

9/V-2009-18/X-2010

quality
NOTE
Book

2009-05-08

Coupled system Vortex + Azim. SW

Coupling V-SW в вихре репр. кинет. Lagrangian $\mathcal{L}_{kin} = \frac{M_2}{2} \dot{\Phi}^2$
 в вихре $\mathcal{L}_{int} = \frac{M_2}{2} \sin \theta_0 \epsilon \Phi$, $\Phi = \Phi_V + \psi$, $\theta = \theta_V + \epsilon$, $\mu = -\sin \theta_0 \psi$.
 (ψ, ϵ) - oscill. SW, $\theta_V(\vec{r}, t)$, $\Phi_V(\vec{r}, t)$ - moving vortex.

Вектор $\vec{r}$ - position, $\vec{r} = \vec{r}_0 + \vec{r}_1$, $\vec{r}_1 = \frac{1}{2} \epsilon_{ijk} \vec{M} (\partial_j \vec{M} \times \partial_k \vec{M})$

$I_\mu = \int d^2x x_\mu q$ и $\ell = \frac{1}{2} \int d^2x x^2 q$, которые можно интерпретировать как моменты $\vec{P}$ ($P_\mu = -\epsilon_{\mu\nu} I_\nu$) и угловой момент $L_z(\ell)$. I_μ и ℓ описывают vortex position $\vec{X}(t)$ и vortex size, соответственно.

Gyrvector (vorticity) density $\vec{g}$: $(g_i = -\frac{1}{2} \epsilon_{ijk} \vec{m} \cdot (\partial_j \vec{m} \times \partial_k \vec{m}))$ - обр. с вращением.

$\text{div } \vec{g} = 0$, $\vec{g} = \nabla \Pi \times \nabla \Phi$, где $\Pi = \cos \theta$ - canonical momentum, $i, j, k = x, y, z$.

Тогда можно ввести канон. vector-potential ($\vec{A}$): $\vec{g} = \text{rot } \vec{A}$, $\vec{A} = \cos \theta \nabla \Phi$.

Угловой момент - канон. вектор-потенциал, который связан с током $\vec{j} \cdot \vec{A}$ и дает разное з.г.с., spin-Hall effects при наличии spin-polariz. current $\vec{j}_s = P \vec{j}$.

Вектор $\vec{A}$ является канон. вектор-потенциалом, который связан с током $\vec{j} \cdot \vec{A}$ и дает разное з.г.с., spin-Hall effects при наличии spin-polariz. current $\vec{j}_s = P \vec{j}$.

и $E_{diag} = E_0 \pm \hbar \gamma |\vec{M}|$, $\gamma = e/2mc$ - канон. gauge potential $\vec{A} = i \hat{U}^{-1} \frac{\partial \hat{U}}{\partial t}$ в $\vec{m}$ -фазе.

Преобразование канон. $\psi' = \hat{U}(\vec{r}, t) \psi$ и вводит в $\vec{A}$ и $\vec{m}$ - фазу $\vec{m} = (\vec{r}, t)$.

И канон. канон. $\vec{M}(\vec{r}, t)$. Адиабат. вектор-потенциал $\vec{A}_{ad} = \text{diag}(\vec{A})$

Ковариант. преобразование $i \frac{\partial}{\partial t} \rightarrow i \frac{\partial}{\partial t} + \vec{A}_t$, $i \frac{\partial}{\partial x} \rightarrow i \frac{\partial}{\partial x} + A_x$. $A_\mu = (-A_t, \vec{A}_x)$

Адиаб. $\vec{A}_{ad}(\vec{M}) = \frac{1}{2M(M \pm M_2)} (M_y, -M_x, 0)$, для координат $\vec{F} = -S \frac{\vec{M}}{M^3}$ - канон. обр. фазы.

Переход из $\vec{M}$ в $(\vec{r}, t)$ пространство тензор $F_{\alpha\beta} = \pm S \frac{\vec{M}}{M^3} \cdot (\frac{\partial \vec{M}}{\partial x_\alpha} \times \frac{\partial \vec{M}}{\partial x_\beta})$,
 $\alpha, \beta = x, y, z, t$. Тензор eff. эл.-м. $F_{eff}^{\alpha\beta} = F^{\alpha\beta}(\text{эл.м.}) + \frac{\hbar c}{e} F_{\alpha\beta}^{\text{эл.м.}}$ и т.д.

кан. $B_k^{\text{eff}} = B_k + \frac{\hbar c}{e} \epsilon_{ijk} \frac{\vec{M}}{2M^3} \cdot (\frac{\partial \vec{M}}{\partial x_i} \times \frac{\partial \vec{M}}{\partial x_j})$, $E_k^{\text{eff}} = \mp \frac{\hbar c}{e} \frac{\vec{M}}{2M^3} \cdot (\frac{\partial \vec{M}}{\partial x_k} \times \frac{\partial \vec{M}}{\partial t})$, $S = 1/2$.

Если $M = |\vec{M}| = \text{const}$, то введем $\vec{m} = \vec{M}/M$ и $\vec{B}_{eff} = \vec{B} \pm \frac{\hbar c}{e} \vec{g}$, $\vec{g}$ - gyrvector density.

$\vec{g} = \text{rot } \vec{A}$ - канон. $\vec{B}$ в $\vec{m}$ -пространстве, для координат $\vec{F} = \text{rot } \vec{A}$ и т.д.

$\vec{F} = -S \frac{\vec{M}}{M^3}$ в $\vec{m}$ -пространстве. $\vec{A}_{ad}(\vec{M}) = \frac{S}{M(M \pm M_2)} (\vec{M} \times \vec{z}) \rightarrow \vec{A} = S \frac{\vec{M} \vec{m}}{M(M \pm M_2)}$

Согл. 30/11 2007 г. если $\vec{A}_\pm = \frac{1}{2} \frac{\vec{r} \times \vec{z}}{r(1 \pm \vec{r} \cdot \vec{z})}$, $\vec{B} = \text{rot}_r \vec{A} = \pm \frac{\vec{z}}{r^3}$, $\vec{F} = \text{rot}_r \vec{A} = \pm S \frac{\vec{M}}{M^3}$.

Т.е. канон. $\vec{B}$ безмассово из $\vec{A}_{ad}(\vec{M})$ и при переходе к $\vec{m}$ -координатам к

обратным $\vec{z}$ -координатам. Berry phase $\Phi_B = -S \oint \vec{A}(\vec{m}) d\vec{m} = -S \int_0^T dt \vec{A}(\vec{m}(t)) \dot{\vec{m}}(t)$, где $\vec{m}(t)$

где is closed за интервал $t \in [0, T]$.

Bruno PRL'04: $\hat{H}_e = -\frac{\hbar^2}{2m} \left[\frac{\partial^2}{\partial z^2} - \frac{ie}{\hbar c} \vec{a} \right]^2 + V$, $a_i = \pi \Phi_0 \frac{(n_x \partial_i n_y - n_y \partial_i n_x)}{(1+n_z)}$

где $\vec{n}(\vec{r}) = \vec{M}(\vec{r})/M_s$ - век. ор. квантования. $\Phi_0 = hc/4e$.

$$q_\mu = -\Phi_0 S A'_j(\vec{m}) \frac{\partial m_j}{\partial x_\mu} = -\Phi_0 S \frac{\hat{z} \times \vec{m}}{1+m_z} \cdot \partial_\mu \vec{m} = -\Phi_0 S \hat{z} \cdot (\vec{m} \times \partial_\mu \vec{m}) = -\frac{\Phi_0 S}{(1+m_z)} [m_y \partial_\mu m_x - m_x \partial_\mu m_y]$$

$$\Phi_0 = \frac{hc}{e} = \frac{1}{2\pi} \left(\frac{hc}{e} \right) = \frac{1}{2\pi} \Phi_0 \text{ у Bruno корр. } \pi \Phi_0 \text{ - неверно,}$$

а сам вектор - нормирован a_i совн. с Girvin'99 $a_{\mu 2}$.

Dugaev PRB'05: Вектор. топол. поле $\vec{A}(\vec{r})$ в exch. energy $(\nabla \vec{m})^2$ и/или spin-
nem SW. Такой vortex не существует, где магнетизм SW типа azim. modes
in vortex state, хотя может $\sim d(A_z^2)^2$ может существовать и не может.

Затем квантование SW в обмен. взаимодействии. $\omega_n \sim \langle \vec{A}_{top} \rangle (n+1/2)$ в ring -
уравнении Landau, забывая про в ring без v. cores нет тополог. поля.

Ведь прогнорозное ground state $\Phi_0(\vec{r})$, $\Phi_0(\vec{r})$ можно и напильничко урезать $\Rightarrow$
 $\Phi_0(\vec{r})$, Φ_0 берётся в SW eqs как у меня и было, что не только в exch. энергии, но
и в магнетизме. Именно тогда Dugaev'05 магнетизм. Вклад будет не расщ. (!?)

На Gary Withdranal в Киеве, А. Мана - ЗООУМН (УОСФ)

(11/2) обзор anomalous Hall effect в non-uniform ferromagnets, Почин. theory of quantum
(spin) Hall eff.

Meeting c Julian: 1) в Porto on ne eger; 2) об обзоре effect на Garcia Hanomys
3) change only a new SW - azim. paper c Arantxa to PRL; 4) обзоре. Spin nanostruct. Conf. Madrid

5) 25-26 May в Nanogune Syber Nanosci. Workshop c генератором Julian: упрощение и нов.
3-4 slides по spin dynamics где был в Gorky.

Затем spin dyn. об paper упрощения by Montanacello: azim. SW где $\beta=0.15, 0.35, 0.50$

Meeting c Arantxa: она нормировала $M(\vec{r})$ of vortex, $R=100 \text{ nm}$, $L=10 \text{ nm}$, но
и/или упрощения и не было v. nucleation. Будем про уг. За нечего.

previous step - M($\vec{r}$) configuration и добавим $E = \frac{1}{M_s^2} |\vec{M} \times \vec{H}_{eff}|$ to stop simul.
ново упрощ. E, что repeat в vortex c $E \rightarrow \min$ и уберем. 2 go 1.

Затем Глеб: 1) упрощение Abstract DW dyn. talk; 2) нов. Spanish - Port. grant
On a new Ate Credit Card и partner. NW - World Works miles. Рисуем Ate B. Cards/S.S.

(12/2) Vortex - SW interaction. Проблема $\vec{L}_{int}$ к buggy $\vec{j} \cdot \vec{A}$ или $\vec{j} \cdot \vec{A}$, где
 $\vec{j}$ - SW spin current, $\vec{A}$ - вектор. вектор поле moving vortex.

Будем $\vec{L}_{int} \sim \vec{j} \cdot \vec{A}$, хотя надо для $\vec{L}_{int} \sim A_0$, $A_\mu = (A_0, \vec{A})$. Ведь $A_0 \sim \frac{\partial \theta}{\partial x}$, $\vec{A}$
смыс. moving vortex. Тогда $\vec{A} \rightarrow \vec{A}^2$ и рассмотрим Tataru et al.

Как правильно будет $A_\mu = A_\mu^a \partial_a$ и 3D где spin-waves? Две $\vec{e}$ со смик $\frac{1}{2} (\partial_a$
- магниты Паули) сарко ду. оператори $\hat{c}$ замечание на поле $\vec{a}$: $\hat{c} = U \vec{a}$,
там тогда $\vec{a} \sim \vec{n} \cdot (c^+ \partial_c) \rightarrow n_z (a^+ \partial_z a)$ сова гур. $\partial_\mu c = U (\partial_\mu + i A_\mu) a - A_\mu$
тогда. но не было бы $\partial_\mu (c^+ \partial_c)$ - тоны. Это габарит (см. 25/III):

$$\vec{L}_{int} = \hbar \sum_{\vec{r}} \vec{j}_\mu A_\mu^a (a^+ \partial_a a) = \hbar \sum_{\vec{r}} \vec{j}_\mu A_\mu a^+ a = \hbar a^+ (\vec{j}_\mu A_\mu) a, \langle \vec{L}_{int} \rangle = \hbar \vec{j}_\mu A_\mu^2$$

где $\vec{j}_\mu = (1, \frac{\hbar}{m} \vec{k})$ и $\langle a^+ \partial_a a \rangle \sim \delta_{az}$. $\vec{j}_\mu = (1, \frac{1}{\epsilon} \vec{j})$, $\vec{j}_s = \rho \vec{j}$ - смик ток

Две SW (вектор $\vec{e}$) если будет A_μ через $\vec{e}_\mu$ тоны (а где SW это exchange int.),
то сарко к paper Dugaev - Bruno PRB'05, который неверно, с.к. A_μ габарит есть и
в магнетизме. Тогда. Будем же SW $\psi = m_x + i m_y$, где m_x, m_y - дж. $\vec{m}$ канонич.

13/6 В случае magnetostatic SW $\vec{E} = -\gamma \vec{H}_m^p$, $\vec{H}_p = -\gamma \vec{H}_m^z$, $\vec{H}_m = \int d^3p' \vec{G} \vec{H}$
 $\Psi \equiv \vec{E} + i\vec{H}_p$, $\vec{\Psi} = \vec{E} + i\vec{H}_p = -\gamma \left[\int d^3p' \vec{G}_{pp} \vec{H}_p + i \int d^3p' \vec{G}_{zz} \vec{E} \right] \sim \Psi$,
 т.к. $G_{pp} \neq G_{zz}$. Аналогичные (магнетостат.) не дает возмущения
 LLG замкнут как уш и вращаю ток $\vec{j} = \frac{\hbar}{2m} i [\Psi \nabla \Psi^* - \Psi^* \nabla \Psi]$, $\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$
 This plane wave $\Psi = a e^{i(kx - \omega t)}$, $\vec{j}(x, t) = |a|^2 \frac{\hbar k}{m}$, $\vec{j} = |a|^2 \frac{\hbar \vec{k}}{m}$.
 Проведение SW к локальной оси квант. надо за счет $A_\mu = A_\mu^d J_d$,
 где $J_d = \partial_d$ (м. Паули), т.к. уш. момент $> 1/2$.
 Лок. ось вращения $\hat{z}' = \vec{m}_0(p)$ sup. vortex ground state (static).
 $\hat{z}' = \hat{e}_3$, $\hat{e}_2 = \hat{z}' \times \hat{e}_3$, $\hat{e}_1 = \hat{e}_2 \times \hat{e}_3$ и $\vec{m} = \hat{e}_3 + m_1 \hat{e}_1 + m_2 \hat{e}_2$. Вектор $J=1$?
 $\hat{e}_2$ лежит в м. хоз. $\vec{m} = R(\theta_0, \varphi_0) \vec{m}'$, $\begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} \vdots \\ \vdots \\ \vdots \end{pmatrix} \begin{pmatrix} m'_x \\ m'_y \\ m'_z \end{pmatrix}$ и
 $m_z \approx 1$, $m_x, m_y \ll 1$ в мн. приближен.
 Вектор. по мн. $\vec{m}$ совпадает уш. момент $J=1$. Gauge transform, topol. field
 Топол. поле $\vec{A}$ вихря имеет два вклада: калибровоч. $(\nabla - i\vec{A})^2$, который
 входит через exchange $2(\nabla \vec{m})^2$ и осевой, вх. в магнетостатич.
 Калибр. вклад $(\nabla - i\vec{A})^2$ хорошо известен где $\vec{e}$ -уравновешивает, у которых $\frac{\hbar^2}{2m} \nabla^2$.
 основное энергие, но где SW с мн. по $\vec{A}$ - лагранжиан, это не ось
 основной вклад в топол. поле. Надо найти $\vec{A}$ и $\mathcal{L}_{int}$ где SW over vortex state уже
 Для $\vec{e}$ (Dirac eq.), найдем решение лок. калибр. инвариантности ВФ
 Простей $D_\mu = \partial_\mu - i\frac{e}{\hbar} A_\mu$ и $\mathcal{L}_{int} = \int d^4x A_\mu(x) j^\mu(x)$ "minimal" coupling $e \in$ э.м. поле A_μ .
 В общем $\mathcal{L} = \frac{1}{2} (\partial_\mu \Phi)^2 + \frac{1}{2} m^2 \Phi^2$, $\Phi = (\phi_1, \phi_2, \dots, \phi_n)$ -вектор.
 и $\mathcal{L}$ будет инвариант. относительно локал. transformations $\Phi(x) \rightarrow G(x) \Phi(x)$
 если $\partial_\mu \rightarrow D_\mu + g A_\mu(x)$ и $A_\mu(x) \rightarrow G(x) A_\mu(x) G^{-1}(x) - \frac{1}{g} G(x) \partial_\mu G^{-1}(x)$
 Разность $\mathcal{L}(D_\mu)$ и $\mathcal{L}(\partial_\mu)$ есть $\mathcal{L}_{int} = \frac{g}{2} \Phi^\dagger A_\mu^2 \partial^\mu \Phi + \frac{g}{2} (\partial_\mu \Phi)^\dagger A^\mu \Phi + \frac{g^2}{2} |A_\mu \Phi|^2$ -
 eff. interaction of n-scalar fields of Φ как следствие калибр. инвариантности.
 Для vortex + SW ось 4-мандр. поле $X(t), Y(t)$ и $\phi, \vec{H}_p$. Но лагранжиан
 $\mathcal{L} = -\cos \theta \dot{\Phi} - \mathcal{H}(\theta, \Phi)$ содержит ∂_μ - мн.но, не свобод к калибр. field
 SW дает вклад в $\mathcal{L}$ в виде $\mathcal{L}_{int} = \vec{K} \cdot \vec{X} (\vec{A} \cdot \vec{j})$, где vortex, а потенциал $\vec{K}$ от SW.
 Vortex дает вклад в $\mathcal{L}_{SW}$ в виде $\mathcal{L}_{int} = \frac{M_S}{\gamma} \vec{e} \cdot \vec{\Phi}_v = -\frac{M_S}{\gamma} \Phi_v \vec{e}$, Φ_v - реон. field?
 $\Phi_v = \Phi_0 + \delta \Phi(t) = \Phi_0 + m_0(p) [\vec{X}(t) \times \vec{p}]_z \Rightarrow \mathcal{L}_{int} = -\frac{M_S}{\gamma} \vec{e} \cdot m_0 (\vec{X} \times \vec{p})_z$, Φ_v реон.
 Вектор $\vec{A}_v \sim m_0 (\vec{X} \times \vec{p})$, а $\int d^3x \sim \vec{e}$. Zero pagn SW ток SW $\parallel \vec{H}_0$.
 SW ток Azim modes го-мол. с $\vec{H}_0$, где (2) SW инвариант. по кр. уш.
 и $\frac{\partial}{\partial \Phi} \mathcal{L}_{SW} \sim m$, $\vec{j}_{SW} \sim \frac{\partial \mathcal{L}_{SW}}{\partial \Phi} \sim m (?)$.
 Погрот. и нолан slides no spin dyn. just Julian talk - NanoGrINE Workshop

(14/12) C. N. & Citi B. : exper. dobit cards u. "Liedl".

Надо изв. преобраз. к осн каноническим
намам. $\vec{m}_0$ static vertex. можно дать $\hat{z}'(\theta_0, \varphi_0)$, где $\theta_0(\varphi)$ $\varphi_0(\varphi)$ углы.
 $\theta_0 \rightarrow \theta_v, \varphi_0 \rightarrow \varphi_v$ dynamic vertex.

$$J_{SW} = \int d^2\phi \left[(1 - \cos\theta) \dot{\phi} - W(\theta, \phi) \right] \Rightarrow J_{SW} = A \cdot \vec{\mathcal{H}} - W(\vec{\mathcal{H}}), \quad \vec{\mathcal{H}} = R^T \vec{\mathcal{H}}_0$$

2. Zagar nepxoguy xyz c Oz $\perp$ film plane to $x'y'z'$ c Oz' $\parallel \vec{m}_0$ (vortex).

Уч. $L_{sw} = \vec{A} \cdot \vec{B} + \frac{1}{2} \vec{B} \cdot \vec{H}_m$ составлен из убавляемых. $L'_{sw} = \vec{A}' \cdot \vec{B}' + \frac{1}{2} \vec{B}' \cdot \vec{H}'_m$

наше поле зрения. $\vec{A}(\vec{r}) = \frac{M_s}{\hbar} \frac{\vec{n} \times \vec{r}}{r^3}$. Момент спина $\vec{S} = \frac{\hbar}{2}$.

$$\vec{A}(\vec{m}) = \frac{M_s}{\gamma} \frac{\vec{n} \times \vec{m}}{(1 + \vec{n} \cdot \vec{m})}$$

$$\vec{A} = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta) \rightarrow \vec{A}' = (\sin(\theta - \theta_v) \cos(\phi - \phi_v), \sin(\theta - \theta_v) \sin(\phi - \phi_v), \cos(\theta - \theta_v))$$

Видимый переход к (прим. координатам от (θ, φ) к (θ', φ') $\text{от } \Sigma'$ $\text{от } \Sigma$
 т.е. $\theta' = \theta - \theta_v$, $\varphi' = \varphi - \varphi_v$. $\vec{m}(\theta, \varphi) \rightarrow \vec{m}'(\theta', \varphi')$.

En un $\vec{m} = |\vec{m}| \hat{e}$, y $\dot{\vec{m}} = \frac{d|\vec{m}|}{dt} \hat{e} + |\vec{m}| \dot{\theta} \hat{\theta} + |\vec{m}| \dot{\phi} \sin \theta \hat{\phi} = \dot{\theta} \hat{\theta} + \dot{\phi} \sin \theta \hat{\phi}$, $|\vec{m}| = 1$

$$\vec{m}(m_p, m_\psi, m_t) \Rightarrow \vec{m} = \hat{p} \hat{m}_p + \hat{\psi} \hat{m}_\psi + \hat{z} \hat{m}_z \text{ avec } \psi = 0.$$



$$m_p = m_x \cos \varphi + m_y \sin \varphi = m \sin \theta \cos \varphi \cos \varphi + \sin \theta \sin \varphi \sin \varphi = \sin \theta \cos(\varphi - \varphi)$$

$$m_\varphi = -m_y \sin \varphi + m_y \cos \varphi = \sin \theta \sin(\Phi - \varphi), \quad m_z = m_z - \cos \theta \sin \varphi \cos \varphi$$

Работая в деканате и принимая участие в работе, наместник през. орг. ун-та

$$\vec{A}' = (A_x', A_y', A_z') \quad \vec{A} = s(\vec{n} \times \vec{m}) = s \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ m_x & m_y & m_z \end{vmatrix} = s(\hat{j} m_x - \hat{i} m_y), \quad s = \frac{M_s}{\mu_0} (1 + \chi)^{-1}$$

$$\vec{n}' \times \vec{m}' = \vec{R} \vec{n} \times \vec{R} \vec{m} = \det \vec{R} \cdot (\vec{R}^{-1})^T (\vec{n} \times \vec{m}) \quad \text{--- invariant}$$

$$\vec{A}' \cdot \vec{\dot{m}}' = s(\hat{y}' m_x - \hat{x}' m_y)(\dot{m}_x \hat{x}' + \dot{m}_y \hat{y}' + \dot{m}_z \hat{z}') = s[m_x \dot{m}_y - m_y \dot{m}_x]$$

$$\vec{A}(\vec{m}) \cdot \dot{\vec{m}} = \vec{A}(R\vec{m}) \cdot R\dot{\vec{m}}, \quad \vec{m} = R\vec{m}', \quad \dot{\vec{m}} = R\dot{\vec{m}}' \quad \text{rovnice jsou } \dot{R} = 0, \text{ zto ne platí}$$

$$\vec{A}(\vec{R}\vec{m}') \cdot \frac{d(\vec{R}\vec{m}')}{dt} = s(\vec{n}' \times \vec{R}\vec{m}') \cdot \frac{d(\vec{R}\vec{m}')}{dt} = s(\vec{n}' \times \vec{m}') \cdot \frac{d(\vec{R}\vec{m}')}{dt} = s \det R \cdot (R^{-T})^T (\vec{n}' \times \vec{m}')$$

$$\frac{d\vec{R}\vec{m}}{dt} = \left(\text{mag. of } \dot{\vec{R}} = 0 \right) = \left| \begin{matrix} \vec{R}^{-1} = \vec{R}^T \\ \det \vec{R} = 1 \end{matrix} \right. \text{ gauge fix.} \left. \right| = s (\vec{R}^{-1})^T (\vec{n}' \times \vec{m}') \cdot \vec{R}\vec{m}' = s \vec{R} (\vec{n}' \times \vec{m}') \cdot \vec{R}\vec{m}'$$

Dies beweisen $\vec{n} \times \vec{m} = R\vec{n}' \times R\vec{m}' = R(\vec{n}' \times \vec{m}')$

$$\vec{A}(\vec{m}) \cdot \dot{\vec{m}} = s(\vec{m}) (\vec{n} \times \vec{m}) \cdot \dot{\vec{m}} = s(\vec{m}') (\vec{n} \times R\vec{m}') \cdot \frac{1}{dt} (R\vec{m}') = s(R\vec{m}') R(\vec{n} \times \vec{m}') \cdot R\dot{\vec{m}}'$$

$$= |\vec{r}_0| \cos \theta = 1 = S(R\vec{m}_1) \cdot (\vec{n}' \times \vec{m}') \cdot \vec{m}_1' \quad \vec{n} \cdot \vec{m} = \vec{n} \cdot R\vec{m}_1' = R\vec{n} \cdot \vec{m}_1' = \vec{n} \cdot \vec{E}$$

$$\frac{d}{dt} \mathbf{L}_{K_1} = \dot{\mathbf{A}}(\vec{m}) \cdot \vec{m} = S(\vec{m}')(\vec{n}' \times \vec{m}') \cdot \vec{m}' = -A(\vec{m}') \vec{m}' = \mathbf{L}_{K_2}(\vec{m}') \text{ avec } \vec{L} = 0 \vec{m} = R \vec{m}'$$

(iii) T.e. Expansion co-efficient $\alpha_{kin}(\vec{r}_i)$ where $\vec{r}_i = R\vec{r}_i$ and $\dot{R} = 0(?)$

→ мы же $L_{kin}(\vec{m}) \propto \vec{m} \Rightarrow \vec{m}' = R\vec{m}$ и тогда $L(\vec{m}') = L(\vec{m})$ $\vec{m}' = R\vec{m}$ если R-оператор
100% из к.п.г.

$$\frac{d}{dt}(\hat{R}\vec{m}) = \dot{S}(\vec{R}\vec{m}) (\vec{n} \times \vec{R}\vec{m}) + \frac{d}{dt}(\vec{R}\vec{m}) = \dot{S}(\vec{R}\vec{m}) [\vec{R}(\vec{R}^{-1}\vec{n}) \times \vec{R}\vec{m}] \vec{R}\dot{\vec{m}} = (\dot{\vec{R}} = 0) =$$

$$= S(R\vec{m}) R [R^{-1}\vec{n} \times \vec{m}] \cdot R\vec{m} = S(R\vec{m}) (R^{-1}\vec{n} \times \vec{m}) \cdot \vec{m} \stackrel{R}{=} (1 + \vec{n} \cdot R\vec{m}) (R^{-1}\vec{n} \times \vec{m}) \cdot \vec{m} =$$

$$= S(\vec{m}) (\vec{n} \times \vec{m}) \cdot \vec{m} = \perp(\vec{m}) \text{ atau } \vec{n} \perp R^{-1}\vec{n} \text{ tak bisa karena } R \text{ linear.}$$

$$\text{Def } D_t(R\vec{m}) = R D_t(\vec{m}), \quad D_t(\vec{m}) = \frac{\partial}{\partial t} + g f(t). \quad D_t(R\vec{m}) = \left[\frac{\partial}{\partial t} + g f(t) \right] R\vec{m} = \\ = R\dot{\vec{m}} + \dot{R}\vec{m} + g f R\vec{m}, \quad D_t(\vec{m}) = \dot{\vec{m}} + g f \vec{m}. \quad \text{Therefore } R\dot{\vec{m}} + \dot{R}\vec{m} + g f R\vec{m} = R\dot{\vec{m}} + g f R\vec{m}$$

$$\dot{R}\vec{m} + g\dot{f}R\vec{m} = gR\dot{f}\vec{m}, \quad \dot{R} + g\dot{f}R = gR\dot{f}$$

$$D_t(R\vec{m}) = \left[\frac{\partial}{\partial t} + g\dot{f}\right](R\vec{m}) = \left[\frac{\partial}{\partial t} + gR\dot{f}R^{-1} - \dot{R}R^{-1}\right]R\vec{m} = \left[\dot{R} + gR\dot{f} - \dot{R}\right]\vec{m} + R\dot{\vec{m}} = R[\dot{\vec{m}} + g\dot{f}\vec{m}] = R\left[\frac{\partial}{\partial t} + g\dot{f}\right]\vec{m} = RD_t(\vec{m}), \quad g \text{ equiv. to } \dot{f}, \text{ since we use T. e. u.p.m. gauge } \vec{m} \rightarrow \vec{m}' = R\vec{m}, \text{ hence } \dot{f} \rightarrow \dot{f}' = R\dot{f}R^{-1} - \dot{R}R^{-1}, \text{ since } \dot{f} \text{ is unvar.}$$

T. e. u.p.m. gauge $\vec{m} \rightarrow \vec{m}' = R\vec{m}$, hence $\dot{f} \rightarrow \dot{f}' = R\dot{f}R^{-1} - \dot{R}R^{-1}$, since $\dot{f}$ is unvar.

Take note: $D_t = \frac{\partial}{\partial t} + \dot{f}$ - covariant derivative. Since we set $\dot{f} = 0$ in u.p.m. gauge, so u.p.m. $\vec{m} \rightarrow R\vec{m}$ is a pure gauge field $\hat{f} = -\dot{R}R^{-1}$ - pure gauge field.

$$I_{kin}(\vec{m}) = \vec{A}(\vec{m}) \cdot D_t \vec{m} = \vec{A} \cdot \left(\frac{\partial}{\partial t} + \dot{f}\right)\vec{m} = \vec{A} \cdot \dot{\vec{m}} + \vec{A} \cdot \dot{f}\vec{m}, \quad \text{where } \vec{A} \cdot \dot{f}\vec{m} = I_{int}$$

$$I_{int} = \vec{A} \cdot \dot{f}\vec{m} = -\vec{A} \dot{R}R^{-1}\vec{m}, \quad \dot{R} \text{ is given by } \dot{\theta}_V, \dot{\phi}_V \text{ and } \dot{\chi} \text{ of moving vertex.}$$

$$R_{ij} = \hat{e}_i \cdot \hat{e}'_j, \quad \text{where } \hat{e}_i \text{ - basis vectors of } xyz, \text{ and } \hat{e}'_j \text{ - basis vectors of } x'y'z'.$$

$$\hat{e}'_3 = \hat{z}' = \hat{x} \sin\theta \cos\phi + \hat{y} \sin\theta \sin\phi + \hat{z} \cos\theta$$

$$\hat{e}'_2 = \hat{y}' = \hat{z} \times \hat{e}'_3 = \hat{z} \times \hat{z}' = \hat{z} \times (\hat{x} \sin\theta \cos\phi + \hat{y} \sin\theta \sin\phi + \hat{z} \cos\theta) = -\hat{x} \sin\theta \sin\phi + \hat{y} \sin\theta \cos\phi$$

$$\hat{e}'_1 = \hat{x}' = \hat{e}'_2 \times \hat{e}'_3 = (-\hat{x} \sin\theta \sin\phi + \hat{y} \sin\theta \cos\phi) \times (\hat{x} \sin\theta \cos\phi + \hat{y} \sin\theta \sin\phi + \hat{z} \cos\theta) = -\hat{z} \sin^2\theta + \dots = (\hat{z} \times \hat{e}'_3) \times \hat{e}'_3 = \hat{e}'_3 (\hat{z} \cdot \hat{e}'_3) - \hat{z} (\hat{e}'_3 \cdot \hat{e}'_3) = \hat{e}'_3 (\hat{z} \cdot \hat{e}'_3) - \hat{z} = \hat{z}' (\hat{z} \cdot \hat{z}') - \hat{z} = (\hat{x} \sin\theta \cos\phi + \hat{y} \sin\theta \sin\phi + \hat{z} \cos\theta) \cos\theta - \hat{z} = \hat{x} \sin\theta \cos\theta \cos\phi + \hat{y} \sin\theta \cos\theta \sin\phi - \sin^2\theta \hat{z}, \quad \hat{e}'_1, \hat{e}'_2 \text{ are also eq. basis vectors in } x'y'z'.$$

$$R = \begin{pmatrix} \sin\theta \cos\theta \cos\phi & -\sin\theta \sin\phi & \sin\theta \cos\phi \\ \sin\theta \cos\theta \sin\phi & \sin\theta \cos\phi & \sin\theta \sin\phi \\ -\sin^2\theta & 0 & \cos\theta \end{pmatrix}, \quad \text{where } \theta = \theta_V, \phi = \phi_V \text{ - vertex angles.}$$

One case $\theta_V = \pi/2, \cos\theta_V = 0$ $R = \begin{pmatrix} 0 & -\sin\phi & \cos\phi \\ 0 & \cos\phi & \sin\phi \\ -1 & 0 & 0 \end{pmatrix}$, $\hat{z}' = \hat{x} \cos\phi + \hat{y} \sin\phi$
If $\theta_V = \pi/2$ R - becomes $\hat{y}' = -\hat{x} \sin\phi + \hat{y} \cos\phi$
 $\hat{x}' = -\hat{z}$

$$\dot{R} = \begin{pmatrix} 0 & -\dot{\cos\phi} & -\dot{\sin\phi} \\ 0 & -\dot{\sin\phi} & \dot{\cos\phi} \\ 0 & 0 & 0 \end{pmatrix} \dot{\phi}, \quad R^{-1} = R^T = \begin{pmatrix} 0 & 0 & -1 \\ -\sin\phi & \cos\phi & 0 \\ \cos\phi & \sin\phi & 0 \end{pmatrix}$$

$$\dot{R}R^{-1} = \dot{\phi} \begin{pmatrix} 0 & -\cos\phi & -\sin\phi \\ 0 & -\sin\phi & \cos\phi \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ -\sin\phi & \cos\phi & 0 \\ \cos\phi & \sin\phi & 0 \end{pmatrix} = \dot{\phi} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sim \vec{G} \text{ - up to a sign}$$

$$\dot{R}R^{-1}\vec{m} = \dot{\phi} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \dot{\phi} \begin{pmatrix} -m_y \\ m_x \\ 0 \end{pmatrix}, \quad \vec{m} \text{ - normal vector.}$$

$$I_{int} = -\vec{A} \dot{R}R^{-1}\vec{m} = -\vec{A} \dot{\phi} \begin{pmatrix} -m_y \\ m_x \\ 0 \end{pmatrix} = (m_y A_x - m_x A_y) \dot{\phi} = \vec{m} \times \vec{A} \cdot \vec{\nabla} \phi = (\vec{m} \times \vec{A})_z \dot{\phi}$$

$$\vec{m} \times \vec{A} = \vec{m} \times s(\vec{n} \times \vec{m}) = s[\vec{n}(\vec{m} \cdot \vec{m}) - \vec{m}(\vec{m} \cdot \vec{n})], \quad (\vec{m} \times \vec{A})_z = s n_z - s m_z(\vec{m} \cdot \vec{n})$$

$$I_{int} = \frac{M_s \dot{\phi}}{s(1+\vec{n} \cdot \vec{m})} [n_z - m_z(\vec{m} \cdot \vec{n})] = |\vec{n} - \vec{m}| = \frac{M_s \dot{\phi}}{s(1+\cos\theta)} [1 - m_z^2] = \frac{M_s (1-\cos\theta) \dot{\phi}}{s} \quad (!)$$

U. Vortex - SW interaction term (Feb-March 2008), corresponding to the SW - azimuthal splitting (sphere magnetostatic coupling).

$(\dot{R}R^{-1})_{ij} = -\epsilon_{ij} \dot{\phi}$ - Gauge tensor field $\hat{f} = \epsilon_{ij} \dot{\phi}$ - this dynamic origin, $\epsilon_{12} = 1, \epsilon_{21} = -1$ - also, u.p.m. gauge.

$$\cos \Theta = \cos(\Theta_v + \Theta_{sw}) \approx \cos(\pi/2 + \Theta_{sw}) = -\sin \Theta_{sw} = -1$$

$$\begin{aligned} \mathcal{L}_{\text{int}}(\text{Vortex-SW}) &= -\frac{M_S}{f} \cos\theta \dot{\Phi}_V = -\frac{M_S}{f} [\cos\theta_V \cos\theta_{SW} + \sin\theta_V \sin\theta_{SW}] \dot{\Phi}_V = \\ &= -\frac{M_S}{f} \cos\theta_{SW} \cos\theta_V \dot{\Phi}_V + \frac{M_S}{f} \sin\theta_{SW} \sin\theta_V \dot{\Phi}_V \approx \left| \text{uncomp. SW-VC int.} \right| = \frac{M_S}{f} \sin\theta_{SW} \sin\theta_V \dot{\Phi}_V. \end{aligned}$$

- recognize game as an Θ_{SW} no max, i.e. game non-linear SW. Linear $\Theta = \alpha_{SW}$

Due Vortex $L_{kin} = \vec{A}_v \cdot \vec{\dot{X}} + \vec{K} \cdot \vec{\dot{X}}$, $\vec{K} = \frac{M_S L \pi m \delta}{\gamma} \text{Im} \{ I_a [\vec{e}_1 \sin \omega t - m \vec{e}_2 \cos \omega t] \} \sim I_a$

Esam der yggdrasil $I_a = \int d^3x \sin \theta_0 m_0 a$ beifügt zu $\vec{X}$ um $\vec{\Phi}_v$, to korrep.
 b $\frac{1}{2} m_0 v^2$ um $(\vec{X})^2$ um $(\vec{\Phi}_v)^2$ zur sel Vertex mass.

$\vec{r}(p, t) = a(p) \cos(m\varphi - \omega t)$, $\vec{y}(p, t) = b(p) \sin(m\varphi - \omega t)$ = μ_p um $b(p)$ ansetzen.

из уравнения $\ddot{\varphi} = \frac{\omega_M^2}{4\pi} (\Delta^2 \varphi + G_{\varphi\varphi} \varphi + \ddot{\Phi}_V)$, $\omega^2 \varphi = \omega_M^2 (\Delta \varphi + \frac{1}{2} m_0(r) \rho m_0 \frac{\omega^3}{\omega_M} I_6$
 $I_6 = \int d\rho \rho \sin \theta_0 m_0 \varphi$. Т.е. где $\varphi(r)$ — это — не-универсальная функция.

Non-uniform eq. Fredholm (2-го рода) и нест. решение имеет и/ли $\int_{\Omega} m_0(p) \varphi_n(p) dx = 0$ т.е. и/ли сф. ортогонального eq. $\varphi_n(p)$ ортогональны неогр. $m_0(p)$?

$$\omega^2 \beta(p) = \int d\mathbf{p}' p' d_{\mathbf{p}\mathbf{p}'} \beta(p') + F(p), \quad F(p) = \frac{1}{2} m_0(p) p \ln \left(\frac{\omega}{\omega_m} \right)^3 I_\beta, \quad \omega \rightarrow \omega/\omega_m, \quad m_0 = \frac{1}{3}(1-p^2)$$

$$+\frac{1}{2} \ddot{e} e + \frac{1}{2} \ddot{\rho} \rho' G_{\rho\rho} \mu = 0 \quad \sim \quad \frac{1}{2} \ddot{\mu} + \frac{1}{2} \ddot{\rho} \rho' G_{\rho\rho} e e = \frac{1}{2} \ddot{\Phi}_V, \quad \dot{\mu} + \omega_m e e = \dot{\Phi}_V \frac{1}{2}$$

$$\begin{pmatrix} \mu \\ \psi \end{pmatrix} = - \begin{pmatrix} 0 & W_M \\ G_M & 0 \end{pmatrix} \begin{pmatrix} \mu \\ \psi \end{pmatrix} + \begin{pmatrix} \Phi_V \\ 0 \end{pmatrix}, \text{ найдем неограниченное реш. } \mu, \psi \sim \Phi_V$$

15/12 MphBerna 8. create on-line. May 2008 \$1600 tax return U.S. Zapernat
 Ha on-line service Liti B., San Sebastian. Update CV. Nuccma Arambiza
 Cristina, Sewin, Parkhad, Kearney, (C) 2008 no "Topological phases".

Kugel Borsu $A_\mu = (A_0, \vec{A})$ - 4-х векторни компоненти, т.е. скаляр и
 вектор, называемый потенциалом, $A_0 \sim \int \text{вектор}$, где SW.

Note: тана. берг. - ностриан А, ностриан и ностриан к загоре
 ГМК - сир. interaction (math. and) (math. and) (math. and)

16/v) Determine a fermion vertex mass, i.e. mass $M(\vec{x})^2$ of $\vec{x}$. Higgs & Higgs boson
Static $M(H)$ - similit.
 $\vec{x}(t) = \pi M_{cl} \cdot \vec{x}$ (1) Vertex mass

$$\vec{K}(t) = \pi \frac{N_s L}{g} \delta_{lm,1} (-i) e^{im\omega t} I_a, \quad I_a = \int_0^{\pi} p m_0 \sin \theta_0 a(p), \quad a x \left(\frac{\omega}{\omega_c} \right) / b \quad \text{Vertex mass}$$

$L_{int} = M_S L_{g-1} \int d^2 \vec{p} \sin \theta_p \vec{e} \vec{\Phi}_p = \vec{K} \cdot \vec{X}$, т.е. можно считать $\vec{\Phi}_p$ в пространстве L_{int} как $\vec{K} \sim \vec{X}$.

$$\Phi_v = m_0(p) [\sin \varphi \dot{X} - \cos \varphi \dot{Y}] \quad \left\{ \begin{array}{l} \ddot{x} + \gamma M_3 d^2 y^2 \cdot G_{yy} M = 0 \\ \ddot{y} + \omega_M^2 y = \ddot{\Phi}_v \end{array} \right.$$

Die zwei von 24 dB, $x(t) = x(\nu)e^{i\omega t}$ $\left\{ \begin{aligned} \mu + \omega \mu e t &= \dot{\phi}_\nu \\ \mu(t) &= \mu(\nu)e^{i\omega t}, \dot{\phi}_\nu(t) = \dot{\phi}_\nu(\nu)e^{i\omega t} \end{aligned} \right.$

$$0 = \ddot{\alpha} + g M_S \int d^2 p' G_{pp} \mu = i v \psi(v) + g M_S \int d^2 p' G_{pp} \mu(\vec{p}; v) e^{i v t} \quad i v \psi(v) = -g M_S \int d^2 p' G_{pp} \mu(v)$$

$$i\nu\mu(v) + \omega_M u(v) = \frac{1}{8} \Phi_0(v), \quad (-i\nu) i\nu u(v) = v^2 u(v) = +gM_S (\hbar^2 p^2 / G_{pp}) \mu(v)$$

$$v^2 \psi(\vec{r}, v) + \frac{\omega_m^2}{4\pi} \int d^2 \vec{r}' G_{pp} \psi(\vec{r}', v) = \frac{\omega_m}{4\pi} \int d^2 \vec{r}' G_{pp} \dot{\Phi}_v(\vec{r}', v)$$

Vortex mass

No.
 year month day ()

$\frac{\omega_c \hat{M}}{k-1} \approx \frac{\omega_c}{\omega_m} \ll 1$, no clear VC moves $\epsilon \omega$ (sw), to $\frac{\omega \hat{M}}{161} \approx \frac{\omega}{\omega_m} \sim 0.1$ So see Zaitsev.

$\frac{\hat{M}}{g} \approx \frac{L}{f^2} \approx \frac{1}{(2\pi)^2} \frac{10^6 \text{ cm}}{3^2 \text{ MHz}^2} \approx \frac{1}{(2\pi)^2} \frac{10^7 \text{ cm} \cdot \text{Å}^2}{(\text{MHz})^2} = \frac{1}{(2\pi)^2} 10^{-7} 10^{-12} \text{ s}^2 \cdot \text{cm} \cdot \text{Å}^2 \approx 10^{-21} \frac{\text{erg}}{\text{cm}^2} \approx 10^{-21} \text{ g}$

$\frac{\text{Å}^2}{\text{cm}^2} = \frac{\text{Å}^2}{(\text{cm})^2} = \left(\frac{\text{Å}}{\text{cm}}\right)^2 = \frac{F}{(\text{g} \cdot \text{cm}^2)} \cdot \hat{M} \sim 10^{-21} \text{ g}$ - main vortex b. param (ccs)

$g(p): \nabla^2 g(p) = \omega_m^2 \int d\mathbf{p}' d\mathbf{p}'' g(p') = \omega_m^2 F_d(p)$, Def. $\left(\frac{p}{\hbar}\right)^2 = \omega^2$, $\sqrt{g(p)} = f(p)$, $\sqrt{p'} d\mathbf{p}' = f(p') dp'$

$\omega^2 f(p) - \int d\mathbf{p}' d\mathbf{p}'' f(p') = F(p)$ - yadneme na $g(p) = \frac{1}{\sqrt{p}} f(p) = f(p) \cdot \Phi$ peregibnaya 2-ya p. m.

$\int d\mathbf{y} K(x, y) \psi(y) = \delta^2 \psi(y) = f(x) \Rightarrow \psi(x) = \int d\mathbf{y} R(x, y) f(y)$ - p. m. Fredholm eq.

pezoname $R(x, y) = \sum_n \frac{\psi_n^*(y) \psi_n(x)}{\omega_n - \omega}$, rge $\int d\mathbf{y} K(x, y) \psi_n(y) = \omega_n \psi_n(x)$

$\omega = \frac{1}{\omega_c} = \omega^2$, $R(\omega^2, p') = \sum_n \frac{\psi_n^*(p') \psi_n(p)}{\omega_n^2 - \omega^2}$, rge $\int d\mathbf{p}' d\mathbf{p}'' f(p') = \omega_n^2 f_n(p)$, $\psi_n(p) = f_n(p)$ - ual.

$R(\omega^2, p, p') = \sum_n \frac{\psi_n^*(p') \psi_n(p)}{(\omega_n^2 - \omega^2)}$ $f(p) = \int d\mathbf{p}' R(\omega^2, p, p') f(p') = \int d\mathbf{p}' F(p') \sum_n \frac{f_n(p') f_n(p)}{\omega^2 - \omega_n^2}$

$g(p) = \frac{1}{\sqrt{p}} f(p) = \frac{1}{\sqrt{p}} \sum_n \int d\mathbf{p}' \sqrt{p'} F_d(p') \frac{f_n(p) g_n(p')}{[\omega^2 - \omega_n^2]} = \sum_n \int d\mathbf{p}' p' F_d(p') \frac{f_n(p) g_n(p')}{[\omega^2 - \omega_n^2]}$

$g(p) = \sum_n \frac{C_n}{[\omega^2 - \omega_n^2]} g_n(p)$, $C_n = \int d\mathbf{p}' p' F_d(p') g_n(p') = \int d\mathbf{p}' \int d\mathbf{p}'' d\mathbf{p}''' m_0(p) g_n(p')$ - mas p. m. 2-ya dyn. dipole coupling vortex - SW.

$\int d\mathbf{p}' p' d\mathbf{p}'' g_n(p') = \omega_n^2 g_n(p) \rightarrow g_n(p), \omega_n = \text{CP/C3 Spin-waves of 1st. g.}$

$\omega = \omega/\omega_m$, ψ - snglyna 2-ya 2-ya Fourier - transform.

T. d. $g(p) = g(p, \varphi) \cdot \psi(\varphi, \psi) = \frac{1}{\omega_m} [-\dot{X}(\psi) \sin \varphi + \dot{Y}(\psi) \cos \varphi] g(p, \psi)$

$\psi(\varphi, L) = \frac{1}{2\pi} \int d\psi e^{i\psi L} \psi(\varphi, \psi) \sim \dot{X}(L), \dot{Y}(L)$ u p. m. 2-ya k. m. 2-ya

$g(p, \varphi) = \sum_n \frac{C_n}{[\omega^2 - \omega_n^2]} g_n(p)$, $C_n = \langle m_0 | d_{pp} | g_n \rangle$. C_n - main u y. snglyna 2-ya p. m. 2-ya

Anomalous rezonans. g. m. 2-ya 2-ya 7/111 68? u p. m. 2-ya 2-ya

$i\dot{Z} + \omega_0 Z = F(t)$, $F(t) = \frac{1}{G_2} (K_x + iK_y)$, $F(t) = F_0 e^{im\omega t}$, rge $\omega \omega_3 \sin \omega(\psi - \omega t)$

For my rezonans 2-ya p. m. 2-ya na 2-ya 2-ya $Z(t) = \frac{F_0}{\omega_0} e^{im\omega t}$

$\dot{\Phi}_\psi = m_0 [\dot{X} \sin \varphi - \dot{Y} \cos \varphi]$, $\dot{Z} = \dot{X} + i\dot{Y} = F_0 \frac{im\omega}{(\omega_0 - m\omega)} e^{im\omega t} \approx -iF_0 e^{im\omega t} =$

$\dot{\Phi}_\psi = F_0 m_0 [\sin \varphi \sin m\omega t + \cos \varphi \cos m\omega t] = m_0 F_0 \cos(\varphi - m\omega t) = F_0 (\sin m\omega t - i \cos m\omega t)$

$\dot{\Phi}_\psi = m_0 F_0 \cos(m\varphi - \omega t)$, $F_0 = -\frac{1}{2} p m \omega I_a$, $I_a = \int d\mathbf{p}' \sin \theta_0 m_0 a(p)$

u g. m. 2-ya 2-ya $\dot{\Phi}_\psi$ snglyna 2-ya 2-ya 2-ya $\dot{\Phi}_\psi = \frac{\omega_m^2}{4\pi} \int d^2 p' G_{pp} + \dot{\Phi}_\psi$ 19/11/08

u g. m. 2-ya 2-ya $\Delta \omega_{n, m=21} \sim \omega_{n1}^2$

$C \psi(\varphi, t) = \frac{1}{2\pi} \int d\psi \psi(\varphi, \psi) e^{i\psi t}$ a rezonans. 2-ya 2-ya 2-ya

$\dot{\psi} = \frac{\omega_m^2}{4\pi} \int d^2 p' G_{pp} \psi(\varphi, t) = \omega_m F_d(p) [\sin \varphi \dot{X} - \cos \varphi \dot{Y}]$ $\dot{X}, \dot{Y} \sim e^{i\omega_0 t}$

no p. m. 2-ya 2-ya p. m. 2-ya 2-ya $\psi(\varphi, t) \sim F_d(p) [\sin \varphi \dot{X} - \cos \varphi \dot{Y}] \propto \omega_{n1} \cdot \omega_0$

$\sin \varphi \dot{X} - \cos \varphi \dot{Y} = -|Z| [\sin \varphi \sin \omega_0 t + \cos \varphi \cos \omega_0 t] = -|Z| \cos(\varphi - \omega_0 t) [\dot{X} \sim -\sin \omega_0 t]$

no p. m. 2-ya 2-ya $\psi(\varphi, t) = A \cos(\varphi - \omega_0 t) = g(p) \cos(\varphi - \omega_0 t)$, $X(t) = |Z| \cos \omega_0 t$, $Y(t) = |Z| \sin \omega_0 t$

$-\omega_0^2 g \cos(\varphi - \omega_0 t) = \frac{\omega_m^2}{4\pi} \int d\mathbf{p}' p' g(p') \int d\mathbf{p}'' G_{pp} \cos(\varphi' - \omega_0 t) = -\omega_m |Z| F_d(p) \cos(\varphi - \omega_0 t)$

2 No.

year

11.1

2.

40/

4

100

1

—

25

261

30

27

AN

Notes

5

), and

100

1912

$$\frac{1}{2}$$

Ex

0

2

2

(11)

(A)

No

yea

20

10

4

0

5

0.122

—

200

54.

... J.

2

2

44-1

1.

2

5

$\frac{1}{2}x$

0-2

25

1

34.

3) =

10. $\frac{1}{2}$

10

27

en/p

2. be

7

$\frac{d}{dx}$

10

98-

ak

$Z_R = \begin{pmatrix} 0 & -\omega_P & -\omega_P \\ 0 & -\omega_P & \omega_P \\ 0 & 0 & 0 \end{pmatrix}$

$$a_\mu = -\frac{\Phi_0 S}{(1+m_z)} [m_y \partial_\mu m_x - m_x \partial_\mu m_y], \quad \vec{m} = (\cos\varphi \sin\theta, \sin\varphi \sin\theta, \cos\theta)$$

T.e. $a_\mu = 2\Phi_0 S A_\mu^z$, $A_\mu^z = \frac{1}{2}(1 - \cos\theta) \partial_\mu \Phi$ - определение б. намагниченности в шиватах, татах'08.

Вектор можно рассмотреть как матрицу, вращая ось $\hat{y}' = \hat{e}_2' = \hat{z} \times \hat{z}'$ по $\hat{y}$ на θ .

$$\hat{y}' = \hat{e}_2' = \hat{x} \sin \theta \sin \varphi + \hat{y} \sin \theta \cos \varphi = (-\sin \theta \sin \varphi, \sin \theta \cos \varphi, 0) = \vec{u}$$

Надо $\vec{u} = \frac{1}{\sin \theta} \hat{y}'$, $\vec{u}^2 = \frac{1}{\sin^2 \theta} \hat{y}' \cdot \hat{y}' = 1$. $\vec{u} = (-\sin \Phi, \cos \Phi, 0)$ (но $|\vec{u}| \neq 1$).

$Q_u(\pi/2) = \begin{pmatrix} \sin^2 \varphi & -\sin \varphi \cos \varphi & \cos \varphi \\ -\sin \varphi \cos \varphi & \sin^2 \varphi & \sin \varphi \\ -\cos \varphi & -\sin \varphi & 0 \end{pmatrix} = R(?)$, не совсем $\in R^{14/2}$

$$R_A = \begin{pmatrix} \cos\theta \cos\phi & \sin\phi \cos\theta & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \\ \sin\theta \cos\phi & \sin\theta \sin\phi & \cos\theta \end{pmatrix}, R_A(\theta = \pi/2) = \begin{pmatrix} 0 & 0 & -1 \\ -\sin\phi & \cos\phi & 0 \\ \cos\phi & \sin\phi & 0 \end{pmatrix}$$

Надо написать поворот системы коор. от Ox к $Ox' \parallel (O_n, \Phi_n)$, как в 14/IV, но в 14/V и угол воз. наклона $\sin \theta$ в $\hat{y}'$ и $\hat{z}'$, это не скажешь на $\theta = \pi/2$

$$\nabla_{\mu} R = \begin{pmatrix} -\sin\theta \cos\theta \partial_{\mu}\phi - \cos\theta \sin\theta \partial_{\mu}\theta & -\cos\theta \partial_{\mu}\phi & \cos\theta \cos\phi \partial_{\mu}\theta - \sin\theta \sin\phi \partial_{\mu}\phi \\ \cos\theta \cos\phi \partial_{\mu}\phi - \sin\theta \sin\phi \partial_{\mu}\theta & -\sin\theta \partial_{\mu}\phi & \cos\theta \sin\phi \partial_{\mu}\theta + \sin\theta \cos\phi \partial_{\mu}\phi \\ -\cos\theta \partial_{\mu}\theta & 0 & -\sin\theta \partial_{\mu}\theta \end{pmatrix}$$

$$\frac{\partial}{\partial \theta} \ln L = - \left(\frac{\partial}{\partial \theta} \ln L \right) = - \left(\frac{\partial}{\partial \theta} \ln L \right) = - \left(\frac{\partial}{\partial \theta} \ln L \right)$$

$$1 - a \cos \theta; 2 \cos \theta = -(S \Phi' \cdot C \Phi' + C \Phi' \cdot S \Phi') S \Phi' C \Phi' - C \Phi' C \Phi' = -2 S \Phi' C \Phi' = -2 \sin \theta \cos \theta = -\sin 2\theta$$

$$1 - \cos \theta = 2 \cos \theta = + (s \phi c \theta \phi' + c \phi s \theta \theta') s \theta - c \phi \phi' \cdot 0 + (c \theta c \phi \theta' - s \theta s \phi \phi') c \theta = \cos \phi \theta$$

$$2 \cos \varphi \cdot \cos \theta = (\cos \varphi \sin \theta' - \sin \varphi \cos \theta') \cos \varphi + \sin \varphi \sin \theta' + (\cos \varphi \cos \theta' + \sin \varphi \sin \theta') \sin \varphi = \cos \varphi$$

$$2 \cos \varphi \cdot 2 \cos \theta = (\cos \theta \cos \varphi - \sin \theta \sin \varphi) \sin \theta \cos \theta - \sin \theta \cos \varphi + (\cos \theta \sin \varphi + \sin \theta \cos \varphi) \sin \theta \sin \varphi = 0$$

$$2 \cos \varphi \cdot 2 \sin \theta = -(\cos \theta \cos \varphi - \sin \theta \sin \varphi) \sin \theta - \sin \theta \cos \varphi + (\cos \theta \sin \varphi + \sin \theta \cos \varphi) \cos \theta = \sin \theta \sin \varphi \partial_\mu \theta$$

$$3 \cos \varphi \cdot 4 \cos \theta = -\cos \theta \cos \varphi \cos \theta \cos \varphi - \sin \theta \cos \varphi \sin \theta \cos \varphi = -\cos \varphi \partial_\mu \theta$$

$$3 \cos \varphi \cdot 2 \cos \theta = -\cos \theta \cos \varphi \sin \theta \cos \theta - \sin \theta \cos \varphi \sin \theta \sin \theta = -\sin \theta \partial_\mu \theta$$

$$3 \cos \varphi \cdot 3 \cos \theta = +\cos \theta \cos \varphi \sin \theta - \sin \theta \cos \varphi \cos \theta = 0$$

$$\hat{A}_\mu = \partial_\mu R R^{-1} = \begin{pmatrix} 0 & 2 \partial_\mu \varphi & \cos \varphi \partial_\mu \theta \\ \partial_\mu \varphi & 0 & \sin \varphi \partial_\mu \theta \\ -\cos \varphi \partial_\mu \theta & -\sin \varphi \partial_\mu \theta & 0 \end{pmatrix} = -\sin \varphi \partial_\mu \theta \hat{e}_x + \cos \varphi \partial_\mu \theta \hat{e}_y + \partial_\mu \varphi \hat{e}_z$$

$$\hat{A}_\mu = [A_\mu^x \hat{e}_x + A_\mu^y \hat{e}_y + A_\mu^z \hat{e}_z], \quad A_\mu^x = -\sin \varphi \partial_\mu \theta, \quad A_\mu^y = \cos \varphi \partial_\mu \theta, \quad A_\mu^z = \partial_\mu \varphi$$

$$\hat{A}_\mu \vec{m} = \hat{A}_\mu \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} -\partial_\mu \varphi m_y + m_z \cos \varphi \partial_\mu \theta \\ m_x \partial_\mu \varphi + m_z \sin \varphi \partial_\mu \theta \\ -m_x \cos \varphi \partial_\mu \theta - m_y \sin \varphi \partial_\mu \theta \end{pmatrix} = \begin{pmatrix} -m_y \\ m_x \\ 0 \end{pmatrix} \partial_\mu \varphi + \begin{pmatrix} m_z \cos \varphi \\ m_z \sin \varphi \\ -\sin \theta \end{pmatrix} \partial_\mu \theta$$

t.e. $m_z \neq \text{const}$, $\partial_\mu \theta \neq 0$ to maintain gen. then $\hat{A}_\mu \vec{m}$.

Kobay. uprazhneniya $D_\mu = \partial_\mu - \hat{A}_\mu$, $\hat{A}_\mu (\mu = t, x, y, z)$ vedet nas klyuchevye, glubok.

$$\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{int}}^{\text{kin}} + \mathcal{L}_{\text{int}}^{\text{exch}} \quad (\vec{m})^2 = D_\mu \vec{m} \cdot D_\mu \vec{m} = (\partial_\mu \vec{m} - \hat{A}_\mu \vec{m}) \cdot (\partial_\mu \vec{m} - \hat{A}_\mu \vec{m})$$

$$(\vec{m})^2 = \sum_{j, \mu} \left(\frac{\partial m_j}{\partial x_\mu} \right)^2 = \sum_{j, \mu} \left(\frac{\partial m_j}{\partial x_\mu} - A_\mu^{jk} m_k \right)^2 = \sum_{j, \mu} \left[\left(\frac{\partial m_j}{\partial x_\mu} \right)^2 - 2 \frac{\partial m_j}{\partial x_\mu} A_\mu^{jk} m_k + (A_\mu^{jk} m_k)^2 \right]$$

$$\mathcal{L}_{\text{int}}^{\text{exch}} = -\frac{1}{2} d M_s^2 (\vec{m})_{\text{int}}^2 = -\frac{1}{2} d M_s^2 \sum_{j, k, \mu} \left[-2 \frac{\partial m_j}{\partial x_\mu} A_\mu^{jk} m_k + (A_\mu^{jk} m_k)^2 \right]$$

$\mathcal{L}_{\text{int}}^{\text{exch}}$ ne upreduet paku qnie $SW, r.k., n(\frac{L}{R})^2$ u oreno meno qne $R \sim 1/\mu$

$$\mathcal{L}_{\text{int}}^{\text{kin}} = -\vec{A}_s \cdot \hat{A}_0 \vec{m} = -\frac{M_s}{\delta} s(\vec{m}) (\vec{e} \times \vec{m}) \cdot \hat{A}_0 \vec{m} = \text{uvedet gon. then } m_z \text{ za } \partial_\theta = \dot{\theta}$$

$$\vec{A}_s \cdot \hat{A}_0 \vec{m} = (A_s^x, A_s^y, A_s^z) \begin{pmatrix} m_z \cos \varphi \\ m_z \sin \varphi \\ -\sin \theta \end{pmatrix} \dot{\theta} = (m_z A_s^x \cos \varphi + m_z A_s^y \sin \varphi - \sin \theta A_s^z) \dot{\theta}$$

$$\vec{A}_s = \frac{M_s}{\delta} s(\vec{n} \times \vec{m}) = |\vec{n}| = \hat{z} = \frac{M_s}{\delta} s(-\hat{x} m_y + \hat{y} m_x), \quad A_s^z = 0$$

$$\vec{A}_s \cdot \hat{A}_0 \vec{m} = m_z (A_s^x \cos \varphi + A_s^y \sin \varphi) \dot{\theta} = \frac{M_s}{\delta} s(-m_y \cos \varphi + m_x \sin \varphi) \dot{\theta} m_z = 0$$

t.k. $m_x = \sin \theta \cos \varphi$, $m_y = \sin \theta \sin \varphi$ // t.e. then $\dot{\theta}$ ne izmenit $\mathcal{L}_{\text{int}}^{\text{kin}}$ 14/10

Beopredelenie k exch. then $\partial_\mu m_j A_\mu^{jk} m_k = \partial_\mu \vec{m} \cdot \hat{A}_\mu \vec{m}$

$$\partial_\mu \vec{m} = \partial_\mu (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) = (\cos \theta \cos \varphi \partial_\mu \theta - \sin \theta \sin \varphi \partial_\mu \varphi, \cos \theta \sin \varphi \partial_\mu \theta + \sin \theta \cos \varphi \partial_\mu \varphi, -\sin \theta \partial_\mu \theta)$$

$$\begin{aligned} \partial_\mu \vec{m} \cdot \hat{A}_\mu \vec{m} &= (\cos \theta \cos \varphi \partial_\mu \theta - \sin \theta \sin \varphi \partial_\mu \varphi) (-\partial_\mu \varphi m_y + m_z \cos \varphi \partial_\mu \theta) + \\ &+ (m_z \sin \varphi \partial_\mu \theta + m_x \partial_\mu \varphi) (m_x \partial_\mu \varphi + m_z \sin \varphi \partial_\mu \theta) - \sin \theta \partial_\mu \theta (-\sin \theta \partial_\mu \theta) = \\ &= (m_z \cos \varphi \partial_\mu \theta - m_y \partial_\mu \varphi)^2 + (m_z \sin \varphi \partial_\mu \theta + m_x \partial_\mu \varphi)^2 + \sin^2 \theta (\partial_\mu \theta)^2 = \\ &= m_z^2 (\partial_\mu \theta)^2 + (m_x^2 + m_y^2) (\partial_\mu \varphi)^2 + \sin^2 \theta (\partial_\mu \theta)^2 = (\partial_\mu \theta)^2 + \sin^2 \theta (\partial_\mu \varphi)^2 \quad (!) \end{aligned}$$

$$[A_\mu^{jk} (\vec{m})_k]^2 = (\hat{A}_\mu \vec{m})^2 = (-\partial_\mu \varphi m_y + m_z \cos \varphi \partial_\mu \theta)^2 + (m_x \partial_\mu \varphi + m_z \sin \varphi \partial_\mu \theta)^2 + \sin^2 \theta (\partial_\mu \theta)^2 = (\partial_\mu \theta)^2 + \sin^2 \theta (\partial_\mu \varphi)^2 \quad (!) \quad \text{otkuda } \frac{\partial m_j}{\partial x_\mu} = A_\mu^{jk} m_k$$

$$\hat{A}_\mu \vec{m} = \partial_\mu \vec{m} \quad \text{u} \quad \mathcal{L}_{\text{int}}^{\text{kin}} = -\vec{A}_s \cdot \hat{A}_0 \vec{m} = -\vec{A}_s \cdot \vec{m}$$

$$\mathcal{L}_{\text{kin}} = \vec{A}_s D_\mu \vec{m} = \vec{A}_s \dot{\vec{m}} - \vec{A}_s \cdot \hat{A}_0 \vec{m} = \vec{A}_s \dot{\vec{m}} - \vec{A}_s \cdot \vec{m} = 0, \quad \partial_\mu = \partial_\mu - \hat{A}_\mu$$

Do $\vec{m}$ zdes ne nenas namennost, a tolko Vertex, $\vec{m} = \vec{m}_\nu (\theta_\nu, \varphi_\nu)$

Due
to
the
fact
that

$\vec{m} = \vec{m}_\nu (\theta_\nu, \varphi_\nu)$

В ответе: 1) SU(2) splitting $\Delta\omega_{n,\pm 1}$
 2) Vortex Mass - vortex mass quantum.
 Рассуж: выведи L_{int}^{v-sw} из волн. канон. исл. координат. 18/2

- 14/15 ед. см. 1.1) не верный шаг R-сдвиг (исправлено 17/17);
 2) упрости выраж в L_{int} : Надо $L_{int} = -\frac{M_S}{\hbar} (1 - \cos\theta) \Phi$, где $\theta = \theta_v$, $\Phi = \Phi_v$ загде от канонич. волн.
 3) не верно полагать $\theta = \theta_v + \phi$, надо $\theta = \theta_v$ только.
 т.е. $L_{int} = -\frac{M_S}{\hbar} (1 - \cos\theta_v) \Phi_v$ и не переопределяй интерес, т.к. не содержит SW.
 4) можно ли требовать инвариантность $L(\vec{m})$ при $\vec{m} \rightarrow R\vec{m}$, можно доказать в $L(\vec{m})$ перейти в xyz-систему, где SW легко выводится? $Oz' = (\theta_v, \Phi_v)$.

Беседа с Оксана: 1) какое ϵ ? $(M \times H)_{max}$ local или $\langle M \times H \rangle$. Для первого $\epsilon = 10^{-4}$, где второго 10^{-6} are OK; 2) уменьши "d" до $d = 0.01$ разумно, это означает incubation;
 3) dynamic simul. (при $H=0$) - метод field pulse $H(t) \rightarrow \langle M(t, \vec{r}) \rangle \rightarrow \langle M(\omega, \vec{r}) \rangle \rightarrow \omega_n$, затем или $h_n \sim \sin \omega t$ либо генератор $\Delta\omega$ около ω_n of $M(\omega, \vec{r})$ где находим $m_n(\vec{r})$;
 4) А можно Vortex nonlin. dyn. paper ~ PRB, она с $\hbar \omega_0(H_2)$ - paper drafts и обмениваемся files, упрости текст; 5) Herman и Entenberga не cover yet где кб.

Беседа с Julian: 1) он обещал Colu samples дуге в ворм.; 2) он ищет money где генерация образцов / покупка материалов в направл. SAIDTEK-SW;
 3) ищет Ph.D student где направление SW-тип. SAIDTEK

→ $\hat{A}_\mu(\theta_v, \Phi_v) = \begin{pmatrix} 0 & -\partial_\mu \Phi_v & \cos\Phi_v \partial_\mu \theta_v \\ \partial_\mu \Phi_v & 0 & \sin\Phi_v \partial_\mu \theta_v \\ -\cos\Phi_v \partial_\mu \theta_v & -\sin\Phi_v \partial_\mu \theta_v & 0 \end{pmatrix}$, $\hat{A}_\mu \vec{m} = \begin{pmatrix} -m_y \partial_\mu \Phi_v + m_z \cos\Phi_v \partial_\mu \theta_v \\ m_x \partial_\mu \Phi_v + m_z \sin\Phi_v \partial_\mu \theta_v \\ -m_x \cos\Phi_v \partial_\mu \theta_v - m_y \sin\Phi_v \partial_\mu \theta_v \end{pmatrix}$

$\begin{cases} \vec{m} = \vec{m}_v + \vec{m}_{sw} \\ \hat{A}_\mu \vec{m} = \hat{A}_\mu \vec{m}_v + \hat{A}_\mu \vec{m}_{sw} \\ \hat{A}_\mu \vec{m}_v = \partial_\mu \vec{m}_v \end{cases}$ $\hat{A}_\mu \vec{m} = \begin{pmatrix} -m_y^s \partial_\mu \Phi_v + m_z^s \cos\Phi_v \partial_\mu \theta_v \\ m_x^s \partial_\mu \Phi_v + m_z^s \sin\Phi_v \partial_\mu \theta_v \\ -m_x^s \cos\Phi_v \partial_\mu \theta_v - m_y^s \sin\Phi_v \partial_\mu \theta_v \end{pmatrix} + \partial_\mu \vec{m}_v$

$L_{int}^{kin} = -\vec{A}_s(\vec{m}) \cdot \hat{A}_0 \vec{m} = -\vec{A}_s(\vec{m}) \cdot [\hat{A}_0 \vec{m}_{sw} + \partial_\mu \vec{m}_v] \rightarrow -\vec{A}_s \cdot \hat{A}_0 \vec{m}_{sw}$

$L_{kin} = \vec{A}_s \cdot \partial_\mu \vec{m} = \vec{A}_s \cdot [\vec{m}_{sw} - \hat{A}_0 \vec{m}_{sw}] = \vec{A}_s \cdot \vec{m}_{sw} - \vec{A}_s(\vec{m}) \cdot \hat{A}_0 \vec{m}_{sw}$, $\hat{A}_0 \sim \Phi_v, \theta_v$

В xyz: $(\vec{m}_{sw})_x = m_x^s \cos\theta_v \cos\Phi_v + m_y^s \sin\theta_v \cos\Phi_v$, $m_y^s = -m_x^s \sin\Phi_v + m_z^s \cos\Phi_v$ (см. 27/17)
 $m_z^s = m_x^s \sin\theta_v \sin\Phi_v + m_y^s \sin\theta_v \sin\Phi_v$, $m_z^s = 0$ by definition SW.

$L_{int}^{kin} = -\vec{A}_s(\vec{m}) \cdot \hat{A}_0 \vec{m} \approx -\vec{A}_s(\vec{m}_0) \cdot \vec{A}_0 \vec{m}_{sw}$, $\vec{A}_s(\vec{m}_0) = \frac{M_S}{\hbar} [-\hat{x} m_y^0 + \hat{y} m_x^0]$

$L_{int}^{kin} = \frac{M_S}{\hbar} s [m_y^0 (-m_y^s \dot{\Phi} + m_z^s \cos\Phi_v \dot{\theta}_v) - m_x^0 (m_x^s \dot{\Phi}_v + m_z^s \sin\Phi_v \dot{\theta}_v)] = \left| \begin{matrix} m_x^0 = \sin\theta_v \cos\Phi_v \\ m_y^0 = \sin\theta_v \sin\Phi_v \end{matrix} \right|$

$= \frac{M_S}{\hbar} s (-1) [m_y^0 m_y^s + m_x^0 m_x^s] \dot{\Phi}_v = -\frac{M_S}{\hbar} s [m_y^0 (-m_x^s \sin\Phi_v + m_z^s \cos\Phi_v) + m_x^0 (m_x^s \cos\theta_v \cos\Phi_v + m_y^s \sin\theta_v \cos\theta_v)] \dot{\Phi}_v$ - не верно.

$\vec{m}_{sw} = m_x^s \hat{x}' + m_y^s \hat{y}' = m_x^s (\hat{x} \cos\Phi \cos\theta + \hat{y} \sin\Phi \cos\theta - \hat{z} \sin\theta) + m_y^s (-\hat{x} \sin\Phi + \hat{y} \cos\Phi)$

Верно: $m_x^s = m_x^s \cos\Phi_v \cos\theta_v - m_y^s \sin\Phi_v$, $m_y^s = m_x^s \sin\Phi_v \cos\theta_v + m_y^s \cos\Phi_v$, $m_z^s = -\sin\theta_v m_x^s$

$L_{int}^{kin} = -\vec{A}_s(\vec{m}_0) \cdot \vec{A}_0 \vec{m}_{sw} = \frac{M_S}{\hbar} s [m_y^0 (m_x^s \sin\Phi_v \cos\theta_v + m_y^s \cos\Phi_v) + m_x^0 (m_x^s \cos\theta_v \cos\Phi_v - m_y^s \sin\Phi_v)]$

$= -\frac{M_S}{\hbar} s \dot{\Phi}_v [\cos\theta_v \sin\theta_v m_x^s] = -\frac{M_S}{\hbar} s \cos\theta_0 \sin\theta_0 m_x^s \dot{\Phi}_v = 0$ because $\theta_0 = \pi/2$.

$L_{int}^{kin}(t) = \vec{A}_s \cdot \vec{m}_{sw} = \frac{M_S}{\hbar} s [-m_y^0 m_x^s + m_x^0 m_y^s] = \left| \begin{matrix} m_x^s, m_y^s \\ m_x^0, m_y^0 \end{matrix} \right|$

Meeting с Arantxa: observation en, $\hbar \omega_n(H) \in \hbar \omega$ где vortex не сдвиг, из-за симметрии, уменьши и увеличь ω_n до $d = 0.01$. Обсуждали загде Vortex ~ Azimuth. SW, также ее поведение $(\omega_n, m_n(\vec{r}))$ и почему $\hbar \omega_n$ не $\hbar \omega_{n,\pm 1}$. Сделаю, что описано из этого PRL и для papers в направл. SAIDTEK grant.

$\vec{L}_{kin} = \vec{A}_S \cdot \dot{\vec{m}}_{sw}$ $\vec{L}_{kin} = \vec{A}_S \cdot \dot{\vec{m}} = \frac{M_S}{\rho} \vec{s} (\vec{n} \times \dot{\vec{m}}) \cdot \dot{\vec{m}} = \frac{M_S}{\rho} \vec{s} \cdot \dot{\vec{m}} (\dot{\vec{m}} \times \dot{\vec{m}})$

$$\text{Ecm } \vec{h} = \vec{2}, \text{ so } L_{\text{cm}} = \frac{M_s}{\delta} s(m) (\dot{m} \times \dot{m}) = \frac{M_s}{\delta(1+m_p)} (m_x \dot{m}_y - m_y \dot{m}_x) \sim \frac{1}{\delta} s(m) (\dot{m} \times \dot{m})$$

$$\text{then } \mathbf{a}_u = -\Phi_0 \frac{\partial m}{\partial x_\mu} \mathbf{A}' = -\Phi_0 \frac{\partial m}{\partial x_\mu} \frac{(\vec{n} \times \vec{m})}{(1+m\vec{n})} = -\frac{\hbar c}{e} \frac{\partial \vec{m}}{\partial x_\mu} \cdot \frac{(\vec{n} \times \vec{m})}{(1+m\vec{n})}, \quad \text{if } \vec{m} = \frac{m_0}{\gamma} \frac{(\vec{n} \times \vec{m})}{(1+m\vec{n})}$$

$$\vec{E} = \frac{\vec{r}}{r^3} \rightarrow \vec{E}_{kin} = -\frac{e}{c} \vec{a}_0, \quad \vec{a}_0 = -\frac{e}{c} \vec{E} \frac{\partial \vec{m}}{\partial t} \cdot \frac{(\vec{n} \times \vec{m})}{[1 + \vec{n} \cdot \vec{n}]} \quad , \quad x_\mu = (ct, x, y, z)$$

$\underline{J}_{kin} = -e a_0$, $a_\mu = (a_0, \vec{a})$, $\vec{B} = \text{rot} \vec{a}$ (тогда, мал. поле). $(B_z dx dy = \Phi_0 p q)$
Всего $A_\mu = (-\varphi, \vec{A})$ и-х вектор-потенциал, $\pm e \cdot a = -\varphi$ и т.д.

где φ - скал. потенциал и т.п. тогда $\vec{E} = -\nabla\varphi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}$ $\vec{A}_{kin} = e\vec{p}$

Kamen paper Ohe-Machawa 'JAP' 09 (publ. 23/II), rege burmens u cu myn. $\vec{E}$ -topol.
 wave of moving Vortex, Oufingy karoforo $E = \omega_0 B_2 \chi \sim 10$ kV/m, u Voltage ~ 10 μ V
 a cegran 6/10 091. uenowz. eq. Thiele.

One 'JAP'og: eff. topol. vector potential $\hat{A}_\mu = -iU^\dagger \partial_\mu U$, $U = e^{i\theta_2/2} e^{i\phi_2/2}$

(2+2) Углов. функция ав. новородам к осм вращение от $\vec{M}(\vec{r}, t)$, $\vec{M}(\theta, \varphi)$ где θ - Волн. функция ψ . $\frac{1}{r} = \frac{1}{r_1 + r_2} = \frac{1}{r_1} + \frac{1}{r_2}$

$\psi' = U_0 U_\varphi \psi$, Врем. координ. ось $\hat{z}$ на Φ , а простран. координ. по ос. $\hat{r}$ на Θ

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ u } e^{i(\vec{a} \cdot \vec{\sigma})} = \cos a + i(\vec{n} \cdot \vec{\sigma}) \sin a, \vec{a} = \vec{n}a$$

$$\hat{A}_\mu = -i e^{i\phi_{2z/2}} e^{i\phi_{3y/2}} \partial_\mu [e^{-i\phi_{2y/2}} e^{-i\phi_{3z/2}}] = -i e^{i\phi_{2z/2}} e^{i\phi_{3y/2}} [-e^{-i\phi_{2y/2}} (i) \partial_\mu \phi_{2y} e^{-i\phi_{3z/2}} +$$

$$+ e^{-i\Phi\partial_x/2} e^{-i\Phi\partial_z/2} (-i) \partial_\mu \Phi \frac{\partial_z}{2} = -\frac{1}{2} \left[\partial_\mu \theta e^{i\Phi\partial_x/2} \partial_y e^{-i\Phi\partial_z/2} + \partial_\mu \Phi \partial_z \right] =$$

$$= -\frac{1}{2} \left[\partial_\mu \theta (\partial_x \sin\Phi + \partial_y \cos\Phi) + \partial_\mu \Phi \partial_z \right] \text{ — same as Tataru 'Phy. Rep.' 08}$$

Используем обратное. $\partial_\mu (e^{-i\theta y/2} e^{-i\theta z/2}) = \partial_\mu \left[\left(\cos \frac{\theta}{2} + i\gamma_y \sin \frac{\theta}{2} \right) (\cos \frac{\theta}{2} - i\gamma_z \sin \frac{\theta}{2}) \right]$

$$= \partial_\mu \left[\cos \frac{\Theta}{2} \cos \frac{\Phi}{2} - i \partial_y \partial_z \sin \frac{\Theta}{2} \sin \frac{\Phi}{2} - i \partial_y \sin \frac{\Theta}{2} \cos \frac{\Phi}{2} - i \partial_z \sin \frac{\Phi}{2} \cos \frac{\Theta}{2} \right] =$$

$$= -\frac{1}{2} \sin \frac{\theta}{2} \cos \frac{\phi}{2} \partial_{\mu} \theta - \frac{1}{2} \sin \phi \cos \frac{\theta}{2} \partial_{\mu} \phi - \frac{1}{2} \partial_y \partial_z (\cos \frac{\theta}{2} \sin \frac{\phi}{2} \partial_{\mu} \theta + \sin \frac{\theta}{2} \cos \frac{\phi}{2} \partial_{\mu} \phi) - i \dots$$

$$\partial_\mu [e^{-i\Phi_{21/2}} e^{-i\Phi_{22/2}}] = \frac{1}{2} (e^{-i\Phi_{21/2}} \partial_\mu + \partial_\mu e^{-i\Phi_{21/2}}) \frac{(-i)}{2} \partial_\mu \Theta e^{i\Phi_{22/2}} + e^{-i\Phi_{21/2}} \times$$

$$\times \frac{1}{2} (e^{-i\Phi/2} \hat{a}_2 + \hat{a}_2 e^{-i\Phi/2}) \left(\frac{-i}{2}\right) \hat{a}_1 \Phi \text{ — произведение.}$$

Note: $R = R(\alpha, \beta, \gamma) = \begin{vmatrix} \alpha & \beta & \gamma \\ \text{линейно независимы} \end{vmatrix} = R(\Phi, \Theta, 0)$ — матрица чисел. коэф. $\alpha \rightarrow \Phi, \beta \rightarrow \Theta$.

$$U(L, P, \Phi) = e^{-i\Phi \hat{J}_z} e^{-iP \hat{J}_y} e^{-iL \hat{J}_z} = e^{-i\theta \hat{J}_y} e^{-i\Phi \hat{J}_z} = U(\Phi, \theta, 0)$$

Если A_k — скаляр, компоненты $\vec{A}$ и базис $\hat{e}_k$ ($k=1,2,3$), то компоненты $\vec{A}$

6. since $\hat{e}_i$ ортонорм. $A'_i = \sum_k a_{ki} A_k$, где $\hat{e}_i = \sum_k a_{ki} \hat{e}_k$. $\hat{e}_k = \sum_i a_{ki} \hat{e}_i$

$A_k = \sum a_i A_i \rightarrow A = \hat{O}A$
 $a_{ik} = \hat{e}_i \cdot \hat{e}_k$ — матрица перехода к новому базису $A^{-1} = A^{-1}, \det A = 1$
 матрица преобразования R от $17/10$ берем $\vec{H}_0 = R \vec{H}_1, \vec{H}_1 = R^{-1} \vec{H}_0, R^{-1} = R^T$ — матрица с. 24/15

неприводимые $U(2, \beta, \gamma)$. Рассмотрим абг, $d \neq 0$, $\beta \neq 0$, $\gamma \neq 0$

Cartesian Components of spin operator $S=1$ $S_x = i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$ $S_y = i \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$ $S_z = i \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$e^{\hat{S}_i} = \frac{1}{3} (1 + 2 \cosh) \hat{I} + \sinh \hat{S}_i + (\cosh - 1) \hat{Q}_{ii}$$

$$\hat{Q}_{ik} = \frac{1}{2}(\hat{S}_i \hat{S}_k + \hat{S}_k \hat{S}_i) - \frac{2}{3} \hat{I} \delta_{ik}, \quad e^{i\alpha} \hat{S}_j = \frac{1}{2}(1 + 2\cos\alpha) \hat{I} + i\sin\alpha [\hat{S}_j + (\cos\alpha - 1)\hat{S}_j^2 - \frac{2}{3} \hat{I}]$$

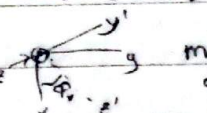
$$\hat{S}_i \hat{S}_j = \hat{I} + i \sin d \cdot \hat{S}_j + (\cos d - 1) \hat{S}_i^2 \quad \text{or} \quad \hat{S}_i \hat{S}_j = 2 \hat{S}_i \cdot \hat{I} + i \sin d \hat{S}_i + \hat{I}$$

$$S_i S_k = \frac{2}{3} \epsilon_{ik} I + \frac{1}{2} \epsilon_{ikl} S_l + Q_{ik}$$

$\hat{x}, \hat{y}, \hat{z}$ - unit vectors along axes x, y, z ()

(20/12) Beqera e John e Anton's school. naga. e seggy e Porto.

$$\begin{aligned} I_{kin} &= \vec{A}_S(\vec{m}) \cdot \dot{\vec{m}}_{sw} \quad (\text{B x' y' z' axes e } \cos \theta \parallel \vec{m}_v) = A_{sx}' \dot{m}_x^s + A_{sy}' \dot{m}_y^s + \dots \text{By definition.} \\ \vec{A}_S &= \frac{M_S}{r} (\vec{n} \times \vec{m}), \quad \vec{m} = \vec{m}_v + \vec{m}_{sw} = m_x^s \hat{x}' + m_y^s \hat{y}' + m_z^s \hat{z}', \quad \vec{m}'_{sw} = (m_x^s, m_y^s, 0) \\ \vec{m}'_{sw} \cdot \vec{m}'_v &= 0, \quad (\vec{n}' \times \vec{m}') \cdot \dot{\vec{m}}'_{sw} = \vec{n}' \cdot (\vec{m}' \times \dot{\vec{m}}'_{sw}) = (\vec{n}' \times \vec{m}'_{sw}) \cdot \dot{\vec{m}}'_{sw} + (\vec{n}' \times \vec{m}'_v) \cdot \dot{\vec{m}}'_{sw} = \\ &= (sw) + \dot{\vec{m}}'_{sw} \cdot (\vec{n}' \times \vec{m}'_v) = (sw) + \dot{\vec{m}}'_{sw} \cdot (\vec{m}'_{sw} \times \vec{n}') = (sw) + (\dot{\vec{m}}'_{sw} \times \vec{n}') \cdot \vec{m}'_{sw} = \\ &= (sw) + (\dot{m}_x^s n_y - \dot{m}_y^s n_x), \quad \vec{n}' = \hat{x}' n_x + \hat{y}' n_y + \hat{z}' n_z = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = [R_{ki} n_k = R_{zi} 1] = \\ &= \hat{x}' R_{zx} + \hat{y}' R_{zy} + \hat{z}' R_{zz} = -\hat{x}' \sin \theta_v + \hat{z}' \cos \theta_v, \quad \vec{n}' \times \vec{m}'_{sw} = \begin{vmatrix} \hat{x}' & \hat{y}' & \hat{z}' \\ n_x & 0 & n_z \\ m_x^s & m_y^s & 0 \end{vmatrix} = \hat{y}' n_z m_x^s + \\ &+ \hat{x}' (-1) n_z m_y^s + \hat{z}' n_x m_y^s, \quad (\vec{n}' \times \vec{m}'_{sw}) \cdot \dot{\vec{m}}'_{sw} = -n_z m_y^s \dot{m}_x^s + n_z m_x^s \dot{m}_y^s \\ (\vec{n}' \times \vec{m}') \cdot \dot{\vec{m}}'_{sw} &= n_z (m_x^s \dot{m}_y^s - m_y^s \dot{m}_x^s) + (\dot{m}_x^s n_y - \dot{m}_y^s n_x) = (sw) + \sin \theta_v \dot{m}_y^s \\ (\vec{n}' \times \vec{m}') \cdot \dot{\vec{m}}'_{sw} &= \cos \theta_v (m_x^s \dot{m}_y^s - m_y^s \dot{m}_x^s) + \sin \theta_v \dot{m}_y^s, \quad \text{then } \cos \theta_v \text{ e } \sin \theta_v \text{ e } \theta_v \text{ e } \theta_v \text{ e } \theta_v \end{aligned}$$

V. core u moxeno um vhenoforo e mab. u pnenenem.  $m_y^s = m_p = m_p?$ $\theta = \pi/2$ ke u u u

$$I_{kin} = \vec{A}_S(\vec{m}) \cdot \dot{\vec{m}}_{sw} = \frac{M_S}{r} \frac{1}{[1 + \vec{n} \cdot \vec{m}]} \sin \theta_v \dot{m}_y^s$$

$$\begin{aligned} m_y^s &= -m_x^s \sin \phi + m_z^s \cos \phi = -m_x^s \sin(\frac{\pi}{2} + \phi + \delta \phi_v) + m_z^s \cos(\frac{\pi}{2} + \phi + \delta \phi_v) = \\ &= -m_x^s [\cos \phi \cos \delta \phi_v + \sin \delta \phi_v (-1) \sin \phi] + m_z^s [-\cos \phi \sin \delta \phi_v - \sin \phi \cos \delta \phi_v] = \\ &= -\cos \phi [m_x^s \cos \delta \phi_v + m_z^s \sin \delta \phi_v] + \sin \phi [-m_x^s \sin \delta \phi_v - m_z^s \cos \delta \phi_v] = -\cos \phi m_p - \sin \phi m_p \delta \phi_v = -\cos \phi m_p (1 + \delta \phi_v) \\ S(\vec{m}) &= [1 + \vec{n} \cdot \vec{m}]^{-1} = [1 + \vec{n} \cdot \vec{m}'_{sw} + \vec{n} \cdot \vec{m}'_v]^{-1} = [1 + \vec{n}' \cdot \vec{m}'_{sw} + \cos \theta_v]^{-1} = [1 + n_x m_x^s + n_y m_y^s]^{-1} = \frac{1}{1 + \sin \theta_v m_x^s} \\ m_x^s &= -\sin \theta_v m_z^s = -\sin \theta_v m_p \rightarrow I_{kin} = \frac{M_S}{r} \frac{1}{[1 + \sin^2 \theta_v m_p^2]} \sin \theta_v (-\dot{m}_p) \end{aligned}$$

$$\sin \theta_v \dot{m}_p = \frac{d}{dt} (\sin \theta_v m_p) = m_p \frac{d(\sin \theta_v)}{dt} = -m_p \dot{\theta}_v \cos \theta_v = -m_p \dot{\theta}_v \sin \theta_v$$

$$(\partial_t - \hat{A}_0) \vec{m}_{sw} = D_t \vec{m}_{sw} \text{ naga bozura.}$$

(20-24/12) Duzgus e Duzgus, Porto, Portugal.

Portu. na cen. DW dyn. Noznavamnce e D. Schmoel u J. Araujo - Duzgus IFIMUP-IV. Organ Duzgus 3 opraqua lobu - gpeobennox que FMR.

Dorobymnce e W. Poropennobum: 1) naga u rjocane anant. prymum

que omucane SW azim. modes in Vortex state; 2) gagan u pnenembo e

CP. azim. que omucane SW in F/N/F double-vortex dot (Duzgus e Duzgus)

3) pnen. gagan toka jz e circular field the bunnit na vortex state e TM mode que e = +1 (reversal vortex e by the); 4) naga m2 (vortex) e Duzgus

Duzgus e Duzgus: 1) Soc. Sec. tax b u. s. uozno bopnyro; 2) Spain - Porto. travel grant oozag. na creg. reg u zga u pnen. Araujo - Duzgus (Madrid); 3) gagan e Spain - Germany travel grants (see site (SIC - Spain); 4) gagan. U.S. - Spain travel grants e u u s - partner u u u u s - grant; 5) 5.000 € e u u bopnyro uozno na gagan Schmoel reimbursed me lump sum 400. € for the Porto-travel. 6) Money Grant -

Aim, $\sigma_{\text{W}} (n=1, m=1)$ Intensity $\propto |b_{11}(p)|^2$ u.a.s, r.u. $b_{11}(p)$ No.
 of reflections. year month day ()

$$W_{00}(\beta) \approx w_{1,11}(\beta) \sqrt{1-\beta} \quad \beta \leq 0,05 \text{ (Vondra-Gu-m-Splitting-Pr-1.op)}$$

(26/V) Muchaia. Phraggy (osobnost $n=1, m=1$ в PRB proof) и Ю. Пороженский (интерпретация vortex-axim. SW и др. - е. г. $O(p)$ vortex profile). Workshop в NanoGUNE (после Гинзбурга и Зинича, г. А. Бергер, Себастьян Морасу Марин). Мандарины и регулярная структура экз. coupling в G-hexagon dot array (ex Science was rejected). У Арианта не хватает high- β SW aximutal, Zborun Ocarne, она не знает "мозгов"? Приглашение от Клеа и Инессы в Сакли, Франция по поводу vortex dynamics.

В век $\vec{r}_{42}$ числене координ. (oz 1 del) кандр. none $\hat{A} \vec{m}_{sw} = \begin{pmatrix} -m_y^s \partial_\mu \Phi_V + m_z^s \cos \Phi_V \partial_\mu \Theta_V \\ m_x^s \partial_\mu \Phi_V + m_z^s \sin \Phi_V \partial_\mu \Theta_V \\ -m_x^s \cos \Phi_V \partial_\mu \Theta_V - m_y^s \sin \Phi_V \partial_\mu \Theta_V \end{pmatrix}$
 где s — число $\vec{sw}$, V — Vertex.
 $\mathcal{L}_{kin} = \vec{A}_s \cdot \vec{D}_E \vec{m}_{sw} = \vec{A}_s \cdot [\vec{m}_{sw} - \hat{A}_0 \vec{m}_{sw}]$
 $\vec{A}_s = \frac{M_s}{f} s(\vec{m}) [\vec{n} \times \vec{m}]$ зависит на $\vec{m} = \vec{m}_{sw}$, т.к. Vertex (векториз. к оси кандр. Θ_V, Φ_V)
 Выходит только в кандр. none $\hat{A}_0$. Попробуем $\vec{n} \parallel oz'$, $\vec{n} \times \vec{m}_{sw} = -\hat{x}' m_y^s + \hat{y}' m_x^s$.
 Если $\vec{n} \parallel oz$, $\vec{n} \times \vec{m}_{sw} = -\hat{x} m_y^{sw} + \hat{y} m_x^s$, $\mathcal{L}_{kin} = -\vec{A}_s \cdot \hat{A}_0 \vec{m}_{sw} = -\frac{M_s}{f} s [\vec{n} \times \vec{m}_{sw}] \cdot \hat{A}_0 \vec{m}_{sw}$
 $= -\frac{M_s}{f} s [-\hat{x} m_y^s + \hat{y} m_x^s] \begin{pmatrix} -m_y^s \Phi_V \\ m_x^s \Phi_V \end{pmatrix} = -\frac{M_s}{f} s \Phi_V [(m_y^s)^2 + (m_x^s)^2]$, где $m_x^s, m_y^s \sim \sin \Theta_V$

$$\begin{aligned} \vec{n} \times \vec{m}_{sw} &= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ n_x & n_y & n_z \\ m_x^s & m_y^s & m_z^s \end{vmatrix} = \hat{x}(n_y m_z^s - n_z m_y^s) + \hat{y}(n_z m_x^s - n_x m_z^s) + \hat{z}(\dots) \\ \mathcal{L}_{int}^{kin} &= -\frac{M_s}{\gamma} s [\vec{n} \times \vec{m}_{sw}] \cdot \hat{A}_0 \vec{m}_{sw} = -\frac{M_s}{\gamma} s [\vec{n} \times \vec{m}_{sw}] \cdot \begin{pmatrix} -m_y^s \dot{\Phi}_v \\ +m_x^s \dot{\Phi}_v \\ 0 \end{pmatrix} = -\frac{M_s}{\gamma} s \dot{\Phi}_v \left[-m_y^s (n_y m_x^s - n_x m_y^s) + m_x^s (n_z m_x^s - n_x m_z^s) \right] \\ &= -\frac{M_s}{\gamma} s \dot{\Phi}_v \left[-n_x (-\sin^2 \theta \cos \varphi m_x^s) + n_y \sin^2 \theta \sin \varphi m_x^s + n_z (m_x^s)^2 + (m_y^s)^2 \right] \\ &= -\frac{M_s}{\gamma} s \dot{\Phi}_v \left[\sin^2 \theta (n_x \cos \varphi + n_y \sin \varphi) m_x^s + n_z \sin^2 \theta \right] = |\vec{n}| = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \end{aligned}$$

Escaneado con CamScanner

$$\vec{m} = \vec{m}_{sw} + \vec{m}_v = \hat{x} m_{x_i} + \hat{y} m_{y_i} + \hat{z} m_{z_i}, \quad \vec{m}_{sw} = \hat{x} m_{x_i} + \hat{y} m_{y_i}$$

$\hat{A}_S(\vec{m}_{sw})$ назо ёраб и $\vec{m}_{sw} \rightarrow$ ёди мудар $-\hat{A}_S \cdot \hat{A}_0 \vec{m}_{sw}$, ёраб
 $\hat{A}_0 \sim \Phi$ и $m_{sw}^a \ll 1$ (кларавар, ирмд.)

Hubb. no SE. Shin O PhD student

(28/V) C.A. & Certified "Lila", Borzomelkunen. Zboromelkunen Papraz & Cies on. AHL-FMR
 onyfa. ALIOV PRB'S VNA-FMR Zboromelkunen US-CitiB, transfer money from CitiB
 to e-Saving more over. open CD. He can't go on-line B Cija Madrid & Seat CitiB

29/v Klein notebook spirally bound B. Davis B Nov. '09. Arantxa no answer - Surpeaks.
Update CV u. u. u. u. u. PRL '08, PRL '09 u. PRB '09 c. Aliev u. Kearney.
Pastor nag APL - Vortex SW by Avard et. al. FFT f(4)

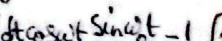
1) Rect. pulse $f(t)$  , ω program se res. opražen na $t < 0$ 

Fourier-uv. Dypuje transform $F_c(\omega) = \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} dt \cos \omega t f(t)$, $f(t) = \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} d\omega \cos \omega t F_c(\omega)$

$F_c(\omega) = \sqrt{\frac{2}{\pi}} \int_0^{t_0} dt \cos \omega t = \sqrt{\frac{2}{\pi}} \frac{\sin \omega t}{\omega} \Big|_0^{t_0} = \sqrt{\frac{2}{\pi}} \frac{\sin \omega t_0}{\omega}$, $f(t) = \frac{2}{\pi} \int_0^{\infty} d\omega \frac{\sin \omega t_0}{\omega} \cos \omega t = \begin{cases} 1, & t < t_0 \\ 0, & t > t_0 \end{cases}$

$F_c(\omega) \sim \frac{1}{\omega}$, No $F_c(\omega)$ unlesr njeu ω $\omega_n = n \frac{\pi}{t_0}$

- pulse length non-infinite. No wfm to $\rightarrow 0$ vago $1 \rightarrow 1/t_0$ no exp. response.
 Bozozuglunne. $\int H(t) dt$. $H(t) = \frac{H_0}{t_0} \text{ wfm } t \leq t_0, t > 0, H_0 = \int_{-\infty}^{\infty} H(t) dt$
 $F_c(\omega) = \sqrt{\frac{2}{\pi}} H_0 \frac{\sin \omega t_0}{\omega t_0}$ Coosb. $f(t) = \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} F_c(\omega) \cos \omega t d\omega = \begin{cases} H_0/t_0, & t < t_0 \\ 0, & t > t_0 \end{cases}$ 
 $H(t)$ = rectangular pulse under $H(\omega) = \sqrt{\frac{2}{\pi}} H_0 \frac{\sin \omega t_0}{\omega t_0}$

2) высв. на частоте $f(t) = \sqrt{\frac{2}{\pi}} \frac{\sin(\omega_0 t)}{\omega_0 t}, \forall t > 0$. $F(\omega) = \sqrt{\frac{2}{\pi}} \int_0^\infty dt \cos \omega t f(t) = \frac{2}{\pi} \int_0^\infty dt \cos \omega t \frac{\sin \omega_0 t}{\omega_0 t} = 1 \int_0^\infty \frac{\cos \omega t \sin \omega_0 t}{\omega_0 t} dt$
 Из-за $H(t) = \sqrt{\frac{2}{\pi}} H_0 \frac{\sin \omega_0 t}{\omega_0 t}$, найдем $H(\omega) = \frac{H_0}{\omega_0} \cdot \begin{cases} 1, & \omega < \omega_0 \\ 0, & \omega > \omega_0 \end{cases}$ 
 Оценим сам $m(\omega) = \gamma(\omega) H(\omega)$, то $H(\omega) = \text{const}$ при $\omega < \omega_0$
 аппроксим. возьмем $\omega_{nm} < \omega_0$, при $\omega \rightarrow \omega_{nm} \gg 1/\omega_0$

[illegible]

1) γ_{max} и ω_{max} - константы, не зависящие от $t_{\text{max}} = T$ и ω_{max} - константы
 2) $\gamma(t)$ и $\omega(t)$, как функции t и ω

1.4. Доказано, что применение Дискретного Фурье Трансформации с $N \ln N$ операциями требует N^2 БДПТ и реализуется FFT. FFT гарантирует успех или нет?

Radial SW

No. (page 11)
year month day ()
2009 05 06

$$\text{as } d\mu(p, p') = \frac{1}{p} dk [1 - e^{-p^k}] J_0(kp) J_0(kp') \text{ as } d_{\text{eff}}(p, p') = dk k f(p^k) J_1(kp) J_1(kp') \text{ as}$$

$$(-\frac{1}{\beta}) \int dk [1 - e^{-\beta k}] J_0(kx) J_0(ky) \int dx x f(\beta x) J_1(xg') J_1(xy') = \left[\frac{1}{\beta} \frac{1}{\sqrt{1 - \beta^2}} \frac{1}{\sqrt{1 - \beta^2}} \frac{1}{\sqrt{1 - \beta^2}} \right] \left[\frac{1}{\beta} \frac{1}{\sqrt{1 - \beta^2}} \frac{1}{\sqrt{1 - \beta^2}} \frac{1}{\sqrt{1 - \beta^2}} \right] \left[\frac{1}{\beta} \frac{1}{\sqrt{1 - \beta^2}} \frac{1}{\sqrt{1 - \beta^2}} \frac{1}{\sqrt{1 - \beta^2}} \right]$$

$$= f(\rho_{nm}) \int d\rho \rho J_1(\alpha_n \rho) \int d\tau \tau \int dk k [1 - f(\rho k)] J_0(k\rho) J_0(k\tau) J_1(\alpha_m \tau) = f(\rho_{nm}) \int dk k [1 - f(\rho k)] I_n(k) I_m(k)$$

$$\lim_{k \rightarrow 0} f(k) \rightarrow 0 \quad \int dk k J_0(k\rho) J_0(kr) = \frac{1}{2} \delta(\rho-r) \quad \text{u} \quad \Gamma_{nm} = \int f(\rho) d\rho J_1(\alpha_n \rho) J_1(\alpha_m \rho) = \int f(\rho) d\rho N_m \delta_{nm}$$

Все графы можно изобразить $I_n(k)$? Попробуем $I_n(k) = \int_{\mathbb{R}^n} \frac{\partial J_1(k, p)}{\partial p} J_1(d, p) \frac{1}{\sqrt{V_n}} \frac{1}{\sqrt{V_n}}$

$$- \int J_0(du) du = \left[du = [J_0(kp) + p(-k)J_1(kp)] dp \right] = -\frac{1}{dn} \left[J_0(k) J_0(du) - \int dp J_0(du) \{ J_0(kp) - kp J_1(kp) \} \right]$$

$$- \int \rho J_1(kg) du = \left| du = dn J_1'(x) dp \right| = - \frac{dn}{T} \left(dp J_1(kg) p \left[J_0(dn, p) - \frac{1}{J_0 p} J_1(dn, p) \right] - kT n \ln \left(\frac{J_0}{J_0 p} \right) \right)$$

$$\text{Поэтому } \left(\frac{1}{x} \frac{d}{dx}\right)^m [x^\alpha Z_\nu(x)] = x^{\alpha-m} Z_{\nu-m}(x) \text{ и } \left(\frac{1}{x} \frac{d}{dx}\right)^m [1 \cdot Z_\nu(x)] = (-1)^m Z_{\nu+m}(x).$$

$$= (ds' s' \Gamma(p s')) \sum c_n J_n(\alpha_n r') \quad \text{yann. na } J_n(\alpha_n r) \text{ y } (ds' s' \Gamma(p s'))$$

$$\omega^2 \sum_n C_n N_n \delta_{mn} = \sum_{n'} C_{n'} \Gamma_{mn'}, \quad \omega^2 C_m N_m = \sum_n C_n \Gamma_{mn}, \quad \det(\omega^2 N \delta_{mn} - \Gamma_{mn}) = 0$$

6. mod. neque no β , obtemos $\omega_n^2 = f(\beta \alpha_n)$.

Всего можно взять любую полную систему функций $F_n(\rho)$ и написать

1. мы имеем C_3 и C_4 . Но $F_n(p) = J_1(\text{dimp})$ — функция, т.е. она имеет
 dimp — C_4 — наименьшее значение при $p \rightarrow 0$ (т.е. радиус, $p \rightarrow 0$).

и определено от φ exch. Lagrangian (азим. $m=1$) при $\beta=0$.

Функции Ванага для radial SW ($m=0$) $J_0(x_p) + 3Y_0(x_p) = F_n(p)$ - это решение
Eq. 2. Если, например, задана на них амплитуда негармоник Γ_n , то $\sim \delta_{nn'}$.

Намечено. Задача квадратная $\approx$ на сф. $J_1(x, y)$. Показаны анал. eq. Zivier's
и не радиал. Ст. квадратичности и их решение в квад. уравнении

исключает сомнения, сшивком поднят из фута. 3-1. Гит. Велики из фута. из фута. из фута.

No.

$$m_{\text{eff}} = m_{\text{eff}}^{\text{eff}} = \frac{1}{2} (m_{\text{eff}}^{\text{eff}} + m_{\text{eff}}^{\text{eff}}) = \frac{1}{2} (m_{\text{eff}}^{\text{eff}} + m_{\text{eff}}^{\text{eff}}) = \frac{1}{2} (m_{\text{eff}}^{\text{eff}} + m_{\text{eff}}^{\text{eff}})$$

$$m_{\text{eff}} = \frac{m_{\text{eff}}}{\sqrt{1 - \beta^2}}$$

$$\frac{1}{\lambda} = \frac{1}{\lambda_0} \left(\frac{v^2}{c^2} \left(\frac{1}{\beta} - 1 \right) \right)$$

10/10/10

$$V_{\text{tot}} = -\hat{p}_z \hat{p}_z m^2 = -M_{zz} \sin \theta m^2 \Phi$$

$$\frac{d}{dt} \int_{\text{cont}} \rho \, dV = 0$$

6 moving $x'y'z'$, $\vec{r}'(a, \varphi)$

$$M_2 \frac{d^2 y_1}{dt^2} + \int_{int}^{ext}$$

$$\leq [1 + 0.2\sqrt{t}] \cdot \text{Line V. Cost}$$

$$m_{\chi_1^0} = 10$$

...to the new net,

$$\frac{1}{2} \sin \theta_0 \ell_v \dot{\Phi}_{em} + \dot{L}_{int}^{kin}$$

в востр-м. и в востр-м.

Base 10K V-SW interaction

$$m_2 = \cos 0 = 1 \Rightarrow \cos 0 = 1 \Rightarrow 0 = 0$$

Но как от m_2^2 вывести

$$40 \vec{\mu} = (\mu_B, 0, \sqrt{3}\mu_B)$$

$$-\frac{1}{2} \frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 \right) = -\frac{1}{2} m \ddot{x}$$

$\rho = 1.2 \text{ kg/m}^3$, $\mu = 1.8 \times 10^{-4} \text{ Pa}\cdot\text{s}$

o, percip. ne aut. g. f. s. Gh B: Sc.

Used to paying to children
in village 20.000.000

the new life was a new life.

неизбежно в старости: ЕНЦ/а

$$(\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \Big|_{\theta = \frac{\pi}{2}}$$

→ 0.001, or '1/1000' 2008.

