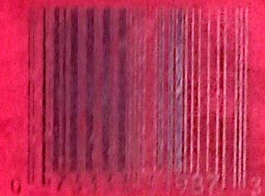
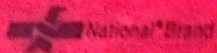


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Загрузка и просмотр Abstracts интересных докладов МММОУ.
Получил на рецензию JAP paper by Крючек - Героден по
SW в многос. вычислениях.
Обсуждаю МММ-paper - репробинг и свои работы к нему по
double-vort. data - Крючек.

$$= \left| \int d\mathbf{r} d\mathbf{r}' e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')} \sum_{\mathbf{l}} f(\mathbf{l}) \right| = R^2 \int_{\mathbf{r}, \mathbf{r}' \in R} d\mathbf{r} d\mathbf{r}' m_p(\mathbf{r}, \mathbf{r}) m_p(\mathbf{r}', \mathbf{r}') \times$$

$$\times \int d\mathbf{k} \sum_{\mathbf{l}} f(\mathbf{l}) \sum_{\mathbf{l}'} e^{i\mathbf{l} \cdot (\mathbf{r} - \mathbf{r}')} J_{\mathbf{l}}(\mathbf{k}, \mathbf{r}) J_{\mathbf{l}'}(\mathbf{k}, \mathbf{r}')$$

$$W_m(S) = R^2 L \int d\mathbf{r} d\mathbf{r}' m_p(R, \mathbf{r}) m_p(R, \mathbf{r}') \int d\mathbf{k} f(\mathbf{k}) \sum_{\mathbf{l}} e^{i\mathbf{l} \cdot (\mathbf{r} - \mathbf{r}')} J_{\mathbf{l}}(\mathbf{k}, \mathbf{r}) J_{\mathbf{l}}(\mathbf{k}, \mathbf{r}') \Big|_{\mathbf{r}, \mathbf{r}' \in R}$$

Если $m_p(R, \mathbf{r})$ не зависит от $\mathbf{r}$ — постоянная концентрация, то

$$W_m(S) = (2\pi)^2 R^2 L m_p^2(R) \int d\mathbf{k} f(\mathbf{k}) J_0^2(kR) \quad V = \pi R^2 L \text{ — объем газа}$$

$$\frac{1}{V} W_m(S) = 2\pi m_p^2(R) \int d\mathbf{k} f(\mathbf{k}) J_0^2(kR)$$

— вычисл. поверхн. плотности энергии для заданной $\mathbf{r}$
 $\mathbf{r} = \mathbf{r}(\rho)$ т.е. $[m_p = f(\rho)]$

$$W_m(S_2) = \frac{1}{2} \oint_S \oint_{S'} \frac{[\pm m_2(\mathbf{r})][\pm m_2(\mathbf{r}')] }{|\mathbf{r} - \mathbf{r}'|} dS dS'$$

$$= \frac{1}{2} \int_{z=L} \int_{z'=L} dS dS' [\dots] - \frac{1}{2} \int_{z=L} \int_{z'=0} dS dS' [\dots] - \frac{1}{2} \int_{z=0} \int_{z'=L} dS dS' [\dots] + \frac{1}{2} \int_{z=0} \int_{z'=0} dS dS' [\dots] =$$

$$= \int dS dS' m_2(\mathbf{r}, \mathbf{r}) m_2(\mathbf{r}', \mathbf{r}') \int d\mathbf{k} [1 - e^{-kL}] \sum_{\mathbf{l}} e^{i\mathbf{l} \cdot (\mathbf{r} - \mathbf{r}')} J_{\mathbf{l}}(\mathbf{k}, \mathbf{r}) J_{\mathbf{l}}(\mathbf{k}, \mathbf{r}') =$$

— так как $m_2 \neq f(z)$, то же 2D распределение $\tilde{m}(\vec{r}) = \tilde{m}(\rho)$.

Если еще $m_2 \neq f(\rho)$, то $|\int dS| = \int d\rho \rho d\varphi$:

$$W_m(S_2) = \int d\rho \rho \int d\varphi \int d\rho' \rho' \int d\varphi' m_2(\rho) m_2(\rho') \int d\mathbf{k} [1 - e^{-kL}] \sum_{\mathbf{l}} e^{i\mathbf{l} \cdot (\mathbf{r} - \mathbf{r}')} J_{\mathbf{l}}(\mathbf{k}, \mathbf{r}) J_{\mathbf{l}}(\mathbf{k}, \mathbf{r}') =$$

$$= \int d\rho \rho \int d\rho' \rho' m_2(\rho) m_2(\rho') (2\pi)^2 \int d\mathbf{k} [1 - e^{-kL}] J_0(k\rho) J_0(k\rho')$$

$$g_{zz}(\rho, \rho') = -\frac{4\pi}{L} \int d\mathbf{k} [1 - e^{-kL}] J_0(k\rho) J_0(k\rho') = \frac{1}{L} \int d\mathbf{r} d\mathbf{r}' \int d\varphi G_{pp}(\vec{r}, \vec{r}')$$

$$g_{zz}(\rho, \rho') = -\frac{4\pi}{\rho} \delta(\rho - \rho') \text{ в пределе } L \rightarrow 0 \text{ (т.е. } L \rightarrow 0 \text{ (т.е. } L \rightarrow 0 \text{))}$$

$$\int_0^R d\rho \rho J_0(k\rho) = \left| k\rho = x \right| = \int_0^{kR} \frac{dx}{k^2} x J_0(x) = \frac{1}{k^2} x J_1(x) \Big|_0^{kR} = \frac{R}{k} J_1(kR)$$

Если в $W_m(S_2)$ $(1 - e^{-kL}) = kL$ (т.е. $L \rightarrow 0$) $\int d\mathbf{k} k J_0(k\rho) J_0(k\rho') = \frac{1}{\rho} \delta(\rho - \rho')$

$$W_m(S_2) = (2\pi)^2 \int d\rho \rho \int d\rho' \rho' m_2(\rho) m_2(\rho') L \frac{1}{\rho} \delta(\rho - \rho') = (2\pi)^2 L \int d\rho \rho m_2^2(\rho)$$

$$\frac{1}{V} W_m(S_2) = 2\pi \int d\rho \rho m_2^2(\rho) \text{ — локальная плотность энергии } \text{при } L \rightarrow 0$$

$$W_m(S_2) = -\pi L \cdot \int d\rho \rho \int d\rho' \rho' g_{zz}(\rho, \rho') m_2(\rho) m_2(\rho'), \text{ но } g_{zz}(\rho, \rho') \text{ зависит от } L.$$

Если же $m_2(\rho) = \text{const}$:

$$W_m(S_2) = (2\pi)^2 m_2^2 \int d\rho \rho \int d\rho' \rho' \int d\mathbf{k} [1 - e^{-kL}] J_0(k\rho) J_0(k\rho') =$$

$$= (2\pi)^2 m_2^2 \int d\mathbf{k} [1 - e^{-kL}] J_1^2(kR) R^3$$

$$\frac{1}{V} W_m(S_z) = 4\pi m_z^2 \frac{1}{L} \int \frac{dk}{k^2} [1 - e^{-kL}] J_1^2(kR) \quad \text{или}$$

$$\frac{1}{V} W_m(S_z) = 4\pi m_z^2 \frac{1}{\beta} \int \frac{dt}{t^2} [1 - e^{-\beta t}] J_1^2(t), \quad \beta = \frac{L}{R} \quad \text{репробавая м. энергию}$$

Впр. $\beta \rightarrow 0$ т.е. $\frac{1}{V} W_m(S_z) = 4\pi m_z^2 \int \frac{dt}{t} J_1^2(t) = 2\pi m_z^2$, но еще
 чл. β т.е. β - расхождение (!), указывающее на сингулярность.

$$\frac{1}{\beta t} (1 - e^{-\beta t}) = 1 - f(\beta t)$$

$$\rightarrow \frac{1}{V} W_m(S_z) = 4\pi m_z^2 \int \frac{dt}{t} [1 - f(\beta t)] J_1^2(t) = 4\pi m_z^2 \left[\frac{1}{2} - N_x(\beta) \right] = 2\pi m_z^2 [1 - 2N_x(\beta)]$$

$$N_z(\beta) = 1 - 2N_x(\beta) \quad \text{так что} \quad \frac{1}{V} W_m(S_z) = 2\pi N_z(\beta) m_z^2 \quad \text{верно.}$$

$$N_x(\beta) = \int \frac{dt}{t} f(\beta t) J_1^2(t) \sim \beta \ln \beta, \text{ см. APL'01.}$$

$$\text{т.е. } W_m(S_z) = 2\pi m_z^2 - O(\beta \ln \beta),$$

а для энергии можно ожидать в более слож. виде:

$$W_m(S) = \frac{1}{2} R^2 \int dz dz' d\varphi d\varphi' m_p(z) m_p(z') \sum e^{i\mu(\varphi - \varphi')} \int dk e^{-k|z - z'|} J_\mu^2(kR)$$

$$\frac{1}{|z - z'|} \Big|_{\substack{p=R, \\ p'=R}} = \frac{1}{\sqrt{p^2 + p'^2 - 2pp' \cos(\varphi - \varphi') + (z - z')^2}} = \frac{1}{\sqrt{2R^2[1 - \cos(\varphi - \varphi')] + (z - z')^2}}$$

