\sin\gamma\gamma' + \frac{2}{3}\frac{1}{r}(\cos\gamma-\rho)\right]P_2(x)$

$\operatorname{div}\vec{m}(\vec{r}) = a_0 P_0(\cos\theta) + a_2 P_2(\cos\theta)$, $a_0 = -\frac{2}{3}\sin\gamma\gamma' + \frac{1}{r}\frac{4}{3}\cos\gamma + \frac{2}{3}\frac{\rho}{r}$

$a_2 = \frac{2}{3}\left[\sin\gamma\gamma' + \frac{1}{r}(\cos\gamma - \rho)\right]$

$q_\ell(r) = \int_0^\pi d\theta \sin\theta\, P_\ell(\cos\theta)\left[a_0 P_0(\cos\theta) + a_2 P_2(\cos\theta)\right]$, $\int_0^\pi d\theta\sin\theta = -\int_0^\pi d(\cos\theta) = \int_{-1}^{1} dx (\dots)$, $x = \cos\theta$

$\to \int_{-1}^{1} dx\, P_\ell(x) P_{\ell'}(x) = \frac{2}{(2\ell+1)}\delta_{\ell\ell'}$

$q_\ell(r) = a_0 2\delta_{\ell 0} + a_2 \frac{2}{5}\delta_{\ell 2}$. $q_\ell(r) = 2\left[\frac{4}{3}\frac{1}{r}\cos\gamma + \frac{2}{3}\frac{\rho}{r} - \frac{2}{3}\sin\gamma\gamma'\right]\delta_{\ell 0} +$

$+ \frac{2}{5}\frac{2}{3}\left[\sin\gamma\gamma' + \frac{1}{r}(\cos\gamma - \rho)\right]\delta_{\ell 2}$. $q_\ell(r) q_{\ell'}(r') = (2a_0\delta_{\ell 0} + \frac{2}{5}a_2\delta_{\ell 2})(2a_0'\delta_{\ell' 0} + \frac{2}{5}a_2'\delta_{\ell' 2}) =$

$= 4a_0 a_0'\delta_{\ell 0}\delta_{\ell' 0} + \frac{4}{25}a_2 a_2'\delta_{\ell 2}\delta_{\ell' 2}$

$W_m(\gamma) = 2\pi^2 M_s^2 \int_0^R dr\, r^2 \cdot \int_0^R dr'\, r'^2 \left[4a_0(r)a_0'(r')\frac{1}{r_>} + \frac{4}{25}a_2(r)a_2(r')\frac{r_<^2}{r_>^3}\right]$

$\int_0^R dr\, r^2 \int_0^R dr'\, r'^2 \frac{1}{r_>} = \int_0^R dr\, r^2 \left[\int_0^r dr'\, r'^2 \frac{1}{r} + \int_r^R dr'\, r'^2 \frac{1}{r'}\right] =$

$= \int dr\, r^2 \left[\frac{1}{r}\frac{r'^3}{3}\Big|_0^r + \frac{1}{2}r'^2\Big|_r^R\right] = \int_0^R dr\, r^2\left[\frac{1}{3}r^2 + \frac{1}{2}(R^2 - r^2)\right]$ — нас, будет $a_{0,2} = f(r)$.

$W_m(\gamma) = 2\pi^2 M_s^2 \int_0^R dr\, r^2 \int_0^R dr'\, r'^2 \left[4a_0(r)a_0(r')\frac{1}{r_>} + \frac{4}{25}a_2(r)a_2(r')\frac{r_<^2}{r_>^3}\right] =$

$= (\cdot)\int dr\, r^2 \left[\int_0^r dr'\, r'^2 \left\{4a_0(r)a_0(r')\frac{1}{r} + \frac{4}{25}a_2(r)a_2(r')\frac{r'^2}{r^3}\right\} + \int_r^R dr'\, r'^2 \times\right.$

$\left. \times \left\{4a_0(r)a_0(r')\frac{1}{r'} + \frac{4}{25}a_2(r)a_2(r')\frac{r^2}{r'^3}\right\}\right]$. $\int_0^r dr'\, r'^2 a_0(r') = \int_0^r dr\, r^2 a_0(r) = f_1(r)$

$\int_0^r dr'\, r'^4 a_2(r') = f_2(r)$, $\int_r^R dr'\, r'^2 a_0(r')\frac{1}{r'} = f_3(r)$, $\int_r^R dr'\, r'^2 a_2(r')\frac{1}{r'^3} = f_4(r)$

$W_m(\gamma) = 2\pi^2 M_s^2 \int_0^R dr\, r^2 \left[4a_0(r)\frac{f_1(r)}{r} + \frac{4}{25}a_2(r)\frac{f_2(r)}{r^3} + 4a_0(r)f_3(r) + \frac{4}{25}a_0(r)r^2 f_4(r)\right]$

каждая из $f_n(r)$ есть функция γ и γ'.

$W_m(\gamma) = 2\pi^2 M_s^2 \int_0^R dr\, r^2 \left[4a_0(r)\left\{\frac{f_1(r,\gamma)}{r} + f_3(r,\gamma)\right\} + \frac{4}{25}a_2(r)\left\{\frac{f_2(r,\gamma)}{r^3} + r^2 f_4(r,\gamma)\right\}\right]$,

итого $W_m[\gamma(r)]$ — функционал. Ищем $\gamma(r) = \text{const}$ — должны получить из $\min W(\gamma)$, $W(\gamma) = W_{ex}(\gamma) + W_m(\gamma)$, $\gamma = \gamma_0 = 112^\circ$ или 113°, Döring '68 Elias + 11.

Проблема $\frac{1}{|\vec{r}-\vec{r}_i|}$ разлагается по $\frac{r_<}{r_>}$ — тяжело.

(15/VIII) Не имеет смысла с тяжелыми $W_m(\gamma)$, а сразу найти $\gamma(r)$ через $H_m(\gamma)$.

$\frac{\partial\vec{m}}{\partial\varphi} = \frac{\partial}{\partial\varphi}(\underset{x}{\sin\theta\cos\varphi}, \underset{y}{\sin\theta\sin\varphi}, \underset{z}{\cos\theta}) = \hat{x}\sin\theta\sin\varphi(-1) + \hat{y}\sin\theta\cos\varphi =$

$= -\sin\theta\sin\varphi\left[\sin\theta\cos\varphi\,\hat{r} + \cos\theta\cos\varphi\,\hat{\theta} - \sin\varphi\,\hat{\varphi}\right] + \sin\theta\cos\varphi\left[\sin\theta\sin\varphi\,\hat{r} + \cos\theta\sin\varphi\,\hat{\theta} + \cos\varphi\,\hat{\varphi}\right] =$

$= -\hat{r}\sin^2\theta\sin(\varphi-\varphi) + \hat{\theta}\sin\theta\cos\theta(-1)\sin(\varphi-\varphi) + \sin\theta\,\hat{\varphi}\cos(\varphi-\varphi) = \left|\varphi = \varphi+\gamma, \varphi = 1\right| =$

$= -\hat{r}\sin^2\theta\sin\gamma + \hat{\theta}(-1)\sin\theta\cos\theta\sin\gamma + \sin\theta\cos\gamma\,\hat{\varphi}$

$\frac{\partial\vec{m}}{\partial\theta} = \frac{\partial}{\partial\theta}(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) = \hat{x}\cos\theta\cos\varphi + \hat{y}\cos\theta\sin\varphi - \hat{z}\sin\theta =$

$= p\cos\theta\cos\varphi(\sin\theta\cos\varphi\,\hat{r} + \cos\theta\cos\varphi\,\hat{\theta} - \sin\varphi\,\hat{\varphi}) + p\cos\theta\sin\varphi(\sin\theta\sin\varphi\,\hat{r} + \cos\theta\sin\varphi\,\hat{\theta} +$

$+ \cos\varphi\,\hat{\varphi}) - \sin\theta(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}) = [p\cos\theta\sin\theta\cos(\varphi-\varphi) - \sin\theta\cos\theta]\hat{r} + [p\cos^2\theta\cos(\varphi-\varphi) +$

$+ \sin^2\theta]\hat{\theta} + p\cos\theta\sin(\varphi-\varphi)\hat{\varphi} = \sin\theta\cos\theta(p\cos\gamma - 1)\hat{r} + [p\cos^2\theta\cos\gamma + \sin^2\theta]\hat{\theta} + p\cos\theta\sin\gamma\,\hat{\varphi}$

Верно

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(14/VIII) $m_z = \sin^2\theta\cos(\varphi-\varphi) + \rho\cos^2\theta$, $m_{\theta\varphi} = \rho\sin\theta\cos\theta\cos(\varphi-\varphi) - \sin\theta\cos\theta = \frac{1}{2}\sin 2\theta\,[\rho\cos(\varphi-\varphi)-1]$

$m_\varphi = \sin\theta\sin(\varphi-\varphi)$ - нет ρ, это не общее свойство $\sin 2\theta = \rho\sin 2\theta$

$\frac{\partial\vec m}{\partial\varphi} = \frac{\partial}{\partial\varphi}(m_z\hat z + m_{\theta\varphi}\hat\theta + m_\varphi\hat\varphi) = \hat z\left[-\sin(\varphi-\varphi)\sin^2\theta\right] + \hat\theta\,\frac{1}{2}\sin 2\theta\,(-\rho)\sin(\varphi-\varphi) +$

$+\hat\varphi\sin\theta\cos(\varphi-\varphi) = \hat z\left[-\sin\gamma\sin^2\theta\right] + \hat\theta\left[-\frac{1}{2}\rho\sin 2\theta\sin\gamma\right] + \hat\varphi\left[\sin\theta\cos\gamma\right]$

$\frac{\partial\vec m}{\partial\theta} = \frac{\partial}{\partial\theta}(m_z\hat z + m_{\theta\varphi}\hat\theta + m_\varphi\hat\varphi) = \hat z\left[\sin 2\theta\cos(\varphi-\varphi) - \rho\sin 2\theta\right] + \hat\theta\left[\cos 2\theta\,(\rho\cos\gamma-1)\right] +$

$+\hat\varphi\,(+\cos\theta)\sin(\varphi-\varphi) = \hat z(\rho\cos\gamma-1)\sin 2\theta + \hat\theta\cos 2\theta(\rho\cos\gamma-1) + \hat\varphi\rho\cos\theta\sin\gamma$

$\hat z$ и $\hat\theta$ можно в $\frac{\partial\vec m}{\partial\theta}$ не сокращать с нужн. разложением.

(*) $\begin{cases} m_z = \sin\theta\sin\theta\cos(\varphi-\varphi) + \cos\theta\cos\theta \\ m_{\theta\varphi} = \sin\theta\cos\theta\cos(\varphi-\varphi) - \cos\theta\sin\theta \\ m_\varphi = \sin\theta\sin(\varphi-\varphi) \end{cases}$ $\xrightarrow[\varphi=\varphi+\gamma]{\theta=\rho\theta+\frac{\pi}{2}(1-\rho)}$ $\begin{cases} m_z = \sin^2\theta\cos\gamma + \rho\cos^2\theta \\ m_{\theta\varphi} = \sin\theta\cos\theta\,(\cos\gamma-\rho) \\ m_\varphi = \sin\theta\sin\gamma \end{cases}$

$\rho = 1$

$\frac{\partial\vec m}{\partial\theta} = \left|\text{подставим ур. (*)}\right| = \hat z\frac{\partial m_z}{\partial\theta} + \hat\theta\frac{\partial m_{\theta\varphi}}{\partial\theta} + \hat\varphi\frac{\partial m_\varphi}{\partial\theta} = \hat z\left[\cos\theta\sin\theta\cos\gamma - \right.$

$\left. - \sin\theta\cos\theta\right] + \hat\theta\left[\cos\theta\cos\theta\cos\gamma + \sin\theta\sin\theta\right] + \hat\varphi\left[\cos\theta\sin\gamma\right] = \left|\text{теперь } \theta=\rho\theta+\frac{\pi}{2}(1-\rho)\right| =$

$= \hat z\left[\rho\sin\theta\cos\theta\cos\gamma - \sin\theta\cos\theta\right] + \hat\theta\left[\rho\cos^2\theta\cos\gamma + \sin^2\theta\right] + \hat\varphi\rho\cos\theta\sin\gamma$ –

– снимаем расчет $\frac{\partial\vec m}{\partial\theta}$ - уже на 15/VIII.

В сферич. координатах $\vec r = (r,\theta,\varphi)$, $\vec m(\vec r) = (m_z, m_{\theta\varphi}, m_\varphi)$ не зависит от φ (!) и зависит от z через $\rho(z)$

Дип. поле $\vec H = (H_z, H_{\theta\varphi}, H_\varphi)$ вводим в УД. $G_{\alpha\beta}(\vec r,\vec r') = -(\nabla_\alpha)(\nabla_{r'})_\beta \frac{\cos\gamma(\alpha)}{|\vec r-\vec r'|}$

$H_z = M_S\int d^3\vec r'\left[G_{zz}m_z(\vec r') + G_{z\varphi}m_\varphi(\vec r') + G_{z\theta\varphi}m_{\theta\varphi}(\vec r')\right]$

$H_{\theta\varphi} = M_S\int d^3\vec r'\left[G_{\theta\varphi z}m_z + G_{\theta\varphi\theta\varphi}m_{\theta\varphi} + G_{\theta\varphi\varphi}m_\varphi\right]$, $\frac{H_\varphi}{M_S} = \int d^3\vec r'\left[G_{\varphi z}m_z + G_{\varphi\theta\varphi}m_{\theta\varphi} + \right.$

$\left. + G_{\varphi\varphi}m_\varphi\right]$

$\frac{1}{|\vec r-\vec r'|} = 4\pi\sum_{lm}\frac{1}{(2l+1)}\frac{r_<^l}{r_>^{l+1}}Y^*_{lm}(\theta'\varphi')Y_{lm}(\theta\varphi)$, $Y_{lm}(\theta,\varphi) = a_{lm}P_l^m(\cos\theta)e^{im\varphi}$

$G_{\theta\varphi}, G_{z\varphi} \sim \frac{\partial}{\partial\varphi}e^{im\varphi'} \sim m'$ $G_{\varphi z}, G_{\theta\varphi\varphi} \sim \frac{\partial}{\partial\varphi}e^{im\varphi} \sim m$, $G_{\varphi\varphi} \sim \frac{\partial^2}{\partial\varphi\partial\varphi}(\dots) \sim mm'$

поэтому $\int_0^{2\pi}d\varphi\,d\varphi'$ для всех $G_{\alpha\beta}$ компонент $= 0$.

$h_z = \langle H_z\rangle_\varphi = \frac{1}{2\pi}\int_0^{2\pi}d\varphi\,H_z$, $\int\frac{d\varphi'}{2\pi}$ в вир. $G_{\alpha\beta}$, $H_\alpha = M_S\sum_\beta\int dV'G_{\alpha\beta}m_\beta$

$h_\alpha = \frac{1}{2\pi}\int d\varphi\,H_\alpha = \frac{1}{2\pi}\int d\varphi\,M_S\sum_\beta\int d^3\vec r\int d\varphi'\,G_{\alpha\beta}m_\beta$, $g_{\alpha\beta}(r\theta,r\theta') = \frac{1}{2\pi}\int d\varphi\,d\varphi'\,G_{\alpha\beta}$

$h_\alpha = M_S\int d^3\vec r'\sum_\beta g_{\alpha\beta}m_\beta(r'\theta')$, где $\int d^3\vec r = \int dr'\int d\theta'\,r'^2\sin\theta'$

$h_z = M_S\int d^3\vec r'\left[g_{zz}m_z + g_{z\theta\varphi}m_{\theta\varphi}\right]$, $h_{\theta\varphi} = M_S\int d^3\vec r'\left[g_{\theta\varphi z}m_z + g_{\theta\varphi\theta\varphi}m_{\theta\varphi}\right]$

$\vec h\cdot\frac{\partial\vec m}{\partial\varphi} = h_z\left(\frac{\partial\vec m}{\partial\varphi}\right)_z + h_{\theta\varphi}\left(\frac{\partial\vec m}{\partial\varphi}\right)_{\theta\varphi} = h_z\cdot(-1)\sin^2\theta\sin\gamma + h_{\theta\varphi}(-1)\sin\theta\cos\theta\sin\gamma$ $\quad h_\varphi = M_S\cdot 0 = 0$

$\vec h\cdot\frac{\partial\vec m}{\partial\theta} = h_z\left(\frac{\partial\vec m}{\partial\theta}\right)_z + h_\theta\left(\frac{\partial\vec m}{\partial\theta}\right)_{\theta\varphi} = h_z\cdot\sin\theta\cos\theta\,(\rho\cos\gamma-1) + h_{\theta\varphi}\left[\rho\cos^2\theta\cos\gamma + \sin^2\theta\right]$

$\rho\cos^2\theta\cos\gamma + \sin^2\theta = \cos^2\theta(\rho\cos\gamma-1) + 1$

$-2A\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\gamma')\sin^2\theta = \vec h M_S\frac{\partial\vec m}{\partial\varphi} = -M_S\left[h_z\sin^2\theta\sin\gamma + h_{\theta\varphi}\sin\theta\cos\theta\sin\gamma\right]$

(1) $\frac{2A}{M_S}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\gamma') = h_z\sin\gamma + h_{\theta\varphi}\operatorname{ctg}\theta\sin\gamma$

$A\left(\frac{1}{r^2\sin^2\theta} + (\gamma')^2\right)\rho\sin 2\theta\cdot\theta - 2A\frac{\rho}{r^2}\operatorname{ctg}\theta = \vec h M_S\frac{\partial\vec m}{\partial\theta} = M_S\left[h_z\sin\theta\cos\theta\,(\rho\cos\gamma-1) + \right.$

$\left. + h_{\theta\varphi}\left(\rho\cos^2\theta\cos\gamma + \sin^2\theta\right)\right]$

(2) $\frac{A}{M_S}(\gamma')^2\rho\sin 2\theta\cdot\theta = h_z\sin\theta\cos\theta\,(\rho\cos\gamma-1) + h_{\theta\varphi}\left(\rho\cos^2\theta\cos\gamma + \sin^2\theta\right)$

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Два магнита - диполь. eqs to find $\gamma = \gamma(r)$. (1), (2).

$$W_m(\gamma) = -\frac{1}{2}\int dV \vec{H}\vec{M} = -\frac{1}{2}M_s \int d^3\vec{r}\ \vec{h}\cdot\vec{m} = -\frac{1}{2}M_s \int d^3\vec{r}\ [h_z m_z + h_{ee} m_{ee}]$$

$$W_m(\gamma) = -\frac{1}{2}M_s^2 \int d^3\vec{r} \int d^2\vec{r}' \left[\left(g_{zz} m_z(r') + g_{zee} m_{ee}(r')\right) m_z(\vec{r}) + \left(g_{eez} m_z + g_{eee} m_{ee}\right) \right]$$

$W_m(\gamma) = W_m(\cos\gamma)$, т.к. $m_z(r, ee)$ зависит лишь от $\cos\gamma$. $\times m_{ee}$]

$$h_z = \frac{1}{2\pi}\int d\varphi\, h_z = M_s \int dr' \int d\theta' r'^2 \sin\theta' \left[g_{zz}(r\theta, r'\theta') m_z(r'\theta') + g_{zee} m_{ee} \right]$$

$$g_{zz}(r\theta, r'\theta') = \frac{1}{2\pi}\int d\varphi d\varphi' G_{zz}(\vec{r}\vec{r}'), \quad G_{zz} = -\frac{\partial^2}{\partial r \partial r'}\frac{1}{|\vec{r}-\vec{r}'|}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\varphi}, \quad g_{zz} = \frac{1}{2\pi}\int d\varphi d\varphi' (-1) \frac{\partial}{\partial r}\frac{\partial}{\partial r'} \times$$

$$\times \sum_{lm} \frac{4\pi}{(2l+1)} \frac{r_<^l}{r_>^{l+1}} Y_{lm}^*(\theta'\varphi') Y_{lm}(\theta\varphi), \quad |\vec{r}-\vec{r}'|^2 = (\vec{r}-\vec{r}')^2 = r^2 + r'^2 - 2rr'\cos\Gamma = R^2$$

$$\cos\Gamma = \cos(\hat{\vec{r}}\hat{\vec{r}}') = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi-\varphi'), \quad D = |\vec{r}-\vec{r}'| = R$$

$$G_{zz} = -\frac{\partial^2}{\partial r \partial r'}\frac{1}{D} = -\frac{\partial}{\partial r}\frac{1}{D^2}(-1) = -\frac{\partial}{\partial r}\frac{\partial}{\partial r'}\frac{1}{R} = -\frac{\partial}{\partial r}\frac{(-1)}{R^2}\frac{\partial R}{\partial r'}$$

Сим. б/г 2018 г. $G_{\alpha\beta}(\vec{r}\vec{r}') = -\frac{\partial^2}{\partial x_\alpha \partial x_\beta'}\frac{1}{|\vec{r}-\vec{r}'|} = -\frac{1}{R^3}\left[\delta_{\alpha\beta} - \frac{3R_\alpha R_\beta}{R^2}\right]$,

где $\vec{R} = \vec{r}-\vec{r}'$. $W_m = \frac{1}{2}M_s^2 \int dV dV' \frac{1}{R^3}\left[\vec{m}(\vec{r})\cdot\vec{m}(\vec{r}') - \frac{3}{R^2}(\vec{R}\cdot\vec{m})(\vec{R}\cdot\vec{m}')\right]$ -

- полная магнитостат. Энергия взаим. объема и зарядов m.charges

Видимо eq. для $G_{\alpha\beta}$ верно если α,β - любые орт-ты. координаты и

$\frac{\partial}{\partial x_\alpha} \to (\nabla\vec{r})_\alpha$, $\alpha = r, \theta, \varphi$ полет в сферич. коорд. $\nabla = \left(\hat{r}\frac{\partial}{\partial r}, \hat{\theta}\frac{1}{r}\frac{\partial}{\partial\theta}, \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\right)$

$$G_{zz}(\vec{r}\vec{r}') = -\frac{1}{R^3}\left[1 - \frac{3}{R^2}(\vec{r}-\vec{r}')_z (\vec{r}-\vec{r}')_z\right]$$

$\vec{r}-\vec{r}'$, $\vec{r} = r\hat{r} + r\theta\hat{\theta} + r\varphi\hat{\varphi}$, $(\vec{r})_r = r$, $(\vec{r}-\vec{r}')_r = r - r'$ (?)

$R_\alpha R_\beta = (\vec{r}-\vec{r}')\cdot\hat{r}\ (\vec{r}-\vec{r}')\cdot\hat{r}' = (\vec{r}\cdot\hat{r} - \vec{r}'\cdot\hat{r})(\vec{r}\cdot\hat{r}' - \vec{r}'\cdot\hat{r}')$

$\alpha = \hat{r}$, $\beta = \hat{r}'$ $\vec{r}'\cdot\hat{r} = r'\hat{r}'\cdot\hat{r} = r'\cos\Gamma$, $\vec{r}\cdot\hat{r}' = |\vec{r}|\hat{r}\hat{r}' = \cos\Gamma\cdot r$

$R_\alpha R_\beta = (r - r'\cos\Gamma)(r\cos\Gamma - r')$

$$\frac{\partial R}{\partial r'} = \frac{\partial}{\partial r'}\sqrt{r^2 + r'^2 - 2rr'\cos\Gamma} = \frac{1}{2R}(2r' - 2r\cos\Gamma) = \frac{1}{R}(r' - r\cos\Gamma)$$

$$G_{rr} = \frac{\partial}{\partial r}\frac{1}{R^2}\frac{\partial R}{\partial r'} = \frac{\partial}{\partial r}\frac{1}{R^3}(r' - r\cos\Gamma) = -\frac{3}{R^4}\frac{\partial R}{\partial r}(r' - r\cos\Gamma) + \frac{1}{R^3}(-\cos\Gamma) =$$

$$= -\frac{\cos\Gamma}{R^3} - \frac{3}{R^4}(r' - r\cos\Gamma)\frac{1}{R}(r\cos\Gamma - r) = -\frac{\cos\Gamma}{R^3} + \frac{3}{R^5}(r - r'\cos\Gamma)(r\cos\Gamma - r')$$

Получили $G_{rr}(\vec{r}\vec{r}') = -\frac{1}{R^3}\left[\cos\Gamma - \frac{3}{R^2}(r\cos\Gamma - r')(r'\cos\Gamma - r)\right]$ и

$$G_{\alpha\beta}(\vec{r}\vec{r}') = -\frac{1}{R^3}\left[\hat{e}_\alpha\hat{e}_\beta - \frac{3}{R^2}(\vec{R}\vec{e}_\alpha)(\vec{R}\cdot\vec{e}_\beta)\right], \quad \hat{e}_\alpha \text{ unit vector } x_\alpha\text{-coord.}$$

$\hat{e}_\beta$ - unit vector x_β' - coord.

$$\int d\varphi d\varphi' \frac{\cos\Gamma}{(r^2 + r'^2 - 2rr'\cos\Gamma)^{3/2}} = \int_0^{2\pi}\int_0^{2\pi} d\varphi\, d\varphi' F(\cos(\varphi-\varphi')) = \left|\varphi - \varphi' = \psi\right| =$$

$$= -\int_0^{2\pi} d\varphi \int_{-\varphi}^{2\pi-\varphi} d\psi\, F(\cos\psi) = -\int_0^{2\pi} d\varphi \int_{-2\pi}^{0} d\psi\, F(\cos\psi) = \int_0^{2\pi} d\varphi \int_0^{2\pi} d\psi\, F(\cos\psi) = 0$$ как

интеграл от период. функции по периоду (?). Пусть $\cos\ldots = u$.

$$\int d\varphi\, Y_{lm}(\theta\varphi) = \int d\varphi\, a_{lm} P_l^m(\cos\theta) e^{im\varphi} = 2\pi\delta_{m0} a_{l0} P_l(\cos\theta)$$

$$g_{zz} = -\frac{\partial^2}{\partial r \partial r'}\frac{1}{2\pi}\int d\varphi \int d\varphi' \sum_{lm}\frac{4\pi}{(2l+1)}\frac{r_<^l}{r_>^{l+1}} Y_{lm}^* Y_{lm} = -\frac{\partial^2}{\partial r \partial r'}\frac{1}{2\pi} \times$$

$$\times \sum_{lm}\frac{4\pi}{(2l+1)}\frac{r_<^l}{r_>^{l+1}} 2\pi\delta_{m0} a_{l0} P_l(\cos\theta) 2\pi\delta_{m0} a_{l0} P_l(\cos\theta'), \quad a_{l0} = \sqrt{\frac{2l+1}{4\pi}}$$

$$g_{rr}(r\theta, r'\theta') = -2\pi\frac{\partial^2}{\partial r \partial r'}\sum_l \frac{r_<^l}{r_>^{l+1}} P_l(\cos\theta) P_l(\cos\theta') \qquad (\nabla)_r (\nabla)_{r'} = \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial\theta'}$$

$$g_{\theta\theta}(r\theta, r'\theta') = -2\pi\frac{1}{rr'}\frac{\partial^2}{\partial\theta\partial\theta'}\sum_l \frac{r_<^l}{r_>^{l+1}} P_l(\cos\theta) P_l(\cos\theta'), \text{ у } \varphi\varphi \text{ аналогично, } g_{\theta\theta}$$

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Def. $F(rr',\theta\theta') = 2\pi \sum_\ell \frac{r_<^\ell}{r_>^{\ell+1}} P_\ell(\cos\theta) P_\ell(\cos\theta')$, $g_{rr} = -\frac{\partial}{\partial r}\frac{\partial}{\partial r'} F(rr',\theta\theta')$

$g_{r\theta} = -\frac{\partial}{\partial r}\frac{1}{r'}\frac{\partial}{\partial\theta'} F(rr',\theta\theta')$, $g_{\theta r} = -\frac{\partial}{r\partial\theta}\frac{\partial}{\partial r'} F$ и $g_{\theta\theta} = -\frac{1}{rr'}\frac{\partial}{\partial\theta}\frac{\partial}{\partial\theta'} F$

$$\begin{cases} h_r = M_s \int_0^R dr' \int d\theta' r'^2 \sin\theta' \left[-\frac{\partial^2 F}{\partial r \partial r'} m_r(r'\theta') - \frac{1}{r'} \frac{\partial^2 F}{\partial r \partial \theta'} m_{\theta\theta}(r'\theta') \right] \\ h_{\theta\theta} = M_s (-1) \int dr' \int d\theta' r'^2 \sin\theta' \left[\frac{1}{r} \frac{\partial^2 F}{\partial\theta \partial r'} m_r(r'\theta') + \frac{1}{rr'} \frac{\partial F}{\partial\theta \partial\theta'} m_{\theta\theta}(r'\theta') \right] \end{cases},$$

где $m_r(r,\theta) = \sin^2\theta\cos\gamma + \cos^2\theta$, $m_{\theta\theta}(r,\theta) = \sin\theta\cos\theta(\cos\gamma - p)$

$F(rr',\theta\theta') = 2\pi \sum_\ell P_\ell(\cos\theta) P_\ell(\cos\theta') \times \left\{ \begin{matrix} r^\ell/r'^{\ell+1}, & r<r' \\ r'^\ell/r^{\ell+1}, & r'<r \end{matrix} \right\} = 2\pi \sum_\ell f_\ell(rr') P_\ell(\cos\theta) \times P_\ell(\cos\theta')$

Если $\gamma(r) = \gamma_0 = \text{const}$, то из (1) $h_r = -h_{\theta\theta} \operatorname{ctg}\theta$. Из eq. (2)

$-h_{\theta\theta}\operatorname{ctg}\theta \sin\theta\cos\theta (p\cos\gamma - 1) + h_{\theta\theta}[\cos^2\theta(p\cos\gamma - 1) + 1] = 0$

$-\cos^2\theta(p\cos\gamma - 1) + \cos^2\theta(p\cos\gamma - 1) + 1 = 0$ не выполн., т.е. либо $\gamma(r) = \text{const}$ не есть решение с $W = W_{ex} + W_m$, либо magnetostatic part в YD Slonczewski 14/VIII неверна ($\frac{\partial m}{\partial\theta}$, $\frac{\partial m}{\partial r}$ - неверны).

Hedgehog BP $m_{\theta\theta} = 0$, $\cos\gamma = p$, $\gamma = 0, \pi$. $m_r = \sin^2\theta\cos\gamma + p\cos^2\theta = p$ (!)

$h_{\theta\theta}(\text{hedgehog}) = -M_s \frac{p}{r} \frac{\partial}{\partial\theta} \int dr' d\theta' r'^2 \sin\theta' \frac{\partial F}{\partial r'}$, $\int d\theta' \sin\theta' F \sim \int_0^\pi d\theta' \sin\theta' P_\ell(\cos\theta') =$

$= \int_{-1}^{1} dx\, P_0(x) P_\ell(x) = \frac{2}{2\ell+1}\delta_{\ell 0} = 2 \neq 0$. Но при $\ell = 0$ $P_\ell(\cos\theta) = 1$, $\frac{\partial}{\partial\theta}(1) = 0$, $h_{\theta\theta} = 0$

т.е. hedgehog $m_{\theta\theta}$, $\cos\gamma = p$, $m_r = p$ - даст $h_{\theta\theta} = 0$ и удовлетв. YD Slonczewski

$h_r(\text{hedgehog}) = -M_s p \frac{\partial}{\partial r} \int dr' d\theta' r'^2 \sin\theta' \frac{\partial F}{\partial r'} = -M_s p \frac{\partial}{\partial r} \int dr' r'^2 \frac{\partial}{\partial r'} \times$

$\times \int d\theta' \sin\theta' F$. $\int d\theta' \sin\theta' F = \int d\theta' \sin\theta' 2\pi \sum_\ell f_\ell P_\ell(\cos\theta) P_\ell(\cos\theta') =$

$= 2\pi \sum_\ell f_\ell P_\ell(\cos\theta) \int d\theta' \sin\theta' P_\ell(\cos\theta') = 2\pi \sum_\ell f_\ell P_\ell(\cos\theta) 2\delta_{\ell 0} = 4\pi f_0 = 4\pi \frac{1}{r_>}$

$h_r(\text{hedehog}) = -M_s p \frac{\partial}{\partial r} \int dr' r'^2 \frac{\partial}{\partial r'} \frac{4\pi}{r_>} = h_r^*(r)$

$h_r^*(r > r') = -M_s p \frac{\partial}{\partial r} \int dr' r'^2 \frac{\partial}{\partial r'} \frac{4\pi}{r} = 0$, $h_r^*(r' > r) = -M_s p \frac{\partial}{\partial r} \int_r^R dr' r'^2 \frac{\partial}{\partial r'} \frac{4\pi}{r'} =$

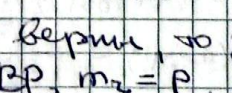
$= -M_s p \frac{\partial}{\partial r} \int_r^R dr' r'^2 \frac{(-1)4\pi}{r'^2} = M_s p \frac{\partial}{\partial r} \int_r^R dr' = M_s p \frac{\partial}{\partial r}(R - r) 4\pi = -4\pi M_s p$

$h_r^*(r < r') = -4\pi M_s p$, $h_r^{(4\pi)}(r > r') = 0$, проверено.

$h_r^+(r) = -M_s p \cdot 4\pi \frac{\partial}{\partial r} \int_0^R dr' r'^2 \frac{\partial}{\partial r'} \frac{1}{r_>} \overset{r>R}{=} -4\pi M_s p \frac{\partial}{\partial r} \left[\int_0^r dr' r'^2 \frac{\partial}{\partial r'} \frac{1}{r} + \int_r^R dr' r'^2 \frac{\partial}{\partial r'} \frac{1}{r'} \right] =$

$= -4\pi M_s \frac{\partial}{\partial r} p \left[0 - \int_r^R dr' \right] = -4\pi M_s p \frac{\partial}{\partial r}[-(R - r)] = -4\pi M_s p$, $r \leq R$.

Hedgehog $h_r(r) = -4\pi M_s p$, $r \leq R$, $h_r(r) = 0$ при $r > R$. $b_r(r) = h_r + 4\pi M_s^2 = 0$, $p = \pm 1$. $M_s^2 = M_s p$. $h_{\theta\theta}(r) \equiv 0$.



Вывод: если eqs Slonczewski дают $\gamma(r)$ верное, то $\gamma(r) = \text{const}$ с W_m возможно лишь для "pure" hedgehog BP, $m_r = p$, $\gamma = 0, \pi$. $\partial\gamma/\partial r \neq 0$ ведет к degree index of BP $Q = \frac{1}{4\pi}\int d^2r\, \gamma'(r) \neq 0$.

$\frac{l_e^2}{r^2}\frac{\partial}{\partial r}(r^2\gamma') = \sin\gamma[h_r + h_{\theta\theta}\operatorname{ctg}\theta]$, $l_e^2 = 2A/M_s^2$, h_r в eq. M_s.

$h_r = h_r[\vec{m}(r,\theta)]$, $h_{\theta\theta} = h_{\theta\theta}[\vec{m}(r,\theta)]$, γ входит в $m_r, m_{\theta\theta}$.

$\int d\theta' \sin\theta' F m_r(r'\theta) = \int d\theta' \sin\theta' 2\pi \sum_\ell f_\ell P_\ell(\cos\theta) P_\ell(\cos\theta') [\cos\gamma + \cos^2\theta'(p - \cos\gamma)] =$

$= 2\pi \sum_\ell P_\ell(\cos\theta) f_\ell \int_{-1}^{1} dx\, P_\ell(x) [\cos\gamma + x^2(p - \cos\gamma)] = 2\pi \sum_\ell f_\ell P_\ell(\cos\theta) \times$

$\times \int_{-1}^{1} dx\, P_\ell(x) [\cos\gamma P_\ell(x) + (p - \cos\gamma)\frac{1}{3}(2P_2(x) + 1)] = (\ldots) \int dx\, P_2 [\{\cos\gamma + \frac{1}{3}(p - \cos\gamma)\} P_0 +$

Escaneado con CamScanner

$+\frac{2}{3}(\rho-\cos\gamma)P_2(x)\Big] = 2\pi\sum_\ell f_\ell P_\ell(\cos\theta)\Big[\{\cos\gamma+\frac{1}{3}(\rho-\cos\gamma)\}2\delta_{\ell0}+\frac{2}{3}(\rho-\cos\gamma)\frac{2}{5}\delta_{\ell2}\Big] =$

$= 4\pi\Big[f_0 P_0(\cos\theta)\{\cos\gamma+\frac{1}{3}(\rho-\cos\gamma)\}+\frac{2}{15}(\rho-\cos\gamma)f_2P_2(\cos\theta)\Big]$, $P_2(x)=\frac{1}{2}(3x^2-1)$

$\int d\theta' \sin\theta' F m_z(r',\theta') = 4\pi\Big[f_0\{\cos\gamma+\frac{1}{3}(\rho-\cos\gamma)\}+\frac{2}{15}(\rho-\cos\gamma)f_2P_2(\cos\theta)\Big]$

$\int d\theta'\sin\theta' \frac{\partial F}{\partial\theta'} m_{\theta\theta}(r',\theta') = \int d\theta'\sin\theta' 2\pi\sum_\ell f_\ell P_\ell(\cos\theta)\frac{\partial}{\partial\theta'}P_\ell(\cos\theta')\sin\theta'\cos\theta'(\cos\gamma-\rho)$

$\frac{dP_n(x)}{dx} = \frac{n}{(x^2-1)}[xP_n(x)-P_{n-1}(x)]$, $\frac{dP_\ell(x)}{dx} = \frac{\ell}{-\sin^2\theta}[\cos\theta P_\ell(x)-P_{\ell-1}(x)]$

$dx = d(\cos\theta) = -\sin\theta\, d\theta$, $-\frac{1}{\sin\theta}\frac{\partial P_\ell(x)}{\partial\theta} = \frac{\ell}{\sin\theta}[\cos\theta P_\ell(x)-P_{\ell-1}(x)]$

$\frac{\partial P_\ell(x)}{\partial\theta} = \frac{\ell}{\sin\theta}[\cos\theta P_\ell(x)-P_{\ell-1}(x)]$

$\int d\theta'\sin\theta' \frac{\partial F}{\partial\theta'} m_{\theta\theta} = 2\pi\sum_\ell f_\ell P_\ell(\cos\theta)\int d\theta'\sin\theta'\frac{\ell}{\sin\theta'}[\cos\theta' P_\ell(x)-P_{\ell-1}(x)]\sin\theta'\cos\theta'\times(\cos\gamma-\rho) =$

$= 2\pi\sum_\ell f_\ell P_\ell(\cos\theta)\ell\int_{-1}^{1}dx\,[xP_\ell(x)-P_{\ell-1}(x)]\times(\cos\gamma-\rho) = 2\pi(\cos\gamma-\rho)\sum_\ell \ell f_\ell P_\ell(\cos\theta)\times$

$\times\int_{-1}^{1}dx\,[x^2P_\ell(x)-xP_{\ell-1}(x)] = (\dots)\int dx\Big[\frac{1}{3}(2P_2+1)P_\ell(x)-P_1(x)P_{\ell-1}(x)\Big] =$

$= (\dots)\Big[\frac{2}{3}\frac{2}{5}\delta_{\ell2}+\frac{1}{3}2\delta_{\ell0}-\frac{2}{3}\delta_{\ell2}\Big] = (\dots)\frac{2}{3}\Big[\delta_{\ell0}-\frac{3}{5}\delta_{\ell2}\Big]$

$\int d\theta'\sin\theta'\frac{\partial F}{\partial\theta'} m_{\theta\theta} = 2\pi(\cos\gamma-\rho)\sum_\ell \ell f_\ell P_\ell(\cos\theta)\frac{2}{3}\Big(\delta_{\ell0}-\frac{2}{5}\delta_{\ell2}\Big) = 2\pi(\cos\gamma-\rho)\frac{2}{3}\times$

$\times\Big[-\frac{2}{5}2f_2P_2(\cos\theta)\Big] = 2\pi\Big(-\frac{8}{15}\Big)(\cos\gamma-\rho)P_2(\cos\theta)f_2$, $\gamma=\gamma(r')$

$h_z[\vec m] = -\int dr' r'^2\Big[\frac{\partial}{\partial z}\frac{\partial}{\partial r'}4\pi\Big[f_0\{\frac{1}{3}\rho+\frac{2}{3}\cos\gamma\}+\frac{2}{15}(\rho-\cos\gamma)f_2P_2(\cos\theta)\Big]+$

$+\frac{1}{r'}\frac{\partial}{\partial z}2\pi\Big(-\frac{8}{15}\Big)(\cos\gamma-\rho)f_2P_2(\cos\theta)\Big] = -\frac{4\pi}{3}\frac{\partial}{\partial z}\int dr' r'^2\Big[\frac{\partial}{\partial r'}\Big[f_0(\rho+2\cos\gamma)+$

$+\frac{2}{5}(\rho-\cos\gamma)f_2P_2(\cos\theta)\Big]-\frac{1}{r'}\frac{4}{5}(\cos\gamma-\rho)f_2P_2(\cos\theta)\Big]$, $f_\ell(rr') = \frac{r_<^\ell}{r_>^{\ell+1}}$

$[f_0(\rho+2\cos\gamma)+\frac{2}{5}(\rho-\cos\gamma)f_2P_2] = f_0\rho+\frac{2}{5}\rho f_2P_2+2f_0\cos\gamma-\frac{2}{5}\cos\gamma f_2P_2 =$

$= \rho(f_0+\frac{2}{5}f_2P_2)+2\cos\gamma(f_0-\frac{1}{5}f_2P_2)$

$h_z[\vec m] = -\frac{4\pi}{3}\frac{\partial}{\partial z}\int dr' r'^2\Big[\frac{\partial}{\partial r'}\rho(f_0+\frac{2}{5}f_2P_2)+\frac{\partial}{\partial r'}2\cos\gamma(f_0-\frac{1}{5}f_2P_2)-\frac{1}{r'}\frac{4}{5}(\cos\gamma-\rho)f_2P_2\Big]$

Найти $h_{\theta\theta}[\vec m]$ и $h_z+h_{\theta\theta}\operatorname{ctg}\theta$.

$h_{\theta\theta} = -\int dr' r'^2\Big[\frac{1}{r}\frac{\partial}{\partial\theta}\frac{\partial}{\partial r'}\int d\theta'\sin\theta' F m_z+\frac{1}{rr'}\frac{\partial}{\partial\theta}\int d\theta'\sin\theta'\frac{\partial F}{\partial\theta'}m_{\theta\theta}\Big] =$

$= -\frac{1}{r}\frac{\partial}{\partial\theta}\int dr' r'^2\Big[\frac{\partial}{\partial r'}\int d\theta'\sin\theta' F m_z+\frac{1}{r'}\int d\theta'\sin\theta'\frac{\partial F}{\partial\theta'}m_{\theta\theta}\Big]$

$h_z[\vec m] = -\frac{\partial}{\partial z}\int dr' r'^2\Big[\frac{\partial}{\partial r'}\int d\theta'\sin\theta' F m_z+\frac{1}{r'}\int d\theta'\sin\theta'\frac{\partial F}{\partial\theta'}m_{\theta\theta}\Big]$

$h_z+h_{\theta\theta}\operatorname{ctg}\theta = -\Big(\frac{\partial}{\partial z}+\frac{\operatorname{ctg}\theta}{r}\frac{\partial}{\partial\theta}\Big)[\dots]$, $[\dots] = \frac{4\pi}{3}\int dr' r'^2\Big[\frac{\partial}{\partial r'}\rho(f_0+\frac{2}{5}f_2P_2)+\dots\Big]$

$\frac{\ell_0^2}{r^2}\frac{\partial}{\partial r}(r^2\gamma') = -\sin\gamma\Big(\frac{\partial}{\partial r}+\frac{\operatorname{ctg}\theta}{r}\frac{\partial}{\partial\theta}\Big)\frac{4\pi}{3}\int dr' r'^2\Big[\rho\frac{\partial}{\partial r'}(f_0+\frac{2}{5}f_2P_2)+2\frac{\partial}{\partial r'}\cos\gamma(f_0-\frac{1}{5}f_2P_2)-$

$-\frac{1}{r'}\frac{4}{5}(\cos\gamma-\rho)f_2P_2\Big]$, $P_2 = P_2(\cos\theta)$

$\frac{\partial}{\partial\theta}P_2(\cos\theta) = \frac{2}{\sin\theta}[\cos\theta P_2(\cos\theta)-P_1(\cos\theta)] = \frac{2}{\sin\theta}\Big[x\frac{1}{2}(3x^2-1)-x\Big] =$

$= \frac{x}{\sin\theta}[3x^2-1-2] = \frac{3x(x^2-1)}{\sqrt{1-x^2}} = -3x\sqrt{1-x^2}$, $\frac{\partial P_2(\gamma)}{\partial\theta} = -\sin\theta\frac{\partial P_2}{\partial x} = -\sin\theta\cdot$

$\operatorname{ctg}\theta\frac{\partial P_2}{\partial\theta} = \operatorname{ctg}\theta(-3)\sin\theta\cos\theta = -3\cos^2\theta$ $+3\cos\theta$

Дело не сделано. Кроме тривиальных $\gamma=0,\pi$ hedgehog - ремешка тяжело не бывает. Можно искать $\gamma(r)=0+\varepsilon(r)$, $\varepsilon\ll1$. $\cos\gamma = 1-\varepsilon^2/2 = 1+O(\varepsilon^2)$

Escaneado con CamScanner

16/VIII Дырка $\gamma = (0, \pi) + \varepsilon$, $\cos\gamma = p - \frac{1}{2}\varepsilon^2$. Для $\varepsilon(r)$ получаем ур-ние $\frac{l_0^2}{r^2}\frac{\partial}{\partial r}(r^2\varepsilon') = \varepsilon f_1(r)$. Но зачем нам $\varepsilon(r)$? Можно найти hedgehog solution $\cos\gamma = p$, hedgehog есть max $W[\gamma]$.

Если предположить $\gamma(r) = \text{const}$, то γ можно найти из $h_z(\gamma) = h_{ee}(\gamma) = 0$

$$S(\gamma) = \int dr' r'^2 [\dots]_{\gamma=\text{const}} = \int dr' r'^2 \left[p \frac{\partial}{\partial r'}\left(f_0 + \frac{2}{5} f_2 P_2\right) + 2\cos\gamma \frac{\partial}{\partial r'}\left(f_0 - \frac{1}{5} f_2 P_2\right) - \right.$$
$$\left. - \frac{1}{r'}\frac{4}{5}(\cos\gamma - p) f_2 P_2 \right] = \int dr' r'^2 \left[(p + 2\cos\gamma)\frac{\partial f_0}{\partial r'} + \frac{2}{5} P_2 (p - \cos\gamma)\frac{\partial f_2}{\partial r'} - \frac{1}{r'}\frac{4}{5}(\cos\gamma - p) f_2 P_2 \right]$$

$$S(\gamma) = \int dr' r'^2 \left[(p + 2\cos\gamma)\frac{\partial f_0}{\partial r'} + \frac{2}{5} P_2 (p - \cos\gamma)\left\{ \frac{\partial f_2}{\partial r'} + \frac{2}{r'} f_2 \right\} \right]$$

$$\int_0^R dr' r'^2 \frac{\partial f_0}{\partial r'} = \left| f_0 = \frac{1}{r_>} \right| = \int_0^r dr' r'^2 \frac{\partial}{\partial r'}\frac{1}{r} + \int_r^R dr' r'^2 \frac{\partial}{\partial r'}\frac{1}{r'} = \int_r^R dr' r'^2 \frac{(-1)}{r'^2} = -\int_r^R dr' = r - R.$$

$$\int_0^R dr' r'^2 \left\{ \frac{\partial f_2}{\partial r'} + \frac{2}{r'} f_2 \right\} = \left| f_2 = \frac{r_<^2}{r_>^3} \right| = \int_0^r dr' r'^2 \underset{(r>r')}{\left\{ \frac{\partial}{\partial r'}\frac{r'^2}{r^3} + \frac{2}{r'}\frac{r'^2}{r^3} \right\}} +$$
$$+ \int_r^R dr' r'^2 \underset{(r'>r)}{\left\{ \frac{\partial}{\partial r'}\frac{r^2}{r'^3} + \frac{2}{r'}\frac{r^2}{r'^3} \right\}} = \int_0^r dr' r'^2 \left\{ 2\frac{r'}{r^3} + \frac{2r'}{r^3} \right\} + \int_r^R dr' r'^2 \left\{ r^2 \frac{(-3)}{r'^4} + 2\frac{r^2}{r'^4} \right\} =$$
$$= \frac{4}{r^3}\int_0^r dr' r'^2 \cdot r' - r^2 \int_r^R dr' r'^2 \frac{1}{r'^4} = \frac{4}{r^3}\frac{r'^4}{4}\Big|_0^r - r^2 \int_r^R dr' \frac{1}{r'^2} = r - r^2 \cdot \left(\frac{-1}{r'}\right)\Big|_r^R =$$
$$= r + r^2\left(\frac{1}{R} - \frac{1}{r}\right) = \frac{r^2}{R}.$$

$$S(\gamma) = (p + 2\cos\gamma)(r - R) + \frac{2}{5} P_2 (p - \cos\gamma)\frac{r^2}{R}, \quad h_z + h_{ee}\operatorname{ctg}\varepsilon_{ee} =$$
$$= -\left(\frac{\partial}{\partial r} + \frac{\operatorname{ctg}\varepsilon_{ee}}{r}\frac{\partial}{\partial \varepsilon_{ee}}\right)\frac{4\pi}{3} S(\gamma) = -\frac{4\pi}{3}\left(\frac{\partial}{\partial r} + \frac{\operatorname{ctg}\varepsilon_{ee}}{r}\frac{\partial}{\partial \varepsilon_{ee}}\right)\left[(p + 2\cos\gamma)(r - R) + \frac{2}{5}(p - \cos\gamma) P_2 \frac{r^2}{R} \right] =$$
$$= -\frac{4\pi}{3}\left[p + 2\cos\gamma + \frac{2}{5R}(p - \cos\gamma)\left(\frac{\partial}{\partial r} + \frac{\operatorname{ctg}\varepsilon_{ee}}{r}\frac{\partial}{\partial \varepsilon_{ee}}\right) P_2(\cos\varepsilon_{ee})\, r^2 \right]$$

$$\left(\frac{\partial}{\partial r} + \frac{\operatorname{ctg}\varepsilon_{ee}}{r}\frac{\partial}{\partial \varepsilon_{ee}}\right) P_2(\cos\varepsilon_{ee})\, r^2 = 2r P_2 + r\operatorname{ctg}\varepsilon_{ee}\frac{\partial P_2(\cos\varepsilon_{ee})}{\partial \varepsilon_{ee}} = 2r\frac{1}{2}(3\cos^2\varepsilon_{ee} - 1) +$$
$$+ r(-3)\cos^2\varepsilon_{ee} = -r, \quad h_z + h_{ee}\operatorname{ctg}\varepsilon_{ee} = -\frac{4\pi}{3}\left[p + 2\cos\gamma + \frac{2}{5}\frac{1}{R}(p - \cos\gamma)(-r) \right]$$

$$h_z + h_{ee}\operatorname{ctg}\varepsilon_{ee} = -\frac{4\pi}{3}\left[p + 2\cos\gamma - \frac{2}{5}\frac{r}{R}(p - \cos\gamma) \right]$$ в предположении $\gamma = \text{const}$.

Если $R \to \infty$ (Dirac case) $h_z + h_{ee}\operatorname{ctg}\varepsilon_{ee} = 0$ при $\cos\gamma = -\frac{p}{2}$, $\gamma_F = \begin{cases} 120^\circ, & p = +1 \\ 60^\circ, & p = -1 \end{cases}$

Pylyavskyi 12 назвала γ_F – Feldkeller's solution.

Моя $\operatorname{div}\vec{m} = -\sin^2\varepsilon_{ee}\sin\gamma\gamma' + \frac{2p}{r} + \frac{1}{r}(\cos\gamma - p)(1 + \cos^2\varepsilon_{ee})$ при $\gamma = \text{const}$

$$\operatorname{div}\vec{m} = \frac{1}{r}\left[2p + (\cos\gamma - p)(1 + \cos^2\varepsilon_{ee}) \right] = \frac{1}{r}\left[p\sin^2\varepsilon_{ee} + \cos\gamma(1 + \cos^2\varepsilon_{ee}) \right]$$

$\left(\rho(\vec{r}) = -M_s \operatorname{div}\vec{m}\right)$ с точностью до множителя $\frac{1}{4\pi}$ совп. с charge density Pylyavskyi 2012

Pylyavskyi 12 для $\gamma = \text{const}$ mag. potential $\psi(r) = p\pi z + \frac{\pi}{3}(3z - 8)\cos\gamma +$
$+ \pi r^2 (p - \cos\gamma)\cos^2\varepsilon_{ee}$, $z \to \frac{r}{R}$.

тогда $h_z = -\frac{\partial \psi}{\partial z} = -p\pi - 3\pi\cos\gamma - \pi(p - \cos\gamma)\cos^2\varepsilon_{ee}$

Моё $h_z = -\frac{4\pi}{3}\frac{\partial}{\partial z} S(\gamma) = -\frac{4\pi}{3}\left[p + 2\cos\gamma + \frac{2}{5}(p - \cos\gamma)\frac{2r}{R}\frac{1}{2} \right] = -\frac{4\pi}{3}\left[p + 2\cos\gamma + \right.$
$\left. + \frac{4}{5}(p - \cos\gamma)\frac{r}{R}\frac{1}{2}(3\cos^2\varepsilon_{ee} - 1) \right] = -\frac{4\pi}{3}\left[p + 2\cos\gamma + \frac{2}{5}(p - \cos\gamma)\frac{r}{R}(3\cos^2\varepsilon_{ee} - 1) \right]$ сильно отличается от h_z – Pylyav. Хотя при $\cos\gamma = p$ оба $h_z = -4\pi p$ совпадают, это странно. Ведь моё $h_z = H_z/M_s$, $h_z(\text{Pyl}) = H_z/4\pi M_s$

$h_\varphi(\text{Pyl}) = -\frac{1}{r}\frac{\partial}{\partial \varepsilon_{ee}}\psi = -\frac{1}{r}\pi r^2 (p - \cos\gamma)(-1)\, 2\sin\varepsilon_{ee}\cos\varepsilon_{ee} = 2\pi(p - \cos\gamma)\sin\varepsilon_{ee}\cos\varepsilon_{ee}$

Pylyav $h_z + h_{ee}\operatorname{ctg}\varepsilon_{ee} = -p\pi - 3\pi\cos\gamma - \pi(p - \cos\gamma)\cos^2\varepsilon_{ee} + 2\pi(p - \cos\gamma)\cos^2\varepsilon_{ee} =$
$= -p\pi - 3\pi\cos\gamma + \pi(p - \cos\gamma)\cos^2\varepsilon_{ee}$ – здесь ε_{ee} и не сокращается ←
моё $h_z + h_{ee}\operatorname{ctg}\varepsilon_{ee}$. Попробуем моё $h_z + h_{ee}\operatorname{ctg}\varepsilon_{ee}$.

Escaneado con CamScanner

Проверим $W_m(\gamma)$ Pylypovskyi given by Eq. (12). $\gamma_{min} \to \frac{3}{R}$.

$$W_m = -\frac{1}{2} M_s^2 \int dV \vec{h}_m \cdot \vec{m} = -\frac{1}{2} M_s^2 \int_0^1 dr \int_0^\pi d\theta \int_0^{2\pi} d\varphi\, r^2 \sin\theta\, \vec{h}_m \cdot \vec{m}$$

$\vec{h}_m \cdot \vec{m} = h_\theta m_\theta + h_z m_z = 2\pi (p - \cos\gamma) \sin\theta\cos\theta \cdot \sin\theta\cos\theta\, (\cos\gamma - p) +$

$+ \left[-p\pi \cdot 3\pi\cos\gamma - \pi (p - \cos\gamma) \cos^2\theta \right] \cdot (\sin^2\theta \cos\gamma + p\cos^2\theta)$

$$W_m(\gamma) = -\frac{1}{2} M_s^2 \cdot 2\pi \int_0^1 dr \int_0^\pi d\theta\, r^2 \sin\theta \left[-2\pi (p - \cos\gamma)^2 \sin^2\theta \cos^2\theta - \right.$$

$$\left. - \pi \{ p + 3\cos\gamma + (p - \cos\gamma)\cos^2\theta \} (\sin^2\theta \cos\gamma + p\cos^2\theta) \right]$$

$\int_0^\pi d\theta \sin\theta \cdot \sin^2\theta \cos^2\theta = \int_{-1}^{1} dx\, (1 - x^2) x^2 = \int dx\, (x^2 - x^4) = \frac{1}{3} x^3 - \frac{1}{5} x^5 \Big|_{-1}^{1} = \frac{2}{3} - \frac{2}{5} = \frac{4}{15}$

$\int_0^\pi d\theta \sin\theta\, (\sin^2\theta \cos\gamma + p\cos^2\theta) = \int_{-1}^{1} dx\, ((1 - x^2)\cos\gamma + p x^2) = \int dx\, (1 - x^2)\cos\gamma +$

$+ \int dx\, x^2 p = \cos\gamma \left(x - \frac{x^3}{3} \right) \Big|_{-1}^{1} + p \frac{x^3}{3} \Big|_{-1}^{1} = \cos\gamma \cdot \frac{2}{3} \cdot 2 + p \frac{2}{3} = \frac{2}{3} (p + 2\cos\gamma)$

$\int_0^\pi d\theta \sin\theta \cos^2\theta\, (\sin^2\theta \cos\gamma + p\cos^2\theta) = \int_{-1}^{1} dx\, x^2 \left[(1 - x^2)\cos\gamma + p x^2 \right] =$

$= \cos\gamma \int dx\, (x^2 - x^4) + p \int dx\, x^4 = \cos\gamma \left(\frac{x^3}{3} - \frac{x^5}{5} \right) \Big|_{-1}^{1} + p \frac{x^5}{5} \Big|_{-1}^{1} = \cos\gamma \cdot \frac{4}{15} + p \frac{2}{5}$

$$W_m(\gamma) = -\pi M_s^2 \frac{1}{3} \left[-2\pi (p - \cos\gamma)^2 \frac{4}{15} - \pi \left\{ (p + 3\cos\gamma) \frac{2}{3} (p + 2\cos\gamma) + \right. \right.$$

$$\left. \left. + (p - \cos\gamma) \frac{2}{5} \left(p + \frac{2}{3}\cos\gamma \right) \right\} \right] = -\pi M_s^2 \frac{1}{3} (-\pi) \frac{2}{15} \left[4 (p - \cos\gamma) + 5 (p + 3\cos\gamma) \times \right.$$

$$\left. \times (p + 2\cos\gamma) + 3 (p - \cos\gamma) \left(p + \frac{2}{3}\cos\gamma \right) \right] = \pi^2 M_s^2 \frac{2}{45} \left[4p - 4\cos\gamma + 5 + 25p\cos\gamma + 30\cos^2\gamma + \right.$$

$$\left. + 3 - p\cos\gamma - 2\cos^2\gamma \right] = \pi^2 M_s^2 \frac{2}{45} \left[4p + 8 - 4\cos\gamma + 24p\cos\gamma + 28\cos^2\gamma \right]$$

$$W_m(\gamma) = \frac{2}{45} \pi^2 M_s^2 \left[8 + 4p - 4\cos\gamma + 24p\cos\gamma + 28\cos^2\gamma \right]$$

$$W_m(\gamma) = \frac{2}{45} \pi^2 M_s^2 \left[8 + 4p + 4 (6p - 1)\cos\gamma + 28\cos^2\gamma \right]$$ совпад. со Eq. (12) Pylypovskyi

$\frac{\partial W_m}{\partial (\cos\gamma)} = (\cdot) \left[4 (6p - 1) + 56\cos\gamma \right] = 0$, $\cos\gamma_0 = -\frac{4(6p-1)}{56} = \begin{cases} -10/28, & p = +1 \\ -1/2, & p = -1 \end{cases}$

$\cos\gamma_0 = \begin{cases} -5/14, & p = +1 \\ +1/2, & p = -1 \end{cases}$ на основе $\psi, \vec{m}$ - Pylypovskyi '12 $\to$ $\gamma_0 = \begin{cases} 111^\circ, & p = +1 \\ 60^\circ, & p = -1 \end{cases}$

А Pylypovskyi Eq. (13) given $\cos\gamma_0 = \begin{cases} -1/4, & p = +1 \\ 1/4, & p = -1 \end{cases}$, $\gamma_0 = \begin{cases} 105^\circ, & p = +1 \\ 75^\circ, & p = -1 \end{cases}$

т.е. $\psi, \vec{m}$ - Pylyp. не Подгут к $W_m(\gamma)$ Eq. (12) Pylypovskyi '12. - Нет, см. 9/10

Тоже $h_z + h_\theta \cot\theta\, e_\theta = -\frac{4\pi}{3} \left[p + 2\cos\gamma - \frac{2}{5} \frac{r}{R} (p - \cos\gamma) \right]$ н/м $r = R$ дает

$p + 2\cos\gamma - \frac{2}{5}(p - \cos\gamma) = \frac{3}{5} p + \left(\frac{2}{5} + 2 \right) \cos\gamma = 0$, $\frac{3}{5} p + \frac{12}{5} \cos\gamma = 0$, $\cos\gamma_0 = -\frac{p}{4}$ -

это в точности Eq. (13) - Pylyp., которое подтверждается by simulations.

Видимо Eq. (12) $W_m(\gamma)$ - Pylyp. написано так чтоб давало $\cos\gamma_0 = -\frac{p}{4}$.

Но $\gamma(r) \neq \text{const}$, и тоже $h_z + h_\theta \cot\theta\, e_\theta = f(r)$ н/м $\gamma = \text{const}$. Это подтверждение.

Ведь если $\gamma(r) = \text{const}$, то $h_z + h_\theta \cot\theta\, e_\theta \equiv 0$ $\forall r$ должно, а нетрив. $f(r)$.

Тоже $h_z + h_\theta \cot\theta\, e_\theta = f(r) = 0$ дает н/м $r = 0$ $\cos\gamma_0$ - Feldtk. а н/м $r = R$ $\cos\gamma_0$ - Pylyp. '12

Дальше если $\psi(r)$ - Pylyp. где $\gamma(r)$ - Верен, то $\psi[\gamma(r)]$, $h_m = h_m[\gamma(r)]$

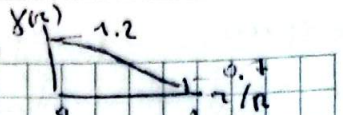
$W_m = (\cdot) \int dV\, \vec{h}_m[\gamma(r)] \cdot \vec{m}(\vec{r}) = \int dV \vec{m}(r) \int dV' f(\gamma(r')) = \int dV \int dV' (\ldots) \to \int dr \int dr'$

- двойной интеграл по r, r'. Поэтому $W_m[\gamma(r)]$ - Pylyp. (17в) Неверно.

$W_m(\gamma)$ Eq. (17в) получено из $W_m(\gamma)$ Eq. (12) путем $\gamma \to \gamma(r)$ и добавл. $\int dV$. Это

математически. Здесь Eq. (18) Pylyp. '12 где $\gamma(r)$ - неверна. В.с. $\frac{\partial \gamma}{\partial r}\Big|_{r=0,R} = 0$ - это exch. в.с

Рубрика 12 simulated $\gamma(z)$ для $R=\frac{3}{2}a_0 \sim 10$ nm



$\Delta\gamma = \gamma(R) - \gamma(0) = (0,7 - 1,2) \simeq -0,5\,(\text{rad}) \approx -29° \approx -\frac{\pi}{6}$; $p=+1$.

$\text{Hopf-}Q_H\,(BP) = \frac{1}{4\pi}\int_S \mathbf{b}\cdot d\mathbf{s}' = \frac{1}{4\pi}[\gamma(R)-\gamma(0)] = \frac{1}{4\pi}\left(-\frac{\pi}{6}\right) \simeq -\frac{1}{24}$, $Q_H \ll 1$

Численно $\gamma(z)$ падает от 69° до 46°. Зато слои Fig.2 – но $Q_H(BP) \neq 0$.

Рубрика 12 где $W_{in}(\gamma)$ имеет min при $\gamma_0 \approx 105°$?

(17/VIII) DW dynamics Mn_2Au. DW Mn_2Au

Для режима III (после быстрого поля) field at $t=100$ ps) distance travelled by DW пропорц. $\frac{1}{c^2\Delta}\ddot{q} + \frac{\gamma}{\Delta}\dot{q} = h(t)$ и $h(t)=0$,

$\dot{q} = -c^2\gamma q$, $c^2\gamma = 2k_c\gamma M_0 A = \frac{\Delta}{\tau}$, $v = -\frac{1}{\tau}v$, $v(t) = v_0 e^{-t/\tau}$

Distance travelled by DW at $t=0$ (когда $H_{sot}(t)=0$) до остановки at t_1, $\frac{t_1}{\tau} \gg 1$

$q(t_1) = \int_0^{t_1} dt\, v(t) = -v_0\tau \Big|_0^{t_1} e^{-t/\tau} = v_0\tau\,(1 - e^{-t_1/\tau})$ или $q(\infty) = v_0\tau$.

$v_0 \simeq 42.6$ m/s – velocity of DW steady motion in regime II где $H_{sot}(t) = \text{const}$ (пока $H_{sot} = 60$ mT)

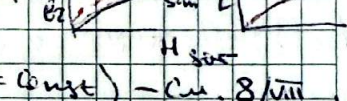
пока $v_0 = v(t)$ в момент $H_{sot}(t) = 0$

$q_{Br} = q(\infty)$ (braking distance) $= v_0(H_{sot})\,\tau$, $\tau = \frac{1}{2k_c\gamma M_0 A}$, $A = \frac{J_{eff}}{M_0^2 V}$

$J_{eff}\,(\text{моё}) = 2|4J_1 + J_2|\frac{1}{4}$, $J_{eff}\,(Ric) = |J_2|S^2$, $\frac{A(Ric)}{A(\text{моё})} = \frac{S^2}{2(4\frac{J_1}{J_2}+1)} = 0,786$

$\frac{q_{Br}(\text{моё})}{q_{Br}(Ric)} = \frac{A(Ric)}{A(\text{моё})} = 0,786$. $q_{Br}(\text{моё})$ длиннее слоя.

c simulations. Но если $q_{Br}(Ric)$ слишком с simulations – даёт очень длинный слой. Это где $v_0(H)$ – моё слой. с $v_0(H)$ – Ricardo.



При $H = \text{const}$ и $t \gg \tau$ (точнее усреднённая $H(t) = H = \text{const}$) – см. 8/VIII, $\tau \simeq 1,8$ ps

то $v_0(H) = \frac{c}{\sqrt{1+v^2}}$ – steady velocity at $t \gg \tau$. $v = \frac{d_0 c}{\Delta_0 \gamma H}$. $v_0(H)$ моё и

$v_0(H)$ Ricardo совпадут, если $\Delta_0(\text{моё}) = \Delta_0(Ric)$, это проверить.

Scale of $H = H_{sot}$ is $H_0 = \frac{d_0 c}{\gamma \Delta_0}$. При разных Δ_0, scales Fig.3c и 3b – Ricardo

Но если Δ_0 и Δ_0 брались из simulations, no problem. $v_0(H)$ are the same.

E-mail to Ricardo и Ruben отм. Revised Figs. 3d, 3e, 4 of the MS.

(18/VIII) Проверка magnetostatics of BP. $Q=1$.

$\frac{\partial \vec{m}}{\partial\theta} = \hat{i}\sin\theta\cos\varphi\,(\cos\chi - 1) + \hat{\theta}(\cos^2\theta\cos\chi + \sin^2\varphi) + \hat{\varphi}\cos\theta\sin\chi$

$\frac{\partial \vec{m}}{\partial\varphi} = -\hat{i}\sin^2\theta\sin\chi - \hat{\theta}\sin\theta\cos\theta\sin\chi + \hat{\varphi}\sin\theta\cos\chi$, $G_{\alpha\beta}(\vec{r},\vec{r}') = -(\nabla_r)_\alpha(\nabla_{r'})_\beta \frac{1}{|\vec{r}-\vec{r}'|}$

$\frac{1}{|\vec{r}-\vec{r}'|} = \sum_{\ell m}\frac{4\pi}{(2\ell+1)} f_\ell(rr')\,Y^*_{\ell m}(\theta'\varphi')\,Y_{\ell m}(\theta\varphi)$, $Y_{\ell m}(\theta\varphi) = a_{\ell m}P_\ell^m(\cos\theta)\,e^{im\varphi}$

$|\vec{r}-\vec{r}'|^{-1} = \sum_{\ell m}\frac{4\pi}{(2\ell+1)} a^2_{\ell m} f_\ell(rr')\,P_\ell^m(\cos\theta)\,P_\ell^m(\cos\theta')\,e^{im\varphi}e^{-im\varphi'}$, $f_\ell(rr') = (r_<)^\ell/(r_>)^{\ell+1} = \begin{cases} r^\ell/r'^{\ell+1}, & r<r' \\ r'^\ell/r^{\ell+1}, & r'<r \end{cases}$

$G_{r\varphi}(\vec{r}\vec{r}') = -\frac{\partial}{\partial r}\frac{1}{r'\sin\theta'}\frac{\partial}{\partial\varphi'}\sum_{\ell m}\frac{4\pi}{(2\ell+1)} a^2_{\ell m} f_\ell P_\ell^m P_\ell^m(\cdot)\,e^{im\varphi}e^{-im\varphi'}$

$G_{\theta\varphi}(\vec{r}\vec{r}') = -\frac{1}{r}\frac{\partial}{\partial\theta}\frac{1}{r'\sin\theta'}\frac{\partial}{\partial\varphi'}\sum_{\ell m}\frac{4\pi}{(2\ell+1)} a^2_{\ell m} f_\ell P_\ell^m P_\ell^m(\cdot)\,e^{im\varphi}e^{im\varphi'}$

$G_{\varphi\beta}(\vec{r}\vec{r}') = -\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(\nabla_{r'})_\beta\sum_{\ell m}\frac{4\pi}{(2\ell+1)} a^2_{\ell m} f_\ell P_\ell^m P_\ell^m(\cdot)\,e^{im\varphi}e^{-im\varphi'}$, $\beta = \hat{r}, \hat{\theta}, \hat{\varphi}$

$H_d(\vec{r}) = M_s\int d^3r'\sum_\beta\left[G_{d\beta}(\vec{r}\vec{r}')\,m_\beta(\vec{r}')\right]$. $\vec{m}(\vec{r})$ где BP не зависит от φ.

$\int d\varphi' G_{d\varphi}(\vec{r}\vec{r}') = (-1)\int_0^{2\pi} d\varphi'\,(\nabla_{r'})_\varphi\frac{1}{r'\sin\theta'}\frac{\partial}{\partial\varphi'}\sum_{\ell m}\frac{4\pi}{(2\ell+1)} a^2_{\ell m} f_\ell P_\ell^m P_\ell^m(\cdot)\,e^{im\varphi}e^{-im\varphi'} = \left|\int_0^{2\pi}(\cdots)e^{-im\varphi'} = 2\pi\delta_{m0}\right|$

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$$= -(2\pi)^2(\nabla_i)_\alpha \int_0^{2\pi} d\varphi' \frac{1}{r'\sin\theta'} \sum_{\ell m} \frac{4\pi}{(2\ell+1)} a_{\ell m}^2 f_\ell P_\ell^m P_\ell^m(\cdot) e^{im\varphi} e^{-im\varphi'} (-im) =$$

$$= -(\nabla_i)_\alpha \frac{1}{r'\sin\theta'} \sum_{\ell m} \frac{4\pi}{(2\ell+1)} a_{\ell m}^2 f_\ell P_\ell^m P_\ell^m(\cdot) e^{im\varphi} (+im) 2\pi\delta_{m0} \equiv 0$$

$$\int_0^{2\pi} d\varphi' G_{\alpha\beta}(\vec{r}\vec{r}') = -\int_0^{2\pi} d\varphi' \frac{1}{r\sin\theta} \frac{\partial}{\partial\varphi}(\nabla_i)_\beta \sum_{\ell m} \frac{4\pi}{(2\ell+1)} a_{\ell m}^2 f_\ell P_\ell^m P_\ell^m(\cdot) e^{im\varphi} e^{-im\varphi'} =$$

$$= -\frac{1}{r\sin\theta}(\nabla_{i'})_\beta \sum_{\ell m} \frac{4\pi}{(2\ell+1)} a_{\ell m}^2 f_\ell P_\ell^m P_\ell^m(\cdot) e^{im\varphi} (im) 2\pi\delta_{m0} \equiv 0.$$

Т.е. $\int d\varphi' G_{\alpha\beta}(\vec{r}\vec{r}') \equiv 0$ если $\alpha = \varphi$, $\beta = \varphi$, или $\alpha = \beta = \varphi$. При $\alpha \neq \varphi$, $\beta \neq \varphi$,

$$\int d\varphi' G_{\alpha\beta}(\vec{r}\vec{r}') = -\int d\varphi' (\nabla_i)_\alpha (\nabla_{i'})_\beta \sum_{\ell m} \frac{4\pi}{(2\ell+1)} a_{\ell m}^2 f_\ell P_\ell^m P_\ell^m(\cdot) e^{im\varphi} e^{-im\varphi'} =$$

$$= -2\pi (\nabla_i)_\alpha (\nabla_{i'})_\beta \sum_\ell \frac{4\pi}{(2\ell+1)} a_{\ell 0}^2 f_\ell P_\ell P_\ell(\cdot), \quad a_{\ell m}^2 = \frac{(2\ell+1)}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}, \quad a_{\ell 0}^2 = \frac{(2\ell+1)}{4\pi}$$

$$\int d\varphi' G_{\alpha\beta}(\vec{r}\vec{r}') = -2\pi (\nabla_i)_\alpha (\nabla_{i'})_\beta \sum_\ell f_\ell(rr') P_\ell(\cos\theta) P_\ell(\cos\theta'), \quad \alpha, \beta \neq \varphi. \quad \vec{h} = \frac{\vec{M}}{M_s}.$$

Def. $g_{\alpha\beta}(r\theta, r'\theta') = \int d\varphi' G_{\alpha\beta}(\vec{r}\vec{r}') = \frac{1}{2\pi}\int d\varphi \int d\varphi' G_{\alpha\beta}(\vec{r}\vec{r}') = -2\pi(\nabla_i)_\alpha(\nabla_{i'})_\beta \sum_\ell f_\ell(rr') P_\ell P_\ell(\cdot)$

Def. (45/VIII) $F(r\theta, r'\theta') = 2\pi \sum_\ell f_\ell(rr') P_\ell(\cos\theta) P_\ell(\cos\theta')$, then $g_{\alpha\beta}(r\theta, r'\theta') = (\nabla_i)_\alpha (\nabla_{i'})_\beta F$ (-1)

Def. $h_\alpha = \frac{1}{2\pi}\int d\varphi H_\alpha \frac{1}{M_s} = \frac{1}{2\pi}\int d\varphi \int d^3r' \sum_\beta G_{\alpha\beta}(\vec{r}\vec{r}') m_\beta(\vec{r}') = \int \frac{d\varphi}{2\pi} \int dr' d\theta' r'^2 \sin\theta' \int d\varphi' \times$

$\times \sum_\beta G_{\alpha\beta}(\vec{r}\vec{r}') m_\beta(r'\theta') = \int dr' \int d\theta' r'^2 \sin\theta' \sum_\beta g_{\alpha\beta}(r\theta, r'\theta') m_\beta(r'\theta')$

$h_\alpha(r,\theta) = \int dr' \int d\theta' \sum_\beta g_{\alpha\beta}(r\theta, r'\theta') m_\beta(\vec{r}'\theta') r'^2 \sin\theta'$, $dV = dr\, d\theta\, d\varphi\, r^2 \sin\theta$, $\alpha, \beta \neq \varphi$

Интеграл $\int \frac{d\varphi}{2\pi}$ в операт. $g_{\alpha\beta}$ не обозначаем, т.к. $\int d\varphi' G_{\alpha\beta}(\vec{r}\vec{r}')$ от φ не зависит.

$\vec{h}\cdot\frac{\partial\vec{m}}{\partial\theta} = h_\theta m_\theta + h_r m_r = h_r[\sin^2\theta\cos\gamma + p\cos^2\theta] + h_{\theta\theta}[\sin\theta\cos\theta(\cos\gamma - p)]$ $h_\varphi \equiv 0$.

$\vec{h}\cdot\frac{\partial\vec{m}}{\partial\varphi} = h_r\left(\frac{\partial\vec{m}}{\partial\varphi}\right)_r + h_{\theta\theta}\left(\frac{\partial\vec{m}}{\partial\varphi}\right)_{\theta\theta} = h_r\cdot(-1)\sin^2\theta\sin\gamma + h_{\theta\theta}(-1)\sin\theta\cos\theta\sin\gamma =$

$= -\sin^2\theta\sin\gamma[h_r + h_{\theta\theta}\operatorname{ctg}\theta]$

$\vec{h}\cdot\frac{\partial\vec{m}}{\partial\theta} = h_r\left(\frac{\partial\vec{m}}{\partial\theta}\right)_r + h_{\theta\theta}\left(\frac{\partial\vec{m}}{\partial\theta}\right)_{\theta\theta} = h_r[\sin\theta\cos\theta(p\cos\gamma - 1)] + h_{\theta\theta}[p\cos^2\theta\cos\gamma + \sin^2\theta]$

→ Slonc. eqs. $-2A(\nabla^2\Phi)\sin 2\theta - 2A(\nabla\theta\cdot\nabla\Phi)\sin 2\theta = M_s^2 h\cdot\frac{\partial\vec{m}}{\partial\Phi}$

$A(\nabla\Phi)^2\sin 2\theta - 2A\nabla^2\theta = M_s^2\frac{\partial\vec{m}}{\partial\theta}\cdot\vec{h}$, где $\theta = p\theta + \frac{\pi}{2}(1-p)$, $\Phi = \varphi + \gamma(r)$

$\sin\theta = \sin\theta$, $\cos\theta = p\cos\theta$, $\sin 2\theta = p\sin 2\theta$, $\nabla\theta = \frac{\hat{\theta}}{r}\frac{\partial}{\partial\theta}(p\theta) = \frac{p}{r}\hat{\theta}$

$\nabla^2\theta = \nabla^2\theta(\theta) = \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\theta(\theta)\right] = \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}[\sin\theta\, p] =$

$= \frac{1}{r^2}\frac{1}{\sin\theta}p\cos\theta = \frac{p}{r^2}\operatorname{ctg}\theta$, $\nabla\Phi = \hat{r}\frac{\partial}{\partial r}(\varphi+\gamma) + \frac{\hat{\varphi}}{r\sin\theta}\frac{\partial}{\partial\varphi}[\varphi+\gamma] =$

$= \hat{r}\gamma' + \frac{\hat{\varphi}}{r\sin\theta}$, $\nabla\theta\cdot\nabla\Phi = 0$. $\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}(\varphi+\gamma)\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}(\varphi+\gamma) =$

$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\gamma')$.

$-2A\sin^2\theta\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\gamma') - 2A\sin 2\theta\cdot p\cdot 0 = M_s^2\vec{h}\cdot\frac{\partial\vec{m}}{\partial\theta} = -M_s^2\sin^2\theta\sin\gamma[h_r + h_{\theta\theta}\operatorname{ctg}\theta]$

$Ap\sin 2\theta\left[(\gamma')^2 + \frac{1}{r^2\sin^2\theta}\right] - 2A\frac{p}{r^2}\operatorname{ctg}\theta = M_s^2\vec{h}\frac{\partial\vec{m}}{\partial\theta} = M_s^2\left[h_{\theta\theta}(\sin^2\theta\cdot\cancel{+p\cos^2\theta}) + \right.$

$\left. + h_r\cdot\sin\theta\cos\theta(\cos\gamma - p)\right]$

$$\frac{2A}{M_s^2}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\gamma') = \sin\gamma[h_r + h_{\theta\theta}\cdot\operatorname{ctg}\theta] \quad (1).$$

$$\frac{A}{M_s^2}p\sin 2\theta(\gamma')^2 = h_{\theta\theta}(\sin^2\theta\cdot\cancel{+p\cos^2\theta}) + h_r\sin\theta\cos\theta(p\cos\gamma - 1) \quad (2).$$

$$\frac{2A}{M_s^2}p(\gamma')^2 = h_{\theta\theta}(\operatorname{tg}\theta + p\cos\gamma\cdot\operatorname{ctg}\theta) + h_r(p\cos\gamma - 1) \quad (2).$$

Решение $\gamma' = 0$ возможно либо $\sin\gamma = 0$ (a), либо $h_r + h_{\theta\theta}\operatorname{ctg}\theta = 0$, $\sin\gamma \neq 0$

(2) при $\sin\gamma = 0$, $\cos\gamma = p$, $h_{\theta\theta}(\operatorname{tg}\theta + \operatorname{ctg}\theta) = 0$, $h_{\theta\theta}/\sin\theta\cos\theta = 0$, что возможно при $h_{\theta\theta} = 0$.

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$h_{ee}=0$, $\cos\gamma=p$ - hedgehog BP globally. $\gamma'=0$ and θ,φ are arbitrary.

$D=h_{ee}(\operatorname{tg}\varepsilon_{ee}+p\cos\gamma\operatorname{ctg}\varepsilon_{ee})+h_z(p\cos\gamma-1)=h_{ee}\operatorname{tg}\varepsilon_{ee}-h_z+p\cos\gamma(h_z+h_{ee}\operatorname{ctg}\varepsilon_{ee})$

When $\gamma'=0$ $h_z+h_{ee}\operatorname{ctg}\varepsilon_{ee}=0$, $D=h_{ee}\operatorname{tg}\varepsilon_{ee}-h_z=h_{ee}\operatorname{tg}\varepsilon_{ee}+h_{ee}\operatorname{ctg}\varepsilon_{ee}=\frac{h_{ee}}{\sin\varepsilon_{ee}\cos\varepsilon_{ee}}$

$D=0$ is possible only when $\cos\gamma=p$ (hedgehog).

(B) $h_z+h_{ee}\operatorname{ctg}\varepsilon_{ee}=0$, but $\sin\gamma\neq0$, then $D=\frac{h_{ee}}{\sin\varepsilon_{ee}\cos\varepsilon_{ee}}=0$, $h_{ee}=0$, which is impossible for a hedgehog $\cos\gamma=p$, $\sin\gamma=0$.

(1) $\int l_e^2\cdot\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\gamma')=\sin\gamma(h_z+h_{ee}\operatorname{ctg}\varepsilon_{ee})$

(2) $\left.\begin{aligned}pl_e^2(\gamma')^2&=p\cos\gamma(h_z+h_{ee}\operatorname{ctg}\varepsilon_{ee})+h_{ee}\operatorname{tg}\varepsilon_{ee}-h_z\end{aligned}\right\}$ - equations

trivial solution $\gamma'=0$, $\cos\gamma=p$ ($h_{ee}=0$ (hedgehog BP), $h_z\neq0$, $\sin\gamma=0$)

If we want $\gamma'=0$, $\sin\gamma\neq0$, then $pl_e^2(\gamma')^2=0$, $h_z+h_{ee}\operatorname{ctg}\varepsilon_{ee}=0$, $h_{ee}\operatorname{tg}\varepsilon_{ee}-h_z=\frac{h_{ee}}{\sin\varepsilon_{ee}\cos\varepsilon_{ee}}=0$, $h_{ee}=0$, $h_z=0$ - this is impossible. Because $h_{ee}(r)=0$, $h_z(r)$ - for different equations, then their roots γ_{ee} and γ_z coincide? $\gamma_{ee}=\text{const}$, $\gamma_z=\text{const}$

$h_z+h_{ee}\operatorname{ctg}\varepsilon_{ee}=-\left(\frac{2}{r}+\operatorname{ctg}\varepsilon_{ee}\frac{\partial}{\partial\varepsilon_{ee}}\right)\int dr'r'^2\left[\frac{2}{r'^3}\int d\varepsilon_{ee}'\sin\varepsilon_{ee}'Fm_z+\frac{1}{r'^2}\int d\varepsilon_{ee}'\sin\varepsilon_{ee}'\frac{\partial F}{\partial\varepsilon_{ee}'}m_{ee}\right]$

→ In the case h_z on 15/VIII I made a mistake, I forgot $\sin^2\varepsilon_{ee}$ in m_z. Now all OK.

$F(rr',\varepsilon_{ee}\varepsilon_{ee}')=2\pi\sum_l f_l(rr')P_l(\cos\varepsilon_{ee})P_l(\cos\varepsilon_{ee}')$, $\left\{\begin{aligned}m_z&=\sin^2\varepsilon_{ee}\cos\gamma+p\cos^2\varepsilon_{ee}\\m_{ee}&=\sin\varepsilon_{ee}\cos\varepsilon_{ee}(\cos\gamma-p)\end{aligned}\right\}$

$m_z=(1-\cos^2\varepsilon_{ee})\cos\gamma+p\cos^2\varepsilon_{ee}=\cos\gamma+(p-\cos\gamma)\cos^2\varepsilon_{ee}$, $P_2(\cos\varepsilon_{ee})=\frac{1}{2}(3\cos^2\varepsilon_{ee}-1)$, $P_0(\varepsilon_{ee})=1$

$m_z=P_0\cos\gamma+(p-\cos\gamma)\frac{1}{3}(2P_2+1)=P_0\left[\cos\gamma+\frac{1}{3}(p-\cos\gamma)\right]+\frac{2}{3}(p-\cos\gamma)P_2(\cos\varepsilon_{ee})$

$\int_{-1}^{1}dx\,P_l(x)P_{l'}(x)=\frac{2}{(2l+1)}\delta_{ll'}$.

$\int d\varepsilon_{ee}'\sin\varepsilon_{ee}'Fm_z=\int d\varepsilon_{ee}'\sin\varepsilon_{ee}'2\pi\sum_l f_lP_l(\cos\varepsilon_{ee})P_l(\cos\varepsilon_{ee}')\left[P_0\left\{\cos\gamma+\frac{1}{3}(p-\cos\gamma)\right\}+\right.$

$\left.+P_2(\cos\varepsilon_{ee}')\frac{2}{3}(p-\cos\gamma)\right]=2\pi\sum_l f_lP_l(\cos\varepsilon_{ee})\int_{-1}^{1}dx\,P_l(x)\left[P_0(x)\left\{\cos\gamma+\frac{1}{3}(p-\cos\gamma)\right\}+\right.$

$\left.+P_2(x)\frac{2}{3}(p-\cos\gamma)\right]\overset{\gamma=\gamma(r)}{=}2\pi\sum_l f_lP_l\left[2\delta_{l0}\left\{\cos\gamma+\frac{1}{3}(p-\cos\gamma)\right\}+\frac{2}{5}\frac{2}{3}(p-\cos\gamma)\delta_{l2}\right]=$

$=4\pi\left[f_0P_0\left\{\cos\gamma+\frac{1}{3}(p-\cos\gamma)\right\}+f_2P_2\frac{2}{15}(p-\cos\gamma)\right]$

$\int d\varepsilon_{ee}'\sin\varepsilon_{ee}'Fm_z=4\pi\left[f_0\left\{\cos\gamma+\frac{1}{3}(p-\cos\gamma)\right\}+\frac{2}{15}f_2(p-\cos\gamma)P_2(\cos\varepsilon_{ee})\right]$ - coeff. on 15/VIII

$\frac{dP_l(x)}{dx}=\frac{l}{(x^2-1)}\left[xP_l(x)-P_{l-1}(x)\right]$, $dx=d(\cos\varepsilon_{ee})=-\sin\varepsilon_{ee}d\varepsilon_{ee}$, $x=\cos\varepsilon_{ee}$.

$\frac{dP_l(\cos\varepsilon_{ee})}{d\varepsilon_{ee}}\frac{(-1)}{\sin\varepsilon_{ee}}=\frac{l}{-\sin^2\varepsilon_{ee}}\left[xP_l(x)-P_{l-1}(x)\right]$, $\frac{dP_l(\cos\theta)}{d\theta}=\frac{l}{\sin\theta}\left[xP_l(x)-P_{l-1}(x)\right]$

$\int d\varepsilon_{ee}'\sin\varepsilon_{ee}'\frac{\partial F}{\partial\varepsilon_{ee}'}m_{ee}=\int d\varepsilon_{ee}'\sin\varepsilon_{ee}'\frac{\partial}{\partial\varepsilon_{ee}'}2\pi\sum_l f_lP_l(\cos\varepsilon_{ee})P_l(\cos\varepsilon_{ee}')\sin\varepsilon_{ee}'\cos\varepsilon_{ee}'(\cos\gamma-p)=$

$=2\pi(\cos\gamma-p)\sum_l f_lP_l\int d\varepsilon_{ee}'\sin\varepsilon_{ee}'\frac{\partial P_l(\cos\varepsilon_{ee}')}{\partial\varepsilon_{ee}'}\sin\varepsilon_{ee}'\cos\varepsilon_{ee}'=2\pi(\cos\gamma-p)\sum_l f_lP_l(\cos\varepsilon_{ee})\times$

$\times\int d\varepsilon_{ee}'\sin\varepsilon_{ee}'\frac{l}{\sin\varepsilon_{ee}'}\left[xP_l(x)-P_{l-1}(x)\right]_{x=\cos\varepsilon_{ee}'}\cos\varepsilon_{ee}'=2\pi(\cos\gamma-p)\sum_l f_lP_l(\cos\varepsilon_{ee})l\times$

$\times\int_{-1}^{1}dx\left[xP_l(x)-P_{l-1}(x)\right]x=(-)\int_{-1}^{1}dx\left[x^2P_l(x)-xP_{l-1}(x)\right]=(-)\int_{-1}^{1}dx\left[\frac{1}{3}(2P_2(x)+1)P_l(x)-\right.$

$\left.-P_1(x)P_{l-1}(x)\right]=(-)\int_{-1}^{1}dx\left[\frac{2}{3}P_2(x)P_l(x)+\frac{1}{3}P_0P_l(x)-P_1(x)P_{l-1}(x)\right]=$ |$\gamma=\gamma(r)$| $=$

$=2\pi(\cos\gamma-p)\sum_l f_lP_l\left[\frac{2}{3}\frac{2}{5}\delta_{l2}+\frac{1}{3}2\delta_{l0}-\frac{2}{3}\delta_{l2}\right]=2\pi(\cos\gamma-p)2f_2P_2\frac{2}{3}\left(\frac{2}{5}-1\right)=$

$=-2\pi\frac{4}{3}\frac{3}{5}(\cos\gamma-p)f_lP_2=-2\pi\frac{4}{5}(\cos\gamma-p)f_2P_2$ - error. on 15/VIII error. 3/2

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$$h_2 + h_{ee}\,ctg\varepsilon\varepsilon = -\left(\frac{\partial}{\partial r} + \frac{ctg\varepsilon\varepsilon}{r}\frac{\partial}{\partial\varepsilon\varepsilon}\right)\int dr' r'^2 \left[\frac{2}{4\pi}\frac{\partial}{\partial r'}\left[\ldots f_0\left\{\cos\gamma + \frac{1}{3}(\rho - \cos\gamma)\right\} + \right.\right.$$

$$\left.\left. + \frac{2}{15}(\rho - \cos\gamma) f_2 P_2\right] + \frac{1}{r'}(-2\pi)\frac{4}{5}(\cos\gamma - \rho) f_2 P_2\right] = -4\pi\left(\frac{\partial}{\partial r} + \frac{ctg\varepsilon\varepsilon}{r}\frac{\partial}{\partial\varepsilon\varepsilon}\right) \times$$

$$\times \int dr' r'^2 \left[\frac{\partial}{\partial r'}\left[f_0\left(\cos\gamma + \frac{1}{3}(\rho - \cos\gamma)\right) + \frac{2}{15}(\rho - \cos\gamma) f_2 P_2\right] - \frac{2}{5}\frac{1}{r'}(\cos\gamma - \rho) f_2 P_2\right]$$

Видим $\cos\varepsilon = \cos\gamma - \rho$, $\rho - \cos\gamma = -\cos\varepsilon$, $\varepsilon' = \gamma'$

$$h_2 + h_{ee}\,ctg\varepsilon\varepsilon = -4\pi\left(\frac{\partial}{\partial r} + \frac{ctg\varepsilon\varepsilon}{r}\frac{\partial}{\partial\varepsilon\varepsilon}\right)\int dr' r'^2 \left[\frac{\partial}{\partial r'}\left[\left(\rho + \cos\varepsilon - \frac{1}{3}\cos\varepsilon\right) f_0 + \frac{2}{15}\cos\varepsilon (-1) f_2 P_2\right] - \frac{2}{5}\frac{1}{r'}\cos\varepsilon f_2 P_2\right]$$

$$h_2 + h_{ee}\,ctg\varepsilon\varepsilon = -4\pi\left(\frac{\partial}{\partial r} + \frac{ctg\varepsilon\varepsilon}{r}\frac{\partial}{\partial\varepsilon\varepsilon}\right)\int dr' r'^2 \left[\frac{\partial}{\partial r'}\left\{\left(\rho + \frac{2}{3}\cos\varepsilon\right) f_0 - \frac{2}{15}\cos\varepsilon f_2 P_2\right\} - \frac{2}{5}\frac{1}{r'}\cos\varepsilon f_2 P_2\right]$$

$$S'(\varepsilon) = \int_0^R dr' r'^2 \left[\frac{\partial}{\partial r'}\left\{\left(\rho + \frac{2}{3}\cos\varepsilon\right) f_0 - \frac{2}{15}\cos\varepsilon f_2 P_2\right\} - \frac{2}{5}\frac{1}{r'}\cos\varepsilon f_2 P_2\right]$$

$$h_2 + h_{ee}\,ctg\varepsilon\varepsilon = -4\pi\left(\frac{\partial}{\partial r} + \frac{ctg\varepsilon\varepsilon}{r}\frac{\partial}{\partial\varepsilon\varepsilon}\right) S(\varepsilon)$$

Расчет. $J = \int dr' r'^2 \frac{\partial}{\partial r'} f_0$. $\cos\varepsilon = \cos\gamma - \rho$ не годится, т.к. $\sin\gamma$ не найти.

$$S(\gamma) = \int dr' r'^2 \left[\frac{\partial}{\partial r'}\left[f_0\left(\frac{1}{3}\rho + \frac{2}{3}\cos\gamma\right) + \frac{2}{15}\rho f_2 P_2 - \frac{2}{15}\cos\gamma f_2 P_2\right] + \frac{2}{5}\frac{\rho}{r'} f_2 P_2 - \frac{2}{5}\frac{1}{r'}\cos\gamma f_2 P_2\right] =$$

$= S_0 + S_1(\gamma)$, S_0 - часть $S(\gamma)$ не зависящая от $\cos\gamma$.

$$S_0 = \int dr' r'^2 \left[\frac{\partial}{\partial r'}\left[\frac{1}{3}\rho f_0 + \frac{2}{15}\rho f_2 P_2\right] + \frac{2}{5}\frac{\rho}{r'} f_2 P_2\right] = \rho \int dr' r'^2 \left[\frac{\partial}{\partial r'}\left(\frac{1}{3} f_0 + \frac{2}{15} f_2 P_2\right) + \right.$$

$$\left. + \frac{2}{5}\frac{1}{r'} f_2 P_2\right]$$

$$I_1 = \int dr' r'^2 \frac{\partial}{\partial r'} f_0 = \int_0^r dr' r'^2 \frac{\partial}{\partial r'}\frac{1}{r} + \int_r^R dr' r'^2 \frac{\partial}{\partial r'}\left(\frac{1}{r'}\right) = \quad (r'<r,\ r'>r)$$

$$= \int_r^R dr' \cdot r'^2 \frac{(-1)}{r'^2} = -\int_r^R dr' = (r - R).$$

$$I_2 = \int dr' r'^2 \frac{\partial}{\partial r'} f_2 = \int_0^r dr' r'^2 \frac{\partial}{\partial r'}\frac{r'^2}{r^3} + \int_r^R dr' r'^2 \frac{\partial}{\partial r'}\frac{r^2}{r'^3} = \frac{1}{r^3}\int_0^r dr' r'^2 \cdot 2r' + r^2 \int_r^R dr' r'^2 \frac{(-3)}{(r')^4} =$$

$$= \frac{2}{r^3}\int_0^r dr' r'^3 - 3r^2 \int_r^R dr' \frac{1}{(r')^2} =$$

$$= \frac{2}{r^3}\frac{1}{4}(r')^4 \Big|_0^r - 3r^2 \frac{(-1)}{r'}\Big|_r^R = \frac{1}{2}r + 3r^2\left(\frac{1}{R} - \frac{1}{r}\right) = -\frac{5}{2}r + 3\frac{r^2}{R},$$

$$I_3 = \int dr' r'^2 \frac{1}{r'} f_2 = \int_0^r dr' r' \frac{r'^2}{r^3} + \int_r^R dr' r' \frac{r^2}{r'^3} = \frac{1}{r^3}\int_0^r dr' r'^3 + r^2 \int_r^R dr' \frac{1}{r'^2} =$$

$$= \frac{1}{r^3}\frac{1}{4}(r')^4 \Big|_0^r + r^2 (-1)\frac{1}{r'}\Big|_r^R = \frac{1}{4}r - r^2\left(\frac{1}{R} - \frac{1}{r}\right) = \frac{5}{4}r - \frac{r^2}{R}$$

$$S_0 = \rho\left[\frac{1}{3} I_1 + \frac{2}{15} P_2 I_2 + \frac{2}{5} P_2 I_3\right] = \rho\left[\frac{1}{3}(r - R) + \frac{2}{15} P_2\left(-\frac{5}{2}r + 3\frac{r^2}{R}\right) + \right.$$

$$\left. + \frac{2}{5} P_2\left(\frac{5}{4}r - \frac{r^2}{R}\right)\right] = \rho\left[\frac{1}{3}(r - R) + P_2\left\{\frac{1}{6}r\right\}\right] = \frac{1}{3}\rho\left[r - R + \frac{1}{2}r P_2\right]$$

$$h_2 + h_{ee}\,ctg\varepsilon\varepsilon = -4\pi\left(\frac{\partial}{\partial r} + \frac{ctg\varepsilon\varepsilon}{r}\frac{\partial}{\partial\varepsilon\varepsilon}\right) S(\gamma), \quad S(\gamma) = S_0 + S_1(\gamma)$$

$$\left(h_2 + h_{ee}\,ctg\varepsilon\varepsilon\right)_0 = -4\pi\left(\frac{\partial}{\partial r} + \frac{ctg\varepsilon\varepsilon}{r}\frac{\partial}{\partial\varepsilon\varepsilon}\right) S_0 = -\frac{4\pi}{3}\rho\left(\frac{\partial}{\partial r} + \frac{ctg\varepsilon\varepsilon}{r}\frac{\partial}{\partial\varepsilon\varepsilon}\right)\left[r - R + \frac{1}{2}r P_2\right] =$$

$$= -\frac{4\pi}{3}\rho\left[1 + \frac{1}{2}P_2 + ctg\varepsilon\varepsilon\frac{\partial}{\partial\varepsilon\varepsilon}\left(\frac{1}{2}r P_2\right)\right] = -\frac{4\pi}{3}\rho\left[1 + \frac{1}{2}P_2 + \frac{1}{2}ctg\varepsilon\varepsilon\frac{\partial P_2}{\partial\varepsilon\varepsilon}\right]$$

$$ctg\varepsilon\varepsilon\frac{\partial P_2}{\partial\varepsilon\varepsilon} = ctg\varepsilon\varepsilon\,(-3)\sin\varepsilon\varepsilon\cos\varepsilon\varepsilon = -3\cos^2\varepsilon\varepsilon, \quad P_2 + ctg\varepsilon\varepsilon\frac{\partial P_2}{\partial\varepsilon\varepsilon} = \frac{1}{2}(3\cos^2\varepsilon\varepsilon - 1) - 3\cos^2\varepsilon\varepsilon =$$

$$= -\frac{3}{2}\cos^2\varepsilon\varepsilon - \frac{1}{2}. \quad \left(h_2 + h_{ee}\,ctg\varepsilon\varepsilon\right)_0 = -\frac{4\pi}{3}\rho\left[1 + \frac{1}{2}\left(-\frac{3}{2}\cos^2\varepsilon\varepsilon - \frac{1}{2}\right)\right] = -\frac{4\pi}{3}\frac{3}{4}\sin^2\varepsilon\varepsilon\rho =$$

$$\left(h_2 + h_{ee}\,ctg\varepsilon\varepsilon\right)_0 = -\pi\rho\sin^2\varepsilon\varepsilon. \qquad = -\pi\rho\sin^2\varepsilon\varepsilon$$

$$S_1(\gamma) = \int dr' r'^2 \left[\frac{\partial}{\partial r'}\left\{\frac{2}{3} f_0\cos\gamma - \frac{2}{15}\cos\gamma f_2 P_2\right\} - \frac{2}{5}\frac{1}{r'}\cos\gamma f_2 P_2\right]$$

Учтем $\cos\gamma = const \Rightarrow$ $S_1(\gamma) = \cos\gamma \int dr' r'^2 \left[\frac{\partial}{\partial r'}\left(\frac{2}{3} f_0 - \frac{2}{15} f_2 P_2\right) - \frac{2}{5}\frac{1}{r'} f_2 P_2\right] =$

$$= \cos\gamma\left[\frac{2}{3} I_1 - \frac{2}{15} P_2 I_2 - \frac{2}{5} P_2 I_3\right] = \cos\gamma\left[\frac{2}{3}(r - R) - \frac{2}{15} P_2\left(-\frac{5}{2}r + 3\frac{r^2}{R}\right) - \frac{2}{5} P_2\left(\frac{5}{4}r - \frac{r^2}{R}\right)\right] =$$

$$= \cos\gamma\left[\frac{2}{3}(r - R) - \frac{1}{3}r P_2\right], \text{ вот наконец! Мы вычислили коэф. в } S_0 \text{ и } S_1(\gamma).$$

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$(h_2 + h_{ee}\operatorname{ctg}\theta\theta)_1 = -4\pi\left(\frac{\partial}{\partial r} + \frac{\operatorname{ctg}\theta\theta}{r}\frac{\partial}{\partial\theta\theta}\right)S_1(\delta) = -4\pi\left(\frac{\partial}{\partial r} + \operatorname{ctg}\theta\theta\frac{\partial}{\partial\theta\theta}\right)\cos\gamma\left|\frac{r}{6}(r-R) - \frac{1}{3}R^3\right|$

$= -4\pi\cos\gamma\left[\frac{2}{3} - \frac{1}{3}P_0 - \frac{1}{3}\frac{\operatorname{ctg}\theta\theta}{r}\frac{\partial}{\partial\theta\theta}(rP_2)\right] = -4\pi\cos\gamma\left[\frac{2}{3} - \frac{1}{3}P_0 - \frac{1}{3}\operatorname{ctg}\theta\theta\frac{\partial P_2}{\partial\theta\theta}\right] =$

$= -4\pi\cos\gamma\left[\frac{2}{3} + \frac{1}{3}\frac{1}{2}(3\cos^2\theta\theta - 1) - \frac{1}{3}(+3)\cos^2\theta\theta\right] = 4\pi\cos\gamma\left[\frac{1}{3} + \frac{1}{6} + \frac{1}{2}\cos^2\theta\theta\right] =$

$= -2\pi\cos\gamma\left[\frac{5}{3} + \cos^2\theta\theta\right]$. $h_2 + h_{ee}\operatorname{ctg}\theta\theta = -\pi p\sin^2\theta\theta - 2\pi\cos\gamma\left(\frac{5}{3} + \cos^2\theta\theta\right)$

Сравним с S_0 есть $f(\theta\theta)$. Или $\cos\gamma = \text{const}$.

$\langle\sin^2\theta\theta\rangle = \frac{1}{V}\int dV\sin^2\theta\theta = \frac{1}{\frac{4}{3}\pi R^3}\int_0^R dr\int_0^{\pi} d\theta\theta\int_0^{2\pi} d\varphi\, r^2\sin\theta\theta\cdot\sin^2\theta\theta =$

$= \frac{3}{4\pi R^3}\cdot\frac{1}{3}R^3\cdot 2\pi\int_0^{\pi} d\theta\theta\sin\theta\theta\cdot\sin^2\theta\theta = \frac{1}{2}\int_{-1}^{1} dx\,(1-x^2) = \frac{1}{2}\left(x - \frac{x^3}{3}\right)\Big|_{-1}^{1} = 2/3$

$\langle\cos^2\theta\theta\rangle = 1 - \langle\sin^2\theta\theta\rangle = 1/3$. Если $h_2 + h_{ee}\operatorname{ctg}\theta\theta$ зависимость $\cos^2\theta\theta$ с $\sin^2\theta\theta$ (или $\sin\theta\theta$)

$\langle h_2 + h_{ee}\operatorname{ctg}\theta\theta\rangle_V = -\pi p\cdot\frac{2}{3} - 2\pi\cos\gamma\left(\frac{5}{3} + \frac{1}{3}\right) = -\frac{\pi}{3}2p - 2\pi\cos\gamma\cdot 2 = 0$

$\cos\gamma_0 = -\frac{2\pi/3\cdot p}{4\pi} = -\frac{1}{6}p$. Note $\left(\frac{\partial}{\partial r} + \frac{\operatorname{ctg}\theta\theta}{r}\frac{\partial}{\partial\theta\theta}\right)r^2P_2(\cos\theta\theta) = -r$ не зависит от $\theta\theta$.

(19/VIII) $h(r,\theta\theta) = h_2(r,\theta\theta) + h_{ee}(r,\theta\theta)\operatorname{ctg}\theta\theta$ не должно зависеть от $\theta\theta$.

Иначе Eq. (1) для $f(r)$ невозможно. $h_1(r,\theta\theta) = h_{ee}\operatorname{ctg}\theta\theta - h_2 +$ не должно зависеть от $\theta\theta$, иначе Eq (2) невозможно.

Пульс.12 (16/VIII) $h(r,\theta\theta) = h_2 + h_{ee}\operatorname{ctg}\theta\theta = -p\pi - 3\pi\cos\gamma + 3\pi(p-\cos\gamma)\cos^2\theta\theta =$

$= -p\pi\sin^2\theta\theta + \pi\cos\gamma\left[\cos^2\theta\theta + 3\right]$

Имеем $h_2 + h_{ee}\operatorname{ctg}\theta\theta = -\pi p\sin^2\theta\theta - 2\pi\cos\gamma\left(\frac{5}{3} + \cos^2\theta\theta\right)$ – решение при $(\cos\gamma)'' = 0$ (при $\gamma = \text{const}$). – обнаружили.

Сделаем аналогию в $S_1(\delta)$ – расчет $\left(\frac{1}{3}P_2\right) \to \left(\frac{1}{6}\right)$ не надо.

$(h_2 + h_{ee}\operatorname{ctg}\theta\theta)_1 = -4\pi\cos\gamma\left(\frac{\partial}{\partial r} + \frac{\operatorname{ctg}\theta\theta}{r}\frac{1}{\partial\theta\theta}\right)\left[\frac{2}{3}(r-R) - \frac{1}{6}rP_2\right] =$

$= -4\pi\cos\gamma\left[\frac{2}{3} - \frac{1}{6}P_2 - \frac{1}{6}\frac{\operatorname{ctg}\theta\theta}{r}\frac{\partial}{\partial\theta\theta}(rP_2)\right] = -4\pi\cos\gamma\left[\frac{2}{3} - \frac{1}{6}P_2 - \frac{1}{6}\operatorname{ctg}\theta\theta\frac{\partial P_2}{\partial\theta\theta}\right] =$

$= -4\pi\cos\gamma\left[\frac{2}{3} - \frac{1}{6}\frac{1}{2}(3\cos^2\theta\theta - 1) - \frac{1}{6}(-3)\cos^2\theta\theta\right] = -4\pi\cos\gamma\left[\frac{2}{3} + \frac{1}{12} + \frac{1}{4}\cos^2\theta\theta\right] =$

$= 4\pi\cos\gamma\left[\frac{3}{4} + \frac{1}{4}\cos^2\theta\theta\right] = -\pi\cos\gamma\,(3 + \cos^2\theta\theta)$

$h_2 + h_{ee}\operatorname{ctg}\theta\theta = -\pi p\sin^2\theta\theta - \pi\cos\gamma(3 + \cos^2\theta\theta)$, что совпадает с $(h_2 + h_{ee}\operatorname{ctg}\theta\theta)$ Пульс. построенным на основе потенциала $\psi(\theta)$ Eq. (11) – Пульсировский 12 (!).

Т.е. оба уравнения верны (имеем в Пульс. 12 или $\gamma = \text{const}$)

Если $\sin^2\theta\theta \to \langle\sin^2\theta\theta\rangle_V = 2/3$, $\cos^2\theta\theta \to \langle\cos^2\theta\theta\rangle_V = 1/3$ $\Big) \to \langle h_2 + h_{ee}\operatorname{ctg}\theta\theta\rangle_V = -\pi p\frac{2}{3} - \pi\cos\gamma\left(3 + \frac{1}{3}\right)$,

$\cos\gamma_0 = -\frac{2/3\,p}{(3 + 1/3)} = -\frac{2p}{10} = -\frac{p}{5}$. У Пульс. 12 $\cos\gamma_0 = -\frac{p}{4}$ Eq. (13).

Если $\sin^2\theta\theta \to 1/2$ $\cos^2\theta\theta \to \frac{1}{2}$ $\langle h_2 + h_{ee}\operatorname{ctg}\theta\theta\rangle = -\pi p\frac{1}{2} - \pi\cos\gamma\left(3 + \frac{1}{2}\right)$

$\cos\gamma_0 = -\frac{p/2}{(3 + 1/2)} = -\frac{p}{7}$. $\cos\gamma_0 = -\frac{p\sin^2\theta\theta}{3 + \cos^2\theta\theta}$ и $\cos\gamma_0 = -\frac{p}{4}$ или разумное значение (не нужно).

$\cos\gamma_0 = -p/5 \to$ мое $\gamma_0 = \begin{cases} 101,5^\circ, & p = +1 \\ 78,5^\circ, & p = -1 \end{cases}$, $\cos\gamma_0 = -\frac{p}{4}$. $\gamma_0 = \arccos\left(-\frac{p}{4}\right) = \begin{cases} 104,1^\circ, & p = +1 \\ 75,5^\circ, & p = -1 \end{cases}$ Пульс. 12

$\cos\gamma = -p/5$ можно разумно согласовать.

Eq. (1)? $-p\cos\gamma(h_2 + h_{ee}\operatorname{ctg}\theta\theta) + h_{ee}\operatorname{tg}\theta\theta - h_2 = p\cos\gamma(\ldots) + 2\pi(p - \cos\gamma)\sin\theta\theta\cos\theta\theta\cdot\operatorname{tg}\theta\theta -$

$- \left[-p\pi - 3\pi\cos\gamma - \pi(p - \cos\gamma)\cos^2\theta\theta\right] = p\cos\gamma\left[-\pi p\sin^2\theta\theta - \pi\cos\gamma(3 + \cos^2\theta\theta)\right] +$

$+ 2\pi(p - \cos\gamma)\sin^2\theta\theta + p\pi + 3\pi\cos\gamma + \pi(p - \cos\gamma)\cos^2\theta\theta =$

$= \pi\left[-\cos\gamma\sin^2\theta\theta - p\cos^2\gamma(3 + \cos^2\theta\theta) + 2(p - \cos\gamma)\sin^2\theta\theta + p + 3\cos\gamma + (p - \cos\gamma)\cos^2\theta\theta\right]$

При $\cos\gamma = -\frac{p}{5}$, $\sin^2\theta\theta = 2/3$, $\cos^2\theta\theta = 1/3$ должно $[\ldots] = 0$?

$\frac{p}{5}\cdot\frac{2}{3} - p\cdot\frac{1}{25}\left(3 + \frac{1}{3}\right) + 2\cdot\frac{6}{5}p\frac{2}{3} + p + 3\left(-\frac{p}{5}\right) + \frac{6}{5}p\frac{1}{3} = p\left[\frac{2}{15} - \frac{1}{25}\frac{10}{3} + \frac{8}{5} + 1 - \frac{3}{5} + \frac{2}{5}\right]$

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$[\dots] = \frac{12}{5}$, т.е. $\cos\gamma_0 = -\frac{p}{5}$ удовл. Eq.(1) при $\gamma' = 0$, но не удовл. Eq.(2).

$[p\cos\gamma(h_r + h_{\theta\theta}\operatorname{ctg}\theta) + h_{\theta\theta}\operatorname{tg}\theta - h_r] = \left|\begin{matrix}\text{hedgehog}\\ \cos\gamma = p\end{matrix}\right| = h_r - h_r = 0$, т.е. hedgehog $\cos\gamma = p$ всегда решение обоих Eq.(1) и Eq.(2) при $\gamma' = 0$.