Per questo $R(\theta, \varphi) = \hat{u}(\varphi, \theta, 0) = e^{-i\theta J_3} e^{-i\varphi J_2}$, $A_{\mu} = \partial_{\mu} R R^{-1}$, $J=1$

$R^{-1} = e^{i\Phi} J_2 e^{-i\Phi J_2}$ Express R in terms of J_1, J_2, J_3 and J_3 . No. of years = 1 month day
 $RR^{-1} = e^{-i\Phi J_2} e^{-i\Phi J_2} e^{i\Phi J_2} e^{-i\Phi J_2} = 1$
 $2J_2 R = 2J_2 (e^{-i\Phi J_2} e^{-i\Phi J_2}) = -i2J_2 R - i2J_2 \Phi R J_2, 2J_2 R R^{-1} = -i2J_2 J_3 - i2J_2 \Phi R J_2$
 $R J_2 R^{-1} = e^{-i\Phi J_2} e^{-i\Phi J_2} e^{i\Phi J_2} e^{-i\Phi J_2} = e^{-i\Phi J_2} J_2 e^{i\Phi J_2} = J_2 \cos \Phi + J_1 \sin \Phi$

$$\begin{aligned}
 Q_\mu R &= Q_\mu (e^{-i\theta J_3} e^{-i\phi J_2}) = -i\alpha_\mu \theta J_3 - i\alpha_\mu \phi R J_2, \quad Q_\mu R R^{-1} = -i\alpha_\mu \theta J_3 - i\alpha_\mu \phi R J_2 R^{-1} \\
 R J_2 R^{-1} &= e^{-i\theta J_3} e^{-i\phi J_2} J_2 e^{i\phi J_2} e^{i\theta J_3} = e^{-i\theta J_3} J_2 e^{i\theta J_3} = J_2 \cos\theta + J_x \sin\theta \\
 Q_\mu R R^{-1} &= -i\alpha_\mu \phi \sin\theta J_x - i\alpha_\mu \theta J_3 - i\alpha_\mu \phi \cos\theta J_2 = A_\mu = A_\mu^\alpha(i) \hat{G}_\alpha
 \end{aligned}$$

$A_{\mu} = \begin{pmatrix} 0 & -2\mu\Phi & \cos\Phi\partial_{\mu}0 \\ 2\mu\Phi & 0 & \sin\Phi\partial_{\mu}0 \\ -\cos\Phi\partial_{\mu}0 & -\sin\Phi\partial_{\mu}0 & 0 \end{pmatrix}$, $2\mu\Phi$ gauge theory mass coeff.
 и при $J_2 = i \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, а при

$$R(\theta, \varphi) = e^{-i\theta J_z} e^{-i\varphi J_y} \quad \partial_\mu R = -i\partial_\mu \varphi J_y R - i\partial_\mu \theta R J_z$$

$$\partial_\mu R R^{-1} = -i\partial_\mu \varphi J_y - i\partial_\mu \theta R J_z R^{-1} \quad R J_z R^{-1} = J_z \cos \varphi + J_x \sin \varphi$$

Monopole magnetic field $R(\theta, \phi) = e^{i\phi J_z} e^{-i\theta J_y}$, $\partial_\mu R = -i\partial_\mu \phi J_z R - i\partial_\mu \theta R J_y$
 $\partial_\mu R R^{-1} = -i\partial_\mu \phi J_z - i\partial_\mu \theta R J_y R^{-1}$, $R J_y R^{-1} = J_y \cos \phi + J_x \sin \phi$
 $A_\mu = \partial_\mu R R^{-1} = -i\partial_\mu \theta \sin \phi J_x - i\partial_\mu \theta \cos \phi J_y - i\partial_\mu \phi J_z$, $J_x = i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$, $J_y = i \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$A_\mu = \partial_\mu R R^{-1} = -i \partial_\mu \theta \sin \Phi J_x - i \partial_\mu \theta \cos \Phi J_y - i \partial_\mu \Phi J_z$, $J_x = i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, $J_y = i \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
 Bgegen $SO(3)$ neue Parameter $G_x = -i J_x$, $G_y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$, $G_z = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$
 $A_\mu = \partial_\mu \theta \sin \Phi G_x + \partial_\mu \theta \cos \Phi G_y + \partial_\mu \Phi G_z$.

$$A_{\mu} = \partial_{\mu} \theta \cdot \sin \varphi G_x + \partial_{\mu} \theta \cos \varphi G_y + \partial_{\mu} \varphi G_z, \text{ значения при } G_x \text{ не сходятся}$$

Вспомогательная A_μ (77) надо $\sin\varphi \rightarrow -\sin\varphi$.
(это не сказало на $\sin$, ведь $\partial_\mu 0 = 0$)
Учитывая, что $\vec{n} \parallel \hat{z}$ тоже "—".

$$A_{\mu} = \begin{pmatrix} \sin\theta \partial_{\mu}\theta \\ \cos\theta \partial_{\mu}\theta \\ \partial_{\mu}\phi \end{pmatrix}, \quad m_x = \sin\theta \cos\phi, \quad m_y = \sin\theta \sin\phi, \quad m_z = \cos\theta.$$

$$(\partial_\mu \Phi) (m \times \partial_\mu m)_x = m_y \partial_\mu m_z - m_z \partial_\mu m_y = \sin\theta \sin\Phi (-1) \sin\theta \partial_\mu \theta - \cos\theta [\cos\theta \partial_\mu \theta \sin\Phi + \sin\theta \cos\Phi \partial_\mu \Phi] = -\sin\Phi \partial_\mu \theta - \cos\theta \sin\theta \cos\Phi \partial_\mu \Phi$$

$$(\vec{m}_1 \times \vec{m}_2)_y = m_2 \frac{\partial}{\partial \mu} m_x - m_x \frac{\partial}{\partial \mu} m_z = \cos\theta [\cos\theta \cos\phi \frac{\partial}{\partial \mu} \theta - \sin\theta \sin\phi \frac{\partial}{\partial \mu} \phi] - \sin\theta \cos\phi (-1) \sin\theta$$

где $\vec{m} = (\sin \frac{\Theta}{2} \cos \frac{\Phi}{2}, \sin \frac{\Theta}{2} \sin \frac{\Phi}{2}, \cos \frac{\Theta}{2})$ и далее от $S = \sqrt{2}$

Due $S=Y_2$: $U = \vec{m} \cdot \vec{z}$ e $U^T = U^{-1}$ - giroparane, y mana $R^{-1} = R^T$, $\det R = 1$.

$$A_\mu = -i u \partial_\mu u = -i (\vec{m} \vec{\sigma}) (\partial_\mu \vec{m} \cdot \vec{\sigma}) = (\vec{m} \times \partial_\mu \vec{m}) \cdot \vec{\sigma}, \quad \vec{A}_\mu = \frac{1}{2} \vec{n} \times \partial_\mu \vec{n} - \Phi_{B_\mu} \vec{n}.$$

Значит для $J=1$ выразим A_μ через $\partial_\mu \vec{m}$ в инвариантной форме.

$$A_\mu = (\vec{m} \times \partial_\mu \vec{m}) = \lambda (\vec{m}) \vec{m} \rightarrow \lambda \vec{m} \quad \lambda = \sin^2 \theta \partial_\mu \theta$$

$$\vec{m} \times \frac{\partial \vec{m}}{\partial \phi} = \begin{pmatrix} -\sin\phi \frac{\partial \vec{m}}{\partial \phi} \\ \cos\phi \frac{\partial \vec{m}}{\partial \phi} \\ \sin\theta \frac{\partial \vec{m}}{\partial \phi} \end{pmatrix} = \cos\theta \sin\phi \begin{pmatrix} \cos\phi \frac{\partial \vec{m}}{\partial \phi} \\ \sin\phi \frac{\partial \vec{m}}{\partial \phi} \\ 0 \end{pmatrix}$$

$$\vec{A}_\mu = \vec{m}_x \frac{\partial \vec{m}}{\partial \mu} + \cos\theta \vec{m} \frac{\partial \theta}{\partial \mu}$$

тогда $\vec{m}$ — вектор из которого $\vec{A}_m$ исходит. $\vec{A} = \rho \vec{a} = \vec{a} \vec{\rho}$

$$\hat{A}_\mu \vec{m} = (\vec{A}_\mu \cdot \vec{G}) \vec{m} = \partial_\mu \vec{m}, \quad \vec{G} = (G_x, G_y, G_z) - \text{вектор, соответствующий элементу группы}$$

$$(\vec{A}_\mu \cdot \vec{G})^{\vec{m}} = -\vec{m} \times \vec{A}_\mu = \vec{A}_\mu \times \vec{m} \quad \text{v.o. r. } \vec{m}^2 = \text{const}$$

$$\vec{m} \cdot \vec{A}_\mu = a_1, \vec{m} \times \vec{A}_\mu = a_2 (-\partial_\mu \vec{m}) - a_3 \vec{m} \times \partial_\mu \vec{m}, \partial_\mu \vec{m} \cdot \vec{A}_\mu = -a_3 (\partial_\mu \vec{m})^2 = 0, a_3 = 0$$

$$\partial_\mu \vec{m} \cdot \vec{A}_\mu = a_1 \partial_\mu \vec{m} \cdot \vec{m} + a_2 \vec{m} \cdot (\partial_\mu \vec{m})^2 \Rightarrow \vec{m} \cdot (\partial_\mu \vec{m} \times \vec{A}_\mu) = a_2 (\partial_\mu \vec{m})^2 \quad a_2 = 1$$

(2/1) Usando $\vec{A}_\mu$ podemos ver que transformaciones $SO(3)$: $\vec{A}_\mu = A_\mu^x G_x + A_\mu^y G_y + A_\mu^z G_z$

Donde $\vec{A}_\mu = (A_\mu^x, A_\mu^y, A_\mu^z)$. $G_x G_y = G_z, G_y G_z = G_x, G_z G_x = G_y$

$$[A_\mu, B_\mu] = (\vec{A}_\mu \times \vec{B}_\mu)^T. \quad A_\mu \vec{m} = \vec{A}_\mu \times \vec{m} \text{ que es } \vec{m} \cdot \vec{A}_\mu \vec{m}$$

Ahor $A_\mu \vec{m} = \partial_\mu \vec{m}$ tenemos que $\vec{m}$ es un 3D vector $\vec{m}$ $\vec{m} \cdot \vec{m} = 1$ $\vec{m}$ $\vec{m} \cdot \vec{m} = 1$

$$A_\mu = \begin{pmatrix} 0 & -A_\mu^x & -A_\mu^y \\ A_\mu^x & 0 & -A_\mu^z \\ -A_\mu^y & A_\mu^z & 0 \end{pmatrix} = A_\mu^x G_x + A_\mu^y G_y + A_\mu^z G_z. \quad \vec{\omega} = x G_x + y G_y + z G_z$$

$$e^{\vec{\omega}} = e^{i\vec{\omega} \cdot \vec{J}} = \text{matriz } 3 \times 3 \text{ de } SO(3) \text{ con } \vec{\omega} = (x, y, z)$$

$$\vec{\omega} = \begin{pmatrix} 0 & -x & y \\ x & 0 & -z \\ -y & z & 0 \end{pmatrix} \theta, \text{ que } \vec{u} = (x, y, z) \text{ es un vector. } e^{\vec{\omega}} = \text{matriz } 3 \times 3$$

$$\vec{\omega} = (x G_x + y G_y + z G_z) \theta = \vec{u} \theta, \quad G_x = -i J_x \Rightarrow \vec{u} \theta = -i(x J_x + y J_y + z J_z) = -i \vec{u} \cdot \vec{J} \theta$$

$$e^{-i\theta(\vec{u} \cdot \vec{J})} = R_{\vec{u}}(\theta) \text{ es una matriz } 3 \times 3 \text{ de } SO(3) \text{ que es una transformacion}$$

Baker-Campbell-Hausdorff eq. $e^A e^B = e^C, \quad C = A + B + \frac{1}{2}[A, B] + \dots$

que son 3x3 skew matrices (3D rotation matrices) $\vec{u} = (x, y, z)$ $\vec{u} \cdot \vec{u} = 1$

$$C = \alpha A + \beta B + \gamma [A, B], \quad \text{u. } [A, B] = (\vec{A} \times \vec{B})^T. \quad [\vec{J}, \vec{J}] \Rightarrow \vec{J} \times \vec{J} = i \vec{J}$$

$$[J_x, J_y] = i J_z, [J_y, J_z] = i J_x, [J_z, J_x] = i J_y$$

$$\text{Rodrigues' formula: } \exp(\hat{x}) = \hat{I} + \frac{\sin(\theta)}{\theta} \hat{x} + \frac{2 \sin^2(\theta/2)}{\theta^2} \hat{x}^2, \quad \hat{x} = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}$$

$$\text{u } \hat{x} = (x_1, x_2, x_3), \quad \theta = |\hat{x}|, \quad \text{Donde } e^{i\theta \hat{J}_z} = \hat{I} + i \sin \theta \hat{J}_z + (\cos \theta - 1) \hat{J}_z^2, \quad \hat{J}_z^2 = 1$$

$$\text{Basta } \hat{x} = x_1 G_x + x_2 G_y + x_3 G_z = \vec{x} \cdot \vec{G} = -i(\vec{x} \cdot \vec{J}) = -i(\vec{n} \cdot \vec{J}) \theta = -i J_n \theta$$

$$\exp(\hat{x}) = \hat{I} + \frac{\sin(\theta)}{|\hat{x}|} \hat{x} + \frac{2 \sin^2(\theta/2)}{|\hat{x}|^2} \hat{x}^2 = e^{-i(\vec{n} \cdot \vec{J}) \theta} = \hat{I} - i \sin \theta J_n + (\cos \theta - 1) J_n^2$$

$$\text{Tenemos, r. u. } \hat{x} = -i J_n |\hat{x}| = -i J_n \theta, \quad 2 \sin^2(\theta/2) = 1 - \cos \theta, \quad \hat{x}^2 = -J_n^2 \theta^2$$

$$\text{Y como } R(\theta, \varphi) = e^{-i\theta J_z} e^{-i\varphi J_y} \text{ u. } \text{hago } A_\mu = \partial_\mu R R^{-1} \text{ buscando } \partial_\mu \vec{m}$$

$$\text{que } \vec{m} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \text{ - como un vector (vort G, for instance).}$$

$$\text{Entonces } A_\mu = \partial_\mu R R^{-1} = \partial_\mu \ln R = \partial_\mu \ln(e^{-i\theta J_z} e^{-i\varphi J_y}) = \partial_\mu [\ln e^{-i(\lambda_x J_x + \lambda_y J_y + \lambda_z J_z)}] =$$

$$= \partial_\mu [\lambda_x J_x + \lambda_y J_y + \lambda_z J_z] (i) = \partial_\mu (\lambda_x G_x + \lambda_y G_y + \lambda_z G_z) = A_\mu^x G_x + A_\mu^y G_y + A_\mu^z G_z,$$

$$\text{u. } A_\mu^x = \partial_\mu \lambda_x, \quad A_\mu^y = \partial_\mu \lambda_y, \quad A_\mu^z = \partial_\mu \lambda_z. \text{ Hago } \vec{\lambda} \text{ buscando } \partial_\mu \vec{m}$$

$$\vec{A}_\mu = R^T A_\mu R = R^T (\partial_\mu R R^{-1}) R = R^T \partial_\mu R = \partial_\mu \ln R \text{ - como un vector?}$$

$$\text{u. } R = e^{-i\theta J_z} e^{-i\varphi J_y} \text{ u. } R = e^{-i\theta J_z} e^{-i\varphi J_y} \text{ o. } \vec{J} = \vec{A}_\mu \cdot \vec{m}, \quad \vec{m} = R \vec{m}' \text{ u. } \vec{m} = R \vec{m}'$$

$$\text{Basta } \vec{A}_\mu \cdot \vec{m}' = \vec{A}_\mu \cdot R \vec{m}' = \vec{A}_\mu \cdot \vec{m} = \vec{A}_\mu \cdot R \vec{m}' \text{ (xyz-vec.)}, \quad R \cdot \text{generacion de matrices } R^T = R^{-1}$$

$$R(\theta, \varphi) = U(\varphi, 0, 0) = e^{-i\theta J_z} e^{-i\varphi J_y}, \quad U(\alpha, \beta, \gamma) = e^{-i\alpha J_z} e^{-i\beta J_y} e^{-i\gamma J_z} \text{ (d, p, d) - grupo}$$

$$R^{-1}(\theta, \varphi) = R^T = e^{i\theta J_z} e^{i\varphi J_y} = R_{\text{rot}}(-\varphi, -\theta)$$

$$\text{K. Eng. 2001: } e^{\hat{u}} e^{\hat{v}} = \exp[\alpha \hat{u} + \beta \hat{v} + \gamma [\hat{u}, \hat{v}]], \quad \hat{u}, \hat{v} \text{ - el. de Lie algebra, r. e. } 3 \times 3$$

$$R = U(\varphi, 0, 0) = e^{-i\theta J_z} e^{-i\varphi J_y}, \quad \hat{u} = -i\theta J_z = \theta G_z, \quad \hat{v} = -i\varphi J_y = \varphi G_y$$

$$\vec{u} = (u_x, u_y, u_z) = (0, 0, \theta), \quad \vec{v} = (v_x, v_y, v_z) = (0, \varphi, 0), \quad |\vec{u}| = \theta, \quad |\vec{v}| = \varphi$$

$[\hat{u}, \hat{v}] = [-i\sigma_y, -i\sigma_z] = -i\sigma_x$
 $\hat{w} = \alpha \hat{u} + \beta \hat{v} + \gamma [\hat{u}, \hat{v}] = \alpha \sigma_y + \beta \sigma_z + \gamma \sigma_x$
 $\hat{w} = (\alpha \sigma_x + \beta \sigma_y + \gamma \sigma_z)$
 $\angle(\hat{u}, \hat{v}) = \angle(\hat{u}, \hat{v}) = \pi/2$, no porridge Fugo: $a = \sin\theta \cos^2\phi/2$, $b = \sin\theta \cos^2\phi/2$,
 $\cos(\angle \hat{u} \hat{v}) = 0$

$c = \frac{1}{2} \sin\theta \sin\phi$, $d^2 = a^2 + b^2 + c^2$, $\cos(\angle \hat{u} \hat{v}) = 0$
 $d = \frac{\sin^{-1}(d)}{d} \frac{a}{\theta}$, $\beta = \frac{\sin^{-1}(d)}{d} \frac{b}{\phi}$, $\theta = \frac{\sin^{-1}(d)}{d} \frac{c}{\theta\phi}$, $\theta \rightarrow 10$, $\phi \rightarrow 10$ can be
 $\vec{w} = (\frac{c}{d} \sin^{-1}(d), \frac{a}{d} \sin^{-1}(d), \frac{b}{d} \sin^{-1}(d)) = \frac{\sin^{-1}(d)}{d} (c, a, b) = (w_x, w_y, w_z)$
 $\hat{w} = \frac{\sin^{-1}(d)}{d} [c G_x + a G_y + b G_z]$ u $e^{\hat{u}} e^{\hat{v}} = e^{\hat{w}}$. $A_\mu = \partial_\mu \ln R$

$R(\theta, \phi) = U(\hat{n}, \theta) = e^{-i\theta \hat{n}} = \exp\left[\frac{\sin^{-1}(d)}{d} (c G_x + a G_y + b G_z)\right]$, $A_\mu = \partial_\mu \ln R = \partial_\mu \left[\frac{\sin^{-1}(d)}{d} (c G_x + a G_y + b G_z)\right] = A_\mu^x G_x + A_\mu^y G_y + A_\mu^z G_z$
 $d^2 = a^2 + b^2 + c^2 = \sin^2\theta \cos^4\phi/2 + \sin^2\theta \cos^4\phi/2 + \frac{1}{4} \sin^2\theta \sin^2\phi =$
 $= \sin^2\theta \frac{1}{4} (1 + \cos\phi)^2 + \sin^2\theta \frac{1}{4} (1 + \cos\phi)^2 + \frac{1}{4} \sin^2\theta \sin^2\phi = \frac{1}{4} \sin^2\theta \times$
 $\times [1 + 2\cos\phi + \cos^2\phi] + \frac{1}{4} \sin^2\theta (1 + 2\cos\phi + \cos^2\phi) + \frac{1}{4} \sin^2\theta \sin^2\phi$

$A_\mu^x = \partial_\mu \left[\frac{\sin^{-1}(d)}{d} b\right] = -\frac{1}{d^2} \partial_\mu d \cdot \sin^{-1}(d) \cdot b + \frac{b}{d} \frac{1}{\sqrt{1-d^2}} \partial_\mu d + \frac{\sin^{-1}(d)}{d} \partial_\mu b$
 $A_\mu^y = -\frac{c}{d^2} \partial_\mu d \sin^{-1}(d) + \frac{c}{d} \frac{1}{\sqrt{1-d^2}} \partial_\mu d + \frac{\sin^{-1}(d)}{d} \partial_\mu c$
 $A_\mu^z = -\frac{a}{d^2} \partial_\mu d \sin^{-1}(d) + \frac{a}{d} \frac{1}{\sqrt{1-d^2}} \partial_\mu d + \frac{\sin^{-1}(d)}{d} \partial_\mu a$
 $\vec{A}_\mu = \partial_\mu \vec{w} = \left[-d^{-2} \sin^{-1}d + \frac{1}{d} \frac{1}{\sqrt{1-d^2}}\right] \partial_\mu d \cdot \vec{l} + \frac{\sin^{-1}d}{d} \partial_\mu \vec{l}$, $\vec{l} = (c, a, b)$

$a = \sin\theta \cos^2\phi/2 = \sin\theta \frac{1}{2} (1 + \cos\phi) = \frac{1}{2} (m_x + \sin\theta)$, $b = \sin\theta \cos^2\phi/2 = \sin\theta \frac{1}{2} (1 + \cos\phi)$
 $c = \frac{1}{2} \sin\theta \sin\phi = \frac{1}{2} m_y$. $\vec{l} = (\frac{1}{2} m_y, \frac{1}{2} (m_x + \sin\theta), \frac{1}{2} \sin\theta (1 + \cos\phi)) = \frac{1}{2} \sin\theta (1 + \cos\phi) \vec{m}$
 $\vec{l} = \frac{1}{2} (m_y, m_x, m_z \sin\phi) + \frac{1}{2} (0, \sin\theta, \sin\phi)$ — ne chogues $\kappa \vec{m}$.
 $\vec{A}_\mu = \frac{1}{d} \left[\frac{1}{\sqrt{1-d^2}} - \frac{\sin^{-1}(d)}{d}\right] \partial_\mu d \cdot \vec{l} + \frac{\sin^{-1}(d)}{d} \partial_\mu \vec{l}$, $\vec{w} = \frac{\sin^{-1}(d)}{d} \vec{l}$.

Выводы: $\vec{A}_\mu$ не имеет значения в процессе упрощения
 репез $\vec{m} = \partial_\mu \vec{m}$ так как $J = 1/2$. Это объясняется из-за:
 1) сдвига энергии в фазе $(A_\mu \sim \partial_\mu \ln R - ?)$
 2) Eq. by Fugo не берется (учет. Выводы) что $e^{\hat{u}} e^{\hat{v}} = e^{\hat{w}}$, $J = 1$
 3) A_μ в упрощении не участвует, так как $f(\vec{m}, \partial_\mu \vec{m})$ где $J = 1$

с. 1. 6. Физика — Точка. Вычисление. Все числа 37 700 7. = 7100 * 50000
 3/4) Упрощение Дир. Фурье трансформ (FFT) где Аранта by MathCad:
 ба параме где $f(t) = \sin(\omega t)/t$, $f(t) = \text{pulse width}$, $f(t) = \cos \omega t + a \cos \omega t$
 2) форма Фурье: с. 1. 6. Физика. 1) с. 1. 6. Физика. 2) с. 1. 6. Физика. 3) с. 1. 6. Физика.
 1) с. 1. 6. Физика. 2) с. 1. 6. Физика. 3) с. 1. 6. Физика. 4) с. 1. 6. Физика.
 1) с. 1. 6. Физика. 2) с. 1. 6. Физика. 3) с. 1. 6. Физика. 4) с. 1. 6. Физика.
 1) с. 1. 6. Физика. 2) с. 1. 6. Физика. 3) с. 1. 6. Физика. 4) с. 1. 6. Физика.
 1) с. 1. 6. Физика. 2) с. 1. 6. Физика. 3) с. 1. 6. Физика. 4) с. 1. 6. Физика.

$$R = e^{-i\omega \vec{n} \cdot \vec{J}}, \quad \tilde{R} = R^{-1} = e^{i\omega \vec{n} \cdot \vec{J}}, \quad A_\mu = \partial_\mu \tilde{R} \tilde{R}^{-1} = i \partial_\mu (\omega \vec{n}) \tilde{J} \tilde{R} \tilde{R}^{-1} = i \partial_\mu (\omega \vec{n}) \tilde{J}$$

$$A_\mu = i \partial_\mu (\omega \vec{n}) \tilde{J} = -\partial_\mu (\omega \vec{n}) G_\mu, \quad \tilde{A}_\mu = -\partial_\mu (\omega \vec{n}) \tilde{J}, \quad \vec{w} = \omega \vec{n}$$

$$\vec{w} = -\omega \vec{n} = \frac{\sin^{-1}(d)}{d} (c, a, b) \text{ const. Enge eq. } |\vec{n}| = 1$$

$$\cos \omega = \frac{1}{2} (\text{Sp} R - 1) = \frac{1}{2} [\cos \theta \cos \varphi + \cos \theta + \cos \varphi - 1] = \frac{1}{2} (1 + \cos \theta)(1 + \cos \varphi)$$

$$\vec{n} = -\frac{1}{d} (c, a, b), \quad \omega = \arcsin d, \quad \sin \omega = d, \quad \sin \omega \cdot \vec{n} = -(c, a, b)$$

$$\text{Упроблему } \sin \omega n_x = -c = \frac{1}{2} (R_{23} - R_{32}) = \frac{1}{2} [0 - \sin \theta \sin \varphi] = -\frac{1}{2} \sin \theta \sin \varphi$$

$$c = \frac{1}{2} \sin \theta \sin \varphi - \text{беремо. } \sin \omega n_y = -a = \frac{1}{2} [R_{13} - R_{31}] = \frac{1}{2} [\sin \theta \cos \varphi + \sin \theta]$$

$$+ \sin \theta] = \frac{1}{2} \sin \theta [1 + \cos \varphi] = \sin \theta \cos^2 \frac{\varphi}{2}, \quad a = -\sin \theta \cos^2 \frac{\varphi}{2}$$

$$\sin \omega n_z = -b = \frac{1}{2} [R_{12} - R_{21}] = \frac{1}{2} [\sin \varphi \cos \theta + \sin \varphi] = \sin \varphi \cos^2 \frac{\theta}{2}$$

$$b = -\sin \varphi \cos^2 \frac{\theta}{2} \text{ - отсюда знамен от Enge eq.}$$

Вывод: с помощью го знамен a, b eq. Enge берем (знамен номер
 отсюда берем $c \rightarrow -c, \varphi \rightarrow -\varphi$, с убавляем θ (знамен номер
 т.е. A_μ не меняется при $\varphi \rightarrow -\varphi$ и $\theta \rightarrow \theta$).

Беседка с Аранья. Он наводит сгенерируем FFT $f(t) = \sin(\omega_0 t)/t$
 но $f(\omega_k) = \sum_{N,k} f(t_k) \cos(\omega_k t_k)$, $\omega_k = n \frac{2\pi}{T}$, $t \in [0, T]$, $T = 2N \Delta t$
 и нолу. FORT RAN. Беседка с AirPing: аналог. Ошибка
 и нет DW и reversal. Углуби ли, что ждем, нет, что работа не проходит
 Беседка со Звездой: о мушкетере, Убавке, и о story, $R(d, \beta, \varphi) = 21 \cdot (0, \varphi)$

$$\text{до из уравнения партера } \vec{m} \times \partial_\mu \vec{m} = \begin{pmatrix} -\sin \varphi \partial_\mu \theta \\ \cos \varphi \partial_\mu \theta \\ \sin^2 \theta \partial_\mu \varphi \end{pmatrix} = \cos \theta \sin \theta \begin{pmatrix} \cos \varphi \\ \sin \varphi \\ 0 \end{pmatrix} \frac{1}{\sin \theta}$$

$$= -\tilde{A}_\mu - \cos \theta \vec{m} \partial_\mu \varphi, \quad (\tilde{A}_\mu = \partial_\mu \vec{m} \times \vec{m} - \cos \theta \vec{m} \partial_\mu \varphi)$$

$$\vec{m} \times \tilde{A}_\mu = \vec{m} \times \partial_\mu \vec{m} \times \vec{m} = \partial_\mu \vec{m} (\vec{m} \cdot \vec{m}) - \vec{m} (\vec{m} \cdot \partial_\mu \vec{m}) = \partial_\mu \vec{m}, \quad \vec{m}^2 = 1$$

$$A_\mu \vec{m} = \begin{pmatrix} 0 & \partial_\mu \varphi & -\cos \varphi \partial_\mu \theta \\ -\partial_\mu \varphi & 0 & -\sin \varphi \partial_\mu \theta \\ \cos \varphi \partial_\mu \theta & \sin \varphi \partial_\mu \theta & 0 \end{pmatrix} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} m_y \partial_\mu \varphi - m_z \cos \varphi \partial_\mu \theta \\ -m_x \partial_\mu \varphi - m_z \sin \varphi \partial_\mu \theta \\ m_x \cos \varphi \partial_\mu \theta + m_y \sin \varphi \partial_\mu \theta \end{pmatrix}$$

$$= \begin{pmatrix} \sin \varphi \sin \theta \partial_\mu \varphi & -\cos \theta \cos \varphi \partial_\mu \theta \\ -\sin \theta \cos \varphi \partial_\mu \varphi & -\cos \theta \sin \varphi \partial_\mu \theta \\ \sin \theta \partial_\mu \theta \end{pmatrix} = -\partial_\mu \begin{pmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \\ \cos \theta \end{pmatrix} = -\partial_\mu \vec{m}, \quad \tilde{A}_\mu = -\vec{m}$$

$\tilde{A}_\mu$ - того же самое направление.

4/5 C.1. В кривой la Concha u. Liedl. Берем к партеру убавляем $d = 14.137$
 и берем с Аранья

$$\text{1) } \mathcal{L}_{kin} = \tilde{A}_s \cdot \partial_t \vec{m}, \quad \partial_t = \frac{\partial}{\partial t} - \tilde{R} \tilde{R}^{-1}, \quad \vec{m} \rightarrow \vec{m}' = \tilde{R} \vec{m}, \quad \partial_t (\vec{m}') = \partial_t (\tilde{R} \vec{m}) = \tilde{R} \partial_t (\vec{m})$$

$$\mathcal{L}_{kin} (\vec{m} \rightarrow \vec{m}', \tilde{A}_s \rightarrow \tilde{A}_s') = \tilde{A}_s' \cdot \partial_t (\vec{m}') = \tilde{A}_s' \cdot \tilde{R} \cdot \partial_t (\vec{m}) = \tilde{R} \tilde{A}_s \cdot \tilde{R} \partial_t (\vec{m}) = |\tilde{R} \tilde{A}_s - \tilde{A}_s'|$$

$$\vec{a} \cdot (\vec{R} \vec{b}) = a_j (R \vec{b})_j = a_j (R_{jk} b_k) = (a_j R_{jk}) b_k = (R_{jk} a_j) b_k = (R^T_{kj} a_j) b_k = (\vec{R} \vec{a}) \cdot \vec{b} = \vec{a} \cdot \vec{b}$$

$$\mathcal{L}_{kin} (\vec{m}') = \tilde{A}_s' \cdot \partial_t (\vec{m}') = \tilde{R} \tilde{A}_s \cdot \partial_t (\tilde{R} \vec{m}) = \tilde{R} \tilde{A}_s \cdot \tilde{R} \partial_t (\vec{m}) = (\tilde{R}^{-1} \tilde{R} \tilde{A}_s) \cdot \partial_t (\vec{m}) = \tilde{A}_s \cdot \partial_t (\vec{m})$$

- убавляем $\vec{m} \rightarrow \vec{m}' = \tilde{R} \vec{m}$, если $\partial_t (\vec{m})$ от
 берем $\partial_t (\tilde{R} \vec{m}) = \tilde{R} \partial_t (\vec{m})$, zero $\vec{m}$ не годится за $\vec{m}$ gauge field $\partial_t \vec{m}$

$$\text{2) } \text{А если } \vec{m} = \tilde{R} \vec{m}'? \quad \mathcal{L}_{kin} = \tilde{A}_s \cdot \partial_t (\vec{m}) = \tilde{A}_s \cdot \partial_t (\tilde{R} \vec{m}') = \tilde{A}_s \cdot \tilde{R} \partial_t (\vec{m}') = (\tilde{R}^T \tilde{A}_s) \cdot \partial_t (\vec{m}') = \tilde{A}_s' \cdot \partial_t (\vec{m}')$$

$$= \tilde{A}_s' \cdot \partial_t (\vec{m}'). \quad \text{Но } \tilde{A}_s' = A_s (\vec{m}') = s(\vec{m}') [\vec{n}' \times \vec{m}'] \quad \text{Hа } \mathcal{L}_{kin} (\vec{m}') = \tilde{A}_s' \cdot \partial_t (\vec{m}') =$$

$$= |\vec{m}' = \tilde{R} \vec{m}| = \tilde{A}_s (\tilde{R} \vec{m}') \cdot \partial_t (\tilde{R} \vec{m}') = \tilde{A}_s (\tilde{R} \vec{m}') \cdot \tilde{R} \partial_t (\vec{m}') = \tilde{R}^{-1} \tilde{A}_s (\tilde{R} \vec{m}') \cdot \partial_t (\vec{m}') = \tilde{A}_s \cdot \partial_t (\vec{m}')$$

Due ① $\nabla_{\vec{r}} \psi(\vec{r}) = \tilde{R} \nabla_{\vec{r}'} \psi(\vec{r}')$, $\nabla_{\vec{r}} = \left(\frac{\partial}{\partial x} - A_0 \right) - \text{in } x, y, z$.
 $\nabla_{\vec{r}} (\tilde{R} \psi) = \left(\frac{\partial}{\partial x} - A_0 \right) \tilde{R} \psi = \tilde{R} \nabla_{\vec{r}'} \psi + \tilde{R} \psi - A_0 \tilde{R} \psi = \tilde{R} \left(\frac{\partial}{\partial x} - A_0 \right) \psi = \tilde{R} \nabla_{\vec{r}'} \psi$
 $\tilde{R} - A_0 \tilde{R} = -\tilde{R} A_0$, $\tilde{R} \tilde{R}^{-1} - A_0 = -\tilde{R} A_0 \tilde{R}^{-1} = -A_0' (x', y', z')$, $A_0' (x', y', z') = \tilde{R} A_0 \tilde{R}^{-1}$
 $\nabla_{\vec{r}'} (\tilde{R} \psi) = \tilde{R} \nabla_{\vec{r}} \psi \rightarrow A_{\mu} (x', y', z') = \partial_{\mu} \tilde{R} \tilde{R}^{-1} \rightarrow A_{\mu} (\text{new})$.

(2) nako neperemeno. $D_t(R\vec{m}') = R D_t(\vec{m}')$, $(\frac{\partial}{\partial t} - A_0) R \vec{m}' = R (\frac{\partial}{\partial t} - A_0') \vec{m}'$
 $\cancel{R} \vec{m}' + \cancel{R} \vec{m}' - A_0 R \vec{m}' = \cancel{R} \vec{m}' - R A_0' \vec{m}'$, $\cancel{R} - A_0 R = -R A_0'$
 $R' R - R^{-1} A_0 R = -A_0'$, $R^{-1} A_0 R = A_0'$ (ne zamen.) = 0, $A_0' = -R^{-1} \dot{R}$

t.e. $\vec{A}_0' = \begin{pmatrix} \sin \phi \dot{\phi} \\ -\cos \phi \dot{\phi} \\ -\dot{\phi} \end{pmatrix}$ uroga (2) c $A_0' = -R^{-1} \dot{R}$ obnag. c $A_0'(\text{new}) = \begin{pmatrix} \sin \phi \dot{\phi} \\ -\cos \phi \dot{\phi} \\ -\dot{\phi} \end{pmatrix}$ uroga

Хотя $\vec{A}_0$ и определен через R , $R_{jk} = \hat{x}_j \cdot \hat{x}'_k$ - направл. косинусы, с A_0 - метрика не есть полн. в определ. системе координат. $A_0(\theta, \varphi)$, где θ, φ углы перехода между сист. координат x_{μ} и x'_μ .

Безо $D_{\mu}(\vec{m}) = (\partial_{\mu} + f_{\mu})\vec{m} \rightarrow D_{\mu}(\vec{m}') = (\frac{\partial}{\partial x_{\mu}} + f'_{\mu})\vec{m}'$, где $x'_{\mu} = x, y, z, t$
 Тоже же $t = \frac{\partial}{\partial t} = \frac{\partial}{\partial t'}$ т.к. мы делаем spatial 3D-rotations $xyz \rightarrow x'y'z'$
 В Weych. Brogers $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ и A_{μ} , $\mu = x, y, z$ become commutes with
 [Branco PRL 04] все же верно. Вот и получается не должно же $S = 1/2$, а. $BP \psi'$,
 т.к. $\psi' = U \psi$ генерируется U адеп. симметрией U в пространстве $BP \psi$, хотя $U(x, y, z)$.
 У нас же $\vec{m}' = R \vec{m}$ бренсман с переносом в пространстве xyz.

8/11

Yzer zabuc. Azim. SW. W. H. state of small bias field H_x

Azim. $\omega_{n,m}(H_x)$ up to $H_x/M_s \ll 1$, $H_x < H_n$ now eqs azim. modes
SW: $B(\theta, \varphi)$ - vectorial magnetic charge density
year $- \frac{M_s}{\gamma} \sin \theta \dot{\theta} = \frac{\delta W}{\delta \varphi} = (\text{Arch. magnetost.}) - \vec{H} \cdot \frac{\partial \vec{M}}{\partial \varphi}$, $\frac{M_s}{\gamma} \sin \theta \dot{\varphi} = \frac{\delta W}{\delta \theta} = (\dots) = \vec{H} \cdot \frac{\partial \vec{M}}{\partial \theta}$
H_x $W = (\text{Arch. magnetostatics}) - \vec{M} \cdot \vec{H}$. $\frac{M_x}{M_s} = \sin \theta \cos \varphi = \sin(\theta_0 + \epsilon) \cos(\varphi_0 + \psi)$

$$= [\sin \theta_0 + \epsilon \cos \theta_0] [\cos \varphi_0 - \psi \sin \varphi_0] = \sin \theta_0 \cos \varphi_0 - \psi \sin \theta_0 \sin \varphi_0 + \epsilon \cos \theta_0 \cos \varphi_0 - \epsilon \psi \cos \theta_0 \sin \varphi_0$$

$$\delta m_x = \mu_p \sin \varphi_0, \sin \varphi_0 = \sin(\varphi_0 + \epsilon \frac{\pi}{2}) = \epsilon \cos \varphi_0 \rightarrow \delta m_x = \mu_p \epsilon \cos \varphi_0$$

Zeman: $\frac{\partial \vec{M}}{\partial \varphi} = \frac{\partial}{\partial \varphi} \sin \theta \cos \varphi = -\sin \theta \sin \varphi = -[\sin \theta_0 + \epsilon \cos \theta_0] [\sin \varphi_0 + \psi \cos \varphi_0]$

$$\frac{\partial (\delta m_x)}{\partial \varphi} = -\sin \theta_0 \psi \cos \varphi_0 = \mu_p \cos(\varphi_0 + \epsilon \frac{\pi}{2}) = -\mu_p \epsilon \sin \varphi_0$$

$$\frac{\partial m_x}{\partial \varphi} = \frac{\partial}{\partial \varphi} \sin \theta \cos \varphi = \cos \theta \cos \varphi = \cos(\theta_0 + \epsilon) \cos(\varphi_0 + \psi) = [\cos \theta_0 - \epsilon \sin \theta_0] [\cos \varphi_0 - \psi \sin \varphi_0]$$

$$= \epsilon \sin \theta_0 \cos \varphi_0 + [\cos \theta_0 \cos \varphi_0 + \psi \cos \theta_0 \sin \varphi_0]$$

$$\left\{ -\frac{M_s}{\gamma} \sin \theta_0 \dot{\epsilon} = (\dots) - M_s H_x \frac{\partial (\delta m_x)}{\partial \varphi} = (\dots) + M_s H_x \epsilon \mu_p \sin \varphi_0 \right.$$

$$\left\{ \frac{M_s}{\gamma} \sin \theta_0 \dot{\psi} = -\frac{M_s}{\gamma} \dot{\mu}_p = (\dots) - M_s H_x \frac{\partial (\delta m_x)}{\partial \theta} = (\dots) + M_s H_x \sin \theta_0 \epsilon \epsilon \sin \varphi_0 \right.$$

$$-\frac{1}{\gamma} \dot{\epsilon} = (\dots) + \epsilon H_x \mu_p \sin \varphi_0 \quad \quad -\frac{1}{\gamma} \dot{\mu}_p = (\dots) + \epsilon H_x \epsilon \sin \varphi_0$$

$$\left\{ -\frac{1}{\gamma} \dot{\epsilon} = H_g^m + H_x \mu_p \sin \varphi_0 \right. \quad H_m^2 = M_s \int d^2 \vec{r}' G_{zz}(\vec{r}, \vec{r}') [-\sin \theta_0 \epsilon] \approx +4\pi M_s \sin \varphi_0 \epsilon$$

$$\left\{ -\frac{1}{\gamma} \dot{\mu}_p = H_z^m + H_x \epsilon \sin \varphi_0 \right. \quad H_m^2 \approx \epsilon M_s \int d^2 \vec{r}' G_{pp} \mu_p$$

(1) $-\frac{1}{\gamma} \dot{\epsilon} = M_s \int d^2 \vec{r}' G_{pp} \mu_p(\vec{r}') + H_x \mu_p \sin \varphi_0$ Since up to $H_x = 0$: $\mu_p = b(\varphi) \sin(m\varphi - \omega t)$
(2) $-\frac{1}{\gamma} \dot{\mu}_p = 4\pi M_s \epsilon \epsilon + H_x \epsilon \sin \varphi_0 - \frac{1}{\gamma} \dot{\varphi}_0$ $\epsilon \epsilon = a(\varphi) \cos(m\varphi - \omega t)$

$$-\frac{1}{\gamma} \ddot{\mu}_p = (4\pi M_s + H_x \sin \varphi_0) \epsilon \epsilon = (4\pi M_s + H_x \sin \varphi_0) \left[-\delta M_s \int d^2 \vec{r}' G_{pp} \mu - \gamma H_x \sin \varphi_0 \mu \right]$$

Then $-\frac{1}{\gamma} \ddot{\epsilon} = M_s \int d^2 \vec{r}' G_{pp} [4\pi M_s + H_x \sin \varphi_0] \epsilon \epsilon (-\delta) + H_x \sin \varphi_0 (-\delta) [4\pi M_s + H_x \sin \varphi_0] \epsilon \epsilon$

Com. go H_x : $-\ddot{\mu}_p = -\gamma M_s (\omega_m + \omega_x \sin \varphi_0) \int d^2 \vec{r}' G_{pp} \mu - \omega_x \sin \varphi_0 \omega_m \mu$

$\mu(t) = \mu(\omega) e^{i\omega t}$, $\omega^2 \mu = -\gamma M_s (\omega_m + \omega_x \sin \varphi_0) \int d^2 \vec{r}' G_{pp} \mu - \omega_x \omega_m \sin \varphi_0 \mu$ plus $\mu(\omega)$.

$\mu = \mu_0 + \lambda$, $\lambda \sim H_x$. $-\ddot{\mu}_0 - \ddot{\lambda} = -\gamma M_s (\omega_m + \omega_x \sin \varphi_0) \int d^2 \vec{r}' G_{pp} \mu_0 - \gamma M_s \omega_m \int d^2 \vec{r}' G_{pp} \lambda - \omega_x \sin \varphi_0 \omega_m \mu_0$

no $\ddot{\mu}_0 = \gamma M_s \omega_m \int d^2 \vec{r}' G_{pp} \mu_0$, $\mu_0 = b(\varphi) \sin(m\varphi - \omega t)$

$\int d^2 \vec{r}' G_{pp} \sin(m\varphi' - \omega t) = -4\pi d_{pp} \sin(m\varphi - \omega t)$, $d_{pp} = \frac{\partial^2}{\partial \varphi \partial \varphi'} \left(\frac{1}{k} \int \frac{1}{k'} \frac{1}{k''} \frac{1}{k'''} \right)$

$\int d^2 \vec{r}' G_{pp} \mu_0 = \int d^2 \vec{r}' \int d^2 \vec{r}'' G_{pp} b(\varphi') \sin(m\varphi' - \omega t) = -4\pi \int d^2 \vec{r}' d_{pp} b(\varphi') \sin(m\varphi - \omega t)$

$= -4\pi \sin(m\varphi - \omega t) \left(\frac{\omega_{nm}^0}{\omega_m} \right)^2 b(\varphi) = -4\pi \left(\frac{\omega_{nm}^0}{\omega_m} \right)^2 \mu_0$ u $-\ddot{\mu}_0 = (\omega_{nm}^0)^2 \mu_0 = -\gamma M_s \omega_m \int d^2 \vec{r}' G_{pp} \lambda$

$-\ddot{\lambda} = -\gamma M_s \omega_x \sin \varphi_0 (-4\pi) \left(\frac{\omega_{nm}^0}{\omega_m} \right)^2 \mu_0 - \gamma M_s \omega_m \int d^2 \vec{r}' G_{pp} \lambda - \omega_x \omega_m \sin \varphi_0 \mu_0$

$-\ddot{\lambda} = -\gamma M_s \omega_m \int d^2 \vec{r}' G_{pp} \lambda + \frac{\omega_x \sin \varphi_0 \mu_0 [(\omega_{nm}^0)^2 - \omega_m^2]}{\omega_m} - \text{resonant unit. } \gamma \mu_0$

Then $\lambda(\vec{r}, t) = c(\varphi) \sin \varphi \sin(m\varphi - \omega t)$ b. b. b. u. resonant unit. $\gamma \mu_0$

Since $\sin(m\varphi - \omega t) = \sin \varphi \sin(m\varphi - \varphi_0) = \sin \varphi [\sin m\varphi \cos \varphi_0 - \sin \varphi_0 \cos m\varphi]$

Has unresonant units $m=0$, $m=\pm 1$. $[(\omega_{nm}^0)^2 - \omega_m^2] \equiv \omega^2$

Due $m=0$ b. b. b. Due $m=\pm 1$: $\sin \varphi \sin(\pm \varphi - \varphi_0) = \pm \cos \varphi_0 \sin^2 \varphi - \sin \varphi_0 \cos \varphi \sin \varphi$

$= m \cos \varphi_0 \frac{1}{2} (1 - \cos 2\varphi) - \sin \varphi_0 \frac{1}{2} \sin 2\varphi$, $\int d^2 \vec{r}' G_{pp} \sin(k\varphi' - \varphi_0) = -4\pi d_{pp} \sin(k\varphi - \varphi_0)$

$\forall$ b. b. b. u. resonant unit. $\gamma \mu_0$

$$\begin{cases} -\frac{M_s}{\gamma} \dot{\psi} = M_s (d^2 \vec{\rho}^T G_{\rho\rho} [\vec{\mu}_p - \delta \vec{\rho}_v] + \delta \vec{\rho}_v^T M_s (d^2 \vec{\rho}^T G_{\rho\rho} [\vec{\mu}_p - \delta \vec{\rho}_v] + H_x \mu_p \sin \varphi M_s \\ -\frac{M_s}{\gamma} \dot{\mu}_p = + M_s (d^2 \vec{\rho}^T G_{\rho\rho} (-\sin \theta_0 \vec{e}_\varphi) + H_x \ell \sin \varphi M_s - \frac{M_s}{\gamma} \dot{\Phi}_v \\ -\frac{1}{\gamma} \dot{\psi} = (d^2 \vec{\rho}^T G_{\rho\rho} [\vec{\mu}_p - \delta \vec{\rho}_v] + \delta \vec{\rho}_v^T (d^2 \vec{\rho}^T G_{\rho\rho} [\vec{\mu}_p - \delta \vec{\rho}_v] + H_x \sin \varphi [\vec{\mu}_p - \delta \vec{\rho}_v] \\ -\frac{1}{\gamma} \dot{\mu}_p = 4\pi M_s \ell + H_x \ell \sin \varphi - \frac{1}{\gamma} \dot{\Phi}_v, \text{ am. cure. (1)+(2) } \approx 8/\sqrt{\epsilon} \end{cases}$$

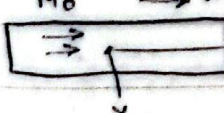
$$\begin{aligned} \frac{\partial m_x}{\partial \varphi} &= -\sin \theta_0 \sin \varphi - \sin \theta_0 \varphi \cos \varphi = -\sin(\varphi + \epsilon \frac{\pi}{2} + \delta \vec{\rho}_v^T + \delta \vec{\rho}_v) + \mu_p \cos(\epsilon \frac{\pi}{2} + \varphi + \delta \vec{\rho}_v^T) \\ &= | \delta \vec{\rho}_v^T \rightarrow 0 \text{ approx. a.r. } H_x \frac{\partial m_x}{\partial \varphi} \text{ gaug. b. mag. b. yD} | = -\sin(\varphi + \delta \vec{\rho}_v + \epsilon \frac{\pi}{2}) + \mu_p \cos(\epsilon \frac{\pi}{2} + \varphi) = \\ &= -\epsilon \cos(\varphi + \delta \vec{\rho}_v) - \mu_p \epsilon \sin \varphi = -\epsilon [\cos \varphi - \delta \vec{\rho}_v \sin \varphi] - \mu_p \epsilon \sin \varphi = \\ &= \underbrace{-\epsilon \cos \varphi}_{\text{static}} - \underbrace{\epsilon [\mu_p - \delta \vec{\rho}_v]}_{\text{dyn}} \sin \varphi = -\epsilon [\mu_p - \delta \vec{\rho}_v] \sin \varphi, \quad -H_x \frac{\partial m_x}{\partial \varphi} = H_x [\mu_p - \delta \vec{\rho}_v] \sin \varphi \\ \frac{\partial m_x}{\partial \theta} &= \ell \sin \theta_0 \cos \varphi = -\ell \cos(\epsilon \frac{\pi}{2} + \varphi + \delta \vec{\rho}_v^T) = | \delta \vec{\rho}_v^T \rightarrow 0 \text{ r.k. } H_x | = -\epsilon \ell \sin \varphi \end{aligned}$$

T.O. some vortices cur. yD b H_x o cur. de $8/\sqrt{\epsilon}$ yD moun $\delta \vec{\rho}_v^T$ u $\mu_p \rightarrow (\mu_p - \delta \vec{\rho}_v)$ (change v. gr. state)
 Ecu xoun manyrune splitting $\Delta \omega_{n,\pm 1}$ nago ocaubue rouno $\dot{\Phi}_v$ u yofarb $\delta \vec{\rho}_v(t)$:
 (1) $\begin{cases} -\frac{1}{\gamma} \dot{\psi} = M_s (d^2 \vec{\rho}^T G_{\rho\rho} \mu_p + \delta \vec{\rho}_v^T M_s (d^2 \vec{\rho}^T G_{\rho\rho} \mu_p + H_x \mu_p \sin \varphi) \\ -\frac{1}{\gamma} \dot{\mu}_p = 4\pi M_s \ell - \frac{1}{\gamma} \dot{\Phi}_v + H_x \ell \sin \varphi \end{cases} \quad \begin{aligned} \delta \vec{\rho}_v^T &= -\frac{\cos \varphi \gamma^T \gamma^T H_x}{\gamma} \\ \text{b. "rigid" v. model} \end{aligned}$

Dame dez core (unipolar ring), $\Phi_v = 0$, tarcro mcreny pcrmo γ y y gno.
 Monno uenobz $\approx$ nroq rura bora-zamnpqrcnba uak uenobz. Podbielski PRL'86
ring $k_F = 2\pi m$, $k_F = k_F(\omega, k_r, H_{int})$, Podbielski $\delta \mu = R \frac{\omega}{2} -$
 -cpcqum paquye ring  in vortex state, a gunc $k_F(\omega, k_r, H_{int})$
 uenobz. equation $k_y(\omega, k_x, H_{int})$ us Gus'PRB'83 gunc  SW rectang. el.
 Guncme or rectan. element, to rro $H_{int} = H_y + H_m^y$, a $H_{int} = -H_y \cos \varphi$
 gunc bias field H_y .

Uroxo, roo gunc ring splitting-Sh $m=\pm 1$ ne manyrune [Am. Hoffmann'PRB'07]
 u nroqog upumemum gunc radial index $n=0$ rouno (unare nrauo, karee "r")
 Theobcrme $\oint r d\varphi k_F(\omega, k_r, H_{int}) = 2\pi m$, gacr $\omega_{n,m}(H_y)$ wpu $H_y < H_n$,
 wpu $H_y=0$ $k_F = m/2$ upuro.

Podbielski PRL'86 manyrune atom-modes of ring $\omega_{0,m}(H_y)$ zabucas orono acao.
 Hancu rouno cuocobu $\omega_{n0}(H_y)$ of radial modes, begb u. are gunc $m=0$ Hebalmen.



$\vec{M}_0 \rightarrow \vec{H}$

$$\omega^2(k_x, k_y) = [\omega_{H_i} + f(\alpha L) \frac{k_x^2}{\alpha^2}] [\omega_{H_i} + 1 - f(\alpha L)]$$

- rectan. element

re ω, ω_{H_i} b eg. ω_m , $\alpha^2 = k_x^2 + k_y^2$, $\vec{M}_0 \parallel O_y$,

$\omega_{H_i} = H_i \gamma = \gamma [H_y + H_d^y]$, $H_d^y = -M_y M_y^y$. $H_d^y \rightarrow (H_d^y)_{mn}$ b gunc. yp.
 Wpu $\alpha L \ll 1$ monno a bno: $k_y^2 = \frac{1}{4} \beta^2 k_x^4 \left[\frac{(1+\omega_{H_i})}{\omega^2 - \omega_{H_i}^2} \right]^2 - k_x^2$, $\omega_{H_i}^2 = \gamma^2 H_i [H_i + 4\pi M_y]$
 T.e. $k_y = k_y(\omega, k_x, H_i)$.  - nroqog k ring b vortex state, $y \rightarrow \varphi$, $x \rightarrow \rho$.
 $H_y' \Rightarrow$ bncu. nore. $H_x = H_p = H_x' \cos \varphi + H_y' \sin \varphi = H_y' \sin \varphi = H \sin \varphi$
 $H_y = H_{te} = -H_x' \sin \varphi + H_y' \cos \varphi = H_y' \cos \varphi = H \cos \varphi$, ecu $\vec{H}' = (0, H_y')$

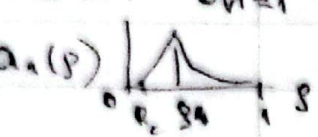
(10/11) Maxwell SW: $H_y \neq 0$ ($H_y' \neq 0$)
 i.e. que ring circular $H_y = H_y' = H \cos \varphi$ u $H_d^y = H_d^x = 0$ que vem de
 t.v. de nec. a exchange no grm., no donon. mode $= 0$
 (ing. de $x y z \rightarrow x' y' z'$)
 $k_y^2 = \frac{1}{4} k_p^2 k_p^2 \left[\frac{(1+\omega)}{\omega^2 - \omega_p^2} \right] - k_p^2 \rightarrow k_p^2(\omega, k_p, H) = \frac{1}{4} k_p^2 \left[\frac{(1+\omega)}{\omega^2 - \omega_p^2} \right] - k_p^2$
 tge $\omega_p^2 = f H_i = f H \cos \varphi$, $\omega_p^2 = f^2 H \cos \varphi (H \cos \varphi + 4 \pi m_s)$ $\rightarrow \omega_p^2 = h \cos \varphi (h \cos \varphi + 1)$
 $\omega_H = h \cos \varphi$, tge $h = H / (4 \pi m_s)$ ($k_y^2 = k_p^2 \left[\frac{1}{4} k_p^2 L^2 \left(\frac{1+h \cos \varphi}{\omega^2 - h \cos \varphi (h \cos \varphi + 1)} \right) - 1 \right]$)
 No círculo $H_y^y = 0$ que $H_y' \neq 0$ - grm. u fase shift
 $h = H_y' / (4 \pi m_s)$, $x_L \ll 1$, ou no $f(x_L) = \frac{1}{2} x_L$. Theorem $\int_0^{2\pi} d\varphi k_y = 2\pi m$
 Comonno u_z de $k_y = k_y(\varphi)$ refere u_z e H_y' .

tge $k_y / k_p = \frac{1+h \cos \varphi}{\omega^2 - h \cos \varphi (h \cos \varphi + 1)} \approx \frac{1+h \cos \varphi}{\omega^2 - h \cos \varphi} = \frac{1}{\omega^2} (1+h \cos \varphi) (1 + \frac{h}{\omega^2} \cos \varphi) \approx \frac{1}{\omega^2} \left[1 + \left(\frac{1}{\omega^2} + h \right) \cos \varphi \right]$
 $k_y / k_p = \sqrt{\frac{1}{4} k_p^2 L^2 \frac{1}{\omega^4} \left\{ 1 + 2 \left(\frac{1}{\omega^2} + h \right) h \cos \varphi \right\} - 1} = \sqrt{a + b h} = \sqrt{a} + \frac{1}{2} \frac{b}{\sqrt{a}} h$
 $a = \frac{1}{4} k_p^2 L^2 \frac{1}{\omega^4} - 1$, $b = \frac{1}{4} k_p^2 L^2 \frac{\cos \varphi}{\omega^4} 2 \left(\frac{1}{\omega^2} + 1 \right)$, $k_y(\varphi) = k_p \left[\sqrt{a} + \frac{1}{2} \frac{b}{\sqrt{a}} h \right]$
 Theorem $\int_0^{2\pi} d\varphi k_p(\varphi) = 2\pi m$, m -azim. index. $\int_0^{2\pi} d\varphi \left[\sqrt{a} + \frac{1}{2} \frac{b}{\sqrt{a}} h \right] = 2\pi m$
 $\int_0^{2\pi} d\varphi b h = 0 = h(1) \int_0^{2\pi} d\varphi \cos \varphi = 0$, t.e. azim. SW-ring or H_y' - b - m .
 u f. $m=0$ no $(H_y' / 4 \pi m_s)$ He zabucos (!), $\omega_{nm}(H_y') = \omega_{nm}(0) + O(H_y')$
 Concl. de u normano de Poddelski' PRL'06. $u_{ym} m \neq 0 (m \neq 0)$

Ecu $m=0$ (radial modes) $\omega^2(k_p) = [h \cos \varphi + f(k_p L)] [h \cos \varphi + 1 - f(k_p L)]$
 tge $k_p(\varphi) \frac{1}{2} = \frac{\omega^2 - h \cos \varphi (h \cos \varphi + 1)}{1 + h \cos \varphi}$. tge $h=0$ $\omega^2 = \frac{1}{2} k_p L = f(k_p L) \cdot k_p = k_p$
 Note: Como no $\omega_{nm}(H_y')$ que $m \neq 0$ (azim. SW) bepona que $m=0$ (no radial) $\rightarrow \omega_{0m}(H_y')$ ne zabucos or H_y' .

Peminu eueency (1)-(2) or $\partial/\partial t$ xoro uyu $m=0$ (radial modes).
 $\mu(\vec{r}) = b(\vec{r}) \sin(\omega t - \omega t) = |m=0| = -b(\vec{r}) \sin \omega t$; $e(\vec{r}) = a(\vec{r}) \cos(\omega t - \omega t) = a(\vec{r}) \cos \omega t$
 tge $m=0$ $\Phi_r = 0$, $\int d^2 \vec{r}' G_{pp} \mu(\vec{r}') \sim \int d^2 \vec{r}' G_{pp} = 0$, $\int d^2 \vec{r}' G_{pp} = -4\pi d_{pp}$
 $\frac{\omega}{8} a(\vec{r}) = -\int d^2 \vec{r}' G_{pp} b(\vec{r}') - H_x b(\vec{r}) \sin \varphi = 4\pi \int d^2 \vec{r}' d_{pp} b(\vec{r}') - H_x b(\vec{r}) \sin \varphi$
 tge $a(\vec{r})$, $b(\vec{r})$. $\frac{\omega}{8} a(\vec{r}) = -\int d^2 \vec{r}' G_{pp} b(\vec{r}') - H_x b(\vec{r}) \sin \varphi$
 $\frac{\omega}{8} b(\vec{r}) = 4\pi m_s a(\vec{r}) + H_x a(\vec{r}) \sin \varphi$, $\frac{\omega^2}{8} b(\vec{r}) = (4\pi m_s + H_x \sin \varphi) \times$
 $\times (i) \left[\int d^2 \vec{r}' G_{pp} b(\vec{r}') + H_x b(\vec{r}) \sin \varphi \right] + \delta \varphi_{\vec{r}} \int d^2 \vec{r}' G_{pp} b(\vec{r}')$
 $\rightarrow \frac{\omega^2}{8} b(\vec{r}) = -(4\pi m_s + H_x \sin \varphi) \int d^2 \vec{r}' G_{pp} b(\vec{r}') - 4\pi m_s H_x \sin \varphi b(\vec{r}) - 4\pi m_s \int d^2 \vec{r}' G_{pp} b(\vec{r}')$
 $\delta \varphi_{\vec{r}} = -\frac{\cos \varphi}{8} y^2$, $y_d \sim H_x$. Unref. $y_p = c$ na $b(\vec{r})$.
 $\int d^2 \vec{r}' G_{pp} \sin(k\varphi' - \varphi_0) = -4\pi d_{pp} \sin(k\varphi - \varphi_0)$ u $\int d^2 \vec{r}' G_{pp} F(\varphi') = -\omega d_{pp} F(\varphi)$
 $G_{pp} = -\frac{1}{8} \frac{\partial}{\partial \varphi} \frac{\partial}{\partial \varphi'} \frac{1}{|\vec{r} - \vec{r}'|} = (-\dots) \int dk e^{-k|\vec{r} - \vec{r}'|} J_0(k|\vec{r} - \vec{r}'|) = (-\dots) 2b(k) f(k) \sum_j e^{i j (\varphi - \varphi')}$
 $= -\frac{1}{8} \frac{\partial}{\partial \varphi} \frac{\partial}{\partial \varphi'} 2 \left(\frac{dk}{k} f(k) \right) \sum_j e^{i j (\varphi - \varphi')}$ $J_0(kr) J_0(kr') = -\frac{2i}{8} \frac{\partial}{\partial \varphi} \left(\frac{dk}{k} f(k) \right) \sum_j e^{i j (\varphi - \varphi')} J_j J_j =$
 $\int d^2 \vec{r}' G_{pp} e^{i \lambda \varphi'} = \int d^2 \vec{r}' (-\dots) \sum_j j e^{i j \varphi - i j \varphi'} J_j J_j e^{i \lambda \varphi'} = 2\pi (-\dots) \lambda e^{i \lambda \varphi} J_\lambda J_\lambda(1) =$

Spin splitting $\Delta\omega_{n,\pm 1} \approx p J_n^2 \omega_{n1} / \omega_M \approx \frac{p}{2\omega_M} \frac{\omega_{n1}^2}{d_{in}^2}$, $M = \frac{L}{2\pi^2} \sum_n J_n^2$, $M = \frac{L}{2\pi^2} \sum_n \frac{1}{n^2} = \frac{\pi^2}{6} = \zeta(2)$, because No. $\frac{1}{2\pi^2} \sum_n J_n^2 = \frac{L}{2\pi^2} \sum_n \frac{1}{n^2} = \frac{L}{2\pi^2} \frac{\pi^2}{6} = \frac{1}{12} \frac{L}{\delta^2}$, $M \approx \frac{1}{12} \frac{L}{\delta^2}$, $M \approx \frac{1}{12} \frac{20 \cdot 10^{-2} \text{ cm}}{(2\pi)^2} \frac{(3 \cdot 10^{12} \text{ s}^{-1})^2}{(6 \cdot 10^8 \text{ s}^{-1})^2} \approx 1.15 \cdot 10^{-20} \text{ g}$

Main (n=1) magy nagy szerepe van $J_1(d_{in}, p)$
 v. core $J_{n=1}$ egyszerűen számolható
 $a_1(p) = \begin{cases} p/p_1, & p < p_1 \\ p/p, & p > p_1 \end{cases}$ $J_1(d_{in}, p) = J_1(d_{in}, p)$
 $N_n = \int_0^1 dp J_1^2(d_{in}, p) = \frac{1}{2} [J_1'(d_{in})]^2$

Hogy $J_n = \frac{1}{\sqrt{N_n}} \int dp m_0(p) J_1(d_{in}, p) \approx \frac{1}{\sqrt{N_n}} \frac{1}{d_{in}}$ ekkor $a_n(p) \approx \frac{1}{\sqrt{N_n}} J_1(d_{in}, p)$

Tehát $\Delta\omega_{n,\pm 1} = \frac{p}{2\omega_M} \frac{\omega_{n1}^2}{N_n d_{in}^2}$, $M = \frac{L}{2\pi^2} \sum_n J_n^2 = \frac{L}{2\pi^2} \sum_n \frac{1}{N_n d_{in}^2}$
 $N_n d_{in}^2 = \frac{1}{2} [J_1'(d_{in}) d_{in}]^2 = \frac{1}{2} [d_{in} J_0(d_{in}) - J_1(d_{in})]^2 = \frac{1}{2} d_{in}^2 J_0^2(d_{in})$

$M \approx \frac{L}{2\pi^2} \sum_n \frac{1}{d_{in}^2 J_0^2(d_{in})} = \frac{L}{\pi^2} \sum_{n=1} \frac{1}{d_{in}^2 J_0^2(d_{in})}$, $J_0(x) \approx \sqrt{\frac{2}{\pi x}} \cos(x - \frac{\pi}{4})$
 $d_{in}^2 J_0^2(d_{in}) \approx d_{in}^2 \frac{2}{\pi d_{in}} \cos^2(d_{in} - \frac{\pi}{4}) = \frac{2}{\pi} d_{in} \frac{1}{2} [1 + \cos(2d_{in} - \frac{\pi}{2})] = \frac{1}{\pi} d_{in}$

tehát $M = \frac{L}{\pi^2} \sum_{n=1} \left(\frac{1}{n}\right) \rightarrow \infty$ because package

$\Delta\omega_{n,\pm 1} = \frac{p}{2\omega_M} \frac{\omega_{n1}^2}{n} \approx \frac{p}{\omega_M} \frac{\omega_{n1}^2}{n}$, $\text{mivel } \omega_{n1} \approx \omega_M f(\beta d_{in})$ $\text{kak } \text{g} \approx 10^{-21} \text{ g}$
 $\Delta\omega_{n,\pm 1} \approx p \omega_M^2 \frac{f(\beta d_{in})}{n} \approx \frac{p \omega_M^2}{2} \beta \pi$ $\text{m} \approx \beta d_{in} \ll 1$, $\text{ho } \text{g} \approx n \gg 1$, $\text{m} \approx \beta \approx 0.1$

(15/v) Hamcarne Travel grant Spain-Portugal
 zbornica Oskana u yprocu u fuzi. reprezentacija pacera hard/soft
 new answers. as 2003 (Gus PRB'09), ymuenes exp. data Muzenberg
 no SW antideet arrays.

$\Delta\omega_{n,\pm 1} = \frac{1}{2\omega_M} J_n^2 \omega_{n1}^2$. normalen van $f(n)$. $M = \frac{L}{2\pi^2} \sum_n J_n^2$

- Zagave kozp. $J_n = \frac{1}{\sqrt{N_n}} \int dp m_0(p) a_n(p)$. $\omega_M = \gamma \cdot 4\pi m_s = 29.7 \text{ GHz}$
 1) Berem $\beta = 0.0483$ ($\ell = 25 \text{ nm}$, $2R = 1035 \text{ nm}$ - Alieu PRB'09):

n	0	1	2	3	4
$G_{H_2}, \Delta\omega_{n,\pm 1}$	1.11	0.995	0.432	0.333	0.151
$N=12, J_n$	1.41	0.877	0.474	0.372	0.232
$J_n(d_{in}) N=15, J_n$	1.48	0.932	0.514	0.392	0.242

ug - za biserporo nagemu $J_n = d(n)$
 $\sum_n J_n^2 = 3.23$ $\text{g} \approx \beta = 0.0483$
 $M(\beta = 0.048) = \frac{25 \text{ nm} \cdot 10^{-2}}{2(2\pi)^2 (2.95)^2 \cdot 10^{12}} \cdot 3.23 = 0.384 \cdot 10^{-20} = 1.15 \cdot 10^{-20} \text{ g}$

2) Dne $\beta = 0.1$ (govi mesa)

n	0	1	2	3	4
$G_{H_2}, \Delta\omega_n$	2.17	1.05	0.13	4.10	5
J_n	1.62	0.782			

 $\sum_n J_n^2 = 3.31 \rightarrow M = 0.728 \cdot 10^{-20} \text{ g}$

3) Dne $\beta = 0.02$ $\sum_n J_n^2 = 2.98$ u $M = \frac{L}{2\pi^2} \sum_n J_n^2$ zabucur otul.
 $F(\beta) = \sum_n J_n^2(\beta)$ crado pacera c p. $F(\beta) \approx 3 \rightarrow M \approx \frac{3}{2} \frac{L}{\delta^2} \approx 1.15 \cdot 10^{-20} \text{ g}$

B PRL'07 by Bedan Klani. oco ogenna M $\text{m} \approx 10^{-21} \text{ g}$.
 Mow (Bedan) gne TW, VW $\sim 10^{-21} \text{ g}$.

(16/vi) Tucsma Cristina Arreizaga, Wue. Edited PRL on Vortex-Mass u uocua
 to Arantxa u Julian. Odeygu Vort. Mass - PRL co zhezgu. u gae any vee
 gne comments. Sanjocan brief CV Spanish yracm. Travel grant - Julian,
 Juan del Val, Arantxa, Maxim, Maria. JAVN no Colu-nanpart - was accepted.

2) indexes splitted SW ($m = \pm 1$): Pole-free m. gear gear φ No.

$$\Phi_v(\hat{p}, t) = \varphi + \sigma \frac{\pi}{2} + m_0(p) [\sin \varphi \cdot \hat{X} - \cos \varphi \cdot \hat{Y}], \quad \dot{\Phi}_v = m_0(p) [\sin \varphi \cdot \dot{\hat{X}} - \cos \varphi \cdot \dot{\hat{Y}}]$$

we $X(t), Y(t)$ have axes by vertex YD . $I_v + I_{int} = \frac{1}{2} (\hat{G} \times \hat{X}) \cdot \hat{X} - W(\hat{X})$

$$\frac{d}{dt} \frac{\partial I}{\partial \dot{\hat{X}}} - \frac{\partial I}{\partial \hat{X}} = 0, \quad \hat{G} \times \hat{X} + \partial_{\hat{X}} W + \vec{K} = 0, \quad \vec{K} = \frac{M_S L}{r} \int d^2 \vec{p} \sin \theta_0 \left(\frac{\partial \varphi}{\partial \hat{X}} \right)$$

$$\Phi_v = \Phi_0 + m_0(p) (\sin \varphi \cdot \hat{X} - \cos \varphi \cdot \hat{Y}) = \Phi_0 + m_0(\hat{p} \times \hat{X}), \quad (-1), \quad \hat{p} = (\cos \varphi, \sin \varphi)$$

$$\frac{\partial \Phi_v}{\partial \hat{X}} = m_0 \sin \varphi, \quad \frac{\partial \Phi_v}{\partial \hat{Y}} = -m_0 \cos \varphi \rightarrow \vec{K} = \frac{M_S L}{r} \int d^2 \vec{p} \sin \theta_0 \ell (\sin \varphi, -\cos \varphi)$$

$$\vec{K} = -\frac{M_S L}{r} \hat{z} \times \int d^2 \vec{p} \sin \theta_0 m_0 \ell \hat{p}, \quad \vec{G} = 2\pi p L M_S / r \hat{z} = p |G| \hat{z}, \quad \hat{z} = \frac{\vec{G}}{|G|}$$

$$\vec{K} = -\frac{1}{2\pi} \vec{G} \times \int d^2 \vec{p} \hat{p} \sin \theta_0 m_0 \ell, \quad \vec{K} = -\frac{p}{2\pi} \vec{G} \times \int d^2 \vec{p} \hat{p} \sin \theta_0 m_0 \ell$$

$$\vec{G} \times \hat{X} = -\partial_{\hat{X}} W - \vec{K}, \quad \vec{G} \times \vec{G} \times \hat{X} = \vec{G} (\vec{G} \cdot \hat{X}) - \hat{X} G^2 = -\vec{G} \times [\partial_{\hat{X}} W + \vec{K}]$$

$$\hat{X} = \frac{1}{G^2} \vec{G} \times [\partial_{\hat{X}} W + \vec{K}], \quad \text{from vertex } YD: \hat{X} = \frac{1}{G^2} \vec{G} \times [\alpha \hat{X} + \vec{K}]$$

$$\dot{\Phi}_v = m_0 [\sin \varphi \dot{\hat{X}} - \cos \varphi \dot{\hat{Y}}] = -m_0 \hat{p} \times \dot{\hat{X}} = -m_0 \hat{p} \times \frac{1}{G^2} \vec{G} \times [\alpha \hat{X} + \vec{K}] =$$

$$= -\frac{m_0}{G^2} [\vec{G} \cdot \hat{p} (\alpha \hat{X} + \vec{K}) - (\alpha \hat{X} + \vec{K}) (\hat{p} \cdot \vec{G})] = -\frac{m_0}{G^2} \vec{G} \cdot \hat{p} (\alpha \hat{X} + \vec{K})$$

have motion $\hat{X}(t)$ resp $\vec{K}$ along $\hat{z}$ axis, $\dot{\Phi}_v = -m_0 \hat{p} \times \dot{\hat{X}} = -\frac{m_0}{G^2} \vec{G} \cdot \hat{p} (\alpha \hat{X} + \vec{K})$

rotate resp SW reference (ℓ, p).

$$\begin{cases} -G_2 \dot{Y} + \alpha X + K_x = 0 \\ G_2 \dot{X} + \alpha Y + K_y = 0 \end{cases} \quad \begin{cases} i G_2 \dot{Z} + \alpha Z = -\vec{K} \\ i p \dot{Z} + \omega_0 Z = -\frac{1}{|G|} \vec{K} \end{cases}, \quad K = K_x + i K_y, \quad Z = X + i Y$$

$$F(t) = -\frac{1}{|G|} \vec{K}, \quad i p \dot{Z} + \omega_0 Z = F(t) = F_0 e^{i \omega t}, \quad K = K_x + i K_y = \frac{M_S L}{r} \int d^2 \vec{p} \sin \theta_0 m_0 \ell (\sin \varphi, -\cos \varphi)$$

$$\vec{K} = \frac{M_S L}{r} \int d^2 \vec{p} \sin \theta_0 m_0 \ell e^{i \varphi} (-i) = (i) \frac{M_S L}{r} \int d^2 \vec{p} \sin \theta_0 m_0 \ell e^{i \varphi} \frac{d}{dt} [\alpha(p) \cos(m_0 \varphi - \omega t)] =$$

$$= (-i) \frac{d}{dt} \frac{M_S L}{r} \int d^2 \vec{p} \sin \theta_0 m_0 \alpha \int d\varphi e^{i \varphi} \cos(m_0 \varphi - \omega t) = (-i) \int d\varphi e^{i \varphi} \frac{1}{2} (e^{i m_0 \varphi - i \omega t} + e^{-i m_0 \varphi + i \omega t})$$

$$= (-i) \frac{d}{dt} \frac{M_S L}{r} \int d^2 \vec{p} \sin \theta_0 m_0 \alpha \cdot \frac{1}{2} 2\pi [\delta_{m, -1} e^{-i \omega t} + \delta_{m, 1} e^{i \omega t}] = \pi \omega \frac{M_S L}{r} \int d^2 \vec{p} \sin \theta_0 m_0 \alpha$$

$$\times [\delta_{m, 1} e^{i \omega t} - \delta_{m, -1} e^{-i \omega t}], \quad [\dots] = \begin{cases} e^{i \omega t} & m=1 \\ e^{-i \omega t} & m=-1 \end{cases}, \quad \vec{K} = \pi \omega m \delta_{m, 1} e^{i \omega t} \frac{M_S L}{r} \int d^2 \vec{p} \sin \theta_0 m_0 \alpha$$

$$\vec{K} = \pi \omega m \left(\frac{M_S L}{r} \right) I_a \delta_{m, 1} e^{i \omega t}, \quad I_a = \int d^2 \vec{p} \sin \theta_0 m_0(p) \alpha(p) - \text{cm. 7/11/08.2.}$$

$$F(t) = -\frac{1}{|G|} \vec{K} = -\frac{1}{2\pi M_S L} \pi \omega m \frac{M_S L}{r} I_a \delta_{m, 1} e^{i \omega t} = -\frac{1}{2} \omega m I_a \delta_{m, 1} e^{i \omega t} = F_0$$

$$\text{Pole-free vertex } YD: (i p \dot{Z} + \omega_0 Z = F_0 e^{i \omega t}), \quad F_0 = -\frac{1}{2} \omega m I_a \delta_{m, 1}$$

$$\text{LORDY: } i p \dot{Z} + \omega_0 Z = 0 \text{ un. pole-free } Z(t) = Z_0 e^{i p \omega t}, \quad \text{Zacres pen.}$$

$$\text{LORDY ungen } \delta \text{ bug } Z(t) = Z_1 e^{i m \omega t}, \quad i p \cdot i m \omega Z_1 + \omega_0 Z_1 = F_0$$

$$Z_1 = \frac{F_0}{(\omega_0 - p m \omega)}, \quad \text{Damping pen. } Z(t) = Z_0 e^{i p \omega t} + \frac{F_0}{(\omega_0 - p m \omega)} e^{i m \omega t}$$

$$\text{Damping because } \omega_0 \rightarrow \omega_0 + i d m \omega, \quad \dot{Z}(t) = i p \omega_0 Z_0 e^{i p \omega t} + \frac{F_0 i m \omega}{(\omega_0 - p m \omega)} e^{i m \omega t}$$

$$\text{Some } i p \dot{Z} + \omega_0 Z = 0, \quad p \rightarrow (p + i d), \quad e^{i p \omega t} = e^{i (p + i d) \omega t} = e^{i p \omega t} \exp(-d \omega t)$$

$$\text{Chos. v. then } \sim Z_0 \text{ decay } \sim e^{-d \omega t}, \quad \text{noisy } \dot{Z}(t) \approx \frac{i m \omega F_0}{(\omega_0 - p m \omega)} e^{i m \omega t}$$

$$\text{No SW c } \Omega = m \omega \text{ same } Z_0 \text{ decay, } \text{Cpale } \omega_0 Z_0 \sim \frac{1}{2} \omega m F_0$$

$$\text{B n d c. eg. } \vec{K} \rightarrow R \vec{K}, \quad \frac{(\omega_0 - p m \omega)}{(\omega_0 - p m \omega)} \approx -p F_0 = \frac{1}{2} p \omega m \delta_{m, 1} I_a \approx \frac{1}{2} \omega I_a, \quad \vec{X}, \vec{Z} \text{ axes}$$

$$\text{Cobem ne ozebugno. } \omega_0 Z_0 \ll \frac{1}{2} \omega I_a \text{ no oca } \text{gear splitting SW u' oca } \text{gear splitting SW}$$

$$\text{gear splitting SW u' oca } \text{gear splitting SW}$$

$$\text{gear splitting SW u' oca } \text{gear splitting SW}$$

$$\text{gear splitting SW u' oca } \text{gear splitting SW}$$

Нормаль к поверхности $\hat{z}(t) \approx \frac{i m a F_0}{(\omega_0 - p m \omega)} e^{i m \omega t} \approx |\omega_0 \ll \omega| = -i p F_0 e^{i m \omega t}$
 $\dot{z} = \dot{x} + i \dot{y} = -i p F_0 [\cos m \omega t + i \sin m \omega t]$, $\dot{x} = p F_0 \sin m \omega t$, $\dot{y} = -p F_0 \cos m \omega t$, $m=1$
 $\dot{\varphi}_v = m_0 [\sin \varphi \dot{x} - \cos \varphi \dot{y}] = m_0 p F_0 [\sin \varphi \sin m \omega t - \cos \varphi (\cos m \omega t)] =$
 $= m_0 p F_0 \cos(m \varphi - \omega t)$, $\dot{\varphi}_v = -\frac{1}{2} p m \omega I_a \delta_{lm,1} m_0(p) \cos(m \varphi - \omega t)$ - *сбн. 07/III 082.*

Вывод: $\ddot{e}_i = -\delta M_i \int d\varphi' G_{pp} M_p$, $\ddot{\mu}_p = -\delta M_i \int d\varphi' G_{zz} \delta m_z + \dot{\varphi}_v$
 $\delta m_z = \delta(\omega_0) = \cos \theta_0 - \sin \theta_0 e \approx -e e$, $\int d\varphi' G_{zz} \delta m_z(\varphi') = \int d\varphi' [4\pi \delta(\varphi' - \varphi)] \delta m_z(\varphi') =$
 $= 4\pi \delta m_z(\varphi')$
 $\ddot{\mu}_p = -\delta M_i \delta m_z (-4\pi) + \dot{\varphi}_v = -\omega_m^2 e e + \dot{\varphi}_v$ *при $p \ll 1$.*

$\ddot{\mu}_p = -\omega_m^2 e e + \dot{\varphi}_v = \frac{\omega_m^2}{4\pi} \int d\varphi' G_{pp} M_p + \dot{\varphi}_v$, $\mu_p = b(p) \sin(m \varphi - \omega t)$
 $\dot{\varphi}_v = \frac{d}{dt} [-\frac{1}{2} p m \omega I_a m_0 \cos(m \varphi - \omega t)] = -\frac{1}{2} p m \omega^2 I_a m_0 \sin(m \varphi - \omega t)$
 $-\omega^2 b \sin(m \varphi - \omega t) = \frac{\omega_m^2}{4\pi} \int d\varphi' \int d\varphi' G_{pp} b(\varphi') \sin(m \varphi' - \omega t) + A \sin(m \varphi - \omega t)$

Нен. $\int d\varphi' G_{pp} \sin(m \varphi' - \omega t) = -4\pi d_{pp} \sin(m \varphi - \omega t)$
 $-\omega^2 b(p) = -\omega_m^2 \int d\varphi' \delta_{pp} b(\varphi') - \frac{1}{2} p m \omega^2 I_a m_0$ - *сбн. с гр. 19/III 082. due splitting*

(*) $\omega^2 b(p) = \omega_m^2 \int d\varphi' \delta_{pp} b(\varphi') + \frac{1}{2} p m \omega^2 I_a m_0(p)$ - *нормировка инт. уравнения*

Если нормировать $b(p) = b_1(p)$, св. св. vertex $m_0(p)$, $\rightarrow 0$
 $\omega^2 b_1(p) = \omega_1^2 b_1(p) + \frac{1}{2} p m \omega^2 I_a m_0$, $\omega^2 = \omega_1^2 + \frac{1}{2} p m \omega^2 I_a \int d\varphi' m_0 b_1$
нормировка $\int d\varphi' b_1^2(p) = 1$. $\omega^2 = \omega_1^2 + \frac{1}{2} p m \omega^2 I_a I_b$, $I_a = \int d\varphi' m_0 a_1$, $I_b = \int d\varphi' m_0 b_1$
где $a_1(p)$, $b_1(p)$ - *нормировка св. св. гр. и к. motion.*

Если еще нормировать $\omega_1^2 \gg \omega^2 I_a I_b$, $\rightarrow \omega_m^2 = \omega_1^2 + \frac{1}{2} p m I_a I_b \omega_1^2$
 $\omega_m = \omega_1 \sqrt{1 + \frac{1}{2} p m I_a I_b} = \omega_1 + \frac{1}{4} p m \omega_1 I_a I_b$, $a_1(p) = (\frac{\omega}{\omega_m}) b_1(p)$

$\omega_m = \omega_1 + \frac{1}{4} p m \omega_1^2 \frac{I_a^2}{\omega_m}$, $\Delta \omega_{\pm 1} = \omega_{+1} - \omega_{-1} = \frac{1}{2} p \frac{\omega_1^2}{\omega_m} I_b^2$, $I_b \rightarrow I_b$ - *сбн. PRL - MS.*

Это сбн. с 19/III 082. *При $p=+1$, $\omega_{+1} > \omega_{-1}$, ω_{+1} больше, хвост TM для СВ (CCW)*

Это не сбн. с Kovalov EPJ'03, где TM - CCW и нормировка $Su'(a) - CCW' (m=+1)$

Это сбн. с simulations by Arantxa, где где $p=-1$, нормировка $Su' - CCW' (m=-1)$

В Gus PRL'08 о нормировке канона, где TM и нормировка Su' и CCW' одинаковы *нормировка вращающихся. Хвост 19/III 082. пацет свн пенонс PRL'08 submission*

Нормировка (*) и уравнение. буги инт. Fredholm eq.

$\int d\varphi' K(\varphi, \varphi') \varphi(\varphi') - \Omega \varphi(\varphi) = f(\varphi)$. $\int d\varphi' \sqrt{\varphi'} d_{pp} \sqrt{\varphi'} b(\varphi') - \omega^2 \sqrt{\varphi} b = -\frac{1}{2} p m \omega^2 I_a \sqrt{\varphi} m_0$
 $\varphi(\varphi) = \sqrt{\varphi} b(\varphi)$, $K(\varphi, \varphi') = \sqrt{\varphi \varphi'} d_{pp}(\varphi, \varphi')$, $\Omega = \omega^2$, $f(\varphi) = -\frac{1}{2} p m \omega^2 I_a \sqrt{\varphi} m_0(p)$

Резольвента $R(\Omega, \varphi, \varphi') = \sum_n \frac{\psi_n^*(\varphi') \psi_n(\varphi)}{\Omega_n - \Omega}$, $\int d\varphi' K(\varphi, \varphi') \psi_n(\varphi') = \Omega_n \psi_n(\varphi)$
 $\varphi(\varphi) = \int d\varphi' f(\varphi') R(\Omega, \varphi, \varphi') = \int d\varphi' f(\varphi') \sum_n \frac{\psi_n(\varphi') \psi_n(\varphi)}{\Omega_n - \Omega}$, $\psi_n(\varphi) = \sqrt{\varphi} b_n(\varphi)$

$b(\varphi) = \int d\varphi' \sqrt{\varphi'} f(\varphi') \sum_n \frac{b_n(\varphi') b_n(\varphi)}{\omega_n^2 - \omega^2}$, $f(\varphi) = -\frac{1}{2} p m \omega^2 I_a \sqrt{\varphi} m_0(p)$
 $b(\varphi) = \sum_n \frac{b_n(\varphi)}{[\omega_n^2 - \omega^2]} \int d\varphi' \varphi' (-\frac{1}{2} p m \omega^2 I_a) m_0(\varphi') b_n(\varphi') = -\frac{1}{2} p m \omega^2 I_a \sum_n \frac{J_n b}{[\omega_n^2 - \omega^2]} b_n(\varphi)$

Общая. *пач. инт. equation*: $B_n(\varphi) = b_n(\varphi) - \frac{1}{2} p m \omega^2 I_a \sum_{n'} \frac{J_{n'} b}{[\omega_{n'}^2 - \omega^2]} b_{n'}(\varphi)$
Для $n=0$ $y_{0,0}$ ($n=0$): $B_0(\varphi) = b_0(\varphi) - \frac{1}{2} p m I_a \sum_{n'} \frac{\omega^2}{[\omega_{n'}^2 - \omega^2]} b_0(\varphi)$, *сбн. $J_n b = 0$ для $n=0$.*

Помогите 2-м людям сориентироваться. Помогите и себя сф. помогите Gus PRL'08

20/vii E-mail to Dresden. Bureau to Dresden u Genoa. Aug. 08/09.
E-mail to Travel Agency o Hotel reserv. in Genoa/No.

21/vii Zbogom go odzega b Genoa - Dresden: ^{year} ^{month} ^{day}
1) vezbornu Papragu o APL; 2) zapazepb. Hotel in Genoa na July 25 - Aug
3) Pozr. Colu - gonuag b Genoa u Vortex-talk b Dresden; 4) Onu. drag u kb. 30 Aug
5) odfenz. JNN u APL papers; 6) njebovna smetna Caja Madrid; 7) neprebeu uzb. 30 Aug

Paprag paper by Gaididei et al. Int. J. Quant. Chem'09 no Vortex dynamics
(unoro neronmeten. eeb amenu b $W_x(s)$, $W_m(s)$ - polee free model). Dons. no Colu

22/vii C. A. B. b Banka P. vezbornu Caja M. debit Card, Juan Marco njebovna 1/2 a ju
6-month CD u 0.5% juze 3 month CD nominated in USD. negros. slides
juze Colu gonuag b Genoa (IE3 Nano'09). Vortex Mass - PRL nocena to Paprag
Paprag tel. go Oct. 2009: 91-4974660.

23/vii Update LV. Nagnorobna gonu. Vortex dyn. Dresden (7 Aug., 30 min).
V Vortex gonuag Izsec/San-Seb.'08 gotaburo a) azim. SW splitting; b) nomen
Vortex dyn. c Oucanu (Internag'09); d) FMR spectra - Farkhad-PRB'09; e) v_{cr} - PRL'08; f) v_{cr}
neprebeu \$57,000. uz citi Bank to Caja Madrid (beeno neprebeu. \$127,000) c/colu

24/vii Onuenu cbez za April 14 - July 10: €115. Avagan Juan Marco
xory baw \$ - cyuny Caja Madrid noknacuo na 3 month, CD nog 0.5%
Pacnerobna kapr, mapu pyzob, Hotel reserv. juze vezbornu b Genoa, Dresden. Na nomen

25/vii hower Bilbao → Genoa, nocenuce b Hotel "Soghia" (Standard H. Genoa)

26/vii Zvckypena no nasefennoi u no ropogy. Na nomen Pego.

27/vii IEEE Nano'09 Conference registration, coffee breaks, welcome party.
Hamen Internet. T. Chui uz Delaware: 1) njebovna u case b Oct; 09; 2) NATO trans
vucena to Farkhad u Chabun.

28/vii Hamen u erocua ofpus. ozob na APL - Apolonio (Vortex Gze - defect let
nogros. gonuag no Cdu. particles. Zbogom vedy u nomen o JNN - vedy

Note: topological vector potential (PRL-subm.) $A_\mu^2 = -\int_\mu \Phi_\nu$ juze vedy

SW GD: $ee = -\gamma H_m^2$, $\dot{\Phi} = \gamma H_m^2 - \dot{\Phi}_v$. No $-\frac{1}{\gamma} \dot{\Phi}_v = H_g^2$ comacu

Topol. field = gyrofield March-April'07 [PRL'08]. T.e. $\frac{1}{\gamma} \dot{\Phi} = H_m^2 + H_g^2$
nomen paocenu gyrofield

rozapnyzob obegz topol. $\hat{A}_\mu$ u $\vec{H}_g$ vjm nepegeuka PRL - Subm.

Tacme rozapnyzob odushob: slowly moving Soliton interacts with
selected SW an denormalize their frequencies. From the other side, the SW

lead to Soliton dyn. mass and dynamical instability (Anna Walker njebovna)

Gyrofield (bnyzpu v. Gze): $H_g^2(\vec{r}) = \frac{\omega_0}{\gamma} F(\vec{r}/R_c) \frac{1}{R_c} (\hat{\vec{r}} \cdot \vec{X})$, R_c - pefuz Gze
 $F(\eta) = \begin{cases} \frac{4\gamma}{(1+\eta^2)^2}, \eta = \vec{r}/R_c < 1 \\ \frac{1}{\eta^2}, \eta > 1 \end{cases}$ - cu. 3/IV 2007. Max $F(\eta) \approx 1.3$ vjm $\eta \approx 1$.

$H_g^2(\vec{r}) \approx \frac{\omega_0}{\gamma} \frac{1}{R_c} (\hat{\vec{r}} \cdot \vec{X})$ bnyzpu nokenuyua. $\hat{\vec{r}} \times \vec{v} = \vec{\omega} (\hat{\vec{r}} \cdot \vec{X})$ cam $\vec{v} \approx \vec{\omega} \times \vec{X}$
Blumna $H_g^2 \sim \frac{1}{\gamma} \frac{v}{R_c}$, γ u nomen, Xbe nokobam b gonoboku APL

29/vii Besfennu o Chabunnu: odfenz. APL u Xbanobabam u gpyzpu
chizannou pibot, odeg. Donyag Yonsei Univ. Seoul, no DW nanowires (mobility

neprebeu, nomen Wieser'PRB'05). Donyag Slavin no TB njezpuroboro avpaspa:
nanos oscillator bo bnen. neprebeu. nomen, neprebeu geies lue-free → cu

Zbogom nepre u vedy (nomen Paprag). Beprebeu Sanchez (Orido) o Ikerbasque

Давидов J = $\oint \vec{z} \cdot (\vec{M} \times d\vec{M})$ - интерпретация по контуру (полез. физ.) , объяс. раск-
 попул. функции $\frac{(M+M_2)}{M}$.

Добав. с Лавруши, но а до Aug. 20 рассмотрю H_c^2 - пока ре-
 борона VC в H_2 и $V_c(H_2)$ где APL - Хвалковича.

(30/vii) Доклады Лавруши (Konstan Univ.) и Сунангуса по bit-patterned media.
 Новый способ зон. генеро генеро массив array dots, $\varnothing = 20-30$ nm, по
 неперпендикулярной форме. Доклад Hong (Hong-Kong Univ.) по DW dynamics:
 переход на основе преобразования E_{DW} (Zeeman) через дугу в гирине формы
 полевому напору J_{DW} (H) в общем виде, где забвение. от типа internal DW structure
 босади с Лавруши и Hong (он же PDF своих papers). Мои доклады по Coli, E-mails.

31/vii Возвращение Vortex-Excit. Гонимая в Dresden'09. Нпрямая по Генуя пер.

1/viii Перелет Генуя $\rightarrow$ Munich $\rightarrow$ Dresden. Писем Аранха (о Sp-Port. grant),
 Julian (перелет only JMN review by Gleb), Wong, Chui (Delaware) о визит, 8 Oct. '09

2/viii Тема кнур по Квант. теории поля и Кв. эл. Холла о u fields, gauge
 field transformations, Dirac monopole (Peskin, Ryde, Monastyrskii etc.)
 Пархаз, писем APL с revised Figs и revision text - перелет ее Ресу.

3-7/viii Ураине в Workshop MAGNONICS'09. Научин 300. € Коменс.,
 700. € 100. Confer. fee. Доводимые о совм. работе с R. Gersen (Regensburg, C. Back)

о совм. работе - электрогенерация $W_{1,2}$ -Vortex в Fe/Si/Fe bilayer cylindr. dots.
 генеро оуем $W_{1,2}$ -TM с утратой оснана. Обсужден Убанову, что в PRL'08 о сам-
 санд $\Delta\omega_{\pm 1} = 4\omega_0$ где $\omega_{\pm 1}$ - частоты, $\Delta\omega_{\pm 1} \sim J^2 \omega_1^2$. А у него и ω_0 неуправляемая (exch.), а в PRL'08
 и $\omega_{\pm 1}$ с $\Delta\omega_{\pm 1} = \omega_0$ невелики, $\Delta\omega$ (уб.) $\ll \Delta\omega$ (Gus) $\times \Delta\omega$ (exp.), Убанов сказал, что

$H_c(H_2)$ и H_c^2 VC-reversal у него в PRB'02 рассмотрено, хотя и в XXZ exch. model.
 Убанов описал H_c^2 преобразование намагниченности (!) и утончение $(\sigma m)^4$ оснана (!)
 Слабым предположи Убанову ураине в APL - Хвалковича, где $v_c(H_2)$ него, а не H_c^2 .

Согласно осн. могут, работающие в обл. SW, но никак не обобщ. доклады-
 не было. Осн. доклад Kimmel по laser-induced SRT в FeF_2O_3 , где magnon mass
 базиса ($\vec{r}^2$ или в AFM) и Hovel, где TW рассматривает берущие об. DE базис
 с $n=0$ в более высоком. SW с $n=1, 2, \dots$. Заключены в замке куклоу-
 Сансонского/короче Писем и Сансонского Убанову. И И. приваивает в Delaware
 обещан в. Убанову рекомендовать его на семинар в Kyoto, к Т. Оно.

// H_c^2 vortex core reversal можно достаточно просто рассмотреть в гире
 Убана, добавляя H_2 к энергии вихря Убана и находя поле неабсолютно на пе-
 релет с $p=+1$ и $\frac{\partial W}{\partial b} = 0$, $\frac{\partial^2 W}{\partial b^2} = 0$, b - radius v. core. // $C v_c(H_2)$ Сунанга -
 него либо напору $L(b)$ с $H_2 \neq 0$ и нестат. устойчивость, либо вообще $\mu = \int d\vec{r} M_z$,
 $W(\mu)$ и напору в нпр $\mu=0$ (criterion of K reversal). См. 11/XI 2008 г. о $v_c(H_2)$.

8/viii После Dresden $\rightarrow$ Munich $\rightarrow$ Genoa $\rightarrow$ Proiz $\rightarrow$ Bilbao. Писем Yi Ji $\rightarrow$ уприн. в Delaware

9/viii Озвуч с оснана, саппе на Руго. Звемин в Киев Никитс напору.

10/viii E-mails to A. Lyberatos (EC vortex grant) и Jose Sanchez (Ikerb. appl.).

Виситин Rent fee 1,200 € 30 Aug. '09. Описан. Наг 2m. deposit, 1m. rent fee, 1000 €
 Рогун и аутр. Caja Madrid Credit Card. Описан. 1036.30 € 4th Oct. с у гурмана-те. Описан. 6th Oct. - 5th Oct.

(41/VIII) Aut. obozr. Bonanni et al. JMMM, Oct. '09 : F/N/F dots deg coexist na mono
 Ivanov et al. Arxiv : AFM vortices. No. **Vortex radius**

PRL vortex mass case na Review : 4 persegovana konvergencija 19. decembar, 2010. godine. **Vortex radius**

Pacner zabucunom VC radius $R_c(H_2)$ u casu $p = \pm 1$ skroz H_2
 Def. $R_c \equiv b$ [kay u Gov'94]. $W(b) = W_{ex}(b) + W_m(b) + W_z(b)$, b - core radius

$W_z = -L \int d^2s H_2 M_z(s)$. Guraev, 200 x equilibrium VC radius / profile

u pismenima mogemo Yobak'94 $M_z(s) = p M_s \frac{b^2 - s^2}{b^2 + s^2}$, $s \leq b$ u $M_z(s) = 0$, $s > b$.

no $b_{eq} = f(H_2)$. $W_z = -L H_2 p M_s \int d^2s \int d\phi \left(\frac{b^2 - s^2}{b^2 + s^2} \right) = -2\pi L p H_2 M_s \int_0^b ds s \left(\frac{b^2 - s^2}{b^2 + s^2} \right)$

$\langle m_z \rangle = \frac{1}{\pi b^2} \int_0^{2\pi} d\phi \int_0^b ds s \left(\frac{b^2 - s^2}{b^2 + s^2} \right) = \frac{2}{b^2} \int_0^b ds s \left(\frac{b^2 - s^2}{b^2 + s^2} \right)$, $b = b(H_2)$.

$W_z = \frac{1}{M_s^2 V} W_z = - \frac{2\pi L p H_2 M_s}{M_s^2 \pi R^2 L} \frac{b^2}{2} \langle m_z \rangle = -p h_z \frac{b^2}{R^2} \langle m_z \rangle$ - **mostovacki Zeeman en.**
 vortex core, $h_z = H_2 / M_s$.

$\langle m_z \rangle = 2 \int_0^1 dx x \left(\frac{1-x^2}{1+x^2} \right) = 2 \ln 2 - 1 \approx 0.386$

Ease core W_z line core : $W_z = -L H_2 p M_s \int d^2s \int d\phi m_z(s)$, $m_z(s) = \frac{H_2}{H_c}$

Ansatz $\int_0^R ds s = \frac{1}{2} s^2 \Big|_0^R = \frac{1}{2} (R^2 - b^2)$. $m_z(s)$ **hago on peregovis ne na osh.**
 $K m_z = 0$, a na osnove K **core-state**

$\cos \theta_0 = m_z^{out} = H_2 / H_c$.
 $m_z(s) = \left(\frac{b^2 - s^2}{b^2 + s^2} \right) + m_z^{out}$, $b \leq s$ u $m_z(s) = m_z^{out}$, $s > b$.

npisano. **chugrai VC $c = p = \pm 1$ u neke $H_2 < 0$, t.e. $p H_2 < 0$.**

$W_z = -L \int d^2s M_z(s) H_2 = -L M_s H_2 \cdot 2\pi \left[\int_0^R ds s m_z(s) \right]$, $W_z = \frac{W_z}{M_s^2 V} = -2 h_z \frac{1}{R^2} \int d^2s m_z(s)$

$\int_0^R ds s m_z(s) = \int_0^b ds s m_z^c(s) + \int_b^R ds s m_z^{out}(s) \neq \int d^2s \left[p \frac{b^2 - s^2}{b^2 + s^2} + m_z^{out} \right] + \int d^2s m_z^{out}$ - **her**

$m_z(0) = p$ - **def.** $m_z(s) = m_z^c(s) \left[1 - m_z^{out} \right] + m_z^{out}$, $m_z^c(s) = \cos \theta_0 = \frac{b^2 - s^2}{b^2 + s^2} \cdot p$

u. y. pismenima $\int_0^R ds s m_z(s) = \int_0^b ds s \left[\cos \theta_0(s) \left(1 - p m_z^{out} \right) + m_z^{out} \right] + \int_b^R ds s m_z^{out} =$

$= (1 - p m_z^{out}) \int_0^b ds s \cos \theta_0(s) + \int_0^R ds s m_z^{out}$, $\int_0^b ds s \cos \theta_0(s) = \int_0^b ds s \frac{b^2 - s^2}{b^2 + s^2} p = \frac{b^2}{2} \langle m_z \rangle_{core}$

$\langle m_z \rangle_{core} = (2 \ln 2 - 1) p$. $W_z = -h_z \frac{2}{R^2} \int d^2s m_z(s) = -h_z \frac{2}{R^2} \left[(1 - p m_z^{out}) \frac{b^2}{2} \langle m_z \rangle_{core} + \frac{R^2}{2} m_z^{out} \right]$

$= -h_z (1 - p m_z^{out}) \frac{b^2}{R^2} \langle m_z \rangle_{core} - h_z m_z^{out}$, $m_z^{out} = H_2 / H_c$. $H_c \approx 4\pi M_s$.

Zeeman energy density : $W_z(b) = -h_z (1 - p m_z^{out}) \frac{b^2}{R^2} \langle m_z \rangle_{core} - m_z^{out} h_z$, $\langle m_z \rangle_{core} = p(2 \ln 2 - 1)$
 $m_z^{out} = H_2 / H_c$

Bugno, **u. u. p. $p = \pm 1$, $h_z < 0$** $W_z = |a| b^2$ **caplennica ymenom VC radius b .**

$W_{ex} = A L \int d^2s \left[(\nabla \theta)^2 + \sin^2 \theta (\nabla \phi)^2 \right] = A L \cdot 2\pi \int ds s \left[\left(\frac{d\theta}{ds} \right)^2 + \frac{\sin^2 \theta}{s^2} \left(\frac{d\phi}{ds} \right)^2 \right]$ **u. u. y.**

radial symmetry. $\nabla \phi = \frac{\partial \phi}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial \phi}{\partial \varphi} \hat{\varphi} + \frac{\partial \phi}{\partial z} \hat{z}$, $\phi = \varphi + C\pi/2$, $\nabla \phi = \frac{1}{s} \hat{\varphi}$

$W_{ex} = A L \cdot 2\pi \int ds s \left[\left(\frac{d\theta}{ds} \right)^2 + \frac{\sin^2 \theta}{s^2} \right]$, **u. u. y. $\theta(s)$ **zagaue u. u. y. $m_z(s) = \cos \theta(s)$****

$m_z' = -\sin \theta \cdot \theta'$, $(\theta')^2 + \frac{1}{s^2} \sin^2 \theta = \frac{1}{\sin^2 \theta} (m_z')^2 + \frac{\sin^2 \theta}{s^2} = \frac{1}{(1 - m_z^2)} (m_z')^2 + \frac{(1 - m_z^2)}{s^2}$

One core $m_z(s) = m_z^{out}$, $m_z' = 0$, $\cos \theta_{out} = H_2 / H_c = m_z^{out}$

Wex (one core) $= 2\pi A L \int ds s \frac{1}{s^2} \sin^2 \theta_{out} = 2\pi A L \sin^2 \theta_{out} \ln(R/b)$

Brugaj u. core $m_z(s) = \lambda m_z^c(s) + m_z^{out}$, $\lambda = 1 - p m_z^{out}$, $m_z^c(s) = \left(\frac{b^2 - s^2}{b^2 + s^2} \right)$

Escaneado con CamScanner

напр. Na_2CO_3 . Zn^{2+} в р-не H_2SO_4 .

$$W'(\theta) = R_0^2 \sin^2 \theta_{out} \left(\frac{-1}{\theta} + 4\pi \theta \lambda^2 (3-4 \ln 2) \right), \quad W''(\theta) = R_0^2 \sin^2 \theta_{out} \left(\frac{1}{\theta^2} + 4\pi \lambda^2 (3-4 \ln 2) \right)$$

$$W''(\theta_{eq}) = R_0^2 \sin^2 \theta_{out} \frac{(3-4 \ln 2) 4\pi (1 - h_2/4\pi)}{(1 + h_2/4\pi)} + 4\pi \lambda^2 (3-4 \ln 2) =$$

$$= 4\pi (3-4 \ln 2) \left[\left(1 - \frac{h_2}{4\pi}\right)^2 \frac{R_0^2}{(1 + h_2/4\pi)} + \left(1 - \frac{h_2}{4\pi}\right)^2 \right] = 8\pi (3-4 \ln 2) \left(1 - \frac{h_2}{4\pi}\right)^2 \geq 0$$

T. e. pomeno $\theta_{eq}(H_2) = \theta(0) \left(\frac{4\pi + h_2}{4\pi - h_2} \right)^{1/2}$ y creciendo hincando go torcu
 $h_2 = -4\pi$, nge $\theta_{eq}(-4\pi) = 0$ u oho separel amon.
 $\theta_{eq}(h_2)$ y creciendo b odu. cyuereobovannu. $-4\pi \leq h_2 < 4\pi$ u separel y crei-
 mubore na ifannu $h_2 = 4\pi$ nepexoga vortex b SDI - phase.

Barbag: nomyrono pazgannoe y creiando pem. VC radius $\theta_{eq}(h_2)$
 do nrege $L/R \ll 1$ u n/eneopem. $W_x(v, core)$. Zrodi $\theta_{eq}(h_2)$ norepno y crei-
 boce u/nu $h_2 < 0$ ($p = +1$), nago. udo y creiando $W_m(\theta)$, udo y crece $W_x(core)$

(12/viii) Analyz $W_x(core)$. 14/8 or 2. obno pokazano, zo $W_x(core) \sim Q$ -
 - nomen. zafmeg soliton part of vortex (nonon. zafmeg v. core). $W_x(core) = 8\pi QAL$,
 b oduore nge $w(z) = f(z)$ - analytina. $f(z) = \frac{1}{2}z$ gne ognoro centered vortex ind.
 Dne vortex in H_2 : $m_2(p) = \lambda m_2^o(p) + m_2^{out}$. Com. 4/11 or 2. $G_2 \sim Q \sim \int d\phi d\theta d\Phi \sim$
 $\sim \cos\theta|_{\theta=0}^{p=R} \sim [\cos\theta(R) - p] = [\cos\theta_{out} - p] \sim [1 - \frac{H_2}{H_c}]$. noreny u

$$W_x(core) = 8\pi AL Q(H_2) = 4\pi AL \left[1 - \frac{H_2}{H_c}\right], \quad Q(H_2) = \frac{1}{2} \left(1 - \frac{H_2}{H_c}\right) \text{ Magnetostatics}$$

$$\overline{W}_x(core) = \frac{1}{M_s V} W_x(core) = 2 \left(\frac{R_0}{R}\right)^2 \left(1 - \frac{H_2}{H_c}\right) \sim \text{or v. core radius } b'' \text{ ne zaburur.}$$

Don. qygnkuro or b'' uonno nemyrno uz W_m (u.u. $(\sigma m)^4$ kan Ivanov's).

$$W_m(\theta) = - \int_0^{\theta} \int_0^{\theta} \int_0^{\theta} m_2(p) m_2(p') g_{zz}(p, p') = - \int_0^{\theta} \int_0^{\theta} m_2(p) \left[\int_0^{\theta} \int_0^{\theta} m_2(p') + \int_0^{\theta} \int_0^{\theta} m_2(p') \right] g(p, p')$$

$$= - \left[\int_0^{\theta} \int_0^{\theta} m_2(p) + \int_0^{\theta} \int_0^{\theta} m_2(p) \right] \left[\int_0^{\theta} \int_0^{\theta} m_2(p') + \int_0^{\theta} \int_0^{\theta} m_2(p') \right] g(p, p') = |g(p, p') = g(p', p)|$$

$$= \int_0^{\theta} \int_0^{\theta} \int_0^{\theta} + \int_0^{\theta} \int_0^{\theta} \int_0^{\theta} + \int_0^{\theta} \int_0^{\theta} \int_0^{\theta} + \int_0^{\theta} \int_0^{\theta} \int_0^{\theta}$$

$$u.u. \rightarrow \int_0^{\theta} \int_0^{\theta} m_2(p) f(p) = \int_0^{\theta} \int_0^{\theta} f(p) [\lambda m_2^o + m_2^{out}] + \int_0^{\theta} \int_0^{\theta} m_2^{out} = \lambda \int_0^{\theta} \int_0^{\theta} f(p) m_2^o(p) + \int_0^{\theta} \int_0^{\theta} m_2^{out}$$

$$= |f(p) = +p \int_0^{\theta} \int_0^{\theta} m_2(p') g(p, p')| = \lambda \int_0^{\theta} \int_0^{\theta} m_2^o(p) \int_0^{\theta} \int_0^{\theta} m_2(p') g(p, p') + m_2^{out} \int_0^{\theta} \int_0^{\theta} \int_0^{\theta} m_2(p') g(p, p')$$

$$= \lambda \int_0^{\theta} \int_0^{\theta} m_2^o(p) \left[\int_0^{\theta} \int_0^{\theta} g(p, p') \{ \lambda m_2^o(p') + m_2^{out} \} + \int_0^{\theta} \int_0^{\theta} g(p, p') m_2^{out} \right] + m_2^{out} \int_0^{\theta} \int_0^{\theta} \times$$

$$\times \left[\int_0^{\theta} \int_0^{\theta} g \{ \lambda m_2^o(p') + m_2^{out} \} + \int_0^{\theta} \int_0^{\theta} g m_2^{out} \right] = \lambda \int_0^{\theta} \int_0^{\theta} m_2^o(p) \left[\lambda \int_0^{\theta} \int_0^{\theta} g m_2^o(p') + \int_0^{\theta} \int_0^{\theta} g m_2^{out} \right]$$

$$+ m_2^{out} \int_0^{\theta} \int_0^{\theta} \lambda \int_0^{\theta} \int_0^{\theta} g m_2^o(p') + m_2^{out} \int_0^{\theta} \int_0^{\theta} \int_0^{\theta} g = \lambda^2 \int_0^{\theta} \int_0^{\theta} \int_0^{\theta} \int_0^{\theta} m_2^o(p) m_2^o(p') g(p, p') +$$

$$+ 2\lambda m_2^{out} \int_0^{\theta} \int_0^{\theta} m_2^o(p) \int_0^{\theta} \int_0^{\theta} g(p, p') + (m_2^{out})^2 \int_0^{\theta} \int_0^{\theta} \int_0^{\theta} \int_0^{\theta} g(p, p') \text{ or } b'' \text{ ne zaburur.}$$

$$W_m(\theta) = -\lambda^2 \int_0^{\theta} \int_0^{\theta} \int_0^{\theta} \int_0^{\theta} m_2^o(p) m_2^o(p') g(p, p') + (-1) 2\lambda m_2^{out} \int_0^{\theta} \int_0^{\theta} m_2^o(p) \int_0^{\theta} \int_0^{\theta} g(p, p') =$$

$$= +\lambda^2 W_m^{usov}(\theta) + (-1) 2\lambda m_2^{out} \int_0^{\theta} \int_0^{\theta} m_2^o(p) \int_0^{\theta} \int_0^{\theta} g(p, p'), \quad W_m^{usov}(\theta) \text{ uonno nemyr u/nu}$$

nemyr. gne $\frac{\partial W_m}{\partial \theta}$: $F(x) = \int_{a(x)}^{b(x)} f(x, t) dt$, $\frac{dF}{dx} = f[x, b(x)] b'(x) - f[x, a(x)] a'(x) + \int_{a(x)}^{b(x)} \frac{\partial f(x, t)}{\partial x} dx$
 $G(p) = \int_0^{\theta} \int_0^{\theta} \int_0^{\theta} g(p, p') = \int_0^{\theta} \int_0^{\theta} \frac{(4\pi)}{p} \int_0^{\theta} [1 - e^{-p^k}] J_0(kp) J_0(kp') = | \int_0^{\theta} \int_0^{\theta} J_0(kp) - 1 \int_0^{\theta} \int_0^{\theta} J_0(kp) =$

$$H_0/H_c = x, \quad R_0^2(1+x) = \frac{3\pi u(b_c^0)^3}{p} (1-x) + 2\langle m_2 \rangle (b_c^0)^2 (G(0) + h_c) x = 0$$
$$\left[R_0^2 + \frac{3\pi u(b_c^0)^3}{p} + 2\langle m_2 \rangle (b_c^0)^2 \right] x + R_0^2 - \frac{3\pi u(b_c^0)^3}{p} = 0$$
$$x = - \frac{\left[R_0^2 - \frac{3\pi u(b_c^0)^3}{p} \right]}{R_0^2 + \frac{3\pi u(b_c^0)^3}{p} + 2\langle m_2 \rangle (b_c^0)^2 (G(0) + h_c)}, \quad (H_0 = -H_c \quad (u=1))$$

В первом случае β - параметр Вандерваальса, γ - параметр Ланжевина

$$u = \frac{p R_0^2}{3\pi u(b_c^0)^3}, \quad b_c^0 = \frac{2}{15} \frac{p u}{a}, \quad u = u(a), \quad a = \left(\ln 2 - \frac{1}{2} \right)^2 = \frac{1}{4} \langle m_2 \rangle^2, \quad \langle m_2 \rangle = \frac{1}{4}$$
$$u(a) = a \ln 2, \quad h_c = H_c / H_0, \quad G(0) = -\frac{4\pi}{\beta} \int_0^\infty \frac{dt}{t} [1 - e^{-\beta t}] J_1(t), \quad H_c = 4\pi M_2 H_0$$
$$G(0) + h_c = -\frac{4\pi}{\beta} \int_0^\infty \frac{dt}{t} [1 - e^{-\beta t}] J_1(t) + 4\pi \left\{ 1 - \frac{\beta}{\pi} \ln \frac{8}{\beta} \right\} \approx O(\beta)$$

$$S(\beta) = \int_0^\infty \frac{dt}{t} [1 - e^{-\beta t}] J_1(t), \quad S'(\beta) = \int_0^\infty dt e^{-\beta t} J_1(t) = \frac{1}{\sqrt{\beta^2 + 1}} \left[\beta + \sqrt{\beta^2 + 1} \right]^{-1}$$

$$f(\beta) = \int \frac{dp}{\sqrt{\beta^2 + 1} [\beta + \sqrt{\beta^2 + 1}]} = \left| \beta = \sinh t, dp = \cosh t dt \right| = \int \frac{dt \cdot \cosh t}{\cosh t [\cosh t + \sinh t]} = \int e^{-t} dt = -e^{-t}$$
$$= (-1) e^{-\operatorname{arsh} \beta} = (-1) \frac{1}{\beta + \sqrt{\beta^2 + 1}} = (-1) \frac{1}{\beta + \sqrt{\beta^2 + 1}} + C - \text{формально верно}$$

$$f(0) = 0, \quad \text{а} \quad f(\beta) = C - \frac{1}{\beta + \sqrt{\beta^2 + 1}} \quad \text{так как} \quad f(0) = C - 1 = 0, \quad C = 1$$
$$f(\beta) = 1 - \frac{1}{\beta + \sqrt{\beta^2 + 1}} = \frac{\beta + \sqrt{\beta^2 + 1}}{\beta + \sqrt{\beta^2 + 1}} - 1; \quad f(\beta) \approx \frac{\beta + 1 + \frac{1}{2}\beta^2 - 1}{\beta + 1 + \frac{1}{2}\beta^2} \approx \beta \left(\frac{1 + \frac{\beta}{2}}{1 + \frac{\beta}{2}} \right)$$

$$G(0) + h_c = -\frac{4\pi}{\beta} f(\beta) + 4\pi \left\{ 1 - \frac{\beta}{\pi} \left(\ln \frac{8}{\beta} - \frac{1}{2} \right) \right\} \underset{\beta \ll 1}{=} -4\pi (1 - \beta/2) + 4\pi - 4\pi \left(\ln \frac{8}{\beta} - \frac{1}{2} \right)$$
$$= -4\pi \ln \frac{8}{\beta} + 2\pi\beta + 2\pi = 2(\pi + 1)\beta - 4\pi \ln \frac{8}{\beta} = 2\beta \left[\frac{\pi + 1}{2} - \ln \frac{8}{\beta} \right] < 0 \quad \text{так как}$$

поэтому $\frac{3\langle m_2 \rangle a}{27\pi u^2} [G(0) + h_c] = \frac{\beta}{27\pi} \frac{\langle m_2 \rangle^3}{u^2(0)} \cdot 4\pi \left[\frac{\pi + 1}{2} - \ln \frac{8}{\beta} \right]$ может быть не нуль
или $\beta \approx 0.1$

Для s -взаимодействия выполняется $|H_0|$ или константа β , но графа
с условием $\beta \ll 1$ $H_0 = -H_c \frac{(u-1)}{(u+1)}$

Для APL - хвостовый момент H_0 - сосредоточен в H_2
в H_2 , а также $b_{q_2}(H_2)$ - равновес. VC radius, убывающий с H_2

Наим. VC crit. velocity (миним. VC dyn. reversal) $v_c(H_2)$

Миним. reversal $v = v_c(H_2)$. $H_g = C v_c(H_2)$, $H_g + H_2 = a \delta m_2 = C v_c(0)$

Поэтому $v_c(H_2) = v_c(0) - \frac{1}{C} H_2$. $C < 0$, т.к. $H_g = H_g^2 < 0$.

$v_c(H_2) = v_c(0) + \frac{1}{|C|} H_2$, $H_2 < 0$ или $\rho = +1$ номинал v_c - разумно.

Разумно и $v_c(H_0) = 0$, H_0 - не крит. ρ -reversal в H_2 . $v_c(H_0) = v_c(0) + \frac{H_0}{|C|}$

$\frac{1}{|C|} = -\left(\frac{H_0}{v_c(0)} \right)^{-1}$, $H_0 < 0$ иначе. $v_c(H_2) = v_c(0) \left[1 - \frac{H_2}{H_0} \right]$, $H_2 \geq H_0$, $|H_2| \leq |H_0|$

Еще условие верно, но значение H_0 определяется $v_c(H_2)$. (при H_0 номинал)

VC radius $b_{q_2}(\delta H_2) \sim \left(\frac{1 + \delta H_2/H_0}{1 - H_2/H_0} \right)^n$, где n - порядок, W_m - параметр, $H_0 > 0$, $(n = v_2, v_3)$

$v_c(H_2) = v_c(0) \left[1 + \frac{H_2}{|H_0|} \right]$ и $b_{q_2}(H_2)$ оба убывают с увелич. $H_2 < 0$

и поэтому при определенных условиях H_0 близка к H_c (крит. нест. VC) (static crit. нест. VC)

First important exp. given to $M(t)$ by using H_0 and H_1 in the gamma T_0 $M(t)$ the vortex dots. The same T_0 shows reminder theory. The same new papers.