→ Не хватает. проверим. заметим (Note: в формуле из б. vortex paper)

$$W_m(S - S_z) = \frac{1}{2} \int_{p=R} dz R d\varphi m_p(z) \int_{z'=0, L} \frac{m_z(z') p' d\varphi' d\varphi'}{|z - z'|} + [\text{аналогич.}] =$$

$$= \frac{1}{2} R \int dz d\varphi \int_{p'=R} dp' p' d\varphi' m_p(z) m_z(z') \int_{z'=0, L} dk e^{-k|z - z'|} \sum e^{i\mu(\varphi - \varphi')} J_\mu(kp) J_\mu(kp') =$$

$$= |\text{pag. cum.}| = \frac{1}{2} R (2\pi)^2 \int dz \int_{p'=R} dp' p' m_p(R, z) m_z(p', z') \int_{(z'=0, L)} dk e^{-k|z - z'|} J_0(kR) J_0(kp')$$

→ bugunno = 0.

$$W_m(V) = \frac{1}{2} \int dV \int dV' \frac{\text{div} \vec{m}(z) \cdot \text{div} \vec{m}(z')}{|z - z'|}, \quad \text{div} \vec{m} = \frac{\partial m_p}{\partial \varphi} + \frac{1}{p} \frac{\partial m_\varphi}{\partial \varphi} + \frac{\partial m_z}{\partial z}$$

Средняя энергия в м. энергии с Eq. Martens (PRL que review, 30/VIII):

$$\text{Use } \frac{1}{R^2 L} W_m^{(S)} = \int d\varphi \int d\varphi' m_p(R, \varphi) m_p(R, \varphi') \int \frac{dk}{k} f(kL) \sum e^{i\mu(\varphi - \varphi')} J_\mu^2(kR)$$

$$\text{Martens: } \frac{1}{R^2 L} E_m(S) = \frac{1}{2\pi} \beta |\log \beta| \int_{\partial \Omega} m_p^2, \quad \partial \Omega - \text{граница } 2D \text{ области } \Omega, \quad d\ell = R d\varphi$$

Независимость по z установлена

интерпр. по z'' , означающая независимость по φ'' .

Может интерпр. по z'' не раскрывается $\frac{1}{|z - z'|}$ в $W_m(S)$ по φ -Бесселя?

$$J_z = \int dz dz' \frac{1}{|z - z'|} = \int dz dz' \frac{1}{\sqrt{2R^2[1 - \cos(\varphi - \varphi')] + (z - z')^2}} = \int_0^L dz \int_0^L dz' \frac{1}{\sqrt{a^2 + (z - z')^2}}$$

$J_z = \int dz \operatorname{Arsh} \frac{z'-z}{a} \Big|_0^L = \int_0^L dz \left[\operatorname{Arsh} \frac{L-z}{a} + \operatorname{Arsh} \frac{z}{a} \right], \quad a^2 = 2R^2[1 - \cos(\varphi - \varphi')] = 4R^2 \sin^2(\frac{\varphi - \varphi'}{2})$

(бегущие на $J_z(\varphi, \varphi') \propto \delta(\varphi - \varphi')$ при $L/R \rightarrow 0$ (упрощение)).

$W_m(S) = \frac{1}{2} R^2 \int d\varphi \int d\varphi' m_p(R, \varphi) m_p(R, \varphi') J_z(\varphi, \varphi')$

Далее считаем $m_p \neq f(\varphi)$:

используя $\frac{1}{(\pi R^2 L)} W_m(S) = 4\pi m_p^2(R) \int_0^1 dt f(\beta t) J_0^2(t) = F(\beta) m_p^2(R), \quad F(\beta) = 4\pi \int_0^1 dt f(\beta t) J_0^2(t)$

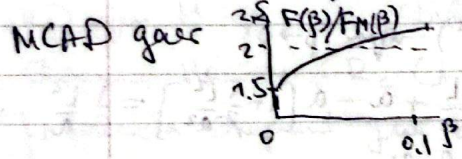
Martens $\frac{E_m(S)}{(\pi R^2 L)} = \frac{\beta}{\pi} |\log \beta| \cdot m_p^2(R)$

Упрощенная $E_m(S)$ и Martens-PRL $\frac{E_m(S)}{m_p^2 R^2} = \frac{L}{R} \int d^2 x m_2^2$

$\frac{E_m(S_2)}{m_p^2 (\pi R^2 L)} = \frac{1}{\pi} \int d^2 x m_2^2 = \frac{1}{\pi} \int_0^1 d\beta \beta \int_0^{2\pi} d\varphi m_2^2 = \frac{1}{\pi} \frac{1}{2} 2\pi m_2^2 = m_2^2 \quad (\beta \rightarrow 0)$

$\frac{W_m(S_2)}{m_p^2 V} = 2\pi m_2^2, \quad \text{т.е.} \quad \frac{W_m(S_2)}{\text{Martens}} = 2\pi \cdot \frac{E_m(S_2)}{\text{Martens}}$

Martens $\Rightarrow \frac{E_m(S)}{V} \rightarrow \frac{W_m(S)}{V} = 2\pi \frac{E_m(S)}{V} = 2\beta |\log \beta| m_p^2(R) = F_M(\beta) m_p^2$



$F(\beta) > F_M(\beta)$ в 1.5-2.5 раза при $\beta < 0.1$

$\frac{F(\beta)}{F_M(\beta)}$ не есть const(β).

У Martens есть из Eq. (6) следствие $m_p = f(\varphi)$ (θ - бегущая волна).

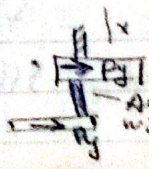
- Balog :
- 1) поведение Z и A при m_p^2 и Martens не зависят от β
 - 2) Martens упрощенная $[1 - 2N_x(\beta)]$ уменьшается при m_2^2 , что дает Z и $A \sim \beta \mu \cdot m_2^2$, хотя при $m_2 = 0$ это не так
 - 3) упрощенная (2π) углы.

Упрощенная bulk term $W_m(V)$ и Martens, которые совпадают.

Выводы $\{dz dz'(\dots)\}$, много р. бегущие (через $\operatorname{Arsh} z$), где упрощенная J_z
Вывод $\beta \rightarrow F(\beta)$ не имеет аналогов.

Group Meeting

- (I) Упрощенная что Sam 2-page proposals go 21/17
то упрощенная не имеет (field), то есть сейчас в мн. плане
"то что сейчас (challenges, links to technology, Refs, Figs)."
- (II) Seminar talk by Yi J. "Lateral spin-values" T_2 ?
См. гон. мн. упрощенная spin-дл. length $\delta_s = \sqrt{2\sigma T_2}$ в Au
 $\delta_s(\text{Au}) = 65 \text{ nm}$ при ток. течение по T_2 - FM(x) induced в Au.



8/1x Belega & Vanden. Описываемое нами ядро, но которое, но мы не ждем, но и когда он происходит не совсем. Описываемое MIM-paper in Double-vortex.

Самое простое и базовое: 1) ring MIM и 2) FMR in double-layer stripes/dots; 3) two-vortex dynamics (H) и динамика double-L dots

8/1x Belega & T. Shima: описываемая для анзаца $K_1(r) \sim M_2^2(r)$.

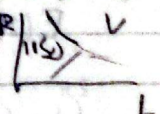
FePt FeRh - нечетко $K_1(r)$ и $M_2(r)$ для случая, у нас описываем нечетко монофункционалов FePt с осью "с" и поперечные

b) Shima - co-автор Goto no FeRh/FePt paper - MIM, генерация излучения $M(H)$ при $\mu_0 H$ и J_2 не фазе

c) Shima генерация излучения TbFe/Fe M_2 , где spin-flop из-за AFM coupling и сформированное намагниченность $\lambda(H)$ $\sqrt{\mu}$ вращение λ H норм. анз. no L phase.

Double-vortex

Belega & Krusten: описываемая vortex stability нечетко (L, R) при $H=0$ и v. stability as function of field $\mu_0 H$ and (L, R) Single-L.