Вывод: исходим $\gamma = \gamma(r)$, тогда $\gamma = \gamma(r,\theta)$, $\Phi = \varphi + \gamma(r,\theta)$

$$\nabla\Phi = \hat r\frac{\partial\Phi}{\partial r} + \frac{\hat\theta}{r}\frac{\partial\Phi}{\partial\theta} + \frac{\hat\varphi}{r\sin\theta}\frac{\partial\Phi}{\partial\varphi} = \hat r\gamma_r' + \frac{\hat\theta}{r}\gamma_\theta' + \frac{\hat\varphi}{r\sin\theta}$$

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\varphi^2} =$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\gamma_r'\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\gamma_\theta'\right),\quad \vec\Omega\cdot\nabla\Phi = \frac{p}{r}\hat\theta\cdot(\nabla\Phi) = \frac{p}{r}\frac{1}{r}\gamma_\theta'$$

$$\begin{cases} -2A\sin^2\theta\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\gamma_r'\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\gamma_\theta'\right)\right] - 2Ap\sin^2\theta\frac{p}{r^2}\gamma_\theta = M_s^2\hat n\cdot\frac{\partial\vec m}{\partial\varphi} \\ Ap\sin^2\theta\left[\left(\gamma_r'\right)^2 + \frac{1}{r^2}\left(\gamma_\theta'\right)^2 + \frac{1}{r^2\sin^2\theta}\right] - 2A\frac{p}{r^2}\operatorname{ctg}\theta = M_s^2\hat n\cdot\frac{\partial\vec m}{\partial\theta} \end{cases}$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\gamma_\theta'\right) = \frac{1}{\sin\theta}\left[\cos\theta\gamma_\theta' + \sin\theta\gamma_\theta''\right] = \operatorname{ctg}\theta\gamma_\theta' + \gamma_\theta''$$

$$-2A\sin^2\theta[\dots] - 2Ap\sin^2\theta\frac{p}{r^2}\gamma_\theta' = M_s^2(-1)\sin^2\theta\sin\gamma\left(h_r + h_\theta\operatorname{ctg}\theta\right)$$

$$\ell_e^2\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\gamma_r'\right) + \frac{\operatorname{ctg}\theta}{r^2}\gamma_\theta' + \frac{1}{r^2}\gamma_\theta''\right] + 2\ell_e^2\operatorname{ctg}\theta\cdot\frac{1}{r^2}\gamma_\theta' = \sin\gamma\left(h_r + h_\theta\operatorname{ctg}\theta\right)$$

$$Ap\sin^2\theta[\dots] - 2A\frac{p}{r^2}\operatorname{ctg}\theta = M_s^2\left[h_r\sin\theta\cos\theta(p\cos\gamma - 1) + h_\theta\left(p\cos\gamma\cos^2\theta + \sin^2\theta\right)\right]$$

$$\ell_e^2 p[\dots] - \ell_e^2 p\frac{1}{r^2}\operatorname{ctg}\theta\frac{1}{\sin\theta\cos\theta} = h_r(\operatorname{ctg}\theta)^2(p\cos\gamma - 1) + h_\theta\left(p\cos\gamma\operatorname{ctg}\theta + \operatorname{tg}\theta\right)$$

$$\begin{cases} \ell_e^2\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\gamma_r'\right) + \frac{\operatorname{ctg}\theta}{r^2}\gamma_\theta' + \frac{1}{r^2}\gamma_\theta''\right] + 2\ell_e^2\frac{\operatorname{ctg}\theta}{r^2}\gamma_\theta' = \sin\gamma\left(h_r + h_\theta\operatorname{ctg}\theta\right) & (1) \\ p\ell_e^2\left[\left(\gamma_r'\right)^2 + \frac{1}{r^2}\left(\gamma_\theta'\right)^2\right] = p\cos\gamma\left(h_r + h_\theta\operatorname{ctg}\theta\right) + h_\theta\operatorname{tg}\theta - h_r & (2) \end{cases}$$

Emergent field $\vec B = \sin\Theta\nabla\Theta\times\nabla\Phi = \sin\theta\frac{p}{r}\hat\theta\times\left[\hat r\gamma_r' + \frac{\hat\theta}{r}\gamma_\theta' + \frac{\hat\varphi}{r\sin\theta}\right] =$

$= \left|\hat\theta\gamma_\theta' \text{ не дает вклад}\right| = \frac{g}{r^2}\hat r + \sin\theta\frac{p}{r}\gamma_r'\hat\varphi = B_r\hat r + B_\varphi\hat\varphi$, $B_r = p/r^2$, $q = 1$

$B_r = \frac{g}{r^2}$, $B_\varphi = -\frac{p}{r}\sin\theta\gamma_r'$, $g = pq$ Hopf index of Bloch point

$A_\varphi = -\frac{g}{r}\frac{1+\cos\theta}{\sin\theta}$, $\frac{\partial A_r}{\partial\theta} = -p\gamma_r'\sin\theta$ из $-\frac{1}{r}\frac{\partial A_r}{\partial\theta} = B_\theta$ при $A_{\theta=0} = 0$.

Но A_r можно как $A_r = \int d\theta\frac{\partial A_r}{\partial\theta} = \int d\theta(-p\gamma_r'\sin\theta) = p\gamma_r'\cos\theta + f(r,\varphi)$ как 13/VIII. Когда $\gamma = \gamma(r,\theta)$ и $\frac{\partial\gamma}{\partial r} = \gamma_r'(r,\theta)$. $A_r = A_r(r,\theta) = -p\int_0^\theta d\theta\gamma_r'\sin\theta +$

$\int dV A_r B_r = \int dr d\theta d\varphi\, r^2\sin\theta\frac{g}{r^2}A_r(r,\theta) = 2\pi g\int_0^\pi dr d\theta\sin\theta A_r(r,\theta) + f(r,\varphi)$

$\int dV A_r B_r = 2\pi g\int dr\int d\theta\sin\theta(-p)\int_0^\theta d\theta'\gamma_r'(r,\theta')\sin\theta'$

$\int dV A_\varphi B_\varphi = \int dr\int d\theta d\varphi r^2\sin\theta\left(-\frac{g}{r^2}\right)\frac{(1+\cos\theta)}{\sin\theta}\frac{p}{r}\sin\theta\gamma_r' = -2\pi g p\times$

$\times\int dr\int_0^\pi d\theta(1+\cos\theta)\sin\theta\gamma_r'(r,\theta)$ - аналог. ф-ле 13/VIII, где $\gamma_r' = \gamma_r'(r)$ и можно $\int d\theta(\dots)$

$\int dV\vec A\cdot\vec B = \int dV A_\varphi B_\theta + \int dV A_r B_r = -2\pi g p\int dr\int d\theta\sin\theta(1+\cos\theta)\gamma_r'(r,\theta) +$

$+ (-2\pi g p)\int dr\int d\theta\sin\theta\int_0^\theta d\theta'\sin\theta'\gamma_r'(r,\theta')$

$$\left(\frac{-1}{2\pi g p}\right)\int dV\vec A\cdot\vec B = \int_0^\infty dr\int_0^\pi d\theta\sin\theta\left[(1+\cos\theta)\gamma_r'(r,\theta) + \int_\theta^\pi d\theta'\sin\theta'\gamma_r'(r,\theta')\right] \quad Q_H\,(BP)$$

Если $\gamma_r' = \gamma_r(r)$ только, то $\int d\theta\sin\theta[1+\cos\theta - \cos\theta] = \int d\theta\sin\theta = 2$

и $\left(-\frac{1}{2\pi g}\right)\int dV\vec A\cdot\vec B = 2\int dr\gamma_r'$, $\int dV\vec A\cdot\vec B = -4\pi q\int dr\gamma_r'$, $Q_H = \frac{q}{4\pi}\int dr\gamma_r'$ - как 13/VIII. Но $A_r = p\gamma'\cos\theta + f(r,\varphi)$ и сумма $\int_0^{2\pi} d\varphi f(r,\varphi) = 0$, т.к. $f(r,\varphi)$ - периодическая с периодом 2π. Но $f(\varphi) = \bar f + \sum_{n=1}^{\infty} a_n\cos(n\varphi) + \sum_{n=1}^{\infty} b_n\sin(n\varphi)$ и $\bar f = \frac{1}{2\pi}\int_0^{2\pi} d\varphi f(\varphi)$ не обращается в 0. Расчет Q_H 13/VIII надо поправить.

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при $\gamma=\gamma(z)$ $\int dV A_z B_z = 2\pi \int dz \int_0^{2\pi} d\varphi f(z,\varphi) = 2\pi \int dz f(z)$ $f(z)=\frac{1}{2\pi}\int_0^{2\pi} d\varphi f(z,\varphi)$

$\int dV A_r B_r = 4\pi \int dr f(r)$ и Q_H не зависит

всегда если $\vec B = \mathrm{rot}\vec A$, то $\vec A \to \vec A + \nabla\psi$ не изменит $\vec B$, $\mathrm{rot}\,\mathrm{grad}(\psi)=0$

Но $\int dV \vec A\cdot\vec B \to \int dV (\vec A+\nabla\psi)\cdot\vec B$, $\int dV \nabla\psi\cdot\vec B = ?$

$\nabla\cdot(\psi\vec B) = \psi\,\mathrm{div}\vec B + \nabla\psi\cdot\vec B = \nabla\psi\cdot\vec B$, $\int dV \nabla\psi\cdot\vec B = \int dV\,\mathrm{div}(\psi\vec B) =$

$= \oint d\vec S\,\psi\vec B = \oint dS\,\psi B_r \cdot 1 \cdot d\vec S = \vec r\,dS$, $B_r = \frac{\mu}{r^2}$

$\oint \frac{dS}{r=R}\psi B_r = \int d\varphi d(\cos\theta) R^2 \psi \frac{\mu}{R^2} = \mu \int d\varphi d\cos\theta\, \psi(R,\varphi,\theta) \neq 0$?

$\left(\frac{-1}{2\pi\mu}\right)\int dV \vec A\cdot\vec B = \int_0^{\pi} d\theta \sin\theta \left[(1+\cos\theta)\int_0^{R} dr \frac{\partial A(r,\theta)}{\partial r} + \int_{\pi}^{0} d\theta \sin\theta \int_0^{R} dr \frac{\partial A(r,\theta)}{\partial r}\right]$ — объем сферы

т.е. $Q_H \sim \int_0^R dr \frac{\partial\gamma}{\partial r}(r,\theta) = \gamma(R,\theta) - \gamma(0,\theta)$ и однозначность $\gamma(r)$

Для вектор-потенциал $\vec A$ должна быть однозначна.

Note. $h_\alpha(\vec r) = \int_V d^3\vec r' \sum_\beta G_{\alpha\beta}(\vec r,\vec r') m_\beta(\vec r')$, $G_{\alpha\beta}(\vec r,\vec r') = -(\nabla_{\vec r})_\alpha (\nabla_{\vec r'})_\beta \frac{1}{|\vec r-\vec r'|}$

Для сферы радиуса R магнитостатическое поле $\vec h = \vec H/\mu_s$ не зависит от R. $\vec h$ может зависеть лишь от aspect ratio (для цилиндра $p=L/R$), но для сферы нет dimensionless aspect ratio. Поэтому

$h_z(r,\theta)$ если $F(\theta)$ только и зависит от r только через $\gamma(r)$.

$h_z(r,\theta) = F(\theta,\gamma)$ и никаких членов типа $\left(\frac{r}{R}\right)$ как $16/\pi^2$ в h_1 быть не может.

Eq. (1) при $\gamma=\gamma(r)$: $F_1(r,\gamma) = \sin\gamma\, F_2(\theta,\cos\gamma)$, при $\gamma=\mathrm{const}$

$\sin\gamma\, F_2(\theta,\cos\gamma)=0 \to \sin\gamma=0$, $\gamma=0,\pi$ — hedgehog BP.

Eq. (1) при $\gamma=\gamma(r,\theta)$: $F_1(r,\gamma,\gamma'_\theta,\theta) = \sin\gamma\, F_2(\theta,\cos\gamma)$, $\cos\gamma(r',\theta)$ под знаком $\int dr'd\theta(\dots)$.

$S(\gamma) = S_0 + S_1(\gamma) = \frac{1}{3}p\left[r-R+\frac{1}{2}rP_2\right] + \cos\gamma\left[\frac{2}{3}(r-R)-\frac{1}{6}rP_2\right]$

$S(\gamma) = \frac{1}{3}(r-R)[p+2\cos\gamma] + \frac{1}{6}(p-\cos\gamma)rP_2(\cos\theta)$, $P_2(\cos\theta)=\frac{1}{2}(3\cos^2\theta-1)$

$h_r = -4\pi\frac{\partial}{\partial r}S(\gamma)$, $h_\theta = -4\pi\frac{1}{r}\frac{\partial}{\partial\theta}S(\gamma)$. $\frac{\partial P_2}{\partial\theta} = -3\sin\theta\cos\theta$

т.е. $4\pi S(\gamma) = \psi(r,\theta)$ — магнитостатический потенциал для $\vec m$ — BP анзатц с $\cos\gamma=\mathrm{const}$.

$\frac{1}{4\pi}h_r = -\frac{\partial}{\partial r}S(\gamma) = -\frac{1}{3}(p+2\cos\gamma) + \frac{1}{6}(p-\cos\gamma)P_2(\cos\theta)(-1)$

$\frac{1}{4\pi}h_\theta = -\frac{1}{r}\frac{\partial}{\partial\theta}S(\gamma) = -\frac{1}{r}\left[\frac{1}{6}(p-\cos\gamma)r\frac{\partial P_2}{\partial\theta}\right] = -\frac{1}{6}(p-\cos\gamma)(-3)\sin\theta\cos\theta =$

$= \frac{1}{2}(p-\cos\gamma)\sin\theta\cos\theta$

$\frac{1}{4\pi}(h_r + h_\theta\,\mathrm{ctg}\theta) = -\frac{1}{3}(p+2\cos\gamma) - \frac{1}{6}(p-\cos\gamma)P_2 + \frac{1}{2}(p-\cos\gamma)\cos^2\theta =$

$= -\frac{1}{3}(p+2\cos\gamma) - \frac{1}{6}(p-\cos\gamma)P_2 + \frac{1}{2}(p-\cos\gamma)\frac{1}{3}(2P_2+1) = -\frac{1}{3}p - \frac{2}{3}\cos\gamma + \frac{1}{6}p - \frac{1}{6}\cos\gamma \cdot 1$

$+\frac{1}{6}(p-\cos\gamma)P_2 = -\frac{1}{6}p - \frac{5}{6}\cos\gamma + \frac{1}{6}(p-\cos\gamma)\frac{1}{2}(3\cos^2\theta-1) =$

$= -\frac{1}{12}p - \frac{11}{12}\cos\gamma - \frac{1}{6}p + \frac{1}{6}\cos\gamma + \frac{1}{2}(p-\cos\gamma)\cos^2\theta = -\frac{1}{4}p - \frac{3}{4}\cos\gamma + \frac{1}{2}(p-\cos\gamma)\cos^2\theta =$

$= -\frac{1}{6}p - \frac{5}{6}\cos\gamma - \frac{1}{12}p + \frac{1}{12}\cos\gamma + \frac{1}{4}(p-\cos\gamma)\cos^2\theta = -\frac{1}{4}p - \frac{3}{4}\cos\gamma + \frac{1}{4}(p-\cos\gamma)\cos^2\theta$

$\frac{1}{4\pi}(h_r + h_\theta\,\mathrm{ctg}\theta) = -\frac{1}{4}p\sin^2\theta + \cos\gamma\frac{1}{4}[-3-\cos^2\theta]$ — Верно.

$\frac{1}{4\pi}h_r = -\frac{1}{3}(p+2\cos\gamma) - \frac{1}{6}(p-\cos\gamma)\frac{1}{2}[3\cos^2\theta-1] = -\frac{1}{4}p - \frac{3}{4}\cos\gamma - \frac{1}{4}(p-\cos\gamma)\cos^2\theta$

Проверим $W_m(\gamma) = -\frac{1}{2}M_s^2\int dV \vec h\cdot\vec m$ от $16/\pi^2$ при $\cos\gamma=\mathrm{const}$. Если он $\min W_m(\gamma)$?

Escaneado con CamScanner

$\int dV \vec{h}\cdot\vec{m} = \int_0^R dr \int_0^\pi d\theta \int_0^{2\pi} d\varphi\, r^2 \sin\theta\, \vec{h}\cdot\vec{m}$, $\vec{h} = \vec{h}(R)$, $m_z = \sin^2\theta\cos\gamma + p\cos^2\theta$

$m_{\theta\theta} = \sin\theta\cos\theta(\cos\gamma - p)$

$\int dV \vec{h}\cdot\vec{m} = \frac{1}{3}R^3 2\pi \int d\theta \sin\theta\, \vec{h}\cdot\vec{m}$

$\frac{1}{4\pi}\int_0^\pi d\theta \sin\theta \Big[-\frac{1}{4}p - \frac{3}{4}\cos\gamma - \frac{1}{4}(p-\cos\gamma)\cos^2\theta \Big]\Big[\sin^2\theta\cos\gamma + p\cos^2\theta \Big] =$ (h_z, m_z)

$= -\frac{1}{4}(p+3\cos\gamma)\int d\theta \sin\theta \big[\sin^2\theta\cos\gamma + p\cos^2\theta\big] - \frac{1}{4}(p-\cos\gamma)\int d\theta \sin\theta\cos^2\theta \times$

$\times\big[\sin^2\theta\cos\gamma + \cos^2\theta\big] = -\frac{1}{4}(p+3\cos\gamma)\int_{-1}^{1} dx \big[(1-x^2)\cos\gamma + px^2\big] - \frac{1}{4}(p-\cos\gamma)\times$

$\times\int_{-1}^{1} dx\, x^2\big[(1-x^2)\cos\gamma + px^2\big] = -\frac{1}{4}(p+3\cos\gamma)\Big[\cos\gamma\Big(x - \frac{x^3}{3}\Big) + p\frac{x^3}{3}\Big]_{-1}^{1} - \frac{1}{4}(p-\cos\gamma)\times$

$\times\Big[\cos\gamma\Big(\frac{x^3}{3} - \frac{x^5}{5}\Big) + p\frac{x^5}{5}\Big]_{-1}^{1} = -\frac{1}{4}(p+3\cos\gamma)\Big[\frac{4}{3}\cos\gamma + \frac{2}{3}p\Big] - \frac{1}{4}(p-\cos\gamma)\times$

$\times\Big[\cos\gamma\cdot\frac{4}{15} + \frac{2}{5}p\Big] = -\frac{1}{6}(p+3\cos\gamma)(p+2\cos\gamma) - \frac{1}{10}(p-\cos\gamma)\Big(p+\frac{2}{3}\cos\gamma\Big)$

$\frac{1}{4\pi}\int d\theta \sin\theta\, h_\theta m_{\theta\theta} = \int d\theta \sin\theta \frac{1}{2}(p-\cos\gamma)\sin\theta\cos\theta \cdot \sin\theta\cos\theta(\cos\gamma - p) =$

$= -\frac{1}{2}(p-\cos\gamma)^2 \int d\theta \sin\theta \sin^2\theta\cos^2\theta = -\frac{1}{2}(p-\cos\gamma)^2 \int_{-1}^{1} dx (1-x^2)x^2 = -\frac{1}{2}(p-\cos\gamma)^2 \times$

$\times\Big[\frac{x^3}{3} - \frac{x^5}{5}\Big]_{-1}^{1} = -\frac{1}{2}(p-\cos\gamma)^2\Big(\frac{1}{3} - \frac{1}{5}\Big)2 = -\frac{2}{15}(p-\cos\gamma)^2$

$\int dV \vec{h}\cdot\vec{m} = \frac{1}{3}R^3 2\pi \int d\theta \sin\theta\big[h_z m_z + h_{\theta\theta} m_{\theta\theta}\big] = \frac{2\pi}{3}R^3 4\pi\Big[-\frac{1}{6}(p+3\cos\gamma)(p+2\cos\gamma)$

$-\frac{1}{10}(p-\cos\gamma)\Big(p+\frac{2}{3}\cos\gamma\Big) - \frac{2}{15}(p-\cos\gamma)^2\Big]$, $[\dots] = \Big[-\frac{1}{6}(\cdot)(\cdot) - \frac{1}{30}(p-\cos\gamma)(3p+2\cos\gamma)$

$-\frac{2}{15}(p-\cos\gamma)^2\Big] = \frac{-1}{30}\big[5(p+3\cos\gamma)(p+2\cos\gamma) + (p-\cos\gamma)(3p+2\cos\gamma) + 4(p-\cos\gamma)^2\big]$

$\int dV \vec{h}\cdot\vec{m} = \frac{2\pi}{3}4\pi R^3\Big(\frac{-1}{30}\Big)\big[5(1+5p\cos\gamma+6\cos^2\gamma) + 3 - p\cos\gamma - 2\cos^2\gamma +$

$+4(1-2p\cos\gamma+\cos^2\gamma)\big] = -\frac{1}{30}\frac{2\pi}{3}4\pi R^3\big[12 + 16p\cos\gamma + 32\cos^2\gamma\big] = -\frac{1}{30}\frac{2\pi}{3}4\pi R^3 \times$

$\times 16\big[\frac{3}{4} + p\cos\gamma + 2\cos^2\gamma\big] = -\frac{8}{15}2\pi V\big[\frac{3}{4} + p\cos\gamma + 2\cos^2\gamma\big]$

$W_m^{(\gamma)} = -\frac{1}{2}M_s^2\int dV \vec{h}\cdot\vec{m} = \frac{8}{15}\pi M_s^2 V\big[\frac{3}{4} + p\cos\gamma + 2\cos^2\gamma\big]$. $\varepsilon_m = W_m/(\frac{1}{4}\pi M_s^2 V)$

$\gamma = \text{const}$ $\frac{1}{4\pi M_s^2 V} W_m(\gamma) = \frac{2}{15}\big[\frac{3}{4} + p\cos\gamma + 2\cos^2\gamma\big]$, $\varepsilon_m(\gamma) = \frac{1}{30}\big[7 + 4p\cos\gamma + 4\cos^2\gamma\big]$

[Pylyp'12]

$\varepsilon_m(\gamma) = \frac{1}{30}\big[7 + 4p\cos\gamma + 4(2\cos^2\gamma - 1)\big] = \frac{1}{30}\big[3 + 4p\cos\gamma + 8\cos^2\gamma\big] = \frac{2}{15}\big[\frac{3}{4} + p\cos\gamma + 2\cos^2\gamma\big]$

мех $W_m(\gamma)$ совп. с $W_m(\gamma) = 4\pi M_s^2 V \varepsilon_m(\gamma)$ – Pylyp'12, Eq. (12).

$\frac{\partial W_m}{\partial\gamma} = (\cdot)\cdot$ $\frac{\partial W_m}{\partial(\cos\gamma)} = (\dots)\big[p + 4\cos\gamma\big] = 0$, $\cos\gamma_0 = -p/4$. $\varepsilon_m(\gamma)$ Pylyp. – верно.

$\frac{\partial^2 W_m}{\partial(\cos\gamma)^2} = (\dots)4 > 0$, т.е. $\cos\gamma_0 = -p/4$ даст min $W_m(\gamma)$.

Формально можно найти $W_m(\gamma)$ при $\gamma = \text{const}$, $\cos\gamma_0 = -p/4$.

Но LLG Eqs. (1), (2) дают, что $\gamma = \text{const}$ возможно лишь при $\gamma = 0, \pi$ – hedgehog. $\cos\gamma = p$

$W_m(\gamma)$ с $\gamma = \text{const} \neq 0, \pi$ – всё фигня. $\varepsilon_m(\cos\gamma = p) = \frac{2}{15}\big(\frac{3}{4} + 1 + 2\big) = \frac{1}{2}$

$\varepsilon_m(\gamma) = \frac{2}{15}\big[\frac{3}{4} + p\cos\gamma + 2\cos^2\gamma\big]$, $\frac{\partial\varepsilon_m}{\partial(\cos\gamma)} = \frac{2}{15}\big[p + 4\cos\gamma\big]$ не имеет экстремума при $\cos\gamma_0 = p$, лишь при $\cos\gamma_0 = -p/4$. Так, $\frac{\partial\varepsilon_m}{\partial\gamma} = \frac{\partial\varepsilon_m}{\partial(\cos\gamma)}\frac{\partial(\cos\gamma)}{\partial\gamma} =$

$= -\sin\gamma\frac{\partial\varepsilon_m}{\partial(\cos\gamma)} = -\sin\gamma\frac{2}{15}\big(p + 4\cos\gamma\big) = 0 \Rightarrow \gamma = 0, \pi$ (max) и $\cos\gamma_0 = -p/4$ (min).

$\gamma(r,\varphi) = \gamma = \text{const}$ – это пробная trial function, но $W = W_{ex} + W_m(\gamma)$ имеет при $\gamma = \text{const}$ минимум. $W(\gamma_0)$ – всё ещё min $W(\gamma)$.

Но $W[\gamma(\vec{r})]$ в более общем $\gamma = \gamma(\vec{r})$ даст $\min W[\gamma(\vec{r})] < \min W(\gamma_0)$, $\gamma_0 = \text{const}$.

Pylyp. и дают $W_m \to \varepsilon_m(\gamma_0) = \frac{1}{12} = 0.083$, а численно $\min \varepsilon_m(\gamma) = 0.03$. Это может быть ложный. Так $\cos\gamma_0 = -p/4$ – грубое приближение.

G.E. Khodenkov, Techn. Phys. 55, 738 (2010). Exch. approxim. для ВР $\vec{m} = \frac{\vec{r}}{r}$ с $M_s(r)$.

Escaneado con CamScanner

(2 фев) Как по $\vec{B} = \sin\theta\, e_\theta \times e_\varphi$ восстановить вектор-пот $\vec{A}$? $\vec{B} = \mathrm{rot}\vec{A}$

Инв. хирал. $Q_H = \int dV \vec{A}\cdot\vec{B}$ и её выражение через $\vec{A}$.

• Y. Liu et al PRB 98, 174433 (2018): Hopf index Q_H is invariant under gauge transformation $\vec{A} \to \vec{A} + \nabla\chi$ only when $\mathrm{div}\vec{B} = 0$.

$\vec{A}$ и Q_H calculated in momentum space using Coulomb gauge $\vec{k}\cdot\vec{A} = 0$.

$\vec{A}(\vec{r}) = \frac{1}{N}\sum_{\vec{k}} \vec{A}(\vec{k}) e^{i2\pi\vec{k}\vec{r}}$, $\vec{B}(\vec{r}) = \frac{1}{N}\sum_{\vec{k}} \vec{B}(\vec{k}) e^{i2\pi\vec{k}\vec{r}}$, $\mathrm{rot}\vec{A} = \vec{B} \to i2\pi\vec{k}\times\vec{A}(\vec{k}) = \vec{B}(\vec{k})$

Use $\vec{k}\cdot\vec{A} = 0$, $\vec{A}(\vec{k}) = -\frac{i}{2\pi}\frac{\vec{k}\times\vec{B}(\vec{k})}{k^2}$, $Q_H \sim \frac{i}{N}\sum_{\vec{k}} \frac{\vec{B}(-\vec{k})\cdot[\vec{k}\times\vec{B}(\vec{k})]}{\mathrm{const}\, k^4}$

• F.N. Rybakov et al. arXiv 1904.00250v1 (2019)

Если $\vec{m}(\vec{r}) \to \vec{m}_0 = \mathrm{const}$ at $|\vec{r}| \to \infty$, then Q_H - Hopf invariant is integer. Т.е. $Q_H = 0, \pm 1, \pm 2, \ldots$ various type localized solitons.

См. 13/фев. Арапинолан '93 - type localized solitons $\vec{B}(r) \sim \frac{1}{r^{3+d}}$ when $r\to\infty$. ($\vec{m}(\vec{r}) \to \vec{m}_0$, $d > 0$.

••• X.S. Wang – A Brataas PRL 123, 147203 (2019)

① $H = \frac{1}{(4\pi)^2}\int dV \vec{A}\cdot\vec{B}$, $B_i = \frac{1}{2}\varepsilon_{ijk}\vec{m}(\partial_j\vec{m}\times\partial_k\vec{m})$, $\vec{B} = \mathrm{rot}\vec{A}$.

Hopf index is well defined for ∞-system and $\mathrm{div}\vec{B} = 0$, $B(r) \sim 1/r^{3+d}$.

$\vec{A}' = \vec{A} + \nabla f$, $H(\vec{A}') = H(\vec{A}) + \frac{1}{(4\pi)^2}\int dV \vec{B}\cdot\nabla\chi$, $\int dV \vec{B}\nabla\chi = \int dV \nabla(\chi\vec{B}) -$

$- \chi^0 \int dV \chi \mathrm{div}\vec{B} = \int dV \mathrm{div}(\chi\vec{B}) = \oint d\vec{S}\cdot\chi\vec{B} = 0$, т.к. в ∞-system

можно S неограниченно at $r\to\infty$, где $\vec{m}(\vec{r}) \to \vec{m}_0$ и $\vec{B}(\vec{r}) \sim \frac{1}{r^{3+d}} \to 0$

$\vec{H}(\vec{A}') = H(\vec{A})$ - invariant gauge transforms и $H(\vec{A})$ может быть integer.

② // Для cylinder Sym. system $\vec{m}(\Theta,\Phi)$, $\Theta = \Theta(r,z)$, $\Phi = q\varphi + \gamma(r,z)$ ($\vec{r} = (r,\varphi,z)$)

$B_r = -\frac{q}{r}\sin\Theta\frac{\partial\Theta}{\partial z}$, $B_\varphi = \sin\Theta\left(\frac{\partial\Theta}{\partial z}\frac{\partial\gamma}{\partial r} - \frac{\partial\Theta}{\partial r}\frac{\partial\gamma}{\partial z}\right)$, $B_z = \frac{q}{r}\sin\Theta\frac{\partial\Theta}{\partial r}$

$A_r = -(1+\cos\Theta)\frac{\partial\gamma}{\partial r}$, $A_\varphi = \frac{q}{r}(1-\cos\Theta)$, $A_z = -(1+\cos\Theta)\frac{\partial\gamma}{\partial z}$

$H = \frac{1}{(4\pi)^2}\int dV \vec{A}\cdot\vec{B} = \frac{q}{4\pi}\int_{-\infty}^{\infty} dz \int_0^\infty dr \sin\Theta\left[\frac{\partial\Theta}{\partial z}\frac{\partial\gamma}{\partial r} - \frac{\partial\Theta}{\partial r}\frac{\partial\gamma}{\partial z}\right]$ — см. 6/фев 20 Якобиан

$\sin\Theta$-number in r-z plane.

А где Dirac string в $\vec{A}$? ($r\to 0$)

$\mathrm{div}\vec{B} = \frac{1}{s}\frac{\partial}{\partial s}(sB_s) + \frac{1}{s}\frac{\partial B_\varphi}{\partial\varphi} + \frac{\partial B_z}{\partial z} = \frac{1}{s}\frac{\partial}{\partial s}\left(-q\sin\Theta\frac{\partial\Theta}{\partial z}\right) + \frac{1}{s}\frac{\partial}{\partial\varphi}\sin\Theta\left(\frac{\partial\Theta}{\partial z}\frac{\partial\gamma}{\partial s} - \frac{\partial\Theta}{\partial s}\frac{\partial\gamma}{\partial z}\right) +$

$+ \frac{\partial}{\partial z}\left(\frac{q}{s}\sin\Theta\frac{\partial\Theta}{\partial s}\right) = \frac{1}{s}(-q)\left[\cos\Theta\frac{\partial\Theta}{\partial s}\frac{\partial\Theta}{\partial z} + \sin\Theta\frac{\partial^2\Theta}{\partial s\partial z}\right] + \frac{q}{s}\left[\cos\Theta\frac{\partial\Theta}{\partial z}\frac{\partial\Theta}{\partial s} + \right.$ (=0)

$\left. + \sin\Theta\frac{\partial^2\Theta}{\partial z\partial s}\right] = 0$, $\mathrm{div}\vec{B} = 0$ без Dirac strings, в отличие от сферич. координат.

Видимо, cyl. coordinates (s,φ,z) подходят для "тороидальных" hopfions, где $\Theta = \Theta(s,z)$, $\Phi = q\varphi + \gamma(r,z)$. ($m_x, m_y, m_z$) См. [J. Hietarinta, P. Salo, Ph. Lett. B 451, 60 (1999)]

Helicity $\gamma(r,z)$ важна для $H \neq 0$; при $\gamma = \mathrm{const}$, $H \equiv 0$ как для BP с $\gamma = \mathrm{const}$.

→ Q_H $H \sim \int dV \vec{A}\cdot\vec{B}$ работает и для non-localized solitons (BP) если $\gamma = \mathrm{const}$. gauge для $\vec{A}$.

Для BP $\vec{r} = (r,\theta,\varphi)$ $A_\varphi = -\frac{q}{r}\frac{1+\cos\theta}{\sin\theta}$, $A_r = -\int_{\theta_0}^{\theta} d\theta \sin\theta\, \gamma'_r(r,\theta) + f(r,\varphi)$

$\mathrm{div}\vec{A}(BP) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r)$ ($A_\theta = 0$, $A_\varphi = 0$)

$\mathrm{div}\vec{A} = 0$ требует $A_r = 0$. Но если $B_r = q/r^2$, $B_\theta = \frac{\gamma}{r}\sin\theta\frac{\partial\gamma}{\partial r}$, $\gamma = \gamma(r,\theta)$, то $\vec{A}$ можно восстановить. Верно ли $\vec{A}(\vec{r}) = \frac{1}{4\pi}\int dV' \frac{\vec{B}'\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|}$ при $\vec{B} = \mathrm{rot}\vec{A}$ и как это согласуется?

Jackson, Electrodynamics, Ch 5 $\nabla\times\vec{B}=\mu_0\vec{J} \to \vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\int dV'\frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}$

$\nabla\cdot\vec{B}=0$ $\vec{B}=\mathrm{rot}\vec{A}$, $\mathrm{div}\vec{A}=0$ (coulomb - space)

Аналог $\vec{A}(\vec{r})=\frac{1}{4\pi}\int dV'\frac{\vec{B}'\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$ или $\mathrm{div}\vec{B}=0$, $\mathrm{div}\vec{A}=0$ - gauge. $\mathrm{div}\vec{A}=0$ - gauge

Для ВР $B_\varphi=\frac{\beta}{r\sin\theta}+B_\varphi\frac{r}{r_0}$ since $\gamma'_{r\theta}$, $B_{\theta\theta}=0$

$\mathrm{rot}\vec{B}=\frac{\hat{r}}{r\sin\theta}\frac{\partial}{\partial\theta}(B_\varphi\sin\theta)+\frac{\hat{\theta}}{r}\left[\frac{1}{\sin\theta}\frac{\partial B_r}{\partial\varphi}-\frac{\partial}{\partial r}(rB_\varphi)\right]-\frac{1}{r}\frac{\partial B_r}{\partial\theta}\hat{\varphi}=$

$=\frac{\hat{r}}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\beta}{r_0}\sin\theta\gamma'_{r\theta}\right)-\frac{\hat{\theta}}{r}\frac{\partial}{\partial\varphi}\left(\beta\sin\theta\gamma'_{r\theta}\right)-\frac{\hat{\varphi}}{r}\frac{\partial}{\partial\theta}\left(\beta/r^2\right)\cdot$

$=\hat{r}\frac{\beta}{r_0\sin\theta}\frac{\partial}{\partial\theta}(\sin^2\theta\gamma'_{r\theta})-\hat{\theta}\frac{\beta}{r}\sin\theta\gamma''_{r\theta}=\hat{r}\frac{\beta}{r_0}\left[2\sin\theta\cos\theta\gamma'_{r\theta}+\sin^2\theta\gamma''_{r\theta\theta}\right]-$

$-\hat{\theta}\frac{\beta}{r}\sin\theta\gamma''_{r\theta}=\hat{r}\frac{\beta}{r_0}\left[2\cos\theta\gamma'_{r\theta}+\sin\theta\gamma''_{r\theta\theta}\right]-\hat{\theta}\frac{\beta}{r}\sin\theta\gamma''_{r\theta}$

$\vec{B}'\times\vec{B}(\vec{r}')=\hat{r}'\frac{\beta}{r_0}\left[2\cos\theta'\gamma'_{r\theta}+\sin\theta'\gamma''_{r\theta\theta}\right]-\hat{\theta}'\frac{\beta}{r}\sin\theta'\gamma''_{r\theta}=(\cdot)\hat{r}'+(\cdot)\hat{\theta}'$

$\frac{1}{|\vec{r}-\vec{r}'|}=\sum_{lm}\frac{4\pi}{(2l+1)}\cdot f_l(r_<r_>)Y^*_{lm}(\theta'\varphi')Y_{lm}(\theta\varphi)$, где $\gamma=\gamma(r,\theta\theta)$ и

$\int d\vec{r}'d\theta'$ не берется, если $\hat{r}=\hat{r}'$, $\hat{\theta}=\hat{\theta}'$. Тогда $\vec{A}$ не имеет θ-комп.: $A_\theta=0$?

А если $\gamma=\mathrm{const}$, $\nabla'\times\vec{B}(\vec{r}')=0 \Rightarrow \vec{A}(\vec{r})=0$ - нонсенс.

Чтобы $\vec{A}=(A_r,A_{\theta\theta},A_\theta)$ бесконечнозначен, но $\vec{B}=\mathrm{rot}\vec{A}$ как $R/\sqrt{R}$

Теперь идея Maxwell eqs. $\nabla\times\vec{B}=\mu_0\vec{J}$, $\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\int dV\frac{\vec{J}(\vec{r})}{R}$

применимы к emergent mag. field $\vec{B}$? Что такое $\vec{J}$ emergent?

Другие eq. $Q_H\sim\int dVdV'\frac{1}{R^3}\vec{B}(\vec{r}')[\vec{R}\times\vec{B}]$ Kundu'82 и Kominea's'96 Boettge не использовали. $\vec{A}(\vec{r})=\frac{1}{4\pi}\int dV'\frac{\nabla'\times\vec{B}(\vec{r}')}{|\vec{r}-\vec{r}'|}$ авт. формулой как эл. динамику, а не через теорему обращения $\vec{B}=\mathrm{rot}\vec{A}$, $\vec{A}=f(\vec{B})$, находя $\vec{A}$.

(2) Выражение $\vec{A}$ через $\vec{B}$ методом Biot-Savart [Kundu, Kominea's] или k-интегрированием связано с gauge $\mathrm{div}\vec{A}=0$. Почему должно выполняться $\mathrm{div}\vec{A}=0$? Для цилиндр [Wang'2019] $\vec{r}=(s,\varphi,z)$. $\theta=\theta(s)$, $\Phi=\varphi+\gamma(s)$

$\mathrm{div}\vec{A}=\frac{1}{s}\frac{\partial}{\partial s}(sA_s)+\frac{1}{s}\frac{\partial A_\varphi}{\partial\varphi}+\frac{\partial A_z}{\partial z}=\frac{1}{s}\frac{\partial}{\partial s}\left[-s(1+\cos\theta)\frac{\partial\varphi}{\partial s}\right]+\frac{1}{s}\frac{\partial}{\partial\varphi}\frac{\partial}{\partial s}(1-\cos\theta)+$

$+\frac{\partial}{\partial z}(-1)(1+\cos\theta)\frac{\partial\varphi}{\partial z}=-\frac{1}{s}(1+\cos\theta)\frac{\partial\varphi}{\partial s}+\sin\theta\frac{\partial\theta}{\partial s}\frac{\partial\varphi}{\partial\varphi}-(1+\cos\theta)\frac{\partial}{\partial s}\frac{\partial\varphi}{\partial s}+$

$+\sin\theta\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial z}-(1+\cos\theta)\frac{\partial\varphi}{\partial z^2}=\sin\theta\left[\frac{\partial\varphi}{\partial s}\frac{\partial\theta}{\partial s}+\frac{\partial\varphi}{\partial z}\frac{\partial\theta}{\partial z}\right]-(1+\cos\theta)\nabla^2\gamma=$

Условие $\mathrm{div}\vec{A}=0$ выполнено (toroidal hopfions) (!)

Helmholtz theorem (частный случай $\Phi=0$): любое поле $\vec{B}$ может быть записано как $\vec{B}=-\nabla\Phi+\mathrm{rot}\vec{A}$. $\vec{B}=\mathrm{rot}\vec{A}$, где $\vec{A}$ is defined as

$\vec{A}(\vec{r})=\frac{1}{4\pi}\int_V\frac{\nabla'\times\vec{B}(\vec{r}')dV'}{|\vec{r}-\vec{r}'|}+\frac{1}{4\pi}\oint_S\frac{\vec{B}(\vec{r}')\times d\vec{S}'}{|\vec{r}-\vec{r}'|}$, $d\vec{S}'=\vec{n}'\cdot dS'$, если V-конечен. $\vec{n}'$ - outward normal

$\vec{A}(\vec{r})=\frac{1}{4\pi}\int dV'\frac{\nabla'\times\vec{B}(\vec{r}')}{|\vec{r}-\vec{r}'|}$ если $V\to\infty$, то $\vec{B}(\vec{r})\sim\frac{1}{r^{1+d}}$, $r\to\infty$. $\vec{B}'=\vec{B}(\vec{r}')$, $\vec{R}=\vec{r}-\vec{r}'$

$\nabla\times\left(\frac{\vec{B}'}{R}\right)=\frac{1}{R}\nabla\times\vec{B}'+\nabla\left(\frac{1}{R}\right)\times\vec{B}'$, $R=|\vec{r}-\vec{r}'|$, $\frac{1}{R}\nabla\times\vec{B}'=\nabla\times\left(\frac{\vec{B}'}{R}\right)-\nabla_r\left(\frac{1}{R}\right)\times\vec{B}'$

$\frac{1}{R}\nabla'\times\vec{B}(\vec{r}')=\nabla'\times\left(\frac{\vec{B}'}{R}\right)-\nabla'\left(\frac{1}{R}\right)\times\vec{B}'$, $\nabla'\left(\frac{1}{R}\right)=-\nabla\left(\frac{1}{R}\right)$.

$\vec{A}(\vec{r})=\frac{1}{4\pi}\int dV'\frac{1}{R}\nabla'\times\vec{B}'+\frac{1}{4\pi}\oint\frac{1}{R}\vec{B}'\times d\vec{S}'=\frac{1}{4\pi}\int dV'\left[\nabla'\times\left(\frac{\vec{B}'}{R}\right)-\nabla'\left(\frac{1}{R}\right)\times\vec{B}'\right]+$

$+\frac{1}{4\pi}\oint\frac{1}{R}\vec{B}'\times d\vec{S}'$, $\nabla'\left(\frac{1}{R}\right)=-\nabla\left(\frac{1}{R}\right)=+\frac{\vec{R}}{R^3}$ $\vec{A}(\vec{r})=-\frac{1}{4\pi}\int dV\frac{\vec{R}\times\vec{B}}{R^3}$

$\vec{A}(\vec{r})=\frac{1}{4\pi}\int dV'\frac{\vec{R}'}{R^3}\times\vec{B}'(-)+\frac{1}{4\pi}\int dV'\nabla'\times\left(\frac{\vec{B}'}{R}\right)+\frac{1}{4\pi}\oint\frac{\vec{B}'}{R}\times d\vec{S}'=$

$=-\frac{1}{4\pi}\int dV'\frac{\vec{R}'}{R^3}\times\vec{B}'+\frac{1}{4\pi}(-1)\oint\frac{\vec{B}'}{R}\times d\vec{S}'+\frac{1}{4\pi}\oint\frac{\vec{B}'}{R}\times d\vec{S}'=-\frac{1}{4\pi}\int dV'\frac{\vec{R}'}{R^3}\times\vec{B}'$

$\int dV\vec{A}\cdot\vec{B}'=\int dV\vec{B}'\left(\frac{1}{4\pi}\right)\int dV'\frac{\vec{R}'}{R^3}\times\vec{B}'=-\frac{1}{4\pi}\int dV\int dV'\frac{1}{R^3}\vec{B}\cdot(\vec{R}'\times\vec{B}')$ как у Kundu'82

Escaneado con CamScanner

Cap. 13/fim Eq. (1) tendo "(0,1) fórmulas" 16 Segunda Eq. gravitacional na
vanetrocrita V e B(r) dadas de n em n. Zero Qn obtém expandindo
-gerador B(r) = 1/(r+1), r+1=n, t=0.
Condição fundamental: m = cosθ = u(r,cc) = p cosec , Φ = φc + γ(r,cc) . Gauge
$\vec{B}_{(fundo)} = -\frac{q}{r^2\sin\theta}\left(\hat{r}\frac{\partial w}{\partial cc} - cc\,\hat{r}\,r\frac{\partial w}{\partial r}\right) - \hat{\varphi}\frac{1}{r}\left(\frac{\partial w}{\partial r}\frac{\partial f}{\partial cc} - \frac{\partial w}{\partial cc}\frac{\partial f}{\partial r}\right) =$ Bφ
$= -\frac{q}{r^2\sin\theta}\hat{r}\,p(-1)\sin cc - \frac{\hat{\varphi}}{r}(1)\,p(-\sin cc)\frac{\partial f}{\partial r} = \frac{qp}{r^2}\hat{r} - \frac{\hat{\varphi}}{r}\sin cc\,p\frac{\partial f}{\partial r}$
$B_r(mac) = \hat{r}\frac{q}{r^2} + \hat{\varphi}\frac{p}{r}\sin cc\frac{\partial f}{\partial r}$, $q = qp$. $B_k^m = B(mac)$ coeficiente e termo. Sigma B_{cc}.
$Q_n(fundo) = \frac{p}{4\pi}\int dr\int dcc\,(1-p\cos cc)\left[\frac{\partial w}{\partial r}\frac{\partial f}{\partial cc} - \frac{\partial w}{\partial cc}\frac{\partial f}{\partial r}\right] = \frac{1}{4\pi}\int dr\int dcc(1-\cos cc\,p)\times$
$\times\left[-p(-\sin cc)\frac{\partial f}{\partial r}\right]_0^{\infty} = \frac{qp}{4\pi}\int dr\int dcc\sin cc\,(1-\cos cc\,p)\frac{\partial f}{\partial r}$ (c. $B_\varphi A_r$)
$Q_n\left(\frac{mac}{fundo}\right) = -\frac{1}{(4\pi)^2}\int dV\vec{A}\cdot\vec{B} = -\frac{1}{(4\pi)^2}(-2\pi pq)\int dr\int dcc\sin cc\left[(1+\cos cc)\frac{\partial f}{\partial r} + \right.$
$\left. + \int_{cc}^{cc} dcc'\sin cc'\frac{\partial f}{\partial r}(r,cc')\right]$ ($A_r B_r$) , $Q_n\left(\frac{mac}{sig\,A_r B_r}\right) = \frac{pq}{8\pi}\int dr\,dcc\sin cc\,(1+\cos cc)\frac{\partial f}{\partial r}(r,cc)$
$Q_n(fundo) = \frac{qp}{4\pi}\int dr\int dcc\sin cc\,(1-p\cos cc)\frac{\partial f}{\partial r}$, $Q_n(mac) = \frac{q}{8\pi}\int dr\,dcc\sin cc(1+\cos cc)\frac{\partial f}{\partial r}$
Buscamos y fundo $A_\varphi = -\frac{q}{r}\frac{(1-p\cos cc)}{\sin cc}$, y mesma $A_\varphi = -\frac{q}{r}\frac{1+\cos cc}{\sin cc}$.
Q_n (fundo) mesmo sig $A_r B_r$ - term. Q_n(fundo) é 2 pq obtemos Q_n(mac).
w_3-zero? Buscamos w_3-ésimo "2" de Eq. (1). Eq. (3) - Bermac, c
gravar $B_k' = 2\vec{B}(mac)$, a Eq. (1) - mac.
$\vec{A}(\vec{r}) = -\frac{1}{4\pi}\int dV'\frac{\vec{R}}{R^3}\times\vec{B}(\vec{r}')$, $\nabla\left(\frac{1}{R}\right) = -\frac{\vec{R}}{R^3}$, $\vec{A}(\vec{r})$ mesmo u mesmo no $\vec{B}(\vec{r})$
$\vec{A}(\vec{r}) = \frac{1}{4\pi}\nabla_r\int dV'\frac{1}{R}\times\vec{B}(\vec{r}') = \frac{1}{4\pi}\int dV'\nabla_r\left(\frac{1}{R}\right)\times\vec{B}(\vec{r}')$
$\nabla_r f\times\vec{B} = \left[\frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial cc}\hat{cc} + \frac{1}{r\sin cc}\frac{\partial f}{\partial\varphi}\hat{\varphi}\right]\times\left[B_r'\hat{r} + B_{cc}'\hat{cc}\right] = \frac{\partial f}{\partial r}B_{cc}'\hat{r}\times\hat{cc} + \frac{1}{r}\frac{\partial f}{\partial cc}B_r'\hat{cc}\times\hat{r} +$
$+\frac{1}{r}\frac{\partial f}{\partial cc}B_{cc}'\hat{cc}\times\hat{cc} + \frac{1}{r\sin cc}\frac{\partial f}{\partial\varphi}\hat{\varphi}\times\hat{r}B_r' = \left|\hat{cc}\times\hat{r} = \hat{\varphi},\ \hat{cc}\times\hat{\varphi} = \hat{r},\ \hat{\varphi}\times\hat{r} = \hat{cc}\right| =$
$= \frac{1}{r}\frac{\partial f}{\partial cc}B_\varphi'\hat{r} + \left[\frac{1}{r\sin cc}\frac{\partial f}{\partial\varphi}B_r' - \frac{\partial f}{\partial r}B_\varphi'\right]\hat{cc} + \frac{1}{r}\frac{\partial f}{\partial cc}B_r'\hat{\varphi}$
$\vec{A}(\vec{r}) = \frac{1}{4\pi}\int dV'\left\{\frac{1}{r}\frac{\partial f}{\partial cc}B_\varphi'\hat{r} + \left[\frac{1}{r\sin cc}\frac{\partial f}{\partial\varphi}B_r' - \frac{\partial f}{\partial r}B_\varphi'\right]\hat{cc} + \frac{1}{r}\frac{\partial f}{\partial cc}B_r'\hat{\varphi}\right\}$
$B_r = q/r^2$, $B_\varphi = (p/r)\sin cc\frac{\partial f}{\partial r}$
$B_\varphi \sim \frac{\partial f}{\partial r}(r,cc)$ gerar A_r e A_{cc} mesmo/regenerando, logo $f(r,cc)$ mesmo Bermac.
$A_\varphi = \frac{1}{4\pi}\int dV'\frac{1}{r}\frac{\partial f}{\partial cc}B_r' = \frac{1}{4\pi r}\int dV'\frac{\partial f}{\partial cc}\frac{q}{r^2} = \frac{q}{4\pi r^2}\int dr'\int dcc'\,d\varphi'\sin cc'\frac{\partial f}{\partial cc}$
$\frac{1}{|\vec{r}-\vec{r}'|} = \sum_{lm}\frac{4\pi}{(2l+1)}f_c(rr')Y_{lm}^*(cc'\varphi')Y_{lm}(cc\varphi)$, $A_\varphi = \frac{q}{4\pi}\frac{1}{r}\frac{\partial}{\partial cc}\int dr'dcc'\int d\varphi'\sin cc'\times$
$\int dcc'\sin cc'\int d\varphi' Y_{lm}^*(cc'\varphi') = \int dV' Y_{lm}^*(cc'\varphi')\sqrt{4\pi}\,Y_{00}(cc'\varphi') = \sqrt{4\pi}\,\delta_{l0}\delta_{m0}$
$A_\varphi = \frac{q}{4\pi}\frac{1}{r}\frac{\partial}{\partial cc}\int dr'dcc'd\varphi'\sin cc'\sum_{lm}\frac{4\pi}{(2l+1)}f_c\,Y_{lm}^*(cc'\varphi')Y_{lm}(cc\varphi) = \frac{q}{4\pi}\frac{1}{r}\frac{\partial}{\partial cc}\int dr'\times$
$\times\sum_{lm}\frac{4\pi}{(2l+1)}f_c\,Y_{lm}(cc,\varphi)\sqrt{4\pi}\,\delta_{l0}\delta_{m0} = \frac{q}{4\pi}\frac{1}{r}\frac{\partial}{\partial cc}\int 4\pi\int_0 dr' \cdot$ mas mesmo, $A_\varphi = 0$.
$\int dV' Y_{lm}^*(cc'\varphi') = 2\pi\delta_{mo}\sqrt{\frac{2l+1}{4\pi}}P_l(\cos cc')$, $A_\varphi = \frac{q}{4\pi}\frac{1}{r}\frac{\partial}{\partial cc}\int dr'dcc'\sin cc'\times 2\pi\times$
$\times\sum_l f_c\,P_l^0(\cos cc')P_l(\cos cc) = \frac{1}{2}\frac{q}{r}\int dr'dcc'\sin cc'\sum_l f_c\,P_l(\cos cc')\frac{\partial P_l}{\partial cc} =$

$= \frac{1}{2}\frac{g}{r}\int d\theta' \sin\theta' \sum_\ell f_\ell P_\ell(\cos\theta') \frac{\ell}{\sin\theta}\left[\cos\theta P_\ell(x) - P_{\ell-1}(x)\right]$, $\int d\theta' \sin\theta' P_\ell(\cos\theta') =$

$= \int dx\, P_\ell(x) P_0 = \frac{2}{2\ell+1}\delta_{\ell 0} = 2\delta_{\ell 0}$ и все равно $A_\ell = 0$ из $\ell \delta_{\ell 0}$ - term.

$\vec{A}(\vec{r}) = -\frac{1}{4\pi}\int dV' \frac{1}{R^3}\vec{R}\times\vec{B}(r')$ просто не вычислить, сингул. $\frac{1}{R}$ бадяга

Оставим $Q_H = \frac{q}{2\pi}\int_0^\infty dr \int_0^\pi d\theta \sin\theta(1+\cos\theta)\frac{\partial\gamma}{\partial z}(r,\theta)$ — дает ВР.

$Q_H \sim \gamma(R,\theta) - \gamma(0,\theta)$. Если предположить $\frac{\partial\gamma}{\partial z}|_{z=0,R} = 0$, $\frac{\partial\gamma}{\partial\theta}|_{z=0,R} = 0$ то eqs. (1)(2) дают $\gamma(r,\theta)$ сводится к $h_z + h_\theta$ с $\text{deg}\theta = 0$ $\int dr' \Gamma(r,r')\gamma(r',\theta') \to$ $\text{deg}\theta \int dr' \Gamma(r,r',\theta,\theta')\cos\gamma(r'\theta') = F(\theta)$, где $r = 0, R$.

Это Fredholm int. equation of the first kind $\to$ найти $\cos\gamma(r,\theta)$.

$\to$ Exchange b.c. $\frac{\partial\gamma}{\partial n}|_{r=R} = 0$, $\frac{\partial\gamma}{\partial n} = (\nabla\gamma\cdot\vec{n}) = (\nabla\gamma)_{\vec{n}=\hat{r}} = \frac{\partial\gamma(r,\theta)}{\partial r}$, $\frac{\partial\gamma}{\partial r}|_{r=R} = 0$, $\frac{\partial\gamma}{\partial\theta}|_R = 0$ требовать нельзя, иначе сведение к ур. Фредгольма не проходит.

Видно, что в ∞-sample Q_H конечно если $\frac{\partial\gamma}{\partial r} \sim \frac{1}{r^{1+\alpha}}$, $r\to\infty$, $\alpha \geq 0$

$\gamma(R) = \int_0^R dr \frac{\partial\gamma}{\partial r} \sim \int dr \frac{1}{r^{1+\alpha}} \sim \frac{1}{r^\alpha}\Big|^R \sim \frac{C}{R^\alpha} \to 0$ при $R\to\infty$. $\gamma(R,\theta) \xrightarrow[R\to\infty]{} 0$.

А как ведет себя $\gamma(r,\theta)$ при $r\to 0$? $\frac{\partial\gamma(r)}{\partial r} \sim \frac{1}{r^{1-\beta}}$, $\beta \geq 0$

$\gamma(r) \sim r^\beta$, $\int dr \frac{\partial\gamma}{\partial r} = C\int \frac{dr}{r^{1-\beta}} = C r^\beta = \gamma(r)$ при $r\to 0$. $\gamma(r) = C r^\beta$ $\beta\geq 0$, $r\to 0$

Это никто не дает, т.к. Возможно $\alpha = 0$, $\beta = 0$ и $\gamma(R)$, $\gamma(0)$ - конечны.

При $r/\ell_e \ll 1$, $\gamma(r) = \gamma(0) + \gamma_1 r^\alpha + \dots$, при $\frac{r}{\ell_e} \gg 1$ $\gamma(r) = \gamma(R) + \frac{C}{r^\alpha}$

Если $\vec{A} \to \vec{A}' + \nabla f$, $Q_H(A') = Q_H(A) - \frac{1}{(4\pi)^2}\oint d\vec{S}\cdot\chi\vec{B}$

$\int dV \chi^2$ - finite $\chi(r) \sim \frac{1}{r^{3/2+\alpha}}$, $B(r) \sim \frac{1}{r^{2+\beta}}$, $\oint d\vec{S}\chi\vec{B} \xrightarrow[r\to\infty]{} 0$, но это для бесконечного Sample (!). Для finite sample $\oint dS\chi\vec{B} \neq 0$ и Q_H перестает сохраняться $\to$ χ нельзя

Alternative: gauge для $\vec{A}$ должна быть fixed для finite sample. (gauge transf.)

$\text{div}\vec{A} = 0$ не помогает, т.к. χ можно выбрать так что $\nabla^2\chi \neq 0$.

$\int_0^R dr \frac{\partial\gamma}{\partial r} = \gamma(R) - \gamma(0)$. Начинаем ВР-paper. G

22/VIII $W_{ext} = (\nabla\Theta)^2 + \sin^2\Theta(\nabla\Phi)^2$, $\Phi = q\varphi + \gamma(r,\theta)$, $\sin\Theta = \sin\theta$, $\cos\Theta = \cos\theta$

$\nabla\Phi = \hat{r}\frac{\partial}{\partial r}[q\varphi+\gamma] + \frac{1}{r}\hat{e}_\theta\frac{\partial}{\partial\theta}[q\varphi+\gamma] + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}[q\varphi+\gamma] = \hat{r}\frac{\partial\gamma}{\partial r} + \hat{e}_\theta\frac{1}{r}\frac{\partial\gamma}{\partial\theta} + \frac{q}{r\sin\theta}\hat{\varphi}$

$\nabla\Theta = \frac{1}{r}\hat{e}_\theta$. $\frac{1}{A}W_{ext} = \frac{1}{r^2} + \sin^2\theta\left[\left(\frac{\partial\gamma}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\gamma}{\partial\theta}\right)^2 + \frac{q^2}{r^2\sin^2\theta}\right] = \frac{(1+q^2)}{r^2} + \sin^2\theta(\nabla\gamma)^2$

$\vec{B} = q\vec{P}\frac{\hat{r}}{r^2} + \frac{P}{r}\sin\theta\frac{\partial\gamma}{\partial r}\hat{\varphi} + \vec{B}_{sing}$, $\vec{B}_{sing} = -4\pi g\Theta(z)\delta(x)\delta(y)\hat{z}$ обеспечивает

$\oint d\vec{s}\cdot\vec{B} = 0$ и $\text{div}\vec{B} = 0$ (см. ВР/VIII). $\int dV \vec{B}_{sing} = -4\pi g\int_{-R}^{R} dx\,dy\,dz\,\Theta(z)\delta(x)\delta(y)\hat{z} =$

$= -4\pi g\int_{-R}^{R} dz\,\Theta(z)\hat{z} = -4\pi g\int_0^R dz\,\hat{z} = -4\pi g\cdot R\hat{z}$

Без $\vec{B}_{sing}$, $\vec{G} = \int dV\vec{B} = \int dV\left[q\frac{\hat{r}}{r^2} + \frac{P}{r}\sin\theta\frac{\partial\gamma(r,\theta)}{\partial r}\hat{\varphi}\right] = \int_0^R dr\int_0^\pi d\theta\int_0^{2\pi} d\varphi\left[q\frac{\hat{r}}{r^2} + \frac{P}{r}\sin\theta\frac{\partial\gamma}{\partial r}\hat{\varphi}\right] =$

$= \int_0^{2\pi} d\varphi\int_0^R dr\int_0^\pi d\theta\sin\theta\left[q\hat{r} + Pr\sin\theta\frac{\partial\gamma}{\partial r}\hat{e}_\theta\right]$, $\int_0^\pi d\theta\sin\theta\int_0^{2\pi} d\varphi\hat{r} = \int_0^\pi d\theta\sin\theta\int_0^{2\pi} d\varphi \times$

$\times[\sin\theta\cos\varphi\hat{x} + \sin\theta\sin\varphi\hat{y} + \cos\theta\hat{z}] = 2\pi\int_0^\pi d\theta\sin\theta\cos\theta\hat{z} = 2\pi\hat{z}\int_0^\pi \times = 0$ G (11/VIII)

$\vec{G} = \int dV\vec{B} = \int_0^{2\pi} d\varphi\int_0^R dr\int_0^\pi d\theta\sin\theta Pr\frac{\partial\gamma}{\partial r}\hat{e}_\theta = \int_0^R dr\int_0^\pi d\theta\sin\theta Pr\frac{\partial\gamma}{\partial r}\int_0^{2\pi} d\varphi\hat{\varphi}$, $\gamma = \gamma(r,\theta)$

$\int_0^{2\pi} d\varphi\hat{\varphi} = \int_0^{2\pi} d\varphi[-\hat{x}\sin\varphi + \hat{y}\cos\varphi] = 0 \Rightarrow \vec{G} = \int dV\vec{B} = 0$

Но $\vec{G}_s = \int dV\vec{B}_{sing} = -4\pi g R\hat{z} \neq 0$ для ВР. Как же $\vec{B}_{sing}$ влияет в Q_H?

Itz

(!)

$\vec{A}\cdot\vec{B}_{sing} = \vec{A}\cdot\vec{B}_3\hat{z} = A_z B_3$, $B_3 = -4\pi g\,\theta(z)\delta(x)\delta(y)$, $\vec{B}_3 = B_3\hat{z}$

$\vec{A}\cdot\hat{z} = (A_r\hat{r} + A_\theta\hat{\theta} + A_\varphi\hat{\varphi})\cdot\hat{z} = A_r\hat{r}\cdot\hat{z} + A_\theta\hat{\theta}\cdot\hat{z} + A_\varphi\hat{\varphi}\cdot\hat{z} = A_r(+1)\cos\theta +$
$+A_\theta(-\sin\theta) = A_r\cos\theta$, если $A_\theta = 0$. $\frac{\partial A_r}{\partial\theta} = +\rho\gamma_z'\sin\theta$, $\vec{A}\cdot\hat{z} = A_r\cos\theta$

$A_r = A_r(r,\theta) = -\rho\int_{\theta_0}^{\theta} d\theta'\sin\theta'\gamma_z'(r,\theta') + f(r,\theta)$

$\int dV\,\vec{A}\cdot\vec{B}_{sing} = \int dV\,A_r\cos\theta\,B_3 = \int dr\int d\theta\int d\varphi\, r^2\sin\theta\cos\theta\left[-\rho\int_{\theta_0}^{\theta} d\theta'\sin\theta'\gamma_z'(r,\theta') + \right.$
$\left. + f(r,\theta)\right]\times\left[-4\pi g\,\theta(z)\delta(x)\delta(y)\right] = 2\pi\int dr\int d\theta\, r^2\sin\theta\cos\theta\left[-\rho\int_{\theta_0}^{\theta} d\theta'\sin\theta'\gamma_z' + \bar{f}(r)\right]\left[-4\pi g\,\theta(z)\delta(x)\delta(y)\right] =$
$+\int_0^{\pi} d\theta\sin\theta\cos\theta\, F(r,\theta)\,\delta(x)\delta(y) = 0$, т.к. $x=0, y=0$ только в z-плоскости, где $\sin\theta = 0$.
$+ x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$ и $x=0, y=0 \to \sin\theta = 0$.

$Q_H(\vec{B}_{sing}) \sim \int dV\,\vec{A}\cdot\vec{B}_{sing} = 0$ – Вывод. Это для любого $\gamma = \gamma(r,\theta)$.

$\int dV A_r B_r = g\int dr\int d\theta\int d\varphi\sin\theta A_r(r,\theta) = g\int dr\int d\theta\int d\varphi\left[-\rho\int_{\theta_0}^{\theta} d\theta'\sin\theta'\gamma_r' + f(r,\theta)\right] =$
$= 2\pi g\int dr\int d\theta\sin\theta\left[-\rho\int_{\theta_0}^{\theta} d\theta'\sin\theta'\gamma_r'(r,\theta') + \bar{f}(r)\right]$, $\bar{f}(r) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, f(r,\theta)$

$\vec{A}' = \vec{A} + \nabla\chi$, $Q_H(\vec{A}') = Q_H(\vec{A}) - \frac{1}{(4\pi)^2}\int dV\,\vec{B}\cdot\nabla\chi$, $f(r,\theta)$ – произвольная функция

$\int dV\,\vec{B}\cdot\nabla\chi = \oint d\vec{S}\cdot\vec{B}\chi = \oint dS_r\hat{r}\cdot\vec{B}\chi = |$граница $BP| = \oint dS_r\frac{g}{r^2}\chi = \int_{r=R} d\theta\, d\varphi\, r^2\sin\theta\,\frac{g}{r^2}\chi(\vec{r}) = g\int d\theta\int d\varphi\sin\theta\,\chi(\vec{r})$, $\int dV\chi^2(\vec{r})$ конечен $\int_0^R dr\, r^2\chi^2(r)$ – конечно при $R\to\infty$

$\chi(\vec{r}) \underset{r\to 0}{\sim} \frac{1}{r^{3/2+\alpha}}$, $\chi^2(\vec{r}) \sim \frac{1}{r^{3+2\alpha}}$, $\int_0^R dr\, r^2\frac{1}{r^{3+2\alpha}} = \int_0^R dr\frac{1}{r^{1+2\alpha}} \sim \frac{1}{r^{2\alpha}}\Big|_0^R \sim \frac{1}{R^{2\alpha}} \to 0$, $R\to\infty$

При таком $\chi(\vec{r}) \sim \frac{1}{r^{3/2+\alpha}}$, $\int dV\,\vec{B}\nabla\chi = g\int_{r=R} d\theta\int d\varphi\sin\theta\,\chi(\vec{r}) \sim \frac{1}{R^{3/2+\alpha}} \to 0$, $R\to\infty$

Gauge добавка к $\vec{A}$ дает добавку к $Q_H \sim \frac{1}{R^{3/2+\alpha}}$, пренебрежимо мало при $R\gg l_e$

$f(r,\theta)$ в A_r это $(\nabla\chi)_r$ – gauge добавка, $f(r,\theta)$ можно пренебречь при $R\to\infty$

$\int dV A_r B_r = -2\pi\rho g\int dr\int d\theta\sin\theta\int_{\theta_0}^{\theta} d\theta'\sin\theta'\gamma_r' = -2\pi\rho g\left[\gamma(R,\theta) - \gamma(0,\theta)\right]\int d\theta\sin\theta\int_{\theta_0}^{\theta} d\theta'\sin\theta'$

$\int dV A_r B_r = -2\pi g\rho\int_0^{\pi} d\theta\sin\theta\int_{\theta_0}^{\theta} d\theta'\sin\theta'\left[\gamma(R,\theta') - \gamma(0,\theta')\right]$, т.е. Q_H (см. VIII) вернулось

$Q_H = -\frac{1}{(4\pi)^2}\int dV\,\vec{A}\cdot\vec{B} = \frac{2}{2\pi}\int_0^R dr\int_0^{\pi} d\theta\sin\theta\left[(1+\cos\theta)\gamma_z'(r,\theta) + \int_{\theta_0}^{\theta} d\theta'\sin\theta'\gamma_z'(r,\theta')\right]$,

Нарушение чётности

A=0 A≠0 если $A_r \neq 0$.

Магнитостатический предел. Если $l_e = 0$, можно ли найти $\gamma(r,\theta)$?

$W_{эл} \sim R$, $W_m \sim R^3$, при больших R, W_m доминирует.

$h_r = -\frac{\partial}{\partial r}\Psi(r,\theta)$, $h_\theta = -\frac{1}{r}\frac{\partial}{\partial\theta}\Psi(r,\theta)$, $h = -\nabla\Psi$, Ψ – магнитостатический потенциал от ВР

$\Psi(r,\theta) = \int dr' r'^2\left[\frac{\partial}{\partial r'}\int d\theta'\sin\theta' F m_r + \frac{1}{r'}\int d\theta'\sin\theta'\frac{\partial F}{\partial\theta'}m_\theta\right]$

$F(r r',\theta\theta') = 2\pi\sum_\ell f_\ell(r r')P_\ell(\cos\theta)P_\ell(\cos\theta')$, $f_\ell(r r') = r_<^\ell/r_>^{\ell+1}$.

Ранее нашли $\gamma = \gamma(\alpha)$ после $\int d\theta'(\dots)$ и $\gamma = \text{const}$ после нашли $W_m(\gamma)$.

$m_r(r\theta) = \sin^2\theta\cos\gamma + p\cos^2\theta = P_0\left[\frac{1}{3}p + \frac{2}{3}\cos\gamma\right] + \frac{2}{3}(p - \cos\gamma)P_2$, $P_\ell = P_\ell(\cos\theta)$

$m_r = p\cos^2\theta + \cos\gamma\sin^2\theta$, $\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{1}{3}(2P_2 + 1)$

$m_r = p\frac{1}{3}(2P_2+1) + \cos\gamma\sin^2\theta = \frac{1}{3}pP_0 + \frac{2}{3}pP_2 + \cos\gamma\sin^2\theta$, см. 18/VIII

$\int d\theta'\sin\theta' F m_r = 2\pi\int d\theta'\sin\theta'\sum_\ell f_\ell P_\ell(\cos\theta)P_\ell(\cos\theta')\left[\frac{1}{3}pP_0 + \frac{2}{3}pP_2 + \cos\gamma\sin^2\theta'\right] =$
$= 2\pi\int d\theta'\sin\theta'\sum_\ell f_\ell P_\ell P_\ell(\cos\theta')\left[\frac{1}{3}pP_0 + \frac{2}{3}pP_2(\cos\theta') + \int d\theta'\sin\theta' F\cos\gamma(\cdot)\sin^2\theta'\right.$

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$m_r = m_r^0 + \cos\gamma \sin^2\theta$, дальше вычисл не зависит от $\cos\gamma$. $x = \cos\theta'$

$$\int d\theta' \sin\theta' F m_r^0 = 2\pi \sum_\ell f_\ell P_\ell(\cos\theta) \left[\tfrac{1}{3} P \int_{-1}^{1} dx\, P_\ell(x) P_0(x) + \tfrac{2}{3} P \int_{-1}^{1} dx\, P_\ell(x) P_2(x) \right] =$$

$$= 2\pi \frac{P}{3} \sum_\ell f_\ell P_\ell \left[2\delta_{\ell 0} + 2 \cdot \frac{2}{5} \delta_{\ell 2} \right] = 4\pi \frac{P}{3} \left[f_0 + \frac{2}{5} f_2 P_2(\cos\theta) \right]$$

$m_{\theta\theta} = \sin\theta\cos\theta (\cos\gamma - P) = -P \sin\theta\cos\theta + \cos\gamma \sin\theta\cos\theta = m_{\theta\theta}^0 + \cos\gamma \sin\theta\cos\theta$

$$\int d\theta' \sin\theta' \frac{\partial F}{\partial \cos\theta'} m_{\theta\theta}^0 = 2\pi \int d\theta' \sin\theta' \sum_\ell f_\ell P_\ell \frac{\partial P_\ell(\cos\theta')}{\partial \theta'} (-P) \sin\theta' \cos\theta' = 2\pi \int d\theta' \sin\theta' \times$$

$$\times \sum_\ell f_\ell P_\ell (-P) \sin\theta' \cos\theta' \frac{\ell}{\sin\theta'} [x P_\ell(x) - P_{\ell-1}(x)] = -2\pi P \sum_\ell f_\ell P_\ell \ell \int_{-1}^{1} dx \left[x^2 P_\ell(x) - x P_{\ell-1}(x) \right] =$$

$$= -2\pi P \sum_\ell f_\ell P_\ell \ell \int_{-1}^{1} \left[\tfrac{2}{3} P_2(x) P_\ell(x) + \tfrac{1}{3} P_0 P_\ell(x) - P_1(x) P_{\ell-1}(x) \right] dx = -2\pi P \sum_\ell f_\ell P_\ell \ell \times$$

$$\times \left[\frac{2}{3} \cdot \frac{2}{5} \delta_{\ell 2} + \frac{1}{3} 2 \delta_{\ell 0} - \frac{2}{3} \delta_{\ell 2} \right] = -2\pi P \frac{2}{3} \sum_\ell f_\ell P_\ell \ell \left[\delta_{\ell 0} - \frac{3}{5} \delta_{\ell 2} \right] =$$

$$= -2\pi P \frac{2}{3} \left(-\frac{3}{5}\right) f_2 P_2 2 = 2\pi P \cdot \frac{4}{5} f_2 P_2 .$$

$$\psi^0(r,\theta) = \int dr' r'^2 \left[\frac{\partial}{\partial r'} 4\pi \frac{P}{3} \left\{ f_0 + \frac{2}{5} f_2 P_2 \right\} + \frac{1}{r'} 2\pi P \frac{4}{5} f_2 P_2 \right] =$$

$$= \frac{4\pi}{3} P \int dr' r'^2 \left[\frac{\partial}{\partial r'} \left\{ f_0 + \frac{2}{5} f_2 P_2 \right\} + \frac{1}{r'} \frac{6}{5} f_2 P_2 \right] = \frac{4\pi}{3} P \left[I_1 + \frac{2}{5} P_2 I_2 + \frac{6}{5} P_2 \cdot I_3 \right] =$$

$$= \frac{4\pi}{3} P \left[r - R + \frac{2}{5} P_2 \left(-\frac{5}{2} r + 3 \frac{r^2}{R} \right) + \frac{6}{5} P_2 \left(\frac{5}{4} r - \frac{r^2}{R} \right) \right] = \frac{4\pi}{3} P \left[r - R + \frac{1}{2} r P_2 \right]$$

$\psi^0(r,\theta) = 4\pi S_0(\gamma)$, $S_0(\gamma) = \frac{P}{8} (r - R + \frac{1}{2} r P_2)$ — совп с 18/VIII

$$\psi^0(r,\theta) = \frac{4\pi}{3} P \left[r - R + \frac{1}{2} r \frac{1}{2} (3\cos^2\theta - 1) \right] = \frac{4\pi}{3} P \left[-R + \frac{3}{4} r + \frac{3}{4} r \cos^2\theta \right]$$

$\psi^0(r,\theta) = \pi P \left[r + \cos^2\theta \cdot r - \frac{4}{3} R \right]$ — задач м. потенциал не зависящая от $\cos\gamma$.

$$h_r^0(r,\theta) = -\frac{\partial}{\partial r} \psi^0 = -\pi P \underset{x(1+\cos^2\theta)}{x}, \quad h_{\theta\theta}^0(r,\theta) = -\frac{1}{r} \frac{\partial}{\partial \theta} \psi^0 = 2\pi P \frac{r}{r} \sin\theta\cos\theta$$

$$h_r^0 + \operatorname{ctg}\theta\, h_{\theta\theta}^0 = -\pi P (1 + \cos^2\theta) + \operatorname{ctg}\theta\, 2\pi P \sin\theta\cos\theta = \pi P \left[-1 - \cos^2\theta + 2\cos^2\theta \right] = -\pi P \sin^2\theta .$$

$h_r^0(r,\theta) = -\pi P (1 + \cos^2\theta)$, $h_\theta^0(r,\theta) = \pi P \sin 2\theta$

$$\psi_\gamma(r,\theta) = \int dr' r'^2 \left[\frac{\partial}{\partial r'} \int d\theta' \sin\theta' F \sin^2\theta' \cos\gamma + \frac{1}{r'} \int d\theta' \sin\theta' \frac{\partial F}{\partial \theta'} \sin\theta' \cos\theta' \cos\gamma \right]$$

$$\psi_\gamma(r,\theta) = \int dr' r'^2 \int d\theta' \sin^3\theta' \left[\frac{\partial}{\partial r'} (F \cos\gamma) + \frac{1}{r'} \operatorname{ctg}\theta' \frac{\partial F}{\partial \theta'} \cos\gamma \right] \qquad \gamma = \gamma(r', \theta')$$

$$\psi(r,\theta) = \pi P \left[r^2 (1 + \cos^2\theta) - \frac{4}{3} R \right] + \int dr' r'^2 \int d\theta' \sin^3\theta' \left[\frac{\partial}{\partial r'} (F \cos\gamma) + \frac{1}{r'} \operatorname{ctg}\theta' \frac{\partial F}{\partial \theta'} \cos\gamma \right]$$

M. potential of BP

$h_r + h_{\theta\theta} \operatorname{ctg}\theta = -\pi P \sin^2\theta + \left(\frac{\partial}{\partial r} + \frac{\operatorname{ctg}\theta}{r} \frac{\partial}{\partial \theta} \right) \psi_\gamma(r,\theta)$, $F = F(r\theta, r'\theta')$

Если учесть $f_2 = 0$, то $h_r = 0$, $h_{\theta\theta} = 0$, т.е. $\psi(r,\theta) = \text{const}$, что есть двухмерное ур-ние неразрывности что нужно на функцию $\cos\gamma(r,\theta)$.

$$\psi_\gamma(r,\theta) = \int dr' r'^2 \int d\theta' \sin^3\theta' \left(\frac{\partial}{\partial r'} + \frac{\operatorname{ctg}\theta'}{r'} \frac{\partial}{\partial \theta'} \right) F \cos\gamma(r'\theta') \quad \text{— нет, } \frac{\partial}{\partial r'} \text{ в } \psi \text{ относится лишь к } F \text{ (см. 15/VIII)}$$

$$\psi(r,\theta) = \pi P \left[r^2 (1 + \cos^2\theta) - \frac{4}{3} R \right] + \int dr' r'^2 \int d\theta' \sin^3\theta' \left[\frac{\partial F}{\partial r'} + \frac{1}{r'} \operatorname{ctg}\theta' \frac{\partial F}{\partial \theta'} \right] \cos\gamma(r', \theta')$$

Corrected m. potential BP

$$\psi(r,\theta) = \pi P \left[r^2 (1 + \cos^2\theta) - \frac{4}{3} R \right] + \int dr' r'^2 \sin\theta' \Gamma(rr', \theta\theta') \cos\gamma(r', \theta')$$

Ядро $\Gamma(rr', \theta\theta') = \sin^2\theta' \left(\frac{\partial F}{\partial r'} + \frac{\operatorname{ctg}\theta'}{r'} \frac{\partial F}{\partial \theta'} \right)$, $F = F(rr', \theta\theta')$ $-\pi P \frac{4}{3} R$ — вычтено из ψ.

$$Q_H = \frac{q}{8\pi} \int_0^\pi d\theta \sin\theta \left[(1 + \cos\theta) \{ \gamma(R,\theta) - \gamma(0,\theta) \} + \int_0^\theta d\theta' \sin\theta' \{ \gamma(R,\theta') - \gamma(0,\theta') \} \right]$$

При $r \to 0$, exch. dominates, $\gamma = \text{const}$, откуда $W_m(\gamma)$ дает $\cos\gamma = -P/4$, $\gamma(0,\theta) = \arccos(-P/4)$. При $r \to \infty$ magnetostatics dominates и $\gamma(r,\theta) = 0, \pi$, $\cos\gamma = P$ — hedgehog (?). $\gamma(0,\theta)$ не зависит от $\theta \to Q_H = \frac{q}{4\pi} [\gamma(R) - \gamma(0)]$

$Q_H = \frac{1}{4\pi} [\arccos(P) - \arccos(-P/4)]$?

Escaneado con CamScanner

(23/VIII) Намираме paper на Q_H - Hopf index of Bloch points (по силен Окс.)
Изчисляваме $Q_H = \frac{1}{4\pi}[\arccos(p) - \arccos(-p/4)]$, ако $\gamma(r=\infty) = 0, \pi$.
For $p=+1$ $\arccos(1) - \arccos(1/4) = -\arccos(1/4)$. For $p=-1$ $\arccos(-1) - \arccos(1/4) = \pi - \arccos(1/4) = \pi + [\arccos(\frac{1}{4}) = \pi - \arccos(-1/4)] = \pi + \arccos(-1/4)$. $Q_H = \frac{1}{4\pi}(-p)\arccos(-1/4)$, $Q_H = -p \cdot 0{,}145$ (!).
При общото $\gamma(R) \in [0, 2\pi]$, $|\gamma(R) - \gamma(0)| \le 2\pi$, $|Q_H| \le 1/2$.
Оценка от сим. Simulations (atomistic с $2R = 35a$, a - lattice period), $p=+1$
$Q_H = \frac{1}{4\pi}[\gamma(R) - \gamma(0)] = \frac{(-1)}{4\pi}\begin{bmatrix} 1{,}18 - 0{,}70 \\ 1{,}18 - 0{,}68 \end{bmatrix} = -(0{,}038 \div 0{,}040)$ [Pylypovskyy 2012] < 0 докар $p=+1$
Също малко Q_H можно обяснено малки $R \approx 18a \approx 5{,}4$ nm дори $a = 0{,}3$ nm $l_{ex} \approx 18$ nm (FeNi), $R \ll l_{ex}$ - dip exch. dominated regime.

(24/VIII) Получих collaboration letter дали „Spin-Hybrids" FS proposal от J. Klos (Poznań), M. Urbaneck (Brno) и O. Dobrovolskiy (Vienna).

Мин Ан DW dynamics, Вчерашно чтение theor. DW braking distance D намного меньше чем D-simulated. $D = V_0 \tau$. Dimensionless eq. for DW position $q(t)$ within rigid DW ansatz.

(25/VIII) Разговор с Оксана о плочи по 3D-charge расчети. Обсъждаме 1) после как Benjamin приключи draft of theor. MS, ще дадавим тази § по расчет Q_H; 2) Оксана нейде сварки или приведе новите BP-DW simulations по cylinder. nanowires of $\vec{m}(\vec{r})$, после нейде $\gamma(\vec{r})$ и Hopf index Q_H; 3) Оксана обещава за разумно време (1 нед.) сделат simulations of $\gamma(\vec{r})$ Bloch points in spherical dots (FeNi), после нейде Q_H. Тогава и ее и Самуелова вече as co-authors in my BP-manuscript. Arantxa to contract? (26/VIII) Писмо до Cristina (UPV/EHU - Arantxa's contract) и Ricardo (Мин Ан).

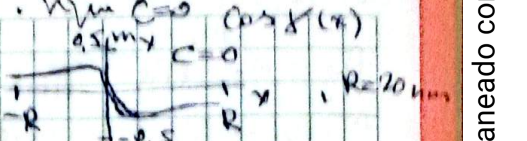
Dirac potential $\vec{A} = \frac{g}{r}\frac{(\vec{r}\times\vec{n})}{r - (\vec{r}\cdot\vec{n})}$. $\vec{n}$ - Dirac string unit vector.
При $\vec{n} = \hat{z}$, $\hat{r}\times\hat{z} = [\sin\theta\cos\varphi\,\hat{x} + \sin\theta\sin\varphi\,\hat{y} + \cos\theta\,\hat{z}] \times \hat{z} = -\sin\theta\cos\varphi\,\hat{x}\times\hat{z} + \sin\theta\sin\varphi\,\hat{y}\times\hat{z} = \sin\theta\cos\varphi\,\hat{y}(-1) + \sin\theta\sin\varphi\,\hat{x}$
При $\hat{r}\times\hat{z} = \hat{r}\times(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}) = -\sin\theta\,\hat{r}\times\hat{\theta} = -\sin\theta\,\hat{\varphi}$
$\vec{A}(\vec{n}=\hat{z}) = \frac{g}{r}\frac{-r\sin\theta\,\hat{\varphi}}{r(1-\cos\theta)} = -\frac{g}{r}\frac{\sin\theta}{1-\cos\theta}\hat{\varphi} = -\frac{g}{r}\frac{1+\cos\theta}{\sin\theta}\hat{\varphi}$
Ако точно $A_\theta \neq 0$, ако $\vec{n} = \hat{z}$ - polar axis of spherical coord. System. Дотук A_θ и намерихме дали реално Hopf index Q_H in the MS-BP.
Flux quantum (Φ_0): in superconductivity $\Phi_0 = \frac{h}{2e}$. Обичайно $\Phi_0 = \frac{h}{e}$; $\oint d\vec{S}\cdot\vec{B} = n\,\mathrm{pq}\,\Phi_0'$, в определени energetic fields $\vec{B}$ за общо имам. Φ_0'? $\Phi_0' = \frac{h}{2e}$, и $\Phi_0 = 4\pi\frac{\hbar}{2e} = \frac{h}{e} = \Phi_0$.

(27/VIII) Намираме BP-manuscript. Лит. обзор. Sk-mass ($\approx m_e$) due to spin-phonon coupling [Chudnovsky-Garanin, PRB'2020]. Smelyukh - 3D-helicons in conical/helix phase of DM ferromagnet [PRL'2020]. E-mails to Julian (consejo depart.), Окс., Maciej (new Ph.D student?)

(28/VIII) E-mails to Angel, Ruben, Julian. Изучаване на ples. Lonsky - APL Mat. Sk-dyn. Tejo simul. BP
BP in Cartesian coordinates: $m_x = \sin\theta\cos\varphi = \sin\theta\cos(\varphi+\gamma)$, Tejo sim.
$m_y = \sin\theta\sin(\varphi+\gamma)$, $m_z = \cos\theta = p\cos\theta$ $\vec{r} = (r, \theta, \varphi)$ сферични координати
$z = r\cos\theta$, $\mathrm{tg}\varphi = \frac{y}{x}$. Сечение by plane $z = C$, $\cos\theta = \frac{C}{r}$. $m_z(x)$ обичайно $\varphi = 0$, т.е. $m_x = \pm\sqrt{1 - C^2/r^2}\cos\gamma$, $m_y = \pm\sqrt{1 - C^2/r^2}\sin\gamma$, $m_z = p\frac{C}{r}$
$\sqrt{1 - C^2/r^2} = \frac{1}{r}\sqrt{r^2 - C^2} = (x^2+y^2)/(\sqrt{x^2+y^2+C^2}) = s/\sqrt{s^2+C^2} = \left|\begin{matrix} y=0 \\ s=x \end{matrix}\right| = x/\sqrt{x^2+C^2}$

Escaneado con CamScanner

$m_d(x, y=0, z=C)$: $m_x = +\frac{x}{\sqrt{x^2+C^2}}\cos\gamma(x,0,C)$, $m_y = +\frac{x}{\sqrt{x^2+C^2}}\sin\gamma$, $m_z = p\frac{C}{\sqrt{x^2+p^2}}$

т.е. $m_x(x) \sim \cos\gamma(x)$. При $C \gtrsim R$ $\cos\gamma(x)$ мал. При $C=0$ $\cos\gamma(x)$ 

меняется от 0.2 до 0.3. $\frac{m_y}{m_x} = \operatorname{tg}\gamma(x)$.

$C=0 \to \cos\varphi = 0$ $\varphi = \pi/2$.

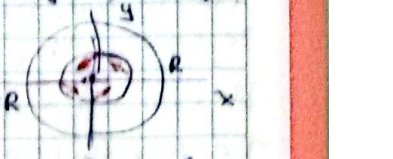
$\gamma(x) \to \gamma(r,\varphi)$ при $\varphi = \pi/2$.

Замечаем тело подвергнуто, что $\vec{m}_{BP}$ не есть функция φ.

$\gamma(r, \pi/2)$: $\gamma(0, \pi/2) = 0$, $\gamma(R, \pi/2) = \arccos(0.2)$.

$\gamma(0) = \pi$ довершает нельзя. $|m_x|_{max} = |\cos\gamma|_{max}$ при $x = 2$ nm, $\cos\gamma(2\text{nm}) = -0.3$

$\cos\gamma(r)$ растет от -0.3 до -0.2 при $r \in [2, 20]$ nm, т.е. $\gamma(r)$ падает с 107.5° до 101.5°, при $\varphi = \pi/2$, $R = 20$ nm. $z = \text{const} = 0$



Из $\vec{B} = \sin\Theta\nabla\Theta\times\nabla\Phi$, подставляя BP ansatz $\Theta\Phi(r)$ - получаем

Sign $A_\varphi = -A$

Верно $\vec{B} = \frac{pz}{r^2}\hat{r} + \frac{p}{r}\sin\varphi\frac{\partial\gamma}{\partial r}\hat{\varphi}$. Дирак potential ($\vec{A}_D = -\frac{qp}{r}\cdot\frac{1+\cos\varphi}{\sin\varphi}\hat{\varphi}$, $\hat{n} = \hat{z}$.

29/VIII (string)

$\operatorname{rot}\vec{A}_D = \frac{\hat{r}}{r\sin\varphi}\frac{\partial}{\partial\varphi}(A_\varphi\sin\varphi) - \frac{\hat{\varphi}}{r}\frac{\partial}{\partial r}(rA_\varphi) = \frac{\hat{r}}{r\sin\varphi}\frac{\partial}{\partial\varphi}\left[-\frac{pz}{r}(1+\cos\varphi)\right] -$

$-\frac{\hat{\varphi}}{r}\frac{\partial}{\partial r}\left[-\frac{qp}{\sin\varphi}(1+\cos\varphi)\right] = -\hat{r}\frac{pz}{r^2}\frac{1}{\sin\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi) = \frac{pz}{r^2}\hat{r} = \vec{B}$ -

- дает только B_r - компонент, А $B_\varphi \sim \partial\gamma/\partial r$ куда делось?

Gus'EPL : $\vec{A} = (1-\cos\Theta)\nabla\Phi$ дает $A_r, A_\varphi, A_\varphi = 0$, но topol. charge density $\vec{A}\cdot\vec{B} = (1-\cos\Theta)\sin\Theta\nabla\Phi\cdot(\nabla\Theta\times\nabla\Phi) \equiv 0$ - for any Θ, Φ (как для BP)

12/VIII Получено $A_r = -p\cos\varphi\frac{\partial\gamma}{\partial r} = -\cos\Theta\frac{\partial\gamma}{\partial r} + f(r,\varphi)$, $f(r,\varphi) \to f(r)$, 19/VIII.

Сом. Gus'EPL'16 : $A_r = (1-\cos\Theta)\frac{\partial\gamma}{\partial r}$, $A_\varphi = (1-\cos\Theta)\frac{1}{r}\frac{\partial\gamma}{\partial\varphi}$, $A_\varphi = (1-\cos\Theta)\frac{q}{r\sin\varphi}$

$\vec{A} = (1-\cos\Theta)\nabla\Phi \to \vec{B} = \operatorname{rot}\vec{A} = \operatorname{rot}[(1-\cos\Theta)\nabla\Phi] = \sin\Theta\nabla\Theta\times\nabla\Phi$, $\Theta = \frac{1}{r}\frac{(1-\rho\cos\varphi)}{\sin\varphi}\rho$

$\vec{A}\cdot\vec{B} = A_r B_r + A_\varphi B_\varphi + A_\varphi B_\varphi = (1-\cos\Theta)\frac{\partial\gamma}{\partial r}\frac{pz}{r^2} + (1-\cos\Theta)\frac{1}{r}\frac{\partial\gamma}{\partial\varphi}\cdot 0 +$

$+\frac{q}{r}\frac{(1-\cos\Theta)}{\sin\varphi}\frac{pz}{r}\sin\varphi\frac{\partial\gamma}{\partial r} \neq 0$, потому. Знаки B_r, B_φ исправить.

$\vec{B} = \sin\Theta\nabla\Theta\times\nabla\Phi = \sin\Theta\rho\frac{1}{r}\hat{e}_\varphi\times\left[\frac{q}{r\sin\varphi}\hat{\varphi} + \frac{\partial\gamma}{\partial r}\hat{r} + \frac{\hat{e}_\varphi}{r}\frac{\partial\gamma}{\partial\varphi}\right] = \frac{p}{r}\sin\varphi\left[\frac{q}{r\sin\varphi}\hat{e}_\varphi\times\hat{\varphi} + \right.$

$\Phi = q\varphi + \gamma(r,\varphi)$ $\left. + \frac{\partial\gamma}{\partial r}\hat{e}_\varphi\times\hat{r}\right] = \left|\begin{matrix}\hat{e}_\varphi\times\hat{\varphi} = \hat{r}\\ \hat{e}_\varphi\times\hat{r} = -\hat{\varphi}\end{matrix}\right| = \frac{pz}{r^2}\hat{r} - \frac{p}{r}\sin\varphi\frac{\partial\gamma}{\partial r}\hat{\varphi} = \vec{B}$.

$\vec{B} = \frac{pz}{r^2}\hat{r} - \frac{p}{r}\sin\varphi\frac{\partial\gamma}{\partial r}\hat{\varphi}$ для BP, $\gamma(r,\varphi)$, $\frac{\partial\gamma}{\partial\varphi}$ не входит.

Если $\vec{A} = (1-\cos\Theta)\nabla\Phi$, то $\vec{A}\vec{B} \equiv 0$ for any $\Theta(\vec{r}), \Phi(\vec{r})$, включая и BP.

Как найти $\vec{A}$, чтоб $\vec{A}\cdot\vec{B} \neq 0$? $\vec{A}_D = A_\varphi^D\hat{\varphi}$ не подходит $\vec{B} = \operatorname{rot}\vec{A}$ не вып.

Хотим $\operatorname{div}\vec{B} = 0$ (калиб.-инв. $\vec{A}$). Поэтому добавим $\vec{B}_S = -4\pi g H(z)\delta(x)\delta(y)\hat{z}$ в $\vec{B}$

$\hat{z} = \cos\varphi\hat{r} - \sin\varphi\hat{e}_\varphi$, $\vec{B}_S = B_S\hat{z} = B_S\cos\varphi\hat{r} - B_S\sin\varphi\hat{e}_\varphi$, $B_S = -4\pi g H(z)\delta(x)\delta(y)$,

Для BP : $\vec{B} = \left[\frac{pz}{r^2} + B_S\cos\varphi\right]\hat{r} - B_S\sin\varphi\hat{e}_\varphi - \frac{p}{r}\sin\varphi\frac{\partial\gamma}{\partial r}\hat{\varphi}$ $g = pz$

Если $\vec{A} = \vec{A}_D$, $\vec{B}_D = \operatorname{rot}\vec{A}_D = \frac{pz}{r^2}\hat{r} - 4\pi g H(z)\delta(x)\delta(y)\hat{z} = \vec{B}_{reg} + \vec{B}_S$, $\vec{B}_{reg} = \frac{pz}{r^2}\hat{r}$.

$\vec{A}_D$ дает оба $\vec{B}_{reg}$ и $\vec{B}_S$, но не дает $\vec{B}_\varphi = -\frac{p}{r}\sin\varphi\frac{\partial\gamma}{\partial r}\hat{\varphi} = B_\varphi\hat{e}_\varphi \sim \frac{\partial\gamma}{\partial r}$.

$\vec{B}_D = \operatorname{rot}\vec{A}_D$ из кн. „Монополь Дирака" 1980.

$B_\varphi = (\operatorname{rot}\vec{A})_\varphi = \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\varphi) - \frac{\partial A_r}{\partial\varphi}\right] = -\frac{p}{r}\sin\varphi\frac{\partial\gamma}{\partial r}$ вернулись к 12/VIII.

Найти A_r, A_φ что дадут это B_φ, но оставят $B_r = \frac{1}{r\sin\varphi}\left[\frac{\partial}{\partial\varphi}(A_\varphi\sin\varphi) - \frac{\partial A_\varphi}{\partial\varphi}\right]$

прежним $B_r = pz/r^2$. Можно $A_\varphi = 0$ и искать только A_r по $\partial A_r/\partial\varphi = p\sin\varphi\frac{\partial\gamma}{\partial r}$. Анизотропия $A_\varphi \neq 0$, $A_\varphi = A_\varphi^D + A'_\varphi$,

Escaneado con CamScanner

$\frac{\partial}{\partial\theta}(\sin\theta A'_\varphi) - \frac{\partial A_\theta}{\partial\varphi} = 0$. $B_z = \frac{pq}{r^2} = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A^D_\varphi \sin\theta)$ – выполнено.

Т.е. или $\begin{matrix}A_\theta=0\\A'_\varphi=0\end{matrix}$, $\frac{\partial A_r}{\partial\theta} = p\sin\theta\frac{\partial\gamma}{\partial r}$ (1), или (2) $\begin{matrix}A_\theta\neq 0\\A'_\varphi\neq 0\end{matrix}$, $\frac{\partial}{\partial\theta}(\sin\theta A'_\varphi) = \frac{\partial A_\theta}{\partial\varphi}$ и $\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta} = -p\sin\theta\times\frac{\partial\gamma}{\partial r}$

$A_\varphi = A^D_\varphi + A'_\varphi$. $\vec{A}\vec{B} = A_\theta B_\theta + A_r B_r$, т.к. $B_\theta = 0$ ($\vec{B}_s$). $\vec{B} = \vec{B}_{reg} + \vec{B}_s$

(1) $A_\theta = 0$, $A'_\varphi = 0$ – нужно, т.к. $A_\theta B_\theta = 0$ из-за $A_\theta = 0$, хотя $B^s_\theta \neq 0$.

(1) $\vec{A} = (A_r, 0, A^D_\varphi)$ и нашли A_r из $\frac{\partial A_r}{\partial\theta} = p\sin\theta\frac{\partial\gamma}{\partial r}$. $\vec{B}_s = B_s\cos\theta\,\hat{r} = B^s_r\hat{r}$

$A^D_\varphi B_\varphi = A^D_\varphi\left[-\frac{p}{r}\sin\theta\frac{\partial\gamma}{\partial r} + \underbrace{0}_{B^s_\varphi=0}\right] = -\frac{p}{r}\sin\theta\frac{\partial\gamma}{\partial r}A^D_\varphi$ – это связано с ВР-ми.

$A_r B_r = \frac{pq}{r^2}A_r$, $\int dV A_r B_r = \int dr\int d\theta d\varphi\, r^2\sin\theta\frac{pq}{r^2}A_r = q\int dr\int d\theta d\varphi\sin\theta A_r$.

Нашли A_r. $A_r = A_r(r,\theta,\varphi)$, хотя $A^D_\varphi = -\frac{q}{r}\frac{1+\cos\theta}{\sin\theta} = A^D_\varphi(r,\theta)$

если еще $B_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA^D_\varphi)\right] = 0$, $\frac{\partial}{\partial r}(rA^D_\varphi) = 0$, $\frac{\partial A_r}{\partial\varphi} = 0$, $A_r(r,\theta)$

но если $B^s_\theta = -B_s\sin\theta$, $\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA^D_\varphi) = -B_s r\sin\theta = 0$, т.к. $B_s\sin\theta \sim \delta(x)\delta(y)\sin\theta = 0$

т.е. $A_r = A_r(r,\theta)$, $A_r(r,\theta) = p\left(\int d\theta'\sin\theta'\frac{\partial\gamma}{\partial r}(r,\theta') + f(r)\right)$ // что с $f(r)$ делать?

$\int dV A_r B^s_r = \int dr\int d\theta\int d\varphi\, r^2\sin\theta A^{\theta}_r B_s\cos\theta = 0$, т.к. $B_s\sin\theta = 0$

$\vec{B}_s = (B^s_r, B^s_\theta, 0)$ не вносит вклада в $\int dV\vec{A}\cdot\vec{B}$ из-за $A_\theta = 0$.

$\int dV A_r B_r = 2\pi q\int dr\int d\theta\sin\theta A_r = 2\pi q\int dr\int d\theta\sin\theta\left[p\int d\theta'\sin\theta'\frac{\partial\gamma}{\partial r} + f(r)\right] =$

$= 2\pi qp\int dr\int_0^{\pi} d\theta\sin\theta\int_0^{\theta} d\theta'\sin\theta'\frac{\partial\gamma}{\partial r}(r,\theta') + 4\pi q\int dr f(r)$

$\operatorname{div}\vec{A} = \operatorname{div}(A_r, 0, A^D_\varphi)^{\theta}_{\varphi} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\theta}\frac{\partial A^D_\varphi}{\partial\varphi}^{0} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r)$

Тем основанием приравняем $\int dV A_r B_r$ к Q_H. //

$\int_0^{\pi} d\theta\sin\theta A_r = \left|\begin{matrix}d\theta\sin\theta = dv\\ v = -\cos\theta\end{matrix}\right| = -\cos\theta A_r\Big|_0^{\pi} - \int(+\cos\theta)\frac{\partial A_r}{\partial\theta}d\theta =$

$= -\cos\theta A_r\Big|_0^{\pi} + \int_0^{\pi} d\theta\cos\theta\, p\sin\theta\frac{\partial\gamma}{\partial r} = f_1(r) + p\int_0^{\pi} d\theta\sin\theta\cos\theta\frac{\partial\gamma}{\partial r}$, $f_1 = -\cos\theta A_r\Big|_0^{\pi}$

$\int dV A_r B_r = 2\pi q\int dr\int d\theta\sin\theta A_r = 2\pi q\int dr\left[p\int d\theta\sin\theta\cos\theta\frac{\partial\gamma}{\partial r} + f_1(r)\right]$

$\int dV A_\theta B_\theta = \int dV\left(-\frac{q}{r^2}\right)\frac{1+\cos\theta}{\sin\theta}\left(-\frac{p}{r}\sin\theta\frac{\partial\gamma}{\partial r}\right) = pq\int dV\frac{1}{r^2}(1+\cos\theta)\frac{\partial\gamma}{\partial r} =$

$= pq\int dr\int d\theta\int d\varphi\, r^2\sin\theta\frac{1}{r^2}(1+\cos\theta)\frac{\partial\gamma}{\partial r} = 2\pi pq\int dr\int d\theta\sin\theta(1+\cos\theta)\frac{\partial\gamma}{\partial r}$

$\int dV\vec{A}\cdot\vec{B} = \int dV A_\theta B_\theta + \int dV A_r B_r = 2\pi q\int dr\int d\theta\sin\theta(1+\cos\theta)\frac{\partial\gamma}{\partial r} +$

$+ 2\pi q\int dr\left[\int d\theta\sin\theta\cos\theta\frac{\partial\gamma}{\partial r} + pf_1(r)\right] = 2\pi q\int dr\int d\theta\sin\theta(1+q\cos\theta)\frac{\partial\gamma}{\partial r} +$

$+ 2\pi qp\int dr f_1(r)$, где $f_1(r) = -\cos\theta A_r(r,\theta)\Big|_0^{\pi}$

$Q_H = -\frac{1}{(4\pi)^2}\int dV\vec{A}\cdot\vec{B} = -\frac{q}{8\pi}\int_0^{R} dr\int_0^{\pi} d\theta\sin\theta(1+2\cos\theta)\frac{\partial\gamma}{\partial r} - \frac{pq}{8\pi}\int_0^{R} dr f_1(r)$, $\vec{A} = (A_r, 0, A^D_\varphi)$

Нет index BP с учетом $A_r \neq 0$ – corrected Q_H (BP) on 22/VIII.

30/VIII (2) $\vec{A} = (A_r, A_\theta, A^D_\varphi + A'_\varphi)$. $\frac{\partial}{\partial\theta}(\sin\theta A'_\varphi) = \frac{\partial A_\theta}{\partial\varphi}$, $\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta} = -p\sin\theta\frac{\partial\gamma}{\partial r}$

В силу аксиальной симм. $\vec{A} = \vec{A}(r,\theta)$. $\frac{\partial A_\theta}{\partial\varphi} = 0$, $\frac{\partial}{\partial\theta}(\sin\theta A'_\varphi) = 0$, $A'_\varphi = \frac{f_1(r)}{\sin\theta}$

$0 = B_\varphi = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\theta)\right] = 0$, $\frac{\partial}{\partial r}\left[r(A^D_\varphi + A'_\varphi)\right] = 0$

$A^D_\varphi + A'_\varphi = -\frac{q}{r}\frac{1+\cos\theta}{\sin\theta} + \frac{f_1(r)}{\sin\theta}$, $r(A^D_\varphi + A'_\varphi) = -q\frac{1+\cos\theta}{\sin\theta} + \frac{rf_1(r)}{\sin\theta}$

$\frac{\partial}{\partial r}(rA_\theta) = \frac{1}{\sin\theta}\frac{\partial}{\partial r}(rf_1(r)) = 0$, $f_1(r) = \frac{C}{r}$, вкл. $C = 0$. $A'_\varphi = \frac{C}{r\sin\theta}$.

Но A_r, A_θ – любые, вообще. $\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta} = -p\sin\theta\frac{\partial\gamma}{\partial r}$. B_z есть $A_\theta = 0$,

$\frac{\partial A_r}{\partial\theta} = p\sin\theta\frac{\partial\gamma}{\partial r}$ – совпадает с разделом. Но $A_\theta = A^D_\varphi + \frac{C}{r\sin\theta} \to B_r = q/r^2$.

$\int dV A_\varphi B_\varphi = \int dV \left(A_\varphi^D + \frac{C}{r\sin\theta}\right)\left(-\frac{\rho}{2}\sin\theta\frac{\partial\psi}{\partial r}\right) = (A_\varphi^D) + \int dV \frac{(-C\rho)}{r^2}\frac{\partial\psi}{\partial r} =$

$= (A_\varphi^D) - C\rho \int d\varphi \int dr \int d\theta\, r^2 \sin\theta \frac{1}{r^2}\frac{\partial\psi}{\partial r} = (A_\varphi^D) - C\rho 2\pi \int dr \int d\theta \sin\theta \frac{\partial\psi}{\partial r}$

$\int dV A_\varphi B_\varphi = 2\pi q \int dr\, d\theta \sin\theta (1+\cos\theta)\frac{\partial\psi}{\partial r} - 2\pi\rho C \int dr \int d\theta \sin\theta \frac{\partial\psi}{\partial r} =$

$= 2\pi \int dr \int d\theta \sin\theta \left[(q-\rho C) + q\cos\theta\right]\frac{\partial\psi}{\partial r}$

$\rightarrow \int dV \vec{A}\cdot\vec{B} = 2\pi \int dr \int d\theta \sin\theta \left[(q-\rho C) + 2q\cos\theta\right]\frac{\partial\psi}{\partial r} + 2\pi q\rho \int dr\, f_1(r)$ ($\uparrow A_z$)

[A_φ' – из минимума]

Т.е. учет $A_\varphi' \neq 0$ в $A_\varphi = A_\varphi^D + A_\varphi'$ и $A_z(r,\theta)$ ведет к изменению Q_H.

$A_\varphi' = \frac{C}{r\sin\theta}$ is singular at $\theta = 0, \pi$. $A_\varphi' \neq 0$ изменит $\vec{B}_3$. $\vec{B}_3(A_\varphi^D)$ подобрано так чтоб $\operatorname{div}\vec{B} = 0$. $\vec{B}_3(A_\varphi^D)$ изменяет топологию $\Rightarrow$ ($A_\varphi' = 0$, $C = 0$, и def. $Q_H(\psi)$ верно.

$f_2(r) = -\cos\theta A_z(r,\theta)\Big|_0^\pi = A_z(r,\pi) + A_z(r,0)$ – как найти?

$A_z(r,\theta) = \rho\left(\int_0^\theta d\theta' \sin\theta' \frac{\partial\psi}{\partial r}(r,\theta') = \left|\int_{\theta_0}^\theta = \int_{\theta_0}^0 + \int_0^\theta\right| = \rho\int_0^\theta d\theta' \sin\theta' \frac{\partial\psi}{\partial r} -\right.$

$- \int_0^{\theta_0} d\theta' \sin\theta' \frac{\partial\psi}{\partial r}\rho = \rho\int_0^\theta d\theta' \sin\theta' \frac{\partial\psi}{\partial r} + f_2(r)$, $A_z(r,0) = f_2(r)$, $A_z(r,\pi) =$

$= \rho\int_0^\pi d\theta' \sin\theta' \frac{\partial\psi}{\partial r} + f_2(r) = f_3(r) + f_2(r)$, $f_{12}(r) = 2f_2(r) + f_3(r)$. $f_2(r) = 0$

$f_3(r) = \rho\int_0^\pi d\theta' \sin\theta' \frac{\partial\psi}{\partial r}$. $f_2(r)$ – просто калибровка, т.к. θ_0 – arbitrary

$f_2(r) = f_3(r) = \rho\int_0^\pi d\theta \sin\theta' \frac{\partial\psi}{\partial r} \rightarrow \int dV \vec{A}\cdot\vec{B} = (\dots) + 2\pi q\rho \int dr\, \rho \int_0^\pi d\theta' \sin\theta' \frac{\partial\psi}{\partial r} =$ Ст. ВЗ/I-2021

Hopf inv. $= (\dots) + 2\pi q \int_0^\pi d\theta \int dr \sin\theta \frac{\partial\psi}{\partial r} = 4\pi q \int dr \int_0^\pi d\theta \sin\theta (1+\cos\theta)\frac{\partial\psi}{\partial r}$

BP → $Q_H = -\frac{1}{(4\pi)^2}\int dV \vec{A}\cdot\vec{B} = -\frac{q}{4\pi}\int_0^R dr \int_0^\pi d\theta \sin\theta (1+\cos\theta)\frac{\partial\psi}{\partial r}$, $\vec{A} = (A_r, 0, A_\varphi^D)$

(где учтена main part $A_z \neq 0$ по отнош. к gauge transformations of A_r.

Рец. MS Lonsky – Hoffmann Sk-dyn. APL Mat. → minor revision.

Подпис. ANEXO I – New contract и подал to Arantxa.

(3/VIII) Отишли в "Кутха" Анкета нашия награден за 2019 (By cash 452 €) перемина с Angel, Arantxa, Ricardo, Ruben.

(1/IX) Phone обсъжд. с Ruben :) 1) new PhD student (име нам-са в PhD project по 3D top. charge / DW dyn. nanowires); 2) M_{n2} R MS for PR Appl. (2-model in main text, E-model in Appendix, Kyle tracking distance vs. Hsat – уточн.); 3) Пакетов нов Skyrmion project (travels, beads, fungibles). С Ю. в „Carrefour Market" Hologic E-mail от Ivan's от институции Depto. Física de Materiales, корпус Sygaz División de Depto. de Polímeros y Materiales Avanzados: Física, Química y Tecnología. Consejo del Departamento с Oct. 2. Abs. Nanospain

(2/IX) Изпратих MS „Walker-like DW breakdown in layered AF..." to Commun. Phys. (Impact 2019 = 12.1). Изпратих отказ APC 2.300 € из своето Sk-project Зборник Гьоб: обещах нови експ. FMR data по Co-nanowires in single-vortex state – parameters как в 2019-paper Ivan Chuvilin. Submitted Skyrmion-review abstract to Nanospain, Bilbao, Sept. 29-Oct. 1.