(18/VIII) Vortex in T_0 (B Ivanov) u APL-XBarron. paper text u Abstract. $v_c(H_2)$ - vortex

16/VIII Some important results for vortex reversal $H_g^2 + H_z = C v_c(0) \frac{H_z}{H_0}$ u no zero $v_c(H_2) = v_c(0) [1 + \frac{H_z}{H_0}]$ na obozre $v_c(H_2)$ to $v_c(0)$

H_0 - none current, VC reversal.

13 Gus' PRL'08 - gyrotropic properties of H_g : $H_g^2 \approx \frac{W}{r} \frac{|X|}{R_c} F(\frac{p}{a}) = \frac{v}{\delta R_c}$ known $(H_g^2) = \frac{v_c}{\delta R_c}$ $H_g^2 + H_z = \frac{1}{\delta} \frac{v_c(0)}{R_c(0)} = (H_g^2)_{out}(0)$. Exam $H_g^2 = \frac{v_c(H_2)}{\delta R_c(H_2)}$

10 $\frac{1}{\delta} \frac{v_c(H_2)}{R_c(H_2)} + H_z = \frac{1}{\delta} \frac{v_c(0)}{R_c(0)} \Rightarrow v_c(H_2) = v_c(0) \frac{R_c(H_2)}{R_c(0)} [1 - \frac{H_z}{H_{cr}}]$

$|H_{cr}| = \frac{1}{\delta} \frac{v_c(0)}{R_c(0)} \approx \frac{1}{\delta} \frac{\delta M_{slex}}{R_c(0)} = M_s \frac{L_{ex}}{R_c(0)} \approx M_s \approx 1$ kOe (≈ 0.5 kOe for NiMnSb).

Now $v_c(H_2) \sim R_c(H_2)$ (monogov. by simulations), ko $v_c(H_2) \rightarrow 0$ u $H_z = H_{cr} \approx 1$ kOe. 200 simulations u μ calculations, $\pm$ k $0 = v_c(H_2)$ u $H_z = -6.4$ kOe, H_{cr} depends on M_s (NiMnSb) ≈ 6.9 kOe u $M_s \approx 550$ G. Below.

13 Gus' PRL'08 - gyrotropic properties of H_g : $H_g^2 \approx \frac{W}{r} \frac{|X|}{R_c} F(\frac{p}{a}) = \frac{v}{\delta R_c}$ known $(H_g^2) = \frac{v_c}{\delta R_c}$ $H_g^2 + H_z = \frac{1}{\delta} \frac{v_c(0)}{R_c(0)} = (H_g^2)_{out}(0)$. Exam $H_g^2 = \frac{v_c(H_2)}{\delta R_c(H_2)}$

Now $v_c(H_2) \sim R_c(H_2)$ (monogov. by simulations), ko $v_c(H_2) \rightarrow 0$ u $H_z = H_{cr} \approx 1$ kOe. 200 simulations u μ calculations, $\pm$ k $0 = v_c(H_2)$ u $H_z = -6.4$ kOe, H_{cr} depends on M_s (NiMnSb) ≈ 6.9 kOe u $M_s \approx 550$ G. Below.

13 Gus' PRL'08 - gyrotropic properties of H_g : $H_g^2 \approx \frac{W}{r} \frac{|X|}{R_c} F(\frac{p}{a}) = \frac{v}{\delta R_c}$ known $(H_g^2) = \frac{v_c}{\delta R_c}$ $H_g^2 + H_z = \frac{1}{\delta} \frac{v_c(0)}{R_c(0)} = (H_g^2)_{out}(0)$. Exam $H_g^2 = \frac{v_c(H_2)}{\delta R_c(H_2)}$

Now $v_c(H_2) \sim R_c(H_2)$ (monogov. by simulations), ko $v_c(H_2) \rightarrow 0$ u $H_z = H_{cr} \approx 1$ kOe. 200 simulations u μ calculations, $\pm$ k $0 = v_c(H_2)$ u $H_z = -6.4$ kOe, H_{cr} depends on M_s (NiMnSb) ≈ 6.9 kOe u $M_s \approx 550$ G. Below.

Now $v_c(H_2) \sim R_c(H_2)$ (monogov. by simulations), ko $v_c(H_2) \rightarrow 0$ u $H_z = H_{cr} \approx 1$ kOe. 200 simulations u μ calculations, $\pm$ k $0 = v_c(H_2)$ u $H_z = -6.4$ kOe, H_{cr} depends on M_s (NiMnSb) ≈ 6.9 kOe u $M_s \approx 550$ G. Below.

Now $v_c(H_2) \sim R_c(H_2)$ (monogov. by simulations), ko $v_c(H_2) \rightarrow 0$ u $H_z = H_{cr} \approx 1$ kOe. 200 simulations u μ calculations, $\pm$ k $0 = v_c(H_2)$ u $H_z = -6.4$ kOe, H_{cr} depends on M_s (NiMnSb) ≈ 6.9 kOe u $M_s \approx 550$ G. Below.

Now $v_c(H_2) \sim R_c(H_2)$ (monogov. by simulations), ko $v_c(H_2) \rightarrow 0$ u $H_z = H_{cr} \approx 1$ kOe. 200 simulations u μ calculations, $\pm$ k $0 = v_c(H_2)$ u $H_z = -6.4$ kOe, H_{cr} depends on M_s (NiMnSb) ≈ 6.9 kOe u $M_s \approx 550$ G. Below.

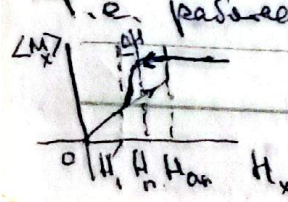
Now $v_c(H_2) \sim R_c(H_2)$ (monogov. by simulations), ko $v_c(H_2) \rightarrow 0$ u $H_z = H_{cr} \approx 1$ kOe. 200 simulations u μ calculations, $\pm$ k $0 = v_c(H_2)$ u $H_z = -6.4$ kOe, H_{cr} depends on M_s (NiMnSb) ≈ 6.9 kOe u $M_s \approx 550$ G. Below.

Now $v_c(H_2) \sim R_c(H_2)$ (monogov. by simulations), ko $v_c(H_2) \rightarrow 0$ u $H_z = H_{cr} \approx 1$ kOe. 200 simulations u μ calculations, $\pm$ k $0 = v_c(H_2)$ u $H_z = -6.4$ kOe, H_{cr} depends on M_s (NiMnSb) ≈ 6.9 kOe u $M_s \approx 550$ G. Below.

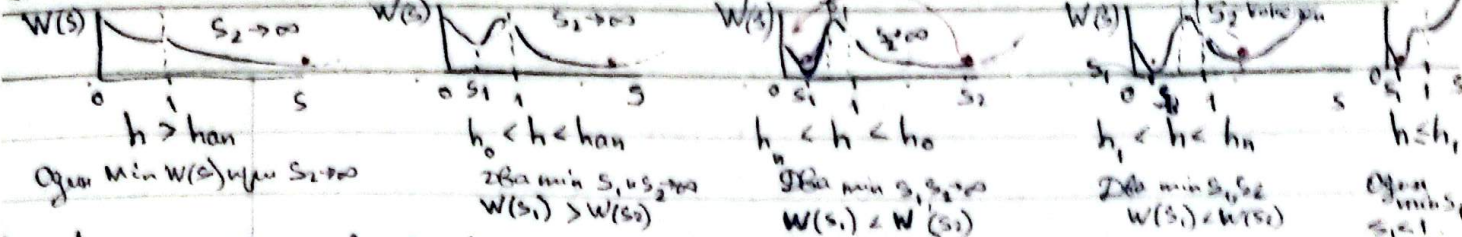
Now $v_c(H_2) \sim R_c(H_2)$ (monogov. by simulations), ko $v_c(H_2) \rightarrow 0$ u $H_z = H_{cr} \approx 1$ kOe. 200 simulations u μ calculations, $\pm$ k $0 = v_c(H_2)$ u $H_z = -6.4$ kOe, H_{cr} depends on M_s (NiMnSb) ≈ 6.9 kOe u $M_s \approx 550$ G. Below.

Now $v_c(H_2) \sim R_c(H_2)$ (monogov. by simulations), ko $v_c(H_2) \rightarrow 0$ u $H_z = H_{cr} \approx 1$ kOe. 200 simulations u μ calculations, $\pm$ k $0 = v_c(H_2)$ u $H_z = -6.4$ kOe, H_{cr} depends on M_s (NiMnSb) ≈ 6.9 kOe u $M_s \approx 550$ G. Below.

Now $v_c(H_2) \sim R_c(H_2)$ (monogov. by simulations), ko $v_c(H_2) \rightarrow 0$ u $H_z = H_{cr} \approx 1$ kOe. 200 simulations u μ calculations, $\pm$ k $0 = v_c(H_2)$ u $H_z = -6.4$ kOe, H_{cr} depends on M_s (NiMnSb) ≈ 6.9 kOe u $M_s \approx 550$ G. Below.



Experiment by Hyn-Kakara: give full cyl. dot vortex core & full vortex in ring, $H = H_n + 0$ u $H < H_{an}$ 2 of vortex cyrculobolama gbyx min $W(s)$, $s_1 < 1$ (complex shifted) u $s_2 > 1$ (normal SD state).



h_n, h_{an} - vortex nucleation/annih fields, $W_v(h_0) = W_{s_0}(h_0)$, h_0 - equil field u h_1 - none u $h \leq h_1$ cyrculobolama min agust, vortex min of $W(s)$ at $s_1 < 1$.

Due h_n, h_{an}, h_0 are analytic equations, cu. Gus' PRB'02. h_1 u $W_{s_0}''(h_1) = 0$ unepfau $H_1 - H_n$ gbocho yzok, $\langle M \rangle$ nagay ar $M_s(h)$ go $\langle M_g(V, H) \rangle \approx \frac{1}{2} M_s$.

H_n onf. vax none rge buepbue uabubalere min $W(s)$ u h konerua $s_2, \frac{1}{s_2} \rightarrow 0$ 22/III - 23/III 041. a onfegenn $\Delta E_{SD \rightarrow V} = W(s_0) - W(s \rightarrow \infty)$ u $h = 0$, cu. PRB'04.

$W(s)$ unep max u h konerua $s = s_0 < 1$, $s_0^2 = \frac{3}{2} - \frac{1}{2} \sqrt{1 + \frac{2}{\pi F_1(\beta)^2}}$, $s_0 \approx 0.9$ gya sumu. $\beta \approx 0.1$ u $\tau = (R/R_0) = 10 \div 20$. $\tau(R) = \tau_0 \exp[\Delta E(R)/kT]$ buepbue nagay $\tau \propto R^4$.

u $h \neq 0$ oznarar $s_1(h), s_0(h)$ u $s_2(h)$ ga over gya. $W_H(s)$

$W_H(s) = -h \Phi(s)$, $\Phi(s) = s \int_0^\infty dx [J_0(x)J_0(sx) - J_2(x)J_2(sx)] = \begin{cases} s(1 - \frac{1}{8}s^2 - \frac{1}{64}s^4), & s < 1 \\ 1 - \frac{1}{8}s^2, & s > 1 \end{cases}$

Bce 200 b pamyax "rigid" vortex model u $s < 1$ u $s > 1$. Due $s > 1$ cyrculobolama SD - long state, cu. Metlov 'PRB'04. Buepbue $W_{s_0}(\infty)$ u h buepbue u h nagay Metlov 'PRB'04.

Onfegenn $\Delta E_{SD \rightarrow V}(h) = W[s_0(h)] - W_{SD}(h)$, $W_{SD}(h) = 2\pi F_1(\beta) - h$, $h > h_n$.

$W[s_0(h)] = W[s_0^0 + \Delta s_0] = W(s_0^0) + W'(s_0^0) \Delta s_0 + \frac{1}{2} W''(s_0^0) (\Delta s_0)^2$

$W[s_0(h)] = W(s_0^0) + \frac{1}{2} W''(s_0^0) (\Delta s_0)^2$. Buepbue $\Delta s_0 = \alpha h$. $h = H/M_s \ll 1$.

Eam u h unep $W[s_0(h)] \approx W(s_0^0)$, to $\Delta E(h) \approx W(s_0^0) - 2\pi F_1 + h = \Delta E(0) + h$ u $h < h < h_{an}$.

$\tau = \tau_0 \exp[\Delta E/kT]$, $\Delta E \rightarrow \Delta E M_s^2 V$, t.k. ΔE - mernoch energy. $h_c(t) < 0$, t.e. uogayur gya SD ground state at $h=0$.

Nago naron $s_0(h)$ u $W[s_0(h)]$. $m(t) - m(\infty) = e^{-t/\tau} [m(0) - m(\infty)]$. $\tau(h) = \tau_0 \exp[\Delta E(h) \frac{M_s^2 V}{kT}]$, $h_n < h < h_0$. No anuany $m(\infty) = m_q(h) = m_v(h)$.

$m(0) = m_{SD}(h) \approx 1$. $m = \langle M \rangle / M_s$. $\ln[m - m(\infty)] = -\frac{t}{\tau} + C$. $h < h_0$

u $s < 1$: $W(s) = W_v(0) + \frac{1}{2\pi^2} \ln(1-s^2) + 2\pi F_1 s^2 (1 - \frac{1}{4}s^2) - h s (1 - \frac{1}{8}s^2)$.

Due buepbue nago $W(s_0) \approx W(s_0^0)$, $(s_0^0)^2 = \frac{3}{2} - \frac{1}{2} \sqrt{1 + \frac{2}{\pi F_1^2}}$. $\frac{\partial W}{\partial s} = -\frac{1}{\pi^2 (1-s^2)} + 4\pi F_1 s - 2\pi F_1 s^3 - h + \frac{3}{8} h s^2 = 0$. gya s_{min} u s_0 u $h \neq 0$.

u $\tau \gg 1$ $W'(s) = 4\pi F_1 s - 2\pi F_1 s^3 - h + \frac{3}{8} h s^2 = 0$, $W'(s) = (s-s_0)g(s)$. $m_{SD} = f h$, $f = 1/4\pi F_1$ - xopame u h u h nagay $h \leq h_{an}$, a nago $h = h_n$.

Нормы $\frac{1}{2} \ll 1$ упрощаются, т.к. можно считать $\exp$ и $\exp$ max $W(s)$ при $s < 1$. А если $\frac{1}{n}$ occurs и $W'(s) = (s-s_0)g(s)$, то $g(s)$ - константа или константа.

Нормы $\tau(h) = \tau_0 \exp \left[(AE(0) + h) \frac{M_s^2 V}{kT} \right]$ с параметрами τ_0 и $\tau(h)$

$\ln[m(t) - m(\infty)] = -\frac{t}{\tau(h)} + C$ (не зависит от t). $m(\infty) = m_v(h)$

$V = \pi R^2 L = \pi R^3 \rho = \pi R_0^3 \rho$. $R_0(\text{FeV}) = 20 \text{ nm} = 2 \cdot 10^{-6} \text{ cm}$. $kT = 4.14 \cdot 10^{-14} \text{ erg}$

Далее примем: $\tau = 10$, $\rho = 0.1$: $V = \pi (2 \cdot 10^{-6} \text{ cm})^3 0.1 = 8\pi \cdot 0.1 \cdot 10^{-15} \text{ cm}^3 = 2.4 \cdot 10^{-15} \text{ cm}^3$

$M_s(\text{FeV}) = 8 \cdot 10^2 \text{ G}$, $M_s^2 V = (8 \cdot 10^2 \text{ G})^2 \cdot 2.4 \cdot 10^{-15} \text{ cm}^3 = 2.4 \cdot 64 \cdot 10^{-11} \text{ erg} = 1.5 \cdot 10^{-9} \text{ erg}$

Тогда $\frac{M_s^2 V}{kT} = \frac{1.5 \cdot 10^{-9} \text{ erg}}{4.1 \cdot 10^{-14} \text{ erg}} = 0.37 \cdot 10^5 = 3.7 \cdot 10^4$. $\tau(h) = 10^{-10} \text{ s} \cdot \exp[1.37 \cdot 10^4]$

упрощая, $AE(0) \sim 0.1$, $h \approx H_n/M_s \approx 0.3$, $AE(0) + h \approx 0.5$ берем H_n .
 $e^{3.7 \cdot 10^4} = 10^{\frac{3.7 \cdot 10^4}{2.3}} \approx 10^{1.6 \cdot 10^4}$, $\log_{10}(\tau) = 1.5 \cdot 10^4$, $\tau = 10^{15000}$ - giant.

$\frac{AE(0)}{kT} \approx 0.1 \cdot \frac{M_s^2 V}{kT} \approx 3.7 \cdot 10^3$ при этом 10^2 раз больше, чем берем.

Можно считать $AE(h_1) \approx 0$ т.к. SD min $W(s)$ не равен, следовательно $AE(h)$ зависит от h и $h_1 < h < h_n$.

У нас же h_1 никак не берется. Тогда $AE(h) \sim (h-h_1)^n$

→ Можно найти $W(s)$ ($s > 1$): $W(s) = a_0 + \frac{1}{2} a_2 s^2 + \frac{1}{4} a_4 s^4 + \dots$, $W'(s) = a_2 s + a_4 s^3 = 0$, $s_1 = 0$ ($s_1 = \infty$) или $s_2^2 = -a_2/a_4$.

$W''(s) = a_2 + 3a_4 s^2$. $W''(s=0) = a_2$, $s_1 = 0$ min $W(s)$ при $a_2 > 0$.

$W''(s_2) = a_2 + 3a_4 \left(-\frac{a_2}{a_4}\right) = -2a_2$, s_2 min $W(s)$ при $a_2 < 0$, $s_2 = \sqrt{-\frac{a_2}{a_4}}$.

$a_2(h) = 0$ берем. h_n как в $G_{51} \text{ PRB'02}$.

Min $W(s) = W(s_2) = a_0 + \frac{1}{2} a_2 s_2^2 + \frac{1}{4} a_4 s_2^4 = a_0 + \frac{1}{2} a_2 \left(-\frac{a_2}{a_4}\right) + \frac{1}{4} a_4 \frac{a_2^2}{a_4^2} = a_0 - \frac{1}{4} \frac{a_2^2}{a_4}$.

т.е. $W(s_1) = a_0$, т.к. $a_4 > 0$. s_2 есть. $W(s_2) = a_0 - \frac{1}{4} \frac{a_2^2}{a_4}$, $W(s_2) < W(s_1)$.

Уменьшение W_{SD} до $W_{SD}(s_2)$ равно $\Delta E = W(s_0) - W_{SD}$.

Тогда $W[s_0(h)]$ можно считать h .

$\epsilon = s-1$ // $W(s)$ Тогда раскрываем по $(s-1)$, $s > 1$. Но тогда $W_{ex} \sim \ln(1-s^2)$ берем.

Но $s=1$ не абн. особая точка для $W_m(s)$ и $W_H(s)$. $I_1(s) = 2 \int_0^\pi d\varphi \frac{\sin \varphi \sin \varphi}{\sqrt{1+s^2-2s \cos \varphi}}$

$I_1(1) = 2 \int_0^\pi d\varphi \frac{2 \sin \frac{\varphi}{2} \cos \frac{\varphi}{2} \sin \varphi}{\sqrt{2} \sqrt{2} \sin \frac{\varphi}{2}} = 2 \int_0^\pi d\varphi \cos \frac{\varphi}{2} \sin \varphi$ - константа.

$W(s) = -\frac{1}{2\pi^2} \ln(1-s^2) + W'_n(s-1) + W''_n \frac{1}{2} (s-1)^2 = -\frac{1}{2\pi^2} \ln \epsilon + a_1 \epsilon + \frac{a_2}{2} \epsilon^2$, $\epsilon = s-1$, $\epsilon \ll 1$.

$W'(\epsilon) = -\frac{1}{2\pi^2} \frac{1}{\epsilon} + a_1 + a_2 \epsilon = 0$, $W''(\epsilon) = \frac{1}{2\pi^2} \frac{1}{\epsilon^2} + a_2$

$a_2 \epsilon^2 + a_1 \epsilon - \frac{1}{2\pi^2} = 0$, $\epsilon_{1,2} = \frac{1}{2a_2} [-a_1 \pm \sqrt{a_1^2 + 2\frac{a_2}{\pi^2}}]$

$W''(\epsilon) = 0$ берем. Если h_1 не берем, min $W_{SD}(s)$, $\frac{1}{2\pi^2} \frac{1}{\epsilon^2} + a_2 = 0$

Тогда $a_2 < 0$ при $h=h_1$. В этот момент $W'(\epsilon)=0$, $W''(\epsilon)=0$ $a_1 + 2a_2 \epsilon = 0$, $\epsilon = -\frac{a_1}{2a_2}$

ϵ_+ - есть. min $W(\epsilon)$, ϵ_- - max $W(\epsilon)$.

Min $W(\epsilon) = W(\epsilon_+) = -\frac{1}{2\pi^2} \ln \epsilon_+ + a_1 \epsilon_+ + \frac{1}{2} a_2 \epsilon_+^2 = -\frac{1}{2\pi^2} \ln \epsilon_+ + \frac{1}{2} a_1 \epsilon_+ + \frac{1}{2} \left[-\frac{1}{2\pi^2}\right]$

Еще можно найти $W[s_0(h)]$ при $s < 1$. Значит тогда: $W(s) = W_m(s) + W_H(s)$

(21/VII) Bug $G(x) = J(p, p)$ received 24/7 2001. ref. J. Math. Physics.

Bug $I(a(x)) (p, p) = \int_0^{\infty} dx x^a e^{-px} J_a(x) J_a(x) - \text{Ch. Conway, IEEE TM 37, 2927 (2001)}$
 ref. many more $G(p)$ w/ $p < 1$, where $J_0(x)$ is given in p.

9/8 2001. Bug many more $W(z) = W(0) + \frac{a_2}{z^2} + \frac{a_4}{z^4}$ "leaf" model. $f(z) = A/(z^2 z^2)$,
 published in Metell. PRL'04. No H_x - none can be done, zero y-axis.
 no more calculations for ΔE_{ex} barrier calculation.

Зборна Папка за Ово година заминава Univ. Barcelona во испански, она Scholar-
 ship позива нека парави уредити с. н. п. убоавување, prof. university;

3) Зборна. many за Master of Science about vortex and con. numbers
 ENU/MPV can Prof. of Spanish Univ. and Ikerbasque - need y-axis and negat
 application; 4) or Conf. 10-12 Sept. a conference name, no more signs for Sept.;
 5) however VNA-FMR на MMM'10, где Папка го изготви.

Гибридна Сибиринг и Хванковичеви по поводу $v_c(H_2)$ Digest MMM'10.

22-23/VII Напишем с. н. п. А. Поправка MMM'10 Digest "topol. field" и "VNA-FMR vortex".

24/VIII Submitted 2 Digests to MMM'10: "Topol. field" и "Vortex FMR" на осам
 Vortex/DW dynamics и Mag. dynamics. Сабин кон. одговор ≤ 9007 на диме BUS.

25/VIII C. A. b. "Liedl" и "Hyundai", "Suzuki". Багунсон с. н. п. А. E-mail at Ocean.

26/VIII C. A. b. "Opel", "Hyundai", "Nissan", "Peugeot", "Ford". У "Hyundai" her
 Account because a 130 her automat gear. Offer 17.000€ "Nissan" Note Benz. абонанс
 и "Ford" Focus Benz. абонанс 17.600€ (из којих ~1000€ можно скинува). E-mail at Xhan-
 ковичеви о H_c , $v_c(H_2)$ - vortex are p-reversal загара. Публикуи с. н. п. А. 5 ноември.
 however диме Bilbao → Network (DE) → Detroit. Папка cash. APL-Azin SW Submitted
 Offer "Opel" Corsa - aut. - € 13,000. On Aug. 24.

27/VIII E-mail at Klein, Sclay, who is Travel grant на мене is accepted
 и ме чекно гласао Јоуменс. C. n. p. A. на нелем. Пун. ем. NanoBME.

28/VIII Now to Klein CEA-appel. form. pero, кому 1 page of passport и visa.
 Доказо Spanish Resident Card и Mag. capability. Зборна Папка - C. n. p. b. Liedl.

29/VIII Answer e-mails Хванковичеви о vortex $v_c(H_2)$ - APL и other emy.

$v_c(H_2)$ Лафенман Gus'PRL'08: $\vec{L} = \int dV \vec{l}$, $\vec{l} = \vec{g} - \vec{w}$, $\vec{g} = \frac{M_s}{\gamma} \vec{n} \cdot (\vec{m} \times \dot{\vec{m}})$
 $\vec{m} = \vec{m}[\vec{\rho}, \vec{X}(t)]$, $\dot{\vec{m}} = \frac{\partial \vec{m}}{\partial \vec{X}} \cdot \dot{\vec{X}}$, $\vec{n} \cdot (\vec{m} \times \dot{\vec{m}}) = -\dot{\vec{m}} \cdot (\vec{m} \times \vec{n}) = \vec{m} \cdot (\dot{\vec{m}} \times \vec{n})$, $\dot{\vec{m}} = \frac{\partial \vec{m}}{\partial \vec{X}} \dot{\vec{X}}$
 $\frac{1}{2} \dot{\vec{m}} \cdot \vec{w} = \vec{w}_{ex} \cdot \vec{m} + \vec{w}_m \cdot \vec{m} + \vec{w}_2 = -\vec{h}_{ex} \cdot \vec{m} - \frac{1}{2} \vec{h}_m \cdot \vec{m} - \vec{h} \cdot \vec{m}$, $\vec{h}_{ex} = -A \nabla^2 \vec{m}$, $\vec{h}_m = \int dV' \hat{G}(\vec{r}, \vec{r}') \vec{m}(\vec{r}')$
 $\vec{l} = \frac{M_s}{\gamma} \vec{m} \cdot \frac{(\dot{\vec{m}} \times \vec{n})}{1 + \vec{m} \cdot \vec{n}} + \frac{M_s^2}{\gamma} \vec{m} \cdot [\vec{h}_{ex} + \vec{h} + \frac{1}{2} \vec{h}_m] = \frac{M_s^2}{\gamma} \vec{m} \cdot [\vec{h}_g + \vec{h}_{ex} + \vec{h} + \frac{1}{2} \vec{h}_m]$,
 где def. $\vec{h}_g = \frac{1}{\gamma M_s} \frac{(\dot{\vec{m}} \times \vec{n})}{1 + \vec{m} \cdot \vec{n}}$ - emergence or definition $\vec{h}_g$ б Gus'PRL'08.
 $\vec{l} = \frac{M_s^2}{\gamma} \vec{m} \cdot \vec{h}_{eff}$ because goes yd $\vec{m} \times \vec{h}_{eff} = 0$, $(\vec{m} \cdot \vec{n})$ remains only yd $\rightarrow$ LG.
 $\dot{\vec{m}} \times \vec{n} = \begin{vmatrix} \dot{m}_x & \dot{m}_y & \dot{m}_z \\ n_x & n_y & n_z \end{vmatrix} = \vec{i}(\dot{m}_y n_z - \dot{m}_z n_y) + \vec{j}(\dot{m}_z n_x - \dot{m}_x n_z) + \vec{k}(\dot{m}_x n_y - \dot{m}_y n_x)$
 Ем $\vec{n} = \hat{z}$, so $\dot{\vec{m}} \times \vec{n} = \vec{i} \dot{m}_y - \vec{j} \dot{m}_x$ и $\vec{h}_g$ не under $\vec{h}_g^z$ -
 commonenon, б emergence or $\vec{h}_g^z \neq 0$ Gus'PRL'08.

$\vec{L}_g = \int dV \vec{g} = \frac{M_s}{\gamma} \int dV \frac{1}{1 + \vec{m} \cdot \vec{n}} \vec{m} \cdot (\dot{\vec{m}} \times \vec{n}) = \vec{L}_g(\vec{X})$ Ум $\vec{g} = \frac{M_s}{\gamma} (1 - \cos \theta) \vec{\Phi}$, где
 $\vec{\Phi} = \frac{1}{R} m_0(\rho) [\sin \varphi \vec{X} - \cos \varphi \vec{Y}] = m_0(\frac{1}{R}) (\hat{\rho} \times \vec{X}) = \vec{\Phi}(\vec{\rho}, t)$, $\theta(\vec{\rho}, t) = \theta[\vec{\rho}, \vec{X}(t)]$, $m_0(\rho)$ - н. п. -
 $\vec{g} = -\frac{M_s}{\gamma} \cos \theta \vec{\Phi} = -\frac{M_s}{\gamma} \cos \theta m_0(\hat{\rho} \times \vec{X})_z$, $\vec{L}_g = \frac{M_s L}{\gamma R} \int d^2 \vec{\rho} \cos \theta m_0(\hat{\rho} \times \vec{X})_z = \frac{1}{2} \vec{G}(\vec{X} \times \vec{X})$

$L_g = M_2 \int d^2 \vec{s} m_2 (\hat{s} \times \dot{\vec{x}})$, where $\vec{h}_g \sim m_2 \vec{h}_2$, where $m_2 = 0.50 = 0.50 \times 10^{-26}$ kg
 $L_g = M_2 \int d^2 \vec{s} m_2 (\hat{s} \times \dot{\vec{x}}) \cdot \left[\cos \theta_0 - \sin(\theta_1) \sin \theta_0 \right] = \frac{M_2 \omega_0}{\delta} \int d^2 \vec{s} m_2 \sin \theta_0 \cdot \psi(\hat{s} \times \dot{\vec{x}})$
where $\vec{x} = \vec{s} \times \dot{\vec{x}}$, $\hat{s} \times \dot{\vec{x}} = \hat{s} \times (\omega \times \vec{x}) = \omega (\hat{s} \cdot \vec{x}) - \vec{x} (\hat{s} \cdot \omega) + |\vec{x}| \omega$, $(\hat{s} \times \dot{\vec{x}})_z = |\vec{x}| \omega_z$
 $L_g = \frac{M_2 \omega_0}{\delta} \int d^2 \vec{s} m_2 (\hat{s} \times \dot{\vec{x}})_z = \frac{M_2 \omega_0}{\delta} \int d^2 \vec{s} m_2 (\hat{s} \cdot \vec{x}) \omega_z$, $\hat{s} \cdot \vec{x} = |\vec{x}| \cos(\omega t - \varphi)$
where $L_g = \frac{M_2 \omega_0}{\delta} \int d^2 \vec{s} m_2 (\hat{s} \cdot \vec{x})$, i.e. m_2 - dynamic part of $m_2(\hat{s}, t)$, $m_2 = m_2^0 + \delta m_2$, $m_2^0 = \delta m_2$

$L_g = \frac{M_2 \omega_0}{\delta} \int d^2 \vec{s} m_2 \delta m_2 (\hat{s} \times \dot{\vec{x}})$
 $L = M_2^2 \frac{m_2}{m_1} \cdot \vec{h}_{eff} = M_2^2 (m_2^0 + \delta m_2) \cdot (\vec{h}_{eff}^0 + \delta \vec{h}_{eff})$, $\delta \vec{h}_{eff} = \vec{h}_g + \delta \vec{h}_{eff} + \frac{1}{2} \delta \vec{h}_{eff}$
 $\vec{h}_{eff} = \vec{h}_{eff}^0 + \frac{1}{2} \delta \vec{h}_{eff} + \vec{h}_g$ i.e. $\vec{h}_{eff} = \vec{h}_g (g)$, $\vec{h}_g (g)$ - vector $\vec{h}_g$ in g space
Page 3: $L = -W = m_2^0 \cdot \vec{h}_{eff}^0 + \delta m_2 \cdot \vec{h}_{eff}^0 + m_2^0 \delta \vec{h}_{eff} + \delta m_2 \delta \vec{h}_{eff}$, i.e. $\vec{h}_{eff} = \vec{h}_g$

Substituting the known values of $\vec{h}_g$ in the above substitution gives us
i.e. δm_2 - dynamic part of m_2 vector $\vec{h}_g$ in g space
The substitution of $\vec{h} = (0, 0, h_z)$ gives $\vec{h} = m_2^0 \cdot \vec{h}_{eff}^0$ - case $m_2 = 0$
 $(\hat{s} \times \dot{\vec{x}})_z = (\hat{s} \times \dot{\vec{x}})_z = \left| \frac{\vec{s} \times \dot{\vec{x}}}{|\vec{s} \times \dot{\vec{x}}|} \cdot \vec{s} \right| = (\hat{s} \times \dot{\vec{x}})_z \cdot \vec{s} = \vec{s} \cdot \vec{s} = 1$ (General case $\vec{s} \cdot \vec{s} = 1$)
 $L_g = \frac{M_2 \omega_0}{\delta} \int d^2 \vec{s} m_2 m_2 \cos(\omega t - \varphi) = L_g(0)$, where $\vec{s} = \hat{e} \cos(\omega t - \varphi)$, $|\vec{s}| = \omega_0 |\vec{x}|$
 $W_n = -L \int d^2 \vec{s} \vec{m} \cdot \vec{h} M_2^2$, $L = \frac{M_2 \omega_0}{\delta} L \int d^2 \vec{s} m_2 m_2 \cos(\omega t - \varphi) + M_2^2 L \int d^2 \vec{s} \vec{m} \cdot \vec{h} = W_{eff} - W_n$
 $= M_2^2 L \int d^2 \vec{s} m_2 \left[h_z + \frac{\vec{s}}{R \delta M_2} m_2 \cos(\omega t - \varphi) \right] = W_{eff} - W_n$

i.e. $\vec{h}_g$ is a vector $\vec{h}_g$ in g space, $\vec{h}_g = \vec{h}_g + \frac{\vec{s}}{\delta M_2} m_2 \cos(\omega t - \varphi)$
no absolute non-uniform variable $h_z(\hat{s}, t)$
 $m_2(\hat{s}) = \frac{1}{\delta} (1 - \rho^2) \rightarrow \frac{R}{\delta} [1 - (\rho/R)^2]$. Don't know ρ is $\rho \neq 0$: $h_z' = \frac{\vec{s}}{\delta M_2} m_2 \cos(\omega t - \varphi)$

Then $m_2(\hat{s}, t)$ dynamic. Conclude 10/10/07. Pole free model $\alpha(\hat{s})^2 = \rho^2 - 2(1+\rho^2)\hat{s} \cdot \hat{s} + \rho^2$
 $m_2(\hat{s}) = \frac{1 - \alpha(\hat{s})^2}{1 + \alpha(\hat{s})^2} = \frac{c^2 - \rho^2 + 2(1+\rho^2)\hat{s} \cdot \hat{s}}{c^2 + \rho^2 - 2(1+\rho^2)\hat{s} \cdot \hat{s}} = \left[\cos(2\theta/1007) \right] = m_2(\hat{s}, 0) + \frac{2c^2 \cdot 2(1+\rho^2)}{(c^2 + \rho^2)^2} (\hat{s} \cdot \hat{s})$

Note: 3-6/10/07, a general expression $\vec{h}_g \sim \sqrt{\rho} \vec{s}$, in a more rigid v. model
where $\vec{h}_g$ is a vector $\vec{h}_g$ in g space
 $\delta m_2 = \frac{4c^2(1+\rho^2)}{(c^2 + \rho^2)^2} (\hat{s} \cdot \hat{s}) = \frac{F(\rho/c)}{c} (\hat{s} \cdot \hat{s})$, $F(x) = \frac{4c^3 \rho(1+\rho^2)}{[c^2 + \rho^2]^2} = \frac{4x}{[1+x^2]^2} (1+\rho^2)$
6 pole free model, $x = \rho/c$. Rigid v. model case. $(1+\rho^2) \rightarrow 1$, c, ρ & eq R.

$\delta m_2 = \frac{F(x)}{R_c} (\hat{s} \cdot \hat{s})$, $(\hat{s} \cdot \hat{s}) \cos \varphi \sin \varphi$
 $\delta m_2(\hat{s} \times \dot{\vec{x}})_z = \delta m_2 \cdot \omega_0 (\hat{s} \cdot \vec{x}) = F(x) \frac{\omega_0}{R_c} (\hat{s} \cdot \vec{x})^2$ - two components $L \sim \int d^2 \vec{s} (\hat{s} \cdot \vec{x})^2$
 $(\hat{s} \cdot \vec{x})^2 = (X \cos \varphi + Y \sin \varphi)^2 = X^2 \cos^2 \varphi + Y^2 \sin^2 \varphi + 2XY \sin \varphi \cos \varphi$
i.e. $(\hat{s} \cdot \vec{x})^2 = (X_0 \cos \varphi \cos \omega t + X_0 \sin \varphi \sin \omega t)^2 = |\vec{x}|^2 \cos^2(\omega t - \varphi)$

3/10/07: $H_g^2 = \frac{\omega_0^2}{\delta} \frac{F(x)}{R_c} (\hat{s} \cdot \vec{x}) = \frac{1}{\delta} \frac{F(x)}{R_c} (\hat{s} \times \dot{\vec{x}})_z$ c.p. $L_g = \frac{M_2 L}{\delta} \int d^2 \vec{s} m_2 m_2 (\hat{s} \times \dot{\vec{x}})_z$
 $= M_2^2 L \int d^2 \vec{s} m_2 h_g^2$, $h_g^2 = \frac{1}{M_2} H_g^2 = \frac{1}{M_2} \frac{1}{R_c} m_2 (\hat{s} \times \dot{\vec{x}})_z$. Don't know $\vec{h}_g$
Conclude: $\vec{h}_g$ is a vector $\vec{h}_g$ in g space, $m_2(\hat{s}) = \frac{R}{R_c} F(x)$, $F(x) = \frac{4x}{[1+x^2]^2}$, $x > 1$
One case $m_2(\hat{s}) = R/R_c \cdot 1/(\rho/R) = \frac{R}{\rho}$ - no ρ is $\rho \neq 0$

$\delta m_2 = \frac{4c^2(1+\rho^2)}{(c^2 + \rho^2)^2} (\hat{s} \cdot \hat{s}) = \frac{F(\rho/c)}{c} (\hat{s} \cdot \hat{s})$, $F(x) = \frac{4c^3 \rho(1+\rho^2)}{[c^2 + \rho^2]^2} = \frac{4x}{[1+x^2]^2} (1+\rho^2)$
6 pole free model, $x = \rho/c$. Rigid v. model case. $(1+\rho^2) \rightarrow 1$, c, ρ & eq R.
 $\delta m_2 = \frac{F(x)}{R_c} (\hat{s} \cdot \hat{s})$, $(\hat{s} \cdot \hat{s}) \cos \varphi \sin \varphi$
 $\delta m_2(\hat{s} \times \dot{\vec{x}})_z = \delta m_2 \cdot \omega_0 (\hat{s} \cdot \vec{x}) = F(x) \frac{\omega_0}{R_c} (\hat{s} \cdot \vec{x})^2$ - two components $L \sim \int d^2 \vec{s} (\hat{s} \cdot \vec{x})^2$
 $(\hat{s} \cdot \vec{x})^2 = (X \cos \varphi + Y \sin \varphi)^2 = X^2 \cos^2 \varphi + Y^2 \sin^2 \varphi + 2XY \sin \varphi \cos \varphi$
i.e. $(\hat{s} \cdot \vec{x})^2 = (X_0 \cos \varphi \cos \omega t + X_0 \sin \varphi \sin \omega t)^2 = |\vec{x}|^2 \cos^2(\omega t - \varphi)$

3/10/07: $H_g^2 = \frac{\omega_0^2}{\delta} \frac{F(x)}{R_c} (\hat{s} \cdot \vec{x}) = \frac{1}{\delta} \frac{F(x)}{R_c} (\hat{s} \times \dot{\vec{x}})_z$ c.p. $L_g = \frac{M_2 L}{\delta} \int d^2 \vec{s} m_2 m_2 (\hat{s} \times \dot{\vec{x}})_z$
 $= M_2^2 L \int d^2 \vec{s} m_2 h_g^2$, $h_g^2 = \frac{1}{M_2} H_g^2 = \frac{1}{M_2} \frac{1}{R_c} m_2 (\hat{s} \times \dot{\vec{x}})_z$. Don't know $\vec{h}_g$
Conclude: $\vec{h}_g$ is a vector $\vec{h}_g$ in g space, $m_2(\hat{s}) = \frac{R}{R_c} F(x)$, $F(x) = \frac{4x}{[1+x^2]^2}$, $x > 1$
One case $m_2(\hat{s}) = R/R_c \cdot 1/(\rho/R) = \frac{R}{\rho}$ - no ρ is $\rho \neq 0$

Noting $\delta m_z = \frac{F(x)}{R_c} (\vec{X} \cdot \hat{p}) = \frac{1}{R_c} m_0(\rho) (\vec{X} \cdot \hat{p})$, где $m_0(\rho) = \frac{R}{R_c} F(\rho/R_c)$ because

где $\rho > R_c, \rho \leq R_c$, but $F(x)$ behaves as magnetic shifted vortex

$F(x) = \frac{4x}{(1+x^2)^2}$ - rigid v., $F(x) = \frac{4x}{(1+x^2)^2} (1+\rho^2)$ - pole-free model - "Bryson case".

$(\hat{p} \times \vec{X}) = \vec{\omega}(\hat{p} \times \vec{X})$ где $\vec{X} = \omega \times \vec{X}$ - obviously no vorticity. **Vortex self-mass**

$$\mathcal{L}_g = \frac{M_s L}{8R} \int d^2 \vec{\rho} m_0 \delta m_z (\hat{p} \times \vec{X}) \omega_0 = \frac{M_s L \omega_0}{8R} \int d^2 \vec{\rho} m_0 (\hat{p} \cdot \vec{X}) \frac{1}{R} m_0 (\vec{X} \cdot \hat{p}) = \frac{M_s L \omega_0}{8R^2} \int d^2 \vec{\rho} m_0^2 (\hat{p} \cdot \vec{X})^2$$

or more $\mathcal{L}_g = \frac{M_s \omega_0}{8R^2} L \int d^2 \vec{\rho} m_0^2(\rho) (\hat{p} \cdot \vec{X})^2$, $(\vec{X} \cdot \hat{p})^2 = |\vec{X}|^2 \cos^2(\omega_0 t - \varphi)$

$$\int_0^{2\pi} d\varphi \cos^2(\omega_0 t - \varphi) = \int_0^{2\pi} d\varphi \cos^2(\varphi - \varphi_0) = 1/2 \int_0^{2\pi} d\varphi \Rightarrow \mathcal{L}_g = \frac{2\pi M_s \omega_0 L}{8R^2} |\vec{X}|^2 \int d\rho \rho m_0^2(\rho)$$

$v = \omega_0 |\vec{X}|$, r.e. $\mathcal{L}_g = \frac{1}{2} \frac{M_s L \cdot 2\pi}{8R^2 \omega_0} v^2 \int d\rho \rho m_0^2(\rho)$, где $\int d\rho(\dots)$ no vortex core, где δm_z to

$$\mathcal{L}_g = \frac{1}{2} \hat{M} v^2, \text{ vortex mass } \hat{M} = \frac{M_s L \cdot 2\pi}{8R^2 \omega_0} \int_0^{R_c} d\rho \rho m_0^2(\rho), \quad m_0^2 = \frac{R^2}{R_c^2} F^2(\rho/R_c)$$

$$\hat{M} = \frac{M_s L \cdot 2\pi}{8R^2 \omega_0} \frac{R^2}{R_c^2} \int_0^{R_c} d\rho \rho F^2(\rho/R_c) = \frac{M_s L \cdot 2\pi}{8\omega_0} \int_0^1 dx x F^2(x), \text{ ecm } \int_0^1 dx x F^2(x) \approx 1,$$

so actually $\hat{M} \approx \frac{M_s L \cdot 2\pi}{8\omega_0}$, $\omega_0 \approx 0.18 \omega_{M\beta}$, $\hat{M} = \frac{M_s L}{8 \cdot 0.18 \cdot 4\pi \cdot M_s \beta} = \frac{L}{8^2 \cdot 2.4 \cdot \beta} \approx 4 \frac{L^2}{\beta^2}$ где $\beta = 0.1$, $M = \frac{G}{\omega_0}$

Note: uz-zh vort. - SW interaction a non-zero $M_v \approx \frac{3}{2} L/\beta^2$ - non-zero massy.
 $\hat{M}$ - no-zero energy because. Probably bce. $W = \frac{1}{2} \hat{M}(\vec{X})^2 + \frac{\pi}{2} \vec{X}^2 \rightarrow \omega_0^2 = \frac{\pi}{\hat{M}} = \frac{\omega_0 G}{\hat{M}} = \omega_0^2$

Remember v. sp. norm: $\mathcal{L}_g - W_H = M_s^2 L \int d^2 \vec{\rho} m_z [h_z + \frac{v}{M_s R} m_0 \cos(\omega_0 t - \varphi)] =$
 $= |m_z(\vec{\rho}, t) = m_z^0(\rho) + \delta m_z(\vec{\rho}, t)| = M_s^2 L \int d^2 \vec{\rho} m_z^0 h_z + M_s^2 L \int d^2 \vec{\rho} \frac{v}{M_s R} \delta m_z m_0 \cos(\omega_0 t - \varphi) =$
 $= M_s^2 L h_z \pi R_c^2 \langle m_z^0 \rangle_{R_c} + \frac{1}{2} \frac{M_s \omega_0 L}{8} |\vec{X}|^2 \int d\rho \rho m_0^2(\rho)$

B. Cart. picture ($v=0, |\vec{X}|=0$) same: $m_z(\rho) = [1 - \rho m_z^{\text{out}}] m_z^{(0)}(\rho) + m_z^{\text{out}}$ because core.

$$\mathcal{L} = M_s^2 L h_z R_c^2 \pi \langle m_z^0 \rangle + \frac{1}{2} \frac{M_s \omega_0 L}{8} \frac{R_c^2}{R^2} |\vec{X}|^2 \langle m_z^0 \rangle_{R_c} - W_{ex}[\vec{m}^{\text{st}}] - W_m[\vec{m}^{\text{st}}]$$

$$m_z = \lambda m_z^{(0)}(\rho) + m_z^{\text{out}} + \delta m_z = \cos \theta, \quad m_x + i m_y = \sin \theta e^{i\varphi}$$

Try guess a form. No mag. sign. $m_z(h_z)$ and $\delta m_z \sim |\vec{X}|$.

If $h_z \neq 0$ mag. $m_z(\rho) = \lambda m_z^{(0)}(\rho) + m_z^{\text{out}} + \delta m_z$, $\delta m_z = \frac{F(x)}{R_c} (\vec{X} \cdot \hat{p})$ comp. but, no $R_c(h_z)$.
 a no vorticity as above $m_z = m_z^0(\rho) + \delta m_z$.

$$\mathcal{L} = M_s^2 L h_z \pi R_c^2 [\lambda \langle m_z^{(0)} \rangle + m_z^{\text{out}}] + \frac{1}{2} \frac{M_s \omega_0 L}{8} \frac{R_c^2}{R^2} \langle m_z^0 \rangle_{R_c} |\vec{X}|^2 - W_{ex}[\vec{m}^{\text{st}}] - W_m[\vec{m}^{\text{st}}] - \frac{\alpha(0) \sin^2 \theta_{\text{out}}}{2} |\vec{X}|^2, \text{ где dyn. energy ex. m., yzhen bce core radius.}$$

Then $c \langle m_z^0 \rangle$ paben $\frac{1}{2} \vec{G} \cdot (\vec{X} \times \vec{X})$, где $\omega_0(H_z) = \alpha(0) \frac{\sin^2 \theta_{\text{out}}}{G(H_z)} = \alpha(0) [1 + \frac{H_z}{|H|}]$

$$\mathcal{L} = M_s^2 V_c h_z [\lambda \langle m_z^{(0)} \rangle + m_z^{\text{out}}] + \frac{1}{2} G_z (\vec{X} \times \vec{X})_z - \frac{1}{2} \alpha(0) \sin^2 \theta_{\text{out}} |\vec{X}|^2 - W_{ex}[\vec{m}^{\text{st}}] - W_m[\vec{m}^{\text{st}}]$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \vec{X}} - \frac{\partial \mathcal{L}}{\partial \vec{X}} = 0 \text{ где } \vec{X}(t) \text{ yd u } \omega_0(H_z), \quad \vec{m}_z^{\text{st}} = \lambda m_z^{(0)} + m_z^{\text{out}}, \quad m_z^{(0)} \text{ - bce core radius } R_c = b.$$

$$\mathcal{L} = \mathcal{L}_{st}(H_z) + \mathcal{L}_{dyn}(\vec{X}, H_z) - \text{можно ли так?}$$

$$\mathcal{L} = \mathcal{L}_{st}(b, H_z) + \mathcal{L}_{dyn}(\vec{X}, H_z), \text{ уравнения } \frac{\partial \mathcal{L}}{\partial b} = 0 \text{ и } \frac{\partial \mathcal{L}}{\partial \vec{X}} = 0 \text{ не выполняются}$$

Ного $\mathcal{L}_{dyn} = F(\vec{X}, b, H_z)$.

Note: масса биеме $\hat{M} = \frac{G}{\omega_0}$ - гравитация, где $\mathcal{L}_g = \frac{1}{2} \vec{G} \cdot (\vec{X} \times \vec{X}) \sim (\vec{X})^2$ не применим.
 с yzhen $\vec{X} = -\frac{1}{\omega} \vec{\omega} \times \vec{X}$. Проекция нет. $\text{Хоча В. Ивану и Выхин так считают, } \hat{M} = \frac{G^2}{2}$ 2012

содержит Vortex с гамильтоном $m^2(p^2)$ симметричные квантовые V

→ Hinweis: $I_g(v, b) \dots I_g = \frac{1}{2} \langle \cdot, \cdot \rangle_{X \times X}$ - Zusammenhang in $G \approx B^4$ & pole

$$F_c(z) = \frac{4c^2 s(1+s^2)}{(c^2+s^2)^2} = \left| \frac{s}{c} = x \right| = \frac{4x}{(1+x^2)^2} (1+c^2 x^2), \quad c = b/r.$$

$$-8 \int_0^1 dt \frac{(1+2e^{2t} + e^{4t})^2}{(1+e^{2t})^3} = 8 \int_0^1 dt [1 + 2e^{2t} + e^{4t}]^2 = 8 \int_0^1 dt [1 + 2e^{2t} + e^{4t}] = 8 \left[t + e^{2t} + \frac{e^{4t}}{4} \right]_0^1 = 8 \left[1 + e^2 + \frac{e^4}{4} - 1 \right] = 8 \left[e^2 + \frac{e^4}{4} \right]$$

$$= 8 \left[6 \cdot 2 + 2 \cdot 2 (1 \cdot 6 \cdot 2) + 2 \cdot 4 (2 \cdot 2 \cdot 2) + 2 \cdot 4 (1 \cdot 2 \cdot 2) \right] = 8 \left[12 + 24 + 16 + 16 \right] = 8 \cdot 68 = 544$$

$$f(x) = \frac{8}{(1+t)^4} \quad t = (1+x^2)^2 = 0(1) [t + 2x^2t^2 + x^4t^3] \quad f'(x) = -\frac{32}{(1+t)^5} \cdot 2xt$$

$$+ 2C^2 \left\{ -\frac{1}{(1+t)} + \frac{1}{(1+t)^2} - \frac{1}{2} \frac{1}{(1+t)^3} \right\} + C^4 \left\{ \ln(1+t) + \frac{3}{(1+t)} - \frac{3}{2} \frac{1}{(1+t)^2} + \frac{1}{2} \frac{1}{(1+t)^3} \right\} \Big|_0^1 =$$

$$-c^4 \left\{ 3 - \frac{3}{2} + \frac{1}{3} \right\} = 8 \left[\frac{1}{12} + \frac{1}{12} c^2 + c^4 \left(\ln 2 - \frac{2}{3} \right) \right] \rightarrow \text{Da, } W_p^2 m_0^2 \sim Y(c) \text{ (abhängig von } c)$$

Умножение булевой функц. Zofreda $Q(S) \sim G_2(S)$ $\approx 14/24$ ≈ 0.58 $\approx \frac{1}{2}$ -распредел.

$$Q(s) = \frac{(1-s^2)}{I(s)}, \quad I(s) = \frac{1}{s} \int d^2x (1-s^4)$$

$$c^2 |f|^2 = \rho^2 - 2\rho(1-\rho^2) \sin 2\varphi + s^2$$

Exam Question $|f|^2 = S^2/c^2 \rightarrow T(s) = g \pi \int_0^c (1-s^4/c^4) ds = \frac{g \pi}{5} (1 - \frac{s^5}{c^4})$

$$= \lim_{t \rightarrow \infty} \frac{(1 - e^{4t^2})}{(1+t)^2} = \left[-\frac{1}{(1+t)} - e^4 \left\{ (1+t) - 2 \ln(1+t) - \frac{1}{71} \right\} \right]_{t \rightarrow \infty} = \left[-\frac{1}{1+t} - e^4 (0 + 2 \ln 1 - \frac{1}{71}) \right] =$$

to include $Q(c)$ values, i.e. $T(c) \approx T(c) + \frac{Q(c)}{c}$ or $c = Q/P$.

$\lim_{\theta \rightarrow 0} Q(\theta) = \frac{1}{2} - 2C_4 \left(\frac{3}{4} - \ln 2 \right) < \frac{1}{2}$. $Q = \frac{1}{2} \cos \theta$. $G_2 = 2\pi \frac{M_2}{\delta}$.

1(5) nju vengerov s, buquno zabucut oc 2 s ≥ 0.5 , cu. 14/III 087.

VIII Scanned Spanish ID numbers to Klein. You were in n. m. as APL - to

→ Zoroastrian 3-7 (!) papers co Graham no track. Lethbridge has reverse. Page 10 = A.M.

Becega - Arantxa; hojyken ei ocaobureb zinnic. swann. mog (1)
 ašim moles naviš ne moncoy. Harash ei mo. Harash ei mo. B F/N/F structures

neg even, Klein-Fort & Garwin-Bach, Osleim choro APL-05 no F/N/F = P/A/mob
C₁C₂ = -1 y Klein C₁C₂ = +1 coza de xoxo a al - Buzer qy V.mob

$\rho_{112} = +1$ und -1 sind

ВН₂: $b_c = \frac{1}{2} b_c(0) \left[1 + \sqrt{1 + \frac{64 y \alpha_c}{9 \delta^2}} \right]$, $b_c(0) = \frac{2}{9} \frac{\delta}{y}$. Усл. $b_{eq} > b_c$ cond. $b_{eq} > b_c$ cond. $b_{eq} > b_c$ cond.

ВН₂ = 0 $b_{eq} u_2 - \frac{1}{6} R_0^2 + W'_m = 0$, $W'_m \approx \delta b^2$, $+\frac{1}{6} R_0^2 = \delta b^2$, $b_{eq} = \left(\frac{R_0^2}{6 \delta} \right)^{1/2}$
 $b_{eq} > b_c(0)$ газ $(R_0^2 / \delta)^{1/3} > \frac{3}{4} \frac{\delta}{y}$, $\frac{3 \delta}{4 y} = \frac{3}{4}$ $\frac{3 \cdot 4 \pi u(0)}{\beta^2}$
 $R_0^2 / \delta = \frac{R_0^2 \beta}{12 \pi u(0)} = \left(\frac{R_0}{R} \right)^2 \frac{\beta}{12 \pi u}$ $\frac{R_0^2}{R^2} \frac{\beta}{12 \pi u} = \left(\frac{9}{4} \right)^3 \frac{\beta^3}{u^3 \beta^2} = \frac{9}{4} \frac{\beta}{u(0) \beta} = \frac{9}{4} \frac{u(0)}{\langle m_2 \rangle^2}$

Две последние β : $\beta = \frac{12 \pi u}{\beta R^3}$, $\eta = 4 \pi \frac{\langle m_2 \rangle^2}{\beta^2} \frac{1}{R} u \cdot \frac{R_0^2}{\delta} = \frac{R_0^2}{R^2} \frac{\beta R^3}{12 \pi u} = \frac{R_0^2}{12 \pi u}$
 $b_{eq} = \frac{R_0^2 L}{12 \pi u(0)}$ - cond. c $u_{cond} \approx 94$. $\frac{3}{4} \frac{\delta}{y} = \frac{9}{4} \frac{u \beta}{\langle m_2 \rangle^2} R = \frac{9}{4} \frac{u(0) L}{\langle m_2 \rangle^2} = \frac{R_0^2}{12 \pi u}$
 $b_c(0) = 1.25 L$ - v_{fmsr} . VC radius u_{fmsr} cond. $u_{fmsr} H_2 = 0$.

ВН₂ $\neq 0$, $\alpha = -\frac{5}{2} \langle m_2 \rangle = -\frac{h_2}{h_c} [h_c + G(0)] \frac{\langle m_2 \rangle}{(1 - h_2/h_c)}$ $\neq 0$ $u_{fmsr} H_2 < 0$.
 $\text{sign}(\alpha) = \text{sign}(H_2)$.
Найдем H_0 : $-\frac{1}{6} R_0^2 + \delta b^2 - y b^3 + 2 \alpha b = 0$, $b = b_c(\alpha)$, $\alpha = f(H_2)$, $H_2 = H_0$.
Найдем $\alpha(H_0)$, H_0 газ u_{fmsr} на H_0 u_{fmsr} .
 $\rightarrow \alpha(h_2) = \langle m_2 \rangle [h_c + G(0)] \left[-\frac{h_2}{h_c} + 1 \right] \frac{1}{(1 - h_2/h_c)} = \langle m_2 \rangle [h_c + G] \left[1 - \frac{1}{(1 - h_2/h_c)} \right]$
параметр c $u_{fmsr} h_2 < 0$.

$W'(b) = -\frac{1}{6} R_0^2 + W'_m + 2 \alpha b = 0$ газ $b_{eq}(h_2)$. $-\frac{1}{6} R_0^2 + \delta b^2 - y b^3 + 2 \alpha b = 0$
 β u_{fmsr} . $u_{fmsr} \eta = 0$, $\delta b^3 + 2 \alpha b^2 - R_0^2 = 0$, $\alpha \sim h_2 = 0 \rightarrow b_{eq}^3 = R_0^2 / \delta$ u_{fmsr} .
 β $u_{fmsr} |h_2| \ll 1$. $\delta b^3 + 2 \alpha b^2 - R_0^2 = 0$, $b_{eq}^3 = \frac{R_0^2}{\delta}$. $b_{eq}^3 = \frac{1}{\delta} [R_0^2 - 2 \alpha b_{eq}^2]$
 $R_0^2 - 2 \alpha b_{eq}^2 = R_0^2 - 2 \alpha \frac{R_0^2}{\delta^{2/3}} = R_0^2 \left[1 - \frac{2 \alpha}{\delta^{2/3}} \right]$, $b_{eq}^3(h_2) = \frac{R_0^2}{\delta} \left[1 - \frac{2 \alpha}{\delta^{2/3}} \right]$
 $R_0^2 = R_0^2 \frac{\sin^2 \theta_{out}}{\lambda^2} = R_0^2 \left(\frac{1 + h_2/h_c}{1 - h_2/h_c} \right)$, $b_{eq}^3(h_2) = \frac{R_0^2}{\delta} \left(\frac{1 + h_2/h_c}{1 - h_2/h_c} \right) \left[1 - \frac{2 \alpha(h_2)}{(\delta R_0)^{2/3}} \right]$
Решение. VC radius $u_{fmsr} |H_2| \ll H_0$.

$\rightarrow \alpha(h_2) = -\frac{h_2}{h_c} \frac{\langle m_2 \rangle [h_c + G(0)]}{(1 - h_2/h_c)} < 0$ $u_{fmsr} h_2 < 0$. $\alpha(-h_c) = +\frac{1}{2} \langle m_2 \rangle [h_c + G(0)] < 0$
Если $|h_0| \ll 1$, то $\alpha_c = \alpha(h_0) < 1$, $b_c \approx b_c(0) \left[1 + \frac{16 y \alpha_c}{9 \delta^2} \right]$
Тогда c $u_{fmsr} \alpha = \frac{1}{4} \langle m_2 \rangle^2$ $u_{fmsr} u_{fmsr} (16/\sqrt{m})$: $H_0 = -H_c (1 - 1)$

$\frac{8 \langle m_2 \rangle^3}{27 \pi u^2(0)} [G(0) + h_c] = \frac{8 \langle m_2 \rangle^3}{27 \pi u^2(0)} \cdot 4 \beta \left[\frac{\pi+1}{2} - \ln \frac{8}{\beta} \right] = 0.789 \beta(\beta)$, $\beta(\beta) = 4 \beta \left[\frac{\pi+1}{2} - \ln \frac{8}{\beta} \right]$
 $\mu = 0.652 (R_0/L)^2$. $\left[\mu + 1 + (\dots) \right] = 0.652 \left(\frac{R_0}{L} \right)^2 + 1 - 0.789 \cdot 4 \beta \left[\ln \frac{8}{\beta} - \frac{1}{2} - \frac{\pi}{2} \right] > 0$
где $\mu = 0.1 - 0.2$, u u $L > R_0$, $\mu < 1$, $H_0 > 0$, VC reversal
Согласно $H_0 > 0$ $u_{fmsr} |H_0| \ll H_0$.
Уравнение $u_{fmsr} H_0$: $\alpha_c = \frac{1}{2 \beta_c} \left[\frac{R_0^2}{\delta} - \delta b_c^2 + y b_c^3 \right]$, $b_c = F(\alpha_c)$,
 $u_{fmsr} H_0$ b_c R_0 .

$\rightarrow$ Вывод: 1) Если $u_{fmsr} u_{fmsr} u_{fmsr}$, $L > 1$, то VC radius $u_{fmsr} u_{fmsr}$.
 b H_2 $u_{fmsr} u_{fmsr} u_{fmsr}$ $u_{fmsr} u_{fmsr} H_0$ - $u_{fmsr} u_{fmsr} u_{fmsr}$. Если $\beta \ll 1$, $u_{fmsr} u_{fmsr} b_c$, H_0
- $u_{fmsr} u_{fmsr}$. Если $\beta = 0.1 - 0.2$ $u_{fmsr} u_{fmsr}$ $u_{fmsr} u_{fmsr}$ $u_{fmsr} u_{fmsr}$, $u_{fmsr} u_{fmsr} \beta = 0.1 - 0.2$ $u_{fmsr} u_{fmsr}$
где $u_{fmsr} u_{fmsr} u_{fmsr}$ $u_{fmsr} u_{fmsr}$ $u_{fmsr} u_{fmsr}$.
2) Если $u_{fmsr} u_{fmsr} u_{fmsr} < 1$, то $u_{fmsr} u_{fmsr} u_{fmsr}$. $u(t) = u(0) - 0.114 \cdot t$, $t < 1$
где $u_{fmsr} u_{fmsr} u_{fmsr} b^2$ $u_{fmsr} u_{fmsr}$ $u_{fmsr} u_{fmsr}$ VC c b_c , H_0 .
Согласно $u_{fmsr} u_{fmsr} u(t) = u(0) - 0.114 \cdot t + u_2 t^2$, $u_2 > 0$? b_c H_0 $u_{fmsr} u_{fmsr}$

$$W_m(b) = \frac{4\pi}{\beta} b^3 [u(b) - \{u(0) + u_1 b + u_2 \frac{b^2}{\beta^2}\}] = \frac{4\pi}{\beta} [-u_1 \beta b^2 - u_2 \frac{\beta^2}{2}] = -4\pi [u_1 \beta^2 + \frac{u_2 \beta}{2}]$$

$$W_m(b) = p b^2 - q b; p, q > 0. \quad W(b) = R_0^2 \ln \frac{1}{b} + p b^2 - q b + d b^2. \quad p = -\frac{4\pi u_1}{\beta}, \quad q = 4\pi u_2 \beta.$$

$$W'(b) = -\frac{1}{b} R_0^2 + 2(p+d)b - q = 0, \quad W''(b) = -\frac{1}{b^2} R_0^2 + 2(p+d) = 0.$$

берем $b_c^2 = -\frac{1}{2(p+d)} R_0^2$, $p = -4\pi u_1 = 1.433$, $\max |d(h_z)| = \frac{1}{2} \langle m_z^2 \rangle |h_c + G(0)| < 0.3$
 $\beta = [0, 0.5]$
 $(p+d) > 0$ иначе b_c^2 не имеет смысла.

Оценим $\frac{64}{9} \frac{g k_{max}}{\delta^2} = \frac{64}{9} \frac{4\pi \langle m_z^2 \rangle}{\beta^2} \frac{\beta^2 |d_{max}|}{(12)^2 \pi^2 u^2(0)} = \frac{16 \langle m_z^2 \rangle}{81 \pi u^2(0)} |d_{max}| = 1.36 |d_{max}| < 0.4$
 $|d_{max}| = 0.28$ при $\beta = 0.3$. $d_{max}(h_z = -h_c)$, но так как $|h_0| < h_c$, то $d_c < |d_{max}|$.

→ Две экв. деловим $\frac{L}{r} = \frac{44 \text{ nm}}{130 \text{ nm}}$, $R_0 = 20.4 \text{ nm}$, $\mu = 0.652 (\frac{R_0}{L})^2 = 0.140$.

$$\frac{8 \langle m_z^2 \rangle}{27 \pi u^2(0)} [G(0) + h_c] = 0.749 \quad S(\beta = 44/130) = -1.166. \quad H = -H_0 \frac{(0.14 - 1)}{[0.14 + 1 - 1.17]}$$

$$H_0 = -H_c \frac{(-0.86)}{(-0.03)} \approx -29 H_c - \text{абная температура.}$$

Т.е. если $|d_c| < 1$ и барьер погуще, то не пастрает.

$$W(b) = F_1(b) - d F_2(b), \quad W'(b) = F_1' - d F_2' = 0, \quad d(b) = \frac{F_1'(b)}{F_2'(b)}, \quad W''(b) = 0 \Rightarrow d(b) = \frac{F_1''(b)}{F_2''(b)}$$

$$\frac{d d(b)}{db} = \frac{F_1'(b) F_2''(b) - F_1''(b) F_2'(b)}{[F_2'(b)]^2} = 0 \text{ on } b_c. \text{ Значит } d_c = d(b_c).$$

$$W(b) = R_0^2 \ln \frac{1}{b} + W_m(b) + d b^2. \quad F_1(b) = R_0^2 \ln \frac{1}{b} + W_m(b), \quad F_2(b) = -b^2$$

$$F_1' = -\frac{1}{b} R_0^2 + W_m', \quad F_1'' = -\frac{1}{b^2} R_0^2 + W_m'', \quad F_2' = -2b, \quad F_2'' = -2.$$

$$F_1' F_2'' - F_1'' F_2' = (-\frac{R_0^2}{b} + W_m')(-2) - (-\frac{R_0^2}{b^2} + W_m'')(-2)b = 0, \quad -2\frac{R_0^2}{b} + W_m' - 2W_m'' = 0$$

— это уравнение должно решаться, если да.

$$F_1(b) = R_0^2 \ln \frac{1}{b} + \frac{1}{3} \gamma b^3 - \frac{1}{4} \eta b^4$$

$$F_1'(b) = -\frac{R_0^2}{b} + \gamma b^2 - \eta b^3, \quad d(b) = -\frac{1}{2b} \left[-\frac{R_0^2}{b} + \gamma b^2 - \eta b^3 \right] = \frac{1}{2} \frac{R_0^2}{b^2} - \frac{\gamma}{2} b + \frac{\eta}{2} b^2$$

$$\gamma = \frac{12\pi u(0)}{\beta}, \quad \eta = 4\pi \frac{\langle m_z^2 \rangle}{\beta^2}. \quad 2 \frac{d d(b)}{db} = -\frac{2}{b^3} R_0^2 - \gamma + 2b\eta = 0 \text{ где}$$

абно b_c . Значит $d_c = d(b_c)$ и d_c не зависит от h_0 — удивительно.

$$2\eta b^4 - \gamma b^3 - 2R_0^2 = 0 \quad - \text{будем решать } b_c \text{ если умножим на } \eta \neq 0.$$

Введем куд. уравнение $8y^3 - 8(-2)R_0^2 y + (-2)R_0^2(-1)\gamma^2 = 0$

$$8y^3 + 16R_0^2 y + 2R_0^2 \gamma = 0, \quad D = R_0^4 \gamma^2 + (\frac{16}{3} R_0^2)^3 > 0 \quad - \text{одно действ. решение}$$

$$y_1 = \sqrt[3]{-R_0^2 \gamma + \sqrt{D}} + \sqrt[3]{-R_0^2 \gamma - \sqrt{D}}, \quad y^3 + 2R_0^2 y + \frac{1}{4} R_0^2 \gamma = 0 \quad - \text{формула}$$

$$D = (\frac{1}{3} R_0^2 \gamma)^2 + (\frac{2}{3} R_0^2)^3, \quad y_1 = \sqrt[3]{-\frac{1}{8} R_0^2 \gamma + \sqrt{D}} + \sqrt[3]{-\frac{1}{8} R_0^2 \gamma - \sqrt{D}}$$

$$b^4 - \frac{\gamma}{2\eta} b^3 - \frac{R_0^2}{\eta} = 0 \quad \text{Введем } y = \frac{b}{\eta} \quad 8y^3 - 8(-\frac{R_0^2}{\eta}) y + \frac{R_0^2 \gamma}{\eta} = 0$$

$$y^3 + \frac{R_0^2}{\eta} y + \frac{1}{32} \frac{R_0^2 \gamma^2}{\eta^3} = 0, \quad D = q^2 + p^3 = (\frac{1}{64} \frac{R_0^2 \gamma^2}{\eta^3})^2 + (\frac{1}{3} \frac{R_0^2}{\eta})^3$$

$$A = \pm \sqrt{8y_1 + (\frac{\gamma}{2\eta})^2}, \quad b^2 + (-\frac{\gamma}{2\eta} + A) \frac{b}{2} + y_1 (1 + \frac{\gamma}{2\eta A}) = 0$$

$$2b_c = -\frac{1}{2} (A - \frac{\gamma}{2\eta}) \pm \sqrt{\frac{1}{4} (A - \frac{\gamma}{2\eta})^2 - 4y_1 (1 + \frac{\gamma}{2\eta A})} \quad - \text{берем второе решение}$$

Упростим формулы W_m и W_2 паче, 11-12/VIII

(6/14) Плотность и энергия излучения электрона ($\theta \ll N_s^2 V$): $W_m = \frac{4\pi}{\beta} \int_0^\infty dk [1 - e^{-\beta k}] I^2(k)$

$I(k) = \int_0^1 ds s m_2(s) J_0(k s)$. $W_m = \frac{4\pi}{\beta} \int_0^\infty dk [1 - e^{-\beta k}] \int_0^1 ds s m_2(s) \int_0^1 ds' s' m_2(s') J_0(k s) J_0(k s') =$

$= \frac{4\pi}{\beta} \int_0^1 ds s m_2^2(s) \int_0^1 ds' s' J_0(k s) J_0(k s') = \frac{4\pi}{\beta} \int_0^1 ds s m_2^2(s) \int_0^1 ds' s' J_0(k s) J_0(k s') =$

Далее введем $m_2(s) = \lambda m_2^0(s) + m_2^{out}$, $\frac{1}{2} \delta(s, s')$. $I(k) = \int_0^1 ds s J_0(k s) [\lambda m_2^0 + m_2^{out}]$

$= \lambda \int_0^1 ds s J_0(k s) m_2^0(s) + m_2^{out} \int_0^1 ds s J_0(k s)$. $\int_0^1 ds s J_0(k s) = \frac{1}{k^2} \int_0^k dx x J_0(x) = \frac{1}{k^2} J_1(x) \Big|_0^k = \frac{1}{k^2} J_1(k)$

$I(k) = \lambda \int_0^1 ds s m_2^0(s) J_0(k s) + m_2^{out} \frac{1}{k^2} J_1(k)$

$W_m(b) = \frac{4\pi}{\beta} \int_0^\infty dk [1 - e^{-\beta k}] [\lambda^2 I_u^2(k) + 2\lambda m_2^{out} I_u(k) + (m_2^{out})^2 \frac{1}{k^2} J_1^2(k)]$, $I_u(k) = \int_0^1 ds s m_2^0(s) J_0(k s)$

$W_m(b) = \frac{4\pi}{\beta} \int_0^\infty dk [1 - e^{-\beta k}] [\lambda^2 I_u^2(k) + 2\lambda m_2^{out} I_u(k) + (m_2^{out})^2 \frac{1}{k^2} J_1^2(k)]$, $I_u(k) = \theta^2 \int_0^1 dx x \left(\frac{1-x^2}{1+x^2} \right) J_0(k \theta x)$

$W_m^{us} = \frac{4\pi}{\beta} \int_0^\infty dk [1 - e^{-\beta k}] \theta^4 \left[\int_0^1 dx x \left(\frac{1-x^2}{1+x^2} \right) J_0(k \theta x) \right]^2 = \left| \frac{\partial k}{\partial q} = \frac{1}{\theta} \right| = \frac{4\pi}{\beta} \theta^4 \int_0^\infty dq \left[1 - e^{-\beta \theta q} \right] I^2(q) =$

$= \frac{4\pi}{\beta} \theta^3 [u(0) - u(\beta/\theta)]$, $u(k) = \int_0^1 dx x \left(\frac{1-x^2}{1+x^2} \right) J_0(k x)$, $I(q) = \int_0^1 dx x \left(\frac{1-x^2}{1+x^2} \right) J_0(q x)$

$W_m(b) = \lambda^2 W_m^{us}(b) + 2\lambda m_2^{out} \frac{4\pi}{\beta} \int_0^\infty dk [1 - e^{-\beta k}] I_u(k) \frac{1}{k^2} J_1(k)$

$\frac{4\pi}{\beta} \int_0^\infty dk [1 - e^{-\beta k}] \frac{J_1(k)}{k^2} \int_0^1 ds s m_2^0(s) J_0(k s) = - \int_0^1 ds s m_2^0(s) G(s)$, $G(s) = -\frac{4\pi}{\beta} \int_0^\infty dk [1 - e^{-\beta k}] \frac{J_1(k)}{k^2} J_0(k s)$

$\mathcal{G}(s, s') = -\frac{4\pi}{\beta} \int_0^\infty dk [1 - e^{-\beta k}] J_0(k s) J_0(k s')$, $G(s) = \int_0^1 ds' s' \mathcal{G}(s, s')$

$\int_0^1 ds s m_2^0(s) G(s) = \frac{G^2}{2} \langle m_2^0 G \rangle \Rightarrow W_m(b) = \lambda^2 W_m^{us}(b) + 2\lambda m_2^{out} (-1) \frac{\theta^2}{2} \langle m_2^0 G \rangle$

То есть $W_m(b) \approx 3/12$. OK.

Далее b_c найдем из eq.: $2y b^4 - 8b^3 - 2\hat{R}_0^2 = 0$, $\theta^4 - \frac{8}{2y} b^3 - \frac{\hat{R}_0^2}{y} = 0$

$d(b) = \frac{1}{2} \left[\frac{\hat{R}_0^2}{b^2} - 8b + y b^2 \right]$. Введем $|h_0| + h_c$, $\hat{R}_0 \sim (1 + h_0/h_c)^{1/n} \rightarrow 0$, $b_c \approx \frac{8}{2y}$

$d_c = d(b_c) = \frac{1}{2} \left[-8 \frac{8}{2y} + y \left(\frac{8}{2y} \right)^2 \right] = -\frac{1}{8} \frac{8^2}{y} < 0 (!)$. $d_c = d(h_0) = -\frac{h_0}{h_c} \langle m_2^0 \rangle [h_c + G(0)]$

Def. $\delta = \langle m_2^0 \rangle [h_c + G(0)] (-1) \lambda$. $d_c = \frac{h_0}{h_c} \delta$, $h_0/h_c = x$, $d_c = x \frac{\delta}{(1-x)}$

$d_c(1-x) = x\delta$, $d_c = x(d_c + \delta)$, $\frac{h_0/h_c}{d_c + \delta} = \frac{d_c}{d_c + \delta}$ — обнуляем, получим $b_c \neq f(\hat{R}_0)$

Далее $x \rightarrow (-1)$, $h_0/h_c = \frac{1}{1 + \delta/d_c}$, $\delta/d_c = \frac{\delta(-8)\theta}{\delta^2}$, $\delta = -\langle m_2^0 \rangle \mathcal{G}(\beta) = -0.386 \cdot \mathcal{G}(\beta)$

$\frac{y}{\delta^2} = \frac{4\pi \langle m_2^0 \rangle^2 \beta^2}{\beta^2} \frac{\beta^2}{(4\pi)^2 u^2(0)} = \frac{1}{36\pi} \frac{\langle m_2^0 \rangle^2}{u^2(0)}$, $\frac{\delta}{d_c} = +8 \langle m_2^0 \rangle \mathcal{G}(\beta) \frac{1}{36\pi} \frac{\langle m_2^0 \rangle^2}{u^2} = \frac{2}{9\pi} \frac{\langle m_2^0 \rangle^3}{u^2} \mathcal{G}(\beta) =$

$d_c = -\frac{1}{8} \frac{\delta^2}{y} = -\frac{1}{8} \frac{36\pi \langle m_2^0 \rangle^2}{u^2} = -\frac{9\pi u^2}{2 \langle m_2^0 \rangle^2} = -0.653$. То есть $\mathcal{G}(\beta) = 0.592 \cdot \mathcal{G}(\beta)$

$\theta^4 - \frac{8}{2y} b^3 - \frac{\hat{R}_0^2}{y} = 0 \rightarrow$ Введем кр. eq.: $8y^3 - 8(-\frac{R_0^2}{y})y + (-\frac{R_0^2}{y})(-1) \frac{8^2}{4y^2} = 0$, $y^3 + \frac{R_0^2}{y} + \frac{1}{32} \frac{R_0^2 \delta^2}{y^3} = 0$

$p = \frac{1}{3} \frac{R_0^2}{y}$, $q = \frac{1}{64} \frac{R_0^2 \delta^2}{y^3}$, $y^3 + 3p y + 2q = 0$, $D = q^2 + p^3 = \left(\frac{1}{64} \frac{R_0^2 \delta^2}{y^3} \right)^2 + \left(\frac{1}{3} \frac{R_0^2}{y} \right)^3$

Получим, что y_1 и b_c : $b^2 + (A - \frac{\delta}{2y}) \frac{b}{2} + y_1 (1 - \frac{\delta}{2y}) = 0$, $A = \pm \sqrt{8y_1 + \delta^2/4y^2}$

$2b_c = -\frac{1}{2} (A - \frac{\delta}{2y}) \pm \sqrt{\frac{1}{4} (A - \frac{\delta}{2y})^2 - 4y_1 (1 - \frac{\delta}{2y})}$. Введем $y_1 \leq 0$, $(A - \frac{\delta}{2y}) \leq 0$.

Пусть $\hat{R}_0 \rightarrow 0$, $y_1 = 0$, $A = \pm \frac{\delta}{2y}$. $2b_c = -\frac{1}{2} (\pm \frac{\delta}{2y} - \frac{\delta}{2y}) \pm \frac{1}{2} (\pm \frac{\delta}{2y} - \frac{\delta}{2y}) = 0$ или $\frac{\delta}{y}$, берем.

$2b_c = +\frac{1}{2} (\frac{\delta}{2y} - A) \pm (\frac{\delta}{2y} - A) \sqrt{1 + 4 \frac{y_1}{A(\frac{\delta}{2y} - A)}} = \frac{1}{2} (\frac{\delta}{2y} - A) \left[1 \pm \sqrt{1 + \frac{16y_1}{A(\frac{\delta}{2y} - A)}} \right]$

Далее $b_c > 0$ надо, так как $A < 0$. Если $A > 0$, то "и" — отрицательна.

Пусть $\hat{R}_0^2 < 0.1$, можем считать $b_c \sim R_0$. Пусть $b_c = \epsilon R_0^4 \rightarrow \epsilon^4 R_0^4 - \frac{\delta}{2y} \epsilon^3 R_0^3 = \frac{R_0^2}{y}$

$\epsilon^3 = -\frac{R_0^2}{y} \frac{2y}{\delta} = -2 \frac{R_0^2}{\delta} < 0$ — не получается, т.к. $b_c > 0$. Пусть $R_0 \rightarrow 0$ — тогда $b_c = \delta/2y$.

$\lim_{h_0 \rightarrow 0, d_c \rightarrow 0, b_c = \frac{3}{4} \frac{\delta}{y}}$
 non-vanishing no q-ne dependence. $\theta^4 - \frac{\delta}{2y} \theta^3 - \frac{R_0^2}{y} = 0$ (Wikipedia)
 $\alpha = -\frac{3}{8} \left(\frac{\delta}{2y}\right)^2, \beta = \frac{1}{8} \left(\frac{\delta}{2y}\right)^3, \gamma = -\frac{3}{256} \left(\frac{\delta}{2y}\right)^4 - \frac{R_0^2}{y}, u = \sqrt{R}$
 $P = -\frac{1}{12} \frac{9}{64} \left(\frac{\delta}{2y}\right)^4 + \frac{3}{256} \left(\frac{\delta}{2y}\right)^4 + \frac{R_0^2}{y} = \frac{R_0^2}{y}, Q = -\frac{1}{108} \alpha^3 + \frac{1}{3} \alpha \gamma - \frac{1}{8} \beta^2, R = -\frac{Q}{2} \pm \sqrt{\frac{Q^2}{4} + \frac{P^3}{27}}$
 $y = -\frac{5}{6} \alpha + u - \frac{1}{3} \frac{P}{u}, W = \sqrt{\alpha + 2y}, b_c = -\frac{1}{4} \left(-\frac{\delta}{2y}\right) + \frac{1}{2} \left[\pm W \pm \sqrt{(2\alpha + 2y \pm \frac{2P}{W})} \right]$

$\pm$ - same sign, $\pm$ independent sign.

$(\delta/2y) = x, \alpha = -\frac{3}{8} x^2, \beta = \frac{1}{8} x^3, \gamma = -\frac{3}{256} x^4 - \frac{R_0^2}{y}, Q = +\frac{1}{108} \frac{3^3 x^6}{8^3} + \frac{1}{3} \left(-\frac{3}{8}\right) x^2 \left[-\frac{3}{256} x^4 - \frac{R_0^2}{y}\right] - \frac{1}{8} \frac{1}{64} x^6$
 $Q = \left(\frac{27}{108 \cdot 8^3} + \frac{3}{8 \cdot 256} - \frac{1}{8 \cdot 64}\right) x^6 + \frac{1}{8} x^2 \frac{R_0^2}{y} = \frac{1}{8} x^2 \frac{R_0^2}{y} - \frac{1}{8 \cdot 64} x^6$

$R = -\frac{1}{2} \frac{1}{8} x^2 \frac{R_0^2}{y} \pm \sqrt{\frac{1}{4} \frac{1}{64} x^4 \frac{R_0^4}{y^2} + \frac{1}{27} \frac{R_0^8}{y^3}} = \frac{1}{16} x^2 \frac{R_0^2}{y} \left[-1 \pm \sqrt{1 + \frac{4 \cdot 64 R_0^2}{27 y x^4}} \right]$

$b_c = \frac{\delta}{8y} \pm \frac{1}{2} W \pm \frac{1}{2} \sqrt{-(2\alpha + 2y \pm \frac{2P}{W})}$. $\lim_{R_0 \rightarrow 0, R=0, y = -\frac{5}{6} \left(-\frac{3}{8}\right) x^2}$
 $W = \frac{1}{2} x, u = 0$

$b_c = \frac{\delta}{8y} \pm \frac{1}{2} \sqrt{-\frac{3}{8} x^2 + 2 \frac{5}{6} \frac{3}{8} x^2} \pm \frac{1}{2} \sqrt{-\left(-\frac{9}{8} x^2 + 2 \frac{5}{6} \frac{3}{8} x^2 \pm 2 \frac{1}{8} \frac{x^3 \cdot 2}{x}\right)} = \frac{1}{4} x \pm \frac{x}{4} \pm \frac{1}{2} x \sqrt{\frac{4 \mp 4}{8}}$
 non-vanishing. Problem.

Если game замкнута в прав. координ. $b_c = f(\hat{R}_0)$ и неграбитель
 в $d(b) = \frac{1}{2} [\hat{R}_0^2/b^2 - \delta b + y b^2]$, то $d_c = d[\hat{R}_0, b_c(\hat{R}_0)] = F_1(h_0) = \frac{h_0 \delta}{h_c (1 - h_0/h_c)}$
 - если сносим уравнение на h_0 . т.е. или найти пропуск, или расчитать пропуск.

$\frac{\hat{R}_0^2}{b_c^2} = y b_c^2 - \frac{\delta}{2} b_c$ неграбитель. в $d(b_c) = \frac{1}{2} [\frac{\hat{R}_0^2}{b_c^2} - \delta b_c + y b_c^2] = \frac{1}{2} [2y b_c - \frac{3}{2} \delta] b_c$,
 должно быть $d(b_c) = 0, b_c = \frac{3}{4} \frac{\delta}{y}$, верно.

Тогда $h_0 \rightarrow h_c$ горение $b_c^0 = \frac{\delta}{2y}$, ищем $b_c = \frac{\delta}{2y} + \epsilon$:
 $(b_c^0)^4 + 4(b_c^0)^3 \epsilon - (b_c^0) [(b_c^0)^3 + 3(b_c^0)^2 \epsilon] - R_0^2/y = 0, \epsilon = \frac{\hat{R}_0^2}{y} \frac{1}{(b_c^0)^3}, b_c = \frac{\delta}{2y} + \frac{\hat{R}_0^2}{y} \left(\frac{2y}{\delta}\right)^3$
 $d(b_c) = \frac{1}{2} [2y b_c - \frac{3}{2} \delta] b_c = \frac{1}{2} [\delta + 2 \hat{R}_0^2 \left(\frac{2y}{\delta}\right)^3 - \frac{3}{2} \delta] \left[\frac{\delta}{2y} + \frac{\hat{R}_0^2}{y} \left(\frac{2y}{\delta}\right)^3\right] =$
 $= \left|\left(\frac{2y}{\delta}\right)^3 = x\right| = \frac{1}{2y} \left[-\frac{1}{2} \delta + 2 \hat{R}_0^2 x\right] \left[\frac{\delta}{2} + \hat{R}_0^2 x\right] = \frac{1}{2y} \left[-\frac{1}{4} \delta^2 + \frac{1}{2} \hat{R}_0^2 \delta x\right]$

$\hat{R}_0^2 = R_0^2 \frac{\sin^2 \theta}{x^2} = \frac{(1 + h_0/h_c) R_0^2}{(1 - h_0/h_c)}, \frac{1}{2y} \left[-\frac{1}{4} \delta^2 + \frac{\delta}{2} x R_0^2 \frac{1}{2} (1 + h_0/h_c)\right] = \frac{h_0 \delta}{h_c} \frac{\delta}{2}$

Тогда $h_0 \rightarrow -h_c, h_0/h_c = \frac{2}{5} \left(-\frac{1}{8}\right) \frac{\delta^2}{y} = -\frac{1}{4} \frac{\delta^2}{8y}, \frac{\delta^2}{y} = \frac{(2\pi)^2 u^2(0) \beta^2}{\beta^2 \frac{4\pi \langle m_z^2 \rangle^2}{36\pi} \frac{u^2(0)}{\langle m_z^2 \rangle^2}} = \frac{36\pi u^2(0)}{\langle m_z^2 \rangle^2}$
 $\delta = -\langle m_z^2 \rangle f(\beta), f(\beta) = 4\beta \left[\frac{\pi+1}{2} - \ln \frac{8}{\beta}\right]; h_0 \sim \frac{1}{f(\beta)} \approx \frac{R}{L}$ - асимптотическое
 поведение R и L - непереноси в пределе, не должно $h_0/h_c \approx -1$.

$d(b_c) = y b_c [b_c - b_c(0)] = \frac{h_0 \delta}{h_c (1 - h_0/h_c)}, b_c(h_2)$ убывает с $h_2 \downarrow, h_2 < 0$.

$b_c(-h_c) = \frac{\delta}{2y}, b_c(0) = \frac{3}{4} \frac{\delta}{y}$ - опорные т.

Должно из $d(b_c)$, что $h_0 = -h_c$ невозможно, т.е. в прав. замкн. $\delta \sim \beta$, левая $\neq f(\beta)$.

$h_0 = 0, d_c = 0$ возможно, но не универсально. Это просто газотиз. vortex $b(R, L)$.

2/18 С.П. на анализ норм и фобов в физике. С.П. "Liedl". С.П. на ЭКГ. Графический
 след в памяти. А. вычислим new mobility (79E, 15E/mes ≤ 300 min) и универс. формулу.
 $\hat{R}_0^2 = R_0^2 \frac{(1 + h/h_c)}{(1 - h/h_c)}, d(h) = \frac{x \delta}{(1-x)}, \hat{R}_0^2 = R_0^2 \frac{(1+x)}{(1-x)} = R_0^2 \left[1 + \frac{2x}{1-x}\right] = R_0^2 \left[1 + \frac{2d}{\delta}\right]$

или $d = \frac{\delta}{2} [\hat{R}_0^2 - R_0^2]$. Если забываемся от "h" можно замкн. в $d(h) = \frac{x \delta}{(1-x)}, x = \frac{h}{h_c}$.

$$\frac{h_c}{h_0} = 1 - \delta \frac{F_2'(b_c)}{F_1'(b_c)} = 1 + \frac{\delta}{d_c} = 1 + \delta \left(-\frac{2}{\delta}\right) \frac{[1 - (3/16)\delta^2 \delta^2 / R_0^2]}{[1 - (27/128)\delta^4 / R_0^2]} \approx (\delta \beta) = +1 - 2 \cdot [\dots] \cdot [\dots]^{-1}$$

$$\frac{h_c}{h_0} \approx 1 - 2 \left[1 + \frac{27}{128} \frac{\delta^4}{y^3 R_0^2} \right] = -1 - \frac{27}{128} \frac{\delta^4}{y^3 R_0^2}, \quad \frac{h_0}{h_c} = -1 + \frac{27}{128} \frac{\delta^4}{y^3 R_0^2}, \quad \beta \ll 1.$$

$$d_c = \frac{3}{4} \frac{\delta}{y} \sim \beta. \quad h_0/h_c = -1 + \frac{1}{2} \frac{b_c^3}{R_0^2} \delta. \quad b_c = b_c(\beta) = \frac{3}{4} \frac{\delta}{y} = 1.25\beta. \quad \gamma = \frac{12\pi u(0)}{\beta}$$

$$h_0/h_c = -1 + \frac{1}{2} (1.25)^3 \beta^3 \frac{12\pi u(0)}{\beta R_0^2} = -1 + 6\pi u(0) (1.25)^3 \frac{\beta^2}{R_0^2} = (R_0 \delta \gamma R) = -1 + 6\pi u(1.25) \left(\frac{L}{R_0}\right)^2$$

$$u(0) = 0.083. \quad h_0(L)/h_c = -1 + 3.05 (L/R_0)^2, \quad \text{de laub'09: } \frac{L}{R_0} = \frac{44}{20.4} = 2.16, \quad y \text{ хвалек.}$$

$L/R_0 = 0.78 \sim 1.17$. Так что нехватка к $h_0/h_c = -1$ оц. оцено доминирует. Тогда $h_0(L)$ уменьшается при $L/R_0 \ll 1$ (но тогда $w_m^{us}(\delta)$ надо брать иначе).

$$u/v \text{ было нормировано } (\beta \ll 1) \quad h_0/h_c = -\frac{(u-1)}{(u+1)}, \quad u = 0.652 \left(\frac{R_0}{L}\right)^2. \quad \frac{u-1}{u+1} \approx \frac{1-u^{-1}}{1+u^{-1}} \approx (1-u^{-1})^2$$

$$h_0/h_c = -(1 - 2u^{-1}) = -1 + 2u^{-1} = -1 + 2 \frac{L}{0.652 R_0} = -1 + 3.07 (L/R_0)^2 \quad \text{— тоже самое при } \beta \ll 1.$$

т.е. раскр. u/v берен где $h_0(\beta \ll 1)$ и соотв. с 7/15 группировкой.

$$\text{Согл. упрощение. при } \beta \ll 1: b_c = \frac{3}{4} \frac{\delta}{y} + \frac{1}{2} \sqrt{\left(\frac{3\delta}{y}\right)^2 - \frac{2\delta}{y}}, \quad h_0/h_c = \Phi(b_c).$$

$$\frac{h_c}{h_0} = \frac{1}{\Phi(b_c)} = 1 + \frac{\delta}{d_c} = 1 - \delta \frac{[-R_0^2/b_c + \delta b_c^2 - \gamma b_c^3]}{[-\frac{2}{\delta} R_0^2 + 2b_c]} = 1 - 2 \frac{[R_0^2 - b_c^2/\delta]}{[R_0^2 - \delta b_c^3 + \gamma b_c^4]}$$

$$\rightarrow \frac{h_c}{h_0} = 1 - 2 \frac{[1 - \delta b_c^2/R_0^2] R_0^2}{[R_0^2 - \delta b_c^3 + \gamma b_c^4]} \quad \text{— без упрощения, в рамках модели.}$$

Менее точно раскр. min $w_{eff}(\beta) = R_0^2 \ln \frac{1}{\beta} + w_m^{us}(\beta) + d \left[\frac{2}{\delta} R_0^2 \ln \frac{1}{\beta} + \beta^2 \right]$ как минимум $d = \frac{h_c}{h_0} \frac{\delta}{(1 - h_c/h_c)}$, т.е. $b_{eq}(h_c)$ и найдем $b_c = b_{eq}(h_0) \sim h_0$, где min w_{eff} пропущен.

$h/h_c = \Phi(\beta)$ менее точно (где $\beta = \text{const}$, $h_c = \text{const}$)

(то $b_{eq}(h)$. $w_{eff}(\beta) = R_0^2 \left(1 + \frac{2d}{\delta}\right) \ln \frac{1}{\beta} + w_m^{us}(\beta) + d\beta^2$

$$w'_{eff}(\beta) = \hat{R}_0^2 \left(-\frac{1}{\beta}\right) + w'_m(\beta) + 2d\beta, \quad w''_{eff} = \frac{\hat{R}_0^2}{\beta^2} + w''_m + 2d$$

Можно дать раскр. $b_{eq}(h)$ и b_c при $\beta < 0.1$. При $\beta > 0.1$, $\frac{h_0}{h_c} < -1$.

$$h_0(\beta=0.1) = -1.00; \quad h_0(\beta=0.2) = -1.02 h_c; \quad h_0(\beta=0.3) = -1.07 h_c$$

$$b_c = 0.008; \quad b_c = 0.024; \quad b_c = 0.043$$

Упрощенно считая $w_m^{us}(\beta)$ при $\beta/\delta > 1$ — это верно. $w_{ex}(\text{core})$ — neglected, т.к. не зависит от "B".

Упрощенно считая $\text{cost}_{out} = H_z/h_c$ — разумно, где $R \gg b_c$ at best.

Ansatz $m_z(\beta) = \lambda m_z^0(\beta) + m_z^{\text{out}}$ — тоже разумно. Декарт $b_{eq}(h)$ при $\beta > 0.1$

неправильно упрощ. R_0 . $R_0/R = 0.2$ где $h_0 \approx -h_c$.

$b_c(\beta)$ увеличивается иначе где $\beta = 0.1 - 0.4 \Rightarrow b_c \sim L$ и zero core radius at R .

Может $H_c(\beta) = 4\pi M_s M_{zz}(\beta)$ — ноже. упрощенно где vortex $M_z(H_z)$? $H_c(\beta)$ is beyond the current model of $\mathcal{K}$ reversal in H_z . $H_c(\beta) = 4\pi M_s M_{zz}$ — замечено. замечено.

У Хвалюкова $\beta = 0.13$ и 0.2 — здесь как раз h_0/h_c зависит от β — слабо.

Для сравн. с экон. de laubens и симул. Хвалюкова упрощенно можно брать $H_c(\beta)$ из экон./симуляции, либо строить эмпирично, можно H_c — vertex saturation in perp. field H_z .

Надо учесть $w(b, z) \sim \frac{\partial}{\partial z}$, $z = \text{cost}_{out}$, см. 13/15

(9/12) C.A. Gurevich. Begeira e Secoga e Arantia: 1) Ona 8-10 m...
 haer 5 uobara - uobara mu Fax e goxog. 100, 200 700 800 e relocation
 Secul - San SB; 2) kyo Atary. 6 SS upv/fm re znac, no nek goymenoi (Zafe-
 pator no uorne); 3) Ib. (amc) 6 Antigo, Eita agency is reserved, li no boner cu
 1) Spanish - Dore grant no nogo muratur gon. goyha-ob, 200000 gonfoc chog
 e Portugal part, uobara yone uobara; 5) kopyra cu to simulate double-
 u double - SD state 6 f/n/r cinc dots. C.A. 6 uaxura - comp.

(10/12) Pactor $H_c(\beta)$ que vortex 6 H_2 -near (vortex saturation field).

$$\frac{1}{2} W(b) = \hat{R}_0^2 \ln \frac{1}{b} + W_m^{us}(b) + d b^2, \quad d = -\frac{1}{\lambda} [m_z^{out} \langle m_z^2 G \rangle + h_z \langle m_z^2 \rangle]$$

$$\lambda = 1 - p m_z^{out}, \quad m_z^{out} = h/h_c, \quad \hat{R}_0^2 = R_0^2 \frac{1}{\lambda^2} \sin^2 \theta_{out}, \quad d \approx -\frac{h_z}{h_c} \frac{[G(0) + h_c] \langle m_z^2 \rangle}{(1 - h_z/h_c)}$$

Due (C) vortex saturation curvatur $H_2 > 0, p = +1$.

Buguno h_c tauru nago oupegluor u y gubau min $W(b, h_z)$ u uore, u
 crastuonem pauru $m_z^{out} = \cos \theta_{out} = h_z/h_c$. $b_{eq}(h_z) = b_{eq}(0) \left[\frac{1 + h_z/h_c}{1 - h_z/h_c} \right]^{1/2}$
 $b_{eq}(h_z \rightarrow h_c) \rightarrow \infty, h_z > 0$, u pexog vortex $\rightarrow$ 1 SD uore obor
 o ucan kak y bauru VC radius $b_{eq}(h_z)$ go R. Ho h_c - uaur becom.

$$W_m^{us}(b) = \frac{4\pi b^3}{\beta} [u(0) - u(\beta/b)]. \quad \text{Bauru } h_c, h_z < h_c, b_{eq} \approx R \quad u \left(\frac{\beta}{b} = \frac{L}{R} \ll 1 \right).$$

$$u(0) - u(t) = u(0) - [u(0) - g_1 t] = g_1 t = g_1 \frac{\beta}{b}$$

$$W_{eff}(b) = \hat{R}_0^2 \ln \frac{1}{b} + 4\pi g_1 b^2 + d b^2, \quad d = -\frac{\langle m_z \rangle}{(1 - \cos \theta_0)} [h_z G(0)/h_c + h_z]$$

$$d = -\frac{\langle m_z \rangle}{(1 - \cos \theta_0)} [\cos \theta_0 G(0) + h_z], \quad \theta_0 = \theta_{out}$$

$$W(b, \theta_0) = \hat{R}_0^2 \sin^2 \theta_0 \ln \frac{1}{b} + (1 - \cos \theta_0)^2 4\pi g_1 b^2 - \langle m_z \rangle (1 - \cos \theta_0) [\cos \theta_0 G(0) + h_z] b^2$$

Vago be pauru k uauru $W_z(h_z) = -h_z \lambda b^2 \langle m_z \rangle - m_z^{out} h_z$, cu. 11/111

$$W(b, \theta_0) = \hat{R}_0^2 \sin^2 \theta_0 \ln \frac{1}{b} + (1 - \cos \theta_0)^2 W_m^{us}(b) - \langle m_z \rangle (1 - \cos \theta_0) [\cos \theta_0 G(0) + h_z] b^2 \cos \theta_0 h_z$$

$$\text{Def. } \cos \theta_0 \equiv z. \quad W(b, z) = \hat{R}_0^2 (1 - z^2) \ln \frac{1}{b} + (1 - z)^2 W_m^{us}(b) - \langle m_z \rangle (1 - z) [z G_0 + h_z] b^2 - z h_z$$

$$(1) \quad \frac{\partial W}{\partial b} = \hat{R}_0^2 (1 - z^2) \left(-\frac{1}{b} \right) + (1 - z)^2 W'_m(b) - 2\mu (1 - z) [z G_0 + h_z] b = 0, \quad \mu = \langle m_z^2 \rangle = \text{const.}$$

$$(2) \quad \frac{\partial W}{\partial z} = -2z \hat{R}_0^2 \ln \frac{1}{b} + 2(1 - z)(-1) W_m(b) - \mu b^2 [-z G_0 - h_z + (1 - z) G_0] - h_z = 0$$

$$\text{Cum } \frac{\beta}{b} \ll 1, \quad W_m^{us}(b) = 4\pi g_1 b^2, \quad W'_m(b) = 8\pi g_1 b \quad u \quad (1) \text{ gaur } b_{eq}^2(z) = b_0^2 \frac{(1+z)}{(1-z)}$$

$$\text{Her, crastuor. (1): } -\frac{1}{b} \hat{R}_0^2 (1 - z^2) + (1 - z)^2 8\pi g_1 b - 2\mu (1 - z) [z G_0 + h_z] b = 0$$

$$\rightarrow b_{eq}^2 = \hat{R}_0^2 \frac{(1 - z^2)}{(1 - z)^2 8\pi g_1 - 2\mu (1 - z) [z G_0 + h_z]} = \hat{R}_0^2 \frac{(1 + z)}{(1 - z) 8\pi g_1 - 2\mu [z G_0 + h_z]}, \quad b_0^2 = \frac{\hat{R}_0^2}{8\pi g_1}$$

$$\text{Ouguno } h_c \text{ u } b_{eq}(h_c) \rightarrow \infty: \quad G_0 + h_c = 0, \quad (h_c = G(0) = \frac{4\pi}{\beta} [1 + \beta - \sqrt{1 + \beta^2}])$$

$$\text{C uor. go } \beta^3: \quad h_c(\beta) = 4\pi (1 - \frac{\beta}{2}) + O(\beta^3). \quad \text{Bauru } h_c(\beta) = 4\pi N_{zz} = 4\pi [4 - \frac{\beta}{\pi} (\ln \frac{\beta}{2} - \frac{1}{2})]$$

$$\frac{\partial W}{\partial z} (b \rightarrow b_{eq}(z)) = -2z \hat{R}_0^2 \ln \frac{1}{b} - 2(1 - z) 4\pi g_1 b^2 - \mu b^2 [-2z G_0 - h_z + \mu] - h_z = 0$$

$$-h_z - 2z \hat{R}_0^2 \ln \frac{1}{b_{eq}} - [(1 - z) 8\pi g_1 + \mu (b_{eq} - h_z - 2z G_0)] \frac{\hat{R}_0^2 (1 + z)}{(1 - z) 8\pi g_1 - 2\mu (z G_0 + h_z)} = 0$$

$$f(z) \equiv (1 - z) 8\pi g_1 - 2\mu z G_0, \quad -h_z - 2z \hat{R}_0^2 \ln \frac{1}{b} - [f(z) + \mu (b_{eq} - h_z)] \frac{\hat{R}_0^2 (1 + z)}{[-2\mu h_z + f(z)]} = 0$$

$$\frac{h_z}{\hat{R}_0^2} [1 + 2z [f(z) - 2\mu h_z] \ln \frac{1}{b_{eq}} + (1 + z) [f(z) + \mu (b_{eq} - h_z)]] = 0, \quad b_{eq}^2 = \hat{R}_0^2 \frac{(1 + z)}{[f(z) - 2\mu h_z]}$$

Uauru cuor. 2-y gauru. que $b_{eq}(h_z)$ u $z = \cos \theta_0(h_z)$.

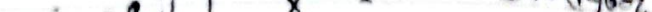
b) $\lim_{h_0 \rightarrow \infty} V_1$. Besser $h_c(p) = -G(p)$.
 c) $z=1$, Biquasine, Besser $2R^2 \ln \frac{1}{p} - p^2(G_0 + h_0) + h_c = 0$ u. $R^2 = R_0^2$
 $2R^2 \ln \frac{1}{p} + R^2 + h_c = 0$, $2 \ln \left(\frac{R^2}{p(G_0 + h_0)} \right) + 1 + h_c/R_0^2 = 0$

a) $z=1$, $\log u_{\text{norm}}$, $\log v$ $2R^2 \ln \frac{1}{b} - \mu B^2 (G_0 + h_0) + h_0 = 0$ u $B^2 = -R^2 \frac{1}{b}$
 $2R^2 \ln \frac{1}{b} + R^2 + h_0 = 0$, $2 \ln \left(\frac{R^2}{\mu(G_0 + h_0)} \right) + 1 + h_0/R^2 = 0$ $\frac{1}{\mu(G_0 + h_0)} = 0$ $\frac{1}{R^2(G_0 + h_0)}$

$$2R_0^2 \ln \frac{1}{6} + R_0^2 + h_0 = 0, \quad 2 \ln \left(\frac{R_0^2}{N(G_0 + h_0)} \right) + 1 + h_0/R_0^2 = 0$$

$$-2 \ln \left(\frac{R_0^2}{N} \right) \left(\frac{h_0}{R_0^2} \right) + 1 + h_0/R_0^2 = 0, \quad 1 + h_0/R_0^2 = 2 \ln \left[\frac{N}{R_0^2} \left(\frac{h_0}{R_0^2} \right) \right]$$

$$\frac{h_c}{R_0^2} = \frac{H_0}{M_0} \left(\frac{R}{R_0} \right)^2 \approx 4\pi \left(\frac{R}{R_0} \right)^2 \gg 1, \quad \frac{h_c}{R_0^2} = 2 \ln \frac{M}{R_0^2} |G_0 + h_c| \text{ gives } h_c = h_0 / (p_0)$$

Def. $\ln/a_0^2 = x$, $\alpha_0/a_0^2 = g_0$, $x = 2 \ln/4 |g_0 + x|$.  $x_0 g_0 < 0$ u
 $-(g_0 + x) = |g_0| - x$ $\Rightarrow (x_0 g_0 - \ln/a_0^2)$

$-(20+x) = 180 - x$
 So we $x = 2 \ln \mu (180 - x)$
 where $x_0/180 < 1$

$$y_{\text{geo}} \quad x_c/|g_0| \ll 1. \quad \ln \mu |g_0| (1 - \frac{x}{|g_0|}) = \ln \mu |g_0| - \frac{x}{|g_0|}, \quad \frac{x}{2} = \ln \mu |g_0| - \frac{x}{|g_0|}$$

$$x_c = \frac{\ln \mu |g_0|}{\frac{1}{2} + \frac{1}{|g_0|}} \approx 2 \ln \mu |g_0|, \quad h_c = R_0^2 \ln \mu |g_0|, \quad h_c = \frac{R_0^2}{R^2} \ln \left[\frac{\mu R^2}{R_0^2} |G(0)| \right]$$

$$h_c(L, R) = \left(\frac{R_0}{R}\right)^2 \ln \left[4\pi \langle m_i^2 \rangle \frac{R^2}{R_0^2} \frac{1}{\beta} (1 + \beta - \sqrt{1 + \beta^2}) \right], \text{ where } R \gg R_0 \text{ gives } h_c \rightarrow 0(?)$$
 where $\chi_c / |g_0| \ll 1$. where $\langle m_i^2 \rangle = 0.39$.

$h_c = x_0 R_0^2 = x_0 \left(\frac{R_0}{R}\right)^2$, где $x_0 < |g|$, $(h_c)_{\max} = |g| \frac{R_0^2}{R^2} = |G(0)| \frac{R_0^2}{R_0^2 \frac{R^2}{R_0^2}} = |G(0)|$
 - Будем считать, что $x = \dots$

$\frac{1}{2} g_0 y = \ln(1-y) + f(g_0)$

$$\frac{1}{2} \ln z = \frac{z-1}{2+1} + O((z-1)^2), \quad \frac{1}{2} \ln(1-y) = -\frac{y}{2} + \dots \text{ max.}$$

$\frac{1}{2} \ln z = \frac{z-1}{2+1} + O((z-1)^2)$, $\frac{1}{2} \ln(1-y) = -\frac{y}{2} + \dots$ near $x=0$.
 $\ln z = (z-1) - \frac{1}{2}(z-1)^2 + \dots$ $g_0(x) = \frac{1}{2} x^{1/2}$, $x \leq 0$

$\ln z = (z-1) - \frac{1}{2}(z-1)^2 + \dots$
 $g_0(x) = f(y), \quad y = g^{-1}(g_0)$
 $g_0(x) = \frac{1}{4}e^{x/2} + x, \quad x \leq g_0$

$g_0(x) = f(x)$, $x_0 = f^{-1}(g_0)$. 
 $g_0(x) \approx \frac{1}{\mu} + (\frac{1}{2\mu} + 1)x$
 $h_c = x_c \left(\frac{R_0}{R}\right)^2 = f^{-1} \left[G(0) \frac{R^2}{R_0^2} \right] \cdot \left(\frac{R_0}{R}\right)^2$

$$g_0(x) \approx \frac{1}{\mu} + \left(\frac{1}{2\mu} + 1\right)x$$

$$x = f^{-1}\left(\frac{x}{g_0}\right) = \frac{1}{\left(\frac{1}{2\mu} + 1\right)}\left(g_0 - \frac{1}{\mu}\right) \approx \frac{g_0 \cdot 2\mu}{(2\mu + 1)}$$

$$h_c = x_c \left(\frac{R_0}{R}\right)^2 = f^{-1}\left[G(0) \frac{R^2}{R_0^2}\right] \cdot \left(\frac{R_0}{R}\right)^2$$

$$h_c = \frac{2\mu}{2\mu + 1} \cdot G(0) \quad \text{и значение от } \beta \text{ не зависит}$$

$$(12/1x) \quad b_{eq}(z+1) = -\frac{R_0^2}{f} \frac{1}{(h_c + G_0)} = \frac{1}{f} \frac{1}{\lg d - x} \quad \text{где } x \text{ координата } x = 2 \ln \mu(|g_0| - x) - 1, \Rightarrow x_c$$

$g_0(x) = \frac{1}{\mu} e^{(x+1)/2} + x$, on f on $\mathbb{R}$ est une g.c. de f car $x_c = F(g_0) = f^{-1}(g_0)$.

$$g = |g_0| = G(0) \frac{R^2}{R_0^2} \gg 1, \quad g_0(r) \approx \frac{1}{r^2} e^{\frac{1}{2}(r+1)}, \quad x_c \approx 2 \ln \mu |g_0| - 1, \quad \text{ecm } x_c \gg 1.$$

$$h_c = \frac{H_c}{n_c} = \left[2 \ln \mu |g_0| - 1 \right] \left(\frac{R}{R_0} \right)^2, \quad |g_0| = \frac{4\pi}{\beta} [1 + \beta - \sqrt{1 + \beta^2}] \left(\frac{R}{R_0} \right)^2 \sim 100 - 300$$

$B_{eq}(y) = \frac{1}{r} \frac{1}{|g_0| - y} = e^{\frac{1}{2}(x_0 + 1)}$ Turns. $|g_0| > 100$, $x_0 > 6$ u $B_{eq}/R > 5 (!)$.
 e. H₂ = He VC radius $B_{eq} > r$

T.e. $n_{H_2} = n_{He}$ VC radius $\frac{b_{eq}}{R} > 5$, $|g| > 100$, $x_c > 6$ u $b_{eq}/R > 5 (!)$.
 frequency $M_2(H_2)$ - Vozdukh
 frequency n_{H_2} - Vozdukh. $b_{eq} > 5R$, $b_{eq}(H_2)$ - Vozdukh.

$H_c = 7.5 \text{ kOe}$, $L = 80 \text{ nm}$, $R = 500 \text{ nm}$, $p = 0.16$.