→ Описываем $R_c(L, d)$ где 11 SD - vortex нечетко $R_c(L, d)$ 

SD нечетко under magnetic $\textcircled{1} \Rightarrow \textcircled{2}$ при $H=0$, нечетко

бухало, нечетко в анзаца $\textcircled{3}$ off-centered vortices в $\textcircled{4}$ $\textcircled{5}$ $\textcircled{6}$ $\textcircled{7}$ $\textcircled{8}$ $\textcircled{9}$ $\textcircled{10}$ $\textcircled{11}$ $\textcircled{12}$ $\textcircled{13}$ $\textcircled{14}$ $\textcircled{15}$ $\textcircled{16}$ $\textcircled{17}$ $\textcircled{18}$ $\textcircled{19}$ $\textcircled{20}$ $\textcircled{21}$ $\textcircled{22}$ $\textcircled{23}$ $\textcircled{24}$ $\textcircled{25}$ $\textcircled{26}$ $\textcircled{27}$ $\textcircled{28}$ $\textcircled{29}$ $\textcircled{30}$ $\textcircled{31}$ $\textcircled{32}$ $\textcircled{33}$ $\textcircled{34}$ $\textcircled{35}$ $\textcircled{36}$ $\textcircled{37}$ $\textcircled{38}$ $\textcircled{39}$ $\textcircled{40}$ $\textcircled{41}$ $\textcircled{42}$ $\textcircled{43}$ $\textcircled{44}$ $\textcircled{45}$ $\textcircled{46}$ $\textcircled{47}$ $\textcircled{48}$ $\textcircled{49}$ $\textcircled{50}$ $\textcircled{51}$ $\textcircled{52}$ $\textcircled{53}$ $\textcircled{54}$ $\textcircled{55}$ $\textcircled{56}$ $\textcircled{57}$ $\textcircled{58}$ $\textcircled{59}$ $\textcircled{60}$ $\textcircled{61}$ $\textcircled{62}$ $\textcircled{63}$ $\textcircled{64}$ $\textcircled{65}$ $\textcircled{66}$ $\textcircled{67}$ $\textcircled{68}$ $\textcircled{69}$ $\textcircled{70}$ $\textcircled{71}$ $\textcircled{72}$ $\textcircled{73}$ $\textcircled{74}$ $\textcircled{75}$ $\textcircled{76}$ $\textcircled{77}$ $\textcircled{78}$ $\textcircled{79}$ $\textcircled{80}$ $\textcircled{81}$ $\textcircled{82}$ $\textcircled{83}$ $\textcircled{84}$ $\textcircled{85}$ $\textcircled{86}$ $\textcircled{87}$ $\textcircled{88}$ $\textcircled{89}$ $\textcircled{90}$ $\textcircled{91}$ $\textcircled{92}$ $\textcircled{93}$ $\textcircled{94}$ $\textcircled{95}$ $\textcircled{96}$ $\textcircled{97}$ $\textcircled{98}$ $\textcircled{99}$ $\textcircled{100}$

Disk/ring magnetostatics $\beta \rightarrow 0$ / Surface anisotropy

$J_z(\varphi, \varphi')$. Него где $m_p \neq f(\varphi')$ $\int d\varphi d\varphi' J_z(\varphi, \varphi') = J_{\text{ex}}$

$$J_{\text{ex}} = \int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi' J_z(\varphi - \varphi') = \int_0^{2\pi} d\varphi' \int_0^{2\pi} d\varphi J_z(x) = \int_0^{2\pi} d\varphi' \int_{-\varphi'}^{2\pi - \varphi'} dx J_z(x) = \int_0^{2\pi} d\varphi' \int_0^{2\pi} dx J_z(x) = 2\pi \int_0^{2\pi} d\varphi J_z(\varphi) = 2\pi J_{\varphi}$$

$$\frac{1}{2L} J_z(\varphi) = \text{Arsh} \left(\frac{\beta}{2 \sinh \varphi} \right) + \frac{2 \sinh \varphi}{\beta} - \sqrt{1 + \left(\frac{2 \sinh \varphi}{\beta} \right)^2} = f(\beta)$$

$$\frac{df}{d\beta} = \frac{1}{\sqrt{1 + \frac{\beta^2}{4 \sinh^2 \varphi}}} \cdot \frac{1}{2 \sinh \varphi} + 2 \sinh \varphi \left(-\frac{1}{\beta^2} \right) - \frac{4 \sinh^2 \varphi \left(\frac{2}{\beta^3} \right)}{2 \sqrt{1 + \frac{4 \sinh^2 \varphi}{\beta^2}}} = \frac{1}{\sqrt{4 \sinh^2 \varphi + \beta^2}} - \frac{2 \sinh \varphi}{\beta^2} +$$

$$+ \frac{4 \sinh^2 \varphi}{\beta^2 \sqrt{\beta^2 + 4 \sinh^2 \varphi}} = \frac{1}{\sqrt{4 \sinh^2 \varphi + \beta^2}} \left[1 + \frac{4 \sinh^2 \varphi}{\beta^2} \right] - \frac{2 \sinh \varphi}{\beta^2} = \frac{1}{\beta^2 \sqrt{\beta^2 + 4 \sinh^2 \varphi}} - \frac{2 \sinh \varphi}{\beta^2}$$

$$f_1(\beta) = \frac{df}{d\beta} = \frac{1}{\beta^2} \left[\sqrt{\beta^2 + 4 \sinh^2 \varphi} - 2 \sinh \varphi \right] = \frac{2}{\beta^2} \left[\sqrt{\beta^2/4 + \sinh^2 \varphi} - \sinh \varphi \right] = f_1(\beta, \varphi) = \frac{d}{d\beta} \left[\frac{1}{2L} J_z(\varphi, \beta) \right]$$

Учтем $f_1(\beta, \varphi)$ по φ .

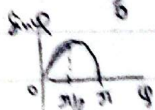
$$\int_0^{2\pi} d\varphi f_1(p, \varphi) = \frac{2}{\beta^2} \int_0^{2\pi} d\varphi \left[\sqrt{\beta^2/4 + \sin^2 \varphi} - \sin \varphi \right] = \frac{2}{\beta^2} \int_0^{2\pi} d\varphi \sqrt{\beta^2/4 + \sin^2 \varphi}$$

Pr. - p. - 2.576. $\int \sqrt{a + b \cos x} dx = (\dots)$ u.m. 2.597. Zurückw. second kind

$$\int_0^{2\pi} d\varphi \sqrt{\beta^2/4 + \sin^2 \varphi} = \frac{\beta}{2} \int_0^{2\pi} d\varphi \sqrt{1 + \left(\frac{\beta}{2}\right)^2 \sin^2 \varphi} = \frac{\beta}{2} \left[\sqrt{1 + \beta^2/4} E\left(\alpha, \frac{\beta/2}{\sqrt{1 + \beta^2/4}}\right) - \frac{\beta^2 \sin \varphi \cos \varphi}{\sqrt{1 + \beta^2 \sin^2 \varphi}} \right]_0^{2\pi}$$

$$= \frac{\beta}{2} \sqrt{1 + \beta^2/4} E\left(\alpha, \frac{1}{\sqrt{1 + \beta^2/4}}\right) \Big|_0^{2\pi} \quad \alpha = \alpha(\varphi) = \arcsin \left(\frac{\beta/2 \sin \varphi}{\sqrt{1 + \beta^2 \sin^2 \varphi}} \right), \quad \beta = \beta/2.$$

$$\int_0^{2\pi} d\varphi F(\sin^2 \varphi) = \int_{-\pi}^{\pi} d\varphi F = 2 \int_0^{\pi} d\varphi F = 4 \int_0^{\pi/2} d\varphi F(\varphi)$$



$$\int_0^{2\pi} d\varphi \sqrt{\beta^2/4 + \sin^2 \varphi} = \frac{\beta}{2} \left[\sqrt{1 + \beta^2/4} E\left(\alpha, \frac{1}{\sqrt{1 + \beta^2/4}}\right) - \frac{\beta^2 \sin \varphi \cos \varphi}{\sqrt{1 + \beta^2 \sin^2 \varphi}} \right]_0^{\pi/2} \cdot 4 =$$

$$= 4 \sqrt{1 + \beta^2/4} E\left(\alpha, \frac{1}{\sqrt{1 + \beta^2/4}}\right) \Big|_0^{\pi/2}, \quad \alpha(\pi/2) = \arcsin \frac{\sqrt{1 + \beta^2}}{\sqrt{1 + \beta^2}} = \arcsin(1) = \frac{\pi}{2}$$

$$\alpha(0) = 0$$

$$\int_0^{2\pi} d\varphi \sqrt{\beta^2/4 + \sin^2 \varphi} = 4 \sqrt{1 + \beta^2/4} \left[E\left(\frac{\pi}{2}, \frac{1}{\sqrt{1 + \beta^2/4}}\right) - E\left(0, \frac{1}{\sqrt{1 + \beta^2/4}}\right) \right] = 4 \sqrt{1 + \beta^2/4} E\left(\frac{\pi}{2}, \frac{1}{\sqrt{1 + \beta^2/4}}\right)$$