(3/IX) Вернение к Wm of Bloch point. BP

$h_z + \frac{\partial_\theta(\sin\theta h_\theta)}{r\sin\theta} = -\frac{\partial}{\partial r}\psi(r,\theta) - \frac{\partial_\theta(\sin\theta)}{r}\frac{\partial}{\partial\theta}\psi(r,\theta)$ – умножим по θ чрез $\int_0^\pi d\theta \sin\theta(\dots)$

$\int_0^\pi d\theta \sin\theta\, \partial_\theta(\sin\theta\frac{\partial\psi}{\partial\theta}) = \int d\theta \frac{\partial\psi}{\partial\theta}\cos\theta = \psi\cos\theta\Big|_0^\pi + \int_0^\pi d\theta \sin\theta\,\psi$

$dv \quad u$

$$\int\left[h_r+\operatorname{ctg}\theta\, h_\theta\right]\sin\theta\, d\theta = -\int d\theta\sin\theta\left[\frac{\partial\psi}{\partial r}+\frac{\operatorname{ctg}\theta}{r}\frac{\partial\psi}{\partial\theta}\right] = -\frac{\partial}{\partial r}\int d\theta\sin\theta\,\psi -$$

$$-\frac{1}{r}\int d\theta\sin\theta\operatorname{ctg}\theta\frac{\partial\psi}{\partial\theta} = -\frac{\partial}{\partial r}\int d\theta\sin\theta\,\psi - \frac{1}{r}\left[\int d\theta\sin\theta\,\psi + \psi\cos\theta\Big|_0^\pi\right] =$$

$$= -\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)\int d\theta\sin\theta\,\psi - \frac{1}{r}\psi(r,\theta)\cos\theta\Big|_0^\pi \qquad \psi(r,\theta)\ \text{boş a-mı. zəpəz}$$

$$\int d\theta\sin\theta\,\Gamma(r,\theta,r',\theta') = \int d\theta\sin\theta\left[\frac{\partial F}{\partial r'}+\frac{\operatorname{ctg}\theta'}{r'}\frac{\partial F}{\partial\theta'}\right] \qquad \Gamma(r,\theta,r',\theta')$$

$$F(r,\theta,r',\theta') = 2\pi\sum_\ell f_\ell P_\ell(\cos\theta)P_\ell(\cos\theta') = 2\pi\sum_\ell f_\ell P_\ell P_\ell(1). \qquad \int_{-1}^{1}dx P_\ell(x)P_{\ell'}(x) = \frac{2}{(2\ell+1)}\delta_{\ell\ell'}$$

$$\int d\theta\sin\theta F(r,\theta,r',\theta') = 2\pi\sum_\ell f_\ell P_\ell(1)\int d\theta\sin\theta P_\ell(\cos\theta) = 2\pi\sum_\ell f_\ell P_\ell(\cos\theta')\times$$

$$\times\int_{-1}^{1}dx P_\ell(x) = 2\pi\sum_\ell f_\ell P_\ell(1)\,2\delta_{\ell 0} = 4\pi f_0 P_0(1) = 4\pi f_0, \quad f_\ell = f_\ell(r,r')$$

$$\int d\theta\sin\theta\frac{\partial F}{\partial\theta'} = \int d\theta\sin\theta\, 2\pi\sum_\ell f_\ell P_\ell(\cos\theta)\frac{\partial P_\ell(\cos\theta')}{\partial\theta'} = 2\pi\sum_\ell f_\ell\int d\theta\sin\theta P_\ell(\cos\theta)\times$$

$$\times\frac{\ell}{\sin\theta'}\left[\cos\theta' P_\ell(\cos\theta') - P_{\ell-1}(\cos\theta')\right] = 2\pi\sum_\ell f_\ell\cdot 2\delta_{\ell 0}+\frac{\ell}{\sin\theta'}[\dots] = \sum_\ell \ell\delta_{\ell 0}(\dots) = 0$$

$$\rightarrow \int d\theta\sin\theta\,\Gamma(r,\theta,r',\theta') = \sin^2\theta'\frac{\partial}{\partial r'}\int d\theta\sin\theta F(r,\theta,r',\theta') = 4\pi\sin^2\theta'\frac{\partial}{\partial r'}f_0(r,r')$$

$$\int d\theta\sin\theta\left[h_r+\operatorname{ctg}\theta h_\theta\right] = -\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)\int d\theta\sin\theta\,\psi(r,\theta) - \frac{1}{r}\psi\cos\theta\Big|_0^\pi = -\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)\int d\theta\sin\theta\times$$

$$\times\left[\pi p r(1+\cos^2\theta)+\int dr' r'^2\int d\theta'\sin\theta'\,\Gamma(r,\theta,r',\theta')\cos\gamma(r',\theta')\right] - \frac{1}{r}\psi(r,\theta)\cos\theta\Big|_0^\pi =$$

$$= -\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)\left[\pi p r\cdot\frac{8}{3}+\int dr' r'^2\int d\theta'\sin\theta'\cos\gamma(r',\theta')\cdot 4\pi\sin^2\theta'\frac{\partial f_0}{\partial r'}\right] - \frac{1}{r}\psi\cos\theta\Big|_0^\pi$$

$$\int d\theta\sin\theta\left[h_r+h_\theta\operatorname{ctg}\theta\right] = -\left[\frac{16\pi}{3}p\right] - \frac{1}{r}\psi\cos\theta\Big|_0^\pi - \left(\frac{\partial}{\partial r}+\frac{1}{r}\right)4\pi\int dr' r'^2\int d\theta'\sin^3\theta'\frac{\partial f_0}{\partial r'}\cos\gamma(r',\theta')$$

$$f_0(r,r') = \frac{r_<^\ell}{r_>^{\ell+1}} = \frac{1}{r_>} = \begin{cases}\frac{1}{r'}, & r'>r\\ \frac{1}{r}, & r>r'\end{cases}\Bigg\}, \quad \frac{\partial f_0}{\partial r'} = \frac{\partial}{\partial r'}\begin{cases}\frac{1}{r'}, & r'>r\\ \frac{1}{r}, & r>r'\end{cases}\Bigg\} = \begin{cases}-1/r'^2, & r'>r\\ 0, & r>r'\end{cases}\Bigg\}$$

$$\int dr' r'^2\frac{\partial f_0}{\partial r'}\cos\gamma(r',\theta') = \int_0^r dr'(\dots) + \int_r^R dr'(\dots) = \int_r^R dr' r'^2\left(-\frac{1}{r'^2}\right)\cos\gamma = -\int_r^R dr'\cos\gamma(r',\theta')$$

$$\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)\int dr' r'^2\frac{\partial f_0}{\partial r'}\cos\gamma(r',\theta') = -\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)\int_r^R dr'\cos\gamma(r',\theta') = \cos\gamma(r,\theta) - \frac{1}{r}\int_r^R dr'\cos\gamma(r',\theta')$$

$$\rightarrow \int_0^\pi d\theta\sin\theta\left(h_r+h_\theta\operatorname{ctg}\theta\right) = -\frac{16}{3}\pi p - \frac{1}{r}\psi\cos\theta\Big|_0^\pi - 4\pi\int d\theta'\sin^3\theta'\left[\cos\gamma(r,\theta') - \frac{1}{r}\int_r^R dr'\cos\gamma(r',\theta')\right]$$

$$\psi(r,\pi) = 2\pi p r^2 + \int dr' r'^2\int d\theta'\sin\theta'\,\Gamma(r,\pi,0;r',\theta')\cos\gamma(r',\theta')$$

$$F(r,\pi,0,r',\theta') = 2\pi\sum_\ell f_\ell P_\ell(\cos\pi)P_\ell(\cos\theta'), \quad P_\ell(\cos\pi) = P_\ell(-1)$$

$$\psi(r,\theta)\cos\theta\Big|_0^\pi = \pi p r^2(1+\cos^2\theta)\cos\theta\Big|_0^\pi + \int dr' r'^2\int d\theta'\sin\theta'\cos\gamma(r',\theta')\Gamma\left(r,\frac{\pi}{\cos\theta'},r',\theta'\right)\Big|_{w=0}^{\pi}$$

$$\Gamma(r,\theta,r',\theta')\cos\theta\Big|_0^\pi = \sin^2\theta'\left(\frac{\partial}{\partial r'}+\frac{\operatorname{ctg}\theta'}{r'}\frac{\partial}{\partial\theta'}\right)\cos\theta F(r,\theta,r',\theta')\Big|_{\theta=0}^{\pi} =$$

$$= \sin^2\theta'\left(\frac{\partial}{\partial r'}+\frac{\operatorname{ctg}\theta'}{r'}\frac{\partial}{\partial\theta'}\right)2\pi\sum_\ell f_\ell P_\ell(\cos\theta')\cos\theta P_\ell(\cos\theta)\Big|_{\theta=0}^{\pi} \sim \left[(-1)P_\ell(-1) - 1\cdot P_\ell(1)\right] =$$

$$= (-1)\left[P_\ell(-1)+P_\ell(1)\right] = (-1)\left[(-1)^\ell+1\right] = (-2) \to \ell - \text{even}, \quad m=0 \quad \ell - \text{nəzər.}$$

$$\psi(r,\theta)\cos\theta\Big|_0^\pi = -4\pi p r^2 + \int dr' r'^2\int d\theta'\sin\theta'\cos\gamma(r',\theta')\sin^2\theta'\left[\frac{\partial}{\partial r'}+\frac{\operatorname{ctg}\theta'}{r'}\frac{\partial}{\partial\theta'}\right](4\pi)\sum_{\ell\text{ even}} f_\ell P_\ell(\cos\theta')$$

$$\rightarrow \int_0^\pi d\theta\sin\theta\left[h_r+h_\theta\operatorname{ctg}\theta\right] = -\frac{4\pi}{3}p + \frac{4\pi}{r}\int dr' r'^2\int d\theta'\sin^3\theta'\cos\gamma(r',\theta')\left[\frac{\partial}{\partial r'}+\frac{\operatorname{ctg}\theta'}{r'}\frac{\partial}{\partial\theta'}\right]\sum_{\ell\text{ even}} f_\ell P_\ell(\cos\theta')$$

$$-4\pi\int d\theta'\sin^3\theta'\left[\cos\gamma(r,\theta') - \frac{1}{r}\int_r^R dr'\cos\gamma(r',\theta')\right]. \qquad \cos\gamma(r,\theta) = \sum_\ell c_\ell(r)P_\ell(\cos\theta) - ?$$

$h_r+h_\theta\operatorname{ctg}\theta = 0$ qaçı mənasında. Integral equation for $\gamma = \gamma(r,\theta)$. Fcn pəyənmə $\cos\gamma(r,\theta) = \sum_\ell c_\ell(r)P_\ell(\cos\theta)$. və unit-yə. görə $c_\ell(r)$.

Varshalovich 1988: $\frac{2c+1}{2}\int_0^\pi d\theta\sin\theta P_a(\cos\theta)P_b(\cos\theta)P_c(\cos\theta) = \left(C^{c0}_{a0b0}\right)^2$

$C^{c0}_{a0b0} = 0$ əgər $a+b+c = 2g+1$, $C^{c0}_{a0b0} = \frac{(-1)^{g-c}\sqrt{2c+1}\,g!}{(g+a)!(g+b)!(g-c)!}\sqrt{\frac{(2g-2a)!(2g-2b)!(2g-2c)!}{(2g+1)!}}$

əgər $a+b+c = 2g$ - even. C_v - Clebsch - Gordan coeffs.

Escaneado con CamScanner

(4/IX) Оксана прислала simulations by F. Teyssandier для nanosphere с R=100 нм. $\gamma(r, \theta=\pi/2)$ падает с $\gamma \approx 111°$ до 90°, $\gamma(r, \theta=0)$ падает от 111° до $\approx 72°$, т.е. $\approx$ на 40°(!). $\gamma(r,\theta)$ анизотропно. $A = 21\ pJ/m$, $M_s = 1700\ G$ (Fe).

Попросим ее найти Q_H интегрируя $\gamma(r,\theta)$ и построить $\gamma(r)$ для $\theta = 0, \pi/4, \pi/2$. $\gamma(r,\theta)$ в сечениях $z = const$ для скирмов не надо.

Звонила Окс. (prorroga SK-project, Q_H, travels, Aurelio M(?)exp.)

Cristina (UPV/EHU) о contract Arantxa и prorroga. Запрос prorroga - согласие у Julio Camarero, переписка с M. Valvidares.

(5/IX) С Л. в "Lyl" Olaberria С Л. и О. в Е. Zubkin Tolosa. Написание BP-Hopf index MS. Papers by Borisov JETPLett. 2008, 2009, PRM'2011.

(6/IX) • Borisov et al. JETP Lett. 2008, 2009 (v. 88, 303 (2008) и v. 90, 593 (2009)) утверждает, что при $\vec{m} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$, $\theta = \theta(\rho, z)$, $\Phi(\vec{r}) = \omega t + q\varphi + \gamma(\rho, z)$ в cyl. coordinates $\vec{r}(\rho, \varphi, z)$, Hopf index $Q_H = qT$, $T = \frac{1}{4\pi}\int_{-\infty}^{\infty} dz \int_0^{\infty} d\rho\, \vec{n}\cdot\left[\frac{\partial \vec{n}}{\partial \rho} \times \frac{\partial \vec{n}}{\partial z}\right]$ = integer где $\vec{n} = (\sin\theta\cos\gamma, \sin\theta\sin\gamma, \cos\theta)$ согласно Kundu'82 и Gladikowski'97. $\vec{n}$ - mapping на helicity $\gamma(\rho, z)$.

• $\frac{\partial \vec{n}}{\partial \rho} \times \frac{\partial \vec{n}}{\partial z} = \hat{x}\left[\frac{\partial n_y}{\partial \rho}\frac{\partial n_z}{\partial z} - \frac{\partial n_z}{\partial \rho}\frac{\partial n_y}{\partial z}\right] + \hat{y}\left[\frac{\partial n_z}{\partial \rho}\frac{\partial n_x}{\partial z} - \frac{\partial n_x}{\partial \rho}\frac{\partial n_z}{\partial z}\right] + \hat{z}\left[\frac{\partial n_x}{\partial \rho}\frac{\partial n_y}{\partial z} - \frac{\partial n_y}{\partial \rho}\frac{\partial n_x}{\partial z}\right] =$

$= \hat{x}\left[(\cos\theta\,\theta'_\rho \sin\gamma + \sin\theta\cos\gamma\,\gamma'_\rho)(-\sin\theta)\theta'_z - (-\sin\theta)\theta'_\rho(\cos\theta\,\theta'_z\sin\gamma + \sin\theta\cos\gamma\,\gamma'_z)\right] + \ldots$

Def. $\zeta = \rho + iz$, $\bar{\zeta} = \rho - iz$, $\frac{\partial}{\partial \rho} = \frac{1}{2}\left(\frac{\partial}{\partial \zeta} + \frac{\partial}{\partial \bar{\zeta}}\right)$, $\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial \rho} - i\frac{\partial}{\partial z}\right)$, $\frac{\partial}{\partial \bar{\zeta}} = \frac{1}{2}\left(\frac{\partial}{\partial \rho} + i\frac{\partial}{\partial z}\right)$

Ирония если $\vec{m}\cdot\left(\frac{\partial \vec{m}}{\partial x} \times \frac{\partial \vec{m}}{\partial y}\right)$ дает $\vec{G} = \hat{z}\int dS \sin\theta\,(\nabla\Phi \times \nabla\theta)_z$ в xy-plane $\vec{m}(x,y)$, то $\vec{n}\cdot\left(\frac{\partial \vec{n}}{\partial \rho} \times \frac{\partial \vec{n}}{\partial z}\right)$ с $\vec{n}(\theta, \gamma)$ дает $\left(\begin{smallmatrix} x\to\rho \\ y\to z \\ \Phi\to\gamma \\ z\to\varphi \end{smallmatrix}\right)$ $\sin\theta\,(\nabla\gamma \times \nabla\theta)_\varphi$

$\sin\theta(\nabla\gamma\times\nabla\theta)_\varphi = \sin\theta\hat{\varphi}\cdot\left(\frac{\partial\gamma}{\partial\rho}\hat{\rho} + \frac{\partial\gamma}{\partial z}\hat{z}\right)\times\left(\frac{\partial\theta}{\partial\rho}\hat{\rho} + \frac{\partial\theta}{\partial z}\hat{z}\right) = \sin\theta\,\hat{\varphi}\cdot$

$\cdot\left[\frac{\partial\gamma}{\partial\rho}\frac{\partial\theta}{\partial z}\hat{\rho}\times\hat{z} + \frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial\rho}\hat{z}\times\hat{\rho}\right] = \left|\hat{z}\times\hat{\rho} = \hat{\varphi}\right| = \sin\theta\hat{\varphi}\cdot\hat{\varphi}\left[\frac{\partial\gamma}{\partial\rho}\frac{\partial\theta}{\partial z} - \frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial\rho}\right]$

$T = \frac{1}{4\pi}\int d\rho\, dz\, \vec{n}\cdot\left(\frac{\partial\vec{n}}{\partial\rho}\times\frac{\partial\vec{n}}{\partial z}\right) = \frac{(\pm 1)}{4\pi}\int_{-\infty}^{\infty} dz \int_0^{\infty} d\rho \sin\theta\left[\frac{\partial\gamma}{\partial\rho}\frac{\partial\theta}{\partial z} - \frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial\rho}\right]$. И как получить

• $T = (\pm 1)$ если $\gamma = \gamma(\rho, z)$ и $\theta = \theta(\rho, z)$ как у Borisov JETP Lett. '08?

У Gladikowski97 этого нет, он сразу переходит к toroidal coordinates

Kundu'82 дает $Q_H = \frac{q}{4\pi}\int_0^{\infty} dr \int_0^{\pi} d\theta\, (1-\cos\theta)\left(\frac{\partial\cos\theta}{\partial r}\frac{\partial\gamma}{\partial\theta} - \frac{\partial\cos\theta}{\partial\theta}\frac{\partial\gamma}{\partial r}\right)$ - в сферич. координатах $\vec{n}(r, \theta, \varphi)$, $m_z = \cos\Theta = m_z(r,\theta)$, $\Phi = q\varphi + \gamma(r,\theta)$ без подробного вывода. Для BP $\Theta(\vec{r}) = p\theta + \frac{\pi}{2}(1-p)$, $\Phi(\vec{r}) = q\varphi + \gamma(r,\theta)$

$Q_H(\text{Kundu}) = \frac{qp}{4\pi}\int_0^{\infty} dr \int_0^{\pi} d\theta\, (1 - p\cos\theta)\sin\theta\frac{\partial\gamma}{\partial r}$ отличается от него

$Q_H(\text{Gus}) = -\frac{q}{4\pi}\int_0^{R} dr \int_0^{\pi} d\theta\, \sin\theta(1+\cos\theta)\frac{\partial\gamma}{\partial r}$ полученного из $\vec{B}$ и $\vec{A}$.

$\vec{B}(\text{Kundu}) = -\nabla(\cos\theta)\times\nabla\Phi = -\frac{q}{r^2\sin\theta}\left[\hat{r}\frac{\partial\cos\theta}{\partial\theta} - \hat{\theta}\,r\frac{\partial\cos\theta}{\partial r}\right] - \frac{\hat{\varphi}}{r}\left[\frac{\partial\cos\theta}{\partial r}\frac{\partial\gamma}{\partial\theta} - \frac{\partial\cos\theta}{\partial\theta}\frac{\partial\gamma}{\partial r}\right]$

или $\frac{\partial\cos\theta}{\partial\theta} = -\sin\theta\frac{\partial\theta}{\partial\theta} = -\sin\theta\,p$, $\frac{\partial\cos\theta}{\partial r} = 0$ (BP ansatz) сводится к

• $\vec{B}(\text{Kundu, BP}) = \frac{qp}{r^2}\hat{r} - \frac{\hat{\varphi}}{r}\sin\theta\frac{\partial\gamma}{\partial r}p$ совпадает с моим $\vec{B}$(Gus, BP).

$\vec{A}$(Kundu) не показан. Q_H(Kundu, BP) отличается от Q_H(Gus), как и близко к нему, Q_H(Kundu) = Q_H(Gus) для $p = -1$. T-integer следует из Q_H-integer in toroid. coord.

→ Вывод: утверждение Borisov JETP'08, что T = integer аналитически не доказано.

Wang-PRB'2019

Escaneado con CamScanner

Для 2D $\vec{m}(x,y)$ $Q = \frac{1}{4\pi}\int dx\,dy\,\vec{m}\cdot\left(\frac{\partial\vec{m}}{\partial x}\times\frac{\partial\vec{m}}{\partial y}\right) = \frac{(\pm1)}{4\pi}\int dx\,dy\,\sin\Theta\,(\nabla\Theta\times\nabla\Phi)_z$

$m_z = \Theta = \Theta(\rho)$, $\Phi = q\varphi + \Phi_0$, $\nabla\Theta\times\nabla\Phi = \frac{\partial\Theta}{\partial\rho}\hat{\rho}\times\frac{\hat{\varphi}}{\rho}\frac{\partial}{\partial\varphi}(q\varphi+\Phi_0) = \frac{q}{\rho}\Theta'_\rho\,\hat{\rho}\times\hat{\varphi} = \frac{q}{\rho}\Theta'_\rho\,\hat{z}$

$Q = \frac{(\pm1)}{4\pi}\int d^2\rho\,\sin\Theta\,\frac{q}{\rho}\frac{\partial\Theta}{\partial\rho} = \frac{(\pm1)}{4\pi}2q\int d\rho\,\rho\int d\varphi\,\sin\Theta\frac{\partial\Theta}{\partial\rho} = (\pm1)\frac{q}{2}\int_0^R d\Theta\sin\Theta =$

$= (\pm1)\frac{1}{2}q\,(-1)\cos\Theta\Big|_0^R = \frac{(\pm1)}{(-1)}\frac{1}{2}q\left[\cos\Theta(R)-\cos\Theta(0)\right]$, $m_z(\rho) = \cos\Theta(\rho) = p$

$\cos\Theta(R) = (\text{skyrmion}) = -p$, $Q = \frac{1}{2}q(-1)(-2p) = qp$ - integer

т.е. 2D charge integer если $\Theta = \Theta(\rho)$, $\Phi = q\varphi+\Phi_0$ тополог. Для arbitrary $\vec{m}(x,y)$ $\Theta(\rho)$, $\Phi(\rho)$ доказать $Q = pq$ - integer - тополог.

- Wang-Brataas '2019 далее в цил. coordinates $\Theta = \Theta(\rho,z)$, $\Phi = q\varphi + \gamma(\rho,z)$, $\vec{r} = (\rho,\varphi,z)$

$Q_H = \frac{1}{(4\pi)^2}\int dV\,\vec{B}\cdot\vec{A} = \frac{q}{4\pi}\int_{-\infty}^{\infty}dz\int_0^\infty d\rho\,\sin\Theta\left[\frac{\partial\Theta}{\partial z}\frac{\partial\gamma}{\partial\rho} - \frac{\partial\Theta}{\partial\rho}\frac{\partial\gamma}{\partial z}\right] = q\cdot T$ - eq. gives Hopf index the same as Borisov '2008. T - skyrmion number in ρ-z plane. Тождество не следует, что T is integer, in general case. Toroidal Hopfion

- Gladikowski '97: $\vec{A}\vec{B} = A_\alpha B_\alpha + A_\beta B_\beta + A_\gamma B_\gamma$, $\vec{r}(\gamma,\beta,\alpha)$, $\alpha=\varphi$ - toroidal coordinates

$\vec{A}\cdot\vec{B} = \frac{\tau}{a\,\mathrm{sh}\gamma}N(w-1)\frac{\tau^2}{a^2}\left[w'(M+\dot{\gamma}) - \gamma' w\right] + \frac{\tau}{a}(M+\dot{\gamma})(w+1)\frac{(-\tau^2)}{a^2\,\mathrm{sh}\gamma}Nw' +$

$+\frac{\tau}{a}\gamma'(w+1)\frac{\tau^2}{a^2\,\mathrm{sh}\gamma}N\dot{w} = \frac{N\tau^3}{a^3\mathrm{sh}\gamma}\left[(w-1)w'(M+\dot{\gamma}) - \gamma' w(w-1) - w'(w+1)(M+\dot{\gamma}) + \right.$

$\left. + \gamma'\dot{w}(w+1)\right] = \frac{N\tau^3}{a^3\mathrm{sh}\gamma}\left[w(w'M + w'\dot{\gamma} - \gamma'\dot{w} - w'M - w'\dot{\gamma} + \gamma'\dot{w}) - w'M - w'\dot{\gamma} + \right.$

$\left. + \gamma'\dot{w} - w'M - w'\dot{\gamma} + \gamma'\dot{w}\right] = \frac{N\tau^3}{a^3\mathrm{sh}\gamma}\left[-2w'M - 2w'\dot{\gamma} + 2\gamma'\dot{w}\right]$, $(\dot{x}) = \frac{\partial x}{\partial\beta}$, $x' = \frac{\partial x}{\partial\gamma}$

$\vec{A}\vec{B} = -2\frac{N\tau^3}{a^3\mathrm{sh}\gamma}\left[w'M + \frac{\partial w}{\partial\gamma}\frac{\partial\gamma}{\partial\beta} - \frac{\partial\gamma}{\partial\gamma}\frac{\partial w}{\partial\beta}\right]$, $\begin{cases} m_1 + i m_2 = \sin\Theta e^{i[N\varphi+\nu]} \\ m_3 = \cos\Theta = w(\gamma,\beta), \ \nu = \nu(\gamma,\beta) \end{cases}$

$\nu(\gamma,\beta) = M\beta + \gamma(\gamma,\beta)$, $\gamma(\gamma,\beta) \approx$ constant map.

Toroidal coordinates $\vec{r}(\gamma,\beta,\varphi)$: $z = \frac{a\sin\beta}{\tau}$, $\rho = \frac{a\,\mathrm{sh}\gamma}{\tau}$, $\tau = \mathrm{ch}\gamma - \cos\beta$

$\varphi\in[0,2\pi]$, $\beta\in(-\pi,\pi]$, $\gamma\in[0,\infty)$, $h_1 = h_2 = \frac{a}{\tau}$, $h_3 = \frac{a\,\mathrm{sh}\gamma}{\tau}$. $dV = \prod_i h_i\,dq^i$

$dV = \frac{a}{\tau}d\beta\cdot\frac{a}{\tau}d\gamma\cdot\frac{a\,\mathrm{sh}\gamma}{\tau}d\varphi = \frac{a^3\,\mathrm{sh}\gamma}{\tau^3}d\gamma\,d\beta\,d\varphi$

$Q_H = -\frac{1}{(4\pi)^2}\int dV\,\vec{A}\cdot\vec{B} = -\frac{1}{(4\pi)^2}\int_0^\infty d\gamma\int_{-\pi}^{\pi}d\beta\int_0^{2\pi}d\varphi\,\frac{a^3\,\mathrm{sh}\gamma}{\tau^3}(-2)\frac{N\tau^3}{a^3\mathrm{sh}\gamma}\left[M\frac{\partial w}{\partial\gamma} + \frac{\partial w}{\partial\gamma}\frac{\partial\gamma}{\partial\beta} - \frac{\partial\gamma}{\partial\gamma}\frac{\partial w}{\partial\beta}\right] =$

$= \frac{1}{4\pi}N\int_0^\infty d\gamma\int_{-\pi}^{\pi}d\beta\left[M\frac{\partial w}{\partial\gamma} + \frac{\partial w}{\partial\gamma}\frac{\partial\gamma}{\partial\beta} - \frac{\partial\gamma}{\partial\gamma}\frac{\partial w}{\partial\beta}\right]$, $\int_0^\infty d\gamma\frac{\partial w}{\partial\gamma} = w(\infty)_\beta - w(0)_\beta = m_z(\infty,\beta) - m_z(0,\beta) = -1-1 = -2$.

$m_z(0,\beta) = +1$, $m_z(\infty,\beta) = -1$ assumption by Gladikowski '97 - core-like, localized soliton approximation $m_z(\vec{r}\to\infty) = -1$.

$Q_H = \frac{1}{4\pi}NM(-2)\int_{-\pi}^{\pi}d\beta + \frac{N}{4\pi}\int_0^\infty d\gamma\int_{-\pi}^{\pi}d\beta\left[\frac{\partial w}{\partial\gamma}\frac{\partial\gamma}{\partial\beta} - \frac{\partial\gamma}{\partial\gamma}\frac{\partial w}{\partial\beta}\right]$, т.е. Hopf index in toroidal coordinates is

Toroid. hopfion

$Q_H = -NM + \frac{N}{4\pi}\int_0^\infty d\gamma\int_{-\pi}^{\pi}d\beta\left[\frac{\partial w}{\partial\gamma}\frac{\partial\gamma}{\partial\beta} - \frac{\partial\gamma}{\partial\gamma}\frac{\partial w}{\partial\beta}\right]$, $w = m_z = \cos\Theta = w(\gamma,\beta)$

helicity γ из $\nu = M\beta + \gamma$

и только если $\gamma(\gamma,\beta) = \text{const}$, $Q_H = -NM$ is integer for axially sym. Hopfion in toroidal coordinates. $N = q$ - winding number по φ, M - winding number по β.

Если $m_z(\infty,\beta) - m_z(0,\beta) \neq \pm 2$ (non-localized soliton) или если $\gamma \neq \text{const}$, то Q_H не будет целым даже для "toroidal" Hopfions (!) с axial sym.

Сам. Kundu '82 в сферич. координатах $\vec{r}(r,\theta,\varphi)$ $Q_H = \frac{q}{4\pi}\int_0^R dz\int_0^\pi d\theta\,(1-\cos\Theta)\left[\frac{\partial w}{\partial z}\frac{\partial\gamma}{\partial\theta} - \frac{\partial w}{\partial\theta}\frac{\partial\gamma}{\partial z}\right]$

$m_z = w(r,\theta) = \cos\Theta$, $\Phi = q\varphi + \gamma(r,\theta)$. По аналогии с "toroidal" Hopfion можно предполож. $\gamma = q'\varphi + \Gamma(r,\theta)$. $Q_H = \frac{q}{4\pi}\int_0^R dz\int_0^\pi d\theta\,(1-w)\left[\frac{\partial w}{\partial z}q' + \frac{\partial w}{\partial z}\frac{\partial\Gamma}{\partial\theta} - \frac{\partial w}{\partial\theta}\frac{\partial\Gamma}{\partial z}\right]$, q' - integer

Escaneado con CamScanner

$\int_0^R dr\,(1-w)\frac{\partial w}{\partial r} = \int d(w)(1-w) = w - \frac{1}{2}w^2 = w(r,\varphi) - \frac{1}{2}w^2(r,\varphi)\Big|_0^R = \pm 4$ (!?)

$Q_H = qq' + \frac{q}{4\pi}\int_0^R dr \int_0^{\pi} d\varphi\,(1-w)\left[\frac{\partial w}{\partial r}\frac{\partial \Gamma}{\partial \varphi} - \frac{\partial w}{\partial \varphi}\frac{\partial \Gamma}{\partial r}\right]$, $Q_H = qq'$ - integer

при $\Gamma(r,\varphi) = const$

Но если дан "toroidal" Hopfions $\int_0^{\infty} dy \frac{\partial w}{\partial y} = m_2(\infty,\beta) - m_2(0,\beta) = \pm 2$ винтообразно, соответствия $m_2(+\infty) = \mp 1$, $m_2(0) = \pm 1$, то $m_2 - \frac{1}{2}m_2^2\Big|_0^R = \pm 4$ винтообразно. Для BP $m_2 = \cos\Theta(r)$ - равен это число винтообразно, $m_2 \neq f(r)$ и $m_2 - \frac{1}{2}m_2^2\Big|_0^R = 0$, $qq' \cdot 0 = 0$, остается только $\Gamma(r,\varphi)$ - term (или сразу $q' = 0$).

$m_2(R) - m_2(0) + \frac{1}{2}\left[m_2^2(0) - m_2^2(R)\right] = (1) + (2)$, $|(1)| \le 2$,

$|(2)| \le 1/2$ и $\left|m_2 - \frac{1}{2}m_2^2\right|_0^R \le 2\,1/2$.

при $\int dw(1+w)\frac{\partial w}{\partial y} = \left(w + \frac{1}{2}w^2\right)\Big|_0^R = m_2(R) - m_2(0) + \frac{1}{2}\left[m_2^2(R) - m_2^2(0)\right]$

$\left|m_2 + \frac{1}{2}m_2^2\right|_0^R \le 2\,1/2$, не противоречит до "4".

Вывод: 1) для toroidal axial sym. Hopfions $Q_H = \pm qq'$ - is integer если $\Phi(\vec{r}) = q\varphi + q'\beta + const$ и soliton is localized $m_2(y,\beta)\Big|_{y=0}^{y=\infty} = \pm 2$

$\vec{r}(y,\beta,\varphi)$ - toroidal coordinates, φ - azimuthal angle.

2) spherical Hopfions (винтообразные Bloch points) с axial symmetry могут иметь $Q_H \neq 0$ только если $\gamma(r,\varphi)$. $\vec{r}(r,\varphi,\varphi)$ - spherical coordinates.

Note: axial symmetry only for $q=1$ Hopfions (BP).

(7/IX) E-mails to Angel, Julian, Oksana. С п.ч. 40. в San Sebastian/Donostia планируем Bloch point MS.

Hopf index Q_H = product of two winding numbers, по одной угловой это меняется в интервале $[0, 2\pi]$. Из всех orthogonal curvilinear coordinates это возможно две "toroidal" и "bispherical" coordinates. Toroidal Hopfions уже известны → Получаем Bispherical Hopfions.

(8/IX) E-mails to Julian (Anexo I) и BP-paper. BP Simul. Tejo

Sim.-Оказано изменение $\gamma(r,\varphi)$ для sphere $R = 50$ и 100 nm, и $\gamma(0,\varphi)$, $\gamma(R,\varphi)$. $\gamma(0,\varphi)$ - слабо зависит от R.

→ $Q_H = -\frac{1}{4\pi}\int_0^{\pi} d\varphi \sin\varphi\,(1+\cos\varphi)\left[\gamma(R,\varphi) - \gamma(0,\varphi)\right]$ - Eq.(6) MS

$\gamma(0,\varphi) \approx a_1 + b_1\sin\varphi$, $\gamma(R,\varphi) \approx a_2 + b_2\sin\varphi$ - approximation. $B_{1,2} = (\dots)$

$\gamma(R,\varphi) - \gamma(0,\varphi) = (a_2 - a_1) + (b_2 - b_1)\sin\varphi = a + b\sin\varphi$. $a_{1,2} = \gamma(\frac{0}{R}, \frac{\pi}{2})$

$Q_H = -\frac{1}{4\pi}\int_0^{\pi} d\varphi \sin\varphi\,(1+\cos\varphi)(a + b\sin\varphi) = -\frac{1}{4\pi}\int_0^{\pi} d\varphi \sin\varphi\left[a + a\cos\varphi + b\sin\varphi + \frac{b}{2}\sin 2\varphi\right] = -\frac{1}{4\pi}\int_0^{\pi} d\varphi \sin\varphi\left[a + b\sin\varphi\right] = -\frac{1}{4\pi}\left[2a + \frac{\pi}{2}b\right]$

$Q_H(BP) \approx -\frac{1}{2\pi}\left[a + \frac{\pi}{4}b\right]$. Для $R = 100$ nm и $R = 50$ nm $a = a_2(R) - a_1$

$a_{1,2} = \gamma(\frac{0}{R}, \varphi = 0, \pi)$, $\gamma(0, \frac{\pi}{2}) = a_1 + b_1$, $b_1 = \gamma(0, \frac{\pi}{2}) - \gamma(0,0) = 110° - 104° \approx 6°$

$b_2(R=50\text{ nm}) = \gamma(R, \frac{\pi}{8}) - \gamma(R,0) \approx 91° - 75° = 16°$ $b_2 = \gamma(R, \pi/2) - \gamma(R,0)$

$b_2(R=100\text{ nm}) = \gamma(R, \frac{\pi}{2}) - \gamma(R,0) \approx 91° - 72° = 19°$. Очевидно a_1, b_1 от R не зависят.

$a_2(R=50\text{ nm}) = \gamma(R,0) = 75°$; $a_2(R=100\text{ nm}) = 72°$. $a_1 = 104°$

Для $R = 50$ nm: $a(R) + \frac{\pi}{4}b(R) = (75° - 104°) + \frac{\pi}{4}(16° - 6°) = -21,8° = -0,380$ rad

Для $R = 100$ nm: $a(R) + \frac{\pi}{4}b(R) = (72° - 104°) + \frac{\pi}{4}(19° - 6°) = -24,8° = -0,433$ rad

$Q_H(R=50\text{ nm}) = -\frac{1\cdot 2}{4\pi}(-0,369) = 2,94 \cdot 10^{-2}$, $Q_H(R=100\text{ nm}) = -\frac{1\cdot 2}{4\pi}(-0,380) = 3,03 \cdot 10^{-2}$.

Т.е. согласно Simul. F. Tejo $Q_H(R) \approx 0,03 \cdot 2\pi$ слабо растет с R↑.

$l_e(\text{Tejo}) = \sqrt{A/2\pi M_s^2} = \sqrt{2,1\cdot 10^6/2\pi(1,7)^2 10^6} = 10^{-6}\text{ cm}\sqrt{\frac{2,1}{2\pi\cdot 2,89}} = 3,40$ nm.

Escaneado con CamScanner

У тейо $\frac{R}{l_e} = \frac{50\text{ нм}}{3.40\text{ нм}} = 14.7$ и $\frac{R}{l_e} = \frac{100\text{ нм}}{3.40\text{ нм}} = 29.4$. $l_e = \sqrt{\frac{A}{2\pi M_s^2}} = 3.40$ нм.

Pylypovskyi, PhD thesis (ch. 2) 2016 : $E_{ex} = \frac{A}{3}\int d^3\vec{r}\,(-\vec{m}\cdot\nabla^2\vec{m})$, $A' = 2A$, $E_{ex} = \int dV\, A(\nabla\vec{m})^2$

Dzyaloshinskii $l_{ex} = \sqrt{\frac{A'}{4\pi M_s^2}} = \sqrt{\frac{A}{2\pi M_s^2}} = l_e$. $A' = 2A$

$2R = 35a$, $l = 3.95a$, $l - ?$ $E_{ex} = 4\pi M_s^2 \frac{4}{3}\pi R^3 \frac{3}{8\pi}\epsilon \int_{(R^3)} d^3\vec{r}\,(\nabla\vec{m})^2$, $\vec{r}$ in units R.

$E_{ex} = 2\pi M_s^2 R^3 \frac{l^2}{R^2}\int d^3\vec{r}\,(\nabla\vec{m})$ и $E_{ex} = AR\frac{R}{l^2}\int d^3\vec{r}\,(\nabla\vec{m})^2$

$2\pi M_s^2 l^2 R = AR$, $l = \sqrt{\frac{A}{2\pi M_s^2}} = l_e$, если A из $E_{ex} = A\int dV(\nabla\vec{m})^2$ – standard A.

$l_e = 3.95a$, $R = \frac{1}{2}35a$, $\frac{R}{l_e} = \frac{35a}{2\cdot 3.95a} = 4.43 = \frac{1}{\varepsilon^2}$ (Pylyp '12)

$\varepsilon = \frac{1}{(4.43)^2} = 0.051$

У Pylypovskyi PRB'12 и thesis'16 simulations give $\frac{R}{l_e} = 4.43$ вихрь, но $\Delta\gamma = \gamma(0) - \gamma(R) \cong 70° - 40° = 30°$ и не зависит от $\frac{R}{l_e}$. $30° = \pi/6$

$Q_H(\text{Pylyp}) = -\frac{1}{4\pi}\int_0^{\pi} d\theta\, d\varphi \sin\theta\,(1+\cos\theta)\left(=\frac{\pi}{6}\right) = \frac{1}{24}\int_0^{\pi} d\theta\, d\varphi \sin\theta = \frac{1}{12} = 0.083$ (thesis)

23/VIII я вычислил Q_H(Pylyp) по Fig. 5, PRB'2012 as $Q_H = (+2)\cdot(-0.38 \div 0.40)_{10^{-1}} = 0.076 \div 0.080$.

Q_H(Pylyp - thesis'16) = 0.083, Q_H(Pylyp: PRB') ≈ 0.080.

Выводы: 1) $\gamma(r, \theta_e)$ give $\frac{R}{l_e} = 4.43$ Pylyp: PRB'12 хорошо ровно тем $\gamma(r, \theta_e)$-Тейо give $\frac{R}{l_e} = 14.7$ и 29.4 simulated; 2) у Pylyp. '2012 $\gamma(r, \theta = \frac{\pi}{4})$ и $\gamma(r, \frac{\pi}{2})$ почти совпадают, а у Тейо $\gamma(r, \frac{\pi}{4})$ и $\gamma(r, \frac{\pi}{2})$ заметно отличаются, на ~10° give $R = 50$ нм. Тем больше R/l_e, тем больше magnetostat. energy и $\gamma(r)$ должно быть ровно, у Pylyp.: PRB'12 – наоборот.

→ Q_H by Eq. (6) give particular Ansatz $\Theta(\vec{r}) = p\theta + \frac{\pi}{2}(1-p)$, $\Phi(\vec{r}) = \varphi + \gamma(r, \theta_e)$, $\vec{r} = (r, \theta_e, \varphi)$ in spherical coordinates. $\Phi(\vec{r}) = \varphi + \gamma$ хорошо всего верно, а $\Theta(\vec{r}) = \Theta(\theta_e) = p\theta + \frac{\pi}{2}(1-p)$ – слишком упрощенное, плохо надо доказать – проверить by simulations. Q_H Eq. (3.7) – Kominear '96 или Kundu '82 через $\vec{B}(\vec{r})$ не зависит от ансатца, но $\vec{m}(\vec{r}) \to \vec{B}(\vec{r})$ надо знать численно из $E[\vec{m}(\vec{r})] \to \min$.

BP Feldtkeller's ansatz: $\Theta(\vec{r}) = p\theta + \frac{\pi}{2}(1-p)$, $\Phi(\vec{r}) = q\varphi + \gamma$, $\vec{m}(\Theta, \Phi)$.

9/IX Запомнить и послать Anexo I to Angel / Iñaki. E-mails to Oksana / Tejo. Обсуждение BP-paper simul. по сеч. с Олег. Skype to Gleb to hire new PhD student João from Porto. Олег. обещали new features и probrosa – Самарского. Оксана предложила убрать DMI-Bulk $(\vec{m}\cdot\mathrm{rot}\vec{m})$ и surface anisotropy to increase $\gamma(R, \theta_e)$ и Q_H. Обсуждение с Антон new enrolement to UPV/EHU.

10/IX E-mail Олег. / Felipe по Q_H spheres. Разбор new laptops с Антоном.

DMI-bulk $W_D = D(\vec{m}\cdot\mathrm{rot}\vec{m})$, $\mathrm{rot}\vec{m} = \frac{\hat{r}}{r\sin\theta_e}\left[\frac{\partial}{\partial\theta_e}(m_\varphi\sin\theta_e) - \frac{\partial m_{\theta_e}}{\partial\varphi}\right] + \frac{\hat{\theta}}{r}\left[\frac{1}{\sin\theta_e}\frac{\partial m_r}{\partial\varphi} - \frac{\partial}{\partial r}(r m_\varphi)\right] + \frac{\hat{\varphi}}{r}\left[\frac{\partial}{\partial r}(r m_{\theta_e}) - \frac{\partial m_r}{\partial\theta_e}\right] =$

$= \frac{\hat{r}}{r\sin\theta_e}\frac{\partial}{\partial\theta_e}(\sin^2\theta_e\sin\gamma) + \frac{\hat{\theta}}{r}(-1)\frac{\partial}{\partial r}(r\sin\theta_e\sin\gamma) + \frac{\hat{\varphi}}{r}\left[\frac{\partial}{\partial r}(r\sin\theta_e\cos\theta_e(\cos\gamma\cdot p)) - \frac{\partial}{\partial\theta_e}(\sin^2\theta_e\cos\gamma + p\cos^2\theta_e)\right] =$

$= \frac{\hat{r}}{r\sin\theta_e}(\sin 2\theta_e\sin\gamma + \sin^2\theta_e\cos\gamma\frac{\partial\gamma}{\partial\theta_e}) - \hat{\theta}\frac{\sin\theta_e}{r}(\sin\gamma + r\cos\gamma\frac{\partial\gamma}{\partial r}) + \frac{\hat{\varphi}}{r}\left[\sin\theta_e\cos\theta_e(\cos\gamma - p - r\sin\theta_e\frac{\partial\gamma}{\partial r}) - \sin 2\theta_e\cos\gamma - \sin^2\theta_e(-\sin\gamma)\frac{\partial\gamma}{\partial\theta_e} + p\sin 2\theta_e\right]$

$(\vec{m}\cdot\mathrm{rot}\vec{m}) = \frac{1}{r}(\sin^2\theta_e\cos\gamma + p\cos^2\theta_e)(2\cos\theta_e\sin\gamma + \sin\theta_e\cos\gamma\frac{\partial\gamma}{\partial\theta_e}) - \sin\theta_e\cos\theta_e(\cos\gamma - p) \times$

$\times \frac{\sin\theta_e}{r}(\sin\gamma + r\cos\gamma\frac{\partial\gamma}{\partial r}) + \frac{\sin^2\theta_e}{r}\sin\gamma\left[\cos\theta_e(\cos\gamma - p - r\sin\theta_e\frac{\partial\gamma}{\partial r}) + \sin\theta_e\sin\gamma\frac{\partial\gamma}{\partial\theta_e} + (p+\cos\gamma)2\cos\theta_e\right]$

Escaneado con CamScanner

$(\vec{m}\cdot \mathrm{rot}\vec{m}) = \frac{2}{r}\cos\theta\sin\gamma\,(\sin^2\theta\cos\gamma + \rho\cos^2\theta) - \sin\theta\cos\theta\,(\cos\gamma-\rho)\frac{\sin\theta}{r}\sin\gamma +$

$+ \frac{1}{r}\sin^2\theta\sin\gamma\left[\cos\theta\,(\cos\gamma-\rho) + 2(\rho-\cos\gamma)\cos\theta\right] + \frac{1}{r}(\sin^2\theta\cos\gamma+\rho\cos^2\theta)\sin\theta\cos\gamma\frac{\partial\gamma}{\partial\theta} -$

$- \sin\theta\cos\theta\,(\cos\gamma-\rho)\frac{\sin\theta}{r}\cos\gamma\frac{\partial\gamma}{\partial r} + \frac{1}{r}\sin\gamma\sin^3\theta\left[-2\cos\theta\frac{\partial\gamma}{\partial r} + \sin\gamma\frac{\partial\gamma}{\partial\theta}\right] =$

$= \frac{1}{r}\cos\theta\sin^2\theta\sin\gamma\cos\gamma + \frac{2}{r}\rho\cos^3\theta\sin\gamma + \frac{\rho}{r}\sin\gamma\cdot\sin^2\theta\cos\theta +$

$+ \frac{1}{r}\sin^2\theta\cos\theta\sin\gamma\,(\rho-\cos\gamma) + \frac{1}{r}\sin\theta\frac{\partial\gamma}{\partial\theta}\left[\cos\gamma(\sin^2\theta\cos\gamma+\rho\cos^2\theta) +\right.$

$\left. + \sin^2\gamma\sin^2\theta\right] - \sin^2\theta\cos\theta\frac{\partial\gamma}{\partial r}\left[\cos\gamma(\cos\gamma-\rho) + \sin\gamma\sin\theta\right]$

$(\vec{m}\cdot\mathrm{rot}\vec{m}) = \frac{\rho}{r}\sin\gamma\cos\theta\left[2\cos^2\theta + \sin^2\theta + \sin^2\theta\right] + \frac{1}{r}\sin\theta\frac{\partial\gamma}{\partial\theta}\left[\sin^2\theta + \rho\cos\gamma\cos^2\theta\right] -$

$- \sin^2\theta\cos\theta\frac{\partial\gamma}{\partial r}\left[\cos\gamma(\cos\gamma-\rho) + \sin\gamma\sin\theta\right]$

$(\vec{m}\cdot\mathrm{rot}\vec{m}) = \frac{2\rho}{r}\cos\theta\sin\gamma + \frac{1}{r}\sin\theta\left[\sin^2\theta + \rho\cos\gamma\cos^2\theta\right]\frac{\partial\gamma}{\partial\theta} - \sin^2\theta\cos\theta\left[\cos\gamma(\cos\gamma-\rho) + \sin\gamma\sin\theta\right]\times\frac{\partial\gamma}{\partial r}$

$W_D = \int dV\, w_D = D\int_0^R dr\int_0^\pi d\theta\int_0^{2\pi} d\varphi\, r^2\sin\theta\,(\vec{m}\cdot\mathrm{rot}\vec{m}) = 2\pi D\int dr\int d\theta\, r^2\sin\theta\cdot\left[\times(\vec{m}\cdot\mathrm{rot}\vec{m})\right]$

→ **Note**: если $\gamma = \gamma(r,\theta) = \mathrm{const}$, $W_D = 2\pi D\int_0^R dr\int_0^\pi d\theta\, r^2\sin\theta\left(\frac{2\rho}{r}\cos\theta\sin\gamma\right) \sim$

$\sim \int_0^\pi d\theta\sin\theta\cos\theta = 0$, $W_D = 0$ (!). (DMI $\neq 0$ только если $\gamma \neq \mathrm{const}(r,\theta)$

Рассм. ВР $\vec{m}(\vec{r})$ в окрестности полюса $z=0$, $\theta = \frac{\pi}{2} \pm \varepsilon$, $\varepsilon \ll 1$.

$$\begin{cases} m_r(r,\theta) = \sin^2\theta\cos\gamma + \rho\cos^2\theta = \sin^2\left(\frac{\pi}{2}\pm\varepsilon\right)\cos\gamma + \rho\cos^2\left(\frac{\pi}{2}\pm\varepsilon\right) = \cos\gamma + \rho\cdot 0 = \cos\gamma + O(\varepsilon^2) \\ m_\theta = \frac{1}{2}\sin 2\theta\,(\cos\gamma-\rho) = \frac{1}{2}\sin(\pi\pm 2\varepsilon)(\cos\gamma-\rho) = \mp\varepsilon(\cos\gamma-\rho) \\ m_\varphi = \sin\theta\sin\gamma = \sin\left(\frac{\pi}{2}\pm\varepsilon\right)\sin\gamma = \sin\gamma + O(\varepsilon^2) \end{cases}$$

$m_r = \cos\gamma$, $m_\theta = \mp(\cos\gamma-\rho)\varepsilon$, $m_\varphi = \sin\gamma$ вблизи $z=0$, $\theta = \frac{\pi}{2}\pm\varepsilon$, плоскость, m_θ changes sign при переходе через плоскость $z=0$.

В cyl. coordinates $\vec{m} = m_r\hat{r} + m_\theta\hat{\theta} + m_\varphi\hat{\varphi} = m_r(\sin\theta\hat{\rho} + \cos\theta\hat{z}) + m_\theta(\cos\theta\hat{\rho} - \sin\theta\hat{z}) + m_\varphi\hat{\varphi} = (m_r\sin\theta + m_\theta\cos\theta)\hat{\rho} + m_\varphi\hat{\varphi} + (m_r\cos\theta - m_\theta\sin\theta)\hat{z} =$

$= \left|\begin{matrix}\sin(\frac{\pi}{2}\pm\varepsilon) = 1+O(\varepsilon^2) \\ \cos(\frac{\pi}{2}\pm\varepsilon) = \pm\varepsilon(-1)\end{matrix}\right| = \left[\cos\gamma + (\mp\varepsilon)(\cos\gamma-\rho)(\mp\varepsilon)\right]\hat{\rho} + m_\varphi\hat{\varphi} + \left[\cos\gamma(\mp\varepsilon) - (\mp\varepsilon)(\cos\gamma-\rho)\right]\hat{z} = \cos\gamma\hat{\rho} + m_\varphi\hat{\varphi} - \varepsilon(\pm 1)\rho\hat{z}$

ВР $\vec{m}(m_\rho, m_\varphi, m_z)$

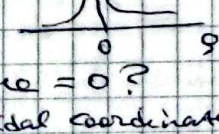
$\vec{m} = \cos\gamma\hat{\rho} + \sin\gamma\hat{\varphi} \mp \varepsilon\rho\hat{z}$, $m_\varphi = \sin\gamma$, $m_\rho = \cos\gamma$, при $\gamma(r,\theta) = \gamma(r,\frac{\pi}{2})$ не меняет знак при пересечении $z=0$ plane. $m_\varphi(z), m_\rho(z)$ – even fn.

$m_z = \mp\varepsilon\hat{z}\rho = |\rho=+1| = \mp\varepsilon\hat{z} = \begin{cases} -\varepsilon, & z<0 \\ +\varepsilon, & z>0 \end{cases}$, $m_z(z)$ – odd func. z – меняет знак при $z=0$. Это особен. similarities тейо $m_z(z)$ – нечет.

$m_z = m_r\cos\theta - m_\theta\sin\theta = \cos\theta\left[\sin^2\theta\cos\gamma + \rho\cos^2\theta\right] - \sin\theta\sin\theta\cos\theta\,(\cos\gamma-\rho) =$

$\vec{m}(\vec{r})$ ВР $= \rho\cos\theta\,(\cos^2\theta + \sin^2\theta) = \rho\cos\theta$, сразу $m_z = \cos\Theta = \cos\left[\rho\theta + \frac{\pi}{2}(1-\rho)\right] = \rho\cos\theta$

$\begin{cases} m_z = \rho\cos\theta, \text{ теремне } z = \mathrm{const}, \ z = r\cos\theta = c, \ m_z(\vec{\rho}) = \rho\frac{c}{\sqrt{c^2+\rho^2}}, \ c = z. \\ m_\varphi = \sin\theta\sin\gamma = \pm\sin\gamma\frac{\rho}{\sqrt{c^2+\rho^2}}, \ m_\rho = \pm\cos\gamma\frac{\rho}{\sqrt{c^2+\rho^2}} \end{cases}$



(11/IX) Gladikowski'97, см. 6/IV. Удовлетворяется ли $\mathrm{div}\vec{B}=0$ для toroidal hopfions? Gyrovector $\vec{G} = \langle\vec{B}\rangle = 0$ или не $= 0$?

Gladikowski: ansatz с $\nu_0 = \gamma(y,\beta) = \mathrm{const}$, $\vec{x} = (y,\beta,\varphi)$ toroidal coordinates.

$B_\varphi = 2\frac{\tau^2}{a^2}M\frac{\partial w}{\partial y}$, $m_z = w(y,\beta)$, $\tau = \cosh y - \cos\beta$, a – toroid. radius.

$B_\beta = -\frac{2\tau^2}{a^2\sinh y}N\frac{\partial w}{\partial y}$, $B_y = 2\frac{\tau^2}{a^2\sinh y}N\frac{\partial w}{\partial\beta}$, $dV = \frac{a^3}{\tau^3}\sinh y\,dy\,d\beta\,d\varphi$.

Escaneado con CamScanner

$\vec{G} = \int dV \vec{B}$, $G_\varphi = \int dV B_\varphi = \int_0^{2\pi} dy \int_{-\pi}^{\pi} d\beta \int_0^{\infty} d\varepsilon \frac{a^3}{\varepsilon^3} \sinh y \cdot 2M \frac{\varepsilon^2}{a^2} \frac{\partial w}{\partial y} = 4\pi M a \times$

$\times \int_0^{2\pi} dy \int_{-\pi}^{\pi} d\beta \frac{\sinh y}{(\cosh y - \cos\beta)} \frac{\partial w}{\partial y}$; $G_\beta = \int dV B_\beta = \int dy \int d\beta d\varepsilon \frac{a^3}{\varepsilon^3} \sinh y \frac{\varepsilon^2}{a^2 \sinh y} \frac{\partial w}{\partial y} =$

$= -4\pi M a \int_0^{2\pi} dy \int_{-\pi}^{\pi} d\beta \frac{1}{(\cosh y - \cos\beta)} \frac{\partial w}{\partial y}$; $G_y = \int dV B_y = \int dy \int d\beta d\varepsilon \frac{a^3}{\varepsilon^3} \sinh y \frac{\varepsilon^2 w}{a^2 \sinh y} \frac{\partial w}{\partial \beta} =$

$= 4\pi M a \int_0^{2\pi} dy \int_{-\pi}^{\pi} d\beta \frac{1}{(\cosh y - \cos\beta)} \frac{\partial w}{\partial \beta}$. $w(0,\beta) = 1$, $w(\infty,\beta) = -1$ assumed B. cond.

$w(y,\beta)$ periodic in β (e.g. on a circle of radius 2π, $M=1$).

$\int_0^{2\pi} dy F(y) \frac{\partial w}{\partial y} = \int dy \frac{\partial w}{\partial y} F(y) = \left| F = w, \; dw = F_y' dy \right| = F(y) w(y,\beta) \Big|_0^{2\pi} - \int w F_y' dy =$

$= F(\infty,\beta)(-1) - F(0,\beta) \cdot 1 - \int dy \, w(y,\beta) \frac{\partial F}{\partial y}$, $F(y,\beta) = \frac{\sinh y}{\cosh y - \cos\beta}$, $F(\infty,\beta) = 1$, $F(0,\beta) = 0$

Generator $\vec{G}$: $G_y = 4\pi M a \int_0^{2\pi} dy \int_{-\pi}^{\pi} d\beta \frac{1}{(\cosh y - \cos\beta)} \frac{\partial w}{\partial \beta}$, $G_\beta = -4\pi M a \int_0^{2\pi} dy \int_{-\pi}^{\pi} d\beta \frac{1}{(\cosh y - \cos\beta)} \frac{\partial w}{\partial y}$

$G_\varphi = 4\pi M a \int_0^{2\pi} dy \int_{-\pi}^{\pi} d\beta \frac{\sinh y}{(\cosh y - \cos\beta)} \frac{\partial w}{\partial y}$. Ικανοποιούν, που $\vec{G}(G_y, G_\beta, G_\varphi) = 0$.

Wang-Braatas '2019 και Göbel '2020 γενίκευσαν, που $\vec{G} = 0$ για hopfion με $Q_H = 1$. Μόνον γ παν hopfions ικανοποιούν ή toroidal hopfion by Gladikowski ?

Wang-Braatas '2019: $m_x = 4 \frac{[2xz - y(r^2-1)]}{(1+r^2)^2}$, $m_y = 4 \frac{[2yz + x(r^2-1)]}{(1+r^2)^2}$, $m_z = 1 - \frac{8(x^2+y^2)}{(1+r^2)^2}$

Σε σύγκριση με [Hietarinta-PLB '99] , όπου δοκ-sample $Q_H = 1$, και $r^2 = x^2+y^2+z^2$.

Göbel '2020: $m_x = \frac{4\xi\rho[\Omega\cos\varphi - (\Omega-1)\sin\varphi]}{(1+\Omega)^2}$, $m_y = \frac{4\xi\rho[\Omega\sin\varphi + (\Omega-1)\cos\varphi]}{(1+\Omega)^2}$, $m_z = 1 - \frac{8\xi^2\rho^2}{(1+\Omega)^2}$,

$\xi = \left[1 + \left(\frac{2z}{L}\right)^2\right] \frac{1}{\cos\left(\frac{\pi\rho}{2L}\right)} \frac{1}{L}$, $\Omega = \operatorname{tg}\left(\frac{\pi z}{L}\right)$, $\Omega = \xi^2\rho^2 + \frac{1}{4}\Omega^2$ για cylinder

with $R = L$ και height L ή cylinder coordinates $\vec{r} = (\rho, \varphi, z)$, σύγκριση με Sutcliffe J. Ph. A 51, 375401 (2018) Math. Theor. , όπου ανσατς για cylinder με $R = \frac{3}{2}L$ και height L (!)

Bound. conditions $\vec{m} = \hat{z}$ at $z = \pm\frac{L}{2}$ και $(\vec{n}\cdot\vec{\nabla})\vec{m} = \frac{2\pi}{L}\vec{n}\times\vec{m}$ - Rohart '2013

bound. conditions at $\rho = R = \frac{3}{2}L$, $L = 4\pi\frac{A}{D}$ ο $E = A(\nabla\vec{m})^2 + D(\vec{m}\cdot\nabla\times\vec{m})$

$\vec{n} = \hat{\rho}$ outward normal: $\frac{\partial\vec{m}}{\partial\rho} = \frac{D}{A}\vec{m}\times\hat{\rho}$. Τα Sutcliffe παίρνουν

αντί $R = \frac{3}{2}L$, L ο Bulk-DMI.

Sutcliffe '18 γενίκευση που δίνει ένα ανσατς (*) για $R = L$, $\vec{m} = \hat{z}$

Βρίσκει και παραγωγούς για $|z| = L/2$ και $\rho = R$, και ποσότητα $Q_H = 1$ (!?)

Εξ ορισμού Q_H. Όταν $|z| = \frac{L}{2}$, $\Omega = \operatorname{tg}\left(\frac{\pi z}{L}\right) \to \infty$, $\Lambda \to 1$ και $\vec{m} \to \hat{z}$, όπως.

Όταν $\rho = \frac{L}{2}$, $z = 0$, $\xi = \frac{1}{L}\frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{L}\sqrt{2}$, $\Omega = 0$, m_x, m_y - κανονικά.

→ Βοηθάει: ανσατς $\vec{m}(x)$ Sutcliffe '18 να εκφράσεις όλες οι γραμμικούς και Fig. 2 → δικό ανσατς δικιάς μας.

Έτσι όταν $\xi = 1$, $\Lambda = \xi^2\rho^2 + \frac{1}{4}\Omega^2 = z^2$, που $m_x = \frac{4[\Omega x - (r^2-1)y]}{(1+r^2)^2}$.

Δικό μας ανσατς Hietarinta '99, όπου $\Omega = z \cdot 2$, $\frac{1}{4}\Omega^2 = z^2$, δικό δικιάς μας.

Και μας $\Omega = \operatorname{tg}\left(\frac{\pi z}{L}\right)$ μας δίνει $\Omega = 2z$, $\lim_{L\to\infty} \operatorname{tg}\left(\frac{\pi z}{L}\right) = \frac{\pi z}{L} = 2z$.

αλλά z σε μονάδες $z_0 = \frac{2}{\pi}L$.

Gladikowski '97: $W(\eta=0,\beta) = 1$, δικό $m_z = 1$ στον θετικό άξονα, $W(\infty,\beta) = -1$, δικό $m_z = -1$ στον αρνητικό άξονα σε κύκλο ακτίνας a στο xy επίπεδο

Ανσατς Hietarinta '99 χρησιμοποιείται στο Wang-Braatas '19 και έχει $m_z = +1$ στον z-άξονα

και $m_z = 1 - \frac{8(x^2+y^2)}{(1+x^2+y^2+z^2)^2} = \left| x^2+y^2 = 1, z = 0 \right| = 1 - \frac{8}{4} = 1 - 2 = -1$ στο

κύκλο $x^2+y^2 = 1$, $z = 0$ στο επίπεδο συμμετρικών σημείων ως προς z.

Escaneado con CamScanner

T-e. $\pi/3$ $(x^2+y^2)\to 0$ u $(x^2+y^2)\to 1$, $z\to 0$ limits Hietarinta'99 ongaar cos+ber. toroidal hopfion By Gladikowski '97.

Hietarinta'2000 : $\vec{m} = \left[\frac{2f}{1+f^2}\cos(m\beta+n\varphi),\ \frac{2f}{1+f^2}\sin(m\beta+n\varphi),\ \frac{f^2-1}{f^2+1}\right]$

in toroidal coordinates $\vec{x}(y,\beta,\varphi)$. $f(y)\to\infty$, $y\to 0$, $m_z\to 1$, oz axis u $f(y)\to 0$, $y\to\infty$, $m_z=-1$ на оф кружност $x^2+y^2=1$, $z=0$, u $Q_H=-nm$ хаос а Bustamante & Burgess Gladikowski '97.

$\vec{m} = \frac{1}{1+|u|^2}\left[u+\bar{u},\ -i(u-\bar{u}),\ |u|^2-1\right]$, $u(y,\beta,\varphi) = f(y)\,e^{i(m\beta+n\varphi)}$

$W(y,\beta) = m_z = \frac{f^2(y)-1}{f^2(y)+1} = F(y)$ solution. $m_x+im_y = \frac{2u}{1+|u|^2}$

$m_z = \frac{f^2-1}{f^2+1} = 1 - \frac{2(x^2+y^2)}{(1+r^2)^2}$. $f^2(y) = \frac{1}{4\rho^2}\left[(1+r^2)^2-4\rho^2\right]$, $\rho^2=x^2+y^2$, $r^2=\rho^2+z^2$

$m_z(\vec{x}) = 1 - \frac{2\rho^2}{(1+r^2)^2} = \cos\Theta(\vec{x}) = W(y,\beta)$ - Gladikowski '97.

Cylindr. coordinates (ρ,z,φ), $\rho = a\frac{\mathrm{sh}y}{\tau}$, $z = \frac{a\sin\beta}{\tau}$, $\varphi=\varphi$, $\tau = \mathrm{ch}y-\cos\beta$

toroid. coordinates (y,β,φ) $r^2 = a^2\frac{\mathrm{ch}y+\cos\beta}{\mathrm{ch}y-\cos\beta} = \frac{a^2}{\tau}(\mathrm{ch}y+\cos\beta)$

$x = a\frac{\mathrm{sh}y\cos\varphi}{\tau}$, $y = \frac{a}{\tau}\mathrm{sh}y\sin\varphi$, $z = \frac{a}{\tau}\sin\beta$.

$r^2 = \rho^2+z^2 = \frac{a^2}{\tau^2}\mathrm{sh}^2y + \frac{a^2}{\tau^2}\sin^2\beta = \frac{a^2}{\tau^2}\left[\mathrm{ch}^2y-\cos^2\beta\right] = \frac{a^2}{\tau}(\mathrm{ch}y+\cos\beta)$

$m_z = W(y,\beta) = 1 - 8\cdot\frac{\mathrm{sh}^2y}{\tau^2}\frac{1}{\left[1+\frac{a^2}{\tau}(\mathrm{ch}y+\cos\beta)\right]^2}$, $a=1$, $\tau+\mathrm{ch}y+\cos\beta = 2\mathrm{ch}y$

$m_z = 1 - 8\frac{\mathrm{sh}^2y}{\tau^2}\frac{1}{\left[\tau+\mathrm{ch}y+\cos\beta\right]^2} = 1 - 8\cdot\frac{\mathrm{sh}^2y}{\left[2\mathrm{ch}y\right]^2} = 1-2\,\mathrm{th}^2y = W(y,\beta)$

$W(0,\beta)=1$, $W(\infty,\beta)=-1$ условия Gladikowski '97 удовл.

$f^2(y) = \frac{1\cdot\tau^2}{4\,\mathrm{sh}^2y}\left[\left\{1+\frac{1}{\tau}(\mathrm{ch}y+\cos\beta)\right\}^2 - 4\frac{\mathrm{sh}^2y}{\tau^2}\right] = \frac{1}{4}\frac{1}{\mathrm{sh}^2y}\left\{\tau+\mathrm{ch}y+\cos\beta\right\}^2 -$

$-1 = \frac{1}{4}\frac{1}{\mathrm{sh}^2y}(2\mathrm{ch}y)^2 - 1 = \mathrm{cth}^2y - 1 = \frac{\mathrm{ch}^2y-\mathrm{sh}^2y}{\mathrm{sh}^2y} = \frac{1}{\mathrm{sh}^2y}$, $\left(f(y)=\frac{1}{\mathrm{sh}y}\right)$

M=1 N=1 $W(y,\beta) = W(y) = 1-2\,\mathrm{th}^2y$, $\frac{\partial W}{\partial\beta}=0$, $\frac{\partial W}{\partial y} = (-2)\,2\,\mathrm{th}y\frac{d(\mathrm{th}y)}{dy} = -4\frac{\mathrm{th}y}{\mathrm{ch}^2y}$ - ф-ла

$G_y\equiv 0$, $G_\beta = -4\pi Na\int_0^\infty dy\int_{-\pi}^{\pi}d\beta\frac{1}{(\mathrm{ch}y-\cos\beta)}(-4)\frac{\mathrm{th}y}{\mathrm{ch}^2y}$, $\mathrm{ch}y\geq 1$. M=1 N=1

$J(y) = \int_{-\pi}^{\pi}d\beta\frac{1}{(\mathrm{ch}y-\cos\beta)} = 2\int_0^{\pi}d\beta\frac{1}{1-a\cos\beta}$ $a = \left|a=\frac{1}{\mathrm{ch}y}\right| = 2a\frac{\pi}{\sqrt{1-a^2}} =$

$= 2\pi\frac{1}{\mathrm{ch}y}\frac{1}{\sqrt{1-1/\mathrm{ch}^2y}} = 2\pi\frac{1}{\sqrt{\mathrm{ch}^2y-1}} = \frac{2\pi}{\mathrm{sh}y}$; $G_\beta = -4\pi Na(-4)\int_0^\infty dy\frac{\mathrm{th}y}{\mathrm{ch}^2y}\cdot\frac{2\pi}{\mathrm{sh}y} =$

$= 216\pi^2Na\int_0^\infty\frac{dy}{\mathrm{ch}^3y} = 32\pi^2Na\cdot\frac{1}{2}B\left(\frac{1}{2},\frac{3}{2}\right) = 3{,}512$, Г.-Роман

$G_\varphi = 4\pi Ma\int_0^\infty dy\int_{-\pi}^{\pi}d\beta\frac{\mathrm{sh}y}{(\mathrm{ch}y-\cos\beta)}\frac{\partial W}{\partial y} = 4\pi Ma\int_0^\infty dy\,\mathrm{sh}y(-4)\frac{\mathrm{th}y}{\mathrm{ch}^2y}\cdot\frac{2\pi}{\mathrm{sh}y} =$

$= -32\pi^2Ma\int_0^\infty dy\frac{\mathrm{th}y}{\mathrm{ch}^2y} = \left|\mathrm{th}y=x,\ \frac{dy}{\mathrm{ch}^2y}=dx\right| = -32\pi^2Ma\int_0^1 dx\,x = -16\pi^2Ma$

Gyrovectors in toroidal coord. $\vec{G} = (G_y, G_\beta, G_\varphi) = 16\pi^2a\left(0, NB\left(\frac{1}{2},\frac{3}{2}\right), -M\right)$ - Тук при M=1, N=1

Вобсе $\vec{G}\neq 0$ хаос degenerate Wang-Brataas '19 и Göbel '2020 give toroidal hopfions. $B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$, $B(1/2,3/2) = \frac{\Gamma(1/2)\Gamma(3/2)}{\Gamma(2)} = \sqrt{\pi}\frac{\sqrt{\pi}}{2} = \frac{\pi}{2}$

$\vec{G} = (G_y, G_\beta, G_\varphi) = 16\pi^2a\left(0, \frac{\pi}{2}N, -M\right)$, a - toroid radius, $x^2+y^2=a^2$.

Wang-Brataas '19 gorcas. $G_z=0$ using (ρ,φ,z) cylindrical coordinates.

$M=q$ - winding no φ, $N=q'$ - winding no β - angle.

Repeiun as $\hat{y},\hat{\beta},\hat{\varphi}$ - toroidal unit vectors и $(\hat{\rho},\hat{\varphi},\hat{z})$ cylindr. orths u u Cartesian $(\hat{x},\hat{y},\hat{z})$ orths. Scale factors $h_1 = h_2 = \frac{a}{\tau}$, $h_3 = \frac{a}{\tau}\mathrm{sh}y$.
(β) (y) (φ)

Cm. G de Vega '78, 12/VIII 2019 G40.

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Use of $\vec{B}$ - Gladikowski '97 to find $\mathrm{div}\vec{B}$ in toroidal coordinates.

$$\mathrm{div}\vec{B} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial\rho}\left(\frac{a^2}{\tau^2}\mathrm{sh}\gamma B_\rho\right) + \frac{\partial}{\partial\gamma}\left(\frac{a^2}{\tau^2}\mathrm{sh}\gamma B_\gamma\right) + \frac{\partial}{\partial\varphi}\left(\frac{a^2}{\tau^2}B_\varphi\right)\right] =$$

$$= \frac{\tau^3}{a^3\mathrm{sh}\gamma}\left[\frac{\partial}{\partial\rho}\left(\frac{a^2}{\tau^2}\mathrm{sh}\gamma\frac{(-2)\tau^2}{a^2\mathrm{sh}\gamma}N\frac{\partial w}{\partial\gamma}\right) + \frac{\partial}{\partial\gamma}\left(\frac{a^2}{\tau^2}\mathrm{sh}\gamma\cdot 0\right) + \frac{\partial}{\partial\varphi}\left(\frac{a^2}{\tau^2}2\frac{\tau^2}{a^2}M\frac{\partial w}{\partial\gamma}\right)\right] =$$

$$= \frac{\tau^3}{a^3\mathrm{sh}\gamma}\left[\frac{\partial}{\partial\rho}\left(-2N\frac{\partial w}{\partial\gamma}\right) + \frac{\partial}{\partial\varphi}\left(2M\frac{\partial w}{\partial\gamma}\right)\right] \equiv 0, \text{ т.к. } \frac{\partial w}{\partial\gamma} = -4\frac{\mathrm{th}\gamma}{\mathrm{ch}^2\gamma} = F(\gamma), \; \frac{\partial w}{\partial\beta}=0$$

$w = w(\gamma)$, $Q_H = 1$

$\mathrm{div}\vec{B} \equiv 0$ для $\vec{B}$-ansatz Gladikowski '97, если $\gamma(\gamma,\beta) = \mathrm{const}$, $v(\gamma,\beta) = M\beta + \mathrm{const}$.

($\mathrm{div}\vec{B}=0$ для $\gamma(\gamma,\beta) = \mathrm{const}$, только если $Q_H = -NM$ - integer.)

– hopfions in toroidal coordinates $\vec{r}(\gamma,\beta,\varphi)$ сопр. $\vec{B}$ – Gladikowski '1997.

Л. 2 нас. Киев: Л.Р. 11 kg + Л. чанта/чене 14 kg – 40€, застрах. рис. мат. 42€

12/IX Среща Гена. Обнаружихме, че в машина сломалка механизъм повдигане/спускане левого перед. стекла.

13/IX Написване BP manuscript. Refs. и figs из simulations by Tejo.

$$w_{ex} = A\left[\frac{2}{\tau^2} + \mathrm{sin}^2\varepsilon(\partial\gamma)^2\right], \; q=1. \quad W_{ex} = \int_R dW\, w_{ex}(\gamma=\mathrm{const}) =$$

BP

$$= \int_0^{2\pi} d\varphi \int_0^{\pi} d\varepsilon \int_0^{\infty} d\tau\, \tau^2 \mathrm{sin}\varepsilon\, A\frac{2}{\tau^2} = 2A\cdot 2\pi\int_0^{\infty} d\tau \int_0^{\pi} d\varepsilon \mathrm{sin}\varepsilon = 8\pi A R, \quad \ell_e^2 = \frac{A}{2\pi M_S^2}$$

Это сов. с E_{ex} (Doring '68) $= A\int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} p^2 \tau^2 d\Omega = 8\pi A$ при $n=1$.

$W_{ex} = \int_0^{\infty} d\tau\, E_{ex} = R\,E_{ex}$ (Döring). $W_{ex} = 8\pi A R$, Рубцов '12 дает $W_{ex} = 4\pi A' R$, но $A' = 2A$ (!?)

$W_m[\vec{m}(\vec{r})] = W_m[\gamma(\vec{r})]$ аналитически не найти.

При helicity $\gamma(\vec{r}) = \mathrm{const}$ $W_m(\gamma) = 4\pi M_S^2 V \frac{2}{15}\left[\frac{3}{4} + p\cos\gamma + 2\cos^2\gamma\right] < W_m \to \min$

при $\cos\gamma_0 = -p/4$. $\mathrm{Min}\, W_m(\gamma_0) = (\ldots)\left[\frac{3}{4} + p\left(-\frac{p}{4}\right) + 2\left(-\frac{p}{4}\right)^2\right] = 4\pi M_S^2 V \frac{2}{15}\cdot\frac{5}{8}$

$W_m(\gamma_0) = \frac{1}{3}\pi M_S^2 V = \frac{1}{3}\pi M_S^2 \frac{4}{3}\pi R^3 = \frac{4}{9}\pi^2 M_S^2 R^3 \sim R^3$.

Ratio $R_e = \frac{W_m(\gamma_0)}{W_{ex}(\gamma_0)} = \frac{4}{9}\frac{\pi^2 M_S^2 R^3}{3\pi A R} = \frac{4}{9}\frac{\pi^2 M_S^2 R^2}{8\pi\cdot 2\pi M_S^2 \ell_e^2} = \frac{1}{36}\left(\frac{R}{\ell_e}\right)^2$

Calc. $R_e(R) = \frac{1}{36}\frac{R^2}{\ell_e^2}$. Дано $R = 175$ nm, $\left(\frac{R}{\ell_e}\right)^2 = \left(\frac{175\,\mathrm{nm}}{3{,}40\,\mathrm{nm}}\right)^2 = 2{,}649\cdot 10^3$

$R_e(175\,\mathrm{nm}) = \frac{1}{36}\cdot 2649 = 73{,}6$. Tejo simulated $R_e(175\,\mathrm{nm}) = \frac{3\cdot 10^{-17}\,\mathrm{J}}{15\cdot 10^{-17}\,\mathrm{J}} = \frac{1}{5}$

→ Giant difference in Re – ($\gamma=\mathrm{const}$) model и Re simulated.

Simulated slope $\frac{E_{ex}}{R} = \frac{15\cdot 10^{-17}\,\mathrm{J}}{175\,\mathrm{nm}} = 8{,}57\cdot 10^{-19}\,\mathrm{J/nm}$.

Calculated slope при $\gamma = \mathrm{const}$ $W_{ex}/R = 8\pi A = 8\pi\cdot 21\cdot p\,\mathrm{J/m} = 8\pi\cdot 21\cdot 10^{-21}\,\frac{\mathrm{J}}{\mathrm{nm}} \Rightarrow$

$\Rightarrow 5{,}278\cdot 10^{-19}\,\mathrm{J/nm}$

Т.е. ($\gamma=\mathrm{const}$) model завишава W_{ex} в $\frac{5{,}278\cdot 10^{-19}}{8{,}57\cdot 10^{-19}} = 61{,}6\cdot 10^{-2} = 0{,}616$ pJ.

Това е ясно, т.к. при $\gamma = \mathrm{const}$ $W_{ex} \to \min$ и чрез $\gamma(r,\varepsilon) \approx \mathrm{const}$ генерира увеличава W_{ex} из-за $(\nabla\gamma)^2$ - term. Corrected, дано и есе.

Calc. $W_m(\gamma_0) = \frac{4}{9}\pi^2 M_S^2 R^3$ при $M_S = 1700$ G, $R = 175$ nm $W_m(\gamma_0) = \frac{4}{9}\pi^2 (1{,}7\cdot 10^3\,\mathrm{G})^2 (1{,}75\cdot 10^{-7})^3$

$\mathrm{nm}^3 = \frac{4}{9}\pi^2 (1{,}7)^2\cdot 10^6\,\mathrm{G}^2 (1{,}75)^3\cdot 10^6\cdot 10^{-21}\,\mathrm{cm}^3 = 67{,}9\cdot 10^{-9}\,\mathrm{erg} = 67{,}9\cdot 10^{-16}\,\mathrm{J} \Rightarrow$

$\Rightarrow = 6{,}79\cdot 10^{-17}\,\mathrm{J}$

Simul. $W_m[\gamma] = 3{,}0\cdot 10^{-17}\,\mathrm{J}$ при $R = 175$ nm.

→ Чрез $\gamma(r,\varepsilon)$ нумбед к увеличение W_m по сравн. с ($\gamma=\mathrm{const}$) model.

Тогава W_{ex} - simulated силно завишена – тъй като функцията $\gamma(r,\varepsilon)$ е по-сложна.

Calc. $\frac{W_{ex}}{R} = 5{,}28$, Sim. $\frac{W_{ex}}{R} = 8{,}57$ (in 10^{-19} J/nm)

Calc. $W(\gamma_0) = 5.278\cdot 10^{-19}\,\frac{\mathrm{J}}{\mathrm{nm}}\cdot 175\,\mathrm{nm} + 6{,}79\cdot 10^{-17}\,\mathrm{J} = (9{,}24 + 6{,}79)\times 10^{-17}\,\mathrm{J} = 16{,}03\cdot 10^{-17}\,\mathrm{J}$

Sim. $W[\gamma(r)] = 8.57\cdot 10^{-19}\,\frac{\mathrm{J}}{\mathrm{nm}}\cdot 175\,\mathrm{nm} + 3{,}0\cdot 10^{-17}\,\mathrm{J} = (15{,}0 + 3{,}0)\times 10^{-17}\,\mathrm{J} = 18{,}0\cdot 10^{-17}\,\mathrm{J}$, $R = 175$ nm

Вот защо $W_{tot}(\gamma_0 = \mathrm{const}) < \mathrm{Sim.}\, W[\gamma(\vec{r})]$. Вероятно, simulations занижават W_{ex} из-за невярното чрез области $r \approx 0$ в sim.

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(14/IX) Edited PoP - Q_H manuscript и послал его в Оксане у Felipe.

• Y. Shnir Topol. Solitons, 2018. $W = \frac{m_x + i m_y}{1 - m_z}$ ⇒ $(m_x, m_y, m_z) = \frac{1}{1+w\bar{w}}\left(w+\bar{w}, i(\bar{w}-w), 1-w\bar{w}\right)$

Toroidal hopfions с Q_H - integer минимизируют Faddeev-Skyrme model energy. The first time $Q_H = 1,2$ soliton solutions were constructed by Gladikowski '1997. (Nicole de Vega '78 pamoue) Hopf map of degree $Q_H = 1$ is

Shnir'18 (*) $m_x = \frac{4}{(1+r^2)^2}\left[2yz + x(1-r^2)\right]$, $m_y = \frac{4}{(1+r^2)^2}\left[-2xz + y(1-r^2)\right]$, $m_z = 1 - \frac{8(x^2+y^2)}{(1+r^2)^2}$

$\vec{m} = \vec{m}(x,y,z)$, $r^2 = x^2+y^2+z^2$.

$\vec{m}(x,y,z)$ can be described as degree-one map $R_3 \to S_3 = C_2$ [Sutcliffe'2007]

$z_1 = 2(x-iy) = 2r\sin\theta e^{i\varphi}$, $z_0 = 1 - r^2 - 2iz = 1 - r^2 - 2ir\cos\theta$

$W = \frac{z_1}{z_0} = \frac{2(x-iy)}{1-r^2-2iz}$, $\vec{m} = (m_x, m_y, m_z) = \frac{1}{1+w\bar{w}}\left(w+\bar{w}, i(\bar{w}-w), 1-w\bar{w}\right)$

$W = \text{const} = C$, $2(x+iy) = (1-r^2-2iz)C$, $2x + C(r^2-1) = +i2(y-zC)$, C - complex

Лучше $|W|^2 = |C|^2$, $|W|^2 = \frac{4(x^2+y^2)}{[(1-r^2)^2+4z^2]} = |C|^2$ are circles на S_3 (x_1, x_2, x_3, x_4) ($\vec{m} = \text{const}$ → circle S_3) $\sum_i x_i^2 = 1$

Note: ansatz (*) отлич. от ansatz Hietarinta'99 заменой $x \to y$ и $m_y \to -m_y$.

(1+ww̄) $m_x + i m_y = 2^0(w + i\bar{w}) + i^1 i(\bar{w} - w) = w + \bar{w} - (\bar{w} - w) = 2w$ - верно.

$1 + m_z = 1 + \frac{(1-w\bar{w})}{(1+w\bar{w})} = \frac{1}{(1+w\bar{w})}\left[(1+w\bar{w}) + (1-w\bar{w})\right] = \frac{2}{(1+w\bar{w})}$

$W = \frac{m_x + i m_y}{1 + m_z} = \frac{2w}{(1+w\bar{w})}\frac{(1+w\bar{w})}{2} = w$, отсюда $W = \frac{m_x + i m_y}{1+m_z}$.

$m_x = \frac{1}{(1+w\bar{w})}(w+\bar{w}) = \frac{1}{(1+w\bar{w})}\left[\frac{2(x-iy)}{1-r^2-2iz} + \frac{(2)(x+iy)}{1-r^2+2iz}\right] = \frac{2}{1+w\bar{w}} \times$

$\times\left[\frac{(x-iy)(1-r^2+2iz)}{(1-r^2)^2+4z^2} + \frac{(x+iy)(1-r^2-2iz)}{(1-r^2)^2+4z^2}\right] = \frac{2}{(1+w\bar{w})}\left[\frac{1}{(1-r^2)^2+4z^2}\right]\left[2x(1-r^2) + 4yz\right]$

$w\bar{w} = \frac{2(x-iy)}{[(1-r^2)-2iz]}\frac{2(x+iy)}{[(1-r^2)+2iz]} = \frac{4(x^2+y^2)}{(1-r^2)^2+4z^2}$

$(1+w\bar{w})\left[(1-r^2)^2+4z^2\right] = \left[1 + \frac{4(x^2+y^2)}{(1-r^2)^2+4z^2}\right]\left[(1-r^2)^2+4z^2\right] = (1-r^2)^2 + 4z^2 + (x^2+y^2)4 =$

$= (1-r^2)^2 + 4r^2 = (1+r^2)^2$

→ $m_x = \frac{4}{(1+r^2)^2}\left[2yz + x(1-r^2)\right]$, что

• Совпадает с (*) Shnir'2018.

т.о. $m_x(x,y,z)$ и/ф $W = \frac{2(x-iy)}{(1-r^2-2iz)}$ верное у Shnir'2018 и неверное у Hietarinta'99.

$m_x = \frac{2}{1+w\bar{w}}\text{Re}\,w$, $m_y = \frac{2}{1+w\bar{w}}\text{Im}\,w = \frac{2}{1+w\bar{w}}\text{Im}\frac{2(x-iy)}{(1-r^2-2iz)}$. $\text{Im}\,w = \frac{1}{2i}(w-\bar{w})$

$m_y = \frac{1}{1+w\bar{w}}\frac{1}{i}(w-\bar{w}) = \frac{1}{(1+w\bar{w})}\frac{1}{i}\frac{2}{[(1-r^2)^2+4z^2]}\left[(x-iy)(1-r^2+2iz) - (x+iy)(1-r^2-2iz)\right] =$

$= \frac{2}{i}\frac{1}{(1+r^2)^2}\left[-iy(1-r^2)\cdot 2 + ixz\cdot 4\right] = \frac{4}{(1+r^2)^2}\left[2zx - y(1-r^2)\right]$

$m_y = \frac{4}{(1+r^2)^2}\left[2zx - y(1-r^2)\right]$ отличается знаком от m_y (*) - Shnir'18.

и/ф $W = \frac{2(x-iy)}{(1-r^2-2iz)}$, $m_x = \frac{4}{(1+r^2)^2}\left[2yz + x(1-r^2)\right]$, $m_y = \frac{4}{(1+r^2)^2}\left[2zx - y(1-r^2)\right]$,

тоже отличается от $\vec{m}(x,y,z)$ Hietarinta'99 by $x \rightleftarrows y$.

$m_z = 1 - \frac{2}{(1+w\bar{w})} = 1 + 2\frac{1}{(1+w\bar{w})} = \frac{1-w\bar{w}}{1+w\bar{w}} = \frac{1 - \frac{4s^2}{(1-r^2)^2+4z^2}}{1 + \frac{4s^2}{(1-r^2)^2+4z^2}} =$

$= \frac{(1-r^2)^2+4z^2-4s^2}{(1-r^2)^2+4z^2+4s^2} = \frac{(1+r^2)^2+4z^2+4s^2-8s^2}{(1+r^2)^2} = 1 - \frac{8(x^2+y^2)}{(1+r^2)^2}$ - верно.

и/ф φ-winding n, а и/ф β-winding m. $\vec{r}(\eta, \beta, \varphi)$ - toroidal coordinates как Gladikowski'97, $\nu(\eta, \beta) = m\beta + \nu_0$, $W = z_1^n / z_0^m$ и $Q_H = -nm$ и/ф $\nu_0 = \text{const}$.

Def. Shnir'2018 $\sin f = \frac{2r}{1+r^2}$, $\cos f = \frac{1-r^2}{1+r^2}$, $f(r)$. $f(\infty)=\pi$, $f(0)=0$



$$\begin{cases} m_x = \frac{8yz}{(1+r^2)^2} + \frac{4x(1-r^2)}{(1+r^2)^2} = 8yz\frac{\sin^2 f}{4r^2} + 2\frac{x}{r}\cdot\frac{(1-r^2)}{(1+r^2)}\frac{r}{(1+r^2)} = \frac{x}{r}\sin 2f + 2\frac{yz}{r^2}\sin^2 f \\ m_y = \frac{8zx}{(1+r^2)^2} - 4y\frac{(1-r^2)}{(1+r^2)^2} = 8zx\frac{\sin^2 f}{4r^2} - 2\frac{y}{r}\frac{(1-r^2)}{(1+r^2)}\frac{2r}{(1+r^2)} = -\frac{y}{r}\sin 2f + 2\frac{zx}{r^2}\sin^2 f \\ m_z = 1 - \frac{8}{(1+r^2)^2}(r^2-z^2) = 1 - \frac{8r^2}{(1+r^2)^2} + 8\frac{z^2}{(1+r^2)^2} = 1 - 2\sin^2 f + 2\frac{z^2}{r^2}\sin^2 f = \cos 2f + 2\frac{z^2}{r^2}\sin^2 f \end{cases}$$

(**) $m_x = \frac{x}{r}\sin 2f + \frac{2yz}{r^2}\sin^2 f$, $m_y = -\frac{y}{r}\sin 2f + 2\frac{zx}{r^2}\sin^2 f$, $m_z = \cos 2f + 2\frac{z^2}{r^2}\sin^2 f$

$m_x^2 + m_y^2 + m_z^2 = \frac{\rho^2}{r^2}\sin^2 2f + 4\frac{z^2}{r^2}\frac{\rho^2}{r^2}\sin^4 f + \cos^2 2f + 4\frac{z^2}{r^2}\cos 2f\sin^2 f + 4\frac{z^4}{r^4}\sin^4 f$

$\sum_i m_i^2 = 1 - \frac{z^2}{r^2}\sin^2 2f + 4\frac{z^2}{r^2}\frac{\rho^2}{r^2}\sin^4 f + 4\frac{z^2}{r^2}\cos 2f\cdot\sin^2 f + 4\frac{z^4}{r^4}\sin^4 f =$

$= 1 + 4\frac{z^2}{r^2}\sin^2 f\left[-\cos^2 f + \frac{\rho^2}{r^2}\sin^2 f + \cos 2f + \frac{z^2}{r^2}\sin^2 f\right] = 1 + (\cdot)[0] = 1$

Shnir 18:
$$\begin{cases} m_x = \frac{x}{r}\sin 2f + \frac{2yz}{r^2}\sin^2 f \\ m_y = -\frac{y}{r}\sin 2f + \frac{2zx}{r^2}\sin^2 f \\ m_z = \cos 2f + \frac{2z^2}{r^2}\sin^2 f \end{cases}$$
$f_s(0)=0$, $f_s(\infty)=\pi$

Liu'PRL'20
$$\begin{cases} m_x = \frac{x}{r}\sin 2f + \frac{2yz}{r^2}\sin^2 f \\ m_y = \frac{y}{r}\sin 2f - \frac{2xz}{r^2}\sin^2 f \\ m_z = \cos 2f + \frac{2z^2}{r^2}\sin^2 f \end{cases}$$
$f_L(0)=\pi$, $f_L(\infty)=0$

β m_y

у Liu

У Liu'2020 изменилось "z" в m_x, m_y.

$\cos f = \frac{r^2-1}{r^2+1}$, $\sin f = \frac{2r}{1+r^2}$

Можно привести $f_S = \pi - f_L$, $\cos f_S = -\cos f_L$, $\sin f_S = \sin f_L$

или $f_S = \pi + f_L$, $\cos f_S = -\cos f_L$, $\sin f_S = -\sin f_L$. При $f_S = \pi + f_L$ $\vec{m}$ - Shnir

Сходится по началам к $\vec{m}$ - Liu, за исключ. m_y (Shnir) $= -m_y$ (Liu).

$\vec{m}(xy) = m_x\hat{x} + m_y\hat{y} = m_x(\cos\varphi\hat{\rho} - \sin\varphi\hat{\varphi}) + m_y(\sin\varphi\hat{\rho} + \cos\varphi\hat{\varphi}) = \hat{\rho}(m_x\cos\varphi + m_y\sin\varphi) + \hat{\varphi}(-m_x\sin\varphi + m_y\cos\varphi)$

m_ρ m_φ

Shnir'18 in cyl. coord. $\vec{r}(\rho,\varphi,z)$
$$\begin{cases} m_x = \frac{\rho}{r}\cos\varphi\sin 2f + 2\frac{z}{r}\sin\varphi\sin^2 f\cdot\frac{\rho}{r} \\ m_y = -\frac{\rho}{r}\sin\varphi\sin 2f + 2\frac{z}{r}\cos\varphi\sin^2 f\frac{\rho}{r} \\ m_z = \cos 2f + 2\frac{z^2}{r^2}\sin^2 f \end{cases}$$

$m_\rho = \frac{\rho}{r}\cos 2\varphi\sin 2f + 2\frac{z}{r}\sin 2\varphi\sin^2 f\frac{\rho}{r}$, $m_\varphi = -\frac{\rho}{r}\sin 2\varphi\sin 2f + 2\frac{z}{r}\cos 2\varphi\sin^2 f\frac{\rho}{r}$ для Shnir'18

$m_\rho = \frac{\rho}{r}\sin 2f$, $m_\varphi = -2\frac{z}{r}\frac{\rho}{r}\sin^2 f$, $m_z = \cos 2f + 2\frac{z^2}{r^2}\sin^2 f$ для Liu'2020.

ВР: $m_\rho = \cos f\frac{\rho}{r}$, $m_\varphi = \sin f\cdot\frac{\rho}{r}$, $m_z = p\frac{z}{r}$, см 10/ix. $r^2 = \rho^2 + z^2$.

→ Ansatz $\vec{m}$ - Liu'20 более правильный: т.к. $\vec{m}(\vec{r}) = \vec{m}(\rho,z)$ не зависит от φ, что-то в $\vec{m}$ - работе Shnir'18 не так. $m_z = 1 - 2\frac{\rho^2}{r^2}\sin^2 f$, как и должно. $\sin f = (\pm 1)\frac{2r}{(1+r^2)}$.

Может надо $W = z_1/z_0 = \frac{2(x+iy)}{(1-r^2+2iz)}$, т.е. $W \to \overline{W}$, при этом $m_y \to -m_y$.

Тонга $Q_H = 1$ hopfion $\vec{m}(\vec{r})$
$$\begin{cases} m_x = \frac{x}{r}\sin 2f + 2\frac{yz}{r^2}\sin^2 f \\ m_y = \frac{y}{r}\sin 2f - 2\frac{xz}{r^2}\sin^2 f \\ m_z = \cos 2f + 2\frac{z^2}{r^2}\sin^2 f \end{cases}$$
- Corrected Liu'20 ansatz, где $\cos f(r) = \frac{r^2-1}{r^2+1}$, $\sin f(r) = \frac{2r}{1+r^2}$, $f(0)=\pi$, $f(\infty)=0$

$W = F(\eta)e^{i(n\varphi + m\beta)}$ больше в тороид. coordinates $\vec{r}(\eta,\beta,\varphi)$

В cyl. coordinates $W = 2\rho e^{i\varphi}\frac{1}{[-\cos f + i\sin f\cos\theta]}\frac{1}{(1+r^2)} = -\frac{2r}{(1+r^2)}\frac{\sin\theta e^{i\varphi}}{[\cos f - i\sin f\cos\theta]}$

где тороид. hopfion.

А какая W для Bloch point ?

$\cos\theta = z/r$, $\sin\theta = \rho/r$.

$\rho = \frac{1}{\tau}\sinh\gamma$, $z = \frac{1}{\tau}\sin\beta$ $(a=1)$, $1 - r^2 + 2iz = 1 - \rho^2 - z^2 + 2iz = 1 - \frac{1}{\tau^2}\sinh^2\gamma - \frac{1}{\tau^2}\sin^2\beta +$

$+ 2i\frac{1}{\tau^2}\sin\beta = \frac{1}{\tau^2}\left[(\cosh\gamma - \cos\beta)^2 - \sinh^2\gamma - \sin^2\beta + 2i\tau\sin\beta\right] = \frac{1}{\tau^2}2\left[\cos^2\beta + i(\cosh\gamma - \cos\beta)\sin\beta -\right.$

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$-ch\gamma\cos\beta] = \frac{2}{\tau^2}[\cos\beta(\cos\beta - i\sin\beta) + ich\gamma\sin\beta - ch\gamma\cos\beta] =$

$= \frac{2}{\tau^2}[\cos\beta e^{-i\beta} - ch\gamma(\cos\beta - i\sin\beta)] = \frac{2}{\tau^2}e^{-i\beta}[\cos\beta - ch\gamma] = -2\frac{1}{\tau}e^{-i\beta}$

$\rightarrow W = \frac{2\rho e^{i\varphi}}{(1-\rho^2-z^2+2iz)} = \frac{2\rho e^{i\varphi}}{(-\frac{2}{\tau})e^{-i\beta}} = -\rho\tau e^{i(\varphi+\beta)} = -sh\gamma e^{i(\varphi+\beta)} = F(\gamma)e^{i(\varphi+\beta)}$ (!)

- в toroid. coordinates $W = F(\gamma)e^{i(\varphi+\beta)}$ упрощалась, $F(\gamma) = -sh\gamma$.

$m_z = \frac{1-|w|^2}{1+|w|^2} = \frac{1-sh^2\gamma}{1+sh^2\gamma} = \frac{1-sh^2\gamma}{ch^2\gamma} = \frac{1}{ch^2\gamma} - th^2\gamma = 1-2th^2\gamma$ (!), также

m_z и ноутбуки 11/12. На 11/12 я один сюрприз. Было $W = f(\gamma)e^{i(m\varphi+n\beta)}$ $\rightarrow W = f(\gamma)e^{i(\gamma,\beta,\varphi)}$

$\vec{m} = \frac{1}{1+|w|^2}(w+\bar{w}, -i(w-\bar{w}), |w|^2-1)$ и $f(\gamma) = 1/sh\gamma$.

Now, according to shnurr's 8-corrected $W(\rho,\varphi,z) = \frac{2\rho e^{i\varphi}}{(1-\rho^2-z^2+2iz)} = F(\gamma)e^{i(\varphi+\beta)} = W_{(\gamma,\beta,\varphi)}$ $(Q_H=1)$

$F(\gamma) = -sh\gamma$, $\vec{m} = \frac{1}{(1+w\bar{w})}(w+\bar{w}, -i(w-\bar{w}), 1-w\bar{w})$, $m_z = \frac{1-|w|^2}{1+|w|^2}$

Однажды $m_z = 1-2th^2\gamma$, if $m_z = \frac{|w|^2-1}{|w|^2+1}$, $|w| = \frac{1}{sh\gamma}$ or $m_z = \frac{1-|w|^2}{1+|w|^2}$, $|w| = sh\gamma$.

m_x, m_y сохраняют свой вид с точностью до знака при $w \to -\frac{1}{w}$, $w \to \frac{1}{w}$

(15/12) E-mails to PRB, Smalyukh, Ruben.

Дис BP: $m_z = p\cos\theta$, $m_\rho = \sin\theta\cos\gamma$, $m_\varphi = \sin\theta\sin\gamma$, $\vec{r}(r,\theta,\varphi)$ сферич. коорд.

$m_x + im_y = m_\rho\cos\varphi - m_\varphi\sin\varphi + i(m_\rho\sin\varphi + m_\varphi\cos\varphi) = m_\rho e^{i\varphi} + im_\varphi(e^{i\varphi}) = (m_\rho + im_\varphi)e^{i\varphi}$

$W = \frac{m_x+im_y}{1+m_z} = \frac{(m_\rho+im_\varphi)e^{i\varphi}}{1+m_z} = \frac{(\sin\theta\cos\gamma + i\sin\theta\sin\gamma)e^{i\varphi}}{(1+m_z)} = \frac{\sin\theta e^{i(\varphi+\gamma)}}{(1+m_z)}$

$W_{BP}(r,\theta,\varphi) = \frac{\sin\theta e^{i(\varphi+\gamma)}}{(1+p\cos\theta)}$ - BP. $W_H = \frac{2z\sin\theta e^{i\varphi}}{(1-r^2+2ir\cos\theta)}$, $m_x+im_y = \frac{2w}{1+w\bar{w}}$

Toroid. hopfion: $m_x = \frac{-sh\gamma\cdot 2}{1+sh^2\gamma}\cos(\varphi+\beta)$, $m_y = \frac{-2sh\gamma}{1+sh^2\gamma}\sin(\varphi+\beta)$, $m_z = 1-2th^2\gamma$

$m_x = -2\frac{sh\gamma}{ch^2\gamma}\cos(\varphi+\beta)$, $m_y = -2\frac{sh\gamma}{ch^2\gamma}\sin(\varphi+\beta)$, $m_z = 1-2th^2\gamma$ $\vec{n}(\gamma,\beta,\varphi)$

$W_H = \frac{2\rho e^{i\varphi}}{[\rho^2+(1+iz)^2]} = -\frac{2\rho e^{i\varphi}}{[\rho^2+(i-z)^2]}$, $W_{BP} = \frac{\rho e^{i(\varphi+\delta)}}{\sqrt{\rho^2+z^2}(1+p\frac{z}{\sqrt{z^2+\rho^2}})}$ BP

Def $\zeta = x+iy$, $|\zeta| = \rho$, $W_{BP} = \frac{\zeta e^{i\gamma}}{[\sqrt{|\zeta|^2+z^2}+pz]} = W_{BP}(\zeta,z)$ - complex function of ζ, depending on z as parameter. При $z=0$, $W_{BP}(\zeta,0) = \frac{\zeta}{|\zeta|}e^{i\gamma}$ - vortex with helicity q, the $0z$ "core". $|W_{BP}(\zeta,0)| = 1$, $m_z = 0$.

При $z \neq 0$ появляется $m_z \neq 0$, $m_z = p\cos\theta = p\frac{z}{\sqrt{z^2+\rho^2}}$, $1+m_z = 1+p\cos\theta \geq 0$.

$|W_{BP}| = \frac{\sin\theta}{(1+p\cos\theta)} = \frac{\rho}{[\sqrt{\rho^2+z^2}+pz]}$, $1+|W|^2 = 1+\frac{\sin^2\theta}{(1+p\cos\theta)^2} = \frac{1}{(1+p\cos\theta)^2} \times$

$\times[1+\cos^2\theta+2p\cos\theta+\sin^2\theta] = \frac{2}{(1+p\cos\theta)}$ BP gyromode frequency

$m_x+im_y = \frac{2W}{1+|W|^2} = \frac{2}{(1+p\cos\theta)}\frac{\sin\theta}{(1+p\cos\theta)}e^{i(\varphi+\gamma)} = \sin\theta e^{i(\varphi+\gamma)}$, $m_\rho+im_\varphi = \sin\theta e^{i\gamma}$

Замена $\zeta \to \zeta - s$ дает сдвиг BP shifted to point $\zeta = s$ in the xy-plane

$\vec{m}(\vec{r}) \to \vec{m}(\vec{r}-\vec{s})$, $s_x+is_y = s$, s_z - медленно меняющаяся переменная.

Можно ли eq. Thiele $\vec{G}\times\dot{\vec{s}} = \frac{\partial W}{\partial \vec{s}}$, $\vec{G}\times\dot{\vec{s}} = G_z\hat{z}\times(\dot{s}_x\hat{x}+\dot{s}_y\hat{y}+\dot{s}_z\hat{z}) = \vec{G}\times\dot{\vec{s}}_{in-plane\ xy}$

$\vec{G}\times\dot{\vec{s}} = \frac{\partial W}{\partial \vec{s}}$ дает $s = s_x+is_y$, и $\frac{\partial W}{\partial s_z} = 0$, Дис sphere $W(\vec{s}) = \frac{1}{2}\varkappa\vec{s}^2R^2$

$\frac{\partial W}{\partial s_z} = \varkappa s_z R^2 = 0$, $s_z = 0$, Oscillations of $s(t)$ in x-y plane. $\omega_G = \frac{\varkappa}{|G|}$ - gyromagn. BP frequency. $\vec{G} = -4\pi pqR\hat{z}$ in units M_s/γ, $|G| = 4\pi\frac{M_s}{\gamma}R$

$\omega_G = \frac{\varkappa R^0}{4\pi\frac{M_s}{\gamma}R} = \frac{\gamma\varkappa R^{-1}}{4\pi M_s}$, т.к. у Дис $\vec{G}\times\dot{\vec{s}} = \frac{1}{R^2}\frac{\partial W}{\partial \vec{s}}$. Найти $\varkappa$ для sphere $W(s) = \frac{1}{2}\varkappa R^2 s^2$.

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(16/IX) С Л. в UPV/EHU: подписан и отдан Антхони factura-Ruben на книги. Зообразование в CFM signed Anexo I to hire Arantxa. С Л. в „Carref. market".

(17/IX) Л. сдала машину в Веазой на ремонт окна. Предв. стат. ~350€. Отправили Anexo I scanned to Cristina, Vicerrectorado UPV/EHU.

(18/IX) Отправили Anexo I to Ana Martikez, будущая Arantxa. Ricardo прислал список вопросов по Mn_2Au DW dynamics paper. Проснулся Julio Camarero и прислал draft „Skyrion" project prorroga application.
→ Cristina: 1) min contract 3 mes, for „researcher within Ministerio project;
2) sept. 21 is deadline Anexos I for October call.

(19-20/IX) Отдых дома. Были с 10. в „Carrefour market". Ужин Оксана и Felipe по проверке BP ansatz by Fehlkeller $\vec{m}(\vec{r})$ и доделка figs. Comments to Cristina об Anexo I.

(21/IX) Corrected FIS 2016 prorroga-application by Camarero (общие расч) и читал Camarero y Valvidares. Запрос сост. гранта FIS 2016 - project.

(22/IX) Принят наш JEMS'2020 abstracts „SW-DW scattering" и „Skyrmions" as orals и Ricardo's „Inertial DW - AF dyn." as poster.
Запрос тарификации 3D textures, от Вани: check topfindex. integer? check $\vec{G}$ to доп 3D $\vec{m}(\vec{r})$ textures.

(23/IX) Запрос Ruben'a to support (letter) his application Royal Soc. Fellowship и напоминание о рек. letter. Ricardo e-mails о stay in Norway (A. Bratkas). Обсуждение вопросов/comments Ricardo по Mn_2Au AF DW dynamics" MS.

(24/IX) С Л. и А. в Donostia. Получен 600€ Santander и отдала А. Испанск в UPV/EHU „Ingeniería Eléctrica" 413.55€; подписан factura на книги Ruben. Беседа с Julian о new Department. С Л. и А. в „Eroski" Urbil. Skype с Глеб: SW dots формулы из „Nanoscale". 1 positive, 1 neg. refer.

(25/IX) Сроно: 1) ответить на comments/questions Ricardo по Mn_2Au DW dyn. MS; 2) recom. letter Ruben - RS Fellowship; 3) prorroga FIS 2016 project; 4) new PhD student; 5) new computers - Min. new project; 6) Block point paper; 7) Ug-modes in thick dots / nanowires; 8) trip Gijon - Santiago.
Написание ответов на вопросы Ricardo по Mn_2Au DW dyn. manuscript.

(26/IX) Домсан ответы Ricardo по Mn_2Au DW и послал to Ricardo y Ruben. Были.

(27/IX) Support letter Ruben для Royal Soc. Ind. Fellowship. Были.

(28/IX) С Л. и А. в Университет. С Л. в „Al Campo" Quartzun. Были.

(29/IX) BP manuscript. Л. здорова с А. машину из ремонта через Зверя. Hotel Gijón.

(30/IX) Meeting с Ruben y Ricardo: расклад или о topfindex index BP разное. С Л. и А. в „Eroski" Urbil. Были. Тарификация принята, ответ о 3D textures.

(1/X) Вопрос Ikerbasque о З/п. Sept. (Santander зарегистрован с 25/IX) Звонил Фархад → сделан и послан ему theory part of MMH - 2020 poster. Написание BP - paper. Visit Santander - Undarreta Bank office. с А (21/X)

(2/X) Переписка с I. Atxutegi: Ikerbasque of Sept. salary. С Л. и А. „Eroski" Urbil. Ricardo прислал revised Mn_2Au DW dyn. paper. 3-model sec. переделать.

(3/X) С Л. „Carrefour", „Lidl" Olaberria. Переделка Mn_2Au paper.

(4/X) Mn_2Au Ист. Ch. 3 - Turov - Antiferromagnets - 2001. M. crystals without центр симметрии

Mn layers ① ② 1 2 (B) ③ ④ 3 4 (A) unit cell Mn_2Au

$$\begin{cases} \vec{L}_A = \vec{M}_1 - \vec{M}_2 - \vec{M}_3 + \vec{M}_4 \\ \vec{L}_B = \vec{M}_1 - \vec{M}_2 + \vec{M}_3 - \vec{M}_4 \\ \vec{L}_C = \vec{M}_1 + \vec{M}_2 - \vec{M}_3 - \vec{M}_4 \\ \vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \vec{M}_4 \end{cases}$$

$\vec{M}_1 \neq \vec{M}_3$, $\vec{M}_4 = \vec{M}_2$, $\vec{M}_1, \vec{M}_4$ - произвольные

Specific Mn_2Au →

$$\begin{cases} \vec{L}_A = 0 \\ \vec{L}_B = 2\vec{M}_3 - 2\vec{M}_2 \\ \vec{L}_C = 0 \\ \vec{M} = 2\vec{M}_3 + 2\vec{M}_2 \end{cases}$$

обычно для центро-аксим. AF

Вектор антиферромагнетизма $\vec{L}_B = \vec{L} \cdot 2 = 2(\vec{M}_3 - \vec{M}_2)$, $\vec{L} = \vec{M}_3 - \vec{M}_2 = \vec{M}_A - \vec{M}_B$

Вектор ферромагнетизма $\vec{M} = 2(\vec{M}_3 + \vec{M}_2) = 2\vec{M}'$, $\vec{M}' = \vec{M}_3 + \vec{M}_2 = \vec{M}_A + \vec{M}_B$

Переделка Mn_2Au DW paper.

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(5/X) E-mails to Iberbasque, Ricardo/Ruben Oyarzabal, Joao Fradet (Porto) PhD
Изучаем FMR – ref. list, paper Dobrovolskiy to Nanoscale.
Разобраться с Δ_0 DW width in Mn_2Au. Согласно 1/VIII для Mn_2Au

$$E_{ex}/u.c. = -2(4J_1+J_2)\vec{m}_A\cdot\vec{m}_B + 2A_3\left(\frac{\partial\vec{m}_A}{\partial x}\right)^2 + 2A_3\left(\frac{\partial\vec{m}_B}{\partial x}\right)^2$$

где $\vec{m}_A$ для $i=3$ Mn layer, $\vec{m}_B$ для $i=2$ Mn layer counted from top

$$\vec{l} = \frac{1}{2}(\vec{m}_A - \vec{m}_B) \quad \vec{m} = \frac{1}{2}(\vec{m}_A + \vec{m}_B) \longrightarrow \left(\frac{E_{ex}}{u.c.}\right. = 4|4J_1+J_2|\vec{m}^2 + 4A_3\left(\frac{\partial\vec{l}}{\partial x}\right)^2, \quad A_3 = J_3a^2.$$

of u.c.
V – unit cell volume

→ 2-model $w_{ex} = \frac{1}{2}A\vec{M}^2 + \frac{1}{2}\alpha\left(\frac{\partial\vec{l}}{\partial x_i}\right)^2 = 2AM_0^2\vec{m}^2 + \frac{1}{2}\alpha M_0^2\left(\frac{\partial\vec{l}}{\partial x_i}\right)^2$

$\vec{M} = 2\vec{m}M_0$, $\vec{L} = 2M_0\vec{l}$. $A = \frac{2}{M_0^2}|4J_1+J_2|$, $\alpha = \frac{8}{M_0^2}A_3 = \frac{8}{M_0^2}J_3a^2$, см. VIII

$J_i \to J_i/V$.

Anisotropy $w_a(\vec{l}) = x_z l_{y'}^2 - x_y l_{z'}^2 = \left|\begin{matrix} y=z', x=x' \\ z=-y' \end{matrix}\right| = x_z l_z^2 - x_y l_y^2$, $x_z, x_y > 0$
$(x'y'z')$

Sin-Gordon для $\theta'(x,t)$ в $x'y'z'$-system, см. 26/VII, 7/VIII.

DW width $\Delta_0^2 = \frac{\alpha M_0^2}{2x_y} = \frac{M_0^2}{2x_y}\frac{8}{M_0^2}J_3a^2 = 4\cdot\frac{J_3a^2}{x_y} = 4\frac{A_3}{x_y}$.

x_y – reduced anisotropy constant || Oy in (xy) – basal plane, только x_y и K_{211} нужны для расчета Δ_0.

Часто м. анизотропу $\frac{W_a}{u.c.} = -K_{211}\sum_i(\vec{m}_i\cdot\hat{y})^2$ related to OAH-eq.

$\vec{m}_i = \vec{m}_A$ для $i=1,3$. $\vec{m}_i = \vec{m}_B$ для $i=2,4$ within Mn_2Au unit cell.

2 f.u. Mn_2Au или 4 Mn atoms per unit cell, 2 атома типа A и два Mn-атома типа B.

$$\frac{W_a}{u.c.} = -2K_{211}\left[(\vec{m}_A\cdot\hat{y})^2 + (\vec{m}_B\cdot\hat{y})^2\right], (m_A^y)^2 + (m_B^y)^2 = 2m_y^2 + 2l_y^2 = 2l_y^2$$

within 2-model, с $\vec{m}^2 \ll \vec{l}^2$, $\vec{m} = \frac{1}{2}(\vec{m}_A + \vec{m}_B)$, $\vec{l} = \frac{1}{2}(\vec{m}_A - \vec{m}_B)$, см. 7/VIII

$\frac{W_a}{u.c.} = -4K_{211}l_y^2 = -x_y l_y^2 = w_a(\vec{l})$, откуда $x_y = 4K_{211}$ и $\Delta_0^2 = \left(\frac{A_3}{K_{211}}\right)$.

$A_3 = J_3a^2/V$ известно. Какое K_{211} (erg/cm^3) выбрать? см. обзор K_{211} 9/VIII

Ricardo references to $K_{211} = 7K_{4111} = 7\cdot1.855\cdot10^{-25}$ J из Otxoa' arXiv' 2020.
В arXiv' Otxoa' 2020 есть $\Delta_0 = 19.8$ nm – simulated. У Ricardo MS $\Delta_0 = 19.7$ nm
Согласно 7/VII тогда $\Delta_0 = a\sqrt{J_3/K_{211}} = 10.77$ nm. Δ_0(calc) заметно $< \Delta_0$(Sim.), $K_4 = 0$
или все же параметры J_1, J_2, J_3, K_{211}. Note: Δ_0(Cal) within 2-model $< \Delta_0$ 11,1

(6/X) Editing Forkhead poster MMM'20 FMR in nanowires. Отдых.
(7/X) подготовка FIS2016 Prorroga appl., новая final version Comerog
Proofs "Dimm modes" Nanoscale paper.
(8/X) Editing/changing Mn_2Au DW manuscript, Бишь.
(9/X) С/А. в „Lidl" „Carrefour" Olaberria. Пос. Клеб Л. пане (23 €) + Лене (11 kg) от 10 min
(10/X – 12/X) Отдых дома. Intro to BP-3D charge paper. Refs. Mn_2Au paper Fiz.
(13/X) План работ с Оксаной. Тел. разговор: 1) напомнить Julio о FIS2016 prorroga application. 2) Simulations of Hopf index by Tk-method (видеи) для BP paper.
3) Thes2 paper domo-shaped dots (обсудили пункты the last version).
Вычитка Proofs AF DW paper Commun. Phys. (equations). Запрос signature Comarog – prorroga.
(14/X) Submitted application на Extension FIS2016 project на полгода.
(15/X) Работа над Mn_2Au DW paper. E-mail to Ricardo/Ruben.
(16/X) Комменты Ruben и Ricardo уравнять DW width $\Delta_0 \propto \sqrt{J_3}$ засчет renorm. J_3, referring to Saporzak' PRB' 2018. Верно ли $J_3 \to (J_3 + 2J_3)$?
(17/X) С/А. в „Carrefour" Diartzen. С/А. и Ю. в San Sebastian, La Perla.
(18/X) С Ю. на 3 нед. горы Izura/Aralar. Бишь. Публикация Δ_0 DW Mn_2Au.
(19/X) Оксана получила грант. $Q_H = 0.04$ C/m^2 (sphere $R = 20$ nm) для Black Point.
WoS – number of citations стало > 7.000. Cost trip отдел для Оксаны.
Cristina (UPV/EHU) подтвердила по PID-2019 есть 6 k€ в 2020. Запрос. Acc. draft Borgano

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Mn_2Au — Mn planes in Mn_2Au unit cell, нумерация как в Ricardo-Overbeaf.
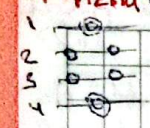


$$E_{ex} = -\sum_{ij} J_{ij}\vec{m}_i\cdot\vec{m}_j = -J_1\vec{m}_1\cdot\vec{m}_2\cdot 4 - \dots = -\sum_{nn';ij} J_{nn'}\vec{m}_n^{\,i}\cdot\vec{m}_{n'}^{\,j}$$

n — layer number, $n = 1,2,3,4$

$$E_{ex} = -\sum_{nn';ij} J_{nn'}\vec{m}_n(x_i)\cdot\vec{m}_{n'}(x_j) = -\sum_{n;ij} J_{nn}\vec{m}_n(x_i)\cdot\vec{m}_n(x_j) -$$

$$-\sum_{n\neq n';ij} J_{nn'}\vec{m}_n(x_i)\vec{m}_{n'}(x_j) = -4J_3\sum_{n;i}\vec{m}_n(x_i)\cdot\vec{m}_n(x_{i+1}) -$$

$$-J_1\sum_{ij}\vec{m}_1(x_i)\cdot\underset{(12)}{\vec{m}_2}(x_j) - J_2\sum_{ij}\vec{m}_2(x_i)\underset{(23)}{\vec{m}_3}(x_j) - J_1\sum_{ij}\underset{(34)}{\vec{m}_3}(x_i)\vec{m}_4(x_j)$$

$$+\tfrac{1}{2}[(14)+(41)]$$



$$\frac{E_{ex}}{u.c.} = -J_3\sum_{n;i}\vec{m}_n(x_i)\vec{m}_n(x_{i+1}) - 2J_1\sum_{i}\vec{m}_1(x_i)\left[\vec{m}_2(x_{i-1})+\vec{m}_2(x_{i+1})\right] -$$

$$-2J_2\sum_i\vec{m}_2(x_i)\cdot\vec{m}_3(x_i) - 2J_1\sum_i\vec{m}_4(x_i)\cdot\left[\vec{m}_3(x_{i-1})+\vec{m}_3(x_{i+1})\right]$$

В одной Mn плоскости ($n=n'$) $x_{i+1}-x_i = a$, вдоль $\parallel Ox$.