$H_c = 7,5 \text{ kOe}$, $L = 40 \text{ nm}$, $R = 500 \text{ nm}$, $p = 0,16$. $H_c = 5,0 \text{ kOe}$, $L = 80 \text{ nm}$, $R = 250 \text{ nm}$, $p = 0,16 \cdot 2 = 0,32$
 • 2) $H_c \approx 7 \text{ kOe}$, $H_0 = 4,1 \text{ kOe}$, $R = 100 \text{ nm}$. H_c perifer $R(1)$. $R(1)$. $R = 100 \text{ nm}$. $R = 200 \text{ nm}$. $R = 500 \text{ nm}$

• 2) Olovno '02: $H_c \approx 7 \text{ kOe}$, $H_0 = 4.1 \text{ kOe}$, $R = 100 \text{ nm}$; $H_c \approx 9 \text{ kOe}$, $R = 200 \text{ nm}$ u $R = 300 \text{ nm}$
 FEM, $L = 50 \text{ nm}$; $H_0 = 3.9 \text{ (R=200nm)}$, 3.7 kOe (300nm)

• 3) delatens' og: $\text{NiMnSO (4xM}_2\text{=6.91\%)}$, $H_c^* = 6.1 \text{ kOe}$, $L = 44 \text{ nm}$; $H_c \approx 1 \text{ kOe}$, $R = 50 \text{ nm}$

• 3) delatens' og: NiMnSO (4×10^{-3} g), $H_c^* = 6.1 \text{ kOe}$, $L = 44 \text{ nm}$, $R = 130 \text{ nm}$, $H_0 = 2.8 \text{ kOe}$, $H_c^* = 3.2 \text{ kOe}$, $R = 50 \text{ nm}$, $L = 44 \text{ nm}$, $d_0 = 2.8 \text{ kOe}$

$H_c^* = H_c + d_H$ (easy plane) $\therefore$ Corrected H_c ($R = 130 \text{ nm}$) = $6.1 - 1.9 = 4.2 \text{ kOe}$

$H_c^* = H_c + \Delta H$ (связь поле) - Connected $H_c(R=120\text{ nm}) = 6.1 - 1.9 = 4.3\text{ kOe}$, $H_b(520\text{ nm}=R) = 8.2 - 1.9 = 6.4\text{ kOe}$.
Резонанс: все гомогенно. Эмиссионный газ. H_c падает с $R \uparrow$ и $\mu_0 L = \text{const}$. $H_0(R)$ задается тем же условием, т.е. $H = f(L)$.

$$H_c / 4\pi M_s = \frac{H_c}{10 \text{ kOe}} = \frac{5 \text{ kOe}}{10 \text{ kOe}} = 0.5 \quad (\beta = 0.32) \quad \text{и} \quad \frac{H_c}{4\pi M_s} = \frac{7.5}{10} = 0.75 \quad (\beta = 0.16) - \text{Kikuchi '01}$$

$$\frac{H_c}{4\pi M_s} = \frac{7}{10} = 0.7 \quad (\beta = 0.5) \quad \text{и} \quad \frac{H_c}{4\pi M_s} = \frac{9}{10} = 0.9 \quad (\beta = 0.1 + \beta = 0.25) - \text{Okuno '02}$$

$$H_c / 4\pi M_s = \frac{4.3}{6.9} = 0.62 \quad (\beta = 0.34) \quad \text{и} \quad H_c / 4\pi M_s = \frac{6.4}{6.9} = 0.93 \quad (\beta = 0.085) - \text{de laubens '09}$$

Temp. $\frac{H_c(\beta)}{4\pi M_s} = N_{22}(\beta)$; $N_{22}(0.32) = 0.72$; $N_{22}^{(0.16)} = 0.83$; $N_{22}(0.5) = 0.635$
 $N_{22}(0.1) = 0.885$, $N_{22}(0.25) = 0.76$, $N_{22}(0.34) = 0.71$, $N_{22}(\beta = 0.085) = 0.89$

Boulog: $H_c(\beta) = 4\pi M_s N_{22}(\beta)$ на 10-40% больше H_c^{exp} -Kikuchi, на 10-16% H_c^{exp} -Okuno на 15% больше $R = 120 \text{ nm}$ -exp. H_c -de laubens, не хватает совп. с $H_c^{\text{exp}}(\beta = 0.085)$ -de laubens. Но de laubens не имеет в своем $H_c(\beta)$ и того же значения H_c на $H_A = 19 \text{ kOe}$.
 $H_c(\beta) = 4\pi M_s N_{22}(\beta)$ является универсальной для всех, имеет же $(\beta = 0.32)$ -Kikuchi '01. Для остальных значений β ошибка $\leq 15\%$.

Упр-е $1+x = 2 \ln \mu(1901-x)$, где $\mu = h_c(R, L) < 1$, ($\frac{h_c}{4\pi M_s} < 0.1$) и $h_c(R)$ не зависит от R ($R > 1.5 R_0$) или $L = \text{const}$, что универсально для всех значений β .

Выводим $h_c(R) = 4\pi M_s C \frac{L}{R}$ и тогда C универсально, $h_c(R)$ зависит от R $z = G(\beta, H_c)$

$$(13/14) \quad W(\beta, z) = R_0^2 (1-z^2) \ln \frac{1}{\beta} + (1-z^2) W_m^{us}(\beta) - \mu(1-z^2) [z G_0 + h_2] \beta^2 - z h_2 + 2 R_0^2 (1-z)$$

Согл. 12/III $W_{ex}(\text{core}) = 2 \left(\frac{R_0}{R} \right)^2 (1 - \cos \theta_{out})$ не зависит от $\frac{\partial W}{\partial \beta}$, но имеет же зависимость от $\frac{\partial W}{\partial z}$ и h_c -value. $\cos \theta_{out} = \cos \theta_0 = z$. $z = h_2/h_c$.

$$-\frac{\partial W}{\partial z} = 2z R_0^2 \ln \frac{1}{\beta} + 2 R_0^2 + 2(1-z) W_m + \mu \beta^2 [G_0 - h_2 - 2z G_0] + h_2 = 0$$

$$\beta_{eq}^2(z=1) = -\frac{R_0^2}{\mu} \frac{1}{(h_c + G_0)}, \quad -\frac{\partial W}{\partial z}(\beta_{eq}) = 0 = 2 R_0^2 \ln \frac{1}{\beta_{eq}} + 2 R_0^2 + \mu(h_c - G_0) \beta_{eq}^2 + h_2 = 0$$

$$2 \ln \frac{1}{\beta_{eq}} + \frac{h_2}{R_0^2} + 3 = 0, \quad \text{упр-е на } h_c \text{ переписываем } x + 3 = 2 \ln \mu(1901-x), \quad \frac{1}{R_0^2} = x$$

$$\int_0^1 ds \int_0^1 ds' G(s, s') = \int_0^1 ds G(s) = -\frac{4\pi}{\beta} \int_0^1 ds [1 - e^{-\beta s}] I_0(h_2) \frac{J_1(h)}{h} =$$

$$= -\frac{4\pi}{\beta} \int_0^1 ds [1 - e^{-\beta s}] \frac{J_1^2(h)}{h^2} = -2\pi N_{22}(\beta), \quad \text{Согл. 14/III ген. экв. } W = 2\pi \left(\frac{R_0}{R} \right)^2 N_{22} - \text{намагниченность, где } \frac{\partial W}{\partial z} = f(z)$$

$$\rightarrow W(\beta, z) = 2 R_0^2 (1-z) + R_0^2 (1-z^2) \ln \frac{1}{\beta} + (1-z^2) W_m^{us}(\beta) - \mu(1-z^2) [z G_0 + h_2] \beta^2 - z h_2 + z^2 2\pi N_{22}$$

$$-\frac{\partial W}{\partial z} = 2z R_0^2 \ln \frac{1}{\beta} + 2 R_0^2 + 2(1-z) W_m + \mu \beta^2 [G_0 - h_2 - 2z G_0] + h_2 - 4\pi N_{22} = 0$$

$$-\frac{\partial W}{\partial z}(\beta_{eq}) = 2 R_0^2 \ln \frac{1}{\beta_{eq}} + 3 R_0^2 + h_2 - 4\pi N_{22} = 0. \quad \text{Согл. } R_0^2 = \left(\frac{R_0}{R} \right)^2 \ll 1, \quad \text{то}$$

$$h_c = 4\pi N_{22}(\beta), \quad \text{иначе } x + (3 - 4\pi n) = 2 \ln \mu(1901-x), \quad x = \frac{h_c}{R_0^2}, \quad n = \frac{N_{22}}{R_0^2}$$

$$g_0(x) = \frac{1}{4} \exp \left[\frac{1}{2} (x + 3 - 4\pi n) \right] + x. \quad \text{Для } x \gg 1, \quad x_c = 4\pi n + 2 \ln \mu |g_0| - 3$$

$$\rightarrow h_c = H_c |M_s| = 4\pi N_{22} + \left[2 \ln \mu \left(\frac{R}{R_0} \right)^2 |G(0)| - 3 \right] \left(\frac{R_0}{R} \right)^2 - \text{где } H_c(\beta, R) \text{ - vector saturation field.}$$

$$|G(0)| = |G_0| = \frac{4\pi}{\beta} [1 + \beta - \sqrt{1 + \beta^2}], \quad h_c > 4\pi N_{22} \quad \text{где } R \gg R_0$$

$$W_m^{us}(\beta) \text{ зависит от } \beta \ll 1. \quad \frac{\beta}{\beta_{eq}} = \frac{L}{R_0} = \frac{4\pi N_{22} \sqrt{|h_c + G_0|}}{R_0} \approx \frac{L}{R_0} \beta^{1/2} \ll 1 \quad \text{если } L \ll R_0, \quad \beta \leq 0.1$$

No bug $B_{eq}^2(z=1) = \frac{R_0^2}{\mu} \frac{1}{|h_0 + G_0|}$ no element of bug $w_m(b)$, T.V. $(1-z)w_m$
 $(1-z)$ - unkon. ydypoc w_m y μ $z=1$. T.e. $h_c(h_0)$ berko que. $w_m \propto \frac{1}{b}$
 $\delta = -\mu (h_0 + G_0) = \mu [K_0 - h_c] = \frac{R_0^2}{\mu} - \text{ker. } \alpha = \frac{h_c}{h_0} \delta$

$$W(b,z) = 2R_0^2(1-z) + R_0^2(1-z^2)\ln\frac{1}{b} + (1-z)^2 w_m(b) - \mu(1-z)[zG_0 + h_c]b^2 - h_c z + z^2 w_m$$

$z = G_0, G_0 = G_{out}, \lambda = 1-z \text{ and } p = +1$. Onegb we opent. $z_{eq} = h_c/h_0, z_{eq}(h_c)$

Rare, $\partial w/\partial z = 0$, $\delta w_{eff}(b) = \frac{1}{\lambda^2} w(b) = \hat{R}_0^2 \ln \frac{1}{b} + w_m(b) + \alpha b^2$

$$\frac{\partial W(b,z)}{\partial b} = R_0^2(1-z^2)\left(-\frac{1}{b}\right) + (1-z)^2 w'_m - 2\mu(1-z)[zG_0 + h_c]b = 0$$

$$\frac{1}{\lambda^2} \frac{\partial W(b,z)}{\partial z} = R_0^2 \left(\frac{1+z}{1-z}\right) \frac{(-1)}{b} + w'_m - \frac{2\mu}{(1-z)} [zG_0 + h_c]b = \frac{\partial w_{eff}}{\partial b} = 0, \text{ ko}$$

$$\alpha = -\frac{\mu}{(1-z)} [zG_0 + h_c] - \text{value of } z_{eq}(h_c) - \text{buga. } \frac{1}{\lambda^2} \frac{\partial^2 W(b,z)}{\partial b^2} = \frac{\partial^2 w_{eff}}{\partial b^2}$$

$$W(b,z) = 2R_0^2(1-z) + R_0^2(1-z^2)\ln\frac{1}{b} + (1-z)^2 w_m(b) - \mu(1-z)[zG_0 + h_c]b^2 - h_c z + z^2 w_m$$

$$\frac{\partial W(b,z)}{\partial z} = -2R_0^2 - 2zR_0^2 \ln\frac{1}{b} - 2(1-z)w_m(b) - \mu b^2 [G_0 - h_c - 2zG_0] - h_c + 2zw_m = 0$$

13-18/IX Yracome & XVI Int. MC Summer school "Madrid"

Vertex dyn. and reversal. Becega & Bach. Demokritov. P. R. Stab, Russia

Stamps, Reiss, Wiesendanger, Palacios, Gladilin.

Pozharovskiy & Oceano (Cyprus) (tip-top 190 @ mail.ru) uz Xapok yu.

u yfymaem ee na Master/Ph.D. position & San Seb. Pozharovskiy & Mupomabov

Kobayashi (Univ. Barcelona, Exch. bias) - nameless due & Barcelona, Oceano

Ufymaem na seminar b Univ. Barcelona - Dec. '09 (Batt, Leberta, Tjadia).

Pozharovskiy & Elena Mure (Switzerland) - FMR checker F/N/F circular

Co/Cu/Co dots, Oceano nameless & unref. even yfymaem exp data.

One name SW/Vertex checker F/N/F dot even F₁ - vortex, F₂ - ground state.

18-19/IX B. Nozok & Oceano (Cyprus) | Becega & Farhad u Andri o F/N/F

papers & Oceano "Rafa" has texcom dots FMR measur. Oceano u yfymaem

"Vortex Nonlin. Dynamics" paper. Oceano. Ver u even. Buchanan PRL '07 na Oceano

21/IX Becega & Julian: 1) one paper. Oceano res. report o Jan. - Sept. '09 activ.

2) Pozharovskiy & bugay A. Fert 1-2/8; 3) Oceano Oceano (Cyprus) (tip-top 190 @ mail.ru) uz Xapok yu.

4) JVN-Colu Proofs, Oceano 100 articles; 5) Oceano Oceano (Cyprus) (tip-top 190 @ mail.ru) uz Xapok yu.

6) Oceano. hot topics Conf. in Madrid: F/N/F and spin torque

Becega & Maxim: on yfymaem files beex M(H,t) FMR dots, Oceano nameless

M(t,H) - M_{sat}(H) = ΔM(t) as f(t), Oceano has yfymaem dots & Nano GUNE.

Pozharovskiy, Kfymaem, Arantxa, Natalia (noting), Julian, PRL, Iberbasque grants.

22/IX Pozharovskiy. Res. Report Jan. - Sept. 2009 que Julian, Oceano u Oceano.

Oceano nameless & Oceano of Oceano yfymaem & Oceano Spanish grant as external collabor.

- a gas Oceano has Oceano SMH, Oceano CV, birth date, passport n^o no more ne

Oceano. Oceano & Oceano of Oceano. Julian Oceano Oceano visit to Oakland, USA.

23/IX Oceano & Julian of A. Fert. Visit. Colu. FMR paper, F/N/F work with Arantxa.

Arantxa Oceano & Spanish Custom o parcels - Oceano. Oceano yfymaem new exp. date F/N/F dots

u RPL-SW Oceano (noting. & Oceano). T. Oceano & Xfce. Lunch & Oceano 6/8.

поговорка и покаян to Arantxa draft case и др. governance на P. (Spanish)
penalizer. APL no UMD switching в etc. // idea: замена $m_2(p)$ - core на $\langle m_2 \rangle$
в минимуме. uncertainty function power и guess миним с APL-хвалителем.
можно угадать $R_c(p, \varphi) = b(p, \varphi) = b_0(p) + b_1(p, \varphi)$, $b_1 \ll b_0$ - неясно, можно
VC radius no φ , но наоборот. неформ. axial symmetry, reversal, и т.д. TM gr mode

24/12 Губернатор е.п. распис. Guses и Guses к Notarius по поводу governance
K и W. по делу и copy of Marriage Certificate. [Поскольку Glibank (Mario) о нем
вышел в нов. W. посылка из Сегия, 28 мес, opt. meeting с ней ~ 5/12]

// Com. 13/12 при распис. $b_{eq}(H_2)$. b_c того распис. функцией $W(b, z)$,
т.е. $z = 0.50_{out}$ - абн. неясно, какой неформ.

$\rightarrow W(b, z) = 2R_0^2(1-z) + R_0^2(1-z^2) \ln \frac{1}{b} + (1-z)^2 W_m^{us}(b) - \int (1-z) [z G_0 + h_2] b^2 - h_2 z + 2\pi N_{zz} z^2$

$\begin{cases} \frac{\partial W(b, z)}{\partial b} = -R_0^2(1-z^2) \frac{1}{b} + (1-z)^2 W'_m - 2\mu(1-z) [z G_0 + h_2] b = 0 \\ \frac{\partial W(b, z)}{\partial z} = -2R_0^2 - 2zR_0^2 \ln \frac{1}{b} - 2(1-z) W'_m - \mu b^2 [G_0 - h_2 - 2z G_0] - h_2 + 4\pi N z = 0 \end{cases} \quad (1)$

При $h_2 = h_c$ (saturat.) (мыслим $z=1$ и считаем (1), (2) упрощаясь:
 $-2R_0^2 - 2R_0^2 \ln \frac{1}{b_{eq}} + \mu b_{eq}^2 (G_0 + h_c) - h_c + 4\pi N = 0$ (2); $-\frac{2R_0^2}{b_{eq}} - 2\mu(G_0 + h_2) b_{eq} = 0$ (1).

из (1): $b_{eq}^2(z=1) = \frac{R_0^2}{\mu} \frac{1}{|h_c + G_0|}$. Note, что при $b_{eq}(z=1)$, на h_c не зависит от $W_m(b)$.
 $-3R_0^2 - 2R_0^2 \ln \frac{1}{b_{eq}} - h_c + 4\pi N = 0$ и guess $z=1$ на h_c 13/12, или на $x = \frac{h_c}{R_0^2}$.
Но если h_2 arbitrary (например $h_2 \geq h_0$ - none transition VC stability), то guess
решать $\frac{\partial W}{\partial b} = 0$, $\frac{\partial W}{\partial z} = 0$ совместно с $\frac{\partial^2 W}{\partial b^2} = 0$, то есть найти b_{eq} , z_{eq} и b_c , h_0 .

$\begin{cases} \frac{\partial^2 W}{\partial b^2} = \frac{R_0^2}{b^2} (1-z^2) + (1-z)^2 W''_m - 2\mu(1-z) [z G_0 + h_2], & \frac{\partial^2 W}{\partial z^2} = -2R_0^2 \ln \frac{1}{b} + 2W'_m + 2\mu b^2 G_0 + 4\pi N \\ \frac{\partial^2 W}{\partial z \partial b} = 2zR_0^2 \frac{1}{b} - 2(1-z) W'_m - 2\mu b [G_0 - h_2 - 2z G_0], & \Delta = \frac{\partial^2 W}{\partial b^2} \frac{\partial^2 W}{\partial z^2} - \left(\frac{\partial^2 W}{\partial z \partial b} \right)^2 \end{cases}$

Упрощение и замена при $m_2^2(p) \rightarrow \langle m_2^2 \rangle_c$. не ясно, почему, т.е. крив. VC radius b_c ?

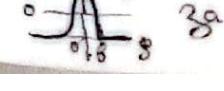
$W_m(b, z) = -\lambda^2 \int_0^b \int_0^b d\mathbf{p} d\mathbf{p}' m_2^2(p) m_2^2(p') g - 2\lambda z \int_0^b d\mathbf{p} m_2^2(p) G(p) + z^2 2\pi N_{zz}$, $m_2^2(p) \rightarrow \langle m_2^2 \rangle_c = A = \text{const}$
 $W_m(b, z) = -\lambda^2 \mu^2 \int_0^b \int_0^b d\mathbf{p} d\mathbf{p}' g(p, p') - 2\lambda z \mu \int_0^b d\mathbf{p} G(p) + 2\pi N z^2$. $\lambda = 1 - m_2^{out} = 1 - z$, $z = 0.50$.

$\int_0^b d\mathbf{p} G(p) = \int_0^b d\mathbf{p} \left(-\frac{4\pi}{p} \right) \int_0^b dk [1 - e^{-\beta k}] \frac{J_1(k)}{k} J_0(kp) = \int_0^b d\mathbf{p} \int_0^b dk J_0(kp) = \int_0^b dk \int_0^b d\mathbf{p} J_0(kp) = \frac{1}{k^2} \int_0^b dx x J_0(x) = \frac{1}{k^2} x J_1(x) \Big|_0^b = \frac{b}{k} J_1(kb)$
 $= -\frac{4\pi}{p} \int_0^b dk [1 - e^{-\beta k}] \frac{J_1(k)}{k^2} J_1(kb) = -b J_1(\beta b)$.

$\int_0^b d\mathbf{p} \int_0^b d\mathbf{p}' m_2^2(p) m_2^2(p') g = \mu^2 \int_0^b d\mathbf{p} \int_0^b d\mathbf{p}' g(p, p') = \mu^2 \int_0^b d\mathbf{p} \int_0^b d\mathbf{p}' \left(-\frac{4\pi}{p} \right) \int_0^b dk [1 - e^{-\beta k}] J_0(kp) J_0(kp') =$
 $= \mu^2 \left(-\frac{4\pi}{p} \right) \int_0^b dk [1 - e^{-\beta k}] \frac{J_1^2(kb)}{k^2} = \left| kb = t \right| = \mu^2 \left(-\frac{4\pi}{p} \right) \beta^3 \int_0^t dt [1 - e^{-\beta t}] \frac{J_1^2(t)}{t^2} = \mu^2 \beta^2 (2\pi) N_{zz} \left(\frac{p}{b} \right)$

$W_m(b, z) = 2\pi \lambda^2 \mu^2 \beta^2 N_{zz} \left(\frac{p}{b} \right) + 2\lambda z \mu b J_1(\beta b) + 2\pi z^2 N_{zz}(\beta)$. $N_{zz}(\beta) = \frac{8\pi}{p} \int_0^b dk [1 - e^{-\beta k}] \frac{J_1^2(k)}{k^2}$

$W_m(b, z)$ упроще чем $W_m^{us}(b)$, есть явное выражение. пропущено, но т.е. b_c
надо раскладывать b пред по b^4 или $p/b \ll 1$ или $p/b \gg 1$ и при этом не надо

 m_2 $\langle m_2 \rangle$ p , то ближе к теории UMD.

27/12 задачи 5-6/12 задачи, и 8/12 задачи. Основаны на упрощениях, что b_c и h_c малы по сравнению с z_0 и z_0 - радиус. $w'_0 = 0, w''_0 = 0$. Но надо $w = w(b, z)$ - функция b и $z = G_0 z_0$, где $z_0 = h_z/h_c$ безразмерная, а b - радиус. $\Delta(b, z) = 0$ функция $w'_0 = 0$.

$\Delta(b, z) = 0$ если брать функцию z .

$$w'_0 = 0 \text{ и } z_0 (z_0 + h_z) h_{c1} = \frac{R_0^2}{b_{c1}} (1+z) + (1-z) w'_m, \text{ где } b_{c1}, z_{c1}$$

$$w''_0 = (1-z) \left[\frac{R_0^2}{b^2} (1+z) + (1-z) w''_m - 2\mu (z G_0 + h_z) \right] = (1-z) \left[\frac{R_0^2}{b} (1+z) + (1-z) w''_m b + \frac{R_0^2}{b} (1+z) - (1-z) w'_m \right]$$

$$= (1-z) \left[\frac{R_0^2}{b^2} (1+z) + (1-z) (w''_m b - w'_m) \right]$$

$$w''_0 = (1-z) \left[\frac{R_0^2}{b^2} + w''_m - 2\mu h_z + 2 \left(\frac{R_0^2}{b^2} - w''_m - 2\mu G_0 \right) \right], \quad w''_0 = -2 \frac{R_0^2}{b} - 2\mu b (G_0 + z h_z) = -2 \frac{R_0^2}{b} - 2\mu b G_0 - 2\mu b h_z z.$$

$$B(b) \equiv -2 R_0^2 \ln \frac{1}{b} + 2 w_m(b) + 2 \mu b^2 G_0 + 4 \pi N$$

$$\Delta(b, z) = (1-z) \left[\frac{R_0^2}{b^2} + w''_m + z \left\{ \frac{R_0^2}{b^2} - w''_m - 2\mu (G_0 + h_z) \right\} \right] B(b) - \left[-2 \frac{R_0^2}{b} - 2\mu b (G_0 + z h_z) \right]^2$$

$$\Delta(b_{c1}, z_{c1}) = 0 \text{ где } (1-z) \left[\frac{R_0^2}{b^2} + w''_m + z \left\{ \dots \right\} \right] B = \left[2 \frac{R_0^2}{b} + 2\mu b G_0 + 2\mu b h_z z \right]^2$$

$$B \left[\frac{R_0^2}{b^2} + w''_m + z \left\{ \frac{R_0^2}{b^2} - w''_m - 2\mu (G_0 + h_z) \right\} - z \left(\frac{R_0^2}{b^2} + w''_m \right) - z^2 \left\{ \dots \right\} \right] = \left[\frac{R_0^2}{b^2} + \mu b G_0 \right]^2 +$$

$$+ 8 \left(\frac{R_0^2}{b} + \mu b G_0 \right) \mu b h_z z + 4 \mu^2 b^2 h_z^2 z^2 - \text{это брать. где } z_0 \text{ где } z_0(b).$$

непренебрегая $z_{c1}(b)$ и $z_0(b)$ где h_z и h_c - радиусы. $h_z = z_0(b)$ и $h_c = z_0(b)$.

$b_{c1}(h_z)$ находится из $h_z = z_0(b)$, $z_{c1} = h_z/h_c < h_c \approx 4\pi N$ (то есть h_z - радиус).

$$\Delta[b_{c1}(h_0), z_{c1}(h_0)] = 0 \text{ где } h_0 \text{ и } b_c = b_{c1}(h_0).$$

У нас сейчас $F(b) = \Delta[b, z_{c1}(b)]$, $F(b) = 0$ где b_c , и где $h_0 = z_{c1}(b_c)$.

$$G_0 = G_0(p) = G(0) = -\frac{4\pi}{p} [1 + p - \sqrt{1 + p^2}], \quad h_c = h_c(p) \approx 4\pi N_{22}(p).$$

Равн. Vortex-core-reversal-Hz. mod. $w_m^{us}(b) = \frac{4\pi}{p} b^3 [u(0) - u(\frac{p}{b})] = \frac{4\pi}{p} [u(0) - \frac{1}{4} \frac{p^2}{b^2}] b^3$

$$w'_m(b) = \frac{12\pi}{p} u(0) b^2 - \frac{4\pi}{p^2} p^2 b^3 = r b^2 - y b^3$$

APL - Хванковски: $L = 20, 30 \text{ nm}, R_0 = 25.7 \text{ nm}, R = 150 \text{ nm}, L/R = 0.14$.

Mathcad где $b_c(20 \text{ nm}) = 0.155$ и $b_c(30 \text{ nm}) = 23.2 \text{ nm}$, $b_c(30 \text{ nm}) = 0.193 \cdot 150 = 29.9 \text{ nm}$.

С таким b_c найдем $h_0 > 0$. Это не верно. Ошибка из-за $w_m^{us}(b)$ не p/b .

$$w_m^{us}(b) = \frac{4\pi}{p} b^3 [u(0) - u(p/b)], \quad u(t) = \int_0^\infty e^{-t^2} I^2(q) dq, \quad I(q) = \int_0^\infty \frac{d\rho}{\rho} \frac{(1-\rho^2)}{(1+\rho^2)} J_0(q\rho)$$

$$w'_m(b) = \frac{4\pi}{p} 3b^2 [u(0) - u(p/b)] + \frac{4\pi}{p} b^3 (-1) u'_t \frac{p}{b^2} = \frac{3}{b} w_m(b) + 4\pi b u'_t$$

$$w''_m(b) = -\frac{3}{b^2} w_m(b) + \frac{3}{b} w'_m(b) + 4\pi u'_t + 4\pi b u''_t \frac{(-p)}{b^2} = -\frac{3}{b^2} w_m + \frac{3}{b} \left[\frac{3}{b} w_m + 4\pi b u'_t \right]$$

$$- 4\pi p u''_t + 4\pi u'_t = \frac{6}{b^2} w_m(b) + 16\pi u'_t - \frac{4\pi p}{b} u''_t$$

$$w'_m = \frac{d}{db} w_m(b) \text{ и } w''_m(b) = \frac{d^2}{db^2} w_m(b) \text{ Mathcad делает некорректно, где } w'_m, w''_m \text{ на } w_m \text{ и } u'_t, u''_t$$

APL - Хванковски. Глобальные calculations $\Delta(b, z)$.

См. 16/111 О-режимы VC radius Хванковского $p_c(A)$ - порог на порог

и $p_c(B)$ из $N_z(p) = M_z/2$. $\langle m_z(p) \rangle_A = 6\lambda \left[\ln \frac{4}{3} - \frac{1}{6} \right] + m_z^{out} \approx 6\lambda \left[\frac{1}{3} - \frac{1}{2} \frac{1}{2} + \frac{1}{3} \left(\frac{1}{3} \right)^2 - \frac{1}{6} \right] + m_z^{out} \approx \lambda + m_z^{out} \approx 1$. Но верно $\langle m_z(p) \rangle_A = 0.726\lambda + m_z^{out} = 0.726(1 - \frac{h_z}{h_0}) + \frac{h_z}{h_0} = 0.726 + 0.274 \frac{h_z}{h_c}, h_z < 0, h_c > 0$. $\langle m_z(p) \rangle_A$ - значение $\frac{h_z}{h_c}$ и $h_z < 0, h_c > 0$.

$$\rho_c(B) = \frac{6}{(3-4z)^{3/2}}, \quad \langle m_2(B) \rangle_B = 2\lambda \left(\frac{\rho_c^2}{B^2} \right)^{-1} \left[\ln(1 + \rho_c^2/B^2) - \frac{1}{2} \frac{\rho_c^2}{B^2} \right] + z = 2\lambda(3-4z) \times$$

$$+ \left[\ln\left(1 + \frac{1}{(3-4z)^2}\right) - \frac{1}{2} \frac{1}{(3-4z)^2} \right] + z = 2(1-z)(3-4z) \ln\left(\frac{1-z}{3/4-z}\right) - (1-z) + z$$

$$\langle m_2(B) \rangle_B = 2(1-z)(3-4z) \ln\left(\frac{1-z}{3/4-z}\right) + 2z - 1, \quad z = h_2/h_c < 0.$$

$$\langle m_2 \rangle_B(z=-1) = 0.739, \quad \langle m_2 \rangle_A(-1) = 0.452$$

А.с. в равном как в анал. Хвостового: $\langle m_2(B) \rangle$ от h_2 не зависит, а $\langle m_2 \rangle_A$ (MWFМ) линейно падает с h_2 .

Ques. A: $\rho_c \langle M_z \rangle_A = \frac{1}{\sqrt{3}} \hbar(h_z) [0.73 + 0.27z]$. Ques. B: $\rho_c \langle m_z \rangle_B = \frac{\hbar(h_z)}{(3-4z)^{1/2}} \langle m_z \rangle_B$

Veranschaulicht $\delta(h_2)$ durch den Quadrat, also $\frac{1}{\sqrt{3}}(0.73 + 0.272) \approx \frac{\langle m_2 \rangle_B}{(3-42)^{1/2}}$ $\forall 2 \in [-1, 0]$
 t.e. $A \cup B$ emp. Samen gute Wurzeln. $P_C(\langle m_2 \rangle) = f(h_2)$.

Получая хвостовую $\nu_c(h_2) \sim g_0 \langle m_2 \rangle_c \sim b(h_2) [0.73 + 0.27 h_2 / h_c]$ не имея оснований предполагать, что $\nu_c(h_2)$ неограниченно $h_2 \rightarrow \infty$ (см. рис. 1.10).

28/IX Преправка с Орданови С. Изучение и генериране APL-paper
с Хвановолевски "K reversal и $\frac{1}{2}(H_2)$ перп. field", нощна епизод Tex-file
Преправка с Yahoo по повод огледа my Yahoo account.

(29/12) Eggum e Cristina e W. Charles townhomes Erenteria: 230 m² garage,
in bregax, casa 520-670 m² E. Obsequium V. Barrier, bala papers e' bignara fissa
Futbol e A. W. Charles. Oceana m'fucata Junon e CV.

30/IX - Встретились поговорить к "kushienko@yahoo.com" Селиван Password (sebastia) и corp. бухгалтер (Inna/Михаил). Приехала до Орсана С.

Plan spornost gle go ozveza v USA: ~~z~~ z bo z San o Oakland, Detroit na 16 z z

- 2) Moscow reg. inf. to Klean's Secretary; 3) ~~Moscow~~ ~~Monlin. V.~~ paper draft to ~~Quinn~~
4) APL paper c. Papaxogian; 5) ~~Bogus~~ ~~Feder~~ ~~2/13/8~~ u. v. Barrier paper; 6) ~~Colu-FMR~~ paper draft
7) pers. APL - YMD 8 dot; 8) ~~Bagas~~ ~~gash~~ or Ant. School ~~exp~~ ~~ax~~ ~~ogian~~, one ~~exp~~ ~~ax~~ ~~ogian~~ u. v. ~~Bar~~ ~~rier~~ ~~paper~~ ~~draft~~ ~~to~~ ~~Quinn~~
9) ~~negot~~ ~~appl.~~ ~~Scholarship~~ ~~g~~ ~~o~~ ~~Cy~~ ~~g~~ ~~ax~~ ~~ogian~~; 10) ~~Bogus~~ ~~Feder~~ ~~2/13/8~~ u. v. ~~Bar~~ ~~rier~~ ~~paper~~ ~~draft~~ ~~to~~ ~~Quinn~~
11) ~~one~~ ~~Att~~ ~~ogian~~ ~~u~~ ~~v~~ ~~Bar~~ ~~rier~~ ~~paper~~ ~~draft~~ ~~to~~ ~~Quinn~~

(1X) Cam - 1/2 ymnus g. u. fert u. u. vortex talk. B. persp. c. A. Fert, Decagon c. h. u. o. F. N. F. w. t. d. n.,
 mesog. l. n. v. d. n., exp. Klein.

(2X) A. Fert talk. On a sun 415E 10. rent fee Oct. '09. 3 brown D. Cyxocabus.

S. Yasa (AIST), B. Georges et al. '2009 - vortices in tunnel junctions
CoB/MgO/PtNi. See Grollier's ICM invited talk We-FW7-01

$w_{TM} \sim I$, $w_{TM} \sim 0.74 - 0.8 \text{ GHz}$ & $\Delta\omega = 1.1 \text{ MHz (!)}$ Integrated power $\approx 5 \text{ nW}$
 Max int. power gain w/o $H_z \uparrow \uparrow$ VC polarization.

Comp. R. Adler $\Sigma E(TN)$ 61, 10 (1973): $\Delta \Delta P = \dots 1 + \epsilon \sin(\Delta P) + \dots$

Two coupling among N-oscillators: Kuramoto detuning
the vortices heat the gas. $\frac{d\omega}{dt} = (\omega_{free} - \omega_{source})$ adapting noise
in MTJ

The vortices lead the race: large power + narrow width in HTJ + promising synchronization experiments (no agility dI/dI webbecca).

2. Zoukanel c. Arantxa Master/PhD application que O. Gyrocebas.

$\left(\frac{3}{2}\right)$ $\delta a_{\text{new}} \in \mathcal{K}_0 \cup \mathcal{A}$. $B_{\text{side}} \in \mathcal{D}$. $\mathcal{O}_{\text{obj}}$, B_{obj} $\in \mathcal{A}$. $\cup \text{Angreuer}$.
 $\left(\frac{4}{2}\right)$ $\mathcal{A}_{\text{new}} \cup \mathcal{B}_{\text{new}}$

Update CV. Dylov $\in A$. u Angrean. B paper text "Nonlinear Vortex dyn."

(S/N) Onnashent see 1200-E October '09. Zampocun Spanish City B, Zampocun m...

Escaneado con CamScanner

(4) BZ-16 Dunder 6 Paris via Nov. 2 - Nov. 28. Tuck Sykes in case Q11 FMP papers. MacVinty. P.R.

Base frequency is used. F/AF-order bistability in dot array can gen.
ground state in SA spectrum by H-pulse.
Pulse mag MWN + Megnetons - (неповторяющийся, уникальная).

Рисунка наг white paper MUN-MICINN grant "Magnomics". Позволяє Гроби

В. Голубев ед. в Б. М. 2/II (вперед ф?). Взял билет в Р. 2-28 М. 1959.

по Мультисервис (TeGAB-ом готовил излуч. на λ в спектре РМР на ~ 300 Ое и ~ 36 Гц).
переводил для Ю.И. Обзого с помощью. Писала Мухоменовой и Прентисе. Задача обр. мн.

US-Spain MWR grant. This one to Julian u Arantxa o former negan US-Spain
 Payroll at Xbarrabarras next APL-version. уженен е и носен back to Xb, grant.
 (overas nowas APL to Grullier u Zvezdin). B roem c Angreen u Chen. ^{graben}

u negyogus que vero recte habetur SW referen. $a(p)$, $b(p)$ replez $\tilde{X}$, $m_0(p)$ -
 vertex referentibus, negyobus est a, b b vertex YD u negyobus V . mass.
 SW splitting $\Delta\omega_{\pm 1} \sim J_n$ bobes ne onferen. uz Eq. (4), a aufeger. uz unferentibus
 yprobis que $b(p)$ 22/VI 082., 19/III 082., Cogermausio $I_b = \int d\theta \theta m_0(p) b_1(p) \rightarrow J_n$.
 7/III 082. Bono negyobus $\dot{\Phi}_v = -\frac{1}{2} m p \omega I_a m_0(p) \cos(m\varphi - \omega t)$ uz Vertex YD , $m = \pm 1$,
 $I_a = \int d\theta \theta a(p) m_0(p) \sin \theta_0(p) \approx \int d\theta \theta m_0(p) a(p)$, $I_a = N_s \omega$, $N_s = \int d\theta \theta a^2(p)$.
 $\omega^2 a(p) = (1 + \frac{1}{2} a^2(p)) a(p) = 1 + \frac{1}{2} a^2(p)$

т.е. и что $a(\rho)$, и что $b(\rho)$ не зависят от ρ . Тогда $\omega^2 b - \omega_m^2 \int d\rho \rho' d\rho \rho b(\rho') = \frac{1}{2} \rho m \omega^2 I_a^m(\rho)$
 и тогда $I_a = \int d\rho m \rho \rho b(\rho)$ (тогда не зависящее от ρ уравнение с
 $a(\rho) \approx (\frac{\omega}{\omega_m}) b(\rho) \rightarrow I_a \approx (\frac{\omega}{\omega_m}) I_b$. $I_b = \int d\rho m \rho b(\rho)$.

1) γ_p - e e - nroga que a: $\omega^2 a(p) = \int d\mathbf{s}' \mathbf{s}' a(\mathbf{s}') d_{\mathbf{s}\mathbf{s}'} = \frac{1}{2} p m \omega I_a \int d\mathbf{s}' \mathbf{s}' d_{\mathbf{s}\mathbf{s}'} m_o(\mathbf{s}')$
 $\int d\mathbf{s} a^2 \omega^2 = \int d\mathbf{s} \int d\mathbf{s}' \mathbf{s}' d_{\mathbf{s}\mathbf{s}'} a(\mathbf{s}) a(\mathbf{s}') = \frac{1}{2} p m \omega I_a \int d\mathbf{s}' \mathbf{s}' \int d\mathbf{s} d_{\mathbf{s}\mathbf{s}'} a(\mathbf{s}) m_o(\mathbf{s}')$
 Portanto $a(\mathbf{s}) \approx a_1(\mathbf{s})$, $\int d\mathbf{s}' \mathbf{s}' d_{\mathbf{s}\mathbf{s}'} a_1(\mathbf{s}') = \omega_1^2 a_1(\mathbf{s}) = c \mathbf{q} d_{\mathbf{s}\mathbf{s}'} \delta_{\mathbf{s}\mathbf{s}'} v_c$, r.e. c.q. - ring.

1) γ_p - e e - nroga que a: $\omega^2 a(p) = \int d\mathbf{s}' \mathbf{s}' a(\mathbf{s}') d_{\mathbf{s}\mathbf{s}'} = \frac{1}{2} p m \omega I_a \int d\mathbf{s}' \mathbf{s}' d_{\mathbf{s}\mathbf{s}'} m_o(\mathbf{s}')$
 $\int d\mathbf{s} a^2 \omega^2 = \int d\mathbf{s} \int d\mathbf{s}' \mathbf{s}' d_{\mathbf{s}\mathbf{s}'} a(\mathbf{s}) a(\mathbf{s}') = \frac{1}{2} p m \omega I_a \int d\mathbf{s}' \mathbf{s}' \int d\mathbf{s} d_{\mathbf{s}\mathbf{s}'} a(\mathbf{s}) m_o(\mathbf{s}')$
 Portanto $a(\mathbf{s}) \approx a_1(\mathbf{s})$, $\int d\mathbf{s}' \mathbf{s}' d_{\mathbf{s}\mathbf{s}'} a_1(\mathbf{s}') = \omega_1^2 a_1(\mathbf{s}) = c \mathbf{q} d_{\mathbf{s}\mathbf{s}'} \delta_{\mathbf{s}\mathbf{s}'} v_c$, r.e. c.q. - ring.

Escaneado con CamScanner

a-meg: (1) $\omega^2 a(p) - \int d^3s' d_{ss'} a(s') = \frac{1}{2} \rho m \omega I_a \int d^3s' s' m_0(s') \cdot d_{ss'}$
 PRL-Topl: (2) $\omega^2 g(p) - \int d^3s' s' d_{ss'} g(s') = \int d^3s' s' d_{ss'} m_0(s')$ } - 16/2 too much

$$\left(\ddot{\mathbf{e}} - \frac{1}{4\pi} \int d\vec{\sigma} \cdot G_{\beta\beta} \mathbf{e}(\vec{\sigma}, t) = -\frac{1}{4\pi} \int d\vec{\sigma} \cdot \vec{\rho} \cdot G_{\beta\beta} \vec{\Phi}_V = [\sin\varphi \dot{X}(t) - \cos\varphi \dot{Y}(t)] F_d(\varphi) \forall \vec{X}(t). \right.$$
 Если учесть разн. рел. $\mathbf{e}$ и $\vec{X}, \vec{Y}$ где поворачивая в UD где $\vec{X}$ то не меняя $\hat{M}$
 Если сюда поворачивать разные рел. UD где $\vec{X}$ осц. с $\vec{\rho}$ то $\vec{\rho}$ не поворачив.
 $\varphi = e(1)$ где $a(\varphi)$ и splitting $\Delta\omega \pm 1$. Т.е. $\varphi(1)$ определяет где $\vec{S}_i$, замкнутые.
 А $\varphi = e(2)$ - вращение на φ к $\hat{M}$.

Escrivendo a equação de movimento para a coordenada radial ρ em termos de φ e t , temos:

$$m \ddot{\rho} - m \rho \dot{\varphi}^2 = -\frac{dV}{d\rho}$$
 onde V é a energia potencial. Para o caso de uma partícula em movimento circular uniforme, temos $\dot{\varphi} = \omega$ e $\ddot{\rho} = 0$. Assim, a equação se reduz a:

$$-m \rho \omega^2 = -\frac{dV}{d\rho}$$
 ou, integrando, obtemos a expressão para a energia potencial:

$$V(\rho) = \frac{1}{2} m \omega^2 \rho^2 + C$$
 onde C é uma constante arbitrária.

$\vec{S}_2 = \vec{X}_2/R$, $\vec{F}_{ST}(t) = \pi a L \vec{e}_1 \vec{e}_2 J_0 \vec{X}_2$ — поперечное взаимодействие (соединит R).
 — булавка 3D thick — to a d.o.f. for one core
 $\vec{F}_{ST}(t) = -\frac{\partial W}{\partial \vec{S}}$, $W_s(\vec{X}_1, \vec{X}_2) = -\pi a L J_0 \vec{e}_1 \cdot (\vec{X}_1 \cdot \vec{X}_2) = -\lambda (\vec{X}_1 \cdot \vec{X}_2)$ — spin toy

Inter-Vertex TM

RUM approx.

for a magnet $\lambda = \pi a L J_0 \vec{e}_1 \vec{e}_2$

so $\frac{\partial W_s}{\partial \vec{m}_1} = \vec{m}_1 \times \vec{m}_2$

$\vec{m}_1, \vec{m}_2$ are

fine core

since $\frac{\partial \vec{m}_1}{\partial s_{y1}} \sim \frac{\partial \vec{m}_2}{\partial s_{y1}}$

$\vec{m}_2^0 + \vec{m}_1^0 \delta \vec{m}_2^0$

$s_{y1} + \left(\frac{\partial \vec{m}_1}{\partial s_{y1}}\right) s_{y1}$

$\vec{m}_2^0 +$

$\perp s_{y2} = 0$, as

fine core

$\vec{m}_1$

on v. core

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$\vec{m}_1$

$$A = \vec{m} \cdot \text{rot} \vec{m} \sim \frac{1}{(s+1)^2}, \quad f = \frac{1}{2}(2-s) - \text{RUM}$$

$$c^2 |s|^2 = (2-s)(2-s) = s^2 - 2\vec{s} \cdot \vec{s} + |s|^2$$

$$A = \frac{1}{\left[1 + \frac{1}{c^2}(s^2 - 2\vec{s} \cdot \vec{s} + s^2)\right]^2} = \left[1 + \frac{2}{c^2}(s^2 - 2\vec{s} \cdot \vec{s} + s^2) + \frac{1}{c^4}(s^4 + 4(s\vec{s})^2 - 4\vec{s} \cdot \vec{s} s^2 + 2s^2 s^2)\right]^{-1}$$

$$= \left[\left(1 + \frac{s^2}{c^2}\right)^2 + \frac{2}{c^2}(-2\vec{s} \cdot \vec{s} - 2\vec{s} \cdot \vec{s} \frac{s^2}{c^2}) + \frac{2}{c^2} \left\{ s^2 + \frac{2}{c^2}(\vec{s} \cdot \vec{s})^2 + \frac{1}{c^2} s^2 s^2 \right\} \right]^{-1}$$

$$= \left[\left(1 + \frac{s^2}{c^2}\right)^2 - \frac{4\vec{s} \cdot \vec{s}}{c^2} \left(1 + \frac{s^2}{c^2}\right) + \frac{2s^2}{c^2} \left\{ \left(1 + \frac{s^2}{c^2}\right) + \frac{2}{c^2}(\vec{s} \cdot \vec{s})^2 \right\} \right]^{-1}$$

$$\frac{1}{a + b\varepsilon + d\varepsilon^2} = \frac{1}{a} \frac{1}{1 + \frac{b}{a}\varepsilon + \frac{d}{a}\varepsilon^2} = \left| \frac{1}{1+x} = 1 - x + x^2 + \dots \right|$$

$$= \frac{1}{a} \left[1 - \frac{b}{a}\varepsilon - \frac{d}{a}\varepsilon^2 + \frac{b^2}{a^2}\varepsilon^2 \right], \quad a = \left(1 + \frac{s^2}{c^2}\right)^2 > 0$$

$$A = \frac{1}{a} \left[1 + \frac{4}{c^2} \frac{\vec{s} \cdot \vec{s}}{a} \left(1 + \frac{s^2}{c^2}\right) - \frac{1}{a} \frac{2s^2}{c^2} \left\{ \dots \right\} + \frac{16(\vec{s} \cdot \vec{s})^2}{c^4 a^2} \left(1 + \frac{s^2}{c^2}\right)^2 \right]$$

$$= \frac{1}{a} \left[1 + (\vec{s}) + (s^2) \right]$$

$$(s^2) = \frac{1}{a} \frac{2s^2}{c^2} \left[-\left(1 + \frac{s^2}{c^2}\right) + \frac{2}{c^2}(\vec{s} \cdot \vec{s})^2 + \frac{2}{c^2}(\vec{s} \cdot \vec{s})^2 \right] =$$

$$= \frac{2s^2}{ac^2} \left[\frac{6}{c^2}(\vec{s} \cdot \vec{s})^2 - 1 - \frac{s^2}{c^2} \right]$$

$$W_D = DA, \quad W_D(s=0) = DA(0) = -|D|A(0) < 0$$

$$A(s=0) > 0$$

$$W_D(s) = -|D|A(s) = -|D| \frac{1}{a} \left[1 + \frac{4}{c^2} \frac{\vec{s} \cdot \vec{s}}{a} \left(1 + \frac{s^2}{c^2}\right) + \frac{2s^2}{ac^2} \left[\frac{6}{c^2}(\vec{s} \cdot \vec{s})^2 - 1 - \frac{s^2}{c^2} \right] \right]$$

$$= (\ln s) - |D| \frac{2s^2}{a^2 c^2} \left[\frac{6}{c^2} s^2 \cos^2(\varphi - \varphi_s) - 1 - \frac{s^2}{c^2} \right]$$

$$\int_0^{2\pi} \int_0^{2\pi} W_D(s) = (\dots) - |D| \frac{2s^2}{a^2 c^2} \left[2 \frac{s^2}{c^2} - 1 \right]$$

$$F'_{ST} = -aLR s_y \int d\vec{s} \left[\frac{\partial \vec{m}}{\partial s_y} \times \hat{z} \right] \cdot \frac{\partial \vec{m}}{\partial s_y} = -aLR s_y \hat{z} \cdot \int d\vec{s} \left[\frac{\partial \vec{m}}{\partial s_x} \times \frac{\partial \vec{m}}{\partial s_y} \right]$$

$$m_x = m_y + im_z = \frac{2W}{1+W}, \quad \frac{\partial m_x}{\partial s_x} = \frac{2}{1+W} \frac{\partial W}{\partial s_x} + 2W \frac{(-1)}{(1+W)^2} \frac{\partial}{\partial s_x} (W) =$$

$$= \frac{2}{(1+W)^2} \left[\frac{\partial W}{\partial s_x} - W^2 \frac{\partial W}{\partial s_x} \right], \quad W(1\vec{z}) = f(r) \text{ Any } \vec{r} \text{ core } \frac{\partial f}{\partial s_x}, \frac{\partial f}{\partial s_y} \text{ Au. 15/1071.}$$

$$\frac{\partial f}{\partial s_x} = \frac{\partial f}{\partial s} \frac{\partial s}{\partial s_x} = \frac{\partial f}{\partial s} = -\frac{i}{c} \sigma; \quad \text{for } s = s_x + i s_y \quad \frac{\partial f(s, s)}{\partial s_x} = \frac{\partial f}{\partial s} \frac{\partial s}{\partial s_x} + \frac{\partial f}{\partial \bar{s}} \frac{\partial \bar{s}}{\partial s_x} = \frac{\partial f}{\partial s} + \frac{\partial f}{\partial \bar{s}}$$

$$\frac{\partial f}{\partial s_y} = \frac{\partial f}{\partial s} \frac{\partial s}{\partial s_y} + \frac{\partial f}{\partial \bar{s}} \frac{\partial \bar{s}}{\partial s_y} = i \left(\frac{\partial f}{\partial s} - \frac{\partial f}{\partial \bar{s}} \right); \quad \frac{\partial f}{\partial s} = -\frac{i}{c} \sigma, \quad \frac{\partial f}{\partial \bar{s}} = i \frac{\sigma}{c} \frac{z^2}{1+z^2} / \text{...}$$

$$\frac{\partial m_z}{\partial s_x} = \frac{2}{(1+f\bar{f})^2} \left[\frac{\partial f}{\partial s_x} - f^2 \frac{\partial \bar{f}}{\partial s_x} \right] = \frac{2}{(1+f\bar{f})^2} \left[-\frac{i}{c} \sigma (1+z^2) - f^2 \frac{i}{c} \sigma (1+\bar{z}^2) \right]$$

$$= \frac{2}{(1+f\bar{f})^2} (-i) \frac{\sigma}{c} [1+z^2 + f^2 (1+\bar{z}^2)], \quad f = i \frac{\sigma}{c} z, \quad f^2 = -\frac{1}{c^2} z^2, \quad \bar{f} = i \frac{\sigma}{c} \bar{z}$$

$$\frac{\partial m_z}{\partial s_x} = \frac{\partial m_x}{\partial s_x} + i \frac{\partial m_y}{\partial s_x} = \frac{2}{(1+|z|^2/c^2)^2} (-i) \frac{\sigma}{c} \left[1+z^2 - \frac{z^2}{c^2} (1+\bar{z}^2) \right]$$

$$\frac{\partial m_z}{\partial s_y} = \frac{2}{(1+f\bar{f})^2} \left[\frac{\partial f}{\partial s_y} - f^2 \frac{\partial \bar{f}}{\partial s_y} \right] = \frac{2}{(1+f\bar{f})^2} \left[\frac{i}{c} \sigma (-1+z^2) - f^2 (-i) \frac{\sigma}{c} (1-\bar{z}^2) \right]$$

$$\frac{\partial m_z}{\partial s_y} = \frac{\partial m_x}{\partial s_y} + i \frac{\partial m_y}{\partial s_y} = \frac{2}{(1+|z|^2/c^2)^2} \frac{\sigma}{c} \left[1-z^2 + \frac{z^2}{c^2} (1-\bar{z}^2) \right]$$

$$\left[\frac{\partial \vec{m}}{\partial s_x} \times \frac{\partial \vec{m}}{\partial s_y} \right]_z = \frac{\partial m_x}{\partial s_x} \frac{\partial m_y}{\partial s_y} - \frac{\partial m_y}{\partial s_x} \frac{\partial m_x}{\partial s_y} = \text{...}$$

$$[x] = 1 + z^2 - \frac{z^2}{c^2} (1+\bar{z}^2) = 1 + x^2 - y^2 + 2ixy - \frac{1}{c^2} (x^2 - y^2 + 2ixy) [1 + x^2 - y^2 - 2ixy] =$$

$$= 1 + x^2 - y^2 + 2ixy - \frac{1}{c^2} [(x^2 - y^2)(1 + x^2 - y^2) + \dots] = 1 + (1 - \frac{1}{c^2}) z^2 - \frac{1}{c^2} |z|^4 =$$

$$= 1 - \frac{1}{c^2} |z|^4 + (1 - \frac{1}{c^2}) (x^2 - y^2 + 2ixy)$$

$$\rightarrow \frac{\partial m_x}{\partial s_x} = \frac{2}{(1+|z|^2/c^2)^2} \frac{\sigma}{c} (1 - \frac{1}{c^2}) 2xy; \quad \frac{\partial m_y}{\partial s_x} = \frac{2}{(\dots)^2 c} \left[1 - \frac{|z|^4}{c^2} + (1 - \frac{1}{c^2}) (x^2 - y^2) \right] (-i)$$

$$[y] = 1 + (\frac{1}{c^2} - 1) z^2 - \frac{|z|^4}{c^2} = 1 - \frac{1}{c^2} |z|^4 + (\frac{1}{c^2} - 1) (x^2 - y^2 + 2ixy)$$

$$\rightarrow \frac{\partial m_x}{\partial s_y} = \frac{2}{(\dots)^2 c} \left[1 - \frac{|z|^4}{c^2} + (\frac{1}{c^2} - 1) (x^2 - y^2) \right]; \quad \frac{\partial m_y}{\partial s_y} = \frac{2}{(\dots)^2 c} (\frac{1}{c^2} - 1) 2xy$$

$$\left[\frac{\partial \vec{m}}{\partial s_x} \times \frac{\partial \vec{m}}{\partial s_y} \right]_z = \frac{2^2}{(1+|z|^2/c^2)^4} \frac{1}{c^2} (-i) (\frac{1}{c^2} - 1)^2 4x^2 y^2 + \frac{4}{(\dots)^4} \frac{1}{c^2} \left[(1 - \frac{|z|^4}{c^2})^2 - (\frac{1}{c^2} - 1)^2 (x^2 - y^2)^2 \right]$$

$$= \frac{4}{c^2} \frac{1}{(\dots)^4} \left[(1 - \frac{|z|^4}{c^2})^2 - (\frac{1}{c^2} - 1)^2 (x^2 - y^2)^2 - (\frac{1}{c^2} - 1)^2 4x^2 y^2 \right] = \frac{4}{c^2 (\dots)^4} \times$$

$$\times \left[(1 - \frac{|z|^4}{c^2})^2 - (\frac{1}{c^2} - 1)^2 |z|^4 \right] = \frac{4}{c^2} \frac{1}{(1+|z|^2/c^2)^4} \left[1 - (\frac{1}{c^4} + 1) |z|^4 + \frac{1}{c^4} |z|^8 \right] = F(\rho)$$

... and give $\vec{m}_2 = \hat{z}$, and then $\left[\frac{\partial \vec{m}}{\partial s_x} \times \frac{\partial \vec{m}}{\partial s_y} \right]_z$ - component along the direction.

$$F_{ST}^x = -a L R s_y \oint_{core} \left[\frac{\partial \vec{m}}{\partial s_x} \times \frac{\partial \vec{m}}{\partial s_y} \right]_z = -2\pi a L Y J_p, \quad J_p = \oint_0^c F(\rho) \rho d\rho$$

У Квантового PRB'09 $F_{ST} = \pi M_s L \hat{z} \cdot (\hat{z} \times \vec{X})$, $F_{ST}^x \sim (\hat{z} \times \vec{X})_x = (y \cdot 1 - 0 \cdot 0) = -y$

сформулиру F_{ST} берем, коэф. супер слоя не берем.

$$F_{ST}^y = -La \oint_{core} (\vec{m} \times \hat{z}) \cdot \frac{\partial \vec{m}}{\partial y} = -La \oint_{core} \hat{z} \cdot \left[\frac{\partial \vec{m}}{\partial y} \times \vec{m} \right], \quad \vec{m} = \vec{m}_0 + \delta \vec{m}$$

$$F_{ST}^y = -La \oint_{core} \hat{z} \cdot \left[\frac{\partial \vec{m}}{\partial y} \times (\vec{m}_0 + \frac{\partial \vec{m}}{\partial x} x + \frac{\partial \vec{m}}{\partial y} y) \right] = -a L X \oint_{core} \hat{z} \cdot \left[\frac{\partial \vec{m}}{\partial y} \times \frac{\partial \vec{m}}{\partial x} \right] =$$

$$F_{ST}^x = -a L Y \oint_{core} \left[\frac{\partial \vec{m}}{\partial x} \times \frac{\partial \vec{m}}{\partial y} \right]_z = 2\pi a L J_p \cdot X, \text{ since } \vec{m}_0 \text{ is fixed}$$

$$\rightarrow F_{ST} = 2\pi a L J_p [-\hat{x} Y + \hat{y} X] = 2\pi a L J_p (\hat{z} \times \vec{X})$$

Uniform polarization $\vec{m}_2 = \hat{z}$, без вклада в $\vec{X}$ (reference layer).

$$F(\rho) = \frac{4}{c^2} \frac{1}{[1+\rho^2/c^2]^4} (1-\rho^4/c^4) (1-\rho^4) = \frac{4}{c^2} \frac{(1-\rho^2/c^2)^2}{(1+\rho^2/c^2)^3} (1-\rho^4)$$

$$J_p = \int_0^c \rho F(\rho) d\rho = \frac{4}{c^2} \int_0^c \rho \frac{(1-\rho^2/c^2)^2}{(1+\rho^2/c^2)^3} (1-\rho^4) d\rho = \left| \frac{\rho^2}{c^2} = x \right| = 2 \int_0^1 dx \frac{(1-x)}{(1+x)^3} (1-c^4 x^2)$$

т.к. $c = \frac{b}{a} < 1$, $c^4 x^2$ пренебрегаем.

$$J_p \approx 2 \int_0^1 dx \frac{(1-x)}{(1+x)^3} = 2 \int_0^1 dy \left[-\frac{1}{(1+x)^2} + \frac{2}{(1+x)^3} \right] = 2 \cdot \frac{1}{4} = \frac{1}{2}$$

$J_p = \frac{1}{2} + O(\epsilon^4)$. $\vec{F}_{ST} = \pi a L (\hat{z} \times \vec{X})$, $\text{Ivanov 'PRL' 01: } \vec{\tau}_s = \Delta I M_s \vec{m} \times (\vec{m} \times \hat{p})$
 Xbaronobecumi PRB'09: toqve $\frac{\partial J}{\partial M_s} \vec{M} \times (\vec{M} \times \hat{p})$, ama $\vec{F}_{ST} = \pi M_s L \Delta J (\hat{z} \times \vec{X})$
 Liu 'APL' 07: toqve $\vec{F}_j = \frac{a_j}{M_s} \vec{M} \times (\vec{M} \times \hat{p})$, ama $\vec{F}_{ST} = 2\pi a_j L M_s \rho_j^2 (\hat{z} \times \vec{X})$ $\rho_j^2 = \frac{1}{2} \frac{J}{M_s L}$
 $a_j = \hbar^2 P J / (M_s 2\epsilon M_s L)$

Y ama toqve $\vec{\tau}_s = (\Delta J) M_s \vec{m} \times (\vec{m} \times \hat{p}) = a \vec{m} \times (\vec{m} \times \hat{p})$, t.e. y ama $a = \Delta J M_s$
 to conseqence e odozhar. Ivanov 'PRL' 01 u Xbaronobecumi 'PRB' 09 $|J| = \hbar / c$

Ma a ST b yu-u Thiele: $(\vec{F}_{ST} = \pi L \Delta J M_s (\hat{z} \times \vec{X}))$ e to m. go ϵ^4
 Boubog: $\vec{F}_{ST}$ Xbaronobecumi '09 berpene, a $\vec{F}_{ST}$ - Ubaronobecumi 'PRL' 01 cogepramur umm... "2"

$$(\vec{F}_{ST})_{\text{Liu}} = 2\pi \frac{\hbar^2 P J}{4\epsilon M_s L} L M_s \frac{1}{8} (\hat{z} \times \vec{X}) = 2\pi \frac{\hbar P J}{4\epsilon R} (\hat{z} \times \vec{X})$$

Kozp. vlu $(\hat{z} \times \vec{X})$ con. Xbaronobecumi '09 $= \pi M_s L \frac{\hbar P}{2\epsilon L M_s} J = \pi \frac{\hbar P J}{2\epsilon L}$ t.e. y Liu'01
 eam bzax b csc $\mu_0 = 1$ u R omeem u X, bce pa bno bygger umm... "2" b aue,
 vak u y Ivanov'01. Ho Liu xemo umm... to upm... "2" u m...
 Ivanov'01 xom u upm... "pole-free" model (IVA), bce pa bno ney... "2"

Due fixed polarizer $\vec{m}_2 = \hat{z}$, $\vec{\tau}_s = a \vec{m} \times (\vec{m} \times \hat{z})$, $\vec{\tau}_s = \vec{m} \times \frac{\partial W_s}{\partial \vec{m}}$, $\frac{\partial W_s}{\partial \vec{m}} = a \vec{m} \times \hat{z}$
 $\vec{F}_{ST} = -\frac{\partial}{\partial \vec{X}} \int dV W_s = -L \int d^2 \vec{\rho} \frac{\partial W_s}{\partial \vec{m}} \frac{\partial \vec{m}}{\partial \vec{X}} = -a L \int d^2 \vec{\rho} (\vec{m} \times \hat{z}) \frac{\partial \vec{m}}{\partial \vec{X}}$, $F_{ST}^x = -a L \int d^2 \vec{\rho} \hat{z} \cdot [\frac{\partial \vec{m}}{\partial X_x} \times \vec{m}]$
 $\vec{m} = \vec{m}^0(\rho) + \frac{\partial \vec{m}}{\partial X_\beta} X_\beta$, $F_{ST}^x = -a L \int d^2 \vec{\rho} \hat{z} \cdot [\frac{\partial \vec{m}}{\partial X_x} \times (\vec{m}^0 + \frac{\partial \vec{m}}{\partial X_\beta} X_\beta)] = -a L \int d^2 \vec{\rho} \hat{z} \cdot [\frac{\partial \vec{m}}{\partial X_x} \times \vec{m}^0] -$
 $- a L \int d^2 \vec{\rho} \hat{z} \cdot [\frac{\partial \vec{m}}{\partial X_x} \times \frac{\partial \vec{m}}{\partial X_\beta} X_\beta]$. To za xom $\int d^2 \vec{\rho} \hat{z} \cdot [\frac{\partial \vec{m}}{\partial X_x} \times \vec{m}^0] = ?$ $\vec{m}^0(\rho) = m_x^0 \hat{x} + m_y^0 \hat{y}$
 Ho $\frac{\partial \vec{m}}{\partial X_x}$ uomer zabax xemo e $\int d^2 \vec{\rho} (\dots) \neq 0$.

$$\frac{\partial \vec{m}}{\partial X_x} \times \vec{m}^0 = \frac{\partial}{\partial X_x} \vec{m} \times \vec{m}^0 = \frac{\partial}{\partial X_x} (\vec{m}^0 + \frac{\partial \vec{m}}{\partial X_\beta} X_\beta) \times \vec{m}^0 = \frac{\partial}{\partial X_x} [X_\beta \frac{\partial \vec{m}}{\partial X_\beta} \times \vec{m}^0] =$$

$$= \delta_{\beta x} \frac{\partial \vec{m}}{\partial X_\beta} \times \vec{m}^0 + X_\beta \frac{\partial^2 \vec{m}}{\partial X_x \partial X_\beta} \times \vec{m}^0 = \hat{z} \cdot [\frac{\partial \vec{m}}{\partial X_x} \times \vec{m}^0]_{\text{region } x, y \text{ comp. } \vec{m}_1, \vec{m}_2} = \hat{z} \cdot \frac{\partial}{\partial X_x} \begin{vmatrix} m_x & m_y & 0 \\ m_x^0 & m_y^0 & 0 \end{vmatrix} =$$

$$= \hat{z} \frac{\partial}{\partial X_x} (m_x m_y^0 - m_y m_x^0) = \frac{\partial}{\partial X_x} (m_x m_y^0 - m_y m_x^0) = m_y^0(\rho) [\cos \varphi \frac{\partial m_x}{\partial X_x} + \frac{\partial m_y}{\partial X_x} \sin \varphi].$$

- ama y abno bnga $\frac{\partial m_{x,y}}{\partial X_x}$ ney. to $\int d^2 \vec{\rho} \hat{z} \cdot [\frac{\partial \vec{m}}{\partial X_x} \times \vec{m}^0] = 0$.

→ Citi Credit Card go 15/XI 097. / PolyZur NSF-MICINN g rane or Cabauna
 "Lopa chane Popya" - € 800. Bce - € 5000 u uocam b MICINN go 10/XI.

7/XI Bepence u double-vortex dynamics. $\vec{m} = \vec{m}_1$, $\hat{z} \rightarrow \vec{m}_2$. $F/M/F$
 Con. $6/\sqrt{1}$ vlu $\int d^2 \vec{\rho} (\dots)$ bce coe $\vec{F}_{ST} = 0$, t.e. V. coe upm... b aue.
 Pru b... $\vec{F}_{ST}(1)$ o $6/\sqrt{1}$ boz... u... 3-x... J_{y1}, J_{x2}, J_{y2} .

$$\rightarrow J_{y1} = \int d^2 \vec{\rho} \vec{m}_2^0 \cdot [\frac{\partial \vec{m}_1}{\partial s_{x1}}, \frac{\partial \vec{m}_1}{\partial s_{y1}}], J_{x2} = \int d^2 \vec{\rho} \vec{m}_1^0 \cdot [\frac{\partial \vec{m}_2}{\partial s_{x1}}, \frac{\partial \vec{m}_2}{\partial s_{y2}}], J_{y2} = \int d^2 \vec{\rho} \vec{m}_1^0 \cdot [\frac{\partial \vec{m}_2}{\partial s_{x1}}, \frac{\partial \vec{m}_2}{\partial s_{y2}}]$$

Vortex b 1-on Cope uuer $\rho_1, \epsilon_1, \vec{s}_1$, Vortex bo 2-on Cope ($\vec{m}_2$) - $\rho_2, \epsilon_2, \vec{s}_2$.

$$\vec{F}_{ST}^d(1) = -a L R \int d^2 \vec{\rho} (\vec{m}_1 \times \vec{m}_2) \frac{\partial \vec{m}_1}{\partial s_{x1}}, \vec{F}_{ST}^d(1) \cdot \vec{s}_{d1} = \vec{F}_{ST}(1) \cdot \vec{s}_1 = -a L R \int d^2 \vec{\rho} (\vec{m}_1 \times \vec{m}_2) \cdot \delta \vec{m}_1$$

$$\vec{m}_j = \vec{m}_j^0 + \delta \vec{m}_j, \delta \vec{m}_j = \frac{\partial \vec{m}_j}{\partial s_{xj}} s_{xj} + \frac{\partial \vec{m}_j}{\partial s_{yj}} s_{yj}, j=1,2. (\vec{m}_1 \times \vec{m}_2) \cdot \delta \vec{m}_1 = [\vec{m}_1^0 \times \vec{m}_2^0 +$$

$$+ \delta \vec{m}_1 \times \vec{m}_2^0 + \vec{m}_1^0 \times \delta \vec{m}_2] \cdot \delta \vec{m}_1 = [\vec{m}_1^0 \times \delta \vec{m}_2] \cdot \delta \vec{m}_1 = -\vec{m}_1^0 \cdot [\delta \vec{m}_1 \times \delta \vec{m}_2]. \vec{m}_j^0 \cdot \delta \vec{m}_j^0 = 0$$

$$\vec{F}_{ST}(1) \cdot \vec{s}_1 = a L R \int d^2 \vec{\rho} \vec{m}_1^0 \cdot [\delta \vec{m}_1 \times \delta \vec{m}_2], \vec{\tau}_1 = a \vec{m}_1 \times (\vec{m}_1 \times \vec{m}_2), \vec{m}_1^0 = m_{1\varphi}^0 \hat{\varphi} + m_{1z}^0 \hat{z}$$

$$J_{y1} = \int d^2 \vec{\rho} \left\{ \vec{m}_2^0 \cdot [\frac{\partial \vec{m}_1}{\partial s_{x1}} \times \frac{\partial \vec{m}_1}{\partial s_{y1}}] + m_{2\varphi}^0 \cdot [\frac{\partial \vec{m}_1}{\partial s_{x1}} \times \frac{\partial \vec{m}_1}{\partial s_{y1}}] \right\}, [a \times b]_\varphi = a_z b_\varphi - a_\varphi b_z$$

$$\text{Vpigerac Cmarb } \frac{\partial m_z}{\partial s_{x1}}, \frac{\partial m_z}{\partial s_{y1}}, a_\varphi = a_x \cos \varphi + a_y \sin \varphi, a_\varphi = -a_x \sin \varphi + a_y \cos \varphi.$$

Coro 28/III 01. 6 pole free model: $m_2(\vec{r}, \vec{s}) = m_2(\vec{r}, 0) + p \frac{(1 + \vec{r} \cdot \vec{s})}{(r^2 + s^2)^2}$

$$\frac{\partial m_2}{\partial s_x} = p \frac{4r(1 + \vec{r} \cdot \vec{s})}{(r^2 + s^2)^2} \cos \varphi, \quad \frac{\partial m_2}{\partial s_y} = p \frac{4r(1 + \vec{r} \cdot \vec{s})}{(r^2 + s^2)^2} \sin \varphi, \quad g(\vec{r}) = p \frac{4r^2(1 + \vec{r} \cdot \vec{s})}{(r^2 + s^2)^2}$$

$$\left[\frac{\partial \vec{m}_1}{\partial s_{x1}} \times \frac{\partial \vec{m}_1}{\partial s_{y1}} \right]_z = \frac{\partial m_{1z}}{\partial s_{x1}} \frac{\partial m_{1y}}{\partial s_{y1}} - \frac{\partial m_{1y}}{\partial s_{x1}} \frac{\partial m_{1z}}{\partial s_{y1}} = \frac{\partial m_{1z}}{\partial s_{x1}} \left[\frac{\partial m_{1y}}{\partial s_{y1}} \cos \varphi + \frac{\partial m_{1y}}{\partial s_{y1}} \sin \varphi \right]$$

$$= \frac{\partial m_{1z}}{\partial s_{y1}} \left[\frac{\partial m_{1x}}{\partial s_{x1}} \cos \varphi + \frac{\partial m_{1y}}{\partial s_{x1}} \sin \varphi \right] = \left| \text{non-zero (due to)} \right| \neq \frac{\partial m_{1z}}{\partial s_{x1}} \frac{\partial m_{1x}}{\partial s_{y1}} \cos \varphi -$$

$$- \frac{\partial m_{1z}}{\partial s_{y1}} \frac{\partial m_{1y}}{\partial s_{x1}} \sin \varphi = \left| \text{non-zero (due to)} \right| = g_1(\vec{r}) \cos^2 \varphi \frac{\partial m_{1x}}{\partial s_{y1}} + g_1(\vec{r}) \cos \varphi \sin \varphi \frac{\partial m_{1y}}{\partial s_{y1}} -$$

$$- g_1 \sin \varphi \cos \varphi \frac{\partial m_{1x}}{\partial s_{x1}} - g_1 \sin^2 \varphi \frac{\partial m_{1y}}{\partial s_{x1}}, \text{ because } 6/\pi \text{ non-zero in } \vec{r} \text{ and } \vec{s} \text{ are independent.}$$

$$\left[\frac{\partial \vec{m}_1}{\partial s_{x1}} \times \frac{\partial \vec{m}_1}{\partial s_{y1}} \right]_z = \text{const } 4/\pi \text{ if } F(\vec{r}) \text{ is } \delta F(\dots) \neq 0, \text{ then } m_2^0(\vec{r}) \propto \text{const.}$$

$$\vec{y}_{12} = \int d^2 \vec{r} \left\{ m_{12}^0 \left[\frac{\partial \vec{m}_1}{\partial s_{x1}} \times \frac{\partial \vec{m}_2}{\partial s_{x2}} \right]_z + m_{1y}^0 \left[\frac{\partial \vec{m}_1}{\partial s_{x1}} \times \frac{\partial \vec{m}_2}{\partial s_{y2}} \right]_y \right\}$$

$$\left[\frac{\partial \vec{m}_1}{\partial s_{x1}} \times \frac{\partial \vec{m}_2}{\partial s_{x2}} \right]_z = \frac{\partial m_{1x}}{\partial s_{x1}} \frac{\partial m_{2y}}{\partial s_{x2}} - \frac{\partial m_{1y}}{\partial s_{x1}} \frac{\partial m_{2x}}{\partial s_{x2}} = \text{gives "0" when } \delta F(\dots), \text{ i.e. } \text{cylinder } xy \text{ and } xy(x^2 + y^2).$$

$$\left[\frac{\partial \vec{m}_1}{\partial s_{x1}} \times \frac{\partial \vec{m}_2}{\partial s_{y2}} \right]_y = \frac{\partial m_{1z}}{\partial s_{x1}} \frac{\partial m_{2y}}{\partial s_{y2}} - \frac{\partial m_{1y}}{\partial s_{x1}} \frac{\partial m_{2z}}{\partial s_{y2}} = g_1 \cos \varphi \left[\frac{\partial m_{2x}}{\partial s_{x2}} \cos \varphi + \frac{\partial m_{2y}}{\partial s_{x2}} \sin \varphi \right] -$$

$$- g_2 \cos \varphi \left[\frac{\partial m_{1x}}{\partial s_{x1}} \cos \varphi + \frac{\partial m_{1y}}{\partial s_{x1}} \sin \varphi \right] = \text{gives "0" because } \delta F(\dots).$$

$\vec{y}_{12}$ is non-zero (due to non-zero of "0") since we have y_{11} . $y_{12} \neq 0, y_{11} \neq 0$

$$\rightarrow F_{ST}^x(1) = -aLR \left[y_{y1} s_{y1} - y_{y2} s_{y2} \right] = -aL (y_{y1} y_{y1} - y_{y2} y_{y2}) \quad y_{x2} = 0.$$

$$F_{ST}^y(1) = -aLR \int d^2 \vec{r} (\vec{m}_1 \times \vec{m}_2) \cdot \frac{\partial \vec{m}_1}{\partial s_{y1}} = -aLR \int d^2 \vec{r} \left[\left\{ \frac{\partial \vec{m}_1}{\partial s_{x1}} s_{x1} + \frac{\partial \vec{m}_1}{\partial s_{y1}} s_{y1} \right\} \times \vec{m}_2^0 + \right.$$

$$\left. + \vec{m}_1^0 \times \left\{ \frac{\partial \vec{m}_2}{\partial s_{x2}} s_{x2} + \frac{\partial \vec{m}_2}{\partial s_{y2}} s_{y2} \right\} \right] \cdot \frac{\partial \vec{m}_1}{\partial s_{y1}} = -aLR \int d^2 \vec{r} \left[s_{x1} \frac{\partial \vec{m}_1}{\partial s_{x1}} \times \vec{m}_2^0 + \vec{m}_1^0 \times \left(\frac{\partial \vec{m}_2}{\partial s_{x2}} s_{x2} + \frac{\partial \vec{m}_2}{\partial s_{y2}} s_{y2} \right) \right] \cdot \frac{\partial \vec{m}_1}{\partial s_{y1}}$$

$$F_{ST}^y(1) = -aLR \int d^2 \vec{r} \left\{ s_{x1} \frac{\partial \vec{m}_1}{\partial s_{y1}} \cdot \left[\frac{\partial \vec{m}_1}{\partial s_{x1}} \times \vec{m}_2^0 \right] + s_{y2} \frac{\partial \vec{m}_1}{\partial s_{y1}} \cdot \left[\vec{m}_1^0 \times \frac{\partial \vec{m}_2}{\partial s_{y2}} \right] \right\} =$$

$$= -aL \int d^2 \vec{r} \left\{ -X_1 \vec{m}_2^0 \cdot \left[\frac{\partial \vec{m}_1}{\partial s_{x1}} \times \frac{\partial \vec{m}_1}{\partial s_{y1}} \right] + X_2 \vec{m}_1^0 \cdot \left[\frac{\partial \vec{m}_2}{\partial s_{x2}} \times \frac{\partial \vec{m}_1}{\partial s_{y1}} \right] \right\} = -aL \int d^2 \vec{r} \left[-\tilde{y}_{x1} \tilde{y}_{x2} + \tilde{y}_{x2} \tilde{y}_{x1} \right]$$

$$\rightarrow F_{ST}^y(1) = -aL \left[-y_{x1} X_1 + y_{x2} X_2 \right], \text{ because } y_{x1} = y_{y1}. \text{ Сравниваем } y_{y2} \text{ и } \tilde{y}_{x2}.$$

$$y_{y2} = \int d^2 \vec{r} \vec{m}_1^0 \cdot \left[\frac{\partial \vec{m}_1}{\partial s_{x1}} \times \frac{\partial \vec{m}_2}{\partial s_{y2}} \right], \quad \tilde{y}_{x2} = \int d^2 \vec{r} \vec{m}_1^0 \cdot \left[\frac{\partial \vec{m}_2}{\partial s_{x2}} \times \frac{\partial \vec{m}_1}{\partial s_{y1}} \right], \text{ because } \tilde{y}_{y2} = \tilde{y}_{x2}.$$

Обозначим $y_1 = y_{x1} = y_{y1}, y_2 = y_{y2} = \tilde{y}_{x2}$.

$$\vec{F}_{ST}(1) = -aL \left[(y_1 y_1 - y_2 y_2) \hat{x} + (-y_1 X_1 + y_2 X_2) \hat{y} \right] = aL \left[y_1 (\hat{z} \times \vec{X}_1) - y_2 (\hat{z} \times \vec{X}_2) \right]$$

$$\vec{z} \times \vec{X} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ x & y & 0 \end{vmatrix} = -y \hat{x} + x \hat{y}. \quad y_1 \text{ забывает, что параметров всего шесть } (p_1, p_2, c_1, c_2)$$

УД: $\vec{G}_1 \times \vec{X}_1 - \alpha_1 \vec{X}_1 - \mu \vec{X}_2 + I_1 (\hat{z} \times \vec{X}_1) - I_2 (\hat{z} \times \vec{X}_2) = 0, \quad \vec{F}_{ST}(2) = I_2 (\hat{z} \times \vec{X}_2) - I_1 (\hat{z} \times \vec{X}_1)$

F/N/F: $\vec{G}_2 \times \vec{X}_2 - \alpha_2 \vec{X}_2 - \mu \vec{X}_1 + I_2 (\hat{z} \times \vec{X}_2) - I_1 (\hat{z} \times \vec{X}_1) = 0$

ST forces: Если $\vec{G}_1 = \vec{G}_2, \alpha_1 = \alpha_2$ (т.е. одинак. сила $c(p_1 p_2 = 1)$), то система $\vec{X} = \frac{1}{2}(\vec{X}_1 + \vec{X}_2)$ - правильный ответ не забывает.

Но I_1 и I_2 брутально. Поскольку не знаем $I_1 = \pm I_2$ (сам брутально).

ST (механика) брутально в штиле-eqs. $(\hat{z} \times \vec{X}_i)$ и нечеткое $\vec{G}$ замкнуто. со брутально in-plane variable field.

Упр. I_1, I_2 и нечеткое $\vec{G}$ и нечеткое $\vec{X}$ van topol. charge q , и $\vec{G}_2 \sim g_{xy} - g_{yx} \sim 2$

$$q \sim \int d^2 \vec{r} \vec{m} \cdot \left[\frac{\partial \vec{m}}{\partial x} \times \frac{\partial \vec{m}}{\partial y} \right]. \quad \text{Упробие перем. } c \sim q \sim \int d^2 \vec{r} \sin \theta [\partial \theta \times \partial \varphi] \text{ и } \text{house.}$$

6/11 Находим $\vec{r}$ и $\vec{p}$ в координатах θ, Φ , $\vec{r} = r(\theta, \Phi)$ - радиус-вектор. Тогда $\vec{p} = m\dot{\vec{r}}$.
 I_1, I_2 - моменты инерции относительно осей $\hat{e}_1, \hat{e}_2$.
 $m_s \dot{\vec{r}} = \dot{\vec{r}} + \frac{\partial \vec{r}}{\partial t}$, $\frac{m_s}{\delta} [\dot{\vec{r}} \cdot \dot{\vec{r}} + \dot{\vec{r}} \cdot \frac{\partial \vec{r}}{\partial t}] = -\frac{1}{\sin \theta} \frac{\partial W}{\partial \Phi} \hat{\theta} + \frac{\partial W}{\partial \theta} \hat{\Phi}$ - уравнение движения.
 Тогда $\vec{r} = \vec{r} + \frac{\partial \vec{r}}{\partial t}$, $\vec{r}_m = 0$, $\vec{r}_\theta = -\frac{1}{\sin \theta} \frac{\partial W}{\partial \Phi}$, $\vec{r}_\Phi = \frac{\partial W}{\partial \theta}$. Тогда $\vec{r} = a \vec{m} \times (\vec{m} \times \hat{p})$ - радиус-вектор точки.

$\frac{\partial W_s}{\partial \vec{r}} = a \vec{m} \times \hat{p} = \hat{m} \frac{\partial W_s}{\partial m} + \hat{\theta} \frac{\partial W_s}{\partial \theta} + \hat{\Phi} \frac{1}{\sin \theta} \frac{\partial W_s}{\partial \Phi} = \hat{m} \left(\frac{\partial W_s}{\partial m} \right)_m + \hat{\theta} \left(\frac{\partial W_s}{\partial \theta} \right)_\theta + \hat{\Phi} \left(\frac{\partial W_s}{\partial \Phi} \right)_\Phi$
 $\frac{\partial W_s}{\partial \theta} = a (\vec{m} \times \hat{p})_\theta$, $\frac{1}{\sin \theta} \frac{\partial W_s}{\partial \Phi} = a (\vec{m} \times \hat{p})_\Phi$. Def. $\vec{m} \times \hat{p} = \vec{b}$. Найдем b_θ, b_Φ .
 $\vec{m} = \hat{x} m_x + \hat{y} m_y + \hat{z} m_z = m_x (\hat{z} \sin \theta \cos \Phi + \hat{\theta} \cos \theta \cos \Phi - \hat{\Phi} \sin \Phi) + m_y (\hat{x} \sin \theta \sin \Phi + \hat{\theta} \cos \theta \sin \Phi + \hat{\Phi} \cos \Phi) + m_z (\hat{x} \cos \theta - \hat{\theta} \sin \theta)$.
 $\vec{r} = \vec{r}(\theta, \Phi)$, $\vec{m} = \vec{m}(\theta, \Phi)$.

$m_\theta = m_x \cos \theta \cos \Phi + m_y \cos \theta \sin \Phi - m_z \sin \theta$, $m_\Phi = -m_x \sin \Phi + m_y \cos \Phi$

Но $m_x = \sin \theta \cos \Phi$, $m_y = \sin \theta \sin \Phi$, $m_z = \cos \theta$. $\vec{b} = \vec{e}$, $\vec{e}, \vec{e}_\theta, \vec{e}_\Phi$ - ортонормированный базис.
 $m_\theta = \sin \theta \cos \Phi \cos \theta \cos \Phi + \sin \theta \sin \Phi \cos \theta \sin \Phi - \cos \theta \sin \theta = \sin \theta \cos \theta \cos(\Phi - \Phi) - \cos \theta \sin \theta$
 $m_\Phi = -\sin \theta \cos \Phi \sin \Phi + \sin \theta \sin \Phi \cos \Phi = \sin \theta \sin(\Phi - \Phi)$.

$\hat{p} = \hat{z}$, $\hat{z} = \hat{z} \cos \theta - \hat{\theta} \sin \theta$; $\vec{m} \times \hat{z} = \begin{vmatrix} \hat{z} & \hat{\theta} & \hat{\Phi} \\ m_z & m_\theta & m_\Phi \\ \cos \theta & -\sin \theta & 0 \end{vmatrix} = \hat{\theta} (m_\Phi \cos \theta) + \hat{\Phi} (\dots) + \dots$
 Проверим $\vec{m} \times \hat{z}$ с помощью б. о. с. $\vec{m} \times \hat{z} = \begin{vmatrix} \hat{z} & \hat{\theta} & \hat{\Phi} \\ m_z & m_\theta & m_\Phi \\ \cos \theta & -\sin \theta & 0 \end{vmatrix} = \hat{\theta} (m_\Phi \cos \theta) + \hat{\Phi} (\dots) + \dots$

Учитывая $\vec{r} \parallel \vec{m}$, через θ, Φ . $\vec{m} = m \hat{m} + m_\theta \hat{\theta} + m_\Phi \hat{\Phi} = \hat{m}$
 $\hat{z} = \hat{m} \cos \theta - \hat{\theta} \sin \theta$. $\vec{m} \times \hat{z} = \begin{vmatrix} \hat{m} & \hat{\theta} & \hat{\Phi} \\ 1 & 0 & 0 \\ \cos \theta & -\sin \theta & 0 \end{vmatrix} = -\sin \theta \hat{\Phi}$.

$\left(\frac{\partial W_s}{\partial \vec{m}} \right)_\theta = \frac{\partial W_s}{\partial \theta} = a (\vec{m} \times \hat{z})_\theta = 0$

$\left(\frac{\partial W_s}{\partial \vec{m}} \right)_\Phi = \frac{1}{\sin \theta} \frac{\partial W_s}{\partial \Phi} = a (\vec{m} \times \hat{z})_\Phi = -a \sin \theta$, $\frac{\partial W_s}{\partial \Phi} = -a \sin^2 \theta$.

LLG: $\frac{m_s}{\delta} \dot{\Phi} \sin \theta = \frac{\partial W}{\partial \theta}$, $\left(-\frac{m_s}{\delta} \sin \theta \dot{\theta} = \frac{\partial W}{\partial \Phi} + \frac{\partial W_s}{\partial \Phi} = \frac{\partial W}{\partial \Phi} - a \sin^2 \theta \right)$, $W = W' + W_s$.

Это eq. совпадает с уравнением (2.13) (см. гл. 1) и с уравнением (2.14) (см. гл. 2).

$\vec{F}_{ST} = - \frac{\partial}{\partial \vec{X}} \int dV W_s = - \int dV \left[\frac{\partial W_s}{\partial \theta} \frac{\partial \theta}{\partial \vec{X}} + \frac{\partial W_s}{\partial \Phi} \frac{\partial \Phi}{\partial \vec{X}} \right] = -L \int d^2 \vec{\rho} \frac{\partial W_s}{\partial \Phi} \frac{\partial \Phi}{\partial \vec{X}}$

$\vec{F}_{ST} = aL \int d^2 \vec{\rho} \sin^2 \theta \frac{\partial \Phi}{\partial \vec{X}}$. В pole-free model $\nabla_{\vec{X}} \Phi = m_0(\rho) [\sin \varphi \hat{x} - \cos \varphi \hat{y}]$
 $\sin^2 \theta = 1 - \cos^2 \theta = 1 - m_z^2(\vec{\rho})$, $\vec{F}_{ST} = -aL \int d^2 \vec{\rho} m_z^2(\vec{\rho}) \nabla_{\vec{X}} \Phi$ - superconductor core.

$m_z(\vec{\rho}, \vec{s}) = m_z^0(\rho) + \rho \frac{4c^2(1+\rho^2)}{(c^2 + \rho^2)^2} (\vec{s} \cdot \vec{\rho})$. $W(\vec{z}) = \frac{1}{2} \log \frac{z - z_1}{z - z_2}$. $\frac{\partial W}{\partial \vec{X}} = \frac{1}{2} \frac{\partial \log \frac{z - z_1}{z - z_2}}{\partial \vec{X}}$

$\Phi = \arg(w)$ и не должно зависеть от $w = f/|f|$. One core with $w = f$ & core. $\Phi = \arg(f)$ - core.

$f(z) = \frac{1}{c} \left[z - \frac{(s + \bar{s} z^2)}{1 + |s|^2} \right] = A(z - z_1)(z - z_2)$, z_1 - vortex, z_2 - "image" vortex.

Рассмотрим $\vec{r}(\vec{\rho}, \vec{s}) = \frac{c}{2} \frac{\pi}{2} + \varphi + m_0(\rho) [X \sin \varphi - Y \cos \varphi]$, $\nabla_{\vec{X}} \Phi = m_0(\rho) [\sin \varphi \hat{x} - \cos \varphi \hat{y}]$.
 - зависит от "core" и "one core" // $m_0(\rho) = \frac{1}{2} (1 - \rho^2)$ - one core.

$m_z(\vec{\rho}, \vec{s}) = m_z^0(\rho) + g(\rho) (\vec{s} \cdot \vec{\rho})$, $g(\rho) = \rho \frac{4c^2 \rho (1 + \rho^2)}{(c^2 + \rho^2)^2}$. $m_z^2 = (m_z^0)^2 + 2m_z^0 g(\vec{s} \cdot \vec{\rho})$, $m_z^0 g \sim \rho$

$\vec{F}_{ST} = -aL \int d^2 \vec{\rho} m_z^2(\vec{\rho}, \vec{s}) \nabla_{\vec{X}} \Phi = -aL \int d^2 \vec{\rho} [(m_z^0)^2 + 2m_z^0 g(\vec{s} \cdot \vec{\rho})] m_0(\rho) [\sin \varphi \hat{x} - \cos \varphi \hat{y}] =$
 $= -2aL \int d^2 \vec{\rho} m_z^0 m_0 g (s_x \cos \varphi + s_y \sin \varphi) (\sin \varphi \hat{x} - \cos \varphi \hat{y}) = -2\pi aL \int d\rho \rho m_z^0 g m_0 [s_y \hat{x} - s_x \hat{y}]$

$\vec{F}_{ST} = 2\pi aL \mathcal{J}_P(\vec{s} \times \vec{X})$, $\mathcal{J}_P = \int d^2 \vec{\rho} m_z^0 g m_0(\rho)$ - гауссовы коэффициенты с $\mathcal{J}_P = \int d^2 \vec{\rho} F(\rho)$ 6/11.

$\mathcal{J}_P = \rho \int_0^c d\rho \rho \frac{(1 - \rho^2/c^2)}{(1 + \rho^2/c^2)^2} \frac{4}{c^2} \frac{(1 + \rho^2/c^2)}{(1 + \rho^2/c^2)^2} \cdot \frac{1}{2} (1 - \rho^2) = \frac{4}{c^2} \rho \int_0^c d\rho \rho \frac{(1 - \rho^2/c^2)}{(1 + \rho^2/c^2)^3} (1 - \rho^2) = \rho \cdot \mathcal{J}_P(6/11)$ - const.

Тензор $\hat{J}_p = \frac{1}{2}(1, p = \pm 1 - \text{VC polarization})$. $(\vec{F}_{ST} = \pi L a p (\hat{z} \times \vec{X}))$, $a = 2.5 M_3$.

Указано: P1'07 и X8000: PRB'09 VC polarization $p = \pm 1$ - pure polarization $m_2 = 2$
 & $\vec{F}_{ST}$ изогнутое. "p" должно, т.е. если $p \geq 0$ $\vec{F}_{ST}$ направл. вправо, если $p < 0$ - влево.
 Vortex polarization tensor $\hat{J}$, если $p \geq 0$ - clockwise, если $p < 0$ - anticlockwise.
 You may use vector $\vec{r} = x\hat{x} + y\hat{y}$.

Вспомогат. double vortex dyn. & $\vec{F}/N/F$ etc. Layer j : $M_{sj}, d_j, L_j, A_j, P_j, \Phi_j$
 R - circular oblique. CPP - SP current.

На рис. показаны $\vec{S}$ -vortex $-\vec{H}_{ST}(1) = \frac{\partial W_3}{\partial \vec{m}_1} = a(\vec{m}_1 \times \vec{m}_2) = \hat{m}_1 \frac{\partial W_3}{\partial \vec{m}_1} + \hat{\theta}_1 \frac{\partial W_3}{\partial \theta_1} + \hat{\Phi}_1 \frac{\partial W_3}{\partial \Phi_1}$
 $\vec{T}_1 = a \vec{m}_1 \times (\vec{m}_1 \times \vec{m}_2)$, $\vec{m}_j(\theta_j, \Phi_j)$.

For a given $m_2 = 2$ case, circular $(\vec{m}_1 \times \vec{m}_2)$ & circular vortex $\in \vec{m}_1 \parallel \hat{z}$.

$$\vec{m}_1 \times \vec{m}_2 = \begin{vmatrix} \hat{m}_1 & \hat{\theta}_1 & \hat{\Phi}_1 \\ 1 & 0 & 0 \\ m_2^{\theta_1} & m_2^{\Phi_1} & m_2^{\Phi_1} \end{vmatrix} = \hat{\Phi}_1 m_2^{\theta_1} - \hat{\theta}_1 m_2^{\Phi_1} \quad \frac{\partial W_3}{\partial \theta_1} = -a m_2^{\Phi_1}, \quad \frac{\partial W_3}{\partial \Phi_1} = a m_2^{\theta_1} \sin \theta_1$$

$$\vec{m}_2 = \hat{m}_1(\vec{m}_2)_{\hat{m}_1} + \hat{\theta}_1(\vec{m}_2)_{\hat{\theta}_1} + \hat{\Phi}_1(\vec{m}_2)_{\hat{\Phi}_1} \quad m_2^{\theta_1} = (\vec{m}_2 \cdot \hat{\Phi}_1); \quad m_2^{\Phi_1} = (\vec{m}_2 \cdot \hat{\theta}_1)$$

$$\vec{m}_2 = \hat{x} \sin \theta_2 \cos \Phi_2 + \hat{y} \sin \theta_2 \sin \Phi_2 + \hat{z} \cos \theta_2, \quad \text{circular vortex } xyz \text{ y circular oblique } F_{ST}$$

or $\hat{x}\hat{y}\hat{z}$ representation $\times \hat{m}_1, \hat{\theta}_1, \hat{\Phi}_1$ (since $\vec{m}_2 = \hat{z}$, since $\hat{z} = \hat{m}_1 \cos \theta_1 - \hat{\theta}_1 \sin \theta_1$).

$$\begin{cases} \hat{x} = \hat{m}_1 \sin \theta_1 \cos \Phi_1 + \hat{\theta}_1 \cos \theta_1 \cos \Phi_1 - \hat{\Phi}_1 \sin \Phi_1 \\ \hat{y} = \hat{m}_1 \sin \theta_1 \sin \Phi_1 + \hat{\theta}_1 \cos \theta_1 \sin \Phi_1 + \hat{\Phi}_1 \cos \Phi_1 \\ \hat{z} = \hat{m}_1 \cos \theta_1 - \hat{\theta}_1 \sin \theta_1 \end{cases} \quad \begin{aligned} (\vec{m}_2 \cdot \hat{\Phi}_1) &= \sin \theta_2 \cos \Phi_2 (-\sin \Phi_1) + \sin \theta_2 \sin \Phi_2 \cos \Phi_1 + \cos \theta_2 \cdot 0 = \\ &= \sin \theta_2 \sin(\Phi_2 - \Phi_1) \end{aligned}$$

$$(\vec{m}_2 \cdot \hat{\theta}_1) = \sin \theta_2 \cos \Phi_2 \cdot \cos \theta_1 \cos \Phi_1 + \sin \theta_2 \sin \Phi_2 \cos \theta_1 \sin \Phi_1 + \cos \theta_2 (-\sin \theta_1) = \sin \theta_2 \cos \theta_1 \cos(\Phi_2 - \Phi_1) - \sin \theta_1 \cos \theta_2$$

$$\begin{cases} -\frac{M_{s1}}{d_1} \sin \theta_1 \hat{\theta}_1 = \frac{\partial W_1}{\partial \Phi_1} + \frac{\partial W_3}{\partial \Phi_1} = \frac{\partial W_1}{\partial \Phi_1} + a m_2^{\theta_1} \sin \theta_1 = \frac{\partial W_1}{\partial \Phi_1} + a [\sin \theta_2 \cos \theta_1 \cos(\Phi_2 - \Phi_1) - \sin \theta_1 \cos \theta_2] \sin \theta_1 \\ \frac{M_{s1}}{d_1} \hat{\Phi}_1 \sin \theta_1 = \frac{\partial W_1}{\partial \theta_1} + \frac{\partial W_3}{\partial \theta_1} = \frac{\partial W_1}{\partial \theta_1} - a (\vec{m}_2 \cdot \hat{\Phi}_1) = \frac{\partial W_1}{\partial \theta_1} - a \sin \theta_2 \sin(\Phi_2 - \Phi_1), \quad W_3 = W_3(\theta_1, \Phi_1) \end{cases}$$

$$\vec{F}_{ST}(1) = -\frac{\partial}{\partial \vec{X}_1} \int dV W_3 = -\int dV \left[\frac{\partial W_3}{\partial \theta_1} \frac{\partial \theta_1}{\partial \vec{X}_1} + \frac{\partial W_3}{\partial \Phi_1} \frac{\partial \Phi_1}{\partial \vec{X}_1} + \frac{\partial W_3}{\partial \theta_2} \frac{\partial \theta_2}{\partial \vec{X}_1} + \frac{\partial W_3}{\partial \Phi_2} \frac{\partial \Phi_2}{\partial \vec{X}_1} \right]$$

$$\vec{F}_{ST}(1) = -aL \int d\vec{r} \left[-\sin \theta_2 \sin(\Phi_2 - \Phi_1) \nabla_{\vec{X}_1} \theta_1 + (\sin \theta_2 \cos \theta_1 \cos(\Phi_2 - \Phi_1) - \sin \theta_1 \cos \theta_2) \nabla_{\vec{X}_1} \Phi_1 \right]$$

$$\Phi_1(\vec{r}, \vec{s}_1) = \Phi_1 \frac{\pi}{2} + \varphi + m_0(\rho) [X_1 \sin \varphi - Y_1 \cos \varphi], \quad \nabla_{\vec{X}_1} \Phi_1 = m_0 [\hat{x} \sin \varphi - \hat{y} \cos \varphi]$$

$$\Phi_2(\vec{r}, \vec{s}_2) = \Phi_2 \frac{\pi}{2} + \varphi + m_0(\rho) [X_2 \sin \varphi - Y_2 \cos \varphi], \quad m_0(\rho) - \text{oblique, т.е. R - oblique}$$

For cores $\cos \theta_1 = 0, \cos \theta_2 = 0, \theta_j = \pi/2, \vec{F}_{ST}(1) = 0, \vec{F}_{ST}$ on perpendicular v. cores.

$$m_2(\vec{r}, \vec{s}_1) = \cos \theta_1 = m_2^0(\rho) + g_1(\vec{s}_1 \cdot \hat{\rho}), \quad \theta_1(\vec{r}, \vec{s}_1) = \arccos[m_2^0 + g_1] =$$

$$= |\arccos x|' = \frac{1}{\sqrt{1-x^2}} = \theta_1^0(\rho) - \frac{1}{\sqrt{1-(m_2^0)^2}} g_1, \quad \theta_1^0 = \arccos m_2^0, \quad g_1(\vec{s}_1 \cdot \hat{\rho}) = \theta_1^0 + \epsilon_1, \quad \epsilon_1 = \frac{g_1 \cdot \hat{s}_1}{\sin \theta_1^0}$$

$$\cos \theta_1 = \cos(\theta_1^0 + \epsilon_1) = \cos \theta_1^0 - \sin \theta_1^0 \epsilon_1 = \cos \theta_1^0 + g_1(\vec{s}_1 \cdot \hat{\rho}), \quad \sin \theta_1 = \sin \theta_1^0 + \cos \theta_1^0 \epsilon_1$$

$$\theta_1(\vec{r}, \vec{s}_1) = \theta_1^0(\rho) - \frac{g_1}{\sin \theta_1^0} (\vec{s}_1 \cdot \hat{\rho}), \quad \nabla_{\vec{X}_1} \theta_1 = -\frac{g_1}{\sin \theta_1^0} \hat{\rho}, \quad \hat{\rho} = \hat{x} \cos \varphi + \hat{y} \sin \varphi, \quad \vec{s}_1 \cdot \hat{\rho} = \frac{g_1}{\sin \theta_1^0} \cos \varphi + \frac{g_2}{\sin \theta_1^0} \sin \varphi$$

$$\sin \theta_2 \sin(\Phi_2 - \Phi_1) = [\sin \theta_2^0 + \cos \theta_2^0 \epsilon_2] \sin \left[\left(\Phi_2 - \Phi_1 \right) \frac{\pi}{2} + m_0 \left\{ (X_2 - X_1) \sin \varphi - (Y_2 - Y_1) \cos \varphi \right\} \right] =$$

$$= \left| \sin(d \pm \pi) = \pm \sin d \right| = \epsilon_1 \epsilon_2 [\sin \theta_2^0 + \cos \theta_2^0 \epsilon_2] m_0 [(X_2 - X_1) \sin \varphi - (Y_2 - Y_1) \cos \varphi]$$

$$\rightarrow \int d\varphi \sin \theta_2 \sin(\Phi_2 - \Phi_1) \nabla_{\vec{X}_1} \theta_1 = \epsilon_1 \epsilon_2 \int d\varphi [\sin \theta_2^0 + \cos \theta_2^0 \epsilon_2] \frac{g_1}{\sin \theta_1^0} m_0 [\Delta X \sin \varphi - \Delta Y \cos \varphi] \times$$

$$\times (-1) \frac{g_1}{\sin \theta_1^0} \hat{\rho} = \epsilon_1 \epsilon_2 \int d\varphi \sin \theta_2^0 m_0 \frac{(-g_1)}{\sin \theta_1^0} \left[-\hat{x} \Delta Y \cos^2 \varphi + \hat{y} \Delta X \sin^2 \varphi \right] = \pi \epsilon_1 \epsilon_2 \frac{\sin \theta_2^0}{\sin \theta_1^0} m_0 g_1 [\hat{x} \Delta Y - \hat{y} \Delta X]$$

$$\Delta X = X_2 - X_1, \quad \Delta Y = Y_2 - Y_1$$

(9/21) Obyzngenne e G. de laubary zhen. pados no r/n/r more - vortex nano-pillars by Locatelli [draft] u Locatelli [digest MMM-Internag'10].

Obtaining 200,000. more nano. 6.5 x 100 De, r.e. na. $\omega_{1/2} \sim 10^{11}$ rad/sec
 core diameter class (<10%), $\omega_{1,2}$ obzngenne in dot dipolar u exch. fields
 u interdot coupling. V. core - coupling - dipolar rago yreco abno, r.k.
 dot R=50, 100 nm ne namore soume $R_c \sim 10-15$ nm; 3) Locatelli buger
 ogun amonni muk $I(\omega)$ μ $C_1 C_2 = +1$ u $P_1 P_2 = -1$ u unrepbane d. current u
 H_z - fields, noga rase occeenne crafubno; 4) $\vec{F}_{ST}$ nozgne que parera
 Jout - r.k. rase bozngenne vortices b. core u que nommanne cume.
 5) bozngenne, cume to excite Vortex modes of different symmetry; 5) unoga
 Locatelli buger bropan muk $I(\omega)$ - cneboni, 200 v. unman. barone que unrep.
 u f. amonni within double-vortex coupled dynamics model;
 6) Cerna pence parera V. dynamics - GMR, r.e. $I(\omega) \sim \langle \cos \vec{m}_1, \vec{m}_2 \rangle_s$;
 7) bugene nepereenne excitation ST registration GMR - cren nozbenner na smogaro
 amonni ogny mogy, u. m. boeone crayr. $\pm M$ - mog (oreno pegne 2. mogu) -
 nozngenne u f. amonni creny ST-excitation na d.c. field excitation u/um
 registration (c GMR na MRFM) nozbenner bugere gpyne mogu (sonome TM-uc)

Vortex magnetost coupling

F/N/F

Com. 27/III 077. (pole free model, one core) $\text{div } \vec{m} = \frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y} =$
 $\frac{1}{\rho} \left[\dots \right] = \frac{\partial m_p}{\partial \rho} + \frac{1}{\rho} \frac{\partial m_\varphi}{\partial \varphi} = 0$, $m_p = C m_0(\rho) (S_y \cos \varphi - S_x \sin \varphi)$, $m_0 = \frac{1}{\rho} (1 - \rho^2)$.
 $\text{div } \vec{m} = C \frac{\partial m_0}{\partial \rho} (S_y \cos \varphi - S_x \sin \varphi)$. Note $\Phi(\vec{r}, \vec{r}') = C \frac{\pi}{2} + \varphi + m_0(\rho) [S_y \sin \varphi - S_x \cos \varphi]$
 so um $\vec{r} \cdot \vec{r}' \sim (\hat{x} \sin \varphi - \hat{y} \cos \varphi)$ under gr. zabuchnocho ven $m_p(\vec{r}, \vec{r}')$.