$E(k) = E\left(\frac{\pi}{2}, k\right)$ - nome da 2.ª esp. de segunda kind. $\beta \rightarrow 0, k \rightarrow 1$

$$E(k) = 1 + \frac{1}{2} k'^2 \left(\ln \frac{4}{k'} - \frac{1}{2} \right) + O(k'^4) \quad \text{u.m.} \quad k' = \sqrt{1 - k^2} \rightarrow 0.$$

$$k^2 = \frac{1}{1 + \beta^2/4}, \quad k'^2 = 1 - k^2 = 1 - \frac{1}{1 + \beta^2/4} = \frac{\beta^2}{4} \frac{1}{1 + \beta^2/4}, \quad k' = \frac{\beta}{2} \frac{1}{\sqrt{1 + \beta^2/4}}$$

$$E\left(\frac{\pi}{2}, \frac{1}{\sqrt{1 + \beta^2/4}}\right) = 1 + \frac{1}{2} \frac{\beta^2}{4} \frac{1}{1 + \beta^2/4} \left[\ln \frac{4}{(\beta/2)(\sqrt{1 + \beta^2/4})} - \frac{1}{2} \right] \approx 1 + \frac{\beta^2}{8} \left[\ln \frac{8}{\beta} - \frac{1}{2} \right]$$

$$\int_0^{2\pi} d\varphi f_1(p, \varphi) = \frac{2}{\beta^2} \cdot 4 \int_0^{\pi/2} d\varphi \left[\sqrt{\beta^2/4 + \sin^2 \varphi} - \sin \varphi \right] = \frac{8}{\beta^2} \left[\sqrt{1 + \beta^2/4} E\left(\frac{\pi}{2}, \frac{1}{\sqrt{1 + \beta^2/4}}\right) - 1 \right] \Big|_{\beta \rightarrow 0}$$

$$\approx \frac{8}{\beta^2} \left[\left(1 + \frac{\beta^2}{8}\right) \left(1 + \frac{\beta^2}{8} \left\{ \ln \frac{8}{\beta} - \frac{1}{2} \right\} \right) - 1 \right] = \frac{8}{\beta^2} \left[1 + \frac{\beta^2}{8} \left(\ln \frac{8}{\beta} - \frac{1}{2} \right) + \frac{\beta^2}{8} - 1 \right] =$$

$$= \frac{8}{\beta^2} \frac{\beta^2}{8} \left[\ln \frac{8}{\beta} + \frac{1}{2} \right] = \ln \frac{8}{\beta} + \frac{1}{2}$$

$$y(\beta) = \int_0^{2\pi} d\varphi f_1(p, \varphi) = \frac{1}{2L} \frac{d}{d\beta} \int_0^{2\pi} d\varphi J_2(\varphi, \beta) = \ln \frac{8}{\beta} + \frac{1}{2}, \quad \frac{dy}{d\beta} = \ln \frac{8}{\beta} + \frac{1}{2} + O(\beta^2) + O(\beta^2 \ln \beta)$$

$$y(\beta) = \int d\beta \left[\ln \frac{8}{\beta} + \frac{1}{2} \right] + C = \int d\beta \left[-\ln \beta + \ln 8 + \frac{1}{2} \right] =$$

$$= -[\beta \ln \beta - \beta] + \beta \left[\ln 8 + \frac{1}{2} \right] + C = \beta \left[-\ln \beta + 1 + \ln 8 + \frac{1}{2} \right] + C = \beta \left[\ln \frac{8}{\beta} + \frac{3}{2} \right] + C$$

$C = 0$ u.m. $y(0) = 0$.

$$y(\beta) = \beta \left[\ln \frac{8}{\beta} + \frac{3}{2} \right]$$

$$\frac{1}{2L} \int_0^{2\pi} d\varphi J_2(\varphi, \beta) = y(\beta) = \beta \left[\ln \frac{8}{\beta} + \frac{3}{2} \right]$$

$$\rightarrow W_m(s) = \frac{1}{2} R^2 \int d\varphi d\varphi' m_p(R, \varphi) m_p(R, \varphi') J_2(\varphi - \varphi') = \frac{1}{2} R^2 m_p^2(R) \int_0^{2\pi} d\varphi d\varphi' J_2(\varphi - \varphi') =$$

$$= \frac{1}{2} R^2 m_p^2 \cdot 2\pi \int_0^{2\pi} d\varphi J_2(\varphi) = \frac{1}{2} R^2 m_p^2 \cdot 2\pi \cdot 2L \cdot \beta \left[\ln \frac{8}{\beta} + \frac{3}{2} \right] = \frac{1}{2} R^2 m_p^2 \cdot 2\pi \cdot 2L \cdot \beta \left[\ln \frac{8}{\beta} + \frac{3}{2} \right]$$

Cylinder $\frac{1}{V} W_m(s) = m_p^2(R) \cdot 2\beta \left[\ln \frac{8}{\beta} + \frac{3}{2} \right] = F(\beta) m_p^2(R)$ поверх. с гранич. усл. $m_p = f(\beta)$

у Mertens $\frac{1}{V} W_m(s) = m_p^2(R) \cdot 2\beta |\log \beta| = F_m(\beta) \cdot m_p^2(R)$

Можно $F(\beta) = 2\beta \left[\ln \frac{8}{\beta} + \frac{3}{2} \right]$, а у Mertens-PRL $F_m(\beta) = 2\beta \ln \frac{1}{\beta}$ (если $\log \beta = \ln \beta$)

$F(\beta) = 2\beta \left[\ln \frac{1}{\beta} + \ln 8 + \frac{3}{2} \right]$ и формально при $\beta \rightarrow 0$ $F(\beta) = 2\beta \ln \frac{1}{\beta}$, но

при всех разумных $\beta = 0.001 - 0.1$ член $(\ln 8 + \frac{3}{2})$ важен.

$\beta = L/R$. И если $R = 1 \mu m = 10^3 nm$, то $\beta = 0.001 = \frac{1 nm}{10^3 nm}$ const. $L = 1 nm$ - тоньше не имеет смысла,

т.е. там нужен густой газ.

$\ln 8 + \frac{3}{2} = 3.58$ и $\ln \frac{1}{\beta_{min}} = \ln \left(\frac{1}{0.001} \right) = 6.91$, т.е. всегда $\ln \frac{1}{\beta}$ сравнимо $(\ln 8 + \frac{3}{2})$

Выводы: 1) поверхн. член Mertens после $(\times 2\pi)$ дает $2\beta \ln \frac{1}{\beta}$, что верно лишь при мал. предель $\beta \rightarrow 0$, но неверно $\forall \beta > 0.001$ - реальных [то при $m_p(\varphi) = const$]

2) то что $J_z(\varphi - \varphi') \sim \delta(\varphi - \varphi')$ строго доказать, т.е. при $m_p = f(\varphi)$ математика $W_m(s)$ некачественна. Важно $m_p(\varphi)$ у Mertens. $J_z(\varphi - \varphi')$ имитирует $\delta(\varphi - \varphi')$ при $\beta \rightarrow 0$.

$\hat{J}_z(\varphi) = \frac{1}{2L} J_z(\varphi) = \text{Arsh} \left(\frac{\beta}{2 \sin \varphi} \right) + \frac{2 \sin \varphi}{\beta} - \sqrt{1 + \frac{4 \sin^2 \varphi}{\beta^2}}$ Cyl-Bound Cond. mod

Если $\hat{J}_z(\varphi) \sim \delta(\varphi)$, то $J_z(0) = \infty$, $J_z(\varphi \neq 0) = 0$, т.е. важно. при $\beta \rightarrow 0$
 $\hat{J}_z(\varphi) = A \delta(\varphi)$ $\int_{-\pi}^{\pi} d\varphi f(\varphi) \frac{\hat{J}_z(\varphi)}{A} = f(0)$, $A = \int_{-\pi}^{\pi} d\varphi \hat{J}_z(\varphi)$ - нормировка

то при $\beta > 0.001$ не выполняется.

Конечно $\hat{J}_z(\varphi)$ имеет острый max при $\varphi = 0$ если $\beta \ll 1$, но само

$J_z(\varphi) \approx \text{Arsh} \left(\frac{\beta}{2\varphi} \right) \approx \ln \frac{\beta}{\varphi}$ - расхождение слабо, логарифмическое.

Для $\delta(x)$ - приближенных представлений характерна более сильная расхождение $\sim \frac{1}{\epsilon}$ или $\sim \frac{1}{x}$. $\delta(x) = \lim_{\epsilon \rightarrow 0} \frac{1}{\pi} \frac{\epsilon}{\epsilon^2 + x^2}$

→ Показывая математика для surface cylinder/ring не проходит даже при $\beta \ll 1$. Mertens не ув. ($p=R$)

$\lambda_{(p)} = \int d\mathbf{p}' g_{pp}(\mathbf{p}, \mathbf{p}') m_p(\mathbf{p}')$, m_p - sw неустойчивость dynamic для $m_p = f(p)$ - разг. ам. в cylinder

$\lambda_{\mu}(p) = \int d\mathbf{p}' g(\mathbf{p}, \mathbf{p}') \mu(\mathbf{p}')$, $g(\mathbf{p}, \mathbf{p}') = (-4\pi) \int dk k f(k) J_1(kp) J_1(kp')$ не есть $g(p-p)$ как в stripes

Разность $\mu(\mathbf{p}) = \mu(p) + \mu'(p)(\mathbf{p} - p) + \dots$ как в stripe где bound. conditions для cylinder бред и неужет.

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$p \rightarrow R, \quad \lambda \mu(p) = \int dg g' g(p, p') [J^\mu(p) + \mu'(p)(p' - p) + \dots] = \mu(p) \int dg g' g(p, p') +$
 $+ \mu'(p) + \int dg g' g(p, p')(p' - p) = 0$ no Ivanov, $d\mu(R) + \mu'(R) = 0$ boundary condition.

Prüfung: $d = R \int_0^R r g(R, r) / \left[\int_0^R r g(R, r) (r - R) \right] = d(\beta)$, $R=1$ und $P/R \rightarrow P$.

$$d(\bar{p}) = \frac{\int_0^1 dp p^2 g(1, p)}{\int_0^1 dp p g(1, p)} - 1 \quad g(1, p) = (-4\pi) \int dt p(p+t) J_1(t) J_1(tp)$$

Из-за малой сходимости $g(p, p')$ МСД очень медленно сходится.