В разных Mn плоскостях ($n\neq n'$), $x_i - x_{i+1} = \mp\frac{a}{2}$, $x_{i+1}-x_{i-1}=a$

$$\vec{m}_n(x_{i-1}) + \vec{m}_n(x_{i+1}) = \vec{m}_n(x_i) - \frac{a}{2}\frac{\partial\vec{m}_n}{\partial x}(x_i) + \frac{1}{2}\frac{a^2}{4}\frac{\partial^2\vec{m}_n}{\partial x^2}(x_i) + \vec{m}_n(x_i) +$$

$$+\frac{a}{2}\frac{\partial\vec{m}_n}{\partial x}(x_i) + \frac{a^2}{4}\frac{1}{2}\frac{\partial^2\vec{m}_n}{\partial x^2}(x_i) = 2\vec{m}_n(x_i) + \frac{a^2}{4}\frac{\partial^2\vec{m}_n}{\partial x^2}(x_i)$$

$(n=2,3)$

$$\frac{E_{ex}}{u.c.} = -J_3\sum_{n;i}\vec{m}_n(x_i)\vec{m}_n(x_{i+1}) - 2J_1\cdot 2\sum_i\vec{m}_1(x_i)\vec{m}_2(x_i) - J_1\frac{a^2}{4}\sum_i\vec{m}_1(x_i)\frac{\partial^2\vec{m}_2(x_i)}{\partial x^2}\cdot 2 -$$

$$-2J_2\sum_i\vec{m}_2(x_i)\vec{m}_3(x_i) - 2J_1\cdot 2\sum_i\vec{m}_4(x_i)\vec{m}_3(x_i) - J_1\frac{a^2}{4}\sum_i\vec{m}_4(x_i)\frac{\partial^2\vec{m}_3}{\partial x^2}(x_i)\cdot 2$$

переходя к 2-sublattice AF model $\vec{m}_1 = \vec{m}_3$, $\vec{m}_2 = \vec{m}_4$ (оставим $\vec{m}_2, \vec{m}_3$):

$$\frac{E_{ex}}{u.c.} = -J_3\sum_{n;i}\vec{m}_n(x_i)\cdot\vec{m}_n(x_{i+1}) - 8J_1\sum_i\vec{m}_2(x_i)\cdot\vec{m}_3(x_i) - 2J_2\sum_i\vec{m}_2(x_i)\vec{m}_3(x_i) -$$

$$-J_1\frac{a^2}{2}\sum_i\vec{m}_3(x_i)\frac{\partial^2\vec{m}_2(x_i)}{\partial x^2} - J_1\frac{a^2}{2}\sum_i\vec{m}_2(x_i)\frac{\partial^2\vec{m}_3}{\partial x^2}(x_i)$$

$$\frac{E_{ex}}{u.c.} = -J_3\sum_{n;i}\vec{m}_n(x_i)\vec{m}_n(x_{i+1}) - 2(4J_1+J_2)\sum_i\vec{m}_2(x_i)\cdot\vec{m}_3(x_i) - J_1\frac{a^2}{2}\sum_i\left[\vec{m}_2(x_i)\frac{\partial^2\vec{m}_3}{\partial x^2} + \vec{m}_3\frac{\partial^2\vec{m}_2}{\partial x^2}\right]_{x=x_i}$$

Сомнение 1/VIII: число J_1 bonds/u.c. $= 8$, число J_2 bonds/u.c. $= 2$.

Сом. 1/VIII, 5/VIII для одного Mn-layer $E_{ex}^n = -J_3\sum_i\vec{m}_i\cdot\vec{m}_{i+1} = J_3a^2\left(\frac{\partial\vec{m}_n}{\partial x}\right)^2$

$$\frac{E_{ex}}{u.c.} = -2(4J_1+J_2)\vec{m}_2\cdot\vec{m}_3 + 2A_3\left(\frac{\partial\vec{m}_2}{\partial x}\right)^2 + 2A_3\left(\frac{\partial\vec{m}_3}{\partial x}\right)^2 - \frac{1}{2}J_1a^2\left[\vec{m}_2\frac{\partial^2\vec{m}_3}{\partial x^2} + \vec{m}_3\frac{\partial^2\vec{m}_2}{\partial x^2}\right]$$

$A_3 = J_3a^2$

При подсчете $\sum_i f\left[\vec{m}_2(x_i), \vec{m}_3(x_i)\right]$ учтем 4 связи $\vec{m}_2 - \vec{m}_3$, каждая на 1/4 принад. ячейке to unit cell.

В $E_{ex}/u.c.$ появился new inhom. term $\sim\left[\vec{m}_2\frac{\partial^2\vec{m}_3}{\partial x^2} + \vec{m}_3\frac{\partial^2\vec{m}_2}{\partial x^2}\right] =$

$$= \vec{m}_A\cdot\frac{\partial^2\vec{m}_B}{\partial x^2} + \vec{m}_B\cdot\frac{\partial^2\vec{m}_A}{\partial x^2} = \left|\vec{m}_A = \vec{l}+\vec{m},\ \vec{m}_B = \vec{m}-\vec{l}\right| = (\vec{m}+\vec{l})\frac{\partial^2}{\partial x^2}(\vec{m}-\vec{l}) + (\vec{m}-\vec{l})\frac{\partial^2}{\partial x^2}(\vec{m}+\vec{l}) =$$

$$= \vec{m}\frac{\partial^2\vec{m}}{\partial x^2} - \vec{m}\frac{\partial^2\vec{l}}{\partial x^2} + \vec{l}\cdot\frac{\partial^2\vec{m}}{\partial x^2} - \vec{l}\frac{\partial^2\vec{l}}{\partial x^2} + \vec{m}\frac{\partial^2\vec{m}}{\partial x^2} + \vec{m}\frac{\partial^2\vec{l}}{\partial x^2} - \vec{l}\frac{\partial^2\vec{m}}{\partial x^2} - \vec{l}\frac{\partial^2\vec{l}}{\partial x^2} =$$

$$= 2\vec{m}\cdot\frac{\partial^2\vec{m}}{\partial x^2} - 2\vec{l}\cdot\left(\frac{\partial\vec{l}}{\partial x}\right)^2 \simeq -2\vec{l}\frac{\partial^2\vec{l}}{\partial x^2}.$$

$$\frac{E_{ex}}{u.c.} = 4\left|4J_1+J_2\right|\vec{m}^2 + 4J_3a^2\left(\frac{\partial\vec{l}}{\partial x}\right)^2 - \frac{1}{2}J_1a^2(-2)\vec{l}\cdot\frac{\partial^2\vec{l}}{\partial x^2}.\quad \vec{l}\cdot\frac{\partial^2\vec{l}}{\partial x^2} = -\left(\frac{\partial\vec{l}}{\partial x}\right)^2 - \vec{l}\cdot\frac{\partial\vec{l}}{\partial x}\Big|_a^b = 0.$$

$$\frac{E_{ex}}{u.c.} = 4\left|4J_1+J_2\right|\vec{m}^2 + (4J_3-J_1)a^2\left(\frac{\partial\vec{l}}{\partial x}\right)^2$$

т.е. E_{ex} (1/VIII) верна, но $J_3 \to \left(J_3 - \frac{1}{4}J_1\right)$

$4J_3 \to (4J_3 - J_1)$, $J_1 < 0$, т.е. $J_3 \to J_3\left(1+\frac{1}{4}\frac{|J_1|}{J_3}\right) = J_3\left(1+\frac{1}{4}\frac{396}{115}\right) = 1{,}261\cdot J_3$

При $J_3 \to 1{,}861\,J_3$, $\Delta_0 \propto \sqrt{J_3}$ возрастет в 1,364 раза и с $\sqrt{J_3}\uparrow$ в 1,364 раза.

Нужно учесть вклад inhom. J_1-term

Pierre-Ruben simulated ultrathin film Mn_2Au, где число слоев. Domain. cocogen группа → 3-moded parameters (A,α) группа, учел Δ_0 группа.

20/X Выбор с A. computers с ИФ/ЕНИ-Site, оформление Васюка, переписка с Сюсина у Антухе с new computers. — 2 „Dell" comp.: Сомнения: Too Dw width Mn_2Au надо переснять, уточнить J_1 (Saporishnik' 2018).

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(21/X) Видях деда. Блиц. Editing Mn_2Au DW dyn. manuscript.

(22/X) С Л. и А. в иРУ. Подписах lectures new comps., Ricardo pg. for Mn_2Au Беседа с Julian С Л. в "Lidl" Hernani. E-mail Ricardo/Rubén area, а на per unit cell volume. Нов translation invariance $\parallel Ox$ - direction of DW motion.

(23/X) С Ю. и Julian в Ayuntamiento Irura: информация Julian за Elvira Skorbasque on-line workshop - Nov. 12, registration, enter "room" Acting.

(24/X) С Л. и Ю. "Carrefour" Oiartzun и "Mercadona". Блиц.

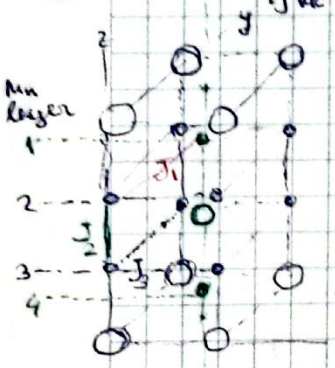
(26/X) E-mail от Зурби: Al. dynamics of ferrimagnets near angular momentum compensation point. Thiele eq. Прочетох Зурби [ref.] Намирах линеен вариант. Деф. на $\vec{G}$? Деф. Kerr in eq's. motion?

Mn_2Au параметър в-model (A,1) от 1/VIII надо перпендикулярно, вадя $\vec{m}_i(x)$ в yz-plane, и уравнявам вторично перпендикулярно на DW propagation direction Ox. Energy density per unit DW square, S

$$E_{ex} = -\frac{1}{S}\sum_{ij} J_{ij}\,\vec{m}_i(x)\vec{m}_j(x), \quad (i,j) - \text{узлов кристал. решетка } Mn_2Au \text{ в yz плоск.}$$

$$E_{ex} = -\sum_{ij,kk'} J_{ij}^{(kk')}\,\vec{m}_i(x_k)\cdot\vec{m}_j(x_{k'}), \quad x_{k+1}-x_k = \frac{a}{2} \text{ за } J_1,\ x_{k+1}-x_k = a \text{ за } J_3$$

k - plane number in x-direction.



• Mn на разбор • Mn върху (001) unit cell
○ Au на разбор ○ Au върху unit cell, Cu. Fig. 1/VIII
•• exchange J_1, •• връзка oz - J_2 •• връзка Ox - J_3
yz - cross-section (или xz) показано на Fig. 1/VIII.

При $x_k = const$, каквито Mn atoms in yz plane имат 2 J_3-bonds и общо J_2 bond. На една rectangular (in-plane yz) unit cell: 4 - Mn atoms, каквито на $\frac{1}{2}$ принадлежат to S_u - testcom. unit cell.

При $x_k = const$ $\quad \frac{E_{ex}}{S_u} = -\sum_{ij} J_{ij}^{(k)}\,\vec{m}_i(x_k)\cdot\vec{m}_j(x_k) = -\frac{2}{2}J_3\vec{m}_2\cdot\vec{m}_2 - J_3\vec{m}_3\cdot\vec{m}_3 - \frac{2}{2}J_2\vec{m}_2\cdot\vec{m}_3 = const - J_2\vec{m}_2\cdot\vec{m}_3, \quad \vec{m}_i = \vec{m}_i(x_k)$

Unit square eqn?

При $x_k \neq x_{k'}$: $\frac{E_{ex}}{S_u} = -2J_1\vec{m}_1\cdot\vec{m}_2(x_k) + \{J_3\} - 2J_1\vec{m}_3(x_k)\cdot m_4(x_k \pm \frac{a}{2}) =$

$(x_{k'} = x_k \pm \frac{a}{2})$ среда $x_k \pm a/2$ дължини на $\vec{m}_2$ и $\vec{m}_3$

$= \{J_3\} - \frac{2}{2^0}J_1\vec{m}_2(x_k)\cdot\vec{m}_1(x_k - \frac{a}{2}) - \frac{2}{2^0}J_1\vec{m}_2(x_k)\cdot\vec{m}_1(x_k+\frac{a}{2}) - \frac{2}{2^0}J_1\vec{m}_3(x_k)\vec{m}_4(x_k-\frac{a}{2}) - \frac{2}{2^0}J_1\vec{m}_3(x_k)\cdot\vec{m}_4(x_k+\frac{a}{2}) = \{J_3\} - 2J_1\vec{m}_2(x_k)\cdot\left[\vec{m}_1(x_k-\frac{a}{2}) + \vec{m}_1(x_k+\frac{a}{2})\right] - 2J_1\vec{m}_3(x_k)\left[\vec{m}_4(x_k-\frac{a}{2}) + m_4(x_k+\frac{a}{2})\right]$.

$\vec{m}(x-\frac{a}{2}) + \vec{m}(x+\frac{a}{2}) = \vec{m}(x) + \frac{\partial\vec{m}}{\partial x}(-\frac{a}{2}) + \frac{\partial^2\vec{m}}{\partial x^2}\frac{1}{2}(\frac{a}{2})^2 + \vec{m}(x) + \frac{\partial\vec{m}}{\partial x}\frac{a}{2} + \frac{1}{2}\frac{\partial^2\vec{m}}{\partial x^2}(\frac{a}{2})^2 = 2\vec{m}(x) + \frac{1}{4}a^2\frac{\partial^2\vec{m}}{\partial x^2}, \quad x = x_k$.

При $x_k \neq x_{k'}$: $\frac{E_{ex}}{S_u} = \{J_3\} - 2J_1\vec{m}_2(x_k)\left[2\vec{m}_1(x_k) + \frac{1}{4}a^2\frac{\partial^2\vec{m}_1}{\partial x^2}\right] - 2J_1\vec{m}_3(x_k)\cdot\left[2\vec{m}_4(x_k) + \frac{1}{4}a^2\frac{\partial^2\vec{m}_4}{\partial x^2}\right]$.

$\{J_3\} = \frac{E_{ex}(J_3)}{S_u} = -J_3\vec{m}_2(x_k)\cdot\vec{m}_2(x_k+a)\frac{2}{2^0} - J_3\vec{m}_3(x_k)\cdot\vec{m}_3(x_k+a)\frac{2}{2^0}$.

$2\vec{m}(x+a) = \vec{m}(x-a) + \vec{m}(x+a) = \vec{m}(x) - \frac{\partial\vec{m}}{\partial x}a + \frac{1}{2}\frac{\partial^2\vec{m}}{\partial x^2}a^2 + \vec{m}(x) + \frac{\partial\vec{m}}{\partial x}a + \frac{1}{2}\frac{\partial^2\vec{m}}{\partial x^2}a^2 = 2\vec{m}(x) + \frac{\partial^2\vec{m}}{\partial x^2}a^2$

$\{J_3\} = -J_3\vec{m}_2(x_k)\frac{2}{2}\left[2\vec{m}_2(x_k) + \frac{\partial^2\vec{m}_2}{\partial x^2}a^2\right] - J_3\vec{m}_3(x_k)\frac{2}{2}\left[2\vec{m}_3(x_k) + \frac{\partial^2\vec{m}_3}{\partial x^2}a^2\right] = -J_3\frac{a^2}{2}\vec{m}_2(x_k)\frac{\partial^2\vec{m}_2}{\partial x^2} - 2J_3\frac{a^2}{2}\vec{m}_3(x_k)\frac{\partial^2\vec{m}_3}{\partial x^2}$

При $x_k \neq x_{k'}$: $\frac{E_{ex}}{S_u} = -2J_3\frac{a^2}{2}\vec{m}_2(x_k)\frac{\partial^2\vec{m}_2}{\partial x^2} - 2J_3\frac{a^2}{2}\vec{m}_3(x_k)\frac{\partial^2\vec{m}_3}{\partial x^2} - 2J_1\vec{m}_2(x_k)\vec{m}_1(x_k) -$

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$-\frac{1}{4}J_1 a^2 \vec{m}_2(x_k)\frac{\partial^2\vec{m}_1}{\partial x^2} - 2J_1\vec{m}_3(x_k)\vec{m}_4(x_k) - \frac{1}{4}a^2 J_1 \vec{m}_3(x_k)\cdot\frac{\partial^2\vec{m}_4}{\partial x^2}$.

Two Sublattice model: $\vec{m}_1 = \vec{m}_3$, $\vec{m}_2 = \vec{m}_4$ (остались $\vec{m}_2, \vec{m}_3$).

$\frac{E_{ex}}{S_u} = -2J_2\vec{m}_2(x_k)\cdot\vec{m}_3(x_k) - 4J_1\vec{m}_2(x_k)\cdot\vec{m}_3(x_k) - \frac{1}{4}a^2J_1\left[\vec{m}_2(x_k)\frac{\partial^2\vec{m}_3}{\partial x^2} + \vec{m}_3(x_k)\frac{\partial^2\vec{m}_2}{\partial x^2}\right] - \frac{1}{2}a^2J_3\left[\vec{m}_2(x_k)\frac{\partial^2\vec{m}_2}{\partial x^2} + \vec{m}_3\frac{\partial^2\vec{m}_3}{\partial x^2}\right]$, $x_k \to x$

$\frac{E_{ex}(x)}{S_u} = -2(4J_1+J_2)\vec{m}_2(x)\vec{m}_3(x) - \frac{1}{4}a^2J_1\left[\vec{m}_2(x)\frac{\partial^2\vec{m}_3}{\partial x^2} + \vec{m}_3(x)\frac{\partial^2\vec{m}_2}{\partial x^2}\right] - \frac{1}{2}a^2J_3\left[\vec{m}_2(x)\frac{\partial^2\vec{m}_2}{\partial x^2} + \vec{m}_3(x)\frac{\partial^2\vec{m}_3}{\partial x^2}\right]$. $\vec{m}_2 = \vec{m}_B$, $\vec{m}_3 = \vec{m}_A$, $\vec{l} = \frac{\vec{m}_A - \vec{m}_B}{2}$, $\vec{m} = \frac{1}{2}(\vec{m}_A + \vec{m}_B)$

Сом. 19/X $\left[\vec{m}_2\frac{\partial^2\vec{m}_3}{\partial x^2} + \vec{m}_3\frac{\partial^2\vec{m}_2}{\partial x^2}\right] \approx -2\vec{l}\cdot\frac{\partial^2\vec{l}}{\partial x^2} = 2\left(\frac{\partial\vec{l}}{\partial x}\right)^2$ т.к. $\int_a^b \frac{\partial}{\partial x}(\dots)$ и $\vec{l}\frac{\partial\vec{l}}{\partial x}\Big|_a^b = 0$.

$\left[\vec{m}_2\frac{\partial^2\vec{m}_2}{\partial x^2} + \vec{m}_3\frac{\partial^2\vec{m}_3}{\partial x^2}\right] = (\vec{m}-\vec{l})\frac{\partial^2}{\partial x^2}(\vec{m}-\vec{l}) + (\vec{m}+\vec{l})\frac{\partial^2}{\partial x^2}(\vec{m}+\vec{l}) = 2\vec{m}\frac{\partial^2\vec{m}}{\partial x^2} + 2\vec{l}\frac{\partial^2\vec{l}}{\partial x^2} \approx 2\vec{l}\frac{\partial^2\vec{l}}{\partial x^2} = -2\left(\frac{\partial\vec{l}}{\partial x}\right)^2$, $\vec{m}_2\cdot\vec{m}_3 = \vec{m}_A\cdot\vec{m}_B = (\vec{m}+\vec{l})(\vec{m}-\vec{l}) =$

$= \vec{m}^2 - \vec{l}^2 = |\vec{m}^2 + \vec{l}^2| = 1 = 1 - 2\vec{l}^2 = 2\vec{m}^2 - 1$

$\frac{E_{ex}(x)}{S_u} = -2(4J_1+J_2)\vec{m}^2(x) - \frac{1}{4}a^2J_1\cdot 2\left(\frac{\partial\vec{l}}{\partial x}\right)^2 + \frac{1}{2}a^2J_3(-2)\left(\frac{\partial\vec{l}}{\partial x}\right)^2$

$\frac{E_{ex}(x)}{S_u} = +2|4J_1+J_2|\vec{m}^2(x) + a^2\left(2J_3 - \frac{1}{2}J_1\right)\left(\frac{\partial\vec{l}}{\partial x}\right)^2$ — AF energy per unit square

$E_{ex}(x) = 2|4J_1+J_2|\vec{m}^2(x) + \left(2J_3 - \frac{1}{2}J_1\right)a^2\left(\frac{\partial\vec{l}}{\partial x}\right)^2$, где $J_i \to J_i/S_u$, $S_u = ac$.

$W_{ex} = \int_a^a dx\, E_{ex}(x)\int dS = \frac{1}{a}\int dV\, E_{ex}(x)$, energy density per unit volume

$\to W_{ex} = \frac{1}{a}E_{ex} = 2|4J_1+J_2|\vec{m}^2 + \left(2J_3 - \frac{1}{2}J_1\right)a^2\left(\frac{\partial\vec{l}}{\partial x}\right)^2$, где $J_i \to J_i/V$, $V = a^2c$.

Сом. Kaneyoshi 1990, Ch. 9: $W_{ex} = \frac{1}{2}A\vec{M}^2 + \frac{1}{8}\alpha\left(\frac{\partial\vec{L}}{\partial x}\right)^2 = 2AM_0^2\vec{m}^2 + \frac{1}{2}\alpha M_0^2\left(\frac{\partial\vec{l}}{\partial x}\right)^2$,

отсюда (см. 5/VIII) $2|4J_1+J_2| = 2AM_0^2$, $A = \frac{1}{M_0^2}|4J_1+J_2|$, $\left(2J_3 - \frac{1}{2}J_1\right)a^2 = \frac{1}{2}\alpha M_0^2$

$\alpha = \frac{2a^2}{M_0^2}\left(J_3 - \frac{1}{2}J_1\right)$. 5/VIII было $A = \frac{2}{M_0^2}|4J_1+J_2|$, $\alpha = \frac{8}{M_0^2}a^2J_3$.

Тогда $c = \gamma M_0\sqrt{\alpha A} = \gamma M_0\sqrt{\frac{|4J_1+J_2|}{M_0^2}\frac{2a^2}{M_0^2}\left(J_3 - \frac{1}{2}J_1\right)} = \frac{\gamma a}{M_0}\sqrt{|4J_1+J_2|\,2\left(J_3 - \frac{1}{2}J_1\right)}$

$J_3 - \frac{1}{2}J_1 = J_3\left(1 + \frac{|J_1|}{2J_3}\right) = J_3\left(1 + \frac{1}{2}\frac{396}{115}\right) = 2.722\cdot J_3$, $\sqrt{J_3} \to 1.650\sqrt{J_3}$.

Было 5/VIII $E_{ex}/V = \dots + 4A_3(\partial\vec{l}/\partial x)^2 = \dots + 4J_3a^2\left(\frac{\partial\vec{l}}{\partial x}\right)^2$. Теперь $W_{ex} = \left(J_3 - \frac{1}{2}J_1\right)a^2\left(\frac{\partial\vec{l}}{\partial x}\right)^2 + \dots$

$J_3 \to \frac{1}{4}\left(J_3 - \frac{1}{2}J_1\right) = \frac{1}{4}2.722\,J_3 = 0.680\,J_3$, $\Delta_0 \sim \sqrt{J_3}$ значит, $\Delta_0^{new} = 0.825\,\Delta_0$ (old)

Old $c = \frac{4\gamma a}{M_0}\sqrt{J_3|4J_1+J_2|}$, $\frac{c_{new}}{c_{old}} = \frac{\gamma a}{M_0}\frac{M_0}{4\gamma a}\frac{\sqrt{|4J_1+J_2|\,2\left(J_3 - \frac{1}{2}J_1\right)}}{\sqrt{|4J_1+J_2|\,J_3}} =$ Сом. 22/X

$= \frac{1}{4}\sqrt{2\left(1 + \frac{|J_1|}{2J_3}\right)} = \frac{1}{4}\sqrt{2\cdot 2.722} = 0.583$. Max Dw velocity увел в $\frac{1}{0.583} = 1.71$ раза. Нов

Проверим расчет $E_{ex}(x)/$ (rectan. unit cell area). Uniform term сом. (1)

Видимо, old (Aug. 1 - Aug. 7) $E_{ex}/V_{u.c.}$ с учетом $\vec{m}_i(x)$ как 19/X верно. Но надо разумеется Mn_2Au-Bulk и Mn_2Au thin film по одному методу сосчет.

22/X получили $E_{ex}/u.c.$ расчет 19/X берем для bulk Mn_2Au. Сделаем это для четырехатомной пленки Mn_2Au с одной unit cell $\parallel z$-thickness. for this film: 2

$E_{ex}/u.c. = -\sum_{n,i\neq j} J_{nn}\,\vec{m}_n(x_i)\cdot\vec{m}_n(x_j) = -J_3\sum_{n,i}\vec{m}_n(x_i)\cdot\vec{m}_n(x_{i+1}) - J_1\sum_{ij}\vec{m}_1(x_i)\vec{m}_2(x_j) -$ (12)

$- J_1\sum_{ij}\vec{m}_3(x_i)\cdot\vec{m}_4(x_j) - J_2\sum_{ij}\vec{m}_2(x_i)\cdot\vec{m}_3(x_j) = \{J_3\} + \{J_1\} - J_2\sum_i\vec{m}_2(x_i)\vec{m}_3(x_i)$,
(34) (23)

т.к. J_2 bonds $(n=1)-(n=4')$ и $(n=1')-(n=4)$ чередуются, т.к. $n=1, n=4'$ Mn слоев нет в thin film. Число J_1 bonds/u.c. и J_3 bonds/u.c. не изменилось для thin film по сравн. с bulk Mn_2Au. Число J_2 bonds/u.c. = 1, для bulk было J_2 bonds/u.c. = 2.

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т.е. в bulk расчет 19/I надо $J_2 \to \frac{J_2}{2}$ из-за перехода к thin film с одной unit cell $\parallel Oz$.

$\frac{E_{ex}}{u.c.}(film) = 4\left|4J_1 + \frac{1}{2}J_2\right|\vec{m}^2 + (4J_3 - J_1)a^2\left(\frac{\partial \vec{l}}{\partial x}\right)^2$, $A = \frac{2}{M_0^2}\left|4J_1 + \frac{1}{2}J_2\right|$, exch.

Stiffness $d = \frac{8}{M_0^2}a^2 J_3' = \frac{8a^2}{M_0^2}J_3\left(1 - \frac{1}{4}\frac{J_1}{J_3}\right) = \frac{8a^2}{M_0^2}J_3\left(1 + \frac{|J_1|}{4J_3}\right)$, $\frac{J_2 \to J_2/2}{J_3 \to J_3' = J_3}$

New $c = \gamma M_0\sqrt{dA} = \gamma M_0\sqrt{\frac{2}{M_0^2}\left|4J_1 + \frac{1}{2}J_2\right|\frac{8a^2}{M_0^2}J_3\left(1 + \frac{|J_1|}{4J_3}\right)} = \frac{4\gamma a}{M_0}\sqrt{\left|4J_1 + \frac{1}{2}J_2\right|J_3\left(1 + \frac{|J_1|}{4J_3}\right)}$

$\frac{c_{new}}{c_{old}} = \frac{c(film)}{c(bulk)} = \sqrt{\frac{\left|4J_1 + \frac{1}{2}J_2\right|J_3\left(1 + \frac{|J_1|}{4J_3}\right)}{|4J_1 + J_2|\,J_3}} = \sqrt{\frac{\left|4J_1 + \frac{1}{2}J_2\right|\left(1 + \frac{|J_1|}{4J_3}\right)}{|4J_1 + J_2|}}$

$\frac{c(film)}{c(bulk)} = \sqrt{\frac{(4\cdot 396 + \frac{1}{2}532)\left(1 + \frac{1}{4}\frac{396}{115}\right)}{(4\cdot 396 + 532)}} = \sqrt{\frac{(1584 + 266)\cdot 1{,}861}{(1584 + 532)}} = \sqrt{\frac{1850\cdot 1{,}861}{2116}} = 1{,}276$

$c(film) = 1{,}276\cdot c(bulk) = 1{,}276\cdot 43{,}0\,\frac{km}{s} = 54{,}85\ km/s$ – max DW velocity thin film.

$\Delta_0(film) = \sqrt{\frac{J_3'}{J_3}}\Delta_0(bulk) = \sqrt{1{,}861}\cdot\Delta_0(bulk) = 1{,}364\cdot 11{,}63\,nm = 15{,}86\ nm$.

теперь для film $c = 54{,}85\ km/s$, $\Delta_0 = 15{,}9$ nm calculated in the 2-model, и max DW velocity $c = 43{,}3$ km/s, $\Delta_0 = 19{,}8$ nm simulated atomistically.

Film = ultrathin Mn_2Au film с thickness = 1 unit cell $\parallel Oz$.

Согласие calc. и simulations не очень хорошее.

Может предположение $\vec{m}_1 = \vec{m}_3$, $\vec{m}_2 = \vec{m}_4$ неточное для $\vec{m}_i(x)$?



Нужно $\begin{cases}\vec{m}_1(x_i) = \frac{1}{2}\left[\vec{m}_3(x_{i-1}) + \vec{m}_3(x_{i+1})\right] \\ \vec{m}_4(x_i) = \frac{1}{2}\left[\vec{m}_2(x_{i-1}) + \vec{m}_2(x_{i+1})\right]\end{cases}$, $x_{i+1} - x_{i-1} = a$

Надо DW $\vec{m}_i(x)$ вдоль Ox.

$\frac{E_{ex}(J_1)}{u.c.} = -2J_1\sum_i \vec{m}_1(x_i)\left[\vec{m}_2(x_{i-1}) + \vec{m}_2(x_{i+1})\right] - 2J_1\sum_i \vec{m}_4(x_i)\left[\vec{m}_3(x_{i-1}) + \vec{m}_3(x_{i+1})\right]$

$\frac{E_{ex}(J_1)}{u.c.} = -\frac{2J_1}{2}\sum_i\left[\vec{m}_3(x_{i-1}) + \vec{m}_3(x_{i+1})\right]\cdot\left[\vec{m}_2(x_{i-1}) + \vec{m}_2(x_{i+1})\right] - \frac{2J_1}{2}\sum_i\left[\vec{m}_2(x_{i-1}) + \vec{m}_2(x_{i+1})\right]\cdot$

$\cdot\left[\vec{m}_3(x_{i-1}) + \vec{m}_3(x_{i+1})\right] = -2J_1\sum_i\left[\vec{m}_2(x_{i-1}) + \vec{m}_2(x_{i+1})\right]\left[\vec{m}_3(x_{i-1}) + \vec{m}_3(x_{i+1})\right] =$

$= -2J_1\sum_i\left[2\vec{m}_2(x_i) + \frac{a^2}{4}\frac{\partial^2\vec{m}_2}{\partial x^2}\right]\left[2\vec{m}_3(x_i) + \frac{a^2}{4}\frac{\partial^2\vec{m}_3}{\partial x^2}\right] = -2J_1\sum_i\left[4\vec{m}_2(x_i)\cdot\vec{m}_3(x_i) + \right.$

$\left. + \frac{a^2}{4}\cdot 2\left\{\vec{m}_3(x_i)\frac{\partial^2\vec{m}_2}{\partial x^2} + \vec{m}_2(x_i)\frac{\partial^2\vec{m}_3}{\partial x^2}\right\} + O(a^4)\right] = -8J_1\sum_i\vec{m}_2(x_i)\vec{m}_3(x_i) -$

$- J_1a^2\sum_i\left[\vec{m}_2(x_i)\frac{\partial^2\vec{m}_3}{\partial x^2} + \vec{m}_3(x_i)\frac{\partial^2\vec{m}_2}{\partial x^2}\right] = -8J_1(\Sigma)^0\vec{m}_2\cdot\vec{m}_3 - J_1a^2\left[\vec{m}_2\frac{\partial^2\vec{m}_3}{\partial x^2} + \vec{m}_3\frac{\partial^2\vec{m}_2}{\partial x^2}\right] =$

$= -8J_1\vec{m}_2\cdot\vec{m}_3 - J_1a^2(-2)\vec{l}\cdot\frac{\partial^2\vec{l}}{\partial x^2} = -8J_1\vec{m}_2\cdot\vec{m}_3 - 2J_1a^2\left(\frac{\partial\vec{l}}{\partial x}\right)^2$.

Для new ansatz перехода к 2-Sublattice AF model $\to \frac{E_{ex}(J_1)}{u.c.} = -8J_1\vec{m}_2\cdot\vec{m}_3 - 2J_1a^2\left(\frac{\partial\vec{l}}{\partial x}\right)^2$.

19/I было $E_{ex}(J_1)/u.c. = -J_1a^2\left(\frac{\partial\vec{l}}{\partial x}\right)^2$, stiffness выросла, Δ_0 – выросла.

$E_{ex}(J_2)/u.c.$ 19/I не изменилось, т.к. они зависят от $\vec{m}_2, \vec{m}_3$ только. $A_3 = J_3a^2$

$\frac{E_{ex}(J_3)}{u.c.} = -J_3\sum_{ni}\vec{m}_n(x_i)\vec{m}_n(x_{i+1}) = +J_3a^2\sum_n\left(\frac{\partial\vec{m}_n}{\partial x}\right)^2 = A_3\left(\frac{\partial\vec{m}_1}{\partial x}\right)^2 + A_3\left(\frac{\partial\vec{m}_2}{\partial x}\right)^2 +$

$+ A_3\left(\frac{\partial\vec{m}_3}{\partial x}\right)^2 + A_3\left(\frac{\partial\vec{m}_4}{\partial x}\right)^2 = A_3\left(\frac{\partial\vec{m}_2}{\partial x}\right)^2 + A_3\left(\frac{\partial\vec{m}_3}{\partial x}\right)^2 + A_3\left[\frac{\partial}{\partial x}\left\{\vec{m}_3 + \frac{a^2}{8}\frac{\partial^2\vec{m}_3}{\partial x^2}\right\}\right]^2 +$

$+ A_3\left[\frac{\partial}{\partial x}\left\{\vec{m}_2 + \frac{a^2}{8}\frac{\partial^2\vec{m}_2}{\partial x^2}\right\}\right]^2 \approx \left|\text{пренебрегая } a^2\frac{\partial^3\vec{m}_n}{\partial x^3}\right| = 2A_3\left(\frac{\partial\vec{m}_2}{\partial x}\right)^2 + 2A_3\left(\frac{\partial\vec{m}_3}{\partial x}\right)^2$,

$E_{ex}(J_3)/u.c. = 4J_3a^2(\partial\vec{l}/\partial x)^2$ совпадает свой вид как 19/I.

Для bulk Mn_2Au теперь $\to E_{ex}/u.c. = 4\left|4J_1 + J_2\right|\vec{m}^2 + 2(2J_3 - J_1)a^2(\partial\vec{l}/\partial x)^2$ с new ansatz.

т.е. $4J_3 \to 4J_3 - 2J_1 = 4J_3\left(1 - \frac{1}{2}J_1/J_3\right) = 4J_3\left(1 + \frac{1}{2}|J_1/J_3|\right)$, $J_3 \to J_3' = J_3\left(1 + \frac{1}{2}\frac{|J_1|}{J_3}\right)$

выросло, Δ_0 выросло. Но увы и с – выросла (по сравн. с $J_3' = J_3 - J_1/4$-case)

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(8/I) Недостаток E_{ex}/V_{uc} подходов с $\vec{m}_i(x)$ от 19/I, 21/I если то, что надо

Описать $\vec{m}_2(x_i)$, $\vec{m}_3(x_i)$, x_i - in unit cell center, где $\vec{m}_2(x)$, $\vec{m}_3(x)$ не существуют. Уран Нам (film состав). $J_2 \to J_2/2$, т.к. число J_2-bonds/u.c. уменьшается с 2 (Bulk) to 1 (Thin film). Uniform $E_{ex} \sim (4J_1 + J_2)$ меняется на $\sim (8J_1 + J_2)$ - film. Исходно (19/I) отмечено, что J_1 - bond (Bulk) короткая,
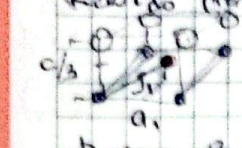


ее длина $\sqrt{\left(\frac{c}{6}\right)^2 + \left(\frac{\sqrt{2}}{2}a\right)^2} = \sqrt{\left(\frac{c}{6}\right)^2 + \frac{1}{2}a^2} = \sqrt{\left(\frac{8.54}{6}\right)^2 + \frac{1}{2}(3.33)^2} =$

$= \sqrt{2.026 + 5.544} = 2.75$ Å где $c = 8.539$ Å, $a = 3.328$ Å

Всего 8 J_1-bonds/u.c. + J_1-bond имеет min длину возможных пересечений. Так $\vec{m}_1$ и $\vec{m}_2$, $\vec{m}_3$ и $\vec{m}_4$ тесно связаны by AF exchange. $\vec{m}_1 \approx -\vec{m}_2$, $\vec{m}_4 \approx -\vec{m}_3$. Model E_{ex} от 19/I дает: Mn_2Au

$$\frac{E_{ex}}{u.c.} = -J_3 \sum_i \vec{m}_n(x_i)\vec{m}_n(x_{i+1}) - 4J_1 \sum_i \vec{m}_1(x_i)\vec{m}_2(x_i) - 4J_1 \sum_i \vec{m}_3(x_i)\cdot\vec{m}_4(x_i) - 2J_2 \sum_i \vec{m}_2(x_i)\vec{m}_3(x_i) -$$

$$- \frac{1}{2}J_1 a^2 \sum_i \vec{m}_1(x_i)\frac{\partial^2 \vec{m}_2}{\partial x^2} - \frac{1}{2}J_1 a^2 \sum_i \vec{m}_4(x_i)\cdot\frac{\partial^2 \vec{m}_3}{\partial x^2}. \quad \text{Через } \vec{m}_1 = -\vec{m}_2,\ \vec{m}_4 = -\vec{m}_3, \text{ то } (\vec{m}_i^2 = 1)$$

$$\frac{E_{ex}}{u.c.} = \{J_3\} - 4J_1 \sum_i (-1) - 4J_1 \sum_i (-1) - 2J_2 \sum_i \vec{m}_2(x_i)\cdot\vec{m}_3(x_i) + \frac{1}{2}J_1 a^2 \sum_i \vec{m}_2 \frac{\partial^2 \vec{m}_2}{\partial x^2} +$$

$$+ \frac{1}{2}J_1 a^2 \sum_i \vec{m}_3(x_i)\frac{\partial^2 \vec{m}_3}{\partial x^2} = \{J_3\} - 2J_2 \vec{m}_2\cdot\vec{m}_3 - \frac{1}{2}J_1 a^2\left[\left(\frac{\partial \vec{m}_2}{\partial x}\right)^2 + \left(\frac{\partial \vec{m}_3}{\partial x}\right)^2\right]$$

$$\left(\frac{\partial \vec{m}_2}{\partial x}\right)^2 + \left(\frac{\partial \vec{m}_3}{\partial x}\right)^2 = \left(\frac{\partial \vec{m}_A}{\partial x}\right)^2 + \left(\frac{\partial \vec{m}_B}{\partial x}\right)^2 = \left| \vec{m}_A = \vec{l} + \vec{m},\ \vec{m}_B = \vec{m} - \vec{l} \right| = \left(\frac{\partial}{\partial x}(\vec{m}+\vec{l})\right)^2 +$$

$$+ \left(\frac{\partial}{\partial x}(\vec{m}-\vec{l})\right)^2 = 2\left(\frac{\partial \vec{m}}{\partial x}\right)^2 + 2\left(\frac{\partial \vec{l}}{\partial x}\right)^2 \approx 2\left(\frac{\partial \vec{l}}{\partial x}\right)^2, \quad \vec{m}_2\cdot\vec{m}_3 = (\vec{m}-\vec{l})(\vec{m}+\vec{l}) = -1 + 2\vec{m}^2$$

$$\to \frac{E_{ex}}{u.c.} = -2J_2 - 2\vec{m}^2 + \{J_3\} - \frac{1}{2}J_1 a^2 \cdot 2\left(\frac{\partial \vec{l}}{\partial x}\right)^2 = -4J_2\vec{m}^2 + (4J_3 - J_1)a^2\left(\frac{\partial \vec{l}}{\partial x}\right)^2 -$$

связано с 19/I, что J_1 вычтено из E_{ex} (uniform). $\Delta_0(J_3') = 15.9$ mm, но $c \sim \sqrt{|J_2|\, J_3\left(1 + \frac{1}{4}\frac{|J_1|}{J_3}\right)}$ очень мало, из-за $|4J_1 + J_2| \to |J_2|$ даже для Bulk. И еще меньше для film где $J_2 \to J_2/2$.

Вывод: assumption $\vec{m}_1 = -\vec{m}_2$, $\vec{m}_4 = -\vec{m}_3$ не подходит, max DW velocity (c) мала.

Su Проверим Su-расчет (на unit cell square S_u) от 26/I.

Так надо на S_u заменить $J_1 \to 2J_1$, $J_3 \to 2J_3$ по числу связей на rectangular unit cell S_u. Но $J_2 \to 2J_2$ не надо, т.к. одна J_2-bond на S_u.

Con. $$\frac{E_{ex}(x)}{S_u} = 2\left|4\cdot 2J_1 + J_2\right|\vec{m}^2(x) + 2a^2\left(J_3 - \frac{1}{2}J_1\right)\left(\frac{\partial \vec{l}}{\partial x}\right)^2 = 2AM_0^2\vec{m}^2 + \frac{1}{2}AM_0^2\left(\frac{\partial \vec{l}}{\partial x}\right)^2$$

$$A = \frac{1}{M_0^2}\left|8J_1 + J_2\right|, \quad \alpha = \frac{4a^2}{M_0^2}\left(J_3 + \frac{1}{2}J_1\right), \quad c = \gamma M_0\sqrt{\alpha A} = \gamma M_0 \frac{2a}{M_0^2}\sqrt{|8J_1 + J_2|\left(J_3 - \frac{1}{2}J_1\right)}$$

DW vel. $$\frac{c_{new}}{c_{old}} = \frac{(2\gamma a/M_0)\sqrt{|8J_1 + J_2|\left(J_3 - \frac{1}{2}J_1\right)}}{(4\gamma a/M_0)\sqrt{|4J_1 + J_2|\, J_3}} = \frac{1}{2}\sqrt{\left(\frac{8J_1 + J_2}{4J_1 + J_2}\right)\left(1 + \frac{1}{2}\frac{|J_1|}{J_3}\right)} = \frac{1}{2}\sqrt{\frac{3700}{2116}\cdot 2.722} = 1.091$$

$$\frac{\Delta_0(new)}{\Delta_0(old)} = \sqrt{\frac{1}{4J_3}\cdot 2\left(J_3 - \frac{1}{2}J_1\right)} = \sqrt{\frac{1}{2}\left(1 + \frac{1}{2}\frac{|J_1|}{J_3}\right)} = 1.167$$

$c_{new} = 1.091 \cdot 42.99$ km/s $= 46.90$ km/s $\quad \Delta_0(new) = 1.167 \cdot 11.63 = 13.57$ nm

→ Как мы сравним, S_u-model (corrected) дает нужную пару (c, Δ_0) для всех расчетов включая наш. J_1 - вывод т.к. при учете J_2 (1-4) и (1-4) bonds не учитывается, что нет различия c, Δ_0 (Bulk) и c, Δ_0 (thin-film).

Имеет смысл рассчитывать E_{ex}/S_u на rectangular unit cell в yz-плоскости с углом $\vec{m}_1(x)$. При этом $A_3 = A_3(J_3, J_1)$ в духе Sapozhnik PRB 2018. Но в Sapozhnik 18 нет J_2, нет model of AF-Mn_2Au.

(29/I) C.D. b-correlation market + Novosa (сегодня Минцифры).

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FiM → A.K. Zvezdin et al. JMMM 509, 166876. Anomalies in the dynamics of ferrimagnets near angular momentum compensation. Lagrangian of FiM со ссылкой на M.D. Davydova et al. JPCM 32, 01LT01 (2019)

$\vec{L} = \vec{M}_1 - \vec{M}_2$ AF vector, Lagrangian $\mathcal{L} = \mathcal{L}[\theta, \varphi, \dot\varphi, \dot\theta, \partial_x\varphi, \partial_x\theta]$.

Walker case $\dot\varphi = 0, \partial_x\varphi = 0 \to$ УД $\frac{\gamma}{\gamma_e^2}\ddot\theta - 2A\frac{\partial^2\theta}{\partial x^2} + \sin 2\theta\,(K_u - \frac{\gamma}{2}H^2) + mH\sin\theta + \frac{\lambda M}{\gamma}\dot\theta = 0$ (*)

$\gamma = \gamma_1 = \frac{M}{|L_{ex}|}$, $m = \frac{1}{2}(M_2 - M_1)$, $M = \frac{1}{2}(M_1 + M_2)$, $\gamma_e = \frac{\gamma M}{(m - m_0)}$, $H = H_z$, K_u – ПМА, xy anis.

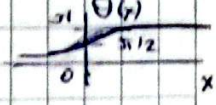
Решение УД: $\theta(x,t) = 2\arctg\exp\left(\frac{x - \dot q t}{\Delta\sqrt{1 - \dot q^2/c^2}}\right)$ domain wall $\dot q \neq 0$, $H \neq 0$?

$\frac{\dot q}{\sqrt{1-\dot q^2/c^2}} = \frac{m}{M}\frac{\gamma}{\lambda}\Delta H$, $\Delta^2 = \frac{A}{(K_u - \frac{\gamma}{2}H^2)}$, $c = \frac{m}{|m - m_0|}$, $c_0 = \gamma\sqrt{\frac{2A}{\chi}}$, $m_0 = M\frac{\gamma_1 - \gamma_2}{\gamma_1 + \gamma_2}$.

$\theta = \theta(y) = 2\arctg e^y$, $\frac{d\theta}{dy} = 2\frac{1}{1+e^{2y}}e^y = \frac{1}{\operatorname{ch} y}$. $x = \ln\operatorname{tg}\left(\frac{\pi}{4} + \frac{\theta}{2}\right) \to \theta = g(x)$

$\theta(x) = 2\arctg e^x - \frac{\pi}{2}$ – Gudermannian. $\theta' = \theta - \pi/2$, $\operatorname{sh} y = \operatorname{tg}\theta' = \operatorname{tg}\left(\theta - \frac{\pi}{2}\right) = -\operatorname{ctg}\theta$

$\operatorname{ch} y = \frac{1}{\cos\theta'} = \frac{1}{\cos(\theta - \pi/2)} = \frac{1}{\sin\theta}$, $\sin\theta = \frac{1}{\operatorname{ch} y}$, $\operatorname{tg}\theta = -\frac{1}{\operatorname{sh} y}$.



$\operatorname{th} y = \sin\theta' = \sin(\theta - \pi/2) = -\cos\theta$.

$\frac{d\theta}{dy} = \frac{1}{\operatorname{ch} y} = \sin\theta$, $\frac{d^2\theta}{dy^2} = \frac{d}{dy}\sin\theta = \cos\theta\frac{d\theta}{dy} = \cos\theta\frac{1}{\operatorname{ch} y} = \sin\theta\cos\theta$

$y = \frac{x - \dot q t}{\Delta\sqrt{1-\dot q^2/c^2}}$, $\ddot\theta = \theta_{yy}(\dot y)^2$. $\dot\theta = \frac{d\theta}{dy}\dot y$, $\ddot\theta = \frac{d^2\theta}{dy^2}(\dot y)^2$, $\frac{d^2\theta}{dx^2} = \frac{d^2\theta}{dy^2}\left(\frac{dy}{dx}\right)^2$

Вращение решение Zvezdin'79 только для $\dot q = v(t) = \text{const}$, steady motion.

$\frac{1}{\Delta^2}\frac{\chi}{\gamma_e^2}\sin\theta\cos\theta\frac{v^2}{(1 - v^2/c^2)} - \frac{2A}{\Delta^2}\sin\theta\cos\theta\frac{1}{(1 - v^2/c^2)} + \sin 2\theta\cdot K + mH\sin\theta + \frac{\lambda}{\gamma}M\frac{\sin\theta(-v)}{\Delta\sqrt{1 - v^2/c^2}} = 0$

Чтобы $v(H)$ из $\frac{v}{\sqrt{1-v^2/c^2}} = \frac{\gamma}{\lambda}\Delta\frac{m}{M}H$. $\frac{1}{\Delta^2(1 - v^2/c^2)}\left[\frac{v^2\chi}{\gamma_e^2} - 2A\right] + 2K = 0$, $\Delta^2 = \frac{A}{K}$.

$K = K_u - \frac{1}{2}\chi H^2$. $\frac{v^2\chi}{\gamma_e^2} - 2A + (1 - v^2/c^2)\,2K\cdot\frac{A}{K} = 0$, $\frac{v^2\chi}{\gamma_e^2} - 2A + \left(1 - \frac{v^2}{c^2}\right)2A = 0$

$\frac{\chi}{\gamma_e^2} = \frac{2A}{c^2}$, $c^2 = \gamma_e^2\frac{2A}{\chi}$, $c = |\gamma_e|\sqrt{\frac{2A}{\chi}}$, $\gamma_e = \frac{\gamma M}{(m - m_0)}$, $c = \frac{m}{|m - m_0|}\gamma\sqrt{\frac{2A}{\chi}}$

→ т.е. решение $\theta(x,t)$ – Zvezdin возможно для FiM, но только при $S(t) = \text{const}$.

// как $\theta(x,t)$ steady-state Walker ($\dot\varphi = 0$) ansatz можно использовать чтобы записать УД для $q(t)$, и $\varphi(t)$? (1D DW motion).

Упр-е для $\varphi(x,t)$ AF DW in MnBi-manuscript linear eq. (*) Zvezdin и подстановка $\varphi(x,t) = 2\arctg\left[\exp\left(\frac{x - q(t)}{\Delta}\right)\right] = \varphi_0[x - q(t)]$ linear ansatz. Заменим переменную $\varphi(x,t) \to q(t)$, сводим УД для $\varphi(x,t)$ к УД для $q(t)$ в рамках ansatz $\varphi(x,t) = \varphi_0(x - q(t))$. $\dot q = v = f(t)$. При $v = \text{const}$ и $v = v(H)$, $\varphi(x,t) = \varphi_0(x - vt)$ есть решение steady-state motion DW.

Конкретный вид $\varphi_0(X) = 2\arctg e^X$ позволяет найти коэффициенты в УД для $q(t)$. Unlike, the coefficients – функционалы от $\varphi_0(X)$.

// Rigid motion approximation/assumption: считаем что $\varphi_0(x - vt) = \varphi(x,t)$ верно для $v = \text{const}$. Сохраняет свой вид для $v \neq \text{const}$ $\varphi(x,t) = \varphi_0(x - q(t))$.

Еще нужно найти $\varphi(x,t) = \varphi_0[x, q(t)]$, где вид $\varphi_0(x, \dot q)$ не очевиден. Аналогично для 2D case $\vec m(\vec r, t)$ или $\vec L(\vec r, t)$. Но для 2D есть gyro term.

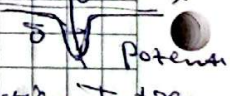
(30/X) С Л. очень холодно Sec. Sec. – 30 Oct. = 36°F, 87°C. Transfer CONSTANTO не работает

// Анализ вывода Iyer Zvezdin'2020, где Ref. to Davydova et al. JPCM'2020 по Davydova'2020 вывод Iyer near $M_1 = M_2$ Comp. point и без non-uniform exchange, как же derived Iyer Eq. (3) Zvezdin'2020 near $\frac{M_1}{\gamma_1} = \frac{M_2}{\gamma_2}$ (angular moment comp. point), обоснование? $\mathcal{H}_{ex}$ – non-uniform должна учитываться при теоретическом вывод. Вблизи $M_1/\gamma_1 = M_2/\gamma_2$ point $|\vec M| \ll |\vec L|$ AF approximation не работает и FiM dynamics не сводится к $\vec L$-dynamics с $\chi_{eff} = \chi_e$ (!)

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Получил запрос от PhD applicant Octave Duros и переписал Julian, Ricardo, перенаправил с Вопросом по Lagrangian FiM - Zvezdin.
Участие в online Perspin conf. (Italy): talks S. Tacchi, L. Aballe, S. Bonetti (Inertial Spin dynamics, no Wergrave; tMi-term LLG. K. Mecej, Nat. Phys. 22)
(13 1/X) С А.и Ю. на Замаг. порту за грибами (Ромашки, Опека). Камень.
(1/XI) плоские model MnzAu - lattice, Вишу. Ортрет.
(2/XI) Papers Zvezdin-IMMM'2020 и Davydova JPCM'02 dynamics FiM, Jeff.
Skype разговор с Игорем - Conversation Skype с Pawel, Maciej, Igor no edge-SW manuscript.
(3/XI) Долгое обсуждение Jeff - Ferrimagnet с Анатолием Звездиным: FiM
1) steady state (χ-$\dot{q}$), rigid DW assumption при выводе YD для $q(t)$, $\Psi(t)$;
2) Почему пренебрежим те, ор terms (отклонение от AF state) при расч. ε и β из Jeff (Звездин. они малы при $ka \ll 1$)
3) $\gamma \to \gamma_{eff}$ в IMMM'20 без обоснования. Использовали и $|\vec{M}| \ll |\vec{L}|$, которое не оправдывается near angular mom. compensation point (не только $m \ll M$).
Рассказал Анатолию о расчетах Hopf index of the Bloch points. Предложил Игорю написать обзор paper по eq. thiele-sky motion in FiM. Email to Duros.
Игорь прислал slides SW - Pawel. Skype meeting: Friday, Nov. 6, 11:30 AM.
Звездин прислал paper A.K. Zvezdin et al. PIT 18 367 (1976) (многослойка) и "Распространение света вдоль DW" и Давыдова PRB 100, 064409 (2019) MPD H-T of FiM near compensation point.

Zvezdin'76. Решение ($n=0$) для Электр.поля $E_y(x)$, ОХ $\perp$ DW, ОZ $\parallel \vec{M}_0$ в DW,
$E_y(x) = E_0 [\sin\theta(x)]^{-s}$, $\sin\theta(x) = \frac{1}{ch(x/\delta)}$. Для n = целая часть $\frac{1}{2}(s - iq\delta)$
$E_y''(x) + \left\{ q^2 + \frac{s(s+1)}{\delta^2 ch^2 \frac{x}{\delta}} \right\} E_y(x) = 0$ - ур-ие для potential Пешля-Теллера
при $q^2 < 0$, ур-ие имеет n - связанных (локализованных на DW) состояний
См. З. Флюгге. Задачи по квант. механике, т. 1, 106 (1974). Частный случай ур-ия Пэшля-Теллера с $s=1$ ($q=-i$?) решил Winter, PR 1961.
S. Flügge, Practical Quantum Mechanics, Springer-Verlag, Berlin, 1999, v. 1, problem 39.
ур-ие $u'' + \left\{ k^2 + \frac{d^2 \lambda(\lambda-1)}{ch^2 dx} \right\} u(x) = 0$, $k^2 = \frac{2m}{\hbar} E$ - energy, $d^{-1} = \delta$ - well width
ур-ие имеет два fund. solutions рег. $u_e(x)$ и нерег. $u_0(x)$
Вопрос: когда интересен фундаментально.



Asymptotic form $|dx| \gg 1$, $u(x) = \begin{cases} e^{ikx} + Re^{-ikx}, & x<0 \\ Te^{ikx}, & x>0 \end{cases}$ R - reflection, T - transmit. coefficients
$|T|^2 = \sin^2(\varphi_e - \varphi_0)$, $|R|^2 = \cos^2(\varphi_e - \varphi_0)$, где $\varphi_e - \varphi_0 = \operatorname{arctg}\left[\frac{sh\,\pi k\delta}{\sin \pi\lambda}\right]$
Если λ - integer, $\sin\pi\lambda = 0$, $\Delta\varphi = \pi/2$, $|T|^2 = 1$, $|R|^2 = 0$ no reflection.
Bound (localized) solutions: $k = i\varkappa$, $E = -\frac{\hbar^2 \varkappa^2}{2m}$, eigenvalues $E_n = -\frac{\hbar^2}{2m}\varkappa^2 (\lambda - 1 - n)^2$
For λ - integer, there is always one eigenvalue $E_n = 0$ at $n = \lambda - 1$. $n \le \lambda - 1$
λ - целочисл. квант. число, $\delta = 1/\alpha$ - ширина DW. $E_n(\lambda)$, n - bound solutions.
Non-localized solution при $\frac{|x|}{\delta} \gg 1$ - plane waves $e^{\pm ikx}$ с $E_n(\lambda) \le 0$ (!) Pöschl-Teller potential [1933].
(4/XI) Оформление travel certificate Ирина - Donostia (Ikerbasque). переписка с Ок. JEAN fee, Options.
(5/XI) Подготовка к Skype - conf.: Edge SW - Pawel, Maciej, Igor. Carrefour" Quantum с Л. Бинш. SW scatter. calc. (Apr 2020) + slides Pawel (May).
(6/XI) Skype conversation с Pawel, Igor, Maciej. Решили писать 2 papers:
1) APL - edge SW + GH effect, draft by Pawel in Nov. 2020; 2) SW - edge SW scattering review. для Phys. Rev. B или Applied. Abs. to INTERMAG'21.
(7/XI) E-mails to Duros (PhD candidate - O. Duros) и Игорю (Сост. план по FiM Skyrmion near angular. moment. comp. point).

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(8/XI) План работы (Сч. 21/I 2020):

- 0) Engineering A_1M_3 + Nanovolcanoes → APL + Dobrovolskiy
- 1) AF DW dynamics Mn_2Au – занимает paper (Ricardo, Ruben) – Subm. PRA
- 2) SW scattering at film edge (Pawel) → APL, PRA? subm. APL 18/XII
- 3) Bloch point paper (Oksana) – занимает, subm. PRL 20/I
- 4) Bergenza – dome shaped dot theory paper / Tejo AE-paper – subm.
- 5) V. gyrovortices thick dots / nanowires – Hannover, Figs. + FMR Physrag 15/III
- 6) Pacer ring / Sk. dot (Zelent)
- 7) 1D DW dynamics, inhom. solution $\theta(s)$, $\lambda \gg 1$
- 8) New PhD student
- 9) "Movilidad" UPV/EHU travel grant (deadline Nov.)
- 10) Ons. JEMS fee + nogroc. gorvagor.
- 11) Skyrmion a_G ferrimagnets calc. (Zukhra) – Lesz (FiM)
- 12) Abs. IMMM'21, Vienna, 31/I – 4/II 2021, abs. go Feb. 28/2021 – postponed
- 13) Abs. PM'21, Poznań, June 28 – July 2, 2021 – online
- 14) Abs. Nano-wires/tubes, Waischenfeld, Germany, Aug. 30 – Sept. 3, 2020, abs. go May 14
- 15) Nogroc. Zorel tells JEMS'20, Session chair + conf. fee
- 16) Bajo S.L., Caldera, ~~rowboat/racing~~, Aramburu – Antxoni, AiXA-SelSkyM
- 17) Justificacion FIS2016 project by 7/III 2021 (31/XII 2020)

K.J. Kim – T. Ono, Nat. Mat. 16, 1187 (2017).

Exp. observation уравнения DW velocity in GdRuCo films near angular moment comp. point ($v_0 \approx 1.7$ km/s).

См. также Kim-Tserkovnyak APL 111, 032401 (2017); C. Zaspel PRA 12, 044019 (2019) и T. Lisenkov PRB 100, 100409 (2019) по vortex-FiM nanooscillators

(9/XI) Собрание Дерк. Phys. Mat. Образование Sección Física de Materiales в составе Depto. Polímeros y Materiales Avanzados – нач. Пограна y Antxoni. Зухра прислала draft skyrmion FiM near comp. point manuscript.

(10/XI) Ответ Зухре, Refs "to Ivanov Tserkovnyak papers по vortex-FiM STNO. Спросила прислал Hopf index simulations, H_{sim}, и в 2.5 раза больше H_{analyt} на основе Felsketter-ansatz. Проверить все, включая коэффициенты. Hopf idx.

Lin PRB'2018 def. Hopf index $H = -\int d^3\vec{r}\, \vec{A}\cdot\vec{B}$, $B_i = \frac{1}{8\pi}\varepsilon_{ijk}\vec{n}\cdot(\partial_j\vec{n}\times\partial_k\vec{n})$

$\vec{B} = \mathrm{rot}\vec{A}$. Если удобно $\frac{1}{4\pi}$ из def. $\vec{B}, \vec{A}$ – Lin, $\vec{B}_{Lin} = \frac{1}{4\pi}\vec{B}$, $\vec{A}_{Lin} = \frac{1}{4\pi}\vec{A}$, то

$H = -\int d^3r \vec{B}_L\cdot\vec{A}_L = -\frac{1}{(4\pi)^2}\int d^3\vec{r}\,\vec{A}\cdot\vec{B} = Q_H$ (ч.т.д.).

$\vec{A}(\vec{r}) = \frac{1}{N}\sum_{\vec{k}}\vec{A}(\vec{k})e^{2\pi i\vec{k}\vec{r}}$, $\vec{B}(\vec{r}) = \frac{1}{N}\sum_{\vec{k}}\vec{B}(\vec{k})e^{2\pi i\vec{k}\vec{r}}$, $\vec{B} = \mathrm{rot}\vec{A} = \mathrm{rot}\frac{1}{N}\sum_{\vec{k}}\vec{A}(\vec{k})e^{2\pi i\vec{k}\vec{r}} =$

$= \frac{1}{N}\sum_{\vec{k}}\vec{A}(\vec{k})\,\mathrm{rot}\,(e^{2\pi i\vec{k}\vec{r}}) = \frac{1}{N}\sum_{\vec{k}}\mathrm{rot}\left[\vec{A}(\vec{k})e^{2\pi i\vec{k}\vec{r}}\right] = \frac{1}{N}\sum_{\vec{k}}\left[e^{2\pi i\vec{k}\vec{r}}\nabla\times\vec{A}(\vec{k}) + \nabla(e^{\dots})\times\vec{A}\right] =$

$= \frac{1}{N}\sum_{\vec{k}} e^{i2\pi\vec{k}\vec{r}}(2\pi i)\vec{k}\times\vec{A}(\vec{k})$, $\vec{B}(\vec{k}) = 2\pi i\vec{k}\times\vec{A}(\vec{k})$, $\vec{k}\times\vec{B}(\vec{k}) = 2\pi i\times$

$\times\vec{k}\times[\vec{k}\times A(\vec{k})] = 2\pi i\left[\vec{k}(\vec{k}\cdot\vec{A}(\vec{k})) - \vec{A}(\vec{k})\vec{k}\cdot\vec{k}\right]$. Если $\vec{k}\cdot\vec{A}(\vec{k}) = 0$, $\mathrm{div}\vec{A} = 0$,

то $\vec{k}\times\vec{B}(\vec{k}) = -2\pi i k^2\vec{A}(\vec{k})$ и $H = -\int d^3\vec{r}\,\vec{B}(\vec{r})A(\vec{r}) = -\int d^3\vec{r}\frac{1}{N^2}\sum_{\vec{k}'}\vec{B}(\vec{k}')e^{i2\pi\vec{k}'\vec{r}}\cdot$

$\cdot\sum_{\vec{k}}\vec{A}(\vec{k})e^{2\pi i\vec{k}\vec{r}} = -\frac{1}{N^2}\int d^3\vec{r}\sum_{\vec{k}\vec{k}'}\vec{B}(\vec{k}')\frac{(-1)}{2\pi i k^2}\vec{k}\times\vec{B}(\vec{k})e^{i2\pi(\vec{k}+\vec{k}')\vec{r}} = \frac{1}{2\pi i N^2}\times$

$\times\sum_{\vec{k}\vec{k}'}\frac{1}{k^2}\vec{B}(\vec{k}')\cdot[\vec{k}\times\vec{B}(\vec{k})]\int d^3\vec{r}\,e^{i2\pi(\vec{k}+\vec{k}')\vec{r}}$, $\int d^3\vec{r}\,e^{i2\pi(\vec{k}+\vec{k}')\vec{r}} = \delta_{\vec{k}+\vec{k}',0}\cdot V$.

$H = \frac{1}{2\pi i N^2}\sum_{\vec{k}\vec{k}'}\vec{B}(\vec{k}')\cdot[\vec{k}\times\vec{B}(\vec{k})]V\delta_{\vec{k}',-\vec{k}} = \frac{V}{2\pi i N^2}\sum_{\vec{k}}\frac{1}{k^2}\vec{B}(-\vec{k})[\vec{k}\times\vec{B}(\vec{k})]$ – Hopf idx k-space

Итого $\vec{A}(\vec{r}) = \frac{1}{V}\sum_{\vec{k}}\vec{A}(\vec{k})e^{2\pi i\vec{k}\cdot\vec{r}}$, $\vec{B}(\vec{r}) = \frac{1}{V}\sum_{\vec{k}}\vec{B}(\vec{k})e^{i2\pi\vec{k}\vec{r}}$, тогда $[\vec{B}]_{(\vec{k})} = \mathrm{cm}\cdot[\vec{B}(\vec{k})] = \frac{1}{\mathrm{cm}^2}$

$H = \frac{-i}{2\pi V}\sum_{\vec{k}}\frac{1}{k^2}\vec{B}(-\vec{k})\cdot[\vec{k}\times\vec{B}(\vec{k})]$ is dimensionless. У Lin'18 размерность H

в Eq. (25) неверная, и знак перепутан в (25), (24), верно $\vec{A}(\vec{k}) = i\frac{1}{2\pi k^2}\vec{k}\times\vec{B}(\vec{k})$

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Главное. Eq. (5) Liu'2018 требует $\mathrm{div}\vec{A}=0$, а это сильное условие.
При нахождении Q_H в $\vec{r}$-space через $\vec{B}(\vec{r})$, используя Helmholtz theorem, см. Kundu'82, Kominear'96 условие $\mathrm{div}\vec{A}=0$ не требуется (!), а лишь $\vec{B}(r)\sim\frac{1}{r^{1+d}}$, $r\to\infty$ (см. 21/VIII).

Мой vector potential $\vec{A}=A_r\hat{r}-\frac{pq}{r}\frac{(1+\cos\theta)}{\sin\theta}\hat{\varphi}$ in spher. coordinates $\vec{r}=(r,\theta,\varphi)$

$\mathrm{div}\vec{A}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}A_\varphi=$

$=\mathrm{div}(A_r,0,A_\varphi)=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r)$. A_r из $\frac{\partial A_r}{\partial\theta}=p\frac{\partial\gamma}{\partial r}\sin\theta$, helicity $\gamma=\gamma(r,\theta)$

Если требовать $\mathrm{div}\vec{A}=0$, то $r^2A_r=\mathrm{const}(r)=f(\theta)$, $A_r(r,\theta)=\frac{1}{r^2}f(\theta)$.

Совместно с $A_r=\frac{1}{r^2}f(\theta)$ с $\frac{\partial A_r}{\partial\theta}=p\frac{\partial\gamma}{\partial r}\sin\theta$ (получен из $\mathrm{div}\vec{B}=0$).

$\frac{\partial A_r}{\partial\theta}=\frac{1}{r^2}\frac{\partial f}{\partial\theta}=p\frac{\partial\gamma}{\partial r}\sin\theta$, $\frac{\partial f(\theta)}{\partial\theta}=pr^2\sin\theta\frac{\partial\gamma}{\partial r}(r,\theta)=f'(\theta)$, $\frac{\partial\gamma}{\partial r}=\frac{f'(\theta)}{p\sin\theta}\frac{1}{r^2}$,

$\frac{\partial\gamma}{\partial r}=c\frac{1}{r^2}$, $c=c(\theta)$, $\gamma(r,\theta)=-\frac{1}{r}c(\theta)+c_1(\theta)$ – singularity при $r\to0$.

Имп. $Q_H=-\frac{q}{4\pi}\int_0^{\pi}d\theta\sin\theta(1+\cos\theta)\int_0^R dr\frac{\partial\gamma}{\partial r}=\left|\text{если }\frac{\partial\gamma}{\partial r}=\frac{c(\theta)}{r^2}\right|=-\frac{q}{4\pi}\times$

$\times\int_0^{\pi}d\theta\sin\theta(1+\cos\theta)\int_0^R dr\frac{c(\theta)}{r^2}=-\frac{q}{4\pi}\int_0^{\pi}d\theta\sin\theta(1+\cos\theta)c(\theta)\left(-\frac{1}{r}\right)\Big|_0^R$ – singular at $r\to0$.

Balog: условие (Coulomb gauge) $\mathrm{div}\vec{A}=0$ несовместимо с моим $\vec{A}(\vec{r})$, приводя к $\gamma(r)\sim1/r$, $Q_H\sim1/r$ singularities at $r\to0$ (BP poisson).

Мои Q_H и Q_H-Liu'2018 получены с разными $A(\vec{r})$ [у Liu $\mathrm{div}\vec{A}=0$], и остаются, поэтому. Helmholtz theorem (21/VIII) $\vec{A}(\vec{r})=-\frac{1}{4\pi}\int dV'\frac{\vec{R}}{R^3}\times\vec{B}(\vec{r}')$ и где Q_H – Kundu'82, Kominear'96 do in the Coulomb gauge $\mathrm{div}\vec{A}=0$, $\vec{A}(\vec{r})=\nabla\times\int dV'\frac{1}{4\pi R}\vec{B}(\vec{r}')$, $\vec{R}=\vec{r}-\vec{r}'$.

Какие основания применять Coulomb gauge $\mathrm{div}\vec{A}=0$ для emergent vector-potential $\vec{A}$?

E-mail to Oksana/Felipe с вопросами /comments по вычислению Q_H in $\vec{r}$-space, между Liu PRB'2018.

Gauge invariance of Q_H – Hopf

(14/XI) Условие $\mathrm{div}\vec{A}=0$ проверим для „toroidal" topologies, где $\vec{A}$ и $\vec{B}$ известны.

См. Gladikowski, Phys. Rev. D '1997 (6/IV, 14/IV): $\vec{A}=(A_\gamma,A_\beta,A_\varphi)$ $\varphi\to\alpha$.

$\vec{r}=\vec{r}(\gamma,\beta,\varphi)$,

$A_\varphi=2\frac{\tau}{a\,\mathrm{sh}\gamma}N(w-1)$, $A_\beta=2\frac{\tau}{a}(M+\upsilon_0')(w+1)$, $A_\gamma=2\frac{\tau}{a}\upsilon_0'(w+1)$, $\tau=\mathrm{ch}\gamma-\cos\beta$, $(\cdot)'=\partial/\partial\beta$

$m_x+im_y=\sin\Theta e^{i(N\varphi+\upsilon)}$, $m_z=\cos\Theta=w(\gamma,\beta)$, $\upsilon=\upsilon(\gamma,\beta)=M\beta+\upsilon_0(\gamma,\beta)$,

$\upsilon_0(\gamma,\beta)=\gamma(\gamma,\beta)$.

$(\cdot)'=\partial/\partial\gamma$.

$\mathrm{div}\vec{A}=\frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial\beta}\left(\frac{a^2}{\tau^2}\mathrm{sh}\gamma A_\beta\right)+\frac{\partial}{\partial\gamma}\left(\frac{a^2}{\tau^2}\mathrm{sh}\gamma A_\gamma\right)+\frac{\partial}{\partial\varphi}\left(\frac{a^2}{\tau^2}A_\varphi\right)\right]=$ (14/XI)

$=\frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial\beta}\left(\frac{a^2}{\tau^2}\mathrm{sh}\gamma\cdot\frac{2\tau}{a}(M+\frac{\partial\gamma}{\partial\beta})(w+1)\right)+\frac{\partial}{\partial\gamma}\left(\frac{a^2}{\tau^2}\mathrm{sh}\gamma\frac{2\tau}{a}\frac{\partial\gamma}{\partial\gamma}(w+1)\right)+\frac{\partial}{\partial\varphi}\left(\frac{a^2}{\tau^2}\frac{2\tau}{a\,\mathrm{sh}\gamma}N(w-1)\right)\right]=$

$=\left|\text{если }\frac{\partial\gamma}{\partial\gamma}=0,\ \frac{\partial\gamma}{\partial\beta}=0,\ \gamma=\upsilon_0=\mathrm{const}\right|=\frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial\beta}\left(\frac{2a}{\tau}\mathrm{sh}\gamma M(w+1)\right)+\frac{\partial}{\partial\varphi}\left(\frac{2a}{\tau}\frac{N}{\mathrm{sh}\gamma}(w-1)\right)\right]=$

$=\frac{1}{h_1h_2h_3}\left[2aM\,\mathrm{sh}\gamma\frac{\partial}{\partial\beta}\frac{(w+1)}{\tau}+0\right]=\frac{2aM\,\mathrm{sh}\gamma}{h_1h_2h_3}\left[-\frac{1}{\tau^2}\frac{\partial\tau}{\partial\beta}(w+1)+\frac{1}{\tau}\frac{\partial w}{\partial\beta}\right]$ *

$\frac{\partial\tau}{\partial\beta}=\sin\beta$, $\mathrm{div}\vec{A}=\frac{2aM\,\mathrm{sh}\gamma}{h_1h_2h_3}\left[-\frac{\sin\beta}{\tau^2}(w+1)+\frac{1}{\tau}\frac{\partial w}{\partial\beta}\right]=\frac{2aM\,\mathrm{sh}\gamma\cdot\tau^3}{\tau^2a^3\,\mathrm{sh}\gamma}\left[\tau\frac{\partial w}{\partial\beta}-\sin\beta(w+1)\right]$

$\mathrm{div}\vec{A}=\frac{2\tau M}{a^2}\left[\tau\frac{\partial m_z}{\partial\beta}-\sin\beta(1+m_z)\right]\neq0$ даже при $\gamma=\upsilon_0=\mathrm{const}$, $\mathrm{div}\vec{B}=0$.

Сравните (11/XI, 14/XI) $m_z=1-2\,\mathrm{th}^2\gamma$, $\frac{\partial m_z}{\partial\beta}=0$, $1+m_z=2-2\,\mathrm{th}^2\gamma=2(1-\mathrm{th}^2\gamma)=\frac{2}{\mathrm{ch}^2\gamma}$

$\mathrm{div}\vec{A}=\frac{2\tau M}{a^2}(-\sin\beta)\frac{2}{\mathrm{ch}^2\gamma}=-\frac{4M\tau\sin\beta}{a^2\mathrm{ch}^2\gamma}\neq0$ даже toroidal topology с $\upsilon_0=\gamma=\mathrm{const}$

Сравн. Wang PRL'2018 (20/VIII), $H(\vec{A})=\frac{1}{(4\pi)^2}\int dV\vec{A}\cdot\vec{B}$, $\vec{B}=\mathrm{rot}\vec{A}$, при gauge $\vec{A}'=\vec{A}+\nabla\chi$ transform

$H(\vec{A}')=H(\vec{A})+\frac{1}{(4\pi)^2}\int dV\vec{B}\cdot\nabla\chi=H(\vec{A})+\frac{1}{(4\pi)^2}\oint dS\cdot\chi\vec{B}$ is gauge invariant $H(\vec{A}')=H(\vec{A})$

если $\oint dS\,\chi\vec{B}\to0$. Пусть $\chi(r)$ – конечна при $r\to\infty$, тогда если $\vec{B}(\vec{r})\sim1/r^{3+d}$, $r\to\infty$

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$\to 0$ $\oint d\vec{S}\cdot[\vec{B}(\vec{r}) \to 0$ при $r\to\infty$. Т.е. Hopf index $H(\vec{A})$ is gauge invariant $\vec{A}\to\vec{A}+\nabla f=\vec{A}'$ для ∞-system. Все это при $\operatorname{div}\vec{B}=0$.

Если $H(\vec{A})$ is gauge invariant, то можно взять any gauge, например $\operatorname{div}\vec{A}'=0$, $\operatorname{div}\vec{A}\neq 0$, но $\operatorname{div}\vec{A}'=\operatorname{div}\vec{A}+\operatorname{div}(\nabla f)=0$ за счет выбора $f(\vec{r})$. // Но для $H(\vec{A})$ invariant надо $\operatorname{div}\vec{B}=0$ и $\vec{B}(\vec{r})\sim 1/r^{3+d}$, $r\to\infty$, т.е. localized soliton (toroidal hopfion, for instance), $r\to\infty$ только infinite sample. У Lin'2018 finite dot $\Rightarrow B(r)\sim 1/r^{3+d}\to 0$, $r\to\infty$ не выполнить, и $Q_H(\vec{A})=H(\vec{A})$ не может быть gauge invariant и брать gauge $\operatorname{div}\vec{A}=0$ нельзя.

// Если $\vec{m}$ не $\to\vec{m}_0=$ const at $r\to\infty$, $B(\vec{r})\to 0$ медленнее чем $\frac{1}{r^{3+d}}$, soliton is non-localized (Bloch point, for instance), будет ли $Q_H(\vec{A})$ gauge invariant? Для BP $B_r(r)\sim 1/r^2$, $B_\theta(r)\sim\frac{1}{r}\frac{\partial f}{\partial r}$. Если $f(r)\sim\frac{1}{r^d}$, $d>0$, то $B_\theta(r)\sim\frac{1}{r}\frac{1}{r^{1+d}}=\frac{1}{r^{2+d}}$ (см. 22/VIII). $d\vec{S}=dS\hat{r}$. Поэтому только $B_r(r)$ важна для расчета $Q_H(\vec{A})$ invariance.

$\vec{A}'=\vec{A}+\nabla f$, $Q_H(\vec{A}')=Q_H(\vec{A})-\frac{1}{(4\pi)^2}\int dV\vec{B}\nabla f$, при $\operatorname{div}\vec{B}=0$. $\int dV\vec{B}\nabla f=\oint_S d\vec{S}f\vec{B}$

$\oint d\vec{S}\vec{B}f=\oint dS\hat{r}\cdot\vec{B}f=\oint dS B_r f=\oint dS f\frac{g}{r^2}=\int_{r=R}\sin\theta\,d\theta\int d\varphi\, r^2 f g\frac{1}{r^2}=$

$=g\int d\theta\sin\theta\int d\varphi f(\vec{r})$, $\int dV\vec{B}\nabla f=g\int d\theta\sin\theta\int d\varphi f(\vec{r})_{r=R}=I(f)$, хотим $I(f)\to 0$, тогда $Q_H(\vec{A}')=Q_H(\vec{A})$ и $Q_H(\vec{A})$ is gauge invariant. $f(\vec{r})=f(r,\theta,\varphi)$.

$I[f]=I[f(\vec{r})]=g\int d\theta\sin\theta\, d\varphi f(R,\theta,\varphi)$. Надо чтоб $f(R)\to 0$ при $R\to\infty$.

Интересно, что для BP in spherical particle $Q_H(\vec{A})$-invariance зависит от B_r только, B_θ и A_θ - не важны. ($d\vec{S}=dS\hat{r}$)

$Q_H(\vec{A})=-\frac{1}{(4\pi)^2}\int dV\vec{A}\cdot\vec{B}=-\frac{1}{(4\pi)^2}\left[\int dV A_r B_r+\int dV A_\theta B_\theta\right]$ для сферы всегда gauge invariant при $\vec{A}\to\vec{A}+\nabla f=\vec{A}'$

$Q_H(\vec{A}')=Q_H(\vec{A})-\frac{1}{(4\pi)^2}\oint_S dS B_r f(\vec{r})|_{r=R}$, $dS=r^2\sin\theta\,d\theta\,d\varphi$

$I[f(\vec{r})]=\int dV\vec{B}\nabla f=\oint_S dS B_r f(\vec{r})_{r=R}=g\int d\theta\sin\theta\int d\varphi f(\vec{r})|_{r=R}$

22/VIII я потребовал $\int dV f^2(\vec{r})$ - конечен, что дало $f(\vec{r})\sim\frac{1}{r^{3/2+d}}$ при $r\to\infty$, $d>0$. $f(\vec{r})=\frac{\Psi(\theta,\varphi)}{r^{3/2+d}}$

$I[f(\vec{r})]=g\int d\theta\sin\theta\int d\varphi\frac{\Psi(\theta,\varphi)}{R^{3/2+d}}=\frac{g}{R^{3/2+d}}\int d\theta\sin\theta\int d\varphi\Psi(\theta,\varphi)\sim\frac{1}{R^{3/2+d}}\to 0$ при $R/l_e\to\infty$.

Но условие $\int dV f^2$ - конечен - сильное требование для BP. Для localized solitons было $f(\vec{r})$ - конечна при $r\to\infty$, т.е. $f(\vec{r})\sim\frac{1}{r^d}$ при $r\to\infty$, $d\geq 0$, более слабое условие.

// Вторым подход для расчета $\vec{A}$ и $Q_H(\vec{A})$ для BP был сделан 29/VIII, на основе Feldkeler'ansatz $f(r,\theta)$ я предположил $A_{\theta\theta}=0$. Известно из Feldkeler's ansatz $\vec{m}(r,\theta)\subset f=f(r,\theta)$ helicity, что $\vec{B}=\frac{PQ}{r^2}\hat{r}-\frac{P}{r}\sin\theta\frac{\partial f}{\partial r}\hat{\theta}=(B_r,0,B_\theta)$, $B_\theta=\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta)-\frac{\partial A_r}{\partial\theta}\right]=-\frac{P}{r}\sin\theta\frac{\partial f}{\partial r}$

т.к. $A_{\theta\theta}=0$, то $\frac{\partial A_r}{\partial\theta}=P\sin\theta\frac{\partial f}{\partial r}$. $\vec{A}=(A_r,A_\theta,A_\varphi)$ - сферич. компоненты $\vec{A}$.

$B_r=\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\varphi\sin\theta)-\frac{\partial A_\theta}{\partial\varphi}\right]=\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\varphi\sin\theta)=\frac{PQ}{r^2}$, что выполнялось для $A_\varphi=A_\varphi^{\mathcal{D}}=-\frac{QP}{r}\frac{(1+\cos\theta)}{\sin\theta}$. $B_{\theta\theta}=\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi}-\frac{\partial}{\partial r}(rA_\varphi)\right]\equiv 0$ для $A_r=A_r(r,\theta)$, и $A_\varphi=A_\varphi^{\mathcal{D}}$. Я взял $\vec{A}=(A_r,A_\theta,A_\varphi)=(A_r,0,A_\varphi^{\mathcal{D}})$, с $\frac{\partial A_r}{\partial\theta}=P\sin\theta\frac{\partial f}{\partial r}$, $f=f(r,\theta)$.

Свобода выбора компоненты $\vec{A}=\vec{A}(r,\theta)$ была лишь в выборе A_r, A_θ как чтобы $B_\theta=\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta)-\frac{\partial A_r}{\partial\theta}\right]=-\frac{P}{r}\sin\theta\frac{\partial f}{\partial r}$.

Можно взять $A_\theta=A_\theta(r,\theta)=\frac{1}{r}\Psi(\theta)\neq 0$, сохраняя $\frac{\partial A_r}{\partial\theta}=P\sin\theta\frac{\partial f}{\partial r}$. Но т.к. $B_{\theta\theta}=0$ для BP ansatz Feldkeler $\vec{m}(\vec{r})=\vec{m}(r,\theta)$, то Q_H из-за $A_\theta\neq 0$ не изменен. Мой выбор gauge is $\frac{\partial}{\partial r}(rA_\theta)=0$, $A_\varphi=A_\varphi^{\mathcal{D}}$, и автоматом отсюда $\partial A_r/\partial\theta=P\sin\theta\,\partial f/\partial r$.

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Альтернативный выбор $A_\theta \neq \frac{\Psi(\theta)}{r}$, $A_\theta = A_\theta^D + A'_\theta$ недопустим, т.к. A'_θ сингулярно (30/VIII)

$\frac{\partial A_r}{\partial \theta} = p \sin\theta \frac{\partial \gamma}{\partial r} \Rightarrow A_r(r,\theta) = p\int_{\theta_0}^{\theta} d\theta' \sin\theta' \frac{\partial \gamma}{\partial r}(r,\theta') + f(r)$, См. 30/VIII

$\int dV A_r B_r = 2\pi q \int dr \int d\theta \sin\theta \cos\theta \frac{\partial \gamma}{\partial r} + 2\pi p q \int dr f_r(r)$, $f_r(r) = -\cos\theta A_r(r,\theta)\Big|_0^\pi$

$f_r(r) = -[\cos\pi A_r(r,\pi) - \cos 0 A_r(r,0)] = A_r(r,\pi) + A_r(r,0)$

$f_r(r) = p\int_{\theta_0}^{\pi} d\theta' \sin\theta' \frac{\partial \gamma}{\partial r}(r,\theta') + p\int_{\theta_0}^{0} d\theta' \sin\theta' \frac{\partial \gamma}{\partial r}(r,\theta') + 2f(r)$.

$\int_{\theta_0}^{\pi} d\theta F(\theta) + \int_{\theta_0}^{0} d\theta F(\theta) = \int_{\theta_0}^{\pi} - \int_{0}^{\theta_0}$. Итак $A_r(r,\theta) = p\int_{\theta_0}^{\theta} d\theta' \sin\theta' \frac{\partial \gamma}{\partial r} - p\int_{0}^{\theta_0} \sin\theta' \frac{\partial \gamma}{\partial r} d\theta' + f(r)$

$f_r(r) = p\int_{\theta_0}^{\pi} d\theta' \sin\theta' \frac{\partial \gamma}{\partial r} - 2p\int_{\theta_0}^{0} d\theta' \sin\theta' \frac{\partial \gamma}{\partial r} + 2f(r) = p\int_{0}^{\pi} d\theta' \sin\theta' \frac{\partial \gamma}{\partial r} + f_1(r)$

$f_1(r) = 2f(r) - 2p\int_{0}^{\theta_0} d\theta' \sin\theta' \frac{\partial \gamma}{\partial r}(r,\theta')$ – gauge function, θ_0 – arbitrary.

$\int dV \vec{A}\cdot\vec{B} = 2\pi q \int dr \int_0^\pi d\theta \sin\theta (1+2\cos\theta) \frac{\partial \gamma}{\partial r} + 2\pi p q \int dr f_r(r) = 2\pi q \int dr \int_0^\pi d\theta \sin\theta \times$

$\times (1+2\cos\theta)\frac{\partial \gamma}{\partial r} + 2\pi p q \int dr \left[p\int_0^\pi d\theta \sin\theta \frac{\partial \gamma}{\partial r} + f_1(r)\right] = 4\pi q \int dr \int_0^\pi d\theta \sin\theta (1+\cos\theta)\frac{\partial \gamma}{\partial r} +$

$+ 2\pi p q \int dr f_1(r)$, где $f_1(r) = 2f(r) - 2p\int_0^{\theta_0} d\theta \sin\theta \frac{\partial \gamma}{\partial r}(r,\theta)$.

Ранее, 30/VIII, при расчете $\int dV \vec{A}\cdot\vec{B} \sim Q_H$ я применил gauge function $f_1(r)$.

Вполне можно $\theta_0 = 0$, оставляя лишь $f(r)$ (где $\theta_0 \neq 0$ удобна), и $f_1(r) = 2f(r)$

$A_r(r,\theta) = p\int_{\theta_0}^{\theta} d\theta' \sin\theta' \frac{\partial \gamma}{\partial r}(r,\theta') + f(r)$, any $f(r)$ дает верное $B_\theta = -\frac{p}{r}\sin\theta\frac{\partial \gamma}{\partial r}$

$A_r \to A'_r = A_r + (\nabla\chi)_r = A_r + \frac{\partial \chi}{\partial r}$, т.е. $f(r) = \frac{\partial \chi}{\partial r}$, $\chi(\vec{r})$ – gauge function

В BP-manuscript, где Q_H Eq. (6) использовался gauge $f(r) = 0$.

$Q_H = -\frac{1}{(4\pi)^2}\int dV \vec{A}\cdot\vec{B} = -\frac{q}{4\pi}\int_0^\pi d\theta \sin\theta (1+\cos\theta)[\gamma(R,\theta) - \gamma(0,\theta)] - \frac{pq}{4\pi}\int dr f(r)$, где

$f(r)$ is const (θ) при интегрировании $\frac{\partial A_r(r,\theta)}{\partial \theta} = p\sin\theta\frac{\partial \gamma}{\partial r}(r,\theta)$

Предположение $f(r) = 0$ согласуется с Kindu' 1982.

В принципе возможно $A_\theta \neq 0$, $A_\theta \neq \frac{\Psi(\theta)}{r}$, и $A_\theta = A_\theta^D = -\frac{q}{r}\frac{(1+\cos\theta)}{\sin\theta}$?

тогда B_θ будет другим. но B_θ посчитано прямо из BP $\vec{m}(\vec{r})$ – Feldkeller, единственно.

но B_θ может измениться, если выбрать другой ansatz $\vec{m}(\vec{r})$.

Т.е. $Q_H(\vec{A})$ is gauge invariant для BP при $\int dV r^2$ – конечен, $R\to\infty$ или $\frac{\partial \chi}{\partial r} = 0$ при $\vec{A}\to\vec{A}' = \vec{A} + \nabla f$, $Q_H(\vec{A}') = Q_H(\vec{A})$.

Можно исп. Helmholtz theorem $\vec{A}(\vec{r}) = -\frac{1}{4\pi}\int dV' \frac{\vec{R}\times\vec{B}(\vec{r}')}{R^3}$, $\vec{R} = \vec{r} - \vec{r}'$,

и $\int dV \vec{A}\cdot\vec{B} = -\frac{1}{4\pi}\int dV dV' \frac{1}{R^3}\vec{B}(\vec{r})\cdot[\vec{R}\times\vec{B}(\vec{r}')] \sim Q_H$, gauge assumed.

$B(\vec{r}) = \vec{B}(r,\theta)$ и только $B_r \neq 0$, $B_\theta \neq 0$, это упрощает $\vec{A}(\vec{r})$ и Q_H – calculations.

$A_r = \vec{A}\cdot\hat{r} = -\frac{1}{4\pi}\int dV' \frac{1}{R^3}[\vec{R}\times\vec{B}(\vec{r}')]\cdot\hat{r} = -\frac{1}{4\pi}\int \frac{dV'}{R^3}(R_\theta B_\varphi - R_\varphi B_\theta) =$

$= -\frac{1}{4\pi}\int dV' \frac{1}{R^3} B_\theta(\vec{r}')(\vec{r} - \vec{r}')\cdot\hat{\varphi}$, $B_\theta = -\frac{p}{r}\sin\theta\frac{\partial \gamma}{\partial r}$, при $\frac{\partial \gamma}{\partial r} = 0$, $B_\theta = 0$ и $A_r = 0$

Т.е. $A_r \sim \partial\delta/\partial r$, поэтому $f(r) = 0$ – единственный выбор of the gauge.

для $\vec{A}$, который следует из Feldkeller ansatz $\vec{m}(r,\theta)$ of Bloch point.

Согласно X.G. Wen, A. Zee PRL 61, 1025 (1988) $Q_H \sim \int dV \varepsilon^{\mu\nu\lambda}(z^+\partial_\mu z)(\partial_\nu z^+\partial_\lambda z)$ is local (!) in terms of spinor $z = (z_1, z_2)$ defined by $\vec{m}(\vec{r}) = z^+(\vec{r})\vec{\sigma} z(\vec{r})$.

Новые вопросы/Comments по Q_H of BP to Oksana и Felipe.

(12/XI) Ikerbasque on-line meeting. New Ikerbasques, Ikerbasque travel program. signed и sent Antxon's doctoral на APC Ruben com. Phys. AP DW paper.

(13/XI) С.Л. и Ю.Б. и Eroski и Garbera. Бинт.

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FiM (14/VI) Намиране MS по Skyrmion dynamics in FiM near ang. mom. compensation T_A point.

oo S.K.Kim, Y.Tserkovnyak, Ferrimag. disk W_c near T_A. APL 111, 032401 (2017).

$\mathcal{L} = -s\vec{a}[\vec{n}]\cdot\dot{\vec{n}} + \frac{1}{2}s(\dot{\vec{n}})^2 - U(\vec{n})$, $\vec{M} = M_s\vec{n}$, $\vec{s} = s\vec{n}$, $\vec{M} = \gamma\vec{s}$, $\vec{s}$ - spin density

Gyrofrequency FM, $f_{FM} \sim \gamma M_s$, AF $f_{AFM} \sim \sqrt{\frac{J}{\hbar}\gamma M_s}$.

thiele eq. с mass M_v-term, $M_v = 2\pi s L \ln R/R_a$, $s \approx \hbar^2/Ja^3$, L,R - dot sizes

$\omega_\pm = \frac{G}{2M_v} \pm \sqrt{\left(\frac{G}{2M_v}\right)^2 + \frac{\varkappa}{M_v}}$, при T_A, $s=0$, $G=0$ и $\omega_\pm = \sqrt{\frac{\varkappa}{M_v}}$ degenerated

Очевидно $\omega_\pm(s=0) = \omega_{AFM} = \sqrt{\frac{\varkappa}{M_v}}$, $M_v \sim \frac{1}{J}$ и $\omega_{AFM} \to f_{AFM} \sim \sqrt{J\gamma M_s}$.

Зад не ден, видя че $\varkappa \sim M_s^2$ и gyrofrequency $\omega_{AFM} \sim \sqrt{JM_s^2} = \sqrt{J}M_s$.

т.е. изучаем gyrofrequency ω_{AFM} и зависимостта AFM-resonance $f_{AFM} \sim \sqrt{J_{ex}H_a}$.

(15/VI) напомняне с Oxe. с Overleaf subscription и Hopf index. Зсомм/Skype Гиев: текущите разбори, предлага Вече chairman M-dyn. Session JEMS 2020, а съответно, приемам предложението. Мне ест. JEMS '20 g. see в 26-27 Nov. Нужна пристигане просто о полнени (money transfer)

(16/VI) JEMS'2020 напомнящи о reg. fee. Приемето 2021 al talks: Skyrmions y Dw-SW scattering. Вчера приемам FiM-T_A paper с особени за мои comments.

(17/VI) Министерство изпраща 200 Skyrmion FIS 2016 програми на Бъдър, Fin. 29/12/2020. напомняне с Oxe. по Skyrmion и Hopf index Sinnk. С Л. „Самолет" Одобрение.

(18/VI) Spintronics Red de España on-line Conf. Factura „Delta" yes to Antxon. Платено 300€.

→ Преводи WU/непредаго cash пари в Киев:

2018: (см. 24/XI 2018) — 2.130 €

2019: 450€ (20/II) + 400€ (16/VII cash) + 300€ (15/X cash) = 1.150 €

2020: 300 € (10/IV) + 300 € (18/VI)

(19/VI) Бегомелка к расчету параметров 2-model Mn_2Au по J_1, J_2, J_3 (28/V)

Вид non-uniform $\alpha(J_1J_2J_3)$ и uniform $A(J_1J_2J_3)$ exchange зависят от числа ближ. сосед Mn-atoms, т.е. от „bulk" или „thin film" sample расстояния.

Bulk Mn_2Au - Crystalographical periodicity $\parallel Ox, Oy, Oz$, $\vec{m}_i(x)$ - Dw.

Roy'PRB'2016 отвал $\approx$ Bulk Mn_2Au, infinite (150 u.c.'s), 150 u.cells $\parallel Oy$, 5 layers $\parallel Oz$.

• Roy/Ruben in the current MS by Ricardo et al. („Inertial Dw motion...") simulated „ultrathin" film: one cell, $c = 0.854$ nm, $\parallel Oz$; one cell with p.b.c. $\parallel Oy$; $6\cdot10^3$ cells $\parallel Ox$ Dw $\vec{m}_i(x)$.

Energy density per unit volume: $W = \frac{1}{2}A\vec{M}^2 + \frac{1}{8}\alpha\left(\frac{\partial\vec{L}}{\partial x}\right)^2 + w_a(\vec{L}) - \vec{L}\cdot\vec{H}_D$

$\vec{L} = \vec{M}_A - \vec{M}_B$, $\vec{M} = \vec{M}_A + \vec{M}_B$, A и B - две подрешетки, с AF упорядочением $\vec{M}_{A,B}$.

$|\vec{M}_A| = |\vec{M}_B| = M_0$, $\vec{L} = 2M_0\vec{l}$, $\vec{M} = 2M_0\vec{m}$, $w_{ex,an} = 2AM_0^2\vec{m}^2 + \frac{1}{2}\alpha M_0^2\left(\frac{\partial\vec{l}}{\partial x}\right)^2 - \varkappa_y l_y^2 + \ldots$

А.Dys поставка в Киев, 10.5 kg, 21 €.

С Dw atomistic simulations сравняваме $v_{max} = c = \gamma M_0\sqrt{\alpha A}$ и $\Delta_0 = \sqrt{\frac{\alpha M_0^2}{2\varkappa_y}}$

Два „bulk" Mn_2Au (7/VIII) anisotropy $\varkappa_y = 4K_{211}$ т.к. 4 Mn atoms / u.c.

Два ultrathin film" $\varkappa_y = \varkappa_y(bulk)$, т.к. също Mn atoms per u.c. = 4.

Насоки към Basenо M_0-value? $A = \frac{1}{M_0^2}\frac{1}{V}F_A(J_i)$, $\alpha = \frac{1}{M_0^2}\frac{a^2}{V}F_d(J_i)$, где

V-unit cell Volume, $V = a^2c$, $F_A(J_i)$, $F_d(J_i)$ linear functions of atomistic exch. integrals J_i, $[J_i] = erg$, $[F_A] = [F_d] = erg$, $[K_{211}] = erg$.

$c = \gamma M_0\sqrt{\frac{a^2F_d}{M_0^2V}\frac{F_A}{M_0^2V}} = \frac{\gamma a}{M_0V}\sqrt{F_dF_A}$, $\Delta_0 = \sqrt{\frac{a^2F_d}{M_0^2V}\frac{M_0^2}{2\cdot4F_{211}V}} = a\sqrt{\frac{F_d}{8K_{211}}}$.

Только „c" Зависи от $M_0V = \mu$.

Diszde $\mu = M_0 V_{u.c.}$ определено с точността до инт. „2“. Simulated $c_s = 43{,}3\ \frac{km}{s}$

Неизвестен вид F_d, F_A. Ако потребуваме $\Delta_0 = a\sqrt{\frac{F_d}{2K_{211}}} = \Delta_s$, то $\Delta_0 = \Delta_s = 19{,}8\,nm$

Вектор $F_d = \frac{1}{a^2}\Delta_s^2 \cdot 8K_{211}$ - известна. Требование $c = \frac{\gamma a}{\mu}\sqrt{F_A F_d} = c_s$ дава $F_A(J_i)$ - вектор

$F_A = \frac{1}{F_d}\frac{\mu^2 c_s^2}{\gamma^2 a^2} = \frac{a^2}{8K_{211}\Delta_s^2}\frac{\mu^2 c_s^2}{\gamma^2 a^2} = \frac{\mu^2}{8K_{211}}\left(\frac{c_s}{\gamma\Delta_s}\right)^2$. Т.е. векторите F_d, F_A определени от c_s, Δ_s simulated $F_d = 8K_{211}\left(\frac{\Delta_s}{a}\right)^2$, $F_A = \frac{\mu^2}{8K_{211}}\left(\frac{c_s}{\gamma\Delta_s}\right)^2$.

$F_A = n_1 J_1 + n_2 J_2$, $F_d = m_1 J_1 + m_3 J_3$.

→ Говорили с Иванов и Илиев по Facebook - online. // Forward Krawczyk info
Переписка с Овчаров по CEMAG - poster Farkhad-wires. // PhD stipends in Spain.

20/IV ~ 17 в „Сарафски“ Диактини. Пълни пациенти / имена „Fem“ - „Erosk“.

21/IV Намиране векторите F_d и F_A. Отново $\mu = M_0 V = 4\mu_B$. Mn_2Au

$\frac{\mu}{\gamma} = 4\cdot\frac{\mu_B}{\gamma} = 4\frac{\mu_B}{g\frac{\mu_B}{\hbar}} = |g=2| = 2\hbar$. $\frac{F_d}{J_3} = \frac{8\cdot 7\cdot 1{,}855\cdot 10^{-25}J}{115K\cdot 1{,}381\cdot 10^{-23}\frac{J}{K}}\frac{(19{,}78)^2}{(0{,}3328)^2} =$

$= \frac{56}{115}\frac{1{,}855}{1{,}381}\cdot 10^{-2}\cdot 3{,}533\cdot 10^3 = 2{,}311\cdot 10^3\cdot 10 = 2{,}311\cdot 10$ (?) $= 23{,}11$.

$J_3 = 115K\cdot k_B = 158{,}82\cdot 10^{-23}J$, $\Delta_0 = a\sqrt{\frac{J_3}{K_{211}}} = 0{,}3328\sqrt{\frac{158{,}82\cdot 10^{-23}J}{7\cdot 1{,}855\cdot 10^{-25}J}} \Rightarrow$ $= 11{,}64\,nm$

$\frac{F_d}{J_3} = 23{,}11$ при подадени Δ_s. 7/VIII Взех $d = \frac{8J_3 a^2}{M_0^2 V} = \frac{a^2}{M_0^2 V}\cdot 8J_3$ (bulk), $d = \frac{a^2}{M_0^2 V}F_d$ - def.,

т.е. $F_d/J_3 = 8$ (bulk Mn_2Au). Simulated $F_d/J_3 = 23{,}11$ ⇒ трябва вместо J_1 в F_d.

$F_d = m_1 J_1 + 8J_3$, $F_d/J_3 = 8 + m_1\frac{J_1}{J_3} = 23{,}11$, $m_1 \approx (23{,}11-8)\frac{J_3}{J_1} = 15{,}11\cdot\frac{115K}{(-396K)} = -4{,}39$

Т.к. m_1 - integer, то берем $m_1 = -4$, $F_d = -4J_1 + 8J_3 = 4(2J_3 - J_1) = 8\left(J_3 + \frac{1}{2}|J_1|\right)$

Note: имаме $\left(J_3 + \frac{1}{2}|J_1|\right)$ вместо J_3 в полученото за двуслойна ultrathin film. Вид F_d одинаков за bulk „и thin film“, т.е. имаме сходящи in xy-plane (J_3) и diagonal (J_1) не променят при „bulk“ ⇒ thin film.

$F_A = \frac{\mu^2}{\gamma^2}\frac{1}{8K_{211}}\left(\frac{c_s}{\Delta_s}\right)^2 = 4\hbar^2\frac{1}{8K_{211}}\left(\frac{c_s}{\Delta_s}\right)^2 = \frac{1}{2K_{211}}\left(\frac{\hbar c_s}{\Delta_s}\right)^2 = \frac{1}{2\cdot 7\cdot 1{,}855\cdot 10^{-25}J}\left(\frac{1{,}055\cdot 10^{-34}Js\cdot 43{,}3\cdot 10^3\frac{m}{s}}{19{,}78\cdot 10^{-9}m}\right)^2$

$= \frac{1}{14\cdot 1{,}855\cdot 10^{-25}J}\left(\frac{1{,}055\cdot 43{,}3}{19{,}78}10^{-22}J\right)^2 = \frac{5{,}334\cdot 10^{-44}J^2}{14\cdot 1855\cdot 10^{-25}J} = 0{,}2054\cdot 10^{-19}J$

$\frac{F_A}{|J_2|} = \frac{0{,}2054\cdot 10^{-19}J}{532K\cdot 1{,}381\cdot 10^{-23}\frac{J}{K}} = 2{,}795\cdot 10^4\frac{10^{-19}}{10^{-23}} = 2{,}795$ (!).

7/VIII (bulk резюме) Имаме $A = \frac{2}{M_0^2 V}|4J_1 + J_2| = \frac{1}{M_0^2 V}F_A$, $F_A = 2|4J_1 + J_2|$, $\frac{F_A}{|J_2|} = 2\left(\frac{4J_1}{J_2} + 1\right) =$

$= 2\left(4\cdot\frac{396}{532} + 1\right) = 7{,}955$. $\frac{F_A}{|J_2|} = n_1\frac{J_1}{|J_2|} + n_2 = 2{,}795$.

Ако bulk (7/VIII) $F_A/|J_2| = 8\frac{J_1}{J_2} + 2 = \frac{1}{|J_2|}(-8J_1 - 2J_2)$, $F_A = -8J_1 - 2J_2$, $n_2 = +2$

Ако thin film имаме J_2 - bonds в 2 реда повече, т.к. (1)-(4), (4)-(1) бонда нез.

Ако thin film отново $F_A = n_1 J_1 - J_2$ ($n_2 = -1$), $\frac{F_A}{|J_2|} = n_1\frac{J_1}{|J_2|} + 1 = 2{,}795$

$n_1 \approx (2{,}795 - 1)\frac{|J_2|}{J_1} = -1{,}795\cdot\frac{532K}{115K} = -8{,}30$ (396 / 115 нас). Берем $n_1 = -8$, $F_A = -8J_1 - J_2$,

или $F_A = |8J_1 + J_2| = 2\left|4J_1 + \frac{1}{2}J_2\right|$. Ако bulk „е верно $F_A = 2|4J_1 + J_2|$

Вариант: от „bulk“ values (без член с J_1 в A_3) 7/VIII надо перейти к „thin film“ values $F_A = 2\left|4J_1 + \frac{1}{2}J_2\right|$, $F_d = 8J_3\left(1 + \frac{|J_1|}{2J_3}\right)$.

$A = \frac{1}{M_0^2 V}2\left|4J_1 + \frac{1}{2}J_2\right|$, $d = \frac{a^2}{M_0^2 V}\cdot 8J_3\left(1 + |J_1|/2J_3\right)$, $J_3 \to J_3' = J_3(1 + |J_1|/2J_3)$

$c = \gamma M_0\sqrt{d\cdot A} = \gamma M_0\sqrt{\frac{2|4J_1 + J_2/2|}{M_0^2 V}\frac{8J_3 a^2}{M_0^2 V}(1 + |J_1|/2J_3)} = \frac{4\gamma a}{M_0 V}\sqrt{|4J_1 + J_2/2|J_3(1 + |J_1|/2J_3)}$

$M_0 V = \mu = \mu_{Mn} = 4\mu_B$, $\gamma/\mu = \frac{1}{2\hbar}$, $c = (2a/\hbar)\sqrt{|4J_1 + J_2/2|\cdot(J_3 + |J_1|/2)}$

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$$c = \frac{2a}{\hbar} k_B \sqrt{(J \cdot \ln K)} = \frac{2 \cdot 0{,}3328\,\text{nm} \cdot 1{,}381 \cdot 10^{-23}\,\text{J/K} \cdot \text{K} \cdot \sqrt{(4.396 + \frac{1}{2}532)(115 + \frac{1}{2}396)}}{1{,}055 \cdot 10^{-34}\,\text{J}\cdot\text{s}} =$$

$$= \frac{2 \cdot 0{,}3328 \cdot 1{,}381 \cdot 10^{-9}\,\text{m} \cdot 10^{-23}}{1{,}055 \cdot 10^{-34}\,\text{s}} \sqrt{1850 \cdot 313} = \frac{2 \cdot 0{,}3328 \cdot 1{,}381}{1{,}055} \cdot 10^{2} \cdot 76035\ \text{m/s} = 66{,}3\ \frac{\text{km}}{\text{s}}.$$

$$\Delta_0 = a\sqrt{\frac{F_d}{8F_{211}}} = a\sqrt{\frac{J_3}{K_{211}}\left(1 + |J_1|/2J_3\right)} = 11{,}64\,\text{nm}\sqrt{1 + \frac{396}{2 \cdot 115}} = 19.20\ \text{nm}.$$

Пробваме подредбата FA. $F_A = 2{,}795\,|J_2|$, $F_d = 23{,}11\,J_3$ - fitted.

$$c = \frac{\sqrt{8}a}{\hbar}\sqrt{F_d F_A} = \frac{a}{2\hbar}\sqrt{23{,}11 \cdot 2{,}795}\, k_B \sqrt{532 \cdot 115}\ \text{K} = \frac{0{,}3328\,\text{nm} \cdot 1{,}381 \cdot 10^{-23}\,\text{J/K}}{2 \cdot 1{,}055 \cdot 10^{-34}\,\text{J s}} \times$$

$$\times 8{,}037 \cdot 247{,}3_k = 433{,}0 \cdot 10^{-9}\,\text{m}\ 10^{-23}\ 10^{+34}\,\frac{1}{\text{s}} = 433{,}0 \cdot 10^{2}\,\frac{\text{m}}{\text{s}} = 43{,}3\ \text{km/s}\ (!).$$

Но сега $F_d = 8(J_3 + |J_1|/2)$ $(n_1 = -4)$, $F_d/J_3 = 8(1 + |J_1|/2J_3) = 21{,}77$;

$F_A\,(n_1 = -8) = 2\,|4J_1 + \frac{1}{2}J_2|$, $F_A/|J_2| = 2(4J_1/J_2 + 1/2) = 6{,}955\ (?) = 2{,}729$.

$F_A/|J_2| = n_1\frac{J_1}{|J_2|} - n_2 = |n_1 = -8.30,\ n_2 = -1| = 1{,}795 + 1 = 2{,}795$.

$F_A/|J_2| = |n_1 = -8,\ n_2 = -1| = 1{,}729 + 1 = 2{,}729$

$c = \frac{2a}{\hbar}(\ldots)$. Ние разчем n_1 а вместо, подсъвайки J_3 вместо J_1, $(n_1 = -2.30$ не работи).

Тогава $n_1 = (2{,}795 - 1)(-1)\frac{J_2}{J_1} = -1{,}795\,\frac{532}{396} = -2{,}411$,

тогава $F_A/|J_2| = -n_1\left(\frac{J_1}{J_2}\right) - n_2 = +2{,}411 \cdot \frac{396}{532} + 1 = 2{,}795$ из fitting c_{max}

Вземем $n_1 = -2$ вместо $n_1 = -2{,}411$, $F_A/|J_2| = 2\frac{396}{532} + 1 = 2{,}489$

Calculated $c = \frac{2a}{\hbar}\sqrt{F_d F_A} = \frac{2a}{\hbar}\sqrt{21{,}77 \cdot J_3 \cdot 2{,}489 \cdot |J_2|} = \frac{2a}{\hbar}\sqrt{21{,}77 \cdot 2{,}489} \cdot \sqrt{J_3 |J_2|}$

Calculated $c = 43{,}3\,\frac{\text{km}}{\text{s}}\sqrt{\frac{21{,}77 \cdot 2{,}489}{23{,}11 \cdot 2{,}795}} = 43{,}3\,\frac{\text{km}}{\text{s}} \cdot 0{,}916 = 39{,}66\,\frac{\text{km}}{\text{s}} = 397\ \text{km/s}$.

$n_1 = -2$ дава $F_A = -2J_1 - J_2 = |2J_1 + J_2|$, тъй като приемаме, че „thin film".

Вариант: 1) разсм. „thin" 1 unit cell ⊥ oz и 1 unit cell ∥ Oy.

тогава няма J_2 свързващи съседни, т.к. edge atoms ∈ 2 unit cells, а не 4-те cells как че „thin film";

2) отказваме се от $\vec{m}_1 = \vec{m}_3$, $\vec{m}_2 = \vec{m}_4$, т.к. (1) layer не еквивалентен на (3) осъществяващ vertical bonds че (1), (4) layers в „thin films".

Пробваме че „thin film" $MnAu_2$ $\vec{m}_4 = -\vec{m}_1$, $\vec{m}_3 = -\vec{m}_2$.

тогава $\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \vec{M}_4 \equiv 0$, и няма $w_{ex} \sim A\vec{M}^2$ не работи.

Оставаме standard „bulk" ansatz $\vec{m}_3 = \vec{m}_1$, $\vec{m}_4 = +\vec{m}_2$, но вземаме $\frac{E_{0x}}{u.c.}(19/\text{Å})$ че „thin film". Тогава няма J_2 - bonds/u.c. бъдат = 1, т.е. $J_2 \to J_2/2$.