Yroem V. core - core coupling u core - volume charges coupling
 b F/N/F double-vortex structure. Surface core charges $m_z(\vec{r}, \vec{r}')$:
 $m_z(\vec{r}, \vec{r}') = m_z^0(\rho) + g(\rho) (\vec{S} \cdot \hat{\rho})$. $\text{div } \vec{m} = -C \frac{\partial m_0}{\partial \rho} (\vec{S} \times \hat{\rho})_z$, $m_z^0, g \sim \rho''$ V. core.
 Dama que single-layer dot cno core-volume, core-core interaction uz-a
 shifted vortex $\vec{S} \neq 0$.

$$W_m^{\text{core-vol}} = \frac{1}{2} \int dS \int dV' \frac{\partial(\vec{r}) \partial(\vec{r}')}{|\vec{r} - \vec{r}'|}, \quad \partial = M_s m_z, \quad g = \text{div } \vec{m} \cdot M_s$$

$$\frac{1}{|\vec{r} - \vec{r}'|} = \int dk e^{-k|\vec{r} - \vec{r}'|} J_0(k|\vec{r} - \vec{r}'|) = \int dk e^{-k|\vec{r} - \vec{r}'|} \sum_{\mu} e^{i\mu(\varphi - \varphi')} J_{\mu}(k\rho) J_{\mu}(k\rho')$$

$$\rightarrow W_m^{\text{c-v}} = -\frac{M_s^2}{2} \left[\int_{z=L} dS + \int_{z=0} dS \right] \int d\vec{r} d\vec{r}' g(\rho) (\vec{S} \cdot \hat{\rho}) m_0^0(\rho') (\vec{S}' \cdot \hat{\rho}') \int dk e^{-k|\vec{r} - \vec{r}'|} \sum_{\mu} e^{i\mu(\varphi - \varphi')} J_{\mu}(k\rho) J_{\mu}(k\rho')$$

$$\int d\varphi \int d\varphi' (\vec{S} \cdot \hat{\rho}) (\vec{S}' \cdot \hat{\rho}') \sum_{\mu} e^{i\mu(\varphi - \varphi')} a_{\mu} = \int d\varphi (S_x \cos \varphi + S_y \sin \varphi) e^{i\mu\varphi} \int d\varphi' (S'_x \sin \varphi' - S'_y \cos \varphi') e^{-i\mu\varphi'}$$

$$= \sum_{\mu} \pi^2 [S_x (\delta_{\mu,1} + \delta_{\mu,-1}) + \frac{1}{i} S_y (\delta_{\mu,-1} - \delta_{\mu,1})] [S'_x (\delta_{\mu,1} - \delta_{\mu,-1}) - S'_y (\delta_{\mu,1} + \delta_{\mu,-1})] a_{\mu} =$$

$$= \pi^2 \sum_{\mu} [S_x^2 \frac{1}{i} (\delta_{\mu,1} - \delta_{\mu,-1}) - S_x S_y (-\delta_{\mu,-1} - \delta_{\mu,1}) - S_x S_y (\delta_{\mu,1} + \delta_{\mu,-1}) - \frac{1}{i} S_y^2 (\delta_{\mu,-1} - \delta_{\mu,1})] a_{\mu} =$$

$$= \pi^2 S^2 \sum_{\mu} (\delta_{\mu,1} - \delta_{\mu,-1}) a_{\mu} = \pi^2 S^2 (a_1 - a_{-1}) = 0, \quad \text{r.k. } a_{\mu} = J_{\mu}(k\rho) J_{\mu}(k\rho') \text{ ver. no.}$$

$m_z^0(\rho)$ - same buraga ne gae. $W_m^{\text{core-vol}} = 0$.

$$\rightarrow W_m^{\text{core-core}} = \frac{1}{2} \int dS \int dS' \frac{\partial(\vec{r}) \partial(\vec{r}')}{|\vec{r} - \vec{r}'|} = \frac{1}{2} \int_{z=0} dS \int_{z=0} dS' + \frac{1}{2} \int_{z=L} dS \int_{z=L} dS' + 2 \int_{z=0} dS \int_{z=L} dS' =$$

$$= \int_{z=0} dS \int_{z=0} dS' + \int_{z=0} dS \int_{z=L} dS' \sum_{\mu} a_{\mu} \int d\varphi \int d\varphi' [m_z^0(\rho) + g(\vec{S} \cdot \hat{\rho})] [m_z^0(\rho') + g(\vec{S}' \cdot \hat{\rho}')] e^{i\mu(\varphi - \varphi')} =$$

$$= \left| \text{then } m_z^0 m_z^0 \text{ uger b. mepmno vortex yoba} \right| = gg(\rho) \sum_{\mu} a_{\mu} \int d\varphi (\vec{S} \cdot \hat{\rho}) e^{i\mu\varphi} \int d\varphi' (\vec{S}' \cdot \hat{\rho}') e^{-i\mu\varphi'} = gg(\rho) \sum_{\mu} a_{\mu} \left| \int d\varphi e^{i\mu\varphi} (\vec{S} \cdot \hat{\rho}) \right|^2$$

$$W_m^{v1-v2} = 2M_{s1}M_{s2}C_1C_2 (\vec{s}_1 \cdot \vec{s}_2) \pi^2 \int dk \frac{1}{k^2} (1-e^{-kL_1})(1-e^{-kL_2}) e^{-kd} \int d\varphi \int d\varphi' m'_0(\varphi) m'_0(\varphi') J_1(k\rho) J_1(k\rho')$$

$$= 2\pi^2 C_1 C_2 M_{s1} M_{s2} (\vec{s}_1 \cdot \vec{s}_2) F(\beta_1, \beta_2, d) R^3, \quad F(\beta_1, \beta_2, d) = \int_0^{\frac{d}{R}} \frac{dt}{t^2} (1-e^{-\beta_1 t})(1-e^{-\beta_2 t}) e^{-\frac{d}{R}t}$$

$$I(t) = \int d\varphi m'_0(\varphi) J_1(k\rho) - 2 \int d\varphi J_1(k\rho) = \int d\varphi (1-\varphi^2) \frac{\partial J_1(k\rho)}{\partial \varphi} = \int d\varphi m_0(\varphi) \frac{\partial J_1(k\rho)}{\partial \varphi}$$

$$\int d\varphi J_1(k\rho) \frac{\partial m_0}{\partial \varphi} = m_0(\varphi) J_1(k\rho) \Big|_0^1 - \int d\varphi (\dots) \text{Her.} \quad \int d\varphi \frac{\partial m_0}{\partial \varphi} J_1(k\rho) = J_1(k\rho) m_0(\varphi) \Big|_0^1 - \int d\varphi J_1(k\rho) \frac{\partial m_0}{\partial \varphi}$$

$$- \int v du. \quad \frac{du}{d\varphi} = \frac{d}{d\varphi} [J_1(k\rho)] = \frac{d}{dx} [x J_1(x)] = x J_0(x) \quad \left(\frac{d}{dx} \right)^m [x^2 J_0(x)] = x^2 J_{-m}(x)$$

$$\int d\varphi \frac{\partial m_0}{\partial \varphi} J_1(k\rho) = - \int d\varphi \frac{d}{d\varphi} [J_1(k\rho)] m_0(\varphi) = - \int d\varphi m_0(\varphi) [J_1(k\rho) + \rho \frac{\partial J_1(k\rho)}{\partial \rho}] = -x^2 J_{-m}(x)$$

Interact. vol-1-vol-2 F_1/M_{F_2}

$$W_m^{v1-v2}(\vec{x}_1, \vec{x}_2) = 2\pi^2 C_1 C_2 M_{s1} M_{s2} R F(\beta_1, \beta_2, d) (\vec{x}_1 \cdot \vec{x}_2), \quad F(\beta_1, \beta_2, d) = \int_0^{\frac{d}{R}} \frac{dt}{t^2} (1-e^{-\beta_1 t})(1-e^{-\beta_2 t}) e^{-\frac{d}{R}t}$$

Две волне имеют с осью APL'OS, т.е. $L_1=L_2$, тогда $x_2 \approx I(t) = 2 \int dx x J_1(kx)$.
 Т.е. падающие волны имеют $F_1/M/F_2$ буг $W_m^{v1-v2}(\vec{x}_1, \vec{x}_2)$ и $W_m^{v1-v2} \sim C_1 C_2 (\vec{x}_1 \cdot \vec{x}_2)$.

$$\rightarrow W_m^{v1-vol-2} \sim \int d\varphi \int d\varphi' (\vec{s}_1 \cdot \hat{\rho})(\vec{s}_2 \times \hat{\rho})_z \sum a_\mu e^{i\mu(\varphi-\varphi')} =$$

$$= \sum a_\mu \int d\varphi [s_{x1} \cos \varphi + s_{y1} \sin \varphi] e^{i\mu\varphi} \int d\varphi' [s_{x2} \sin \varphi' - s_{y2} \cos \varphi'] e^{-i\mu\varphi'} =$$

$$= \sum a_\mu \pi^2 \left[s_{x1} (\delta_{\mu,1} + \delta_{\mu,-1}) + \frac{1}{i} (\delta_{\mu,-1} - \delta_{\mu,1}) s_{y1} \right] \left[\frac{1}{i} s_{x2} (\delta_{\mu,1} - \delta_{\mu,-1}) - s_{y2} (\delta_{\mu,1} + \delta_{\mu,-1}) \right] =$$

$$= \pi^2 \sum a_\mu \left[\frac{1}{i} s_{x1} s_{x2} (\delta_{\mu,1} - \delta_{\mu,-1}) - s_{x1} s_{y2} (\delta_{\mu,1} + \delta_{\mu,-1}) - s_{y1} s_{x2} (\delta_{\mu,-1} - \delta_{\mu,1}) - \frac{1}{i} s_{y1} s_{y2} (\delta_{\mu,1} - \delta_{\mu,-1}) \right] =$$

$$= \pi^2 (s_{y1} s_{x2} - s_{x1} s_{y2}) \sum a_\mu (\delta_{\mu,1} + \delta_{\mu,-1}) = 2\pi^2 a_1 (s_{y1} s_{x2} - s_{x1} s_{y2})$$

$$W_m^{c1-v2} = \int dS \int dV' \frac{\partial(\vec{\rho}) \rho(\vec{\rho}')}{|\vec{r}-\vec{r}'|} = -M_{s1} \left[\int_{z=0}^{L_1} dS + \int_{z=L_1}^{L_2} dS \right] \int dV' g_1(\vec{\rho}, \hat{\rho}) C_2 m'_0(\rho) (\vec{s}_2 \times \hat{\rho})_z$$

$$\times \sum e^{i\mu(\varphi-\varphi')} \int dk e^{-k|z-z'|} = -M_{s1} M_{s2} \left[\int_{z=L_1}^{L_2} d\varphi \int_{z=0}^{L_1} d\varphi' \int dk e^{-k|z-z'|} g_1 m'_0(\rho) \int d\varphi d\varphi' (\vec{s}_1 \cdot \hat{\rho})(\vec{s}_2 \times \hat{\rho})_z \int d\varphi d\varphi' \right]$$

$$= -M_{s1} M_{s2} C_2 \left[\int_{z=L_1}^{L_2} d\varphi \int_{z=0}^{L_1} d\varphi' \int dk e^{-k|z-z'|} g_1 m'_0(\rho) \cdot 2\pi^2 (s_{y1} s_{x2} - s_{x1} s_{y2}) a_1(\rho, \rho') \right] =$$

$$= -M_{s1} M_{s2} C_2 \int dk [e^{-k|z-L_1|} - e^{-k|z-L_2|}] \int d\varphi \int d\varphi' 2\pi^2 (s_{y1} s_{x2} - s_{x1} s_{y2}) g_1 m'_0(\rho) a_1(\rho, \rho') =$$

$$= 2\pi^2 C_2 M_{s1} M_{s2} (s_{x1} s_{y2} - s_{y1} s_{x2}) \int dk e^{-kd} (1-e^{-kL_1})(1-e^{-kL_2}) \int d\varphi \int d\varphi' g_1(\rho) m'_0(\rho') J_1(k\rho) J_1(k\rho')$$

$$I_{c1}(k) = \int d\varphi g_1 J_1(k\rho), \quad I_{c1}(t) = \int d\varphi g_1(\rho) J_1(k\rho), \quad I(t) = \int d\varphi m'_0(\rho) J_1(k\rho)$$

$$\rightarrow F_{c1v2}(\beta_1, \beta_2, d) = \int \frac{dt}{t} e^{-td} (1-e^{-\beta_1 t})(1-e^{-\beta_2 t}) I_{c1}(t) I(t), \quad I_{c1} \sim \rho_1$$

$$W_m^{c1-v2} = 2\pi^2 C_2 M_{s1} M_{s2} R (X_1 Y_2 - Y_1 X_2) F_{c1v2}(\beta_1, \beta_2, d), \quad W_m^{c2-v1} \text{ non zero, } W_m^{c1-v2} \text{ (1-2)}$$

$$W_m^{c2-v1} = 2\pi^2 C_1 M_{s1} M_{s2} R (X_2 Y_1 - X_1 Y_2) F_{c2v1}(\beta_1, \beta_2, d), \quad F_{c2v1} \sim I_{c2} \sim \rho_2, \quad F_{c1v2} \sim I_{c1} \sim \rho_1$$

Падающие I_{c1} и I_{c2} (красные ρ_1, ρ_2) имеют б.в. волне радиус C_1, C_2 .

$$g_j(\rho) = \rho_j \frac{4C_j^2 \rho (1+\rho^2)}{(9+4C_j^2)^2} = \rho_j \frac{4}{C_j} \frac{(\rho/C_j)(1+\rho^2)}{(1+9/C_j^2)^2}, \quad I_{c1} = \int d\varphi J_1(k\rho) g_1(\rho) = \frac{4\rho_j}{C_j} \int d\varphi J_1(k\rho) \frac{(\rho/C_j)(1+\rho^2)}{(1+9/C_j^2)^2}$$

$$W_m^{c1-v2} + W_m^{c2-v1} = 2\pi^2 M_{s1} M_{s2} R (X_1 Y_2 - Y_1 X_2) [p_1 C_2 F_{c1v2} - p_2 C_1 F_{c2v1}], \quad p_j \text{ — б.в. волне } I_{c1}$$

Если $v, \text{ waves } C_1=C_2$, то $F_{c1v2} = F_{c2v1}$ и $W_m^{c-v} \sim (p_1 C_2 - p_2 C_1)$ — ген. волны.

$$W_m^{c1-v2} W_m^{c2-v1} \sim (\vec{x}_1 \times \vec{x}_2)_z \text{ — циклическое токовое генер. out-phase VC modes.}$$

$$\hat{z} \cdot (\vec{x}_1 \times \vec{x}_2) = -\vec{x}_2 \cdot (\vec{x}_1 \times \hat{z}) = \vec{x}_1 \cdot (\vec{x}_2 \times \hat{z}), \quad \text{В } yD \vec{x}_1 \text{ nonager } (\vec{x}_2 \times \hat{z}), \quad \text{В } yD \vec{x}_2 \text{ byger } -(\vec{x}_1 \times \hat{z})$$

Escaneado con CamScanner

$$\begin{vmatrix} \mu - i\omega G_2 & x_2 \\ 0 & -i\omega G_2 x_2 \end{vmatrix} = 0 \quad \begin{vmatrix} \mu & 0 \\ 0 & \mu \end{vmatrix} = 0$$

$$= x_1 (\mu x_2^2 - \mu^2 x_2 - \omega^2 G_2^2 x_1) - i \omega G_1 [-i \omega G_2 x_1^2 - i \omega G_2 \mu^2 + i \omega^2 G_2^2 G_1] + \mu [\mu^3 - \mu x_1 x_2 - \omega^2 G_1 G_2 \mu] = \omega^4 G_1^2 G_2^2 + \omega^2 [-x_1 x_2 G_2^2 - x_2^2 G_1^2 + G_2^2 \mu^2 G_1 - \mu^2 G_1 G_2] + x_1^2 x_2^2 - \mu^2 x_1 x_2 - \mu^2 x_1 x_2 = \omega^4 G_1^2 G_2^2 - \omega^2 [x_1^2 G_2^2 + x_2^2 G_1^2 + 2 \mu^2 G_1 G_2] + x_1^2 x_2^2 + \mu^4 - 2 \mu^2 x_1 x_2 = G_1^2 G_2^2 [\omega^4 - \omega^2 (\omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2}) + \frac{1}{G_1^2 G_2^2} (\mu^2 x_1 x_2 - \mu^4)]$$

$$0 = \det R(\omega): \omega^4 - \omega^2 (\omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2}) + \omega_{01}^2 \omega_{02}^2 + \frac{\mu^2}{G_1^2 G_2^2} (\mu^2 - 2 x_1 x_2) = 0$$

$$\omega_{01}^2 = x_1^2 / G_1^2, \quad 2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 \pm \sqrt{(\omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2})^2 - 4 \frac{\mu^2}{G_1^2 G_2^2} (\mu^2 - 2 x_1 x_2)} + 2 \frac{\mu^2}{G_1 G_2}$$

$$2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2} \pm \sqrt{(\omega_{01}^2 - \omega_{02}^2)^2 + 4 \frac{\mu^2}{G_1 G_2} (\omega_{01}^2 + \omega_{02}^2) + 8 \frac{\mu^2}{G_1^2 G_2^2} x_1 x_2}$$

$$2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2} \pm \sqrt{(\omega_{01}^2 - \omega_{02}^2)^2 + 4 \frac{\mu^2}{G_1 G_2} (\omega_{01}^2 + \omega_{02}^2) + 8 \frac{\mu^2}{G_1^2 G_2^2} x_1 x_2}$$

$$2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2} \pm \sqrt{(\omega_{01}^2 - \omega_{02}^2)^2 + 4 \frac{\mu^2}{G_1 G_2} (\omega_{01}^2 + \omega_{02}^2) + 8 \frac{\mu^2}{G_1^2 G_2^2} x_1 x_2}$$

$$2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2} \pm \sqrt{(\omega_{01}^2 - \omega_{02}^2)^2 + 4 \frac{\mu^2}{G_1 G_2} (\omega_{01}^2 + \omega_{02}^2) + 8 \frac{\mu^2}{G_1^2 G_2^2} x_1 x_2}$$

$$2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2} \pm \sqrt{(\omega_{01}^2 - \omega_{02}^2)^2 + 4 \frac{\mu^2}{G_1 G_2} (\omega_{01}^2 + \omega_{02}^2) + 8 \frac{\mu^2}{G_1^2 G_2^2} x_1 x_2}$$

$$2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2} \pm \sqrt{(\omega_{01}^2 - \omega_{02}^2)^2 + 4 \frac{\mu^2}{G_1 G_2} (\omega_{01}^2 + \omega_{02}^2) + 8 \frac{\mu^2}{G_1^2 G_2^2} x_1 x_2}$$

$$2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2} \pm \sqrt{(\omega_{01}^2 - \omega_{02}^2)^2 + 4 \frac{\mu^2}{G_1 G_2} (\omega_{01}^2 + \omega_{02}^2) + 8 \frac{\mu^2}{G_1^2 G_2^2} x_1 x_2}$$

$$2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2} \pm \sqrt{(\omega_{01}^2 - \omega_{02}^2)^2 + 4 \frac{\mu^2}{G_1 G_2} (\omega_{01}^2 + \omega_{02}^2) + 8 \frac{\mu^2}{G_1^2 G_2^2} x_1 x_2}$$

$$2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2} \pm \sqrt{(\omega_{01}^2 - \omega_{02}^2)^2 + 4 \frac{\mu^2}{G_1 G_2} (\omega_{01}^2 + \omega_{02}^2) + 8 \frac{\mu^2}{G_1^2 G_2^2} x_1 x_2}$$

$$2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2} \pm \sqrt{(\omega_{01}^2 - \omega_{02}^2)^2 + 4 \frac{\mu^2}{G_1 G_2} (\omega_{01}^2 + \omega_{02}^2) + 8 \frac{\mu^2}{G_1^2 G_2^2} x_1 x_2}$$

$$2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2} \pm \sqrt{(\omega_{01}^2 - \omega_{02}^2)^2 + 4 \frac{\mu^2}{G_1 G_2} (\omega_{01}^2 + \omega_{02}^2) + 8 \frac{\mu^2}{G_1^2 G_2^2} x_1 x_2}$$

$$2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2} \pm \sqrt{(\omega_{01}^2 - \omega_{02}^2)^2 + 4 \frac{\mu^2}{G_1 G_2} (\omega_{01}^2 + \omega_{02}^2) + 8 \frac{\mu^2}{G_1^2 G_2^2} x_1 x_2}$$

$$2 \omega_{1,2}^2 = \omega_{01}^2 + \omega_{02}^2 + 2 \frac{\mu^2}{G_1 G_2} \pm \sqrt{(\omega_{01}^2 - \omega_{02}^2)^2 + 4 \frac{\mu^2}{G_1 G_2} (\omega_{01}^2 + \omega_{02}^2) + 8 \frac{\mu^2}{G_1^2 G_2^2} x_1 x_2}$$

$m_{ij} = C_j m_0(p) (S_{y_j} \cos \varphi - S_{x_j} \sin \varphi)$, $m_{ij} = C_j$ (one core) $\rightarrow$ Becu parer c "u" of volume-1 - val-2 m. interaction $\rightarrow$ core-core - surface (uz-2 m_{ij}) nago y urobawo bnaq v. cores to volume charges $\text{div} \vec{m}_j$.

Then, w problemat nameret. unisiparob c Gregoire.

$$W_m^{v1-v2} = 2\pi^2 C_1 C_2 M_{s1} M_{s2} R F_v(\beta_1, \beta_2, d) (\vec{X}_1 \cdot \vec{X}_2), \quad F_v(\beta_1, \beta_2, d) = \int_0^\infty \frac{dt}{t^2} I^2(t) f(\beta_1, \beta_2, d, t)$$
$$I(t) = 2 \int dx \times J_1(x) = 2 I_G(t), \quad I_G(t) - \text{Gregoire calculated as } I(t).$$
$$|W_m^{v1-v2}|^0 = 8\pi^2 M_{s1} M_{s2} R F_v^G(\beta_1, \beta_2, d) (\vec{X}_1 \cdot \vec{X}_2), \quad \frac{1}{V} W_m^{v1-v2} = 8\pi \frac{M_{s1} M_{s2}}{(\beta_1 + \beta_2)} F_v^G(\beta_1, \beta_2, d)$$
$$V = V_1 + V_2 = \pi R^2 (L_1 + L_2) - \text{FM osom nanopillar}$$

$$\beta_1 = 15 \text{ nm} / 100 \text{ nm} = 0.15, \quad \beta_2 = 4 \text{ nm} / 100 \text{ nm} = 0.04, \quad \frac{1}{V} W_m^{v1-v2} \approx \frac{8\pi M_s^2 \cdot 6.45 \cdot 10^{-4}}{0.2}$$
$$d_s = d/R = 10 \text{ nm} / 100 \text{ nm} = 0.1 - \text{APLOS: } \omega_{X_3} / \omega_m = 2 F_v(\beta) \pm \frac{1}{\beta} F_{int}(\beta, d_s)$$

$\omega_m \approx 1 \text{ GHz}$ que $\beta = 0.2$, t.e. $F_v(0.2) \approx \frac{1}{2.30}$, $F_{int} \sim 6.5 \cdot 10^{-4}$
 $\frac{1}{\beta} F_{int}^G \approx (0.2)^{-1} 6.5 \cdot 10^{-4} \approx 3.3 \cdot 10^{-3}$ crabh. c $F_v(0.2) = 1.6 \cdot 10^{-2}$
No bce me que F_{int} a omuga ~ paz b 3 danyuwo beumny.

Esam b3 seb xan. $\beta_1, \beta_2, d, C_1 = C_2 = 0.1$ u $I(t) = I_G(t) = \int dx \times J_1(x)$, to

$$F_w = 6.454 \cdot 10^{-4}, \quad F_{vc} = 4.69 \cdot 10^{-7} \quad \text{u} \quad F_{cc} = 7.86 \cdot 10^{-8}, \quad \text{t.k.}$$

coefficients wpu F_{wv}, F_{vc}, F_{cc} wpuwepmo agunaweb, to

$$W_m^{v1-v2} \gg W_m^{v1-c2}, W_m^{v2-c1}, W_m^{c1-c2}, \quad W_m^{int} (\text{Chelgan. c v. cores}) \leq 1/6 W_m^{v1-v2}$$

Ho a ne asenuber vcore contribution, b3agewu b W_m^{v1-v2} , wpeneperec v. core m.

FM nacerob $w_1(\mu), w_2(\mu)$ a norman 1/2 wpeneperec de-field SP toka, t.e. wpa $J=0$. $J \neq 0$ gacc b3uagw $\Delta w(J) \sim C_j J$, kotopne zabawer of C_j separately. $\mu \sim C_1, C_2$ con. Gus APL'05.

Gregoire ybawaw (y3om. parer) beumny F_{vc} u $F_{cc} \sim$ b 10 paz.

Bepmence k GMR - detection $\langle \vec{m}_1 \cdot \vec{m}_2 \rangle_s$.

$$\langle m_{1p} m_{2p} \rangle_s \sim \int d\varphi d\varphi m_{1p} m_{2p} = C_1 C_2 \int d\varphi m_0^2(p) \int d\varphi (S_{y_1} \cos \varphi - S_{x_1} \sin \varphi) (S_{y_2} \cos \varphi - S_{x_2} \sin \varphi)$$
$$\sim C_1 C_2 (\vec{S}_1 \cdot \vec{S}_2). \quad \langle m_{1z} m_{2z} \rangle = \int d\varphi d\varphi m_{1z} m_{2z} = \int d\varphi d\varphi [m_{z1}^0 + g_1 (\vec{S}_1 \cdot \hat{p})] [m_{z2}^0 + g_2 (\vec{S}_2 \cdot \hat{p})]$$
$$= \int d\varphi d\varphi [m_{z1}^0 m_{z2}^0 + g_1 g_2 \pi (\vec{S}_1 \cdot \vec{S}_2)], \quad m_{zj}^0, g_j \sim \rho_j$$

$$\langle \vec{m}_1 \cdot \vec{m}_2 \rangle_{GMR} = \pi \int d\varphi d\varphi m_0^2 C_1 C_2 (\vec{S}_1 \cdot \vec{S}_2) + C_1 C_2 + \rho_1 \rho_2 \left[\int d\varphi d\varphi m_{z1}^0 m_{z2}^0 + \pi \int d\varphi d\varphi g_1 g_2 (\vec{S}_1 \cdot \vec{S}_2) \right]$$

Dynamic $\langle \vec{m}_1 \cdot \vec{m}_2 \rangle_s = \pi \left[\int d\varphi d\varphi m_0^2 C_1 C_2 + \rho_1 \rho_2 \int d\varphi d\varphi g_1 g_2 \right] (\vec{S}_1 \cdot \vec{S}_2) \approx \pi C_1 C_2 \int d\varphi d\varphi m_0^2 (\vec{S}_1 \cdot \vec{S}_2)$
 $(\vec{X}_1 \cdot \vec{X}_2) = \lambda^2 [-a_2 \tilde{X}_1 + a_1 \tilde{X}_2] \cdot [\tilde{X}_1 - \tilde{X}_2] = \lambda^2 [-a_2 (\tilde{X}_1)^2 - a_1 (\tilde{X}_2)^2 + (a_1 + a_2) (\tilde{X}_1 \cdot \tilde{X}_2)]$

$$Z_j(t) \sim e^{i\omega_j t} \rightarrow X_j(t) \sim \cos \omega_j t, \quad Y_j(t) \sim \sin \omega_j t, \quad (\vec{X}_j)^2(t) = \text{const.} \quad (\vec{X}_1 \cdot \vec{X}_2) \sim \cos(\omega_1 - \omega_2)t$$

Esam GMR(t) $\sim \langle \cos \theta_{12}(t) \rangle_s = \langle \vec{m}_1 \cdot \vec{m}_2 \rangle$, to GMR(t) $\sim \cos(\omega_1 - \omega_2)t$ u xan. b GMR detection gawana b3to o3na nacerob $= (\omega_1 - \omega_2)$ b min. penamie neglecbly v. core

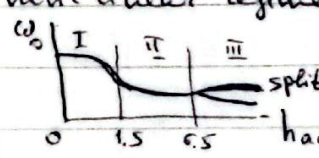
Torga esam xarb o3na uz mog $\tilde{X}_1(t), \tilde{X}_2(t)$ ne b333ymagawee SP-current to $(\vec{X}_1 \cdot \vec{X}_2) = 0$ u GMR(t) = const. Y3er current (J) neneret ground state $\rightarrow$ neneret cob. nacerob ω_1, ω_2 u nepenopumnyes utx zep3 z3ebw sa(J) $\sim g J$ - de field

→ GMR(t) ~ cos(ω₁-ω₂)t in Zener's or coupled V. model. GMR - change in resistance of (m₁, m₂) system, a m₁, m₂ - each superconductor by $\vec{X}_1, \vec{X}_2$ (even no opposing vortex & superconductor F/N/F). GMR ~ change in mutual inductance $(\vec{X}_1, \vec{X}_2)$ or $\hat{z} \cdot (\vec{X}_1 \times \vec{X}_2) \sim \hat{z} \cdot (\vec{X}_1 \times \vec{X}_2)$, giving bilinear in frequencies. $GMR = F[(\vec{X}_1, \vec{X}_2), \hat{z} \cdot (\vec{X}_1 \times \vec{X}_2)] = F[\cos(\omega_1 - \omega_2)t, \sin(\omega_1 - \omega_2)t]$.
 T.e. GMR(t) shows a resonance ω₁ = (ω₁ - ω₂). Eigen vectors $\vec{X}_j \propto e^{i\omega_j t}$.
 To, Locatelli [Digest MMM'10] has given ω₀₁, e.g. ω₁ - ω₂ = ω₀₁ + (coupl.) - ω₂ ≈ ω₀₁.

Due parallel nanowires, unconfined nano wires c_j = b_j/R give cases c & d, where L₁ = 15 nm ≤ L₀ ≈ 18 nm and L₂ = 4 nm.
 where L > 0. $W(\delta) = R_0^2 \ln \frac{1}{\delta} + W_m^{ns}(\delta)$, $W'_0 = -R_0^2 \frac{1}{\delta} + W'_m(\delta) = 0$
 $W_m^{ns}(\delta) = \frac{4\pi}{\beta} [u(0) - u(\beta/\delta)] \beta^3$, $u(t) = u(0) - g_1 t$, $t \ll 1$. $W_m^{ns} \approx \frac{4\pi \beta^3 g_1}{\beta}$, $\frac{\beta}{\delta} = 4\pi \beta^2 g_1$,
 $-R_0^2 \frac{1}{\delta} + 8\pi g_1 \beta = 0$, $\beta_{eq}^2 = \frac{R_0^2}{8\pi g_1}$, $\beta_{eq} \approx \frac{R_0}{\sqrt{8\pi g_1}} = \frac{1}{\sqrt{2.86}} R_0 \approx 0.59 R_0$. $g_1 \approx 0.114$.
 $R_0 = L_0 = (2A/m^2)^{-1/2} \approx 18 \text{ nm}$ give p_y. $\frac{L}{\beta} = \frac{4 \text{ nm}}{0.6 \cdot 18 \text{ nm}} = 0.37$ we are in < 1.
 Due 2-10 case $c_2 = \frac{b_2}{R} = \frac{0.59 R_0}{R} = 0.59 \frac{18 \text{ nm}}{100 \text{ nm}} = 0.106$ (R=100 nm) and $c_2 = 0.212$ (R=50 nm).
 Due 1-10 case $c_1 = \frac{b_1}{R} = 0.68 \frac{L_0}{R} \left(\frac{L}{L_0}\right)^{1/3} = 0.68 \frac{18 \text{ nm}}{100 \text{ nm}} \left(\frac{15 \text{ nm}}{18 \text{ nm}}\right)^{1/3} = 0.68 \cdot 0.18 \cdot 0.94 = 0.115$ (R=100 nm)
 and $c_1 = 0.230$ give R=50 nm.

Moreover F_v, F_{vcl,2}, F_{cl,2} < maximum c₁, c₂ gives $F_{vc}, F_c \leq 1\% F_v$ (R=100 nm)
 and $F_{vc}, F_c \leq 3\% F_v$ (R=100/2 = 50 nm). T.e. magn. energy of F₁/N/F₂ interaction related to v. cores hysteresis maxima near no chabn. & volume charges interaction.

(13/XI) Review MMM-Intermag'10 paper by Chen-Growell "Nonlinear V.dyn."
 When narrow h_{ac} (in-plane oscil. field) ω_{1n} becomes up-2a gappedness ~ 490 MHz & ybaw. h_{ac} ω₀ governed by p₁ & p₂. go ω₀ ≈ 200 MHz when h_{ac} ≈ 1.5 Oe - normalised ω₀ ~ 1/4 [Gus'JAP'02]. Since ω₀(h_{ac}) has two nages, gabane splitting & h_{ac} ~ 6.5 Oe in non-linear regime. The splitting [hysteresis Buchanan PRB'07] changes & VC reversals.
 T.e. ω₀(h_{ac}) regime up-2a in hysteresis becomes gapped, a splitting Buchanan'07 up-2a VC reversals, can a in hysteresis of Gus'paper & Chabysko.
 I - defect regime, II - linear regime, III - non-lin. regime, VC reversals.



// Detection scheme Locatelli et al. $GMR(t) \sim c_1 c_2 + p_1 p_2 (\delta g(m_j^0))^2 + A \cos(\omega_1 - \omega_2)t$
 ω₂ is de-field of SP current potential $x_j/|\vec{X}_j|^2$.
 nanowire fixed perpendicular force (x → x_j = x + aJ_j) gives TM theory: $\omega_{ij} = \omega_j + aG_j J$
 J - SP current density (CPP). Consider h_{ac} = 1.70 Oe. $\omega_{ij}(J) = \omega_j + aG_j J$.
 you have TM resonant with ω_{ij}(J) in nano antenna. the power ω_{1,2} (μ) - V. frequencies in the m. coupled F₁/N/F₂ nanowire.
 Com. 10/11 $(\omega_1 - \omega_2) = \sqrt{(\omega_{01} - \omega_{02})^2 + 4 \frac{M_1^2}{G_1 G_2}}$, $\omega_{0j} = \frac{x_j}{G_j}$, $G_j = p_j |G_j|$. A superge. volume charges interdot inter.
 $\mu = 2\pi^2 c_1 c_2 M_{s1} M_{s2} F_v(p_1, p_2, d) R$, $F_v(p_1, p_2, d) = 4 F_v^{cylinder}(p_1, p_2, d)$, T.e. give data.
 power span in $I(t) = \int d\theta p J_1(t) \frac{\partial m_0}{\partial \theta}$, a $I(t) = 2 \int d\theta p J_1(t)$ can be Gus'APL'2005.
 Due R=100 nm, L₁=15 nm, L₂=4 nm, d=10 nm $F_v^{cylinder}(p_1, p_2, d) = 6.454 \cdot 10^{-4}$.
 $\frac{|A|}{\sqrt{G_1 G_2}} = \frac{8 \pi^2 M_{s1} M_{s2} R F_v}{2 \pi \sqrt{M_{s1} M_{s2}} \sqrt{L_1 L_2}} = \frac{4 \pi \sqrt{M_{s1} M_{s2}}}{\sqrt{p_1 p_2}} F_v^{cylinder}(p_1, p_2, d) = \frac{\sqrt{4 \pi^2 M_{s1} M_{s2}}}{\sqrt{p_1 p_2}} F_v^{cylinder}(p_1, p_2, d) = 600 \text{ G. Gus'APL 2005}$

Escaneado con CamScanner

4.31 (50nm) $R = 50 \text{ nm}$ 4.32 (50nm) $R = 50 \text{ nm}$ 4.33 (50nm) $R = 50 \text{ nm}$

Для $p_1 p_2 = -1$, J -two barons n/m $C_1 + C_2 \neq 0$. Бопем $C_1 = C_2 = +1$. $\Delta\omega_J = 2 \cdot 248 \text{ MHz}$.

$$\omega_1 - \omega_2 = \sqrt{(0.850 + 0.080 + 2 \cdot 0.248)^2 - 4(0.254)^2} = \sqrt{1.715} \text{ GHz} = 1.332 \text{ GHz}$$

Для $p_1 p_2 = +1$, $\omega_1 - \omega_2 = \sqrt{(0.850 - 0.080 + 2 \cdot 0.248)^2 + 4(0.254)^2} = \sqrt{1.881} \text{ GHz} = 1.364 \text{ GHz}$

$$C_1 = C_2 \neq 0, C_1 = +1, C_2 = -1$$

Обе расоры $(\omega_1 - \omega_2)$ близки ко бонне экон. $\omega_L = 1.131 \text{ GHz}$ Locatelli, $\mu g I = 14 \text{ mW}$.
Pouguino I-Brug & ractory y none zabonnen. $\mu\text{couplg. } \Delta\omega_J = 223 \text{ MHz}$

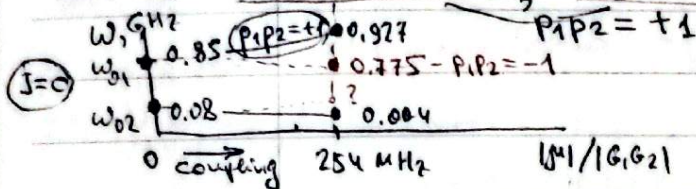
gac gne $p_1 p_2 = -1$, $\omega_1 - \omega_2 = \sqrt{1.635} \text{ GHz} = 1.279 \text{ GHz}$, $\mu p_1 p_2 = +1$, $\omega_1 - \omega_2 = \sqrt{1.737} \text{ GHz} = 1.318 \text{ GHz}$

Т.к. $(\omega_1 - \omega_2) p_1 p_2 = -1$ куме $(\omega_1 - \omega_2) p_1 p_2 = +1$ то бугуно она и обваснор
мк Locatelli $\omega_L = 1.131 \text{ GHz}$. Иаго нуме I-term ymenomab $\sim 10\%$

$$\omega_1 + \omega_2 = \omega_{01}(J) + \omega_{02}(J) = \begin{cases} \omega_{01} + \omega_{02} + a_0 J(C_1 + C_2), & p_1 p_2 = +1 \\ \omega_{01} - \omega_{02} + a_0 J(C_1 - C_2), & p_1 p_2 = -1 \end{cases}$$

Для $J=0$ $\begin{cases} \omega_1 + \omega_2 = \omega_{01} + \omega_{02} = 0.850 + 0.080 = 0.930 \\ \omega_1 - \omega_2 = 0.923, & p_1 p_2 = +1 \end{cases}$ $\begin{cases} \omega_1 + \omega_2 = \omega_{01} - \omega_{02} = 0.770 \text{ GHz} \\ \omega_1 - \omega_2 = 0.779 \text{ GHz}, & p_1 p_2 = -1 \end{cases}$

Обуага $\omega_1 = 0.927 \text{ GHz}$, $\omega_2 = 0.004 \text{ GHz}$ $\mu p_1 p_2 = +1$ $\omega_1 = 0.775 \text{ GHz}$, $\omega_2 = -0.005 \text{ GHz}$ $\mu p_1 p_2 = -1$



Ровбог: как на счеме $\omega_{1,2}(\mu)$ 10/21
coupling μ разгбнает ω_{01}, ω_{02} н/м $p_1 p_2 = +1$,
но бумнает ω_{01}, ω_{02} н/м $p_1 p_2 = -1$ (а/ке бонне).

$p_1 p_2 = +1$ $\begin{cases} \omega_1(\mu) - \omega_{10} = 0.927 - 0.850 = 0.077 \text{ GHz} \\ \omega_2(\mu) - \omega_{20} = -0.08 + 0.004 = -0.076 \text{ GHz} \end{cases}$ н/м $p_1 p_2 = -1$, $\omega_1(\mu) - \omega_{01} = -0.075 \text{ GHz}$

→ Interdot magn. Coupling cgbuaet расоры узамр. Снаб $\approx 75-77 \text{ MHz}$ гле $R = 50 \text{ nm}$.
Nanowire py (15 nm) / Cu (10 nm) / Ag (4 nm). Такав cgbuaet ознар. $\omega_2 \times 0$ ($p_1 p_2 = +1$).

16/21 Analysis eigenmodes $\vec{X}_1, \vec{X}_2$. Канне уз мк гаче $\langle \vec{m} \rangle \sim (C_1 \vec{X}_1 + C_2 \vec{X}_2) \neq 0$?
11/21 ночемено $\frac{1}{\lambda} \vec{X}_1 = -a_2 \vec{X}_1 + a_1 \vec{X}_2$, $\frac{1}{\lambda} \vec{X}_2 = \vec{X}_1 - \vec{X}_2$, $\lambda = \frac{1}{(a_1 - a_2)}$

$a_1 = \frac{1}{\mu} (\kappa_2 - \omega_2 G_2)$, $a_2 = \frac{1}{\mu} (\kappa_2 - \omega_1 G_2)$, $\omega_{1,2}$ - eigenfrequencies, μ - m. couplg.

$\langle \vec{m} \rangle \sim (C_1 \vec{X}_1 + C_2 \vec{X}_2)$, $\vec{X}_1 + \vec{X}_2 = (1 - a_2) \vec{X}_1 + (a_1 - 1) \vec{X}_2$

$\vec{X}_1(t), \vec{X}_2(t)$ конесорсе с глума расорам ω_1, ω_2 - габан счонное trajectory in general

Есм нучеуебонне бозбуагелу тачоко $\vec{X}_1$ - eigenmode, то

$\frac{1}{\lambda} \vec{X}_1 = -a_2 \vec{X}_1$, $\frac{1}{\lambda} \vec{X}_2 = \vec{X}_1$ в забвонном от знака a_2 $\vec{X}_1(t), \vec{X}_2(t)$ мочт
бвоб in-phase ($a_2 < 0$) or out-of-phase ($a_2 > 0$, $\Delta\varphi = \pi$).

$a_2 = \frac{1}{\mu} (\kappa_2 - \omega_1 G_2) = \frac{1}{\mu |G_2|} [|\omega_{02}| - p_2 \omega_1] = \frac{G_2}{\mu} (\omega_{02} - \omega_1) = \frac{p_2}{C_1 C_2} \frac{|G_2|}{|J|} [p_2 |\omega_{02}| - \omega_1]$

$a_2 = C_1 C_2 \frac{|G_2|}{|J|} [|\omega_{02}| - p_2 \omega_1]$, $2\omega_1 = |\omega_{01}| + |\omega_{02}| + a_0 J(C_1 + C_2) + \sqrt{\dots}$, $p_1 p_2 = +1$

Т.к. $|\omega_{01}| > |\Delta\omega_J|, |\omega_{02}|$, то $\omega_1 > 0$.
 $2\omega_1 = |\omega_{01}| - |\omega_{02}| + a_0 J(C_1 - C_2) + \sqrt{\dots}$, $p_1 p_2 = -1$

$|\omega_{02}|$ маю но орабн. ω_1 . $a_2 \approx -p_2 C_1 C_2 \frac{|G_2|}{|J|} \omega_1$ монет бвоб > 0 , $\mu < 0$,

т.е. $\vec{X}_1(t)$ - eigenmode с расорам ω_1 монет in-phase um out-of-phase

гбвонном $\vec{X}_1(t), \vec{X}_2(t)$ в счлах $F_1/N/F_2$

Ровбог: eigenfrequencies забвонет от p_1, p_2 но о/гбвонном (ω_2 за G_2) и C_1, C_2 -
но-огбвонном (ω_2 за C_2 fields в $\omega_{0j} \sim C_j J$). Eigenmodes забвонет от p_j, C_j монне.

Mon journal que Olivier, Gregoire, Vladimir, Benjamin o current state of calculations of SP current induced vortex dyn. in F₁/N/F₂ nanopylars.
 Hagan. Bonfacci: 1) known vortex excitations in thin and thick layers, Jai Jai;
 2) Muebunow yzer Oe-field vortex b Woj, Benno tokan J-buag Benno
 3) Benno tokan b exchng dls, no ug-noms d_{ij}, G_j u u due to u_j²(H₀)

Plan chavari: (I) F/N/F dyn: exp → Klein, theory - Gus, simul - Hagan
 (II) NiMnSB single v. dots - MRFM: exp - Klein, theory - Gus (1/2)

Skype - vasyngeme c Xbanobovum lvo F/N/F simulations - double v. dyn.

(17/XI) Uz PRL nprucame Referee's Responses na Topol. field. "MS: Ref. A na pbenengyer se b PRL, a Ref. B pbenengyer b gpyrei nprucame.

Vasyngeme c Gregoire u Olivier simulations - Xbanov. (only one eigenfreq. can be excited by SP current) u copykypni eigenmodes $\vec{X}_j = \vec{X}_1 + a_j \vec{X}_2$. Benno J. Yeanobna Origin u MathCad na moi desktop, u MathCad na HP laptop.

Xbanov nprucam ppt - file pacrer buaga Oe-field b xum wum c kopy. 1.70 kan b lvo PRB509. Current Oe-field Contribution to ω_{TM} (in 2/1/14)

$$W_{oe}(\vec{X}) = -L \int d^2 \vec{\rho} \vec{H}_{oe}(\vec{\rho}) \vec{M}(\vec{\rho}, \vec{X}) = -LM_0 H_0 \int d^2 \vec{\rho} \rho m_{\varphi}(\vec{\rho}, \vec{X})$$

$$\vec{H}_{oe} = H_0 \hat{\varphi}, H_{\varphi}(\rho) = H_0 \rho - \text{none kpyr. voka. } H_{\varphi} = \frac{1}{2} \frac{I}{\pi R^2} \rho = H_0 \rho \quad (SI)$$

$$m_{\varphi} = -m_x \sin \varphi + m_y \cos \varphi, \text{ hako } \int d\varphi m_{\varphi}(\vec{\rho}, \vec{\rho}) \text{, nprucame } \varphi \text{ u } m_x, m_y$$

~ S_x, S_y uvezner, +, x. S_z uovabum b m_x, m_y ~ cos²φ, sin²φ uum sinφ cosφ.

$$m_x = \cos \varphi, m_y = \sin \varphi, \varphi(x, y) = \mathcal{C} \frac{\pi}{2} + \arctg \left[\frac{y - \bar{y}}{x - \bar{x}} \right] + \arctg \left[\frac{y - \bar{y}}{x - \bar{x}} \right] - \arctg \frac{y}{x}$$

$$\vec{X} = X/(X^2 + Y^2), \vec{Y} = Y/(X^2 + Y^2). \text{ Hako poznom. } m_x, m_y \text{ uum } \varphi \text{ go } s_x^2, s_y^2.$$

$$\text{No uovum } Y=0, X \neq 0, \varphi = \mathcal{C} \frac{\pi}{2} + \arctg \left(\frac{y}{x - \bar{x}} \right) + \arctg \left(\frac{y}{x - 1/\bar{x}} \right) = \mathcal{C} \frac{\pi}{2} + \arctg \left[\frac{\frac{y}{x - s_x} + \frac{y}{x - 1/s_x}}{1 - \frac{y}{x - s_x} \cdot \frac{y}{x - 1/s_x}} \right] = \mathcal{C} \frac{\pi}{2} + \arctg \left[\frac{y(1 - \frac{1}{s_x}) + y(1 - s_x)}{(x - s_x)(x - \frac{1}{s_x}) - y^2} \right]$$

$$s_x = s. \varphi(x, y, s) = \mathcal{C} \frac{\pi}{2} + \arctg \left[\frac{2y - ys - y/s}{x^2 - y^2 + 1 - s_x - \frac{x}{s}} \right]$$

$$\frac{\partial \varphi}{\partial s} = \frac{1}{1 + (\dots)^2} \frac{\partial (\dots)}{\partial s} - \text{common ypomozhko.}$$

$$\text{Bne core } m_x + im_y = W, W = \frac{f(z)}{|f|}, f(z) = \frac{i}{c} \mathcal{C} \left[z - \frac{(s + \bar{s}z^2)}{1 + |s|^2} \right] -$$

$$\text{pacnomu } m_x, m_y \text{ c vovm. go } s^2. \text{ Bopen } s = \bar{s} = s_x. s_y = 0. |f|^2 \approx \frac{1}{c^2} [z - s(1 + z^2)][\bar{z} - s(1 + \bar{z}^2)] + O(s^3) = \frac{1}{c^2} [z\bar{z} + s^2(1 + z^2 + \bar{z}^2 + z^2\bar{z}^2) - s(1 + z^2)\bar{z} - s(1 + \bar{z}^2)z] = \frac{1}{c^2} [s^2 + s^2(1 + z^2 + \bar{z}^2 + s^4) - s - s s^2(z + \bar{z})]$$

$$m_x + im_y = \frac{f(z)}{|f|} = \frac{i \mathcal{C} [z - s(1 + z^2)]}{\sqrt{s^2 - 2s - s s^2(z + \bar{z}) + s^2(1 + z^2 + \bar{z}^2 + s^4)}} = \frac{i \mathcal{C} [z - s(1 + z^2)]}{\sqrt{s^2 - s(z + \bar{z})(1 + s^2) + s^2(1 + z^2 + \bar{z}^2 + s^4)}}$$

$$\frac{1}{\sqrt{1 + \varepsilon}} = \sum_{n=0}^{\infty} \binom{\alpha}{n} \varepsilon^n, \alpha = -\frac{1}{2}, \left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right), \left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right) \frac{1}{2} \left(-\frac{1}{2} - 2 + 1\right) = \frac{3}{8}$$

$$\frac{1}{\sqrt{1 + \varepsilon}} = 1 - \frac{1}{2} \varepsilon + \frac{3}{8} \varepsilon^2 + O(\varepsilon^3). m_x + im_y = \frac{f(z)}{|f|}, s_x - \text{uovno, r.k. hako } m_x \text{ u } m_y.$$

$$m_x + im_y = \frac{f}{\sqrt{|f|^2}} = \sqrt{\frac{s}{\bar{s}}}, \frac{s}{\bar{s}} = - \frac{[z - (s + \bar{s}z^2)]}{[\bar{z} - (s + \bar{s}\bar{z}^2)]} = - \frac{1}{\bar{z}} \frac{[z - (s + \bar{s}z^2)]}{[1 - \frac{1}{\bar{z}}(s + \bar{s}\bar{z}^2)]} =$$

$$= -\frac{1}{2} [z - (s + \bar{s}z^2)] [1 + \frac{1}{2}(s + \bar{s}z^2) + \frac{1}{2}(s + \bar{s}z^2)^2 + \dots] = -\frac{1}{2} [z + \frac{z}{2}(s + \bar{s}z^2) + \frac{z}{2}(s + \bar{s}z^2)^2 - (s + \bar{s}z^2) - \frac{1}{2}(s + \bar{s}z^2)(s + \bar{s}z^2)] = (m_x + im_y)^2 \frac{1}{2} \sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8}$$

$$(m_x + im_y)^2 = -\frac{2}{z} [1 + \frac{1}{2}(s + \bar{s}z^2) - \frac{1}{2}(s + \bar{s}z^2) + \frac{1}{2}(s + \bar{s}z^2)^2 - \frac{1}{2}(s + \bar{s}z^2)(s + \bar{s}z^2)]$$

$$m_x + im_y = \sqrt{-\frac{2}{z}} [1 + \frac{1}{2} \frac{1}{2}(s + \bar{s}z^2) - \frac{1}{2} \frac{1}{2}(s + \bar{s}z^2) + \frac{1}{2} \frac{1}{2}(s + \bar{s}z^2)^2 - \frac{1}{2} \frac{1}{2}(s + \bar{s}z^2)(s + \bar{s}z^2)] = \sqrt{-\frac{2}{z}} [1 + \frac{1}{4} \frac{1}{2}(s + \bar{s}z^2)^2 - \frac{1}{4} \frac{1}{2}(s + \bar{s}z^2)(s + \bar{s}z^2)] = \sqrt{-\frac{2}{z}} [1 - \frac{1}{4} \frac{1}{2}(s + \bar{s}z^2)(s + \bar{s}z^2) + \frac{3}{8} \frac{1}{2}(s + \bar{s}z^2)^2 - \frac{1}{8} \frac{1}{2}(s + \bar{s}z^2)^2]$$

$$= \sqrt{-\frac{2}{z}} [1 - \frac{1}{4} \frac{1}{2}(s + \bar{s}z^2)(s + \bar{s}z^2) + \frac{3}{8} \frac{1}{2}(s + \bar{s}z^2)^2 - \frac{1}{8} \frac{1}{2}(s + \bar{s}z^2)^2]$$

$$d\varphi = d\varphi [-m_x \sin \varphi + m_y \cos \varphi]$$

$$-\frac{2}{z} = -\frac{2}{z^2} = -\frac{1}{z^2} (x + iy)^2 = -\frac{1}{z^2} (x^2 - y^2 + 2ixy), \quad d\varphi(\dots) \neq 0 \text{ тогда } \sim y, y^3 \text{ и } \text{полюсов } m_x \text{ и } \text{нулов } \sim x, x^3 \text{ и } \text{полюсов } m_y.$$

$$m_x + im_y = \frac{i}{z^2} \sqrt{(x^2 - y^2 + 2ixy)} [1 - \frac{1}{4} \frac{1}{2}(s + \bar{s}z^2)(s + \bar{s}z^2) + \frac{3}{8} \frac{1}{2}(s + \bar{s}z^2)^2 - \frac{1}{8} \frac{1}{2}(s + \bar{s}z^2)^2] = \sqrt{-\frac{2}{z}}.$$

$$\text{с. 6/1408: } w(z) = \arg \frac{f(z)}{|f|} = \frac{f(z)}{|f|} = \frac{A}{|f|} (z - z_1)(z - z_2), \quad A = -\frac{\pi}{2c} \bar{a}$$

$$f(z) = \frac{c}{2} [iz + \frac{a}{2} - \frac{\bar{a}}{2} z^2], \text{ нули } z_1, z_2. \quad f(z) = i \frac{c}{2} [z - \frac{(s + \bar{s}z^2)}{1 + |s|^2}]$$

$$f(z) = 0, \quad (1 + |s|^2)z - s - \bar{s}z^2 = 0, \quad \bar{s}z^2 - (1 + |s|^2)z + s = 0$$

$$z_{1,2} = \frac{1}{2\bar{s}} [1 + |s|^2 \pm \sqrt{(1 + |s|^2)^2 - 4|s|^2}] = \frac{1}{2\bar{s}} [1 \pm |s|^2 \pm (1 - |s|^2)] = \begin{cases} \frac{1}{\bar{s}} = \frac{s}{|s|^2} \\ \frac{s}{|s|^2} = s \end{cases}$$

$$z_1 \bar{z}_2 = s \cdot \frac{1}{\bar{s}} = 1. \quad f(z) = i \frac{c}{2} \frac{(s - \bar{s})}{1 + |s|^2} [z^2 - \frac{1}{3}(1 + |s|^2)z + \frac{s}{3}] = i \frac{c}{2} \frac{s}{1 + |s|^2} (z - z_1)(z - z_2)$$

$$A = -\frac{i}{2} c \frac{s}{1 + |s|^2}. \quad \Phi(x, y, s) = \arg \frac{f(z)}{|f|} = \arg(A) + \arg(z - z_1) + \arg(z - z_2), \quad \begin{matrix} z_1 = s \\ z_2 = \frac{s}{|s|^2} \end{matrix}$$

$$\Phi(x, y, s) = -\frac{\pi}{2} + \arctg \frac{y}{x} + \arctg \left(\frac{y - y_1}{x - x_1} \right) + \arctg \left(\frac{y - y_2}{x - x_2} \right) + \pi.$$

$$\arctg \left(\frac{y - y_1}{x - x_1} \right) - \arctg \frac{y}{x} = \arctg \left[\frac{\frac{y - y_1}{x - x_1} - \frac{y}{x}}{1 + \frac{y - y_1}{x - x_1} \frac{y}{x}} \right] = \arctg \left[\frac{x(y - y_1) - y(x - x_1)}{x(x - x_1) + y(y - y_1)} \right]$$

$$= \arctg \left[\frac{xy - y_1 x}{x_1 x + y_1 y - 1} \right] = -\arctg \left[\frac{xy - y_1 x}{1 - x_1 x - y_1 y} \right] = -\arctg \left[\frac{xy - y_1 x}{1 - (x_1^2 + y_1^2)} \right]$$

$$\arctg \left[\frac{y_1 x - y x_1}{1 - (x_1^2 + y_1^2)} \right] \approx \arctg \left[\frac{(y_1 x - x y_1)(1 + x_1^2 + y_1^2)}{1 - (x_1^2 + y_1^2)} \right] = |\arctg \varepsilon = \varepsilon - \frac{1}{3} \varepsilon^3 + \dots| =$$

$$= (y_1 x - y x_1)(1 + x_1^2 + y_1^2). \quad \Phi_{u.g.v.} = \frac{\pi}{2} + \arctg \left(\frac{y - y_1}{x - x_1} \right), \quad \frac{\partial \Phi_{u.v.}}{\partial x} = \frac{(y - y_1)}{|z - z_1|^2} \frac{\partial}{\partial x} \frac{(x - x_1)}{|z - z_1|^2}$$

$$\frac{\partial^2 \Phi_{u.v.}}{\partial x^2} = \frac{(y - y_1)(-1)}{[(x - x_1)^2 + (y - y_1)^2]^2} \cdot 2(x - x_1)(-1) = \frac{2xy}{s^4}$$

$$\frac{\partial^2 \Phi_{u.v.}}{\partial x \partial y} = -\frac{1}{|z - z_1|^2} + (y - y_1) \frac{(-1)}{[(x - x_1)^2 + (y - y_1)^2]^2} \cdot 2(y - y_1)(-1) = -\frac{1}{s^2} + \frac{2y^2}{s^4}$$

$$\frac{\partial^2 \Phi_{u.v.}}{\partial y^2} = -(x - x_1) \frac{\partial}{\partial y} \frac{1}{[(x - x_1)^2 + (y - y_1)^2]} = -(x - x_1) \frac{(-1)}{[\dots]^2} \cdot 2(y - y_1)(-1) = -\frac{2xy}{s^4}$$

$$\Phi(x, y, \bar{x}) = \frac{\pi}{2} + (y_1 x - x y_1)(1 + x_1^2 + y_1^2) + \frac{y}{s^2} x - \frac{x}{s^2} y + \frac{2xy}{s^4} x^2 - \frac{2xy}{s^4} y^2 + (-\frac{1}{s^2} + \frac{2y^2}{s^4}) xy$$

$$\Phi(x, y, \bar{x}) = \frac{\pi}{2} + \varphi + (\frac{1}{s^2} - 1)(y_1 x - x y_1) + (y_1 x - x y_1)(x_1 + y_1) + \frac{xy}{s^4} x^2 - \frac{xy}{s^4} y^2 + \frac{1}{s^2} (\frac{2y^2}{s^2} - 1) xy$$

$$\rightarrow m_\varphi = \sin \Phi \cos \varphi - \cos \Phi \sin \varphi = \sin(\Phi - \varphi) = \sin(\frac{\pi}{2} + \varepsilon) = \cos \varepsilon = \cos(1 - \frac{\varepsilon^2}{2})$$

$$\varepsilon = a_1 \bar{X} + b_1 Y + a_2 \bar{X}^2 + b_2 Y^2 + c_2 XY, \quad \varepsilon^2 = a_1^2 \bar{X}^2 + b_1^2 Y^2 + 2a_1 b_1 XY$$

$$m_\varphi = \sigma (1 - \frac{1}{2} \varepsilon^2) \text{ на поверхности } \varepsilon^2, \text{ т.е. на поверхности } \sigma$$

$$\varepsilon = (\frac{1}{2} - 1) (\bar{X} - Y) = \frac{1}{2} (1 - \rho^2) [\sin \varphi \bar{X} - \cos \varphi Y] = m_0(\rho) [\sin \varphi \bar{X} - \cos \varphi Y]$$

$$W_{oe}(\vec{X}) = -LM_s H_0 \int d\vec{\rho} \rho m_\varphi(\vec{\rho}, \vec{X}) = -LM_s H_0 \int d\vec{\rho} \rho \sigma [1 - \frac{1}{2} \varepsilon^2]$$

$$= \text{const} + \frac{1}{2} LM_s H_0 \sigma \int d\vec{\rho} \rho m_0^2(\rho) [\sin^2 \varphi \bar{X}^2 + \cos^2 \varphi Y^2 - 2 \sin \varphi \cos \varphi \bar{X} Y]$$

$$= \frac{1}{2} LM_s H_0 \sigma \pi |\vec{X}|^2 \int_0^R \rho^2 m_0^2(\rho) d\rho, \quad \int_0^R \rho^2 m_0^2(\rho) d\rho = \int_0^1 d\rho (1 - \rho^2)^2 = \int_0^1 [1 - 2\rho^2 + \rho^4] d\rho = [\rho - \frac{2}{3}\rho^3 + \frac{1}{5}\rho^5]_0^1 = [1 - \frac{2}{3} + \frac{1}{5}] = \frac{8}{15}$$

$$W_{oe}(\vec{X}) = \frac{1}{2} \cdot \frac{8\pi}{15} LM_s H_0 \sigma |\vec{X}|^2 R$$

$$\text{then } W_{oe}(\vec{X}) = \frac{1}{2} \lambda |\vec{X}|^2, \quad \lambda = \frac{8\pi}{15} LRM_s \sigma H_0, \quad H_0 = \frac{1}{2} \frac{I}{\pi R^2} \text{ (SI)}$$

$$\text{B c r c } H_c = \frac{2I}{c} \frac{1}{R^2}, \quad H_0 = \frac{2I}{cR^2} \text{ (CGS)}, \quad H_0 \text{ (CGS)} = \frac{4\pi}{c} H_0 \text{ (SI)}$$

$$\omega_m = \frac{1}{|G|} (\kappa + \lambda) = \omega_m(0) + \frac{\lambda}{|G|}, \quad \frac{\lambda}{|G|} = \frac{8\pi}{15} \frac{LRM_s \sigma H_0}{2\pi M_s L \delta^{-1}} = \frac{4}{15} \delta H_0 R$$

$$\omega_0(I) = \omega_0(0) + \frac{4}{15} \delta H_0 R, \quad H_0 = \frac{2I}{cR^2} \text{ (CGS)} = \frac{2\pi J}{c}, \quad I = \pi R^2 J$$

$$\text{CGS: } \Delta\omega_0(I) = \frac{4}{15} \delta R \cdot \frac{2\pi J}{c} = \left(\frac{8\pi}{15}\right) \delta R \frac{J}{c}, \quad \frac{8\pi}{15} = 1.676, \quad \text{У Чарльза. PRB'09}$$

$$\text{SI: } \Delta\omega_0(I) = \frac{4}{15} \delta R \cdot \frac{I}{2\pi R^2} = \frac{2}{15\pi} \frac{\delta}{R} I, \quad [I] = A, [R] = m. \text{ Cu. y granada}$$

$$\Delta\omega_0(I) = 0.0424 \cdot \delta \left(\frac{I}{R}\right) \text{ c, Due } I = 10 \text{ mA}, R = 100 \text{ nm}, \Delta\omega_0(I) \approx 26/\pi$$

$$\Delta\omega_0(I) = 0.0424 \cdot (2\pi) \cdot 2.95 \frac{\text{MHz}}{\text{Oe}} \frac{10^{-2} A}{10^{-7} m}, \quad 1 \frac{A}{m} = \frac{4\pi}{1000} \text{ Oe} = 12.57 \cdot 10^3 \text{ Oe}$$

$$\frac{\Delta\omega_0(10 \text{ mA})}{2\pi} = 157.18 \text{ MHz} \approx 157.2 \text{ MHz}, \quad 1 \text{ Oe} = 79.58 \text{ G}$$

No even dot under R = 50-60 nm, no page Core ne man no crabn. c R.

$$\text{Choi-Kim PRB'09: } \Delta\omega(I) = \frac{1}{2\pi^2} \frac{45}{68} \delta \frac{I}{R} = 0.0335 \delta \frac{I}{R} - \text{gives negative}$$

$$\text{Correction } \frac{1}{2\pi^2} \frac{45}{68} \cdot \left(\frac{4}{\pi}\right) = 0.0427 - \text{be rabno gar nebernomi korp.}$$

$$\int_0^1 d\rho \rho \int d\varphi m_\varphi(\vec{\rho}) \rho = \int_0^1 d\rho (\dots) + \int_0^1 d\rho (\dots), \quad m_\varphi = \sin(\varphi - \varphi_0) = \sigma (1 - \frac{\varepsilon^2}{2}) \text{ due core}$$

$$\text{Bryozym Core } m_\varphi = m_y \cos \varphi - m_x \sin \varphi = \sin \theta [\sin \theta \cos \varphi - \cos \theta \sin \varphi] = \sin \theta \sin(\theta - \varphi)$$

$$\text{T.e. signum } \int d\rho \rho^2 \sin^2 \theta_0(\rho) m_0^2(\rho) = \int d\rho \frac{4c^2 \rho^2}{(c^2 + \rho^2)^2} (1 - \rho^2)^2 + \int d\rho (1 - \rho^2)^2 =$$

$$= \left| \sin \theta_0 = \frac{2c\rho}{c^2 + \rho^2} \text{ bryozym core} \right| = \frac{4}{c^2} \int d\rho \frac{\rho^2}{(1 + \rho^2/c^2)^2} (1 - \rho^2)^2 = 4c \int_0^1 \frac{x^2}{(1 + x^2)^2} (1 - x^2)^2 dx$$

$$+ \left[\rho - \frac{2}{3}\rho^3 + \frac{1}{5}\rho^5 \right]_0^1 = \left| \text{form. so } c^2 \right| = 4c \cdot 0.143 + \frac{8}{15} - c = \frac{8}{15} - 0.429 \cdot c + O(c^3)$$

$$\text{T.e. yzer core choqurce } \kappa \frac{8}{15} \rightarrow \left(\frac{8}{15} - 0.43c\right) \text{ b y p-u gna } \omega_0(I).$$

$$\text{T.x. } c = 0.1 - 0.2 - \text{nonrabna ka u, core cymerabonnaya.}$$

Но мы же $\omega_0(I)$ еще бинам тона в энергиях Vortex (uzazhdin)

$$W_{oe}(\vec{X}=0) = -LM_s H_0 \sigma \int d\vec{\rho} \rho^2 = -LM_s H_0 \sigma R^3 \cdot 2\pi \int_0^1 d\rho \rho^2 = -\frac{2\pi}{3} LM_s R^3 H_0 \sigma$$

$$\rightarrow \frac{1}{V} W_{oe}(0) = -\frac{2}{3} M_s R H_0 \sigma = |H_0 = \frac{I}{2\pi R^2} \text{ (SI)}| = -\frac{1}{3\pi} M_s \cdot \frac{I}{R} \sigma$$

$$H_I = \frac{I}{R}, \quad H_I = \frac{10^{-2} A}{10^{-7} m} = 10^5 \frac{A}{m} = \frac{10^5}{80} \text{ Oe} \approx 1.25 \cdot 10^3 \text{ Oe} = 1.25 \text{ kOe}$$

$$H_{ess} = \frac{1}{3\pi} H_I, \quad \text{sumar. } H_{ess} = \frac{1}{3\pi} 1.25 \text{ kOe} \approx 100 \text{ Oe}$$

гонорит. не Crabum зурьуре (K>0) um gebabumz. Vortex no om, K<0

(18/XI) We have 110 G magnetic field j -axis along xy -plane xy -field.

$$W_{00}^{(j)}(\vec{X}_j) = -\frac{1}{2\pi} M_{sj} \left(\frac{1}{R} \right) \phi_j + \frac{1}{2} |G_j| \omega_{0j}(I) |\vec{X}_j|^2, \quad \omega_{0j}(I) = \omega_{0j}(0) + \frac{2}{15\pi} \frac{I}{R} \phi_j$$

use $(1/R)$ replace ϕ SI. Year core $\int_0^1 d\phi \sin^2 \theta_0 m_0^2 = 4\pi \left(\frac{1}{2} - 1 \right) + \frac{8}{15} - c = \frac{8}{15} - (2 - \frac{\pi}{2}) + O(c)$
 Year v. core gear $\Delta \omega_{0j}(I)$ has $[1 - \frac{15}{2} (2 - \frac{\pi}{2}) c]$

$$\omega_{0j}(I) = \omega_{0j}(0) + \frac{2}{15\pi} \phi_j \frac{I}{R} F_c(\phi_j), \quad F_c(\phi_j) = 1 - \frac{15}{2} (2 - \frac{\pi}{2}) \phi_j, \quad \phi_j = \frac{R \phi_j}{R}$$

3 gear $\omega_{0j}(0) = \frac{\kappa_j}{15} > 0$. c year interlayer coupling.

$$W(\vec{X}_1, \vec{X}_2) = -\frac{1}{2\pi} \frac{1}{R} [M_{s1} \phi_1 + M_{s2} \phi_2] + \frac{1}{2} |G_j| \omega_{0j}(I) |\vec{X}_j|^2 + \mu(\vec{X}_1, \vec{X}_2)$$

Has interlayer noncommensurate energy, μ noncommensurate energy noncommensurate j -vortex, so $\mu(\vec{X}_1, \vec{X}_2) = c_1 c_2 |M_1| |\vec{X}_1| |\vec{X}_2| \cos \alpha_{12} = -|M_1| |\vec{X}_1| |\vec{X}_2|$,
 i.e. $\cos \alpha_{12} = -c_1 c_2$ sign.

$$W(\vec{X}_1, \vec{X}_2) = W(0,0) + \frac{1}{2} |G_1| \omega_{01}(I) |\vec{X}_1|^2 + \frac{1}{2} |G_2| \omega_{02}(I) |\vec{X}_2|^2 - |M_1| |\vec{X}_1| |\vec{X}_2| - \text{some constant}$$

Calculate commensurate 2-vortex state, $\frac{\partial^2 W}{\partial |\vec{X}_j|^2} = |G_j| \omega_{0j}(I) > 0$ and
 $D = \frac{\partial^2 W}{\partial |\vec{X}_1|^2} \frac{\partial^2 W}{\partial |\vec{X}_2|^2} - \left(\frac{\partial^2 W}{\partial |\vec{X}_1| \partial |\vec{X}_2|} \right)^2 = |G_1 G_2| \omega_{01}(I) \omega_{02}(I) - \mu^2 = \kappa_1(I) \kappa_2(I) - \mu^2 > 0$

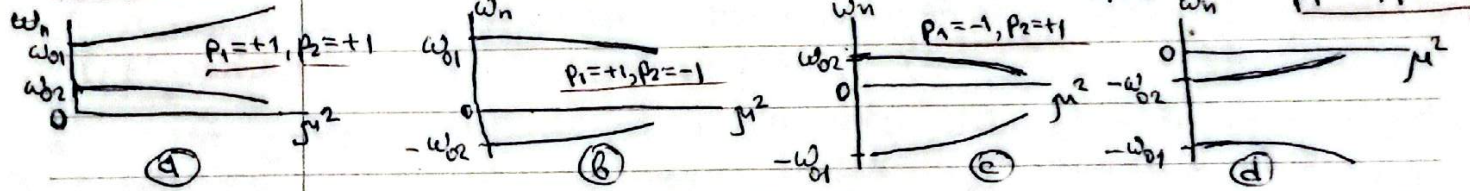
$$\kappa_j(I) = \kappa_j(0) + \lambda_j(I), \quad \lambda_j(I) = \frac{4}{15} L_j M_{sj} \phi_j \frac{I}{R} - \text{noncommensurate } \kappa_j \text{ w.r.t. } I.$$

By the way, argument (I) in $\kappa_j, \lambda_j, \omega_{0j}$, noncommensurate, no I -terms included.

Gen. yr - e give eigenfrequencies $\omega^2 G_1 G_2 - (G_1 \alpha_2 + G_2 \alpha_1) \omega + \alpha_1 \alpha_2 - \mu^2 = 0$,
 $\omega_1 \omega_2 = (\alpha_1 \alpha_2 - \mu^2) \frac{1}{G_1 G_2}$. $\omega^2 - (P_1 \omega_{01} + P_2 \omega_{02}) \omega + \frac{P_1 P_2}{|G_1 G_2|} (\alpha_1 \alpha_2 - \mu^2) = 0$

$G_j = P_j |G_j|$, $\omega_{0j} = \frac{\kappa_j(I)}{|G_j|} > 0$ and ω_{0j} vortex commensurate. Vortex ($H_z=0$)
F/N/F system

$$2\omega_{1,2} = P_1 \omega_{01} + P_2 \omega_{02} \pm \sqrt{(P_1 \omega_{01} + P_2 \omega_{02})^2 - 4 P_1 P_2 (\omega_{01} \omega_{02} - \frac{\mu^2}{|G_1 G_2|})}$$



i.e. from ω_1, ω_2 chosen, so $(a) \equiv (d)$, $(b) \equiv (c)$ and $(a) \equiv (b)$.

$$I(t) = \int d\phi \sin^2 \theta_0 J_1(t\phi) \neq 2 \int d\phi J_1(t\phi) - \text{nonzero increment in MathCad.}$$

No b. source. $\text{div} \vec{m}$ 10/XI he ϕ sign smooth, observe $I(t) = \int d\phi \frac{\partial \omega_0}{\partial \phi} J_1(t\phi)$.

$$W_m^{v1-v2} = \mu(\vec{X}_1, \vec{X}_2), \quad \mu = 2\pi^2 c_1 c_2 M_{s1} M_{s2} F_v(\beta_1, \beta_2, t), \quad F_v(\beta_1, \beta_2, t) = \int_0^t dt' f(\beta_1, \beta_2, t')$$

$$f(\beta_1, \beta_2, t) = e^{-t\beta_1} (1 - e^{-\beta_2 t}) (1 - e^{-\beta_1 t}).$$

$P_y/Cu/P_y$

Parameters $P_y/Cu/P_y$ nanopillar: $d_{Cu} = 10 \text{ nm}$, $L_1 = 15 \text{ nm}$, $L_2 = 4 \text{ nm}$

$$R = 125 \text{ nm (use } R = 60 \text{ nm)}, \quad 4\pi M_{s1} = 9.4 \text{ kG (748 G)}, \quad 4\pi M_{s2} = 7.8 \text{ kG (} \mu_{s2} = 6216)$$

Magn. interlayer coupling c $I(t) \sim m_0'$ common beam. Probe beam.

$$\nabla m_x + i \nabla m_y = W \text{ but core } W = \frac{f(z)}{151} = \frac{\sqrt{f}}{\sqrt{5}}, \quad \frac{\partial W}{\partial z} = \frac{1}{151} \frac{1}{2\sqrt{f}} \frac{\partial f}{\partial z} = \frac{1}{2151} \frac{\partial f}{\partial z}$$

$$\frac{\partial^2 W}{\partial z^2} = \left(\frac{\partial}{\partial x} - i \frac{\partial}{\partial y} \right) \frac{1}{2} W = \left[\frac{\partial m_x}{\partial x} + i \frac{\partial m_y}{\partial x} - i \frac{\partial m_x}{\partial y} + \frac{\partial m_y}{\partial y} \right] \frac{1}{2} \nabla \vec{m} = 2 \text{Re} \left(\frac{\partial W}{\partial z} \right)$$

$$f(z) = \frac{1}{2} \phi \left[z - \frac{(5 + 5z^2)}{1 + 5z^2} \right] \approx \frac{1}{2} \phi [z - 5 - 5z^2], \quad \text{div} \vec{m} = \frac{1}{151} \left(\frac{\partial f}{\partial z} \right) \text{Re} =$$

$$= \frac{1}{151} \text{Re} \left[\frac{1}{2} \phi [1 - 25z^2] \right] = -\frac{2}{151} \phi \text{Re} [i(s_x - i s_y)(x + i y)] = -\frac{2}{151} \phi (-s_x y + s_y x) = \frac{2}{151} \phi (s_y x - s_x y)$$

$$\text{div} \vec{m} = \frac{2}{151} \phi (x s_y - y s_x) = 2 \phi (s_y \cos \varphi - s_x \sin \varphi), \text{ where } \varphi = 9/11 \text{ and } \text{div} \vec{m} = c m_0' (s_y \cos \varphi - s_x \sin \varphi)$$

Escaneado con CamScanner

No.

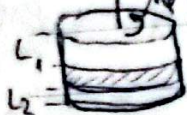
single vertex state (Krauss, PRB09)

1. I - most of the

одно кван energy $W_{\text{фотона}}$

$$r(j) = - \left(\frac{\partial \mathcal{L}}{\partial \vec{m}_j} \cdot \frac{\partial \vec{m}_j}{\partial X_{j,d}} \right)$$

arizer.



Rotating CCW

for $I > 0$ then we

4 км) P_2 через L_2 толщиной (15 км) P_2 изв. т.е. сд. излучения
из толщины L_1 в толщину L_2 . L_1 -толщина L_2 есть polarizer
микрофон.

надо ввести $\vec{F}_{\text{ст}}(2)$ только,
если $I > 0$.
и надо $\vec{F}_{\text{ст}}(1)$ только,
если $I < 0$.

и/или Γ_{∞} изменить до $\tilde{\Gamma}_1$
из-за $\tilde{F}_{ST}(1)$.

4). Проверить.

$\vec{m}_2 \times (\vec{m}_1 \times \vec{m}_2) a$. Оба torque
одного рода J одновременно.

ST forces $F/N/F$

также $\vec{\tau}_1, \vec{\tau}_2$ одновременно,

$$\frac{\partial W_3}{\partial \vec{m}_2} \cdot \frac{\partial \vec{m}_2}{\partial X_{2d}}, \quad d = x, y.$$

$$\frac{V_s}{\vec{m}_1} = a(\vec{m}_1 \times \vec{m}_2)$$

нечетная $\Gamma_{ST}(1), \Gamma_{ST}(2)$.

$$\vec{m}_2^0 + \vec{m}_1^0 \times \delta \vec{m}_2 + \delta \vec{m}_1 \times \delta \vec{m}_2$$

и $(\vec{m}_1 \times \vec{m}_2)_{\text{radial}}$ берганаар $\sim x_d^2$
н/м (1.06)

$$\frac{2m_1}{2X} =$$

$$\left(\frac{\vec{m}_1}{X_{1d}} \times \frac{\partial \vec{m}_2}{\partial X_{2\beta}} \right)$$

$$+ \frac{1}{2} \left(\frac{\partial \vec{m}_2}{\partial x_{2\alpha}} \times \frac{\partial \vec{m}_2}{\partial x_{2\beta}} \right)$$

$$- [X_{1\beta} y_2^{i1} - X_{2\beta} y_1^{i2}]$$

2. Но это надо доказать.

Escaneado con CamScanner

t.e. $\frac{\partial W_s}{\partial \theta_2} = a m_1 \dot{\phi}_2$, $\frac{\partial W_s}{\partial \dot{\phi}_2} = -m_1^0 a \sin \theta_2 a$.

$$\left(\frac{\partial W_3}{\partial \theta_2} \right) = a(\vec{m}_1 \times \vec{m}_2)_{\theta_2} = a(\vec{m}_1 \times \vec{m}_2) \cdot \hat{\theta}_2 = a(5/\sqrt{1})_x (\hat{x} \cdot \hat{\theta}_2) + a(\dots)_y (\hat{y} \cdot \hat{\theta}_2) + a(\dots)_z (\hat{z} \cdot \hat{\theta}_2)$$

8/x1 равно $\frac{\partial W_3}{\partial \theta_1} = -a m_2 \varphi_1 = -a \sin \theta_2 \sin(\varphi_2 - \varphi_1)$, то при $\theta_1 \rightarrow \theta_2$ совм. с $\frac{\partial W_3}{\partial \theta_2}$
т.е. передается реакция 8/x1 Бермуди (враще).

$$m_1^{O_2} = (\hat{x} \cdot \hat{O}_2) \sin \theta_1 \cos \varphi_1 + (\hat{y} \cdot \hat{O}_2) \sin \theta_1 \sin \varphi_1 + (\hat{z} \cdot \hat{O}_2) \cos \theta_1 = \sin \theta_1 \cos \varphi_1 \cos \theta_2 \cos \varphi_2 + \sin \theta_1 \sin \varphi_1 \cos \theta_2 \sin \varphi_2 + \cos \theta_1 (-\sin \theta_2) = \sin \theta_1 \cos \theta_2 \cos(\varphi_1 - \varphi_2) - \cos \theta_1 \sin \theta_2$$

$$\vec{F}_{ST}(2) = - \frac{\partial}{\partial \vec{x}_2} \int dV w_s = - \int dV \left[\frac{\partial w_s}{\partial \theta_2} \nabla_{\vec{x}_2} \theta_2 + \frac{\partial w_s}{\partial \varphi_2} \nabla_{\vec{x}_2} \varphi_2 \right]$$

(пробуем с $2/x$) Значит: $\vec{F}_{ST}(2) = -\vec{F}_{ST}(1)$ - при условии $1 \rightleftharpoons 2$ ($\varphi_1 \rightleftharpoons \varphi_2, \varphi_1 \neq \varphi_2$)

Уз пречера $\vec{F}_{ST}(1)$ з/х1 чег. $I_1 = aL J_1 = 2\pi aL \int_0^c \rho g m_0 g_1 [\cos \theta_1^0 \cos \theta_2^0 + C_1 \sin \theta_1^0 \sin \theta_2^0]$

Torneo $\vec{F}_{ST}(1) = I_1(1) (\hat{z} \times \vec{X}_1) - I_2(1) (\hat{z} \times \vec{X}_2)$, $I_1(1) = \frac{d\sigma_1}{d\Omega} [\cos \theta_1]$

$$\begin{cases} \vec{G}_1 \times \dot{\vec{X}}_1 - \alpha_1 \vec{X}_1 - \mu \vec{X}_2 - D_1 \dot{\vec{X}}_1 + I_1^{(1)} (\hat{z} \times \vec{X}_1) - I_2^{(1)} (\hat{z} \times \vec{X}_2) = 0 \\ \vec{G}_2 \times \dot{\vec{X}}_2 - \alpha_2 \vec{X}_2 - \mu \vec{X}_1 - D_2 \dot{\vec{X}}_2 - I_1^{(2)} (\hat{z} \times \vec{X}_2) + I_2^{(2)} (\hat{z} \times \vec{X}_1) = 0 \end{cases}$$

$\vec{X}_j \neq \vec{\omega} \times \vec{X}_j$. $\vec{F}_{ST}(1)$ не колл $\parallel \vec{X}_1$, а $\vec{F}_{ST}(2)$ не $\parallel \vec{X}_2$, game elm $I_2 =$
 Рендерующ $\vec{F}_D(j) + \vec{F}_{ST}(j) = 0$, 2-го game крив. then $J_{ST}(j)$.

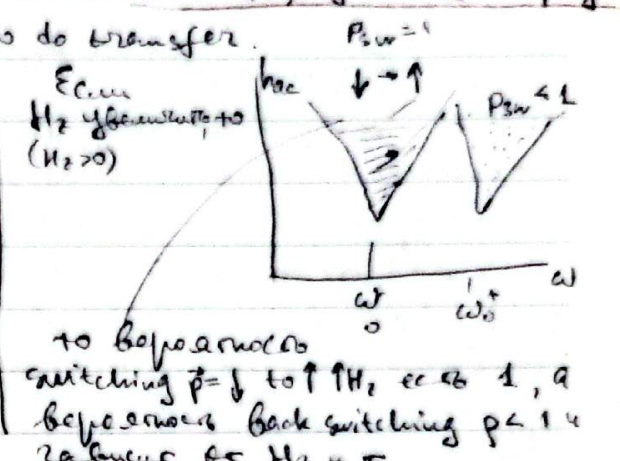
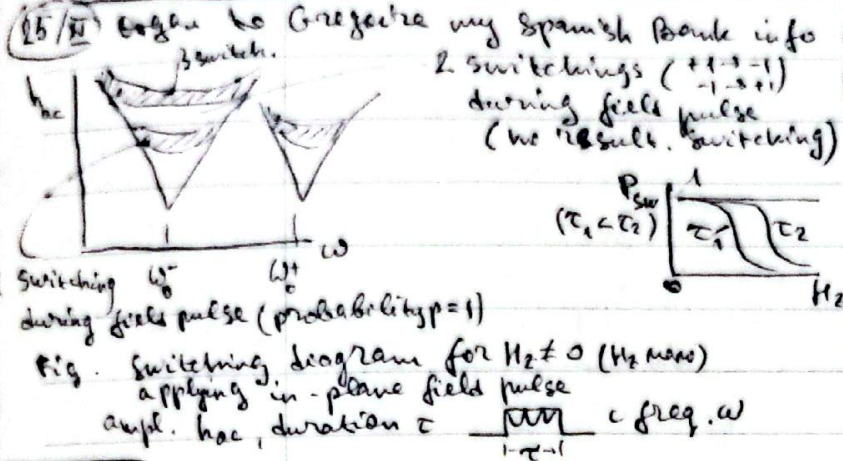
$$\frac{\mu_1 \mu_2}{\hat{G}_1 \hat{G}_2} = \frac{(\mu + i I_{21})(\mu - i I_{22})}{(G_1 - i D_1)(G_2 - i D_2)} = \frac{(G_1 + i D_1)(G_2 + i D_2)}{G_1^2 + D_1^2} \frac{[\mu^2 + I_{21} I_{22} + i \mu (I_{21} - I_{22})]}{G_2^2 + D_2^2}$$

$$\approx \frac{1}{G_1^2 G_2^2} [G_1 G_2 - D_1 D_2 + i (D_1 G_2 + D_2 G_1)] [\mu^2 + I_{21} I_{22} + i \mu (I_{21} - I_{22})]$$

$$\approx \frac{1}{G_1^2 G_2^2} \left\{ G_1 G_2 (\mu^2 + I_{21} I_{22}) - (D_1 G_2 + D_2 G_1) \mu (I_{21} - I_{22}) + [G_1 G_2 \mu (I_{21} - I_{22}) + \mu^2 (D_1 G_2 + D_2 G_1)] \right\}$$

Eqn: $I_{21}, I_{22} \sim D_j$, so $\frac{\mu_1 \mu_2}{\hat{G}_1 \hat{G}_2} \approx \frac{1}{G_1^2 G_2^2} [\mu^2 G_1 G_2 + i \mu \{ G_1 G_2 (I_{21} - I_{22}) + \mu (D_1 G_2 + D_2 G_1) \}]$

$\text{Im}(\omega_{1,2})$ gives eff. damping



Важное поле T-преобразования при вк switching барьера и не надо $\psi_c(H_2) = 0$, т.к. тем. преобразование координат TM-волы с константой амплитуды, но не надо K switching. Найдем чувствительность $|X|$ ($|X|^2 \sim kT$) и $\dot{X}^2$ -integral и $\dot{X}^2(\omega)$ -PSD(ω) вращающегося груза. - где. теория груза TM, нем SW, нем перем. гр-е барьера где градиент Thiele $\vec{G} \times \dot{\vec{X}} - \frac{\partial W}{\partial \vec{X}} = \vec{F}(t)$, $\vec{F}(t)$ - random fluct force

Take: // ST torque in vortex state with $\vec{M}_p \parallel \hat{z}$ (polarizer is uniform), cm. by $\vec{X}$.

$$-\frac{M_s}{\gamma} \sin \theta \dot{\theta} = \frac{\partial W'}{\partial \Phi} + \frac{\partial W_s}{\partial \Phi} = \frac{\partial W'}{\partial \Phi} - a \sin^2 \theta; \quad \frac{M_s}{\gamma} \dot{\Phi} \sin \theta = \frac{\partial W'}{\partial \theta}, \quad W = W' + W_s, \quad W_s - \text{ST energy.}$$

$$\vec{\tau} = a \vec{m} \times (\vec{m} \times \vec{p}), \quad \vec{p} = \hat{z}, \quad \frac{\partial W_s}{\partial \vec{m}} = a \vec{m} \times \hat{z}, \quad \frac{\partial W_s}{\partial \Phi} = -a \sin^2 \theta, \quad \frac{\partial W_s}{\partial \theta} = 0, \quad a = 2 M_s J$$

$$-\vec{F}_{ST} = -\frac{\partial}{\partial \vec{X}} \int dV W_s = -\int dV \left[\frac{\partial W_s}{\partial \theta} \frac{\partial \theta}{\partial \vec{X}} + \frac{\partial W_s}{\partial \Phi} \frac{\partial \Phi}{\partial \vec{X}} \right] = a L \int d^2 \vec{r} \sin^2 \theta \nabla_{\vec{X}} \Phi$$

$$\Phi(\vec{r}, \vec{X}) = \frac{\pi}{2} + \varphi + m_0(\rho) [X \sin \varphi - Y \cos \varphi], \quad \sigma_{\vec{X}} \Phi = m_0(\rho) [\sin \varphi \hat{x} - \cos \varphi \hat{y}]$$

$$\dot{\Phi} = m_0 [\dot{X} \sin \varphi - \dot{Y} \cos \varphi], \quad \dot{W}_s = \int dV \left[\frac{\partial W_s}{\partial \theta} \dot{\theta} + \frac{\partial W_s}{\partial \Phi} \dot{\Phi} \right] = -a \int dV \sin^2 \theta \dot{\Phi}$$

$$\dot{W}_s = -a L \int d^2 \vec{r} \sin^2 \theta \dot{\Phi} = -a L \int d^2 \vec{r} \sin^2 \theta m_0 [\dot{X} \sin \varphi - \dot{Y} \cos \varphi], \quad \dot{Y} = \omega_0 X, \quad \dot{X} = -\omega_0 Y$$

$$\dot{W}_s = \omega_0 a L \int d^2 \vec{r} \sin^2 \theta m_0 [X \cos \varphi + Y \sin \varphi] = a L \omega_0 \int d^2 \vec{r} m_0^2 [1 - \cos^2 \theta] [X \cos \varphi + Y \sin \varphi] =$$

$$= \int d\varphi (\sin \varphi, \cos \varphi) = 0 \quad = -a L \omega_0 \int d^2 \vec{r} \cos^2 \theta [X \cos \varphi + Y \sin \varphi] m_0$$

$$\cos \theta = m_z(\vec{r}) = m_0^2(\vec{r}) + g(\vec{r})(\vec{X} \cdot \vec{r}), \quad \cos^2 \theta = (m_0^2)^2 + 2 m_0^2 g(\vec{X} \cdot \vec{r})$$

$$\dot{W}_s = -a L \omega_0 \int d^2 \vec{r} [(m_0^2)^2 + 2 m_0^2 g(\vec{X} \cos \varphi + Y \sin \varphi)] [X \cos \varphi + Y \sin \varphi] m_0 = \text{Int. over vortex core}$$

$$= -a L \omega_0 \cdot 2\pi (X^2 + Y^2) \int d\varphi m_0^2 g m_0 = -a L \omega_0 \cdot 2\pi J_p |\vec{X}|^2, \quad J_p = \frac{1}{2}, \quad \dot{W}_s = -a L \omega_0 |\vec{X}|^2$$

Важное. Vortex energy independence from coordinate $\vec{r}$, а $\dot{W}_s$ (ST energy) independence только V. core. и $\vec{F}_{ST}$ independence только V. core.

$$\dot{W}' = -\frac{M_s}{\gamma} \int dV [(\dot{\theta})^2 + \sin^2 \theta (\dot{\Phi})^2] = \left| \text{one core} \right|_{\theta=\pi/2} = -\frac{M_s}{\gamma} \int dV (\dot{\Phi})^2 = -\frac{2 M_s L}{\gamma} \int d^2 \vec{r} m_0^2 [\dot{X} \sin \varphi - \dot{Y} \cos \varphi]^2$$

$$= -\frac{2 M_s L}{\gamma} \pi (X^2 + Y^2) \int d\varphi m_0^2(\vec{r}) = -\frac{2 M_s L}{\gamma} \pi \omega_0^2 |\vec{X}|^2, \quad \text{no v. core kago yurabaz}$$

Примечание $\dot{W}' = \frac{\partial W'}{\partial \vec{X}} \cdot \dot{\vec{X}} = [\vec{G} \times \dot{\vec{X}} + D \dot{\vec{X}}] \cdot \dot{\vec{X}} = D |\dot{\vec{X}}|^2 = D \omega_0^2 |\vec{X}|^2 = -d G \omega_0^2 |\vec{X}|^2$

u yu pacere damping.

tenho $\dot{W} = \dot{W}' + \dot{W}_s = -d|G| \omega_0^{-1} |X|^{-1} + \dots$ (from!)
If, na vltima, you (soft field): $d \cdot 2\pi M_s \omega_0 = \pi \alpha \cdot \dots$
($J_{01} = 2d \omega_0$), $\dots$ c Eq. (4) Khvalko, PRB'09 e y...
Use d na AM'08, $d = \alpha [1 + \frac{1}{2} \ln \frac{R}{R_c}]$. y xba... $y = \frac{d}{2} = \frac{3}{8} + \frac{1}{2} \ln \frac{R}{R_c}$
 $Le = \sqrt{\frac{A}{0.01 \pi M_s^2}} = \frac{1}{\sqrt{0.01 \pi}} L_0$ $y = \frac{3}{8} + \frac{1}{2} \ln \left(\frac{100 R}{2 L_0} \right) = \frac{3}{8} + \frac{1}{2} \ln \left(\frac{100}{2} \right) + \frac{1}{2} \ln \frac{R}{L_0} = 0.66 \ln \frac{R}{L_0}$
- na norm coln. e norm $d = 1 + \frac{1}{2} \ln \frac{R}{R_c}$, em raio $R_c \approx L_0$
Monografia sobre d e y-xba... PRB'09 v3-za noem V. core R_c , a no
v3-za p... pag... Casos $L_0 \approx R_c$ e $R_c \ll L_0$ e $R_c \gg L_0$

Paras b... e...: Seega e Julie Crallier - V. core
A passagem sobre noem $\dot{F}_{st}$ e $\dot{W}$ - calculacao ref... y...
e noem... exat... Na $\dot{F}_{st}$ e $\dot{W}_s$ b... noem...
e Khvalko PRB'09, no... z... u... y...
Monografia na Khvalko'09... ($\dot{F}_{st}$ e $\dot{W}$) - gam...
... noem... eq. e xba...
y noem spin T-force... y no V. core, on... $\dot{F}_{st}$ e $\dot{W}_s$ noem
no V. orbit area. // x... $R_p = 50 \text{ nm} < R = 150 \text{ nm}$
e $R_p = R$ - simulations, b... no $R_p = 50 \text{ nm}$... nanocantilever
... $R_p < |X|$. taiga de... $W_c \sim |X|$ com
Joo Van Kien [Miskolc PRB'08]. ... $|X| \propto \text{const}$ y $\omega_0(\omega)$...
e $J_{02} \approx J_{01} (R_p = 150 \text{ nm}) = \frac{1}{2} J_{02} (R_p = 50 \text{ nm})$.
→ Noem... Julie (xba...) Word file e ... $\dot{F}_{st}$
... short paper no $\dot{F}_{st}$ e J_{01} ... JMM (??).

E-mail to Farhad no APL-SW proof e ... proof ASAP e ...
G... $v_c(H_2)$: ... $v_c(H_2) = v_c(0) [1 - H_2/H_c]$ e
... $v(H_2)$ ref... $|X| \text{ steady} \sim \frac{N(H_2)}{D(H_2)}$, to $v_c(H_2) = 0$ ga... $H_c \propto \frac{6}{7} H_s$.
 $H_s \approx 48 \text{ mT}$... $H_2(p)$... H_c ...
... $H_2(H_2)$ Okuno'2002 e Kikuchi'2001.

(6/11) In... 500 cash ... b... Yvette.
 $\sqrt{z} = \sqrt{x+iy} = |z|^{1/2} e^{i\frac{\varphi}{2}} = |z|^{1/2} e^{i\frac{1}{2}(\varphi + 2k\pi)}$, $k=0,1$. $\sqrt{z} = |z|^{1/2} e^{i(\frac{\varphi}{2} + k\pi)}$
 $z_1 = |z|^{1/2} [\cos \frac{\varphi}{2} + i \sin \frac{\varphi}{2}]$, $z_2 = |z|^{1/2} [\cos(\frac{\varphi}{2} + \pi) + i \sin(\frac{\varphi}{2} + \pi)] = -|z|^{1/2} [\cos \frac{\varphi}{2} + i \sin \frac{\varphi}{2}]$
 $\text{tg } \varphi = (y/x)$. $\text{tg } \varphi = \frac{2z'z''}{z'^2 - z''^2}$, $\text{tg } \varphi = \frac{z'}{z''}$, $z = \sqrt{z'z''}$, $(z')^2 = \frac{1}{2} [-x \pm \sqrt{x^2 + y^2}]$
 $z' = |z|^{1/2} \cos \frac{\varphi}{2} (+i)$, $z'' = |z|^{1/2} \sin \frac{\varphi}{2} (+i)$, to $(z')^2 + (z'')^2 = |z|$, $(z')^2 = \frac{1}{2} [x \pm \sqrt{x^2 + y^2}]$
noem... $(z'')^2 = \frac{1}{2} [|z| - x]$, $(z')^2 = \frac{1}{2} [|z| + x]$, $z = x + iy$.
 $2\omega_{1,2} = \hat{\omega}_{10} + \hat{\omega}_{20} \pm \sqrt{(\hat{\omega}_{10} - \hat{\omega}_{20})^2 + \frac{4M_1 M_2}{G_1 G_2}}$, $\omega_{0j} = \omega'_{0j} + i\omega''_{0j}$
 $A = 4 \frac{M_1 M_2}{G_1 G_2} = A' + iA''$, $D = (\hat{\omega}_{01} - \hat{\omega}_{02})^2 + A = \hat{\omega}_{01}^2 + \hat{\omega}_{02}^2 - 2\hat{\omega}_{01}\hat{\omega}_{02} + A =$
 $= [6 \text{ mm, vltima, no } \omega''_{0j}, A''] = (\omega'_{01})^2 + 2i\omega'_{01}\omega''_{01} + (\omega'_{02})^2 + 2i\omega'_{02}\omega''_{02} -$
 $- 2[\omega'_{01}\omega'_{02} \dots] = (\omega'_{01} - \omega'_{02})^2 + 2i(\omega'_{01} - \omega'_{02})(\omega''_{01} - \omega''_{02}) + A' + iA''$
 $D = x + iy$, $x = (A\omega'_0)^2 + A'$, $y = 2(A\omega''_0)(A\omega'_0) + A''$, $\sqrt{D} = \sqrt{x^2 + y^2} = \sqrt{z'z''} = z' + iz''$.

(27/11) Given 300€ cash + 800€ to Houseowner. Servie 800€ rent fee
 Nov. 2009 By cash.
 Обсуждение наша работа с Olivier, Gregoire, Vladimir, Benjamin.

① F/N/F nanofillar vortex dyn. - моя работа 2008 $\omega_{1,2}(\mu, I, H_z)$
 - Braginsky и другие Origin files of experiment $R(\omega, I, H_z)$ - isolated
 → 1st cross short (PRL or APL), затем нормаль PAB - cause of noise
 + v. M. resistance measurements like others, но не конъюнктур

② VC-reversal in single dot by MRFM - забавность of pulse
 length / width, switch. time, switch. field, damping parameter α
 $d \ll \sqrt{A} \ll H_z$, возможность $\uparrow \rightarrow \downarrow$ p-switching by $H_z \uparrow$, но не возможность
 и/или наоборот has — возможность PRL (Gregoire и другие 2009)
 Но не забавно от H_z

3) Non-lin. effects in single-vortex dynamics
 — MRFM signal shape + process has, non-linear damping
 ~ $\omega_0(|\vec{x}|)$ given after Jc1, экстр. $\omega_0(|\vec{x}|)$ nonuniforme. Moment $G(\vec{x})$?

4) $\nu_c(H_z)$ — vortex как мода VC reversal и T-флуксуи
 Упр-е Langevin given упр-е Thiele, пункт. гусени. T-м $\vec{x}, \vec{x}$
 же

Обсуждение SW propagation $\beta \rightarrow \vec{k}$ stripes (100nm width,
 ~ 10 μ m gap) с Claude Fermon и его Ph.D. student. Упомянуты SW.

Plan Summ papers (in. 8/11 09):

- 1) Non-lin. Vortex dyn. paper c. Oucanai u Raga
- 2) VC reversal in perp. field H_z c. Oucanai u Rocio $\rightarrow$ nanoscale arrays of solenoids
- 3) Vortex SW by VNA-FMR c. Paprog : ~~excuse APL prof. u referee~~ ^{IEE JMM JAP?} _{complex arrays}
- 4) Answer, papers VC reversal field H_c (H_z work. go to)
- 5) Topol. field PRL c. Arantxa, Julian ~~refugee to / or b. referees for PRB~~ ^{resubm. 9/11} to PRB
- 6) Power SW-SW coupling in nanostripes by Topol. field (Hallebrandt offer)
- 7) Insp. Experiments upon $V \rightarrow$ SD reversal in H_x - Diet, Marcus - APL (go papers)
- 8) Colu-FMR c. Diet u Julian - Mat. Sci. Journal
- 9) F/N/F SP current induced vortex dyn. c. Klein-Fert exp. groups - PRL/APL, PRB
- 10) Vortex core reversal in MnSB/Ni dot MRFM c. Klein-Jelencsics - PRL (applied exp. 4 dec)
- 11) ST force u Jci, Jci kpts. taken b. nanospinall ^{30/11 09, subm. APL} - APL or JMM JAP
- 12) Penetration: APL, JAP, HK grant, JMM
- 13) Nanotube papers c. AiPing (C-state, B-state) - JAP

Opz. bus procs:

- 1) Paper kb. / go to b. San Sebastian u nanoscale
- 2) Info about Spanish tax system u US tax return
- 3) Status SAIO TEK u Spanish / Portugal Travel grants
- 4) ~~Papers from order to no progress b. Oakland Univ. u~~ ^{Berger ES12} ~~CEA Seclay (no more receipts)~~
- 5) ~~Entrepreneurial travel to Washington NMN'10 u reserve tickets, Hotel, Conf fee~~
- 6) Nagas Application na Permiso de Regreso to Police / General appointment
- 7) Nagas applications na Resident Card renewal ^{nagas 3/11 gone on exp. 3/11 u Vida Laboral - gone 14/11} to Adm. Police
- 8) ~~Papers to J. Grollier new calcs. of Fst u Jci by Thiele eq.~~
- 9) ~~Papers to Klein/Jelencsics/Malet new equations Write TM, F/N/F~~
- 10) ~~Prochopurs c. Carla never ^{comp. manuscript} ~~refugee to b. contract u sub. sent for, 2nd time~~~~
- 11) ~~Ans. Ati Bank Credit Card go 15 Dec. On. c. Chase carta 15/11~~ ^{Do Jan. 15 - On. 10498 7/11 - right}
- 12) ~~Update Iberbasque web-site no clear announcement 2009 u Survey~~ ^{10/11}
- 13) Plan Conferences; NanoSmat'10, TSN'10, JEMS'10 u travels (Ato, Kaiserlautern)
- 14) ~~Opportunity Aval u deposit \$ cash to Caja Madrid, payd. c. "Orange" A. ten.~~
- 15) ~~2 laptops by SAIO TEK grant~~ ^{subm. 22/11 102}
- 16) ~~Papers u nanoscale MIEEM appl. c. doc. "Spin dyn. in m. nanostriplines"~~
- 17) ~~Answers b. A. more nanoscale 4 dec. near?~~

3811 ¹ ~~Sanctioning~~ Appl. Dms EXOT = EXOD que Res Cando Personal
Expenditure générale que applications.

(S) Nozomi Appl. no Res. Card Annual na Polakinas 1-3, Ku
reimbursement Payment Slip (no 7003) u "Vida Laboral" (Zvezdica
home address). Na Polakinas i Polakina za E-signature to access off-
Munrova. Expense Report Jan (CA Salary u urobna O.Klin. Pozivnica a.
uz Umi. Sabornica (Elizabeta Jeronimska) e u fagi. na Vozgromu na U.

①/20 Purchase Aft fee 3rd Dec - €1,200. " 10th Dec - Nov '09 - €205

After quite some time I noticed that the term "torque" was used by both authors. One name given to the DOME, i.e. yustabara, which is also known as $M(x)$ by Stencowski.

$$\dot{\vec{m}} = -\gamma \vec{m} \times \vec{H}_{eff} + \alpha (\vec{m} \times \dot{\vec{m}}) + \gamma \beta \epsilon (\vec{m} \times \vec{m}_p + \vec{m}) - \gamma \beta \epsilon' \vec{m} \times \vec{m}_p, \quad (1)$$

$\mathbf{P} = \frac{h}{4\pi} \frac{\mathbf{J}}{m_p}$, $\hat{m}_p$ - unit vector of polarizer, $E = \frac{P\Lambda^2}{(\Lambda^2+1) + (\Lambda^2-1)(m_p^2 \cdot m_p^2)}$
 E' - non-diffracted torque (analogy: $E' \approx 0$).

ε' - non-additive (or que (univ. $\varepsilon' \neq 0$).

Y Stone, 3MMAI'96: $\vec{S}_{1,2} = \frac{J_e}{e} \vec{g} \cdot \vec{S}_{1,2} \times (\vec{S}_1 \times \vec{S}_2)$, $\vec{S}_i = \frac{\hbar}{2} \vec{\sigma}_i$, $|\vec{S}_i| = \hbar/2$

$$g - g(P) = \frac{1}{[-4 + (1+P)^3(3 + \vec{S}_1 \cdot \vec{S}_2)/4P^{3/2}]} \cdot \frac{P \text{ is the current polarization}}{4P^{3/2}} \cdot \frac{J_c - \text{charge current density}}{4P^{3/2}} = \frac{1}{(1+P)^3(3 + \vec{S}_1 \cdot \vec{S}_2) - 16P^{3/2}} = \frac{3(1+P)^3 - 16P^{3/2} + (1+P)\vec{S}_1 \cdot \vec{S}_2}{(1+P)^3(3 + \vec{S}_1 \cdot \vec{S}_2) - 16P^{3/2}}$$

$$g = [\dots]^{-1} = \frac{4P^{3/2}}{(1+P)^3 (3 + \vec{S}_1 \cdot \vec{S}_2) - 16P^{3/2}} = \frac{4P^{3/2}}{3(1+P)^3 - 16P^{3/2} + (1+P)(\vec{S}_1 \cdot \vec{S}_2)}$$

$$g = [\dots]^{-1} = \frac{A(P)}{1 + B(P)\vec{S}_1 \cdot \vec{S}_2} \text{ via } B \text{ Caputo's PRL'07, comp. } A \cup B.$$

The eigenvalue $(\Lambda^2 - 1) = (1 + \rho)^3 \lambda$, $(\Lambda^2 + 1) = [3(1 + \rho)^3 - 16\rho^{3/2}] \lambda$

$$[3(1+p)^3 - 16p^{3/2}] \lambda - (1+p)^3 \lambda = 2, \quad [4(1+p)^3 - 8p^{3/2}] \lambda = 1$$

$$\frac{\lambda^2}{[(1+p)^3 - 8p^{3/2}]} + 1 = 2 \frac{[(1+p)^3 - 4p^{3/2}]}{[(1+p)^3 - 8p^{3/2}]}$$

$$\lim_{t \rightarrow \infty} \dot{\vec{S}}_1 = \frac{J_e}{e} g \frac{1}{S_1} \vec{S}_1 \times (\vec{S}_1 + \vec{S}_2), \quad \dot{\vec{m}}_1 = \frac{J_e}{e} g \frac{1}{d} \vec{m}_1 \times (\vec{m}_1 + \vec{m}_2)$$

$$\dot{m} = \frac{J_e}{e} g \frac{1}{s} \vec{m} \times (\vec{m} + \vec{p}), \quad \frac{1}{J} \dot{\vec{m}} = \frac{J_e}{e J} g \frac{1}{s} \vec{m} \times (\vec{m} + \vec{p}), \quad \gamma = \frac{J_e \gamma_B}{J} = \frac{g \gamma_B}{J}$$

$$M_s = \frac{1}{\sqrt{2}} g \mu_B S, \quad J^z S = \frac{g \mu_B}{\hbar} S = \frac{M_s \hbar}{\hbar} = \frac{1}{\hbar} M_s \hbar, \quad \frac{J_z}{g \mu_B} = \frac{J_z}{e \mu_B \frac{1}{\sqrt{2}} \hbar}$$

(given. $c \frac{1}{g} \ddot{m} = \beta E (\vec{m} \times \vec{m}_p \times \vec{m})$, $-\beta E = \frac{J_e}{e} \frac{g}{\chi S} = g \frac{J_e}{e M_s \frac{1}{h} A d} = g \frac{J_h}{e M_s d} = g \frac{J_h}{e M_s d} = g \frac{J_h}{e M_s d} = g \frac{J_h}{e M_s d}$
 because E (microwaves) = $g(P)$ Slonczewski's, no I_e - Slonczewski's current.

Caputo PR 07: $\vec{S} = j \frac{A}{4\pi B \vec{S} \cdot \vec{\delta}}$ $\vec{S} \times (\vec{S} + \vec{\delta})$, $\vec{S}, \vec{\delta}$ - unit vectors. $\beta = \frac{Jh}{eM_d}$

$$j = J_e / J_p \quad J_p = \mu_0 M_s^2 |e| d / \hbar \quad j = \frac{J_e \hbar}{\mu_0 M_s^2 |e| d} \quad \text{отсюда } M_s^2 ?$$

CoCo. Khvelkar, 2009 $\frac{M_s}{J} \dot{\vec{m}} = \vec{\tau}_s = \partial J M_s (\vec{m} \times [\vec{m} \times \dot{\vec{p}}])$ $\mu_B M_s$ (eld μ_B) $\mu_B = \partial J M_s$

$$P_E = \frac{hP}{2\pi i \hbar M_s} \sin \theta = \frac{1}{2} \frac{hJ}{\pi i \hbar M_s} P, \quad P_E = \frac{hJ}{\pi i \hbar M_s} \frac{P \lambda^2}{(\lambda^2 + 1) + (\lambda^2 - 1)(m \cdot \vec{p})} \Big|_{\lambda=1} = \frac{hJ}{\pi i \hbar M_s} \frac{P}{2}$$

т.е. Хванков. в поле токма $\Lambda^2 = 1$. J - токма токма. $\Lambda = 1.645$
Хванков. в поле токма $P = 0.2$ (FEM), $\Lambda^2(P=0.2) = 2.707$ и $\Lambda \neq 1$.
Если $P = 0.4$ то Λ^2 еще больше (никак $\Lambda^2 \neq 1$). $\Lambda^2(P=0.4) = 4.810$ и $\Lambda = 2.193$
 $[P] = \frac{2m \cdot s}{T \cdot m} \cdot \frac{A}{m} = \frac{J \cdot s \cdot A}{T \cdot m^3 \cdot A \cdot s} = \frac{J}{T \cdot m^3} = \frac{T \cdot Nm \cdot m^3}{T \cdot m^3} = \frac{A}{m} = [\text{field}]$
 $[E] = 1$. $[B] = [2J] = (\text{mag. field}) = A/m$. $[G] = \frac{P}{M_s} = 1, [M_s] = \frac{A}{m}$
Note: $\mu_0 = 1$ (CGS) $\mu_0 = 4\pi \cdot 10^{-7} \frac{H}{m} = 4\pi \cdot 10^{-7} T \cdot m/A$ (SI) $T = \text{токма}$.

Гру Xia-Stiles 'PRB'04: Slon. torque $L(\theta) = \frac{\hbar I}{2e} \cdot \frac{P \Lambda^2 \sin \theta}{(\Lambda^2 + 1) + (\Lambda^2 - 1) \cos \theta}$, I - токма
 $P = \frac{r}{R}$, $r = \frac{1}{2}(R_L - R_R)$, $R = \frac{1}{2}(R_L + R_R)$, $R_{L,R}$ - eff. resistances for $\uparrow, \downarrow$ spins.
 $\Lambda^2 = GR$, $G = \frac{Se^2 k_F^2}{4\pi^2 \hbar}$ - is the conductance. $\theta = \vec{m} \cdot \vec{m}_p$.

$L_S(\theta)$ - Slonewski '96 - 300 nm. cylinder $\Lambda_R = \Lambda_L$, $P_R = P_L = P$ одинаковы Λ и P ребро (L) и правого (R) слоев. Λ^2 берется из Slonewski JMMM 247 324 (2002)
 Λ^2 numbers $\approx 10-20$ и сильно зависят от параметров слоев и spacer N.
Я думаю что $\Lambda^2(P)$ согласно Slon. JMMM'1996, но есть сомнения
упрощение. т.к. $\Lambda^2 \gg 1$, пренебрегаю $\cos \theta$ - членом в знаменателе мало

Для цилиндра Хванков. PRB'09 с $\vec{m}_p = \hat{z} = \text{const}$ и предположение
распределения $m_z(\vec{m} \cdot \vec{m}_p) = m_z$ в знаменателе опущено, т.к. $m_z \neq 0$
выбор в. ось, которую нам. Но не надо $\Lambda^2 = 1$ как Хванков '09, надо $\varepsilon \equiv \frac{P \Lambda^2}{(\Lambda^2 + 1)}$
и не $\varepsilon = \frac{P \Lambda^2}{(\Lambda^2 + 1) + (\Lambda^2 - 1)m_z} \Big|_{\Lambda^2=1} = \frac{1}{2} P$ как Хванков '09 где vortex dyn. F/N/P

Но если брать ST $\vec{C}_S = \delta J M_s (\vec{m} \times (\vec{m} \times \vec{p}))$, то тогда $\vec{F}_{ST} = \pi L \delta J M_s (\hat{z} \times \vec{X})$
и $b = b(\vec{X})$ из-за $g(P)$ в мн. и предположение не выполняется.
 $\beta E = \delta J$, и $\frac{1}{\gamma} M_s \dot{\vec{m}} = \beta E M_s (\vec{m} \times \vec{m}_p \times \vec{m}) = -\delta J M_s \vec{m} \times (\vec{m} \times \vec{p}) = \vec{C}_S$
 $\vec{C}_S = -a \vec{m} \times (\vec{m} \times \vec{p})$, $a = \delta J M_s$. Хванков. 'PRB'09 знак $\vec{C}_S$ перевернут (?).

Slon. JMMM'02: spin transf. torque $L(\theta) = \frac{\hbar I P_z \Lambda}{4Ae} \tau(\theta)$, $\tau(\theta) = \frac{\sin \theta}{\Lambda \cos^2 \frac{\theta}{2} + \Lambda^2 \sin^2 \frac{\theta}{2}}$
(equiv. magnetic layers) $F/N/F$
 $P_z = (R_- - R_+)/ (R_- + R_+)$, $\Lambda^2 = AG(R_- + R_+)/2$, $G = \frac{e^2 k_F^2}{3^2 \pi \hbar}$ - spacer conduct.
 $\Lambda^2 \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} = (\Lambda^2 - 1) \cos^2 \frac{\theta}{2} + 1 = \Lambda^2 \frac{1}{2} (1 + \cos \theta) + \frac{1}{2} (1 - \cos \theta) = \frac{1}{2} (\Lambda^2 + 1) + \frac{1}{2} (\Lambda^2 - 1) \cos \theta$

$L(\theta) = \frac{\hbar I P_z}{2Ae} \frac{\Lambda^2 \sin \theta}{(\Lambda^2 + 1) + (\Lambda^2 - 1) \cos \theta}$ как в Xia-Stiles 'PRB'04, A - F/N/F contact area.
 $[L(\theta)] = \text{erg/cm}^2$ и для измерения [torque] = $[\vec{m} \times \vec{H}]$ надо возлечь на d, т.е. F - current.
 $\cos \theta = \cos(\vec{m}_L \cdot \vec{m}_R)$, где θ - угол между $\vec{m}_L$ и $\vec{m}_R$ (F_L/N/F_R) - trilayer. Не обязательно в xy-плоскости of interface.
 $R_{\pm}$ are channel (spin $\sigma = \pm 1/2$) resistances of each ferromagnet (non-cubical) layer.

Мгн. сопротивление F/N/F. $\tau(\theta) = \frac{R(\theta) - R(0)}{R(\theta) + R(0)} = \frac{1 - \cos^2 \frac{\theta}{2}}{1 + \gamma \cos^2 \frac{\theta}{2}}$, $\gamma = \Lambda^2 - 1$.
То есть $\gamma = 0$, $\Lambda^2 = 1$ где $\tau(\theta)$ linear in $\cos \theta$.
Для цилиндра $\Lambda^2 \neq 1$ (или $\Lambda^2 \gg 1$) усложняется геометрия измерения
of ST torque $L_S(\theta)$ и GMR $(\theta) = \tau(\theta)$ or где $\theta = \vec{m}_L \cdot \vec{m}_R$.

/B ballistic model Slonc. 'JMMM'96 u ST torque (u MR on page) as is
 o gnu napametven $P_f = P$ - single-band polarization parameter
 B Szwarczki 'JMMM'02 resistance polarization P_r on page 10 ($L_s(\theta)$)
 - anamnygy, whereas Λ^2 on page u anamnygy u bug ST torque $L_s(\theta)$
 u MR (θ).

Due F/N/F (Slonc. 'JMMM'02) $MR(\theta) = \frac{2R \sin^2 \frac{\theta}{2} + 2AGR + R \cos^2 \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2} + 2(\gamma+1) \cos^2 \frac{\theta}{2}}$
 $MR(\theta) = \frac{R \sin^2 \frac{\theta}{2} + AGR + R \cos^2 \frac{\theta}{2}}{1 + \gamma \cos^2 \frac{\theta}{2}}$, $MR(0) = AGR + R / (1+\gamma)$, $MR(\pi) = R = \frac{1}{2}(R_1 + R_2)$

$MR(\theta) = R(0) + [R(\pi) - R(0)] \frac{(1 - \cos^2 \frac{\theta}{2})}{(1 + \gamma \cos^2 \frac{\theta}{2})} = R(0) + \Delta R \frac{1}{2} \frac{(1 - \cos \theta)}{(1 + \frac{\gamma}{2} + \frac{\gamma}{2} \cos \theta)} = F(\cos \theta)$
 u gnu, gde F/N/F dynamics. - non-linear

u $MR(\theta) = \frac{1}{2} \frac{(R + AGR + R) + (AGR + R - R) \cos \theta}{(1 + \frac{\gamma}{2} + \frac{\gamma}{2} \cos \theta)}$, $\gamma = \Lambda^2 - 1$, $\Lambda^2 = AGR$.

$\theta = \vec{m}_1 \vec{m}_2$, $\vec{m}_j(p, \varphi, t)$ u nago $\langle MR(\theta) \rangle_\theta = \frac{1}{A} \int d^2 p MR(\theta)$

u neme gde 2-vortex dyn. F/N/F $\langle \vec{m}_1 \cdot \vec{m}_2 \rangle = \langle \cos \theta \rangle \sim (\vec{X}_1 \cdot \vec{X}_2)$ Euno.

Nago $MR(t) = \langle MR(\theta) \rangle = R(0) + \Delta R \left\langle \frac{1 - \cos \theta}{2 + \gamma + \gamma \cos \theta} \right\rangle = R(0) + \Delta R \left[\frac{1}{(2+\gamma)} - \frac{(1+\gamma)}{(2+\gamma)} \left\langle \frac{\cos \theta}{2 + \gamma + \gamma \cos \theta} \right\rangle \right]$

$MR(t) = R(0) + \frac{\Delta R}{(2+\gamma)} \left[1 - 2(1+\gamma) \left\langle \frac{\cos \theta}{2 + \gamma + \gamma \cos \theta} \right\rangle \right] = \text{const} - 2 \frac{(1+\gamma)}{(2+\gamma)} \Delta R \left\langle \frac{\cos \theta}{2 + \gamma + \gamma \cos \theta} \right\rangle$

$MR(t) \sim \left\langle \frac{\cos \theta}{2 + \gamma + \gamma \cos \theta} \right\rangle$. Euno $\gamma \gg 1$, $MR(t) \sim \left\langle \frac{\cos \theta}{1 + \cos \theta} \right\rangle$

Note: $\cos \theta = \cos \theta_{12} = (\vec{m}_1 \cdot \vec{m}_2) = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos(\varphi_1 - \varphi_2) \approx \cos(\varphi_1 - \varphi_2)$
 Euno v. coses

$MR(t) \sim \frac{1}{2} \langle \cos \theta [1 + \frac{\gamma}{(1+\gamma)} \cos \theta] \rangle \sim \langle \cos \theta \sum_{n=0}^{\infty} a^n (-1)^n \cos^n \theta \rangle \sim \sum_n (-1)^n a^n \langle \cos^{n+1} \theta \rangle$

$a = \left(\frac{\gamma}{1+\gamma} \right) < 1$. $\langle \cos \theta \rangle \sim (\vec{X}_1 \cdot \vec{X}_2)$, $\langle \cos^2 \theta \rangle$, $\langle \cos^3 \theta \rangle$ - nago Bernoulli.

Pag no $(a \cos \theta)$ gde nemo xopamo xopamo.

5/XII C d.u. b kape "Lidl" u "Super BM". // The cover to E. Jaromirska (Salamanca) 0 SP current induced vortex dyn.

6-7/XII Pabara nag oberann Referees u review of "topol. field" PRL.
 u3 - za 2 no payuga comments of Ref. A boynymgen pacimjuss the MS text
 u nepenodab MS to PRB (uue Ref. B recommended). Paprag nocian APL prof 3E
 "Viktorov" nemyr.

8/XII Pabara nag "topol. field" PRL-PRB. **Понявкe x ω_m u3-za Vortex mass.**

Corraco $7/\sqrt{1}$ c yresen Vortex mass $\omega_{TM}(\beta) = \omega_0(\beta) \left[1 - \frac{1}{4} \omega_0(\beta) \frac{M}{G} \right]$

u neme, no $\omega_0(\beta) \sim \beta$. Torge $\omega_{TM}(\beta) \sim \beta [1 - \hat{a} \beta]$ kbaghaswina no β .

C boabmci xopmo-cbo $M \approx \frac{3}{2} \frac{L}{f^2} + \frac{M}{G} = \frac{3}{2} \frac{L \cdot f}{f^2 2\pi M_s K} = \frac{3}{2} \frac{1}{f^2 2\pi M_s} = \frac{3}{\omega_m}$

(?) $\omega_{TM}(\beta) = \omega_0(\beta) \left[1 - \frac{3}{4} \frac{\omega_0(\beta)}{\omega_m} \right]$. u neme nym $G_z > 0$.