M. Stripes Какое поведение млекопитающих анисотропно и stripes?

$$g_{xx}(x, x') = -4\pi\delta(x-x') - g_{zz}(x, x'), \quad g_{zz}(x, x') = \frac{2}{L} \ln \frac{(x-x')^2}{(x-x')^2 + L^2}$$

$$W_m(s) = \frac{1}{2} \iint \frac{d\vec{S} \cdot \vec{m}(r) d\vec{S}' \cdot \vec{m}(r')}{|r-r'|} = \frac{1}{2} \iint_{\text{surf. of } \Omega} \frac{m_x(r) m_x(r')}{|r-r'|} =$$

$$g_{xx}(x, x') = - \left\langle \frac{\partial^2}{\partial x \partial x'} \frac{1}{|r - r'|} \right\rangle_{y, z} \quad \text{Bug } J_2(a) \text{ comparison, } J_2 = \int_0^a dz dz' \frac{1}{|r - r'|},$$

$$\text{no } a^2 = (x - x')^2 + (y - y')^2 \text{ then.}$$

Усложно не "y" и в том, что $\vec{V}$ - негладкое, так как $\delta(k_y)$ близки $k \rightarrow \infty$, отсутствует не "y".

$$J = \int dz dz' \int dy dy' \frac{1}{|z-z'|} = \int_0^L dz dz' \int_{-L/2}^{L/2} dy dy' \int d^3 \vec{k} \frac{e^{i \vec{k} \cdot (z-z')}}{k^2}$$

$\Rightarrow$ Current I_d на поверхности stripe $x = \frac{w}{2}$ $\Delta H_d = 2\pi M_x - (-2\pi M_x) = 4\pi M_x = 8$

Stripe $\rightarrow$ Вектор упругости $H_c(t)$ - stripe exp., возмущенный by $e^{-T/T_0} (?)$ (transverse) sharrack eq.

Boundary conditions give thin stripes/surface anisotropy.

Underpin by Weyl:

$$\frac{e^{ik_0 r}}{r} = \frac{i}{2\pi} \int d^2 \vec{k} \frac{e^{i\vec{k} \cdot \vec{r}} e^{ik_z |z|}}{k_z}, \quad k_z^2 + \vec{k}^2 = k_0^2$$

ФГ que obreiro cria:

$$G_{\alpha\beta}(\vec{p}, \vec{p}') = -\frac{1}{\pi} \int d^2k \frac{k_\alpha k_\beta}{k^2} f(k) e^{i\vec{k}(\vec{p}-\vec{p}')} \\ \alpha, \beta = x, y$$

$$G_{zz}(\vec{p}, \vec{p}') = -\frac{1}{\pi} \int d^2k [1 - f(k)] e^{i\vec{k}\vec{r}_{pp'}}$$

$$G_{\alpha\beta}(\vec{r}, \vec{r}') = \frac{1}{L} \int_0^L dz dz' \Gamma_{\alpha\beta}(\vec{r}, \vec{r}')$$

$$\Gamma_{\alpha\beta}(r, r') = -\frac{1}{2\pi} \int d^2k \frac{k_\alpha k_\beta}{k}$$

- измер/у/р/е/н/и/е по K_2 и $\rho_{\text{всего}}$.

$$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{2\pi} \int d^2\vec{k} \frac{1}{k} e^{-k|\vec{r}-\vec{r}'|} e^{i\vec{k}(\vec{\rho}-\vec{\rho}')}.$$

$$\int_0^L dz dz' \frac{1}{|z-z'|} = \frac{L}{\pi} \int_0^{\pi} d\vec{k} \frac{1}{k} f(kL) \rho(\vec{k}) (\vec{p}-\vec{p}')$$

$$J = \int d^2z d^2z' \int_{-l/2}^{l/2} dy dy' \frac{1}{|z-z'|} = \int dy dy' \frac{L}{\pi} \int d^2k \frac{1}{k^2} f(kL) e^{ik(\vec{r}-\vec{r}')} - \text{см. Zenerovoy "8w-pz61.doc"}$$

$$\int_{-l/2}^{l/2} dy dy' e^{iky(y-y')} = 4 \frac{\sin^2(ky \frac{l}{2})}{ky^2} \xrightarrow{l \rightarrow \infty} 4 \cdot \frac{\pi l}{2} \delta(ky)$$

$$J = J(x, x') = \frac{L}{\pi} \int d^2k \frac{1}{k^2} f(kL) e^{ik_x(x-x')} \int dy dy' e^{iky(y-y')} = \frac{L}{\pi} \int d^2k \frac{1}{k^2} f(kL) e^{ik_x x} (2\pi l) \delta(k_y) =$$

$$= 2Ll \int dk_x \frac{1}{k^2} f(kxL) e^{ik_x(x-x')} - \text{срочно, расхожимся при } k_x \rightarrow 0? \rightarrow \text{см.}$$

→ T. Rosenbaum (Uni. Chicago) seminar: "Quantum annealing".

LiHoFe_4 , $T_c = 1.53 \text{ K}$ — singlet

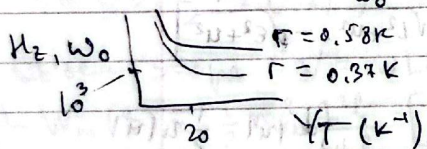
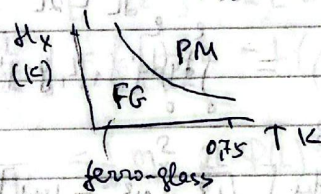
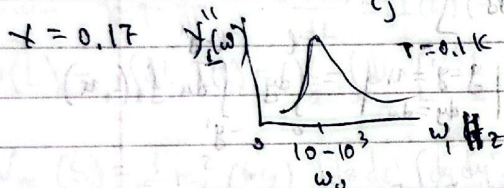
$\text{LiHo}_x\text{Y}_{1-x}\text{Fe}_4$

substitution Y by Ho, $x < 0.01\%$

$9.4 \text{ K} \uparrow$ Ising doublet, $g_{||} = 14, g_{\perp} = 0$

Disordered system. H_x — anisotropy constant ($\vec{H} \perp \text{axis of symmetry}$)

$$\mathcal{H} = - \sum_{ij} J_{ij} \hat{S}_i^z \hat{S}_j^z - H_x \sum_i \hat{S}_i^x, \quad H_x \sim \Gamma, \quad [\mathcal{H}, \hat{S}^z] \neq 0$$



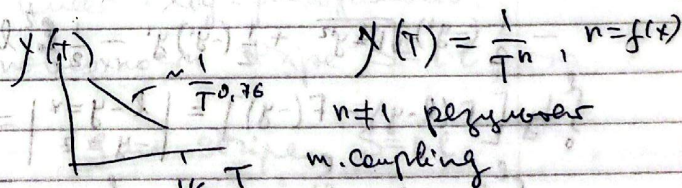
$\omega_0 = \frac{1}{\tau}$ — relax. process, $\tau \sim 10^{-2} - 10^{-4} \text{ s (!)}$

Tunneling of DW:

DW mass $m = \frac{\hbar^2}{2a^2(\Gamma + \Gamma_2)} \sim 10$ Ising spin chain (какое-то значение k масса DW доминирует?)

Santoro, Science.

$\chi(\tau)$ of LiHoYF $x = 4\%$



$\chi''(\omega)$ characterizes non-equilibrium, viz. the change in magnetization.

→ C. T.F. Rosenbaum: "Nature" 425, 6953 (2003), 419, 6901, p. 459 (2002) и 413, 6256, 610 (2001).

Tunneling of DW & Ising magnet: nonferrous case H_x ,

государственный DW и соседней ее сгруппировка (tunneling) на расстоянии

$\uparrow \uparrow \uparrow \downarrow \downarrow \rightarrow H_x \uparrow \uparrow \uparrow \downarrow \downarrow$ при этом $H_z = 0$ и взаимно близкая форма Zeeman interaction.

$$J = \int dz dz' \int dy dy' \frac{1}{|z-z'|} \text{ надо переписать на } \begin{array}{c} z \\ \swarrow \searrow \\ \frac{w}{2} \quad x \end{array} \text{ поверхность}$$

 Strip $x = \frac{w}{2}, x' = \frac{w}{2}, x-x' = 0, \text{ т.е. } J = \int dz dz' \int dy dy' \frac{1}{\sqrt{(y-y')^2 + (z-z')^2}}$

Note: где unknown. goes

$$G_{\alpha\beta}(\vec{r}, \vec{r}') = -2 \frac{\partial^2}{\partial x_\alpha \partial x_\beta} \Phi\left(\frac{|\vec{r}-\vec{r}'|}{L}\right), \quad \Phi(x) = \operatorname{Arsh} \frac{1}{x} + x - \sqrt{1+x^2} - \text{"Sw-p'bl, do"}$$

$$\frac{1}{2L} J_z(a) = \operatorname{Arsh} \frac{L}{a} + \frac{a}{L} - \sqrt{1 + (a/L)^2} \text{ равно } J_z(a/L) \text{ считаем } \Phi(x).$$

то $a^2 = (x-x')^2 + (y-y')^2 = (y-y')^2$ и $x, x' = \frac{w}{2}$ на док. поверхности.

$$\frac{1}{2L} J_z(y, y') = \frac{1}{2L} J_z(a) = \operatorname{Arsh} \frac{L}{|y-y'|} + \frac{|y-y'|}{L} - \sqrt{1 + \left(\frac{|y-y'|}{L}\right)^2}, \quad a = |y-y'|.$$