Няма J_1 bonds/u.c. = 8 и J_3 - bonds/u.c. запазват се (19/Å).

$$\frac{E_{0x}}{u.c.}(\text{film}) = 4\left|4J_1 + \frac{1}{2}J_2\right|\vec{m}^2 + 4\left(J_3 - \frac{1}{4}J_1\right)\left(\frac{\partial \vec{L}}{\partial x}\right)^2,\quad J_2 \to \frac{1}{2}J_2,\ \text{no сравн. с bulk}$$

$$A = \frac{1}{M_0^2 V}\,2\left|4J_1 + \frac{1}{2}J_2\right|,\quad \alpha = \frac{8a^2}{M_0^2 V}J_3' = \frac{8a^2}{M_0^2 V}\left(J_3 + \frac{1}{4}|J_1|\right)\quad J_3 \to J_3' = J_3 + \frac{1}{4}|J_1|$$

$$\Delta_0 = a\sqrt{\frac{J_3}{K_{211}}}\cdot\sqrt{\left(1 + \frac{1}{4}\frac{|J_1|}{J_3}\right)} = 11{,}64\,\text{nm} \cdot \sqrt{1 + \frac{1}{4}\frac{396}{115}} = 11{,}64\,\text{nm}\sqrt{1{,}861} = 15{,}88\ \text{nm}$$

Film $c = \frac{2a}{\hbar}\sqrt{\left|4J_1 + \frac{1}{2}J_2\right| J_3\left(1 + \frac{1}{4}\frac{|J_1|}{J_3}\right)} = \frac{2 \cdot 0{,}3328 \cdot 1{,}381 \cdot 10^{-9} \cdot 10^{-23}\,\text{m}}{1{,}055 \cdot 10^{-34}\,\text{s}}\sqrt{1850 \cdot 214} = 54{,}82\ \frac{\text{km}}{\text{s}}$ (thin film)

Ако bulk + същото J_1 вместо в J_3 $\Delta_0 = 15{,}9$ nm запазва се.

$c\,(\text{bulk}) = (\ldots)\sqrt{|4J_1 + J_2|J_3\left(1 + \frac{1}{4}\frac{|J_1|}{J_3}\right)} = (\ldots)\sqrt{(4.396 + 532) \cdot 214} = (\ldots)\sqrt{2116 \cdot 214} = 58{,}63\ \frac{\text{km}}{\text{s}}$

$c = 0{,}8713 \cdot 10^{2}\sqrt{|J_{2\,eff}|J_3'}\ \frac{\text{m}}{\text{s}}$, J_2, J_3 in K, $J_{2\,eff} = \begin{cases} 4J_1 + J_2\,(\text{bulk}) \\ 4J_1 + J_2/2\,(\text{film}) \end{cases}$, $J_3' = J_3 + \frac{1}{4}|J_1|$

$\to \Delta_0\,(\text{bulk/film}) = 15{,}9$ nm, $c\,(\text{film}) = 54{,}8$ km/s, $c\,(\text{bulk}) = 58{,}6\ \frac{\text{km}}{\text{s}}$ и $\vec{m}_3$ - ansatz.

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(22/XI) E-mail to Ricardo y Ruben с обяснением перенормировки A_3 из-за $J_3 \to J_3' = J_3 + \frac{1}{4}|J_1|$ и $J_2 \to J_2/2$ (ultra-thin film), и 2 new projects: a) BP DW in nanowires; b) FiM dynamics near angular momentum comp. point. На восточную горку с Ю. и Л. Pawel прислал SW edge scatter. Over-leaf. Цифра сообщения, 200 (ужарила малина.

(23/XI) Поправели в Киев Инне 500 € (W.U.) на похороны мамы и поминки. С А. в „Al Campo" Quartzun. Вухра прислала e-mail о расчет FiM dyn. Skype с Игорь по Pawel-SW-APL paper (это не часть of the mult. DW-e SW scattering" MS, a new paper). Поминки после похорон мамы „в Киеве.

(24/XI) Работа над Pawel-SW-APL paper. Послал comments to Pawel, Igor, Maciej Skype с Вухрой по new Skyrmion FiM manuscript. Gain H=47.

(25/XI) Получил в ИФМ/ЕНИ new comp., small Dell Latitude 2 in 1"; СЛ. „Eroski" Urbil tracks Бизне Urbil повредили нам левое зеркало заднего вида. E-mails to Ruben, Oksana, Pawel, Igor.

FiM dyn ••C.D. Stanciu – T. Rasing. GdFeCo ultrafast dyn.: angular momentum comp. PRB 73, 220402(2006)

Две моды FMR: $\omega_{FMR} = \gamma_{eff} |\vec{H}_{eff}|$, $\vec{H}_{eff}$ через $\vec{H}_{eff}$ of LLG для $\vec{M}_1$, $\vec{M}_2$

$$\gamma_{eff} = \frac{M_1 - M_2}{\left(\frac{M_1}{\gamma_1} - \frac{M_2}{\gamma_2}\right)} = \frac{M_1(T)}{A(T)} \quad \text{и} \quad \omega_{ex} = \lambda(\gamma_2 M_1 - \gamma_1 M_2) = \lambda \gamma_1 \gamma_2 A(T),$$

$\vec{H}_{ex}(i) = -\lambda \vec{M}_{j,i}$

$\begin{cases} \omega_{FMR}(T_A), \gamma_{eff}(T_A) \to \infty, \text{ т.к. } A(T_A) \to 0, \text{ T-angular moment comp. temperature.} \\ \omega_{ex}(T_A) \to 0 \end{cases}$

Exp. $d_{eff}(T)$ – max при T_A. Но $\omega_{ex} \to 0$, $\omega_{FMR} \to \infty$ вблизи T_A нет, т.к. ω_{ex} и ω_{FMR} „mix". $\omega_{FMR} \sim \gamma_{eff} \sim M(T) \to 0$ такие exp. не наблюдался (!).

$T \to T_M$, $M(T_M) = 0$

Хорошее согласие с Kittel (1949, 1953) theory линии при $T > T_A$ ($T_A > T_M$)

••M. Binder. PRB 74, 134404(2006). Dyn. GdCo FiM near T_M и T_A.

Exp (pump-probe) видны плавные линии $\gamma_{eff}(T)$, $d_{eff}(T)$ при $T \simeq T_A$. Частота FMR при $T \to T_A$ растет незначительно, с 10-14 GHz до ≃ 15 GHz.

••T. Kato et al. IEE Trans. Magn. 44, 3380 (2008). $Gd_x(FeCo)_{1-x}$ films, FMR/pump-probe

$\gamma_{eff}(x)$, $L_{eff}(x)$ Smooth max при $x \simeq x_A$, $\omega_{FMR}(x)$ 16 GHz, 9 GHz, 0, x_A, x_M, x

••D.H.-Kim – T. Ono. Low damping of FiM GdFeCo. PRL'2019

SW mobility $\mu(T)$ имеет max при T_A а $L_{eff}(T)$ – нет и $\alpha = 0{,}008 \simeq$ const. Значит „все обычная FiM-theory не применима к L_{eff} GdFeCo, damping α мало и нет e⁻-mechanism.

(26/XI) Теория FiM films by Зубкова и обсужд. Skype.

(27/XI) В ремонт. Массировку Astigarraga (связ. с AXA ser.) „Carrefour" Quartzun.

(28-29/XI) Приготов. slides „DW-SW scatter." для JEMS'2020 доклада.

(30/XI) Обсужд. FiM dyn. MS с Зухрой. Zoom conf. по Nanowires project (Оксана, Madrid, Oviedo teams). Зарегистр. на JEMS и опл. 300 € Conf. fee.

•Booked Hotel in Vienna, May 30 – June 3 (ИНМ'2021). Отменить до 29/V 2021.

Edited SW-edge scattering с Pawel.

(1/XII) Edited SW-edge scattering MS и послал comments соавторам. Близ.

(2/XII) Sent file to Ricardo с моим выводом E_{ex} Mn_2Au. Предложил Ikeriba-Tsu (IBT-теория) много Mn_2Au Commun. Phys.'2020 paper для highlights. → Зухра прислала о new „Dyn-SW modes" paper submitted to APL, предложил Вухре со-author и Вчера срочно пробную в текст. одним.

(3/XII) Разбор file Ricardo по $E_{ex}(J_1 J_2 J_3)$ Mn_2Au, послал подробный ответ с комментариями

(4/XII) Skype обсуждение с Зухрой вывода eq. Thiele для FiM из L_{eff}, отличие от Cala, Tserkovnyak'2017 и Inoue'2019. Разобрались с χ_1 – Loss' PRB'1997.

JEMS обсуж. session для session Chairmans (Zoom) с 11., 10. Carrg. „Lidl" Palermo

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(5/VII) Norman Guidelines JEMS session chair to Anton.

FiM • E. Schlömann J. Phys Chem. Sol. 2, 214 (1957)

Obe подрешетки дают 2 sublattice FiM $\omega_0 = \gamma H$ и $\omega_1 = \lambda(\gamma_B M_a + \gamma_a M_B)$

• при разложении dispersion eq. по $\frac{1}{\lambda}$ ($\lambda \sim 10^3$ inter-sull. exchange parameter),

$\gamma = \gamma_{eff} = \frac{M_B - M_a}{(M_B/\gamma_B) - (M_a/\gamma_a)}$. При angular momentum comp. point $S_a \simeq S_B$, разложение по $\gamma\lambda$ невозможно, $\omega_1 \approx \omega_0$, и γ_{eff} не вводим

При $S_a = S_B$, $\gamma_a = \gamma_B = \gamma$ (AF case) точное решение $\frac{\omega_{0,1}}{\gamma} = H \pm \sqrt{H_{an}(2M_a + H_{an})}$

Без ТВ по $\frac{1}{\lambda}$; $U(\omega) = \gamma_B M_B[\gamma_a(H - \lambda M_B) - \omega] -$

$- \gamma_a M_a[\gamma_B(H + \lambda M_B) - \omega] + 2\lambda\gamma_a\gamma_B M_a M_B$ (Ham преобразуем)

Т.е. $\gamma_{eff} \to \infty$ более не дает $\omega_0 = \gamma_{eff} H \to \infty$, т.к. при T_A ω_0, ω_1 другие. ($S_a = S_B$)

$U(\omega) = [\gamma_a(H - \lambda M_B)][\gamma_B(H + \lambda M_a) - \omega] + \lambda^2\gamma_a\gamma_B M_a M_B = 0$ секулярное ур-е.

$\omega^2 - \omega[(\gamma_a + \gamma_B)H - \lambda(\gamma_a M_B - \gamma_B M_a)] + \gamma_a\gamma_B(H - \lambda M_B)(H + \lambda M_a) + \lambda^2\gamma_a\gamma_B M_a M_B = 0$

$\omega^2 - \omega[\gamma_+ H - \lambda\gamma_a\gamma_B(\frac{M_B}{\gamma_B} - \frac{M_a}{\gamma_a})] + \gamma_a\gamma_B[H^2 - \lambda(M_B H - M_a H)] = 0$

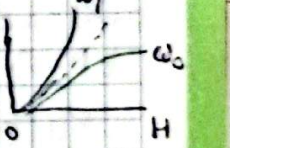
$\frac{S_B}{\gamma_B} - \frac{S_a}{\gamma_a} = \frac{M_B}{\gamma_B} - \frac{M_a}{\gamma_a} = S$, $M_B - M_a = m$, $\omega^2 - \omega[\gamma_+ H - \lambda\gamma_a\gamma_B S] + \gamma_a\gamma_B H(H - \lambda m) = 0$

$\gamma_+ = \gamma_a + \gamma_B$. $2\omega_{0,1} = \gamma_+ H - \lambda\gamma_a\gamma_B S \pm \sqrt{(\gamma_+ H - \lambda\gamma_a\gamma_B S)^2 - 4\gamma_a\gamma_B H(H - \lambda m)}$

1) Angular comp. $S = 0$, $2\omega_{0,1} = \gamma_+ H \pm \sqrt{\gamma_+^2 H^2 - 4\gamma_a\gamma_B H(H - \lambda m)}$, $\omega_{0,1}$ – точечные

$\gamma_+^2 - 4\gamma_a\gamma_B = (\gamma_a + \gamma_B)^2 - 4\gamma_a\gamma_B = (\gamma_a - \gamma_B)^2 = \gamma_-^2$.

$2\omega_{0,1} = \gamma_+ H \pm \sqrt{\gamma_-^2 H^2 + 4\lambda\gamma_a\gamma_B H m} \approx \gamma_+ H \pm 2\sqrt{\lambda\gamma_a\gamma_B H m}$,



2) Magn. Compensation or AF, $m = 0$

$2\omega_{0,1} = \gamma_+ H - \lambda\gamma_a\gamma_B S \pm \sqrt{(\gamma_+ H - \lambda\gamma_a\gamma_B S)^2 - 4\gamma_a\gamma_B H^2}$ при $\approx$ при $\lambda\gamma_a\gamma_B S \gg \gamma_+ H$

$2\omega_{0,1} = -\lambda\gamma_a\gamma_B S \pm |\lambda\gamma_a\gamma_B S| \underset{S>0}{=} \begin{cases} -2\lambda\gamma_a\gamma_B S \\ 0 \end{cases}$, $\omega_0 = -\lambda\gamma_a\gamma_B S$, $\omega_1 = 0$, $S > 0$.

при при $S < 0$, $\omega_0 = 0$, $\omega_1 = \lambda\gamma_a\gamma_B |S|$. $\gamma_{eff} = m/S$.

3) ТВ по $\gamma\lambda$, $\sqrt{(\dots)} = \lambda\sqrt{(\frac{\gamma_+}{\lambda} H - \gamma_a\gamma_B S)^2 - 4\gamma_a\gamma_B \frac{H}{\lambda}(\frac{H}{\lambda} - m)} \approx$

$\approx \lambda\sqrt{(\gamma_a\gamma_B S)^2 - 2\gamma_a\gamma_B S\gamma_+ \frac{H}{\lambda} + \frac{\gamma_+^2}{\lambda^2} H^2 + 4\gamma_a\gamma_B \frac{H}{\lambda} m - 4\gamma_a\gamma_B \frac{H^2}{\lambda^2}} =$

$= \lambda\gamma_a\gamma_B S\sqrt{1 + 2\gamma_a\gamma_B \frac{H}{\lambda}[2m - S\gamma_+] \frac{1}{(\gamma_a\gamma_B S)^2} + \frac{H^2}{\lambda^2}\gamma_-^2 \frac{1}{(\gamma_a\gamma_B S)^2}} = |\frac{1}{\lambda} = \varepsilon| =$

$= \lambda\gamma_a\gamma_B S\sqrt{1 + a_1\varepsilon + a_2\varepsilon^2}$, $\sqrt{1 + x} = 1 + \frac{x}{2} - \frac{1}{8}x^2 = 1 + \frac{1}{2}(a_1\varepsilon + a_2\varepsilon^2) -$

$- \frac{1}{8}(a_1\varepsilon + a_1\varepsilon^2)^2 = 1 + \frac{1}{2}a_1\varepsilon + \frac{1}{2}a_2\varepsilon^2 - \frac{1}{8}a_1\varepsilon^2 = 1 + \frac{1}{2}a_1\varepsilon + \frac{1}{2}\varepsilon^2(a_2 - \frac{1}{4}a_1)$

$a_1 = \frac{2\gamma_a\gamma_B H(2m - S\gamma_+)}{(\gamma_a\gamma_B S)^2}$, $a_2 = \frac{\gamma_-^2 H^2}{(\gamma_a\gamma_B S)^2}$, $a_2 - \frac{1}{4}a_1 = \frac{1}{(\gamma_a\gamma_B S)^2}[\gamma_-^2 H^2 - \frac{1}{4}2\gamma_a\gamma_B H(2m - S\gamma_+)$

$= \frac{H}{(\gamma_a\gamma_B S)^2}[\gamma_-^2 H - \frac{1}{2}\gamma_a\gamma_B(2m - S\gamma_+)]$, $\sqrt{(\dots)} = \lambda\gamma_a\gamma_B S[1 + \frac{1}{2}\frac{1}{\lambda}\frac{2\gamma_a\gamma_B H(2m - S\gamma_+)}{(\gamma_a\gamma_B S)^2} + O(\frac{1}{\lambda^2})]$

$\sqrt{(\dots)} = \lambda\gamma_a\gamma_B S[1 + \frac{1}{\lambda}\frac{H(2m - S\gamma_+)}{\gamma_a\gamma_B S^2} + \frac{1}{2}\frac{1}{\lambda^2}\frac{H\{\gamma_-^2 H - \frac{1}{2}\gamma_a\gamma_B(2m - S\gamma_+)\}}{(\gamma_a\gamma_B S)^2}] \approx$

$\approx \gamma_a\gamma_B S + H\frac{(2m - S\gamma_+)}{S} = \gamma_a\gamma_B S + H(2\frac{m}{S} - \gamma_+)$

$\to 2\omega_{0,1} = \gamma_+ H - \lambda\gamma_a\gamma_B S \pm [\gamma_a\gamma_B S + H(2\frac{m}{S} - \gamma_+)]$, $2\omega_0 = \gamma_+ H - \lambda\gamma_a\gamma_B S - \gamma_a\gamma_B S -$

$- H(2\frac{m}{S} - \gamma_+) = 2\gamma_+ H - 2H\frac{m}{S} - 2\lambda\gamma_a\gamma_B S$, $\omega_0 = -\lambda\gamma_a\gamma_B S + \gamma_+ H - H\frac{m}{S}$.

$2\omega_1 = \gamma_+ H - \lambda\gamma_a\gamma_B S + \gamma_a\gamma_B S + H2\frac{m}{S} - \gamma_+ H$, $\omega_1 = \frac{m}{S} H = \gamma_{eff} H$.

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$\omega_0 \to \omega_1$, $\omega_0 = \gamma_{eff} H$, $\omega_1 = -\lambda\gamma_a\gamma_b s + \gamma_+ H - \gamma_{eff} H$, $\gamma_{eff} = \frac{m}{s} = \frac{M_b - M_a}{(M_b/\gamma_b) - \frac{M_a}{\gamma_a}}$

Решение Schlömann'57, $\omega_0 = \gamma_{eff} H$, $\omega_1 = \lambda(\gamma_b M_a - \gamma_a M_b) = -\lambda\gamma_a\gamma_b s$.

т.е. наше решение ТВ по $\frac{1}{\lambda}$ при $H \ll \lambda s \gamma_{a,b}$ переходит в решение Schlömann.

Очевидно $A(t)$ (Slonczewski'2006) $= \frac{M_1}{\gamma_1} - \frac{M_2}{\gamma_2} = -s$ и $\omega_{ex} = -\lambda\gamma_1\gamma_2 s = \omega_1$ (Schlöm.)

ТВ по $\frac{1}{\lambda}$ есть ТВ по $\frac{1}{\lambda s^2}$ и не работает вблизи angul. compensation $s = \frac{M_b}{\gamma} - \frac{M_a}{\gamma_a} \approx 0$.

γ_{eff} вблизи $s=0$ не работает (!)

- R.K. Wangsness, PR 86, 146 (1952), Eqs. FiM дают $\omega_0 = \omega_{FMR}$ и ω_{ex} с γ тензором намагничивания, в духе Kittel-FMR eq. γ_{eff} не вводится.
- F. Keffer, C. Kittel, PR 85, 329 (1952). Фундамент. paper по AFMR.
- T. Nagamiya, Prog. Th. Phys. VI, 350 (1951). теория AFMR – красиво, в духе 2-model.
- J. Kaplan, C. Kittel, J. Chem. Phys. 21, 760 (1953). Впервые выведена FiM частота $\omega_{ex} = \lambda(\gamma_a M_b - \gamma_b M_a) = \lambda\gamma_a\gamma_b s$, $\lambda \to -\lambda$, $s = \frac{M_b}{\gamma_b} - \frac{M_a}{\gamma_a}$, γ_{eff} нет. Возбуждается лишь при $\gamma_a \neq \gamma_b$, т.е. $f(\omega) \sim (\gamma_a - \gamma_b)^2$.
- R.K. Wangsness, PR 91, 1085 (1953). Phenom. theory of mag. resonance of 2- (and 4-) sublattice ferrimagnet. Общая формула для ω_0, ω_{ex}, при $\frac{R}{F^2} \ll 1$ $\gamma_{eff} = \frac{M_1 + M_2}{(M_1/\gamma_1) + (M_2/\gamma_2)}$, $M_{1z} = M_{iz}$.

$\frac{R}{F^2} = \frac{P^2(\gamma^2 - \delta^2)^2 Q_x Q_y}{\frac{1}{4}(\gamma P - \delta\lambda\Delta)^4}$, $\gamma = \frac{1}{2}(\gamma_1 + \gamma_2)$, $\delta = \frac{1}{2}(\gamma_1 - \gamma_2)$, $M = M_1 + M_2$, $\Delta = M_1 - M_2$

Без намагничивания $Q_{x,y} = H$, $P = H + \lambda M$, $\frac{R}{F^2} = \frac{4P^2(\gamma^2 - \delta^2)^2 H^2}{(\gamma P - \delta\lambda\Delta)^4}$

$\gamma P - \lambda\delta\Delta = \frac{1}{2}(\gamma_1 + \gamma_2)[H + \lambda(M_1 + M_2)] - \frac{\lambda}{2}(\gamma_1 - \gamma_2)(M_1 - M_2) =$

$= \gamma H + \frac{\lambda}{2}(\gamma_1 + \gamma_2)(M_1 + M_2) - \frac{\lambda}{2}(\gamma_1 - \gamma_2)(M_1 - M_2) = \gamma H + \lambda(\gamma_2 M_1 + \gamma_1 M_2)$

$\gamma P - \lambda\delta\Delta = \gamma H + \lambda\gamma_1\gamma_2\left(\frac{M_1}{\gamma_1} + \frac{M_2}{\gamma_2}\right)$. При angular m. compensation $\frac{M_1}{\gamma_1} + \frac{M_2}{\gamma_2} = s = 0$

и $\frac{R}{F^2} = 4\frac{(H + \lambda M)^2(\gamma^2 - \delta^2)^2 H^2}{\gamma^4 H^4} = 4\frac{(\gamma^2 - \delta^2)^2}{\gamma^4}\frac{(H + \lambda M)^2}{H^2}$ перестает быть $\ll 1$, т.к. $M \neq 0$ ($R/F^2 \ll 1$)

Schlömann'57 воспроизвел ТВ Wangsness'53 в области Wangsness'53 в точке углов частота $\omega_{ex} = \lambda(\gamma_2 M_1 + \gamma_1 M_2)$ by Kaplan'53.

т.е. γ_{eff} имеет смысл вдали от angul. momentum comp. point $s=0$

Несколько papers by Wangsness в 1954–1960 к которым Sa. Hub не дает доступа. (no FiM resonance)

- R.K. Wangsness, PR 93, 68 (1954). FiM частоты для $M=0$ и $s=0$ special cases. $s=0$ compens. point не разрешает γ_{eff}.
- R.K. Wangsness, Am. J. Phys. 24, 60 (1956) Обширная paper по FiM mag. resonance. γ_{eff} при $s=0$ не $\to \infty$, хотя may be extremely large, Ref to Wangsness' 1953.
- R.K. Wangsness, PR 111, 813 (1958) Effective parameters in FiM resonance. При angul. m. compensation $s=0$, $s_2 = -s_1$, $s_1 = M/\gamma_1$, $M = M_{1z} + M_{2z} \neq 0$

$\gamma_{eff}(s=0) = \frac{(\gamma_1 - \gamma_2)}{(d_1 + d_2)}\frac{1}{s_1}$ определяется damping d_1, d_2; $d_{eff}(s=0) = \frac{1}{(d_1 + d_2)}\frac{1}{s_1^2}$

$\gamma_{eff}(s=0)$ и $\delta_{eff}(s=0)$ конечны, хотя могут быть очень велики.

- G.G. Scott, Rev. Mod. Phys. Review of Gyromagnetic Ratio Experiments, 34, 102 (1962)

$1/g' + 1/g = 1$. For classical rotating body $\gamma = M/L = (e/2m)$. SI.

For e^- $|\gamma_e| = |e|g_e/2m_e$, $g_e = 2{,}002$. History γ exp. values g, g' for magn. mat.

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(6/XII) Подготовка „Skyrmion" talk slides и отработка „SW-DW" talk for JEMS'2020. Написан Zukhre об эффективности near s=0 mechanical moment compensation point.

(7/XII) Online oral talks „SW-DW scattering" и „Skyrmions in restricted geometry" на JEMS-2020. Вопрос Pirro о „SW-DW scat.": не замечаем ли эффект сохран. углового момента 3-магнон SW-DW scattering? Skype с Зухрой и А.К. Звездин о конкурентности Igor о эфф. near ang. momentum compensation $s=0$. Послал Lo A.K. Wangeness'53 paper по ω_0, ω_{ex} и Hsu'2020 по наблюдению Kaplan-Kittel resonance mode с $\omega_{ex}/2\pi \simeq 8$ ТГц. Skype с Igor по вопросам „DW-SW" talk и Pawel-APL manuscript. JEMS talks

(8/XII) JEMS talks по ultrafast M-dynamics (Gomonay, Olleros, etc.). Бинц.

(9/XII) JEMS talks - Spintronics.

FiM

••• K.J. Kim ... T. Ono. Nat. Mat. 16, 1187 (2017). DW motion near T_A - FiM.

Energy dissipation in steady motion DW regime = уменьшение Zeeman energy

$$P = 2(\alpha_1 S_1 + \alpha_2 S_2)\frac{A}{\Delta}(V^2 + \Delta^2\Omega^2) = 2(M_1 - M_2)HAV, \quad \dot{X} = V, \ \dot{\Phi} = \Omega$$

Gyrotorque Thiele = damping force $\Rightarrow$ $(\alpha_1 S_1 + \alpha_2 S_2)\Delta\Omega = (S_1 - S_2)V$.

Отсюда V и Ω: $V = \dfrac{\Delta(\alpha_1 S_1 + \alpha_2 S_2)(M_1 - M_2)}{(\alpha_1 S_1 + \alpha_2 S_2)^2 + (S_1 - S_2)^2}H$, $\Omega = \dfrac{(S_1 - S_2)(M_1 - M_2)}{(\alpha_1 S_1 + \alpha_2 S_2)^2 + (S_1 - S_2)^2}H$

$V = V_{max}$ при ангул. момент compensation $S_1 = S_2$

$V_{max} = \dfrac{(M_1 - M_2)\Delta}{(\alpha_1 S_1 + \alpha_2 S_2)}H = \dfrac{1}{S_1}\dfrac{(M_1 - M_2)}{(\alpha_1 + \alpha_2)}\Delta H$. $V(H)$ equation Kim'2017

Ближе к Eq. (11) Звездин JMMM'2020 $\dfrac{\dot{q}}{\Delta} = \dfrac{m}{M}\gamma\dfrac{\alpha}{\chi^2 + \left(\frac{m - m_0}{M}\right)^2}H$, $m = m_0$ coord. T_A.

•• Kim 'Nat. Mat. 2017 Suppl.

Spin density $\vec{S}_1 = S_1\vec{n}_1$, $\vec{S}_2 = S_2\vec{n}_2$, $\vec{n}_i^2 = 1$, $S_1 = M_1/\gamma_1$, $S_2 = M_2/\gamma_2$

$\mathcal{L}_{kin} = S_1\vec{a}_1(\vec{n}_1)\cdot\dot{\vec{n}}_1 + S_2\vec{a}_2(\vec{n}_2)\cdot\dot{\vec{n}}_2$, $\vec{a}_1, \vec{a}_2$ - Dirac monopoles potentials

Expanding $\mathcal{L}_{kin}$ to 2nd order in $\vec{m}, \vec{n}$ ($\vec{m} = \vec{n}_1 + \vec{n}_2$, $\vec{n} = \frac{1}{2}(\vec{n}_1 - \vec{n}_2)$) we get

$\mathcal{L}_{kin} = S\dot{\vec{n}}\cdot(\vec{n}\times\vec{m}) + \delta\vec{a}(\vec{n})\cdot\dot{\vec{n}} + \frac{1}{4}\delta\dot{\vec{m}}\cdot(\vec{n}\times\vec{m})$, $S = \frac{1}{2}(S_1 + S_2)$, $\delta = S_1 - S_2$.

Осадим $\mathcal{L}_{kin} = S\dot{\vec{n}}\cdot(\vec{n}\times\vec{m}) + \delta\vec{a}(\vec{n})\cdot\dot{\vec{n}}$, $\vec{n} = \frac{1}{2}(\vec{n}_1 - \vec{n}_2)$ AF vector [SKKim PRB'14].

$\mathcal{L}(\vec{n}, \dot{\vec{m}}) = \mathcal{L}_{kin} - \frac{1}{2}A(\nabla\vec{n})^2 \mp \frac{1}{2\chi}\vec{m}^2 \mp W_{an,z}(\vec{n})$, накинули $\vec{m} = S\gamma\dot{\vec{n}}\times\vec{n}$ как в 2-model AF

$\mathcal{L}(\vec{n}) = \frac{1}{2}\chi S^2(\dot{\vec{n}})^2 + \delta\vec{a}(\vec{n})\cdot\dot{\vec{n}} - \frac{1}{2}A(\nabla\vec{n})^2 \mp W_{an,z}(\vec{n})$ с Refs. to $\vec{m}\cdot\vec{n} = 0$ — ? для FiM

(V.S. Gerasimchuk, A.A. Shitov, Low Temp. Phys. 27, 125 (2001)
BA Ivanov, AL Sukstanskii, Sov Phys. JETP 57, 214 (1983)

Note: FM kinetic term Kim'2017 ~ $\delta = (S_1 - S_2)$, $\mathcal{L}_{kin}$ (FM) Звездин ~ $m = \frac{1}{2}(M_2 - M_1)$

$W = \frac{1}{2}A(\nabla\vec{n})^2 + \frac{1}{2\chi}\vec{m}^2 + W_{an,z}$, $\mathcal{L} = \mathcal{L}_{kin} - W$, Kim'17 знаки $\frac{1}{2\chi}\vec{m}^2$, $W_{an,z}$ —

$S\dot{\vec{n}}\cdot(\vec{n}\cdot\vec{m}) - \frac{1}{2\chi}\vec{m}^2 = \chi S^2(\dot{\vec{n}})^2 - \frac{1}{2\chi}\chi^2 S^2\dot{\vec{n}}^2 \simeq \frac{1}{2}\chi S^2(\dot{\vec{n}})^2$ неучтен.

Два фронта при вводе $\mathcal{L}$ (FiM): 1) разложение $\mathcal{L}_{kin}$; 2) $W(\vec{m}) = \frac{1}{2\chi}\vec{m}^2$

•• S.K. Kim et al. Phys. Rev. B 90, 104406 (2014). DW in AF.

Но в этой paper далее $\mathcal{L}_{kin}$ (FiM) via $\vec{a}_1, \vec{a}_2$, но разложение по $\vec{m}$ приводится лишь для AF, приводя к $\mathcal{L}_{kin}$ (AF) 2-model.

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FiM

• S.H.Oh, ... Y.Tserkovnyak, K.J. Lee. PRB 96, 100407(2017)

Инвариант $\mathcal{I}_{kin}(FiM) = \frac{1}{I} S(\dot{\vec{n}})^2 + \delta\vec{a}(\vec{n})\cdot\dot{\vec{n}}$ со словами, что он вовлечен в ... S.K. Kim et al. PRB 95, 140404(2017) и • A.F. Andreev, V.I. Marchenko, УФН 80 (!?)

↪ $\mathcal{I}_{kin} = -S\vec{a}(\vec{n})\cdot\dot{\vec{n}} + \frac{1}{2} S(\dot{\vec{n}})^2$, S is the net spin density along $\vec{n}$,

$\mathcal{I}_{kin}$ не вовлечен, а имеется $\mathcal{I}_{kin}$ модели с законом $(\dot{\vec{n}})^2$ из $\mathcal{I}_{kin}(\vec{m}_1), \mathcal{I}_{kin}(\vec{m}_2)$ - подробное следуя Marchenko 1980.

1D для $\vec{n}$ сохран. $\mathcal{I}_{kin}$ (без $\alpha, \vec{u}\vec{j}=0$) получено Ivanov JETP 57, 214 (1983).

Eqn eq. Thiele 2D soliton $\vec{n}(\vec{r},t) = \vec{n}_0[\vec{r} - \vec{X}(t)]$ с $\vec{G} = -4\pi S\hat{z}$ гирвектор.

Урок $\vec{H}$ инвариант к $S(\dot{\vec{n}})^2 \to \frac{1}{2}S[\dot{\vec{n}} - g_t\vec{n}\times\vec{H}]^2$, g_t - transverse gyromag. ratio (?!)

... B.A. Ivanov, A.L. Sukstanskii, Sov. Phys. JETP 57, 214 (1983), Sol. St. Com. 50, 523 (1984)

$g_1 = g_2$, $\gamma_1 = \gamma_2$ → mag. compensation $M_1 = M_2$ и angul. compensation points $M_1/\gamma_1 = M_2/\gamma_2$ или $S_1 = S_2$ совпадают. $\vec{L} = \vec{M}_1 - \vec{M}_2$, $\vec{M} = \vec{M}_1 + \vec{M}_2$

$W = \int dV [\delta\vec{M}_1\vec{M}_2 + W_1(\vec{M}_1) + W_2(\vec{M}_2)]$, $\dot{\vec{M}}_i = -\gamma\vec{M}_i\times\vec{H}_i^{eff}$, $\vec{H}_i^{eff} = -\frac{\delta W}{\delta\vec{M}_i}$

Вводя $\vec{L}$ и $\vec{M}$, LLG eqs. подробнее сводятся к $\frac{\partial\vec{L}}{\partial t} = \gamma\delta\vec{L}\times\vec{M}$, откуда

$\vec{M} = \frac{\vec{L}}{L^2}(M_1^2 - M_2^2) + \frac{1}{\gamma^2 L^2}\dot{\vec{L}}\times\vec{L}$, $\vec{M} = M_s\vec{l} + \frac{1}{\gamma\delta}\dot{\vec{l}}\times\vec{l}$, $\vec{l} = \vec{L}/|\vec{L}|$

1D для $\vec{L}$: $-\frac{M_s}{2\gamma M_0^2}\dot{\vec{l}} = \alpha\vec{l}\times[\nabla^2\vec{l} - \frac{1}{c^2}\ddot{\vec{l}}] - \frac{1}{2M_0^2}\vec{L}\times\frac{\partial w_a}{\partial\vec{l}}$, $M_0^2 = \frac{1}{2}(M_1^2 + M_2^2)$, $M_s = \frac{(M_1^2 - M_2^2)}{|\vec{L}|} \simeq$

или $-\frac{1}{\gamma M_0}\dot{\vec{l}} = \vec{l}\times\left[\alpha\nabla^2\vec{l} - \frac{\alpha}{c^2}\ddot{\vec{l}} - \frac{\partial w_a}{\partial\vec{l}}\right]$, $c = \gamma M_0\sqrt{2A\delta}$, $\lambda = \frac{1}{2M_0}(M_1 - M_2)$ $\simeq (M_1 - M_2)$

Ср. 25/VII 1D σ-model по Kosevich 1990, $E = \int dV\, W$, energy density

$W = \frac{1}{2}A\vec{M}^2 - \vec{M}\vec{H} + \frac{\delta}{8}(\nabla\vec{L})^2 + w_a(\vec{L})$ с грехом $\vec{M}^2 << \vec{L}^2$ $|\vec{L}| \sim M_0$ $(A \to \delta)$.

$-\frac{1}{\gamma}\dot{\vec{M}} = \vec{L}\times[\vec{H} + \vec{H}_a + \frac{1}{4}\alpha\nabla^2\vec{L}]$, подставим FiM $\vec{M} = M_s\vec{l} + \frac{1}{\gamma A}\dot{\vec{l}}\times\vec{l}$

$-\frac{1}{\gamma}\frac{d}{dt}\left[M_s\vec{l} + \frac{1}{\gamma A}\dot{\vec{l}}\times\vec{l}\right] = -\frac{M_s}{\gamma}\dot{\vec{l}} - \frac{1}{\gamma^2 A}\ddot{\vec{l}}\times\vec{l} = L\vec{l}\times\left[\vec{H} + \vec{H}_a + \frac{1}{4}\alpha L\nabla^2\vec{l}\right]$

$L^2 = \vec{L}^2 = (\vec{M}_1 - \vec{M}_2)^2 = M_1^2 + M_2^2 - 2\vec{M}_1\vec{M}_2 \rightleftharpoons M_1^2 + M_2^2 + 2M_1M_2 = (M_1 + M_2)^2$

$-\frac{M_s}{\gamma L}\dot{\vec{l}} = \vec{l}\times\left[\alpha'\nabla^2\vec{l} - \frac{\alpha'}{c^2}\ddot{\vec{l}} - \frac{\partial w_a}{\partial\vec{l}}\right]$ совп. с Ivanov'83 при подборе коэффициентов.

$-\frac{M_s}{\gamma}\dot{\vec{l}} = -\frac{1}{\gamma^2 A}\vec{l}\times\ddot{\vec{l}} + L\vec{l}\times\left[\vec{H} - \frac{1}{L}\frac{\partial w_a}{\partial\vec{l}} + \frac{1}{4}\alpha L\nabla^2\vec{l}\right] = (*) + \vec{l}\times\left[L\vec{H} - \frac{\partial w_a}{\partial\vec{l}} + \frac{1}{4}\alpha L^2\nabla^2\vec{l}\right]$

$-\frac{M_s}{\gamma L^2}\dot{\vec{l}} = -\frac{1}{\gamma^2 A L^2}\vec{l}\times\ddot{\vec{l}} + \vec{l}\times\left[\frac{1}{L}\vec{H} - \frac{1}{L^2}\frac{\partial w_a}{\partial\vec{l}} + \frac{1}{4}\alpha\nabla^2\vec{l}\right]$

$-\frac{M_s}{\gamma L^2}\dot{\vec{l}} = \vec{l}\times\left[\frac{1}{4}\alpha\nabla^2\vec{l} - \frac{1}{\gamma^2 A L^2}\ddot{\vec{l}} - \frac{1}{L^2}\frac{\partial w_a}{\partial\vec{l}} + \frac{1}{L}\vec{H}\right]$ и $-\frac{M_s}{2\gamma M_0^2}\dot{\vec{l}} = \vec{l}\times\left[\alpha'\nabla^2\vec{l} - \frac{\alpha'}{c^2}\ddot{\vec{l}} - \right.$

$\frac{\alpha'}{c^2} = \frac{\alpha'}{\gamma^2 A\alpha' M_0'^2} = \frac{1}{\gamma^2 A M_0^2}$, $\left. -\frac{1}{2M_0^2}\frac{\partial w_a}{\partial\vec{l}}\right]$, $-\frac{M_s}{2\gamma M_0^2}\dot{\vec{l}} = \vec{l}\times\left[\alpha\nabla^2\vec{l} - \frac{1}{\gamma^2 A M_0^2}\ddot{\vec{l}} - \frac{1}{2M_0^2}\frac{\partial w_a}{\partial\vec{l}}\right]$

$W_{ex}(I.'83) = \frac{\alpha'}{2}\left[(\nabla\vec{M}_1)^2 + (\nabla\vec{M}_2)^2\right]$, $\vec{M}_1 = \frac{1}{2}(\vec{M} + \vec{L})$, $\vec{M}_2 = \frac{1}{2}(\vec{M} - \vec{L})$

$W_{ex}(I.'83) = \frac{\alpha'}{2}\left[\left\{\frac{\partial}{\partial x}\frac{1}{2}(\vec{M} + \vec{L})\right\}^2 + \left\{\frac{\partial}{\partial x}\frac{1}{2}(\vec{M} - \vec{L})\right\}^2\right] = \frac{\alpha'}{8}\left[(\nabla\vec{M})^2 + (\nabla\vec{L})^2\right] \simeq \frac{\alpha'}{8}(\nabla\vec{L})^2$

α' - exch. Ivanov '83, α - exch. Kosevich 90, $\alpha' = \alpha$.

Iv.83: $\delta\vec{M}_1\cdot\vec{M}_2 = \frac{\delta}{4}(\vec{M} + \vec{L})(\vec{M} - \vec{L}) = \frac{\delta}{4}(\vec{M}^2 - \vec{L}^2)$, $\vec{M}^2 + \vec{L}^2 = 4M_0^2$

$\delta\vec{M}_1\cdot\vec{M}_2 = \frac{\delta}{4}[\vec{M}^2 - 4M_0^2 + \vec{M}^2] = \frac{\delta}{2}\vec{M}^2$, $W_{ex}(Kos'90) = \frac{1}{2}A\vec{M}^2 \Rightarrow A = \delta$

Iv.'83 $c^2 = \gamma^2 M_0'^2 2\alpha\delta$, Kos'90: $-c^2 = \gamma^2 M_0^2 \cdot 2A$, $\Rightarrow 2M_0'^2 = M_1^2 + M_2^2 = M_0^2$

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α', δ, M_0' - в Ivanov'83 α, A, M_0 в Kosevich'90, $|\vec{M}_1| = |\vec{M}_2| = M_0$, $|\vec{L}| = 2M_0$

$2M_0'^2 = M_1^2 + M_2^2$.

УД Kosevich $\dot{\vec{M}} = M_s \dot{\vec{l}} + \frac{1}{\gamma A} \dot{\vec{l}} \times \vec{l}$ осн. $-\frac{M_s}{\gamma} \dot{\vec{l}} - \frac{1}{\gamma^2 A} \ddot{\vec{l}} \times \vec{l} = L \vec{l} \times \left[-\frac{1}{L} \frac{\partial w_d}{\partial \vec{l}} + \frac{d}{4} L \nabla^2 \vec{l} \right]$

(II) $-\frac{M_s}{\gamma} \dot{\vec{l}} = \vec{l} \times \left[\frac{1}{4} d L^2 \nabla^2 \vec{l} - \frac{1}{\gamma^2 A} \ddot{\vec{l}} - \frac{\partial w_d}{\partial \vec{l}} \right]$

УД-Ivanov'83: $-\frac{M_s}{2\gamma M_0'^2} \dot{\vec{l}} = \vec{l} \times \left[\alpha' \nabla^2 \vec{l} - \frac{\alpha'}{c'^2} \ddot{\vec{l}} + \frac{1}{2M_0'^2} \frac{\partial w_d}{\partial \vec{l}} \right]$

Они совпадут если $\alpha' \cdot 2M_0'^2 = \frac{1}{4} d L^2$, $\frac{1}{\gamma^2 A} = \frac{\alpha'}{c'^2} 2M_0'^2$, $c'^2 = \gamma^2 M_0'^2 \cdot 2d\delta$

$L^2 = 8M_0'^2$, $d = \alpha'$, $\frac{1}{\gamma^2 A} = \frac{\alpha \cdot 2M_0'^2}{\gamma^2 M_0'^2 \, 2\alpha\delta}$ совп. при $A = \delta$.

Возможно ли $L^2 = 8M_0'^2 = 4(M_1^2 + M_2^2)$? При том, что $L^2 \approx (M_1 + M_2)^2$.

Видимо надо $L^2 = 4M_0'^2 = 2(M_1^2 + M_2^2)$, тогда при $M_1 = M_2$, $L^2 = 4M_1^2$,

→ Вывод: коэффициенты в УД Ivanov'83 сравнимы, не совп. с 3-model Kosevich УД от 25/XII. Именно в УД Ivanov'83 входит M_0^2 сравнимо недостаток Ivanov'83 assumption $\gamma_1 = \gamma_2$.

→ Note: в Zagel-Ivanov 'PRA'2019 и Ivanov-LTP'2019 никакое M_0^2 не входит. Там все через $\vec{m} = \frac{\vec{S}_1 + \vec{S}_2}{(S_1 + S_2)}$ и $\vec{l} = \frac{\vec{S}_1 - \vec{S}_2}{S_1 + S_2}$ и $S_0 = \frac{1}{2}(S_1 + S_2)$, где $\vec{S}_1, \vec{S}_2$ - spin-volume densities подрешеток.

→ Если $\vec{M} = M_s \vec{l} + \frac{1}{\gamma A} \dot{\vec{l}} \times \vec{l}$ не надо сравн. с $\vec{L}$, то $W(\vec{L})$ надо изменить

Надо получить $\mathcal{L}_{kin}(\vec{l}, \vec{M})$ из равенства $\mathcal{L}_{kin}(\vec{M}_1) + \mathcal{L}_{kin}(\vec{M}_2)$. из

(10/XII) Session chair 4.C. Magn. dynamics, JEMS. Зачет зарплата - 180€

Skype с Лёва, Юра Погорелов, Зухра. Л. новыми Киев: Л. напр. - 11kg.

Лора D. - 9 kg, Anna - 5kg, Dana - 5kg - 30 kg. 45€ + 20€ = 65€

(11/XII) $\mathcal{L}_{kin}(\vec{m}_1) = S_1 \frac{(\vec{n}_1 \times \vec{m}_1)}{(1 + \vec{n}_1 \cdot \vec{m}_1)} \cdot \dot{\vec{m}}_1$, $\mathcal{L}_{kin}(\vec{m}_2) = S_2 \frac{(\vec{n}_2 \times \vec{m}_2) \cdot \dot{\vec{m}}_2}{(1 + \vec{n}_2 \cdot \vec{m}_2)}$, $S_i = \frac{M_i}{\gamma_i}$ FiM

$\vec{n}_i$ - Dirac string direction unit vectors, $\vec{m}_i = \vec{M}_i / |\vec{M}_i|$, $M_i = |\vec{M}_i|$.

Считаем $\vec{n}_1 = \vec{n}_2 = \hat{z}$. Def. $\vec{L} = (\vec{M}_1 - \vec{M}_2)$, $\vec{M} = (\vec{M}_1 + \vec{M}_2)/2$.

$\vec{M}_1 = \vec{M} + \vec{L}$, $\vec{M}_2 = \vec{M} - \vec{L}$, $\vec{M} \cdot \vec{L} = \frac{1}{4}(\vec{M}_1 - \vec{M}_2)(\vec{M}_1 + \vec{M}_2) = \frac{1}{4}(M_1^2 - M_2^2)$

$\vec{M}^2 + \vec{L}^2 = \frac{1}{4}(\vec{M}_1 + \vec{M}_2)^2 + \frac{1}{4}(\vec{M}_1 - \vec{M}_2)^2 = \frac{1}{2}(M_1^2 + M_2^2)$

$\mathcal{L}_{kin}(FiM) = \mathcal{L}_{kin}(\vec{m}_1) + \mathcal{L}_{kin}(\vec{m}_2) = S_1 \cdot \frac{(\hat{z} \times \vec{m}_1) \cdot \dot{\vec{m}}_1}{(1 + \hat{z} \cdot \vec{m}_1)} + S_2 \frac{(\hat{z} \times \vec{m}_2) \dot{\vec{m}}_2}{(1 + \hat{z} \cdot \vec{m}_2)}$

$\mathcal{L}_{kin} = \frac{S_1}{M_1^2} \frac{(\hat{z} \times \vec{M}_1) \cdot \dot{\vec{M}}_1}{(1 + \hat{z} \cdot \vec{m}_1)} + \frac{S_2}{M_2^2} \frac{(\hat{z} \times \vec{M}_2) \cdot \dot{\vec{M}}_2}{(1 + \hat{z} \cdot \vec{M}_2)}$. Def. $a_i = \frac{S_i}{M_i^2} \frac{1}{(1 + \hat{z} \cdot \vec{m}_i)}$

$\mathcal{L}_{kin} = a_1 (\hat{z} \times \vec{M}_1) \dot{\vec{M}}_1 + a_2 (\hat{z} \times \vec{M}_2) \dot{\vec{M}}_2 = a_1 \hat{z} \cdot (\vec{M}_1 \times \dot{\vec{M}}_1) + a_2 \hat{z} \cdot (\vec{M}_2 \times \dot{\vec{M}}_2)$

$a_1 (\vec{M}_1 \times \dot{\vec{M}}_1) + a_2 (\vec{M}_2 \times \dot{\vec{M}}_2) = a_1 \left[(\vec{M} + \vec{L}) \times (\dot{\vec{M}} + \dot{\vec{L}}) \right] + a_2 \left[(\vec{M} - \vec{L}) \times (\dot{\vec{M}} - \dot{\vec{L}}) \right] =$

$= a_1 \left[\vec{M} \times \dot{\vec{M}} + \vec{L} \times \dot{\vec{M}} + \vec{M} \times \dot{\vec{L}} + \vec{L} \times \dot{\vec{L}} \right] + a_2 \left[\vec{M} \times \dot{\vec{M}} - \vec{L} \cdot \dot{\vec{M}} - \vec{M} \times \dot{\vec{L}} + \vec{L} \times \dot{\vec{L}} \right]$

$\vec{M} \times \vec{L} = \frac{1}{4}(\vec{M}_1 + \vec{M}_2) \times (\vec{M}_1 - \vec{M}_2) = \frac{1}{4}(-\vec{M}_1 \times \vec{M}_2 + \vec{M}_2 \times \vec{M}_1) = \frac{1}{2}(\vec{M}_2 \times \vec{M}_1) = \vec{V}$

$\mathcal{L}_{kin} = (a_1 + a_2) \left[\vec{M} \times \dot{\vec{M}} + \vec{L} \times \dot{\vec{L}} \right] \hat{z} + (a_1 - a_2) \left[\vec{L} \times \dot{\vec{M}} + \vec{M} \times \dot{\vec{L}} \right] \hat{z}$, $\vec{L} \times \dot{\vec{M}} + \vec{M} \times \dot{\vec{L}} = ?$

Skype обсуждение FiM dyn. с А.К. Звездой. Он согласился. Это

Zvezdin 'JMMM содержит wrong DW mass $m_{DW}(S \to 0) \to 0$. Но сказал, что верный Lagr с двумя fields есть в его new unpublished paper, которую пришлет. Согласился, что FiM energy of "strong" FiM не сводится к $W(\vec{L})$, т.к. $|\vec{M}|$ не мал по сравнению с $|\vec{L}|$. Вдали от коллинеарности

После А.К. papers by Tserkovnyak 2014-2017 по FiM dyn. + Chen EPL 2016.

(12/XII) Reseng. manuscript Yoo-Kim 'JAP' 2020 Vortex mass. C n. Carrefour "Cadberria.
(13/XII) Rev. MS Yoo 'JAP' 2020 Vortex mass. E-mail Quaque Q_H - sphere Simul.
(14/XII) FiM Budag $\mathcal{L}_{kin}(F;M)$. $a_i = \frac{S_i}{M_i^2}\frac{1}{(1+\hat{z}\cdot\vec{m}_i)}$, $S_i = \frac{M_i}{\gamma_i}$, $M_i = |\vec{M}_i|$

$\mathcal{L}_{kin} = (a_1+a_2)\hat{z}\cdot[\vec{M}\times\dot{\vec{M}} + \vec{L}\times\dot{\vec{L}}] + (a_1-a_2)[\vec{L}\times\dot{\vec{M}} + \vec{M}\times\dot{\vec{L}}]$

Numerical approx. Ivanov '83: $\vec{M} = M_S\vec{l} + \frac{1}{\gamma A}\vec{l}\times\dot{\vec{l}}$, $\vec{l} = \frac{\vec{L}}{|\vec{L}|}$, $M_S = \frac{M_1^2 - M_2^2}{|\vec{L}|}$.

$\dot{\vec{M}} = M_S\dot{\vec{l}} + \frac{1}{\gamma A}\frac{d}{dt}(\vec{l}\times\dot{\vec{l}})$, $\frac{d}{dt}(\vec{l}\times\dot{\vec{l}}) = \dot{\vec{l}}\times\dot{\vec{l}} + \vec{l}\times\ddot{\vec{l}} = \vec{l}\times\ddot{\vec{l}}$, $L = |\vec{L}|$

$\mathcal{L}_{kin}(F;M) = (a_1+a_2)\hat{z}\cdot\left[(M_S\vec{l} + \frac{1}{\gamma A}\dot{\vec{l}}\times\vec{l})\times(M_S\dot{\vec{l}} + \frac{1}{\gamma A}\ddot{\vec{l}}\times\vec{l})\right] + (a_1-a_2)\times$

$\times\hat{z}\cdot\left[L\vec{l}\times(M_S\dot{\vec{l}} + \frac{1}{\gamma A}\ddot{\vec{l}}\times\vec{l}) + (M_S\vec{l} + \frac{1}{\gamma A}\dot{\vec{l}}\times\vec{l})\times L\dot{\vec{l}}\right] = (a_1+a_2)\hat{z}\cdot\left[M_S^2\vec{l}\times\dot{\vec{l}} + \right.$

$+ \frac{M_S}{\gamma A}\vec{l}\times(\ddot{\vec{l}}\times\vec{l}) + \frac{M_S}{\gamma A}(\dot{\vec{l}}\times\vec{l})\times\dot{\vec{l}} + \frac{1}{(\gamma A)^2}(\dot{\vec{l}}\times\vec{l})\times(\ddot{\vec{l}}\times\vec{l})\Big] + (a_1-a_2)\hat{z}\cdot\Big[M_S\vec{l}\times\dot{\vec{l}} +$

$+ \frac{1}{\gamma A}\vec{l}\times(\ddot{\vec{l}}\times\vec{l}) + M_S\vec{l}\times\dot{\vec{l}} + \frac{1}{\gamma A}(\dot{\vec{l}}\times\vec{l})\times\dot{\vec{l}}\Big]L$. Пусть в ansatz Ivanov '83 то

$\vec{l}\times(\ddot{\vec{l}}\times\vec{l}) = \ddot{\vec{l}}(\vec{l}\cdot\vec{l}) - \vec{l}(\dot{\vec{l}}\cdot\ddot{\vec{l}})$, $(\dot{\vec{l}}\times\vec{l})\times\dot{\vec{l}} = \vec{l}\times(\dot{\vec{l}}\times\dot{\vec{l}}) = \vec{l}(\dot{\vec{l}}\cdot\dot{\vec{l}}) - \dot{\vec{l}}(\dot{\vec{l}}\cdot\vec{l})$ то

$\mathcal{L}_{kin}(F;M) \approx (a_1+a_2)\hat{z}\cdot\left[M_S^2\vec{l}\times\dot{\vec{l}} + \frac{M_S}{\gamma A}\left(\ddot{\vec{l}} - \vec{l}(\dot{\vec{l}}\cdot\ddot{\vec{l}})\right) + \frac{M_S}{\gamma A}\vec{l}(\dot{\vec{l}})^2\right] +$

$+ (a_1-a_2)\hat{z}\cdot\left[2M_S\cdot\vec{l}\times\dot{\vec{l}} + \frac{1}{\gamma A}\left(\ddot{\vec{l}} - \vec{l}(\vec{l}\cdot\ddot{\vec{l}})\right) + \frac{1}{\gamma A}\vec{l}(\dot{\vec{l}})^2\right]L$

$\mathcal{L}_{kin}(F;M) = M_S\hat{z}\left[(a_1+a_2)M_S + (a_1-a_2)2L\right]\cdot(\vec{l}\times\dot{\vec{l}}) + (a_1+a_2)\hat{z}\frac{M_S}{\gamma A}\left[\ddot{\vec{l}} - \vec{l}(\vec{l}\cdot\ddot{\vec{l}}) + \vec{l}\dot{l}^2\right] +$

$+ (a_1-a_2)\hat{z}\cdot\frac{L}{\gamma A}\left[\ddot{\vec{l}} - \vec{l}(\vec{l}\cdot\ddot{\vec{l}}) + \vec{l}\dot{l}^2\right]$. При $A\to\infty$, получим $\approx$

$\mathcal{L}_{kin}(F;M) \approx M_S\left[(a_1+a_2)M_S + 2(a_1-a_2)L\right]\hat{z}\cdot(\vec{l}\times\dot{\vec{l}})$

$[\ldots] = (a_1+a_2)M_S + 2(a_1-a_2)L = \left[\frac{S_1}{M_1^2}\frac{1}{(1+\hat{z}\cdot\vec{m}_1)} + \frac{S_2}{M_2^2}\frac{1}{(1+\hat{z}\cdot\vec{m}_2)}\right]\cdot\frac{(M_1^2-M_2^2)}{L} +$

$+ 2L\left[\frac{S_1}{M_1^2}\frac{1}{(1+\hat{z}\cdot\vec{m}_1)} + \frac{S_2}{M_2^2}\frac{1}{(1+\hat{z}\cdot\vec{m}_2)}\right]$, $L^2 = M_1^2 + M_2^2 - 2\vec{M}_1\vec{M}_2 \approx (M_1+M_2)^2$.

Eg. $\vec{M}_1 = \vec{M} + \vec{L}$, $\vec{M}_2 = \vec{M} - \vec{L}$, $\vec{m}_1 = \frac{\vec{M}_1}{M_1} = \frac{1}{M_1}(\vec{M}+\vec{L})$, $\vec{m}_2 = \frac{1}{M_2}(\vec{M}-\vec{L})$

Чтобы $\vec{m}_2 \approx -\vec{m}_1$ (нужно при $M_1 \neq M_2$). и т.д. FiM dyn

(15/XII) Нужно разделить $\mathcal{L}_{kin}(F;M) = \mathcal{L}_{kin}(\vec{m}_1) + \mathcal{L}_{kin}(\vec{m}_2)$. Но это не нужно, т.к. $\delta\mathcal{L}_{kin}/\delta\vec{m}_i$ известно и просто → надо исходить из УД - LLG.

$\mathcal{L} = \mathcal{L}_{kin} - W = \mathcal{L}(\vec{M}_1,\vec{M}_2)$ дает УД $\frac{1}{\gamma_1}\dot{\vec{M}}_1 = \vec{M}_1\times\frac{\delta W}{\delta\vec{M}_1}$, $\frac{1}{\gamma_2}\dot{\vec{M}}_2 = \vec{M}_2\times\frac{\delta W}{\delta\vec{M}_2}$

$W = A\vec{M}_1\cdot\vec{M}_2 + W_1(\vec{M}_1) + W_2(\vec{M}_2)$, $W_i(\vec{M}_i) = -\vec{M}_i\vec{H} + \frac{d_i}{2}\left(\frac{\partial\vec{M}_i}{\partial x}\right)^2 + W_{ai}(\vec{M}_i)$

$\frac{\delta W}{\delta\vec{M}_1} = -\vec{H} + \frac{\partial W_{a1}}{\partial\vec{M}_1} - d_1\frac{\partial^2\vec{M}_1}{\partial x^2} + A\vec{M}_2$, $W(\vec{M})$, $\frac{\delta W}{\delta\vec{M}} = \frac{\partial W}{\partial\vec{M}} - \frac{\partial}{\partial x}\frac{\partial W}{\partial(\partial\vec{M}/\partial x)}$

$\frac{\delta W}{\delta\vec{M}_2} = -\vec{H} + \frac{\partial W_{a2}}{\partial\vec{M}_2} - d_2\frac{\partial^2\vec{M}_2}{\partial x^2} + A\vec{M}_1$.

$\frac{1}{\gamma_1}\dot{\vec{M}}_1 = \vec{M}_1\times\left[A\vec{M}_2 - \vec{H} - d_1\frac{\partial^2\vec{M}_1}{\partial x^2} + \frac{\partial W_{a1}}{\partial\vec{M}_1}\right]$, $\frac{1}{\gamma_2}\dot{\vec{M}}_2 = \vec{M}_2\times\left[A\vec{M}_1 - \vec{H} - d_2\frac{\partial^2\vec{M}_2}{\partial x^2} + \frac{\partial W_{a2}}{\partial\vec{M}_2}\right]$

New variables $\vec{L} = \frac{1}{2}(\vec{M}_1 - \vec{M}_2)$, $\vec{M} = \frac{1}{2}(\vec{M}_1 + \vec{M}_2)$, $\vec{M}_1 = \vec{M} + \vec{L}$, $\vec{M}_2 = \vec{M} - \vec{L}$.

$\frac{1}{\gamma_1}\dot{\vec{M}}_1 + \frac{1}{\gamma_2}\dot{\vec{M}}_2 = +\vec{M}\times(\vec{M}_1 + \vec{M}_2) + \vec{M}_1\times\left[\frac{\partial W_{a1}}{\partial\vec{M}_1} - d_1\frac{\partial^2\vec{M}_1}{\partial x^2}\right] + \vec{M}_2\times\left[\frac{\partial W_{a2}}{\partial\vec{M}_2} - d_2\frac{\partial^2\vec{M}_2}{\partial x^2}\right]$

Хотим и УД в виде $\dot{\vec{L}} = \ldots$, $\dot{\vec{M}} = \ldots$.

$2\dot{\vec{M}} = \gamma_1\vec{M}_1\times\frac{\delta W}{\delta\vec{M}_1} + \gamma_2\vec{M}_2\times\frac{\delta W}{\delta\vec{M}_2}$, $2\dot{\vec{L}} = \gamma_1\vec{M}_1\times\frac{\delta W}{\delta\vec{M}_1} - \gamma_2\vec{M}_2\times\frac{\delta W}{\delta\vec{M}_2}$

$\frac{\delta W}{\delta\vec{M}_1} = \frac{\delta W(\vec{L},\vec{M})}{\delta\vec{M}_1} = \frac{\delta W}{\delta\vec{L}}\frac{\partial\vec{L}}{\partial\vec{M}_1} + \frac{\delta W}{\delta\vec{M}}\frac{\partial\vec{M}}{\partial\vec{M}_1} = \frac{\delta W}{\delta\vec{L}}\frac{1}{2} + \frac{\delta W}{\delta\vec{M}}\frac{1}{2}$

$\frac{\delta W}{\delta\vec{M}_2} = \frac{\delta W}{\delta\vec{L}}\frac{\partial\vec{L}}{\partial\vec{M}_2} + \frac{\delta W}{\delta\vec{M}}\frac{\partial\vec{M}}{\partial\vec{M}_2} = \frac{\delta W}{\delta\vec{L}}\left(-\frac{1}{2}\right) + \frac{\delta W}{\delta\vec{M}}\frac{1}{2}$.

Escaneado con CamScanner

$$2\dot{\vec{M}} = \gamma_1 (\vec{M}+\vec{L}) \times \frac{1}{2}\left(\frac{\delta W}{\delta \vec{L}} + \frac{\partial W}{\partial \vec{M}}\right) + \gamma_2 (\vec{M}-\vec{L}) \times \frac{1}{2}\left(\frac{\delta W}{\delta \vec{M}} - \frac{\delta W}{\delta \vec{L}}\right)$$

$$4\dot{\vec{M}} = \gamma_1 (\vec{M}+\vec{L}) \times \left(\frac{\delta W}{\delta \vec{L}} + \frac{\partial W}{\partial \vec{M}}\right) + \gamma_2 (\vec{M}-\vec{L}) \times \left(\frac{\delta W}{\delta \vec{M}} - \frac{\delta W}{\delta \vec{L}}\right)$$

$$2\dot{\vec{L}} = \gamma_1 (\vec{M}+\vec{L}) + \frac{1}{2}\left(\frac{\delta W}{\delta \vec{L}} + \frac{\partial W}{\partial \vec{M}}\right) - \gamma_2 (\vec{M}-\vec{L}) \times \frac{1}{2}\left(\frac{\delta W}{\delta \vec{M}} - \frac{\delta W}{\delta \vec{L}}\right)$$

$$4\dot{\vec{M}} = \left[\gamma_1 (\vec{M}+\vec{L}) + \gamma_2 (\vec{M}-\vec{L})\right] \times \frac{\delta W}{\delta \vec{M}} + \left[\gamma_1 (\vec{M}+\vec{L}) - \gamma_2 (\vec{M}-\vec{L})\right] \times \frac{\partial W}{\partial \vec{L}}$$

$$4\dot{\vec{L}} = \left[\gamma_1 (\vec{M}+\vec{L}) - \gamma_2 (\vec{M}-\vec{L})\right] \times \frac{\partial W}{\partial \vec{M}} + \left[\gamma_1 (\vec{M}+\vec{L}) + \gamma_2 (\vec{M}-\vec{L})\right] \times \frac{\delta W}{\partial \vec{L}} \quad (4)$$

При $\gamma_1 = \gamma_2 = \gamma$ УД (а) сводится к УД Косевича'90, (9.6) - (9.7)

$\gamma_\pm = \frac{1}{2}(\gamma_1 \pm \gamma_2) \rightarrow$

$$2\dot{\vec{M}} = \left[\gamma_+ \vec{M} + \gamma_- \vec{L}\right] \times \frac{\delta W}{\delta \vec{M}} + \left[\gamma_- \vec{M} + \gamma_+ \vec{L}\right] \times \frac{\partial W}{\partial \vec{L}}$$

$$2\dot{\vec{L}} = \left[\gamma_- \vec{M} + \gamma_+ \vec{L}\right] \times \frac{\delta W}{\delta \vec{M}} + \left[\gamma_+ \vec{M} + \gamma_- \vec{L}\right] \times \frac{\delta W}{\delta \vec{L}}$$

$$\frac{\delta W}{\delta \vec{M}} = \frac{\delta W}{\delta \vec{M}_1}\frac{\delta \vec{M}_1}{\partial \vec{M}} + \frac{\delta W}{\delta \vec{M}_2}\frac{\delta \vec{M}_2}{\partial \vec{M}} = \frac{\delta W}{\delta \vec{M}_1} + \frac{\delta W}{\delta \vec{M}_2} = A(\vec{M}_1 + \vec{M}_2) - 2\vec{H} + \frac{\partial W_{a1}}{\partial \vec{M}_1} + \frac{\partial W_{a2}}{\partial \vec{M}_2} -$$

$$- d_1 \frac{\partial^2 \vec{M}_1}{\partial x^2} - d_2 \frac{\partial^2 \vec{M}_2}{\partial x^2} = 2A\vec{M} - 2\vec{H} + \frac{\partial W_{a1}}{\partial \vec{M}_1} + \frac{\partial W_{a2}}{\partial \vec{M}_2} - d_1 \frac{\partial^2 \vec{M}_1}{\partial x^2} - d_2 \frac{\partial^2 \vec{M}_2}{\partial x^2}$$

$$\frac{\partial W_{a1}}{\partial \vec{M}_1} + \frac{\partial W_{a2}}{\partial \vec{M}_2} = \frac{1}{2}\left(\frac{\partial W_{a1}}{\partial \vec{L}} + \frac{\partial W_{a1}}{\partial \vec{M}}\right) + \frac{1}{2}\left(\frac{\partial W_{a2}}{\partial \vec{L}}(+) \frac{\partial W_{a2}}{\partial \vec{M}}\right), \quad W_{a\pm} = (W_{a1} \pm W_{a2})\frac{1}{2}$$

$$\frac{\delta W}{\delta \vec{M}} = 2A\vec{M} - 2\vec{H} + \frac{1}{2}\left(\frac{\partial W_{a-}}{\partial \vec{L}} + \frac{\partial W_{a+}}{\partial \vec{M}}\right) - 2d_+ \frac{\partial^2}{\partial x^2}(\vec{M}+\vec{L}) - d_2 \frac{\partial^2}{\partial x^2}(\vec{M}-\vec{L}) =$$

$$= 2A\vec{M} - 2\vec{H} + \frac{1}{2}\left(\frac{\partial W_{a-}}{\partial \vec{L}} + \frac{\partial W_{a+}}{\partial \vec{M}}\right) - 2d_+ \frac{\partial^2 \vec{M}}{\partial x^2} - 2d_- \frac{\partial^2 \vec{L}}{\partial x^2}, \quad (d_\pm = \frac{1}{2}(d_1 \pm d_2))$$

$$\frac{\delta W}{\delta \vec{L}} = \frac{\delta W}{\delta \vec{M}_1}\frac{\partial \vec{M}_1}{\partial \vec{L}} + \frac{\delta W}{\delta \vec{M}_2}\frac{\partial \vec{M}_2}{\partial \vec{L}} = \frac{\delta W}{\delta \vec{M}_1} - \frac{\delta W}{\partial \vec{M}_2} = -\vec{H} + \frac{\partial W_{a1}}{\partial \vec{M}_1} - d_1 \frac{\partial^2 \vec{M}_1}{\partial x^2} + A\vec{M}_2 -$$

$$- \left[-\vec{H} + \frac{\partial W_{a2}}{\partial \vec{M}_2} - d_2 \frac{\partial^2 \vec{M}_2}{\partial x^2} + A\vec{M}_1\right] = \frac{\partial W_{a1}}{\partial \vec{M}_1} - \frac{\partial W_{a2}}{\partial \vec{M}_2} - d_1 \frac{\partial^2 \vec{M}_1}{\partial x^2} + d_2 \frac{\partial^2 \vec{M}_2}{\partial x^2} +$$

$$+ A(\vec{M}_2 - \vec{M}_1) = \frac{1}{2}\left(\frac{\partial W_{a1}}{\partial \vec{L}} + \frac{\partial W_{a1}}{\partial \vec{M}}\right) - \frac{1}{2}\left(\frac{\partial W_{a2}}{\partial \vec{L}}(-1) + \frac{\partial W_{a2}}{\partial \vec{M}}\right) - d_1 \frac{\partial^2}{\partial x^2}(\vec{M}+\vec{L}) +$$

$$+ d_2 \frac{\partial^2}{\partial x^2}(\vec{M}-\vec{L}) = 2A\vec{L} = \frac{\partial W_{a+}}{\partial \vec{L}} + \frac{\partial W_{a-}}{\partial \vec{M}} - 2d_- \frac{\partial^2 \vec{M}}{\partial x^2} - 2d_+ \frac{\partial^2 \vec{L}}{\partial x^2} - 2A\vec{L}$$

$$\frac{\delta W}{\delta \vec{L}} = -2A\vec{L} + \frac{\partial W_{a+}}{\partial \vec{L}} + \frac{\partial W_{a-}}{\partial \vec{M}} - 2d_- \frac{\partial^2 \vec{M}}{\partial x^2} - 2d_+ \frac{\partial^2 \vec{L}}{\partial x^2}$$

$$2\dot{\vec{M}} = \left[\gamma_+ \vec{M} + \gamma_- \vec{L}\right] \times \left[2A\vec{M} - 2\vec{H} + \frac{\partial W_{a-}}{\partial \vec{L}} + \frac{\partial W_{a+}}{\partial \vec{M}} - 2d_+ \frac{\partial^2 \vec{M}}{\partial x^2} - 2d_- \frac{\partial^2 \vec{L}}{\partial x^2}\right] +$$

$$+ \left[\gamma_- \vec{M} + \gamma_+ \vec{L}\right] \times \left[-2A\vec{L} + \frac{\partial W_{a+}}{\partial \vec{L}} + \frac{\partial W_{a-}}{\partial \vec{M}} - 2d_- \frac{\partial^2 \vec{M}}{\partial x^2} - 2d_+ \frac{\partial^2 \vec{L}}{\partial x^2}\right] =$$

$$= \left[\gamma_+ \vec{M} + \gamma_- \vec{L}\right] \times \left[2A\vec{M} - 2\vec{H} + \vec{H}_M\right] + \left[\gamma_- \vec{M} + \gamma_+ \vec{L}\right] \times \left[-2A\vec{L} + \vec{H}_L\right]$$

$$\vec{H}_M = \frac{\partial W_{a+}}{\partial \vec{M}} + \frac{\partial W_{a-}}{\partial \vec{L}} - 2d_+ \frac{\partial^2 \vec{M}}{\partial x^2} - 2d_- \frac{\partial^2 \vec{L}}{\partial x^2}; \quad \vec{H}_L = \frac{\partial W_{a+}}{\partial \vec{L}} + \frac{\partial W_{a-}}{\partial \vec{M}} - 2d_+ \frac{\partial^2 \vec{L}}{\partial x^2} - 2d_- \frac{\partial^2 \vec{M}}{\partial x^2}$$

$$2\dot{\vec{M}} = -2\gamma_+ \vec{M} \times \vec{H} + \gamma_+ \vec{M} \times \vec{H}_M + 2A\gamma_- \vec{L} \times \vec{M} - 2\gamma_- \vec{L} \times \vec{H} + \gamma_- \vec{L} \times \vec{H}_M - 2A\gamma_- \vec{M} \times \vec{L} +$$

$$+ \gamma_- \vec{M} \times \vec{H}_L + \gamma_+ \vec{L} \times \vec{H}_L = -2\gamma_+ \vec{M} \times \vec{H} - 2\gamma_- \vec{L} \times \vec{H} + (\gamma_+ \vec{M} + \gamma_- \vec{L}) \times \vec{H}_M + 4A\gamma_- \vec{L} \times \vec{M} +$$

$$+ (\gamma_- \vec{M} + \gamma_+ \vec{L}) \times \vec{H}_L, \quad \dot{\vec{M}} = -(\gamma_+ \vec{M} + \gamma_- \vec{L}) \times \vec{H} + 2A\gamma_- \vec{L} \times \vec{M} + \frac{1}{2}(\gamma_+ \vec{M} + \gamma_- \vec{L}) \times \vec{H}_M + (\gamma_- \vec{M} + \gamma_+ \vec{L}) \times \vec{H}_L \frac{1}{2}$$

$$2\dot{\vec{L}} = (\gamma_- \vec{M} + \gamma_+ \vec{L}) \times \left[2A\vec{M} - 2\vec{H} + \vec{H}_M\right] + (\gamma_+ \vec{M} + \gamma_- \vec{L}) \times \left[-2A\vec{L} + \vec{H}_L\right] = +2(-\gamma_- \vec{M} \times \vec{H} + \gamma_- \vec{M} \times \vec{H}_M +$$

$$+ 2A\gamma_+ \vec{L} \times \vec{M} - 2\gamma_+ \vec{L} \times \vec{H} + \gamma_+ \vec{L} \times \vec{H}_M - 2A\gamma_+ \vec{M} \times \vec{L} + \gamma_+ \vec{M} \times \vec{H}_L + \gamma_- \vec{L} \times \vec{H}_L = -2(\gamma_- \vec{M} + \gamma_+ \vec{L}) \times \vec{H} +$$

$$+ (\gamma_- \vec{M} + \gamma_+ \vec{L}) \times \vec{H}_M + 4A\gamma_+ \vec{L} \times \vec{M} + (\gamma_+ \vec{M} + \gamma_- \vec{L}) \times \vec{H}_L$$

FEM УД →

$$\left\{\begin{aligned} \dot{\vec{L}} &= -(\gamma_- \vec{M} + \gamma_+ \vec{L}) \times \vec{H} + 2A\gamma_+ \vec{L} \times \vec{M} + \frac{1}{2}(\gamma_- \vec{M} + \gamma_+ \vec{L}) \times \vec{H}_M + \frac{1}{2}(\gamma_+ \vec{M} + \gamma_- \vec{L}) \times \vec{H}_L \\ \dot{\vec{M}} &= -(\gamma_+ \vec{M} + \gamma_- \vec{L}) \times \vec{H} + 2A\gamma_- \vec{L} \times \vec{M} + \frac{1}{2}(\gamma_+ \vec{M} + \gamma_- \vec{L}) \times \vec{H}_M + \frac{1}{2}(\gamma_- \vec{M} + \gamma_+ \vec{L}) \times \vec{H}_L \end{aligned}\right\}$$

Это система УД для $\vec{M} = \frac{1}{2}(\vec{M}_1 + \vec{M}_2)$ и $\vec{L} = \frac{1}{2}(\vec{M}_1 - \vec{M}_2)$ без членов неравновесности.

Частный случай $\gamma_1 = \gamma_2 = \gamma = \gamma_+$, $\gamma_- = 0 \rightarrow \dot{\vec{L}} = -\gamma \vec{L} \times \vec{H} + 2A\gamma \vec{L} \times \vec{M} + \frac{\gamma}{2}\vec{L} \times \vec{H}_M + \frac{1}{2}\gamma \vec{M} \times \vec{H}_L$

$\dot{\vec{M}} = -\gamma \vec{M} \times \vec{H} + \frac{1}{2}\gamma \vec{M} \times \vec{H}_M + \frac{1}{2}\gamma \vec{L} \times \vec{H}_L$ и $d_- = 0$, $W_{a-} = 0$ сводится к Косевичу'90

Escaneado con CamScanner

Если считать, что A - term (intersublattice uniform exchange) доминирует, то
$\dot{\vec{L}} = 2A\gamma_+ \vec{L}\times\vec{M}$ и $\dot{\vec{M}} = 2A\gamma_- \vec{L}\times\vec{M}$, $\frac{\dot{\vec{L}}}{\gamma_+} = \frac{\dot{\vec{M}}}{\gamma_-}$, $\gamma_- = \frac{1}{2}(\gamma_1 - \gamma_2) \neq 0$.

$\vec{M} = a(\vec{L} + \dot{\vec{L}})$, $\dot{\vec{L}} = 2A\gamma_+ \vec{L}\times(\vec{L}+\dot{\vec{L}})a = 2Aa\gamma_+\left[\vec{L}(\dot{\vec{L}}\cdot\vec{L}) - \dot{\vec{L}}(\vec{L}\cdot\vec{L})\right]$

$\vec{L}^2 = L^2 = \frac{1}{4}(\vec{M}_1 - \vec{M}_2)^2 = \frac{1}{4}(M_1^2 + M_2^2 - 2\vec{M}_1\vec{M}_2) \approx \frac{1}{4}(M_1^2 + M_2^2 + 2M_1M_2) = \frac{1}{4}(M_1+M_2)^2$

$\vec{L}\cdot\dot{\vec{L}} = \frac{1}{2}\frac{d}{dt}(\vec{L})^2 \neq 0$, т.к. $|\vec{L}| \approx \frac{1}{2}(M_1+M_2) = \text{const}$. $M_i = |\vec{M}_i|$.

$\dot{\vec{L}} = 2Aa\gamma_+(-1)L^2\dot{\vec{L}}$, $a = -\frac{1}{2\gamma_+ A L^2}$ и $\vec{M} \approx -\frac{1}{2\gamma_+ A L^2}(\vec{L} + \dot{\vec{L}})$ - добавка term $\sim \dot{L}$

Если учесть при $\gamma_- = 0$, $\gamma_+ = \gamma$, $\dot{\vec{M}} = -\gamma\vec{M}\times\vec{H} + \frac{1}{2}\gamma\vec{M}\times\vec{H}_M + \frac{1}{2}\gamma\vec{L}\times\vec{H}_L$ и $|\vec{M}| \ll |\vec{L}|$, то приходим к 2-model of AF.

Но $\vec{M}^2 = \frac{1}{4}(\vec{M}_1 + \vec{M}_2)^2 = \frac{1}{4}(M_1^2 + M_2^2 + 2\vec{M}_1\vec{M}_2) \approx \frac{1}{4}(M_1 - M_2)^2$, т.к. $\vec{M}_1 \approx -\vec{M}_2$, хотя $\vec{M}_1$ и $\vec{M}_2$ антипараллельны $\uparrow\downarrow$ в главном по A - приближении.

и $\vec{M}^2 \ll \vec{L}^2$ нельзя использовать, далеко через mag. compensation $|\vec{M}| \neq 0$.

$\vec{M}\cdot\vec{L} = \frac{1}{2}(\vec{M}_1 + \vec{M}_2)\frac{1}{2}(\vec{M}_1 - \vec{M}_2) = \frac{1}{4}(M_1^2 - M_2^2) \neq 0$ в общем случае

$\vec{M}^2 + \vec{L}^2 = \frac{1}{2}(M_1^2 + M_2^2) = \text{const}$, $\vec{L}^2 = \frac{1}{4}(M_1^2 + M_2^2 - 2\vec{M}_1\vec{M}_2) \neq \text{const}$

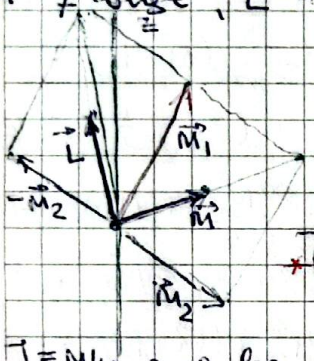
$\vec{L}\cdot\dot{\vec{L}} = \frac{1}{2}\frac{d}{dt}L^2 = 2A\gamma_+\vec{L}\cdot(\vec{L}\times\vec{M}) = 0$ в обменном $A\to\infty$ пределе. $\vec{M}^2 \neq \text{const}$.

$\vec{M}^2 \neq \text{const}$, $\vec{L}^2 \neq \text{const}$, зато $\vec{M}^2 + \vec{L}^2 = \text{const}$. $\vec{M}\cdot\vec{L} = \frac{1}{4}(M_1^2 - M_2^2) \neq 0$

$\vec{M}$ и $\vec{L}$ в одной плоскости, заданной $\vec{M}_1, \vec{M}_2$.

Удобно чтоб Oz-оси также в этой плоскости,

т.е. плоскость $\vec{M}_1\vec{M}_2$ → ограничение на вид W_{a1}, W_{a2}.



$\frac{d(\vec{L}^2)}{dt} = -\frac{1}{2}\frac{d}{dt}(\vec{M}_1\cdot\vec{M}_2) = -\frac{1}{2}\frac{d}{dt}(\vec{M}^2 - \vec{L}^2)$ - тождество т.к. $\vec{M}^2 + \vec{L}^2 = \text{const}$

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* Аналогично для Lin — Wess — Zumino action (WZ)

$\frac{M}{\gamma}\dot{\vec{m}}\times\vec{m} + \frac{\delta U}{\delta\vec{m}} = 0$ LLG eq. из $S_{WZ} = J\int dt\int_0^1 \vec{m}\cdot\left(\frac{\partial\vec{m}}{\partial\tau}\times\frac{\partial\vec{m}}{\partial\tau}\right)$

$J = M/\gamma$ angular momentum density. $S_{WZ} = \frac{J}{M^3}\int dt\int_0^1 d\tau\,\vec{M}\cdot\left(\frac{\partial\vec{M}}{\partial t}\times\frac{\partial\vec{M}}{\partial\tau}\right)$ - плотность Lin - Lagrangian

$\mathcal{L}_{WZ} = \frac{J}{M^3}\int_0^1 d\tau\,\vec{M}\cdot\left(\frac{\partial\vec{M}}{\partial t}\times\frac{\partial\vec{M}}{\partial\tau}\right)$

$\mathcal{L}_{WZ}(F{=}M) = \mathcal{L}_{Lin}(F{=}M) = \frac{S_1}{M_1^3}\int_0^1 d\tau\,\vec{M}_1\cdot\left(\vec{M}_1\times\frac{\partial\vec{M}_1}{\partial\tau}\right) + \frac{S_2}{M_2^3}\int_0^1 d\tau\,\vec{M}_2\cdot\left(\vec{M}_2\times\frac{\partial\vec{M}_2}{\partial\tau}\right)$, $S_i = \frac{M_i}{\gamma_i}$

$\mathcal{L}_{Lin}(F{=}M) = \int_0^1 d\tau\left[b_1\vec{M}_1\cdot\left(\vec{M}_1\times\frac{\partial\vec{M}_1}{\partial\tau}\right) + b_2\vec{M}_2\cdot\left(\vec{M}_2\times\frac{\partial\vec{M}_2}{\partial\tau}\right)\right]$, $b_i = S_i/M_i^3$.

$b_1\vec{M}_1\cdot\left(\vec{M}_1\times\frac{\partial\vec{M}_1}{\partial\tau}\right) + b_2\vec{M}_2\left(\vec{M}_2\times\frac{\partial\vec{M}_2}{\partial\tau}\right) = b_1(\vec{M}+\vec{L})\cdot\left[(\dot{\vec{M}}+\dot{\vec{L}})\times\left(\frac{\partial\vec{M}}{\partial\tau}+\frac{\partial\vec{L}}{\partial\tau}\right)\right] +$

$+ b_2(\vec{M}-\vec{L})\cdot\left[(\dot{\vec{M}}-\dot{\vec{L}})\times\left(\frac{\partial\vec{M}}{\partial\tau}+\frac{\partial\vec{L}}{\partial\tau}\right)\right]$, см. M. Stone PRB'96, Dasgupta PRB'18.

т.к. $\vec{M}$ и $\vec{L}$ одного порядка, что не видно уменьшение в $\mathcal{L}_{WZ}(F{=}M)$.

// Для FiM TB $M^2/L^2 \ll 1$ нельзя использовать, остается TB по $\frac{1}{A}$.

При $A\to\infty$ (в бездемпфирной оси Oz из-за H_z или W_{ai}) $W_{ex} = A\vec{M}_1\vec{M}_2$ предпочитает $\vec{M}_1 \uparrow\downarrow \vec{M}_2$. $\vec{M}_1 = M_1\hat{z}$, $\vec{M}_2 = -M_2\hat{z}$, $\vec{L} = \frac{1}{2}(\vec{M}_1 - \vec{M}_2) = \frac{1}{2}(M_1+M_2)\hat{z}$, $\vec{M} = \frac{1}{2}(M_1 - M_2)\hat{z}$

$\vec{L}\times\vec{M} = 0$ и $\dot{\vec{L}} = 0$, $\dot{\vec{M}} = 0$, mag. dynamics подавлена.

При A большом, но конечном: $\vec{L} = \vec{L}_0 + \vec{\ell}$, $\vec{M} = \vec{M}_0 + \vec{m}$, $\vec{L}_0 = \frac{1}{2}(M_1+M_2)\hat{z}$, $\vec{M}_0 = \frac{1}{2}(M_1 - M_2)\hat{z}$

$\dot{\vec{\ell}} = -(\gamma_-\vec{m} + \gamma_+\vec{\ell})\times\vec{H} + 2A\gamma_+\left[\vec{L}_0\times\vec{m} + \vec{\ell}\times\vec{M}_0\right] + \frac{1}{2}\left[\gamma_-(\vec{M}_0+\vec{m}) + \gamma_+(\vec{L}_0+\vec{\ell})\right]\times$

$\times\left[-2d_+\frac{\partial^2\vec{m}}{\partial x^2} - 2d_-\frac{\partial^2\vec{\ell}}{\partial x^2}\right] + \frac{1}{2}\left[\gamma_+(\vec{M}_0+\vec{m}) + \gamma_-(\vec{L}_0+\vec{\ell})\right]\times\left[-2d_+\frac{\partial^2\vec{\ell}}{\partial x^2} - 2d_-\frac{\partial^2\vec{m}}{\partial x^2}\right]$

$\dot{\vec{\ell}} = -(\gamma_-\vec{m} + \gamma_+\vec{\ell})\times\vec{H} + 2A\gamma_+\left[\vec{L}_0\times\vec{m} + \vec{\ell}\times\vec{M}_0\right] + \frac{1}{2}\left[\gamma_-\vec{M}_0 + \gamma_+\vec{L}_0\right]\times(-2)\left[d_+\frac{\partial^2\vec{m}}{\partial x^2} + d_-\frac{\partial^2\vec{\ell}}{\partial x^2}\right] +$

$+ \frac{1}{2}\left[\gamma_+\vec{M}_0 + \gamma_-\vec{L}_0\right]\times(-2)\left[d_+\frac{\partial^2\vec{\ell}}{\partial x^2} + d_-\frac{\partial^2\vec{m}}{\partial x^2}\right]$, (см. также $\vec{m}, \vec{\ell} \sim \frac{1}{A}$, $\vec{H} = H\hat{z}$, $W_{ai} = 0$ пока

$\gamma_-\vec{M}_0 + \gamma_+\vec{L}_0 = \frac{1}{2}(\gamma_1 - \gamma_2)\frac{1}{2}(M_1 - M_2)\hat{z} + \frac{1}{2}(\gamma_1+\gamma_2)\frac{1}{2}(M_1+M_2)\hat{z} = \frac{1}{2}(\gamma_1 M_1 + \gamma_2 M_2)\hat{z}$

$\gamma_+\vec{M}_0 + \gamma_-\vec{L}_0 = \frac{1}{2}(\gamma_1+\gamma_2)(M_1 - M_2)\hat{z} + \frac{1}{2}(\gamma_1 - \gamma_2)\frac{1}{2}(M_1+M_2)\hat{z} = \frac{1}{2}(\gamma_1 M_1 - \gamma_2 M_2)\hat{z}$

Escaneado con CamScanner

$$\dot{\vec{l}} = -(\gamma_- \vec{m} + \gamma_+ \vec{l}) \times \vec{H} + 2A\gamma_+ \left[\vec{L}_0 \times \vec{m} + \vec{l} \times \vec{M}_0\right] - \frac{1}{2}(\gamma_1 M_1 + \gamma_2 M_2)\hat{z} \times \left[d_+ \frac{\partial^2 \vec{m}}{\partial x^2} + d_- \frac{\partial^2 \vec{l}}{\partial x^2}\right] -$$
$$- \frac{1}{2}(\gamma_1 M_1 - \gamma_2 M_2)\hat{z} \times \left[d_+ \frac{\partial^2 \vec{l}}{\partial x^2} + d_- \frac{\partial^2 \vec{m}}{\partial x^2}\right]$$

$$\dot{\vec{m}} = -(\gamma_+ \vec{m} + \gamma_- \vec{l}) \times \vec{H} + 2A\gamma_- \left[\vec{L}_0 \times \vec{m} + \vec{l} \times \vec{M}_0\right] + \frac{1}{2}(\gamma_+ \vec{M}_0 + \gamma_- \vec{L}_0) \times (-2)\left[d_+ \frac{\partial^2 \vec{m}}{\partial x^2} + d_- \frac{\partial^2 \vec{l}}{\partial x^2}\right] +$$
$$+ \frac{1}{2}(\gamma_- \vec{M}_0 + \gamma_+ \vec{L}_0) \times (-2)\left[d_+ \frac{\partial^2 \vec{l}}{\partial x^2} + d_- \frac{\partial^2 \vec{m}}{\partial x^2}\right], \quad (\gamma_+ \vec{M}_0 + \gamma_- \vec{L}_0) = \frac{1}{2}(\gamma_1 + \gamma_2)\frac{1}{2}(M_1 - M_2)\hat{z} +$$
$$+ \frac{1}{2}(\gamma_1 - \gamma_2)\frac{1}{2}(M_1 + M_2)\hat{z} = \frac{1}{2}(\gamma_1 M_1 - \gamma_2 M_2)\hat{z}, \quad (\gamma_- \vec{M}_0 + \gamma_+ \vec{L}_0) = \frac{1}{2}(\gamma_1 M_1 + \gamma_2 M_2)\hat{z}$$

$$\dot{\vec{m}} = -(\gamma_+ \vec{m} + \gamma_- \vec{l}) \times \vec{H} + 2A\gamma_- \left[\vec{L}_0 \times \vec{m} + \vec{l} \times \vec{M}_0\right] - \frac{1}{2}(\gamma_1 M_1 - \gamma_2 M_2)\hat{z} \times \left[d_+ \frac{\partial^2 \vec{m}}{\partial x^2} + d_- \frac{\partial^2 \vec{l}}{\partial x^2}\right] -$$
$$- \frac{1}{2}(\gamma_1 M_1 + \gamma_2 M_2)\hat{z} \left[d_+ \frac{\partial^2 \vec{l}}{\partial x^2} + d_- \frac{\partial^2 \vec{m}}{\partial x^2}\right], \quad \text{Опр. } \Gamma_\pm = \frac{1}{2}(\gamma_1 M_1 \pm \gamma_2 M_2)$$

$$\begin{cases} \dot{\vec{l}} = -(\gamma_- \vec{m} + \gamma_+ \vec{l}) \times \vec{H} + 2A\gamma_+ \left[\vec{L}_0 \times \vec{m} + \vec{l} \times \vec{M}_0\right] - \Gamma_+ \hat{z} \times \left[d_+ \frac{\partial^2 \vec{m}}{\partial x^2} + d_- \frac{\partial^2 \vec{l}}{\partial x^2}\right] - \Gamma_- \hat{z} \times \left[d_+ \frac{\partial^2 \vec{l}}{\partial x^2} + d_- \frac{\partial^2 \vec{m}}{\partial x^2}\right] \\ \dot{\vec{m}} = -(\gamma_+ \vec{m} + \gamma_- \vec{l}) \times \vec{H} + 2A\gamma_- \left[\vec{L}_0 \times \vec{m} + \vec{l} \times \vec{M}_0\right] - \Gamma_- \hat{z} \times \left[d_+ \frac{\partial^2 \vec{m}}{\partial x^2} + d_- \frac{\partial^2 \vec{l}}{\partial x^2}\right] - \Gamma_+ \hat{z} \times \left[d_+ \frac{\partial^2 \vec{l}}{\partial x^2} + d_- \frac{\partial^2 \vec{m}}{\partial x^2}\right] \end{cases}$$

$$A\vec{L} \times \vec{M} = A(\vec{L}_0 + \vec{l}) \times (\vec{M}_0 + \vec{m}) = A\left[\vec{L}_0 \times \vec{m} + \vec{l} \times \vec{M}_0 + \vec{l} \times \vec{m}\right], \text{ т.к. } |\vec{l}| \sim \frac{1}{A}, \ |\vec{m}| \sim \frac{1}{A}$$

Чтоб не нарушить ТВ по $\frac{1}{A}$, требуем $\vec{L}_0 \times \vec{m} + \vec{l} \times \vec{M}_0 = 0$

$$\begin{cases} \dot{\vec{l}} = -(\gamma_- \vec{m} + \gamma_+ \vec{l}) \times \vec{H} + 2A\gamma_+ (\vec{l} \times \vec{m}) - \hat{z}(\Gamma_+ d_+ + \Gamma_- d_-) \times \frac{\partial^2 \vec{m}}{\partial x^2} - \hat{z}(\Gamma_+ d_- + \Gamma_- d_+) \times \frac{\partial^2 \vec{l}}{\partial x^2} \\ \dot{\vec{m}} = -(\gamma_+ \vec{m} + \gamma_- \vec{l}) \times \vec{H} + 2A\gamma_- (\vec{l} \times \vec{m}) - \hat{z}(\Gamma_- d_+ + \Gamma_+ d_-) \frac{\partial^2 \vec{m}}{\partial x^2} - \hat{z}(\Gamma_- d_- + \Gamma_+ d_+) \times \frac{\partial^2 \vec{l}}{\partial x^2} \end{cases}$$

$$\begin{cases} \dot{\vec{l}} = -(\gamma_- \vec{m} + \gamma_+ \vec{l}) \times \vec{H} + 2A\gamma_+ (\vec{l} \times \vec{m}) - \hat{z} G_1 \times \frac{\partial^2 \vec{m}}{\partial x^2} - G_2 \hat{z} \times \frac{\partial^2 \vec{l}}{\partial x^2} \\ \dot{\vec{m}} = -(\gamma_+ \vec{m} + \gamma_- \vec{l}) \times \vec{H} + 2A\gamma_- (\vec{l} \times \vec{m}) - G_2 \hat{z} \times \frac{\partial^2 \vec{m}}{\partial x^2} - G_1 \hat{z} \times \frac{\partial^2 \vec{l}}{\partial x^2} \end{cases} \quad \begin{aligned} G_1 &= \Gamma_+ d_+ + \Gamma_- d_- \\ G_2 &= \Gamma_+ d_- + \Gamma_- d_+ \end{aligned}$$

(16/XII) $\vec{L}_0 \times \vec{m} + \vec{l} \times \vec{M}_0 = 0$ Инфо о $\vec{m}$ и $\vec{l}$? $\vec{m} \times \vec{L}_0 \times \vec{m} + \vec{m} \times \vec{l} \times \vec{M}_0 = \vec{L}_0(\vec{m}^2) - \vec{m}(\vec{L}_0 \cdot \vec{m}) +$
$+ \vec{l}(\vec{m} \cdot \vec{M}_0) - \vec{M}_0(\vec{m} \cdot \vec{l}) = 0$, $\frac{1}{2}(M_1 + M_2)m^2 = \frac{1}{2}(M_1 - M_2)(\vec{m} \cdot \vec{l})$, если $\vec{m} \cdot \vec{L}_0 = 0$, $\vec{m} \perp \hat{z}$
$(\vec{m} \cdot \vec{l}) = m^2 \frac{(M_1 + M_2)}{(M_1 - M_2)} > 1$ (!). $\vec{l} \times (\vec{L}_0 \times \vec{m}) + \vec{l} \times (\vec{l} \times \vec{M}_0) = \vec{L}_0(\vec{l} \cdot \vec{m}) - \vec{m}(\vec{l} \cdot \vec{L}_0) + \vec{l}(\vec{l} \cdot \vec{M}_0) -$
$- \vec{M}_0 l^2 = 0$, $\vec{L}_0(\vec{l} \cdot \vec{m}) = \vec{M}_0 l^2$, $\frac{1}{2}(M_1 + M_2)(\vec{l} \cdot \vec{m}) = \frac{1}{2}(M_1 - M_2)l^2$, $(\vec{l} \cdot \vec{m}) = l^2 \frac{(M_1 - M_2)}{(M_1 + M_2)}$
$m^2 \frac{(M_1 + M_2)}{(M_1 - M_2)} = l^2 \frac{(M_1 - M_2)}{(M_1 + M_2)}$, $\frac{m^2}{l^2} = \frac{(M_1 - M_2)^2}{(M_1 + M_2)^2}$, $(m/l)^2 \ll 1$ для магн. компенсации $M_0 = 0$, $M_1 = M_2$.

$\frac{\dot{\vec{L}}}{\gamma_+} = \frac{\dot{\vec{M}}}{\gamma_-} \rightarrow \dot{\vec{L}} = (\gamma_+ / \gamma_-)\dot{\vec{M}}$ где $\vec{L} = \vec{L}_0 + \vec{l}$, $\vec{M} = \vec{M}_0 + \vec{l}$

$\dot{\vec{L}} = 2A\gamma_+ \vec{L} \times \vec{M}$ далее $\vec{M} = \vec{M}_0 + a(\vec{L} + \dot{\vec{L}})$, если $\vec{L} \times \vec{M}_0 = 0 = (\vec{L}_0 + \vec{l}) \times \vec{M}_0 = \vec{L}_0 \times \vec{M}_0 + \vec{l} \times \vec{M}_0 = 0$

Нужно знать при $A \to \infty$, $\vec{M}$ в виде $\vec{M} = b\dot{\vec{L}} + a(\vec{L} + \dot{\vec{L}})$, $a \approx -\frac{1}{2\gamma_+ A L^2}(\vec{L} \times \dot{\vec{L}})$
Найти b? $\dot{\vec{M}} = b\ddot{\vec{L}} + a\frac{d}{dt}(\vec{L} + \dot{\vec{L}}) = b\ddot{\vec{L}} + a(\dot{\vec{L}} + \ddot{\vec{L}})$, $\dot{\vec{M}} = 2\gamma_- A \vec{L} \times \vec{M}$.

BP

Skype обсуждение с Оксаной simulations of Hopf index BP in sphere
Проблема t' - метода Q_H calculations [Liu 18] выбор gauge $\operatorname{div}\vec{A} = 0$, т.к. для finite particle и non-localized Soliton Q_H is gauge-dependent. мой выбор gauge of $\vec{A}$, $\frac{\partial}{\partial z}(\times A_{eff}) = 0$ более обоснован (см. 11/XI). Two toroidal Hopfions $\operatorname{div}\vec{A} \neq 0$ [using $\vec{A}$-components by Gladikowski 1997]. Я не получала 600 €
E. Tarbokovskaya переслала ссылку из Poland-NCN 250 млн PLN Cash "Santander"
Hopfion project (200 k€, 2 years) принимает!! Мое grant 6 month с salary 60 k PLN ≈ 13 5 k€
(17/XII) Skype с Ларой Т.: Вторая одобрена visa на Poland-trip и принимает grant.
Обсуждение FiM dynamics с Зухрой. А.К. и Зухра приглашают
Left (FiM) из Zvezdin' JMMM 20 телефон, там надо чел. 2 разных χ_{eff} и $\bar{\chi}_{eff}$
$W_{FiM}, W_G \sim \chi_{eff}$ и $m_{DW} \sim \frac{1}{\chi_{eff}^2}$ будут конкурировать при angular compensation
range $\chi_{eff} \to 0$ $\omega \sim \chi_{eff} \to \infty$, $m_{DW} \sim 1/\chi_{eff}^2 \to 0$ при angul. compensation $\chi_{eff} \to \infty$.
переписка с Зухрой, Maciej и Pawel (APL). 1. Эпос. Киев: 3g./name - 10 kg, 1 л. - 10 kg
Зухра приглашает Lara co-author of corrected name - 5 kg - 50 €.
FiM DW dynamics paper (ArXiv?). Кратко [Liu-18] t'-method для Occ. Q_H.

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FiM Инвар (в пределе $A \to \infty$) $\vec{M} = b\dot{\vec{L}} + a(\vec{L}\times\dot{\vec{L}})$, $\dot{\vec{M}} = b\ddot{\vec{L}} + a(\dot{\vec{L}}\times\ddot{\vec{L}})$ и

$\dot{\vec{M}}(\text{из ур-я Л}) = 2Ak_0(\vec{L}\times\vec{M}) = 2Ak_0\vec{L}\times[b\dot{\vec{L}} + a(\vec{L}\times\dot{\vec{L}})] = 2k_0Aa\,\vec{L}\times(\vec{L}\times\dot{\vec{L}}) = 2k_0Aa\times$
$\times[\vec{L}(\vec{L}\cdot\dot{\vec{L}}) - \dot{\vec{L}}(L^2)]$, $2k_0Aa\left[\vec{L}\frac{d\vec{L}^2}{dt}\frac{1}{2} - L^2\dot{\vec{L}}\right] = b\ddot{\vec{L}} + a(\dot{\vec{L}}\times\ddot{\vec{L}})$.

$b = -2L^2k_0Aa$, $k_0A\vec{L}\frac{d(\vec{L}^2)}{dt} = \vec{L}^2\times\ddot{\vec{L}}$, если $a = -\frac{1}{2k_0AL^2}$, то для b получим $b = -2L^2k_0A\left(-\frac{1}{2}\right)\frac{1}{k_0AL^2} = \frac{\gamma_-}{\gamma_+}$ и $\vec{M} = \frac{\gamma_-}{\gamma_+}\dot{\vec{L}} - \frac{1}{2\gamma_+AL^2}(\vec{L}\times\dot{\vec{L}})$

Но условие $k_0A\vec{L}\frac{d}{dt}(\vec{L}^2) = \vec{L}\dot{\vec{L}}^2$ выполняется только при $\dot{\vec{L}} = 0$.

$\dot{M} = (\gamma_-/\gamma_+)\dot{\vec{L}} + \ldots$ – спранное ур., по-новому на Иванов'83.

→ // Zvezdin JETP Lett'79, Bar.-Ivanov ФНТ'79, Ivanov JETP'83 – это все dynamics of weak FM, AF и FiM near mag. compensation point $M_s = 0$ с учетом необменных тв по γ/γ_{ex} и $|\vec{M}|^2 \ll |\vec{L}|^2$, с $\gamma_1 = \gamma_2$ ($g_1 = g_2$)

A.B. Borisov, V.V. Kiselev, G.G. Tabuts. Sol. St. Com. 44, 411 (1982)

Учтено $\gamma_1 \neq \gamma_2$, но $\vec{m}^2 \ll \vec{l}^2$ обменный предел AF, или FiM (compensat. point)

$\vec{L} = \vec{M}_1 - \vec{M}_2$, $\vec{M} = \vec{M}_1 + \vec{M}_2$, $\vec{L}^2 + \vec{M}^2 = 2(M_1^2 + M_2^2) = 4M_0^2$ def. M_0

$\vec{m} = \frac{1}{2M_0}(\vec{M}_1 + \vec{M}_2)$, $\vec{l} = \frac{1}{2M_0}(\vec{M}_1 - \vec{M}_2)$. При $|\vec{m}|^2 \ll |\vec{l}|^2$ и energy density

$$w = \delta(\vec{M}_1\vec{M}_2) + \frac{d_1}{2}(\partial_x\vec{M}_1)^2 + \frac{d_2}{2}(\partial_x\vec{M}_2)^2 + d_3(\partial_x\vec{M}_1\cdot\partial_x\vec{M}_2)$$

$$\vec{m} = \frac{1}{M_0\delta}\frac{1}{(\gamma_1+\gamma_2)}(\vec{l}\times\dot{\vec{l}}) + \frac{(M_1^2-M_2^2)}{4M_0^2}\vec{l} - \frac{1}{2}[\gamma_1(d_1-d_3) - \gamma_2(d_2-d_3)]\frac{1}{\delta(\gamma_1+\gamma_2)}\vec{l}\times(\vec{l}\times\partial_x^2\vec{l})$$

→ $\vec{l}\times\left(\frac{1}{c^2}\ddot{\vec{l}} - \partial_x^2\vec{l}\right) + \frac{\beta}{c}\dot{\vec{l}} = 0$. $c^2 = M_0^2\delta\gamma_1\gamma_2(d_1+d_2-2d_3)$, $c_{AF}^2 = M_0^2\gamma^2\delta\cdot 2d$

$$\beta = \left[\frac{(M_1^2-M_2^2)}{4M_0^2}(\gamma_1+\gamma_2) + \gamma_2 - \gamma_1\right]\sqrt{\frac{\delta}{\gamma_1\gamma_2(d_1+d_2-2d_3)}}$$ – неправильно, его нет у Иванов JETP'83

$\beta = 0$ только при $M_1 = M_2$ (AF) и $\gamma_1 = \gamma_2$. $\beta \neq 0$ получает β-model.

Иванов ($\gamma_1 = \gamma_2$)'83: $\vec{M} = \frac{(M_1^2-M_2^2)}{|\vec{L}|}\vec{l} + \frac{1}{\gamma\delta}(\dot{\vec{l}}\times\vec{l})$, $|\vec{L}|^2 = 4M_0^2 - |\vec{M}|^2 \approx 4M_0^2$, $|\vec{L}| \approx 2M_0$

Борисов'82 ($\gamma_1 = \gamma_2$, $d_1 \neq d_2$, $d_3 = 0$): $\vec{m} = \frac{\vec{M}}{2M_0} = \frac{1}{M_0\delta\cdot 2\gamma}(\vec{l}\times\dot{\vec{l}}) + \frac{(M_1^2-M_2^2)}{4M_0^2}\vec{l} - \frac{1}{4}\frac{(d_1-d_2)}{\delta}\vec{l}\times(\vec{l}\times\partial_x^2\vec{l})$

Борисов'82 $\vec{m}$ сохранен и Иванов'83 при $d_1 = d_2$ и $(\vec{l}\times\dot{\vec{l}}) \to (\vec{l}\times\dot{\vec{l}})$, $|\vec{l}| = 2M_0$

$$\beta(\gamma_1 = \gamma_2, d_1 = d_2, d_3 = 0) = \left[\frac{M_1^2-M_2^2}{4M_0^2}\cdot 2\gamma\right]\sqrt{\frac{\delta}{\gamma^2 2d}} = \frac{M_1^2-M_2^2}{4M_0^2}\sqrt{2}\sqrt{\frac{\delta}{d}}$$

$$\frac{\beta}{c} = \frac{1}{\gamma M_0\sqrt{2d\delta}}\frac{(M_1^2-M_2^2)}{4M_0^2}\sqrt{2}\sqrt{\frac{\delta}{d}} = \frac{1}{\gamma M_0 d}\frac{(M_1^2-M_2^2)}{4M_0^2}$$. Сравн. с $\nu = \frac{M_s}{2M_0} = \frac{(M_1^2-M_2^2)}{|\vec{L}|\cdot 2M_0}$ ч.м.

$\beta/c\ (\gamma_1 = \gamma_2) = \frac{1}{d\gamma M_0}\nu$. $\nu = \frac{(M_1^2-M_2^2)}{|\vec{L}|2M_0} \approx \frac{(M_1^2-M_2^2)}{|\vec{L}|^2} = \frac{(M_1^2-M_2^2)}{(M_1+M_2)^2} = \frac{(M_1-M_2)}{(M_1+M_2)}$

$\nu = \left(\frac{s_1-s_2}{s_1+s_2}\right)$ в обозначениях Иванов, LTP'2004 через spin density s_1, s_2

т.е. $\beta \sim \nu$ задает FM-like dynamics для FiM.

Пусков $d = \frac{1}{2}(d_1+d_2-2d_3)$. $\frac{\beta}{c} = [\ldots]\sqrt{\frac{\delta}{\gamma_1\gamma_2 2d}}\frac{1}{M_0\sqrt{\delta\gamma_1\gamma_2 2d}} = [\ldots]\frac{1}{2dM_0\gamma_1\gamma_2}$

$$\beta/c = \left[\frac{(M_1^2-M_2^2)}{4M_0^2}(\gamma_1+\gamma_2) + \gamma_2 - \gamma_1\right]\frac{1}{2dM_0\gamma_1\gamma_2}$$, $4M_0^2 = 2(M_1^2+M_2^2)$

(a) Маг. compensation $M_1 = M_2$, $\beta/c = \frac{(\gamma_2-\gamma_1)}{2dM_0\gamma_1\gamma_2}$, $M_0 = M_1$, $\beta/c \neq 0$, т.е. β-term $\mathcal{L}_{Skin}^{FM}$ есть даже для AF при $\gamma_1 \neq \gamma_2$.

(b) Angular momentum compens. ($M_1/\gamma_1 = M_2/\gamma_2$, если $m^2 \ll l^2$) $M_1/\gamma_1 = M_2/\gamma_2$, $\beta/c = \left[\frac{M_1^2(1-\gamma_2^2/\gamma_1^2)(\gamma_1+\gamma_2)}{2M_1^2(1+\gamma_2^2/\gamma_1^2)} + \gamma_2 - \gamma_1\right]\frac{1}{2dM_0\gamma_1\gamma_2}$

$\beta/c = \left[\frac{(1-\gamma_2^2/\gamma_1^2)\cdot(\gamma_1+\gamma_2)}{2(1+\gamma_2^2/\gamma_1^2)(\gamma_1+\gamma_2)} + \gamma_2 - \gamma_1\right]\frac{1}{2dM_0\gamma_1\gamma_2} = \frac{(x-1)^3}{2(1+x^2)}\frac{1}{2dM_0\gamma_2}$, $x = \gamma_2/\gamma_1$

$\beta/c = \frac{\sqrt{2}}{4}\frac{(x-1)^3}{(1+x^2)^{3/2}}\frac{1}{dM_1\gamma_2} \neq 0$ при $\gamma_2 \neq \gamma_1$, $x \neq 1$. FM-like skin есть $\neq 0$. $M_0 = \frac{1}{\sqrt{2}}M_1(1+x^2)^{1/2}$

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(18/XII) Тех. семир. начинает в Шилеба. Конференция от А. Грозу UPV/EHU в Толоса и Сан Себастьян. В „Нам" Ондарета С-л. и Ю.
Paper submitted "Edge on scattering (Kink)" to APL.

(19/XII) D. Loss et al. PRL 1992. Записан Н ферм системи AF chain/ring ($\vec{\ell}$) и central excess spin $\vec{S}$ $S_E = \int_0^\beta d\tau \left[\frac{\chi_\perp}{2\mu_B^2} (\vec{\ell})^2 + J_c \vec{\ell}\cdot\vec{S} + iS\dot{\varphi}_S(1-\cos\theta_S) + \dots \right]$ - „generalized σ-model in terms of Neel vector $\vec{\ell}$ и central spin $\vec{S}$, $S=|\vec{S}|$.

Публикация Kim - Mat. Mat. Jorn Suppl. Работа Ivanov ферм. FiM (см. II/XII)
$\vec{s}_i = \vec{n}_i S_i$, $\vec{n}_2 S_2 = \vec{S}_2$ - spin density of sublattice i, $S_i = M_i/\gamma_i$, $\vec{n}_i$ - unit vectors.

$\mathcal{L}_{kin}(FiM) = S_1 \vec{a}_1(\vec{n}_1)\cdot\dot{\vec{n}}_1 + S_2 \vec{a}_2(\vec{n}_2)\cdot\dot{\vec{n}}_2 = S_1 \frac{\hat{z}_1 \times \vec{n}_1}{1+\hat{z}_1\vec{n}_1}\cdot\dot{\vec{n}}_1 + S_2 \frac{\hat{z}_2\times\vec{n}_2}{1+\hat{z}_1\vec{n}_2}\cdot\dot{\vec{n}}_2$

$\vec{m} = \vec{n}_1 + \vec{n}_2$, $\vec{n} = \frac{1}{2}(\vec{n}_1 - \vec{n}_2)$, $\vec{n}_1 = \frac{1}{2}\vec{m} + \vec{n}$, $\vec{n}_2 = \frac{1}{2}\vec{m} - \vec{n}$.

$\mathcal{L}_{kin}(FiM) = S_1 \vec{a}_1(\vec{n} + \frac{1}{2}\vec{m})\cdot(\dot{\vec{n}} + \frac{1}{2}\dot{\vec{m}}) + S_2 \vec{a}_2(-\vec{n} + \frac{1}{2}\vec{m})\cdot(-\dot{\vec{n}} + \frac{1}{2}\dot{\vec{m}})$

$\vec{a}(\vec{n} + \frac{1}{2}\vec{m}) = \vec{a}(\vec{n}) + \frac{\partial \vec{a}}{\partial \vec{n}}\frac{1}{2}\vec{m}$, $a_i(\vec{n} + \frac{1}{2}\vec{m}) = a_i(\vec{n}) + \frac{\partial a_i}{\partial \vec{n}}\cdot\frac{1}{2}\vec{m}$

$\mathcal{L}_{kin}(FiM) = S_1\left[a_{1i} + \frac{\partial a_{1i}}{\partial \vec{n}}\cdot\frac{1}{2}\vec{m}\right](\dot{n}_i + \frac{1}{2}\dot{m}_i) + S_2\left[a_{2i} + \frac{\partial a_{2i}(-\vec{n})}{\partial(-\vec{n})}\cdot\frac{1}{2}\vec{m}\right](-\dot{n}_i + \frac{\dot{m}_i}{2})$

$= S_1 a_{1i}\dot{n}_i + S_1 \frac{\partial a_{1i}}{\partial \vec{n}}\cdot\frac{1}{2}\vec{m}\dot{n}_i + S_2 a_{2i}(-\dot{n}_i) + S_2 \frac{\partial a_{2i}(\vec{x})}{\partial \vec{x}}\Big|_{x=-\vec{n}}\cdot\frac{1}{2}\vec{m}(-\dot{n}_i)$

$\mathcal{L}_{kin}(FiM) = S_1 \vec{a}_1(\vec{n})\cdot\dot{\vec{n}} + S_2 \vec{a}_2(-\vec{n})\cdot(-\dot{\vec{n}}) + \frac{1}{2}S_1 \frac{\partial a_{1i}(\vec{x})}{\partial \vec{x}}\Big|_{x=\vec{n}}\cdot\vec{m}\dot{n}_i - \frac{1}{2}S_2 \frac{\partial a_{2i}(\vec{x})}{\partial \vec{x}}\Big|_{x=-\vec{n}}\vec{m}\dot{n}_i$

$\vec{a}_1(\vec{n}) = \frac{\hat{z}_1\times\vec{n}_1}{1+\hat{z}_1\cdot\vec{n}}$, $\vec{a}_2(-\vec{n}) = -\frac{\hat{z}_2\times\vec{n}}{1+z_2(-\vec{n})}$ $= |\hat{z}_2 = -\hat{z}_1| = \frac{\hat{z}_1\times\vec{n}}{1+\hat{z}_1\vec{n}} = \vec{a}_1(\vec{n})$

$\frac{\partial}{\partial x}a_{1i} = \frac{\partial}{\partial x}\frac{(\hat{z}_1\times\vec{n}_1)_i}{1+\hat{z}_1\cdot\vec{n}_1} = \frac{\partial}{\partial x}\frac{(\hat{z}_1\times\vec{x})_i}{1+\hat{z}_1\vec{x}} = \left(\frac{\partial}{\partial x}\frac{\hat{z}_1\times\vec{x}}{1+\hat{z}_1\vec{x}}\right)_i = \left(\frac{(\hat{z}_1\times\vec{n}_x)\cdot(1+\hat{z}_1\vec{x}) - (\hat{z}_1\times\vec{x})\cdot\hat{z}_1}{(1+\hat{z}_1\vec{x})^2}\right)_i$.