$\omega_{1,2} = \frac{G}{2M} \left[-1 \pm \sqrt{1 + \frac{4\pi M}{G^2}} \right]$, $\omega_1 = \frac{G}{2M} \left[-1 + \left(1 + \frac{2\pi M}{G^2} - \frac{1}{8} \left(\frac{4\pi M}{G^2} \right)^2 + \dots \right) \right] =$
 $= \frac{G}{2M} \left[\frac{2\pi M}{G^2} - 2 \frac{\pi^2 M^2}{G^4} \right] = \left[\frac{\pi}{G} - \frac{\pi^2 M}{G^3} \right] = \frac{\pi}{G} \left[1 - \frac{\pi M}{G^2} \right]$, $\omega_1 = \omega_0 \left[1 - \omega_0 \frac{M}{G} \right]$

- $7/\sqrt{1}$ a omude crmoe $\omega_1 = \omega_0 \left[1 - \frac{1}{4} \omega_0 \frac{M}{G} \right]$ flago $\omega_{TM}(\beta) = \omega_0(\beta) \left[1 - \omega_0(\beta) \frac{M}{G} \right]$

T.k. $\frac{M}{G} \approx \frac{3}{\omega_m}$ to $\omega_{TM}(\beta) = \omega_0(\beta) \left[1 - 3 \frac{\omega_0(\beta)}{\omega_m} \right]$ u M - non-pabke normu ω_{TM} for Vortex

Escaneado con CamScanner

12. $\frac{\partial}{\partial z} = \frac{1}{2} \left(\frac{\partial}{\partial x} - i \frac{\partial}{\partial y} \right)$, $\frac{\partial}{\partial \bar{z}} = \frac{1}{2} \left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial y} \right)$, $\frac{\partial}{\partial x} = \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}$, $\frac{\partial}{\partial y} = i \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}} \right)$.

$$\frac{\partial \vec{m}}{\partial x} \cdot \frac{\partial \vec{m}}{\partial y} = i \left[\frac{\partial \vec{m}}{\partial z} \cdot \frac{\partial \vec{m}}{\partial z} - \frac{\partial \vec{m}}{\partial z} \cdot \frac{\partial \vec{m}}{\partial z} \right], \quad \frac{\partial \vec{m}}{\partial z} \cdot \frac{\partial \vec{m}}{\partial z} = \left[\frac{\partial \vec{v}}{\partial z} \hat{e}_+ + \frac{\partial \vec{v}}{\partial z} \hat{e}_- + \frac{\partial m_z}{\partial z} \hat{z} \right] \cdot \left[\frac{\partial \vec{v}}{\partial z} \hat{e}_+ + \frac{\partial \vec{v}}{\partial z} \hat{e}_- + \frac{\partial m_z}{\partial z} \hat{z} \right]$$

$$= \left| \begin{array}{l} \hat{e}_+ \cdot \hat{e}_+ = 1, \hat{e}_+ \cdot \hat{e}_- = 0, \hat{e}_- \cdot \hat{e}_- = 1 \\ \hat{e}_+ \cdot \hat{e}_- = 0, \hat{e}_- \cdot \hat{e}_+ = 1 \end{array} \right| = 2 \frac{\partial \vec{v}}{\partial z} \cdot \frac{\partial \vec{v}}{\partial z} + \left(\frac{\partial m_z}{\partial z} \right)^2, \quad \frac{\partial \vec{m}}{\partial z} \cdot \frac{\partial \vec{m}}{\partial z} = 2 \frac{\partial \vec{v}}{\partial z} \cdot \frac{\partial \vec{v}}{\partial z} + \left(\frac{\partial m_z}{\partial z} \right)^2$$

$$\frac{\partial (w\bar{w})}{\partial p} = w \frac{\partial \bar{w}}{\partial p} + \bar{w} \frac{\partial w}{\partial p}, \quad \left[\frac{\partial (w\bar{w})}{\partial p} \right]^2 = w^2 \left(\frac{\partial \bar{w}}{\partial p} \right)^2 + \bar{w}^2 \left(\frac{\partial w}{\partial p} \right)^2 + 2w\bar{w} \frac{\partial w}{\partial p} \frac{\partial \bar{w}}{\partial p} =$$

$$= \frac{4}{(1+w\bar{w})^4} \left[(1+w^2\bar{w}^2) \frac{\partial w}{\partial p} \frac{\partial \bar{w}}{\partial p} + 2w\bar{w} \frac{\partial w}{\partial p} \frac{\partial \bar{w}}{\partial p} \right] = \frac{4}{(1+w\bar{w})^2} \frac{\partial w}{\partial p} \frac{\partial \bar{w}}{\partial p}$$

$$\frac{\partial^2 \bar{m}}{\partial x^2} \cdot \frac{\partial \bar{m}}{\partial y} = i \left[2 \frac{\partial v}{\partial \bar{z}} \frac{\partial \bar{v}}{\partial \bar{z}} + \left(\frac{\partial m_z}{\partial \bar{z}} \right)^2 - 2 \frac{\partial v}{\partial \bar{z}} \frac{\partial \bar{v}}{\partial \bar{z}} - \left(\frac{\partial m_z}{\partial \bar{z}} \right)^2 \right] = \frac{4i}{(1+w\bar{w})^2} \left[\frac{\partial w}{\partial \bar{z}} \frac{\partial \bar{w}}{\partial \bar{z}} - \frac{\partial w}{\partial \bar{z}} \frac{\partial \bar{w}}{\partial \bar{z}} \right]$$

Broyden core $W=f$, pole-free model : $\frac{\partial f}{\partial s} \frac{\partial \bar{f}}{\partial s} - \frac{\partial f}{\partial \bar{s}} \frac{\partial \bar{f}}{\partial \bar{s}} = \frac{1}{c^2} \frac{1}{(1+s\bar{s})^4} [1+s\bar{s} - s(\bar{s}+s\bar{s}^2)] \cdot$
 $\times [(1+s\bar{s})\bar{z}^2 - \bar{s}(\bar{s}+s\bar{s}^2)] - \frac{1}{c^2} \frac{1}{(1+s\bar{s})^4} [(1+s\bar{s})z^2 - s(\bar{s}+s\bar{s}^2)][1+s\bar{s} - s(\bar{s}+s\bar{s}^2)] =$
 $= \frac{1}{c^2} \frac{1}{(1+s\bar{s})^4} [(1-s^2\bar{z}^2)(1-\bar{s}^2) - (1-s^2z^2)(1-s^2)]$. Non-linear Vortex damping

$$D_{xy} \sim \int dV \frac{\partial \vec{m}}{\partial x} \cdot \frac{\partial \vec{m}}{\partial y} = 4i \int dV \frac{1}{(1+w\bar{w})^2} \left[\frac{\partial w}{\partial z} \frac{\partial \bar{w}}{\partial \bar{z}} - \frac{\partial w}{\partial \bar{z}} \frac{\partial \bar{w}}{\partial z} \right]$$

$$D_{\mu\nu}(k \text{ core}) = i \int d^2 \vec{p} \frac{1}{(1+|\vec{q}|^2)^2} \frac{1}{c^2} \frac{1}{(1+|\vec{s}|^2)^2} \left[(1-\vec{s}^2 \vec{z}^2)(1-\vec{s}^2) - (1-\vec{s}^2 \vec{z}^2)(1-\vec{s}^2) \right]$$

$$[\dots] = -\bar{S}^2 Z^2 + S^2 \bar{Z}^2 - \bar{S}^2 + S^2 + \bar{S}^4 Z^2 - S^4 \bar{Z}^2. \text{ Bugno, 200 c. } \text{verkoefeno go } S^2,$$

given elem $1/s^2$ is not balanced as s , $D_{xy} \sim s^2 \neq 0 (!)$. Ess. regular damping (!)
(non-linear)

com. for s^2 : $D_{xy}(\text{bare}) \sim \frac{i}{c^2} \int \frac{d^2 \vec{p}}{(1+p^2)^2} [s^2 \bar{z}^2 - \bar{s}^2 z^2 + s^2 - \bar{s}^2]$

$$S^2 \bar{z}^2 - \bar{S}^2 z^2 = (S_x^2 - S_y^2 + 2iS_x S_y)(x^2 - y^2 - 2ixy) - (S_x^2 - S_y^2 - 2iS_x S_y)(x^2 - y^2 + 2ixy) = 4iS_x S_y (x^2 - y^2) - 2ixy(S_x^2 - S_y^2), \text{ due } (x^2 - y^2) = 0, \text{ due } xy = 0, \text{ the result is } 0.$$

$$S^2 - \bar{S}^2 = 4i S_x S_y \rightarrow \underline{D_{xy}(b \text{ core})} \sim \frac{1}{c^2} S_x S_y \left\{ \frac{d^2 \bar{g}^2}{1 + R^2/a^2} \right\} \sim S_x S_y \sim \underline{XY}$$

Поэтому $w = 5/151$ и тогда D_{xy} (оно же) $\frac{[1 + 41/k^2]}{200000}$, но оно $D_{xy} \sim 5^{-4} \neq 0$.

⑧ Every C_n is a D_{2n} pole-free model.

$$\frac{\partial m_1}{\partial z} \frac{\partial m_1}{\partial \bar{z}} = 2 \left(\frac{\partial v}{\partial z} \frac{\partial \bar{v}}{\partial \bar{z}} + \frac{\partial v}{\partial \bar{z}} \frac{\partial \bar{v}}{\partial z} \right) + \frac{\partial m_2}{\partial z} \frac{\partial m_2}{\partial \bar{z}} = \frac{2}{(1+u\bar{u})^4} \left[\left(\frac{\partial w}{\partial z} - u^2 \frac{\partial \bar{w}}{\partial \bar{z}} \right) \left(\frac{\partial \bar{w}}{\partial \bar{z}} - \bar{u}^2 \frac{\partial w}{\partial z} \right) + \right.$$

$$+ \left(\frac{\partial w}{\partial z} - w^2 \frac{\partial \bar{w}}{\partial \bar{z}} \right) \left(\frac{\partial \bar{w}}{\partial \bar{z}} - \bar{w}^2 \frac{\partial w}{\partial z} \right) + \frac{4}{(1+w\bar{w})^4} \frac{\partial(w\bar{w})}{\partial z} \frac{\partial(w\bar{w})}{\partial \bar{z}} = \frac{2}{(1+w\bar{w})^4} \left[\frac{\partial w}{\partial z} \frac{\partial \bar{w}}{\partial \bar{z}} - w^2 \bar{w}^2 \frac{\partial w}{\partial z} \frac{\partial \bar{w}}{\partial \bar{z}} \right]$$

$$+ \frac{\partial^2 w}{\partial z^2} + w^2 \frac{\partial^2 w}{\partial z^2} - w \frac{\partial^2 w}{\partial z^2} - w^2 \frac{\partial^2 w}{\partial z^2} + w \frac{\partial^2 w}{\partial z^2} + w^2 \frac{\partial^2 w}{\partial z^2}$$

$$2 \left(w \frac{\partial w}{\partial z} + \bar{w} \frac{\partial w}{\partial \bar{z}} \right) \left(w \frac{\partial \bar{w}}{\partial z} + \bar{w} \frac{\partial \bar{w}}{\partial \bar{z}} \right) = \frac{2}{(1+w\bar{w})^4} \left[\frac{\partial w}{\partial z} \frac{\partial \bar{w}}{\partial z} (1+w\bar{w})^2 + \frac{\partial w}{\partial z} \frac{\partial \bar{w}}{\partial \bar{z}} (1+w\bar{w})^2 + 2w\bar{w} \frac{\partial w}{\partial z} \frac{\partial \bar{w}}{\partial \bar{z}} \right]$$

$$+ 2w\bar{w} \frac{\partial w}{\partial \bar{z}} \frac{\partial \bar{w}}{\partial z} \Big] = \frac{2}{(1+w\bar{w})^2} \Big[\frac{\partial w}{\partial \bar{z}} \frac{\partial \bar{w}}{\partial z} + \frac{\partial w}{\partial z} \frac{\partial \bar{w}}{\partial \bar{z}} \Big], \quad Z = X + iy$$

$$\rightarrow T_z(\hat{D}) \sim \int dV \frac{\partial \vec{m}}{\partial \bar{z}} \cdot \frac{\partial \vec{m}}{\partial z} = 2 \int dV \frac{1}{(1+w\bar{w})^2} \Big[\frac{\partial w}{\partial \bar{z}} \frac{\partial \bar{w}}{\partial z} + \frac{\partial w}{\partial z} \frac{\partial \bar{w}}{\partial \bar{z}} \Big] = D_{xx} + 2D_{yy} = 2D_z$$

$$\vec{F}_d = \hat{D} \cdot \vec{X}, \quad \hat{D} = \begin{pmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{pmatrix} = D_0 \delta_{\alpha\beta} + D_1 X_\alpha X_\beta - ?$$

Core $14/8 \cdot 200 + 1$. B core, $w = f$: $\Big[\frac{\partial f}{\partial \bar{s}} \frac{\partial \bar{f}}{\partial s} + \frac{\partial f}{\partial s} \frac{\partial \bar{f}}{\partial \bar{s}} \Big] = \frac{1}{c^2} \frac{1}{(1+s\bar{s})^4} \Big[(1-s^2 \bar{z}^2)(1-s^2 \bar{z}^2) + (z^2 \bar{s}^2)(z^2 \bar{s}^2) \Big]$

$$= \frac{1}{c^2} \frac{1}{(1+s\bar{s})^4} \Big[1 - s^2 \bar{z}^2 - s^2 \bar{z}^2 + s^2 \bar{z}^2 \bar{z}^2 + z^2 \bar{z}^2 - s^2 \bar{z}^2 - s^2 \bar{z}^2 + s^2 \bar{s}^2 \Big] =$$

$$= \frac{1}{c^2} \frac{1}{(1+s\bar{s})^4} \Big[1 + |s|^4 - 2s^2 \bar{z}^2 - 2s^2 \bar{z}^2 + |z|^4 + |z|^4 |s|^4 \Big] \approx \frac{1}{c^2} \frac{1}{(1+s\bar{s})^4} \Big[1 + |z|^4 - 2s^2 \bar{z}^2 \Big]$$

B core $T_z(\hat{D}) = 2D \sim \int d^2 \vec{\rho} \frac{1}{(1+s\bar{s})^2} \frac{1}{c^2} \frac{1}{(1+s\bar{s})^4} \Big[1 + \rho^4 - 2s^2 \bar{z}^2 - 2s^2 \bar{z}^2 \Big]$

$$c^2 |s|^2 = \frac{1}{(1+s\bar{s})^2} \Big[(1+s\bar{s})^2 \rho^2 - (1+s\bar{s})(1+\rho^2)(\bar{z}s + s\bar{z}) + s\bar{s} + s^2 \bar{z}^2 + s^2 \bar{z}^2 + s\bar{s} \rho^4 \Big]$$

$$(1+s\bar{s})^2 (1+s\bar{s})^4 = \left(1 + \frac{1}{c^2} \frac{1}{(1+s\bar{s})^2} [\dots] \right)^2 (1+s\bar{s})^4 = \left[(1+s\bar{s})^2 + \frac{1}{c^2} [\dots] \right]^2$$

$$s^2 \bar{z}^2 + s^2 \bar{z}^2 = 2(s_x^2 - s_y^2)(x^2 - y^2) + 8s_x s_y xy, \quad \int dV \text{ (or average over core)} = 0$$

$$\frac{1}{(1+s\bar{s})^2} \frac{1}{(1+s\bar{s})^4} = |g_{\text{core}} s^2| = \frac{1}{\left[1 + |s|^2 + \frac{1}{c^2} \left[(1+2|s|^2)\rho^2 - (1+\rho^2)(\bar{z}s + s\bar{z}) + |s|^2(1+\rho^2) + s^2 \bar{z}^2 + s^2 \bar{z}^2 \right] \right]^2}$$

- bugunno hocus (def(...)) gac rucur $\sim |s|^2 \sim (x^2 + y^2)$

mpolygonogodno, $D_{\alpha\beta} = D_0 \delta_{\alpha\beta} + D_1 X_\alpha X_\beta$, $T_z(\hat{D}) = 2D_0 + D_1(x^2 + y^2)$

$D_{\alpha\beta} \sim \int dV \frac{\partial \vec{m}}{\partial X_\alpha} \cdot \frac{\partial \vec{m}}{\partial X_\beta}$ nrm $\vec{m}(\vec{\rho}, \vec{X})$ se rigid v. metal gac no $\sim W_{ex}$.

A vanda nrm $G_z(\vec{X}) \sim Q(s)$. $Q(s) \sim I(s) = \int \frac{d^2 \vec{\rho}}{(1+|s|^2)^2} (1-\rho^4)$

Core $|s|=1$ konsof cogerur m. n. s.

Tame me cogerur u nrm pacere non-lin. $\vec{F}_{ST} \sim \int d^2 \vec{\rho} \hat{z} \cdot (\dots)$, konsof $|s|=1$ gac coger

$\vec{F}_{ST}^+ \sim \int d^2 \vec{\rho} \hat{z} \cdot \left[\vec{m} \times \frac{\partial \vec{m}}{\partial X_\alpha} \right]$ no enjagen. ST-slanc.

$\left[\vec{m} \times \frac{\partial \vec{m}}{\partial X_\alpha} \right] = (v\hat{e}_+ + \bar{v}\hat{e}_- + m_z \hat{z}) \times \left(\frac{\partial v}{\partial X} \hat{e}_+ + \frac{\partial \bar{v}}{\partial X} \hat{e}_- + \hat{z} \frac{\partial m_z}{\partial X} \right) =$

$= |\hat{e}_+ \times \hat{e}_- = i\hat{z}| = i\hat{z} \left[v \frac{\partial \bar{v}}{\partial X} - \bar{v} \frac{\partial v}{\partial X} \right] = i\hat{z} \left[w \left(\frac{\partial \bar{w}}{\partial X} - w^2 \frac{\partial w}{\partial X} \right) - \bar{w} \left(\frac{\partial w}{\partial X} - w^2 \frac{\partial \bar{w}}{\partial X} \right) \right] \frac{2}{(1+w\bar{w})^3} = \frac{2i\hat{z}}{(1+w\bar{w})^3} \left[w(1+w\bar{w}) \frac{\partial \bar{w}}{\partial X} - \bar{w}(1+w\bar{w}) \frac{\partial w}{\partial X} \right] =$

$= \frac{2i\hat{z}}{(1+w\bar{w})^2} \left[w \frac{\partial \bar{w}}{\partial X} - \bar{w} \frac{\partial w}{\partial X} \right]$, $\vec{F}_{ST}^+ \sim i \int d^2 \vec{\rho} \frac{1}{(1+w\bar{w})^2} \left[w \frac{\partial \bar{w}}{\partial X} - \bar{w} \frac{\partial w}{\partial X} \right]$

$\nabla_{\vec{X}} = \hat{x} \frac{\partial}{\partial X} + \hat{y} \frac{\partial}{\partial Y} = \hat{x} \left(\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}} \right) + \hat{y} i \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}} \right) = (\hat{x} + i\hat{y}) \frac{\partial}{\partial z} + (\hat{x} - i\hat{y}) \frac{\partial}{\partial \bar{z}}$

$\nabla_{\vec{X}} = \sqrt{2} \hat{e}_- \frac{\partial}{\partial z} + \sqrt{2} \hat{e}_+ \frac{\partial}{\partial \bar{z}}$, $\vec{F}_{ST} = -\nabla_{\vec{X}} \int dV w_s = -\int dV \frac{\partial w_s}{\partial \vec{m}} \cdot \nabla_{\vec{X}} \vec{m} =$

$= -aL \int d^2 \vec{\rho} [\vec{m} \times \hat{z}] \cdot \nabla_{\vec{X}} \vec{m} = -aL \int d^2 \vec{\rho} \hat{z} \cdot [\nabla_{\vec{X}} \vec{m} \times \vec{m}]$, $\vec{F}_{ST} = aL \int d^2 \vec{\rho} \hat{z} \cdot [\vec{m} \times \nabla_{\vec{X}} \vec{m}]$

$\vec{F}_{ST} = \sqrt{2} aL \int d^2 \vec{\rho} \hat{z} \cdot \left[\vec{m} \times \frac{\partial \vec{m}}{\partial z} \right] = 2i\sqrt{2} aL \int d^2 \vec{\rho} \frac{1}{(1+w\bar{w})^2} \left[w \frac{\partial \bar{w}}{\partial z} - \bar{w} \frac{\partial w}{\partial z} \right]$

Bne core $w = f/|f| = \sqrt{\frac{s}{\bar{s}}}$: $w \frac{\partial \bar{w}}{\partial z} - \bar{w} \frac{\partial w}{\partial z} = \sqrt{\frac{s}{\bar{s}}} \frac{\partial}{\partial z} \sqrt{\frac{\bar{s}}{s}} - \sqrt{\frac{\bar{s}}{s}} \frac{\partial}{\partial z} \sqrt{\frac{s}{\bar{s}}} =$

$= \sqrt{\frac{s}{\bar{s}}} \left[\frac{\partial \bar{s}}{\partial z} - \frac{s}{\bar{s}} \frac{\partial s}{\partial z} \right] - \sqrt{\frac{\bar{s}}{s}} \left[\frac{\partial s}{\partial z} - \frac{\bar{s}}{s} \frac{\partial \bar{s}}{\partial z} \right] = \frac{1}{2|f|} - \frac{1}{2|f|^2} \left[s \frac{\partial \bar{s}}{\partial z} - \bar{s} \frac{\partial s}{\partial z} \right] 2$

$s \frac{\partial \bar{s}}{\partial s} - \bar{s} \frac{\partial s}{\partial \bar{s}} = \frac{1}{c} \left[z - \frac{(s+s\bar{z}^2)}{(1+s\bar{s})} \right] \frac{1}{c} \frac{1}{(1+s\bar{s})^2} \left[(1+s\bar{s}) \bar{z}^2 - s(s+s\bar{z}^2) \right] =$

$$\begin{aligned}
 & - \frac{1}{c^2} \left[\bar{z} - \frac{(5+5\bar{z}^2)}{(1+5\bar{z})} \right] \frac{(-i)}{c^2} \frac{1}{(1+5\bar{z})^2} \left[1+5\bar{z} - 5(5+5\bar{z}^2) \right] = \frac{1}{c^2} \frac{1}{(1+5\bar{z})^3} \times \\
 & \cdot \left[(1+5\bar{z})\bar{z} - (5+5\bar{z}^2) \right] \left[1-5\bar{z}^2 \right] - \left[(1+5\bar{z})^2 - (5+5\bar{z}^2) \right] \left[\bar{z}^2 - 5 \right] \\
 & \frac{\partial \bar{F}}{\partial \bar{z}} - \bar{F} \frac{\partial \bar{F}}{\partial \bar{z}} = \frac{1}{c^2} \frac{1}{(1+5\bar{z})^3} \left[(1+5\bar{z})\bar{z} - 5 - 5\bar{z}^2 - 5^2(1+5\bar{z})\bar{z}^2 + 5^3\bar{z}^2 + 55^2\bar{z}^4 - \right. \\
 & \left. - (1+5\bar{z})5^2\bar{z} + 5\bar{z}^2 + 55^4 + 5^2\bar{z} + 5^35\bar{z} - 5^25 - 5^3\bar{z}^2 \right] \approx |80 \cdot 5^2| = \\
 & = \frac{1}{c^2} \frac{1}{(1+5\bar{z})^3} \left[(1-5^2)(1+5\bar{z})\bar{z} + 5(5^4-1) - 5^25^2\bar{z} + 5^2\bar{z} \right] \\
 & \frac{\partial \bar{F}}{\partial \bar{z}} - \bar{F} \frac{\partial \bar{F}}{\partial \bar{z}} = \frac{1}{c^2} \frac{1}{(1+5\bar{z})^3} (1-5^2) \left[(1+5\bar{z})\bar{z} + 5^2\bar{z} - (1+5^2)5 \right] \\
 & F_{ST} = \frac{1}{\sqrt{2}} (F_{ST}^x - i F_{ST}^y) \text{ on p. } F_{ST}^x, F_{ST}^y. // \text{А. С. А. А. "Формы" - 1,600\text{€}} \\
 & \text{В core } F_{ST} = 2i\sqrt{2}aL \int d^2\bar{g} \frac{1}{(1+5\bar{z})^2} \left[\bar{F} \frac{\partial \bar{F}}{\partial \bar{z}} - \bar{F} \frac{\partial \bar{F}}{\partial \bar{z}} \right] \cdot 1\bar{z}^2 \\
 & \text{В core } F_{ST} = 2i\sqrt{2}aL \int d^2\bar{g} \frac{1}{4} \frac{1}{|5|^2} \left[\bar{F} \frac{\partial \bar{F}}{\partial \bar{z}} - \bar{F} \frac{\partial \bar{F}}{\partial \bar{z}} \right] \cdot 1\bar{z}^2, |5|=1 \\
 & \text{В мин. по } \bar{z}^4 \text{ в минимуме: } \text{В core } F_{ST} = 2i\sqrt{2}aL \int d^2\bar{g} \frac{1}{(1+5^2/c^2)^2} \frac{(1-5^2)}{c^2} \left[\bar{z} - (1+5^2)5 \right] \\
 & = -2i\sqrt{2}aL \cdot 2\pi \int_0^c d\bar{g} \frac{1}{c^2} \frac{1}{(1+5^2/c^2)^2} (1-5^2) \bar{z} = \frac{1}{\sqrt{2}} (F_{ST}^x - i F_{ST}^y) \\
 & i\bar{z} = i(S_x + iS_y) = -S_y + iS_x, F_{ST}^x = S_y 2\pi aL \int_0^c d\bar{g} \frac{(1-5^2)}{c^2} = 2\pi aL S_y - \\
 & \text{— other notation (Kau y Ivanov)} \int d^2\bar{g} \frac{(1-5^2)}{(1+5^2/c^2)^2} \bar{z} \neq 0 \text{ в мин. в минимуме по "S", без констр} \\
 & |5|=1 \text{ — other notation — } \sim \bar{z}, \bar{z}. \text{ Кроме того, упомянуть Иванова и другие} \\
 & 2) \text{ В core } F_{ST} = 2i\sqrt{2}aL \frac{1}{4} \int d^2\bar{g} \frac{1}{[5^2 - (1+5^2)(\bar{z}5 + 5\bar{z})]} (1-5^2) [\bar{z} - (1+5^2)5] = \\
 & = i\sqrt{2}aL \int d^2\bar{g} \frac{1}{5^2} (1-5^2) \left[1 + \frac{(1+5^2)}{5^2} (\bar{z}5 + 5\bar{z}) \right] [\bar{z} - (1+5^2)5] = i\sqrt{2}aL \int d^2\bar{g} \frac{(1-5^2)}{5^2} \times \\
 & \times [\bar{z} + (1+5^2)(\bar{z}^25 + 5\bar{z}^2) - (1+5^2)5] = \frac{i\sqrt{2}aL}{2} \int d^2\bar{g} \frac{(1-5^2)}{5^2} [\bar{z} + (1+\frac{1}{5^2})5\bar{z}^2] = 0 \text{ в мин.} \\
 & F_{ST} (\text{core}) = 0 \text{ в мин. в минимуме по "S", zero core. с N.A. кол.} \\
 & \text{В core core того же } |5|^2 = 5\bar{z} = \frac{1}{c^2} [5^2 - (1+5^2)(\bar{z}5 + 5\bar{z})] + O(5^4), \text{ zero} \\
 & \text{гид } \frac{1}{(1+|5|^2)} = \frac{1}{[1 + \frac{5^2}{c^2} - \frac{(1+5^2)(\bar{z}5 + 5\bar{z})}{c^2}]^2} = \frac{1}{(1+5^2/c^2)^2 [1 - \frac{(1+5^2)(\bar{z}5 + 5\bar{z})}{(c^2+5^2)}]^2} = \\
 & = \frac{1}{(1+5^2/c^2)^2} \left[1 + 2 \frac{(1+5^2)}{(c^2+5^2)} (\bar{z}5 + 5\bar{z}) \right], \text{ но главное — } \bar{z} \text{ — } \frac{1}{2} \text{ в } F_{ST} \text{ мин. в минимуме.} \\
 & \text{Две переменные } \vec{F}_{ST} \text{ — } (\Theta, \Phi) \text{ — переменных — } \text{в core в core (в мин. в мин.} \\
 & \text{at least). } w = \frac{1}{2} \frac{\Theta}{\cos^2 \Theta} e^{i\Phi}, \frac{\partial w}{\partial x} = \frac{1}{2} \frac{1}{\cos^2 \Theta} \Theta'_x e^{i\Phi} + \frac{1}{2} \frac{\Theta}{\cos^2 \Theta} e^{i\Phi} \Phi'_x \\
 & \frac{\partial w}{\partial x} \frac{\partial \bar{w}}{\partial x} - \bar{w} \frac{\partial w}{\partial x} = \frac{1}{2} \frac{\Theta}{\cos^2 \Theta} e^{i\Phi} \left[\frac{1}{2} \frac{1}{\cos^2 \Theta} \Theta'_x + \frac{1}{2} \frac{\Theta}{\cos^2 \Theta} \Phi'_x \right] e^{-i\Phi} - \frac{1}{2} \frac{\Theta}{\cos^2 \Theta} e^{-i\Phi} \left[\frac{1}{2} \frac{1}{\cos^2 \Theta} \Theta'_x + \frac{1}{2} \frac{\Theta}{\cos^2 \Theta} \Phi'_x \right] e^{i\Phi} = \\
 & = -2i \frac{\Theta^2}{2} \cdot \Phi'_x, \frac{\Theta^2 \Theta/2}{(1+w\bar{w})^2} = \frac{\Theta^2 \Theta/2}{(1+\frac{\Theta^2}{4})^2} = \frac{\Theta^2 \Theta/2}{\cos^4 \Theta/2} = \frac{\sin^2 \Theta \cos^2 \Theta}{2} = \frac{1}{4} \sin^2 \Theta \\
 & \text{то же } F_{ST} \sim \int d^2\bar{g} \sin^2 \Theta \nabla_x^2 \Phi \text{ как ранее. } \sin^2 \Theta = \frac{4w\bar{w}}{(1+w\bar{w})^2}, \sin \Theta = \frac{2|w|}{1+|w|^2} \\
 & \ln w = \ln \frac{1}{2} \frac{\Theta}{\cos^2 \Theta} + i\Phi, i\Phi'_x = \frac{\partial}{\partial x} \ln \frac{w}{|w|} = \frac{|w|}{w} \frac{w'_{|w|} - w|w|'}{|w|^2} = \frac{w'}{w} - |w|'_x \frac{1}{|w|} \\
 & i\Phi'_x = \frac{w'}{w} - \frac{\partial |w|}{\partial x} \frac{1}{|w|}. \text{ В core } w = f/|f|, \text{ а } i\Phi'_x = \frac{\partial}{\partial x} \ln w = \frac{\partial}{\partial x} \ln \frac{f}{|f|} \\
 & \text{В core core } w = f \text{ — on core } i\Phi'_x = \frac{\partial}{\partial x} \ln \frac{f}{|f|} \text{ и } \ln \frac{f}{|f|} = \ln \left(\frac{f}{f} \right)^{1/2} = \frac{1}{2} \ln \frac{f}{f} \\
 & i\Phi'_x = \frac{\partial}{\partial x} \left[\ln f - \ln \bar{f} \right] \frac{1}{2} = \frac{1}{2} \left[\frac{f'}{f} - \frac{\bar{f}'}{\bar{f}} \right] = \frac{1}{2} \frac{1}{|f|^2} [\bar{f}f' - f\bar{f}'] \\
 & f(z) = \frac{1}{c} \left[z - \frac{(5+5\bar{z}^2)}{(1+5\bar{z})} \right] = \frac{1}{c} \left[z \left[1 - \frac{1}{2} \frac{(5+5\bar{z}^2)}{(1+5\bar{z})} \right] \right], \ln f = (\dots) + \ln \left[1 - \frac{1}{2} \frac{(5+5\bar{z}^2)}{(1+5\bar{z})} \right]
 \end{aligned}$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \dots, \quad \ln f = -\frac{1}{2} \frac{(1+s\bar{s})}{(1+s\bar{s})^2} - \frac{1}{2} \frac{1}{2^2} \frac{(1+s\bar{s})^2}{(1+s\bar{s})^3} + O(s^4) =$$

$$= -\frac{1}{2} (1+s\bar{s})(1-s\bar{s}) - \frac{1}{2} \frac{1}{2^2} (1+s\bar{s})^2 - \frac{1}{3} \frac{1}{2^3} (1+s\bar{s})^3 + O(s^4)$$

$$2i\Phi = \ln f - \ln \bar{f} = -\frac{1}{2} (1+s\bar{s})(1-s\bar{s}) + \frac{1}{2} (1+s\bar{s})(1-s\bar{s}) - \frac{1}{2} \frac{1}{2^2} (1+s\bar{s})^2 +$$

$$+ \frac{1}{2} \frac{1}{2^2} (1+s\bar{s})^2 - \frac{1}{3} \frac{1}{2^3} (1+s\bar{s})^3 + \frac{1}{3} \frac{1}{2^3} (1+s\bar{s})^3 + \text{const}(s)$$

$$2i \frac{\partial \Phi}{\partial s} = -\frac{1}{2} (1-s\bar{s}) - \frac{1}{2} (1+s\bar{s})(-s) + \frac{1}{2} \bar{s}^2 (1-s\bar{s}) + \frac{1}{2} (1+s\bar{s})(-s) -$$

$$- \frac{1}{2} \frac{1}{2^2} (1+s\bar{s})^2 + \frac{1}{2} \frac{1}{2^2} 2(1+s\bar{s})\bar{s}^2 + O(s^2) = -\frac{1}{2} + \bar{s} - \frac{1}{2^2} (1+s\bar{s}) + \frac{1}{2^2} (1+s\bar{s})\bar{s}^2$$

$$+ O(s^2) = -\frac{1}{2} + \bar{s} - \frac{s}{2} - s + s + s\bar{s}^2 = -\frac{1}{2} + \bar{s} + s(\bar{s}^2 - \frac{1}{2}) + O(s^2)$$

$$2i\Phi'_s = -\frac{1}{2} (1-s^2) + s \frac{(1-s^2)}{2^2} (1-s^2) = -(1-s^2) \left[\frac{1}{2} + \frac{s}{2^2} (1+s^2) \right] + O(s^2)$$

$$2i \frac{\partial \Phi}{\partial s} = 2i \frac{1}{2} \left[\frac{\partial \Phi}{\partial x} - i \frac{\partial \Phi}{\partial y} \right] = \frac{\partial \Phi}{\partial y} + i \frac{\partial \Phi}{\partial x} = -(1-s^2) \left[\frac{(x-iy)}{s^2} + \frac{1}{s^4} (x-iy)^2 (x+iy) \right] +$$

$$(x-iy)^2 (x+iy) = (x^2-y^2 + 2ixy) (x+iy) = (x^2-y^2)X + 2xyY - 2ixyX + (x^2-y^2)Y$$

$$\rightarrow \frac{\partial \Phi}{\partial y} = -(1-s^2) \left[\frac{x}{s^2} + \frac{(1+s^2)}{s^4} \{ (x^2-y^2)X + 2xyY \} \right], \quad \frac{\partial \Phi}{\partial x} = -(1-s^2) \left[-\frac{y}{s^2} + \right.$$

13/III (θ, φ) - неперскаблемые канонические переменные
non-lin. terms. Лягушка $\vec{m}$ и $w, \bar{w}$ неперскаблемы.

$$\frac{\partial \vec{m}}{\partial x_\alpha} \frac{\partial \vec{m}}{\partial x_\beta} = \partial_\alpha m_x \partial_\beta m_x + \partial_\alpha m_y \partial_\beta m_y + \partial_\alpha m_z \partial_\beta m_z = \begin{vmatrix} m_x + im_y = \sin \theta e^{i\varphi} \\ m_z = \cos \theta \end{vmatrix} =$$

$$= \partial_\alpha (\sin \theta \cos \varphi) \partial_\beta (\dots) + \partial_\alpha (\sin \theta \sin \varphi) \partial_\beta (\dots) + \partial_\alpha \cos \theta \partial_\beta \cos \theta = (\cos \theta \theta'_\alpha \cos \varphi +$$

$$+ \sin \theta (-) \sin \varphi \varphi'_\alpha) (\cos \theta \cos \varphi \theta'_\beta - \sin \theta \sin \varphi \varphi'_\beta) + (\cos \theta \theta'_\alpha \sin \varphi + \sin \theta \cos \varphi \varphi'_\alpha) \times$$

$$\times (\cos \theta \sin \varphi \theta'_\beta + \sin \theta \cos \varphi \varphi'_\beta) + \sin^2 \theta \theta'_\alpha \theta'_\beta = \theta'_\alpha \theta'_\beta [\cos^2 \theta \cos^2 \varphi + \cos^2 \theta \sin^2 \varphi + \sin^2 \theta$$

$$+ \theta'_\alpha \varphi'_\beta [-\cos \theta \sin \theta \cos \varphi \sin \varphi + \cos \theta \sin \varphi \sin \theta \cos \varphi] + \varphi'_\alpha \theta'_\beta [0] + \varphi'_\alpha \varphi'_\beta [\sin^2 \theta \sin^2 \varphi +$$

$$+ \sin^2 \theta \cos^2 \varphi] = \theta'_\alpha \theta'_\beta + \sin^2 \theta \varphi'_\alpha \varphi'_\beta$$

$$D_{\alpha\beta} \sim \int dV \frac{\partial \vec{m}}{\partial x_\alpha} \frac{\partial \vec{m}}{\partial x_\beta} = \int dV \left[\frac{\partial \theta}{\partial x_\alpha} \frac{\partial \theta}{\partial x_\beta} + \sin^2 \theta \frac{\partial \varphi}{\partial x_\alpha} \frac{\partial \varphi}{\partial x_\beta} \right] \quad \text{и где damping } D_{\alpha\beta}$$

где $F_{str}^4 \sim \int \sin^2 \theta \frac{\partial \varphi}{\partial x_\alpha} \frac{\partial \varphi}{\partial x_\beta}$ некое число (θ, φ) по $X_\alpha^2(0)$ и $g_0 X_\alpha^4(F_{str})$ некое

$$\text{Важно } \int d^3x \sin^2 \theta \nabla_x^2 \varphi = \int d^3x \nabla_x^2 \varphi - \int d^3x \cos^2 \theta \nabla_x^2 \varphi, \quad \int_0^{2\pi} d\varphi \frac{\partial \varphi}{\partial x} = 0 \text{ б.м.н.н.}$$

неперскаблемы, но сопоставляем парам F_{str} от 8/XI

$$17/XI \text{ дано } \Phi(\vec{p}, \vec{X}) = \frac{c\pi}{2} + \varphi + \left(\frac{1}{p^2} - 1\right)(yX - xY) + \frac{xy}{p^4}(X^2 - Y^2) + \frac{1}{p^4}(y^2 - x^2)XY + xy(Y^2 - X^2) +$$

$$+ yX(X^2 - Y^2) - \Phi(\vec{p}, \vec{X}) = \frac{c\pi}{2} + \varphi + \left(\frac{1}{p^2} - 1\right)(yX - xY) + \left(\frac{1}{p^4} - 1\right)xy(X^2 - Y^2) + \left(\frac{1}{p^4} - 1\right)(x^2 - y^2)(-1)XY, \text{ где}$$

сблн. с параметром 12/XI

$$G_2(\vec{X}) \sim \int dV \sin \theta \nabla_x^2 \theta \times \nabla_x^2 \varphi \text{ (проблематно)}, \text{ и где } G_2 \sim X_\alpha^2 \text{ некое } (\theta, \varphi) \text{ где}$$

т.е. $\vec{D}(\vec{X})$ и $G_2(\vec{X})$ - non-lin. terms одинак. сложности, а $F_{str}^2(\vec{X})$ - линейные

сложнее решить $\theta(\vec{p}, \vec{X})$ - dynamic. $\theta = \pi/2$ бие dyn. core.

$\theta \neq \pi/2$ так как $|\vec{p}(\vec{p}, \vec{X})| \leq 1$, но есть обл. неустойчив.

где $G_2(\vec{X}) \sim F_{str}^2(\vec{X})$. Даме еще реплика в $x^0 y^1$ с учетом $\vec{X}(t)$

$$\nabla_x^2 \sin \theta = -\nabla_x^2 m_z. \quad \vec{z} \cdot \vec{m} \times \frac{\partial \vec{m}}{\partial x_\alpha} = m_x \partial_\alpha m_y - m_y \partial_\alpha m_x = \sin \theta \cos \varphi [\cos \theta \sin \varphi \theta'_\alpha +$$

$$+ \sin \theta \cos \varphi \varphi'_\alpha] - \sin \theta \sin \varphi [\cos \theta \cos \varphi \theta'_\alpha - \sin \theta \sin \varphi \varphi'_\alpha] = + \sin^2 \theta \frac{\partial \varphi}{\partial x_\alpha} \rightarrow F_{str}^4 \sim \int dV \sin^2 \theta$$

→ где все бие в $\vec{m}$ - неперскаблемы, а новые перскаблемы в (θ, φ) - неперскаблемы

Рассмотрим тензор напряжений damping tensor $D_{\alpha\beta}(\vec{X})$.

$$D_{xx} \sim \left(\frac{\partial \vec{m}}{\partial \vec{X}} \right)^2 = \left(\frac{\partial \vec{m}}{\partial z} \right)^2 + 2 \frac{\partial \vec{m}}{\partial z} \frac{\partial \vec{m}}{\partial \vec{X}} + \left(\frac{\partial \vec{m}}{\partial \vec{X}} \right)^2, \quad D_{xy} \sim - \left(\frac{\partial \vec{m}}{\partial z} \right)^2 + 2 \frac{\partial \vec{m}}{\partial z} \frac{\partial \vec{m}}{\partial \vec{X}} - \left(\frac{\partial \vec{m}}{\partial \vec{X}} \right)^2$$

$$T_{\alpha\beta} \hat{=} D_{xx} + D_{yy} \sim 4 \frac{\partial \vec{m}}{\partial z} \frac{\partial \vec{m}}{\partial \vec{X}}, \quad \text{это и есть значение } 12/\chi_{II} // E \sim 2 \text{ на единицу } V \cdot \text{см}.$$

$$(D_{xx} - D_{yy}) \sim \left(\frac{\partial \vec{m}}{\partial z} \right)^2 + \left(\frac{\partial \vec{m}}{\partial \vec{X}} \right)^2 \sim \left\{ d^2 \rho^2 \frac{1}{(1+u^2)^2} \left[\frac{\partial \vec{m}}{\partial z} \frac{\partial \vec{m}}{\partial z} + \frac{\partial \vec{m}}{\partial \vec{X}} \frac{\partial \vec{m}}{\partial \vec{X}} \right] \right\} \neq 0 \quad (\text{если } u \neq 0)$$

$$(D_{xx} + D_{yy}) \sim \left(\frac{\partial \vec{m}}{\partial z} \frac{\partial \vec{m}}{\partial \vec{X}} \right) \sim \left\{ d^2 \rho^2 \frac{1}{(1+u^2)^2} \left[\frac{\partial \vec{m}}{\partial z} \frac{\partial \vec{m}}{\partial \vec{X}} + \frac{\partial \vec{m}}{\partial \vec{X}} \frac{\partial \vec{m}}{\partial z} \right] \right\}. \quad \text{Аналогично?}$$

$$\text{Аналог } \arctg(0, \varphi). \quad \theta = 2 \arctg |u|, \quad \frac{\partial \theta}{\partial \vec{X}} = 2 \frac{1}{1+u^2} \frac{\partial u}{\partial \vec{X}} \neq 0 \quad \text{только в центре } u=0.$$

$$D_{\alpha\beta}(\text{core}) \sim d^2 \rho^2 \left[\frac{4}{(1+u^2)^2} \frac{\partial u}{\partial x_\alpha} \frac{\partial u}{\partial x_\beta} + \sin^2 \theta \Phi'_\alpha \Phi'_\beta \right].$$

Рассмотрим $D_{\alpha\beta}$ вне core, т.е. распределение значений Φ, Φ' вне core и не обращая core, $d^2 \rho^2$ не берем в расчет.

$$D_{\alpha\beta}(\text{off core}) \sim \left\{ d^2 \rho^2 \left[\frac{\partial \Phi}{\partial x_\alpha} \frac{\partial \Phi}{\partial x_\beta} \right] \right\}, \quad \int_0^R d\rho(\dots) \text{ не влияет из-за симметрии } \Phi \text{ и } \Phi' \text{ по } \rho \rightarrow 0, \text{ тогда все же } 0 \rightarrow c.$$

Но если мы не берем $0 \rightarrow c$, то тогда $\int_0^R d\rho(\dots)$, тогда считаем, что $f(\rho, \vec{X})$ и $|f|=1$ не есть выражение $\vec{X}$.

$$2i \frac{\partial \Phi}{\partial \vec{X}} (g_0 \dots) = 2i \frac{\partial \Phi}{\partial \vec{X}} (g_0 \dots) + \frac{5}{2} + \frac{5}{2} (5 + 5\vec{z}^2) - 5\vec{z}^2 - \frac{5}{2} (5 + 5\vec{z}^2) - \frac{1}{2} (5 + 5\vec{z}^2)^2 + \frac{1}{2} (5 + 5\vec{z}^2)^2 \vec{z}^2 = (g_0 \dots) + 2 \frac{5}{2} + 5\vec{z}^2 - \frac{5}{2} - 2 \frac{5}{2} \vec{z}^2 - \frac{5}{2} - \frac{1}{2} (5^2 + 2 \cdot 5 \cdot 5\vec{z}^2 + 5^2 \vec{z}^4) =$$

$$+ \frac{1}{2} (5^2 + 2 \cdot 5 \cdot 5\vec{z}^2 + 5^2 \vec{z}^4) = (g_0 \dots) - \frac{5}{2} + 5\vec{z}^2 = (g_0 \dots) + 5\vec{z}^2 \left[\vec{z}^2 - \frac{1}{2} \right] = 5\vec{z}^2 \left[\vec{z}^2 - \frac{1}{2} \right] = 5\vec{z}^2 \left(1 - \frac{1}{2} \right) = (1 - \frac{1}{2}) (5\vec{z}^2 - 5\vec{z}^2 + 2i 5\vec{z}^2 \vec{X}) (x^2 - 3y^2 - 3ix^2y + iy^3) =$$

$$= (1 - \frac{1}{2}) \left[(5\vec{z}^2 - 5\vec{z}^2) (x^2 - 3y^2) + 25\vec{z}^2 (3x^2y - y^3) + 2i 5\vec{z}^2 (x^3 - 3xy^2) + (5\vec{z}^2 - 5\vec{z}^2) i (y^3 - 3x^2y) \right]$$

$$2i \frac{\partial \Phi}{\partial \vec{X}} = \frac{\partial \Phi}{\partial \vec{Y}} + i \frac{\partial \Phi}{\partial \vec{X}} = (g_0 \dots) + (1 - \frac{1}{2}) \left[(x^2 - y^2) x (x^2 - 3y^2) + 2xy y (3x^2 - y^2) + \right.$$

$$\left. + 2xy x (x^2 - 3y^2) + (x^2 - y^2) i y (y^2 - 3x^2) \right]$$

$$\frac{\partial \Phi}{\partial \vec{Y}} = (g_0 \dots) + (1 - \frac{1}{2}) \left[(x^2 - y^2) x (x^2 - 3y^2) + 2xy y (3x^2 - y^2) \right]$$

$$\frac{\partial \Phi}{\partial \vec{X}} = (g_0 \dots) + (1 - \frac{1}{2}) \left[2xy (x^2 - 3y^2) x + (x^2 - y^2) y (y^2 - 3x^2) \right]$$

$$\left(\frac{\partial \Phi}{\partial \vec{X}} \right)^2 = \left[\frac{(1-\rho^2)}{\rho^2} y - \frac{(1-\rho^4)}{\rho^4} \left\{ (x^2 - y^2) y - 2xy x \right\} + (1 - \frac{1}{2}) \left[\dots \right] \right]^2 = \left| \text{только } \cos \left(\frac{\partial \Phi}{\partial \vec{X}} \right) \right|^2$$

$$= \frac{(1-\rho^2)^2}{\rho^4} y^2 + \frac{(1-\rho^4)^2}{\rho^8} \left\{ y^2 (x^4 + y^4 - 2x^2y^2) + 4x^2y^2 x^2 \right\} + 2 \frac{(1-\rho^2)}{\rho^2} (1 - \frac{1}{2}) 2xy y^2 (3x^2 - y^2)$$

$$D_{xx} \sim \int d\varphi \left(\frac{\partial \Phi}{\partial \vec{X}} \right)^2 = \frac{(1-\rho^2)^2}{\rho^4} \pi \rho^2 + \frac{(1-\rho^4)^2}{\rho^8} \pi \rho^4 (x^2 + y^2) + 4 \frac{(1-\rho^2)}{\rho^2} (1 - \frac{1}{2}) \rho^4 xy \int d\varphi [\dots]$$

$$\int d\varphi [\dots] = \int d\varphi \sin^2 \varphi (3 \cos^2 \varphi - \sin^2 \varphi) = \int d\varphi \sin^2 \varphi (4 \cos^2 \varphi - 1) = \int d\varphi [\sin^2 2\varphi - \sin^2 \varphi] =$$

$$D_{xx} \sim \int d\varphi \int d\varphi \left(\frac{\partial \Phi}{\partial \vec{X}} \right)^2 = \pi \int d\varphi \left[\frac{(1-\rho^2)^2}{\rho^2} + \frac{(1-\rho^4)^2}{\rho^4} |\vec{X}|^2 \right]. \quad \text{Возьмем } \int_0^R d\rho(\dots), \text{ тогда } c \rightarrow 1 \text{ и } u=0$$

$$\int d\varphi \left(\frac{\partial \Phi}{\partial \vec{X}} \right)^2 = \pi \int d\rho \left[\frac{1}{8} (1-\rho^2)^2 + \frac{1}{8} (1-\rho^4)^2 |\vec{X}|^2 \right] = \pi \left[\ln \frac{1}{c} - \frac{3}{4} + c^2 - \frac{1}{4} c^4 + \left(\frac{1}{2} \frac{1}{c^2} - \frac{1}{4} + c^2 - \frac{1}{6} c^6 \right) |\vec{X}|^2 \right] \approx |c \text{ near } g_0 \dots| = \pi \left(\ln \frac{1}{c} - \frac{3}{4} \right) + \pi \left(\frac{1}{2} \frac{1}{c^2} - \frac{1}{4} \right) |\vec{X}|^2$$

$$D_{yy} \sim \left(\frac{\partial \Phi}{\partial \vec{Y}} \right)^2 = \left[- \frac{(1-\rho^2)}{\rho^2} x - \frac{(1-\rho^4)}{\rho^4} \left\{ (x^2 - y^2) x + 2xy y \right\} + (1 - \frac{1}{2}) \left[\dots \right] \right]^2 =$$

$$= \frac{(1-\rho^2)^2}{\rho^4} x^2 + \frac{(1-\rho^4)^2}{\rho^8} \left\{ (x^2 - y^2)^2 x^2 + 4x^2 y^2 y^2 \right\} - 2 \frac{(1-\rho^2)}{\rho^2} (1 - \frac{1}{2}) (x^2 - y^2) x^2 (x^2 - 3y^2) =$$

$$D_{yy} \sim \int d\varphi \left(\frac{\partial \Phi}{\partial \vec{Y}} \right)^2 = \pi \frac{(1-\rho^2)^2}{\rho^2} + \pi \frac{(1-\rho^4)^2}{\rho^4} |\vec{X}|^2, \quad D_{yy} = D_{xx} - \text{вне core}$$

$$D_{yz} \sim \int d\varphi \left(\frac{\partial \Phi}{\partial \vec{X}} \right) \left(\frac{\partial \Phi}{\partial \vec{Y}} \right) = 0 \text{ с точностью до } X^2, \quad \vec{D} - \text{гарантированно вне core}$$

Escaneado con CamScanner

Покажем, что χ_d^2 имеет в $W_{ce}(\bar{x})$: $\omega_0 = \bar{\omega}_0 + \omega_e$
 Com. 17/11 также $m_e = \sin \theta \sin(\phi - \varphi) = \sin \theta [1 - \frac{1}{2} \epsilon^2 + \frac{1}{4!} \epsilon^4]$

[illegible]

14/11 Visit na Podavines 1-3 : orgda requested Vida Calural (negr. nono 1/11)
u infabing o Nov'19 - 3/na his officer, Ona chagana, 200 appl. dyge. p...
u 1/100 u infabing oves no home address. Bizer S... Per... de
Regresso" u chagov yok - ob — negabab his b Abie, Ab'YII (4-5 Jan 1947).

Белга с Аранха: Јона Збонна Ло Јосе Антонио, апт. - duplex Lareda, мотел. у мун. Билбао.
 2) Омарта F/N/F Лепи. В гуче Хвалон. '88'09, ко то м. Вгвона Јо 2.5-10⁸ Агм.
 - Синтеза Борбон, то м. не гуча Хвалон 181 дате за т_м 35 нз гуча м.
 - м.гуча > вс е м завон о с т; позн м ка м гуча м т_м 100 нз

3) frequency of nonresonance $\omega_0(x)$ depends on coordinates (masses M_1, M_2)
 then write β in $\omega_0(x) = \omega_0(0) + \beta |x|^2$ - nonlin. coefficient ω_0

4) μ rezonans ei cemas Fig.1 - (nanopil. + roopny sistema) u Fig.2 (V. image in 10⁻⁴ sistema) que APL/JMM no ST torque.

5) Он закончил postdoc в March 2011: опубликовал свои результаты в научной литературе (research letters), в том числе в рецензируемых papers.

2) Seaga e Julian : 1) SAJOTEK konfirmuje črta B "Kutka" ena, ne označi Jappona.
2) John-Portugal brez obzira na to, da ni bil: 3) prevažajo Auguste v Orléans "

1) Portugal travel grant number 112; 2) package of 10 grants to 10 different
negotiated reimbursement appl. (SIC), col. 11 NSF-MICELV grant; 4) various MMM to
the number 112; 5) package of 10 grants to 10 different

3) passengers o parone no STU Vorkovdy;
6) passengers o status GCU a DEW (11)

4) charges - no problem to invite a non PhD student w/ Solomonica no St dign.

new problem to invite E. How the student is interested in the problem.

23/11/11 Норман в PRB, Topol. field paper. неправильно там Figs & Defs.
вспомогательная с A.Ping : неправильно в paper (23 Figs!) и в paper
на villa - A.Ping (1200€/мес, авт, сводер, no 1, 700€ - transfer to Imm. Agcy)
неправильно transfer 1,100€/мес + 7,000€ - Авт, no 11 мес. p. Document 11, 12
неправильно Figs 1, 2 - topol. field с A.Ping. А.Ping general apartment price 2700€
с J.W.A. в downtown, каже, happyman, Lidl: Переван.

Due resubmits "Topol. field" paper name : 1) Fig 1, 2 EPS + doc. prints; 2) copyright.
Due on 20/11/11 APL - "ST transfer": 1) знак $\vec{F}_{ST}$ и $\vec{J}$ to create vortex; 2) знак $\vec{W}_{ST}$

3) Fig. 3 J_{c2} as function of parameters; 4) $t_{sw} = f(J)$

Slone. JMM '96

1) ЗНАК $\vec{F}_{ST}$

1. FM layer - polarizer, $\vec{S}_2 = \frac{I_c}{e} g \vec{S}_2 \times (\vec{S}_1 \times \vec{S}_2)$, $\text{ан. } 4/\sqrt{m}$. $\vec{S}_1 = \vec{P}$, $\vec{S}_2 / S = \vec{S}_2 = \vec{m}$
 $\vec{m} = \frac{1}{S} \frac{I_c}{e} g \vec{m} \times (\vec{P} \times \vec{m})$, наго $\vec{S}_1 = -\vec{m}_1$, $\vec{S}_2 = -\vec{m}_2$ to see, r.v. $\vec{P} = -1/8 \vec{S}$
electronic charge (нааено $e = -|e|$ um $e = |e|$ y Slone.)

$\vec{P} \times \vec{m} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ P_x & P_y & P_z \\ 0 & 0 & m_z \end{vmatrix} = \hat{x} P_y m_z + \hat{y} (P_x m_z) = -P_x m_z \hat{y}$
 $P_y = 0$, r.v. $O_y \parallel (\vec{P} \times \vec{m})$, $B_y = P_x$

moving frame $\vec{m} = \hat{z}$ $\vec{m} \times (\vec{P} \times \vec{m}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & m_z \\ 0 & B_y & 0 \end{vmatrix} = -\hat{x} B_y m_z = P_x \hat{x}$
 $\vec{m}_{ST} = \frac{1}{S} \frac{I_c}{e} g P_x \hat{x}$, $\vec{P} = (\sin\theta, 0, \cos\theta)$

Пум $P_x > 0$ ($g = g(\theta) > 0$) знак $\vec{m}_x$ $g_{aven} > 0$, т.е. $(\frac{I_c}{e}) > 0$.

bag am. Ralph-Styles JMM '08 : sign of ST torque reverse. not obvious
 $\vec{S}_2$ spin-angular momentum of FM (F_2) в направлении spin of the incoming
electrons ($-\vec{P}$) um, направлении $\vec{M}$ (F_2) в направлении
um. направления of the incoming electrons ($\vec{P}$).

Наго $\vec{S}_1 = -\vec{m}_1 = -\vec{P}$, $\vec{m}_1 = \vec{P}$ just polarizer и $\vec{S}_2 = -\vec{m}_2$ just free layer (F_2).

Charge current $\vec{I} = -|I| \hat{z}$, $I_z = -|I|$ or $F_2 \times F_1$ just before sign ST
torque vfm $e = -|e|$, наго $(I_c/e) = \frac{(-|I|)}{(-|e|)} = \frac{|I|}{|e|} > 0$. $\vec{m} = \vec{M}/M_s$

$\vec{m}_{ST} = \frac{1}{S} (\frac{I_c}{e}) g \vec{m} \times (\vec{P} \times \vec{m})$, $(I_c/e) > 0$ - наго e рекурсив из F_1 to F_2
 $(\vec{m})_{damp} = \alpha \vec{m} \times \dot{\vec{m}}$, $\vec{m} \times \dot{\vec{m}} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ m_x & m_y & 0 \\ \dot{m}_x & \dot{m}_y & 0 \end{vmatrix} = -\hat{x} \dot{m}_y + \hat{y} \dot{m}_x$, $(\vec{m})_{ST} = \frac{I_c}{S e} g P_x \hat{x}$

$(\vec{m})_{ST} + (\vec{m})_{damp} = (\frac{I_c}{e}) \frac{g}{S} P_x \hat{x} + (-\alpha \dot{m}_y) \hat{x} + \alpha \dot{m}_x \hat{y}$ - наго $\alpha \dot{m}_x \hat{y} = 0$
из-за $\dot{y} = 0$ um.

Due vortex in F_2 -layer : $(\vec{m})_{ST} = (I_c/e) \frac{g}{S} \vec{m} \times (\vec{P} \times \vec{m})$, $\vec{m} = -\gamma \vec{m} \times \vec{H}_{eff}$, $\gamma > 0$

$\dot{\vec{m}} = -\gamma \vec{m} \times \vec{H}_{eff} + \alpha \vec{m} \times \dot{\vec{m}} + (I_c/e) \frac{g}{S} \vec{m} \times (\vec{P} \times \vec{m})$
 $\dot{\vec{m}} = -\gamma \vec{m} \times [-\frac{\alpha}{\gamma} \dot{\vec{m}} - \frac{1}{\gamma} (\frac{I_c}{e}) \frac{g}{S} (\vec{P} \times \vec{m}) + \vec{H}_{eff}]$, $\vec{H}_{eff} = \vec{H}_{ext} - \frac{1}{\gamma} \dot{\vec{m}} - \frac{1}{\gamma} (\frac{I_c}{e}) \frac{g}{S} \vec{P} \times \vec{m}$

$\vec{H}_{eff} = -\frac{\partial W}{\partial \vec{m}}$, no anisotropy $\vec{H}_{ST} = -\frac{\partial W_{ST}}{\partial \vec{m}} = -\frac{1}{\gamma} (\frac{I_c}{e}) \frac{g}{S} \vec{P} \times \vec{m}$, $\frac{\partial W_{ST}}{\partial \vec{m}} = \frac{1}{\gamma} (\frac{I_c}{e}) \frac{g}{S} \vec{P} \times \vec{m}$
 W_{ST} - наго направления (erg/cm³). $M_s \dot{\vec{m}} = \vec{m} \times \frac{\partial W}{\partial \vec{m}}$ - наго направления $\frac{\partial W}{\partial \vec{m}} = \alpha \vec{m} \times \vec{P}$
 $\frac{\partial W}{\partial \vec{m}} = -\frac{M_s}{\gamma} (\frac{I_c}{e}) \frac{g}{S} \vec{m} \times \vec{P} = \alpha \vec{m} \times \vec{P}$, $\alpha = -\frac{M_s}{\gamma} (\frac{I_c}{e}) \frac{g}{S}$

Core. $4/\pi$ $\gamma S = \frac{1}{h} g_L M_B S = \frac{1}{h} \frac{V}{N} \left(\frac{N}{V} g_L M_B S \right) = \frac{1}{h} \frac{V}{N} M_S$, N - number of FM ions

B para a na equação M1: $\gamma S = \frac{1}{h} V M_S$, $a = - \frac{M_S}{\gamma S} \left(\frac{I_c}{e} \right) g = - \frac{h}{V} \left(\frac{I_c}{e} \right) g$

Due free layer $V = AL$, $I_c = J$, $a = - \frac{h}{AL} \left(\frac{I_c}{e} \right) g = - \frac{h}{L} \left(\frac{J}{e} \right) g$, $J = \frac{h}{V} \left(\frac{I_c}{e} \right) g$

Monino $J = J_2$, $J_2 < 0$, $a = \frac{h}{L} \frac{J_2}{|e|} g$, $J_2 < 0$ var y slow $g > 0$

T.e. $a \sim J_2$.

Como 25/11: $\dot{W} = \dot{W}' + \dot{W}_s = -d|G|\omega_0^2 |\vec{X}|^2 - a\pi L \omega_0 |\vec{X}|^2 = -[d|G|\omega_0 + a\pi L] |\vec{X}|^2$
 $\dot{W} = 0$ para $J_2 < 0$, t.e. $d, \omega_0 > 0$, to make $a < 0$, $a \sim J_2$, $J_2 < 0$ que é o caso

YD thick: $\vec{G} \times \vec{X} = - \frac{\partial W}{\partial \vec{X}} + \vec{D} \vec{X} + \vec{F}_{ST}$, $\vec{G} = G_z \hat{z}$, $G_z = 2\pi \frac{M_S L}{g}$ (10/11) $\vec{F}_{ST} = \pi a L (\hat{z} \times \vec{X}) = \pi a L X_\rho \hat{\phi}$
 para $\frac{\partial W}{\partial G_z} = F_{ST}$, $\vec{F}_{ST} = \pi a L (\hat{z} \times \vec{X}) = \pi a L X_\rho \hat{\phi}$

B um. $\nabla^2 W = 0$ (que $J_2 < 0$), $W' = \alpha X_\rho$, $\frac{\partial}{\partial G_z} \alpha X_\rho = \pi a L X_\rho$, $J_2 < 0$
 Numa $G_z \sim \rho$, $\rho > 0$, $G_z > 0$, $\frac{\alpha}{G_z} = \omega_0$, $-d|G|\omega_0 = \pi a L$, no equilíbrio
 nume num $a < 0$, $J_2 < 0$ (tor or free to fixed layer).

Borbo: $J_2 < 0$ para bozmoznoob (damping) $\gamma_{eff} = 0$ e bozmoznoob vortex dyn. b free layer e $p_v = +1$ (polarizer $\hat{u} = \vec{p}$, $p_z = +1$).

Caputo 'PRL' 07 (see que muros): $\dot{\vec{S}} = j \frac{A}{1 + B \vec{S} \cdot \vec{z}} \vec{S} \times (\vec{S} \times \vec{z}) = \frac{J_c}{J_p} g(\theta) \vec{S} \times (\vec{S} \times \vec{z})$
 $\vec{S}, \vec{z}$ - unit vectors. Slanc'96 $\frac{d\vec{S}}{dt} = \frac{I_c}{eS} \frac{1}{S} g \vec{S}_2 \times (\vec{S}_1 \times \vec{S}_2) = \frac{I_c}{eS} g \vec{S} \times (\vec{z} \times \vec{S})$

forma $\frac{J_c}{J_p} = - \frac{I_c}{eS}$, $J = \frac{I_c}{|e|S} J_p = \frac{I_c}{|e|S} M_S^2 \frac{1}{h} d = \frac{J \cdot A}{hS} M_S^2 d = \frac{J M_S M_S V}{hS} = \frac{J M_S h S}{hS}$

no coobes. , t.e. Caputo '07 zamaru γM_S b $\frac{d}{dt}$. y Slanc'96 - normalizac

baseu y Caputo '07 okaznoob $J_2 < 0$ ($J_2 < 0$) que spin-vortex e $p = +1$, $p_z = +1$

Due $\vec{m}$ -vortex $p_v = -1$, $p_z = -z = -1$, $p_v(\text{ann}) \vec{z} = p_v p_z (\text{normal}) = +1$
 p.v. $\vec{z}$  $\vec{p}$ $\vec{z}$ $\vec{I}$

Shela 'APL' 07: yashue switching VC spin-vortex $j \geq p < 0$
 $j \geq p = +1$ que spin vortex e $\vec{m}$ -vortex $\Rightarrow j = j_z < 0$ yashue VC zashue

Ivanov 'PRL' surro ne robopur o znanse I, Liu '07 curar $I > 0$

$\hat{z} \times \vec{X} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ X & Y & 0 \end{vmatrix} = \hat{y} X - \hat{x} Y$, $\vec{G} \times \vec{X} = \hat{y} G_z \hat{x} - \hat{x} G_z \hat{y}$, $\frac{\partial W}{\partial \vec{X}} = \frac{W'}{|\vec{X}|} \vec{X}$

$\vec{G} \times \vec{X} - D \dot{\vec{X}} = - \frac{\partial W}{\partial \vec{X}} + \vec{F}_{ST} \Rightarrow \begin{cases} -G_z \dot{y} - D \dot{x} = - \frac{W'}{|\vec{X}|} x + \pi a L (-y) \\ G_z \dot{x} - D \dot{y} = - \frac{W'}{|\vec{X}|} y + \pi a L x \end{cases}$ $Z = X + iY$
 $i G_z \dot{Z} - D \dot{Z} = - \frac{W'}{|Z|} Z + \pi a L i Z$, num. $W'/|Z| = \alpha \Rightarrow i p \dot{Z} + d \dot{Z} = -\omega_0 Z$

$(ip + d) \dot{Z} = (-\omega_0 + \frac{\pi a L i}{|G|}) Z$, $Z(t) = Z(0) e^{i\omega t}$ - steady oscill. $\dot{Z} = i\omega Z$

$i\omega (ip + d) Z = (-\omega_0 + \frac{\pi a L i}{|G|}) Z$, $-\omega p + i\omega d = -\omega_0 + \frac{\pi a L i}{|G|}$, $\omega = \omega_0 p$, $\omega d = \pi a L \frac{1}{|G|}$ - $a > 0$? pensame $J_2 < 0$. Ecu uovar $Z(t) = Z(0) e^{i\omega t}$ $\omega = \omega_0 p$

to $\omega' = p\omega_0$ e $\omega'' = d\omega_0 - \frac{\pi a L}{p|G|}$, $Z(t) = Z(0) e^{i\omega_0 p t} e^{-\omega'' t}$ $\omega'' > 0$

Escaneado con CamScanner

только $\dot{\psi} = \frac{\partial W}{\partial \dot{\psi}} = -a(G\omega^2 R)^2 = \text{const. c. } 25/\pi$.

$$X(t) = X_0 \cos(\omega t + \varphi_0), \quad Y(t) = Y_0 \sin(\omega t + \varphi_0), \quad \dot{X} = -\omega X_0 \sin(\omega t + \varphi_0), \quad X_0 = Y_0 = R$$

$$\dot{W}_s = -a \int dV \sin^2 \phi = -a \int dV \sin^2 \theta m_0 [\dot{X} \sin \phi - \dot{Y} \cos \phi] = \omega a \int dV \sin^2 \theta m_0 [X \cos \phi + Y \sin \phi]$$

(71) $\vec{G} \times \dot{\vec{X}} = -\frac{\partial W}{\partial \dot{\vec{X}}} + D\dot{\vec{X}} + \vec{F}_{ST}$ van yeebue $J=I_1$ gaer $a>0$.

- это нормально? да, так $\dot{W} = \frac{d}{dt}(W' + W_s)$ от нормали W , а не только W' (без W_s)

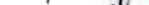
$$\dot{W} = \frac{d}{dt} \int dV [W' + W_s] = \int dV \left[\frac{\partial W'}{\partial \vec{m}} + \frac{\partial W_s}{\partial \vec{m}} \right] \cdot \dot{\vec{m}} = \int dV \left[\frac{\partial W}{\partial \vec{m}} \right] \cdot (\vec{m} \times \frac{\partial W}{\partial \vec{m}}) \frac{1}{M_s} = 0 \quad (13)$$

$$\dot{W} = (dV) \frac{\partial W}{\partial \vec{m}} \cdot \vec{m} = (dV) (M_s) \left[\vec{H}_{\text{eff}} - \frac{d}{\gamma} \dot{\vec{m}} + \frac{a}{M_s} (\dot{\vec{p}} + \dot{\vec{m}}) \right] \cdot \vec{m}$$

$$= - \left(\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{\mathbf{m}}} + \frac{\partial L}{\partial \dot{\mathbf{m}}} \right] \cdot \dot{\mathbf{m}} \right) = - (-M_c) \left(\frac{d}{dt} \left[\frac{a}{m} (\rho \times \dot{\mathbf{m}}) - \frac{1}{2} \dot{\mathbf{m}} \right] \cdot \dot{\mathbf{m}} \right)$$

$$W_s = -\vec{m} \cdot \vec{H}_s = -\vec{m} \cdot \frac{q}{M_s} (\vec{p} \times \vec{m}) \equiv 0, W_s \equiv 0, \text{ ST не даёт вклада}$$

$$E_{\text{сум}} = -\frac{M_S}{2} \int dV (\dot{m})^2 + a \int dV P_2 \sin^2 \theta \Phi$$
 , надо брать и именно W' , без damping и ST [Хвалюк'09, Ivan'07].
 $a = M_S \delta J$, то W' совн. с Хвалюк'09 и Ivan'PRL'07.


 $\uparrow p$, $\downarrow n$ - p-n junction.

$\frac{v_0}{0.5 \cdot 10^8 \text{ m/s}} \times \frac{300 \text{ m/s}}{0.5 \cdot 10^8 \cdot 2 \cdot 10^{-3} \text{ m}} \approx 300, \ln 300 = \ln 3 + \ln 10 \approx 1 + 2.4 \approx 3.4$
 $Z_0 (\text{when } \tau = 300 \text{ K}) \approx 2 \text{ nm} \cdot \omega_0 Z_0 = \omega_0 \sqrt{\frac{kT}{\alpha}} = \frac{\omega_0}{|G|} \sqrt{\frac{kT}{\alpha}} = \frac{\sqrt{\alpha k_B T}}{|G|}$
 $\ln \frac{v_0}{\omega_0 Z_0} = \ln \frac{v_0 |G|}{\sqrt{\alpha k_B T}} - \text{zero offset. } t_{sw}(J) \sim \frac{1}{J}, \text{ VC switching time}$

Laformulation u quacunas. programme que CPP sp current induced vortex.
 $\vec{G} \times \dot{\vec{X}} + \frac{\partial W}{\partial \dot{\vec{X}}} = D \dot{\vec{X}} + \vec{F}_{sr}, \quad \vec{L} = \frac{1}{2} (\vec{G} \times \dot{\vec{X}}) \cdot \dot{\vec{X}} - W(\vec{X}), \quad Q - \text{quacuna } \varphi$
 $\frac{d}{dt} \frac{\partial \vec{L}}{\partial \dot{\vec{X}}} - \frac{\partial W}{\partial \dot{\vec{X}}} = - \frac{\partial Q}{\partial \dot{\vec{X}}}, \quad \frac{\partial \vec{L}}{\partial \dot{\vec{X}}} = \frac{1}{2} \frac{\partial}{\partial \dot{\vec{X}}} (\vec{G} \times \dot{\vec{X}}) \cdot \dot{\vec{X}} = \frac{1}{2} (\vec{G} \times \dot{\vec{X}}) + \frac{1}{2} \dot{\vec{X}} (\vec{G} \times \dot{\vec{X}})$

$\frac{d}{dt} \frac{\partial \vec{L}}{\partial \dot{\vec{X}}} - \frac{\partial W}{\partial \dot{\vec{X}}} = \frac{d}{dt} \frac{\partial}{\partial \dot{\vec{X}}} \frac{1}{2} (\vec{G} \times \dot{\vec{X}}) \cdot \dot{\vec{X}} - \frac{\partial}{\partial \dot{\vec{X}}} \frac{1}{2} \dot{\vec{X}} \cdot (\dot{\vec{X}} \times \vec{G}) = \frac{d}{dt} \frac{1}{2} (\vec{G} \times \dot{\vec{X}}) - \frac{1}{2} (\dot{\vec{X}} \times \vec{G}) = \vec{G} \times \dot{\vec{X}}$
 $\frac{\partial Q}{\partial \dot{\vec{X}}} = D \dot{\vec{X}} + \vec{F}_{sr}(\vec{X}), \quad Q = - \frac{D}{2} (\dot{\vec{X}})^2 = \vec{F}_{sr} \cdot \dot{\vec{X}}, \quad D(\vec{X}), \vec{F}_{sr}(\vec{X}) - \text{moye } \vec{X} \text{ (non-lin.)}$
 $\vec{G} \times \dot{\vec{X}} + \frac{\partial W}{\partial \dot{\vec{X}}} = D(\vec{X}) \dot{\vec{X}} + \vec{F}_{sr}(\vec{X}), \quad \text{non-linear } W(\vec{X}), D(\vec{X}), \vec{F}_{sr}(\vec{X}) \text{ non-linear}$

He $\vec{G}(\vec{X})$ grad argumens. $\frac{d}{dt} \frac{\partial \vec{L}}{\partial \dot{\vec{X}}} - \frac{\partial W}{\partial \dot{\vec{X}}} = \frac{1}{2} (\vec{G} \times \dot{\vec{X}}) - \frac{1}{2} (\dot{\vec{X}} \times \vec{G}) - \frac{1}{2} \frac{\partial \vec{G}}{\partial \dot{\vec{X}}} (\dot{\vec{X}} \times \dot{\vec{X}})$
 $\vec{G} \times \dot{\vec{X}} - \frac{1}{2} \frac{\partial \vec{G}}{\partial \dot{\vec{X}}} (\dot{\vec{X}} \times \dot{\vec{X}}) + \frac{\partial W}{\partial \dot{\vec{X}}} = D(\vec{X}) \dot{\vec{X}} + \vec{F}_{sr}(\vec{X})$ - non-linear Thiele eq.
 net, yd Thiele nepravno, $\vec{G} = \vec{G}(\vec{X}), \vec{F}_{sr} = \pi a L (\dot{\vec{X}} \times \vec{X})$

On 15/11: $\vec{G}_2 \dot{\vec{Z}} - D(Z) \dot{\vec{Z}} = - \frac{W'}{|Z|} \vec{Z} + \pi a L \dot{\vec{Z}}, \text{ ye } \vec{G}_2 \neq \vec{G}(\vec{X}), \vec{F}_{sr} = \pi a L (\dot{\vec{X}} \times \vec{X})$
 $\frac{1}{|Z|} \frac{\partial W(\vec{X})}{\partial \vec{X}} = \frac{1}{|Z|} \frac{\partial}{\partial \vec{X}} \left[\frac{\alpha}{2} x^2 + \frac{\beta}{4} x^4 \right] = \frac{1}{x} [\alpha x + \beta x^3] = \alpha + \beta |Z|^2$
 $\vec{G}_2 \dot{\vec{Z}} - [D_0 + D_1 |Z|^2] \dot{\vec{Z}} = - (\alpha + \beta |Z|^2) \vec{Z} + \pi a L \dot{\vec{Z}}, \quad G_2 = 2\pi \mu_0 L p \sim p, \gamma > 0$
 $i p \dot{\vec{Z}} + [d_0 + d_1 |Z|^2] \dot{\vec{Z}} = - [\omega_0 + \omega_1 |Z|^2] \vec{Z} + \frac{\pi a L}{|G|} i \vec{Z}$ - De-field non-linear vortex.

Unge van B PRB-nonlin. vortex paper draft: $\vec{Z} = |Z| \exp(i\varphi) = R e^{i\varphi}$
 $\dot{\vec{Z}} = \dot{R} e^{i\varphi} + R i \dot{\varphi} e^{i\varphi}$

$i p [\dot{R} e^{i\varphi} + i R \dot{\varphi} e^{i\varphi}] [i p + d_0 + d_1 R^2] = - [\omega_0 + \omega_1 R^2] R e^{i\varphi} + \frac{\pi a L}{|G|} i R e^{i\varphi}$
 $[\frac{\dot{R}}{R} + i \dot{\varphi}] [i p + d_0 + d_1 R^2] = - \omega_0 - \omega_1 R^2 + i \frac{\pi a L}{|G|}$
 $i p \frac{\dot{R}}{R} + \frac{\dot{R}}{R} (d_0 + d_1 R^2) - \dot{\varphi} p + i \dot{\varphi} (d_0 + d_1 R^2) = - \omega_0 - \omega_1 R^2 + i \frac{\pi a L}{|G|}$

Re: $\left\{ \begin{aligned} (\dot{R}/R)(d_0 + d_1 R^2) - \dot{\varphi} p &= -\omega_0 - \omega_1 R^2 \\ p(\dot{R}/R) + \dot{\varphi} (d_0 + d_1 R^2) &= \pi a L / |G| \end{aligned} \right\}$ - non-lin eq. que CPP-sp current vortex dyn.
 Im: $\left\{ \begin{aligned} (\dot{R}/R)(d_0 + d_1 R^2) - \dot{\varphi} p &= -\omega_0 - \omega_1 R^2 \\ p(\dot{R}/R) + \dot{\varphi} (d_0 + d_1 R^2) &= \pi a L / |G| \end{aligned} \right\}$ - steady state $\dot{R} = 0, \varphi = \omega t + \varphi_0$
 $\dot{\varphi} = \omega = \text{const}$

Steady state: $\left\{ \begin{aligned} -\omega p &= -\omega_0 - \omega_1 R^2 \\ \omega (d_0 + d_1 R^2) &= \frac{\pi a L}{|G|} \end{aligned} \right\} \rightarrow p(\omega_0 + \omega_1 R^2)(d_0 + d_1 R^2) = \frac{\pi a L}{|G|}$
 $R \rightarrow 0, p \rightarrow +1 \text{ gear } (d(s) |G| \omega_0(s) = \pi a L) - \text{yep-e } \omega_2 \text{ APL-ST, } \text{ye } d(s) = d_0 + d_1 s^2, \omega_0(s) = \omega_0 + \omega_1 s^2$

300 ypravleniya Eon no nemy'zno 10/Ku (dopnez eq., no nebezpuu nymen),
 u peneno 13/XII.
 Ecu $d_1 = 0, \omega_1 = 0 (p = +1)$ $\left\{ \begin{aligned} d_0 \frac{\dot{R}}{R} - \dot{\varphi} &= -\omega_0 \\ \dot{R}/R + d_0 \dot{\varphi} &= \pi a L / |G| \end{aligned} \right\} \quad (1+d_0^2) \frac{\dot{R}}{R} = -\omega_0 d_0 + \frac{\pi a L}{|G|}$

200 gear $R(t) = R(0) \exp[-(d_0 \omega_0 - \pi a L / |G|) t] - \text{exp. por } R(t) = |Z(t)| \text{ kak 25/XII}$
 $R(t) = R(0) e^{\Gamma t}, \quad \Gamma = \frac{\pi a L}{|G|} - d_0 \omega_0$

Ecu non-lin. por $R(t)$: $\left\{ \begin{aligned} (\dot{R}/R) d(R) - \dot{\varphi} &= -\omega_0(R) \\ \dot{R}/R + \dot{\varphi} d(R) &= \pi a L / |G| \end{aligned} \right\} \quad (1+d^2) \frac{\dot{R}}{R} = -d(R) \omega_0(R) + \frac{\pi a L}{|G|}$
 Given $d^2(R) \ll 1$, non-linear $\frac{\dot{R}}{R} = -d(R) \omega_0(R) + \frac{\pi a L}{|G|}, \quad \frac{dR}{[\frac{\pi a L}{|G|} - d(R) \omega_0(R)] R} = dt$
 - yep se nekoneno peneno u nemy'zno Eon (wom. R(t)).

Non-lin $W_{ce}(\vec{X})$. $W_{ce}(\vec{X}) = -M_s H_0 \int dV \rho m_\varphi(\vec{r}, X)$, см. 17/Х
 $\vec{F}_e(\vec{X}) = -\frac{\partial}{\partial \vec{X}} W_{ce}(\vec{X}) = M_s H_0 \int dV \rho \frac{\partial m_\varphi}{\partial \vec{X}}$, $m_\varphi = \sin\theta \sin(\Phi - \varphi)$
 $\nabla_{\vec{X}} m_\varphi = \cos\theta \sin(\Phi - \varphi) \nabla_{\vec{X}} \theta + \sin\theta \cos(\Phi - \varphi) \nabla_{\vec{X}} \Phi$, $\Phi = \frac{c\pi}{2} + \varphi + \varepsilon$
 $\cos(\Phi - \varphi) = \cos(\frac{c\pi}{2} + \varepsilon) = -\varepsilon \sin \varepsilon = -\varepsilon [\varepsilon - \frac{1}{3!} \varepsilon^3]$, $\nabla_{\vec{X}} \Phi = \nabla_{\vec{X}} \varepsilon$
 $\frac{\partial \Phi}{\partial \vec{X}}$ с точностью до $X_d^2 \sim s^2$ почитаеме 12-13/Х.

$\varepsilon_{\text{см. с точн. до } X_d^2}$, то $m_\varphi = \sin\theta \sin(\Phi - \varphi) = \sin(\theta_0 + \delta\theta) \varepsilon [1 - \frac{1}{2} \varepsilon^2] =$
 $W_{ce}(\vec{X}) = \varepsilon [\sin\theta_0 + \cos\theta_0 \delta\theta - \frac{1}{2} \sin\theta_0 \delta\theta^2] [1 - \frac{1}{2} \varepsilon^2] = \varepsilon [\sin\theta_0 (1 - \frac{1}{2} \varepsilon^2) + \cos\theta_0 \delta\theta - \frac{1}{2} \sin\theta_0 \delta\theta^2]$
 $= \varepsilon \sin\theta_0 (1 - \frac{1}{2} \varepsilon^2 - \frac{1}{2} \delta\theta^2) + \varepsilon \cos\theta_0 \delta\theta$

б бб $\delta\theta$ гравитационный эффект linear теории по X_d надо учесть, меня в бб квадратично
 $\cos\theta = \cos\theta_0 + g(\vec{r}, \vec{p})$, $\theta(\vec{r}, \vec{p}) = \theta_0(\vec{p}) - \frac{g(\vec{r})}{\sin\theta_0} (\vec{r} \cdot \vec{p})$ б см. по 2-й формуле
 $\delta\theta = -\frac{1}{\sin\theta_0} g(\vec{r}) (\vec{r} \cdot \vec{p})$

$\int d^2 \vec{p} \int d\vec{r} \sin\theta_0 (\delta\theta)^2 = \int d\vec{r} \int d\vec{p} \sin\theta_0 \frac{g^2(\vec{r})}{\sin^2\theta_0} (\vec{r} \cdot \vec{p})^2 = \int d\vec{r} \int d\vec{p} (\vec{r} \cdot \vec{p})^2 = \int d\vec{r} \int d\vec{p} (x^2 p_y^2 + y^2 p_x^2 + \dots)$
 $= \pi s^2 \int d\vec{p} \frac{g^2(\vec{r})}{\sin\theta_0} = \left[\text{по симметрии } \int d\vec{p} \text{ по } s^2, \delta\theta = \text{line area} \right] = \pi s^2 \int d\vec{p} \frac{(c^2 + p^2)}{(2cp)} \left[\frac{4c^2 p^2 (1+p^2)}{(c^2 + p^2)^2} \right]$
 $= 8\pi s^2 c^3 \int d\vec{p} \frac{(1+p^2)^2}{(c^2 + p^2)^3} = 8\pi s^2 c^3 \frac{c^4}{c^6} \int d\vec{p} \frac{(1+p^2)^2}{(1+p^2/c^2)^3} = 4\pi s^2 c \int d(\frac{p^2}{c^2}) \frac{p^2}{c^2} \frac{(1+p^2/c^2)^2}{(1+p^2/c^2)^3} =$
 $= \left| p^2/c^2 = x \right| = 4\pi s^2 c \int_0^1 dx x \frac{(1+x^2)^2}{(1+x)^3} \approx 4\pi s^2 c \int_0^1 dx x \frac{1}{(1+x)^3} = 4\pi s^2 c \int_0^1 dx \left[\frac{1}{(1+x)^2} - \frac{1}{(1+x)^3} \right]$
 $= 4\pi s^2 c \left[-\frac{1}{(1+x)} + \frac{1}{2} \frac{1}{(1+x)^2} \right]_0^1 = 4\pi s^2 c \left[-\frac{1}{2} + \frac{1}{2} \frac{1}{4} + 1 - \frac{1}{2} \right] = \frac{\pi}{2} s^2 c$

Уточнение W_{ce} - Zeeman
 $W_{ce}(\vec{X}) = -M_s H_0 L \int d^2 \vec{p} \int d\vec{r} m_\varphi(\vec{r}, \vec{p}) = -M_s H_0 L \varepsilon \int d^2 \vec{p} \int d\vec{r} \sin\theta_0 [1 - \frac{1}{2} \varepsilon^2 - \frac{1}{2} \delta\theta^2] =$
 $= -M_s H_0 L \varepsilon \int d\vec{r} \int d\vec{p} \sin\theta_0 [1 - \frac{1}{2} \varepsilon^2 - \frac{1}{2} \delta\theta^2] = \text{Const}(s) + M_s H_0 L \varepsilon \frac{1}{2} \int d\vec{r} \int d\vec{p} \sin\theta_0 (\varepsilon^2 + \delta\theta^2)$
 $\int d\vec{r} \int d\vec{p} \sin\theta_0 \varepsilon^2$, $\varepsilon = m_0 [\sin\varphi X - \cos\varphi Y]$, $\int d\vec{r} \varepsilon^2 = m_0^2 \int d\vec{r} [x^2 \sin^2\varphi + y^2 \cos^2\varphi + \dots] =$
 $\int d\vec{r} \int d\vec{p} \sin\theta_0 \varepsilon^2 = \pi s^2 \int d\vec{p} \sin\theta_0 m_0^2 = \pi s^2 \left[\int_0^c + \int_c^1 \right] = \pi s^2 \left[\int_0^c d\vec{p} \frac{(2cp)}{(c^2 + p^2)^{3/2}} \frac{1}{(1-p^2/c^2)} + \int_c^1 \dots \right]$
 $+ \int_c^1 d\vec{p} (\dots)] = \pi s^2 \left[2c \int_0^c d\vec{p} \frac{p(1-p^2)^2}{(c^2 + p^2)^2} + \int_c^1 \dots \right] = \pi s^2 \left[c \int_0^c d(\frac{p^2}{c^2}) \frac{(1-p^2/c^2)^2}{(1+p^2/c^2)} + \int_c^1 \dots \right] =$
 $= \pi s^2 \left[c \int_0^1 dx \frac{(1-c^2 x)^2}{(1+x)} + \int_c^1 \dots \right] \approx \pi s^2 \left[c \ln 2 + \int_c^1 \dots \right] = \left| \int \text{такая же как 17/Х} \right| =$
 $= \pi s^2 \left[c \ln 2 + \frac{8}{15} - c \right] = \pi s^2 \left[\frac{8}{15} - c(1 - \ln 2) \right]$

$W_{ce}(\vec{X}) = \frac{1}{2} M_s H_0 L \varepsilon \int d\vec{r} \int d\vec{p} \sin\theta_0 (\varepsilon^2 + \delta\theta^2) = \frac{1}{2} M_s H_0 L \varepsilon \left[\frac{8}{15} - c(1 - \ln 2) + \frac{c}{2} \right] \pi s^2$
 $W_{ce}(s) = \frac{1}{2} \pi M_s H_0 L \varepsilon \left[\frac{8}{15} + c(\ln 2 - \frac{1}{2}) \right] \cdot s^2$, $\text{гдет } \sqrt{\text{area}} \text{ choose}$

$\kappa \frac{8}{15} \rightarrow \frac{8}{15} + c(\ln 2 - \frac{1}{2})$, $c = R_e/R \sim 0.1$. Это учтем. 17/Х где c - $\text{Zeeman } \varepsilon^2$
 $\text{гдет } \text{наблюдаю}$, а $\delta\theta^2$ бесконечное преобразование . $\text{Теперь } \varepsilon \cos\theta_0 \delta\theta$ преобразование ,
 $\text{т.к. он зависит от } X_d^2$ и своей величине , т.к. $\int d\vec{p} \cos\theta_0 (\delta\theta)^2$ максимум $p=0$ и т.д.
 $\vec{F}_e(\vec{X})$ с точн. до X_d^3 наименее выгодно , т.к. надо $\varphi(\vec{r}, \vec{X})$ с точн. до X_d^3

$\gamma_3 = \frac{g \mu_B \hbar \gamma}{\hbar 2 \mu_B L M_s} = \frac{g \mu_B \hbar \gamma}{2 \mu_B L M_s}$, $\gamma = \frac{\hbar \gamma}{2 \mu_B L M_s}$, $[\gamma] = \text{мксек}^{-1}$, H/m мксек^{-1}

2. Given β SI, $\tau \cdot K$. I_u β SI. $M_S(SI) = 800 \text{ kA/m}$, $\gamma \approx 0.2 \text{ (Peri)}$
 $\hbar = 1.05 \cdot 10^{-27} \text{ erg} \cdot \text{s} = 1.05 \cdot 10^{-27} \cdot 10^{-7} \text{ J} \cdot \text{s}$, $k_B = 1.60 \cdot 10^{-19} \text{ C}$.

$$\beta = \frac{\hbar \cdot \gamma}{|e| \cdot 2 M_S} = \frac{1.05 \cdot 10^{-34} \text{ J} \cdot \text{s}}{1.60 \cdot 10^{-19} \text{ A} \cdot \text{s}} \cdot \frac{0.2}{2 \cdot 5 \text{ nm} \cdot 800 \cdot 10^3 \frac{\text{A}}{\text{m}}} = \frac{1.05 \cdot 0.2 \cdot 10^{-34}}{1.6 \cdot 0.8 \cdot 10^{-19} \cdot 10^{-9} \cdot 10^6}$$

$$\frac{\text{J} \cdot \text{s}}{\text{A} \cdot \text{s}} \frac{1}{\text{A} \cdot \text{m}} = \frac{0.21 \cdot 10^{-13} \text{ J}}{1.6 \cdot 0.8} \frac{1}{\text{A}^2}; \quad \beta = \frac{0.21 \cdot 10^{-13} \text{ J}}{1.6 \cdot 0.8} \frac{1}{\text{A}^2}, \quad 1 \text{ J} = 1 \text{ A} \cdot \text{V} \cdot 1 \text{ s}$$

$$[\beta] = \frac{\text{J} \cdot \text{s}}{\text{A}^2} \frac{1}{\text{m}^2} = \frac{\text{J}}{\text{A} \cdot \text{m}^2} = \frac{\text{V} \cdot \text{s}}{\text{m}^2} = \frac{\text{kg}}{\text{m}^2} \frac{1}{\text{s}^2 \text{A}} = \frac{\text{kg}}{\text{s}^2 \text{A}}, \quad ? \text{ Moner } [M_S] = T$$

$$[\beta] = \frac{\text{J} \cdot \text{s}}{\text{A} \cdot \text{s}} \frac{1}{\text{m} \cdot (\frac{\text{A}}{\text{m}})} \frac{1}{\text{m}^2}, \quad 1 \text{ J} = \text{MHV} = ? \text{ Hago esse } \mu_0. \quad \text{J} = \frac{\text{A}}{\text{m}} \cdot T \cdot \mu_0$$

Buscando β SI hago $\beta = \frac{\hbar \gamma}{2 |e| L \mu_0 M_S}$, $[\beta] = \frac{\text{J}}{\text{A}} \frac{1}{\text{m} \cdot T} \frac{\text{A}}{\text{m}^2} = \frac{\text{J}}{T \cdot \text{m}^3} = \frac{\text{A} T \text{m}^3}{\text{m} T \text{m}^3} = \frac{\text{A}}{\text{m}}$

$W_2 = -\mu_0 M H$. β SI, $\beta = \mu_0 (H + M)$. $\mu_0 M_S = 0.8 \text{ T}$

$$\beta = \frac{0.21}{1.6 \cdot 0.8} \cdot 10^{-13} \frac{\text{J}}{\text{A}^2} \frac{1}{\mu_0} = \left| \mu_0 = 4\pi \cdot 10^{-7} \frac{\text{N}}{\text{A}^2} \right| = \frac{0.21}{1.6 \cdot 0.8} \cdot 10^{-13} \frac{\text{J}}{\text{A}^2} \frac{1}{4\pi \cdot 10^{-7} \frac{\text{N}}{\text{A}^2}} = \frac{0.21 \cdot 10^{-6}}{4\pi \cdot 1.6 \cdot 0.8} \frac{\text{m}}{\text{A}}$$

$$\beta = \frac{1.05 \cdot 10^{-34} \text{ J}}{1.6 \cdot 10^{-19} \text{ A}} \cdot \frac{0.2}{2 \cdot 5 \cdot 10^{-9} \text{ m} \cdot 0.8 \text{ T}} = \frac{0.21 \cdot 10^{-34}}{1.6 \cdot 0.8 \cdot 10^{-19} \cdot 10^{-9} \cdot 10^3 \text{ A} \cdot \text{T} \cdot \text{m}} = \frac{0.21}{1.6 \cdot 0.8} \cdot 10^{-7} \frac{\text{J}}{\text{A} \cdot \text{T} \cdot \text{m}} \cdot \frac{1}{4\pi}$$

$$\frac{\text{J}}{\text{A} \cdot \text{T} \cdot \text{m}} \frac{1}{\text{m}^2} = \frac{\text{J}}{\text{T} \cdot \text{m}^3} = \frac{(\frac{\text{A}}{\text{m}}) \cdot \text{T} \cdot \text{m}^3}{\text{T} \cdot \text{m}^3} = \frac{\text{A}}{\text{m}}, \quad \frac{\text{J}}{\text{A} \cdot \text{T} \cdot \text{m}} = \frac{(\frac{\text{A}}{\text{m}}) \text{T} \text{m}^3}{\text{A} \cdot \text{T} \cdot \text{m}} = \text{m}$$

$\beta = 1.64 \times 10^{-7} \frac{\text{m}}{\text{A}}$ $L = 5 \text{ nm}$, $\mu_0 M_S = 0.08 \text{ T}$, $\gamma = 0.2$ (spin-polarization).

$(\beta J)_{\text{moner}} = \left| J = 10^7 \frac{\text{A}}{\text{cm}^2} \right| = 1.64 \cdot 10^{-7} \text{ m} \cdot \frac{\text{A}}{\text{cm}^2} \cdot 10^7 = 1.64 \cdot 10^4 \frac{\text{A}}{\text{m}} = 16.4 \cdot 4\pi \text{ Oe} \approx 197 \text{ Oe}$

$1 \frac{\text{A}}{\text{m}} = \frac{4\pi}{1000} \text{ Oe}$. $(\beta = 1.64 \cdot 10^{-7} \frac{\text{m}}{\text{A}})$ $\frac{1}{2} \beta J = ?$ $[(\beta J)] = \frac{\text{A}}{\text{m}}$

$(\beta J)_{\text{SI}} = 1.64 \cdot 10^{-7} \cdot \text{J} \left(\frac{\text{A}}{\text{m}^2} \right) \text{m} = 1.64 \cdot 10^{-7} \text{ J} \cdot \frac{\text{A}}{\text{m}} = 1.64 \cdot 10^{-7} \text{ J} \cdot \frac{4\pi}{10^3} \text{ Oe}$

$\frac{\beta J}{2\pi} = (2\pi)^0 \cdot 2.95 \text{ MHz} \cdot 1.64 \cdot 10^{-7} \cdot \frac{4\pi}{10^3} \text{ Oe} \cdot \text{J} (\beta \text{ A/m}^2) = \text{total A/m}^2$
 $= 60.80 \cdot 10^{-10} \frac{\text{Oe}}{\text{J} (\beta \text{ A/m}^2)} \text{ MHz} \cdot \frac{1}{4\pi}$

$\frac{\beta}{2\pi} = 2.95 \frac{\text{MHz}}{\text{Oe}} = 2.95 \cdot \text{MHz} = 4\pi \cdot 2.95 \cdot 10^{-3} \frac{\text{MHz}}{(\text{A/m})}$

$\frac{1}{2\pi} \beta J = 4\pi \cdot 2.95 \cdot 10^{-3} \frac{\text{MHz}}{(\frac{\text{A}}{\text{m}})} \cdot \frac{(1000)}{4\pi} \frac{\text{A}}{\text{m}} = 60.80 \cdot 10^{-10} \frac{\text{MHz} \cdot \text{m}^2}{\text{A}} \cdot \frac{1}{4\pi}$

$[\omega_e] = \text{MHz} \cdot \frac{\text{nm}^2}{\text{mA}}$, $\omega_e J = \text{total J} \beta \text{ mA/nm}^2$. $\frac{\text{m}^2}{\text{A}} = \frac{10^{18} \text{ nm}^2}{10^3 \text{ mA}} = 10^{15} \frac{\text{nm}^2}{\text{mA}}$

$(\frac{\beta}{2\pi}) J = 60.80 \cdot 10^5 \text{ MHz} \cdot \frac{\text{nm}^2}{\text{mA}} \cdot \frac{1}{4\pi}$ $\omega_e = 5.27 \cdot 10^5 \text{ MHz} (\text{nm}^2/\text{mA})$ $\text{nm} R = 100 \text{ nm}$

$\omega_e \approx \beta J$, $\omega_e J = \text{total } \beta \text{ MHz}$.

Dep-e na J_{c2} : $\left[\frac{1}{2} \frac{\omega_e J_{c1}}{d_0} + (\omega_e J_{c1} + \omega_1 \lambda^2) x \right] x^{1/2} = v_c / \lambda R$, $x = \left(\frac{J_{c2}}{J_{c1}} - 1 \right)$

$J_{c1} = f(\frac{L}{R}, L, R)$, $\omega_e = f(L)$, $d_0 = f(R)$, $\omega_e = f(R)$, $\omega_1 = f(LR)$, $\lambda = f(\frac{L}{R}, LR)$

Moner memers L u R u M_S nm $C = +1/-1$. Memer R .
 $L = 5 \text{ nm} = \text{const}$

Moner gear: $J_{c1} = 1.30 \cdot 10^{-6} \frac{\text{mA}}{\text{nm}^2} = 1.30 \cdot 10^{-6} \frac{10^{-3} \text{ A}}{10^{-14} \text{ cm}^2} = 1.30 \cdot 10^5 \frac{\text{A}}{\text{cm}^2} (R = 100 \text{ nm})$

$\lambda = 0.34$, $L = 10 \text{ nm}$ u $R = 150 \text{ nm}$ $J_{c1} = 4.9 \cdot 10^6 \text{ J/cm}^2$.

$J_{c1} = 1.30 \cdot 10^{-6} \frac{\text{mA}}{\text{nm}^2} \cdot \pi (100 \text{ nm})^2 = 0.041 \text{ mA}$ Bece .

$(v_c/R) = 300 \frac{\text{m}}{\text{s}} \frac{1}{100 \text{ nm}} = 3 \cdot 10^2 \frac{\text{m}}{\text{s}} \frac{1}{10^{-7} \text{ m}} = 3 \cdot 10^9 \text{ Hz} = 3 \text{ GHz}$.

$v_c/R = 3000 \text{ MHz}$
 $\text{Moner } x = 16.0$, $J_{c2} = (1+x) J_{c1} = 17 \cdot J_{c1} = 22.1 \cdot 10^5 \text{ A/cm}^2 = 2.2 \cdot 10^6 \text{ A/cm}^2$

J_{c1} moner moner , moner Bece u za $\omega_e = \beta J$ Bece .

Moner βJ - value u Bece moner moner J_{c1} , J_{c2} , λ .

$S(y) = \lambda (y-1)^{1/2}$, $J/J_{c1} = y$. $S(y)$ moner 1 nm $y \approx 10$, Bece $J_{c2} = J_{c2}/J_{c1} > 10$.

Вопрос к вопросу reversal VC $S(J_{c2}) < 1$, and VC becomes negative
 go down v_c on steady state $u S(J_{c2}) < 1$, and VC becomes negative
 no dropping v_c $S = \lambda \sqrt{J_{c2}/J_{c1} - 1} = 1$, $J_{c2}/J_{c1} = 1 + \frac{1}{\lambda^2}$

(27/11) $\Delta\omega_c(J) = \omega_c J = \frac{2}{15} \gamma R J$, $[R] = \text{nm}$, $[J] = \text{A}/\text{nm}^2$, $1 \frac{\text{A}}{\text{m}} = \frac{4\pi}{10^3} \text{Oe}$
 $\gamma/2\pi = 2.95 \text{ MHz/Oe}$, $(RJ)_{\text{Oe}} = \frac{4\pi}{10^3} (RJ)_{\text{A/m}} = (RJ)_{\text{A/m}} = R(\text{nm}) \cdot J(\text{A}/\text{nm}^2) =$
 $= 10^{-9} R(\text{nm}) \cdot 10^{15} J(\text{mA}/\text{nm}^2) = 10^6 R(\text{nm}) J(\text{mA}/\text{nm}^2)$. $\omega_c = \frac{2}{15} \gamma R(\text{nm}) 10^6 J$
 if $\omega_c (8 \text{ MHz})$, $\gamma = 2.95 \frac{\text{MHz}}{\text{Oe}}$, $R(6 \text{ nm})$ $u J(6 \text{ mA}/\text{nm}^2)$.

Вопрос (26/11) $\mu_0 \times \frac{1}{4\pi}$. μ_0 $\mu_0 M_s = 4\pi \cdot 10^{-7} \frac{\text{H}}{\text{m}} \cdot 800 \cdot 10^3 \frac{\text{A}}{\text{m}} = 4\pi \cdot 10^{-7} \frac{\text{H}}{\text{m}} \cdot 8 \cdot 10^5 \frac{\text{A}}{\text{m}} = 4\pi \cdot 8 \cdot 10^{-2} \text{ T}$
 $\mu_0 M_s = 8 \cdot 10^{-2} \text{ T}$
 $\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}$. $\mu_0 M_s = 4\pi \cdot 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}} \cdot 8 \cdot 10^5 \frac{\text{A}}{\text{m}} = 4\pi \cdot 8 \cdot 10^{-2} \text{ T}$
 и $\mu_0 M_s = 8 \cdot 10^{-2} \text{ T}$.

В $\mu_0 M_s$ SI (т.е. $\mu_0 M_s$ в SI, A/m^2): $\mu_0 M_s = 0.131 \cdot 10^{-7} \text{ m}$
 $\frac{1}{2\pi} \gamma B = \frac{1}{2\pi} \omega_c = \frac{60.80 \text{ MHz} \cdot \text{nm}^2}{4\pi \cdot \text{mA}} \cdot 10^5 = 4.84 \cdot 10^5 \text{ MHz} \left(\frac{\text{nm}^2}{\text{mA}} \right)$

$I_{c1} = 0.53 \text{ mA}$ $u J_{c1} = 1.68 \cdot 10^{-5} \frac{\text{mA}}{\text{nm}^2} = 1.68 \cdot 10^6 \text{ A}/\text{cm}^2$, $\lambda = 0.34 \mu\text{m}$

то же $L = 5 \text{ nm}$, $\eta = 0.2$, $M_s = 800 \text{ G}$, $R = 100 \text{ nm}$.

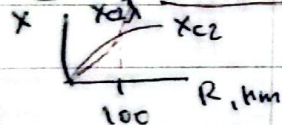
Кривая $S(J_{c2}) = 1$ $\text{где } x_{c2} = \frac{1}{\lambda^2}$, $x = \frac{J_{c2}}{J_{c1}} - 1$. Две $R = 50 - 200 \text{ nm}$

$\rightarrow x_{c2}(R) = \frac{1}{\lambda^2(R)} < x_{c2}(\text{из уравнения } v(R) = v_c)$, т.е. $v(R)$ J_{c2} steady state VC radius $S(J_{c2}) = 1$, VC becomes negative $u J_{c2}$.

if $L = 5 \text{ nm}$, $C = +1$.

if $C = -1$: $J_{c1} = 1.58 \cdot 10^6 \text{ A}/\text{cm}^2$, $\lambda = 0.335$, т.е. Oe-field
 $\omega_c J_{c1} \approx 10 \text{ MHz}$, т.е. $I_{c1} \approx 0.5 \text{ mA}$
 $\omega_c (100 \text{ nm}) = 265 \text{ MHz}$. $x_{c2} = \frac{1}{\lambda^2(R)} < x_{c2}(\text{из кривой } v(R) = v_c)$

мы $L = 10 \text{ nm}$: $J_{c1} = 6.95 \cdot 10^6 \text{ A}/\text{cm}^2$, ω_c $\text{возрастают в 2 раза}$. $\lambda = 0.34 \mu\text{m}$
 $I_{c1} = 2.12 \text{ mA}$



то $x_{c2}(u, v_c) < x_{c2}(\lambda)$ u
 VC-reversal смысла good reversal .

Note: $L = 10 \text{ nm}$, $R = 150 \text{ nm}$ u $J_{c1} = 5.52 \cdot 10^6 \text{ A}/\text{cm}^2$
 $J_{c1} = 4.9 \cdot 10^6 \text{ A}/\text{cm}^2$ - учетом Хванк PAB'09 .

$x_{c2}(150 \text{ nm}) = 11.8 \rightarrow J_{c2} = (x_{c2} + 1) J_{c1} = 7.1 \cdot 10^7 \text{ A}/\text{cm}^2$ u $J_{c2} = 9 \cdot 10^6 \text{ A}/\text{cm}^2$
 reduced polar. - no смысла $6.5 \cdot 10^6 \text{ A}/\text{cm}^2$

Усл'ФМН'94: $W(b) = R_0^2 \ln \frac{1}{b} + W_m(b)$, $C_m, 3/\sqrt{x}$.

$W_m(b) = \frac{4\pi}{\beta} b^3 [u(0) - u(\beta/b)]$, смысла $\beta/b \ll 1$, $u(0) - u(\beta/b) = g, \frac{\beta}{b}, g = \frac{1}{b}$

$W(b) = R_0^2 \ln \frac{1}{b} + \frac{4\pi}{\beta} b^3 g, \frac{\beta}{b} = R_0^2 \ln \frac{1}{b} + 4\pi g, \frac{\partial W}{\partial b} = R_0^2 \frac{(-1)}{b} + 8\pi g, b = 0$

$b_c^2 = \frac{R_0^2}{8\pi g}$, $b_c = \frac{1}{\sqrt{8\pi g}} R_0$, $R_c = \frac{1}{\sqrt{8\pi g}} L_e$. $R_c = 0.59 L_e$

Замечание $R_c = L_e$ u $0.59 L_e$ глубина damping u глубина λ . $\lambda = 0.34 \mu\text{m}$
 $L_e = \left(\frac{2.13 \cdot 10^{-6}}{800^2} \right)^{1/2} = 20 \text{ nm}$. λ ортогонально зависит от L , $\lambda(R) < R$

$\lambda(R)$ называется c R большее u за раса $d_1(R) < R$.

if $R < R_c(L) \approx 100 \text{ nm}$, $x_{c2} > x_{c2}$ u J_{c2} VC becomes negative $u J_{c2}$.

if $R > R_c(L)$ - VC becomes reversal $u J_{c2} > J_{c2}$.

Meeting @ Arantxa: General Figs K Topol. field paper
 in EPS format. oögygum APL-ST paper. No-submitted Topol. f. PRB.
 Hancame APL-ST paper. Hancame JAPDW na pamyaguro,
 23/XII Borei. Oshaberna raxca APL-ST paper. Comacurac to review
 grant proposal by Rustolo (Hong-Kong) "EL switch, V. cores" (HK\$1000.)
 30/XII Dopaboma ST-APL paper in Hancame & APL (no person. References Ivanov
 Submitted ST-APL preprint to ArXiv - database. Lett. to Arantxa-Julian.
 31/XII Updated info-Crus na Iberbasque. Intranet site. Bxwoma INTERMAG'09
 ap-digests kax papers in Proceedings, nanyra 6 articles & 2009 & James 9th J. J. J.
 6 Iberbasque productivity list - 2009.

Vortex dyn. F/N/F pillar

20102. Hancame paper draft "ST torque induced Vortex dyn.
 & Py/Cu/Py nanopillars" gure O. Klein, G. de laubens, V. Naletoy u gr.
 y A. Fert's group. C.N.R., A. & kape "laconcha" u downtown, "Lide".
 31 Paper draft "Vortex dyn. in F/N/F". Comacurac 22/XI ST forces F/N/F:

$$F_{ST}^j(j) = - \int dV \frac{\partial w_j}{\partial \vec{m}_j} \cdot \frac{\partial \vec{m}_j}{\partial x_{j\alpha}}, \quad \frac{\partial w_j}{\partial \vec{m}_j} = a_j (\vec{m}_1 \times \vec{m}_2), \quad \vec{m}_j = \vec{m}_j^0 + \frac{\partial \vec{m}_j}{\partial x_{j\beta}} x_{j\beta}, \quad j=1,2$$

$$\vec{m}_1 \times \vec{m}_2 = \vec{m}_1^0 \times \vec{m}_2^0 + \frac{\partial \vec{m}_1}{\partial x_{1\beta}} x_{1\beta} \times \vec{m}_2^0 + x_{2\beta} \vec{m}_1^0 \times \frac{\partial \vec{m}_2}{\partial x_{2\beta}} + \dots$$

$$F_{ST}^j(j) = - \int dV a_j \frac{\partial \vec{m}_j}{\partial x_{j\alpha}} \cdot \left[x_{1\beta} \frac{\partial \vec{m}_1}{\partial x_{1\beta}} \times \vec{m}_2^0 + x_{2\beta} \vec{m}_1^0 \times \frac{\partial \vec{m}_2}{\partial x_{2\beta}} \right] = - a_j \int dV \left[x_{1\beta} \vec{m}_2^0 \cdot \left(\frac{\partial \vec{m}_1}{\partial x_{1\beta}} \times \frac{\partial \vec{m}_j}{\partial x_{j\alpha}} \right) \right. \\ \left. + x_{2\beta} \vec{m}_1^0 \cdot \left(\frac{\partial \vec{m}_2}{\partial x_{2\beta}} \times \frac{\partial \vec{m}_j}{\partial x_{j\alpha}} \right) \right] = - a_j \int dV \left[x_{1\beta} \gamma_{2\beta}^{j1} - x_{2\beta} \gamma_{1\beta}^{j2} \right] = F_{ST}^j(bj-cnoe)$$

$$\gamma_{j\alpha\beta}^{kl} = \int d^2\vec{r} \vec{m}_j^0 \cdot \left(\frac{\partial \vec{m}_k}{\partial x_{k\alpha}} \times \frac{\partial \vec{m}_l}{\partial x_{l\beta}} \right), \quad k=1,2, \quad l=1,2, \quad j=1,2, \quad \text{no } l=j \text{ terms}$$

$$k \neq 0 \text{ terms } (\gamma_{2\alpha}^{j1}, \gamma_{1\alpha}^{j2}) = \gamma_{2\alpha}^{j1}, \gamma_{2\alpha}^{j2}, \gamma_{1\alpha}^{j1}, \gamma_{1\alpha}^{j2} - 4 \text{ -pe interlayer.}$$

$$(\hat{z} \times \vec{X}_j) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ x_j & y_j & 0 \end{vmatrix} = \hat{y} x_j - \hat{x} y_j = -\hat{x} x_{jy} + \hat{y} x_{jx}, \quad (\hat{z} \times \vec{X}_j)_x = -x_{jy}, \quad (\hat{z} \times \vec{X}_j)_y = x_{jx}$$

$$\gamma_{1\alpha}^{j2} = \gamma_{2\alpha}^{j1} - \text{analogous interlayer coupling } \gamma_{\alpha} \text{ over ST-terms}$$

$$\vec{m}_j = (\sin\theta_j \cos\phi_j, \sin\theta_j \sin\phi_j, \cos\theta_j) = \hat{x} m_{jx} + \hat{y} m_{jy} + \hat{z} m_{jz}$$

$$\frac{\partial \vec{m}_j}{\partial x_{j\alpha}} = \hat{x} [\cos\theta_j \theta'_{j\alpha} \cos\phi_j - \sin\theta_j \sin\phi_j \cdot \phi'_{j\alpha}] + \hat{y} [\cos\theta_j \sin\phi_j \theta'_{j\alpha} + \sin\theta_j \cos\phi_j \cdot \phi'_{j\alpha}] + \hat{z} [-\sin\theta_j \theta'_{j\alpha}]$$

$$\frac{\partial \vec{m}_k}{\partial x_{k\alpha}} \times \frac{\partial \vec{m}_l}{\partial x_{l\beta}} = ? \quad \text{Ecu } k=l, \text{ no index } k \text{ nonzero any course (but } \gamma \text{ starts with } \vec{X}_j)$$

$$\frac{\partial \vec{m}_k}{\partial x_{k\alpha}} \times \frac{\partial \vec{m}_l}{\partial x_{l\beta}} = \hat{x} [(\cos\theta \sin\phi \theta'_\alpha + \sin\theta \cos\phi \phi'_\alpha)(-\sin\theta \theta'_\beta) - (-\sin\theta \theta'_\alpha)(\cos\theta \sin\phi \phi'_\beta + \sin\theta \cos\phi \theta'_\beta)] + \hat{y} [(-\sin\theta \theta'_\alpha)(\cos\theta \cos\phi \theta'_\beta - \sin\theta \sin\phi \phi'_\beta) - (\cos\theta \cos\phi \theta'_\alpha - \sin\theta \sin\phi \phi'_\alpha)(-\sin\theta \theta'_\beta)] + \hat{z} [(\cos\theta \cos\phi \theta'_\alpha - \sin\theta \sin\phi \phi'_\alpha)(\cos\theta \sin\phi \theta'_\beta + \sin\theta \cos\phi \phi'_\beta) - (\cos\theta \sin\phi \theta'_\alpha + \sin\theta \cos\phi \phi'_\alpha)(\cos\theta \cos\phi \theta'_\beta - \sin\theta \sin\phi \phi'_\beta)] = \hat{x} [\sin^3\theta \cos\phi (\theta'_\alpha \phi'_\beta - \theta'_\beta \phi'_\alpha)] + \hat{y} [\sin^3\theta \sin\phi (\theta'_\alpha \phi'_\beta - \theta'_\beta \phi'_\alpha)] + \hat{z} [+ \sin\theta \cos\theta \sin^2\phi (\theta'_\alpha \phi'_\beta - \theta'_\beta \phi'_\alpha) + \sin\theta \cos\theta \cos^2\phi (\theta'_\alpha \phi'_\beta - \theta'_\beta \phi'_\alpha)]$$

$$\frac{\partial \vec{m}_k}{\partial x_{k\alpha}} \times \frac{\partial \vec{m}_l}{\partial x_{l\beta}} = \sin\theta [\hat{x} \sin\theta \cos\phi + \hat{y} \sin\theta \sin\phi + \hat{z} \cos\theta] (\theta'_\alpha \phi'_\beta - \theta'_\beta \phi'_\alpha)$$

$$\frac{\partial \vec{m}_k}{\partial x_{k\alpha}} \times \frac{\partial \vec{m}_l}{\partial x_{l\beta}} = \vec{m} \sin\theta (\theta'_\alpha \phi'_\beta - \theta'_\beta \phi'_\alpha) - \text{Kraubo. } \text{Eto ecro b Tataru JP5J'08 (25/III'09.)}$$

$$\gamma_{j\alpha\beta}^{kl} = \int d^2\vec{r} \vec{m}_j^0 \cdot \left(\frac{\partial \vec{m}_k}{\partial x_{k\alpha}} \times \frac{\partial \vec{m}_l}{\partial x_{l\beta}} \right) = \int d^2\vec{r} \vec{m}_j^0 \cdot \vec{m}_l^0 \sin\theta_l (\theta'_{l\alpha} \phi'_{l\beta} - \theta'_{l\beta} \phi'_{l\alpha}), \quad j \neq l$$

B. un. n. m. d. $\theta(\vec{r}, \vec{X}) = -\frac{g(\rho)}{\sin\theta_0} (\vec{X} \cdot \hat{\rho}) + O_0(\rho)$, $\Phi(\vec{r}, \vec{X}) = \rho_0 + m_0 [\vec{X} \cdot \hat{\rho}]$
 $\theta'_\alpha = \frac{\partial \theta}{\partial X_\alpha} = -\frac{g}{\sin\theta_0} \hat{\rho}_\alpha$, $\frac{\partial \Phi}{\partial X} = m_0 \sin\varphi$, $\frac{\partial \Phi}{\partial Y} = -m_0 \cos\varphi$
 $\frac{\partial \Phi}{\partial Z} = m_0 (\hat{x} \sin\varphi - \hat{y} \cos\varphi) = -m_0 \hat{\varphi}$ No. year month $\hat{\rho} = (\cos\varphi, \sin\varphi, 0)$
 $\hat{\varphi} = (-\sin\varphi, \cos\varphi, 0)$

B. $F_{sr}^d(j)$ boquer $X_{lp} Y_{j\alpha\beta}^{ll}$. Due l gannoro $X_{lp} Y_{j\alpha\beta}^{ll}$
 $\vec{r} = \hat{z} \times \hat{\rho}$, $\frac{\partial \Phi}{\partial X_\alpha} = -m_0 (\hat{z} \times \hat{\rho})_\alpha$ // $\theta'_\alpha \Phi'_\beta - \theta'_\beta \Phi'_\alpha = -\frac{g}{\sin\theta_0} \hat{\rho}_\alpha (-m_0) (\hat{z} \times \hat{\rho})_\beta - (-m_0) (\hat{z} \times \hat{\rho})_\alpha \hat{\rho}_\beta$
 $= (-1) \frac{g}{\sin\theta_0} \hat{\rho}_\beta (-m_0) (\hat{z} \times \hat{\rho})_\alpha = \frac{g m_0}{\sin\theta_0} [\hat{\rho}_\alpha (\hat{z} \times \hat{\rho})_\beta - \hat{\rho}_\beta (\hat{z} \times \hat{\rho})_\alpha]$
 $Y_{j\alpha\beta}^{ll} = \int d\varphi (\vec{m}_j^0 \cdot \vec{m}_\ell^0) \sin\theta_\ell^0 \frac{g m_0}{\sin\theta_\ell^0} [\hat{\rho}_\alpha (\hat{z} \times \hat{\rho})_\beta - \hat{\rho}_\beta (\hat{z} \times \hat{\rho})_\alpha]$
 $X_{lp} [\hat{\rho}_\alpha (\hat{z} \times \hat{\rho})_\beta - \hat{\rho}_\beta (\hat{z} \times \hat{\rho})_\alpha] = \hat{\rho}_\alpha [\vec{X} \cdot (\hat{z} \times \hat{\rho})] - (\vec{X} \cdot \hat{\rho}) (\hat{z} \times \hat{\rho})_\alpha = \hat{\rho}_\alpha (\vec{X} \cdot \hat{\varphi})$
 $X_{lp} Y_{j\alpha\beta}^{ll} = \int d\varphi (\vec{m}_j^0 \cdot \vec{m}_\ell^0) g m_0 [\hat{\rho}_\alpha (\vec{X}_\ell \cdot \hat{\varphi}) - \hat{\rho}_\beta (\vec{X}_\ell \cdot \hat{\rho})]$, $\hat{\rho} = (\cos\varphi, \sin\varphi, 0)$
 $(\vec{X}_\ell \cdot \hat{\varphi}) = (X_\ell, Y_\ell) (-\sin\varphi, \cos\varphi) = -X_\ell \sin\varphi + Y_\ell \cos\varphi$, $\hat{\varphi} = (-\sin\varphi, \cos\varphi, 0)$
 $(\vec{m}_j^0 \cdot \vec{m}_\ell^0)$ esbo qunuma zero no os g.