Due to symmetry $J_z(a, L)$ unknown very group. no L or $8/\pi$.

$$f(L, yy') = f(L) = \frac{1}{2L} J_z(a, L). \quad \text{Consider. } \begin{array}{l} L \leftrightarrow \frac{\beta}{2} \\ |y-y'| \rightarrow \sin(\varphi-\varphi') \end{array}$$

$$f_1(L) = \frac{df}{dL} = \frac{2}{(2L)^2} \left[\sqrt{L^2 + (y-y')^2} - |y-y'| \right] - \text{нужно интегрировать по } y, y'.$$

$$I = \left[\frac{1}{2L} J_z \right]. \quad f_1(L, yy') = \frac{d}{dL} \left[\frac{1}{2L} J_z(L, yy') \right]$$

$$I = \int_{-l/2}^{l/2} dy \int_{-l/2}^{l/2} dy' f_1(L, yy') = \int_0^l dy \int_0^l dy' f_1(L, y-y') = \left| y-y' = u \right| = \int_0^l dy' \int_{-y'}^{l-y'} du f_1(L, u)$$

$$\int du f_1(L, u) = \frac{1}{2L^2} \int_0^l du \left[\sqrt{L^2 + u^2} - |u| \right] = \frac{1}{2L^2} \int_{\epsilon \rightarrow 0}^l du \left[\sqrt{L^2 + u^2} - \sqrt{\epsilon^2 + u^2} \right]$$

$$\int du \sqrt{L^2 + u^2} = \frac{1}{2} u \sqrt{L^2 + u^2} + \frac{1}{2} L^2 \ln \left[u + \sqrt{L^2 + u^2} \right], \quad \int du |u| = \frac{1}{2} u |u|$$

$$\rightarrow \int du f_1(L, u) = \frac{1}{2L^2} \left[\frac{1}{2} u \sqrt{L^2 + u^2} + \frac{1}{2} L^2 \ln \left[u + \sqrt{L^2 + u^2} \right] - \frac{1}{2} u |u| \right] = \frac{1}{2L^2} F(u)$$

$$I = \int_0^l dy' \frac{1}{2L^2} \left[\frac{1}{2} (l-y') \sqrt{L^2 + (l-y')^2} - \frac{1}{2} (l-y')^2 + \frac{1}{2} L^2 \ln \left[l-y' + \sqrt{L^2 + (l-y')^2} \right] - \right. \\ \left. - \frac{1}{2} (-y') \sqrt{L^2 + y'^2} + \frac{1}{2} (-y') y' - \frac{1}{2} L^2 \ln \left[-y' + \sqrt{L^2 + y'^2} \right] \right] = \frac{1}{2L^2} \int_0^l dy' \left[F(l-y') - F(-y') \right]$$

$$\int_0^l dy \left[F(l-y) - F(-y) \right] = \left| \begin{array}{l} l-y = x \\ -y = z \end{array} \right| = - \int_0^l dx F(x) - (-1) \int_0^l dz F(z) = \int_0^l dx F(x) + \int_0^l dz F(z) = \\ = \int_0^l dx F(x) - \int_0^l dy F(-y) = \int_0^l dy \left[F(y) - F(-y) \right] - \text{нужно отразить } F(u).$$

$$I = \frac{1}{2L^2} \int_0^l dy \left[y \sqrt{L^2 + y^2} - y |y| + \frac{1}{2} L^2 \ln \left[y + \sqrt{L^2 + y^2} \right] - \frac{1}{2} L^2 \ln \left[-y + \sqrt{L^2 + y^2} \right] \right]$$

$$\int_0^l dy y \sqrt{L^2 + y^2} = \frac{1}{2} \int_0^l d(y^2) \sqrt{L^2 + y^2} = \frac{1}{2} \int_0^l dt \sqrt{L^2 + t} = \frac{1}{2} \cdot \frac{2}{3} (L^2 + t)^{3/2} \Big|_0^l = \frac{1}{3} \left[(L^2 + l^2)^{3/2} - L^3 \right]$$

$$\int_0^l dy y |y| = \int_0^l dy y^2 = \frac{1}{3} y^3 \Big|_0^l = \frac{1}{3} l^3$$

$$\ln[+] - \ln[-y] = \ln \frac{y + \sqrt{L^2 + y^2}}{\sqrt{L^2 + y^2} - y} - \text{можно упростить.}$$

$$\ln[u + \sqrt{1+u^2}] = \ln\left[\frac{u}{L} + \sqrt{1 + \frac{u^2}{L^2}}\right] + \ln L = \operatorname{Arsh} \frac{u}{L} + \ln L$$

$$[F(u) - F(-u)]_{\ln} = \frac{1}{2L^2} [\ln[+] - \ln[-]] = \frac{1}{2L^2} \cdot 2 \operatorname{Arsh} \frac{u}{L} = \frac{1}{L^2} \operatorname{Arsh} \frac{u}{L}$$

$$I = \frac{1}{2L^2} \left[\frac{1}{3} \left\{ (L^2 + l^2)^{3/2} - L^3 - l^3 \right\} + \int_0^L dy L^2 \operatorname{Arsh} \frac{y}{L} \right]$$

$$\int_0^L dy \operatorname{Arsh} \frac{y}{L} = L \int_0^{l/L} dz \operatorname{Arsh} z = L \left[z \operatorname{Arsh} z - \sqrt{1+z^2} \right]_0^{l/L} = L \left[\frac{l}{L} \operatorname{Arsh} \frac{l}{L} - \sqrt{1 + \frac{l^2}{L^2}} + 1 \right]$$

$$I = \frac{1}{2L^2} \left[\frac{1}{3} \left[(L^2 + l^2)^{3/2} - L^3 - l^3 \right] + L^2 \left[\frac{l}{L} \operatorname{Arsh} \frac{l}{L} + L - \sqrt{L^2 + l^2} \right] \right]$$

теневое $L/l \rightarrow 0$, L - толщина stripe, l - ширина.

$$(L^2 + l^2)^{3/2} = L^3 \left(1 + \frac{l^2}{L^2} \right)^{3/2} = L^3 \left[1 + \frac{3}{2} \frac{l^2}{L^2} \right], \quad \operatorname{Arsh} \frac{l}{L} = \ln \frac{2l}{L}$$

$$I = \frac{1}{2L^2} \left[\frac{1}{3} \left[L^3 + \frac{3}{2} L^2 l - l^3 \right] + L^2 l \left[\ln \frac{2l}{L} - 1 \right] \right] = \frac{1}{2L^2} \left[\frac{1}{2} L^2 l + L^2 l \left(\ln \frac{2l}{L} - 1 \right) \right] =$$

$$= \frac{l}{2} \left[\ln \frac{2l}{L} - \frac{1}{2} \right] = I(L)$$

$$\rightarrow I = \int_0^L dy dy' f_1(L, yy') = \frac{d}{dL} \int dy dy' \frac{1}{2L} J_2(L, yy') = \frac{d}{dL} \Phi(L), \quad \Phi = \frac{1}{2L} \int dy dy' \frac{1}{|r-r'|}$$

Решим. гур. гр. $\frac{d\Phi(L)}{dL} = I(L) = \frac{l}{2} \left[\ln \frac{2l}{L} - \frac{1}{2} \right] = \frac{l}{2} \left[-\frac{1}{2} + \ln 2l - \ln L \right]$

$$\Phi(L) = \int dL I(L) = \frac{l}{2} \int dL \left[-\frac{1}{2} - \ln \frac{L}{2l} \right] = \frac{l}{2} \left[-\frac{1}{2} L - L \left(\ln \frac{L}{2l} - 1 \right) \right] = \frac{1}{2} l L \left[\frac{1}{2} - \ln \frac{L}{2l} \right]$$

$$W_m(S) = \frac{1}{2} m_x^2 (w/2) \int dz dz' \int dy dy' \frac{1}{|r-r'|_{w/2}} = \frac{1}{2} m_x^2 2L \cdot \Phi(L) = \frac{1}{2} m_x^2 2L \cdot \frac{1}{2} l L \left[\frac{1}{2} - \ln \frac{L}{2l} \right]$$

Объем stripe: $V = wLl$

$$\frac{1}{V} W_m(S) = \frac{1}{2} m_x^2 \frac{L^2 l}{wLl} \left[\frac{1}{2} + \ln \frac{2l}{L} \right] = \frac{1}{2} m_x^2 p \left[\ln \frac{2l}{L} + \frac{1}{2} \right], \quad p = l/w.$$

Stripe $\frac{1}{V} W_m(S) = \frac{1}{2} p \left[\ln \frac{2l}{L} + \frac{1}{2} \right] m_x^2$ — поверхн. намагнотес. энергия докової поверхнової stripe.

До границ stripe $l/L \rightarrow \infty$, $\frac{W_m(S)}{V}$ имеет log-расхождение.

Если нормировать на эг. докової поверхнової stripe $S = L \cdot l$, то

$$\rightarrow \frac{1}{S} W_m(S) = \frac{1}{2} m_x^2 L \left[\ln \frac{2l}{L} + \frac{1}{2} \right] \text{ — для stripe, } S = L \cdot l \text{ — } \frac{W_m}{S} \text{ константа}$$

$$\rightarrow \frac{1}{S} W_m(S) = L \left[\ln \frac{8}{\beta} + \frac{3}{2} \right] m_p^2 \text{ — для нанопровода, } S = \pi R L.$$

Справедливо: R нанопровод и $\beta = l/R$ экв. аналогичным границе stripe l и $\frac{l}{\beta}$ ratio.