$\mathcal{L}_{kin}(FiM) = (S_1 - S_2)\vec{a}(\vec{n})\cdot\dot{\vec{n}} + \frac{1}{2}\left[S_1\frac{\partial a_{1i}(\vec{x})}{\partial x}\Big|_{x=\vec{n}} - S_2\frac{\partial a_{2i}(\vec{x})}{\partial x}\Big|_{x=-\vec{n}}\right]\vec{m}\dot{n}_i$

$\mathcal{L}_{kin}(FiM) = (S_1-S_2)\vec{a}(\vec{n})\cdot\dot{\vec{n}} + \frac{1}{2}(S_1+S_2)\frac{\partial a_i(\vec{n})}{\partial \vec{n}}\cdot\vec{m}\dot{n}_i$, $\vec{a}(\vec{n}) = a_1(\vec{n})$.

Kim' Nat. Mat. 2017: $\mathcal{L}_{kin}(FiM) = (S_1-S_2)\vec{a}(\vec{n})\cdot\dot{\vec{n}} + \frac{1}{2}(S_1+S_2)\vec{n}\cdot(\vec{n}\times\dot{\vec{m}})$

$\mathcal{L}_{kin}(FiM)$ собствено чрез $\frac{\partial a_i(\vec{n})}{\partial \vec{n}}\cdot\vec{m}\dot{n}_i = \dot{\vec{n}}\cdot(\vec{n}\times\vec{m})$, $\vec{a}(n) = \frac{\hat{z}_1\times\vec{n}}{1+\hat{z}_1\cdot\vec{n}}$

$\dot{\vec{n}}\cdot(\vec{n}\times\vec{m}) = \vec{m}\cdot(\vec{n}\times\dot{\vec{n}})$, $\frac{\partial a_i(\vec{n})}{\partial \vec{n}}\dot{n}_i = \dot{\vec{n}}\times\vec{n}$.

$\frac{\partial}{\partial \vec{n}}\vec{a}_i(\vec{n})\cdot\dot{\vec{n}}_i = \frac{\partial}{\partial \vec{n}}\vec{a}\cdot\dot{\vec{n}} = \frac{\partial}{\partial \vec{n}}\frac{(\hat{z}\times\vec{n})\cdot\dot{\vec{n}}}{1+\hat{z}\cdot\vec{n}} = \frac{\partial}{\partial \vec{n}}\frac{\vec{n}\cdot(\dot{\vec{n}}\times\hat{z})}{1+\hat{z}\cdot\vec{n}} = \frac{(\dot{\vec{n}}\times\hat{z})(1+\hat{z}\vec{n}) - \vec{m}(\dot{\vec{n}}\times\hat{z})\hat{z}}{(1+\hat{z}\cdot\vec{n})^2}$

Лагранжиан $\mathcal{L}(FiM)$ преминава в $\mathcal{L}(\vec{n})$ при $(\vec{m}^2/\vec{n}^2) \ll 1$, с $\vec{m} = \vec{n}_1 + \vec{n}_2$ - total spin moment density. $\vec{n}_i = \vec{S}_i/|\vec{S}_i|$ работата с ЯД-Борисов 82 из-за $\vec{n}_i$ нормировка.
Отчет: Anony UPV/EHU credits on-line (www.euskadi.eus) - 595,23€ получ. с Л. в „Carrefour" Обаберия.

(20/XII) Все модели FiM dynamics published содержат разложение по $|\vec{M}|/|\vec{L}| \ll 1$ близо magnetic compensation point $|\vec{M}| = 0$ или по $\frac{|\vec{m}|}{|\vec{n}|} \ll 1$ близо angular momentum comp. point $|\vec{m}| = |\vec{S}_1 + \vec{S}_2| \sim (S_1 - S_2) = 0$.
Формално σ-model (разложение по $|\vec{M}|/|\vec{L}| \ll 1$) може бъде extended on FiM, где $\vec{M}_1 + \vec{M}_2 \neq 0$, но $\frac{|\vec{m}|}{|\vec{n}|} \sim (S_1 - S_2) \ll 1$, $S_1 \approx S_2$.

Борисов' 82: $\vec{m} = \frac{(M_1^2 - M_2^2)}{4M_0^2}\dot{\vec{\ell}} + \frac{1}{M_0 B}\frac{1}{(\gamma_1+\gamma_2)}(\vec{\ell}\times\dot{\vec{\ell}}) + (\lambda_x\vec{\ell})$

$(\vec{m}\cdot\vec{\ell}) = \frac{(M_1^2 - M_2^2)}{4M_0^2}\vec{\ell}^2 = \frac{1}{2M_0}(\vec{M}_1 + \vec{M}_2)\frac{1}{2M_0}(\vec{M}_1 - \vec{M}_2) = \frac{1}{4M_0^2}(M_1^2 - M_2^2)$, $\vec{\ell}^2 = 1$,

т.е. коэффициентът пред $\dot{\vec{\ell}}$ в разложението $\vec{m}$ всъщност е $(\vec{m}\cdot\vec{\ell})$.
$\vec{\ell}^2 = \frac{1}{4M_0^2}(\vec{M}_1 - \vec{M}_2)^2 = \frac{1}{4M_0^2}(M_1^2 + M_2^2 - 2\vec{M}_1\vec{M}_2)$, $L^2 \approx 4M_0^2$, $\vec{\ell} = \vec{L}/|\vec{L}| \approx 1$.

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(21/XII) Фархад прислал MS draft „FMR in cylindrical magn. nanowires". Анализ и обсуждение Фархаду (вкл. Onc. и добавить Theory sec.). Прислали APL - Dobrovolskij.

(22/XII) Анализ PhD thesis by T. Okuno · 2020 „$\vec{M}$-dynamics in AF Coupled Ferrimagnets" Хорошо описана FiM DW dynamics при $\xi = (S_1 - S_2) \ll 1$ с аккуратн. Refs.

Dmytriieva et al. JPCM'2020 Вывели $\mathcal{L}_{eff}$ для FiM с $\gamma_1 = \gamma_2 = \gamma$ FiM

Comp. Compensation point $m = M_1 - M_2 = 0$ (magnetic compensation). т.к. $\gamma_1 = \gamma_2$ это и mechanical (Spin) compensation point. $m = (M_1, M_2) < dl = M_1 + M_2$, и т.к. $|\vec{M}| \ll |\vec{L}|$ standard approximation, $\vec{L} = \vec{M}_1 - \vec{M}_2$, $\vec{M} = \vec{M}_1 + \vec{M}_2$, $\vec{L} = \vec{L}(\theta, \varphi)$; (θ, φ) method исключение $\vec{M}$ из УД.

→ $\mathcal{L}_{eff} = \frac{1}{2}\frac{\chi_\perp}{\gamma^2}(\dot{\theta})^2 + m\cos\theta\left(H - \frac{\dot{\varphi}}{\gamma}\right) + \frac{1}{2}\chi_\perp \sin^2\theta\left(H - \frac{\dot{\varphi}}{\gamma}\right)^2 + K(\vec{l}\cdot\vec{n})^2$ получено из $\mathcal{L} = -\frac{M_1}{\gamma}\cos\theta\dot{\varphi}_1 - \frac{M_2}{\gamma}\cos\theta_2\dot{\varphi}_2 - \Phi(\vec{M}_1, \vec{M}_2, \vec{H})$ в Sublattice FiM.

$\Phi(\vec{M}_1, \vec{M}_2, \vec{H}) = -(\vec{M}_1 + \vec{M}_2)\vec{H} + \lambda\vec{M}_1\vec{M}_2 + \Phi_{a1}(\vec{M}_1) + \Phi_{a2}(\vec{M}_2)$. $\chi_\perp = \frac{2M^2}{\delta}$, $M = M_1 + M_2$, $\delta = 2\lambda M_1 M_2$

$\Phi_{eff} = -mH\cos\theta - \frac{1}{2}\chi_\perp H^2\sin^2\theta - K(\vec{l}\cdot\vec{n})^2$, $\vec{l} = \vec{L}/L$. $\vec{l} = \vec{l}(\theta, \varphi)$, $\vec{l}^2 = 1$.

Найдем $\mathcal{L}_{eff}^{kin}$. $\mathcal{L}_{eff} = \mathcal{L}_{eff}^{kin} - \Phi_{eff}$ ⇒ effective kinetic Lagrangian

$\mathcal{L}_{eff}^{kin} = \frac{1}{2}\frac{\chi_\perp}{\gamma^2}(\dot{\theta})^2 - \frac{m}{\gamma}\cos\theta\dot{\varphi} + \frac{1}{2}\chi_\perp\sin^2\theta\left[\frac{\dot{\varphi}^2}{\gamma^2} - 2H\frac{\dot{\varphi}}{\gamma}\right]$. $\vec{H} = H\hat{z}$.

$\mathcal{L}_{eff}^{kin} = \frac{1}{2}\frac{\chi_\perp}{\gamma^2}\left[(\dot{\theta})^2 + \sin^2\theta(\dot{\varphi})^2\right] - \frac{m}{\gamma}\cos\theta\dot{\varphi} - \chi_\perp\sin^2\theta H\frac{\dot{\varphi}}{\gamma}$, $\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2 = (\dot{\vec{l}})^2$

AF F

Bugunno, yob. Dmytriieva'2020 „the Lagrangian and ... preserve the same form if we substitute the parameters γ and α with T-dependent factors γ_{eff} and α_{eff}"

$\frac{1}{\gamma_{eff}} = \frac{1}{\gamma}\left[1 - \frac{dl}{m}\frac{(\gamma_1 - \gamma_2)}{(\gamma_1 + \gamma_2)}\right] = \frac{M_1/\gamma_1 - M_2/\gamma_2}{(M_1 - M_2)}$, $\frac{1}{\gamma} = \frac{1}{2}\left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right)$ — неверно (!).

γ_{eff} использ. в Zvezdin JMMM'2020. Затем Звездин прислал Word file с γ_{eff} и $\bar{\gamma}_{eff}$ разные :)

New $\mathcal{L}_{eff} = \frac{1}{2}\frac{\chi}{(\bar{\gamma}_{eff})^2}\dot{\theta}^2 + m\cos\theta\left(H - \frac{\dot{\varphi}}{\gamma_{eff}}\right) + \frac{1}{2}\chi_\perp\sin^2\theta\left(H - \frac{\dot{\varphi}}{\gamma_{eff}}\right)^2 \to W_n - W_{ex} - W_D$

$\bar{\gamma}_{eff} = \gamma\left(1 - \frac{mm_0}{M^2}\right)^{-1}$, $\gamma_{eff} = \gamma\frac{m}{(m - m_0)}$, $m = \frac{1}{2}(M_2 - M_1)$, $M = \frac{1}{2}(M_1 + M_2)$,

$m_0 = M\frac{(\gamma_1 - \gamma_2)}{(\gamma_1 + \gamma_2)}$ значение „m" в точке угловой компенсации T_A.

При T_A $m = m_0$ $\bar{\gamma}_{eff} = \bar{\gamma}\left(1 - \frac{m_0^2}{M^2}\right)^{-1} = \bar{\gamma}\frac{M^2}{(M^2 - m_0^2)} = \bar{\gamma}\left(1 - \frac{(\gamma_1 - \gamma_2)}{(\gamma_1 + \gamma_2)}\right)^{-1} = \bar{\gamma}\left(\frac{2\gamma_2}{\gamma_1 + \gamma_2}\right)^{-1} =$

$= 2\frac{\gamma_1\gamma_2}{(\gamma_1 + \gamma_2)}\frac{(\gamma_1 + \gamma_2)}{2\gamma_2} = \gamma_1$ (аккомпл. γ_1 и γ_2), $\gamma_{eff} \to \infty$, т.к. $m \to m_0$ при T_A.

$m = m_0$, $\frac{1}{2}(M_2 - M_1) = \frac{1}{2}(M_2 + M_1)\frac{(\gamma_1 - \gamma_2)}{\gamma_1 + \gamma_2}$, $(M_2 - M_1)(\gamma_1 + \gamma_2) = (M_2 + M_1)(\gamma_1 - \gamma_2)$

$M_2\gamma_1 + M_2\gamma_2 - M_1\gamma_1 - M_1\gamma_2 = M_2\gamma_1 + M_1\gamma_1 - M_2\gamma_2 - M_1\gamma_2$, $M_2\gamma_2 = M_1\gamma_1$ (?)

надо $-(M_2 - M_1)(\gamma_1 + \gamma_2) = (M_2 + M_1)(\gamma_1 - \gamma_2) \Rightarrow -M_2\gamma_1 - M_2\gamma_2 + M_1\gamma_1 + M_1\gamma_2 =$

$M_2\gamma_1 = M_1\gamma_2$, $M_1/\gamma_1 = M_2/\gamma_2$ (верно). $= M_2\gamma_1 + M_1\gamma_1 - M_2\gamma_2 - M_1\gamma_2$

→ т.е. в Звездин' JMMM'20 надо либо $m \to -m$, либо $m_0 \to -m_0$, чтоб $m = m_0$ давало T_A.

$\mathcal{L}_{eff}(H=0) = \frac{1}{2}\frac{\chi_\perp}{(\gamma_{eff})^2}\left[(\dot{\theta})^2 + \sin^2\theta(\dot{\varphi})^2\right] - \frac{m}{\gamma_{eff}}\cos\theta\dot{\varphi}$, $\frac{m}{\gamma_{eff}} = \frac{1}{\gamma}(m - m_0) = 0$ при T_A,

// FM $\mathcal{L}_{kin} = -\frac{m}{\gamma_{eff}}\cos\theta\dot{\varphi} \to 0$ при T_A, $S_1 = S_2$, как у Tserkovnyak'2017 и Ivanov 2019.

γ_{eff} в new $\mathcal{L}_{eff}$ - Zvezdin'20 верно, а в $\bar{\gamma}_{eff}$ надо доказывать.

Проверить в new - $\mathcal{L}_{eff}$ (Zvezdin), γ_e, γ_e в Звездин TM (mag. compensation) и T_A!

- 1) сравнить УД $\mathcal{L}_{eff}$ - Borisov'22 с УД, $\mathcal{L}_{eff}$ - Zvezdin;
- 2) решить наш общий УД 15/XII при T_M и/или T_A.

Сбоку УД через $\vec{l}$ и θ, φ: $+\nu\frac{\partial\vec{l}}{\partial t} = \frac{1}{\omega_{ex}}\vec{l}\times\left[\ddot{\vec{l}} - c^2\nabla^2\vec{l}\right] + \ldots$ comp. $-\nu\sin\theta\dot{\varphi} = \frac{1}{\omega_{ex}}\left[\ddot{\theta} - c^2\nabla^2\theta\right] - \sin\theta\cos\theta\frac{1}{\omega_{ex}}\left[(\dot{\varphi})^2 - c^2(\nabla\varphi)^2\right] + \ldots$

[Ivanov '83, 2019]

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$\frac{m}{\gamma_e} = \frac{m}{\gamma_{eff}} = \frac{1}{\delta}(m-m_0) = 0$ при $m = m_0$, T_A. $2A\left(\frac{\beta}{c}\right)_{T_A} = 2A\frac{\sqrt{2}}{4}\frac{(x-1)^3}{(1+x^2)^{3/2}}\frac{1}{2M_1\gamma_2}$, $x = \frac{\gamma_2}{\gamma_1}$

$\frac{A}{2} = \frac{1}{4}d^2(M_1+M_2)^2$, $\frac{1}{4}(M_1+M_2)^2\frac{\sqrt{2}}{2}\frac{(x-1)^3}{(1+x^2)^{3/2}}\frac{1}{M_1\gamma_2} = -\frac{(\gamma_1+\gamma_2)}{2\gamma_1\gamma_2}(m-m_0)$

$m - m_0 = \frac{1}{2}(M_1-M_2) - \frac{1}{2}(M_1+M_2)\frac{(\gamma_1-\gamma_2)}{(\gamma_1+\gamma_2)} = \frac{1}{2(\gamma_1+\gamma_2)}\left[(M_1-M_2)(\gamma_1+\gamma_2) - (M_1+M_2)(\gamma_1-\gamma_2)\right] =$

$= \frac{1}{2(\gamma_1+\gamma_2)}[2M_1\gamma_2 - 2M_2\gamma_1] = \frac{\gamma_1\gamma_2}{(\gamma_1+\gamma_2)}\left[\frac{M_1}{\gamma_1} - \frac{M_2}{\gamma_2}\right] = \frac{1}{2}\delta\left(\frac{M_1}{\gamma_1} - \frac{M_2}{\gamma_2}\right)$

$\frac{m}{\gamma_e} = \frac{1}{\delta}(m-m_0) = \frac{1}{\delta}\frac{1}{2}\delta\left(\frac{M_1}{\gamma_1} - \frac{M_2}{\gamma_2}\right)\frac{(\gamma_1+\gamma_2)}{} = \frac{1}{2}\left(\frac{M_1}{\gamma_1} - \frac{M_2}{\gamma_2}\right)$

$\frac{\sqrt{2}}{8}\frac{(x-1)^3}{(1+x^2)^{3/2}}\frac{(M_1+M_2)^2}{M_1\gamma_2} = -\frac{1}{2}\left(\frac{M_1}{\gamma_1} - \frac{M_2}{\gamma_2}\right)$ не выполнено. Выполняется лишь при $x = \frac{\gamma_2}{\gamma_1} = 1$, T_A, или $x \ll 1$, T_A, при $\left(\frac{\beta}{c}\right)$ при T_A. Для обычно $\left(\frac{\beta}{c}\right)$. $2A\frac{\beta}{c} = -\frac{m}{\gamma_e}$, $\gamma_e \neq \gamma_{eff}$, не выполняется.

→ Вывод: γ_{eff} (Zvezdin+new) и γ_{eff} (Borisov) несовместимы даже при $\gamma_1 \approx \gamma_2$.

$\frac{1}{\gamma_e} = \frac{1}{\gamma}\left(1 - \frac{mm_0}{M^2}\right)$, $\left(1 - \frac{mm_0}{M^2}\right) = \frac{(\gamma_1 M_1 + \gamma_2 M_1)}{(\gamma_1+\gamma_2)M}$, при $m = \frac{1}{2}(M_1-M_2)$, что дает γ_e при $m = m_0$.

Если $m = -\frac{1}{2}(M_1-M_2)$, то $\left(1 - \frac{mm_0}{M^2}\right) = \frac{1}{2(\gamma_1+\gamma_2)M}\left[(\gamma_1+\gamma_2)(M_1+M_2) + (M_1-M_2)(\gamma_1-\gamma_2)\right] =$

$= \frac{1}{2(\dots)}\left[2M_1\gamma_1 + 2\gamma_2 M_2\right] = \frac{(M_1\gamma_1 + M_2\gamma_2)}{(\gamma_1+\gamma_2)M}$. Все равно не выходит ←

$\frac{1}{\gamma_{eff}} = \frac{(M_1/\gamma_1 - M_2/\gamma_2)}{(M_1-M_2)}$ из Davydova, 2020. $\frac{1}{\gamma_{eff}}(\dot\theta)^2$ [Davdova '20], $\frac{1}{\gamma_e}(\dot\theta)^2$ [Zvezdin-new]

→ также определяет DW (Soliton) mass.

$\frac{m}{\gamma_e} = \frac{1}{2}\left(\frac{M_1}{\gamma_1} - \frac{M_2}{\gamma_2}\right)$, $\frac{1}{\gamma_e} = \frac{(M_1/\gamma_1 - M_2/\gamma_2)}{(M_1-M_2)} = \frac{1}{\gamma_{eff}}$ из [Davydova '20].

Zvezdin-new предлагает γ_{eff} на $\gamma_{eff} = \gamma_e = \gamma\left(1 - \frac{mm_0}{M^2}\right)$, γ_e не имеет сингулярности ни при T_A, ни при T_M, но не согласуется с γ_{eff} (Borisov '82).

(23/XII) Вернемся к общим УД для $\vec L$ и $\vec M$ от 15/XII.

ТВ по $1/A$: $\vec L = \vec L_0 \times \vec l$, $\vec M = \vec M_0 + \vec m$, $\vec L_0 = \frac{1}{2}(M_1+M_2)\hat z$, $\vec M_0 = \frac{1}{2}(M_1-M_2)\hat z$

упрощено при $H=0$, $d_i=0$ (1) $\dot{\vec l} = 2A\gamma_+(\vec l \times \vec m)$, (2) $\dot{\vec m} = 2A\gamma_-(\vec l \times \vec m)$, $\vec l \times \dot{\vec l} = 2A\gamma_+\,\vec l \times (\vec l \times \vec m) =$

$= 2A\gamma_+\left[\vec l(\vec l\cdot\vec m) - \vec m l^2\right]$, требуем для ТВ $\vec L_0 \times \vec m + \vec l \times \vec M_0 = 0$

$0 = \vec l \times (\vec L_0 \times \vec m) + \vec l \times (\vec l \times \vec M_0) = \vec L_0(\vec l\cdot\vec m) - \vec m(\vec l\cdot\vec L_0) + \vec l(\vec l\cdot\vec M_0) - \vec M_0(\vec l\cdot\vec l)$, или $\vec l \perp \vec L_0$, $\vec l \perp \vec M_0$

$\vec L_0(\vec l\cdot\vec m) = \vec M_0 l^2$, $(\vec l\cdot\vec m) = \frac{M_0}{L_0}l^2$, $(\vec l\cdot\vec m) = \frac{M_0}{L_0}l^2 = \frac{(M_1-M_2)}{(M_1+M_2)}l^2 = \frac{M_-}{M_+}l^2$, $\vec l \perp \hat z$

(1) $\dot{\vec l} = 2A\gamma_+\left[\vec l\frac{M_-}{M_+}l^2 - \vec m l^2\right] = 2A\gamma_+ l^2\left[\frac{M_-}{M_+}\vec l - \vec m\right]$, находим $\vec m$:

$\vec m = \frac{M_-}{M_+}\vec l - \frac{1}{2A\gamma_+ l^2}(\vec l \times \dot{\vec l})$. $\dot{\vec m} = \frac{M_-}{M_+}\dot{\vec l} - \frac{1}{2A\gamma_+ l^2}(\vec l \times \ddot{\vec l})$, если $\frac{d}{dt}(\vec l^2) = 0$

(2) eq. $\dot{\vec m} = \frac{M_-}{M_+}\dot{\vec l} - \frac{1}{2A\gamma_+ l^2}(\vec l \times \ddot{\vec l}) = 2\gamma_- A\,\vec l \times \left[\frac{M_-}{M_+}\vec l - \frac{1}{2A\gamma_+ l^2}(\vec l \times \dot{\vec l})\right] =$

$= -\frac{\gamma_-}{\gamma_+}\frac{1}{l^2}\vec l \times (\vec l \times \dot{\vec l}) = -\frac{\gamma_-}{\gamma_+}\frac{1}{l^2}\left[\vec l(\vec l\cdot\dot{\vec l}) - \dot{\vec l}\,l^2\right] = \frac{\gamma_-}{\gamma_+}\dot{\vec l}$

$\frac{M_-}{M_+}\dot{\vec l} - \frac{1}{2A\gamma_+ l^2}(\vec l \times \ddot{\vec l}) = \frac{\gamma_-}{\gamma_+}\dot{\vec l}$, $\vec l\left(\frac{M_-}{M_+} - \frac{\gamma_-}{\gamma_+}\right) = \vec l \times \left[\frac{1}{2A\gamma_+ l^2}\dddot{\vec l} + \dots\right]$

$\frac{M_-}{M_+} - \frac{\gamma_-}{\gamma_+} = \frac{1}{\gamma_+ M_+}\left[M_-\gamma_+ - \gamma_- M_+\right] = \frac{1}{\gamma_+ M_+}\left[(M_1-M_2)\frac{1}{2}(\gamma_1+\gamma_2) - \frac{1}{2}(\gamma_1-\gamma_2)(M_1+M_2)\right] =$

$= \frac{1}{\frac{1}{2}(\gamma_1+\gamma_2)}\frac{1}{(M_1+M_2)}\gamma_1\gamma_2(M_1/\gamma_1 - M_2/\gamma_2) = \delta\frac{(S_1+S_2)}{(M_1+M_2)}$, $\delta = \frac{2\gamma_1\gamma_2}{(\gamma_1+\gamma_2)}$, $S_i = \frac{M_i}{\gamma_i}$

→ $\delta\frac{(S_1-S_2)}{(M_1+M_2)}\dot{\vec l} = \vec l \times \left[\frac{1}{2A\gamma_+ l^2}\ddot{\vec l} + \dots\right]$, УД для $\vec l$ оставили только $\dot{\vec l}$ как $|\vec l| \approx 1/A$, $|\dot{\vec l}| \approx 1/A$ ТВ по $1/A$.

Вот так да!! Предположение $|\vec m| \ll |\vec l|$, $S_1 \approx S_2$ не делали, лишь ТВ по $\frac{1}{A}$.

$\vec l \cdot \dot{\vec l} = 2A\gamma_+\,\vec l\cdot(\vec l \times \vec m) = 0$, $\vec l^2 = \text{const}$ в главном приближении. Еще в духе ТВ потребовали $\vec L_0 \times \vec m + \vec l \times \vec M_0 = 0$, что дало $(\vec m\cdot\vec l) = \frac{M_-}{M_+}l^2$ явно.

Ни δ как у (S_1-S_2) как у Ivanov '2019, ни коэффициенты другие и в самом ЭЧ Tszkovnyak '2017, Ivanov '2019, а $(S_1-S_2) \ll (S_1+S_2)$ не предполагали.

Escaneado con CamScanner

$\frac{\gamma(S_1-S_2)}{(M_1+M_2)}\dot{\vec{l}} = \vec{l}\times\left[\frac{1}{2A\gamma_+ l^2}\ddot{\vec{l}}+\dots\right]$ совместно с $l \sim \frac{1}{A}$. $W = A\vec{M}_1\cdot\vec{M}_2 + \dots$

Видно, что $l^2 + m^2 = 0$ или $(\vec{m}\cdot\vec{M}_0) = 0$, $(\vec{l}\cdot\vec{L}_0) = 0$, $\vec{m}\perp\hat{z}$, $\vec{l}\perp\hat{z}$.

$\vec{L}_0(\vec{l}\cdot\vec{m}) = \vec{m}(\vec{l}\cdot\vec{L}_0) - \vec{l}(\vec{l}\cdot\vec{M}_0) + \vec{M}_0 l^2$, $L_0^2(\vec{l}\cdot\vec{m}) = m_2(\vec{l}\cdot\vec{L}_0) - l_2(\vec{l}\cdot\vec{M}_0) + M_0^2 l^2$

$(\vec{l}\cdot\vec{m}) = m_2 l_2 - l_z^2\frac{M_0^2}{L_0^2} + \frac{M_0^2}{L_0^2}l^2$. Но $\vec{L}_0\times\vec{m} + \vec{l}\times\vec{M}_0 = 0$ только не говорит о l_z, m_z.

$\vec{L}^2 = (\vec{L}_0+\dot{\vec{l}})^2 = L_0^2 + 2\dot{\vec{L}}_0\dot{\vec{l}} + \dot{\vec{l}}^2$, $\vec{M}^2 = (\vec{M}_0+\vec{m})^2 = M_0^2 + 2\vec{M}_0\vec{m} + \vec{m}^2$

$\vec{L}^2 + \vec{M}^2 = \frac{1}{2}(\vec{M}_1^2+\vec{M}_2^2) = L_0^2 + M_0^2 + 2\vec{L}_0\cdot\dot{\vec{l}} + 2\vec{M}_0\cdot\vec{m} + l^2 + m^2$, $L_0^2 + M_0^2 = \frac{1}{2}(M_1^2+M_2^2)$

$M_+ l_z + M_- m_z + l^2 + m^2 = 0$ и $(\vec{l}\cdot\vec{m}) = m_z l_z - l_z^2\frac{M_-}{M_+} + \frac{M_-}{M_+}l^2$ – ? Оставим $\vec{m}, \vec{l}\perp\hat{z}$ ($m_z=0, l_z=0$).

Если $\vec{n} = \frac{\vec{l}}{|\vec{l}|}$, $\vec{n}^2 = 1$, то $\left(\frac{\gamma}{M_+}(S_1-S_2)\,l\,\dot{\vec{n}} = \vec{n}\times\left[\frac{1}{2A\gamma_+}\ddot{\vec{n}}+\dots\right]\right)$, l^2 – произвольно.

// Остановимся пока от приближения $\vec{L} = \vec{L}_0 + \vec{l}$, $\vec{M} = \vec{M}_0 + \vec{m}$ в общем УД 15/III

Упрощение, $H=0, \alpha_i=0, W_{ai}=0$ $\dot{\vec{L}} = 2A\gamma_+\vec{L}\times\vec{M}$ (1) и $\dot{\vec{M}} = 2A\gamma_-\vec{L}\times\vec{M}$ (2).

$\vec{L}\times\dot{\vec{L}} = 2A\gamma_+\vec{L}\times(\vec{L}\times\vec{M}) = 2A\gamma_+[\vec{L}(\vec{L}\cdot\vec{M}) - \vec{M}L^2]$

$(\vec{L}\cdot\vec{M}) = \frac{1}{2}(\vec{M}_1-\vec{M}_2)\frac{1}{2}(\vec{M}_1+\vec{M}_2) = \frac{1}{4}(M_1^2-M_2^2) = \frac{1}{4}(M_1-M_2)(M_1+M_2) = \frac{1}{4}M_+M_-$

$M_\pm = \frac{1}{2}(M_1\pm M_2)$ – теперь. $\frac{1}{2A\gamma_+}\vec{L}\times\dot{\vec{L}} = \vec{L}M_+M_- - \vec{M}L^2$, $\vec{M} = \frac{M_+M_-}{L^2}\vec{L} - \frac{1}{2A\gamma_+L^2}\vec{L}\times\dot{\vec{L}}$

Def. $\vec{l} = \vec{L}/L$, $\vec{l}^2 = 1$, $\vec{M} = \frac{M_+M_-}{L}\vec{l} - \frac{1}{2A\gamma_+}\vec{l}\times\dot{\vec{l}}$,

но $|\vec{L}| = L$ не есть const (+). $\vec{L}^2 + \vec{M}^2 = \frac{1}{2^2}(\vec{M}_1-\vec{M}_2)^2 + \frac{1}{4}(\vec{M}_1+\vec{M}_2)^2 = \frac{1}{2}(M_1^2+M_2^2) = M^2$ есть const.

не так просто $\vec{l} = \vec{L}/L$. $\frac{d}{dt}\vec{L} = \frac{d}{dt}(\vec{l}L) = L\dot{\vec{l}} + \vec{l}\dot{L} = \dot{\vec{L}}$

$\vec{M} = \frac{M_+M_-}{L}\vec{l} - \frac{1}{2A\gamma_+}\vec{l}\times\frac{1}{L}(L\dot{\vec{l}} + \vec{l}\dot{L}) = \frac{M_+M_-}{L}\vec{l} - \frac{1}{2A\gamma_+}\vec{l}\times\dot{\vec{l}}$, т.к. $\vec{l}\times\vec{l} = 0$.

$\dot{\vec{M}} = M_+M_-\frac{d}{dt}\left(\frac{1}{L}\vec{l}\right) - \frac{1}{2A\gamma_+}\dot{\vec{l}}\times\dot{\vec{l}} = M_+M_-\left[\frac{1}{L}\dot{\vec{l}} + \vec{l}\left(-\frac{1}{L^2}\right)\dot{L}\right] - \frac{1}{2A\gamma_+}\dot{\vec{l}}\times\dot{\vec{l}}$

$M_+M_-\left[\frac{1}{L}\dot{\vec{l}} - \frac{1}{L^2}\dot{L}\vec{l}\right] = 2A\gamma_-\vec{L}\times\left[\frac{M_+M_-}{L}\vec{l} - \frac{1}{2A\gamma_+}\vec{l}\times\dot{\vec{l}}\right] = -\frac{\gamma_-}{\gamma_+}L\vec{l}\times(\vec{l}\times\dot{\vec{l}})$

$-\frac{1}{2A\gamma_+}\vec{l}\times\ddot{\vec{l}}$ $M_+M_-\left[\frac{1}{L}\dot{\vec{l}} - \frac{\dot{L}}{L^2}\vec{l}\right] - \frac{1}{2A\gamma_+}\dot{\vec{l}}\times\ddot{\vec{l}} = -\frac{\gamma_-}{\gamma_+}L\dot{\vec{l}}\times(\vec{l}\times\dot{\vec{l}})$

$\vec{l}\times(\vec{l}\times\dot{\vec{l}}) = \vec{l}(\vec{l}\dot{\vec{l}}) - \dot{\vec{l}}(\vec{l}\cdot\vec{l}) = \vec{l}\,\frac{1}{2}\frac{d}{dt}(\vec{l}^2) - \dot{\vec{l}}(\vec{l}^2) = |\vec{l}^2 = 1| = -\dot{\vec{l}}$

$M_+M_-\left[\frac{1}{L}\dot{\vec{l}} - \frac{\dot{L}}{L^2}\vec{l}\right] - \frac{1}{2A\gamma_+}\dot{\vec{l}}\times\ddot{\vec{l}} = \frac{\gamma_-}{\gamma_+}L\dot{\vec{l}}$, но сравн. $\vec{L}_0\vec{M}_0$ расчетом есть нет $\dot{L}$-терм.

$\left[\frac{M_+M_-}{L} - \frac{\gamma_-}{\gamma_+}L\right]\dot{\vec{l}} - \frac{M_+M_-}{L^2}\dot{L}\vec{l} = \frac{1}{2A\gamma_+}\dot{\vec{l}}\times\ddot{\vec{l}}$, $\vec{l} = \vec{L}/L$, $\vec{l}^2 = 1$.

$L^2 = \frac{1}{4}(\vec{M}_1-\vec{M}_2)^2 = \frac{1}{4}(M_1^2+M_2^2-2\vec{M}_1\cdot\vec{M}_2)$, $2L\dot{L} = -\frac{1}{2}\frac{d}{dt}(\vec{M}_1\cdot\vec{M}_2) = -\frac{1}{2}(\dot{\vec{M}}_1\vec{M}_2 + \vec{M}_1\dot{\vec{M}}_2)$

$\dot{\vec{M}}_1 = \gamma_1\vec{M}_1\times A\vec{M}_2$, $\dot{\vec{M}}_2 = \gamma_2\vec{M}_2\times A\vec{M}_1$, $\dot{\vec{M}}_1\cdot\vec{M}_2 = 0$ и $\dot{\vec{M}}_2\cdot\vec{M}_1 = 0$ в главном по A прибл.

Считаем $|\dot{\vec{L}}| \approx 0$ в главном приближении по $1/A$

$\rightarrow \left[M_+M_-\gamma_+ - \gamma_-L^2\right]\frac{1}{\gamma_+L}\dot{\vec{l}} = \vec{l}\times\left[\frac{1}{2A\gamma_+}\ddot{\vec{l}}+\dots\right]$, $L^2 = \frac{1}{4}(M_1^2+M_2^2+2M_1M_2-2M_1M_2-2\vec{M}_1\vec{M}_2)$

$= \frac{1}{4}(M_1+M_2)^2 - \frac{1}{2}M_1M_2\left(1+\frac{\vec{M}_1\vec{M}_2}{M_1M_2}\right)$, $\frac{\vec{M}_1\vec{M}_2}{M_1M_2} = \cos(\hat{\vec{M}_1\vec{M}_2}) = \cos\beta$, $1+\cos\beta = 2\cos^2\frac{\beta}{2}$

$\vec{L}^2 = L^2 = M_+^2 - \frac{1}{2}M_1M_2\cdot 2\cos^2\frac{\beta}{2}$, $\beta = \hat{\vec{M}_1\vec{M}_2} \approx 0$.

$\left[M_+M_-\gamma_+ - \gamma_-\left(M_+^2 - M_1M_2\cos^2\beta/2\right)\right]\frac{1}{\gamma_+L}\dot{\vec{l}} = \vec{l}\times\left[\frac{1}{2A\gamma_+}\ddot{\vec{l}}+\dots\right]$

$M_+M_-\gamma_+ - \gamma_-M_+^2 = M_+(M_-\gamma_+ - \gamma_-M_+) = M_+\gamma_1\gamma_2(M_1\gamma_1 - M_2/\gamma_2) = M_+\gamma_1\gamma_2(S_1-S_2)$

$\frac{\gamma_1\gamma_2}{\gamma_+} = \frac{2\gamma_1\gamma_2}{(\gamma_1+\gamma_2)} = \gamma$, $\left\{M_+\gamma_1\gamma_2(S_1-S_2) + \gamma_-M_1M_2\cos^2\frac{\beta}{2}\right\}\frac{1}{\gamma_+L}\dot{\vec{l}} = \vec{l}\times\left[\frac{1}{2A\gamma_+}\ddot{\vec{l}}+\dots\right]$

$L^2 = M_+^2 - M_1M_2\cos^2\frac{\beta}{2} = M_+^2\left[1 - \frac{M_1M_2}{M_+^2}\cos^2\beta/2\right] = M_+^2\left[1-\mu\cos^2\frac{\beta}{2}\right]$, $\mu = \frac{M_1M_2}{M_+^2}$

$\{\dots\}\frac{1}{\gamma_+L} = M_+\left\{\gamma_1\gamma_2(S_1-S_2) + \gamma_-M_+\mu\cos^2\beta/2\right\}\frac{1}{\gamma_+M_+\sqrt{1-\mu\cos^2\beta/2}}$, $\gamma_+ = \frac{1}{2}(\gamma_1+\gamma_2)$

$\left[\gamma(S_1-S_2) + \frac{\gamma_-}{\gamma_+}\mu M_+\cos^2\beta/2\right]\dot{\vec{l}} = \vec{l}\times\left[\frac{1}{2A\gamma_+}\ddot{\vec{l}}+\dots\right]$

$\sqrt{1-\mu\cos^2\beta/2}$

Кинетика part УД FIM.

Escaneado con CamScanner

При слое подложек $\beta = \pi - \varepsilon$, $\cos\frac{\beta}{2} = \cos\left(\frac{\pi}{2} - \frac{\varepsilon}{2}\right) = \sin\frac{\varepsilon}{2} \simeq \frac{\varepsilon}{2}$

$\cos^2\beta/2 = \frac{1}{4}\varepsilon^2$. Очевидно $\varepsilon \sim \frac{1}{A}$, $\cos^2\frac{\beta}{2} \sim 1/A^2 \ll 1$.

Т.е. если $(S_1 - S_2) \neq 0$, вдали от T_A: $\bar{\gamma}(S_1 - S_2)\dot{\vec{l}} = \vec{l} \times \left[\frac{1}{2A\gamma_+}\ddot{\vec{l}} + \dots\right]$

при угловой компенсации $S_1 = S_2$ $\frac{\gamma - \mu M_+ \cos^2\frac{\beta}{2}}{\gamma_+}\dot{\vec{l}} = \vec{l} \times \left[\frac{1}{2A\gamma_+}\ddot{\vec{l}} + \dots\right]$,

но $\cos^2\beta/2 \sim \frac{1}{A^2}$ и $\cos^2\beta/2$-член можно γ_+ пренебречь при $1/A \ll 1$.

$W(\vec{M}_1, \vec{M}_2) = A\vec{M}_1 \cdot \vec{M}_2 + W_{ex} + W_a + W_z + W_m$, все $W_n = W_n(\vec{L}, \vec{M})$ и если хотим $W_n(\vec{L}) = W_n(\vec{l})$ для замкнутого УД для $\vec{l}(\vec{r}, t)$, то надо $|\vec{M}|^2 \ll |\vec{L}|^2$ приближение.

Note: УД записано для $\vec{l} = \frac{\vec{L}}{L}$, $\vec{L} = \frac{1}{2}(\vec{M}_1 - \vec{M}_2)$ через магнетизации подрешеток $\vec{M}_i$.

У Звездкиной, Иванова '19 с $(S_1 - S_2)\dot{\vec{l}}$ членом в УД, предполагалось $(S_1 - S_2) \ll (S_1 + S_2)$, близко к T_A и $\vec{l} = \vec{S}_1 - \vec{S}_2$, $\vec{m} = \vec{S}_1 + \vec{S}_2$ через $\vec{S}_i = \vec{M}_i/\gamma_i$ при деление подрешеток.

У меня ур. для $\vec{l} \sim (\vec{M}_1 - \vec{M}_2)$ записывается near T_M, $M_1 \neq M_2$, $|\vec{M}| \ll |\vec{L}|$, т.е. далеко при $\neq T_M$, kinetic term в УД $\sim (S_1 - S_2)\dot{\vec{l}}$.

→ Вывод: FM-kinetic term в УД для $\vec{l}$ всегда $\sim (S_1 - S_2)\dot{\vec{l}}$, без зависимости от близости к T_M, или T_A.

$L_{eff}^{kin} = \frac{\beta}{c}\cos\theta\dot{\varphi} + \frac{1}{2c^2}(\dot{\vec{l}})^2$ дает УД $-\frac{\beta}{c}\dot{\vec{l}} = \frac{1}{c}\vec{l} \times [\ddot{\vec{l}} + \dots]$

Мой $L_{eff}^{kin} = -\bar{\gamma}(S_1 - S_2)\cos\theta\dot{\varphi} + \frac{1}{4A\gamma_+}(\dot{\vec{l}})^2$ с точностью до множителя размерности $(1/\gamma)$.

L_{eff}^{kin} (Zvezdin-new) $= -\frac{m}{\gamma_e}\cos\theta\dot{\varphi} + \frac{1}{2}\frac{\chi}{(\bar{\gamma}_e)^2}(\dot{\vec{l}})^2$, $\chi = \frac{(M_1+M_2)^2}{\lambda M_1 M_2}$

$\frac{m}{\gamma_e} = \frac{1}{\bar{\gamma}}(m - m_0)$, $m - m_0 = \frac{1}{2}(M_1 - M_2) - \frac{1}{2}(M_1 + M_2)\frac{(\gamma_1 - \gamma_2)}{(\gamma_1 + \gamma_2)} = \frac{1}{2(\gamma_1 + \gamma_2)}(\dots) = \frac{1}{2}\bar{\gamma}(S_1 - S_2)$

Zvezdin $\frac{m}{\gamma_e} = \frac{1}{\bar{\gamma}}(m - m_0) = \frac{1}{\bar{\gamma}}\frac{1}{2}\bar{\gamma}(S_1 - S_2) = \frac{1}{2}(S_1 - S_2)$, у меня $\bar{\gamma}(S_1 - S_2)$ при $\cos\theta\dot{\varphi}$.

Мой L_{kin} (Corrected) $= -\frac{1}{2}(S_1 - S_2)\cos\theta\dot{\varphi} + \frac{1}{8A\gamma_+\bar{\gamma}}(\dot{\vec{l}})^2$, умнож. на $\frac{1}{2\bar{\gamma}}$.

Должно (коэф. при $(\dot{\vec{l}})^2$): $\frac{\chi}{(\bar{\gamma}_e)^2} = \frac{1}{4A\gamma_+\bar{\gamma}}$, $\chi = \frac{4M_+^2}{AM_1M_2}$, $A = \lambda$. $M_+ = \frac{1}{2}(M_1 + M_2)$

$(\bar{\gamma}_e)^2 = \frac{\chi}{(4A)^{-1}}\gamma_+\bar{\gamma} = \chi \cdot 4A\,\gamma_+\bar{\gamma} = \frac{4M_+^2}{AM_1M_2}4A\gamma_+\frac{\gamma_1\gamma_2}{\gamma_+} = 16\frac{M_+^2}{M_1M_2}\gamma_1\gamma_2$

У звездин (new files) $\bar{\gamma}_e = \bar{\gamma}\left(1 - \frac{mm_0}{M}\right)^{-1}$, $(\bar{\gamma}_e)^2 = \bar{\gamma}^2\frac{(\gamma_1+\gamma_2)^2M^2}{(\gamma_1M_2 + \gamma_2M_1)^2} = 4\gamma_1^2\gamma_2^2\frac{M^2}{(\gamma_1M_2 + \gamma_2M_1)^2}$

$M_+ = M$. $4\gamma_1^2\gamma_2^2\frac{M^2}{(\gamma_1M_2 + \gamma_2M_1)^2}$ (Zvezdin) $\neq 16\frac{M^2}{M_1M_2}\gamma_1\gamma_2$ (мое). FiM

→ Вывод: коэффициенты при $\cos\theta\dot{\varphi}$ в L_{kin} у меня (после перенормировки) и Zvezdin-new совпадают. Коэффициенты при $(\dot{\vec{l}})^2$ не совпадают. Видимо $\bar{\gamma}_e$ - Zvezdin-new неверное.

2) мой коэф. при $\dot{\vec{l}}$ в УД не совпадает с коэф. β/c Рапизова '82.

(24/XII) Записать УД через $\vec{S}_i = \vec{M}_i/\gamma_i$, чтоб сравнить с Иванов '19 Звездин-ЭММММ'20 (статье с А. Звездин. Он обещал прислать: 1) new Calcs with FiM, 2) new Calcs for dynamics FiM; 3) вывод L_{eff} - PRB зима. АК предлагал написать совм. paper по FiM dynamics

А. К. сообщил, что $\gamma_\perp$ (new) $= (1/\chi)\cdot(1 - m^2/M^2)^{-1}$ в $(\dot{\vec{l}})^2$-терм.

Статья с Зуевой: обсудили свой вывод L_{kin} и просил ее проверить вывод $\bar{\gamma}_{eff}$ в L_{eff} (Zvezdin-new).

А.К. предлагал делать изучение $\bar{\gamma}_{eff}$ по Calcs. resonance frequencies FiM из его L_{eff}: две обменные частоты (!) FiM through разложение $L_{eff}(\theta, \varphi)$ по θ, $\theta_i = \theta_0 + \theta_i$, $\theta_i(S_w) \ll 1$, $\theta_0 = 0$ - ground state. Обсуждение $\bar{\gamma}_{eff}$ по статье, решили сделать.

Escaneado con CamScanner

(25/XII) Отдых дома. Кафе. Бильярд. Фильм.

(26/XII) СП. и Ю. в Донецке: downtown, "Nam", Drozki, Bibil, Фильм.

(27/XII) Проверить $\bar{\gamma}_{eff}$ в χ_{eff}(Zvezdin new) или $\chi_1 = \frac{1}{\lambda(1-\frac{m^2}{M^2})}$ new FiM

Поправки 23/XII УД FiM $\dot{\vec{l}} = \vec{l} \times \left[\frac{1}{2A\chi_1} \ddot{\vec{l}} + \dots \right]$, где $\vec{l} = \frac{\vec{L}}{L}$, $\vec{L} = \frac{1}{2}(\vec{M}_1 - \vec{M}_2)$ – магнитный вектор AF, $W = A\vec{M}_1\vec{M}_2$, $A \to \sigma$.

До УД получено из LL-eqs, подписанных 15/XII $\frac{1}{\gamma_i}\dot{\vec{M}}_i = \vec{M}_i \times \frac{\delta W}{\delta \vec{M}_i}$.

Кинетическая часть УД обычная, но чтоб $W(\vec{M}_1, \vec{M}_2) \to W(\vec{L})$, $W(\vec{l})$ надо предположить близость к FM, чтоб $|\vec{M}| \ll |\vec{L}|$, AF чувствует, $|\vec{m}| \ll |\vec{l}|$.

Альтернатива: ввести "spin" $\vec{l}$ и "spin" $\vec{m}$ [Ivanov LTP'2019].

Spin density $\vec{S}_i = +\vec{M}_i/g_i\mu_B$, магнетизм $\vec{M}_i = -g_i\mu_B\vec{S}_i = -\gamma_i\hbar\vec{S}_i$.

LL для $\vec{M}_i$, $\frac{1}{\gamma_i}\dot{\vec{M}}_i = \vec{M}_i \times \frac{\delta W}{\delta \vec{M}_i}$. Запишем как $-\hbar\dot{\vec{S}}_i = \vec{S}_i \times \frac{\delta W}{\delta \vec{S}_i}$, $W(\vec{S}_1, \vec{S}_2)$

$\dot{\vec{S}}_i = \vec{S}_i \times \frac{\delta W'}{\delta \vec{S}_i}$ – "spin" LL eq., где $W' = -\frac{1}{\hbar}W$.

$W = A\vec{M}_1\vec{M}_2 = A(-\gamma_1\hbar\vec{S}_1)(-\gamma_2\hbar\vec{S}_2) = \gamma_1\gamma_2\hbar^2 A\,\vec{S}_1\cdot\vec{S}_2 = A'\vec{S}_1\vec{S}_2$

$\frac{1}{\gamma_i}\dot{\vec{M}}_i = \vec{M}_i \times \frac{\delta W}{\delta \vec{M}_i}$ дало $\dot{\vec{l}}(\vec{S}_1 - \vec{S}_2) = \vec{l} \times \left[\vec{l} \frac{1}{2A\chi_1} + \dots \right]$, $W = A\vec{M}_1\vec{M}_2$, $\vec{l} = \frac{\vec{L}}{L}$, $\vec{L} = \frac{1}{2}(\vec{M}_1 - \vec{M}_2)$

$\dot{\vec{S}}_i = \vec{S} \times \frac{\partial W}{\partial \vec{S}_i}$, $W = A'\vec{S}_1\vec{S}_2$ должно дать $(\vec{S}_1 - \vec{S}_2)\dot{\vec{l}} = \vec{l} \times \left[\frac{1}{2A'}\ddot{\vec{l}} + \dots \right]$, т.к. при $\gamma_1 = \gamma_2 = 1$, $\gamma = \frac{2\gamma_1\gamma_2}{\gamma_1+\gamma_2} = 1$, $\chi_1 = \frac{1}{2}(\chi_1 + \chi_2) = 1$, $\vec{l} = \frac{\vec{L}}{L}$, но $\vec{L}$ иначе.

Было $\vec{L} = \frac{1}{2}(\vec{M}_1 - \vec{M}_2)$, сделали $\vec{M}_1 = \gamma_1\vec{S}_1$, $\vec{M}_2 = \gamma_2\vec{S}_2$, $\gamma_i = 1$, $\vec{L} = \frac{1}{2}(\vec{S}_1 - \vec{S}_2)$

$\vec{l} = \vec{L}/L$, $\vec{l} = \frac{(\vec{S}_1 - \vec{S}_2)}{|\vec{S}_1 - \vec{S}_2|}$, $A' = A\gamma_1\gamma_2\hbar^2$, $L = |\vec{S}_1 - \vec{S}_2|$, $L = 0$.

Ivanov'19 описал $|\vec{L}| = L = S_1 + S_2$, и $\vec{l} = \frac{(\vec{S}_1 - \vec{S}_2)}{(S_1 + S_2)}$.

$4\vec{L}^2 = S_1^2 + S_2 - 2\vec{S}_1\vec{S}_2$, $\vec{L}^2 = \frac{1}{4}(S_1+S_2)^2 - S_1S_2\cos\beta \approx \frac{1}{4}(S_1+S_2)^2$, $\beta = \vec{S}_1\vec{S}_2$

$L = \frac{1}{2}(S_1+S_2)$ и $\vec{l} = \frac{\vec{S}_1 - \vec{S}_2}{(S_1+S_2)}$, сохр. с Ivanov'19 в главном члене, но $\frac{1}{A_1}$.

В "spin" variables $\vec{l} = \frac{\vec{L}}{L}$, $\vec{L} = \frac{1}{2}(\vec{S}_1 - \vec{S}_2)$, $(\vec{S}_1 - \vec{S}_2)\dot{\vec{l}} = \vec{l} \times \left[\frac{1}{2A'}\ddot{\vec{l}} + \dots \right]$. УД

Чтоб замкнуть УД $(\dots) = \frac{\partial W}{\partial \vec{l}} = F(\vec{l}, \vec{m}) \to F(\vec{l})$, надо предположить близость к T_A, $|\vec{m}| \ll \vec{l}$, $\vec{m} = \frac{(\vec{S}_1 + \vec{S}_2)}{|\vec{S}_1 - \vec{S}_2|} \approx \frac{(\vec{S}_1 + \vec{S}_2)}{(S_1 + S_2)}$, $\vec{m}^2 \approx \frac{(\vec{S}_1 - \vec{S}_2)^2}{(S_1+S_2)^2}$

$\vec{m}^2 + \vec{l}^2 = 1 + \left(\frac{S_1 - S_2}{S_1 + S_2}\right)^2$. При $T \cong T_A$, $S_1 \approx S_2$ и $\vec{m}\cdot\vec{l} = \frac{(\vec{S}_1+\vec{S}_2)}{(S_1+S_2)} \to 0$, $\vec{m}^2 + \vec{l}^2 \approx 1$.

W[Ivanov'19] $= \frac{1}{2}E_{ex}\vec{m}^2 = \frac{1}{2}E_{ex}\frac{1}{(S_1+S_2)^2}(\vec{S}_1 + \vec{S}_2)^2 = \frac{1}{2}\frac{E_{ex}}{(S_1+S_2)^2}[\vec{S}_1^2 + \vec{S}_2^2 + 2\vec{S}_1\vec{S}_2] =$

$= \frac{E_{ex}}{(S_1+S_2)^2}\vec{S}_1\vec{S}_2 = W(\text{мой}) = A'\vec{S}_1\vec{S}_2$, $A' = E_{ex}/(S_1+S_2)^2$.

$\frac{(S_1 - S_2)}{(S_1+S_2)}\dot{\vec{l}} = \vec{l} \times \left[\frac{(S_1+S_2)}{2A'}\ddot{\vec{l}} + \dots \right]$, $\frac{(S_1+S_2)}{2A'} = \frac{1}{2}\frac{(S_1+S_2)^3}{E_{ex}}$ (?)

Иванов вводит $\omega_{ex} = \frac{E_{ex}}{\hbar}\frac{1}{(S_1+S_2)}$ и $\to \dot{\vec{l}} = \vec{l} \times \left[\frac{1}{2A\omega_{ex}}\ddot{\vec{l}} + \dots \right]$. $\to \gamma = \frac{(S_1 - S_2)}{(S_1+S_2)}$.

$\frac{(S_1+S_2)}{2A'} = \frac{1}{2}\frac{(S_1+S_2)^3}{\hbar\omega_{ex}(S_1+S_2)} = \frac{(S_1+S_2)^2}{2\hbar\omega_{ex}} \to$ мое $\to \dot{\vec{l}} = \vec{l} \times \left[\frac{(S_1+S_2)^2}{2\hbar\omega_{ex}}\ddot{\vec{l}} + \dots \right]$

Почему разница между УД "spin" $\vec{l}$ и УД "spin" $\vec{l}$ Ivanov LTP'19? Найти ошибку (ошибки)?

(28/XII) Переписка с Оксаной по поводу A_1-gauge и Q_{11} of Bloch walls.

A. Zvezdin прислал new corrected calculations of DW dynamics FiM. FiM

Обсуждение χ_{eff} и DW-FiM dynamics с Anatoly по Skype. Зухра прислала вывод χ_{eff} с $\bar{\gamma}_{eff}$, χ_{eff} методом (E, β) – Zvezdin (new Word file) с чуб. $\bar{\gamma}_{eff} = \bar{\gamma}\left(1 + \frac{mm_0}{M^2}\right)^{-1}$, а не $\bar{\gamma}_{eff} = \bar{\gamma}\left(1 - \frac{mm_0}{M^2}\right)^{-1}$ как в предыд. Word file, это я проверил. Оба Word files неправильные χ_{eff} из Zvezdin, JMMM'2020. Неверное $\bar{\gamma}_{eff} = \bar{\gamma}(1 - mm_0/M^2)^{-1}$ было в file Zvezdin от 24/XII 20..

Escaneado con CamScanner

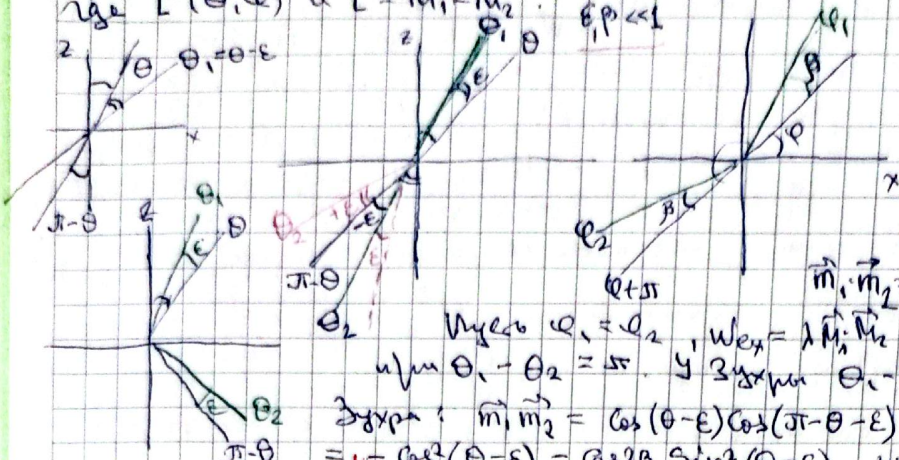
(29/XII) Проверка вывода Less(FiM) by (ε,β) method by Zakhira. F;M

Пока игнорируем $H=0$ и k_1, k_2, non-uniform exchange $\alpha_1, \alpha_2 = 0$.

$\mathcal{L} = \frac{M_1}{\gamma_1}(1-\cos\theta_1)\dot\varphi_1 + \frac{M_2}{\gamma_2}(1-\cos\theta_2)\dot\varphi_2 - \lambda \vec M_1 \vec M_2$, $\vec M_1(\theta_1,\varphi_1)$, $\vec M_2(\theta_2,\varphi_2)$, $\lambda>0$.

Предполагаем $\lambda \gg 1$ ($\lambda \to \infty$) $\theta_1 = \theta - \varepsilon$, $\theta_2 = \pi - \theta - \varepsilon$, $\varphi_1 = \varphi + \beta$, $\varphi_2 = \pi + \varphi - \beta$,

где $\vec L(\theta,\varphi)$ и $\vec L = \vec M_1 - \vec M_2$.



Знак ε в θ_1, θ_2 такой что θ_1 и θ_2 antiparallel.

Знак β at any β удвоит. сумм φ_1 и φ_2, т.е. $\vec M_1$ и $\vec M_2$.

$\vec m_1 \vec m_2 = \cos\theta_1\cos\theta_2 + \cos(\varphi_1-\varphi_2)\sin\theta_1\sin\theta_2$

Пусть $\varphi_1 = \varphi_2$, $W_{ex} = \lambda \vec M_1 \vec M_2 = \lambda M_1 M_2 \cos(\theta_1-\theta_2) \to \min$

при $\theta_1 - \theta_2 = \pi$. У Зухры $\theta_1 - \theta_2 = 2\theta - \pi$

Зухра: $\vec m_1 \vec m_2 = \cos(\theta-\varepsilon)\cos(\pi-\theta-\varepsilon) + \cos(2\beta-\pi)\sin(\theta-\varepsilon)\sin(\pi-\theta-\varepsilon) =$

$= -\cos^2(\theta-\varepsilon) - \cos 2\beta \sin^2(\theta-\varepsilon)$. При $\beta=0$, $\varepsilon=0$, $\vec m_1 \vec m_2 = -1$

Note: $W_{ex} \to \min$ при $\beta = 0$ for any ε. Зачем ε? $W_{ex} = \lambda M_1 M_2 (-1) \to \min$.

При $\lambda \to \infty$ всегда можно выбрать систему координ., где xOz plane = $\vec m_1 \vec m_2$ plane.

В этой системе $\varphi_1 = \varphi_2 = 0$ и $\vec m_1 \cdot \vec m_2 = \cos(\theta_1 - \theta_2) \to \min$ при $\theta_1 - \theta_2 = \pi$ (±)

$\theta_2 = \theta_1 + \pi$. Это alternative approach.

Зухра: $W_{ex} = \lambda M_1 M_2 \vec m_1 \cdot \vec m_2 = -\lambda M_1 M_2 \left[\cos^2(\theta-\varepsilon) + \cos 2\beta \sin^2(\theta-\varepsilon)\right] =$

$= \left|\varepsilon,\beta \ll 1\right| = -(\dots)\left[1 + \cos 2(\theta-\varepsilon) + \cos 2\beta\left\{1 - \cos 2(\theta-\varepsilon)\right\}\right] = -\frac{1}{2}(\dots)\left[1 + \cos 2\beta + \right.$

$\left. + (1-\cos 2\beta)\cos 2(\theta-\varepsilon)\right] = -(\dots)\left[\cos^2\beta + \sin^2\beta \cos 2(\theta-\varepsilon)\right] = -(\dots)\left[1 - \frac{1}{2}\beta^2 + \beta^2\cos 2\theta\right]$ Нет

$W_{ex} = -\lambda M_1 M_2 \left[1 - \frac{1}{2}\beta^2 + \beta^2\cos 2\theta\right]$ и от ε не зависит.

Зухра дает $W_{ex} = +\lambda M_1 M_2 (\varepsilon^2 + \beta^2 \sin^2\theta)$ (?) Откуда это? У меня неверно.

$\cos(\pi-\theta-\varepsilon) = \cos[\pi - (\theta+\varepsilon)] = -\cos(\theta+\varepsilon)$, $\sin(\pi-\theta-\varepsilon) = \sin(\theta+\varepsilon)$

$\vec m_1 \cdot \vec m_2 = -\cos(\theta-\varepsilon)\cos(\theta+\varepsilon) - \cos 2\beta \sin(\theta-\varepsilon)\sin(\theta+\varepsilon)$. При $\varepsilon=0$, $\beta=0$, $\vec m_1 \vec m_2 = -1$

$\vec m_1 \cdot \vec m_2 = -\cos(\theta-\varepsilon)\cos(\theta+\varepsilon) - \sin(\theta-\varepsilon)\sin(\theta+\varepsilon)\left[1 - \frac{1}{2}(2\beta)^2\right] = -\cos[(\theta-\varepsilon)-(\theta+\varepsilon)] -$

$- \sin^2\theta \cdot (-2\beta^2) = -\cos 2\varepsilon + 2\beta^2 \sin^2\theta = -\left(1 - \frac{1}{2}(2\varepsilon)^2\right) + 2\beta^2\sin^2\theta = 2\varepsilon^2 + 2\beta^2\sin^2\theta + \text{const}$

$W_{ex} = \lambda M_1 M_2 \vec m_1 \vec m_2 = 2\lambda M_1 M_2 (\varepsilon^2 + \beta^2 \sin^2\theta)$. У Зухры $W_{ex} = \lambda(\varepsilon^2 + \beta^2\sin^2\theta)$

т.е. в Eq. (3) Зухра надо $\lambda \to 2\lambda$.

$\mathcal{L} = \frac{M_1}{\gamma_1}(-1)\cos\theta_1\dot\varphi_1 - \frac{M_2}{\gamma_2}\cos\theta_2\dot\varphi_2 - W_{ex} = -\frac{M_1}{\gamma_1}\cos(\theta-\varepsilon)(\dot\varphi+\dot\beta) + \frac{M_2}{\gamma_2}\underbrace{\left(-\cos(\theta+\varepsilon)\right)}\left(\cos(\pi-\theta-\varepsilon)\right)(\dot\varphi-\dot\beta) -$

$- W_{ex} = -\frac{M_1}{\gamma_1}(\dot\varphi+\dot\beta)\left[\cos\theta + \varepsilon\sin\theta\right] - \frac{M_2}{\gamma_2}(\dot\varphi-\dot\beta)(+1)\left[\cos\theta - \varepsilon\sin\theta\right] - W_{ex} =$

$= -\left(\frac{M_1}{\gamma_1} - \frac{M_2}{\gamma_2}\right)\cos\theta\,\dot\varphi - \frac{M_1}{\gamma_1}\left[\dot\varphi\varepsilon\sin\theta + \dot\beta\cos\theta + \dot\beta\varepsilon\sin\theta\right] + \frac{M_2}{\gamma_2}\left[-\dot\varphi\varepsilon\sin\theta - \dot\beta\cos\theta + \right.$

$\left. + \dot\beta\varepsilon\sin\theta\right] - W_{ex} = -(S_1 - S_2)\cos\theta\,\dot\varphi + \dot\varphi\varepsilon\sin\theta(-1)(S_1+S_2) + \dot\beta\cos\theta(-1)(S_1+S_2) +$

$+ \dot\beta\varepsilon\sin\theta(-S_1+S_2) - W_{ex}$. $S_1 = M_1/\gamma_1$, $S_2 = M_2/\gamma_2$.

$\mathcal{L} = -(S_1-S_2)\left[\cos\theta\,\dot\varphi + \dot\beta\varepsilon\sin\theta\right] - (S_1+S_2)\left[\dot\varphi\varepsilon\sin\theta + \dot\beta\cos\theta\right] - W_{ex}$.

$\frac{d}{dt}(\beta\cos\theta) = \dot\beta\cos\theta + \beta(-\sin\theta)\dot\theta$, $\frac{d}{dt}(\dots)$ не важна в $\mathcal{L}$. $\dot\beta\cos\theta = \sin\theta\,\beta\dot\theta$

$\beta\varepsilon$ - term выпадает. $\mathcal{L} = -(S_1-S_2)\cos\theta\,\dot\varphi - (S_1+S_2)\left[\dot\varphi\varepsilon\sin\theta + \beta\sin\theta\dot\theta\right] - W_{ex}$

$\left(\mathcal{L} = \mathcal{L}_0 - (S_1+S_2)\sin\theta\left[\dot\varphi\varepsilon + \beta\dot\theta\right] + 2\lambda M_1 M_2(\varepsilon^2 + \beta^2\sin^2\theta)\right.$ Применимо по ε,β-form похоже на Solitons-like interaction

Берем $\beta\dot\theta$ - term отличается от Зухры.

$\frac{\partial\mathcal{L}}{\partial\varepsilon} = \frac{\partial\mathcal{L}}{\partial\beta} = -(S_1+S_2)\sin\theta\,\dot\theta - 4\lambda M_1 M_2 \beta\sin^2\theta = 0$, $\beta\sin\theta = -\frac{(S_1+S_2)}{4\lambda M_1 M_2}\dot\theta$

$\frac{\delta \mathcal{L}}{\delta \varepsilon} = \frac{\partial \mathcal{L}}{\partial \varepsilon} = -(S_1+S_2)\sin\theta\dot{\varphi} - 4\lambda M_1 M_2 \varepsilon = 0, \quad \varepsilon = -\frac{(S_1+S_2)\sin\theta}{4\lambda M_1 M_2}\dot{\varphi}$

$\mathcal{L}(\text{сами } \varepsilon, \beta) = \mathcal{L}_0 - (S_1+S_2)\sin\theta\left[\dot{\varphi}\frac{(-1)\sin\theta\dot{\varphi}(S_1+S_2)}{4\lambda M_1 M_2} + \dot{\theta}\frac{(-1)(S_1+S_2)}{4\lambda M_1 M_2}\frac{\dot{\theta}}{\sin\theta}\right] -$

$-2\lambda M_1 M_2\left[\frac{(S_1+S_2)^2\sin^2\theta}{(4\lambda M_1 M_2)^2}\dot{\varphi}^2 + \frac{(S_1+S_2)^2}{(4\lambda M_1 M_2)^2}\dot{\theta}^2\right]$

$\mathcal{L}(\varepsilon,\beta \to \min) = \mathcal{L}_0 + \frac{1}{8}\frac{(S_1+S_2)^2}{\lambda M_1 M_2}\left[(\dot\theta)^2 + \sin^2\theta(\dot\varphi)^2\right]$

$\mathcal{L} = \mathcal{L}_{kin} = -(S_1-S_2)\cos\theta\dot{\varphi} + \frac{1}{8}\frac{(S_1+S_2)^2}{\lambda M_1 M_2}\left[(\dot\theta)^2 + \sin^2\theta(\dot\varphi)^2\right]$

Def. $\gamma_\perp = \frac{(M_1+M_2)^2}{4\lambda M_1 M_2}$. $\frac{1}{2}\frac{\gamma_\perp}{(\bar\gamma_e)^2} = \frac{1}{8}\frac{(S_1+S_2)^2}{\lambda M_1 M_2}$.

$\frac{1}{(\bar\gamma_e)^2} = \frac{1}{4}\frac{1}{\gamma_\perp}\frac{(S_1+S_2)^2}{\lambda M_1 M_2} = \frac{\lambda}{4}\frac{(M_1+M_2)^2}{(4\lambda M_1 M_2)^{-1}}\frac{(S_1+S_2)^2}{\lambda M_1 M_2}$, $\frac{1}{(\bar\gamma_e)^2} = \frac{(S_1+S_2)^2}{(M_1+M_2)^2}$

$\left(\frac{1}{\bar\gamma_e} = \frac{S_1+S_2}{M_1+M_2}\right)$. Def. $(S_1-S_2) = \frac{m}{\bar\gamma_e}$, $\frac{m}{M} = M_1-M_2$, $\left(\frac{\lambda}{\bar\gamma_e} = \frac{(S_1-S_2)}{(M_1-M_2)}\right)$

T.e. new definition of $(\gamma_\pm)^{-1} = \frac{(S_1\pm S_2)}{(M_1\pm M_2)}$, $\bar\gamma_e = \gamma_+$, $\gamma_e = \gamma_-$.

$\frac{1}{\gamma_e} = \frac{1}{\bar\gamma}\frac{m-m_0}{m}$ $\left(\frac{1}{\bar\gamma} = \frac{\gamma_1+\gamma_2}{2\gamma_1\gamma_2} = \frac{1}{2}\left(\frac{1}{\gamma_1}+\frac{1}{\gamma_2}\right)\right)$, $\frac{m}{\bar\gamma_e} = (S_1-S_2) = \frac{1}{\bar\gamma}(m-m_0)$ $m_0 = M\frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}$

$m-m_0 = M_1-M_2 - (M_1+M_2)\frac{(\gamma_1-\gamma_2)}{(\gamma_1+\gamma_2)} = \frac{1}{(\gamma_1+\gamma_2)}\left[(M_1-M_2)(\gamma_1+\gamma_2) -\right.$

$\left.-(M_1+M_2)(\gamma_1-\gamma_2)\right] = \frac{1}{(\gamma_1+\gamma_2)}\left[2M_1\gamma_2 - 2M_2\gamma_1\right] = \frac{2\gamma_1\gamma_2}{(\gamma_1+\gamma_2)}\left[\frac{M_1}{\gamma_1}-\frac{M_2}{\gamma_2}\right] = \left(\frac{1}{\bar\gamma}\right)(S_1-S_2)$

$\frac{m}{\bar\gamma_e} = \frac{1}{\bar\gamma}(m-m_0) = \frac{1}{\bar\gamma}\bar\gamma(S_1-S_2) = (S_1-S_2)$. Ok, combines.

Вывод. $\frac{1}{\bar\gamma_e} = \frac{1}{\bar\gamma}\left(1+\frac{mm_0}{M^2}\right)$? $\left(1+\frac{mm_0}{M^2}\right) = \frac{1}{M}\left[M + m\frac{m_0}{M}\right] =$

$= \frac{1}{M}\left[M_1+M_2 + (M_1-M_2)\frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\right] = \frac{1}{M(\gamma_1+\gamma_2)}\left[(M_1+M_2)(\gamma_1+\gamma_2) + (M_1-M_2)(\gamma_1-\gamma_2)\right] =$

$= \frac{1}{M(\gamma_1+\gamma_2)}\left[2M_1\gamma_1 + 2M_2\gamma_2\right]$, не combines. Надо $\frac{1}{\bar\gamma_e} = \frac{1}{\bar\gamma}\left(1-\frac{mm_0}{M^2}\right)$

$\left(1-\frac{mm_0}{M^2}\right) = \frac{1}{M(\gamma_1+\gamma_2)}\left[2M_1\gamma_2 + 2M_2\gamma_1\right] = \frac{2\gamma_1\gamma_2}{M(\gamma_1+\gamma_2)}\left(\frac{M_1}{\gamma_1}+\frac{M_2}{\gamma_2}\right) = \frac{1}{\left(\frac{1}{\bar\gamma}\right)M}(S_1+S_2)$

$\frac{1}{\bar\gamma_e} = \frac{1}{\bar\gamma}\left(1-\frac{mm_0}{M^2}\right) = \frac{1}{\bar\gamma}\frac{\bar\gamma}{M}(S_1+S_2) = \frac{(S_1+S_2)}{(M_1+M_2)}$.

→ T.e. набор $m = M_1-M_2$, $M = M_1+M_2$ дает $\frac{1}{\bar\gamma_e} = \frac{1}{\bar\gamma}\frac{(m-m_0)}{m}$ и $\frac{1}{\bar\gamma_e} = \frac{1}{\bar\gamma}\left(1-\frac{mm_0}{M^2}\right)$,

или $\bar\gamma_e = \frac{M}{s}$, $s = S_1+S_2$, $\gamma_e^{-1} = \frac{(S_1-S_2)}{(M_1-M_2)}$, $\gamma_e = \frac{M_1-M_2}{(S_1-S_2)}$, $m_0 = M\left(\frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\right)$.

$\mathcal{L}_{kin} = -\frac{m}{\bar\gamma_e}\cos\theta\dot{\varphi} + \frac{1}{2}\frac{\gamma_\perp}{(\bar\gamma_e)^2}\left[(\dot\theta)^2 + \sin^2\theta(\dot\varphi)^2\right]$, $\frac{m}{\bar\gamma_e} = (S_1-S_2) = 0$ при angular compons.

$\frac{1}{\bar\gamma_e} = \frac{(S_1+S_2)}{(M_1+M_2)}$ не равен ни "0", ни "∞" при TM или TA. $\bar\gamma_e = \frac{(M_1-M_2)}{(S_1-S_2)}$ имеет $\begin{cases}0 & (TM)\\ \infty & (TA)\end{cases}$

Описано Zvezdin exch. frequencies FiM: $\omega_e \sim \frac{m}{\bar\gamma_e}\bar\gamma_e = (S_1-S_2)\bar\gamma_e \to 0$ при TA.

$\omega_0 = \bar\gamma_e\sqrt{\frac{2K_u}{\gamma_\perp} + \frac{1}{4\gamma_\perp^2}\bar\gamma_e^2\left(\frac{m}{\bar\gamma_e}\right)^2} = \bar\gamma_e\sqrt{\frac{2K_u}{\gamma_\perp} + \frac{\bar\gamma_e^2}{4\gamma_\perp^2}(S_1-S_2)^2}$ имеет min при TA.

Сравнить $\bar\gamma_e$ с вектор-calculations of $\mathcal{L}_{kin}$.

(30/XII) Мое YD 23/XII $\bar\gamma(S_1-S_2)\dot{\vec\ell} = \vec\ell\times\left[\frac{1}{2A\gamma_+}\ddot{\vec\ell}\,t + \dots\right]$ совпадение (not normalized)

Lagrangian $\mathcal{L}_{kin}(Gus) = -(S_1-S_2)\cos\theta\dot{\varphi} + \frac{1}{4A\gamma_+\bar\gamma}(\dot{\vec\ell})^2$. $A = \lambda$.

$\mathcal{L}_{kin}(Zvezdin) = -\frac{m}{\bar\gamma_e}\cos\theta\dot{\varphi} + \frac{1}{2}\frac{\gamma_\perp}{(\bar\gamma_e)^2}(\dot{\vec\ell})^2$. Они совпадают если $\frac{m}{\bar\gamma_e} = (S_1-S_2) = 0$ k 4

$\frac{\gamma_\perp}{(\bar\gamma_e)^2} = \frac{1}{2\lambda\gamma_+\bar\gamma}$, $(\bar\gamma_e)^2 = 2\lambda\gamma_+\bar\gamma\gamma_\perp = 2\lambda\frac{1}{2}(\gamma_1+\gamma_2)\frac{2\gamma_1\gamma_2}{(\gamma_1+\gamma_2)}\gamma_\perp = 2\lambda\gamma_1\gamma_2\gamma_\perp$

$(\bar\gamma_e)^2 = 2\lambda\gamma_1\gamma_2\frac{(M_1+M_2)^2}{4\lambda M_1 M_2}$. Согласно Zvezdina $\bar\gamma_e = \frac{(M_1+M_2)}{(S_1+S_2)}$ или $(\bar\gamma_e)^2 = \gamma_1^2\gamma_2^2\frac{(M_1+M_2)^2}{(M_1\gamma_2+M_2\gamma_1)^2}$

Очевидно мое $\bar\gamma_e$ ($\vec\ell$-method) не совп. с $\bar\gamma_e$ (λ-) ($\varepsilon\beta$)-method.

Escaneado con CamScanner

Вчера посвящение $\mathcal{H}_{kin}$, вывели. Векторным методом 23/XII и (ε,β)-методом Звездин сведено с fixed gauge $\vec{n}=\hat{z}$ в $-\cos\theta\dot{\varphi}$-term в векторном методе нет Dirac string $\vec{n}$, и нет ее gauge, $\vec{n}$-direction. С п. в. "Hell", "Carefour" Olaberria.

31/XII 20.. и А. сделали bajo guia CONSTANTO – Agra – autonoma. Вышло. Перечислил ЗЗО € Гамбурги за год – Dec. 2020.

1/I 2021. Добровольский числа proofs-APL. Выслать и send Back. Update Ikerbasque CV, citations, new papers и Ikerbasque web-site.

2/I $\mathcal{H}_{kin}$ – ferromagnet через Dirac string potential (см. 11/XII 002.)

$\mathcal{H}_{kin}(\vec{m}) = s\,\frac{(\vec{n}\times\vec{m})}{(1+\vec{n}\cdot\vec{m})}\cdot\dot{\vec{m}}$, $s=M_s/\gamma$, $\vec{m}=\vec{M}/M_s$, $\vec{n}$ – unit vector Dirac string.

$\vec{A}_D(\vec{m}) = \frac{\vec{n}\times\vec{m}}{(1+\vec{n}\cdot\vec{m})}$, $\mathrm{rot}_{\vec{m}}\vec{A} = -\vec{m}$.

Papers by Valenti-Lax'1977, Klauder'1979, K.B. Vlasov'84 (нет $\mathcal{H}_{kin}$), M. Stone'96, Papanicolaou, Tserkovnyak.

$\vec{A}_D = \frac{\vec{n}}{(1+\vec{n}\cdot\vec{m})}\times\vec{m} = \vec{a}(\vec{n})\times\vec{m}$. Что если усреднить $\vec{a}(\vec{n})$ по направлениям $\vec{n}$, а не брать $\vec{n}=\hat{z}$ и $\vec{m}(\theta,\varphi)$ – representation сферич. угол с Oz?

J. Mathews, R.L. Walker, Math. methods of physics, 1971.

$\vec{a}(\vec{n}) = \frac{\vec{n}}{(1+\vec{n}\cdot\vec{m})} \Rightarrow I_1(\vec{k}) = \int\frac{d\Omega_2}{(1+\vec{k}\cdot\hat{r})} = \frac{2\pi}{k}\ln\left(\frac{1+k}{1-k}\right)$, $d\Omega_2 = d\varphi\sin\theta\,d\theta$

$\hat{r} = (\hat{r}_x,\hat{r}_y,\hat{r}_z) = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$.

$\vec{J}(\vec{k}) = \int d\Omega_n\frac{\hat{r}}{(1+\vec{k}\cdot\hat{r})} = A'\vec{k}$, $A' = \frac{4\pi}{k^2}\left[1-\frac{1}{2k}\ln\left(\frac{1+k}{1-k}\right)\right]$, $\hat{r}\to\vec{n}$, $\vec{k}\to\vec{m}$

$\vec{J}(\vec{m}) = \int d\Omega_n\frac{\vec{n}}{(1+\vec{n}\cdot\vec{m})} = \vec{m}\cdot A'(|\vec{m}|)$, $A'(|\vec{m}|) = A'(m) = \frac{4\pi}{m^2}\left[1-\frac{1}{2m}\ln\left(\frac{1+m}{1-m}\right)\right]$

$A'(m) = \frac{4\pi}{m^2}\left[1-\frac{1}{m}\,\mathrm{Arth}\,m\right]$, $\vec{m}^2=1$, $|\vec{m}|=m=1$. $A'(m\to1) = 4\pi\left[1-\ln\frac{2}{(1-m)}\right]\to\infty$ как $\ln(1-m)$. $A'(m\to1)\cong 4\pi\ln(1-m)$.

$\vec{J}(\vec{m}) = \int d\Omega_n\,\vec{a}(\vec{n}) = \vec{m}\,A'(m)$. Dirac potential $\vec{A}_D(\vec{m},\vec{n}) = \frac{\vec{n}\times\vec{m}}{(1+\vec{n}\cdot\vec{m})} = \vec{a}(\vec{n})\times\vec{m}$

$\int d\Omega_n\,\vec{A}_D(\vec{m},\vec{n}) = \int d\Omega_n\frac{\vec{n}}{(1+\vec{n}\cdot\vec{m})}\times\vec{m} = A'(m)\,\vec{m}\times\vec{m} = 0$, и $A'(m\to1)\to\infty$ ($+\infty$).

Вывод: усреднение Dirac potential $\vec{A}_D(\vec{m},\vec{n})$ по направлениям string $\vec{n}$ дает singular zero (zero with singular coefficient).

$\mathcal{H}_{kin} = s\,\vec{A}_D(\vec{m},\vec{n})\cdot\dot{\vec{m}}$, $\langle\mathcal{H}_{kin}\rangle_{\vec{n}} = s\dot{\vec{m}}\cdot\langle\vec{A}_D(\vec{m},\vec{n})\rangle_{\vec{n}} = s\dot{\vec{m}}\cdot\frac{1}{4\pi}\int d\Omega_n\,\vec{A}_D(\vec{m},\vec{n}) =$

$= \frac{1}{4\pi}s\dot{\vec{m}}\cdot\left[A'(m)\,\vec{m}\times\vec{m}\right] = 0$, и $A'(m\to1)\to\infty$.

$I_1(\vec{m},\vec{b}) = \int d\Omega_n\frac{\vec{b}\cdot\vec{n}}{(1+\vec{m}\cdot\vec{n})} = \int d\Omega_n\,\vec{b}\cdot\vec{a}(\vec{n}) = \frac{4\pi}{m^2}\vec{b}\cdot\vec{m}\left[1-\frac{1}{2m}\ln\left(\frac{1+m}{1-m}\right)\right] = \vec{b}\cdot\vec{m}\,A'(m)$

FiM

3/I Какой же расчет $\mathcal{H}_{kin}$ верный? Вектор-расчет 23/XII? Или (ε,β) расчет Звездин? Надо проверить мой вектор расчет 23/XII и (ε,β) расчет 29/XII Звездин-Гарсиева.

Расчет вектор-метод 23/XII: $\dot{\vec{L}} = 2A\gamma_+\vec{L}\times\vec{M}$, $\vec{M} = 2A\gamma_-\vec{L}\times\vec{M}$

Все зависит от $\dot{\vec{L}}$ – отлично. $\dot{\vec{L}}=0$ или нет? $L^2 = \vec{L}^2 = \frac{1}{4}(\vec{M}_1-\vec{M}_2)^2$

$2L\dot{L} = -\frac{1}{2}\frac{d}{dt}(\vec{M}_1\cdot\vec{M}_2) = -\frac{1}{2}\left[\dot{\vec{M}}_1\vec{M}_2+\vec{M}_2\dot{\vec{M}}_1\right] = -\frac{1}{2}\left[-\vec{M}_2\gamma_1A\cdot\vec{M}_2\times\vec{M}_1+\vec{M}_1\gamma_2\cdot\vec{M}_2\times\vec{M}_1\right]\equiv 0$,

$L\neq0\Rightarrow\dot{L}=0$. $L^2 = M_+^2 - M_1M_2\cos^2\frac{\beta}{2}$, $\beta = \vec{M}_1\hat{\vec{M}}_2$, $2L\dot{L} = +M_1M_2\sin\beta\cdot\dot{\beta}\cdot\frac{1}{2} = 0$

$\sin\beta\,\dot{\beta}=0$, $\beta=\pi-\varepsilon$, $\varepsilon\dot{\varepsilon}=0$, $\frac{d}{dt}(\cos\beta)=0$, $\frac{d}{dt}(w_{ex})=0$, $w_{ex} = AM_1M_2 = AM_1M_2\cos\beta$

$\frac{d}{dt}(w_{ex})=0$ без damping, т.к. $E = w_{ex}$. $L^2 = \mathrm{const}$ (4).

$1-\mu\cos^2\frac{\beta}{2} = 1-\mu\cos^2\left(\frac{\pi}{2}-\frac{\varepsilon}{2}\right) = 1-\mu\sin^2\frac{\varepsilon}{2}\cong 1-\frac{1}{4}\mu\varepsilon^2\cong 1$, $\mu = \frac{M_1M_2}{M_+^2}$.

$\left[\vec{\gamma}(\vec{s}_1-\vec{s}_2) + \frac{\gamma_-}{\gamma_+}\mu M_+ + \frac{1}{4}\varepsilon^2\right]\vec{L} = \vec{L}\times\left[\frac{1}{2\gamma_+A}\dot{\vec{L}}+\dots\right]$, УД 23/XII условно $\beta=\pi-\varepsilon$, $\varepsilon\ll1$, $\varepsilon\cong\frac{1}{A}$.

$\sqrt{1-\frac{1}{4}\mu\varepsilon^2}$

Может в Звездин-Гарсиева расчет ansatz $\theta_1=\theta-\varepsilon$, $\theta_2=\pi-\theta-\varepsilon$ неверен? См. 29/XII Figs. Должно $\theta_1=\theta-\varepsilon$, $\theta_2=\pi-\theta+\varepsilon$, тогда $\varepsilon\neq0$ даст сход $\vec{M}_1$ и $\vec{M}_2$.

Escaneado con CamScanner

$\vec{m}_1 \cdot \vec{m}_2 = \cos\theta_1 \cos\theta_2 + \cos(\varphi_1 - \varphi_2) \sin\theta_1 \sin\theta_2 = \cos(\theta-\varepsilon)\cos(\pi-\theta+\varepsilon) + \cos(\pi-2\beta) \times$
$\times \sin(\theta-\varepsilon)\sin(\pi-\theta+\varepsilon) = \cos(\theta-\varepsilon)\cos[\pi-(\theta-\varepsilon)] - \cos 2\beta \sin(\theta-\varepsilon)\sin[\pi-(\theta-\varepsilon)] =$
$= \left| \begin{matrix} \cos(\pi-\alpha) = -\cos\alpha \\ \sin(\pi-\alpha) = \sin\alpha \end{matrix} \right| = -\cos^2(\theta-\varepsilon) - \cos 2\beta \sin^2(\theta-\varepsilon), \quad \cos 2\beta = 1 - 2\sin^2\beta$

$\vec{m}_1 \cdot \vec{m}_2 = -\cos^2(\theta-\varepsilon) - [1 - 2\sin^2\beta]\sin^2(\theta-\varepsilon) = -1 + 2\sin^2\beta \sin^2(\theta-\varepsilon) \approx$
$\approx -1 + 2\beta^2 \sin^2\theta$. У Зухры $\vec{m}_1 \cdot \vec{m}_2 = -1 + 2\beta^2 \sin^2\theta + 2\varepsilon^2$

$W_{ex} = A M_1 M_2 \vec{m}_1 \vec{m}_2$, т.е. W_{ex} (Зубра) > W_{ex} (Моа) из членов ε^2 у Зухры.

$\mathcal{L} = -\frac{M_1}{\gamma_1}\cos\theta_1 \dot{\varphi}_1 - \frac{M_2}{\gamma_2}\cos\theta_2 \dot{\varphi}_2 - W_{ex} = -S_1\cos(\theta-\varepsilon)(\dot{\varphi}+\dot{\beta}) - S_2\cos(\pi-\theta+\varepsilon)(\dot{\varphi}-\dot{\beta}) -$
$-W_{ex} = \varphi - S_1(\dot{\varphi}+\dot{\beta})[\cos\theta + \sin\theta\,\varepsilon] - S_2(\dot{\varphi}-\dot{\beta})[\cos(\pi-\theta) - \sin(\pi-\theta)\varepsilon] - W_{ex} =$
$= -S_1(\dot{\varphi}+\dot{\beta})(\cos\theta + \varepsilon\sin\theta) - S_2(\dot{\varphi}-\dot{\beta})[-\cos\theta - \varepsilon\sin\theta] - W_{ex} =$
$= -(S_1-S_2)\cos\theta\dot{\varphi} - (S_1+S_2)\cos\theta\dot{\beta} - (S_1-S_2)\dot{\varphi}\varepsilon\sin\theta - (S_1+S_2)\dot{\beta}\varepsilon\sin\theta - W_{ex}$

$\beta\varepsilon$-члены выбросим. $\mathcal{L} = -(S_1-S_2)[\cos\theta\dot{\varphi} + \dot{\varphi}\varepsilon\sin\theta] - (S_1+S_2)\cos\theta\dot{\beta} - W_{ex}$, $\dot{\beta}\cos\theta = \sin\theta\dot{\theta}\beta$

$\mathcal{L} = -(S_1-S_2)[\cos\theta\dot{\varphi} + \varepsilon\sin\theta\dot{\varphi}] + (S_1+S_2)\sin\theta\dot{\theta}\beta - A M_1 M_2[-1 + 2\sin^2\theta\beta^2]$, $A = \lambda$.

$\mathcal{L} = \mathcal{L}_0 - (S_1-S_2)\sin\theta\dot{\varphi}\varepsilon - (S_1+S_2)\sin\theta\dot{\theta}\beta - 2\lambda M_1 M_2 \sin^2\theta\beta^2$.

$\frac{\delta\mathcal{L}}{\delta\varepsilon} = \frac{\partial\mathcal{L}}{\partial\varepsilon} = -(S_1-S_2)\sin\theta\dot{\varphi} = 0 \overset{(?)}{\dot{\varphi}=0}$, $\frac{\delta\mathcal{L}}{\delta\beta} = \frac{\partial\mathcal{L}}{\partial\beta} = -(S_1+S_2)\sin\theta\dot{\theta} - 4\lambda M_1 M_2 \sin^2\theta\beta$

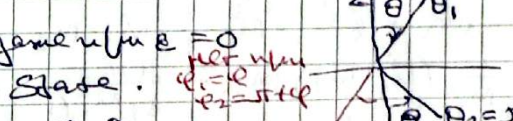
$\beta\sin\theta = -\frac{(S_1+S_2)}{4\lambda M_1 M_2}\dot{\theta}$. $\mathcal{L}_{min} = \mathcal{L}_0 - (S_1+S_2)\dot{\theta}(-1)\frac{(S_1+S_2)}{4\lambda M_1 M_2}\dot{\theta} - 2\lambda M_1 M_2 \frac{(S_1+S_2)^2}{16\lambda^2 M_1^2 M_2^2}\dot{\theta}^2$

$\mathcal{L}_{min} = \mathcal{L}_0 + \frac{1}{8}\frac{(S_1+S_2)^2}{2M_1 M_2}\dot{\theta}^2 = -(S_1-S_2)\cos\theta\dot{\varphi} + \frac{1}{8}\frac{(S_1+S_2)^2}{\lambda M_1 M_2}\dot{\theta}^2$, такой же $\mathcal{L}_{kin}$ как и 29/XII зон., но $\delta\varepsilon_3$ $\sin^2\theta(\dot{\varphi})^2$ - член. более точно по $\varepsilon \ll 1$.

$\mathcal{L} = -S_1\cos(\theta-\varepsilon)(\dot{\varphi}+\dot{\beta}) + S_2\cos(\theta-\varepsilon)(\dot{\varphi}-\dot{\beta}) - W_{ex}$

$\mathcal{L} = -(S_1-S_2)\cos(\theta-\varepsilon)\dot{\varphi} - (S_1+S_2)\cos(\theta-\varepsilon)\dot{\beta} - W_{ex}$. Видно, что ε - любое.

Ansatz Zvezdin $\theta_1 = \theta$, $\theta_2 = \pi - \theta$ неверен даже при $\varepsilon = 0$, см 29/XII.



→ Надо $\theta_1 = \theta$, $\theta_2 = \pi + \theta$, что даёт ↓↑ State.

$\theta_1 = \theta - \varepsilon$, $\theta_2 = \pi + \theta + \varepsilon$, $\varphi_1 = \varphi + \beta$, $\varphi_2 = \varphi - \beta$.

$\vec{m}_1 \cdot \vec{m}_2 = \cos\theta_1\cos\theta_2 + \cos(\varphi_1 - \varphi_2)\sin\theta_1\sin\theta_2 = \cos(\theta-\varepsilon)\cos(\pi+\theta+\varepsilon) + \cos(0-2\beta) \times$
$\times \sin(\theta-\varepsilon)\sin(\pi+\theta+\varepsilon) = \cos(\theta-\varepsilon)(-1)\cos(\theta+\varepsilon) + \cos 2\beta \sin(\theta-\varepsilon)(-1)\sin(\theta+\varepsilon) =$
$= -\cos(\theta-\varepsilon)\cos(\theta+\varepsilon) - \sin(\theta-\varepsilon)\sin(\theta+\varepsilon)\left[1 - \frac{1}{2}(2\beta)^2\right] = -\cos[(\theta-\varepsilon)+(\theta+\varepsilon)] +$
$+ \frac{4}{2}\beta^2\sin^2\theta = -\cos 2\varepsilon + 2\beta^2\sin^2\theta = -1 + 2\varepsilon^2 + 2\beta^2\sin^2\theta$ как у Зухры 29/XII.

$\vec{m} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$, $\vec{m} \to -\vec{m}$ или ① $\theta \to \pi+\theta$, $\varphi \to \varphi$ или ② $\theta \to \pi-\theta$, $\varphi \to \pi+\varphi$,

1) $\sin(\pi+\theta) = -\sin\theta$, $\cos(\pi+\theta) = -\cos\theta$

2) $\sin(\pi-\theta) = \sin\theta$, $\sin(\pi+\varphi) = -\sin\varphi$, $\cos(\pi+\varphi) = -\cos\varphi$, $\cos(\pi-\theta) = -\cos\theta$.

$\mathcal{L}_{kin} = -S_1\cos\theta_1\dot{\varphi}_1 - S_2\cos\theta_2\dot{\varphi}_2 = -S_1\cos(\theta-\varepsilon)(\dot{\varphi}+\dot{\beta}) - S_2\cos(\pi+\theta+\varepsilon)(\dot{\varphi}-\dot{\beta}) =$
(новый ansatz) $= -S_1\cos(\theta-\varepsilon)(\dot{\varphi}+\dot{\beta}) + S_2\cos(\theta+\varepsilon)(\dot{\varphi}-\dot{\beta}) = -S_1(\cos\theta + \varepsilon\sin\theta)(\dot{\varphi}+\dot{\beta}) +$
$+ S_2(\cos\theta - \varepsilon\sin\theta)(\dot{\varphi}-\dot{\beta}) = -(S_1-S_2)\cos\theta\dot{\varphi} - (S_1+S_2)\varepsilon\sin\theta\dot{\varphi} - (S_1+S_2)\cos\theta\dot{\beta} -$
$- (S_1-S_2)\sin\theta\varepsilon\dot{\beta}$, что точно совпадает с $\mathcal{L}_{kin}(\varepsilon,\beta)$ Зухры из 29/XII.

→ Вывод: ansatzes $[\theta_1 = \theta - \varepsilon, \theta_2 = \pi - \theta - \varepsilon, \varphi_1 = \varphi + \beta, \varphi_2 = \pi + \varphi - \beta]$ и $[\theta_1 = \theta - \varepsilon, \theta_2 = \pi + \theta + \varepsilon, \varphi_1 = \varphi + \beta, \varphi_2 = \varphi - \beta]$ эквивалентны при выводе $\mathcal{L}_{kin}$.

? Если $\mathcal{L}_{kin}(\varepsilon,\beta)$ by Zvezdin - Зухра 29/XII верен, то почему он не совпадает с векторным расчётом почти из 23/XII?

Согласуется ли $\mathcal{L}_{kin}(\varepsilon,\beta)$ с $\vec{L} = \vec{L}_0 + \vec{l}$, $\vec{M} = \vec{M}_0 + \vec{m}$ - vector calc. из 23/XII? Вроде и там и там выбрана ось $\hat{z}$ для расчёта $\vec{L}$ и $\vec{M}$, $\vec{L}_0$ и $\vec{M}_0 \parallel \hat{z}$. // Проверить Heff Zvezdin у Ivanov.

Escaneado con CamScanner

4/II two possible references reports на APL - Pavel. Inocian сбои проверочные
Мейлинская статья с Зубаревым про Lagg F:M. Бумага
Предложили to Olong и Масой поехать AGU 3Dmagn: project on July 1, 2021.
5/I Edited / revised Вендина abs. INTERMAG и после Океана.
Anomaly F:M dynamics (почему Lagg (Kuznetsov) и Lagg (Gus) разные?)
Интервью (вопросы) Вилюйская линия в Monthly Highlights (Commun. Phys. 3 2020)
6/I Отдел дома. Розыгрыш. Бумаги
7/I Review INTERMAG digest Gruzdeva. Нефрос и шпинель Parking Theory Sec.
Д.на APL (?) написание по Цепях ФМ наночастиц.
8/I Векторная форма УД, Lagg F:M. Согласно 15/XII уравнение в форме Ландау:
$\frac{1}{\gamma_1}\dot{\vec{M}}_1 = A\vec{M}_1\times\vec{M}_2$, $\frac{1}{\gamma_2}\dot{\vec{M}}_2 = A\vec{M}_2\times\vec{M}_1$, $\frac{1}{\gamma_1}\dot{\vec{M}}_1 + \frac{1}{\gamma_2}\dot{\vec{M}}_2 = 0$.

$\frac{1}{\gamma_1}\ddot{\vec{M}}_1 = A\frac{d}{dt}(\vec{M}_1\times\vec{M}_2) = A[\dot{\vec{M}}_1\times\vec{M}_2 + \vec{M}_1\times\dot{\vec{M}}_2] = A[\gamma_1 A(\vec{M}_1\times\vec{M}_2)\times\vec{M}_2 + \vec{M}_1\times(\vec{M}_2\times\dot{\vec{M}}_1)\gamma_2 A]$

$= A^2[-\gamma_1\vec{M}_2\times(\vec{M}_1\times\vec{M}_2) + \gamma_2\vec{M}_1\times(\vec{M}_2\times\vec{M}_1)] = A^2[-\gamma_1\{\vec{M}_1(\vec{M}_2\cdot\vec{M}_2) - \vec{M}_2(\vec{M}_1\cdot\vec{M}_2)\} +$

$+\gamma_2\{\vec{M}_2(\vec{M}_1\cdot\vec{M}_1) - \vec{M}_1(\vec{M}_1\cdot\vec{M}_2)\}] = A^2[-\gamma_1 M_2^2\vec{M}_1 + \gamma_1\vec{M}_2(\vec{M}_1\cdot\vec{M}_2) + \gamma_2 M_1^2\vec{M}_2 - \gamma_2\vec{M}_1(\vec{M}_1\cdot\vec{M}_2)]$

$\frac{1}{\gamma_1}\ddot{\vec{M}}_1 = A^2[\gamma_2 M_1^2\vec{M}_2 - \gamma_1 M_2^2\vec{M}_1 + (\gamma_1\vec{M}_2 - \gamma_2\vec{M}_1)(\vec{M}_1\cdot\vec{M}_2)]$

Исключим $\vec{M}_2$ из $\frac{1}{\gamma_1}\dot{\vec{M}}_1 = A\vec{M}_1\times\vec{M}_2$, $\vec{M}_2 = a\vec{M}_1 + b\vec{M}_1\times\dot{\vec{M}}_1$

$\frac{1}{\gamma_1}\dot{\vec{M}}_1 = A\vec{M}_1\times[a\vec{M}_1 + b\vec{M}_1\times\dot{\vec{M}}_1] = Ab\vec{M}_1\times(\vec{M}_1\times\dot{\vec{M}}_1) = Ab[\vec{M}_1(\vec{M}_1\cdot\dot{\vec{M}}_1) -$

$-\dot{\vec{M}}_1(\vec{M}_1\cdot\vec{M}_1)] = Ab(-1)M_1^2\dot{\vec{M}}_1$, $\frac{1}{\gamma_1} = -bAM_1^2$, $b = -\frac{1}{\gamma_1 M_1^2 A}$

а из $\vec{M}_1\cdot\dot{\vec{M}}_1 = 0$, $\vec{M}_2 = a\vec{M}_1$, $\vec{M}_2\parallel\vec{M}_1$, а a не определено.

$\vec{M}_2^2 = M_2^2 = [a\vec{M}_1 + b\vec{M}_1\times\dot{\vec{M}}_1]^2 = a^2M_1^2 + 2ab\vec{M}_1\cdot(\vec{M}_1\times\dot{\vec{M}}_1) + b(\vec{M}_1\times\dot{\vec{M}}_1)^2$

$(\vec{a}\times\vec{b})^2 = a^2b^2\sin^2\theta = a^2b^2 - a^2b^2\cos^2\theta = a^2b^2 - (\vec{a}\cdot\vec{b})^2$

$(\vec{M}_1\times\dot{\vec{M}}_1)^2 = M_1^2|\dot{\vec{M}}_1|^2 - (\vec{M}_1\cdot\dot{\vec{M}}_1)^2 = M_1^2|\dot{\vec{M}}_1|^2$ или $(\vec{M}_1\times\dot{\vec{M}}_1)^2 = [\vec{M}_1\times\gamma_1 A\vec{M}_1\times\vec{M}_2]^2$

$M_2^2 = a^2M_1^2 + bM_1^2|\dot{\vec{M}}_1|^2$. Хотим $a = a(\vec{M}_1)$ только.

$\vec{M}_2 = a\vec{M}_1 + b\vec{M}_1\times\dot{\vec{M}}_1$, $\vec{M}_1\cdot\vec{M}_2 = aM_1^2 + b\vec{M}_1\cdot(\vec{M}_1\times\dot{\vec{M}}_1) = aM_1^2$, $a = \frac{1}{M_1^2}(\vec{M}_1\cdot\vec{M}_2)$

$\vec{M}_2 = \frac{1}{M_1^2}(\vec{M}_1\cdot\vec{M}_2)\vec{M}_1 - \frac{1}{\gamma_1 M_1^2 A}(\vec{M}_1\times\dot{\vec{M}}_1)$ подставь в $\frac{1}{\gamma_2}\dot{\vec{M}}_2 = A\vec{M}_2\times\vec{M}_1$

$\dot{\vec{M}}_2 = \frac{1}{M_1^2}[(\dot{\vec{M}}_1\cdot\vec{M}_2)\vec{M}_1 + (\vec{M}_1\cdot\dot{\vec{M}}_2)\vec{M}_1 + (\vec{M}_1\cdot\vec{M}_2)\dot{\vec{M}}_1 + b\frac{d}{dt}(\vec{M}_1\times\dot{\vec{M}}_1)] - \gamma_2 A\cdot$

$\cdot[\frac{1}{M_1^2}(\vec{M}_1\cdot\vec{M}_2)\vec{M}_1 + b(\vec{M}_1\times\dot{\vec{M}}_1)]\times\vec{M}_1$, $\frac{1}{M_1^2}[(\dot{\vec{M}}_1\cdot\vec{M}_2)\vec{M}_1 + bM_1^2(\vec{M}_1\times\ddot{\vec{M}}_1)] =$

$= \gamma_2 A\,b(\vec{M}_1\times\dot{\vec{M}}_1)\times\vec{M}_1$, $(\vec{M}_1\times\dot{\vec{M}}_1)\times\vec{M}_1 = -M_1^2(\dot{\vec{M}}_1\times\vec{M}_1) = -[\vec{M}_1(\vec{M}_1\cdot\dot{\vec{M}}_1) - \dot{\vec{M}}_1M_1^2]$

$= +M_1^2\dot{\vec{M}}_1$, $\frac{1}{M_1^2}[(\dot{\vec{M}}_1\cdot\vec{M}_2)\vec{M}_1 + b(\vec{M}_1\times\ddot{\vec{M}}_1)M_1^2] = \gamma_2 Ab M_1^2\dot{\vec{M}}_1$

$a\dot{\vec{M}}_1 + \frac{bM_1^2}{M_1^2}(\vec{M}_1\times\ddot{\vec{M}}_1) = \gamma_2 AbM_1^2\dot{\vec{M}}_1$ ур-е на a.

Или со же время $\vec{M}_2 = a\vec{M}_1 + b\vec{M}_1\times\dot{\vec{M}}_1$, $\dot{\vec{M}}_2 = -\frac{\gamma_2}{\gamma_1}\dot{\vec{M}}_1$, $\dot{\vec{M}}_2 = a\dot{\vec{M}}_1 + b\dot{\vec{M}}_1\times\ddot{\vec{M}}_1 =$

$= -\frac{\gamma_2}{\gamma_1}\dot{\vec{M}}_1$, $(a + \frac{\gamma_2}{\gamma_1})(\cdot)\cdot a\dot{\vec{M}}_1 = -\frac{\gamma_2}{\gamma_1}\dot{\vec{M}}_1 - b\dot{\vec{M}}_1\times\ddot{\vec{M}}_1$. Умн. скалярно на $\dot{\vec{M}}_1$

$a(\dot{\vec{M}}_1)^2 = -\frac{\gamma_2}{\gamma_1}(\dot{\vec{M}}_1)^2 - b(\vec{M}_1\times\ddot{\vec{M}}_1)\cdot\dot{\vec{M}}_1 \neq -\frac{\gamma_2}{\gamma_1}(\dot{\vec{M}}_1)^2 - b[\ddot{\vec{M}}_1(\vec{M}_1\cdot\dot{\vec{M}}_1) - \dot{\vec{M}}_1(\cdot\cdot)]$

$a = -\frac{\gamma_2}{\gamma_1} - \frac{b}{(\dot{\vec{M}}_1)^2}(\vec{M}_1\times\ddot{\vec{M}}_1)\cdot\dot{\vec{M}}_1$. И что толку?

$\frac{1}{\gamma_1 A^2}\ddot{\vec{M}}_1 = \gamma_2 M_1^2\vec{M}_2 - \gamma_1 M_2^2\vec{M}_1 + (\gamma_1\vec{M}_2 - \gamma_2\vec{M}_1)(\vec{M}_1\cdot\vec{M}_2) = \gamma_2 M_1^2[a\vec{M}_1 + b\vec{M}_1\times\dot{\vec{M}}_1] -$

$-\gamma_1 M_2^2\vec{M}_1 + aM_1^2\gamma_1[a\vec{M}_1 + b\vec{M}_1\times\dot{\vec{M}}_1] - \gamma_2\vec{M}_1 aM_1^2 = \gamma_2 M_1^2 b\,\vec{M}_1\times\dot{\vec{M}}_1 -$

$-\gamma_1 M_2^2\vec{M}_1 + a^2M_1^2\gamma_1\vec{M}_1 + \gamma_1 M_1^2 ab\,\vec{M}_1\times\dot{\vec{M}}_1 = (\gamma_1 M_1^2 a^2 - \gamma_1 M_2^2)\vec{M}_1 +$

$+ bM_1^2(\gamma_2 + \gamma_1 a)(\vec{M}_1\times\dot{\vec{M}}_1)$, $\frac{1}{\gamma_1^2 A^2}\ddot{\vec{M}}_1 = (M_1^2a^2 - M_2^2)\vec{M}_1 + bM_1^2(\frac{\gamma_2}{\gamma_1} + a)(\vec{M}_1\times\dot{\vec{M}}_1)$,

т.к. $a = a(\vec{M}_1)$ получили замкнутое нелинейное уравнение на $\vec{M}_1(t)$.

Его линеаризация должна дать Kaplan - Kittel '53 exchange frequency $\omega_{ex} \sim A$

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Если ввести (15/XII) $\vec{L} = \frac{1}{2}(\vec{M}_1 - \vec{M}_2)$, $\vec{M} = \frac{1}{2}(\vec{M}_1 + \vec{M}_2)$, $\vec{M}_1 = \vec{M} + \vec{L}$, $\vec{M}_2 = \vec{M} - \vec{L}$

$\dot{\vec{M}}_1 = \dot{\vec{M}} + \dot{\vec{L}} = \gamma_1 A(\vec{M}_1 \times \vec{M}_2) = \gamma_1 A\,(\vec{M}+\vec{L}) \times (\vec{M}-\vec{L}) = \gamma_1 A\,[-\vec{M}\times\vec{L} + \vec{L}\times\vec{M}]$

$\dot{\vec{M}}_2 = \dot{\vec{M}} - \dot{\vec{L}} = \gamma_2 A\,(\vec{M}_2 \times \vec{M}_1) = \gamma_2 A\,(\vec{M}-\vec{L}) \times (\vec{M}+\vec{L}) = \gamma_2 A\,[\vec{M}\times\vec{L} - \vec{L}\times\vec{M}]$

$2\dot{\vec{M}} = \gamma_1 A\, 2(\vec{L}\times\vec{M}) + 2\gamma_2 A(\vec{M}\times\vec{L}) = 2(\gamma_1 - \gamma_2)A(\vec{L}\times\vec{M})$, $\dot{\vec{M}} = (\gamma_1 - \gamma_2) A\, \vec{L}\times\vec{M}$

$2\dot{\vec{L}} = \gamma_1 A\, 2\vec{L}\times\vec{M} + 2\gamma_2 A\,\vec{L}\times\vec{M} = 2(\gamma_1 + \gamma_2)A(\vec{L}\times\vec{M})$, $\dot{\vec{L}} = (\gamma_1 + \gamma_2) A\,\vec{L}\times\vec{M}$.

$\dot{\vec{M}} = 2\gamma_- A(\vec{L}\times\vec{M})$ и $\dot{\vec{L}} = 2\gamma_+ A(\vec{L}\times\vec{M})$, $\gamma_\pm = \frac{1}{2}(\gamma_1 \pm \gamma_2)$ получим УД 15/XII 20г.

$(\vec{M}\cdot\vec{L}) = \frac{1}{4}(M_1^2 - M_2^2)$, $\vec{M}^2 + \vec{L}^2 = \frac{1}{2}(M_1^2 + M_2^2) = \text{const}$. Def. $M_\pm = \frac{1}{2}(M_1 \pm M_2)$

$\frac{\dot{\vec{L}}}{\gamma_+} = \frac{\dot{\vec{M}}}{\gamma_-}$ в силу УД$(\vec{M}\vec{L})$, $\vec{L}\times\dot{\vec{L}} = 2\gamma_+ A\,\vec{L}\times(\vec{L}\times\vec{M}) = 2\gamma_+ A\,[\vec{L}(\vec{L}\cdot\vec{M}) - \vec{M}L^2]$, см. 23/XII

$\vec{L}\times\dot{\vec{L}} = 2\gamma_+ A\,[\vec{L}M_+M_- - L^2\vec{M}]$, $\vec{M} = \frac{M_+M_-}{L^2}\vec{L} - \frac{1}{2\gamma_+ A}\vec{L}\times\dot{\vec{L}}\frac{1}{L^2}$ точное ур. при $A\to\infty$

$L = L(t)$, т.к. $M^2 + L^2 = \text{const}(t)$. $\vec{M} = \frac{1}{L^2}\left[M_+M_-\vec{L} - \frac{1}{2\gamma_+ A}\vec{L}\times\dot{\vec{L}}\right]$

$\dot{\vec{M}} = \frac{d}{dt}\left(\frac{1}{L^2}\right)\left[M_+M_-\vec{L} - \frac{1}{2\gamma_+ A}\vec{L}\times\dot{\vec{L}}\right] + \frac{1}{L^2}\left[M_+M_-\dot{\vec{L}} - \frac{1}{2\gamma_+ A}\frac{d}{dt}(\vec{L}\times\dot{\vec{L}})\right]$.

$\vec{L} = L\vec{\ell}$. $\frac{d}{dt}(\vec{L}\times\dot{\vec{L}}) = \vec{L}\times\ddot{\vec{L}}$, $\dot{\vec{M}} = -\frac{2}{L^3}\dot{L}\left[M_+M_-\vec{L} - \frac{1}{2\gamma_+ A}\vec{L}\times\dot{\vec{L}}\right] + \frac{1}{L^2}\left[M_+M_-\dot{\vec{L}} - \frac{1}{2\gamma_+ A}\vec{L}\times\ddot{\vec{L}}\right]$

Исп. $\dot{\vec{M}} = 2\gamma_- A(\vec{L}\times\vec{M})$, $\vec{L}\times\vec{M} = \vec{L}\times\frac{1}{L^2}\left[M_+M_-\vec{L} - \frac{1}{2\gamma_+ A}\vec{L}\times\dot{\vec{L}}\right] = -\frac{1}{2\gamma_+ AL^2}\vec{L}\times(\vec{L}\times\dot{\vec{L}}) =$

$= -\frac{1}{2\gamma_+ AL^2}\left[\vec{L}(\vec{L}\cdot\dot{\vec{L}}) - \dot{\vec{L}}L^2\right]$

$-\frac{2}{L^3}\dot{L}\left[M_+M_-\vec{L} - \frac{1}{2\gamma_+ A}\vec{L}\times\dot{\vec{L}}\right] + \frac{1}{L^2}\left[M_+M_-\dot{\vec{L}} - \frac{1}{2\gamma_+ A}\vec{L}\times\ddot{\vec{L}}\right] = 2\gamma_- A\,\frac{(-1)}{2\gamma_+ AL^2}\left[\vec{L}(\vec{L}\cdot\dot{\vec{L}}) - \dot{\vec{L}}L^2\right]$

23/XII сразу вводим $\vec{\ell} = \vec{L}/L$ и считаем $\dot{\ell} = \dot{L} \approx 0$.