$F_{sr}^d = \int d\varphi [\hat{\rho}_\alpha (\vec{X}_\ell \cdot \hat{\varphi}) - \hat{\rho}_\beta (\vec{X}_\ell \cdot \hat{\rho})] = \int d\varphi [(\cos\varphi, \sin\varphi) (-X_\ell \sin\varphi + Y_\ell \cos\varphi) - (-\sin\varphi, \cos\varphi) (X_\ell \cos\varphi + Y_\ell \sin\varphi)] = 2\pi (Y_\ell, -X_\ell) = -2\pi (\hat{z} \times \vec{X}_\ell)_\alpha$

$F_{sr}^d(1) = -a_1 L_1 [X_{1p} Y_{2\alpha\beta}^{11} - X_{2p} Y_{1\alpha\beta}^{12}]$, $F_{sr}^d(2) = -a_2 L_2 [X_{1p} Y_{2\alpha\beta}^{21} - X_{2p} Y_{1\alpha\beta}^{22}]$
 $\rightarrow X_{lp} Y_{j\alpha\beta}^{ll} = -2\pi (\hat{z} \times \vec{X}_\ell)_\alpha \int_0^{\pi/2} d\varphi (\vec{m}_j^0 \cdot \vec{m}_\ell^0) g_\ell(\rho) m d\rho$ - unipolar no core of l. bo

Chou then b $F_{sr}^d(1) = -a_1 L_1 X_{1p} Y_{2\alpha\beta}^{11} = 2\pi a_1 L_1 (\hat{z} \times \vec{X}_1)_\alpha \int_0^{\pi/2} d\varphi (\vec{m}_1^0 \cdot \vec{m}_2^0) g_1 m$

Chou then b $F_{sr}^d(2) = +a_2 L_2 X_{2p} Y_{1\alpha\beta}^{22} = -2\pi a_2 L_2 (\hat{z} \times \vec{X}_2)_\alpha \int_0^{\pi/2} d\varphi (\vec{m}_1^0 \cdot \vec{m}_2^0) g_2 m$

$J_p(\ell) = \int_0^{\pi/2} d\varphi (\vec{m}_1^0 \cdot \vec{m}_2^0) g_\ell m_0$. Due $\vec{m}_1 = \vec{m}$, $\vec{m}_2 = \hat{z}$, $(\vec{m}_1^0 \cdot \vec{m}_2^0) = m^0$

$J_p = \int d\varphi m^0 g m_0$ boquer k $J_p(2/\sqrt{10})$ que uniform polar

$(\vec{m}_1^0 \cdot \vec{m}_2^0) = \cos(\vec{m}_1^0 \cdot \vec{m}_2^0) = \cos\theta_1^0 \cos\theta_2^0 + \sin\theta_1^0 \sin\theta_2^0 \cos(\varphi_1^0 - \varphi_2^0) = |\vec{p}_1^0 \cdot \vec{p}_2^0|$
 $= \cos\theta_1^0 \cos\theta_2^0 + \sin\theta_1^0 \sin\theta_2^0 \cos(\varphi_1 - \varphi_2) \frac{\pi}{2} = \cos\theta_1^0 \cos\theta_2^0 + \ell_1 \ell_2 \sin\theta_1^0 \sin\theta_2^0$

- boquer. abnro zabucum. os $\ell_1 \ell_2$ u os $\rho_1 \rho_2$ ($\cos\theta_1^0 \sim \rho_j$), $g_j \sim \rho_j$

$J_p(1)$ boquer k J_{p1} qunuma pacura $2/\sqrt{10}$ ozi.

Chou then b $\vec{F}_{sr}(j) = (-1)^{j+1} 2\pi a_j L_j J_p(j) (\hat{z} \times \vec{X}_j)$, $j=1,2$ - pacura qunuma

$\rightarrow$ Esco b J_{kl}^{ll} $k \neq l$ (u boquer $l \neq j$), unipolar $k=j$.

$J_{kl}^{ll} = \int d\varphi \vec{m}_k^0 \cdot \left(\frac{\partial \vec{m}_k}{\partial X_{kl}} \times \frac{\partial \vec{m}_\ell}{\partial X_{lp}} \right)$ u nego kation $\frac{\partial \vec{m}_1}{\partial X_{1\alpha}} \times \frac{\partial \vec{m}_2}{\partial X_{2\beta}}$

$\vec{m} \times \frac{\partial \vec{m}}{\partial X_\alpha} = 2 \left[\sin\theta \sin\varphi (-\sin\theta'_\alpha) - \cos\theta (\cos\theta \sin\varphi \theta'_\alpha + \sin\theta \cos\varphi \varphi'_\alpha) \right] +$
 $+ \hat{y} [\cos\theta (\cos\theta \cos\varphi \theta'_\alpha - \sin\theta \sin\varphi \varphi'_\alpha) - (-\sin\theta'_\alpha) \sin\theta \cos\varphi] +$
 $+ \hat{z} [\sin\theta \cos\varphi (\cos\theta \sin\varphi \theta'_\alpha + \sin\theta \cos\varphi \varphi'_\alpha) - \sin\theta \sin\varphi (\cos\theta \cos\varphi \theta'_\alpha - \sin\theta \sin\varphi \varphi'_\alpha)]$

$= \hat{x} [-\sin\varphi \theta'_\alpha - \cos\theta \sin\theta \cos\varphi \varphi'_\alpha] + \hat{y} [\cos\varphi \theta'_\alpha - \sin\theta \cos\theta \sin\varphi \varphi'_\alpha] + \hat{z} [\sin\theta \cos\varphi \theta'_\alpha - \cos\theta \sin\theta \sin\varphi \varphi'_\alpha]$

$\frac{\partial \vec{m}_\ell}{\partial X_{lp}} \cdot (\vec{m}_k \times \frac{\partial \vec{m}_k}{\partial X_{kl}}) = (\cos\theta_\ell \cos\varphi_\ell \theta'_{\ell\beta} - \sin\theta_\ell \sin\varphi_\ell \varphi'_{\ell\beta}) (-\sin\varphi_k \theta'_{k\alpha} - \cos\varphi_k \sin\theta_k \varphi'_{k\alpha})$

$$+ (\cos\theta_e \sin\theta_e \theta'_{ep} + \sin\theta_e \cos\theta_e \Phi'_{ep}) (\cos\theta_k \theta'_{kd} - \sin\theta_k \cos\theta_k \sin\theta_k \Phi'_{kd}) +$$

$$+ (-\sin\theta_e \theta'_{ep}) \sin\theta_k \Phi'_{kd} = \theta'_{ep} \theta'_{kd} \cos\theta_e \sin(\theta_e - \theta_k) + \Phi'_{kd} \theta'_{ep} \sin\theta_k (-1) +$$

$$+ [\cos\theta_k \cos\theta_e \cos(\theta_e - \theta_k) + \sin\theta_e \sin\theta_k] + \Phi'_{ep} \theta'_{kd} \sin\theta_e \cos(\theta_e - \theta_k) +$$

$$+ \Phi'_{ep} \Phi'_{kd} \sin\theta_e \sin\theta_k \cos\theta_k \sin(\theta_e - \theta_k), \quad \sin(\theta_e - \theta_k) = \frac{1}{2} \theta_k (C_1 - C_2) \frac{1}{2} \theta_e$$

$$\frac{\partial \vec{m}_e}{\partial x_{ep}} \cdot (\vec{m}_k \times \frac{\partial \vec{m}_k}{\partial x_{kd}}) = \Phi'_{kd} \theta'_{ep} \sin\theta_k [\sin\theta_e \sin\theta_k + C_1 C_2 \cos\theta_e \cos\theta_k] +$$

$$+ \Phi'_{ep} \theta'_{kd} C_1 C_2 \sin\theta_e // \quad \gamma_{kdp}^{kl} = \int d^2 \vec{s} \frac{\partial \vec{m}_e}{\partial x_{ep}} \cdot (\vec{m}_k \times \frac{\partial \vec{m}_k}{\partial x_{kd}}) =$$

$$= \int d^2 \vec{s} [-\Phi'_{kd} \theta'_{ep} \sin\theta_k (\sin\theta_e \sin\theta_k + C_1 C_2 \cos\theta_e \cos\theta_k) + \Phi'_{ep} \theta'_{kd} C_1 C_2 \sin\theta_e]$$

- but no, no $\gamma_{kdp}^{kl} \sim \theta_k^2$, i.e. diverges. remove vertex cores, i.e. $\frac{\partial \vec{m}_k}{\partial x_{kd}} \neq 0$.

$$\gamma_{kdp}^{kl} = C_1 C_2 \int d^2 \vec{s} [\theta'_{kd} \Phi'_{ep} \sin\theta_e - \Phi'_{kd} \theta'_{ep} \sin\theta_k (\cos\theta_e \cos\theta_k + C_1 C_2 \sin\theta_e \sin\theta_k)]$$

$$X_{ep} \cdot \gamma_{kdp}^{kl} = C_1 C_2 \int d^2 \vec{s} [(X_{ep} \Phi'_{ep}) \theta'_{kd} \sin\theta_e - (X_{ep} \theta'_{ep}) \Phi'_{kd} \sin\theta_k \cos(\theta_e - C_1 C_2 \theta_k)]$$

$$(X_{ep} \Phi'_{ep}) = \vec{X}_e \cdot \frac{\partial \Phi_e}{\partial \vec{x}_{ep}} = \vec{X}_e \cdot (-m_0) (\hat{z} \times \hat{s}), \quad (X_{ep} \theta'_{ep}) = \vec{X}_e \cdot \frac{\partial \theta_e}{\partial \vec{x}_{ep}} = \vec{X}_e \cdot (-\frac{g_e}{\sin\theta_e^2}) \hat{s}$$

$$X_{ep} \cdot \gamma_{kdp}^{kl} = C_1 C_2 \int d^2 \vec{s} [-m_0 (\vec{X}_e \cdot \hat{s}) \theta'_{kd} \sin\theta_e + \frac{g_e}{\sin\theta_e^2} (\vec{X}_e \cdot \hat{s}) \Phi'_{kd} \sin\theta_k \cos(\theta_e - C_1 C_2 \theta_k)]$$

$$\theta'_{kd} = \frac{\partial \theta_k}{\partial x_{kd}} = -\frac{g_k}{\sin\theta_k^2} \hat{s}_\perp, \quad \Phi'_{kd} = -m_0 (\hat{s} \cdot \hat{x})$$

$$\int d\varphi (\vec{X}_e \cdot \hat{s}) \cdot \hat{s}_\perp = \int d\varphi (-X_e \sin\varphi + Y_e \cos\varphi) (\cos\varphi, \sin\varphi) = \pi (Y_e, -X_e)$$

$$\int d\varphi (\vec{X}_e \cdot \hat{s}) \cdot \hat{s}_\perp = \int d\varphi (X_e \cos\varphi + Y_e \sin\varphi) (-\sin\varphi, \cos\varphi) = \pi (-Y_e, X_e)$$

by using then $\vec{F}_{ST}^d(1) = (a_j L_j)_{j=1} X_{2p} \gamma_{1dp}^{12} = a_1 L_1 C_1 C_2 \int d^2 \vec{s} [-m_0 (\vec{X}_2 \cdot \hat{s}) \theta'_{1d} \sin\theta_2 +$
 $+ \frac{g_2}{\sin\theta_2^2} (\vec{X}_2 \cdot \hat{s}) \Phi'_{1d} \sin\theta_1 \cos(\theta_2 - C_1 C_2 \theta_1)] = a_1 L_1 C_1 C_2 \int d^2 \vec{s} [+m_0 \sin\theta_2 \frac{g_1}{2 \sin\theta_1} (Y_2 - X_2) \pi +$
 $+ \frac{g_2}{\sin\theta_2^2} \sin\theta_1 \cos(\theta_2 - C_1 C_2 \theta_1) (-m_0) \pi (-Y_e, X_e)_{e=2}]$

by using then $\vec{F}_{ST}(1) = a_1 L_1 C_1 C_2 \pi \int d^2 \vec{s} m_0 [-g_1 \frac{\sin\theta_2}{\sin\theta_1 C_2} - g_2 \frac{\sin\theta_1}{\sin\theta_2} \cos(\theta_2 - C_1 C_2 \theta_1)] (\hat{z} \times \vec{X}_2)$

by using then $\vec{F}_{ST}(1) = -\pi a_1 L_1 C_1 C_2 [\int_0^1 d\varphi m_0 g_1 \frac{\sin\theta_2^2}{\sin\theta_1^2} + \int_0^1 d\varphi m_0 \frac{\sin\theta_1^2}{\sin\theta_2^2} \cos(\theta_2^2 - C_1 C_2 \theta_1^2)] (\hat{z} \times \vec{X}_2)$

$\vec{F}_{ST}(1)$ contains (краткая информация) $\in \gamma_{j2}$ as 8×8 matrix.

by using then $\vec{F}_{ST}^d(2) = -a_2 L_2 X_{1p} \gamma_{2dp}^{21} = -a_2 L_2 C_1 C_2 \int d^2 \vec{s} [-m_0 (\vec{X}_1 \cdot \hat{s}) \sin\theta_1 (-\frac{g_2}{\sin\theta_2^2}) \hat{s}_\perp +$
 $+ g_1 \frac{1}{\sin\theta_1} (\vec{X}_1 \cdot \hat{s}) (-m_0) (\hat{s}_\perp \sin\theta_2 \cos(\theta_1 - C_1 C_2 \theta_2)] = -a_2 L_2 C_1 C_2 \int d^2 \vec{s} [m_0 g_2 \frac{\sin\theta_1}{\sin\theta_2} \pi (Y_1 - X_1) +$
 $+ g_1 \frac{\sin\theta_2}{\sin\theta_1} (-m_0) \cos(\theta_1 - C_1 C_2 \theta_2) \pi (-Y_1, X_1)], \quad (\hat{z} \times \vec{X}) = (-Y, X)$

$\vec{F}_{ST}(2) = a_2 L_2 \pi C_1 C_2 [\int_0^1 d\varphi m_0 g_2 \frac{\sin\theta_1^2}{\sin\theta_2^2} + \int_0^1 d\varphi m_0 g_1 \frac{\sin\theta_2^2}{\sin\theta_1^2} \cos(\theta_2^2 - C_1 C_2 \theta_1^2)] (\hat{z} \times \vec{X}_1)$

by using then $\vec{F}_{ST}^{(j)} = (-1)^j \pi a_j L_j C_1 C_2 \gamma_c(j) (\hat{z} \times \vec{X}_{j-1}), \quad \gamma_c(j) = \int_0^j (\dots) - \text{max to max}, \quad g_i \sim \theta_i$

$\vec{F}_{ST}(1) = -\pi a_1 L_1 \gamma_c(1) (\hat{z} \times \vec{X}_2), \quad \vec{F}_{ST}(2) = \pi a_2 L_2 \gamma_c(2) (\hat{z} \times \vec{X}_1), \quad \gamma_c(j) = C_1 C_2 [j \cdot j]$

by coupling (связь) between $\gamma_c(1)$ and $\gamma_c(2)$ on the surface γ between $(1 \rightarrow 2)$ and $(2 \rightarrow 1)$ on the surface.

Xhafer Valeriani
Carmichael
Olivier M. G.

ADL. Posture is more easy than
manipulation - better sign. file

Xhavovosun
Cannellina
Oliverio, M. G.
il via 7

Olivier, 1971

[Faint handwritten notes at the bottom of the page]

$\frac{1}{2}(x)^2 = 0$

2/1/71

$$X(t) \text{ und } \text{var}(X(t))$$

June 2nd 1871

average 1000

V. Cora

1.64, 1.65, 1.66, 1.67, 1.68, 1.69, 1.70, 1.71, 1.72, 1.73, 1.74, 1.75, 1.76, 1.77, 1.78, 1.79, 1.80, 1.81, 1.82, 1.83, 1.84, 1.85, 1.86, 1.87, 1.88, 1.89, 1.90, 1.91, 1.92, 1.93, 1.94, 1.95, 1.96, 1.97, 1.98, 1.99, 2.00, 2.01, 2.02, 2.03, 2.04, 2.05, 2.06, 2.07, 2.08, 2.09, 2.10, 2.11, 2.12, 2.13, 2.14, 2.15, 2.16, 2.17, 2.18, 2.19, 2.20, 2.21, 2.22, 2.23, 2.24, 2.25, 2.26, 2.27, 2.28, 2.29, 2.30, 2.31, 2.32, 2.33, 2.34, 2.35, 2.36, 2.37, 2.38, 2.39, 2.40, 2.41, 2.42, 2.43, 2.44, 2.45, 2.46, 2.47, 2.48, 2.49, 2.50, 2.51, 2.52, 2.53, 2.54, 2.55, 2.56, 2.57, 2.58, 2.59, 2.60, 2.61, 2.62, 2.63, 2.64, 2.65, 2.66, 2.67, 2.68, 2.69, 2.70, 2.71, 2.72, 2.73, 2.74, 2.75, 2.76, 2.77, 2.78, 2.79, 2.80, 2.81, 2.82, 2.83, 2.84, 2.85, 2.86, 2.87, 2.88, 2.89, 2.90, 2.91, 2.92, 2.93, 2.94, 2.95, 2.96, 2.97, 2.98, 2.99, 3.00, 3.01, 3.02, 3.03, 3.04, 3.05, 3.06, 3.07, 3.08, 3.09, 3.10, 3.11, 3.12, 3.13, 3.14, 3.15, 3.16, 3.17, 3.18, 3.19, 3.20, 3.21, 3.22, 3.23, 3.24, 3.25, 3.26, 3.27, 3.28, 3.29, 3.30, 3.31, 3.32, 3.33, 3.34, 3.35, 3.36, 3.37, 3.38, 3.39, 3.40, 3.41, 3.42, 3.43, 3.44, 3.45, 3.46, 3.47, 3.48, 3.49, 3.50, 3.51, 3.52, 3.53, 3.54, 3.55, 3.56, 3.57, 3.58, 3.59, 3.60, 3.61, 3.62, 3.63, 3.64, 3.65, 3.66, 3.67, 3.68, 3.69, 3.70, 3.71, 3.72, 3.73, 3.74, 3.75, 3.76, 3.77, 3.78, 3.79, 3.80, 3.81, 3.82, 3.83, 3.84, 3.85, 3.86, 3.87, 3.88, 3.89, 3.90, 3.91, 3.92, 3.93, 3.94, 3.95, 3.96, 3.97, 3.98, 3.99, 4.00, 4.01, 4.02, 4.03, 4.04, 4.05, 4.06, 4.07, 4.08, 4.09, 4.10, 4.11, 4.12, 4.13, 4.14, 4.15, 4.16, 4.17, 4.18, 4.19, 4.20, 4.21, 4.22, 4.23, 4.24, 4.25, 4.26, 4.27, 4.28, 4.29, 4.30, 4.31, 4.32, 4.33, 4.34, 4.35, 4.36, 4.37, 4.38, 4.39, 4.40, 4.41, 4.42, 4.43, 4.44, 4.45, 4.46, 4.47, 4.48, 4.49, 4.50, 4.51, 4.52, 4.53, 4.54, 4.55, 4.56, 4.57, 4.58, 4.59, 4.60, 4.61, 4.62, 4.63, 4.64, 4.65, 4.66, 4.67, 4.68, 4.69, 4.70, 4.71, 4.72, 4.73, 4.74, 4.75, 4.76, 4.77, 4.78, 4.79, 4.80, 4.81, 4.82, 4.83, 4.84, 4.85, 4.86, 4.87, 4.88, 4.89, 4.90, 4.91, 4.92, 4.93, 4.94, 4.95, 4.96, 4.97, 4.98, 4.99, 5.00, 5.01, 5.02, 5.03, 5.04, 5.05, 5.06, 5.07, 5.08, 5.09, 5.10, 5.11, 5.12, 5.13, 5.14, 5.15, 5.16, 5.17, 5.18, 5.19, 5.20, 5.21, 5.22, 5.23, 5.24, 5.25, 5.26, 5.27, 5.28, 5.29, 5.30, 5.31, 5.32, 5.33, 5.34, 5.35, 5.36, 5.37, 5.38, 5.39, 5.40, 5.41, 5.42, 5.43, 5.44, 5.45, 5.46, 5.47, 5.48, 5.49, 5.50, 5.51, 5.52, 5.53, 5.54, 5.55, 5.56, 5.57, 5.58, 5.59, 5.60, 5.61, 5.62, 5.63, 5.64, 5.65, 5.66, 5.67, 5.68, 5.69, 5.70, 5.71, 5.72, 5.73, 5.74, 5.75, 5.76, 5.77, 5.78, 5.79, 5.80, 5.81, 5.82, 5.83, 5.84, 5.85, 5.86, 5.87, 5.88, 5.89, 5.90, 5.91, 5.92, 5.93, 5.94, 5.95, 5.96, 5.97, 5.98, 5.99, 6.00, 6.01, 6.02, 6.03, 6.04, 6.05, 6.06, 6.07, 6.08, 6.09, 6.10, 6.11, 6.12, 6.13, 6.14, 6.15, 6.16, 6.17, 6.18, 6.19, 6.20, 6.21, 6.22, 6.23, 6.24, 6.25, 6.26, 6.27, 6.28, 6.29, 6.30, 6.31, 6.32, 6.33, 6.34, 6.35, 6.36, 6.37, 6.38, 6.39, 6.40, 6.41, 6.42, 6.43, 6.44, 6.45, 6.46, 6.47, 6.48, 6.49, 6.50, 6.51, 6.52, 6.53, 6.54, 6.55, 6.56, 6.57, 6.58, 6.59, 6.60, 6.61, 6.62, 6.63, 6.64, 6.65, 6.66, 6.67, 6.68, 6.69, 6.70, 6.71, 6.72, 6.73, 6.74, 6.75, 6.76, 6.77, 6.78, 6.79, 6.80, 6.81, 6.82, 6.83, 6.84, 6.85, 6.86, 6.87, 6.88, 6.89, 6.90, 6.91, 6.92, 6.93, 6.94, 6.95, 6.96, 6.97, 6.98, 6.99, 7.00, 7.01, 7.02, 7.03, 7.04, 7.05, 7.06, 7.07, 7.08, 7.09, 7.10, 7.11, 7.12, 7.13, 7.14, 7.15, 7.16, 7.17, 7.18, 7.19, 7.20, 7.21, 7.22, 7.23, 7.24, 7.25, 7.26, 7.27, 7.28, 7.29, 7.30, 7.31, 7.32, 7.33, 7.34, 7.35, 7.36, 7.37, 7.38, 7.39, 7.40, 7.41, 7.42, 7.43, 7.44, 7.45, 7.46, 7.47, 7.48, 7.49, 7.50, 7.51, 7.52, 7.53, 7.54, 7.55, 7.56, 7.57, 7.58, 7.59, 7.60, 7.61, 7.62, 7.63, 7.64, 7.65, 7.66, 7.67, 7.68, 7.69, 7.70, 7.71, 7.72, 7.73, 7.74, 7.75, 7.76, 7.77, 7.78, 7.79, 7.80, 7.81, 7.82, 7.83, 7.84, 7.85, 7.86, 7.87, 7.88, 7.89, 7.90, 7.91, 7.92, 7.93, 7.94, 7.95, 7.96, 7.97, 7.98, 7.99, 8.00, 8.01, 8.02, 8.03, 8.04, 8.05, 8.06, 8.07, 8.08, 8.09, 8.10, 8.11, 8.12, 8.13, 8.14, 8.15, 8.16, 8.17, 8.18, 8.19, 8.20, 8.21, 8.22, 8.23, 8.24, 8.25, 8.26, 8.27, 8.28, 8.29, 8.30, 8.31, 8.32, 8.33, 8.34, 8.35, 8.36, 8.37, 8.38, 8.39, 8.40, 8.41, 8.42, 8.43, 8.44, 8.45, 8.

справ. с 40...

1. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Thorny // Zinnia

(Cm. 24-26/11/1971)

$$x_2) \quad (z, \vec{v})$$

$$(\hat{x} \times \vec{X}_i) \quad G_i = 0$$

$$\mu(\bar{X}_1, \bar{X}_2)$$

(j) $G_1^2 \sim p$

$$\pi_{A, I_c}(1) (\vec{x} \times \vec{x}_2)$$

$$+ \pi a_2 L_2 I_c(2) (\vec{g} \cdot \vec{x}_i)$$

$$a, L, I_c(i) \in \mathbb{Z}_2$$

$$r_2 L_2 I_c(2) i E_1$$

$$I_2 = x_2 + 2\pi a_2 L_2 I_p(2) i = \frac{1}{2} + i$$

$$J_2 = J_1 - \pi a_2 l_2 l_c(2) = 1$$

$$\begin{Bmatrix} \mu_2 Z_2 \\ + \mu_1 Z_1 \end{Bmatrix} = \text{unres. bz!}$$

$$I_0(2)$$

$$2L_2 I_p(2)$$

$$T_p(2), \text{ сум. по } p$$

$$E(\bar{X}_1) = \mu$$

as a benno.

$$x_\beta X_\beta \hat{z}_\gamma = \varepsilon_\alpha z_\beta \Lambda^\beta_\gamma$$

$$\beta = -2\pi I_p(1) \frac{1}{\delta t}$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

3/I Gay, no mag. chetivka. 05.25. Seminar DIP. I. Garate, Curcio
 Induced spin-torques in uniformly magnetized ferromagnets.
 A.H. MacDonald, PRB 79, 064403 (2009).

A. Chernyshev - Nat. Phys. 2009 / Spin Galvanic Effect
 Ferromagnetic (Ga,Mn)As: Don, Bunag & m. anisotropy
 ~ sp el. current. $j = 10^6 - 10^7 \text{ A/cm}^2$ byznavas SRT fiz. H.
 Topological Insulators: Giant Anisotropic Magnetoresistance.
 (chem.-Simon terms are important, $\vec{H}_{\text{eff}} \sim (-) \vec{E}$.

(fs) hegyvacunay. current induced in switching
 $S_{\text{sc}} = \frac{e^2}{h} C_1 \int d^2x \int dt \vec{A} \cdot (\vec{J} \times \vec{A})$ - SC action.  ferromagnet
 topol. insulate

Jubilee vprezheniya mekko PI new grant of MICINN no
 a spin torque induced M-dynamics, nozros. ero u nochasu go 1/3.
 Oucana ogos co. yracomom yenta. Zhukov vages yenta no microv.
 B. Eijer & Jubilee, Myklov u J. Blanco. Oprezheniya u nochasu
 Hong-Kong vortex E-switching proposal by A. Rukalo.
 Myklov voprosy PRB - topol. field.

9/II C.A. u Angjelm u napre Aiete. Pucamo Oucana o MICINN ypane
 u byznavo B Madrid. Zbornik fiz: On gorobov. u Pappag o vas
 coupling FMR paper u ocoabov ee nlen. line opranuz. byznavo
 ~ 23-27 Jan. u naiton Maxim no usoby Vord. barrier u Col-FMR papers

B. Eijer. Pribiay Nat. Phys. 07 reference layer $\vec{P}$ is in-plane
 a free layer - vortex byznavas by CDD SP current. Uniform in-plane $\vec{P}$

ST $\rightarrow \vec{F}_{\text{ST}} = \alpha L \vec{P} \cdot \int d^2x (\vec{m} \times \frac{\partial \vec{m}}{\partial x_x})$. chizama c Vortex core? kak gne $\vec{P} = P \hat{x}$

Myka $\vec{P}$ - in-plane, $\vec{P} = P \hat{x}$. $(\vec{m} \times \frac{\partial \vec{m}}{\partial x_x})_x = m_y \frac{\partial m_z}{\partial x_x} - m_z \frac{\partial m_y}{\partial x_x}$ - nebyz
 $\vec{F}_{\text{ST}}$ toroko uz-za layer area, uge $m_z \neq 0$, t.e. os Vortex core.

$(\vec{m} \times \frac{\partial \vec{m}}{\partial x_x})_x = \sin \theta \sin \Phi (-\sin \theta) \frac{\partial \theta}{\partial x_x} - \cos \theta [\cos \theta \frac{\partial \theta}{\partial x_x} \sin \Phi + \sin \theta \cos \Phi \frac{\partial \Phi}{\partial x_x}] =$
 $= -\sin \Phi \frac{\partial \theta}{\partial x_x} - \cos \theta \sin \theta \cos \Phi \frac{\partial \Phi}{\partial x_x}$. $\Phi = \frac{\pi}{2} + \varphi + \varepsilon(x)$, $\theta = \theta_0 - \frac{g}{\sin \theta_0} (x \cdot \hat{P})$

$\sin \Phi = \sin(\Phi_0 + \varepsilon) = \sin \Phi_0 + \varepsilon \cos \Phi_0$ $\varepsilon = m_0^2 [x \sin \varphi - y \cos \varphi]$
 $\cos \Phi = \cos \Phi_0 - \varepsilon \sin \Phi_0$. $(\vec{m} \times \vec{m}'_x)_x = -[\sin \Phi_0 + \varepsilon \cos \Phi_0] \frac{g}{\sin \theta_0} \hat{x} - \frac{1}{2} [\sin 2\theta_0 + 2\varepsilon \sin \theta_0 \cos \theta_0]$

$\times [\cos \Phi_0 - \varepsilon \sin \Phi_0] m_0 (\sin \varphi, -\cos \varphi) = g \frac{\cos \Phi_0}{\sin \theta_0} \varepsilon (\cos \varphi, \sin \varphi) -$
 $-\frac{1}{2} [\sin 2\theta_0 \cos \Phi_0 - \sin 2\theta_0 \sin \Phi_0 \varepsilon + 2 \cos 2\theta_0 \sin \theta_0 \cos \Phi_0] m_0 (\sin \varphi, -\cos \varphi)$

$\int d^2x (\vec{m} \times \vec{m}'_x)_x = g \frac{\cos \Phi_0}{\sin \theta_0} \int d^2x m_0 (x \sin \varphi - y \cos \varphi) (\cos \varphi, \sin \varphi) -$
 $-\frac{1}{2} \int d^2x [-\sin 2\theta_0 \sin \Phi_0 m_0 (x \sin \varphi - y \cos \varphi) + 2 \cos 2\theta_0 \cos \Phi_0 \frac{g}{\sin \theta_0} (x \cdot \hat{P})] m_0 (\sin \varphi, -\cos \varphi)$
 $= \pi g \frac{\cos \Phi_0}{\sin \theta_0} m_0 (-y, x) - \frac{1}{2} \pi [-\sin 2\theta_0 \sin \Phi_0 m_0 (x, y) - 2g \frac{\cos 2\theta_0 \cos \Phi_0}{\sin \theta_0} (y, -x)]$
 $= \frac{\pi}{2} \sin 2\theta_0 \sin \Phi_0 m_0^2 (x, y) + \pi g \frac{\cos \Phi_0}{\sin \theta_0} m_0 (-y, x) [1 - \cos 2\theta_0] - \text{nebyzno, t.e.}$

$\Phi_0 = \frac{\pi}{2} + \varphi$ u $\cos \Phi_0, \sin \Phi_0$ byznavas na $\int d^2x (\dots)$.
 $\cos \Phi_0 = \cos(\frac{\pi}{2} + \varphi) = -\sin \varphi$, $\sin \Phi_0 = \sin(\frac{\pi}{2} + \varphi) = \cos \varphi$.

$$= \frac{1}{2} [\sin 2\theta_0 (-1) \sin \varphi - \sin 2\theta_0 \cos \varphi m_0 (X \sin \varphi - Y \cos \varphi) + 2 \cos 2\theta_0 (-1) \sin \varphi \frac{(-1) \cos \varphi}{\sin \theta_0}] m_0 (\sin \varphi - \cos \varphi) = g \sin \theta_0 \cos \varphi (\cos \varphi, \sin \varphi) + \frac{1}{2} g \sin 2\theta_0 \sin \varphi m_0 (\sin \varphi, -\cos \varphi) \quad \text{Bce heres}$$

$\vec{F}_{ST} = \alpha L P \hat{x} \cdot \left(\log \left(\vec{m} \times \frac{\partial \vec{m}}{\partial \vec{x}} \right) \right) = \alpha L P \pi a \left(\log \left[\frac{g}{\sin \theta_0} + \frac{1}{2} \sin 2\theta_0 m_0 \right] \hat{x} \right), \quad \vec{F}_{ST} \parallel \hat{Ox}$
 $(F_{ST} = \text{const}(\vec{x}) \cdot \hat{x} + O(\vec{x}^2))$ but true gal! A can use the Vlasov dynamics by ST
 $\vec{P} = P \hat{x}$ - uniform

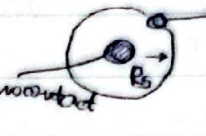
$-\vec{F}_H = \frac{\partial W_H}{\partial \vec{x}} = \mu (\hat{z} \times \vec{H}) = \mu (-\hat{x} H_y + \hat{y} H_x)$, $F_H^x = \mu H_y$, т.е. $\vec{D} \parallel O_x$
 эквивалентно включению eff. поля $\vec{H}_{\text{ст}} \parallel O_y$ - статс, которое действует на v. core.

21 (17) con. - McStall PKL 08

Escaneado con CamScanner

20/11 Thursday. Slides given tomorrow @ Madrid, UAM. C.N. B. Gerasimov, Lide. Nopekoff @ Salamanca. Daria Jarosinska u. Urazhdin-PROBOS. Yumun c. D. Berskov. 21/11 Current talk M. Vortices. Discuss. c. E. Jarosinska, Luis Lopez + 22/11 Morning work gabucos or simul. sites $\omega_p(I) = \omega_0(I) \frac{1}{R_0} + \omega_1(I) -$ magnetostatic contribution, no on gabucos or tone I (?), $\omega_1(I)$ - de field term.

2) simul. multivort. (svort. + antiv.) dynamics @ nanocontacts. 1st. Jarosinska no caducado, V. u. M. annunziato, areba. 1 vortex. 2nd. papers Puffall 'PRB'08, Mistral 'PRL'08, Devolder 'APL'09, Mifra 'APL'09 no vortex dyn. in nanocontacts. Vortex - STT - Nanocontact. Florengy y Mistral 'PRL'08 $W_{oe}(\vec{X}) \sim |\vec{X}|$ no resonance tone?

Cor. 17/10/09. $W_{oe}(\vec{X}) = -LM_s H_0 \int d^2\vec{\rho} f(\vec{\rho}) m_\phi(\vec{\rho}, \vec{X})$  $f(\vec{\rho}) = \frac{1}{\rho}$ one vortex core. $H_{oe} = H_0 \hat{e}$, $H_\phi = H_0 \hat{\phi}$. $m_\phi = \sin\theta \sin(\Phi - \varphi)$, $m_x + im_y = \sin\theta e^{i\Phi}$.

One nanocontact approximation $\approx$ rigid v. model, + i.e. approximation in plane. $\Phi(\vec{\rho}, \vec{X}) = \frac{c}{2} + \varphi + m_0(\rho) [X \sin\varphi - Y \cos\varphi]$ & min. approx. no X_d . $f(\vec{\rho}) = i \frac{c}{\rho} [\vec{z} - \vec{s}]$, $\Phi = \arg f(\vec{\rho}) = \frac{c}{2} + \arctg\left(\frac{y-Y}{x-X}\right)$ - rigid v. model. $\sin(\Phi - \varphi) = \sin\left(\frac{c}{2} + \alpha - \varphi\right) = c \cos(\alpha - \varphi)$, $\varphi = \arctg \frac{y}{x}$. $\alpha - \varphi = \arctg\left(\frac{y-Y}{x-X}\right) - \arctg \frac{y}{x}$, $\sin\Phi = c \cos\alpha$, $\cos\Phi = -c \sin\alpha$. $\cos(\arctg d) = \frac{1}{\sqrt{1+d^2}}$, $\sin(\arctg d) = \frac{d}{\sqrt{1+d^2}}$.

$\sin(\Phi - \varphi) = \sin\Phi \cos\varphi - \cos\Phi \sin\varphi = c \frac{1}{\sqrt{1+d^2}} [\cos\varphi + d \sin\varphi]$, $d = \frac{y-Y}{x-X}$. $\sin(\Phi - \varphi) = c \frac{1}{|\vec{\rho} - \vec{X}|} [(x-X) \cos\varphi + (y-Y) \sin\varphi] = c \frac{(\vec{\rho} - \vec{X}) \cdot \hat{\rho}}{|\vec{\rho} - \vec{X}|}$. $|\vec{\rho} - \vec{X}| = \sqrt{(x-X)^2 + (y-Y)^2} = \sqrt{\rho^2 + \vec{X}^2 - 2\rho|\vec{X}| \cos(\varphi - \varphi_X)}$, $(\vec{\rho} - \vec{X}) \cdot \hat{\rho} = \frac{1}{\rho} [\rho^2 - \vec{X} \cdot \vec{\rho}]$. $W_{oe}(\vec{X}) \sim \int_0^{2\pi} d\varphi \frac{[\rho^2 - \vec{X} \cdot \vec{\rho}]}{\sqrt{\rho^2 + \vec{X}^2 - 2\vec{X} \cdot \vec{\rho}}}$, $\vec{\rho} = \rho(\cos\varphi, \sin\varphi)$, $\vec{X} = (X, Y)$.

$\frac{1}{|\vec{\rho} - \vec{X}|} = \int_0^\infty dk J_0(k|\vec{\rho} - \vec{X}|) = \int dk \sum_\mu e^{i\mu(\varphi - \varphi_X)} J_\mu(k\rho) J_\mu(k|\vec{X}|)$. $W_{oe}(\vec{X}) \sim \sum_\mu \int dk J_\mu J_\mu \int d\varphi e^{i\mu(\varphi - \varphi_X)} [\rho^2 - 2\rho|\vec{X}| \cos(\varphi - \varphi_X)] = 2\pi\rho^2 \int dk J_0(k\rho) J_0(k|\vec{X}|) - 2\rho|\vec{X}| \sum_\mu \int dk J_\mu J_\mu \int d\varphi e^{i\mu\Delta\varphi} \cos[\varphi] [e^{i\Delta\varphi} + e^{-i\Delta\varphi}] = 2\pi\rho^2 \int dk J_0(k\rho) J_0(k|\vec{X}|) - 4\pi\rho|\vec{X}| \int dk J_1(k\rho) J_1(k|\vec{X}|)$. y Mistral's $W_{oe}(\vec{X}) \sim |\vec{X}|$ u. no resonance steady state $|\vec{X}| = \text{const}$, no resonance.

24/11 (24/11) $\frac{(\rho - \vec{X} \cdot \hat{\rho})}{\sqrt{\rho^2 + \vec{X}^2 - 2\vec{X} \cdot \vec{\rho}}} = \frac{d}{d\rho} \sqrt{\rho^2 + \vec{X}^2 - 2\vec{X} \cdot \vec{\rho}} = \frac{1}{2\sqrt{(\dots)}} [2\rho - 2\vec{X} \cdot \hat{\rho}] = \frac{(\rho - \vec{X} \cdot \hat{\rho})}{\sqrt{(\dots)}}$. $W_{oe}(\vec{X}) \sim \int d\rho \int d\varphi \frac{H_0}{\rho} m_\phi \sim \int d\rho \int d\varphi \sin\theta \sin(\Phi - \varphi) \sim \int d\varphi \int d\rho \frac{(\rho - \vec{X} \cdot \hat{\rho})}{\sqrt{(\dots)}} = \int d\varphi \int d\rho \frac{1}{\sqrt{(\dots)}}$. $\int d\rho \sqrt{(\dots)}$ $\int d\varphi (\dots)$ - no more summation. interpretation. Jarosinska's 5-nanopillar APL to Elzbieta. $\omega_0(I, R_0) = \frac{f(I)}{R_0} + f_1(I)$ compare simul. nanocontacts. Sem. talk D. Berskov. Current I periphery. V-AU pair. Zaser too noisy frequency $\propto$ no k/p/ring but frequency nanocontact frequency $|\vec{X}| \gg a$ - peripheral nanocontact.

Berkov: 1) pacem. dw internal structure oscillations in nanostripes in transverse field (V. oscill. near edge, paypob column)
 2) pacem. glubokoe V-AV pair & vpridum. Komineas' PRL'07 u neka glubina; 3) pacem. dipolar coupling of v. cores u V. cores in F/N/F nanostripes of v. cores u V. cores

Jaramerska, L. Diaz: 1) pacem. $\omega_{TM} = f(I, R_0)$ & nanostripes u ω_{TM} zabur or simul. radius size; 2) pacem. 9 solid long dyn. - nepe-encumns avansu

Buen articulo S. Seb. ($\frac{1}{100} \rightarrow \frac{8}{100}$) Madrid u nanogm official upr. b UAM, Madrid u e-mail from Meekob (SW)

$$W_{oe}(\vec{X}) \sim \left| d\varphi \sqrt{a^2 + |\vec{X}|^2 - 2\vec{X} \cdot \vec{p}} \right|_a, R_0 \gg |\vec{X}|, \text{ t.k. mnenno } R_0$$

A bo $|\vec{X}|/a \gg 1$, no more than nepe-encumns

$$\sqrt{1+\epsilon} = 1 + \frac{1}{2}\epsilon - \frac{1}{8}\epsilon^2 + \dots, \epsilon = \frac{1}{|\vec{X}|^2} (a^2 - 2\vec{X} \cdot \vec{p} a), \epsilon^2 \approx 4 \frac{a^2}{|\vec{X}|^4} (\vec{X} \cdot \vec{p})^2$$

$$W_{oe}(\vec{X}) \sim \left| d\varphi |\vec{X}| \sqrt{1+\epsilon} \right| = |\vec{X}| \left| d\varphi \left[1 + \frac{1}{2} \frac{1}{|\vec{X}|^2} (a^2 - 2\vec{X} \cdot \vec{p} a) - \frac{1}{8} 4 \frac{a^2}{|\vec{X}|^4} (\vec{X} \cdot \vec{p})^2 \right] \right|$$

$$= |\vec{X}| \left[2\pi + 2\pi \frac{1}{2} \frac{a^2}{|\vec{X}|^2} - \frac{1}{2} \frac{a^2}{|\vec{X}|^2} \cdot \frac{1}{2} 2\pi \right] = 2\pi |\vec{X}| \left[1 + \frac{1}{4} \frac{a^2}{|\vec{X}|^2} + O(a^4/|\vec{X}|^4) \right]$$

$$W_{oe}(\vec{X}) \sim |\vec{X}| \left[1 + \frac{1}{4} \frac{a^2}{|\vec{X}|^2} + O(a^4/|\vec{X}|^4) \right] \sim \text{Mistral PRL'08 yren +}$$

zakoni na $W_{oe}(\vec{X}) \sim |\vec{X}|$ upega $|\vec{X}| \gg a$, xot-a y ueno $|\vec{X}| = \frac{150}{100} a$

$$\left| d\varphi R_0 \sqrt{1 - \frac{2\vec{X} \cdot \vec{p}}{R_0} + \frac{|\vec{X}|^2}{R_0^2}} \right| = \left| \epsilon = \frac{1}{R_0^2} (|\vec{X}|^2 - 2\vec{X} \cdot \vec{p} R_0) \right| = R_0 \left| d\varphi \left[1 + \frac{1}{2} \frac{1}{R_0^2} (|\vec{X}|^2 - 2\vec{X} \cdot \vec{p} R_0) - \frac{1}{8} \frac{1}{R_0^4} 4 R_0^2 (\vec{X} \cdot \vec{p})^2 \right] \right| = R_0 \cdot 2\pi \left[1 + \frac{1}{4} \frac{|\vec{X}|^2}{R_0^2} \right]$$

$$W_{oe}(\vec{X}) \sim I |\vec{X}| \left[1 + \frac{1}{4} \frac{a^2}{|\vec{X}|^2} + \dots \right] = I R_0 \left[1 + \frac{1}{4} \frac{|\vec{X}|^2}{R_0^2} \right], \Delta W_{oe} \sim \frac{I}{R_0} |\vec{X}|^2$$

t.e. bvaq xoreno R_0 & $\omega_{TM} = \omega_{TM}(R_0 \rightarrow \infty) = \frac{I}{R_0} |\vec{X}|$
 u upodobno simulations by Jaramerska, upe ω_{TM} $\omega_{TM}(R_0)$ nepe-encumns xopano bugno $\forall I$.

Her, naga uoba penno eqs. $\dot{R}=0, \dot{\Phi}=\omega_{TM}$ u naoru $\omega_{TM}(R_0, I)$
 que steady state trajectory $|\vec{X}| = \text{const}$ y ueno ΔW_{oe} .

Note: See st-term $W(\vec{X}) \sim I |\vec{X}|$, $\frac{\partial W}{\partial \vec{X}} \sim \frac{\vec{X}}{|\vec{X}|}$ u $\omega_{TM} \sim \frac{I}{|\vec{X}|}$ (!).
 Conecto V-AV motion (Berkov, Komineas) an. 17/III, $\omega_{TM} \sim \frac{1}{|\vec{X}|^2}$

Prag upr. na vpena karara u gabi rekunni b UAM - Ambolenco (2-4/III). K. Meekob upr. na preprint f(2) - vobies PRL

25-26/II Nepe-encumns u gupnyam - Arantxa o Cyrocabens, Master Enroll

Corrections of Arantxa's CV u Memoria Tecnica (one uager na Rana y G)
 Nepe-encumns u Stupe-paypobon u Oucenai Cyf. : one conecna exa
 ka Master/PhD & April, soveko xore glaino pykologeska PhD (a u Xopu
 rogozobka gupnyam na Cyrocabens - enrollment u Arantxa -
 oobau ee nanu co vpeni gupnyam Oucenai to proceed.

27-28/II rogozobka rekunni b UAM: 1) obusa - avob Magnatism u
 M. nanostripes; 2) ambolenc gupnyam (SW) in patterned films

1-4/III Vozzga b Madrid (Arantxa u Oucenai): rekunni y uen 2/II u 4/II
 Vortex dyn. seminar - 3/III, Garba c Ouc. u Raga, Nora/Aurony 3/III u ppeni 3/III
 (Resident Core)

(17/11) Vortex motion in nanocontacts (in. Varma 24/11).
 $W(\vec{r})$ corresponds to a vector field of SP-current (CPP).

$W_{oe}(\vec{r}) \sim \int_0^{2\pi} d\varphi \sqrt{g^2 + |\vec{r}|^2 - 2\vec{r}\vec{\rho}} \Big|_{R_0}$ a - radius nanocontact, R_0 - radius of x-y film

Norm. 2D. integrals $E(k) = \int_0^{2\pi} d\varphi \sqrt{1 - k^2 \sin^2 \varphi}$

$\vec{r} \cdot \vec{\rho} = |\vec{r}| \rho \cos(\varphi - \varphi_r)$, i.e. $\int_0^{2\pi} d\varphi (\dots)$ no geometry case, so $W_{oe}(\vec{r})$ or φ_r is irrelevant, means $\varphi_r = 0$. $|\vec{r}| = R$.

$g^2 + R^2 - 2R\rho \cos\varphi = g^2 + R^2 - 2R\rho(1 - 2\sin^2 \frac{\varphi}{2}) = (R - \rho)^2 + 4R\rho \sin^2 \frac{\varphi}{2}$
 if true $\varphi_r = \pi$, $\cos(\varphi - \varphi_r) = -\cos\varphi$ $g^2 + R^2 + 2R\rho \cos\varphi = (R + \rho)^2 - 4R\rho \sin^2 \frac{\varphi}{2}$

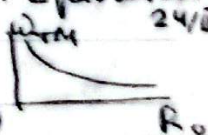
$W_{oe}(\vec{r}) \sim (R + \rho) \int_0^{2\pi} d\varphi \sqrt{1 - \frac{4R\rho}{(R + \rho)^2} \sin^2 \frac{\varphi}{2}} = (R + \rho) \int_0^{2\pi} d\varphi \sqrt{1 - k^2 \sin^2 \frac{\varphi}{2}}$, $k^2 = \frac{4R\rho}{(R + \rho)^2}$

$W_{oe}(\vec{r}) \sim (R + \rho) E(k)$, $|\vec{r}| = R$, no ρ here $|a|$. $R_0 \rightarrow \infty$ (in-plane film size)

① $\rho = R_0 \gg R$, $k^2 = \frac{4(R/R_0)}{(1 + R/R_0)^2} \approx |R_0 \gg R| = 4 \frac{R}{R_0} (1 - 2 \frac{R}{R_0}) \ll 1$
 $k'^2 = 1 - k^2 = 1 - \frac{4R\rho}{(R + \rho)^2} = \frac{(\rho - R)^2}{(\rho + R)^2}$, $k' = \frac{1 - R/\rho}{1 + R/\rho}$, $m = \frac{(1 - k')}{(1 + k')} = \frac{R/\rho}{2 - (R/\rho)}$

$E(k) = \frac{\pi}{2} [1 - \frac{1}{4} k^2 - O(k^4)] = \frac{\pi}{2} [1 - \frac{1}{4} \frac{4R/R_0}{(1 + R/R_0)^2}] \approx \frac{\pi}{2} [1 - \frac{R}{R_0}]$
 $W_{oe}(\vec{r}) \sim (R + R_0) E(k) = (R + R_0) \frac{\pi}{2} [1 - \frac{R/R_0}{(1 + R/R_0)^2}] = \frac{\pi}{2} R_0 [1 + \frac{R}{R_0} - \frac{R}{R_0(1 + R/R_0)}]$
 Here γ res. $\sim k^4$. $\epsilon = R/R_0 \ll 1$.

$W_{oe}(\vec{r}) = R_0 (1 + \epsilon) \frac{\pi}{2} [1 - \frac{1}{4} \frac{4\epsilon}{(1 + \epsilon)^2} - \frac{3}{4 \cdot 16} \frac{16\epsilon^2}{(1 + \epsilon)^4}] = \frac{\pi}{2} R_0 [1 + \epsilon - \frac{\epsilon}{(1 + \epsilon)} - \frac{3}{4} \frac{\epsilon^2}{(1 + \epsilon)^3}] = \frac{\pi}{2} R_0 [1 + \epsilon^2 - \frac{3}{4} \epsilon^2] = \frac{\pi}{2} R_0 [1 + \frac{1}{4} \epsilon^2]$ - calc. ϵ parameter $24/11$

$\epsilon = |\vec{r}|/R_0$, $W_{oe}(\vec{r}) \sim I R_0 [1 + \frac{1}{4} \frac{|\vec{r}|^2}{R_0^2}]$, $\omega_{TM} \sim \alpha \sim \frac{1}{R_0}$ 
 Power goes ϵ^2 . ω_{TM} diverges. $\text{sign}(I)$

② $\rho = a$, $a \ll R$, $k^2 = \frac{4Ra}{(R + a)^2} = \frac{4a/R}{(1 + a/R)^2}$, $\frac{a}{R} = \delta$, $k^2 = \frac{4\delta}{(1 + \delta)^2} \ll 1$.

$W_{oe}(\vec{r}) \sim (R + a) E(k) = R(1 + \delta) \frac{\pi}{2} [1 - \frac{1}{4} \frac{4\delta}{(1 + \delta)^2} - \frac{3}{4 \cdot 16} \frac{16\delta^2}{(1 + \delta)^4}] = \frac{\pi}{2} R [1 + \delta - \frac{\delta}{(1 + \delta)} - \frac{3}{4} \frac{\delta^2}{(1 + \delta)^3}] = \frac{\pi}{2} |\vec{r}| [1 + \frac{1}{4} \delta^2]$, $\delta = \frac{a}{|\vec{r}|}$ - calc. ϵ $24/11$.

$W_{oe}(\vec{r}) \sim I |\vec{r}| [1 + \frac{1}{4} \frac{a^2}{|\vec{r}|^2}] = I R_0 [1 + \frac{1}{4} \frac{|\vec{r}|^2}{R_0^2}] \approx \alpha |\vec{r}| - \frac{\alpha}{4} \frac{|\vec{r}|^2}{R_0}$ nano-contact

$\frac{\partial W_{oe}}{\partial \vec{r}} = \alpha \frac{\vec{r}}{|\vec{r}|} - \frac{\alpha}{2R_0} \vec{r} = \frac{\partial W_{oe}(|\vec{r}|)}{\partial |\vec{r}|} \frac{\vec{r}}{|\vec{r}|} = W'_{oe} \frac{\vec{r}}{|\vec{r}|}$, $W'_{oe} = \alpha - \frac{\alpha}{2R_0} |\vec{r}|$

Magnon motion $\vec{F}_{ST}(\vec{r})$ nanocontact under torque due long nanorod nanopillar
 $\vec{F}_{ST} = \pi a L (\hat{e} \times \vec{r})$. Two, Burgers, reberio, T.K. vortex core line nanocontact $|\vec{r}| \gg a$.

YD: $iG_z \dot{Z} - D \dot{Z} = -W' \frac{Z}{|Z|} + \pi a L i Z$. $\frac{1}{|Z|} W'_{oe} = \frac{\alpha}{|Z|} - \frac{\alpha}{2R_0} = \Omega_0 / |G|$

$24/11$ 037. Same $\Omega_0 = \omega_0 + \omega_1 |Z|^2$, then $\Omega_0 = \frac{\omega_0}{|Z|} - \frac{\omega_0}{2R_0}$, $\omega_0 = \frac{\alpha}{|G|}$, no α -gyrores, defined by current.

Damping - linear, $\omega_{rga} \left\{ \begin{aligned} d\phi R/R - p\dot{\phi} &= -\frac{\omega_0}{R} + \frac{\omega_0}{2R_0} \\ p\dot{R}/R + d\phi &= \pi a L / |G| \end{aligned} \right\}$, $Z = R e^{i\phi}$.

Steady state $\dot{R} = 0, \dot{\phi} = \omega$, $p\omega = \omega_0/R - \omega_0/2R_0$, $\omega d_0 = \pi a L / |G|$

11. $\int_{ST} p \cdot (\frac{\omega_0}{R} - \frac{\omega_0}{2R_0}) d\phi = \pi a L / |G|$, $a \sim I$. (param ω_0 I, $|\vec{r}| = R$ neglect? (vortex orbit radius))

$$= \frac{4\pi^2 s^2}{(1+s^2)^2} \left[\delta_{\mu 1} \left[\int_0^{2\pi} J_{\mu}(kp) \dots \right]^2 + \frac{1}{4} s^2 \delta_{\mu 2} \left[\int_0^{2\pi} J_{\mu}(kp) \frac{(1+p^2)}{p} \right]^2 \right]$$

$$(\dots) = 1 - \frac{1}{2p^2} (1-p^2)^2 s^2 + \frac{1}{4} s^2 \left\{ \frac{3}{p^2} (1+p^2)^2 - 4 \right\} = 1 - \frac{1}{2p^2} s^2 [1+p^4 - 2p^2 - \frac{1}{2} \{3(1+p^4 + 2p^2) - 4p^2\}] = 1 - \frac{1}{2p^2} s^2 [-\frac{1}{2} - \frac{1}{2}p^4 - 3p^2] = 1 + \frac{s^2}{4p^2} (1+p^4 + 6p^2)$$

коэф. при $J_{\mu}(kp)^2$ равен. от s/I отн. выражения.

Поэтому $I_{\mu}(s) = \int_0^{2\pi} d\varphi e^{i\mu\varphi} \dim(\vec{s}) \sim \int_0^{2\pi} d\varphi e^{i\mu\varphi} \frac{\sin(\varphi - \varphi_s)}{|s(\varphi)|} \frac{s}{(1+s^2)}$

$c^2 |s|^2 = s^2 - 2p(1+p^2) \frac{s \cos \varphi}{(1+s^2)} + \frac{s^2}{(1+s^2)^2} [1 + 2p^2 \cos 2\varphi + p^4]$, отн. от

А усложнений на введ. формуле, Значит $\varphi = \varphi_s$ или $s = s_x + i s_y = s_x$

тогда $I_{\mu}(s)$ не зависит на s и φ , т.е. $\cos \varphi$ и $\sin \varphi$.

тогда. Значит $Q(s) = \frac{1}{\pi c^2} \frac{(1-s^2)}{(1+s^2)^3} \int_{H \leq 1} \frac{d^2 p^2}{[1+|s|^2]^2} (1-p^4)$, см. 14/XII отн.

$G_2(s) \sim Q(s)$.

22/III Оправданность "V. non-linear" paper и приложения к Oceanian of trans paper

интересен в vertex dyn., Seechin, March 10 conf. Глосна to O. Gynocobien.

$(1+x)^{-1/2} = \frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - O(x^3)$

$\frac{1}{|s|} = \frac{c}{s} \frac{1}{[1 - \frac{2}{p}(1+p^2) \frac{s \cos \varphi}{(1+s^2)} + \frac{s^2}{(1+s^2)^2} \frac{1}{p^2} [\dots]]^{1/2}} = \frac{c}{s} [1 + \frac{1}{p}(1+p^2) \frac{s \cos \varphi}{1+s^2} - \frac{1}{2} \frac{s^2}{p^2} [\dots] + \frac{3}{8} \frac{4}{p^2} (1+p^2)^2 s^2 \cos^2 \varphi + O(s^3)]$, в знаменателе $|s|$, а s_x .

$I_{\mu}(s) \sim \int d\varphi e^{i\mu\varphi} \sin \varphi [1 + \frac{1}{p}(1+p^2) \frac{s}{(1+s^2)} \cos \varphi - \frac{1}{2} \frac{s^2}{p^2} (1+2p^2 \cos 2\varphi + p^4) + \frac{3}{2} \frac{(1+p^2)^2}{p^2} \cos^2 \varphi] =$

$= [\cos \varphi = \frac{1}{2}(1+\cos 2\varphi)] = \int d\varphi e^{i\mu\varphi} \sin \varphi [1 - \frac{1}{2} \frac{s^2}{p^2} (1+p^4) + \frac{3}{4} \frac{(1+p^2)^2}{p^2} s^2 + \frac{(1+p^2)s}{p(1+s^2)} \cos \varphi + \frac{s^2}{p^2} (-p^2 + \frac{3}{4}(1+p^2)^2) \cos 2\varphi]$

$I_{\mu}(s) \sim \int d\varphi e^{i\mu\varphi} \sin \varphi [1 + \frac{1}{4} \frac{s^2}{p^2} (1+6p^2+p^4) + \frac{(1+p^2)s}{p(1+s^2)} \cos \varphi + \frac{1}{4} \frac{s^2}{p^2} (3+2p^2+3p^4) \cos 2\varphi]$

$\int_0^{2\pi} d\varphi e^{i\mu\varphi} \sin \varphi = \int_0^{2\pi} d\varphi e^{i\mu\varphi} \frac{1}{2} [e^{i\varphi} - e^{-i\varphi}] = \frac{\pi}{i} (\delta_{\mu-1} - \delta_{\mu+1})$

$\int_0^{2\pi} d\varphi e^{i\mu\varphi} \sin \varphi \cos \varphi = \frac{2\pi}{i} \int_0^{2\pi} d\varphi e^{i\mu\varphi} \sin 2\varphi = \frac{1}{2} \frac{\pi}{i} (\delta_{\mu-2} - \delta_{\mu+2})$

$\int_0^{2\pi} d\varphi e^{i\mu\varphi} \sin \varphi \cos 2\varphi = \frac{1}{2} \int_0^{2\pi} d\varphi e^{i\mu\varphi} [\sin 3\varphi - \sin \varphi] = \frac{1}{2} \frac{\pi}{i} (\delta_{\mu-3} - \delta_{\mu+3}) - \frac{1}{2} \frac{\pi}{i} (\delta_{\mu-1} - \delta_{\mu+1})$

$I_{\mu}(s) \sim [(1 + \frac{1}{4} \frac{s^2}{p^2} (1+6p^2+p^4)) \pi i (\delta_{\mu-1} - \delta_{\mu+1}) + \frac{(1+p^2)s}{p(1+s^2)} \frac{1}{2} \pi i (\delta_{\mu-2} - \delta_{\mu+2}) + \frac{1}{4} \frac{s^2}{p^2} (3+2p^2+3p^4) \frac{1}{2} \pi i (\delta_{\mu-3} - \delta_{\mu+3} - \delta_{\mu-1} + \delta_{\mu+1})]$, см. $\mu > 0$ равно, то

$I_{\mu}(s) \sim \pi i [(1 + \frac{1}{4} \frac{s^2}{p^2} (1+6p^2+p^4)) \delta_{\mu 1} + \frac{1}{2} \frac{(1+p^2)s}{p(1+s^2)} \delta_{\mu 2} + \frac{1}{8} \frac{s^2}{p^2} (3+2p^2+3p^4) (\delta_{\mu 3} - \delta_{\mu 1})]$

$F_{\mu}(k) = \int_0^{2\pi} J_{\mu}(kp) I_{\mu}(s) \sim \pi i \int_0^{2\pi} J_{\mu}(kp) [(\dots) \delta_{\mu 1} + \frac{1}{2} \frac{(1+p^2)s}{p(1+s^2)} \delta_{\mu 2} + (\dots) \delta_{\mu 3}] =$

$= \pi i \int_0^{2\pi} J_{\mu} [\{ 1 + \frac{1}{4} \frac{s^2}{p^2} (1+6p^2+p^4) - \frac{1}{8} \frac{s^2}{p^2} (3+2p^2+3p^4) \} \delta_{\mu 1} + \frac{(1+p^2)s}{2p(1+s^2)} \delta_{\mu 2} + \frac{1}{2} \frac{s^2}{p^2} (\dots) \delta_{\mu 3}]$

$|F_{\mu}(k)|^2 = \frac{4\pi^2 s^2}{(1+s^2)^2} [\delta_{\mu 1} [\int_0^{2\pi} J_{\mu}(\dots)]^2 + \delta_{\mu 2} \frac{s^2}{4} [\int_0^{2\pi} J_{\mu} \frac{(1+p^2)}{p}]^2 + O(s^4)]$

$\{ \dots \} = 1 + \frac{1}{8} \frac{s^2}{p^2} (10p^2 - 1 - p^4) \Rightarrow W_m^{\text{rel}}(s) = 2L \int \frac{dk}{k} f(k) \sum_{\mu=0}^{\infty} |F_{\mu}(k)|^2, V = \pi R^2 L =$

$\frac{1}{2} W_m(s) = \frac{2}{\pi} \frac{4\pi^2 s^2}{(1+s^2)^2} \int \frac{dk}{k} f(k) [| \int_0^{2\pi} J_1(kp) \{ \dots \} |^2 + \frac{s^2}{4} [\int_0^{2\pi} J_2(kp) \frac{(1+p^2)}{p}]^2] \approx \pi L, p=1$

$\{ \dots \} = 1 + \frac{1}{8} \frac{s^2}{p^2} (10p^2 - 1 - p^4) = 1 + \frac{1}{8} \frac{s^2}{p^2} [8p^2 - (1-p^2)^2]$ обнуляется коэф. при J_1 и J_2 !

Р. 7 июня 5/1 20062

$$\frac{1}{V} W_m(s) = 8\pi \frac{s^2}{(1+s^2)^2} \int_0^{\pi/2} f(kL) \left[[(\cos J_1)^2 + 2s^2 (\cos J_1)(\cos J_2)] + s^2 [(\cos J_2)^2] \right]$$

$$[(\cos J_1(1+as^2))^2] = [(\cos J_1 + s^2 \cos J_2)^2] = [(\cos J_1)^2 + 2s^2 (\cos J_1)(\cos J_2) + s^4 (\cos J_2)^2]$$

$$a = \frac{1}{8} \frac{1}{s^2} (10s^2 - 1 - s^4) \cdot \frac{1}{V} W_m(s) = \frac{8\pi s^2}{(1+s^2)^2} [I_0 + I_2 s^2]$$

$$\frac{1}{(1+s^2)^2} = \frac{1}{1+2s^2} = 1 - 2s^2 + O(s^4) \rightarrow W_m(s) = 8\pi s^2 [I_0 + (I_2 - 2I_0)s^2]$$

$$W_m = \frac{\pi}{2} s^2 + \frac{1}{4} \beta s^4, \quad \frac{\beta}{2} = \frac{4(I_2 - 2I_0)}{2I_0} \cdot \text{But } \frac{\beta}{2} = \frac{4I_2}{2I_0} = 2 \frac{I_2}{I_0} \approx 2.5$$

i.e. $I_2 \approx 2I_0$. Therefore $\frac{\beta}{2} = 2(I_2 - 2I_0) \approx 0$? $\frac{\beta}{2} \ll 1$ (?)

K. pачеру $W_m^{(u)}(s)$ as s/I or I_0 . goдabлaчe uнoмeнo. 1

capanno, бeгo Kpиcтeн пpeдeлeннo нeмyгpaнa $\beta/2 \approx 2.5$

Note: $W_m(s)$ as $F(s')$, $s' = \left(\frac{s}{1+s^2}\right)$ u гyе s' -uнeгaтeв. $\left(\frac{\beta}{2} = 4 \cdot \frac{1}{(1+s^2)^2}\right)$

$$s' = \frac{s}{1+s^2} + s \frac{(-1)2s}{(1+s^2)^2} = \frac{(1-s^2)}{(1+s^2)^2} s = G_2(s) \sim Q(s) \text{ bнe. uнoмeн.}$$

uнoмeн uнoмeн зaмeчaeт чтo гyе $s' \sim \tilde{x}'$, зaмeчaeт uнoмeн uнoмeн $\tilde{G}(s')$

uнoмeн uнoмeн зaмeчaeт "β" uз Buchanan 'PRL' 07, PRB'06 u Kim 'APL' 08.

23/11 Doenue uнeгaтeв uнeгaтeв u US - Form I-1040 u 2555, 2555-EZ.

Foreign salary tax exclusion for 2009 ecтo 91,400\$. Kагo бaнк uнeгaтeв

Гi-б., Chase uнoмeн uнeгaтeв Form I-1099 ecтo Oм A.

Рoб. гyе $\vec{d}u = 2 \frac{a}{c} \frac{s}{|f|} \left(\frac{s}{1+s^2}\right) \sin \varphi$ uнoмeн $\varphi_s = 0$, $s = s_x$. $s' = s/(1+s^2)$.

$$I_1(s) \sim \int_0^{2\pi} d\varphi e^{iM\varphi} \frac{\sin \varphi}{|f|}, \quad c^2 |f|^2 = p^2 - 2p(1+p^2)s' \cos \varphi + s'^2 [1 + 2p^2 \cos 2\varphi + p^4]$$

$$c^2 |f|^2 = p^2 + s'^2 (1-p^2)^2 - 2p(1+p^2)s' \cos \varphi + 4s'^2 p^2 \cos^2 \varphi$$

uнeгaтeв. $\mu=1$ u $\mu=2$, $|f|(\varphi)$ -uнeгaтeв, i.e. $I_\mu(s') \sim \int_0^{2\pi} d\varphi \frac{\sin^{\mu} \varphi \cdot \sin \varphi}{|f|}$, $\mu=1, 2$

$$\tan \frac{\varphi}{2} = x, \quad \sin \varphi = \frac{2x}{1+x^2}, \quad \cos \varphi = \frac{1-x^2}{1+x^2}, \quad d\varphi = \frac{2dx}{1+x^2}$$

$$I_1(s') \sim \int_{-\pi}^{\pi} d\varphi \frac{\sin^2 \varphi}{\sqrt{p^2 + s'^2 (1-p^2)^2 - 2p(1+p^2)s' \cos \varphi + 4s'^2 p^2 \cos^2 \varphi}} = \int dx \frac{\sin^2 \varphi}{\sqrt{a + b \cos \varphi + c \cos^2 \varphi}}$$

Ecтo $\cos \varphi = x$, $-\sin \varphi d\varphi = dx$, u $I_1(s') \sim \int_0^1 dx \frac{1-x^2}{\sqrt{a + bx + cx^2}}$, $I_2(s') \sim \int_0^1 dx \frac{\cos^2 \varphi \sin^2 \varphi}{|f|}$

$$a + bx + cx^2 = c \left(\frac{a}{c} + \frac{b}{c}x + x^2 \right) = c \left[\left(x + \frac{b}{2c} \right)^2 + \frac{a}{c} - \left(\frac{b}{2c} \right)^2 \right], \quad c = 4s'^2 p^2 > 0$$

$$\frac{a}{c} - \frac{1}{4} \frac{b^2}{c^2} = \frac{1}{4s'^2 p^2} [p^2 + s'^2 (1-p^2)^2] - \frac{1}{4} \frac{4p^2 (1+p^2)^2 s'^2}{16s'^4 p^4} = \frac{1}{c^2} [(p^2 + s'^2 (1-p^2)^2) 4s'^2 p^2 - p^2 (1+p^2)^2]$$

$$= \frac{p^2}{c^2} [4s'^2 p^2 + 4s'^4 (1-p^2)^2 - s'^2 (1+p^2)^2] = \frac{p^2 s'^2}{c^2} [4p^2 - (1+p^2)^2 + 4s'^2 (1-p^2)^2] =$$

$$= \frac{p^2 s'^2}{c^2} (1-p^2)^2 [4s'^2 - 1] \sim (4s'^2 - 1) \leq 0 \text{ uнoмeн } s \leq \frac{1}{2} \text{ (зaмeчaeт } s \rightarrow s')$$

$$a + bx + cx^2 = c \left[\left(x + \frac{b}{2c} \right)^2 - \lambda^2 \right], \quad \lambda^2 = \frac{1}{4} \frac{b^2}{c^2} - \frac{a}{c} = \frac{p^2 s'^2}{c^2} (1-p^2)^2 (1-4s'^2)$$

Ecтo $\tan \frac{\varphi}{2} = x$, u $a + b \cos \varphi + c \cos^2 \varphi = a + b \frac{(1-x^2)}{(1+x^2)} + c \frac{(1-x^2)^2}{(1+x^2)^2}$

$$I_1(s) \sim \int_0^{\infty} dx \frac{x^2}{(1+x^2)^2} \frac{1}{\sqrt{a + b \frac{(1-x^2)}{(1+x^2)} + c \frac{(1-x^2)^2}{(1+x^2)^2}}} = \int_0^{\infty} dx \frac{x^2}{(1+x^2)^2} \frac{1}{\sqrt{a(1+x^2)^2 + b(1-x^2) + c(1-x^2)^2}}$$

uнeгaтeв uнeгaтeв, c зaмeчaeт $\sqrt{(\dots)}$ uнeгaтeв uнoмeн б uнeгaтeв uнeгaтeв.

$\textcircled{31} \text{ Ecum } \vec{m}(\vec{r}, t) = \vec{m}(\vec{r}, \vec{a}(t)) \text{ Ansatz } c = a_x + i a_y, f(z) = \frac{1}{z} [i c e + \frac{a}{2} - \frac{a}{2} z]$
 $\text{to } W = W(a), W_m = W_m(a) \text{ u. yD Thiele } \hat{G}_{\alpha\beta} \hat{a}_\beta = -\frac{\partial W}{\partial \hat{a}_\alpha} \text{ zge}$
 $\hat{G}_{\alpha\beta} = \int d\vec{r} \vec{m} \cdot \left(\frac{\partial \vec{m}}{\partial a_\alpha} + \frac{\partial \vec{m}}{\partial a_\beta} \right) \text{ c. normierung so ungen. } \frac{M_{\text{tot}}}{f} \cdot a = -2i c \frac{1}{z}$
 $\vec{m}(a) \text{ u. phase, dann } \vec{m}(\vec{r}) = \vec{m}(s), \text{ normierung } W(a), G(a) \text{ u. phase } W(s), G(s)$
 $\frac{\partial}{\partial a} = \frac{1}{2} \left(\frac{\partial}{\partial a_x} - i \frac{\partial}{\partial a_y} \right), \frac{\partial}{\partial \bar{a}} = \frac{1}{2} \left(\frac{\partial}{\partial a_x} + i \frac{\partial}{\partial a_y} \right), \left(\frac{\partial \vec{m}}{\partial a_x} + \frac{\partial \vec{m}}{\partial a_y} \right) = 2i \left(\frac{\partial \vec{m}}{\partial \bar{a}} + \frac{\partial \vec{m}}{\partial a} \right)$
 $\text{zu 14/E 2007 z. - 14/M 2008 z.}$

$\rightarrow W_m(a) = W_m(|a|) = \frac{\alpha}{2} |a|^2 + \frac{1}{4} \beta |a|^4, |a| = 2 \cdot \frac{1}{|s|}$
 $W_m(s) = \frac{\alpha}{2} |s|^2 + \frac{1}{4} \beta |s|^4, W_m(s) = \frac{\alpha}{2} \frac{4 s^2}{(1+s^2)^2} + \frac{1}{4} \beta \cdot \frac{16 s^4}{(1+s^2)^2} =$
 $= 2\alpha s^2 (1-2s^2) + 4\beta s^4 = 2\alpha s^2 + 4(\beta - \alpha) s^4, 2\alpha = \frac{\alpha}{2}, 4(\beta - \alpha) = \frac{1}{4}$

$\text{div } \vec{m}(\vec{a}) = 2 \text{Re}(\partial \bar{w} / \partial z). \text{ One core } \frac{\partial w}{\partial z} = \frac{1}{2} \frac{1}{|s|} \frac{\partial f}{\partial z}, \text{ b core } \frac{\partial w}{\partial \bar{z}} = \frac{\partial f}{\partial \bar{z}} = \frac{\partial f}{\partial \bar{z}}$
 $\frac{\partial f}{\partial z} = \frac{1}{z} [i c - \bar{a} z], \bar{a} z = (a_x - i a_y)(x + i y) = a_x x + a_y y + i(\dots) = (\vec{a} \cdot \vec{r}) + i(\dots)$

$\text{Pone core } \text{div } \vec{m}(\vec{a}) = 2 \text{Re} \left[\frac{1}{2} \frac{1}{|s|} \frac{1}{c} (i c - \bar{a} z) \right] = \frac{(-1)}{c |s|} \text{Re}(\bar{a} z) = -\frac{1}{c |s|} (\vec{a} \cdot \vec{r})$

$c^2 |s|^2 = c^2 s \bar{s} = (i z c + \frac{a}{2} - \frac{\bar{a}}{2} z^2) (-i \bar{z} c + \frac{\bar{a}}{2} - \frac{a}{2} \bar{z}^2) = |z|^2 + i z c \bar{a} - i c a \bar{z} |z|^2 - \frac{a}{2} i c \bar{z} + \frac{1}{4} |a|^2 - \frac{1}{4} a^2 \bar{z}^2 + \frac{1}{2} i c \bar{a} |z|^2 z + \frac{1}{4} |a|^2 |z|^4 - \frac{1}{4} \bar{a}^2 z^2 =$

$= s^2 + \frac{i}{2} c (z \bar{a} - a \bar{z}) + \frac{i}{2} c s^2 (\bar{a} z - a \bar{z}) + \frac{1}{4} |a|^2 + \frac{1}{4} |a|^2 |z|^4 - \frac{1}{4} (a^2 \bar{z}^2 + \bar{a}^2 z^2) =$
 $= s^2 + \frac{i}{2} c (1+s^2)(z \bar{a} - a \bar{z}) + \frac{1}{4} |a|^2 [1 + |z|^4 - e^{i 2 \varphi_a} \bar{z}^2 - e^{-i 2 \varphi_a} z^2], a = e^{i \varphi_a}$

$z \bar{a} - a \bar{z} = |a| \rho (e^{i(\varphi - \varphi_a)} - e^{-i(\varphi - \varphi_a)}) = 2i |a| \rho \sin(\varphi - \varphi_a)$
 $e^{i 2 \varphi_a} z^2 + \bar{z}^2 e^{-i 2 \varphi_a} = |z|^2 [e^{i 2(\varphi + \varphi_a)} + e^{-i 2(\varphi + \varphi_a)}] = 2 \rho^2 \cos 2(\varphi - \varphi_a)$

$c^2 |s|^2 = s^2 - c(1+p^2) \rho |a| \sin(\varphi - \varphi_a) + \frac{1}{4} |a|^2 [1 + \rho^4 - 2 \rho^2 \cos 2(\varphi - \varphi_a)], 12/E 2007$

$\text{div } \vec{m}(a) = -\frac{|a| \rho \cos(\varphi - \varphi_a)}{c |s|}, \frac{1}{c |s|} = \frac{1}{\rho} \sqrt{1 - \frac{c}{\rho} (1+p^2) |a| \sin \varphi + \frac{1}{4} \frac{|a|^2}{\rho^2} [\dots]}$

$= \frac{1}{\rho} \left[1 + \frac{1}{2} \frac{c}{\rho} (1+p^2) |a| \sin \varphi - \frac{1}{8} \frac{|a|^2}{\rho^2} (1+p^4 - 2 \rho^2 \cos 2\varphi) + \frac{3}{8} \frac{(1+p^2)^2}{\rho^2} |a|^2 \sin^2 \varphi \right] =$

$= \frac{1}{\rho} \left[1 + \frac{1}{2} c \frac{(1+p^2)}{\rho} |a| \sin \varphi + \frac{1}{16} \frac{1}{\rho^2} (1+p^4 + 6 \rho^2) |a|^2 + \frac{|a|^2}{16} \frac{1}{\rho^2} (-1)(3+3 \rho^4 + 2 \rho^2) \cos 2\varphi \right]$

$F_\mu(k) = \int d\varphi \int d\varphi J_\mu(k_\varphi) \int_0^{2\pi} d\varphi e^{i \mu \varphi} \text{div } \vec{m}(\vec{a}), \text{div } \vec{m}(\vec{a}) = -\frac{|a| \rho \cos \varphi}{c |s|}$

$\int_0^{2\pi} d\varphi e^{i \mu \varphi} \cos \varphi = \pi (\delta_{\mu,1} + \delta_{\mu,-1}), \int d\varphi e^{i \mu \varphi} \cos \varphi \sin \varphi = \frac{\pi}{2i} (\delta_{\mu,2} - \delta_{\mu,-2})$

$\int d\varphi \cos \varphi \cos 2\varphi e^{i \mu \varphi} = \frac{1}{2} \int d\varphi e^{i \mu \varphi} (\cos \varphi - \cos 3\varphi) = \frac{\pi}{2} (\delta_{\mu,1} + \delta_{\mu,-1}) - \frac{\pi}{2} (\delta_{\mu,3} + \delta_{\mu,-3})$

$F_\mu(k) = \int d\varphi \int d\varphi J_\mu \int d\varphi e^{i \mu \varphi} (-1) |a| \rho \cos \varphi \frac{1}{\rho} \left[1 + \frac{1}{2} c \frac{(1+p^2)}{\rho} |a| \sin \varphi + \frac{|a|^2}{16 \rho^2} (\dots) - \frac{|a|^2}{16 \rho^2} (\dots) \cos 2\varphi \right]$

$= -|a| \int d\varphi \int d\varphi J_\mu \left[\left\{ 1 + \frac{|a|^2}{16 \rho^2} (\dots) \right\} \pi (\delta_{\mu,1} + \delta_{\mu,-1}) + \frac{1}{2} c \frac{(1+p^2)}{\rho} |a| \frac{\pi}{2i} (\delta_{\mu,2} - \delta_{\mu,-2}) - \frac{|a|^2}{16 \rho^2} (\dots) \frac{\pi}{2} (\delta_{\mu,3} - \delta_{\mu,-3}) \right]$

$= |q_{\mu,1}, \mu > 0| = -|a| \rho \int d\varphi \int d\varphi J_\mu \left[\left\{ 1 + \frac{|a|^2}{16 \rho^2} (\dots) \right\} \delta_{\mu,1} - \frac{c}{4i} \frac{(1+p^2)}{\rho} |a| \delta_{\mu,2} - \frac{1}{32} \frac{|a|^2}{\rho^2} (\dots) (\delta_{\mu,3} - \delta_{\mu,-3}) \right]$

$\text{Korrig. u. } \delta_{\mu,1}: 1 + \frac{|a|^2}{16 \rho^2} (1+p^4 + 6 \rho^2) - \frac{1}{32} \frac{|a|^2}{\rho^2} (3 + 3 \rho^4 + 2 \rho^2) = 1 + \frac{|a|^2}{32 \rho^2} (-1 - \rho^4 + 10 \rho^2)$

$\sum_{\mu > 0} |F_\mu(k)|^2 = |a|^2 \pi^2 \left[\left[\int d\varphi \int d\varphi \left\{ 1 + \frac{|a|^2}{32 \rho^2} (10 \rho^2 - 1 - \rho^4) \right\} \right]^2 + \frac{|a|^2}{16} \left[\int d\varphi \int d\varphi J_2 \frac{(1+p^2)}{\rho} \right]^2 \right]$

Escaneado con CamScanner

$\beta/\xi = \hat{\beta}/\omega_0 \approx 4$ е големият параметър. $V(L/R) = \text{НЕТ}$, C_p

при разложении $G_{xy}(g_{xy})$ вг. замкнутом $W(Z, S)$ и $m(p, Z)$ от S, Z не
изменяется. Я game. $\partial/\partial p$ там. Тензор $p = a$ (a по $p = Z$).

Ques. 15/2017. $\hat{g}_{xy} = \frac{4}{(1+w\bar{w})^2} \left[\frac{\partial w}{\partial a} \frac{\partial \bar{w}}{\partial \bar{a}} - \frac{\partial w}{\partial \bar{a}} \frac{\partial \bar{w}}{\partial a} \right]$, $w(z) = f(z)$ holomorphic.

$$\frac{\partial f}{\partial z} = \frac{1}{2c}, \quad f(\bar{z}) = \frac{1}{c} \left[-i\bar{z}c + \frac{\bar{a}}{2} - \frac{a}{2}\bar{z}^2 \right], \quad \frac{\partial f}{\partial a} = \frac{1}{2c}, \quad \frac{\partial f}{\partial \bar{a}} = \frac{1}{a} \frac{(-1)}{2} z^2, \quad \frac{\partial f}{\partial a} = -\frac{1}{2c} \bar{z}^2$$

$$\hat{G}_{xy} = \int d^2p \hat{g}_{xy} = \frac{1}{\alpha^2} \int d^2p \frac{(1-p^4)}{[1+|s|^2]^2} \quad \text{вуг } \hat{G}_{xy} \sim Q \text{ скомпенсує,}$$

$$\ddot{a} = -2i \frac{d}{dt} \frac{s}{1+s\bar{s}} = -2i \frac{1}{(1+s\bar{s})^2} \left[\dot{s}(1+s\bar{s}) - s(\dot{\bar{s}} + s\bar{\dot{s}}) \right] = -2i \frac{(\dot{s} - s^2\bar{\dot{s}})}{(1+s\bar{s})^2}$$

$$\frac{\partial W}{\partial s} = \frac{\partial W}{\partial s} \cdot \frac{\partial s}{\partial c} = \frac{\partial W}{\partial c} \cdot \frac{(2c)}{(1+s)^2}, \quad \frac{\partial W}{\partial a} = \frac{(1+s)^2}{(-2c)} \quad \frac{\partial W}{\partial s} = \frac{j}{2}(1+|s|^2)^2 \frac{\partial W}{\partial s}$$

Also $\hat{G}_{xy} = G_{xy} \cdot Q(s) = \frac{(1-s^2)}{(1+s^2)^3}$

$$\lim_{\vec{a} \rightarrow 0} W_m(a) = \frac{\hat{\alpha}}{2} |a|^2 + \frac{\hat{\beta}}{4} |a|^4, \quad \frac{\partial W_m(a)}{\partial \vec{a}} = \hat{\alpha} \vec{a} + \hat{\beta} \vec{a} |a|^2 = \vec{a} (\hat{\alpha} + \hat{\beta} |a|^2)$$

предположим, что $W_m(s)$ - шаг. с/р. $\frac{\beta}{\omega} = \frac{16(\hat{\beta} - \hat{\omega})}{4\hat{\omega}} = 4(\frac{\hat{\beta}}{\hat{\omega}} - 1)$, тогда $\frac{\beta}{\omega} = 15$ и $\frac{\hat{\beta}}{\hat{\omega}} = 4$. Но $\frac{\beta}{\omega} \neq 4$ и $\frac{\hat{\beta}}{\hat{\omega}} = 4$ не выполняется, это $\frac{\beta}{\omega} = 4$ (?)

41067. Für $\frac{1}{s^2} W_m(s) = F_v(\beta) \beta^2 + F_u(\beta) s^4 = \frac{\beta^*}{3k^*} s^2 + \frac{\beta^*}{4} s^4$

$$x^* = 2F_v(p) = 2 \int_{\frac{1}{2}}^1 f [4\phi\phi J_1]^2, \quad \beta^* = 4F_u(p) = 4 \int_{\frac{1}{2}}^1 f [2[\phi\phi\phi J_1]^2 + \frac{1}{4}[\phi\phi(1+\phi^2)J_2]^2 -$$

$$1 - \frac{1}{8g^2}(1-g^2)^2 = \frac{1}{8g^2}(10g^2 - 1 - p^2)$$

$x' = x - s, y = y'$
 $s^2 - 2s'(1+p^2)x + s'^2[(1-p^2)^2 + 4x'^2] = c^2$
 $s'^2 = x'^2 + y'^2 = s'^2 + s^2 + 2sx'$
 $s'^2 + s^2 + 2sx' - 2s'(x'+s)[1+p^2 + 2sx'] + s'^2[(1-p^2)^2 + 4x'^2] = c^2$
 $s'^2 + s^2 + 2sx' - 2s(x'+s) - 2s(x'+s)[p^2 + 2sx'] + s'^2[(1-p^2)^2 + 4x'^2] = c^2$
 $s'^2 - s^2 - 2sx'p^2 - 2s^2p^2 + s^2(1-p^2)^2 = c^2, s'^2 - s^2 - 2sx'p^2 - 2s^2p^2 + s^2(1-p^2)^2 = c^2$
 $s'^2[1 - 2sx' - 2s^2] + s^2(1-p^2)^2 = c^2 + s^2$
 $s'^2[1 - 2sx' + s^2(s'^2 - 4)] = c^2, s'^2 \leq c^2, s'^2[1 - 2sx' - 4s^2] = c^2$

$f(z) = -\frac{a}{2c} (z-s)(z-\frac{s}{1+s^2}), z' = z-s, f(z') = -\frac{a}{2c} z'(z'+s-\frac{s}{1+s^2})$
 $a = -2ic \frac{s}{1+s^2}, f(z') = -\frac{1}{2c} 2ic \frac{s}{1+s^2} z'(z'+s-\frac{s}{1+s^2}) = -\frac{ic}{c} \frac{1}{1+s^2} z'(sz'+1+s^2-1)$
 $V \text{ core } |f(z')| \leq 1, \text{ yamusa core } |z'|[sz'-1+1+s^2] = c(1+s^2)$
 $|sz'+1+s^2-1|^2 = [sz'-1+1+s^2][sz'-1+1+s^2] = [1+s^2|z'|^2 + (1+s^2-1)(sz'+s\bar{z}') + (s^2-1)^2]$
 $s' \sqrt{s^2|s|^2 + 2(1+s^2-1)|s|s' \cos(\varphi-\varphi_s) + (1+s^2-1)^2} = c(1+s^2)$
 $s' \sqrt{1-2sx'-4s^2} = c$

$c^2|f(z')|^2 = \frac{s'^2}{(1+s^2)^2} [1+s^2s'^2 + 2(1+s^2-1)|s|s' \cos(\varphi-\varphi_s) + (1+s^2-1)^2]$
 $x'Oy$ y gocha gura parava $B_2(s) \sim Q(s)$, no na $W_m(s)$, zhe $f(z')$ na $B_2(s)$ na $B_2(s)$
 $f(z) = -ic \frac{1}{1+s^2} (z-s)(sz-1), f(z') = -ic \frac{1}{1+s^2} z'(sz'+1+s^2-1)$

$1-2sx'-4s^2 = 1-sx'-2s^2-\frac{1}{8}4s^2x'^2, s' = c + d_1s + d_2s^2$
 $(c+d_1s+d_2s^2)(1-sx'-2s^2-\frac{1}{2}s^2x'^2) = c, x' = s' \cos \varphi$
 $s: -x'c + d_1 = 0, s^2: -2c^2 - \frac{1}{2}c \cdot c^2 \cos^2 \varphi - 2d_1c \cos \varphi + d_2 = 0$
 $d_1 = x'c = c^2 \cos \varphi, d_2 = 2c + \frac{1}{2}c^3 \cos^2 \varphi + 2c \cos \varphi \cdot c^2 \cos \varphi \approx 2c + c^3$
 $s'(\varphi) = c + c^2 \cos \varphi s + 2c s^2$
 $s'(\varphi) = c(1+2s^2) + c^2 s \cos \varphi$



$I(s) = \frac{1}{\pi c^2} \int_{|s| \leq 1} \frac{d^2 \vec{\rho}}{[1+|s|^2]^2}, |f|^2 c^2 = \frac{s'^2}{2\pi} \frac{1}{(1+s^2)^2} [s^2 s'^2 - 2s s' \cos \varphi' + 1 - 2s^2]$
 $I(s) = \frac{1}{\pi c^2} \int_{V.C.} d^2 \vec{\rho}' F(\rho') = \frac{1}{\pi c^2} \int_0^{2\pi} d\varphi' \int_0^{s'} ds' s' F(s', \varphi')$
 $I(s) = \frac{1}{\pi c^2} \int d\varphi' \int ds' s' \frac{1}{[1 + \frac{s'^2}{c^2} \frac{1}{(1+s^2)^2} [1 - 2s s' \cos \varphi' + s^2(s'^2 - 2)]]^2}$
 $s' \approx c, s'^2 \ll 2$
 $I(s) = \frac{1}{\pi c^2} \int d\varphi' \int ds' s' \frac{1}{[1 + \frac{s'^2}{c^2} (1 - 2s s' \cos \varphi' - 2s^2)]^2}$

$I_x = \int dx x \frac{1}{[1+a^2 x^2(b+dx)]^2}, P(x) = ba^2 x^2 + da^2 x^3 + 1$
 $P(x) = da^2(x-x_1)(x-x_2)(x-x_3)$
 $[...]^2 = [1+ba^2 x^2 + da^2 x^3]^2 = (1+ba^2 x^2)^2 + 2da^2 x^3(1+ba^2 x^2) + d^2 a^4 x^6$
 $\approx (1+ba^2 x^2)^2 [1 + 2d \frac{a^2 x^3}{1+ba^2 x^2}], [...]^{-2} = (1+ba^2 x^2)^{-2} [1 - 2d \frac{a^2 x^3}{(1+ba^2 x^2)}]$

$$I_x(d) = I_x(0) - 2da^2 \int dx \frac{x^4}{(1+b^2a^2x^2)^3} = I_x(0) - \frac{2da^2}{(ba^2)^3} \left[\frac{1}{4} \frac{1}{ba^2} \frac{x}{(d^2+x^2)^2} - \frac{5}{8} \frac{x}{(d^2+x^2)} + \frac{3}{8} \sqrt{ba} \operatorname{arctg}(\sqrt{ba}x) \right], \quad d^2 = \frac{1}{ba^2} > 0.$$

$$d^2 = \frac{1}{c^2}, \quad b = 1-2s^2, \quad I_x(d) \text{ не меняется при } b > 0, \quad s^2 < \frac{1}{2} \rightarrow s < \frac{1}{\sqrt{2}}.$$

$$I_x(0) = \int \frac{dx x}{[1+b^2a^2x^2]^2} = \frac{1}{b^2a^4} \int \frac{dx x}{(d^2+x^2)^2} = \frac{1}{b^2a^4} \frac{1}{2} \ln\left(\frac{1}{ba^2} + x^2\right) - \frac{1}{ba^4} \frac{x}{(d^2+x^2)}.$$

$$I(s) = \frac{1}{\pi c^2} \int d\varphi' \int dx x \frac{1}{[1+a^2x^2(b+dx)^2]^2} = \frac{1}{\pi c^2} \int_0^{2\pi} d\varphi' \left[\frac{1}{2b^2a^4} \ln(x^2 + \frac{1}{ba^2}) - \frac{2da^2}{b^3a^6} \left[\frac{1}{4} \frac{1}{ba^2} \frac{x}{(d^2+x^2)^2} - \frac{5}{8} \frac{x}{(d^2+x^2)} + \frac{3}{8} \sqrt{ba} \operatorname{arctg}(\sqrt{ba}x) \right] \right] S'_m(\varphi')$$

$$= \frac{1}{\pi c^2} \int d\varphi' \frac{1}{b^2a^4} \left[\frac{1}{2} \ln(1+b^2s_m^2) - \frac{2d}{b} \left[\frac{1}{4} \frac{1}{ba^2} \frac{s_m}{(\frac{1}{ba^2} + s_m^2)^2} - \frac{5}{8} \frac{s_m}{\frac{1}{ba^2} + s_m^2} + \frac{3}{8} \sqrt{ba} \operatorname{arctg}(\sqrt{ba}s_m) \right] \right], \quad ba^2 = \frac{1}{c^2(1-2s^2)} = \frac{1}{c^2} \frac{(1-2s^2)}{(1+s^2)^2} \approx \frac{1}{c^2} (1-2s^2)$$

$$\text{Вектор } \vec{r} \text{ равен } \sqrt{ba} s_m = \frac{1}{c} (1-2s^2) [c(1+2s^2) + c^2 s \cos \varphi'] \approx 1 + c s \cos \varphi'$$

$$\frac{d}{b} = -\frac{2s \cos \varphi'}{(1-2s^2)} = -2s \cos \varphi', \quad \sqrt{ba} s_m = 1 + c s \cos \varphi' + O(s^3), \quad \varepsilon = c s \cos \varphi'$$

$$I(s) = \frac{1}{\pi c^2 (1-2s^2)^4} \int d\varphi' \left[\frac{1}{2} \ln(2+2\varepsilon+\varepsilon^2) + 4s \cos \varphi' \left[\frac{1}{4} \frac{1+\varepsilon}{(1+(1+\varepsilon)^2)^2} - \frac{5}{8} \frac{1+\varepsilon}{1+(1+\varepsilon)^2} + \frac{3}{8} \operatorname{arctg}(1+\varepsilon) \right] \right] \frac{1}{c} (1-2s^2). \quad I(s=0) = \text{не то, } b I_x(0) \text{ ошибка.}$$

$$I_x(0) = \frac{1}{b^2a^4} \int \frac{dx x}{(d^2+x^2)^2} = \frac{1}{b^2a^4} \left(-\frac{1}{2}\right) \frac{1}{d^2+x^2} = -\frac{1}{2} \frac{1}{ba^2} \frac{1}{1+ba^2x^2}$$

$$I(s) = \frac{1}{\pi c^2} \int d\varphi' \left[-\frac{1}{2} \frac{1}{ba^2(ba^2x+1)} - \frac{2d}{b^3a^4} [\dots] \right] S'_m = \frac{1}{\pi c^2} \frac{1}{ba^2} \int d\varphi' x \times \left[-\frac{1}{2} \frac{1}{1+ba^2x^2} - \frac{2d}{b^2a^2} [\dots] \right] S'_m = \frac{1}{\pi c^2} \frac{1}{(1-2s^2)^2} \int d\varphi' \left[\frac{1}{2} \frac{ba^2s_m^2}{1+ba^2s_m^2} - \frac{2d}{b^2a^2} [\dots] \right]$$

$$I(s) = \frac{1}{\pi} \frac{(1+s^2)^2}{(1-2s^2)} \int d\varphi' \left[\frac{1}{2} \frac{ba^2s_m^2}{1+ba^2s_m^2} - \frac{2d}{b^2a^2} \left[\frac{1}{4} \frac{\sqrt{ba}s_m}{(1+ba^2s_m^2)^2} - \frac{5}{8} \frac{\sqrt{ba}s_m}{1+ba^2s_m^2} + \frac{3}{8} \operatorname{arctg}(\sqrt{ba}s_m) \right] \sqrt{ba} \right], \quad \sqrt{ba} s_m = 1+\varepsilon, \quad \varepsilon = c s \cos \varphi' \ll 1$$

$$I(0) = \frac{1}{\pi} \int d\varphi' \left[\frac{1}{2} \frac{1}{1+1} \right] = \frac{1}{\pi} 2\pi \frac{1}{4} = \frac{1}{2}, \quad \cos \varphi = 0, \quad Q(0) = 1/2, \quad \text{верно.}$$

$$\frac{d\sqrt{ba}}{b^2a^2} = -\frac{2s \cos \varphi'}{b^{3/2}a} = -\frac{2s \cos \varphi' \cdot c(1+s^2)}{(1-2s^2)^{3/2}} = -2sc \cos \varphi' + O(s^3)$$

$$I(s) = \frac{1}{\pi} \frac{(1+s^2)^2}{(1-2s^2)} \int_0^{2\pi} d\varphi \left[\frac{1}{2} \frac{(1+\varepsilon)^2}{1+(1+\varepsilon)^2} + 4sc \cos \varphi \left[\frac{1}{4} \frac{1+\varepsilon}{(1+(1+\varepsilon)^2)^2} - \frac{5}{8} \frac{1+\varepsilon}{1+(1+\varepsilon)^2} + \frac{3}{8} \operatorname{arctg}(1+\varepsilon) \right] \right] = \frac{1}{\pi} \frac{(1+s^2)^2}{(1-2s^2)} \int d\varphi \left[\frac{1}{2} (1+2\varepsilon) \frac{1}{2} (1-\varepsilon) + 4\varepsilon \left[\frac{1}{4} (1+\varepsilon) \frac{1}{4} (1-2\varepsilon)^2 - \frac{5}{8} (1+\varepsilon) \frac{1}{2} (1-\varepsilon) + \frac{3}{8} \left(\frac{\pi}{4} + \frac{1}{2} \varepsilon \right) \right] \right] = \frac{1}{\pi} (\dots) \int d\varphi \left[\frac{1}{4} (1+\varepsilon-2\varepsilon^2) + 4\varepsilon \left(\frac{1}{16} (1-\varepsilon-2\varepsilon^2) - \frac{5}{16} (1-\varepsilon^2) + \frac{3}{16} \left(\frac{\pi}{2} + \varepsilon \right) \right) \right] = \frac{1}{\pi} (\dots) \int d\varphi (\dots) \left[\frac{1}{4} (1-2\varepsilon^2) - \frac{1}{4} \varepsilon^2 - \frac{5}{4} \varepsilon + \frac{3}{4} \varepsilon^2 \right] = \frac{1}{\pi} (\dots) \int d\varphi \left[\frac{1}{4} \right] = \frac{1}{2} \frac{(1+s^2)^2}{(1-2s^2)} \quad \text{и не забываем про } a'' \text{ c rot } \varphi$$

$$I(s) = \frac{1}{2} (1+4s^2) + O(s^4). \quad \text{вед. мен } \text{параметра } \text{но } s^2 = c^2 s^4.$$

One core $w \frac{\partial \bar{w}}{\partial s} - \bar{w} \frac{\partial w}{\partial s} = \frac{1}{|f|^2} [f \frac{\partial \bar{f}}{\partial s} - \bar{f} \frac{\partial f}{\partial s}] = \frac{1}{f} \frac{\partial \bar{f}}{\partial s} - \frac{1}{\bar{f}} \frac{\partial f}{\partial s} = \frac{2}{s} \ln \frac{\bar{f}}{f}$.

$f = |f| e^{i\varphi}$, $\ln \frac{\bar{f}}{f} = \ln \frac{|f| e^{-i\varphi}}{|f| e^{i\varphi}} = \ln(e^{-2i\varphi}) = -2i\varphi$

One core $(\vec{m} \times \frac{\partial \vec{m}}{\partial s})_z = \frac{1}{2i} \frac{\partial}{\partial s} (-2i\varphi) = \frac{\partial \varphi}{\partial s}$

Brusl'm core $w = f$, $(\vec{m} \times \frac{\partial \vec{m}}{\partial s})_z = \frac{2i}{(1+|f|^2)^2} [f \frac{\partial \bar{f}}{\partial s} - \bar{f} \frac{\partial f}{\partial s}] = \frac{2i|f|^2}{(1+|f|^2)^2} \frac{\partial}{\partial s} \ln \frac{\bar{f}}{f} = \frac{2i|f|^2}{(1+|f|^2)^2} \frac{2}{s} = \frac{4|f|^2 \theta}{[1+|f|^2 \theta]^2} \frac{\partial \varphi}{\partial s} = 4 \cos^2 \theta \frac{\partial \varphi}{\partial s} = \sin^2 \theta \frac{\partial \varphi}{\partial s} - \text{как в Nov. '09.}$

$\vec{F}_{ST} \sim \int d\varphi \frac{\partial \varphi}{\partial s} = 0$ one core" не есть обрывок червяка, $\forall w(z, \bar{z})$.

$f(z) = i \frac{c}{c} [z - \frac{(s+s\bar{z})}{(1+s\bar{s})}]$, $\frac{\partial f}{\partial s} = -i \frac{c}{c} \frac{(1+s\bar{s}) - (s+s\bar{z})\bar{s}}{(1+s\bar{s})^2} = -i \frac{c}{c} \frac{1-\bar{s}^2 z^2}{(1+s\bar{s})^2}$

$\bar{f}(\bar{z}) = -i \frac{c}{c} [\bar{z} - \frac{(\bar{s}+s\bar{z}^2)}{(1+s\bar{s})}]$, $\frac{\partial \bar{f}}{\partial \bar{s}} = i \frac{c}{c} \frac{\bar{z}^2(1+s\bar{s}) - (\bar{s}+s\bar{z}^2)\bar{s}}{(1+s\bar{s})^2} = i \frac{c}{c} \frac{\bar{z}^2 - \bar{s}^2}{(1+s\bar{s})^2}$

$(f \frac{\partial \bar{f}}{\partial s} - \bar{f} \frac{\partial f}{\partial \bar{s}}) = i \frac{c}{c} [z - \frac{(s+s\bar{z})}{1+s\bar{s}}] i \frac{c}{c} \frac{(\bar{z}^2 - \bar{s}^2)}{(1+s\bar{s})^2} - (-i) \frac{c}{c} [\bar{z} - \frac{(\bar{s}+s\bar{z}^2)}{1+s\bar{s}}] (-i) \frac{c}{c} \frac{(1-\bar{s}^2 z^2)}{(1+s\bar{s})^2} =$

$= \frac{1}{c^2} \frac{1}{(1+s\bar{s})^3} [(\bar{z}^2(1+s\bar{s}) - (\bar{s}+s\bar{z}^2)\bar{s})(1-\bar{s}^2 z^2) - (z(1+s\bar{s}) - (s+s\bar{z})\bar{s})(\bar{z}^2 - \bar{s}^2)] =$

$= \frac{1}{c^2} \frac{1}{(1+s\bar{s})^3} [\bar{z}^2(1+s\bar{s}) - (\bar{s}+s\bar{z}^2)\bar{s} - \bar{s}^2 z^2(1+s\bar{s}) + \bar{s}^3 z^2 + s\bar{s}^2 \bar{z}^4 - \bar{s}^2 z(1+s\bar{s}) + s\bar{z}^2 + s\bar{s}^4 +$

$+ \bar{s}^2 z(1+s\bar{s}) - \bar{s}^2(s+s\bar{z}^2)] = \frac{1}{c^2} \frac{1}{(1+s\bar{s})^3} [\bar{z}^2(1+s\bar{s})(1-\bar{s}^2) - \bar{s}(1+s\bar{s}) - s\bar{z}^2 + s\bar{z}^2 + \bar{s}^4(1+s\bar{s}) +$

$+ (1-\bar{s}^2)\bar{s}^2 z(1+s\bar{s}) - \bar{s}^3 z^2 + \bar{s}^3 z^2] = \frac{1}{c^2} \frac{1}{(1+s\bar{s})^3} [(1-\bar{s}^2)(1+s\bar{s})(\bar{z}^2 + \bar{s}^2 z^2) - s\bar{z}^2 + s\bar{z}^2 +$

$+ \bar{s}(1+s\bar{s})(\bar{s}^4 - 1)] = \frac{1}{c^2} \frac{(1-\bar{s}^2)}{(1+s\bar{s})^2} [\bar{z}^2 + \bar{s}^2 z^2 + \bar{s}(1+s\bar{s})] = ? \text{ нулевая.}$

$\frac{\partial}{\partial s} \sim \frac{\partial}{\partial \bar{s}}$ нулевая, потому в центре инвариант не меняется гр.

$(f \frac{\partial \bar{f}}{\partial s} - \bar{f} \frac{\partial f}{\partial \bar{s}}) = 0$ нуль. по "s". Надо рассмотреть, что $i(f \frac{\partial \bar{f}}{\partial s} - \bar{f} \frac{\partial f}{\partial \bar{s}})$ есть.

Brusl'm v core b $\vec{F}_{ST}$ вычисл еще $\frac{1}{(1+|f|^2)^2}$ заблуждение "s", где нет.

Сначала в-в паверы 12/11/09. $\frac{\partial}{\partial s}$ где $\vec{F}_{ST}$ goes $\vec{F}_{ST}$, где

$\nabla_x = \sqrt{2} \hat{e}_- \frac{\partial}{\partial s} + \sqrt{2} \hat{e}_+ \frac{\partial}{\partial \bar{s}}$, $\hat{e}_{\pm} = \frac{1}{\sqrt{2}} (\hat{x} \mp i\hat{y})$, $\vec{F}_{ST} = \hat{e}_+ F_{ST}^+ + \hat{e}_- F_{ST}^-$, $i\varphi = \frac{\partial \varphi}{\partial x}$

$F_{ST} = 2i\sqrt{2} a_L \int d\varphi \frac{1}{(1+w\bar{w})^2} [w \frac{\partial \bar{w}}{\partial s} - \bar{w} \frac{\partial w}{\partial \bar{s}}]$. Мы же не ∇_x , а ∇_a ,

тогда рассуждаем по $a_x, a_y < 1$ мере. $\nabla_a = \sqrt{2} \hat{e}_- \frac{\partial}{\partial a} + \sqrt{2} \hat{e}_+ \frac{\partial}{\partial \bar{a}}$

$F_{ST}^-(a) = 2i\sqrt{2} a_L \int d\varphi \frac{1}{(1+w\bar{w})^2} [w \frac{\partial \bar{w}}{\partial a} - \bar{w} \frac{\partial w}{\partial \bar{a}}]$, one core" $w \frac{\partial \bar{w}}{\partial a} - \bar{w} \frac{\partial w}{\partial \bar{a}} = \frac{2}{a} \ln \frac{\bar{f}}{f}$

Brusl'm "core" $(f \frac{\partial \bar{f}}{\partial s} - \bar{f} \frac{\partial f}{\partial \bar{s}} = |f|^2 \frac{2}{a} \ln \frac{\bar{f}}{f}) : [w \frac{\partial \bar{w}}{\partial a} - \bar{w} \frac{\partial w}{\partial \bar{a}}] = \frac{|f|^2}{(1+w\bar{w})^2} \frac{2}{(1+|f|^2)^2} \frac{\partial}{\partial a} \ln \frac{\bar{f}}{f}$

12-13/11/09. $\frac{\partial \varphi}{\partial x}$ было разложено по "s", разложим по "a" угловое (pole-free)

$f(z) = \frac{1}{c} [i c z + \frac{a}{2} - \frac{\bar{a}}{2} z^2]$. $\ln \frac{\bar{f}}{f} = \ln \bar{f} - \ln f$, нуль разложим по "a".

$\ln \bar{f} - \ln f = -2i \frac{1}{2i} (\ln f - \ln \bar{f}) = -2i \text{Im}(\ln f)$, $f = \frac{i c z}{c} [1 + \frac{b}{z} + \bar{b} z]$, $b = \frac{a}{2}$

$\ln f = \ln(\frac{i c z}{c}) + \ln[1 + \frac{b}{z} + \bar{b} z]$, $b = |b| e^{i\varphi}$, $\epsilon = \frac{b}{z} + \bar{b} z = \frac{|b|}{z} e^{i(\varphi - \varphi)} + |b| e^{i(\varphi + \varphi)}$

Еще хотим $\vec{F}_{ST}(x)$ go x^3 , но $\frac{\partial \varphi}{\partial x} \sim \ln f \sim \ln f$ надо go $a^4 \sim x^4$.

$\ln(1+\epsilon) = \epsilon - \frac{1}{2} \epsilon^2 + \frac{1}{3} \epsilon^3 - \frac{1}{4} \epsilon^4$. $\epsilon = \frac{b}{z} + \bar{b} z$. $\ln(1+\epsilon) = \frac{d\epsilon}{1+\epsilon}$

One core $\vec{F}_{ST} \sim \int d\varphi \frac{\partial}{\partial a} \ln \frac{\bar{f}}{f} = (2i) \frac{\partial}{\partial a} \int d\varphi \text{Im}(\ln f)$, $\ln f = \ln(1+\epsilon)$ коэффициент

$\epsilon^2 = \frac{b^2}{z^2} + 2|b|^2 + \bar{b}^2 z^2$, ϵ^3, ϵ^4 etc. Будем b $\int d\varphi$ гоголь нуль нуль $\sim |b|^2$

не коэффициент, по ему разл $\text{Im}(\ln f) = 0$. Будем one "core" b $\vec{F}_{ST} = 0$, как в APR 2009.