Оценка поверхн. анизотропии намагнотес. и/или exchange:

$$\frac{1}{S} W_m(S) = K_s m_x^2 \text{ или } K_s m_p^2. \quad K_s \sim M_s^2 L \sim 10^6 \frac{\text{erg}}{\text{cm}^3} 10 \text{ nm} \sim 1 \frac{\text{erg}}{\text{cm}^2} \text{ — разумно.}$$

K_s — обменная константа. Но для log для stripe, $\ln d \sim \ln \frac{l}{L}$ — $\ln$ (Pinning)?

$$h_s^z = -m_x \frac{1}{2L} \int dy dy' \int dz' \frac{1}{|z-z'|} \Big|_{z=0}^{z=L}$$

$$\int_0^L dz' e^{-k|z-z'|} = \int_0^z dz' e^{-k(z-z')} + \int_z^L dz' e^{-k(z'-z)} = e^{-kz} \int_0^z e^{kz'} dz' + e^{-kz} \int_z^L e^{-kz'} dz' =$$

$$= e^{-kz} \left[\frac{1}{k} e^{kz'} \Big|_0^z + e^{-kz} \left(-\frac{1}{k} \right) e^{-kz'} \Big|_z^L \right] = e^{-kz} \left[\frac{1}{k} (e^{kz} - 1) - \frac{1}{k} e^{-kz} (e^{-kL} - e^{-kz}) \right] =$$

$$= \frac{1}{k} [2 - e^{-kz} - e^{-k(L-z)}]$$

$$\int_0^L dz' e^{-k|z-z'|} \Big|_{z=0}^{z=L} = \frac{1}{k} [2 - e^{-kL} - 1 - 2 + 1 + e^{-kL}] = 0(!), \text{ т.е. } h_s^z(x) = 0 \text{ из-за } \text{условия на } z=0 \text{ и } z=L$$

Осуществим расчет $h_s^x(x)$ для упр. условия stripe при $x=w/2$. Но есть $h_s^x(w/2)$.

У нас есть $\vec{h}_m(r) = \int dV' \hat{G}(r, r') \vec{m}(r')$ на поверхности и объемные заряды

$$h_x = \int dx' g_{xx}(x, x') m_x(x') = \int dx' g(x, x') \left[m_x(x) + \frac{\partial m_x}{\partial x} \Big|_{x'=x} (x'-x) + \dots \right] =$$

$$= m_x(x) \int dx' g(x-x') + \frac{\partial m_x}{\partial x} \Big|_{x'=x} \int dx' g(x-x') (x'-x), \quad x-x' = \xi, \quad x, x' \in [-\frac{w}{2}, \frac{w}{2}]$$

$$\rightarrow h_x(x) = m_x(x) \int_{-w/2-x}^{w/2-x} d\xi g(\xi) + m'_x(x) \int_{-w/2-x}^{w/2-x} d\xi g(\xi) \xi$$

А в PRB'02 было $x = \frac{w}{2}$ и $h_x(\frac{w}{2}) = m_x(\frac{w}{2}) \int_0^w d\xi g(\xi) - \int_0^w d\xi \xi g(\xi) \cdot m'_x(w/2)$, $g(\xi) = g_{xx}(x-x')$

Далее по условию $h_x(w/2) = 0$ или $\lambda(p) m_x(w/2) = h_x(w/2)$ - невыполнимо

упр. условие из PRB'02 - stripe.

Bound. Conditions (10/17)

$$I(L) = \int_0^\infty \frac{dk}{k^2} [1 - e^{-kL}] \sin k\xi = \text{[p.p. 3.945, где } a=b]$$

$$a \int_0^\infty \frac{dx}{x^2} [e^{-\beta x} - e^{-\gamma x}] \sin ax = a^2 \left[\frac{1}{2} \ln \frac{a^2 + \gamma^2}{a^2 + \beta^2} + \frac{\gamma}{a} \operatorname{arctg} \frac{\gamma}{a} - \frac{\beta}{a} \operatorname{arctg} \frac{\beta}{a} \right]$$

$$I(L, \xi) = \int \frac{dk}{k^2} [1 - e^{-kL}] \sin k\xi = \frac{1}{2} \xi \ln \frac{\xi^2 + L^2}{\xi^2} + L \operatorname{arctg} \frac{L}{\xi}, \quad \frac{dI}{dL} = \operatorname{arctg} \frac{\xi}{L}$$

Упр. $\frac{dI}{dL} = \frac{3}{2} \frac{\xi^2}{\xi^2 + L^2} \cdot \frac{1}{\xi^2} 2L + \operatorname{arctg} \frac{L}{\xi} + L(-1) \cdot \frac{1}{1 + L^2/\xi^2} \cdot \frac{1}{\xi} = \operatorname{arctg} \frac{L}{\xi}$, бермо с учетом $\operatorname{arctg} x = \operatorname{arctg} x^{-1}, x > 0$

$$h_s^x(x) = 2\pi m_x \theta(x - \frac{w}{2}) + 4m_x L^{-1} \left[\frac{1}{2} \xi \ln \frac{\xi^2 + L^2}{\xi^2} + L \operatorname{arctg} \frac{\xi}{L} \right] \Big|_{\xi = \frac{w}{2} - x > 0}$$

$$h_s^x(x + \frac{w}{2} - 0) = -2\pi m_x \text{ при } L \text{ константа.}$$

$$\operatorname{arctg} x = \begin{cases} \operatorname{arctg} \frac{1}{x}, & x > 0 \\ \pi + \operatorname{arctg} \frac{1}{x}, & x < 0 \end{cases}$$

$$\operatorname{arctg} x + \operatorname{arctg} x = \pi/2$$

$$h_s^x(x + \frac{w}{2} - 0) = 0 \text{ так как } m_x(w/2) = 0$$

т.е. упр. $\frac{\partial m_x}{\partial x} \Big|_{x=w/2}$

$$\xi = \frac{w}{2} - x$$

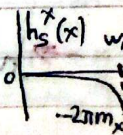
Возьмем $\frac{\partial m_x}{\partial x}$ и проинтегрируем по "volume" charges.

$$\text{при } x < \frac{w}{2} \quad h_s^x(x) = -2\pi m_x + 4m_x \left[\frac{1}{2} \frac{\xi}{L} \ln \frac{\xi^2 + L^2}{\xi^2} + \operatorname{arctg} \frac{\xi}{L} \right] = -2\pi m_x + m_x F(\xi/L)$$

т.е. величина $h_s^x(x)$ задается $F(\xi/L)$, т.е. меняется на границе $x \sim L$.

$$F(\eta) = \frac{4}{2} \eta \ln(1 + \frac{1}{\eta^2}) + \operatorname{arctg} \eta$$

$$F(0) = 0 \text{ при } \xi=0, x=\frac{w}{2}, \quad F(\eta \gg 1) = \frac{4}{2} \eta \left[\frac{1}{\eta^2} \right] + \frac{\pi}{2} = \frac{2}{\eta} + \frac{\pi}{2}, \quad h_s^x(\frac{\xi}{L} \gg 1) = -2\pi m_x + m_x \left[2\pi + \frac{2}{\eta} \right] = \frac{2}{\eta} m_x$$



$$\arctg x = \frac{\pi}{2} - \frac{1}{x}, \quad x \rightarrow \infty, \quad \eta = \frac{1}{L}$$

$$F(\eta) = 2\eta \ln(1 + \frac{1}{\eta^2}) + 4 \arctg \eta = |\eta \rightarrow \infty| = 2\eta \left[\frac{1}{\eta^2} \right] + O(\frac{1}{\eta^3}) + 4 \left[\frac{\pi}{2} - \frac{1}{\eta} \right] = 2\pi - \frac{2}{\eta}$$

$$h_s^{\pi}(\frac{1}{2}L \gg 1) = -2\pi m_x + m_x [2\pi - \frac{2}{j}] = -\frac{2}{j} m_x < 0, \text{ Vermo. - keine } \pi \text{ Zust.}$$

$h_s(x)$ непрерывна в $x = w/2$ и $|x - w/2| \leq L$. В $g(x)$ берём

где λ — параметр, удовлетворяющий условию $\int_{-\infty}^{+\infty} dx h_3(x) = 1$.