Хотим привести УД к виду $a\ddot{\vec{\ell}} = \vec{\ell} + \left[\frac{1}{d^2}\ddot{\vec{\ell}} + \dots\right]$ где ед. вектора $\vec{\ell} = \frac{\vec{L}}{L}$

$-\frac{2}{L^3}\dot{L}\left[M_+M_-\vec{L} - \frac{1}{2\gamma_+ A}\vec{L}\times\dot{\vec{L}}\right] + \frac{\gamma_-}{\gamma_+}\frac{1}{L^2}\vec{L}(\vec{L}\cdot\dot{\vec{L}}) = -\frac{M_+M_-}{L^2}\dot{\vec{L}} + \frac{1}{2\gamma_+ AL^2}\vec{L}\times\ddot{\vec{L}} + \frac{\gamma_-}{\gamma_+}\dot{\vec{L}}$

$-\left(\frac{M_+M_-}{L^2} - \frac{\gamma_-}{\gamma_+}\right)\dot{\vec{L}} + \frac{1}{2\gamma_+ AL^2}\vec{L}\times\ddot{\vec{L}} = -\frac{2\dot{L}}{L^3}\left(M_+M_-\vec{L} - \frac{1}{2\gamma_+ A}\vec{L}\times\dot{\vec{L}}\right) + \frac{\gamma_-}{\gamma_+}\vec{L}\frac{\dot{L}}{L}$

$\frac{1}{2\gamma_+ AL^2}\vec{L}\times\ddot{\vec{L}} - \left(\frac{M_+M_-}{L^2} - \frac{\gamma_-}{\gamma_+}\right)\dot{\vec{L}} = \frac{\dot{L}}{L}\left[\frac{\gamma_-}{\gamma_+}\vec{L} - \frac{2}{L^2}M_+M_-\vec{L} + \frac{1}{2\gamma_+ A}\vec{L}\times\dot{\vec{L}}\frac{2}{L^2}\right]$

(*) $\frac{1}{2\gamma_+ AL^2}\vec{L}\times\ddot{\vec{L}} - \left(\frac{M_+M_-}{L^2} - \frac{\gamma_-}{\gamma_+}\right)\dot{\vec{L}} = \frac{\dot{L}}{L}\left[\left(\frac{\gamma_-}{\gamma_+} - \frac{2M_+M_-}{L^2}\right)\vec{L} + \frac{1}{\gamma_+ AL^2}\vec{L}\times\dot{\vec{L}}\right] - \frac{2\dot{L}}{L}\dot{\vec{L}}$

Если считать приближенно $\dot{L} = 0$, $L^2 = \text{const}$, то

$\frac{1}{2\gamma_+ AL^2}\vec{L}\times\ddot{\vec{L}} - \left(\frac{M_+M_-}{L^2} - \frac{\gamma_-}{\gamma_+}\right)\dot{\vec{L}} = 0$, $\left(\frac{M_+M_-}{L^2} - \frac{\gamma_-}{\gamma_+}\right)L\dot{\vec{\ell}} = \frac{1}{2\gamma_+ A}\vec{\ell}\times\ddot{\vec{\ell}}$, УД

с учетом $L^2 \approx M_+^2$, $M_+M_-\gamma_+ - \gamma_- L^2 = M_+\gamma_1\gamma_2(s_1 - s_2)$ сведется к УД 23/XII для $\vec{\ell}$.

Теперь в УД (*) перейдем к unit-vector $\vec{\ell} = \vec{L}/L$, $\vec{L} = L\vec{\ell}$, $L(t)$, $\vec{\ell}(t)$.

$\dot{\vec{L}} = \frac{d}{dt}(\vec{L}) = \frac{d}{dt}(L\vec{\ell}) = \dot{L}\vec{\ell} + L\dot{\vec{\ell}}$, $\vec{L}\times\dot{\vec{L}} = \vec{L}\times(\dot{L}\vec{\ell} + L\dot{\vec{\ell}}) = \vec{L}L\dot{\vec{\ell}} = L^2\vec{\ell}\times\dot{\vec{\ell}}$

$\ddot{\vec{L}} = \frac{d}{dt}(\dot{\vec{L}}) = \frac{d}{dt}(\dot{L}\vec{\ell} + L\dot{\vec{\ell}}) = \ddot{L}\vec{\ell} + \dot{L}\dot{\vec{\ell}} + \dot{L}\dot{\vec{\ell}} + L\ddot{\vec{\ell}} = \ddot{L}\vec{\ell} + 2\dot{L}\dot{\vec{\ell}} + L\ddot{\vec{\ell}}$

$\vec{L}\times\ddot{\vec{L}} = \vec{L}\times\ddot{\vec{L}} = L\vec{\ell}\times(\ddot{L}\vec{\ell} + 2\dot{L}\dot{\vec{\ell}} + L\ddot{\vec{\ell}}) = 2L\dot{L}\vec{\ell}\times\dot{\vec{\ell}} + L^2\vec{\ell}\times\ddot{\vec{\ell}}$

$\frac{1}{2\gamma_+ AL^2}\left[2L\dot{L}\vec{\ell}\times\dot{\vec{\ell}} + L^2\vec{\ell}\times\ddot{\vec{\ell}}\right] - \left(\frac{M_+M_-}{L^2} - \frac{\gamma_-}{\gamma_+}\right)(\dot{L}\vec{\ell} + L\dot{\vec{\ell}}) = \frac{\dot{L}}{L}\left[\left(\frac{\gamma_-}{\gamma_+} - \frac{2M_+M_-}{L^2}\right)L\vec{\ell} + \right.$

$\left. + \frac{1}{\gamma_+ AL^2}L^2\vec{\ell}\times\dot{\vec{\ell}}\right]$, $\frac{1}{2\gamma_+ A}\vec{\ell}\times\ddot{\vec{\ell}} - \left(\frac{M_+M_-}{L^2} - \frac{\gamma_-}{\gamma_+}\right)L\dot{\vec{\ell}} = \frac{\dot{L}}{L}\left[\left(\frac{\gamma_-}{\gamma_+} - \frac{2M_+M_-}{L^2}\right)L\vec{\ell} + \right.$

$\left. + \frac{1}{\gamma_+ A}\vec{\ell}\times\dot{\vec{\ell}}\right] - \frac{1}{\gamma_+ A}\frac{\dot{L}}{L}\vec{\ell}\times\dot{\vec{\ell}} + \left(\frac{M_+M_-}{L^2} - \frac{\gamma_-}{\gamma_+}\right)L\dot{\vec{\ell}}$

(**) $\frac{1}{2\gamma_+ A}\vec{\ell}\times\ddot{\vec{\ell}} - \left(\frac{M_+M_-}{L^2} - \frac{\gamma_-}{\gamma_+}\right)L\dot{\vec{\ell}} = \frac{\dot{L}}{L^2}(-1)M_+M_-\vec{\ell}$, в правой части по сравнению с 23/XII есть член $\sim\dot{L}$.

т.е. УД FiM не сводится при $A\to\infty$ к УД для unit vector $\vec{\ell}$ ($\vec{\ell}^2 = 1$). Надо добавить УД для $L = |\vec{L}| = L(t)$. $\vec{L} = \frac{1}{2}(\vec{M}_1 - \vec{M}_2)$ – вектор AF.

$\vec{L}^2 + \vec{M}^2 = \frac{1}{2}(M_1^2 + M_2^2)$, $\vec{L}\dot{\vec{L}} + \vec{M}\dot{\vec{M}} = 0$, $\vec{L}\dot{\vec{L}} = -\vec{M}\cdot\dot{\vec{M}} = -\left[\frac{M_+M_-}{L^2}\vec{L} - \frac{1}{2\gamma_+ AL^2}\vec{L}\times\dot{\vec{L}}\right]\cdot\dot{\vec{M}}$

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$\vec{M}^2 = \left(\frac{1}{L^2} M_+ M_- \vec{L} - \frac{1}{2\gamma_+ A L^2}\vec{L}\times\dot{\vec{L}}\right)^2 = \frac{1}{L^2} M_+^2 M_-^2 + \frac{1}{4\gamma_+^2 A^2 L^4}(\vec{L}\times\dot{\vec{L}})^2 \approx \frac{1}{L^2} M_+^2 M_-^2$ если $\vec{L}\dot{\vec{L}} \approx 0$.

$L^2 + \frac{1}{L^2} M_+^2 M_-^2 = \frac{1}{2}(M_1^2+M_2^2)$, $M_+ M_- = \frac{1}{4}(M_1^2 - M_2^2)$ нет

$L^4 - \frac{1}{2} L^2 (M_1^2+M_2^2) + M_+^2 M_-^2 = 0$, $2L^2 = \frac{1}{2}(M_1^2+M_2^2) \pm \sqrt{\frac{1}{4}(M_1^2+M_2^2)^2 - 4\cdot\frac{1}{16}(M_1^2-M_2^2)^2}$

$2L^2 = \frac{1}{2}(M_1^2+M_2^2) \pm \frac{1}{2}\sqrt{4 M_1^2 M_2^2} = \frac{1}{2}(M_1^2+M_2^2 \pm 2M_1M_2)$, $L^2 = \frac{1}{4}(M_1+M_2)^2$.

Выводится в главном по $1/A \ll 1$ путём измерения $L = \frac{1}{2}(M_1+M_2)$, $\dot{L} = 0$. нет

$\dot{\vec{L}} + \dot{\vec{L}} = \dot{\vec{L}} \times 2\gamma_+ A (\vec{L}+\vec{M}) = 2\gamma_+ A [\vec{L}(\vec{L}\vec{M}) + \vec{M} L^2] = 2\gamma_+ A [\vec{L} M_+ M_- + \vec{M} L^2]$

$\frac{1}{4\gamma_+^2 A^2 L^4}(\dot{\vec{L}}+\dot{\vec{L}})^2 = \frac{1}{4\gamma_+^2 A^2 L^4} 4\gamma_+^2 A^2 (\vec{L}^2 M_+ M_- + \vec{M} L^2)^2 = \left(\vec{L}\frac{M_+M_-}{L^2} + \vec{M}\right)^2$ по если $\ll 1$.

$\vec{L}^2(t) + \vec{M}^2(t) = \frac{1}{4}(M_1^2+M_2^2)$ и если же $\neq 0$, $\dot{L}, \dot{M}$. $L\dot{L} + M\dot{M} = 0$, но $|\vec{M}|$ не обязательно $\ll L^2$ для FiM, $|\vec{M}|$ и $|\vec{L}|$ сравнимы и $\dot{L} = -\frac{M}{L}\dot{M} \neq 0$.

Нельзя просто $\dot{L} = 0$ в векторных УД.

Видимо при выводе Less (Zvezdin) условие $\dot{L} \neq 0$ было (упущено) неявно, назвать вектор УД или $L \neq 0$ (23/XII) но сводится к УД-Звездин с β_e, β_e.

Звездин использовал УД $\vec{L}(\theta,\varphi)$ и приближение $\frac{1}{A} = \frac{1}{\lambda} \ll 1$.

Kinetic part of УД FiM: $\left(\frac{M_+M_-}{L^2} - \frac{\gamma_-}{\gamma_+}\right)L\dot{\vec{L}} - \frac{\dot{L}}{L^2} M_+ M_- \vec{L} = \vec{L}\times\left[\frac{1}{2\gamma_+ A}\ddot{\vec{L}} + \dots\right]$ (**)

$L = L(t)$, $\vec{L} = \vec{L}/L$.

– это пара eqs для θ, φ, $\vec{L}(\theta,\varphi)$, когда входит явно $L(t)$ зависимость. Надо первое eq. для $L(t)$.

Способ вычисления $W(\vec{L})$, $\mathcal{L} = \mathcal{L}_{kin} - W$ предполагает $|\vec{M}|^2 \ll |\vec{L}|^2$, где $\vec{M}$ (magnetization) или $\vec{m}$ (spin density). Но тогда $\dot{L} = 0$, т.к. $\vec{L}^2 + \vec{M}^2 = \text{const}$.

Верна ли $\chi_\perp = \frac{(M_1+M_2)^2}{\lambda M_1 M_2}$ – Davydova'2020 или $\chi_\perp = \frac{(M_1+M_2)^2}{2\lambda M_1 M_2}$ из Ward Sole Zutkra?

Здесь приведена вывод $\left(\chi_\perp = \frac{(M_1+M_2)^2}{4\lambda M_1 M_2} = \frac{1}{4}\cdot\chi_\perp\right.$ (Davydova'20), что согл. с моим $\left.\chi_\perp\right.$ (29/XII 20г.)

→ Версии Less (Gus), Less (Gareev) и Less (Ivanov 2019) или свежие УД.

(9/I) Corrected Theory Sec. in Overleaf file by Farkhad «Vortex GM nanowires»

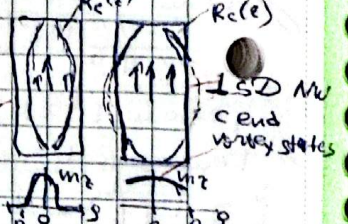
Прочла Gleb и предложила свою коррекцию in Dobrovolskiy MS «Nanovolcanoes». Edited the MS и послала его to Oleksandr у Gleb. (N. «Carrefour» Olaberria. Cher.

(10/I) E-mail to Ricardo (meeting ~ Jan.18). Были с 10 фев. и «In-time».

(11/I) Анализ MS по nanowires by Farkhad и вопросы ему.

Vortex core radius $R_c(z)$. Если $R_c(z) < R$ by z-single vortex NW.

Если $R_c(z) < R$ только при $z \approx \pm\frac{L}{2}$, и $R_c(z) > R$ – 1SD nanowire single vortex NW – или назад (Ends) NW length L (central z) с vortex ends



Edited NW – Farkhad file и послал ему вопросы. Nanowires

Skype с Оксаной: 1) BP – topo index – Oxc. обещала написать calcs Q_H by $\vec{k}$-method, где Dirac-string (должно $\operatorname{div}\vec{B} = 0$) обязательно, зачем $Q_H \neq 0$, и обещала проверить Q_H, роль B_S и Jülich для PRL; Оксана – finish Q_H by k-method

2) Oxc. не знает какие FeCoCu – nanowires первым Фаркад, видимо F/N/F/N/... слишком $t_F \approx 200$ nm, $t_N \approx 50$ nm. послал ей PDF paper by Фаркад.

Skype с Gleb: Nowak первая в Porto FMR – nanodots (FeCoCu), писала о APL-Dobrovol.

(12/I) APL – Dobrovolskiy «Nanovolcanoes» comments/changes. Были. BP

(13/I) Согл. план MS 8, 2020 самое важное/срочное: BP-paper, $MnBi_2Te_4$ и FiM dyn.

BP – stopped change – назад Aug. 10 2020. $\vec{r}(r, e_\theta, e_\varphi)$ сферич. координаты.

12/VIII Брал gauge $A_{cc} = 0$. Feldkeller ansatz: $\Theta(\vec{r}) = p e_\theta + \frac{\pi}{2}(1-p)$, $\Phi(\vec{r}) = q\varphi + \gamma$ с $p = \text{const}$.

Мое предположение $\gamma = \gamma(r, e_\theta)$ – helicity. $\vec{m}(\vec{r}) = (m_r, m_\theta, m_\varphi) = F(e_\theta, \gamma(r, e_\theta))$ – не зависит от φ (axial sym.) и зависит от $r = |\vec{r}|$ только через $\gamma(r, e_\theta)$

→ Вычислить: 1) роль $\vec{B}_{sing}$ для Q_H; 2) выбор gauge для vector-potential $\vec{A}$.

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24/VIII Сем. $\vec{B}_{string}(\vec{r}) = \vec{B}_s(\vec{r}) = -4\pi pq\, H(z)\,\delta(x)\delta(y)\hat{z}$, $Q_H^{(s)} \sim \int dV\, \vec{A}\cdot\vec{B}_s$, $\vec{A}\cdot\vec{B}_s = (A_r\hat{r} + A_\theta\hat{\theta} + A_\varphi\hat{\varphi})B_s\hat{z} = B_s(A_r\hat{r}\cdot\hat{z} + A_\theta\hat{\theta}\cdot\hat{z} + A_\varphi\hat{\varphi}\cdot\hat{z})$, $\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$ Singular $\vec{B}_s$

$\vec{A}\cdot\vec{B}_s = B_s(A_r\cos\theta - A_\theta\sin\theta)$, $B_s = -4\pi pq\, H(z)\,\delta(x)\delta(y)$, $\vec{B}_s = B_s\hat{z}$

$Q_H \sim \int dV\,\vec{A}\cdot\vec{B}_s = \int_0^\infty dr\int_0^\pi d\theta\int_0^{2\pi} d\varphi\, r^2\sin\theta\,[A_r(r,\theta)\cos\theta - A_\theta(r,\theta)\sin\theta](-4\pi pq)\,H(z)\,\delta(x)\delta(y)$

$\int_0^\pi d\theta\sin\theta\,\delta(x)\delta(y)F(\theta) \equiv 0$, т.к. $x = r\sin\theta\cos\varphi = 0$, $y = r\sin\theta\sin\varphi = 0$ одновр. $\sin\theta = 0$. Дирак-струна не попадает $z>0$. $Q_H \sim \int dV\,\vec{A}\cdot\vec{B}_s = 0$. Д. струна не дает вклада в Q_H. Это так, если $A_r(r,\theta)$ и $A_\theta(r,\theta)$ не сингулярны при $\theta = 0$

$A_r(r,\theta) = \frac{1}{r}p\int_{\theta_0}^{\theta} d\theta'\sin\theta'\frac{\partial\gamma(r,\theta')}{\partial r} + f(r)$ - начальная сингулярность при $\theta = 0$ не будет

Можно $\oint_S d\vec{S}\cdot\vec{B}_s = \oint_S dS_r\,\hat{r}\cdot\vec{B}_s = \oint_S dS_r B_s(\vec{r}\cdot\hat{z}) = \oint_S dS_r B_s\cos\theta = \oint_S dS_r(-4\pi pq)H(z)\delta(x)\delta(y) = (-4\pi pq)\int_S dS_r\,H(z)\delta(x)\delta(y) = (-4\pi pq)\int dS_r\,\delta(x)\delta(y) = -4\pi pq$

$\int_{S_+} dS_r\,\delta(x)\delta(y) = 1$, S_+ полусферы $z>0$. $dS_r = r^2\sin\theta\,d\theta\,d\varphi$.

$\int_{S_+} div(\vec{B}) = div(\vec{B}_{string} + \vec{B}_s) = 0$, т.к. поток $\vec{B}_{string} = pq\frac{\hat{r}}{r^2}$ компенсируется потоком $\vec{B}_s = B_s\hat{z}$ (через замкн. поверхность S). См. «Монополь Дирака» 1948.

Но $\int dV = \int dr\int dS_r$ и если $\int dS_r\,\delta(x)\delta(y) = 1$, то $\int dV\,\vec{A}\cdot\vec{B}_s$ - перестанет быть $A_\theta = 0$. $\int dV\,\vec{A}\cdot\vec{B}_s = \int dr\int d\theta\int d\varphi\, r^2\sin\theta A_r\cos\theta(-4\pi pq)\delta(x)\delta(y) =$

$= (-4\pi pq)\int dr\oint dS_r A_r(r,\theta)\cos\theta\,H(z)\delta(x)\delta(y) = (\dots)\int dr\int_{S_+} dS_r A_r(r,\theta)\cos\theta\times$

$\times\delta(x)\delta(y) = (\dots)\int dr\,A_r(r,0)\int_{S_+} dS_r\,\delta(x)\delta(y) = (-4\pi pq)\int dr\,A_r(r,0)$

$\int dV\,\vec{A}\cdot\vec{B}_s = (-4\pi pq)\int dr\,A_r(r,0)$ $\quad \int_{S_+} = 1$

$A_r(r,\theta) = \frac{1}{r}p\int_{\theta_0}^{\theta} d\theta'\sin\theta'\frac{\partial\gamma}{\partial r}(r,\theta') + f(r)$

т.к. A_θ входит в $\int dV$ как комбинация $A_\theta\sin\theta$, то $\sin\theta = 0$, на θ_0 не влияет.

$\int dr\,A_r(r,0) = \int dr\,(-\frac{1}{r})\int_{\theta_0}^{0} d\theta\sin\theta\frac{\partial\gamma}{\partial r}(r,\theta) + \int dr\,d(r) =$ Б(о)льший рост

$= (-\frac{1}{r})\int_{\theta_0}^{0} d\theta\sin\theta\int_0^R dr\frac{\partial\gamma}{\partial r}(r,\theta) + C = (\frac{1}{r})\int_{\theta_0}^{0} d\theta\sin\theta\,[\gamma(R,\theta) - \gamma(0,\theta)] + C$

Сначала 29/VIII надо $\vec{B} = B_r\hat{r} + B_\theta\hat{\theta}$ (регулярно), $B_\theta = -\frac{p}{r}\sin\theta\frac{\partial\gamma}{\partial r}$

$B_\theta = (rot\vec{A})_\varphi = \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right] = -\frac{p}{r}\sin\theta\frac{\partial\gamma}{\partial r}$, что при $A_\theta = 0$

даст $-\frac{1}{r}\frac{\partial A_r}{\partial\theta} = -\frac{p}{r}\sin\theta\frac{\partial\gamma}{\partial r}$, $\frac{\partial A_r}{\partial\theta} = +p\sin\theta\frac{\partial\gamma}{\partial r}$

$\int dV A_r B_r = 2\pi g\int dr\int_0^\pi d\theta\sin\theta A_r(r,\theta)$ для регулярного $\vec{B}$, $\int_0^\pi d\theta\sin\theta A_r = p\int_0^\pi d\theta\sin\theta\cos\theta\frac{\partial\gamma}{\partial r} +$

$f_r(r) = -\cos\theta A_r(r,\theta)\Big|_0^\pi = A_r(r,\pi) + A_r(r,0)$, $g = pq$. $+ f_r(r)$

$A_r(r,\theta) = p\int_{\theta_0}^{\theta} d\theta'\sin\theta'\frac{\partial\gamma}{\partial r}(r,\theta') = p\int_0^\theta d\theta'\sin\theta'\frac{\partial\gamma}{\partial r}(r,\theta) - p\int_0^{\theta_0} d\theta'\sin\theta'\frac{\partial\gamma}{\partial r}$

$\int dV A_r B_r = 2\pi g\int dr\left[p\int_0^\pi d\theta\sin\theta\cos\theta\frac{\partial\gamma}{\partial r} + A_r(r,\pi) + A_r(r,0)\right]$

$\int dV A_r B_r^s$, $\vec{B}^s = B_s\hat{z} = B_s\cos\theta\,\hat{r} - B_s\sin\theta\,\hat{\theta}$, $\vec{A}\cdot\vec{B}_s = \frac{B_s A_r\cos\theta}{A_r B_r^s} - \frac{B_s A_\theta\sin\theta}{A_\theta B_\theta^s}$

$\int dV\,\vec{A}\cdot\vec{B}_s = |A_\theta\sin\theta$ не дает вклада$| = \int dV A_r B_r^s = (-4\pi g)\int dr\,A_r(r,0)$

$\int dV A_r(B_r + B_r^s) = 2\pi g\int dr\left[p\int_0^\pi d\theta\sin\theta\cos\theta\frac{\partial\gamma}{\partial r} + A_r(r,\pi) + A_r(r,0)\right] - 4\pi g\int dr\,A_r(r,0) =$

$= 2\pi q\int dr\int_0^\pi d\theta\sin\theta\cos\theta\frac{\partial\gamma}{\partial r} + 2\pi g\int dr\left[A_r(r,\pi) - A_r(r,0)\right]$

$A_r(r,\pi) - A_r(r,0) = A_r(r,\theta)\Big|_{\theta=0}^{\theta=\pi} = p\int_0^\pi d\theta\sin\theta\frac{\partial\gamma}{\partial r}(r,\theta)$, так как $f_r(r) = 0$ 29/VIII

$\int dV A_r(B_r + B_r^s) = 2\pi q\int dr\int_0^\pi d\theta\sin\theta\cos\theta\frac{\partial\gamma}{\partial r} + 2\pi q\int dr\int_0^\pi d\theta\sin\theta\frac{\partial\gamma}{\partial r}$.

$\int dV A_\theta B_\theta = |B_\varphi^s = 0| = 2\pi q\int dr\int_0^\pi d\theta\sin\theta(1+\cos\theta)\frac{\partial\gamma}{\partial r}$, См. 29/VIII, 19/III.

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$\int dV \vec{A}\cdot(\vec{B}+\vec{B}_s) = \int dV A_r B_r + \int dV A_r (B_z + B_r^s) = 2\pi q \int dz \int_0^\pi d\theta \sin\theta (1+\cos\theta) \frac{\partial f}{\partial z} +$

$+ 2\pi q \int dz \int_0^\pi d\theta \sin\theta (1+\cos\theta) \frac{\partial f}{\partial z} = 4\pi q \int dz \int_0^\pi d\theta \sin\theta (1+\cos\theta) \frac{\partial f}{\partial z}$

$Q_H = -\frac{1}{(4\pi)^2} \int dV \vec{A}\cdot(\vec{B}+\vec{B}_s) = -\frac{q}{4\pi} \int dz \int_0^\pi d\theta \sin\theta (1+\cos\theta) \frac{\partial f}{\partial z} -$

→ это в отличие Q_H Eq. (6) из BP-Hopf manuscript (!)

Удивительно, в расчете Q_H 29-30/VIII я пренебрег вкладом $B_r^{sing} = B_r^s$ и взял произвольно $f_2(r)=0$, $f_r = p \int_0^\pi d\theta \sin\theta \frac{\partial f}{\partial z}$, сделал две ошибки, которые компенсировали друг друга.

$\int dV A_r B_r^s = -4\pi q \int dz A_z(r,0)$, $A_z(r,0) = -p \int_0^{\theta_0} d\theta \sin\theta \frac{\partial f}{\partial z} = f_2(z)$

Взяв 30/VIII $f_2(z)=0$, $\theta_0=0$, $\Rightarrow \int dV A_r B_r^s = 0$, $A_z(r,0)=0$, но $\int dV A_r B_r^s \neq 0$ я компенсировал B_z $f_z(r) = \int_0^\pi d\theta \sin\theta \frac{\partial f}{\partial z}$ членом в Q_H - расчете. $W_z = \frac{\partial f}{\partial z}$, B_z^s

→ Вывод: расчет Q_H Eq. (6) удовлетворительно учтен $\vec{B}_{sing}$. $\vec{B}_{sing}$ дает вклад в Q_H.

Все это при $A_{\theta\theta}=0$. $A_{\theta\theta}$ не дает вклада в $\int dV A_{\theta\theta} B_{\theta\theta}^{sing} \sim Q_H^s$, ибо A_r дает. $A_{\theta\theta}$ не дает вклада в $\int dV A_{\theta\theta} B_{\theta\theta}$, т.к. $B_{\theta\theta}=0$. Выбор $A_{\theta\theta}=0$ разумен при расчете Q_H, т.к. $A_{\theta\theta}\neq 0$ не влияет Q_H. (если $A_{\theta\theta}$ конечно при $\theta=0$)

Gauge для $\vec{A}$ - vector potential при расчете Q_H: $A_{\theta\theta}=0$, $\vec{A} = (A_r, 0, A_\varphi) = \vec{A}(r,\theta)$

$\operatorname{div}\vec{A}\,(Gus) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) \neq 0$ (Gus; $A_{\theta\theta}=0$, $\frac{\partial A_\varphi}{\partial\varphi}=0$)

Gauge-Gus $A_{\theta\theta}=0$ несовместимо с $\operatorname{div}\vec{A}=0$ - Gauge.

(14/I) Revision BP-Hopf manuscript. Переписка с Окс./Раха, Dell/Antxon. Обсужд. с Гибом и corrections MS „Nanovolcanoes" - Добровольку.

(15/I) Published APL „Engineered Co-dots". Submitted APL „Nanovolcanoes".

(16/I) Revision GM Nanowires manuscript с Пархад. Написание BP paper. С Л. в „Carrefour" Quartzum. Сломался замок блокнота. Блин.

(17/I) Пархад прислал revised GM-Nanowire MS всем Co-авторам. Domcan Abstract, Introduction и Summary BP-Hopf paper и послал Окс. Рафе, Химия с А.

(18/I) Semi AXA incident doc to IO. Запрос link PDLS on-line info meeting. Переписка с Оксаной и Пархадом по Nanowires gyromodes paper: разбивать ее на 2 части) exp 25 nm NW + simul.; 2) short NW 1 nm simul. + new theory.

(19/I) Работа над „Nanowires MS и переписка с Окс., Пархад. В „Нам" Одиссея и начало с Л. и Ю. „Erotski + Urbil с Л. IVA „Constantino" 2020.

(20/I) PDLS meeting. Написала семинар с Ricardo/Ruben на 22/I: 1) BP; 2) Mn_2Au. Блин.

(21/I) Работа с А. над Powell-SW paper APL и Letter Editor. Ю. перевел 4000 € зубы.

(22/I) Meeting с Ricardo и Ruben: 1) BP Hopf index (MS с Окс.); 2) Mn_2Au-DW. Угол не зависит от J_2 (?!) - разобрались. „Carrefour" Quartzum с Л., Блин. Обсужд. с Пархад GM-Nanowire manuscript: ещё разобрались с $R_0(z)$ для $L \leq 1$ μm, мне - написать качеств. результаты при и $\vartheta_n(z)$ для vortex NW. С Л. в „Carrefour" Quartzum. Начало new version Nanowires MS.

(23/I) Послал to Ruben y Ricardo draft Bloch point paper (comments?). Ответы Оксаны (Fähnle, Nowak) на вопрос о vortex m-ansatz Usov.

(24/I) Дозвон paper IVA „Constantino" 2020 и заполнил декларацию. Receipt IVA 57,20 €.

(25/I) Оксана прислала changes to BP-paper. Skype/phone с Оксаной: 1) change Bloch-point MS, мне late-to Editor и 100 words; 2) MS Nanowires с Пархад - нелинейные simulations для static $m(r)$ Ларе и dynamics. Обсудили Окс. Что вернуться в мою model Zeeman's Zakon от pC-product и описать ang. low-f mode с учетом/неучетом dot thickness/nanowire length. Работа над BP-MS.

(26/I) Работа над Bloch-point MS. Послал Окс., Раха, Felipe MS draft + cover letter PRL. Переписка с Окс., Пархад о $m(r)$ NW. Фото „Otras ayudas" PID 2019.

(27/I) Skype с Окс., Пархад, Diego: up-date to simul. for L=250 nm NW, где

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моа Vortex GM model разработ; 2) Запросить Gaistika to produce short π-Ni NW и segmented FeCo/Cu NW.

(22/I) Skype - обсуждение с Ruben BP paper: 1) насколько Q_H зависит от Dirac string direction $\vec{n}$? не зависит? System - spherical particle - is isotropic. Выбираем $\vec{n} \parallel \hat{z}$ и $\hat{z}$ - polar axis сферич. координаты $\vec{m}(\theta,\phi)$; 2) будет ли $\vec{G} = \int dV \vec{B} \neq 0$ для symmetric Dirac string for all z (Schwinger string) и что следствие асимметрии string (только $z>0$ или только $z<0$)?

Skype с Фархад, он simulated vortex string dynamics для $L = 250, 500$ nm, где $R_c(z) < R$, wire is in a single vortex state.

Skype с Тарасовской: docs для AOLS 3D $\vec{m}$ и project, Burgers vector index для toroidal hopfion (Zhang'18, Braras'19, Gladikowski'97) и Bloch point (присл. file). Обсуждение BP и hopfion dynamics и применение к BP-DW in nanowires с A. Erokhin Wbil.

→ Gyrovector of toroidal hopfion: $\vec{G}$ - BP 12/VIII 19г.

1) deVega'78 $\vec{G} = \int dV \vec{\gamma} = -\pi c \left[\pi \hat{e}_\beta + 2\hat{e}_\varphi\right] \neq 0$, $c = a$ = hopfion radius.

2) Gladikowski'97, Hietarinta'2000 $\vec{G} = \int dV \vec{B} = (G_\gamma, G_\beta, G_\varphi) = 16\pi^2 a \left(0, \frac{\pi}{2} N, -M\right)$ $\vec{G} \neq 0$

Gladikowski'97: $\operatorname{div} \vec{B} = 0$ при $w = w(\gamma)$ и $\vec{\gamma}_0 = \gamma(\gamma,\beta) = \text{const}$, $\operatorname{div} \vec{A} \neq 0$; 11/IX 2020г.

это вып. для того, что $Q_H = 1$.

→ Bloch point: $\vec{B}_s = -4\pi g H(z)\delta(x)\delta(y)\hat{z}$, см. 22/VIII 20г. $\int dV \vec{B}_s = -4\pi g \int dx\,dy\,dz \times$

$\times H(z)\delta(x)\delta(y)\hat{z} = -4\pi g \int_{-R}^{R} dz\, H(z)\hat{z} = \left| \begin{matrix} H(z)=1, z>0 \\ H(z)=0, z<0 \end{matrix} \right| = -4\pi g \hat{z} \int_0^R dz = -4\pi g R \hat{z}$

$\vec{G} = \int dV \vec{B}_s = -4\pi g R \hat{z}$, $g = p/2$ для Dirac string $z>0$

Если Dirac string is symmetric $\vec{B}_s = -4\pi g \delta(x)\delta(y)\hat{z}$, то $\vec{G} = \int dV \vec{B}_s =$

$= -4\pi g \int_{-R}^{R} dz\, \hat{z} = -4\pi g \hat{z}(2R) \neq 0$. $\vec{A}_{sym} = \frac{1}{2}\left[\vec{A}(\vec{n}) + \vec{A}(-\vec{n})\right]$, $\vec{A}(\vec{n}) = \frac{g}{2} \frac{\vec{n}\times\vec{r}}{|\vec{r} - \vec{n}\cdot\vec{r}|}$

и $\vec{B}^s_{sym} = \operatorname{rot} \vec{A}_{sym} = \frac{1}{2}\vec{B}_s(\vec{n}) + \vec{B}_s(-\vec{n})\frac{1}{2} = \frac{1}{2}\left[B_s(z>0) + B_s(z<0)\right] = \frac{1}{2}\left(-4\pi g \delta(x)\delta(y)\hat{z}\right)$

$\vec{G}_{sym} = \int dV \vec{B}^s_{sym} = \frac{1}{2}(-4\pi g)\int dx\,\delta(x)\int dy\,\delta(y)\int_{-R}^{R} dz\,\hat{z} = (\dots)\frac{1}{2}2R = (\dots)R = -4\pi g R\hat{z}$

Вывод: $\vec{G} = -4\pi g R \hat{z}$ Сохраняет свои св-ва и для symmetric Dirac string вдоль всей оси z.

$\vec{G} \neq 0$ для BP.

$\vec{B}_{s(\vec{n})}$ не дает вклада в Q_H, значит $\vec{n}$ - Dirac string direction не влияет на Q_H? Только $\vec{B}_s(\hat{z})$ не дает вклад в Q_H если $\vec{B}_s \sim \hat{z}$. $\vec{B}_s(\vec{n}) = c\vec{n}$

Если Dirac-string $\vec{n}$, то $\vec{A}_D = -\frac{g}{r}\frac{\vec{n}\times\vec{r}}{(r - \vec{n}\cdot\vec{r})}$, $\vec{B}_s(\vec{n})\cdot\vec{A}_D(\vec{n}) = c\vec{n}\cdot\frac{g}{r}\frac{(\vec{n}\times\vec{r})}{(r-\vec{n}\cdot\vec{r})} = 0$

т.к. $\vec{n}\cdot(\vec{n}\times\vec{r}) = 0$.

$\vec{B} = \vec{B}_0 + \vec{B}_s = \frac{g}{r^2}\hat{r} - p\frac{\partial\gamma}{\partial r}\frac{\sin\theta}{r}\hat{\varphi} + c\vec{n}$, $\vec{A} = A_r\hat{r} + \vec{A}_D(\vec{n})$, $\frac{\partial A_r}{\partial\theta} = p\frac{\partial\gamma}{\partial r}\sin\theta$

$\vec{B}\cdot\vec{A} = \left[\frac{g}{r^2}\hat{r} - p\frac{\gamma_2}{r}\sin\theta\,\hat{\varphi} + c\vec{n}\right]\cdot\left[A_r\hat{r} + \vec{A}_D(\vec{n})\right] =$ и $\vec{B} = \operatorname{rot} A$.

$= A_r\frac{g}{r^2} + \frac{g}{r^2}\hat{r}\cdot\vec{A}_D(\vec{n}) - p\frac{1}{r}\frac{\partial\gamma}{\partial r}\sin\theta\left[\hat{\varphi}\cdot\vec{A}_D(\vec{n})\right] + cA_r\vec{n}\cdot\hat{r} + c\vec{n}\cdot\vec{A}_D(\vec{n})$, $\vec{A}_D(\vec{n}) = -\frac{g}{r}\frac{\vec{n}\times\hat{r}}{(\dots)}$

$\vec{B}\cdot\vec{A} = \frac{gA_r}{r^2} - p\frac{1}{r}\frac{\partial\gamma}{\partial r}\sin\theta\,\hat{\varphi}\cdot\vec{A}_D(\vec{n}) + cA_r\vec{n}\cdot\hat{r}$, $\hat{\varphi}\cdot(\vec{n}\times\hat{r}) = \vec{n}\cdot(\hat{r}\times\hat{\varphi}) = -\vec{n}\cdot\hat{e}_\theta$

$\vec{B}\vec{A} = \frac{gA_r}{r^2} - p\frac{1}{r}\frac{\partial\gamma}{\partial r}\sin\theta\,g\frac{\vec{n}\hat{e}_\theta}{(r - \vec{n}\cdot\vec{r})} + cA_r\vec{n}\cdot\hat{r}$, $\vec{B}_s = -\vec{n}\,2\pi g H(\vec{n}\cdot\hat{r})\,\delta\left(r^2 - (\vec{n}\cdot\vec{r})^2\right) = c\vec{n}$

при $\vec{n} = \hat{z}$, $\vec{B}_s = -\hat{z}\cdot 2\pi g H(z)\,\delta(r^2 - z^2) = -\hat{z}\cdot 4\pi g H(z)\delta(x)\delta(y)$, $\delta(g^2) = 2\delta(x)\delta(y)$

$r - \vec{n}\cdot\vec{r} = r(1 - \vec{n}\cdot\hat{r}) = r(1 - \hat{z}\cdot\hat{r}) = r(1 - \cos\theta)$, $\hat{z} = -\sin\theta\,\hat{e}_\theta + \cos\theta\,\hat{r}$, $\hat{z}\cdot\hat{e}_\theta = -\sin\theta$

$\vec{A}\cdot\vec{B} = \frac{gA_r}{r^2} - pg\frac{1}{r}\frac{\partial\gamma}{\partial r}\sin\theta\frac{(-\sin\theta)}{r(1-\cos\theta)} + cA_r\cos\theta = \frac{gA_r}{r^2} + \frac{g}{r^2}\frac{\partial\gamma}{\partial r}(1+\cos\theta) + cA_r\cos\theta$ $\hat{z}\cdot\hat{r} = \cos\theta$

$c = -4\pi g H(z)\delta(x)\delta(y)$, т.к. $\vec{B}_s = c\hat{z}$ не дает вклад в $\int dV \vec{A}\cdot\vec{B}$, а остальные должны вместе и $\int dV \vec{A}\cdot\vec{B}$ 13/I.

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(29/I) Обработка BP-paper для PRL, Cover lett. Бинц. переписка с Окс.

(30/I) Submitted BP manuscript to PRL как обычно PDF file (text, Figs, Suppl.) Рецензенты: Chernyshev, Hoffmann, Nowak, Jiadong Zang, Smaliyukh.

(31/I) Уборка дома. Футбол. "Tata Steel" chess tournament.

(1/II) переписка с PRL, Отдали the article main text и Suppl. Mat.
Проверка Ricardo's calculations $E_{ex}(J_i)$ in file „Simplified pot. energy..." для Mn_2Au DW, посылал questions to Ricardo. // Ricardo считал $E_{ex}(J_i)$ только для одного атома (1) Mn-layer и затем ×2, чтоб учесть (3)-layer.

(2/II) Проверка $E_{ex}(J_3)$ file Ricardo 13/I. Возился и послал Raúl проф SW-ARL. Участие в 2/1-аутинге „Gran Prix Invierno 2021", 2-ое место из ≈50 участников.

(3/II) Анализ $E_{ex}(J_1 J_2 J_3)$ of Mn_2Au. послал comments to Ricardo/Ruben: если chain Mn_2Au с периодом 1-unit cell была simulated, как было это DW dynamics (v_{max}) не зависит от J_2, который учитывает слои в Mn_2Au unit cell. Зачем $2J_1, 2J_2, 2J_3$ брали в simulations? Рафаэл прислал simulated dynamics $L = 250$ и 500 nm nanowires — странно! сначала starting с $f \approx 30$ GHz (?). 10. вернулся 2006 домой 800€. // Nanowires

(4/II) Skype session с Окс., Рафаэл, Диего — simulations vortex state NW:
1) Рафаэлу проверить Fourier-transform, увеличить амплитуду rf-excitation field (сейчас 10 Oe, это очень мало по сравн. с $H_n \approx 2$ kOe и vortex $|\vec{x}| < 1$ nm будет мало и плохо детектируемо в simulations);
2) спросить зависимость ω_n - simulated от small damping $\alpha = 10^{-3} - 10^{-4}$
3) смогут ли NW $L = 250$ nm колебаться с $f = 31$ GHz — сейчас, где low frequency modes?
4) Окс. обещала прислать Ref. to Hertel, где он в vortex state NW искал simulated dynamics, зависящую от pCC-product (?) — Оксана перешлет. Есть только Hertel PRL по vortex nanotubes, с $\omega(k_z) \neq \omega(-k_z)$.

(5/II) послал co-authors rejection PRL/Tejo по Bloch point, переписка с Окс. куда слать BP manuscript? Работа по уборке дома.

(6/II) А. день рождения. Отдых. Бинц.

(7/II) переделал BP manuscript под Nat. Com. и послал co-authors.

(8/II) Работа над Overleaf new paper Tejo et al. „Skyrmion en. barriers"

(9/II) переделал Introduction BP manuscript and submitted to Nat. Commun. „Skyrmion en. barriers" MS Overleaf.

(10/II) update SolSkyMag web-site с А. СЛ. в „Carefour Market". „Sk_en. Barriers".

(11/II) Nat. Com. rejected BP manuscript, transferred it to Commun. Phys.
E-mails to Invited Speakers SolSkyMag'2020 to confirm their talks at On-line Sol-SkyMag 2021, June 21-24. Обещали, подтвердили участие 14 invited sp.

(12/II) Skype session с Farkhad, Диего, Оксана: dynamic of nanowires.
Последние simulations Рафаэла, для main W_0 NW $L = 250$ nm, $R = 60$ nm.
SK-barrier paper с Окс и Тимофеем. С Л. в „Mercadona-Hernani".

(13/II) С Л. в „Lidl" Olaberria. Бинц. Отдых дома.

(14/II) Отчет FIS2016 (до 1/III) написал. С Л. и Ю. в Zarautz „Carlos Argüiano".

(15/II) Отчет FIS 2016-project: переписка с Ricardo/Ruben. SolSkyMag'21.

(16/II) checked and updated Dept. Mat. Phys publ. list 2020 (Angel). Бинц.

(17/II) Skype с Ruben: спросил про J_2 на Canino для DW dyn. in Mn_2Au, это за пределами 3-model, где $E_{ex} = A\vec{m}^2$ → ... и $A \sim (J_2 + nJ_1)$.
Запрос на Subsanación MAT2013-project. С Л. и Я. в „Eroski" Urbil. Coffee с А. Бинц.

(18/II) Reminder to Commun. Phys. Собрал Angel о переезде в gp. office. E-mail Busch о postdoc. Бинц.

(19/II) переписался с рефери о G-function. В университете: забрал бумаги по MAT2013-project. С Л. и А. в „Eroski" Urbil.

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(20/II) Отчет FIS2016 Skyrmion project. Работа в саду. File Tapp.

(21/II) Skype с таранеховской: 1) ей посчитать $\vec{G}$-components на eq. Glatkowski и сравнить с Nistazinta $m(\vec{H})$ расчетом; 2) найти есть ли min $E(q)$, q - toroidal hopfion radius в cyl. dot.
Новая Таппан. BP - Hopf charge manuscript.

(22/II) Ждан в своем office Angel для re-organization (ничего не изменил).
С Л. в "Carrefour Market" Tolosa. Отчет FIS2016 „Skyrmions"
Замен в „JP Morgan Chase" bank и обнаружил что мои счета in „dormant" переписке" с „Chase" bank. Запросил info о conf. у Окс. Ricardo. State

(23/II) переписка с Chase", asked to re-activate 6 accounts.
Запрос Cristina о salary Arambya. Лена прислала $\vec{G}$-hopfion calc.
Взяли BP paper на рец. Comm. Phys. Запрос JMMM to revisar Fabrele MS.

(24/II) Chase" bank re-activated my accounts: они с March 2020 снимали 12" $/month с checking account (т.к. min > 1,500 $ to be free).
Skype с Окс, Farkhad, Diego: simul. NW dynamics. Skype с Глеб: рец. APL Nanovolcanoes
С Л. в „Alcampo" Oiartzun. Пос. Кнеб Л.D./пана - 11 kg - 22 €

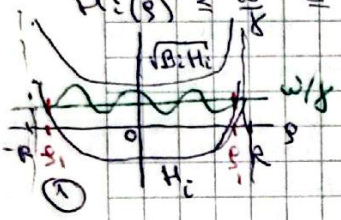
(25/II) D.p. дома. New deadline FIS2016 - Арх. 8. перенос HMM-2021 (Vienna)

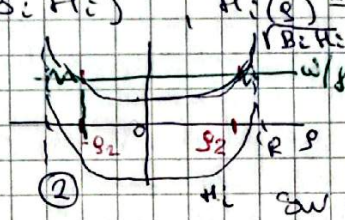
(26/II) Transfer всех $ Chase bank на один xx8597 checking account.
Вернулся тел. 4,900 $ „Chase" и deposited it to $ account „Bankia".
С Л. и Ю. в „Nam" Oudarreta.

(27/II) Ответы Reviewer APL/Nanovolcanoes о SW. Блин. SW "блин"

(28/II) См. анализ dipolar SW in Mo11H 27/IV 2019. Damon'1965, Eshabadi'1963, 1/I 2019. Yukawa'TH, Ikuzawa'1977.
Yukawa'TH: magnetostatic SW существуют в районе q, где Bulk

$$H_i(q) \le \frac{\omega}{\gamma} \le (B_i H_i)^{1/2}, \quad H_i(q) = H_z - 4\pi M_s N(q), \quad B_i = H_i + 4\pi M_s$$





① при малых $\frac{\omega}{\gamma}$, есть turn point q_1 near dot edge, и SW сущ. при $q \in [0, q_1]$

② при больших $\frac{\omega}{\gamma}$, таких что $\frac{\omega}{\gamma} > \sqrt{B_i H_i(0)}$, SW сущ. при $q > q_2$ near dot edge.

Low ω/γ turn. point опреден. eq. $\frac{\omega}{\gamma} = H_i(q_1)$. High ω turn. point опр. $\frac{\omega}{\gamma} = \sqrt{B_i H_i(q_2)}$

Для nanovolcano 

$H_i(q)$ увеличенного ring, q_1 падает, q_2 падает по сравнению с cylindrical dot.

● Wave vector $k(q) = \frac{2}{L}\sqrt{\frac{1}{p(\omega)}}\operatorname{arctg}\frac{1}{p(\omega)}$, $p(\omega) = \frac{B_i H_i - \omega^2/\gamma^2}{\omega^2/\gamma^2 - H_i^2}$ is real for volume MSW. $H_i = H_i(q)$, $B_i = B_i(q) = H_i(q) + 4\pi M_s$.
Это forward magnetostatic SW (FMSW), существ. при $\omega_H \le \omega \le \omega_1$
$\omega_H = \gamma H$, $\omega_1 = \gamma\sqrt{H(H+4\pi M_s)}$, $H \to H_i(q) = H_z + H_m^z(q)$.
Написан draft ответов to Referee APL-Nanovolcanoes и послан co-authors.

(1/III) Skype с Глеб: обсужд. ответов Referee APL-Nanovolcanoes. Блин.
С Л. „Carrefour", „Lidl" Oiartzun. SolSkyMag переписка.

(2/III) Skype с Ruben: Mn_2Au DW dynamics. Решили добавить new sec. в Mn_2Au paper. 3-model is general and correct, но ее недостаточно, чтоб объяснить DW dynamics simulations (где $\vec{m}(x)$ в 2-м Mn layer осциллировала). Надо разбить special model (β_{eff}, J_2), чтобы объяснить Ruben's simulations. V_{max} из DW simulations и SW $\omega(k)$ simulations не зависят от J_2. E-mail to Miguel Palacios - проверка MAT2013.

(3/III) Написал to Elena „Bankia": почему „Chase" check 4,900 $ еще не deposited? 10. visit to Elena: на подтверждение того сделала что нужно по check deposit. Вечером 3/III увидел on-line „Chase", что check deposited и 4,900 $ вышли с # 8597 Check. account. Остаток 30,26 $. MAT2013 project проверка.

(4/III) Написал sec. message to „Chase" о закрытии всех счетов // Из office убрали мои вещи. 2 стола и Книги, никакие бумаги на них.

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5/III E-mails Carletti, Finocchio. Cramp-Lorraine docs MAT 2013 (Arantxa nomina). Напомнить с new Secretary Dept., Maite.

6/III Updating SolSkyMag web-site. Edited Tejo/Sk. Carrier Overleaf manuscript. С А. в „Lidl", „Carrefour" Olaberria.

7/III Нотация to Miguel Lyon 2020 Lectures MAT 2013.

8/III Отсканировал и послал to Miguel Lyon 2020 Lectures MAT 2013, мои explanations и notes excel. Skype с Maciej, Igor, Pawel o SW-edge scattering" MS (see Overleaf file). Беседа с Julian: 1) online SolSkyMag'21 ic Ok; 2) новое явление в Pt/Co Fert им Якубов. С А. в „Mercadona". Отсканировал и послал Thes, Cama Replies to APL - manuscript MS.

9/III Meeting с Ruben y Ricardo в Университете: theme of DW dyn. Mn_2Au paper. Ограничение 0D 2-model с J_2 для DW dynamics, т.к. simulations показали, что на v_{max} (DW), тип acoustic SW dispersion $\omega(k)$ не зависит от J_2. Ricardo прислал Ref. Caretta Science 2020 o DW in Bi-YIG - разобрались (relativistic soliton?). Подверстал Iberfasque promotion docs by June 15.

10/III Обсудил Ricardo о возможности one-parametric 2-model и составили план: структура типа Mn_2Au, где несколько exch. integrals J_1, J_2, J_3 разного знака + biq + changing „SW-edge scattering" MS с Pawel.

11/III Microsoft Team conversation с Oxe и Jose (Oviedo): new exp. на Ni-nanowires на ALBA. BP-DW движется против тока $\vec{J}$ если Hor-sign совп. с chirality sign of DW. Если sign $\vec{C}$ и Hor - разные, то DW движется вдоль $\vec{J}$, также чувствует reversal с DW движется вдоль $\vec{J}$ немного, также ослабевание. Видимо new BP-DW appearance от интерфейса BP-DW (движущегося против $\vec{J}$) какая роль $\vec{G}$-vector и v_{DW} для new BP-DW motion. Mass of BP-DW - Sonin 1989?

12/III Editing „SW edge scat." MS. С А. и Ю. в рес. „La Perla" и „Carrefour" Olaberria. Camarero прислал „Skyrmion" project final report. Окс. сем. Conf. 2019-20.

13-14/III Написание отчета по FIS 2018 - Camarero.

15/III С А. в Университете - Теме Caretta et al. Relativistic DW dyn. Science 2020.

16/III Обсуждение Caretta paper с Thes. Обсуждение FiM dynamics, Mn_2Au, paper by Caretta с B. Ivanov. Обсуждение Caretta' 2020 с A. Zvezdin.

17/III Теме Suppl Mat. к Caretta' 2020 Science и написание Comments. Com. Galkina JETP Lett 2019 max DW velocity FiM $v_c = \frac{c v_w}{\sqrt{c^2 + v_w^2}}$ DW FiM

$v_w = \frac{\sqrt{AK}}{\hbar(S_1+S_2)}\left[\sqrt{1+\lambda} - 1\right] = v_c(\text{FiM})$ $\lambda = \frac{K_x}{K_z}$ - modified Walker velocity,

$c^2 = \frac{A\omega_{ex}}{\hbar(S_1+S_2)}$ - „antiferromagnetic" velocity, or min $v_{ph}(k)$ for acoustic SW

$W_{ex} = \hbar\omega_{ex}\frac{1}{2}(S_1+S_2)\vec{m}^2 + \frac{A}{2}(\nabla\vec{l})^2$

When $S_1 = S_2$ angular compensation, $v_w \to \infty$, $v_w \gg c$ и $v_c = c$.

Вдали от $S_1 = S_2$ point, $v_w \ll c$ и $v_c = v_w$ - как в обычном ferromagnet.

Ferrimagnet → ferromagnet with χ_{eff}, d_{eff}.

Письмо to B. Ivanov Mn_2Au paper (совместно с Ruben обзвон и Caretta' 2020) и Comments to Caretta - Science.

18/III Замечания и письмо в Science Letter (comments к Caretta '2020 paper). Skype с Igor (Pawel edge SW). послал A. Bezginu Mn_2Au paper. Skype с Zyuzin: FiM dynamics и Caretta paper.

19/III Science Letter was rejected. Предложили e-Letter, Resubmitted, published. Skype с B. Ivanov: FiM dyn, Mn_2Au, спасибо за Simul. Ruben где v_{max} ? $f(J_2)$ невероятно. Domino effect ч-з SW в обе стороны, opt. и 2 acoustic, проблемы, предложили проверить SW - Mn_2Au in Arana 'APL' 2017.

20/III Proof APL - Nanowidenings. E-mail Turchuk o F/N/F vortex dyn. где ссылка Ref. Guo 'APL' 2005. Arxiv preprint '2020 by O. Gomonay (ypork '2020) - по exch. Skyrmion - SW solution. С А. в „Carrefour" Olaberria.

New preprint V. Turchuk „Double vortex dyn." E-mail to authors to cite Guo 'APL' 2005.

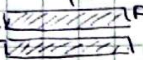
(21/III) План работ (Сн. 8/XI 2020)

1) AF DW dynamics paper Mn_2Au (Ruben, Ricardo)
2) ~~SW scattering at film edge (Pawel) subm. APR 20/V 21~~
3) Vortex gyrotropies thick dots/nanowires - manuscr. (Farkhad)
4) Berganza - dome-shaped dots theory paper - PRX? Nanoscale subm.
5) Skyrmion ω_G ferrimagnets - Less (FiM) - Зубков, Аналоги
6) ~~Abs. PM'21, Poznan, June 28 - July 2, 2021, on-line, abs. до 24/III~~ subm. 13/IV
7) Abs Nanowires/-tubes, Woischan feld, Germany, Aug. 30 - Sept. 3, 2021, abs. до 14 May
8) TMAG, Cefalu-Palermo, Sept. 6-10, 2021, abs. sent in 2020 (?), new deadline 28/V 2021
9) ~~SMM25, Grenoble, France, Sept. 13-16, 2021, abs. до Apr. 4, 2021 - postponed to May 2022~~
10) change AXA, new home insurance до Apr. 15
11) ~~Solsken dag 2021 - подробно, Session chairs, Zoom~~
12) justification FIS2016 project до 7/IV 2021 (Canarero)
13) Ikerbasque promotion report до 14/VI 21
14) ~~Cita Tax office в Бильбао, Lora 10-1 - 12/I ала, 11:00 (?)~~

(29/III) Взели на рец. Nano Letters Li et al. „SW in vortex networks"
Final report FIS2016. Вопросы о TMAG (Palermo) to Finocchio. Orc.-dip. interact.

(23/III) Скана прислала Berganza - theory MS (Google drive). Changed, edited the manuscript „3D magn. configurations in Py dots"

(24/III) Mn_2Au - layered AFM. Вернемся к Arana et al. APL'2017 - SW spectra Mn_2Au и Zvezdin-Kostyuchenko'99 - DW in artificial AFM.

• Zvezdin'99 рис. 

double layer FM film с AF ($J_1>0$) coupling between FM layers J_1, далее при $J_1 >> 4\pi M_s^2$ sine-Gordon рис. in-plane angle φ ($\vec{l}(\theta,\varphi)$, $\theta=\pi/2$) $\frac{1}{c^2}\ddot{\varphi} = \nabla^2\varphi + \frac{1}{2\Delta^2}\sin 2\varphi = 0$, $c = \frac{\gamma}{M_s}\sqrt{2AJ_1}$,

$$E = \sum_i \left[\frac{1}{2}A(\nabla\vec{m}_i)^2 + \frac{1}{2}K(m_i^y)^2 + 2\pi M_s^2 (m_i^z)^2\right] + \frac{1}{2}J_1\vec{m}_1\cdot\vec{m}_2$$

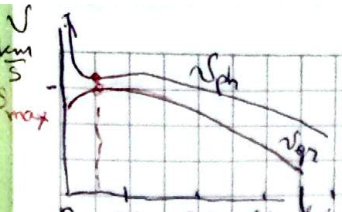
т.е. Zvezdin'99 DW motion eq. такое же как рис. AFM (weak FM), с $v_{DW}^{max} = c \sim \sqrt{AJ_1}$. Это странно, т.к. с определяется min v_{ph}(SW) рис. acoustic SW branch, которая от interlayer coupling не зависит (Gus, Gerlich papers по double-layer films, FMR). Написал вопрос to A.K. Звездин.

Ruben, Fig. 2b Mn_2Au manuscript: $\omega_{ac}(k)$, $\omega_{opt}(k)$. The lowest SW branch $\omega_{ac}(k)$ почти не имеет frequency gap $\omega_{ac}(k=0)\approx 0$. $\omega_{ac}(0)$ определяется маг. анизотропией in the basal plane: $K_2 \parallel (01\bar{1}0 \parallel 0y)$ и K_1, которые малы по сравн. с K_{21} (easy plane xy) и по сравнению с J_1, J_2, J_3 exchange integrals.

$v_{gr}(k) = \frac{d\omega}{dk} \approx \frac{\omega}{k} = v_{ph}$ (т.к. $\omega_{ac}(k)$ почти линейна). $v_{gr} \cong 2\pi\frac{\Delta f}{\Delta k} = 2\pi\frac{20\,THz}{0.5\,nm^{-1}} =$
$= 2\pi\cdot 40\,\frac{10^{12}\,1/s}{10^9\,1/m} = 2\pi\cdot 40\,\frac{km}{s}$. Почему Ruben дает $v_{gr}^{max} = 43\frac{km}{s}$? 2π - где?

Arana - Rezende 'APL'2017: BLS/Raman exp. дает при $k=0$ $\frac{\omega_1(0)}{2\pi} = 121\,GHz$ и $\frac{1}{2\pi}\omega_2(0) = 3.72\,THz$ (Сн. 24/VII, 27/VII 2020). Оценка Arana $v_{gr} \approx 36.6\,km/s$ по $\omega_1(k)$ simulated с J, D_1, D_2 parameters. В теории $\omega_{1,2}(k)$ вместо J_1, J_2, J_3 входит только один AFM integral J - intersublattice и z - nearest neighbors. $H_{ex} = 25\,zJ/g\mu_B$ только и фигурирует в theor. $\omega_{1,2}(k)$.

Если $H_{a_2}(D_2) << H_{a_1}(D_1)$, H_{a_2} пренебрежем, то $\omega_1^2(k) = H_{ex}\left[H_{a_1}(1-\gamma_k) + H_{ex}(1-\gamma_k^2)\right]$
$\gamma_k = \frac{1}{z}\sum_{\delta} e^{i\vec{k}\vec{\delta}}$ (N.N.). $H_{a_1} << H_{ex} \rightarrow \omega_1^2(k) = \gamma^2 H_{ex}^2(1-\gamma_k^2) \approx \frac{1}{2}\gamma H_{ex} ak$, откуда $v_{gr} \cong \frac{1}{2}\gamma H_{ex} a \sim J$ (как в s-model с одним J). Оценка $v_{gr} \approx 38.1\,km/s$ при $H_{ex} = 1.3\cdot 10^4\,kOe$ (!? giant).



$k_{BZ} = \frac{\pi}{a} = \frac{\pi}{0{,}33\ nm} \simeq 9{,}5\ nm^{-1}$, $v_{gr}(k)$ имеет max и/ли

$k_m \approx 2\pi \cdot 0{,}1 \simeq 0{,}6\ nm^{-1}$, $v_{gr}(max) \approx 43\ km/s$

$v_{ph}(k)$ имеет horizontal min и/ли $k \approx k_m$.

$\omega_1/2\pi\,(0) = 340\ GHz$, $\omega_2/2\pi\,(0) = 27{,}28\ THz$ Gaps at wavevector $k=0$ для Mn_2Au

M_S (Barthem'13) $= 780\ G$, $K_{211} = 7 \cdot 1{,}855 \cdot 10^{-25}\ J$. $\frac{\mu_{Mn}}{a^2 c} = \frac{4\mu_B}{a^2 c} = 391{,}7\ G$

2 f.u./unit cell → 4 Mn ions/unit cell, 2 Mn(→), 2 Mn(←) V_0

Rescaling M_S (ferromagn.) $= 2\frac{\mu_{Mn}}{V_0} = 783{,}4\ G$ близко к Barthem'13.

Com. 17/VIII 2020 $W_K = -\frac{4K_{211}}{V_0} l_y^2 = -\chi_y l_y^2$. uniaxial anisotropy constant in the basal xy-plane $\chi_y = 4 \cdot \frac{7 \cdot 1{,}855 \cdot 10^{-25}\ J}{V_0} = 4\frac{K_{211}}{V_0}$

$H_a = \frac{2\chi_y}{M_S} = 2\frac{4K_{211}}{V_0}\frac{1}{2\frac{\mu_{Mn}}{V_0}} = 4\frac{K_{211}}{\mu_{Mn}} = \frac{K_{211}}{\mu_B}$ ← anisotropy field in the basal plane.

$\mu_{Mn} = 4\mu_B$ $H_a = \frac{7 \cdot 1{,}855 \cdot 10^{-25}\ J}{9{,}274 \cdot 10^{-24}\ \frac{J}{T}} = \frac{7 \cdot 1{,}855}{9{,}274} 10^{-1}\ T = 1{,}40 \cdot 10^{-1}\ T = 1{,}40\ kOe$

Gap acoustic branch $\omega_1(0)/2\pi = (\gamma/2\pi)\, p\sqrt{H_{ex} H_a}$, p – пл. множитель $\sim 1\ (>1)$

$H_{ex} \simeq \frac{1}{H_a p^2}\left[\frac{f_1(0)}{(\gamma/2\pi)}\right]^2 = \frac{1}{p^2}\frac{1}{1{,}400\ kOe}\left[\frac{340\ GHz}{3\ GHz/kOe}\right]^2 = \frac{1}{p^2}\frac{1}{1{,}40\ kOe} \cdot 12{,}8 \cdot 10^3\ (kOe)^2 =$

$= \frac{1}{p^2} 9{,}17 \cdot 10^3\ kOe$. Arana 2017 дает $H_{ex} = 13{,}0 \cdot 10^3\ kOe$. Т.к. $p > 1$, мое / Ruben - $H_{ex} < H_{ex}$ (Arana'2017).

Если $\omega_1(0) = p\gamma\sqrt{H_{ex} H_a}$, $\omega_2(0) = p\gamma\sqrt{H_{ex} H_{a1}}$, $H_{a1} = \frac{2K_{21}}{M_S}$ easy plane anis. field, с одним и тем же H_{ex}, то $\frac{\omega_2^2(0)}{\omega_1^2(0)} = \frac{H_{ex(2)}}{H_{ex(1)}}\frac{H_{a1}}{H_a} = \frac{H_{a1}}{H_a}$.

$K_{21} = 1{,}303 \cdot 10^{-22}\ J$ $H_{a1} = 2\frac{\chi_1}{M_S}$, $\chi_1 = 4 \cdot \frac{K_{21}}{V_0}$, $H_{a1} = \frac{2}{M_S}\frac{4K_{21}}{V_0} = \frac{2}{\left(\frac{2\mu_{Mn}}{V_0}\right)}\frac{4K_{21}}{V_0} =$

$= \frac{4K_{21}}{\mu_{Mn}} = \frac{4K_{21}}{4\mu_B} = \frac{K_{21}}{\mu_B}$. $H_{a1} = \frac{K_{21}}{\mu_B} = \frac{1{,}303 \cdot 10^{-22}\ J}{9{,}274 \cdot 10^{-24}\ \frac{J}{T}} \simeq 0{,}141 \cdot 10^2\ T = 14{,}1\ T = 1410\ kOe$

$\frac{H_{a1}}{H_a} = \frac{141{,}0\ kOe}{1{,}40\ kOe} \approx 100{,}7$, $\left[\frac{\omega_2(0)}{\omega_1(0)}\right]^2 = \left[\frac{27{,}28\ THz}{0{,}340\ THz}\right]^2 = 6{,}438 \cdot 10^3$

H_{ex} (optic) $\neq H_{ex}$ (acoust), $\frac{H_{ex(2)}}{H_{ex(1)}} = \left[\frac{\omega_2(0)}{\omega_1(0)}\right]^2 \frac{H_a}{H_{a1}} = \frac{6{,}438 \cdot 10^3}{100{,}7} \simeq 63{,}9$ (!?)

+ а если брать и/ли $k=0$ gaps $\omega_n(0) = \gamma p\sqrt{H_{ex(n)} H_{a(n)}}$, $n=1,2$, $H_{a(1)} = H_a$, $H_{a(2)} = H_{a1}$,

$\frac{H_{ex(2)}}{H_{ex(1)}} \approx 64$ тогда тоже так! Int. J_2 входит в $H_{ex(2)}$, но не входит в $H_{ex(1)}$. Странно.

→ Выводы: 1) $f_{J_2}(0) = 27{,}28\ THz$ ← frequency gap оптической ветви. Чего бы неупругими мне за нее ухватиться J_2 в $\omega_2(k)$-optic branch.

2) $f_{J_2}(0) = 27{,}28\ THz$ противоречит exp. Arana'2017, где $f_2(0) = 3{,}72\ THz$.

Написал e-mail с этими выводами to Ruben/Ricardo.

(25/III) С М. в "Hôtel" Oleberria. Arana'2017 дает g_{eff}, referred to S. Rezende PRB 93, 054412 (2016)

Rezende рассм. SW in cubic f.c.c. NiO AFM

$E = 2\sum_{ij} J_{ij}\vec{S}_i\vec{S}_j + \sum_i D_x (S_i^x)^2 + \sum_i D_y (S_i^y)^2$

Ox – hard axis

Spins in (-111) plane

Spins of other sublattices are oppositely aligned in neighboring (111)-planes. Layered AFM, 2 sublattices. J – is AFM intersublattice exchange, ось z – sublattice exch. interac. не учитывается (Ono не учитывал)

$\delta_{\vec{k}} = \frac{z}{2}\frac{J}{S}e^{i\vec{k}\vec{\delta}}$, δ – сдвиги to nearest neighbors из разных (111)-planes

и/ли $k=0$, $\omega_\alpha(0) = \gamma(2H_{ex} H_{ax})^{1/2}$, $\omega_\beta(0) = \gamma(H_{ex} H_{ay})^{1/2}$, $H_{ax,y} = 2\delta_{x,y}/d^2$

Escaneado con CamScanner

Ночью e-mail to Ricardo/Ruben о величине v_{max}. Ок PhD student advertisement by Ruben (3D magnetism, analyt. theory + simulations).

19/IV Переписка с Самарего о FIS2016 Informe final. СП. в "Lidl" Hernani.

20/IV Прочел Sec. III theory framework MnzAu article. Сделал, что при $J_2=0$ оптич SW branch $\omega_2(k)$ совпадает с acoustic branch $\omega_1(k)$, $\omega_2(k)=\omega_1(k)$. ω_1 не зависит от $J_2 \Rightarrow v_{max}$, которая определяется как $v_{max}=\max\left(\frac{d\omega_1(k)}{dk}\right)$ не зависит от J_2, хотя A (2-model) зависит от J_2. $v_{max} \sim \sqrt{aA}$. См. анализ SW $\omega_{1,2}(k)$ и сравнение с Arona 2017 от 24/III. Skype разговор с Игорем (Pawel edge SW MS и разное). Заполнил пункты filled in Secs. D, F, I. Закончил Informe final и послал в Osc. Skype с Игорем по SW scatter. послал Wang "Magn. frequency comb" article.

21/IV SolSkyMag - Conf. See. Закончил Secs. J, K и послал to Cristina, UPV/EHU.

22/IV СП. "Lidl" Olaberria и "Carrefour Market". Skype с Pawel, Maciej, Igor: SW edge scattering (учесть что Splitting $\xrightarrow{k} \xrightarrow{ms} \vec{k}$, x-edge SW, проходит на grove existing edge SW (ω_e, k) excited by antenna. Правки в SW-edge article.

23/IV Принял заказ. Купил очки Терраса, но, прочел известие о нашем Генеральном Spanish citizenship. Лёг спать в час. ночи. СП. Ю. в Ñam.

Cristina одобрила FIS2016 Informe final (minor revision).

24/IV СП. и Ю. в "La Perla" "La Concha" рестораны (но не обед). Отдых дома.

25/IV Ответил to Elena, Bankia, update conf. see table. Вышел.

26/IV Переписка с Cristina. Закончил FIS2016 Informe final и uploaded it to Ministerio web-site.

27/IV Cristina UPV/EHU: 1) Informe final подписано. 2) мне должны гонорары 2020 и Table Viajes 2017-2020 для отчета. Вышел.

28/IV Вторая Session Chairs (10 час.) SolSkyMag и послал им offers. 8 час. из 10 согласились. Jiadong Zang и Montefiz не ответили. СП. в "Eroski" Urbil.

29/IV СП. в "Lidl" Olaberria и "Carrefour Market". Skype session с Оксаной, Сергеем, Diego по SW-vortex state nanowires. Нужны simulations, хотя localization law-β modes ($L=250$ nm, $R=50$ nm) похожа на analyt. calcs. мои.

30/IV Сделал: ~~проверил~~ Angel о PhD; 2) отв. Cristina по Viajes-2017-20; 3) ~~horarios~~ 2020 и correct horarios 2019 – e-mail Вопросы к Cristina. Пос. Кнеб 1.D. 8kg, 17€.

1/V На Пасх. службу в San Sebastian (номеров. 35€ черкнул)

2/V Проверил tax Propuesta. Возврат 2,900€, а не 3,500€ из-за ASEPEO перечисл. 3,100€, которые посчитали как 3/IV. 2020. Принял tax Propuesta on-line. Праздничный обед дома.

3/V Закончил Partes horarios FIS2016 project, две - 2019, 2020, Julian - 2019 и послал to Cristina, UPV/EHU.

4/V Changes to edge-SW manuscript with Pawel, Invit. Letter for M. Zelant for an internship San Sebastian in Sept.-Dec. 2021. СП. в "Lidl" Hernani. Оборудование, мебель, металлич. AF, JAP2020 и Rezende, AF magnons, JAP 2019.

5/V В университет, копия old Travel reports. Папка "Viajes" по FIS2016 project

6/V В унив. copied old Travel reports. Закончил и послал отчет "Viajes" FIS 2016 to Cristina. СП. Ю. в "Mazda" Gros и Ondarreta "Ñam."

7/V Online conf. Peta spin-Finocchio (Demokritov) и ответил 2nd referee Rivera SK-barriers MS Phys. Rev. Appl.

8/V СП. и Ю. в ресторане Zarautz. СП. в "Carrefour Market".

Вопрос и послал to Ricardo/Ruben Abs. Sec. I, Sec. II of MnzAu manuscript. Skype с А.К. Zvezdin: Jon сказал проверить Eq. (2) of MnzAu Paper – может быть $\frac{d}{dt}\left(\frac{1}{\Delta}\dot{q}\right)$, а не $\frac{1}{\Delta}\ddot{q}$ и Eq. (6) для $\varphi(z,t)$, нет ли там h-terms?

9/V Прочел Согласился на Zvezdin – 1979, где Eq. (6) с damping впервые получено

(9/5) Исследуем Mn_2Au SW spectrum vector для one case Mn_2Au
Граничные: thin film thickness = unit cell $k_x = \infty$, $k_y = \infty$ (P.B.C.)
Решение в рамках 18/пп, $E_{cr}(\vec{m}_i)$, учитываем M_4 layer - нумерация для обращения с матрицей Mn_2Au магнитные, Fig.1. Au can regularly ignore сверху.
Ряды 1, 2, 3, 4.

$E_{cr}(k=0)$ / u.c. $= -\sum_{ij} J_{ij}\vec{m}_i\cdot\vec{m}_j = -\vec{m}_1\left[J_3\vec{m}_1(0x)\,2\cdot\frac{1}{2} + J_3\vec{m}_4(0y)\,2\cdot\frac{1}{2}\right] -$
$-\vec{m}_1\left[4J_1\vec{m}_2\right] - \vec{m}_2\left[J_3\vec{m}_2(0x)\,2\cdot\frac{1}{2} + J_3\vec{m}_2(0y)\,2\cdot\frac{1}{2}\right] - \vec{m}_2\cdot\left[J_2\vec{m}_3\cdot 4\frac{1}{4}\right] -$
$-\vec{m}_3\left[J_3\vec{m}_3(0x)\,2\cdot\frac{1}{2} + J_3\vec{m}_3(0y)\,2\cdot\frac{1}{2}\right] - \vec{m}_3\cdot\left[J_1\vec{m}_4\cdot 4\right] - \vec{m}_4\cdot\left[J_3\vec{m}_4(0x)2\frac{1}{2} + J_3\vec{m}_4(0y)\,2\frac{1}{2}\right]$

$E_{cr}(k=0)$ / u.c. $= -2J_3\vec{m}_1\cdot\vec{m}_1 - 4J_1\vec{m}_1\cdot\vec{m}_2 - 2J_3\vec{m}_2\cdot\vec{m}_2 - J_2\vec{m}_2\vec{m}_3 -$
$-2J_3\vec{m}_3\cdot\vec{m}_3 - 4J_1\vec{m}_3\cdot\vec{m}_4 - 2J_3\vec{m}_4\cdot\vec{m}_4$ SW (FMR) $\vec{m}_i = \vec{\mu}_i + \delta\vec{m}_i$ (1)
$\vec{m}_i\cdot\vec{m}_i = (\vec{\mu}_i + \delta\vec{m}_i)^2 = \mu_i^2 + \vec{\mu}_i\cdot\delta\vec{m}_i + \delta m_i^2 = \mu_i^2 + O(\delta m_i^2)$ (1) $|\vec{\mu}_i| = 1$

$E_{cr}(k=0)$ / u.c. $= -4J_1\vec{m}_1\cdot\vec{m}_2 - J_2\vec{m}_2\cdot\vec{m}_3 - 4J_1\vec{m}_3\cdot\vec{m}_4$ (J_3 из-за easy FMR-плоскости)

E_{an}/u.c. $= K_2\sum(\vec{m}_i\cdot\hat{z})^2 - K_y\sum(\vec{m}_i\cdot\hat{y})^2 = K_2\left[(m_1^z)^2 + (m_4^z)^2 + 4\cdot\frac{1}{4}(m_2^z)^2 + 4\cdot\frac{1}{4}(m_3^z)^2\right] -$
$-K_y\left[(m_1^y)^2 + (m_4^y)^2 + 4\cdot\frac{1}{4}(m_2^y)^2 + 4\frac{1}{4}(m_3^y)^2\right] = K_2\sum(m_i^z)^2 - K_y\sum(m_i^y)^2$ -
анизотропия easy plane yOx + easy axis Oy (equilibrium direction i - Oy).

LL eq. $-\frac{1}{\gamma}\dot{\vec{m}}_i = \vec{m}_i\times\vec{H}_{i\,eff}$, $\vec{H}^{(i)}_{eff} = -\frac{1}{M_s}\frac{\delta E}{\delta\vec{m}_i}$.

$E/u.c. = -4J_1\vec{m}_1\cdot\vec{m}_2 - J_2\vec{m}_2\cdot\vec{m}_3 - 4J_1\vec{m}_3\cdot\vec{m}_4 + K_2\sum_i(m_i^z)^2 - K_y(m_i^y)^2$, $K_2>0$, $K_y>0$ thin film

$M_s\vec{H}^{(1)}_{eff} = -\frac{\delta E}{\delta\vec{m}_1} = 4J_1\vec{m}_2 - 2K_2m_1^z\hat{z} + 2K_ym_1^y\hat{y}$;
$M_s\vec{H}^{(2)}_{eff} = -\frac{\delta E}{\delta\vec{m}_2} = 4J_1\vec{m}_1 + J_2\vec{m}_3 - 2K_2m_2^z\hat{z} + 2K_ym_2^y\hat{y}$
$M_s\vec{H}^{(3)}_{eff} = -\frac{\delta E}{\delta\vec{m}_3} = J_2\vec{m}_2 + 4J_1\vec{m}_4 - 2K_2m_3^z\hat{z} + 2K_ym_3^y\hat{y}$
$M_s\vec{H}^{(4)}_{eff} = -\frac{\delta E}{\delta\vec{m}_4} = 4J_1\vec{m}_3 - 2K_2m_4^z\hat{z} + 2K_ym_4^y\hat{y}$

$-\frac{M_s}{\gamma}\dot{\vec{m}}_1 = \vec{m}_1\times\left[4J_1\vec{m}_2 - 2K_2m_1^z\hat{z} + 2K_ym_1^y\hat{y}\right]$, $-\frac{M_s}{\gamma}\dot{\vec{m}}_2 = \vec{m}_2\times\left[4J_1\vec{m}_1 + J_2\vec{m}_3 - 2K_2m_2^z\hat{z} + 2K_ym_2^y\hat{y}\right]$
$-\frac{M_s}{\gamma}\dot{\vec{m}}_3 = \vec{m}_3\times\left[J_2\vec{m}_2 + 4J_1\vec{m}_4 - 2K_2m_3^z\hat{z} + 2K_ym_3^y\hat{y}\right]$, $-\frac{M_s}{\gamma}\dot{\vec{m}}_4 = \vec{m}_4\times\left[4J_1\vec{m}_3 - 2K_2m_4^z\hat{z} + 2K_ym_4^y\hat{y}\right]$

Без K_z, K_y: $-\frac{M_s}{\gamma}\dot{\vec{m}}_1 = 4J_1\vec{m}_1\times\vec{m}_2$, $-\frac{M_s}{\gamma}\dot{\vec{m}}_2 = \vec{m}_2\times\left[4J_1\vec{m}_1 + J_2\vec{m}_3\right]$
$-\frac{M_s}{\gamma}\dot{\vec{m}}_3 = \vec{m}_3\times\left[J_2\vec{m}_2 + 4J_1\vec{m}_4\right]$, $-\frac{M_s}{\gamma}\dot{\vec{m}}_4 = 4J_1\vec{m}_4\times\vec{m}_3$

$\vec{m}_{1,3} = \pm\hat{y} + \delta\vec{m}_{1,3}$, $\vec{m}_{2,4}(t) = -\pm\hat{y} + \delta\vec{m}_{2,4}(t)$, AF ground state $\vec{m}_1 = \vec{m}_3 = \pm\hat{y}$, $\vec{m}_2 = \vec{m}_4 = \mp\hat{y}$
$\mu = 1$ in reduced units $\vec{m}_i^2 = 1$.
$\vec{m}_1\times\vec{m}_2 = (\hat{y} + \delta\vec{m}_1)\times(-\hat{y} + \delta\vec{m}_2) = \hat{y}\times\delta\vec{m}_2 + \hat{y}\times\delta\vec{m}_1$, $\delta\vec{m}_i = (v_i^x, 0, v_i^z) = \vec{v}_i$
$\vec{m}_2\times\vec{m}_3 = (-\hat{y} + \delta\vec{m}_2)\times(\hat{y} + \delta\vec{m}_3) = -\hat{y}\times\delta\vec{m}_3 - \hat{y}\times\delta\vec{m}_2$, $\vec{m}_3\times\vec{m}_4 = \hat{y} + \delta\vec{m}_4 + \hat{y} + \delta\vec{m}_3$

$\hat{y}\times\delta\vec{m}_i = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 0 & 1 & 0\\ v_i^x & 0 & v_i^z\end{vmatrix} = \hat{x}v_i^z - \hat{z}v_i^x$.

$-\frac{M_s}{\gamma}\dot{\vec{v}}_1 = 4J_1\left[\hat{x}v_2^z - \hat{z}v_2^x + \hat{x}v_1^z - \hat{z}v_1^x\right]$, $-\frac{M_s}{\gamma}\dot{\vec{v}}_4 = 4J_1(-1)\left[\hat{x}v_4^z - \hat{z}v_4^x + \hat{x}v_3^z - \hat{z}v_3^x\right]$
$-\frac{M_s}{\gamma}\dot{\vec{v}}_2 = -4J_1\left[\hat{x}v_2^z - \hat{z}v_2^x + \hat{x}v_1^z - \hat{z}v_1^x\right] + J_2\left[\hat{x}v_3^z - \hat{z}v_3^x + \hat{x}v_2^z - \hat{z}v_2^x\right](-1)$
$-\frac{M_s}{\gamma}\dot{\vec{v}}_3 = -J_2\left[\hat{x}v_3^z - \hat{z}v_3^x + \hat{x}v_2^z - \hat{z}v_2^x\right](-1) + 4J_1\left[\hat{x}v_4^z - \hat{z}v_4^x + \hat{x}v_3^z - \hat{z}v_3^x\right]$

Получили систему однородных ур-й для $\vec{v}_i = \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$, $\vec{v}_i = (v_i^x, 0, v_i^z)$
$M_s/\gamma = 8$.

Escaneado con CamScanner

$-g\dot{v}_1^x = 4J_1(v_2^z + v_1^z)$, $-g\dot{v}_1^z = 4J_1(v_2^x + v_1^x)(-1)$, $g\dot{v}_4^x = 4J_1(v_4^z + v_3^z)$, $g\dot{v}_4^z = (v_4^x + v_3^x)4J_1(-1)$

$-g\dot{v}_2^x = -4J_1(v_2^z + v_1^z) - J_2(v_3^z + v_2^z)$, $-g\dot{v}_2^z = 4J_1(v_2^x + v_1^x) + J_2(v_3^x + v_2^x)$

$-g\dot{v}_3^x = J_2(v_3^z + v_2^z) + 4J_1(v_4^z + v_3^z)$, $-g\dot{v}_3^z = -J_2(v_3^x + v_2^x) - 4J_1(v_4^x + v_3^x)$

Обозначим $\vec{n}(v_1^x, v_1^z, v_2^x, v_2^z, v_3^x, v_3^z, v_4^x, v_4^z)$ – 8-мерный вектор.

$\frac{d}{dt} v_i^{x,z} = \frac{d}{dt} v_i^{x,z}(0) e^{i\omega t} = i\omega v_i^{x,z}$. Тогда $g\omega = \omega \cdot -i\omega \vec{n} = A\vec{n}$.

Сек. ур. $\det(A_{nn'} + i\omega\delta_{nn'}) = 0$. $+i\lambda = i\omega$ $4J_1 = J$, $J + J_2 = J_+$

$$\begin{pmatrix} \lambda & J & . & . & . & . & . & . \\ -J & \lambda & -J & . & . & . & . & . \\ . & -J & \lambda & -J-J_2 & . & -J_2 & . & . \\ J & . & J+J_2 & \lambda & J_2 & . & . & . \\ . & . & . & J_2 & \lambda & J+J_2 & . & J \\ . & . & -J_2 & . & -J-J_2 & \lambda & -J & . \\ . & . & . & . & . & J & -\lambda & J \\ . & . & . & . & -J & . & -J & -\lambda \end{pmatrix} \begin{pmatrix} v_1^x \\ v_1^z \\ v_2^x \\ v_2^z \\ v_3^x \\ v_3^z \\ v_4^x \\ v_4^z \end{pmatrix} = 0 \;,\quad \det(A_{nn'} + \lambda\delta_{nn'}) = 0$$

$1x\ 1z\ 2x\ 2z\ 3x\ 3z\ 4x\ 4z$

Закон. $\dot{\vec{m}}_1 + \dot{\vec{m}}_2 + \dot{\vec{m}}_3 + \dot{\vec{m}}_4 = 0$
в exch. approximation.

$$\det(\hat{A} + \lambda I) = \lambda \begin{vmatrix} J & 0 & J & 0 & 0 & 0 & 0 \\ \lambda & -J & 0 & 0 & 0 & 0 & 0 \\ -J & \lambda & -J-J_2 & 0 & -J_2 & 0 & 0 \\ 0 & J+J_2 & \lambda & J_2 & 0 & 0 & 0 \\ 0 & 0 & J_2 & \lambda & J_+ & 0 & J \\ 0 & -J_2 & 0 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & 0 & 0 & J & -\lambda & J \\ 0 & 0 & 0 & -J & 0 & -J & -\lambda \end{vmatrix} + \ldots = \lambda \begin{vmatrix} \lambda & -J & 0 & 0 & 0 & 0 & 0 \\ -J & \lambda & -J-J_2 & 0 & -J_2 & 0 & 0 \\ 0 & J+J_2 & \lambda & J_2 & 0 & 0 & 0 \\ 0 & 0 & J_2 & \lambda & J_+ & 0 & J \\ 0 & -J_2 & 0 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & 0 & 0 & J & -\lambda & J \\ 0 & 0 & 0 & -J & 0 & -J & -\lambda \end{vmatrix} +$$

$$+(-1)J \begin{vmatrix} -J & -J & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & -J_+ & 0 & -J_2 & 0 & 0 \\ J & J_+ & \lambda & J_2 & 0 & 0 & 0 \\ 0 & 0 & J_2 & \lambda & J_+ & 0 & J \\ 0 & -J_2 & 0 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & 0 & 0 & J & -\lambda & J \\ 0 & 0 & 0 & -J & 0 & -J & -\lambda \end{vmatrix} + (-1)J \begin{vmatrix} -J & \lambda & -J & 0 & 0 & 0 & 0 \\ 0 & -J & \lambda & 0 & -J_2 & 0 & 0 \\ J & 0 & J_+ & J_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & J_+ & 0 & J \\ 0 & 0 & -J_2 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & 0 & 0 & J & -\lambda & J \\ 0 & 0 & 0 & -J & 0 & -J & -\lambda \end{vmatrix} =$$

$$= \lambda^2 \begin{vmatrix} \lambda & -J_+ & 0 & -J_2 & 0 & 0 \\ J_+ & \lambda & J_2 & 0 & 0 & 0 \\ 0 & J_2 & \lambda & J_+ & 0 & J \\ -J_2 & 0 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & 0 & J & -\lambda & J \\ 0 & 0 & -J & 0 & -J & -\lambda \end{vmatrix} + \lambda J(-1) \begin{vmatrix} -J & -J_+ & 0 & -J_2 & 0 & 0 \\ 0 & \lambda & J_2 & 0 & 0 & 0 \\ 0 & J_2 & \lambda & J_+ & 0 & J \\ 0 & 0 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & J & 0 & -\lambda & J \\ 0 & 0 & -J & 0 & -J & -\lambda \end{vmatrix} +$$

$$+(-1)J \begin{vmatrix} -J & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & -J_+ & 0 & -J_2 & 0 \\ J & J_2 & \lambda & J_2 & 0 & 0 \\ 0 & 0 & J_2 & \lambda & J_+ & J \\ 0 & -J_2 & 0 & -J_+ & \lambda & -J \\ 0 & 0 & 0 & J & -\lambda & J \\ 0 & 0 & -J & 0 & -J & -\lambda \end{vmatrix} + (-1)J \begin{vmatrix} -J & \lambda & -J & 0 & 0 & 0 & 0 \\ 0 & -J & \lambda & 0 & -J_2 & 0 & 0 \\ 0 & 0 & \lambda & J_2 & J_2 & 0 & 0 \\ 0 & 0 & 0 & \lambda & J_+ & 0 & J \\ 0 & 0 & -J_2 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & 0 & 0 & J & -\lambda & J \\ 0 & 0 & 0 & -J & 0 & -J & -\lambda \end{vmatrix}$$

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$$\det(\hat{A}+\lambda\hat{I}) = \lambda^2 D_2 + \lambda J^2 \begin{vmatrix} \lambda & J_2 & 0 & 0 & 0 \\ J_2 & \lambda & J_+ & 0 & J \\ 0 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & J & -\lambda & J \\ 0 & -J & 0 & -J & -\lambda \end{vmatrix} + J^2 \begin{vmatrix} \lambda & -J_+ & 0 & -J_2 & 0 & 0 \\ J_2 & \lambda & J_2 & 0 & 0 & 0 \\ 0 & J_2 & \lambda & J_+ & 0 & J \\ -J_2 & 0 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & 0 & J & -\lambda & J \\ 0 & 0 & -J & 0 & -J & -\lambda \end{vmatrix} + J^2 \begin{vmatrix} -J & \lambda & 0 & -J_2 & 0 & 0 \\ \lambda & J_2 & J_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda & J_+ & 0 & J \\ 0 & -J_2 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & 0 & J & -\lambda & J \\ 0 & 0 & -J & 0 & -J & -\lambda \end{vmatrix} + d_2$$

$$D(\lambda J^2) = \begin{vmatrix} \lambda & J_2 & 0 & 0 & 0 \\ J_2 & \lambda & J_+ & 0 & J \\ 0 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & J & -\lambda & J \\ 0 & -J & 0 & -J & -\lambda \end{vmatrix} = \lambda \begin{vmatrix} \lambda & J_+ & 0 & J \\ -J_+ & \lambda & -J & 0 \\ 0 & J & -\lambda & J \\ -J & 0 & -J & -\lambda \end{vmatrix} -$$

$$- J_2 \begin{vmatrix} J_2 & J_+ & 0 & J \\ 0 & \lambda & -J & 0 \\ 0 & J & -\lambda & J \\ 0 & 0 & -J & -\lambda \end{vmatrix} = \lambda^2 \begin{vmatrix} \lambda & -J & 0 \\ J & -\lambda & J \\ 0 & -J & -\lambda \end{vmatrix} - \lambda J_+ \begin{vmatrix} -J_+ & -J & 0 \\ 0 & -\lambda & J \\ -J & -J & -\lambda \end{vmatrix} - \lambda J \begin{vmatrix} -J_+ & \lambda & -J \\ 0 & J & -\lambda \\ -J & 0 & -J \end{vmatrix} -$$

$$- J_2^2 \begin{vmatrix} \lambda & -J & 0 \\ J & -\lambda & J \\ 0 & -J & -\lambda \end{vmatrix} = \lambda^2 \left[\lambda^3 - \lambda J^2 + \lambda J^2\right] - \lambda J_+ \left[-\lambda^2 J_+ + J^3 - J_+ J^2\right] - \lambda J \left[J^2 J_+ + \lambda^2 J - J^3\right] -$$

$$- J_2^2 \left[\lambda^3 - \lambda J^2 + \lambda J^2\right] = \lambda^5 - \lambda J_+ \left[-\lambda^2 J_+ - J_2 J^2\right] -$$

$$- \lambda J^2 \left[J J_+ + \lambda^2 - J^2\right] - \lambda^3 J_2^2 = \lambda^5 + \lambda J_+ (\lambda^2 J_+ + J_2 J^2) - \lambda J^2 (\lambda^2 + J_2 J) - \lambda^3 J_2^2 =$$

$$= \lambda^5 + \lambda^3 (J_+^2 - J^2 - J_2^2) + \lambda J^2 J_2 (J_+ - J) = \lambda^5 + 2 J J_2 \lambda^3 + \lambda J^2 J_2^2 = \lambda (\lambda^2 + J J_2)^2$$

$$\det(\hat{A}+\lambda\hat{I}) = \lambda^2 D_2 + \lambda^2 J^2 (\lambda^2 + J J_2)^2 + J^2 d_1 + J^2 d_2 \,, \quad D(\lambda J^2) = \lambda (\lambda^2 + J J_2)^2$$

$$D_2 = \begin{vmatrix} \lambda & -J_+ & 0 & -J_2 & 0 & 0 \\ J_+ & \lambda & J_2 & 0 & 0 & 0 \\ 0 & J_2 & \lambda & J_+ & 0 & J \\ -J_2 & 0 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & 0 & J & -\lambda & J \\ 0 & 0 & -J & 0 & -J & -\lambda \end{vmatrix} = \lambda \begin{vmatrix} \lambda & J_2 & 0 & 0 & 0 \\ J_2 & \lambda & J_+ & 0 & J \\ 0 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & J & -\lambda & J \\ 0 & -J & 0 & -J & -\lambda \end{vmatrix} - J_+ (-1) \begin{vmatrix} J_+ & J_2 & 0 & 0 & 0 \\ 0 & \lambda & J_+ & 0 & J \\ -J_2 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & J & -\lambda & J \\ 0 & -J & 0 & -J & -\lambda \end{vmatrix} -$$

$$- J_2 (-1) \begin{vmatrix} J_+ & \lambda & J_2 & 0 & 0 \\ 0 & J_2 & \lambda & 0 & J \\ -J_2 & 0 & -J_+ & -J & 0 \\ 0 & 0 & 0 & -\lambda & J \\ 0 & 0 & -J & -J & -\lambda \end{vmatrix} = \lambda \cdot D(\lambda J^2) + J_+^2 \begin{vmatrix} \lambda & J_+ & 0 & J \\ -J_+ & \lambda & -J & 0 \\ 0 & J & -\lambda & J \\ -J & 0 & -J & -\lambda \end{vmatrix} - J_+ J_2 \begin{vmatrix} 0 & J_+ & 0 & J \\ -J_2 & \lambda & -J & 0 \\ 0 & J & -\lambda & J \\ 0 & 0 & -J & -\lambda \end{vmatrix} =$$

$$= \lambda \cdot \lambda (\lambda^2 + J J_2)^2 + J_+^2 \left[\lambda^4 + J_+ (\lambda^2 J_+ + J_2 J^2) - J^2 (\lambda^2 + J J_2)\right] + J_+^2 J_2 \begin{vmatrix} -J_2 & -J & 0 \\ 0 & -\lambda & J \\ 0 & -J & -\lambda \end{vmatrix} +$$

$$+ J_+ J_2 J \begin{vmatrix} -J_2 & \lambda & -J \\ 0 & J & -\lambda \\ 0 & 0 & -J \end{vmatrix} = \lambda^2 (\lambda^2 + J J_2)^2 + J_+^2 \left[\lambda^4 + \lambda^2 (J_+^2 - J^2) + J_+ J_2 J^2 - J_2 J^3\right] +$$

$$+ J_+^2 J_2 (-J_2) (\lambda^2 + J^2) + J_+ J_2 J (-J_2) (-J^2) = \lambda^2 (\lambda^2 + J J_2)^2 + J_+^2 \left[\lambda^4 + \lambda^2 (J_+^2 - J^2) +\right.$$

$$\left. + J^2 J_2 (J_+ - J) - J_2^2 (\lambda^2 + J^2)\right] + J_+ J_2^2 J^3 = \lambda^2 (\lambda^2 + J J_2)^2 + J_+^2 \left[\lambda^4 + \lambda^2 (J_+^2 - J^2 - J_2^2) +\right.$$

$$\left. + J^2 J_2^2 - J_2^2 J^2\right] + J_+ J_2^2 J^3 = \lambda^2 (\lambda^2 + J J_2)^2 + J_+^2 \lambda^2 \left[\lambda^2 + 2 J J_2\right] + J_+ J_2^2 J^3$$

$$d_1 = \begin{vmatrix} \lambda & -J_+ & 0 & -J_2 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix} = \lambda \begin{vmatrix} \lambda & J_2 & 0 & 0 & 0 \\ J_2 & \lambda & J_+ & 0 & J \\ 0 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & J & -\lambda & J \\ 0 & -J & 0 & -J & -\lambda \end{vmatrix} + J_+ \begin{vmatrix} J_2 & J_2 & 0 & 0 & 0 \\ 0 & \lambda & J_+ & 0 & J \\ -J_2 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & J & -\lambda & J \\ 0 & -J & 0 & -J & -\lambda \end{vmatrix} +$$

$$+ J_2 \begin{vmatrix} J_2 & \lambda & J_2 & 0 & 0 \\ 0 & J_2 & \lambda & 0 & J \\ -J_2 & 0 & -J_+ & -J & 0 \\ 0 & 0 & 0 & -\lambda & J \\ 0 & 0 & -J & -J & -\lambda \end{vmatrix} = \lambda D(\lambda J^2) + J_+ \begin{vmatrix} J_2 & J_2 & 0 & 0 & 0 \\ 0 & \lambda & J_+ & 0 & J \\ 0 & -J & \lambda & -J & 0 \\ 0 & 0 & J & -\lambda & J \\ 0 & -J & 0 & -J & -\lambda \end{vmatrix} + J_2 \begin{vmatrix} J_2 & \lambda & J_2 & 0 & 0 \\ 0 & J_2 & \lambda & 0 & J \\ 0 & \lambda & -J & -J & 0 \\ 0 & 0 & 0 & -\lambda & -J \\ 0 & 0 & -J & -J & -\lambda \end{vmatrix} =$$

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$$= \lambda^2(\lambda^2+JJ_2)^2 + J_+J_2\begin{vmatrix} \lambda & J_+ & 0 & J \\ -J & \lambda & -J & 0 \\ 0 & J & -\lambda & J \\ -J & 0 & -J & \lambda \end{vmatrix} + J_2^2\begin{vmatrix} J_2 & \lambda & 0 & J \\ \lambda & -J & -J & 0 \\ 0 & 0 & -\lambda & J \\ 0 & -J & -J & -\lambda \end{vmatrix} = \lambda^2(\lambda^2+JJ_2)^2 +$$

$$+ J_+J_2\lambda\begin{vmatrix} \lambda & -J & 0 \\ J & -\lambda & J \\ 0 & -J & \lambda \end{vmatrix} - J_+^2J_2\begin{vmatrix} -J & -J & 0 \\ 0 & -\lambda & J \\ -J & -J & \lambda \end{vmatrix} - J_+J_2J\begin{vmatrix} -J & \lambda & -J \\ 0 & J & -\lambda \\ -J & 0 & -J \end{vmatrix} + J^2\begin{vmatrix} -J & -J & 0 \\ 0 & -\lambda & -J \\ -J & -J & -\lambda \end{vmatrix}$$

$$- \lambda J_2^2\begin{vmatrix} \lambda & -J & 0 \\ 0 & -\lambda & -J \\ 0 & J & -\lambda \end{vmatrix} - JJ_2^2\begin{vmatrix} \lambda & -J & -J \\ 0 & 0 & -\lambda \\ 0 & J & -J \end{vmatrix} = \lambda^2(\lambda^2+JJ_2)^2 + J_+J_2\lambda\cdot\lambda^3 - J_+^2J_2\begin{vmatrix} 0 & 0 & -\lambda \\ 0 & \lambda & J \\ J & -J & -\lambda \end{vmatrix} -$$

$$- J_+J_2J\begin{vmatrix} 0 & \lambda & 0 \\ 0 & J & -\lambda \\ -J & 0 & -J \end{vmatrix} + J_2^3\begin{vmatrix} 0 & 0 & \lambda \\ 0 & -\lambda & J \\ -J & -J & -\lambda \end{vmatrix} = \lambda^2J_2^2(\lambda^2-J^2) - JJ_2^2\lambda(-J)\lambda = \lambda^2(\lambda^2+JJ_2)^2 +$$

$$+ \lambda^4J_+J_2 + \lambda J_+^2J_2(-J)\lambda + \lambda J_+J_2J(-J\lambda) + J_2^3\lambda(-\lambda J) - \lambda^2J_2^2(\lambda^2-J^2) + \lambda^2J^2J_2^2 =$$

$$= \lambda^2(\lambda^2+JJ_2)^2 + \lambda^4J_+J_2 - \lambda^2J_+^2J_2J - \lambda^2J_+J_2J^2 + \lambda J_2^3(-\lambda J) - \lambda^2J_2^2(\lambda^2-J^2) + \lambda^2J^2J_2^2 =$$

$$d_1 = \lambda^2(\lambda^2+JJ_2)^2 + \lambda^4J_+J_2 - \lambda^2J_+^2J_2J - \lambda^2J_+J_2J^2 - \lambda^2J_2^3J - \quad (= d_1)$$

$$- \lambda^2J_2^2(\lambda^2-J^2) + \lambda^2J^2J_2^2 = \lambda^2(\lambda^2+JJ_2)^2 + \lambda^4J_+J_2 - \lambda^4J_2^2 + 2\lambda^2J_2^2J^2 -$$

$$- \lambda^2J_+^2J_2J - \lambda^2J_+J_2J^2 - \lambda^2J_2^3J = \lambda^2(\lambda^2+JJ_2)^2 + \lambda^4J_2J + \lambda^2J_2J \times$$

$$\times\left[2J_2J - J_+^2 - J_+J - J_2^2\right] = \lambda^2(\lambda^2+JJ_2)^2 + \lambda^4J_2J - \lambda^2J_2J(2J^2+2J_2^2+JJ_2)$$

Having d_2. Term $\lambda^0 = \det(\hat{A})$. $J=4J_1$, $J_+ = J+J_2$. Without λ^2-term.

10/V $d_2 = \begin{vmatrix} -J & \lambda & 0 & -J_2 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix} = (-J)\begin{vmatrix} J_2 & J_2 & 0 & 0 & 0 \\ 0 & \lambda & J_+ & 0 & J \\ -J_2 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & J & -\lambda & J \\ 0 & -J & 0 & -J & \lambda \end{vmatrix} + (-\lambda)\begin{vmatrix} -J & 0 & -J_2 & 0 & 0 \\ 0 & \lambda & J_+ & 0 & J \\ 0 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & J & -\lambda & J \\ 0 & -J & 0 & -J & -\lambda \end{vmatrix} =$

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$$= (-J)\begin{vmatrix} 0 & J_2 & 0 & 0 & 0 \\ -\lambda & \lambda & J_+ & 0 & J \\ J & -J_+ & \lambda & -J & 0 \\ 0 & 0 & J & -\lambda & J \\ J & -J & 0 & -J & -\lambda \end{vmatrix} + (-\lambda)|\cdots| = JJ_2\begin{vmatrix} -\lambda & J_+ & 0 & J \\ J & \lambda & -J & 0 \\ 0 & J & -\lambda & J \\ J & 0 & -J & -\lambda \end{vmatrix} + \lambda J\begin{vmatrix} \lambda & J_+ & 0 & J \\ -J_+ & \lambda & -J & 0 \\ 0 & J & -\lambda & J \\ -J & 0 & -J & -\lambda \end{vmatrix} =$$

$$= JJ_2\begin{vmatrix} -\lambda & J_+ & 0 & J \\ 0 & \lambda & -J & 0 \\ -\lambda & J & -\lambda & J \\ 0 & 0 & -J & -\lambda \end{vmatrix} + \lambda J\left[\lambda^4 + J_+(\lambda^2J_+ + J_2J^2) - J^2(\lambda^2+JJ_2)\right] =$$

$$= JJ_2(-\lambda)\begin{vmatrix} \lambda & -J & 0 \\ J & -\lambda & J \\ 0 & -J & -\lambda \end{vmatrix} - \lambda JJ_2\begin{vmatrix} J_+ & 0 & J \\ \lambda & -J & 0 \\ 0 & -J & -\lambda \end{vmatrix} + \lambda J[\cdots] = -\lambda JJ_2\left[\lambda^3 - \lambda J^2 + \lambda J^2 + \right.$$

$$\left. + \lambda JJ_+ - \lambda J^2\right] + \lambda J\left[\lambda^4 + \lambda^2(J_+^2 - J^2) + J_+J_2J^2 - J^3J_2\right] =$$

$$= -\lambda^2JJ_2\left[\lambda^2 + JJ_2\right] + \lambda J[\cdots]$$, коэффициент при $(-\lambda)$ в мат. d_2 неверен (нет минора)

$$d_2 = -\lambda^2JJ_2(\lambda^2+JJ_2) + (-\lambda)\begin{vmatrix} \lambda & 0 & -J_2 & 0 & 0 \\ 0 & \lambda & J_+ & 0 & J \\ -J_2 & -J_+ & \lambda & -J & 0 \\ 0 & 0 & J & -\lambda & J \\ 0 & -J & 0 & -J & -\lambda \end{vmatrix} = -\lambda^2JJ_2(\cdots) - \lambda^2\begin{vmatrix} \lambda & J_+ & 0 & J \\ -J_+ & \lambda & -J & 0 \\ 0 & J & -\lambda & J \\ -J & 0 & -J & -\lambda \end{vmatrix} +$$

$$+ \lambda J_2\begin{vmatrix} 0 & -J_2 & 0 & 0 \\ -J_+ & \lambda & -J & 0 \\ 0 & J & -\lambda & J \\ -J & 0 & -J & -\lambda \end{vmatrix}$$

$$d_2 = -\lambda^2JJ_2(\lambda^2+JJ_2) - \lambda^2\lambda\begin{vmatrix} \lambda & -J & 0 \\ J & -\lambda & J \\ 0 & -J & -\lambda \end{vmatrix} - \lambda^2(-J_+)\begin{vmatrix} -J_+ & -J & 0 \\ 0 & -\lambda & J \\ -J & -J & -\lambda \end{vmatrix} + \lambda^2J\begin{vmatrix} -J_+ & \lambda & -J \\ 0 & J & -J \\ -J & 0 & -J \end{vmatrix} +$$

$$+ \lambda J_2^2\begin{vmatrix} -J_+ & -J & 0 \\ 0 & -\lambda & J \\ -J & -J & -\lambda \end{vmatrix} = -\lambda^2JJ_2(\lambda^2+JJ_2) - \lambda^3\left[\lambda^3 - \lambda J^2 + \lambda J^2\right] + \lambda^2J_+\left[-\lambda^2J_+ + J^3 - J_+J^2\right] +$$

$$+ \lambda^2J\left[J_+J^2 + \lambda^2J - J^3\right] + \lambda J_2^2\left[-\lambda^2J_+ + J^3 - J^2J_+\right] = -\lambda^2JJ_2(\lambda^2+JJ_2) - \lambda^6 +$$

$$+ \lambda^2J_+\left[-\lambda^2J_+ - J_2J^2\right] + \lambda^2J^2\left[\lambda^2 + JJ_2\right] + \lambda J_2^2\left[-\lambda^2J_+ - J_2J^2\right]$$

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$$d_2 = -\lambda^6 - \lambda^2 JJ_2(\lambda^2 + JJ_2) - \lambda^2 J_+(\lambda^2 J_+ + J_2 J^2) + \lambda^2 J^2(\lambda^2 + JJ_2) - \lambda J_2^2(\lambda^2 J_+ + J_2 J^2)$$

Член в d_2 $\lambda J_2 |4\times4|$ неверен. Надо $\lambda J_2 \begin{vmatrix} 0 & \lambda & 0 & J \\ -J_2 & -J_+ & -J & 0 \\ 0 & 0 & -\lambda & J \\ 0 & -J & -J & -\lambda \end{vmatrix} = \lambda J_2^2 \begin{vmatrix} \lambda & 0 & J \\ 0 & -\lambda & J \\ -J & -J & -\lambda \end{vmatrix} =$

$= \lambda J_2^2 [\lambda^3 - \lambda J^2 + \lambda J^2] = \lambda^4 J_2^2$

$d_2 = -\lambda^6 - \lambda^2 JJ_2(\lambda^2 + JJ_2) - \lambda^2 J_+(\lambda^2 J_+ + J_2 J^2) + \lambda^2 J^2(\lambda^2 + JJ_2) + \lambda^4 J_2^2 =$

$= -\lambda^6 + \lambda^4[-JJ_2 - J_+^2 + J^2 + J_2^2] + \lambda^2[-J^2 J_2^2 - J^2 J_+ J_2 + J^3 J_2] =$

$= -\lambda^6 + \lambda^4[-JJ_2 - 2JJ_2] + \lambda^2 J^2 J_2[-J_2 - J_+ + J] = -\lambda^6 - 3JJ_2\lambda^4 - 2\lambda^2 J^2 J_2^2$

d_1 - неверно. $d_1 = \lambda^2(\lambda^2 + JJ_2)^2 + J_+ J_2 \begin{vmatrix} \lambda & J_+ & 0 & J \\ -J & \lambda & -J & 0 \\ 0 & J & -\lambda & J \\ -J & 0 & -J & -\lambda \end{vmatrix} + J_2^2 \begin{vmatrix} J_2 & \lambda & 0 & J \\ \lambda & -J & -J & 0 \\ 0 & 0 & -\lambda & -J \\ 0 & -J & -J & -\lambda \end{vmatrix} =$

$= \lambda^2(\dots) + J_+ J_2 \begin{vmatrix} \lambda & J_+ & 0 & J \\ -J & \lambda & -J & 0 \\ 0 & J & -\lambda & J \\ 0 & -\lambda & 0 & -\lambda \end{vmatrix} + J_2^2 \begin{vmatrix} J_2 & \lambda & 0 & J \\ \lambda & -J & -J & 0 \\ 0 & 0 & -\lambda & -J \\ -\lambda & 0 & 0 & -\lambda \end{vmatrix} =$

$= \lambda^2(\dots) + J_+ J_2 \begin{vmatrix} \lambda & J_2 & 0 & J \\ -J & \lambda & -J & 0 \\ 0 & 0 & -\lambda & J \\ 0 & 0 & 0 & -\lambda \end{vmatrix} + J_2^2 \begin{vmatrix} J_2 - J & \lambda & 0 & J \\ \lambda & -J & -J & 0 \\ J & 0 & -\lambda & -J \\ 0 & 0 & 0 & -\lambda \end{vmatrix} = \lambda^2(\dots) - \lambda J_+ J_2 \begin{vmatrix} \lambda & J_2 & 0 \\ -J & \lambda & -J \\ 0 & 0 & -\lambda \end{vmatrix} +$

$+ J_2^2(-\lambda) \begin{vmatrix} J_2 - J & \lambda & 0 \\ \lambda & -J & -J \\ J & 0 & -\lambda \end{vmatrix} = \lambda^2(\lambda^2 + JJ_2)^2 - \lambda J_+ J_2[-\lambda^3 - \lambda JJ_2] - \lambda J_2^2[\lambda J(J_2 - J) -$

$- \lambda J^2 + \lambda^3] = \lambda^2(\lambda^2 + JJ_2)^2 + \lambda^2 J_+ J_2(\lambda^2 + JJ_2) - \lambda^2 J_2^2[JJ_2 - J^2 - J^2 + \lambda^2] =$

$= \lambda^2(\lambda^2 + JJ_2)^2 + \lambda^4[J_+ J_2 - J_2^2] + \lambda^2[J_+ JJ_2^2 - J_2^2 J(J_2 - 2J)] =$

$= \lambda^2(\lambda^2 + JJ_2)^2 + \lambda^4 J J_2 + 3J^2 J_2^2 \lambda^2$, $d_1 = \lambda^2(\lambda^2 + JJ_2)^2 + JJ_2\lambda^4 + 3J^2 J_2^2 \lambda^2$

$\det(\hat{A} + \lambda\hat{I}) = \lambda^2[\lambda^2(\lambda^2 + JJ_2)^2 + J_+^2\lambda^2(\lambda^2 + 2JJ_2) + J_+ J^3 J_2^2] + \lambda^2 J^2(\lambda^2 + JJ_2)^2 +$

$+ J^2[\lambda^2(\lambda^2 + JJ_2)^2 + JJ_2\lambda^4 + 3J^2 J_2^2\lambda^2 - \lambda^6 - 3JJ_2\lambda^4 - 2\lambda^2 J^2 J_2^2]$

$\to \frac{1}{\lambda^2}\det(\hat{A} + \lambda\hat{I}) = \lambda^2(\lambda^2 + JJ_2)^2 + J_+^2\lambda^2(\lambda^2 + 2JJ_2) + J_+ J^3 J_2^2 + J^2(\lambda^2 + JJ_2)^2 +$

$+ J^2[\lambda^2(\lambda^2 + JJ_2)^2 - 2JJ_2\lambda^2 - \lambda^4 + J^2 J_2^2] = \lambda^2(\lambda^2 + JJ_2)^2 + J_+^2\lambda^2(\lambda^2 + 2JJ_2) +$

$+ J_+ J^3 J_2^2 + J^2(\lambda^2 + JJ_2)^2 + 2J^4 J_2^2$ (!) Неверно D_2. $\lambda^2 = 0$ всегда корень. $(\omega^2 = 0)$

При $J_2 = 0$, $J_+ = J$ $\frac{1}{\lambda^2}\det(\hat{A} + \lambda\hat{I}) = \lambda^6 + 2\lambda^4 J^2 = \lambda^4(\lambda^2 + 2J^2)$, есть 2-х кратный корень $\lambda^2 = 0$, и один ненулевой корень $\lambda^2 = -2J^2$, $\lambda = i\omega$, $\omega^2 = 2J^2$.

Цепь без Ean. Сколько корней сек. ур.? Ean учесть по ТВ.

Ясно, что при $J_2 = 0$ два корня $\omega_1 = 0$ и $\omega_2 = \sqrt{2}J$ для FMR ($E = 0$).

В расчете D_2 учтены $J_2 |5\times5|$-член и неверно для $J_+ J_2 |4\times4|$-член.