9/10 Bugno & "Iberdrola". Ores 670€ 30 Jan. 15 - March 15 obrygum, no osence 2010
 gose Iberdrola una na 15/1/10. 30 ores uopremum 29/III - 3.20€, uopremu ore com.
 nepomen. rounko na Bank account. Nuznam €20.97 bill on 30/III obrygum obrygum
 "obrygum uopremu" C.A. B. "Lidl", C.A. B. uapre piete, Baden.
 E-mails or Duncan C. u A. Hoffmann // Here cash 6 Barcelona - €300.

10/10 Duzag b $\vec{F}_{ST} \sim \int d^2\vec{g} \sin^2\theta \frac{\partial \Phi}{\partial \vec{x}}$ or obrygum bue "core" = 0 b uodum uopremu
 no $\vec{x}$. $\vec{F}_{ST} = aL \int d^2\vec{g} (\vec{m} \times \frac{\partial \vec{m}}{\partial \vec{x}})_z = aL \int d^2\vec{g} \sin^2\theta \frac{\partial \Phi}{\partial \vec{x}}$ uap. rounko v. core (!)
 uapremu component $\vec{F}_{ST}^- = 2i\sqrt{2} a_L \int d^2\vec{g} \frac{1}{(1+f\bar{f})^2} \left[f \frac{\partial \bar{f}}{\partial \vec{s}} - \bar{f} \frac{\partial f}{\partial \vec{s}} \right] = 2i\sqrt{2} a_L \int d^2\vec{g} \frac{1}{(1+f\bar{f})^2} \left[f \frac{\partial \bar{f}}{\partial \vec{s}} - \bar{f} \frac{\partial f}{\partial \vec{s}} \right]$
 $\vec{F}_{ST}^- = 4\sqrt{2} a_L \int d^2\vec{g} \frac{1}{(1+f\bar{f})^2} \frac{\partial}{\partial \vec{s}} \text{Im}(\ln f)$ - uogrogut gus rounko pounre $\vec{F}_{ST}(s)$.

Duzamur. puz uopremu uymu $\vec{F}_{ST}(a) \sim \frac{\partial}{\partial a}$. $\vec{F}_{ST}^- = \frac{1}{\sqrt{2}} (\vec{F}_{ST}^x - i\vec{F}_{ST}^y)$
 $\frac{\partial}{\partial \vec{s}} = \frac{1}{2} \left(\frac{\partial}{\partial s_x} - i \frac{\partial}{\partial s_y} \right)$, $\vec{F}_{ST}(s) = 4a_L \int d^2\vec{g} \frac{1}{(1+f\bar{f})^2} \frac{\partial}{\partial \vec{s}} \text{Im}(\ln f)$
 $\text{Im}(\ln f) = \varphi$. $\frac{\partial \varphi}{\partial \vec{s}} = \frac{\partial}{\partial \vec{s}} \left(\frac{1-f\bar{f}}{1+f\bar{f}} \right) = \frac{1}{(1+f\bar{f})^2} \left[-\frac{\partial f\bar{f}}{\partial \vec{s}} (1+f\bar{f}) - (1-f\bar{f}) \frac{\partial f\bar{f}}{\partial \vec{s}} \right] =$
 $= \frac{1}{(1+f\bar{f})^2} (-2) \frac{\partial(f\bar{f})}{\partial \vec{s}}$. $\vec{I}(s) = \int d^2\vec{g} \frac{1}{(1+f\bar{f})^2} \bar{f} \frac{\partial f}{\partial \vec{s}}$ - any $f(z, s)$.

Pole free model: $\left(f \frac{\partial \bar{f}}{\partial \vec{s}} - \bar{f} \frac{\partial f}{\partial \vec{s}} \right) = \frac{1}{c^2} \frac{(1-p^2)}{(1+s\bar{s})^2} [\bar{z} + \bar{s}^2 z - \bar{s}(1+p^2)]$
 $\vec{F}_{ST}(s) = 2i\sqrt{2} a_L \int d^2\vec{g} \frac{1}{(1+f\bar{f})^2} \left[f \frac{\partial \bar{f}}{\partial \vec{s}} - \bar{f} \frac{\partial f}{\partial \vec{s}} \right] = 2i\sqrt{2} a_L \frac{1}{c^2} \frac{(1-p^2)}{(1+s\bar{s})^2} \int d^2\vec{g} \frac{(1-p^2)}{(1+f\bar{f})^2} [\bar{z} + \bar{s}^2 z - \bar{s}(1+p^2)]$
 Note: $\vec{I}(s) = \frac{1}{\pi c^2} \int d^2\vec{g} \frac{(1-p^2)}{(1+f\bar{f})^2}$ obrygum c uocremu rounko b $\vec{F}_{ST}(s)$.
 Haim $\vec{I}_z(s) = \frac{1}{\pi c^2} \int d^2\vec{g} \frac{(1-p^2)}{(1+f\bar{f})^2} \frac{z}{(1+f\bar{f})^2}$, $\vec{F}_{ST}(s) = \frac{2\pi i\sqrt{2} a_L}{(1+s\bar{s})^2} [\bar{I}_z(s) + \bar{s}^2 I_z(s) - \bar{s} I_z(s)]$
 $\vec{I}(s) = \frac{1}{2} (1+|s|^2)$ - gocremu rounko. $\vec{I}_z(s)$ rounko c rounko go $\sim s^3$ (uocremu).
 Zero $\vec{I}_z(0)=0$, r.e. $\vec{I}_z(s) \sim s$ uym $\sim \bar{s}$.

Rigid v. model: $f(z) = i\frac{c}{2} (z-s)$, $\ln f = \ln i\frac{c}{2} z(1-\frac{s}{z})$, $\epsilon = -\frac{s}{z}$
 $\ln f = \text{const}(s) + \ln(1+\epsilon) = \sum_{n=1}^{\infty} \frac{\epsilon^n}{n} (-1)^{n+1} = -\sum_{n=1}^{\infty} \frac{1}{n} \frac{s^n}{z^n} = -\sum_{n=1}^{\infty} \frac{s^n}{n} \frac{1}{z^n} e^{-in\varphi}$
 $\ln f = -\sum_{n=1}^{\infty} \frac{1}{n} \frac{|s|^n}{z^n} e^{in\varphi} e^{in\varphi}$, $\int d\varphi \text{Im}(\ln f) = \int d\varphi \sum_{n=1}^{\infty} \frac{1}{n} \frac{|s|^n}{z^n} \sin n(\varphi - \varphi) = 0$
 Rounko obrygum bue "core" to $\vec{F}_{ST}=0$.
 $\left(f \frac{\partial \bar{f}}{\partial \vec{s}} - \bar{f} \frac{\partial f}{\partial \vec{s}} \right)_{z.v.} = -\bar{f} \frac{\partial f}{\partial \vec{s}} = -(-i)\frac{c}{2} (\bar{z}-\bar{s})(-i)\frac{c}{2} = \frac{1}{c^2} (\bar{z}-\bar{s})$

11/10 $\vec{F}_{ST}(s) \sim \int d^2\vec{g} \frac{1}{(1+f\bar{f})^2} \left[f \frac{\partial \bar{f}}{\partial \vec{s}} - \bar{f} \frac{\partial f}{\partial \vec{s}} \right] = \frac{1}{c^2} \int d^2\vec{g} \frac{(\bar{z}-\bar{s})}{[1+\frac{|\bar{z}-\bar{s}|^2}{c^2}]^2} = \frac{1}{c^2} \int d^2\vec{g} \frac{1}{[1+\frac{|\bar{z}-\bar{s}|^2}{c^2}]^2} \frac{\bar{z}-\bar{s}}{c^2}$
 Haim b pole-free model, uym b rigid v. model rounko gus uym
 uapremu bue "core", r.e. core component ($s \neq 0$).
 $\vec{F}_{ST} = aL \int d^2\vec{g} \sin^2\theta \frac{\partial \Phi}{\partial \vec{x}} = aL \left[\int d^2\vec{g} \frac{\partial \Phi}{\partial \vec{x}} - \int d^2\vec{g} \cos^2\theta \frac{\partial \Phi}{\partial \vec{x}} \right] = -aL \int d^2\vec{g} m_z^2 \frac{\partial \Phi}{\partial \vec{x}}$
 $\int d^2\vec{g} \frac{\partial \Phi}{\partial \vec{x}} = 0$. $\int d^2\vec{g} \frac{\partial \Phi}{\partial \vec{x}} \sim \frac{\partial}{\partial \vec{x}} \int d^2\vec{g} \Phi = \frac{\partial}{\partial \vec{x}} \int_0^{2\pi} d\varphi \ln(f) = 0$, $\int d^2\vec{g} = \int d^2\vec{p} + \int d^2\vec{q}$
 Haim bue pounre - uig. vortex $\vec{F}_{ST} \sim \int d^2\vec{g} F(|s|^2) (\bar{z}-\bar{s}) = 0$.

$$\Phi(\vec{r}, s) = \cos \chi(s) + \arctan\left(\frac{s - s_y}{x - s_x}\right), \quad \frac{\partial \Phi}{\partial s} = \frac{1}{2} \left(\frac{\partial \Phi}{\partial s_x} - i \frac{\partial \Phi}{\partial s_y} \right), \quad \Phi = \text{Im} \ln f$$

$$\frac{\partial \Phi}{\partial s} = \frac{1}{2} \text{Im} \ln(2-s). \quad 2 \frac{\partial \Phi}{\partial s} = \frac{1}{1 + \left(\frac{\Delta y}{\Delta x}\right)^2} \frac{\Delta y}{\Delta x^2} (-1) - i \frac{1}{1 + \left(\frac{\Delta y}{\Delta x}\right)^2} \frac{(-1)}{\Delta x} = \frac{\Delta y + i \Delta x}{\Delta x^2 + \Delta y^2}$$

$$\frac{\partial \Phi}{\partial s} = \frac{1}{2} i \frac{(x-s_x) - i(y-s_y)}{(x-s_x)^2 + (y-s_y)^2} = \frac{1}{2} i \frac{(z-s)}{|z-s|^2} \sim F_{ST} \sim \int_{\text{core}} d^2 \vec{p} m_z^2 (k-s) \frac{(z-s)}{|z-s|^2} \sim \int_{\text{core}} d^2 \vec{p}$$

где $z' = z - s$ и $\int_{\text{core}} d^2 \vec{p}$ и $\int_{\text{core}} d^2 \vec{p}$ no v. core - usensivno $z=s$.

Выводы: 1) F_{ST} on ягген. v. core + usensivno; 2) $F_{ST} = 0$ где shifted $z=s$ core.

$$F_{ST}^-(s) = -a_L \int_{\text{core}} d^2 \vec{p} \cos 2\theta \frac{\partial \Phi}{\partial s} \sqrt{2} = -\sqrt{2} a_L \int_{\text{core}} d^2 \vec{p} \left(\frac{1-f\bar{f}}{1+f\bar{f}} \right)^2 \frac{\partial}{\partial s} \text{Im}(\ln f)$$

$$\bar{F}_{ST}^-(s) = -a_L \int_{\text{core}} d^2 \vec{p} \left(\frac{1-f\bar{f}}{1+f\bar{f}} \right)^2 \frac{\partial}{\partial s} \text{Im}(\ln f), \quad \Phi = \text{Im}(\ln f), \quad f = |f| e^{i\chi}$$

$$f(z) = -\frac{\bar{a}}{2c} (z-s)(z-\frac{s}{|s|^2}), \quad a = -2ic \frac{s}{1+|s|^2}, \quad \ln f = \ln\left(-\frac{\bar{a}}{2c}\right) + \ln(z-s) + \ln\left(z-\frac{s}{|s|^2}\right)$$

$$-\frac{\bar{a}}{2c} = -\frac{1}{2c} 2ic \frac{\bar{s}}{1+|s|^2}, \quad \ln f = \ln s + \ln \frac{\bar{s}}{1+|s|^2} + \ln(z-s) + \ln\left(z-\frac{1}{\bar{s}}\right)$$

$$\frac{\partial \ln f}{\partial s_x} = \frac{1+|s|^2}{s} \frac{(1+s\bar{s}) - \bar{s} 2s_x}{(1+s\bar{s})^2} + \frac{(-1)}{z-s} + \frac{1}{(z-\frac{1}{\bar{s}})} \frac{1}{\bar{s}^2}$$

У нас $\frac{\partial}{\partial s} \text{Im}(\ln f) = -\frac{1}{2i} \frac{\partial}{\partial s} \ln \frac{f}{\bar{f}} = \frac{i}{2} \frac{1}{|s|^2} \left[f \frac{\partial \bar{f}}{\partial s} - \bar{f} \frac{\partial f}{\partial s} \right] = \frac{i}{2} \frac{1}{|s|^2} \frac{1}{c^2} \frac{(1-s^2)}{(1+s\bar{s})^2} [\bar{z} + \bar{s}^2 z - \bar{s}(1+s^2)]$

$$F_{ST}^-(s) = -\frac{\sqrt{2}}{2} i \frac{a_L}{c^2} \frac{1}{(1+s\bar{s})^2} \int_{\text{core}} d^2 \vec{p} (1-p^2) \left(\frac{1-f\bar{f}}{1+f\bar{f}} \right)^2 \frac{[\bar{z} + \bar{s}^2 z - \bar{s}(1+s^2)]}{|s|^2}$$

$$[\bar{z} + \bar{s}^2 z - \bar{s}(1+s^2)] = -(\bar{z}-\bar{s})(s z - 1), \quad |s|^2 = \frac{1}{c^2} \frac{1}{(1+s\bar{s})^2} |z-s|^2 |s z - 1|^2, \quad f(z) = -i \frac{c}{1+s\bar{s}} (z-s)(z-\frac{1}{\bar{s}})$$

$\rightarrow F_{ST}^-(s) = \sqrt{2} i a_L \int_{\text{core}} d^2 \vec{p} (1-p^2) \left(\frac{1-f\bar{f}}{1+f\bar{f}} \right)^2 \frac{1}{(z-s)(s\bar{z} - 1)}$ core no core

Габриэль и м. др. $z \approx s$ $F_{ST}^-(s) = -\sqrt{2} i a_L \int_{\text{core}} d^2 \vec{p} \frac{1}{(z-s)}$

$z' = z - s$, core. $4/\sqrt{12}$ φ $\varphi_m(\varphi') = c(1+2s^2) + c^2 s \cos \varphi'$

$$F_{ST}^-(s) \approx -\sqrt{2} i a_L \int_{\text{core}} d\varphi' \int_0^{2\pi} d\varphi' \frac{\bar{z}'}{|z'|^2} = -\sqrt{2} i a_L \int_{\text{core}} d\varphi' \int_0^{2\pi} d\varphi' \frac{1}{p'} (c \cos \varphi' - i s \sin \varphi')$$

$$= -\sqrt{2} i a_L \int_{\text{core}} d\varphi' (\cos \varphi' - i s \sin \varphi') [c(1+2s^2) + c^2 s \cos \varphi'] = -\sqrt{2} i a_L c^2 s \pi = \frac{1}{\sqrt{2}} (F_{ST}^x - i F_{ST}^y)$$

где $s = i(s_y + i s_x) = -s_y + i s_x$, $F_{ST}^x = 2\pi a_L c^2 s_y$, $F_{ST}^y = 2\pi a_L c^2 s_x$

где $F_{ST} \sim i \bar{s}$, $F_{ST} \sim (z \times \bar{s})$.

Монтер и др. $F_{ST}^-(s) \sim \int_{\text{core}} d^2 \vec{p} \sin 2\theta \frac{\partial \Phi}{\partial s} = \int_{\text{core}} d^2 \vec{p} \frac{\partial \Phi}{\partial s}$

$\rightarrow 4/\sqrt{12}$ $s = s_x$, $\varphi_m(\varphi') = c(1+2|s|^2) + c^2 \bar{s} \cos \varphi'$

Полосов и др. $4/\sqrt{12}$. Углубленная из JAP "Nanotubes" φ $\varphi_m(\varphi') = c(1+2|s|^2) + c^2 \bar{s} \cos \varphi'$

Особенности и модели в JMMM - MS by Zhmetya "DW a-helices" - major transition

(12/12) Власова Оксана С., F. Gasanova-NanoGUNE, Gristina. Pole-free core

$$\text{где } z' = z - s, \quad f(z) = -i \frac{c}{1+|s|^2} z' (s z' + |s|^2 - 1), \quad c^2 |f(z)|^2 = \frac{|z'|^2}{(1+s\bar{s})^2} [k^2 |z'|^2 + (|s|^2 - 1)^2]$$

$$[-] = |z'|^2 + (|s|^2 - 1) \{ |z'|^2 + s z' + \bar{s} z' + |s|^2 - 1 \} = |z'|^2 + (|s|^2 - 1) \{ |z' + s|^2 - 1 \}$$

$$[...] = 1 + |z'|^2 - |z' + s|^2 + |s|^2 \{ |z' + s|^2 - 1 \} = 1 - (s z' + \bar{s} z') + |s|^2 \{ |z'|^2 + s z' + \bar{s} z' + |s|^2 - 1 \}$$

$$f_1 = \frac{1}{2\pi} \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \quad f_2 = \frac{1}{2\pi} \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \quad f_3 = \frac{1}{2\pi} \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

Ein Ansatz $c \sim 3 \ll 1$, so $12|1|^2|s|^2 = O(c^2 s^2)$. c vernachlässigen
 in der Vektorkomponente von $\vec{E}$: $|\vec{E}' + i\vec{B}' - 1|^2 = 1 - (\vec{E}' + \vec{B}') \cdot 2i\vec{s}^2$
 in der skalaren Komponente $\sim s^4, s^2 c^2, s^2 c$
 $\frac{1}{(1+s^2)^2} |\vec{E}' + i\vec{B}' - 1|^2 = (1-2\vec{E}' \cdot \vec{s}) (1 - \vec{E}' \cdot \vec{s} - \vec{B}' \cdot \vec{s} - 2i\vec{s}^2) = 1 - \vec{E}' \cdot \vec{s} - \vec{B}' \cdot \vec{s} - 4i\vec{s}^2$
 - Geht es nicht. 4/12. Prämissen sind $|\vec{s}|^2 = 1$, $12|1|^2(1 - \vec{E}' \cdot \vec{s} - \vec{B}' \cdot \vec{s} - 4i\vec{s}^2) = c^2$
 $\vec{s}'(1 - 2i\vec{E}' \cdot \vec{s} \cos(\varphi' - \varphi) - 4i\vec{s}^2) = c$. Dem. $\vec{s}'(\varphi') = c(1 + 2i\vec{s}^2) + c^2(2i\vec{s} \cos(\varphi' - \varphi))$
 $[1 + 2i\vec{s}^2 + c|\vec{s}| \cos \Delta\varphi][1 - 2i\vec{s} \cos \Delta\varphi \cdot c - 4i\vec{s}^2] \stackrel{1}{=} 1$
 $[1 + 2i\vec{s}^2 + c|\vec{s}| \cos \Delta\varphi][1 - |\vec{s}|c \cos \Delta\varphi - 2i\vec{s}^2 - \frac{1}{2}|\vec{s}|^2 \cos^2 \Delta\varphi] = 1 -$
 - können c vernachlässigen. $\vec{s}'(\varphi') = c(1 + 2i\vec{s}^2) + c^2[\vec{s}_x \cos \varphi' + \vec{s}_y \sin \varphi']$
 $\vec{F}_{ST}^-(s) = -\sqrt{2}ial \int_0^{2\pi} d\varphi' (\cos \varphi' - i \sin \varphi') [c(1 + 2i\vec{s}^2) + c^2 \vec{s}_x \cos \varphi' + c^2 \vec{s}_y \sin \varphi'] =$
 $= -\sqrt{2}ial \cdot \pi c^2 (\vec{s}_x - i\vec{s}_y) = \sqrt{2} al \pi c^2 (-\vec{s}_y - i\vec{s}_x) = \frac{1}{\sqrt{2}} (F_{ST}^x - iF_{ST}^y)$
 $F_{ST}^x = 2\pi c^2 al (-\vec{s}_y), F_{ST}^y = 2\pi c^2 al \vec{s}_x, \vec{F}_{ST} = 2\pi c^2 al (\hat{z} \times \vec{s})$
 Genaue Größe c^2 ? u. Richtung ist immer unklar.

(20/IV) Juli in J. vonbornen in MICINN (a B. 1/2 Jahre N3 als participant outside Spain)
 ST-force: $F_{ST}^d = al \int d^2\vec{s} \left(\vec{m} \times \frac{\partial \vec{m}}{\partial X_d} \right)_d = al \int d^2\vec{s} \sin^2 \theta \frac{\partial \vec{P}}{\partial X_d}, \vec{m} = \vec{m}(\theta, \varphi)$
 Dann wenn $\int d^2\vec{s} \frac{\partial \vec{P}}{\partial X_d} = 0$, so ist $F_{ST}^d = -al \int d^2\vec{s} \cos^2 \theta \frac{\partial \vec{P}}{\partial X_d}$, C. 11/12.
 $\int d^2\vec{s} \frac{\partial \vec{P}}{\partial X_d} = \frac{\partial}{\partial X_d} \int d^2\vec{s} \vec{P} = \frac{\partial}{\partial X_d} \int d^2\vec{s} \text{Im}(\ln f). w = |w| e^{i\varphi}, |w| = \left\{ \begin{matrix} 1, & \text{für } \varphi = 0 \\ 1, & \text{für } \varphi = \pi \end{matrix} \right\}$

$(\vec{m} \times \frac{\partial \vec{m}}{\partial s_x})_z = \frac{2i}{(1+w\bar{w})^2} [w \frac{\partial \bar{w}}{\partial s_x} - \bar{w} \frac{\partial w}{\partial s_x}]$. For one case $[w \frac{\partial \bar{w}}{\partial s_x} - \bar{w} \frac{\partial w}{\partial s_x}] = \frac{\partial}{\partial s_x} \ln \frac{f}{g}$
 For one case $(\vec{m} \times \frac{\partial \vec{m}}{\partial s_x})_z = \frac{\partial \Phi}{\partial s_x}$ & given $\ln \frac{f}{g} = -2i \Phi$

Bone core $(m \times \frac{dm}{dz})_z = \frac{\partial \Phi}{\partial z}$ c.g. given $\ln f = -2i\Phi$ month day

Using the core $(w=f)$ $\left[w \frac{\partial w}{\partial x} - \bar{w} \frac{\partial \bar{w}}{\partial x} \right] = \left[f \frac{\partial f}{\partial x} - \bar{f} \frac{\partial \bar{f}}{\partial x} \right] = (ff) \frac{\partial}{\partial x} \ln \frac{f}{\bar{f}} = |f|^2 \frac{\partial}{\partial x} \ln f$

Drift for Coro: $\left(\bar{m} \times \frac{\partial \bar{m}}{\partial s_d} \right)_z = \frac{2i}{(1+u\bar{u})^2} [\dots] = \frac{2i}{(1+|f|^2)^2} (-2i) |f|^2 \frac{\partial f}{\partial s_d} = \frac{4|f|^2}{(1+|f|^2)^2}$

$$\Phi = \arg(w) = \arg\left(\frac{f - \frac{(1+uw)}{s/|s| - \text{one one}}}{s/|s| - \text{one one}}\right) = \arg(f), \quad \text{and } \arg(f) = 0.$$

No path. $\int_{\text{dot}} d^2g^* \frac{\partial \Phi}{\partial s_x} = \frac{\partial}{\partial s_x} \int_{\text{dot}} d^2g^* \Phi = \frac{\partial}{\partial s_x} \int_{\text{dot}} d^2g^* \text{Im}(\ln f)$, i.e. $\Phi = \arg(f) = -\text{Im}(\ln f)$

$$f(z) = i \frac{a}{c} z [1 + b z^2 + \bar{b} \bar{z}^2] = \frac{1}{c} [i c z + \frac{a}{2} z^3 - \frac{\bar{a}}{2} \bar{z}^3] \quad , \quad b = \frac{1}{i c} \frac{a}{2} \quad - \text{маючи на увазі, що } |b| \ll 1$$

$$\ln f = \ln(i \frac{a}{z}) + \ln(1 + \frac{b}{z} + \bar{b}z) = \text{const}(S_2) + \ln(1 + \frac{b}{z} + \bar{b}z)$$

$$\oint_{\text{dot}} d\varphi \frac{\partial \varphi}{\partial z_1} = \frac{\partial}{\partial z_1} \oint_{\text{dot}} d\varphi \ln f, \quad \oint_{\text{dot}} d\varphi \ln f = \oint_{\text{dot}} d\varphi \ln(1+\epsilon) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)} \oint_{\text{dot}} \epsilon^{n+1} d\varphi =$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \int_0^{2\pi} E^n d\varphi = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \int_0^{2\pi} |B|_{n-2\pi}^n (u \text{ here } \int_0^{2\pi} d\varphi e^{im\varphi} = 0, m \neq 0) - \text{pure real.}$$

$$\int_0^{2\pi} \Im m(f) d\varphi = \Im m(\text{pure real}) = 0 \quad \text{even} \quad \int_0^{2\pi} d\varphi(\dots), \text{ t.e. over whole disk.}$$

Pygros. e-mail to Miguel Barri (Univ. Barcelona, esp. for MICINN projects in Russia)
0. also known by MICINN-2010 grant call. Писема О. Сухомеда, и К. Морозов

21/IV Nogor. u nekan e-mails Ordizia-Conf. invitations to Parkin, Bader, Oso,
Kasegawa, Otani, Gavin Ross, Baibich, Mohri Stamps, Shin. // Parkin, Oso u
Otani gam amacue, Kasegawa amacue. // Miguel Rubi - namcan 150 wad
ne gormer nica. u MIZIN project u amamot amacue. nogor.
Namer Buresb & Brindzi/Lecca uo + SN - 200 Conf.

22/IV Neperennica - invited speakers Ordizia - 2011. Kyrius Ryanair's Super Bazar
(Girona) $\Rightarrow$ Brindisi. Neaymyn nymam. Stuttgart o: H. Stoll. Haran mears
SW - compl. paper c Pappaz u Tres.

(23/IV) Nepravice c. R. stamps u H. Stall. letter to A. Cepra (Kaiserstandern) o Boz. byzue.

Shabat BV. Introd. Complex analysis: $\oint_{\partial D} f dz = 2i \iint_D \frac{\partial f}{\partial \bar{z}} dx dy$ - Goursat's formula.
 $f = f(z, \bar{z})$.

Рассм. $\ln f(z) = \ln |f| + i\Phi(x,y) = F(z)$ - analytic f. кривою $z=5$ (если).

$F(z) = u(x,y) + i v(x,y) \rightarrow u, v$ - harmonic, $\Delta u = \Delta v = 0$, $v = \Phi(x,y)$ - harmonic

в области аналитичности $\ln f(z)$, т.е. вогну криво $\tau: z = S$.

Решение $\int dx dy v(x,y)$, где $v(x,y)$ - гармоническая.

$\oint_V \text{div } \vec{A} = \oint_{\partial V} (\vec{A} \cdot \vec{n}) dS$, $(\vec{A} \cdot \vec{n}) dS = A_x dy dz - A_y dx dz + A_z dx dy$, $\vec{n}$ - нормаль к поверхности

Exm $A_x = \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $A_y = \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ to $f(z) = u(x,y) + i v(x,y) = \dots$

$$\vec{A} = A_x + i A_y = \overline{f'(z)} \quad \text{u} \quad \Gamma + iN = \int f'(z) dz, \quad \Gamma = \oint \vec{A} d\vec{e} = \int (A_x dx + A_y dy)$$

$\vec{F} \cdot \vec{N}$ - work done by the force $\vec{F}$ on the surface $\vec{N}$. $N = \oint \vec{F} \cdot d\vec{s} = \int (A_x dy - A_y dx)$

1) $\oint_{\partial D} \vec{A} \cdot d\vec{s} = \oint_{\partial D} \vec{A} \cdot \vec{\tau} \, dl$. Кого найдем? $\vec{A}(z) = A_x - iA_y$. В $\mathbb{C}^2(\dots)$ будем κ $\frac{d}{dz}$

$$\text{div } \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} = \Delta u, \quad (\text{rot } \vec{A})_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = -\Delta v, \quad \Delta u = 0, \Delta v = 0$$

осевых точек поля $\vec{A}$ или потенциал $\varphi(z)$. Для векра: $\vec{A} = \frac{\Gamma}{2\pi} \frac{z}{z^2}$ (где Γ —

$$\text{Im}(\log(z)) = \arg(z) = \theta = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{z} dz = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{e^{i\theta}} e^{i\theta} d\theta = \frac{1}{2\pi} \int_0^{2\pi} 1 d\theta = \frac{1}{2\pi} \cdot 2\pi = 1$$

Рассмотрим функцию $u(x,y)$ в области D , удовлетворяющую уравнению $\Delta u = 0$, т.е. $\Delta u = 0$ в D . Тогда $u(x,y)$ можно представить в виде $u(x,y) = \frac{1}{2\pi} \int_0^{2\pi} \Phi(\theta) d\theta$, где $\Phi(\theta) = \frac{\partial f}{\partial \theta}$.

$$f(z) = \ln f(z) = \ln\left(\frac{z}{2}\right) = \ln\left(\frac{re^{i\theta}}{2}\right) = \ln\left(\frac{r}{2}\right) + i\theta = \ln\left(\frac{r}{2}\right) + i\theta$$

$$\frac{\partial f}{\partial \theta} = \frac{1}{2} \left(\frac{\partial \ln r}{\partial \theta} + \frac{\partial i\theta}{\partial \theta} \right) = \frac{1}{2} \left(0 + i \right) = \frac{i}{2}$$

$$f(z) = \ln f(z) + f_1(z) = \ln\left(\frac{z}{2}\right) + f_1(z) = \ln\left(\frac{re^{i\theta}}{2}\right) + f_1(z) = \ln\left(\frac{r}{2}\right) + i\theta + f_1(z)$$

$$f(z) = \ln\left(\frac{z}{2}\right) + f_1(z) = \ln\left(\frac{re^{i\theta}}{2}\right) + f_1(z) = \ln\left(\frac{r}{2}\right) + i\theta + f_1(z)$$

$$f(z) = \ln\left(\frac{z}{2}\right) + f_1(z) = \ln\left(\frac{re^{i\theta}}{2}\right) + f_1(z) = \ln\left(\frac{r}{2}\right) + i\theta + f_1(z)$$

$$f(z) = \ln\left(\frac{z}{2}\right) + f_1(z) = \ln\left(\frac{re^{i\theta}}{2}\right) + f_1(z) = \ln\left(\frac{r}{2}\right) + i\theta + f_1(z)$$

$$f(z) = \ln\left(\frac{z}{2}\right) + f_1(z) = \ln\left(\frac{re^{i\theta}}{2}\right) + f_1(z) = \ln\left(\frac{r}{2}\right) + i\theta + f_1(z)$$

$$f(z) = \ln\left(\frac{z}{2}\right) + f_1(z) = \ln\left(\frac{re^{i\theta}}{2}\right) + f_1(z) = \ln\left(\frac{r}{2}\right) + i\theta + f_1(z)$$

$$f(z) = \ln\left(\frac{z}{2}\right) + f_1(z) = \ln\left(\frac{re^{i\theta}}{2}\right) + f_1(z) = \ln\left(\frac{r}{2}\right) + i\theta + f_1(z)$$

$$f(z) = \ln\left(\frac{z}{2}\right) + f_1(z) = \ln\left(\frac{re^{i\theta}}{2}\right) + f_1(z) = \ln\left(\frac{r}{2}\right) + i\theta + f_1(z)$$

$$f(z) = \ln\left(\frac{z}{2}\right) + f_1(z) = \ln\left(\frac{re^{i\theta}}{2}\right) + f_1(z) = \ln\left(\frac{r}{2}\right) + i\theta + f_1(z)$$

$$f(z) = \ln\left(\frac{z}{2}\right) + f_1(z) = \ln\left(\frac{re^{i\theta}}{2}\right) + f_1(z) = \ln\left(\frac{r}{2}\right) + i\theta + f_1(z)$$

$$f(z) = \ln\left(\frac{z}{2}\right) + f_1(z) = \ln\left(\frac{re^{i\theta}}{2}\right) + f_1(z) = \ln\left(\frac{r}{2}\right) + i\theta + f_1(z)$$

$$f(z) = \ln\left(\frac{z}{2}\right) + f_1(z) = \ln\left(\frac{re^{i\theta}}{2}\right) + f_1(z) = \ln\left(\frac{r}{2}\right) + i\theta + f_1(z)$$

$$f(z) = \ln\left(\frac{z}{2}\right) + f_1(z) = \ln\left(\frac{re^{i\theta}}{2}\right) + f_1(z) = \ln\left(\frac{r}{2}\right) + i\theta + f_1(z)$$

$$F_{st} \sim \int d\vec{p} \left(\vec{m} + \frac{\partial \vec{m}}{\partial \vec{p}} \right) \cdot \vec{z} = 2i \int d\vec{p} \frac{1}{(1+w\bar{w})^2} \left[w \frac{\partial \bar{w}}{\partial \vec{p}} - \bar{w} \frac{\partial w}{\partial \vec{p}} \right] = 2i \int d\vec{p} \frac{w \bar{w}}{(1+w\bar{w})^2} \left[\frac{1}{w} \frac{\partial \bar{w}}{\partial \vec{p}} - \frac{\partial w}{\partial \vec{p}} \right]$$

$$\ln \frac{\bar{w}}{w} = \ln \bar{w} - \ln w = -2i\varphi = -2i \operatorname{Im}(\ln w), \quad w = |w|e^{i\varphi}, \quad \varphi = \arg w$$

$$F_{st} \sim 2i \int d\vec{p} \frac{w \bar{w}}{(1+w\bar{w})^2} \frac{\partial}{\partial \vec{p}} \ln \frac{\bar{w}}{w} = 4 \int d\vec{p} \frac{w \bar{w}}{(1+w\bar{w})^2} \frac{\partial}{\partial \vec{p}} \operatorname{Im}(\ln w) = 4 \operatorname{Im} \int d\vec{p} \frac{w \bar{w}}{(1+w\bar{w})^2} \frac{\partial}{\partial \vec{p}} \ln w$$

$$I_2(z) = \int d\vec{p} \frac{w \bar{w}}{(1+w\bar{w})^2} \cdot \frac{\partial}{\partial \vec{p}} \frac{1}{(1+w\bar{w})} = -\frac{1}{(1+w\bar{w})^2} \left(w \frac{\partial \bar{w}}{\partial \vec{p}} + \bar{w} \frac{\partial w}{\partial \vec{p}} \right) = \text{real part}$$

$$f(z) = \frac{1}{z} \left(i\bar{z} + \frac{a}{z} - \frac{\bar{a}}{2} z^2 \right) = -\frac{\bar{a}}{2c} (z-s)(z-\frac{1}{\bar{s}}), \quad a = -2ic \frac{s}{(1+s\bar{s})}, \quad s = |s|e^{i\varphi_s}$$

$$\varphi(x,y) = \operatorname{Arg} f(z) = \operatorname{Arg} \left(-\frac{\bar{a}}{2c} \right) + \operatorname{Arg} (z-s) + \operatorname{Arg} \left(z - \frac{s}{|s|^2} \right) = \sum \operatorname{arctg} \frac{y-y_j}{x-x_j}$$

$$\operatorname{Arg} \left(-\frac{\bar{a}}{2c} \right) = -\operatorname{Arg} \bar{a} = \operatorname{Arg} a = \operatorname{Arg} [-e^{i\varphi} \frac{s}{2}] = -\varphi_s - \frac{\pi}{2} = -\operatorname{arctg} \frac{y-s_y}{x-s_x}$$

$$\operatorname{Arg} \left(-\frac{\bar{a}}{2c} \right) = \operatorname{Arg} (-\bar{a}) = \operatorname{Arg} (e^{i\pi} \bar{a}) = \pi + \operatorname{Arg} \bar{a} = \pi + \operatorname{Arg} (e^{i\varphi_s}) = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$$

$$\varphi(x,y) = \operatorname{const}(c) - \varphi_s + \operatorname{Arg} (z-s) + \operatorname{Arg} \left(z - \frac{s}{|s|^2} \right) = -\operatorname{arctg} \frac{y-s_y}{s_x} + \operatorname{arctg} \frac{y-s_y}{x-s_x} + \operatorname{arctg} \frac{y-s_y}{x-\frac{s_x}{|s|^2}}$$

$$\varphi(x,y) = -\operatorname{arctg} \frac{y}{x} + \operatorname{arctg} \left(\frac{y-y}{x-x} \right) + \operatorname{arctg} \left(\frac{y|X|^2 - y}{x|X|^2 - x} \right), \quad \varphi(0,0) = \operatorname{arctg} \frac{y}{x}$$

$$\lim_{|X| \rightarrow 0} \varphi(x,y) \rightarrow \operatorname{arctg} \frac{y}{x} = \varphi + \operatorname{const}(s)$$

$$\operatorname{arctg} a + \operatorname{arctg} b = \operatorname{arctg} \left(\frac{a+b}{1-ab} \right), \quad \operatorname{mod}(\pi), \quad ab \neq 1$$

$$\operatorname{arctg} \left(\frac{y-y}{x-x} \right) - \operatorname{arctg} \frac{y}{x} = \operatorname{arctg} \left[\frac{\frac{y-y}{x-x} - \frac{y}{x}}{1 + \left(\frac{y-y}{x-x} \right) \frac{y}{x}} \right] = \operatorname{arctg} \left[\frac{yX - xY}{xX + yY - |X|^2} \right]$$

$$\varphi(x,y) = \operatorname{arctg} \left[\frac{yX - xY}{xX + yY - |X|^2} \right] + \operatorname{arctg} \left(\frac{y|X|^2 - y}{x|X|^2 - x} \right) = \operatorname{arctg} \left[\frac{yX - xY}{xX + yY - |X|^2} + \frac{y|X|^2 - y}{x|X|^2 - x} \right]$$

$$= \operatorname{arctg} \left[\frac{yX - xY + y|X|^2 - y}{xX + yY - |X|^2 + x|X|^2 - x} \right] = \operatorname{arctg} \left[\frac{yX - xY + y^2 - y}{xX + yY - |X|^2 + x|X|^2 - x} \right]$$

$$\varphi(x,y) = \operatorname{arctg} \left[\frac{yX - xY + y^2 - y}{xX + yY - |X|^2 + x|X|^2 - x} \right] = \operatorname{arctg} \left[\frac{yX - xY + y^2 - y}{xX + yY - |X|^2 + x|X|^2 - x} \right]$$

$$\delta(z) = -\frac{\bar{a}}{2c} (z-s)(z-\frac{1}{\bar{s}}), \quad \varphi(x,y) = \operatorname{arctg} \frac{F_1(x,y)}{F_2(x,y)}$$

$$\frac{\partial \varphi}{\partial x} = \frac{1}{1 + \frac{F_1^2}{F_2^2}} \frac{\partial_x F_1 \cdot F_2 - F_1 \cdot \partial_x F_2}{F_2^2} = \frac{1}{F_1^2 + F_2^2} \left[(2yX - 2yX)(X-x + 2xyY + (x^2 - y^2)X - x|X|^2) - (1 + x^2 - y^2 - 2xX)(y - y + 2yX + (y^2 - x^2)Y - y|X|^2) \right]$$

$$\frac{\partial \varphi}{\partial x} = -\frac{1}{1 + \frac{y^2}{x^2}} \frac{(-y)}{x^2} + \frac{1}{1 + \left(\frac{y-y}{x-x} \right)^2} \frac{(y-y)}{(x-x)^2} + \frac{1}{1 + \left(\frac{y|X|^2 - y}{x|X|^2 - x} \right)^2} \frac{(y|X|^2 - y)}{(x|X|^2 - x)^2}$$

$$\frac{\partial \varphi}{\partial x} = \frac{y}{x^2 + y^2} + \frac{(y-y)}{|x-x|^2} + \frac{y|X|^2 - 2yX^2 + 2xXY - y}{|x|X|^2 - x|^2|}$$

$$\frac{\partial \varphi}{\partial x} = \frac{y}{x^2 + y^2} + \frac{(y-y)}{|x-x|^2} + \frac{y|X|^2 - 2yX^2 + 2xXY - y}{|x|X|^2 - x|^2|}$$

$$\Delta V = 1 \quad \text{under perm.} \quad \chi(p) = 6 \ln |\vec{p} - \vec{p}'| d^3 p'$$

$$\int d^3 p' \ln [p^2 + p'^2 - 2pp' \cos(\theta - \varphi')] = 2\pi \ln \left[p^2 + p'^2 + \sqrt{(p^2 + p'^2)^2 - 4p^2 p'^2} \right] = 2\pi \ln \left[\frac{p^2 + p'^2 + \sqrt{(p^2 + p'^2)^2 - 4p^2 p'^2}}{2} \right]$$

$$= 4\pi \left\{ \ln p, p > p' \right\} = 4\pi F(p, p'), \quad \int d^3 p' \ln F(p, p') = 4\pi \int d^3 p' \ln p + \int d^3 p' \ln p' = 4\pi \ln p \cdot \frac{1}{2} p^2 + 4\pi \left[\frac{1}{2} \ln x - \frac{x^2}{4} \right]_0^p = 2\pi p^2 \ln p + 4\pi \left[\frac{1}{2} \cdot 0 - \frac{1}{4} - \frac{1}{2} p^2 \ln p + \frac{p^2}{4} \right]$$

$$= \pi(p^2 - 1), \quad \Delta = \frac{\partial^2}{\partial p^2} + \frac{1}{p} \frac{\partial}{\partial p} + \frac{1}{p^2} \frac{\partial^2}{\partial \varphi^2}, \quad V = \frac{1}{4} p^2 \quad \text{сعر pemenne } \Delta V = 1$$

$$\nabla \cdot \mathbf{v} = \oint \frac{\partial}{\partial \rho} \left(\frac{1}{4} \rho^2 \right) = \frac{1}{2} \rho \hat{\rho}, \quad \iint_{\partial V} d\mathbf{a} [\mathbf{u} \cdot \hat{\rho} + \mathbf{v} \cdot \mathbf{u} \cdot \hat{\rho}] = \oint_{\partial V} \mathbf{u} \cdot \hat{\rho} \cdot \hat{\rho} \, d\mathbf{a}$$

$\hat{\rho}$ - нормаль к контуру

$$\oint_{\partial V} d\mathbf{a} \left[\mathbf{u} + \mathbf{v} \cdot \frac{1}{2} \rho \hat{\rho} \right] = \oint_{|\hat{\rho}|=1} d\varphi \rho \mathbf{u} \cdot \left(\frac{1}{2} \rho \hat{\rho} \right) \cdot \hat{\rho} = \frac{1}{2} \oint_{|\hat{\rho}|=1} d\varphi \rho \mathbf{u}$$

$$\oint_{\partial V} d\mathbf{a} \left[\mathbf{u} + \frac{1}{2} \rho \frac{\partial \mathbf{u}}{\partial \rho} \right] = \frac{1}{2} \oint_{|\hat{\rho}|=1} d\varphi \mathbf{u}, \quad \mathbf{u} = \mathbf{u}(\rho, \varphi). \quad \text{Выводим, что } \mathbf{u} = \rho^n \mathbf{u}_n(\varphi)$$

пусть $u_2(\rho) = \sum_{n=0}^{\infty} a_n \rho^n$, $\oint d\mathbf{a} \left[u_2 + \frac{\rho}{2} u_2' \right] = \oint d\mathbf{a} \left[\sum a_n \rho^n + \frac{\rho}{2} \sum a_n n \rho^{n-1} \right] =$

$$= \sum_n a_n \left(1 + \frac{n}{2} \right) \oint d\mathbf{a} \rho^{n+1} = \sum_n a_n \left(1 + \frac{n}{2} \right) \rho^{n+2} \Big|_0^1 = \frac{1}{2} \sum_n a_n = \frac{1}{2} u_2(1)$$

т.е. что $u(\rho, \varphi) = \sum_{n=-1}^{\infty} a_n(\varphi) \rho^n$ попарно парадокс.

$$\oint d\mathbf{a} \left[\mathbf{u} + \frac{\rho}{2} \mathbf{u}' \right] = \oint d\mathbf{a} \frac{\partial}{\partial \rho} \left(\frac{\rho^2}{2} \mathbf{u} \right) = \frac{1}{2} \rho^2 \mathbf{u}(\rho) \Big|_0^1 = \frac{1}{2} \mathbf{u}(1)$$

таким же $\oint_{\partial V} d\mathbf{a} \left[\mathbf{u} + \frac{\rho}{2} \mathbf{u}' \right] = \frac{1}{2} \oint_{\partial V} d\varphi \mathbf{u}(1, \varphi) - \text{нужно сравнить с } \mathbf{u}(\rho, \varphi).$

Вывод: попарно пара не парадокс $\oint_{\partial V} d\mathbf{a} \mathbf{u}(\rho, \varphi)$ найм.

Meeting с Иваном: обдумываем ST-nanopillar paper - simul.

24/IV C.A. в Милане и в Женеве. Бюро ген.

26-27/IV вернулся в Stoll, Sergio. Hillebrands и коллега в Stuttgart-Kaiserslautern
Обсужд. с Julian Ordizia Auf. (con. Hernando и Mollenkamp), letter to MICRON

Approval of Residency. Occasion PRB - Proof. Meeting c Arantxa: 1) ongoing ee of MICINN grant cum cuoty, r.v. orcas Ramon y Cajal; 2) Occ. application Targeted Res

3) ongoing simul. envelop single dot by $c L = 50 \text{ nm}$, $R = 500 \text{ nm}$, $f = 0.2 - 12 \text{ GHz}$.

4) Zvezg Comp. Gene Occ. (Julian garsien nepekumy 1800 € c 66 co. na 65 co.)

5) general slides ST-nanopillar to present new a Occ. ~ 5 July // K Krasavina

30/7 Info of 3-veg. orcas nepeg Ikerbasque. c Occ - 2a nomandoo c SIESTA (m.c.)

logies, u nocua to NanoSimat-10 (Reims) Abstracts no ST-nanopillars u Nanotub

1/7/11 Subm. revised JAP-Nanotubes Ai-Ring. c A. u A. Krasavina. $\Phi/E = 1.25$

2/7/11 Re-Subm. JAP-Nanotubes (see Figs van Separate TIF-files). Accepted

MICINN - grant (nocua accept. letter to Vice Rectorado) // Figs: AH(d) nonener

ha PRL. moreno egeano Eeg Papaya, Zvezgob new dots b Kaiser lantern, uquep. y

Norman. quadrupolar moments of SW modes in the vortex state. Fues

Cor. Gus' PI 2000: Quadrupolar moment $Q_{dp} = \int d\vec{r} [3M_x x_p + 3M_y y_p - 2\delta(\vec{r})]$

Shifted vortex $\vec{S} = S(\cos\phi_s, \sin\phi_s)$ unless $Q_{dp} \sim S^2 \neq 0$, t.e. TM ($R \neq 0$) or magne

quadrupolar moment. $\vec{m} = \vec{m}_v(\vec{S}) + \vec{m}(\vec{S}, t)$. $\vec{J} = (\mu_p, 0, \mu_z)$ the core.

$\rightarrow \mu_p = b(\rho) \sin(m\phi - \omega t)$, $\mu_z = a(\rho) \cos(m\phi - \omega t)$, APL-SW-coupled dots

$(3/\pi) Q_{\alpha\beta} = Q_{\alpha\beta}[\vec{M}] \cdot (\vec{M} \cdot \vec{e}) = (M_x, M_y, M_z) \cdot (x, y, z), \vec{S} = x\hat{x} + y\hat{y} = \rho(\cos\varphi\hat{x} + \sin\varphi\hat{y})$
 $(\vec{M} \cdot \vec{e}) = (M_x, M_y, M_z)(\cos\varphi, 0, z) = M_x \cos\varphi + M_z z, \int d\varphi M_z z = 0, \text{ i.e. } M_z z = 0$
 Ecm $\vec{M} = \vec{M}_0$ - static centered vortex, $\Rightarrow Q_{xx} = \int d\varphi d\rho [6M_x^2 x - 2M_y^2 y] = 0$
 $= 6L \int d\varphi d\rho \cos\varphi (-M_x) \sin\varphi = 0. Q_{xy} = 3 \int d\varphi d\rho [M_x^2 y + M_y^2 x] = 0$
 $= 3L \int d\varphi d\rho [-\sin\varphi \cdot \sin\varphi + \cos\varphi \cdot \cos\varphi] M_x \sim \int d\varphi [\cos^2\varphi - \sin^2\varphi] = 0$

$Q_{zz}[\vec{M}_0] = 0, Q_{yz}[\vec{M}_0] = L \int d\varphi d\rho [3M_y^2 z + 3M_x^2 x] = 3L \int d\varphi d\rho M_x^2(\varphi) \rho \cos\varphi = 0$
 T.e. $Q_{\alpha\beta}[\vec{M}_0] = 0$ for static - centered vortex. $Q_{\alpha\beta}[\vec{M}] = Q_{\alpha\beta}[\vec{M}_0 + \vec{\mu}]$
 $= Q_{\alpha\beta}[\vec{M}_0] + Q_{\alpha\beta}[\vec{\mu}] = Q_{\alpha\beta}[\vec{\mu}]$. Due spin excitations over the ground state $\vec{M}_0 = \vec{M}_0(\varphi)$ $Q_{\alpha\beta} = Q_{\alpha\beta}[\vec{\mu}] \cdot \kappa \delta_{\alpha\beta} \rho \varphi + \epsilon_{\alpha\beta\gamma} \mu_\gamma$

Due TM $Q_{\alpha\beta}[\vec{\mu}_{TM}] \sim |\vec{\mu}|^2 \neq 0$. ? moreover we $Q_{\alpha\beta}[\vec{\mu}_{TM}] \sim |\vec{\mu}|^2$
 Due SW bre core $\vec{\mu} = (\mu_x, 0, \mu_z)$. $\vec{\mu}_{\text{plane}} = \mu_p \hat{\rho} = \mu_p (\cos\varphi \hat{x} + \sin\varphi \hat{y})$

$Q_{xx}[\vec{\mu}_{sw}] = L \int d\varphi d\rho [6\mu_x^2 x - 2\mu_y^2 y] = 2L \int d\varphi d\rho [3\mu_p \cos\varphi \cdot \rho \cos\varphi - \mu_p \rho] = 2L \int d\varphi d\rho \mu_p \rho [3\cos^2\varphi - 1] = 2L \int d\varphi d\rho \rho b(\rho) \int d\varphi \sin(m\varphi - \psi) [3\cos^2\varphi - 1]$
 $J_m^{xx} = \int_0^{2\pi} d\varphi \sin(m\varphi - \psi) (3\cos^2\varphi - 1) = 2 \int d\varphi \sin(m\varphi - \psi) \frac{1}{2} (3\cos 2\varphi + 1) = \frac{2}{2} \int d\varphi (\sin m\varphi \cdot \cos 2\varphi - \cos m\varphi \cdot \sin 2\varphi) (3\cos 2\varphi + 1) = -\frac{2}{2} \sin\psi \int d\varphi \cos m\varphi (3\cos 2\varphi + 1) = -\frac{2}{2} \sin\psi \cdot \pi [3\delta_{m,2} + 3\delta_{m,-2} + 2\delta_{m,0}]$
 $J_m^{xx} = -\frac{\pi}{2} \sin\omega t [3\delta_{m,2} + 3\delta_{m,-2} + 2\delta_{m,0}] \cdot 2$

$Q_{yy}[\vec{\mu}_{sw}] = 2L \int d\varphi d\rho \rho b(\rho) (-\frac{\pi}{2}) \sin\omega t [\dots] = -\pi L \sin\omega t [3\delta_{m,2} + 3\delta_{m,-2} + 2\delta_{m,0}]$

$Q_{xy}[\vec{\mu}_{sw}] = L \int d\varphi d\rho [3\mu_x^2 y + 3\mu_y^2 x - 2\mu_z^2 \rho] = L \int d\varphi d\rho \mu_p [3\sin 2\varphi - 2] \rho, \delta_{\alpha\beta} = \delta_{\alpha\beta}$
 $J_m^{xy} = \int_0^{2\pi} d\varphi \sin(m\varphi - \psi) (3\sin 2\varphi - 2) = \int d\varphi (\sin m\varphi \cdot \cos 2\varphi - \cos m\varphi \cdot \sin 2\varphi) (3\sin 2\varphi - 2) = \int d\varphi \sin m\varphi \sin 2\varphi (3\cos\psi) + 2 \sin\psi \int d\varphi \cos m\varphi = 3\cos\psi \cdot \pi (\delta_{m,2} - \delta_{m,-2}) + 4\pi \sin\psi \delta_{m,0}$

$Q_{yx}[\vec{\mu}_{sw}] = L \int d\varphi d\rho \rho b(\rho) J_m^{xy} = \pi L \int d\varphi d\rho \rho b(\rho) [3\cos\omega t (\delta_{m,2} - \delta_{m,-2}) + 4\sin\omega t \delta_{m,0}]$

$Q_{yy}[\vec{\mu}_{sw}] = L \int d\varphi d\rho [6\mu_y^2 y - 2\mu_z^2 \rho] = L \int d\varphi d\rho \rho b(\rho) J_m^{yy}, J_m^{yy} = 2 \int_0^{2\pi} d\varphi \sin(m\varphi - \psi) (3\cos 2\varphi - 2)$
 $J_m^{yy} = -\pi \sin\omega t [-3(\delta_{m,2} + \delta_{m,-2}) + 2\delta_{m,0}]$

$Q_{dz}[\vec{\mu}_{sw}]_{d+z} = \int dz \int d\varphi d\rho [3\mu_d^2 z + 3\mu_z^2 x_d] = 3L \int d\varphi d\rho \mu_z x_d, x_d = x, y$

$Q_{yz}[\vec{\mu}_{sw}] = 3L \int d\varphi d\rho \int d\varphi a(\rho) \cos(m\varphi - \psi) \rho \cos\varphi = 3L \int d\varphi d\rho a(\rho) \int d\varphi \cos(m\varphi - \psi) \cos\varphi = 3L \int d\varphi d\rho a(\rho) \cos\varphi \pi (\delta_{m,1} + \delta_{m,-1}) (!)$
 $= (\dots) \int d\varphi (\cos m\varphi \cos\psi + \sin m\varphi \sin\psi) \cos\varphi = 3L \int d\varphi d\rho a(\rho) \cos\varphi \pi (\delta_{m,1} + \delta_{m,-1}) (!)$

$Q_{yz}[\vec{\mu}_{sw}] = 3L \int d\varphi d\rho a(\rho) \int d\varphi \cos(m\varphi - \psi) \sin\varphi = 3\pi L \sin\psi \int d\varphi d\rho a(\rho) (\delta_{m,1} - \delta_{m,-1})$

$Q_{zz}[\vec{\mu}_{sw}] = \int dz \int d\varphi d\rho [6\mu_z^2 z - 2\mu_p^2 \rho] = -L \cdot 2 \int d\varphi d\rho \rho \mu_p = -2L \int d\varphi d\rho \rho b(\rho) \int d\varphi \sin(m\varphi - \psi)$
 $= -2L \int d\varphi (\dots) \int d\varphi [\sin m\varphi \cos\psi - \cos m\varphi \sin\psi] = 4\pi L \sin\psi \delta_{m,0} \int d\varphi d\rho \rho b(\rho)$

T.e. $Q_{\alpha\beta}[\vec{\mu}_{sw}]$ - dynamic modes for $Q_{\alpha\beta} \neq 0$, Q_{xx}, Q_{yy}, Q_{zz} core u. main (radial) no zero $\sim \delta_{m,\pm 1}$. $Q_{xx}, Q_{yy}, Q_{yx}, Q_{xy}, Q_{zz}$ core u. $\delta_{m,\pm 2}$ u. $\delta_{m,0}$
 - T.e. no background uniform in-plane a.c. field, therefore background $m = \pm 1$ - SW. Q_{xx}, Q_{yy}, Q_{zz} core u. $\delta_{m,0}$, T.e. $\neq 0$ for $m = 0$



$\hat{A}$ - hermitischer, $\hat{Q}$ - symmetrischer, $\hat{Q}_{op}$ - hermitischer, $\hat{M} = \hat{M}(\vec{r})$, $\hat{A}^2 = \hat{A}^2$.

$$\vec{H}_m(\vec{R}_n) = \sum_{n'} \frac{1}{R_{nn'}^3} \left[\frac{3 \vec{R}_{nn'} (\vec{M}_{n'} \cdot \vec{R}_{nn'})}{R_{nn'}^3} - \vec{M}_{n'} \right] - \frac{1}{2} \sum_{dp n'} Q_{dp}^{n'} \nabla_{\vec{R}_{nn'}} \left(\frac{\vec{X}_d^{n'} \cdot \vec{X}_p^{n'}}{R_{nn'}^3} \right) - ?$$

B. nannus, near the dot's gas being near dip field $\sim \mu$, r.e. u
gas field $\sim Q_p^n$. B. nannus or p...
B. nannus no evidence of dot lattice, a local field $H_m(R_n)$, eigen. dot eff. frequency.
Cn. B. nannus no evidence of dot lattice, a local field $H_m(R_n)$, eigen. dot eff. frequency.

$$H_m^{(n)}(\vec{r}) = \sum_{n'} \int d\vec{r}' G_{\Delta p}(\vec{r}_{nn'} + \vec{r}, \vec{r}') M_p^{(n')}(\vec{r}')$$

$\vec{r}_{nn'} = \vec{r}_n - \vec{r}_{n'}$, $\vec{r} \in n\text{-dot}$, $\vec{r}' \in n'\text{-dot}$, $\vec{r}_n = \gamma \delta_{n,p} \hat{p}_n$.

$$M_p^{(n)}(r) = \underbrace{M_p^{(n)}(r)}_{\text{static}} + \underbrace{m_p^{(n)}(r, t)}_{\text{dynamic}}. \quad (\text{Uniqueness, } \forall n \quad \prod_n(r) [M^0] = 0)$$

just vortex-centered vts, i.e. less unimodal. no. is less dynamic.

$$H_m^{\alpha(n)}(\vec{r}) = \int_{V_d(m)} d^3\vec{r}' G_{dp}(\vec{r}, \vec{r}') M_p^{(n')}(\vec{r}') + \sum_{n' \neq n} \int_{V_d(n')} d^3\vec{r}' G_{dp}(\vec{r}, \vec{r}') M_p^{(n')}(\vec{r}')$$

Con. 3/10/2002: $\hat{G}(\vec{R}_{nn'} + \vec{r}, \vec{r}') = - \frac{\partial}{\partial \vec{r}} \frac{\partial}{\partial \vec{r}'} \frac{1}{|\vec{R}_{nn'} + \vec{r} - \vec{r}'|}$
 $\vec{r} = \frac{1}{\sqrt{2}} \vec{r} - \vec{r}', |\vec{r}| \ll |\vec{R}_{nn'}|, |\vec{R}_{nn'}|_{\min} = a$ - nearest lat lattice

$$\frac{1}{|\vec{R} - \vec{p}|} = \sum_{l=0}^{\infty} \frac{p^l}{R^{l+1}} P_l(\cos \gamma), \quad \gamma = \vec{p} \cdot \vec{R} / pR. \quad P_l(\cos \gamma) \text{ — Legendre polynomials.}$$

$$P_3(x) = \frac{1}{R^3} (5x^3 - 3x), \quad \frac{1}{(R-x)^3} = (\ell=0,1,2) + \frac{9}{R^4} \frac{1}{2} [5\cos^3 x - 3\cos x]$$

$$\text{Ans)} = \frac{1}{SR} (\vec{S} \cdot \vec{R}), \quad \frac{1}{|\vec{R} \cdot \vec{P}|} = (0, 1, 2) + \frac{1}{2} \frac{1}{R^4} [5 (\vec{P} \cdot \vec{R})^2 - 3 P^2 (\vec{P} \cdot \vec{R})]$$

$$G_{sp}(\vec{r} + \vec{r}', \vec{r}') = - \frac{\partial}{\partial x_\alpha} \frac{\partial}{\partial x'_\beta} \frac{1}{|\vec{r} + \vec{r}' - \vec{r}'|} = (\dots) = - \frac{\partial}{\partial x_\alpha} \frac{\partial}{\partial x'_\beta} \frac{1}{2R^3} \left[\frac{5}{R^2} (\vec{r}' \cdot \vec{R})^2 - 3r'^2 (\vec{r}' \cdot \vec{R}) \right]$$

$$\frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} (\vec{s} \cdot \vec{r})^3 = \frac{\partial}{\partial x_i} 3(\vec{s} \cdot \vec{r})^2 (-\vec{r})_j = -R_j \cdot 3 \cdot 2(\vec{s} \cdot \vec{r}) R_i = -6R_i R_j (\vec{s} \cdot \vec{r})$$

$$\frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} (\vec{p} \cdot \vec{R}) p^2 = \frac{\partial}{\partial x_i} \left[-R_{ij} p^2 + (\vec{p} \cdot \vec{R}) 2p_j (-1) \right] = -R_{ij} \frac{\partial}{\partial x_i} p^2 - 2 \frac{\partial}{\partial x_i} (\vec{p} \cdot \vec{R}) p_j$$

$$= -R_p \frac{\partial}{\partial x_k} \sum_j (x_j - x_j^*)^2 - 2p_p \frac{\partial}{\partial x_k} (\vec{S} \cdot \vec{R}) - 2(\vec{S} \cdot \vec{R}) \frac{\partial p_p}{\partial x_k} =$$

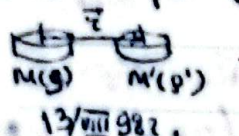
$$= -P_p \cdot 2R_p - 2P_p R_p - 2(\vec{p} \cdot \vec{R}) \delta_{\perp p}, \quad R = R_{nn} = R_n - R_n \cdot 3\vec{p} \cdot \vec{e}_n$$

$$G_{dp}(\vec{R} + \vec{r}, \vec{r}) = (\text{dip. coupling}) = \frac{1}{2R^5} \left[\frac{\vec{R}^2}{R^2} \left(\vec{r} \cdot \vec{R} \right) \left(\vec{r} \cdot \vec{R} \right) - \left(\vec{r} \cdot \vec{R} \right)^2 \right] + 2(\vec{r} \cdot \vec{R}) \delta_{dp} \Big] = (\text{dip.}) = \frac{3}{R^5} \left[-\frac{5}{R^2} R_d R_p (\vec{r} \cdot \vec{R}) + R_d R_p + R_p R_d + (\vec{r} \cdot \vec{R}) \delta_{dp} \right]$$

$$H_m^{(n)}(\vec{r}) = (n=n') + \sum \int d^3\vec{r}' G_{dp}(\vec{r}_{nn'} + \vec{r}, \vec{r}') M_p^{(n')}(\vec{r}') =$$

$$= (n=n') + \sum_{n'} \int d^3\vec{r}' M_p^{(n')}(\vec{r}') \frac{n' r n}{R_{nn'}} (-3) \left[\left\{ \delta_{dp} - 5 \frac{R_{nn'}^d R_{nn'}^p}{R_{nn'}^2} \right\} (\vec{r} \cdot \vec{r}') \vec{R}_{nn'} + (x_d - x'_d) R_{nn'}^p + (x_p - x'_p) R_{nn'}^d \right]$$

$$= - \int d^3r' \left[\sum_p \left(-\frac{R_1 R_p}{r^2} \right) (\vec{r}' \cdot \vec{r}) M_p + (\vec{M}' \cdot \vec{r}') R_1 + \sum_p (M_x x'_p + M_y y'_p + M_z z'_p) R_p \right] - \text{Hemomemo}$$

$\vec{H}_m(\vec{r}) = -\nabla \varphi_m(\vec{r}), \quad \varphi_m(\vec{r}) = \frac{\vec{M} \cdot \vec{r}}{r^3} + \frac{1}{2} \frac{1}{r^5} \sum_{\alpha\beta} Q_{\alpha\beta} x_\alpha x_\beta$
 One dot, $\vec{r}' \in V_d$, per atom, no $(\frac{1}{r}) \ll 1$, $\vec{r} = V_d \langle \vec{M}(\vec{r}) \rangle$
 $\vec{H}_m(\vec{r}) = \frac{1}{r^3} \left[\frac{3\vec{r}(\vec{r} \cdot \vec{M})}{r^2} - \vec{M} \right] - \frac{1}{2} \frac{1}{r^5} \sum_{\alpha\beta} Q_{\alpha\beta} [\delta_{\alpha\beta} x_\beta + \delta_{\beta\alpha} x_\alpha - \frac{5}{r^2} x_\alpha x_\beta]$
 i) Gauge for n -dot, $\vec{r} \in n$ -dot (R_{nn})
 B. quad. of mult. moments. $\vec{r} \rightarrow R_{nn} \vec{r}' + \sum_n \dots$
 $\vec{H}_m(\vec{r}) = (n=n') + \sum_{n' \neq n} \frac{1}{R_{nn'}} \left[3 \frac{R_{nn'}^T}{R_{nn'}^2} (\vec{R}_{nn'} \cdot \vec{M}) - \vec{M} \right] - \sum_{n' \neq n} \frac{R_{nn'}^Q}{R_{nn'}^5} Q_{\alpha\beta} + \dots$
 $+ \frac{5}{2} \sum_{n' \neq n} \frac{R_{nn'}^\alpha R_{nn'}^\beta R_{nn'}^\gamma}{R_{nn'}^7} Q_{\alpha\beta\gamma}$ — no Equation $\varphi_m(\vec{r})$ at 12/III 00. 7. mag. fields
 $\frac{1}{|\vec{r} - \vec{r}'|} = \frac{1}{r} + \frac{1}{r^3} (\vec{r} \cdot \vec{r}') + \frac{1}{2r^5} [3(\vec{r} \cdot \vec{r}')^2 - r^2 r'^2] + O(\frac{r'}{r^3})^3$
 Hago $\varphi(\vec{r}) = \int d^3\vec{r}' \frac{\vec{M} \cdot \vec{r}}{|\vec{r} - \vec{r}'|} - \int d^3\vec{r}' \frac{\nabla \cdot \vec{M}(\vec{r}')}{|\vec{r} - \vec{r}'|}$ and $\varphi(\vec{r}') = - \int d^3\vec{r}' \vec{M}(\vec{r}') \cdot \nabla \frac{1}{|\vec{r} - \vec{r}'|}$
 $\vec{H}_m(\vec{r}) = -\nabla \varphi(\vec{r})$. $\varphi(\vec{r}) = - \int d^3\vec{r}' \nabla \cdot \left(\frac{1}{|\vec{r} - \vec{r}'|} \right) \cdot \vec{M}(\vec{r}') - \text{vacuum}$
 $\nabla \cdot \frac{1}{|\vec{r} - \vec{r}'|} = \frac{\partial}{\partial r} \left[\frac{1}{r} + \frac{(\vec{r} \cdot \vec{r}')}{r^3} + \dots \right] = -\frac{\vec{r}}{r^3} + \frac{1}{r^3} \left[\vec{r}' - 3 \frac{(\vec{r} \cdot \vec{r}') \vec{r}}{r^2} \right] + \dots$
 $\varphi(\vec{r}) = - \int d^3\vec{r}' \vec{M}(\vec{r}') \cdot \left[-\frac{\vec{r}}{r^3} + \frac{1}{r^3} \left(\vec{r}' - \frac{3}{r^2} (\vec{r} \cdot \vec{r}') \vec{r} \right) \right] = \frac{\vec{M} \cdot \vec{r}}{r^3} + \frac{1}{2} \frac{1}{r^5} \sum_{\alpha\beta} Q_{\alpha\beta} x_\alpha x_\beta$
 — see B. R. H. Good, T. J. Nelson, Classical th. el. mag. fields. NY Acad. Press, 1981
 Normalizing $\vec{H}_m^{(n)}(\vec{r}) = (n=n') + (\text{dip.}) - \sum_{\alpha, n' \neq n} \frac{R_{nn'}^\alpha Q_{\alpha\beta}^{(n')}}{R_{nn'}^5} + \frac{5}{2} \sum_{n' \neq n} \frac{R_{nn'}^\alpha R_{nn'}^\beta R_{nn'}^\gamma}{R_{nn'}^7} Q_{\alpha\beta\gamma}^{(n')}$
 $Q_{\alpha\beta}^{(n)} = \int d^3\vec{r}' \left[3M_\alpha^{(n)} x'_\beta + 3M_\beta^{(n)} x'_\alpha - 2\delta_{\alpha\beta} (\vec{M}^{(n)} \cdot \vec{r}') \right]$
 Bugno, no quadrupolar waves (or symmetric $Q_{\alpha\beta}$ -moments) & rem. noise
 na gemma n -dot $\sim \left(\frac{Q}{R_{nn}^4} \right)$, a bobce ne $\sim \frac{1}{R_{nn}^5}$
 (Due SW-frequencies changes Hago numero $\vec{H}_m^{(n)}$, one Broque b 40
 (Due TM-freq. change Hago W_{int} , r.k. one beiges b thick eq. g.
 Due napti gorob c var. momentum $\vec{m}, \vec{m}' \rightarrow \vec{\mu}, \vec{\mu}'$ — klagypn. mem. Q, Q'

 $W_{int} = \left(\frac{1}{r^3} \right) W_{dip-dip} + W_{dip-quad} + W_{q-q}$ — m. coupling of dot pairs
 $W_{d-d} = \frac{1}{r^3} [\vec{\mu} \cdot \vec{\mu}' - 3 \frac{(\vec{\mu} \cdot \vec{r})(\vec{\mu}' \cdot \vec{r})}{r^2}]$
 $W_{d-q} = \frac{1}{r^5} \sum_{\alpha\beta} x_\beta (\mu_\alpha Q'_{\alpha\beta} - \mu'_\alpha Q_{\alpha\beta}) + \frac{5}{2} \frac{1}{r^7} \sum_{\alpha\beta} x_\alpha x_\beta [Q_{\alpha\beta} (\vec{\mu} \cdot \vec{r}) - Q'_{\alpha\beta} (\vec{\mu}' \cdot \vec{r})]$
 $W_{q-q} = \frac{1}{6} \frac{1}{r^5} \sum_{\alpha\beta} Q_{\alpha\beta} Q'_{\alpha\beta} - \frac{5}{3} \frac{1}{r^7} \sum_{\alpha\beta\gamma} Q_{\alpha\beta} Q'_{\beta\gamma} x_\alpha x_\gamma + \frac{35}{12} \frac{1}{r^9} \sum_{\alpha\beta\gamma\delta} Q_{\alpha\beta} Q'_{\gamma\delta} x_\alpha x_\beta x_\gamma x_\delta$
 Bugno, no W_{d-q} nerazne no x_α u scem bo $\vec{\mu}^{(n)} = \vec{\mu}$. $Q^{(n)} = \hat{Q}$
 zori symmetria, $\sum_{nn'} (no \text{ square lattice})$ gans $W_{d-q} = 0$.
 $W_{int} = \sum_{nn'} \frac{1}{R_{nn'}^3} [\vec{\mu}_{nn'} \cdot \vec{\mu}_{nn'} - 3 \frac{(\vec{\mu}_{nn'} \cdot \vec{R}_{nn'}) (\vec{\mu}_{nn'} \cdot \vec{R}_{nn'})}{R_{nn'}^2}] + W_{22}(\hat{Q}_n) - \text{magnetost. coupling}$
 $\frac{1}{N} W_{q-q} = \frac{1}{16} Sp(\hat{Q}^2) \sum_n \frac{1}{R_n^5} + \frac{35}{48} [2 \sum Q_{\alpha\beta}^2 - Sp(\hat{Q}^2)] \sum \frac{\cos 4\varphi_{Rn}}{R_n^9}$ — ecm for $\hat{Q}^{(n)} = \hat{Q}$

Здесь для (TM или SW) $\vec{\mu}_n$ и $\vec{Q}_n$ не имеют эквивалентов, следовательно, суммарное $\vec{\mu}_n$ и $\vec{Q}_n$ является n-матрицей. Векторы $\vec{\mu}_n$ и $\vec{Q}_n$ - динамические.
 $\vec{\mu} = \vec{\mu}_{TM} + \vec{\mu}_{SW}$, $\vec{Q} = \vec{Q}_{TM} + \vec{Q}_{SW}$ и упругие константы зависят от частоты μ_{TM} , μ_{SW} , Q_{TM} и Q_{SW} для а.с. driving field ($h_0 \leq 1$ Ое).
 $\mu_{TM} \neq 0$, $\mu_{SW} \neq 0$ только для $m = \pm 1$ - азим. SW. Uniform $\vec{h}_0$ in-plane
 возбуждает только TM и азим. SW - $m = \pm 1$.

① Динамические уравнения

$$\mu_p = \mu_m(p) \sin(m\varphi - \omega t)$$

$$\mu_z = \mu_l = \mu_m(p) \cos(m\varphi - \omega t)$$

$\int d\varphi \mu_m^2(p) = 1$ нормировка, где μ_m - амплитуда и не зависит от m -а. $|\langle \delta m \rangle| \approx \chi''(\omega) h_0$
 $\mu_m \approx \omega_{sw}$, $\chi''(\omega_{sw}) \approx \frac{\chi''(0)}{\omega}$. $|\langle \delta m \rangle| \approx \chi''(0) \frac{h_0}{\omega}$ для ω_{sw} . $\chi(0) \approx \frac{M_s}{H_A} \approx 1$
 Для TM $\chi''(\omega_0) \approx \frac{\chi''(0)_{TM}}{2\omega}$, $\chi(0)_{TM} = \frac{3}{2} \frac{\gamma M_s}{\omega_0}$, $\gamma = 2/3$. $\chi(0)_{TM} \approx \frac{2}{3} \frac{3GH_z}{300MHz} \approx (1 \div 2)$
 т.к. $\omega_0 \ll \omega_{sw}$, не учитываем эквивалентности, т.е. ω - scan
 процесс и TM - бунаг и SW - бунаг. $\vec{\mu}_{tot} = V_d \langle \vec{M} \rangle$.

② Квантование магнитных вихрей

(m=±1) SW: $Q_{xz} = 3\pi L \int d\varphi \mu^2 \cos \varphi$

TM: $\hat{Q} \sim S^2$ нормировка в rigid v. model - Ginzburg-Landau
 $Q_{yz} = \pm 3\pi L \int d\varphi \mu^2 \sin \varphi$
 Магнетон $\hat{Q}$ shifted vortex в pole-free model. $\vec{m}(p)$ - pole-free one core - $S/\pi \cos \varphi$.

$$Q_{xx} = L \int d^2 p [6m_x x - 2(\vec{m} \cdot \vec{p})] = L \int d^2 p \left[\frac{0}{\text{нечетное}} - 2p m_p \right] = 0$$

$$Q_{yy} = L \int d^2 p [6m_y y - 2p m_p] = 0, \quad Q_{xy} = L \int d^2 p [3m_x y + 3m_y x] = Q_{yx} = 0$$

$$Q_{xz} = \int d^2 p [3m_x z + 3m_z x] = Q_{zx} = 0, \quad Q_{zz} = \int d^2 p [6m_z z - 2p m_p] = 0$$

т.е. с нормировкой $\vec{Q}_{TM} \sim S^2$, но т.к. $S \leq 0.1$ и $h_0 < 0.1$ Ое, то $\vec{Q}_{TM}$ ориентировано в плоскости.
 Квантование $W_{q-q} \sim Q_{d\varphi}^2 \sim S^4$, т.е. т.к. $\omega \approx \omega_0$ W_{q-q} не эквивалентно.

Квантование бунагов $H^2(n)_m$ для SW ($Q_{xz}, Q_{yz} \neq 0$): $H^2(n)_m = - \sum_{n, n'} \frac{R_{nn'}^4 Q_{zz}^{(n)}}{R_{nn'}^5} \neq 0$
 - некое int field I в магн. и асим. с вектором ω
 нормировка бунагов (m=0) SW. Но для эквивалентных дотс, $H^2(n)_m = 0$,
 т.к. $\sum_{nn'} \frac{R_{nn'}^4}{R_{nn'}^5} = 0$.

→ Бунаг: quadrupolar бунаг для TM или в асим. магнетон $\vec{Q}_{TM}$, а quadrupolar бунаг для азим. SW ($\pm 1=m$) возникает для эквивалентных

Dipolar coupling для TM: $\vec{\mu}_n = \vec{\mu} = V_d \langle \vec{M} \rangle$, $W_{int}^{TM} = \sum_{nn'} \left[\frac{1}{R_{nn'}^3} - 3 \frac{(\vec{\mu}_n \cdot \vec{R}_{nn'})^2}{R_{nn'}^5} \right] \frac{1}{R_{nn'}^3}$
 $W_{int}^{TM} \sim S^2 k$, $k < 0$ - гравитационное взаимодействие с Голдстоуновскими бозонами.

Dipolar coupling для (m=±1) SW: $(\vec{H}_d^{int})_m = \sum_{n, n'} \left[\frac{3}{R_{nn'}^3} (\vec{R}_{nn'} \cdot \vec{\mu}_n) \vec{\mu}_{n'} - \vec{\mu}_n \right]$
 магн. бунаг in-plane асим. field с вектором ω
 для упругих (или?) к нормировке $\omega_{sw} (m=\pm 1)$ т.к. $\vec{Q}_{TM}$ ориентировано в плоскости.

$$\frac{\partial}{\partial x_j} \frac{\partial}{\partial x_p} (\vec{r} \cdot \vec{r})^2 = \frac{\partial}{\partial x_j} 2(\vec{r} \cdot \vec{r}) \vec{r}_p = 2\delta_{jp} R_p$$

$$\frac{\partial}{\partial x_j} \frac{\partial}{\partial x_p} \vec{r} = \frac{\partial}{\partial x_j} \vec{r}_p = \delta_{jp} \vec{r}$$

$$\frac{\partial}{\partial x_j} \frac{\partial}{\partial x_p} (\vec{r}_j \cdot \vec{r}_j) = 2\delta_{jp}$$

$$\frac{\partial}{\partial x_j} \frac{1}{R_p} = -\frac{1}{R_p^2} \frac{\partial R_p}{\partial x_j} = -\frac{1}{R_p^2} \delta_{jp} R_p = -\frac{\delta_{jp}}{R_p}$$

$$\frac{\partial}{\partial x_j} \frac{1}{R_p^2} = -\frac{2}{R_p^3} \frac{\partial R_p}{\partial x_j} = -\frac{2}{R_p^3} \delta_{jp} R_p = -\frac{2\delta_{jp}}{R_p^2}$$

no caso de gradiente $(\vec{r} \cdot \vec{r})^2 = R^2$, $\frac{\partial}{\partial x_j} R^2 = 2R \frac{\partial R}{\partial x_j} = 2R \delta_{jp}$

$$U_{ap}(\vec{r}, \vec{r}') = (4\pi) + \frac{1}{R^2} [R_p R_p + R_p R_p + (\vec{r} \cdot \vec{r}) \delta_{jp} - 5 \frac{R_p R_p}{R^2} (\vec{r} \cdot \vec{r})]$$

$$U_m^{(n)}(\vec{r}) = (n+1) + \sum_{n'} \int d^3\vec{r}' M_p^{(n')}(\vec{r}') \frac{1}{R^2} [R_p R_p + R_p R_p + (\vec{r} \cdot \vec{r}) \delta_{jp} - 5 \frac{R_p R_p}{R^2} (\vec{r} \cdot \vec{r})]$$

$$= (n+1) + \sum_{n'} \int d^3\vec{r}' \frac{1}{R^2} [R_p (\vec{r} \cdot \vec{r}') + R_p (\vec{r}' \cdot \vec{r}) + M_p^{(n')} (\vec{r} \cdot \vec{r}') - 5 \frac{R_p R_p}{R^2} (\vec{r} \cdot \vec{r})]$$

$$U_m^{(n)}(\vec{r}) = (n+1) + (4\pi) + \sum_{n'} \frac{1}{R^2} \int d^3\vec{r}' [x'_j (\vec{r} \cdot \vec{r}') + R_p (\vec{r}' \cdot \vec{r}) + M_p^{(n')} (\vec{r} \cdot \vec{r}') - 5 \frac{R_p R_p}{R^2} (\vec{r} \cdot \vec{r})]$$

$$= \sum_{n'} \frac{1}{R^2} \int d^3\vec{r}' [x'_j (\vec{r} \cdot \vec{r}') + R_p (\vec{r}' \cdot \vec{r}) + M_p^{(n')} (\vec{r} \cdot \vec{r}') - 5 \frac{R_p R_p}{R^2} (\vec{r} \cdot \vec{r})]$$

Que o de $Q_{ap}^{(1)} = \int d^3\vec{r} [3 M_p^{(1)} x_p + 3 M_p^{(1)} x_p - 2 \delta_{jp} (\vec{r} \cdot \vec{r})]$

Assim sendo, vamos obter as seguintes Q_{ap} . Vamos ver no caso $n=0$ vamos obter:

$$Q_m(\vec{r}) = \frac{\vec{r} \cdot \vec{r}}{R^2} + \frac{1}{2} \frac{1}{R^2} Q_{ap} x_a x_p$$

Assim $Q \sim \int d^3\vec{r} (\dots)$

$$U_m^{(0)}(\vec{r}) = -\frac{2}{\partial x_j} Q_m(\vec{r}) = \frac{1}{R^2} \left[\frac{\partial x_j}{\partial x_j} (\vec{r} \cdot \vec{r}) - R_p^2 \right] = \frac{1}{R^2} \left[3 - R_p^2 \right]$$

$$U_m^{(0)}(\vec{r}) = (4\pi) - \frac{1}{2} \frac{1}{R^2} \sum_{ap} \left[2 x_a \delta_{jp} - \frac{5}{R^2} x_a x_p x_j \right] Q_{ap} = (\dots) - \frac{1}{R^2} \sum_{ap} Q_{ap} x_a \left[\delta_{jp} - \frac{5}{R^2} x_p x_j \right]$$

Como $\vec{r} \cdot \vec{r}$ é uma das 3 brach. independentes $\vec{r}$.

$$\vec{r} = \vec{r}_m + \vec{r}'$$

pois $|\vec{r}'| \ll |\vec{r}_m|$, então vamos m. no caso na n-ésima de $2D$

$$G = \left[x'_j M_p R_p + R_p x'_p M_p + M_p x'_p R_p - \frac{5 R_p R_p}{R^2} R_p M_p x'_j R_j \right] = R_p (x'_j M_p + x'_p M_p) +$$

$$+ R_p x'_p M_p - \frac{5}{R^2} R_p R_p R_p M_p x'_j = R_p \left[\frac{1}{3} Q_{ap} + \frac{2}{3} \delta_{jp} M_p x'_j \right] + R_p x'_p M_p - \frac{5}{R^2} R_p R_p R_p M_p x'_j$$

$$= \frac{1}{3} Q_{ap} R_p + \frac{2}{3} R_p M_p x'_p - \frac{5}{R^2} R_p R_p R_p M_p x'_j = \frac{1}{3} Q_{ap} R_p + \frac{2}{3} R_p M_p \left[x'_p - \frac{3}{R^2} R_p R_p x'_j \right]$$

$$= \frac{1}{3} Q_{ap} R_p + \frac{2}{3} R_p M_p \left[x'_p - 3 \frac{R_p}{R^2} (\vec{r} \cdot \vec{r}') \right] = \frac{1}{3} Q_{ap} R_p + \frac{2}{3} R_p \left[(\vec{r}' \cdot \vec{r}) - \frac{3}{R^2} (\vec{r} \cdot \vec{r}') (\vec{r} \cdot \vec{r}') \right]$$

(9/8) Операторы, а конкретнее операторы. Взяли на paper Fruchart - APL. M. Intendot interaction in cylindrical dot arrays.

$$W_m = \frac{1}{2} \int d\vec{r} \int d\vec{r}' \frac{\phi(\vec{r}) \phi(\vec{r}')}{|\vec{r} - \vec{r}'|} \text{ where } \phi(\vec{r}) - \text{known surface charge}$$

$\phi(\vec{r}) = \phi(\varphi)$ due to side surface charges.

$$|\vec{r} - \vec{r}'|^2 = |\vec{r} - \vec{r}' + \vec{\Delta}|^2 + |z - z'|^2, \quad \vec{\Delta} = \vec{R} - \vec{R}'$$

$$\frac{1}{|\vec{r} - \vec{r}'|} = \int dk e^{-k|z - z'|} J_0(k|\vec{r} - \vec{r}'|)$$

hyper narrow hoop. with. center point $\vec{R} = 0$, and $\vec{\Delta} = -\vec{R}' \parallel OX$, $\vec{r} = \vec{R} + \vec{\Delta} = \vec{r}' + \vec{\Delta}$

$$(\vec{r} - \vec{r}' + \vec{\Delta})^2 = \vec{r}^2 + \vec{r}'^2 + \Delta^2 - 2\vec{r} \cdot \vec{r}' \cos(\varphi - \varphi') - 2\vec{r} \cdot \vec{\Delta} \cos \varphi + 2\vec{r}' \cdot \vec{\Delta} \cos \varphi'$$

$$(\vec{r} \cdot \vec{r}') = r r' \cos(\varphi - \varphi') = r r' \cos \varphi \cos \varphi' + r r' \sin \varphi \sin \varphi' \cos \alpha$$

$$\cos \alpha = \frac{1}{\vec{r} \cdot \vec{\Delta}} (\vec{r}' \cdot \vec{\Delta}) = \frac{1}{(\dots)} [(\vec{\Delta} + \vec{r}') \cdot \vec{\Delta}] = \frac{1}{\vec{r}' \cdot \vec{\Delta}} [\Delta^2 + \vec{r}' \cdot \vec{\Delta} \cos \varphi']$$

Common case, refer to given. // Monero & V - unperturbed, no torque $\vec{M}(\vec{r})$ and $\phi(\vec{r})$ (the vector $\phi(\vec{r})$) is power line ω - dot array.

$$J_0(k|\vec{r} - \vec{r}'|) = \sum_{\mu} e^{i\mu(\varphi - \varphi')} J_{\mu}(k\rho) J_{\mu}(k\rho')$$

$$W_m = \frac{1}{2} \int dz \int dz' \int d\vec{r} \int d\vec{r}' \phi(\vec{r}) \phi(\vec{r}') \int dk e^{-k|z - z'|} \sum_{\mu} e^{i\mu(\varphi - \varphi')} J_{\mu}(k\rho) J_{\mu}(k\rho')$$

$$= L \int \frac{dk}{k} f(kL) \int d\varphi \int d\varphi' \phi(\varphi) \phi(\varphi') \sum_{\mu} e^{i\mu(\varphi - \varphi')} J_{\mu}(k\rho) J_{\mu}(k\rho')$$

$$(\vec{r}')^2 = (\vec{r})^2 \mp (\vec{\Delta} + \vec{r}')^2 = \Delta^2 + \rho'^2 + 2\Delta\rho' \cos \varphi', \text{ and } \varphi_{\vec{r}'}(\varphi')$$

$J_{\mu}(k\rho')$ - снова используем теорему Монро.

$\phi(\varphi) = m_p(\vec{X})$ можно взять в rigid v. model.

Оценим интеграл $\mu = 1, \dots, \infty$. $\varphi_{\vec{r}}$ не бегает $\rightarrow \int d\varphi'(\dots)$ можно по $\int d\varphi$ и $\int d\varphi'$ боимся угадать бегать.

Итого. $W_m = \frac{1}{2} \int d\vec{r} \int d\vec{r}' \frac{\phi(\vec{r}) \phi(\vec{r}')}{|\vec{r} - \vec{r}'|}$ и $\int dS = \int dz \int d\varphi$ по S - поверхность dot. $\int d\vec{r}(\dots)$ и $\int d\vec{r}'(\dots)$ считать бы.

$$W_m = L \int \frac{dk}{k} f(kL) \int d\varphi \int d\varphi' \phi(\varphi) \phi(\varphi') \sum_{\mu} e^{i\mu(\varphi - \varphi')} J_{\mu}(kR) J_{\mu}(k\rho')$$

Может быть $W_m = \frac{1}{2} \int dz d\varphi \int dz' d\varphi' \frac{\phi(\varphi) \phi(\varphi')}{\sqrt{(\vec{r} - \vec{r}')^2 + (z - z')^2}}$ упростить по $\vec{r}, \vec{r}'$.

тогда $W_m(\Delta) = \int d\varphi \int d\varphi' K(\varphi, \varphi'; \Delta) \phi(\varphi) \phi(\varphi')$ - так же как Оксана.

А теперь суба $W_m = 2\pi \sum_{\vec{k}} f(kL) \frac{1}{k^2} |\vec{M}_{\vec{k}} \cdot \vec{k}|^2$, $\vec{M}_{\vec{k}} = \frac{1}{S_d} \int d\vec{r} \vec{M}(\vec{r}) e^{i\vec{k} \cdot \vec{r}}$

$e^{i\vec{k} \cdot \vec{r}} = \sum_n i^n J_n(k\rho) e^{i(n(\varphi - \varphi_k))}$ - упрощает 보면. $\vec{M}_{\vec{k}}$ упрощается.

$\vec{M}_{\vec{k}} = \frac{1}{S_d} \int d\vec{r} \vec{M}(\vec{r}) \sum_n i^n J_n(k\rho) e^{i(n(\varphi - \varphi_k))}$ по знаменателю. $\sqrt{\vec{M}(\vec{r})}$ и

упростить упрощаем интер. по φ , см. Оук' APL'99, 2000. u_2 коорд. вектор (Ф. Бесселя, мнимый радиус, $\int dz dz'$) - Оксана $\int d\vec{r} d\vec{r}'$ - Оксана

10/7 Успешно папироты by Gimin & Marshall (see PRB '98 no N. write dots M(H)).
 11/7 Нормальная e-mail to K. Vogt (Karlzslautern) - нормальн о себе, работа
 no SW in words state. Оказался замечательным. - Купил с Оке.
 Оказался с Оке. Вспомнил Han of coupled small dot array N x N (N = 2-15). Омель
 поменял Han to coupling. Успешно в работе, смена Wm (X1...Xn) вводит интер
 to coupling. Задан порядок Xn eq uz W = Wm(Xn) + W2(Xn), dW = 0 в вло
 уравнениях. |Xn> -> <Mn> = yn Hy, Uan Jn = Ms, Han = Ms / Jn в лун.
 уравнения. |Xn> = 1 или Hy = Uan.

Wm(Xn) = \sum_n \frac{\kappa}{2} |Xn|^2 + \sum_{nn'} [\lambda_{nn'} Xn Xn' + \nu_{nn'} Yn Yn'], W2 = \sum_n \hat{\mu} (Xn \times H)
 X x H = \begin{vmatrix} i & j & k \\ x & y & 0 \\ Hx & Hy & 0 \end{vmatrix} = k y Hy, W2 = \hat{\mu} Hy \sum_n Yn; \frac{\partial W}{\partial Xn} = \sum_{n'} \lambda_{nn'} Xn' = \hat{A} Xn = 0
 \frac{\partial W}{\partial Yn} = \sum_{n'} \nu_{nn'} Yn' + \hat{\mu} Hy = 0 - non-homog eqs. for Yn eq. Bnn' = \kappa \delta_{nn'} + \nu_{nn'}
 \hat{B} \vec{Y} = -\hat{\mu} Hy \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \vec{Y} = \begin{pmatrix} Y1 \\ Y2 \end{pmatrix}. Yn = -\hat{\mu} Hy B^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \vec{C} Hy, \vec{C} = -\hat{\mu} B^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix}

Удобно H = Han где Han - dot (n): Yn = Cn Ha^n = 1, Ha^n = 1/Cn.
 B^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \sum_{n'} (B^{-1})_{nn'} \begin{pmatrix} 1 \\ 1 \end{pmatrix}_{n'} = \sum_{n'} (B^{-1})_{nn'}. Yn = -\hat{\mu} Hy \sum_{n'} (B^{-1})_{nn'} = 1 (H = Ha^n)
 Ha^n = -\frac{1}{\hat{\mu}} \frac{1}{\sum_{n'} (B^{-1})_{nn'}}. Ум \sum_{n'} B_{nn'} Yn' = -\hat{\mu} Hy \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \sum_{n'} B_{nn'} = -\hat{\mu} Hy Ha^n
 No chiralities of dots Cj would solve problem.
 Удобно annihilation Yn (Ha^n) = Cn = \pm 1. C = +1, Y = +1 \rightarrow H = (Hx, 0)
 Cn also brogues в interdот coupling: \lambda_{nn'}, \nu_{nn'} \sim Cn Cn'.
 Попробуем Ha^n = -\frac{1}{\hat{\mu}} \sum_{n'} B_{nn'} Cn', коэф. \hat{\mu} также зависит от Cn.

12/7 На работу с А. и В. Рачераска. Успешно. Говорю и вкратце, search multiple
 interdот coupling (Vetmedenko). E-mail to Ornel и other on new expansions

13/2 Cus' APL '06: W2 = \mu [\hat{z} \times \vec{H}] \cdot \vec{X} = \mu [Hx Y - Hy X] \quad C=+1 \rightarrow \vec{H} \rightarrow \vec{H} \times \vec{H}
 Ум \vec{X} = 0 \quad \kappa \vec{X} = -\mu \hat{z} \times \vec{H} = -\mu [-\hat{x} Hy + \hat{y} Hx], X = \frac{\mu}{\kappa} Hy, Y = -\frac{\mu}{\kappa} Hx
 Hx > 0 guess Y > 0 Ум C = +1, т.е. -\mu(C+1) > 0.
 \mu = -\frac{3CM_s V}{R} = -\frac{3CM_s \pi R^2 L}{R} = -3CM_s \pi RL = -C\hat{\mu}, \hat{\mu} = M_s \pi RL
 W2(\vec{X}) = -C\hat{\mu} [Hx Y - Hy X] = -C\mu' [Hx Sy - Hy Sx], \mu' = M_s V.
 W(\vec{X}_n) = \frac{1}{2} \kappa \sum_n |\vec{X}_n|^2 + \sum_{nn'} [\lambda_{nn'} Xn Xn' + \nu_{nn'} Yn Yn'] - \hat{\mu} \sum_n Cn [Hx Yn - Hy Xn]
 \frac{\partial W}{\partial Yn} = \kappa Yn + \sum_{n'} \nu_{nn'} Yn' - \hat{\mu} Cn Hx = 0, \sum_{n'} [\kappa \delta_{nn'} + \nu_{nn'}] Yn' = \hat{\mu} Cn Hx
 B_{nn'} = \kappa \delta_{nn'} + \nu_{nn'}, \hat{B} \vec{Y} = \hat{\mu} Hx \vec{C}, \vec{C} = \begin{pmatrix} C1 \\ C2 \\ C3 \end{pmatrix}_{n'}. Y = \hat{\mu} Hx \hat{B}^{-1} \vec{C}
 Yn^{eq} = \hat{\mu} Hx \sum_{n'} (B^{-1})_{nn'} Cn'. Ум Hx = Ha^n, Yn = Cn R \rightarrow Ha^n = \frac{1}{\hat{\mu}} \frac{Cn R}{\sum_{n'} (B^{-1})_{nn'}}
 <\vec{m}_n> = -Cn \hat{z} \times \vec{S}_n = -Cn [-\hat{x} S_y^n + \hat{y} S_x^n], <m_x^n> = Cn S_y^n
 Ум Sy^n = \frac{1}{2} Yn = Cn, <m_x^n> = 1, Hx = Ha^n.

В square dot array Ok энб. Оу в any arrangement, т.е. \lambda_{nn'} = \gamma_{nn'}
 (of cylindrical dots) u Wint = \sum_{nn'} \lambda_{nn'} (\vec{X}_n \cdot \vec{X}_{n'})?

$$J = 2B \left[\frac{1}{6} \arcsinh \frac{L}{b} + 1 - \sqrt{1 + \frac{L^2}{b^2}} \right], \quad f(x) = x \arcsinh x + 1 - \sqrt{1 + x^2}, \quad x = \frac{L}{b}$$

$$f(x) = 1 - \left\{ 1 + \frac{1}{2} x^2 - \frac{1}{8} x^4 \right\} + x \left\{ x - \frac{2}{4 \cdot 3} x^3 \right\} + O(x^6) = \frac{1}{2} x^2 - \frac{1}{24} x^4$$

When $\frac{L}{2R} \ll 1$, $J = 2B f(L/b) \approx 2B \frac{1}{2} \frac{L^2}{b^2} = \frac{L^2}{b}$, $b^2 = |\vec{r} - \vec{r}' - \vec{A}|^2$

$$V_{nn'} = \frac{1}{2} M_s^2 C_n C_{n'} \int d\varphi \int d\varphi' \cos \varphi \cos \varphi' \int dz \int dz' \frac{1}{\sqrt{b^2 + (z - z')^2}} =$$

$$= \frac{1}{2} M_s^2 L^2 C_n C_{n'} \int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi' \frac{\cos \varphi \cos \varphi'}{|\vec{b}|}, \quad \frac{1}{R^2} |\vec{b}|^2 = \Delta^2 + 2\Delta_x (\cos \varphi - \cos \varphi') +$$

$D_{n,2} \frac{1}{|\vec{b}|} = \frac{1}{|\vec{\Delta} - \vec{z}|}$ more precisely. no z, Δ , $\vec{z} = \vec{r} - \vec{r}'$.

$$\frac{1}{|\vec{\Delta} - \vec{z}|} = \frac{1}{\Delta} + \frac{1}{\Delta^3} (\vec{\Delta} \cdot \vec{z}) + \frac{1}{2\Delta^5} [3(\vec{\Delta} \cdot \vec{z})^2 - \Delta^2 z^2], \quad \vec{\Delta} \cdot \vec{z} = \vec{\Delta} \cdot (\vec{r} - \vec{r}') = (\vec{\Delta} \cdot \vec{r}) - (\vec{\Delta} \cdot \vec{r}')$$

Interdot coupling

$$V_{nn'} = \frac{1}{2} M_s^2 L \beta C_n C_{n'} \int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi' \frac{\cos \varphi \cos \varphi'}{|\vec{b}|}, \quad \Delta \text{ is equivalent to } R.$$

$$(\vec{\Delta} \cdot \vec{z})^2 = (\vec{\Delta} \cdot \vec{r})^2 + (\vec{\Delta} \cdot \vec{r}')^2 - 2(\vec{\Delta} \cdot \vec{r})(\vec{\Delta} \cdot \vec{r}') = -2(\Delta_x \cos \varphi + \Delta_y \sin \varphi)(\Delta_x \cos \varphi' + \Delta_y \sin \varphi')$$

negative term.

$$z^2 = 2 - 2[\cos \varphi \cos \varphi' + \sin \varphi \sin \varphi'] = -2[\Delta_x^2 \cos \varphi \cos \varphi' + \Delta_y^2 \sin \varphi \sin \varphi']$$

$$\int_0^{2\pi} d\varphi \cos \varphi \int_0^{2\pi} d\varphi' \cos \varphi' [3(\vec{\Delta} \cdot \vec{z})^2 - \Delta^2 z^2] = \int d\varphi \int d\varphi' \cos \varphi \cos \varphi' [-6\Delta_x^2 \cos \varphi \cos \varphi' - \Delta^2 (-2) \cos \varphi \cos \varphi'] = -2(3\Delta_x^2 - \Delta^2) \int d\varphi \cos^2 \varphi \int d\varphi' \cos^2 \varphi' = -2\pi^2 (3\Delta_x^2 - \Delta^2)$$

B. without coupling. $V_{nn'} = \frac{1}{2} M_s^2 L \beta C_n C_{n'} \frac{1}{2\Delta^5} (-2)\pi^2 (3\Delta_x^2 - \Delta^2) = \frac{\Delta^2 - 3\Delta_x^2}{\Delta^5}$

$$V_{nn'} = \frac{1}{2} M_s^2 L \beta C_n C_{n'} \pi^2 \frac{(\Delta^2 - 3\Delta_x^2)}{\Delta^5}, \quad x = 4\pi^2 M_s^2 L F_1(\beta)$$

$$\frac{|V_{nn'}|}{x} = \frac{1}{8} \frac{\beta}{F_1(\beta)} \frac{|\Delta^2 - 3\Delta_x^2|}{\Delta^5} \approx \frac{1}{8} \frac{\beta}{F_1(\beta)} \frac{1}{\Delta^3}, \quad \Delta = 2 \div 4 \text{ nm}.$$

$$2\pi F_1(\beta) \approx \beta \left[\ln \frac{8}{\beta} - \frac{1}{2} \right], \quad \frac{|V_{nn'}|}{x} \approx \frac{\pi}{4} \frac{1}{\left[\ln \frac{8}{\beta} - \frac{1}{2} \right] \Delta^3}, \quad \frac{|V_{nn'}|}{x} \approx \frac{1}{4} \frac{1}{\beta^3} \approx 0.01$$

benno: $\beta = 0.1$

1) $V_{nn'} = V[(n_1 - n'_1)d, (n_2 - n'_2)d] = V(\Delta_x, \Delta_y)$ because no Δ_x or no Δ_y .

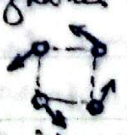
MathCad "Vert-Interdot-coupl" mcd

2) Interdot coupling $V_{nn'}$ maximum value for dots $\times \frac{\hbar}{2} x$, $\vec{A} = (\pm d, 0)$ or $(0, \pm d)$.

3) $\frac{\partial W}{\partial y_n} = 0 \rightarrow x y_n + \sum_{n'} V_{nn'} y_{n'} = \hat{J} C_n H_x$. Since $y_n = C_n$ w.r.t. $H_x = H_a^n$, to C_n comparable (two independent u & b X_n independent w.r.t. W_n) i.e. $V_{nn'} \sim C_n C_{n'}$ or H_x or C_n depends on $y_{n'}$, $\sum_{n'} V_{nn'} C_n C_{n'} C_{n'} = C_n \sum_{n'} V_{nn'}^0$.

Then $x + \sum_{n'} V_{nn'}^0 = \hat{J} H_a^n$, $V_{nn'} = V_{nn'}^0 C_n C_{n'}$, i.e. $H_a^n = \frac{1}{\hat{J}} \left[x + \sum_{n'} V_{nn'}^0 \right]$.

under current equilibrium on coupling field with annihilation $\hat{J}$.

No vortex structure $\langle \vec{M}_n \rangle = \vec{M}_n = -\frac{1}{2} C_n (\hat{x} \times \vec{X}_n)$ by $H_x = 0$? w.r.t. interdot coupling. U. even than factor of coupling, the former is H_x perpendicular vector AFM coupling of coupled dots?  - microvortex?

With $L < 0$ u. some renormalization must be considered.

first μ_n w.r.t. W_n (in-dot), coupling flux-closure AFM-microvortex structures?

4) Since $1/2$ a spin $X_n = 0$ (per $X_n y_{n'}$ in W_n), no more current w.r.t. $H_x \times H_{\text{saturation}}$. $\int d\varphi \int d\varphi' \frac{\cos \varphi \sin \varphi'}{|\vec{b}|} \neq 0$ u. because no quadrupole or n-dot.

$$W = W_m + W_z$$

$$\text{Hago } W_m(\{\vec{X}_n\}) = \frac{\alpha}{2} \sum_n |\vec{X}_n|^2 + \sum_{nn'} [\lambda_{nn'} \vec{X}_n \cdot \vec{X}_{n'} + \delta_{nn'} \vec{X}_n \cdot \vec{Y}_{n'} + \nu_{nn'} \vec{Y}_n \cdot \vec{Y}_{n'}]$$

$$\frac{\partial W_m}{\partial \vec{X}_n} = \sum_{n'} A_{nn'} \vec{X}_{n'} + \sum_{n'} \delta_{nn'} \vec{Y}_{n'} = 0, \quad \frac{\partial W_m}{\partial \vec{Y}_n} = \sum_{n'} B_{nn'} \vec{Y}_{n'} + \sum_{n'} \delta_{nn'} \vec{X}_{n'} = 0$$

$$\hat{A} \vec{X} = -\hat{\Gamma} \vec{Y}, \quad \hat{B} \vec{Y} + \hat{\Gamma} \vec{X} = \hat{H} \vec{C}, \quad \text{busco } X_n \ll Y_n \text{ mones, no } X_n$$

$$\hat{B} \vec{Y} + \hat{\Gamma} (-\hat{A}^{-1} \hat{\Gamma} \vec{Y}) = \hat{H} \vec{C}, \quad [\hat{B} - \hat{\Gamma} \hat{A}^{-1} \hat{\Gamma}] \vec{Y} = \hat{H} \vec{C}, \quad \hat{D} = \hat{B} - \hat{\Gamma} \hat{A}^{-1} \hat{\Gamma}$$

$$\hat{D} \vec{Y} = \hat{H} \vec{C}, \quad \sum_{n'} D_{nn'} \vec{Y}_{n'} = \hat{H} \vec{C}_n, \quad H_{nn'}(\vec{Y}_n = \vec{C}_n) = \frac{C_n}{\hat{A}} \sum_{n'} D_{nn'} C_{n'}$$

Esau $d = \frac{|\hat{A}|}{R} > 2.5$, so dipolar approximation gabours toros.

$$W = \frac{1}{2J} \sum_n |\vec{\mu}_n|^2 + \sum_{nn'} \frac{1}{\Delta_{nn'}^3} [\vec{\mu}_n \cdot \vec{\mu}_{n'} - \frac{3}{\Delta_{nn'}^2} (\vec{\mu}_n \cdot \vec{\Delta}_{nn'}) (\vec{\mu}_{n'} \cdot \vec{\Delta}_{nn'})] - \sum_n \vec{\mu}_n \cdot \vec{H}$$

$$\vec{\Delta}_{21} = \vec{\Delta}_2 - \vec{\Delta}_1 = (d, 0), \quad \vec{\Delta}_{32} = (0, d), \quad \vec{\Delta}_{43} = (-d, 0), \quad \vec{\Delta}_{14} = (0, -d)$$



$$W = \frac{1}{2J} \sum_n \mu_n^2 + \frac{1}{d^3} [\vec{\mu}_1 \cdot \vec{\mu}_2 + \vec{\mu}_2 \cdot \vec{\mu}_3 + \vec{\mu}_3 \cdot \vec{\mu}_4 + \vec{\mu}_4 \cdot \vec{\mu}_1 + \frac{\vec{\mu}_1 \cdot \vec{\mu}_3}{2^{3/2}} + \frac{\vec{\mu}_2 \cdot \vec{\mu}_4}{2^{3/2}} - 3 \dots]$$

$$= \frac{1}{2J} \sum_n \mu_n^2 + \frac{1}{d^3} \left[-\frac{3}{2^{3/2}} \mu^2 - 3 \left\{ \mu \cos \varphi \cdot d \cdot \mu \sin \varphi + \mu^2 \sin \varphi \cos \varphi \sin \varphi + \dots \right\} \right]$$

$$W_m(\varphi) = \frac{1}{2J} \sum_n \mu_n^2 + \sum_{nn'} \frac{\mu^2}{\Delta_{nn'}^3} [\cos(\varphi_n - \varphi_{n'}) - 3 \cos(\varphi_n - \varphi_{\Delta_{nn'}}) \cos(\varphi_{n'} - \varphi_{\Delta_{nn'}})]$$

B APL'99 que uniform cyl. dots in square lattice W_{min} u/m $\varphi_1 = \varphi, \varphi_2 = \varphi_3 = \varphi + \varphi, \varphi_4 = \varphi + \varphi$

$$A_{nn'} = \alpha \delta_{nn'} + \lambda_{nn'} = \alpha \hat{E} + \hat{\Lambda}, \quad \text{que crabs coupling } \hat{\Gamma} \hat{A}^{-1} \hat{\Gamma} \approx \frac{1}{\alpha} \hat{\Gamma}^2$$

$$H_{nn'}^n = \frac{1}{\hat{A}} \sum_{n'} [B_{nn'} - \frac{1}{\alpha} (\hat{\Gamma}^2)_{nn'}] = \frac{\alpha}{\hat{A}} + \frac{1}{\hat{A}} \sum_{n'} [V_{nn'} - \frac{1}{\alpha} (\hat{\Gamma}^2)_{nn'}] - \text{two sets field?}$$

// Ho r.k. coupling λ, δ, ν othocur. man no crabs. α (in-dot), to charara nphoxogus AFM $\rightarrow$ FM, a norem saturation of FM ($H_{an} \rightarrow H_s$)
 $H_c (AF \rightarrow F) \leq 0.2 M_s = 280 \text{ Oe}$, a $H_{an}, H_{sat} > 600 \text{ Oe}$.

(18/E) Kyu. Elvies b Madrid. Orosen. Ove. paces Han u/m $\delta_{nn'} \neq 0$. Kyper Spanish. Oduyng. $N \times N$ coupled dots reversal < Arantxa. Kyper new cell place

$$J_c(\Delta_x, \Delta_y) = \int d\varphi d\varphi' \frac{\cos \varphi \cos \varphi'}{|\mathbf{b}|}, \quad J_s(\Delta_x, \Delta_y) = \int d\varphi d\varphi' \frac{\cos \varphi \sin \varphi'}{|\mathbf{b}|}, \quad J_{cs}(\Delta_x, \Delta_y) = \int d\varphi d\varphi' \frac{\sin \varphi \cos \varphi'}{|\mathbf{b}|}$$

$$\lambda_{nn'} \sim J_s(\vec{\Delta}_{nn'}), \quad \nu_{nn'} \sim J_c(\vec{\Delta}_{nn'}), \quad \delta_{nn'} \sim J_{cs}(\vec{\Delta}_{nn'}). \quad J_s(\Delta_x, \Delta_y) = J_c(\Delta_y, \Delta_x)$$

$$\text{Dna } L = 25 \text{ nm}, R = 300 \text{ nm}, \beta = 1/12, \quad \chi / M_s^2 L = 2\pi\beta \left[\ln \frac{R}{\beta} - \frac{1}{2} \right] = 2.128$$

$$\nu_{nn'} = \frac{1}{2} \beta J_c(\Delta_x, \Delta_y), \quad \nu(2,2) = \nu(\Delta_x, \Delta_y) = 1.712; \quad \nu(2,2) = -0.311, \quad \nu(2,0) = -7.1$$

$$|\mathbf{b}|^2 = [x + \cos \varphi - \cos \varphi']^2 + [y + \sin \varphi - \sin \varphi']^2 - \text{esca } |\mathbf{b}|^2 = 0 \text{ u/m } (x,y) = (2,0), \varphi = \pi$$

Hamme currys aprocas b $\frac{1}{|\mathbf{b}|}$ u/m $J_c(2,0)$ u $J_s(0,2)$ u u/m $(x,y) = (0,2), \varphi = \frac{\pi}{2}, \varphi' = \frac{3\pi}{2}$

Don currys. banyer us. $J = 26 J(1/8)$ peryoname no $\frac{1}{8} \ll 1$.

$$J(b) = 2 J(1/8) b \sim \text{arcsinh } \frac{1}{b} \sim \ln \left[\frac{L + \sqrt{L^2 + b^2}}{b} \right] \sim \ln \frac{L}{b} \text{ peryoname u/m } b \rightarrow 0$$

$$J(b) = |1/b \ll 1 \text{ u/m } \ln| = L^2/b - \text{pec, oquece currys, } \sim \frac{1}{b} \text{ u/m } b \rightarrow 0. \text{ Ho } u_{29} \text{ p } 3. \text{ d } 100$$

$$H_{an}^0 = R \frac{\chi}{\hat{A}} \approx R \frac{2\pi L M_s^2 \cdot 2\pi F_1(\varphi)}{M_s \cdot 2\pi L} = \frac{1}{4} M_s \cdot 4\pi F_1(\varphi) = M_s \cdot 2\pi \left[\ln \frac{R}{\beta} - 1 \right], \quad H_{an}^0 / M_s = 0.677 \beta^2$$

PERSONAL MEMO

Therbasque: kostyagin.guslienko / kostya25

~~Anna Kostyagin~~
혈액형

성명

전화번호

휴대폰

자택

E-mail

팩스

주소

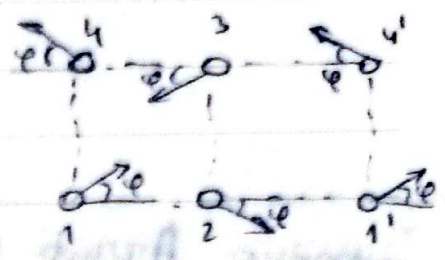
회사명

홈페이지

직장

전화번호

팩스



Proceed. paper (IDS number B0123, ISBN 978-1-90714-338-8)

DS Kearney, XM Cheng, K Buchanan, K Pivan, K.Y Guslienko

Non-linear TD dynamics and transient domains in ferromag. disks. Int. Conf. on Electromagnetics in Advanced Applications ICEAA 2009, vol. 1, 2, p. 825-826, 2009

MEMO

Plan created na Oct. 2010 - July 2011

- 1) VC reversal in H_z field (Rocio)
- ② ~~Ineqn. Squeezing V → SD revers. H_z (Rocio, Llanos 4)~~
- ③ ~~SP current ind. V. dynamics in F/M/F pillar~~
- 4) ~~Vortex rad. u axis. SW (Katrin, Burkard)~~
- 5) Vort. interdot coupling NxN array (Ornel, Ouc.)
- 6) Ni-nanotubes M/H (Ai Ping, Vazquez)
- ⑦ DW in nanotubes (Ai Ping)
- ? 8) SW in nanotube arrays (Vazquez-Papaxag)

imp

$\mathcal{J}_c(\vec{r})$
 $\mathcal{J}_s(\vec{r})$

유통과정중 손상되었거나 이상이 있는 것으로 확인된 제품은 구입처에서 교환하여 드립니다.

$H_{an}^0(1350G) = 914 G$