$D_2 = \lambda D(\lambda J^2) + J_+ |5\times5| + J_2 |5\times5| = \lambda D(\lambda J^2) + J_+^2 \begin{vmatrix} \lambda & J_+ & 0 & J \\ -J_+ & \lambda & -J & 0 \\ 0 & J & -\lambda & J \\ -J & 0 & -J & \lambda \end{vmatrix} - J_+ J_2 \begin{vmatrix} 0 & J_+ & 0 & J \\ -J_2 & \lambda & -J & J \\ 0 & J & -\lambda & 0 \\ 0 & 0 & -J & -\lambda \end{vmatrix} +$

$+ J_2 J_+ \begin{vmatrix} J_2 & \lambda & 0 & J \\ 0 & -J_+ & -J & 0 \\ 0 & 0 & -\lambda & J \\ 0 & -J & -J & -\lambda \end{vmatrix} - J_2^2 \begin{vmatrix} \lambda & J_2 & 0 & 0 \\ J_2 & \lambda & 0 & J \\ 0 & 0 & -\lambda & J \\ 0 & -J & -J & -\lambda \end{vmatrix} = \lambda D(\lambda J^2) + J_+^2[\lambda^4 + J_+(\lambda^2 J_+ + J_2 J^2) - J^2(\lambda^2 + JJ_2)] -$

$- J_+ J_2^2 \begin{vmatrix} J_+ & 0 & J \\ J & -\lambda & 0 \\ 0 & -J & -\lambda \end{vmatrix} + J_+ J_2^2 \begin{vmatrix} -J_+ & -J & 0 \\ 0 & -\lambda & J \\ -J & -J & -\lambda \end{vmatrix} - J_2^2 \lambda \begin{vmatrix} \lambda & 0 & J \\ 0 & -\lambda & J \\ -J & -J & -\lambda \end{vmatrix} + J_2^3 \begin{vmatrix} J_2 & 0 & J \\ 0 & -\lambda & J \\ 0 & -J & -\lambda \end{vmatrix} =$

$= \lambda D(\lambda J^2) + J_+^2[\lambda^4 + \lambda^2(J_+^2 - J^2) + J_+ J_2 - J^3 J_2] - J_+ J_2^2[\lambda^2 J_+ - J^3] + J_+ J_2^2[-\lambda^2 J_+ + J^3 -$

$- J_2^2\lambda[\lambda^3 - \lambda J^2 + \lambda J^2] + J_2^3[\lambda^2 J_2 + J^2 J_2] = \lambda D(\lambda J^2) + J_+^2[\lambda^4 + \lambda^2(J_+^2 - J^2) + J^2 J_2^2] + [-J^2 J_+] -$

$+ J_+ J_2^2[-2\lambda^2 J_+ + 2J^3 - J^2 J_+] - J_2^2\lambda^4 + J_2^4(\lambda^2 + J^2)$

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$$D_2 = \lambda^2(\lambda^2+JJ_2)^2 + J_+^2\left[\lambda^4 + \lambda^2(J_+^2-J^2) + J^2J_2^2\right] + J_+J_2^2\left[-2\lambda^2J_+ + 2J^3 - J^2J_+\right] - J_2^2\lambda^4 +$$

$$D_2 = \lambda^2(\lambda^2+JJ_2)^2 + \lambda^4(J_+^2-J_2^2) + \lambda^2\left[J_+^2(J_+^2-J^2) - 2J_+^2J_2^2 + J_2^4\right] +$$
$$+ J_+^2J^2J_2^2 + J_+J_2^2J^2(2J-J_+) + J_2^4J^2 = \lambda^2(\lambda^2+JJ_2)^2 + \lambda^4(J_+^2-J_2^2) +$$
$$+ \lambda^2\left[J_+^4 + J_2^4 - J_+^2J^2 - 2J_+^2J_2^2\right] + J^2J_2^2\left[J_+^2 + J_+(2J-J_+) + J_2^2\right]$$

$$D_2 = \lambda^2(\lambda^2+JJ_2)^2 + \lambda^4(J_+^2-J_2^2) + \lambda^2\left[J_2^2 + 2J_+J_2\right]J^2 + J^2J_2^2\left[2JJ_+ + J_2^2\right]$$

$$\det(\hat{A}+\lambda\hat{I}) = \lambda^2\Big[\lambda^2(\lambda^2+JJ_2)^2 + \lambda^4(J_+^2-J_2^2) + \lambda^2J^2J_2(J_2+2J_+) + J^2J_2^2(2JJ_+ + J_2^2)\Big] +$$
$$+\lambda^2J^2(\lambda^2+JJ_2)^2 + J^2\lambda^2\left[(\lambda^2+JJ_2)^2 - 2JJ_2\lambda^2 - \lambda^4 + J^2J_2^2\right]$$

$$\frac{1}{\lambda^2}\det(\hat{A}+\lambda\hat{I}) = \lambda^2(\lambda^2+JJ_2)^2 + \lambda^4(J_+^2-J_2^2) + \lambda^2J^2J_2(J_2+2J_+) + J^2J_2^2(2JJ_+ + J_2^2) +$$
$$+ J^2(\lambda^2+JJ_2)^2 + J^2\left[(\lambda^2+JJ_2)^2 - \lambda^4 - 2JJ_2\lambda^2 + J^2J_2^2\right] =$$
$$= \lambda^2(\lambda^2+JJ_2)^2 + \lambda^4J(J+2J_2) + \lambda^2J^2J_2(2J+3J_2) + J^2J_2^2(2J^2+2JJ_2+J_2^2) +$$
$$+ J^2(\lambda^2+JJ_2)^2 + 2J^4J_2^2 = \lambda^2\left[\lambda^4 + 2JJ_2\lambda^2 + J^2J_2^2\right] + \lambda^4J(J+2J_2) + \lambda^2J^2J_2(2J+3J_2) +$$
$$+ J^2\left(\lambda^4 + 2JJ_2\lambda^2 + J^2J_2^2\right) + J^2J_2^2(4J^2+2JJ_2+J_2^2)$$

$$\frac{1}{\lambda^2}\det(\hat{A}+\lambda\hat{I}) = \lambda^6 + \lambda^4\left[2JJ_2 + J(J+2J_2)\right] + \lambda^2\left[J^2J_2^2 + J^2J_2(2J+3J_2) +\right.$$
$$\left.+ J^2 2JJ_2\right] + J^2\cdot J^2J_2^2 + J^2J_2^2(4J^2+2JJ_2+J_2^2)$$

$$F(\lambda) = \frac{1}{\lambda^2}\det(\hat{A}+\lambda\hat{I}) = \lambda^6 + \lambda^4J(J+4J_2) + \lambda^2JJ_2\left[JJ_2 + J(2J+3J_2) + 2J^2\right] +$$
$$+ J^2J_2^2(5J^2+2JJ_2+J_2^2);$$

$$F(\lambda) = \frac{1}{\lambda^2}\det(\hat{A}+\lambda\hat{I}) = \lambda^6 + J(J+4J_2)\lambda^4 + 4J^2J_2(J+J_2)\lambda^2 + J^2J_2^2(5J^2+2JJ_2+J_2^2)$$

$\lambda^2 = -\omega^2 = x$, $F(x) = x^3 + J(J+4J_2)x^2 + 4J^2J_2(J+J_2)x + J^2J_2^2(5J^2+2JJ_2+J_2^2) = 0$

$J = 4J_1 < 0$, $J_2 < 0$, $F(x) = ax^3 + bx^2 + cx + d$ – где корни? и сколько их?
a, b, c, d все > 0.

$x = J^2y$, $x^3 = J^6y^3$, $x^2 = J^4y$, $J_2/J = \varepsilon > 0$.

$$F(y) = J^6y^3 + J(J+4J_2)J^4y^2 + 4J^2J_2(J+J_2)yJ^2 + J^2J_2^2(5J^2+2JJ_2+J_2^2)$$

$$\frac{1}{J^6}F(y) = y^3 + (1+4\varepsilon)y^2 + 4\varepsilon(1+\varepsilon)y + \varepsilon^2(5+2\varepsilon+\varepsilon^2), \quad y \to x$$

$$F(x) = x^3 + (1+4\varepsilon)x^2 + 4\varepsilon(1+\varepsilon)x + \varepsilon^2(\varepsilon^2+2\varepsilon+5) = 0$$

Замена $x = t - \frac{1}{3}\frac{b}{a} = t - \frac{1}{3}(1+4\varepsilon)$ даст $F(t) = t^3 + pt + q = 0$ reduced eq.

$a = 1$, $b = 1+4\varepsilon$, $c = 4\varepsilon(1+\varepsilon)$, $d = \varepsilon^2(\varepsilon^2+2\varepsilon+5)$ $\left(\varepsilon = J_2/J = J_2/(4J_1)\right)$

$$p = \frac{1}{3}(3c - b^2) = \frac{1}{3}\left[12\varepsilon(1+\varepsilon) - (1+4\varepsilon)^2\right] = \frac{1}{3}\left[12\varepsilon+12\varepsilon^2-1-16\varepsilon^2-8\varepsilon\right] = \frac{1}{3}\left[-1+4\varepsilon-4\varepsilon^2\right]$$

$$p = -\frac{1}{3}(2\varepsilon-1)^2 \le 0, \quad q = \frac{1}{27}\left[2b^3 - 9bc + 27d\right] = \frac{1}{27}\Big[2(1+4\varepsilon)^3 - 9(1+4\varepsilon)4\varepsilon(1+\varepsilon) +$$
$$+27\varepsilon^2(\varepsilon^2+2\varepsilon+5)\Big] = \frac{1}{27}\left[2(1+64\varepsilon^3+3\cdot4\varepsilon+3\cdot16\varepsilon^2) - 9(1+5\varepsilon+4\varepsilon^2)4\varepsilon + 27\varepsilon^2(\varepsilon^2+2\varepsilon+5)\right] =$$
$$= \frac{1}{27}\left[2+128\varepsilon^3+12\varepsilon+48\varepsilon^2 - 36\varepsilon - 160\varepsilon^2 - 144\varepsilon^3 + 27\varepsilon^4 + 54\varepsilon^3 + 135\varepsilon^2\right] = \frac{1}{27}\left[2-24\varepsilon-23\varepsilon^2+38\varepsilon^3+27\varepsilon^4\right]$$

$$q = \frac{1}{27}\left[2-24\varepsilon-23\varepsilon^2+38\varepsilon^3+27\varepsilon^4\right]$$ – ? нулей.

Для Mn_2Au $\varepsilon = \frac{J_2}{4J_1} = \frac{532\,K}{4\cdot396\,K} = 0{,}336$. Если $4p^3+27q^2 > 0$ – one real root
$4p^3+27q^2 < 0$ – 3 real roots.

$\frac{\lambda^2}{J^2} = x = t - \frac{1}{3}(1+4\varepsilon) = t - 0{,}781$, $x = -\frac{\omega^2}{J^2}$ MathCad даёт $p = -0{,}0353$, $q = 0{,}2039$

$F(t) = 0$ имеет корень $t_1 = -0{,}609$ и $4p^3+27q^2 = 1{,}123 > 0 \Rightarrow$ один корень $F(x) = 0$

$-\frac{\omega_1^2}{J^2} = t_1 - 0{,}781 = -1{,}390$, $\omega_1^2/J^2 = 1{,}390$ – частота FMR, кроме $\omega_0 = 0$.

т.е. при $E_{an} = 0$ (exch. approximation) для Mn_2Au есть две FMR частоты
$\omega_0 = 0$ и $\omega_1 = 1{,}18J$. ω_1 – finite несмотря на $E_{an} = 0$. Это верно, если $F(t) = 0$ eq. берём

Escaneado con CamScanner

Проверка ур $F(\lambda)=0$ численно в MathCad можно даже выполнить.

Note: $J_2=0$, $\varepsilon=0$, далее $b=1$, $c=0$, $d=0$, $p=-\frac{1}{3}$, $q=\frac{2}{27}\beta^3=\frac{2}{27}$ $F(t)=t^3-\frac{1}{3}t+\frac{2}{27}$

далее $4p^3+27q^2=0$, имеет корень при $t_0>0$, но $-\frac{\omega_1^2}{J^2}=t_0+0{,}721\approx -0{,}5<0$.

Корень (t_1) $-\frac{\omega_1^2}{J^2}\approx +1{,}50$, т.е. ω_1 (FMR) начнёт расти при $J_2=0$

т.е. при $\varepsilon=0$, $J_2=0$ если при FMR заложили $\omega_0=0$, $\omega_1\neq 0$, $\omega_2\neq 0$.

Это верно если аналитически ур. $F(x)=0$ верное. (Разделили ур на $J=4J_1$)

(11/v) Проверим ур LLG MuMax film ас 3/с и поиграем FMR-спектр by MathCad.

AF ground state of 4-layer Mn2Au film: $\vec{m}_1^0=\vec{m}_3^0=\hat{y}$; $\vec{m}_2^0=\vec{m}_4^0=-\hat{y}$

Anisotropy in MAU x_y (K_2), OMM - 0_y (K_y). $K_2\gg K_y$ $4J_1=J$

M_s ω $J\to\omega$, $J\to J/V_{u.c.}$ $\dot{\vec{m}}_i=i\omega\vec{m}_i$. $J_2/J=\varepsilon$. $2K_2/J=\varkappa_2$, $\frac{2K_y}{J}=\varkappa_y$

$$\begin{cases} -i\omega\vec{m}_1=\vec{m}_1\times\left[\vec{m}_2-\varkappa_2 m_1^z\hat{z}+\varkappa_y m_1^y\hat{y}\right], & -i\omega\vec{m}_2=\vec{m}_2\times\left[\vec{m}_1+\varepsilon\vec{m}_3-\varkappa_2 m_2^z\hat{z}+\varkappa_y m_2^y\hat{y}\right] \\ -i\omega\vec{m}_3=\vec{m}_3\times\left[\varepsilon\vec{m}_2+\vec{m}_4-\varkappa_2 m_3^z\hat{z}+\varkappa_y m_3^y\hat{y}\right], & -i\omega\vec{m}_4=\vec{m}_4\times\left[\vec{m}_3-\varkappa_2 m_4^z\hat{z}+\varkappa_y m_4^y\hat{y}\right] \end{cases}$$

$\vec{m}_{1,3}=\hat{y}+\vec{v}_{1,3}$, $\vec{m}_{2,4}=-\hat{y}+\vec{v}_{2,4}$, $\omega\vec{m}_i=\omega\vec{v}_i$ - diff. pairs.

$\vec{m}_1\times\vec{m}_2=(\hat{y}+\vec{v}_1)\times(-\hat{y}+\vec{v}_2)=-\vec{v}_1\times\hat{y}+\hat{y}\times\vec{v}_2=\hat{y}\times\vec{v}_1+\hat{y}+\vec{v}_2=\hat{x}(v_1^z+v_2^z)-\hat{z}(v_1^x+v_2^x)$

$\hat{y}\times\vec{v}_i=\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\0&1&0\\v_i^x&0&v_i^z\end{vmatrix}=\hat{x}v_i^z-\hat{z}v_i^x$. $\vec{m}_2\times\vec{m}_1^4\hat{z}=m_1^4(\hat{y}+\vec{v}_1)\times\hat{z}=v_1^z\hat{y}\times\hat{z}=v_1^z\hat{x}$

$\vec{m}_3\times m_3^z\hat{z}=v_3^z\hat{x}$

$\vec{m}_2\times m_2^z\hat{z}=v_2^z(-\hat{y}+\vec{v}_2)\times\hat{z}=v_2^z(-1)\hat{y}\times\hat{z}=-v_2^z\hat{x}$, $\vec{m}_4\times m_4^z\hat{z}=-v_4^z\hat{x}$

$\vec{m}_1\times m_1^y\hat{y}=m_1^y\vec{m}_1\times\hat{y}=(1+v_1^y)(\hat{y}+\vec{v}_1)\times\hat{y}=(1+v_1^y)\vec{v}_1\times\hat{y}=(v_1^x\hat{x}+v_1^z\hat{z})\times\hat{y}=v_1^x\hat{z}-v_1^z\hat{x}$

$\vec{m}_2\times m_2^y\hat{y}=m_2^y\vec{m}_2\times\hat{y}=(-1+v_2^y)(-\hat{y}+\vec{v}_2)\times\hat{y}=(-1)(v_2^x\hat{x}+v_2^z\hat{z})\times\hat{y}=-v_2^x\hat{z}+v_2^z\hat{x}$

$\vec{m}_3\times m_3^y\hat{y}=v_3^x\hat{z}-v_3^z\hat{x}$, $\vec{m}_4\times m_4^y\hat{y}=-v_4^x\hat{z}+v_4^z\hat{x}$

$\vec{m}_2+\vec{m}_3=(-\hat{y}+\vec{v}_2)+(\hat{y}+\vec{v}_3)=-\hat{y}\vec{v}_3+\vec{v}_2\times\hat{y}=-\hat{y}\times(\vec{v}_2+\vec{v}_3)=\left[\hat{x}(v_2^z+v_3^z)-\hat{z}(v_2^x+v_3^x)\right](-1)$

$\vec{m}_3\times\vec{m}_4=(\hat{y}+\vec{v}_3)\times(-\hat{y}+\vec{v}_4)=\hat{y}\times(\vec{v}_3+\vec{v}_4)=\hat{x}(v_3^z+v_4^z)-\hat{z}(v_3^x+v_4^x)$

$$\begin{cases} -i\omega\vec{v}_1=\hat{x}(v_1^z+v_2^z)-\hat{z}(v_1^x+v_2^x)-\varkappa_z\left[v_1^z\hat{x}\right]+\varkappa_y\left[v_1^x\hat{z}-v_1^z\hat{x}\right] \\ -i\omega\vec{v}_2=-\hat{x}(v_1^z+v_2^z)+\hat{z}(v_1^x+v_2^x)-\varkappa_z\left[-v_2^z\hat{x}\right]+\varkappa_y\left[-v_2^x\hat{z}+v_2^z\hat{x}\right]+\varepsilon\left[-\hat{x}(v_2^z+v_3^z)+\hat{z}(v_2^x+v_3^x)\right] \\ -i\omega\vec{v}_3=-\varepsilon\left[-\hat{x}(v_2^z+v_3^z)+\hat{z}(v_2^x+v_3^x)\right]+\hat{x}(v_3^z+v_4^z)-\hat{z}(v_3^x+v_4^x)-\varkappa_z\left[v_3^z\hat{x}\right]+\varkappa_y\left[v_3^x\hat{z}-v_3^z\hat{x}\right] \\ -i\omega\vec{v}_4=-\hat{x}(v_3^z+v_4^z)+\hat{z}(v_3^x+v_4^x)-\varkappa_z\left[-v_4^z\hat{x}\right]+\varkappa_y\left[-v_4^x\hat{z}+v_4^z\hat{x}\right] \end{cases}$$

$$\begin{pmatrix} 0 & 1-\varkappa_z-\varkappa_y & 0 & 1 & 0 & 0 & 0 & 0 \\ -1+\varkappa_y & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1+\varkappa_z+\varkappa_y & 0 & -\varepsilon & 0 & 0 \\ 1 & 0 & 1-\varkappa_y+\varepsilon & 0 & \varepsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & +\varepsilon & 0 & 1+\varepsilon-\varkappa_z & 0 & 1 \\ 0 & 0 & -\varepsilon & 0 & -\varepsilon-1+\varkappa_y & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1+\varkappa_z+\varkappa_y \\ 0 & 0 & 0 & 0 & 1 & 0 & 1-\varkappa_y & 0 \end{pmatrix} \begin{pmatrix} v_1^x \\ v_1^z \\ v_2^x \\ v_2^z \\ v_3^x \\ v_3^z \\ v_4^x \\ v_4^z \end{pmatrix} = i\omega \begin{pmatrix} v_1^x \\ v_1^z \\ v_2^x \\ v_2^z \\ v_3^x \\ v_3^z \\ v_4^x \\ v_4^z \end{pmatrix}$$

1x 1z 2x 2z 3x 3z 4x 4z

Численная часть совп. с $J/\hbar$.

Параметры:

$\varepsilon=\frac{J_2}{4J_1}$

$\varkappa_z=\frac{2K_z}{4J_1}$, $\varkappa_y=\frac{2K_y}{4J_1}$.

$\varepsilon=0{,}336$.

$\varkappa_z=\frac{2\cdot 1{,}303\cdot 10^{-22}J}{4{,}396\cdot 10^{-23}\cdot 1{,}381}=0{,}012$

$\varkappa_y=\frac{2\cdot 7\cdot 1{,}855\cdot 10^{-25}}{4\cdot 396\cdot 1{,}381\cdot 10^{-23}}=0{,}012$ $\cdot 10^{-2}$

Численно ω в единицах $\omega_J=\frac{\gamma}{M_s}\frac{J}{V_{u.c.}}=\frac{\gamma}{M_s}\frac{4J_1}{V_{u.c.}}$, $M_s=\frac{\mu_{Mn}}{V_1}$; V_1 - volume per 1 Mn atom

$V_1=\frac{1}{2}V_{u.c.}$, Cm. 24/III. $M_s=\frac{2\mu_{Mn}}{V_{u.c.}}$, $M_sV_{u.c.}=2\mu_{Mn}=2\cdot 4\mu_B=8\mu_B$. одной подрешетки

$\omega_J=\frac{\gamma}{2\mu_B}4J_1=\frac{1}{2}\gamma\frac{J_1}{\mu_B}$, $\omega_J/2\pi=\frac{1}{2}\left(\frac{\gamma}{2\pi}\right)\frac{396\,K\cdot 1{,}381\cdot 10^{-23}J/K}{9{,}274\cdot 10^{-24}J/T}=\frac{1}{2}\left(\frac{\gamma}{2\pi}\right)\frac{396\cdot 1{,}381}{9{,}274}\cdot 1T$

$\omega_J/2\pi=\frac{1}{2}(\gamma/2\pi)589{,}7T\approx\frac{1}{2}28{,}02\frac{GHz}{T}\cdot 589{,}7T=8.261{,}7GHz=8{,}262THz$, Cm 30/E

Mathcad „Mn2Au_EMR_frequencies.mcd" дает в exch. approximation 8 корней сек. ур. $\det(A - \omega\delta_{nn'}) = 0$, $\omega_0 = 0$ (4-х крат.), $\omega_1 = \pm 0{,}820$ (2-х кратн.)

ω_1 (абс. ед.) $= 0{,}820 \cdot \omega_J = 0{,}820 \cdot 8{,}262$ ТГц $= 6{,}77$ ТГц (optic branch)

10/V в постоян. аналитически $\omega_1/\omega_J = 1{,}18$ несколько меньше численного $\frac{\omega_1}{\omega_J} = 0{,}820$.

Аналит. сек. ур-е 10/V неверно $F(\lambda) = \frac{1}{\lambda^2}\det(\hat{A} + \lambda\hat{I})$ — неверно.

$\omega_1(0)/2\pi$ [Arana '2017 – exp] $= 3{,}72$ ТГц, различие с 6.77 ТГц можно объяснить different definitions of H_{ex}.

Но у Arana'17 $\omega_1 = \gamma\sqrt{H_{ex} H_{a2}}$, $H_{a2} = 2K_2/M_S$ – easy plane field.

У меня $\frac{\omega_1}{2\pi} = 6{,}77$ ТГц – чисто exchange.

Включили K_2 (χ_2) – Arana-case neglecting K_4: $\omega_0 = 0$, $\frac{\omega_1}{2\pi} = 0{,}805$, $\frac{\omega_2}{2\pi} = 0{,}815$ – частоты чуть понизились по сравн. с $\omega_1(0)/2\pi = 0{,}820$.

Включили $K_2(\chi_2)$ и $K_4(\chi_4)$ – Arana-case без K_4: $\frac{\omega_1}{2\pi} = 0{,}805$, $\frac{\omega_2}{2\pi} = 0{,}815$ Сохранились. Но корень $\omega_0 = 0$ пропал.

$\frac{\omega_1}{2\pi} = 0{,}805 \cdot 8{,}262$ ТГц $= 6{,}65$ ТГц, $\frac{\omega_2}{2\pi} = 0{,}815 \cdot 8{,}262$ ТГц $= 6{,}73$ ТГц.

Вывод: частоты ω_1, ω_2 видимо exchange dominated, но слабо расщеплены из-за K_2 (easy plane) anisotropy. (см. 30/V)

(Arana'17 exp. измерили 3 frequency Mn_2Au bulk: 121 GHz, 3,24 ТГц, 4,20 ТГц (film $t = 0.25$ мкм)

а постоян. ($\chi_4 = K_4 = 0$) при частоте $f_0 = 0$, $f_1 = 6{,}65$ ТГц и $f_2 = 6{,}73$ ТГц

// При $\varepsilon = J_2/J = 0$ частоты FMR вообще нет: $J_2 = 0$ ведет к unstable AF state $\vec{m}_1 = \vec{m}_3 = \hat{y}$, $\vec{m}_2 = m_4 = -\hat{y}$? При $\varepsilon = 0$ (1-2) и (3-4) слои никак не связываются и сек. ур. упрощается.

$$\begin{pmatrix} 0 & 1-\chi_2-\chi_y & 0 & 1 \\ -1+\chi_y & 0 & -1 & 0 \\ 0 & -1 & 0 & -1+\chi_2+\chi_y \\ 1 & 0 & 1-\chi_y & 0 \end{pmatrix} \begin{pmatrix} v_1^x \\ v_1^z \\ v_2^x \\ v_2^z \end{pmatrix} = \lambda(-1)\begin{pmatrix} v_1^x \\ v_1^z \\ v_2^x \\ v_2^z \end{pmatrix}, \quad \det(\hat{A} + \lambda\hat{I}) = \det\begin{pmatrix} \lambda & 1-\chi_2-\chi_y & 0 & 1 \\ -1+\chi_y & \lambda & -1 & 0 \\ 0 & -1 & \lambda & -1+\chi_2+\chi_x \\ 1 & 0 & 1-\chi_y & \lambda \end{pmatrix} =$$

$$= \lambda\begin{vmatrix} 1-\chi_2-\chi_y & 0 & 1 \\ \lambda & -1 & 0 \\ -1 & \lambda & -1+\chi_2+\chi_y \\ 0 & 1-\chi_y & \lambda \end{vmatrix} - \begin{vmatrix} -(1-\chi_2-\chi_y) & -1+\chi_y & 0 \\ 0 & \lambda & -1+\chi_2+\chi_y \\ 1 & 1-\chi_y & \lambda \end{vmatrix} - \begin{vmatrix} -1+\chi_y & \lambda & -1 \\ 0 & -1 & \lambda \\ 1 & 0 & 1-\chi_y \end{vmatrix} =$$

$$= \lambda\left[\lambda^3 - \lambda - \lambda(-1+\chi_2+\chi_y)(1-\chi_y)\right] - (1-\chi_2-\chi_y)\left[\lambda^2(\chi_y-1) + (1-\chi_2-\chi_y) + (\chi_y-1)^2(-1+\chi_2+\chi_y)\right] -$$

$$-\left[(1-\chi_y)^2 + \lambda^2 - 1\right] = \lambda^4 + \lambda^2\left[-1 + (1-\chi_2-\chi_y)(1-\chi_y) + (1-\chi_2-\chi_y)(1-\chi_y) - 1\right] +$$

$$+ (-1)(1-\chi_2-\chi_y)^2\left[1 - (\chi_y-1)^2\right] - \left[(1-\chi_y)^2 - 1\right] = \lambda^4 + 2\lambda^2\left[(1-\chi_2-\chi_y)(1-\chi_y) - 1\right] +$$

$$+ \left[(\chi_y-1)^2 - 1\right]\left[(1-\chi_2-\chi_y)^2 - 1\right] = |\chi_y^2 \ll 1 \text{ вообщем}| = \lambda^4 + 2\lambda^2\left[\chi_2\chi_y - \chi_y - \chi_2 - \chi_y\right] +$$

$$+ (-2\chi_y)\left[\chi_2^2 - 2\chi_2 - 2\chi_y + 2\chi_2\chi_y\right] = \lambda^4 + 2\lambda^2\left[-2\chi_y - \chi_2\right] +$$

$$+ (-2)\chi_y\left[\chi_2^2 - 2\chi_2 - 2\chi_y\right] = 0, \quad \lambda^4 - 2\lambda^2(\chi_2 + 2\chi_y) - 2\chi_y(\chi_2^2 - 2\chi_2 - 2\chi_y) = 0$$

$$D = 4(\chi_2 + 2\chi_y)^2 + 8\chi_y(\chi_2^2 - 2\chi_2 - 2\chi_y) = 4\chi_2^2 + 16\chi_2\chi_y + 8\chi_y\chi_2^2 - 16\chi_y\chi_2 =$$

$$= 4\chi_2^2(1 + 2\chi_y) > 0, \quad D > 0, \quad \lambda \text{ – корни вещ.}$$ $\lambda = i\omega$, ω – pure imaginary

AF case $\vec{m}_1 = \hat{y}$, $\vec{m}_2 = -\hat{y}$ 2-х слойки (1-2) имеют корень $\lambda = 0$ не устойчиво. нет частот FMR?

Announce project meeting on-line June 1, 10^{00}.

(12/V) E-mails to Ruben, Pawel, Jaroslaw, Elena. Бланк. с „Arbeitsmarkt".

(13/V) Скорое обсуждение SW спектра Mn_2Au с Ruben. Бланк. Просим Ruben сделать численно влияние $K_{2,1}$ на AF FMR spectra ω_1, ω_2

(14/V) Переписка с Oxel.: Nanomaterials и IWMMT 2021 Conf. Приняли invited talk „Magic" workshop, July 5-7, 2021. Res. Report Ikerbasque promotion.

Escaneado con CamScanner

15/2 C A «Сигнал» Обработка МФ А → Ю интенсивности флуоресценции. АФМР $M_n A_n$.

16/2 EW spectra of $M_n A_n$. Почему в одну матрицу $\hat{A}$ входят антисимметричные куски АФМ? Как убрать? $+\hat{g}$ и $-\hat{g}$ дают разный порядок: $+(1,3),-(2,4)$

$\omega_3 = 4\gamma_n (J_1/M_s V_{uc}) < 0$, т.е. $J_1 < 0$; надо сделать на $|J_1|$, т.е. $+\to -$

$+$ (3 / 13) $\to$ $-$, $\varepsilon = J_2/4J_1 \to -\varepsilon$ ($\varepsilon = J_2/4|J_1|$), $x_z = \frac{K_z}{2J_1} \to +x_z$ (т.к. $J_1 < 0$), $x_y \to +x_y$

Если матрица $\hat{A}$ меняется, тогда $\omega \to -\omega$ (это соответствует $\vec{H} \to -\vec{H}$) т.к. ур. $F(\omega^2)$, т.е. $\omega \to -\omega$ не меняет вид уравнения.

Можно смотреть ω/ω_3, где $\omega_3 = 4\gamma J_1/M_s V$ (< 0) и обозначать всё матрицей $\hat{A}$

Если делать на J_1, то $x_z = \frac{K_z}{2J_1} < 0$, $x_y = \frac{K_y}{2J_1} < 0$, $(4J_1 + J_2)/4J_1 = 1 + \varepsilon$ ($\omega_3 < 0$). $\varepsilon = J_2/4|J_1| > 0$

Если делать на $|2J_1|$, $\omega_3 > 0$, $x_z = \frac{K_z}{2|J_1|} > 0$, $x_y = \frac{K_y}{2|J_1|} > 0$, $\frac{4J_1 + J_2}{4|J_1|} = -1 + \varepsilon$

т.е. при $\omega_3 = 4\gamma J_1/M_s V < 0$ (обязательно! matter Cad 11/6), $\varepsilon = \frac{J_2}{4|J_1|} > 0$, $\varepsilon = \frac{-J_2}{4|J_1|} > 0$

надо $x_z < 0$, $x_y < 0$.

Именно знаки x_z, x_y в «$M_n A_n$ АФМР-частотах мод» и дают всю экзотику (!!).

1) Exchange approximation ($x_z = 0, x_y = 0$) $\omega_0 = 0$, $\omega_1 = \pm 0.820$ по однородности

2) $x_z \neq 0$, $x_y = 0$ (можно $x_z \neq 0$), $\omega_0 = 0$, $\omega_1 = 0.154$, $\omega_2 = 0.825$, $\omega_4 = 0.834$

3) $x_z \neq 0$, $x_y \neq 0$, ($\omega_0 = 0$) вырождено, $\omega_1 = 0.0155$, $\omega_2 = 0.155$, $\omega_3 = 0.825$, $\omega_4 = 0.834$

($\omega_3 = 4\gamma J_1/M_s V_{uc} = -8.262$ ТГц scale. Частоты в units of ω_3.

4) $\varepsilon = 0$, $J_2 = 0$, $\omega_1 = 0.0155$, $\omega_2 = 0.155$ (обе корня ω_1^2, ω_2^2 – двукратные).

Неустойчивость $J_2 = 0$ или x_z? Ур. $\lambda^4 - 2\lambda^2 x_z + 4 x_y x_z = 0$, $x_y^2 \ll 1$, $x_y \ll x_z$

$D = 4x_z^2 - 4 \cdot 4 x_y x_z = 4x_z^2 (1 - 4\frac{x_y}{x_z}) \approx 4x_z^2 > 0$, $D(11/6) \approx 4x_z^2 > 0$.

При $x_y = 0$, $\lambda^4 - 2\lambda^2 x_z = 0$, $\lambda = 0$ и $\lambda^2 = 2x_z$. 11/6 если $x_z > 0$, $\lambda^2 > 0$, λ – вещ. Надо $x_z = K_z/2J_1 < 0$, $\lambda^2 = 2x_z < 0$, и $\lambda = i\omega$ – мнимый, ω – real.

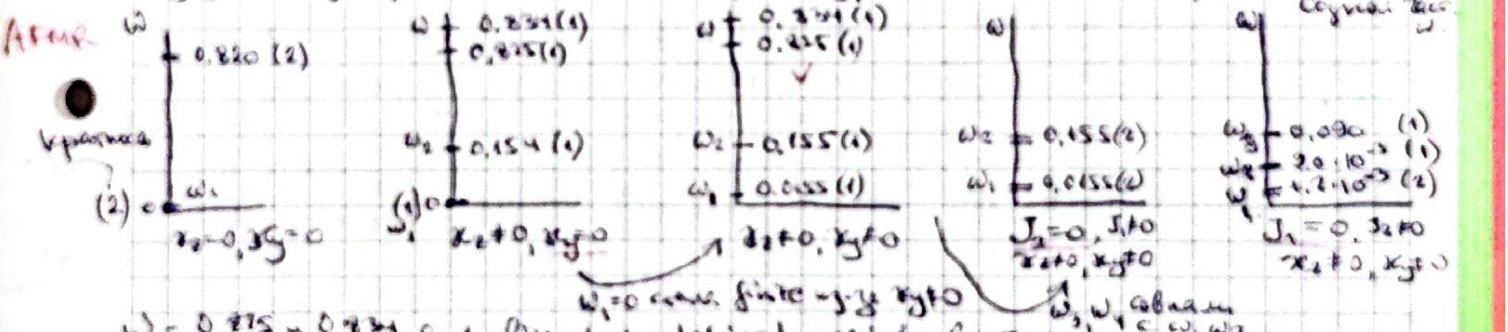
Вывод: включение $x_y \neq 0$ приводит к $\omega_0^2 = 0 \to 0.0155$ (0.128 ТГц = 128 ГГц)

3) при $J_2 = 0$, частоты $\omega_1 = 0.0155$ и $\omega_2 = 0.155$ сохраняются неодинаковыми (с общими точками)

4) при $x_z \neq 0$, $x_y \neq 0$, $J_2 \neq 0$ – вторые (частоты $\neq 0$).

Можно будет нумеровать ω_n по мере возрастания, $n = 1, 2, 3, 4$. ЕW спектры

АФМР



$\omega_1 = 0$ сами близко $x_y \neq 0$ $x_z \neq 0$

ω_3, ω_4 сближаются с ω_1, ω_2

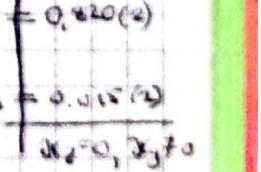
$\omega_{3,4} = 0.825 - 0.834$ Exch. Branches defined mainly by J_2

$\omega_{1,2} = 0.016 - 0.155$ Exch.-anisotropy branches determined $\omega_1(x_y J_1) \to \omega_1(x_z J_1)$ – «anisotropic» modes. ω_3, ω_4 – «optical» modes

Low-frequency part of AFMR spectrum strongly sensitive к анизотропии x_z, x_y. Уравнение 4-го порядка биквадратное $M_n A_n$.

Будем $\omega_2^2 = (a_1 x_z + a_2 x_y) J_{eff}$, $J_{eff} = \text{linear } f(J_1, J_2)$

$\omega_1^2 = b_1 x_y J_{eff}$. J_{eff} in units $|J_1| \cdot 4$.



При $J_2 = 0$: $\lambda^4 - 2\lambda^2 x_z + 4 x_y x_z = 0$, $2\lambda_{1,2}^2 = 2x_z \pm \sqrt{4x_z^2 - 16 x_y x_z}$

$\lambda_{1,2}^2 = -\omega_{1,2}^2 = x_z \pm 2|x_z|\sqrt{1 - 4\frac{x_y}{x_z}}$, $x_z = \frac{K_z}{2J_1} < 0$. $\sqrt{1 - 4\frac{x_y}{x_z}} \approx 1 - \frac{2x_y}{x_z} + O(\frac{x_y^2}{x_z^2})$

$-\omega_1^2 \approx x_z + |x_z|(1 + 2\frac{x_y}{x_z}) = |x_z| - |x_z| = +2\frac{|x_z|}{x_z} x_y = 2x_y = -2|x_y|$, $\beta_1 = -2$

$\omega_1^2 \approx 2|x_y|$, $\omega_1 = \sqrt{2|x_y|} \approx 0.0154$ (1). $-\omega_2^2 = x_z - |x_z|(1 - 2\frac{x_y}{x_z}) = -2|x_z| - 2x_y$

$\omega_2^2 \approx 2|x_z| - 2|x_y| \approx 2|x_z|$ ($\omega_2 = \sqrt{2|x_z|} = 0.154$ (1)) $J_{eff} = 1$, $J_{eff} = 4|J_1|$

Расчеты в abs. units ($J_2=0$) $\omega_1/\omega_J \cong \sqrt{2|\varkappa_z|}$, $\omega_1 = \omega_J\sqrt{2|\varkappa_z|} = \frac{4|J_1|}{M_s V}\sqrt{2\cdot\frac{2K_y}{4|J_1|}}$, $J_2\neq 0$

$\omega_1 = \frac{\gamma}{M_s V} 4|J_1|\sqrt{\frac{K_y}{|J_1|}} = \frac{\gamma}{M_s V}\cdot 4\sqrt{K_y|J_1|}$, $\omega_2 \cong \frac{\gamma}{M_s V} 4\sqrt{K_z|J_1|}$ when $J_2=0$.

$$E_{an} = K_{2\perp}\sum_i (\vec{m}_i\cdot\hat{z})^2 - K_{2\parallel}\sum_i (\vec{m}_i\cdot\hat{y})^2 + \frac{K_{4\perp}}{2}\sum_i (\vec{m}_i\cdot\hat{z})^4 - \frac{K_{4\parallel}}{2}\sum_i\left[(\vec{m}_i\cdot\hat{u}_1)^4 + (\vec{m}_i\cdot\hat{u}_2)^4\right]$$

$K_{2\perp} = K_z > 0$, $K_{2\parallel} = K_y > 0$, Cm. 9/V. $\hat{u}_1 = \frac{1}{\sqrt{2}}[1\,1\,0]$, $\hat{u}_2 = \frac{1}{\sqrt{2}}[1\,\bar{1}\,0]$

$K_{4\perp} = \frac{1}{351}K_{2\perp} = \frac{1}{351}K_z$, $\varkappa_{4z} = \frac{1}{351}\varkappa_z = -3{,}39\cdot10^{-5}$, $\varkappa_y = -1{,}19\cdot10^{-4}$

$K_{4\parallel} = \frac{1}{7}K_{2\parallel} = \frac{1}{7}K_y$, $\varkappa_{4\parallel} = \frac{1}{7}\varkappa_y = -1{,}70\cdot10^{-5}$

$K_{4\perp} = 2K_{4\parallel} > 0$, $K_{4\perp} = 1{,}855\cdot10^{-25}$ J. Знак $K_{4\perp}$ чувствует, это $+|K_{4\perp}|$ или $-|K_{4\perp}|$?





$\hat{u}_1 = \frac{1}{\sqrt{2}}(\hat{x}+\hat{y})$, $\hat{u}_2 = \frac{1}{\sqrt{2}}(\hat{x}-\hat{y})$

$(\vec{m}\cdot\hat{u}_1)^4 + (\vec{m}\cdot\hat{u}_2)^4 = \frac{1}{4}\left[(m_x+m_y)^4 + (m_x-m_y)^4\right] = \frac{1}{4}\left[m_x^4 + 4m_x^3m_y + 6m_x^2m_y^2 + 4m_xm_y^3 + m_y^4 + m_x^4 - 4m_x^3m_y + 6m_x^2m_y^2 - 4m_xm_y^3 + m_y^4\right] =$

$= \frac{1}{4}\left[2m_x^4 + 2m_y^4 + 12m_x^2m_y^2\right] = \frac{1}{2}\left[m_x^4 + m_y^4 + 6m_x^2m_y^2\right]$

$\vec{m}^4 = 1 = (m_x^2+m_y^2+m_z^2)^2 = m_x^4 + m_y^4 + m_z^4 + 2m_x^2m_y^2 + 2m_x^2m_z^2 + 2m_y^2m_z^2$

E_{an} (4th order) $= -\frac{1}{2}K_{4\perp}\sum_i(\vec{m}_i\cdot\hat{z})^4 - \frac{1}{2}K_{4\parallel}\sum_i\left[(\vec{m}_i\cdot\hat{u}_1)^4 + (\vec{m}_i\cdot\hat{u}_2)^4\right] = |K_{4\perp} = 2K_{4\parallel}| =$

$= -\frac{K_{4\parallel}}{2}\sum_i\left[2(m_i^z)^4 + \frac{1}{2}\left\{(m_i^x)^4 + (m_i^y)^4 + 6(m_i^x)^2(m_i^y)^2\right\}\right] =$

$= -\frac{1}{2}K_{4\parallel}\sum_i\left[2(m_z)^4 + \frac{1}{2}\left\{1 - m_z^4 - 2m_x^2m_y^2 - 2m_x^2m_z^2 - 2m_y^2m_z^2 + 6m_x^2m_y^2\right\}\right] =$

$= \text{Const} - \frac{1}{2}K_{4\parallel}\sum_i\left[\frac{3}{2}m_z^4 + 2m_x^2m_y^2 - m_x^2m_z^2 - m_y^2m_z^2\right] = |m_x^2+m_y^2 = 1-m_z^2| =$

$= -\frac{1}{2}K_{4\parallel}\sum_i\left[\frac{3}{2}m_z^4 + 2m_x^2m_y^2 - m_z^2(1-m_z^2)\right] = -\frac{1}{2}K_{4\parallel}\sum_i\left[\frac{5}{2}m_z^4 + 2m_x^2m_y^2 - m_z^2\right]$

Нужное $(\vec{m}\cdot\hat{u}_1)^4 + (\vec{m}\cdot\hat{u}_2)^4 = \frac{1}{2}\left[m_x^4 + m_y^4 + 6m_x^2m_y^2\right] = \frac{1}{2}\left[(m_x^2+m_y^2)^2 + 4m_x^2m_y^2\right] =$

$= \frac{1}{2}\left[(1-m_z^2)^2 + 4m_x^2m_y^2\right]$ и $E_{an} = -\frac{1}{2}K_{4\perp}\sum_i m_z^4 - \frac{1}{4}K_{4\parallel}\sum_i\left[(1-m_z^2)^2 + 4m_x^2m_y^2\right]$

// т.к. 4th order anisotropy $\varkappa_{4z}$, $\varkappa_{4\parallel} \sim 10^{-5}$ и $< \varkappa_y = -1{,}19\cdot10^{-4}$, то сильного влияния на AFMR расчеты она не окажут. Спектр ✓ экспериментальных.

→ Вывод: при учете $\varkappa_z, \varkappa_y$ в спектре AFMR ($\vec{k}=0$) теории расчеты Mn_2Au:

$\omega_1/2\pi = 128$ GHz ($\sim\sqrt{K_y}$), $\frac{\omega_2}{2\pi} = 1{,}281$ THz ($\sim\sqrt{K_z}$), $\frac{\omega_3}{2\pi} = 6{,}816$ THz / $\frac{\omega_4}{2\pi} = 6{,}891$ THz

Cm. 30/V

„acoustic" or anis. exch. Branches — „optics" or Exchange Branches

Model: Ultrathin Mn_2Au film ($L=0{,}85$nm (1 unit cell)), infinite in x, y directions.

// Если $J_1\neq0$ ($J_1 = -396$ K), то при $J_2=0$ exch. branches frequencies 6,82, 6,89 THz резко падают и опускаются к anisotropy-exch. „acoustic" branches

Скорости v_{gr}, v_{ph} of the lowest „acoustic" frequency $\nu_{gr,ph} \sim \frac{\omega_1}{k_{BZ}}$, $k_{BZ} = \frac{\pi}{a} = \frac{\pi}{0{,}333\text{ nm}}$

$v_{gr,ph} \approx \frac{2\pi f_1}{\pi/2} = 2af_1 = 2\cdot0{,}333\text{ nm}\cdot128\cdot10^9\frac{1}{s} = 2\cdot0{,}333\cdot128\cdot10^{-9}\text{ m}\cdot10^9\frac{1}{s} = 85\frac{m}{s}$ —

↳ невероятно. Надо дать оценки $v_{gr,ph} = \frac{\omega_1(k_{BZ}) - \omega_1(k=0)}{k_{BZ}}$, а не брать через $\omega_1(k=\pi)$.

→ Вывод: ω_1, ω_2 „acoustic" frequencies от J_2 не зависят → $v(k) = \frac{d\omega_{1,2}}{dk}$ не зависит от J_2

exchange „optic" ω_3, ω_4 frequencies зависят от J_2 как $\sim\sqrt{J_2}$. при $J_2=0$, exch. ω_3, ω_4 сходятся к „acoustic" ω_1, ω_2. Это AFMR, $\vec{k}=0$.

(17/V) Послал Summary AFMR - Mn_2Au Calc. to Ruben и last version BP paper to Oksana.

(18/V) Оксана вернула BP-article with revisions. Подписал Comp. documents у Elena (new Secretary), нашел ее в/о Cubry. Meeting (важного содержания) in my office и вопросы по каким нормативным актам должна Spanish trips (MV/Elena и на Real Sociedad).

(19/V) Работа над BP manuscript (послал to Co-authors). Оформил trip to Alco Bendas.

Вторая Cover lett. и edge SW manuscript (Appl. Ph. Rev.) и послан to Pavel.

(20/V) Ответы Referees Sk-barriers/Riveros/PRA — послан Оксане.
С Л. В «Aldi» Diartzum.

(24/V) Edited BP manuscript, wrote cover lett. и submitted manuscript to Sc. Rep.
Анализ цунами micromag. approach к singular Block points.
Zoom и b. card terminal Solchaga Mag. 2021 — обсуждение с Антоном.

(24/V) Checked BP manuscript и re-subm. to Sc. Rep. С Л., Ю. В Толоза

(25/V) Анализ APL'2021 Han-Im по V-AV pairs у BP. Ермиз. "Hyundai", "KIA".

(26/V) С Л. Diartzum «Aldi», «Al Campo». Физика с Ант.

(27/V) С Л. и Ю. в Olaberria «Promodès» KIA Hyundai, «Carrefour». Рев, JAP
Cho-Kim coupl. vortex dots dynamics. Ответы Referees to Bergenza-Skyrm. Nanoscale и letter to co-authors.

(28/V) С Л. и Ю. в Lasarte «Mitsubishi» Eclipse, Outlander и «Nissan» Ondarroa. Физика с Ант.

(29/V) Slides Nanowire project meeting. Физика с Ант.

(30/V) 9-13/V Вычисление factors AFMR of Mn_2Au ultrathin film с толщиной $L = c$ - one lattice period. Four layers → Four frequencies.
Если есть неколлинеарность ⊥ Oz (ось c), то у 1st layer Mn и 4th layer Mn появляется gandum. J_2 - bond, которая принадлежит to two unit cells. $E_{ex}(J_2) = -\vec{m}_2 \cdot [J_2 \vec{m}_3 \cdot 4] - \vec{m}_1 J_2 \vec{m}_0 \cdot \frac{1}{2} - \vec{m}_4 \cdot J_2 \cdot \vec{m}_5 \cdot \frac{1}{2}$
$\vec{m}_0$ - Mn layer над 1st Mn, $\vec{m}_5$ - Mn layer под 4th Mn layer. $\vec{m}_0 = \vec{m}_4$, $\vec{m}_5 = \vec{m}_1$ (неколлинеарность ⊥ Oz)

$E_{ex}(J_2) = -J_2 \vec{m}_2 \cdot \vec{m}_3 - J_2 \vec{m}_1 \cdot \vec{m}_4$. Было $E_{ex}(J_2) = -J_2 \vec{m}_2 \cdot \vec{m}_3$ (thin film)

$\underline{E_{ex}(Bulk)}_{u.c.} = -4J_1 \vec{m}_1 \cdot \vec{m}_2 - 4J_1 \vec{m}_3 \cdot \vec{m}_4 - J_2 \vec{m}_2 \cdot \vec{m}_3 - J_2 \vec{m}_1 \cdot \vec{m}_4$

Для Bulk Mn_2Au $\vec{m}_1 = \vec{m}_3$, $\vec{m}_2 = \vec{m}_4$, $\vec{m}_i$ - magnetization of Mn atom in i-layer, $i = 1,2,3,4$

$E_{ex}(bulk) = -4J_1 \vec{m}_2 \cdot \vec{m}_3 - 4J_1 \vec{m}_2 \cdot \vec{m}_3 - J_2 \vec{m}_2 \cdot \vec{m}_3 - J_2 \vec{m}_2 \cdot \vec{m}_3$

$E_{ex}(bulk) = -2(4J_1 + J_2) \vec{m}_3 \cdot \vec{m}_2$, $\frac{E_{an}}{u.c.} = 2K_z \left[(m_2^z)^2 + (m_3^z)^2\right] - 2K_y \left[(m_2^y)^2 + (m_3^y)^2\right]$

$\underline{E(bulk)}_{u.c.} = -2(4J_1 + J_2) \vec{m}_2 \cdot \vec{m}_3 + 2K_z \left[(m_2^z)^2 + (m_3^z)^2\right] - 2K_y \left[(m_2^y)^2 + (m_3^y)^2\right]$ (AFMR)

Определить $E(film)$ 9/V или $\vec{m}_1 = \vec{m}_3$, $\vec{m}_2 = \vec{m}_4$, и $J_2 \to 2J_2$ сводится к $E(bulk)$
Уравнение движения 4-х УД для $\vec{V}_i$ от 11/V, $\vec{m}_{1,3} = \hat{y} + \vec{V}_{1,3}$, $\vec{m}_{2,4} = -\hat{y} + \vec{V}_{2,4}$
взяв как $\vec{V}_1 = \vec{V}_3$, $\vec{V}_2 = \vec{V}_4$:

$-i\omega \vec{V}_2 = -\hat{x}(V_3^z + V_2^z) + \hat{z}(V_3^x + V_2^x) + \varkappa_z V_2^z \hat{x} + \varkappa_y \left[-V_2^x \hat{z} + V_2^z \hat{x}\right] + \varepsilon\left[-\hat{x}(V_2^z + V_3^z) + \hat{z}(V_2^x + V_3^x)\right]$

$-i\omega \vec{V}_3 = \hat{x}(V_3^z + V_2^z) - \hat{z}(V_3^x + V_2^x) - \varkappa_z V_3^z \hat{x} + \varkappa_y \left[V_3^x \hat{z} - V_3^z \hat{x}\right] - \varepsilon\left[-\hat{x}(V_2^z + V_3^z) + \hat{z}(V_2^x + V_3^x)\right]$

$$\begin{pmatrix} 0 & -1+\varkappa_z+\varkappa_y-\varepsilon & 0 & -1-\varepsilon \\ 1-\varkappa_y+\varepsilon & 0 & 1+\varepsilon & 0 \\ 0 & 1+\varepsilon & 0 & 1-\varkappa_z-\varkappa_y+\varepsilon \\ -1-\varepsilon & 0 & -1+\varkappa_y-\varepsilon & 0 \end{pmatrix} \begin{pmatrix} V_2^x \\ V_2^z \\ V_3^x \\ V_3^z \end{pmatrix} i = \omega \begin{pmatrix} V_2^x \\ V_2^z \\ V_3^x \\ V_3^z \end{pmatrix}, \quad \hat{A}\vec{V} = -\lambda \vec{V}, \ \lambda = i\omega$$

$1+\varepsilon = \beta$

$\det\left(A_{nn'} + i\omega \delta_{nn'}\right) = 0$

$$\begin{vmatrix} \lambda & \varkappa_z+\varkappa_y-\beta & 0 & -\beta \\ -\varkappa_y+\beta & \lambda & \beta & 0 \\ 0 & \beta & \lambda & \beta-\varkappa_z-\varkappa_y \\ -\beta & 0 & -\beta+\varkappa_y & \lambda \end{vmatrix} = \lambda \begin{vmatrix} \lambda & \beta & 0 \\ \beta & \lambda & \beta-\varkappa_z-\varkappa_y \\ 0 & \varkappa_y-\beta & \lambda \end{vmatrix} - (\varkappa_z+\varkappa_y-\beta) \begin{vmatrix} \beta-\varkappa_y & \beta & 0 \\ 0 & \lambda & \beta-\varkappa_z-\varkappa_y \\ -\beta & \varkappa_y-\beta & \lambda \end{vmatrix} +$$

$$+ \beta \begin{vmatrix} \beta-\varkappa_y & \lambda & \beta \\ 0 & \beta & -\beta+\varkappa_y \\ -\beta & 0 & \lambda \end{vmatrix} = \lambda\left[\lambda^3 - \lambda\beta^2 - \lambda(\beta-\varkappa_z-\varkappa_y)(\varkappa_y-\beta)\right] - (\varkappa_z+\varkappa_y-\beta)\times$$

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$\times\left[\lambda^2(\beta-x_y)-\beta^2(\beta-x_2-x_y)+(\beta-x_y)^2(\beta-x_2-x_y)\right]+\beta\left[-(\beta-x_y)^2\beta-\beta\lambda^2+\beta^3\right]=$

$=\lambda^4+\lambda^2\left[-\beta^2-(\beta-x_2-x_y)(x_y-\beta)-(x_2+x_y-\beta)(\beta-x_y)\right]+\beta^2\left[\beta^2-(\beta-x_y)^2\right]-$

$-(x_2+x_y-\beta)(\beta-x_2-x_y)\left[(\beta-x_y)^2-\beta^2\right]=\lambda^4+\lambda^2\left[-2\beta^2+2(\beta-x_y)(\beta-x_2-x_y)\right]+$

$+\left[\beta^2-(\beta-x_y)^2\right]\left[\beta^2+(x_2+x_y-\beta)(\beta-x_2-x_y)\right]$

$\det(\hat{A}+\lambda\hat{I})=\lambda^4-2\lambda^2\left[\beta^2-(\beta-x_y)(\beta-x_2-x_y)\right]+\left[\beta^2-(\beta-x_y)^2\right]\left[\beta^2-(\beta-x_2-x_y)^2\right]$

$x_y<0,\ x_2<0.\quad 2\lambda_{1,2}^2=2\left[\beta^2-(\beta-x_y)(\beta-x_2-x_y)\right]\pm\sqrt{D},\quad \beta=1+\varepsilon$

$D=4\left[\beta^2-(\beta-x_y)(\beta-x_2-x_y)\right]^2-4\left[\beta^2-(\beta-x_y)^2\right]\left[\beta^2-(\beta-x_2-x_y)^2\right]=$

$=4\left[\beta^4+(\beta-x_y)^2(\beta-x_2-x_y)^2-2\beta^2(\beta-x_y)(\beta-x_2-x_y)\right]-4\left[\beta^4-\beta^2(\beta-x_2-x_y)^2-\right.$

$\left.-\beta^2(\beta-x_y)^2+(\beta-x_y)^2(\beta-x_y-x_2)^2\right]=4\left[-2\beta^2(\beta-x_y)(\beta-x_2-x_y)+\beta^2(\beta-x_2-x_y)^2+\right.$

$\left.+\beta^2(\beta-x_y)^2\right]=4\beta^2\left[(\beta-x_y)+(\beta-x_y-x_2)\right]^2=4\beta^2x_2^2,\quad x_2<0.$

$2\lambda_{1,2}^2=2\left[\beta^2-(\beta-x_y)(\beta-x_2-x_y)\right]\pm\sqrt{4\beta^2x_2^2}=2\left[\beta^2-\beta^2+\beta(x_2+x_y)+x_y\beta-\right.$

$\left.-x_y(x_y+x_2)\right]\pm2\beta|x_2|,\quad \lambda_{1,2}^2=\beta(x_2+2x_y)\pm\beta|x_2|$

$\lambda_1^2=\beta(x_2+2x_y)-\beta|x_2|=\beta(x_2+2x_y+x_2)=2\beta(x_2+x_y)<0$

$\lambda_2^2=\beta(x_2+2x_y)+\beta|x_2|=\beta(x_2+2x_y-x_2)=2\beta x_y<0,\quad \lambda^2=-\omega^2.$

$1\rightleftarrows2$ Законы AFMR bulk Mn_2Au $\omega_1^2=2\beta|x_y|,\ \omega_2^2=2\beta|x_2+x_y|$ AFMR

Разобраться с нормировкой констант и x_2, x_y (как $\propto SJ_1$?) из $E(bulk)$.

$\beta=1+\varepsilon,\ \varepsilon=J_2/(4J_1)>0$. Вывод: 1) $\varepsilon=0, J_2=0$ отсюда нет щели законы $\omega_1,\omega_2\neq0$, not degenerated; 2) the lowest gap $\omega_1\sim\sqrt{|x_y|}$ is determined by x_y, highest gap $\omega_2\sim\sqrt{|x_2|}$ is determined by x_2 ($|x_2|>>|x_y|$)

Bulk ω_1,ω_2 согласуются с thin film ω_1,ω_2 от 16/V (except normalization)

→ Сравнить их с exp. Arana '2017 (x_2, x_y домножить).

Bulk $\omega_{1,2}$ зависят от β, т.е. от $J_{eff}=(4J_1+J_2)$.

Atomistic LLG eq. $\frac{d\vec{S}_i}{dt}=-\gamma\vec{S}_i\times\vec{H}_{eff}^{\,i},\ \vec{S}_i=\frac{\vec{M}_s}{\mu_s}$ - unit m. vector in site i,

$\vec{H}_{eff}^{\,i}=-\frac{1}{\mu_s}\frac{\partial E}{\partial\vec{S}_i}$, E - energy, $\mu_s=|\vec{\mu}_s|$ - atomic mag. moment.

Уравнение ЛЛГ $\partial/\partial\tau\ -\frac{1}{\gamma}\frac{d\vec{m}_i}{dt}=\vec{m}_i\times\vec{H}_{eff}^{(i)},\ H_{eff}^{(i)}=-\frac{1}{\mu_s}\frac{\partial E}{\partial\vec{m}_i}$, E - total energy

$\vec{m}_i$ - unit vector на сайте i:

$E=\left(\frac{E}{V_{uc}}\right)V_{uc}$ $\quad\mu_s\vec{H}_{eff}^{(i)}=-\frac{\partial E}{\partial\vec{m}_i}=-V_{uc}\frac{\partial(E/V_{uc})}{\partial\vec{m}_i}$, $M_s'\vec{H}_{eff}^{(i)}=-\frac{\partial(E/V_{uc})}{\partial\vec{m}_i}$

$M_s'=\mu_s/V_{uc}$, $\mu_s=\mu_{Mn}=4\mu_B$. Для 11/V для одной $M_s\vec{H}_{eff}^{(i)}=-\frac{\partial}{\partial\vec{m}_i}(E/V)$

$-\frac{M_s}{\gamma}\dot{\vec{m}}_i=\vec{m}_i\times\vec{M}_s\vec{H}_{eff}^{(i)}=\vec{m}_i\times\frac{\partial}{\partial\vec{m}_i}\left(\frac{E}{V}\right)(-1)$, а надо $-\frac{M_s}{\gamma}\dot{\vec{m}}_i=\vec{m}_i\times\frac{\partial E}{\partial\vec{m}_i}(-1)$

или $-\frac{M_s'}{\gamma}\dot{\vec{m}}_i=-\vec{m}_i\times\frac{\partial}{\partial\vec{m}_i}\left(\frac{E}{V}\right)$. $M_s'=\mu_s/V$, $M_s(11/V)=2\mu_s/V=2M_s'$

$M_s'=\mu_s/V$, $V=V_{uc}$, M_s' не есть M_s - magnetization of Sublattice как 11/V

Frequency scale $\omega_J=\frac{\gamma}{M_s'}\frac{(4J_1)}{V}$ будет $\omega_J'=\frac{\gamma}{M_s'}\cdot\frac{(4J_1)}{V}=\frac{1}{2}\omega_J$

$\frac{1}{2\pi}\omega_J'=\frac{1}{2}(\omega_J/2\pi)=\frac{1}{2}2,262\ THz=4,131\ THz$ (?)

→ Все частоты thin film равны 16/V надо умножить на V/2 (!)

$\frac{\omega_1}{2\pi}=64\ GHz$, $\frac{\omega_2}{2\pi}=0,64\ THz$, $\frac{\omega_3}{2\pi}=3,41\ THz$, $\frac{\omega_4}{2\pi}=3,45\ THz$.

2) Будет ω_3,ω_4 близки к эксп. Данным Arana '2017 : 3,24 и 4,20 THz

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Bulk Mn_2Au (или $\vec{m}_1 = \vec{m}_3$, $\vec{m}_2 = \vec{m}_4$). Это частота ω_1, ω_2 as above
нолуч. из $\vec{J}_1 = \vec{J}_3$, $\vec{J}_2 = \vec{J}_4$ соотв., но уже большие. J_2 bonds nonbasic, $J_2 \approx 2J_2$
Frequency scale $\omega_J' = 1/2 \cdot \omega_J = 4.131$ THz, $\gamma_1, \gamma_2 \to$ то же

$(\omega_1/\omega_J')^2 \Rightarrow 2\beta|\gamma_1|$, $(\omega_2/\omega_J')^2 = 2\beta|\gamma_1+\gamma_2| = 2\beta|\gamma_2|$, $\beta = 1 + \frac{2J_2}{4J_1} = 1+\varepsilon'$

Bulk freq. AFMR, $\omega_1 = \omega_J'\sqrt{2\beta|\gamma_1|}$, $\omega_2 = \omega_J'\sqrt{2\beta|\gamma_2|}$ $\varepsilon' = 2\varepsilon = 2\frac{J_2}{4J_1}$

Thin film при $J_2 = 0$ (16/2), $\omega_1 = \omega_J\sqrt{2|\gamma_1|}$, $\omega_2 = \sqrt{2|\gamma_2|}\,\omega_J$, где $\omega_J \to \omega_J'$. или $\vec{m}_1 = \vec{m}_3$, $\vec{m}_2 = \vec{m}_4$

При J_2 bulk и thin film ω_1, ω_2 совпадают. При $J_2 = 0$, $\omega_{1,2}$ (bulk) надо $\times\sqrt{1+\varepsilon'} = \sqrt{1+\frac{2J_2}{4J_1}} = \sqrt{1+\frac{1}{2}\frac{532}{356}} = 1.29$.

Для bulk Mn_2Au предполагаемое $\vec{m}_1 = \vec{m}_3$, $\vec{m}_2 = \vec{m}_4$, ведущее к двум AFMR frequencies измерено. Отклонение от него.

Было thin film $-E_{ex}(J_2) =$ стало bulk $-E_{ex}(J_2) = -J_2\vec{m}_2\cdot\vec{m}_3 - J_2\vec{m}_1\cdot\vec{m}_4$ extra bond bulk
$= -J_2\vec{m}_2\vec{m}_3$

$M_S'\vec{H}_{eff}^{(1)} = -\frac{\partial E}{\partial \vec{m}_1}\frac{1}{V} = (old) + J_2\vec{m}_4$, $M_S'\vec{H}_{eff}^{(4)} = -\frac{1}{V}\frac{\partial E}{\partial \vec{m}_4} = J_2\vec{m}_1 + (old)$

$-\frac{M_S'}{\gamma}\dot{\vec{m}}_1 = \vec{m}_1 \times M_S'\vec{H}_{eff}^{(1)} = \vec{m}_1 \times [(old) + J_2\vec{m}_4]$, $-\frac{M_S'}{\gamma}\dot{\vec{m}}_4 = \vec{m}_4 \times [(old) + J_2\vec{m}_1]$

$\vec{m}_1 \times \vec{m}_4 = (\hat{y} + \vec{v}_1) \times (-\hat{y} + \vec{v}_4) = \hat{y}\times\vec{v}_4 + \hat{y}\times\vec{v}_1 = \hat{y}\times(\vec{v}_1 + \vec{v}_4) = \hat{y}\times\vec{v}$

$\hat{y}\times\vec{v} = \hat{y}\times(v_x\hat{x} + v_z\hat{z}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 1 & 0 \\ v_x & 0 & v_z \end{vmatrix} = \hat{x}v_z - \hat{z}v_x = \hat{x}(v_1^z + v_4^z) - \hat{z}(v_1^x + v_4^x)$

$-\frac{M_S'}{\gamma}\dot{\vec{m}}_1 = -\frac{M_S'}{\gamma}\dot{\vec{v}}_1 = (old) + J_2\vec{m}_1\times\vec{m}_4 = (old) + \hat{x}J_2(v_1^z + v_4^z) - \hat{z}J_2(v_1^x + v_4^x)$

$-\frac{M_S'}{\gamma}\dot{\vec{m}}_4 = -\frac{M_S'}{\gamma}\dot{\vec{v}}_4 = (old) + J_2\vec{m}_4\times\vec{m}_1 = (old) - \hat{x}J_2(v_1^z + v_4^z) + \hat{z}J_2(v_1^x + v_4^x)$ $J_2/4J_1 = \varepsilon$

$-\frac{M_S'}{\gamma}\dot{v}_{1x} = \varepsilon(v_1^z + v_4^z)$, $-\frac{M_S'}{\gamma}\dot{v}_{1z} = (old) + (-\varepsilon)(v_1^x + v_4^x)$

$-\frac{M_S'}{\gamma}\dot{v}_{4x} = (old) - \varepsilon(v_1^z + v_4^z)$, $-\frac{M_S'}{\gamma}\dot{v}_{4z} = (old) + \varepsilon(v_1^x + v_4^x)$

Matrix $\hat{A}$ (11/2) для thin film использовался to describe bulk Mn_2Au. Добавился
$A_{1x,1z} = A_{1x,4z} = \varepsilon$, $A_{4x,1z} = A_{4x,4z} = -\varepsilon$
$A_{1z,1x} = A_{1z,4x} = -\varepsilon$ и $A_{4z,1x} = A_{4z,4x} = \varepsilon$.

$$\begin{array}{c} \\ 1x \\ 1z \\ \vdots \\ 4y \\ 4z \end{array}\begin{pmatrix} 0 & \varepsilon & 0 & 0 & 0 & 0 & 0 & \varepsilon \\ -\varepsilon & 0 & 0 & 0 & 0 & 0 & -\varepsilon & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -\varepsilon & 0 & 0 & 0 & 0 & 0 & -\varepsilon \\ \varepsilon & 0 & 0 & 0 & 0 & 0 & \varepsilon & 0 \end{pmatrix}\begin{pmatrix} v_1^x \\ v_1^z \\ \vdots \\ v_4^x \\ v_4^z \end{pmatrix} i = \omega\begin{pmatrix} v_1^x \\ v_1^z \\ \vdots \\ v_4^x \\ v_4^z \end{pmatrix}$$
$$\begin{array}{cccccccc} 1x & 1z & 2x & 2z & 3x & 3z & 4x & 4z \end{array}$$

Спектр AFMR bulk Mn_2Au: $\frac{\omega_1}{2\pi} = 0.0179$ $\frac{\omega_2}{2\pi} = 0.179$ $\frac{\omega_3}{2\pi} = 1.1627$; $\frac{\omega_4}{2\pi} = 1.1694$
Абсолютные частоты при $\omega_J' = 4.131$ THz: $f_1 = 74$ GHz, $f_2 = 0.740$ THz, $f_3 = 4.80$ THz, $f_4 = 4.83$ THz
Для bulk Mn_2Au ω_1 зависит от J_2, поэтому 3-модель может работать только как ω_1 (thin film) от J_2 не зависит, как у Ruben в simulations. Т.е. $\omega_1 \neq f(J_2)$ – особенность ultrathin film, the lowest ω_1, $\Rightarrow$ $\nu_{max} \neq f(J_2)$ DW

(31/X) переписка с Ruben y Flore. С п. в. "Lidl" Hernani и Carrefour на Рапуха в Ann.

(1/VI) Online Nanowire project meeting: 1) analog. FMR spectra of nanowires by Navas; 2) Block point DW velocity up to 14 km/s simulations by Oksana.
3) multisegmented F/N (3)(5)(7)(9) nanowires, vortex state – excl. spectrum Oks.

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(6/IX) Запис Cristina UAH/EIMI о new PhD position. E-mail R. Zeballos.

(7/IX) с А. и А. В Donostia, Ondarreta, Ñam. Article Dobrovolskiy "Road map SW logics" / IEEE VM – corrections, 1 page.

(8/IX) Skype с Ruben: position для Ricardo с Jan. 2022 на max срок, ≥ 6 мес. E-mail to Cristina, new position.

(9/IX) подготовил и submitted Anexo I для Ricardo's position – 50% Investigador / No doctor, Jan. – Dec. 2022.

Обсуждение с Farkhad по simulations of nanowire with BP domain wall. BP-DW

Note: про new spacer = d_s загадка сборника в Gus APL'2005 (later в APL'2005 v. core was ignored, в nanowires $R_c \approx \frac{1}{2}R$)

$p_1 p_2 = -1$, но $c_1 c_2 = +1$ у Фархад – DW похожа на vortex Bloch point.

Gus' APL (F/N/F): $W_{int}^{dyn} = (\dots) c_1 c_2$, min at $c_1 c_2 = -1$, opposite vortex chiralities only $p_1 p_2 = +1$ case was considered. Фархад для BP-DW

Видно $p_1 p_2 = -1$ и $c_1 c_2 = +1$ для centered DW and $d_s = 0$, что объясн. результаты Фархад BP existance at $d_s = 0$. Возможно, учет finite v. core R_c и gaps W_{int} → min at $c_1 c_2 = +1$ и $p_1 p_2 = -1$ дают без BP.

BP-DW state is stable – при L (wire length) > $L_c(R)$ и metastable – at $L < L_c(R)$, short nanowires.

Thiaville ansatz BP-DW (2003) cylindrical det: $m_x = -\frac{y}{\varrho}\sqrt{1-m_z^2}$, $m_y = \frac{x}{\varrho}\sqrt{1-m_z^2}$; $m_z = \frac{z}{r}\frac{b^2}{b^2+\varrho^2}$

$r^2 = x^2+y^2+z^2$, $\varrho^2 = x^2+y^2$, $b \approx 10$ nm. $m_\varrho = m_x \cos\varphi + m_y \sin\varphi$, $m_\varphi = m_y \cos\varphi - m_x \sin\varphi$

$m_\varrho = -\frac{y\cos\varphi}{\varrho}\sqrt{(\cdot)} + \frac{x}{\varrho}\sin\varphi\sqrt{(\cdot)} = 0$, $m_\varphi = \frac{x}{\varrho}\cos\varphi\sqrt{(\cdot)} + \frac{y}{\varrho}\sqrt{(\cdot)}\sin\varphi = \sqrt{1-m_z^2}$

Thiaville ansatz cyl. coordinates: $m_\varphi = \sqrt{1-m_z^2}$, $m_\varrho = 0$, $m_z = \frac{z}{r}\frac{b^2}{(b^2+\varrho^2)}$

$\vec{m} = (m_\varrho, m_\varphi, m_z) = \vec{m}(\vec{r}) = \vec{m}(\varrho, z)$, axial symmetry, no dependence on φ.

Вообще, Thiaville ansatz с $m_\varphi = m_\varphi(z^2, \varrho^2)$, $m_\varphi(z>0) = m_\varphi(z<0)$, $m_\varphi = m_\varphi(z)$ even относительно vortex-BP (BP-DW). $m_z(z)$ is odd.

Соавтор Wang-Brataas PRB'2020 Hopf index (Hopfion) $Q_H = \frac{1}{(4\pi)^2}\int dV \vec{B}\cdot\vec{A} = \frac{n}{4\pi}\int_{-\infty}^{\infty} dz \int_0^{\infty} dr \sin\Theta \left[\frac{\partial\Theta}{\partial z}\frac{\partial\Psi}{\partial r} - \frac{\partial\Theta}{\partial r}\frac{\partial\Psi}{\partial z}\right]$, где $\vec{m} = \vec{m}(\varrho, z)$, $r \to \varrho$, $\vec{r}(\varrho, \varphi, z)$, $\vec{m}(\Theta, \Phi)$,

$\Phi(\varrho, \varphi, z) = n\varphi + \gamma(\varrho, z)$ γ-helicity. Hopfion $n = 1$, axially sym. $\vec{m} = \vec{m}(\varrho, z)$

$Q_H = \frac{1}{4\pi}\int dz\, d\varrho\, \vec{m}\cdot\left(\frac{\partial\vec{m}}{\partial z}\times\frac{\partial\vec{m}}{\partial\varrho}\right)$, $\vec{m} = m_\varrho\hat{\varrho} + m_\varphi\hat{\varphi} + m_z\hat{z} = (0, m_\varphi, m_z)$

$\frac{\partial\vec{m}}{\partial z}\times\frac{\partial\vec{m}}{\partial\varrho} = \left(\frac{\partial m_\varphi}{\partial z}\hat{\varphi} + \frac{\partial m_z}{\partial z}\hat{z}\right)\times\left(\frac{\partial m_\varphi}{\partial\varrho}\hat{\varphi} + \frac{\partial m_z}{\partial\varrho}\hat{z}\right) = \frac{\partial m_\varphi}{\partial z}\frac{\partial m_z}{\partial\varrho}\hat{\varphi}\times\hat{z} +$

$+ \frac{\partial m_z}{\partial z}\frac{\partial m_\varphi}{\partial\varrho}\hat{z}\times\hat{\varphi} = |\hat{\varphi}\times\hat{z} = \hat{\varrho}| = \left[\frac{\partial m_\varphi}{\partial z}\frac{\partial m_z}{\partial\varrho} - \frac{\partial m_z}{\partial z}\frac{\partial m_\varphi}{\partial\varrho}\right]\hat{\varrho} = [\dots]\hat{\varrho}$

$Q_H = \frac{1}{4\pi}\int dz\int d\varrho\, \vec{m}\cdot\left(\frac{\partial\vec{m}}{\partial z}\times\frac{\partial\vec{m}}{\partial\varrho}\right) = \frac{1}{4\pi}\int dz\, d\varrho\, \vec{m}\cdot[\dots]\hat{\varrho} = \frac{1}{4\pi}\int dz\, d\varrho\, m_\varrho[\dots]$

$Q_H = 0$, т.к. $m_\varrho = 0$ для Thiaville-BP ansatz.

→ Вывод: $Q_H = 0$ для Thiaville vortex-Block point. Нет. Hopfion Q_H не совпад. Q_H-BP

Поэтому Thiaville BP simulated by Farkhad, где $m_z(z) \sim z$ odd, $m_\varphi(z) \sim z^2$ even

если они Gus APL'ов BP ($d_s \neq 0$): $m_z(z)$ – even, $p_1 p_2 = +1$, $m_\varphi(z)$ odd, т.е. $c_1 c_2 = -1$. Энергия какая BP ближе? Thiaville BP или Gus BP?

В Gus' APL только $p_1 p_2 = +1$ case, $W_{int}^{dyn} \sim (\dots) c_1 c_2$ и оба dyn. cases $c_1 c_2 = +1$, $c_1 c_2 = -1$ are considered. W_{int} (static) аналогичен по

(10/IX) Отмечаю, что в simulations показано $W(C_1C_2=-1) < W(C_1C_2=+1)$ и dynamics исследована для $P_1P_2=+1$, $C_1C_2=-1$ case. F/N/F dot.
При $d_s \to 0$ (сильная нанопроволока/dot) $C_1C_2=-1$ соответствует giant exch. на граничных layers и исчезает. Остается $C_1C_2=+1$. Это BP-DW отражается только для $P_1P_2=-1$. Case $P_1P_2=-1$, $C_1C_2=+1$ и отмечен Thiaville'2003 и Парраг. у Паррага $L=250-300$ nm, $R=60$ nm — pillar, FeCo.
// у Паррага „DW pinning" — $d_s=2.5$ nm и $P_1P_2=-1$, $C_1C_2=+1$ dynamics layers.
Хотя в APL'2005 показано, что static state $P_1P_2=+1$, $C_1C_2=-1$ имеет min W. Состояние $P_1P_2=+1$, $C_1C_2=-1$ изобретено (без core). Состояние $P_1P_2=-1$, $C_1C_2=+1$ нестабильно рассчитано. Это R_c сравнимо с R, т.е. core надо учитывать явно.

BP in spherical particle: $\Theta = \rho\theta + \frac{\pi}{2}(1-\rho)$, $\Phi(\vec{r}) = \varphi + \gamma(r,\theta)$, $\vec{r}(r,\theta,\varphi)$

Sph-BP in cyl. coordinates (10/IX 2020): $m_z = \rho\cos\theta$, $m_\varphi = \sin\theta\sin\gamma$, $m_s = \sin\theta\cos\gamma$

Sph-BP: $m_s = \sin\theta\cos\gamma$, $m_\varphi = \sin\theta\sin\gamma$, $m_z = \rho\cos\theta$, $\cos\theta = \frac{z}{r}$
Thiaville-BP (Cylinder): $m_s = 0$, $m_\varphi = \sqrt{1-m_z^2}$, $m_z = \frac{z}{r}\frac{\beta^2}{(\beta^2+s^2)}$

m_z - sph и m_z - cyl. похожи, а вдоль оси $\beta z, s=0$, совпадают $m_z = \pm\frac{z}{r}$.
При $\gamma = \pi/2$ Sph-BP → Vortex-BP (!) $m_s=0$, $m_\varphi = \sin\theta$, $m_z = \rho\cos\theta$.
$Q_H(\gamma=\text{const}) = 0$, поэтому Q_H (Thiav-BP) $=0$.
В плоскости $z=0$, $\theta=\pi/2$, Sph-BP: $m_s=\cos\gamma$, $m_\varphi=\sin\gamma$, $m_z=0$.
$\gamma = \gamma(r,\theta) = \gamma(r,\pi/2) \neq \pi/2$ Thiav.-BP: $m_s=0$, $m_\varphi=1$, $m_z=0$
Из-за $\gamma(r,\theta) \neq \pi/2$, Sph-BP сложнее, чем Thiaville cyl. BP и имеет $Q_H \neq 0$.

(11-12/IX) Отдых дома. Были в Getaria/Zarautz с Л. и Ю.

(13/IX) Переписка с Ricardo о Mn_2Au article и Окс. о BP-DW ansatz in cylindrical nanowires.

(14-19/IX) Поездка с Л. в Oviedo/Gijon. 2 seminar talks „BP" и „Discussions" на семинаре Чудновского Victor в la Prida в Univ. Oviedo.
Обсуждения с Aurelio X-ray measurements of 3D $\vec{m}(\vec{r})$ in thin films (BP, topol. charge через em.mag. field (flux) — Donelli
Package Aurelio о 3D topol. charge (Hopf index) of Bloch points. Paper.

(20/IX) Послал Ricardo/Ruben свое мнение о Mn_2Au AFM model, com. to submit the manuscript to Phys. Rev. Appl.
// Zoom discussion of Felipe/Oksana of F/N/F/N/... Segmented FeCo/Cu nanowires. Похоже, что все observed SW modes in interval 0-20 GHz — vortex gyromodes related to motion of v. core. $L=150$ nm, $R=30$ nm. $\Delta L=50$ nm (spacer). M. coupling is small, поэтому не сдвиг на 0,1 GHz. Мне (1) посмотреть dyn. F/N/F coupling; 2) найти gyro-tropic band $\omega(k)$ для ∞-ensemble of cylindrical dots.

(21/IX) Meeting с Аратха, обсуждение discrete calc. BP-energy, top. charge. Evgeni Elena Travel reports - Oviedo. Послал signed Anexo I to Cristina.

(22/IX) Переписка с M. Krawczyk. Нужно сделать в Poznan Oct. 2/Nov. 2.

(23-24/IX) Skype с Парраг и Диего BP DW dynamics. Отдых. Были.

(25-26/IX) С Л. в Carrefour-Market: Отдых дома.

(27/IX) Ikerbasque meeting in Getaria. Беседа с Джунов о new GB project. Переписка о Экзамен/Nanoscale MS (rejected).

(28/IX) Послал GV+ group project (Zhukov), переписка с Ricardo y Elena.
Запрос Парраги проблемы BP-DW ansatz in cyl. nanowire

29/IX) Переписка с Ингрид и Аркадием (GV-project).

30/IX) Antigen test в Domostia. С Л.и Ю. на обед в La Perla. Ruben - pre-enrolment Javier to UPV/EHU PhD program.

1/X) Переписка с Марией и Лена Т. о поездке в Poznan.

2/X) Перелет в Poznan (BIO → MUC → POZ „Lufthansa"). В „Aldi" с Ланой. Расчет W_G - finite dot

3/X) С Ланой в Puntarenas „Zindow" rest. Расчет W_G cyl. dot.

4/X) UAM cafeteria - обед. С Л. в „Kaufland" (former „TESCO"). W_G - cyl. dot - сворачение eq. Thiele с eq. Mathieu, функции магнитостатики.

5/X) Расчет gyrotropic vortex dynamics thick cyl. dot, см. 15-16/IX 2020 и ранее.

Ding' 2014: $W[\vec{S}(z)] = \frac{1}{2}V\int dz\left[y\left(\frac{\partial\vec{S}}{\partial z}\right)^2 - \mu\vec{S}^2\right] + \frac{1}{2}V\int dz\,dz'\,\chi(zz')\vec{S}(z)\cdot\vec{S}(z')$

$y = M_S^2\left(Le/L\right)^2\left\{\ln\left[R/R_c\right] + 5/4\right\}$, $\mu = 2M_S^2(Le/R)^2$, $Le^2 = 2A/M_S^2$, $R_c(L) = 0.68\,Le\,(L/Le)^{1/3}$

$\rightarrow \chi(zz') = 8\pi M_S^2\beta\int_0^\infty dk\, e^{-\beta k|z-z'|}\,I^2(k)$, $I(k) = \int_0^1 ds\, s\, J_1(ks)$, $V = \pi R^2 L$.

$W_{ex} = \frac{1}{2}yV\int dz\left(\frac{\partial\vec{S}}{\partial z}\right)^2$, $W_m = \frac{1}{2}V\int dz\,dz'\,\chi(zz')\vec{S}(z)\vec{S}(z')$.

Тогда как в Gus' Sci. Rep. 2015: $W_{ex} = \frac{\lambda}{2}R^2L\int dz\left(\frac{\partial\vec{S}}{\partial z}\right)^2$, $W_m = \frac{1}{2}R^2L\int dz\,dz'\,\chi_{Gus}(zz')\vec{S}(z)\vec{S}(z')$

$\chi_{Gus} = 8\pi^2 M_S^2\beta\int dk\, e^{-\beta k|z-z'|}\,J^2(k) = \pi\chi_{Ding}$, $\lambda = \pi y$.

Ding'14: $g_{\alpha\beta}\frac{dX_\beta}{dt} = -\frac{\delta E}{\delta X_\alpha}$, $E = \int dz\,\varepsilon$ - energy per unit dot thickness.

$g_{\alpha\beta} = \frac{M_S}{\gamma}\int dz\,\vec{m}\left(\partial_\alpha\vec{m}\times\partial_\beta\vec{m}\right)$, $\partial_\alpha = \partial/\partial X_\alpha$, $g_{\alpha\beta}$ - это $G_{\alpha\beta}$ per unit thickness $g = G/L$.

Mag. note energy $W = L\int dz\,E = L\int dz\int d^2s\,\varepsilon$, z-dimensionless, in units of L.

$G_{\alpha\beta}\frac{dX_\beta}{dt} = -\frac{\delta W}{\delta X_\alpha}$, $\frac{\delta W}{\delta X_\alpha} = \frac{\partial W}{\partial X_\alpha} - \frac{\partial}{\partial z}\frac{\partial W}{\partial(\partial X_\alpha/\partial z)}$

Skew-symmetric matrix $\hat{G} = [G_{\alpha\beta}] = \begin{pmatrix} 0 & -G_z & G_y \\ G_z & 0 & -G_x \\ -G_y & G_x & 0 \end{pmatrix} = \begin{pmatrix} 0 & -G_z & 0 \\ G_z & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\hat{G}\vec{X} = \vec{G}\times\vec{X}$, $\vec{G} = (G_x, G_y, G_z)$ $\rightarrow\!\!\rightarrow \vec{G} = (0,0,G_z)$, $G_{xy} = -G_z$, $G_{yx} = G_z$.

$G_{\alpha\beta}\dot{X}_\beta = \hat{G}\dot{\vec{X}} = \vec{G}\times\dot{\vec{X}}$

$\vec{G}\times\dot{\vec{X}} = -\frac{\delta W}{\delta\vec{X}}$ - eq. Thiele. // Аппроксимация от Gus' JAP'or, где $\vec{G}\times\dot{\vec{X}} = \frac{\partial W}{\partial\vec{X}}$, $\vec{G} = -G_z$, $G = 2\pi p\frac{M_S L}{\gamma}$.

$\vec{G} = G_{xy}\hat{z} = -G_z\hat{z}$, $G_{xy} = G_z$ - неверно. $G_z = -2\pi p\frac{M_S}{\gamma}L$.

В Ding'14 в аналоге gyrovector в знаке, в eq. Thiele, и в $G_z = G_{xy}$, это сокращалось. $W = R^2L\int dz\int d^2s\,w$, w - energy density ($w = W/V$).

$L\vec{g}\times\dot{\vec{s}}R = -\frac{1}{R}\frac{\delta W}{\delta\vec{s}} = -\frac{1}{R}\frac{\partial W}{\partial\vec{s}}$, $\vec{g}\times\dot{\vec{s}} = -\frac{1}{R^2L}\frac{\delta W}{\delta\vec{S}} = -\frac{1}{R^2L}\frac{\delta}{\delta\vec{S}}R^2L\times$

$\times\int dz\,d^2s\,w = -\frac{\delta W}{\delta\vec{s}}$, $\vec{g}\times\dot{\vec{s}} = -\frac{\delta W}{\delta\vec{s}}$, $\frac{\delta W}{\delta\vec{s}} = \frac{\partial w}{\partial\vec{s}} - \frac{\partial}{\partial z}\frac{\partial w}{\partial(\partial\vec{s}/\partial z)}$ - обоз. с Ding' 2014.

Ding'14: $W = L\int dz\,E = L\int dz\,R^2w = R^2L\int dz\,w$, $\frac{\delta W}{\delta\vec{s}} = \frac{\delta}{\delta\vec{s}}R^2L\int dz\,w = R^2L\frac{\delta w}{\delta\vec{s}}$

Note: $w = w(\vec{s}, \frac{\partial\vec{s}}{\partial z})$ и $\int dz\,d^2s\,w = w\int dz\,d^2s = w\int_0^1 ds\,s\int_0^{2\pi} d\varphi = w\cdot 1\cdot 2\pi = \pi w$ - сразу ясно.

$W = R^2L\int dz\,w$ и $W = \int dV\,w = \int dz\,ds\,w = R^2L\int dz\,d^2s\,w = \pi R^2L\int dz\,w = \pi W$ - разумеется.

$\vec{g}\times\dot{\vec{s}} = \vec{g}\times\vec{\omega}\times\vec{s} = \vec{\omega}(\vec{g}\cdot\vec{s}) - \vec{s}(\vec{g}\cdot\vec{\omega}) = -\vec{s}(g_z\omega_z) = -\vec{s}\,g\omega$.

$-g\omega\vec{s} = -\frac{\delta W}{\delta\vec{s}} = -\frac{\partial W}{\partial\vec{s}} + \frac{\partial}{\partial z}\frac{\partial W}{\partial(\partial\vec{s}/\partial z)}$, $w = \frac{W}{V} = R^2L\int dz\,w$ но $W = R^2L\int dz\int d^2s\,\varepsilon$

ε - volume energy density, $w = \pi\varepsilon$.

$W = \pi R^2L\int dz\,\varepsilon = V\int dz\,\varepsilon = V\frac{1}{2}\int dz\,dz'\,\chi(zz')\vec{S}(z)\vec{S}(z') + V\frac{1}{2}y\int dz\left(\frac{\partial\vec{S}}{\partial z}\right)^2$.

Наконец $-g\frac{\omega}{\pi}\vec{S} = -\frac{\partial\varepsilon}{\partial\vec{S}} + \frac{\partial}{\partial z}\frac{\partial\varepsilon}{\partial(\partial\vec{S}/\partial z)}$, $\varepsilon = \frac{1}{2}\int dz'\,\chi(zz')\vec{S}(z)\vec{S}(z') + \frac{1}{2}y\left(\frac{\partial\vec{S}}{\partial z}\right)^2$

$-g\frac{\omega}{\pi}\vec{S} = -\int dz'\,\chi(zz')\vec{S}(z') + y\frac{\partial^2\vec{S}}{\partial z^2}$, $\frac{g\omega}{\pi}\vec{S} = -y\frac{\partial^2\vec{S}}{\partial z^2} + \int dz'\,\chi(zz')\vec{S}(z') -$

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– Это совпадает с Eq. (M2) Ding, Sci. Rep. 2014.

$$\gamma\frac{\partial^2\vec S}{\partial z^2} - \int dz' \varkappa(zz')\vec S(z') + \frac{g\omega}{\pi}\vec S = 0, \quad g = 2\pi M_s/\gamma \text{ (с точн. до знака, p)}.$$

1) Приближение $\int dz' \varkappa(zz')\vec S(z') \approx \vec S(z)\int dz' \varkappa(zz') = J(z)\vec S(z)\cdot 8\pi M_s^2\beta$.

$J(z) = \frac{2}{\beta}\int_0^\infty \frac{dk}{k} I^2(k)\left[1 - e^{-\beta k/2}\operatorname{ch}\beta k z\right]$, см. 16/IV 2020г. Ранее 16/IV 2020

2) Приближение $J(z) = J(0)\cos pz$ с подгонкой p. $J(z) \to F(z)$.

$$\gamma\frac{\partial^2\vec S}{\partial z^2} - J(0)\cos pz\cdot\vec S(z) + \frac{g\omega}{\pi}\vec S = 0, \text{ точнее } \frac{d^2S}{d\xi^2} + [a - 2q\cos 2\xi]S(\xi) = 0$$

eq. Mathieu

$$pz = 2\xi,\ z = \frac{2}{p}\xi,\ \frac{\partial}{\partial z} = \frac{\partial}{\partial(\frac{2}{p}\xi)} = \frac{p}{2}\frac{\partial}{\partial\xi},\ \frac{\partial^2}{\partial z^2} = \frac{\partial}{\partial z}\frac{\partial}{\partial z} = \left(\frac{p}{2}\right)^2\frac{\partial^2}{\partial\xi^2}$$

$$\gamma\left(\frac{p}{2}\right)^2\frac{\partial^2\vec S}{\partial\xi^2} + \left[\frac{g\omega}{\pi} - J(0)\cos 2\xi\right]\vec S(\xi) = 0, \text{ канон. ур. Mathieu.}$$

$$\frac{\partial^2\vec S}{\partial\xi^2} + [a - 2q\cos 2\xi]\vec S(\xi) = 0,\ a = \frac{g\omega}{\pi}\left(\frac{2}{p}\right)^2\frac{1}{\gamma},\ q = \frac{1}{2}J(0)\left(\frac{2}{p}\right)^2\frac{1}{\gamma}\cdot 8\pi M_s^2\beta.$$

Ding $f_0(k) = \frac{1}{k}\left[1 + \frac{1}{k}(1-e^{-k})(-1)\right]$, $\frac{\omega_0}{\omega_M} = 2\beta\int dk\, f_0(\beta k)I^2(k) =$

$= 2\beta\int dk\, I^2(k)\frac{1}{\beta k}\left[1 - \frac{1}{\beta k}(1-e^{-\beta k})\right] = 2\int\frac{dk}{k}I^2(k)f(\beta k)$, $f(x) = 1 - \frac{1}{x}(1-e^{-x})$.

$\omega_0 = \omega_M\cdot 2\int\frac{dk}{k}f(\beta k)I^2(k)$. В Gus'2002 $\omega_0 = \frac{1}{2}\gamma M_s\frac{1}{J(0)} = \frac{1}{2}\gamma M_s\left(\frac{2}{\beta}\right)^2\times$

$\times 8\cdot 4\pi F_v(\beta) = 2\cdot\gamma M_s\cdot 4\pi\int\frac{dt}{t}f(\beta t)I^2(t) = 2\omega_M\int\frac{dt}{t}f(\beta t)I^2(t)$.

$\Rightarrow$ Множитель $\lambda = \frac{g\omega}{\pi}$ в Ding'ом формуле. Он дает $\omega_0 = \omega_k$ из Gus'JAP02.

Точнее $\int dz' \varkappa(zz') = 8\pi M_s^2\beta\int dz' K(|z-z'|) \approx 8\pi M_s^2\beta F(z)$ $F(z)$ – 15/IV 2020.

до сих пор $F(z) \to J(z)$. $J(0) = F(0) = \frac{2}{\beta}\int_0^\infty\frac{dk}{k}I^2(k)\left[1 - e^{-\beta k/2}\right]$

$$\gamma = M_s^2\left(\frac{L_e}{L}\right)^2\left[\ln\frac{R}{R_c} + \frac{5}{4}\right] = M_s^2\frac{1}{\beta^2}\left(\frac{L_e}{R}\right)^2\left[\ln\frac{R}{R_c} + \frac{5}{4}\right]$$

$$\to a = \frac{g\omega}{\pi}\left(\frac{2}{p}\right)^2\frac{1}{\gamma} = \frac{\omega}{\omega_\gamma};\ \omega_\gamma = \frac{\pi}{g}\left(\frac{p}{2}\right)^2\gamma = \frac{\pi}{2M_s\pi}\frac{1}{\gamma}\left(\frac{p}{2}\right)^2\cdot\frac{M_s^2}{\beta^2}\left(\frac{L_e}{R}\right)^2\left[\ln\frac{R}{R_c} + \frac{5}{4}\right]$$

$$\omega_\gamma = \frac{1}{2}\gamma M_s\left(\frac{p}{2}\right)^2\frac{1}{\beta^2}\left(\frac{L_e}{R}\right)^2\left[\ln\frac{R}{R_c} + \frac{5}{4}\right],\ \omega = a(L)\cdot\omega_\gamma - \text{ГГц}.$$

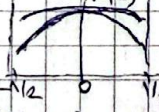
$$\to q = \frac{1}{2}F(0)\left(\frac{2}{p}\right)^2\frac{8\pi M_s^2\beta}{M_s^2\frac{1}{\beta^2}\left(\frac{L_e}{R}\right)^2\left[\ln\frac{R}{R_c} + \frac{5}{4}\right]} = \frac{1}{2}F(0)\left(\frac{2}{p}\right)^2\left(\frac{R}{L_e}\right)^2\frac{8\pi\beta^3}{\left[\ln\frac{R}{R_c} + \frac{5}{4}\right]}$$

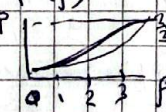
$$F(z) = \frac{2}{\beta}\int_0^\infty\frac{dk}{k}I^2(k)\left[1 - e^{-\beta k/2}\operatorname{ch}\beta k z\right],\ F(0) = \frac{2}{\beta}\int\frac{dk}{k}I^2(k)\left(1 - e^{-\beta k/2}\right)$$

Сом. уточ. см $R_c(L) = 0.68L_e(L/L_e)^{1/3}$, $L_e^2 = 2A/M_s^2$.

R – не fixed $q(\beta) \sim \beta^3 F(0,\beta)\frac{1}{p^2(\beta)}$, при $\beta\gg 1$, $F(0,\beta)\sim\frac{1}{\beta}$, $q(\beta)\sim\frac{\beta^2}{p^2(\beta)}$

$F(z) = F(0)\cos pz$





$p(\beta)$ он подгоняем из условия $F(z=1/2) = F(0)\cos p/2$.

$$\cos\frac{1}{2}p(\beta) = \frac{F(1/2,\beta)}{F(0,\beta)} = \int\frac{dk}{k}I^2(k)\left[1 - e^{-\beta k/2}\operatorname{ch}\beta k/2\right]\Big/\int\frac{dk}{k}I^2(k)\left(1 - e^{-\beta k/2}\right)$$

$$\cos\frac{1}{2}p(\beta) = \frac{1}{2}\cdot\frac{\int\frac{dk}{k}I^2(k)\left(1 - e^{-\beta k}\right)}{\int\frac{dk}{k}I^2(k)\left(1 - e^{-\beta k/2}\right)}.$$

If $\beta\ll 1$, $\cos\frac{1}{2}p \approx 1/2\ \frac{\beta}{\beta/2} \approx 1$, $p(\beta) = 0$.

If $\beta\gg 1$, $\cos\frac{1}{2}p \approx \frac{1}{2}$, $p(\beta) \approx \frac{2}{3}\pi$.

$q(\beta) \sim \frac{\beta^2}{p(\beta)^2} \sim \beta^2$ at $\beta\gg 1$. При $\beta\ll 1$, $q(\beta) \sim \frac{\beta^3}{p^2(\beta)}$, $p(\beta)\sim\beta^n$, найти n.

При $q\gg 1$ $a(q) \approx -2q$, $\beta\gg 1$. $\omega(\beta\gg 1, q\gg 1) = a(q)\cdot\omega_\gamma \sim q\frac{1}{\beta^2} \sim \frac{\beta^2}{\beta^2} = \text{const}(\beta)$

Все $\omega_n(\beta)$ at $\beta\gg 1$ are saturated, $\omega_n(\beta) \approx C_n$ at $\beta\gg 1$.

MathCad дает $p(\beta) \approx 2\sqrt{\beta}$ при $\beta\to 0$. поэтому $q(\beta) \sim \frac{\beta^3}{p^2(\beta)} = \frac{\beta^3}{\beta} \sim \beta^2 \ll 1$ при $\beta\to 0$

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(6/X) В „Carrefour". Работа над Thick vortex ω_G расчетом методом ф. Mathieu.

(7/X) Maciej дал мне contract draft, где нужно PESEL (tax number) и Bank account number. Заполн. application PESEL. Поход в Santander – Kaufland. Заполн. справки, подготовка работы в Univ. Adam Mick. Paweł submitted Edge-ом" MS to Phys. Rev. Appl.

(8/X) Mini-Seminar: "P. Graczyk o simulations of toroidal skyrmions in cyl. dots. Initial state Sutcliffe'2018 ansatz с $R = \frac{3}{2}L$, $L = 70$ nm, $K_s = 10 \frac{mJ}{m^2}$ – simulation creates к toroidal hopfion" которой удалось $H_z \approx 3$ kOe. При $R = 150$ nm ($\approx 2L$) уже нет stable „toroidal" hopfion. Аналог. расчет $\omega_G(L)$ in thick cylindrical dot.

(9/X) Асимптотики $\beta << 1$ и $\beta >> 1$ для Mathieu подхода к alt. Ур-е Mathieu имеет собств. Значения $a_n(\beta)$ для рав. СФ и $b_n(\beta)$ для нечетных СФ. Рав. СФ $ce_n(z)$, нечет. СФ – $se_n(z)$, $z = \frac{\rho}{2} z$, $z \in [\frac{1}{2}, \frac{1}{2}]$

(1) Предел $\beta << 1$, $q = 4\pi F(0,\beta) \frac{4}{\rho^2(\beta)} \left(\frac{R}{Le}\right)^2 \frac{\beta^3}{\left[\ln \frac{R}{R_c} + \frac{5}{4}\right]}$, $\ln \frac{R}{R_c} + \frac{5}{4} = c$, weak funct. of β

$q = 16\pi F(0,\beta) \frac{R^2}{Le^2} \frac{1}{c} \frac{\beta^3}{\rho^2(\beta)}$. $a_0(q) \approx -\frac{1}{2} q^2 + O(q^4)$ at $q \to 0$.

Frequency scale $\omega_y(\beta) = \frac{1}{2} \gamma M_s \frac{1}{4} \rho^2(\beta) \frac{1}{\beta^2} \left(\frac{Le}{R}\right)^2 c$.

$\rho(\beta) \approx 2\sqrt{\beta}$ at $\beta \to 0$ и $\omega_y(\beta \to 0) = \frac{1}{2} \gamma M_s \left(\frac{Le}{R}\right)^2 c \cdot \frac{1}{\beta}$. Absolute frequency

$\omega_0(\beta \to 0) = a_0(q) \cdot \omega_y(\beta \to 0) = -\frac{1}{2} \left[16\pi F(0,\beta) \left(\frac{R}{Le}\right)^2 \frac{1}{c} \frac{\beta^3}{4\beta}\right]^2 \frac{1}{2} \gamma M_s \left(\frac{Le}{R}\right)^2 c \frac{1}{\beta}$

$F(0,\beta) \approx 0.18$ при $\beta \to 0$, $\omega_0(\beta \to 0) = \frac{1}{2} \left[4\pi F(0,\beta) \left(\frac{R}{Le}\right)^2 \frac{1}{c} \beta^2\right]^2 \frac{1}{2} \gamma M_s \left(\frac{Le}{R}\right)^2 \frac{c}{\beta} \sim \beta^3$

$\omega_0(\beta \to 0) \sim \left(\frac{R}{Le}\right)^2 \frac{1}{c} \beta^3$ – неверная однородная ω_G thick dot (ее magnetostatic part)

Magnetostatic part $\omega_G \sim \beta$ согласно Gus' JAP'02. $\omega_0(\beta \to 0) \sim R^2 \beta^3 = \frac{L^3}{R}$ – неверно

→ Предел $\beta \to 0$ у метода Mathieu неверен.

(2) Предел $\beta >> 1$. $\rho(\beta) \to \frac{2}{3}\pi$ при $\beta \to \infty$. $\rho(\beta) \approx 2$ уже при $\beta \in [1, 5]$ – разумных.

$q = 16\pi F(0,\beta) \frac{\beta^3}{\rho^2(\beta)} \left(\frac{R}{Le}\right)^2 \frac{1}{c(\beta)} = 4\pi \left(\frac{R}{Le}\right)^2 F(0,\beta) \frac{\beta^3}{c(\beta)}$, $c(\beta) = \ln \frac{R}{R_c} + \frac{5}{4}$.

$R/R_c = R / \left[0.68\, Le \left(L/Le\right)^{1/3}\right] = \left|L = \beta R\right| = \frac{R}{0.68\, Le \left(\frac{R}{Le}\right)^{1/3} \beta^{1/3}} = \frac{1}{0.68} \left(\frac{R}{Le}\right)^{2/3} \frac{1}{\beta^{1/3}}$.

$c(\beta) = \ln \left[\frac{1}{0.68} \left(\frac{R}{Le}\right)^{2/3} \frac{1}{\beta^{1/3}}\right] + 5/4 = \ln \left[\frac{1}{0.68} \left(\frac{R}{Le}\right)^{2/3}\right] + \frac{5}{4} + \frac{1}{3} \ln \beta$ – weak function of β

$q = 4\pi \tau^2 F(0,\beta) \beta^3 \frac{1}{c(\beta)}$, $c(\beta) = \ln \left(\frac{1}{0.68} \tau^{2/3}\right) + \frac{5}{4} + \frac{1}{3} \ln \beta$, $\tau = R/Le \approx 5 \div 6$.

$\beta F(0,\beta)$ (0.12 … 0.16; 0 2 4 6 β) $F(0,\beta) \approx \frac{(0.12 \div 0.16)}{\beta}$ at $\beta > 2$. $F(0,\beta)\beta^3 \sim \beta^2$ at $\beta >> 1$, $q(\beta) \sim \beta^2$ at $\beta >> 1$

Note: at $\tau = \left(\frac{17\,nm}{100\,nm}\right)^{-1} = (0.17)^{-1}$, $q(\beta, \tau)$ (1, 5, 10, β)

$a_1(q) = 0$ при $q \approx 7.5$. $q(\beta) \leq 5$ при $\beta \leq 10$.

Если $q \leq 5$ for typical $\beta >> 1$, то корень $q_1 = 7.5$ не достижим (!). неверно из-за wrong τ!

при $q >> 1$, $a_0(q) = -2q + 2\sqrt{q} - \frac{1}{4}$. $q = 13 \div 240$ для $\beta = 1 \div 3$.

$\omega_y = \frac{1}{8} \gamma M_s \frac{1}{\tau^2} \frac{\rho^2(\beta)}{\beta^2} c(\beta) \approx \left|\beta >> 1,\ \rho(\beta) \approx 2\beta\right| = \frac{1}{2} \gamma M_s \frac{1}{\tau^2} c(\beta)$ at $\beta >> 1$.

$\omega_0(q) = \omega_y(\beta) \cdot a_0(q)$, где $q = q(\beta)$.

Note: Все корни $a_n(q)$ обращаются в „0" для больших q. т.е. $\omega_n^G(q) = \omega_y a_n(q)$ может $= 0$ для $q >> 1$. Это немного по-другому Mathieu eq. расчет. // Note: $q = 17.1$ для $L = 100$ nm (30/VII 2014).

(10/X) Lindo test, осмотр Temski. Проверка χ ($12 \cdot 21$) при $z \to z'$. Ожидаем $\sim \ln|z - z'|$.

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(11/X) Approx. $F(z) = F(0)[1 - p^2 z^2]$ (16/XII 2020) нужно $y'' + \left[\frac{1}{4}x^2 - a\right] y(x) = 0$

Eq. parabolic cylinder, $a = a(\omega)$. Ищем дал. зав. ω_n, не нашел a_n.

Переформулируем в виде eq. Mathieu $\frac{d^2\tilde{S}}{dz^2} + [a - 2q\cos 2z]\tilde{S}(z) = 0$.

$\int dz' \chi(z,z') = 8\pi M_s^2 \beta \int dz' K(|z-z'|) = 8\pi M_s^2 \beta F(z)$, z in units of L, $z \in [-\frac{1}{2}, \frac{1}{2}]$

$F(z) = \frac{2}{\beta}\int_0^\infty \frac{dk}{k} I^2(k)\left[1 - e^{-\beta k/2}\cos\beta k z\right]$, $I(k) = \int_0^1 dz\, z\, J_1(kz)$, z в eq. R

В духе 15/XII 2020 попробуем $\tilde{F}(z) = \frac{1}{2}A_0 + A_1\cos\beta z$, $\frac{1}{2}A_0 = \int_{-1/2}^{1/2} dz F(z) = \frac{2}{\beta}\int \frac{dk}{k} f(\beta k) I^2(k)$

Но ведь точно $\omega_0(\beta \ll 1) = 2\omega_M \int \frac{dk}{k} f(\beta k) I^2(k)$, т.к. $\frac{1}{2}A_0 = \frac{1}{\beta}\frac{\omega_0(\beta\ll1)}{\omega_M}$

($\tilde{S}(z) = \text{const}$)

Учтем сказанное $f(\beta k) \approx \frac{1}{2}\beta k$ и $\omega_0(\beta \ll 1) = 2\omega_M \frac{1}{2}\beta \int dk\, I^2(k) = 0.177 \cdot \beta\omega_M$.

$K(z) = \int_0^\infty dk\, e^{-\beta k^2} I^2(k)$, $z \geq 0$. $F(z) = \int dz' K(|z - z'|)$

$y\frac{\partial^2 \tilde{S}}{\partial z^2} - \int dz' \chi(zz')\tilde{S}(z') + \frac{g\omega}{\pi}\tilde{S}(z) = 0$, нужно $\int dz' \chi(zz')\tilde{S}(z') = \tilde{S}(z)\int dz' \chi(zz')$

$y\frac{\partial^2 \tilde{S}}{\partial z^2} + \left[\frac{g\omega}{\pi} - \int dz' \chi(zz')\right]\tilde{S}(z) = 0$, $\int dz' \chi(zz') = 8\pi M_s^2 \beta F(z) =$

$= 8\pi M_s^2 \beta\left[\frac{1}{2}A_0 + A_1\cos\beta z\right]$; $\lim_{\beta z \ll 1} A_1\cos\beta z \to 0$ и $\frac{g\omega_0}{\pi} = \int dz' \chi(zz') = 8\pi M_s^2 \frac{1}{2}A_0$

$\tilde{S}(z) = \text{const}$

$\omega_0(\beta) \underset{\beta\ll1}{=} \frac{\pi}{2\pi M_s y} 8\pi M_s^2 \beta \cdot \frac{1}{2}A_0 = 4\cdot 4\pi M_s \beta \cdot \frac{2}{\beta}\int \frac{dk}{k} f(\beta k) I^2(k) = 2\omega_M \int \frac{dk}{k} f(\beta k) I^2(k) =$ — верно.

A_1 и β найдем из $F(0)$ и $F(1/2)$. $F(0) = \frac{1}{2}A_0 + A_1$, $F(\frac{1}{2}) = \frac{1}{2}A_0 + A_1\cos\frac{\beta}{2}$.

$A_1 = F(0) - \frac{1}{2}A_0 = \int \frac{dk}{k} I^2(k)(1 - e^{-\beta k/2})\frac{2}{\beta} - \frac{2}{\beta}\int \frac{dk}{k} f(\beta k) I^2(k)$, $f(x) = 1 - \frac{1}{x}(1 - e^{-x})$

$A_1(\beta \to 0) = \int \frac{dk}{k} I^2 \frac{\beta k}{2}\frac{2}{\beta} - \frac{2}{\beta}\int \frac{dk}{k}\frac{1}{2}\beta k\, I^2 = \int dk\, I^2(k) - \int dk\, I^2(k) = 0$.

$\cos\frac{\beta}{2} = \frac{1}{A_1}\left[F(1/2) - \frac{1}{2}A_0\right] = A_1^{-1}\frac{2}{\beta}\int \frac{dk}{k} I^2(k)\left[\frac{1}{2}(1 - e^{-\beta k}) - f(\beta k)\right]$

$A_1 = \frac{2}{\beta}\int \frac{dk}{k} I^2(k)\left[1 - e^{-\beta k/2} - f(\beta k)\right]$, $A_1\cos\frac{\beta}{2} = \frac{2}{\beta}\int \frac{dk}{k} I^2(k)\left[\frac{1}{2}(1 - e^{-\beta k}) - f(\beta k)\right]$

$F(z,\beta) = \frac{2}{\beta}\int \frac{dk}{k} I^2(k) f(\beta k) + \left(A_1\cos\frac{\beta}{2}\right)\frac{\cos\beta z}{\cos\beta/2}$, $\beta = \beta(\beta)$.

$y\frac{\partial^2 \tilde{S}}{\partial z^2} + \left[\frac{g\omega}{\pi} - 8\pi M_s^2 \beta F(z,\beta)\right]\tilde{S}(z) = y\frac{\partial^2 \tilde{S}}{\partial z^2} + \left[\frac{g\omega}{\pi} - 8\pi M_s^2\beta\left\{\frac{1}{2}A_0 + A_1\cos\beta z\right\}\right]\tilde{S}(z) = 0$

$\beta z = 2\xi$, $y\left(\frac{\beta}{2}\right)^2\frac{\partial^2 \tilde{S}}{\partial \xi^2} + \left[\frac{g\omega}{\pi} - 8\pi M_s^2\beta\frac{A_0}{2} - 8\pi M_s^2\beta A_1\cos\beta z\right]\tilde{S}(z) = 0$

$\frac{\partial^2 \tilde{S}}{\partial \xi^2} + [a - 2q\cos 2\xi]\tilde{S}(\xi) = 0$. $a = \left(\frac{2}{\beta}\right)^2\frac{1}{y}\left[\frac{g\omega}{\pi} - 8\pi M_s^2\beta\frac{A_0}{2}\right]$, $q = \frac{1}{2}\frac{8\pi M_s^2\beta A_1}{y}\left(\frac{2}{\beta}\right)^2$

$q = \frac{4\pi M_s^2\beta}{M_s^2\left(\frac{L_e}{L}\right)^2 c}A_1\frac{4}{\beta^2} = \frac{16\pi\beta^3}{\left(\frac{L_e}{R}\right)^2 c}\frac{A_1}{\beta^2}$, $c = \ln\frac{R}{R} + \frac{5}{4}$. $A_1 = F(0) - \frac{1}{2}A_0$

Более ранее $q = \frac{16\pi\beta^3}{\left(\frac{L_e}{R}\right)^2 c}\frac{F(0,\beta)}{\beta^2}$, т.е. $F(0,\beta) \to A_1(\beta) = F(0,\beta) - \frac{1}{2}A_0(\beta)$.

Чуть, Матвей дал $\cos\beta/2 = -2, -3$, $|\cos\beta/2| > 1$ для $\beta = 0,1 \div 3$.

$\cos\frac{\beta}{2} = \frac{F(1/2) - \frac{1}{2}A_0}{F(0) - \frac{1}{2}A_0}$, $F(1/2) < F(0)$, $F(1/2) < \frac{1}{2}A_0$,

Приближение local magnetostatics с $F(z) = \frac{1}{2}A_0 + A_1\cos\beta z$.

$\frac{1}{2}A_0 = \frac{2}{\beta}\int \frac{dk}{k} f(\beta k) I^2(k) + \omega_0(\beta)$ на разделе.

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