Оценим по $\frac{w/2-L}{\sigma}$ от $m_X(x)$ на поверхности $x = w/2$:

Определим поле $\vec{h}_m(x)$ на поверхности $x = w/2$:

$$\psi(x) = \psi_s(x) + \psi_v(x), \quad \vec{h}_s(x) = -\nabla_x \psi_s(x), \quad \vec{h}_v(x) = -\nabla_x \psi_v(x), \quad \vec{h}_m = \vec{h}_s + \vec{h}_v$$

$$\vec{h}_v(r) = + \nabla_r \int dV' \frac{\text{div } \vec{m}(r')}{|r-r'|}, \quad \chi \approx W/2, \quad r \in S_{\text{DW}}, \quad \text{div } \vec{m}(r') = \frac{\partial m_x(r')}{\partial x'} + \frac{\partial m_z(r')}{\partial z'}, \quad m_y = 0$$

$$\vec{h}_v(x) = \frac{1}{L} \int d\vec{z} \frac{1}{\epsilon} \int d\vec{x}' d\vec{y}' d\vec{z}' \operatorname{div} \vec{m}(\vec{r}') \cdot \nabla_{\vec{r}'} \frac{1}{|\vec{r} - \vec{r}'|} \quad \text{ycl pegrana no zu y.}$$

$$h_v^x(x) = \frac{1}{L^2} \int dz dz' \int dy dy' \int dx' \left[\frac{\partial m_x(r')}{\partial x'} + \frac{\partial m_z(r')}{\partial z'} \right] \frac{\partial}{\partial x} \frac{1}{|z - z'|} \quad \text{--- нужна пока.}$$

$$h_v^2(x) = \frac{1}{4\ell} \int dz dz' \int dy dy' \int dx' \left[\frac{\partial m_1(r')}{\partial x'} + \frac{\partial m_2(r')}{\partial z'} \right] \frac{\partial}{\partial z} \frac{1}{|r-r'|}$$

Größen $m_2 \neq f(z) \rightarrow h_v^x(x) = \frac{1}{L} \frac{\partial}{\partial x} \int dz dz' \int dy dy' \int dx' \frac{\partial m_x(r')}{\partial x'} \cdot \frac{1}{|r-r'|}, \quad x \approx w/2.$

$$\frac{\partial}{\partial x} \int dz dz' \int dy dy' \frac{1}{|x-x'|} = \frac{\partial}{\partial x} \int_{(0/x)} (x, x') =$$

$$= \frac{\partial}{\partial x} (2Le) \int dk_x \frac{1}{k_x^2} f(k_x L) \cos k_x (x-x') = (2Le) \int_{-\infty}^{\infty} \frac{dk_x}{k_x} f(k_x L) \sin k_x (x-x') = -4Le \int_{-\infty}^{\infty} \frac{dk}{k} f(kL) \sin k (x-x')$$

$$h_v(x) = \frac{1}{Ll} \int dx' \frac{\partial m_x(x')}{\partial x'} \cdot (-4Ll) \int_0^{\infty} \frac{dk}{k} f(kL) \sin k(x-x') = (-4) \int_{-w/2}^{w/2} dx' \frac{\partial m_x(x')}{\partial x'} \int_0^{\infty} \frac{dk}{k} f(kL) \sin k(x-x')$$

3-е слагае в фин. формуле. h_3^x root $\int_0^\infty \frac{dk}{k} f(kL) \sinh k(x-x') = \frac{h_3(x)}{\frac{4m}{(w/2-x')}} = \frac{\pi}{2} \Theta(x-x') +$
 $+ L^{-1} \left[\frac{1}{2} \zeta \ln \frac{\zeta^2 + L^2}{\zeta^2} + L \operatorname{arctg} \frac{L}{\zeta} \right] \zeta = x - x' \cdot$ Ноуе. знае ζ , веде $x \geq w/2$.

$$\int_0^{\infty} \frac{dk}{k} f(kL) \sin kz = \int_0^{\infty} \frac{dk}{k} \left[1 - \frac{1}{kL} (1 - e^{-kL}) \right] \sin kz = \int_0^{\infty} \frac{dk}{k} \sin kz - \frac{1}{L} \int_0^{\infty} \frac{dk}{k^2} [1 - e^{-kL}] \sin kz =$$

$$= \frac{\pi}{2} \theta(z) - \frac{1}{L} I(L, z) \quad , \quad I(L, z) = \int_0^{\infty} \frac{dk}{k^2} [1 - e^{-kL}] \sin kz \quad - \text{never. p. 9 or "3"}$$

$\forall z \in \mathbb{C} \setminus \{0\} \quad I(L, z) = \frac{1}{2} z \ln \frac{z^2 + L^2}{z^2} + \text{Larctg} \frac{L}{z} = \frac{z}{2} \ln \frac{z^2 + L^2}{z^2} + \text{Larctg} \frac{z}{L}, \quad I(L, -z) = -I(L, z)$

$$h_v^x(x) = (-4) \int dx' \frac{\partial m_x(x')}{\partial x'} = \left[+\frac{\pi}{2} \Theta\left(\frac{z}{L}\right) - \frac{1}{2} \frac{z}{L} \ln \frac{z^2 + L^2}{L^2} + \arctan \frac{z}{L} \right]_{z=x-x'}$$

$\lim_{L \rightarrow 0} \{x - x' \neq 0\} \quad \arctan \frac{z}{L} = \frac{\pi}{2} \theta(z) \quad \text{u} \quad \frac{\partial m(x')}{\partial x'} = j(x')$

$$\rightarrow h_V(x) = \frac{2}{L} \int dx' \frac{\partial \mu(x')}{\partial x'} \cdot \left\{ \ln \left(\frac{z^2 + L^2}{z^2} \right) \right\}_{z=x-x'} = \frac{2}{L} \int dx' \mu(x') (x-x') \ln \frac{z^2 + L^2}{z^2} = \frac{2}{L} x \int dx' \mu(x') \ln \frac{z^2 + L^2}{z^2} -$$

$$-\frac{2}{L} \int dx' m(x') x' \ln \frac{z^2 + L^2}{z^2} - \text{не важно как упрощать.}$$

поверхн. torque $\vec{M}_0 \times \vec{h}_s = i M_s h_s^z - \vec{k} M_s h_s^x$ и упр. условие без exchange
 $h_s^z = 0$ и $h_s^x = 0$ упр. $x \neq w/2$.

$h_s^x(w/2) = -2\pi m_x(w/2) = 0$ так как $m_x(w/2) = 0$. Но $h_v(x \neq w/2)$ тоже
 должно быть равно 0.

$$h_v^x(w/2) = \frac{w}{L} \int dx' \frac{\partial m_x(x')}{\partial x'} \ln \frac{z^2 + L^2}{z^2} - \frac{2}{L} \int dx' x' \frac{\partial m_x(x')}{\partial x'} \ln \frac{z^2 + L^2}{z^2}, \quad z = x - x' = \frac{w}{2} - x'.$$

$$\rightarrow \text{В PRB02 - стр. 12: } \lambda m(x) = \int dx' g(x, x') m(x'), \quad g(z) = \frac{2}{p} \ln \frac{z^2}{z^2 + p^2}, \quad z = x - x'.$$

$$\text{Упр. упр. } x = -\frac{w}{2} \quad I_1(-\frac{1}{2}, p) \frac{\partial m_x}{\partial x} + [I_0(-\frac{1}{2}, p) - \lambda] m_x(-\frac{1}{2}) = 0, \quad \frac{\partial m}{\partial x} - d(p) m = 0 \Big|_{x=-\frac{w}{2}}$$

$$d(p) = \frac{\lambda - I_0}{I_1}, \quad I_0(-\frac{1}{2}, p) = -2\pi + 2p, \quad \lambda = -4\pi + Ap \quad I_1(-\frac{1}{2}, p) = p(2\ln p - 1)$$

$$d(p) = \frac{-4\pi + Ap + 2\pi - 2p}{p(2\ln p - 1)} = -\frac{2\pi}{p(2\ln p - 1)} = \frac{2\pi}{p(1 + 2\ln p/p)} \quad \text{лим. по "p" равен в } I_0 \text{ и } \lambda$$

$$\lambda = -4\pi \text{ упр. } p \rightarrow 0 \text{ const. } g(x, x') = \frac{2}{p} \ln \frac{z^2}{z^2 + p^2} = -4\pi \delta(x - x').$$

$$\text{т.е. упр. } p = \frac{1}{w} \rightarrow 0 \quad \ln \frac{z^2 + p^2}{z^2} \cong 2\pi p \delta(z), \quad z = \frac{(x - x')}{w}. \quad \left\{ \begin{array}{l} I_0 \text{ const. } h_s^x \\ I_1 \text{ const. } h_v^x \end{array} \right\}$$

$$\ln \frac{z^2 + L^2}{z^2} = 2\pi L \delta(z)$$

$$h_v^x(x) \sim \frac{w}{L} \int dx' \frac{\partial m_x(x')}{\partial x'} \ln \frac{z^2 + L^2}{z^2} = \frac{w}{L} \int dx' \frac{\partial m_x(x')}{\partial x'} 2\pi L \delta(x - x') = 2\pi w \frac{\partial m_x(x)}{\partial x}, \quad x \neq w/2.$$

— почему h_v^x не $\sim p$.

$$\rightarrow \text{Может надо } \ln \frac{z^2 + p^2}{z^2} = 2\pi p \delta(z) + O(\sqrt{z}) \quad \text{уменьш. сдв. или разном.}$$

$$\text{Ведь } z \delta(z) = 0.$$

$$\text{Рык. менков: } H_{ex} = q \nabla^2 M = \frac{2A}{M_s} \nabla^2 m, \quad q M_s = \frac{2A}{M_s}, \quad q = d = \frac{2A}{M_s^2}, \quad A = \frac{1}{2} d M_s^2$$

$$\text{Закрепление } \zeta = \frac{2K_s}{q M_s^2} = K_s / A, \quad [\zeta] = \text{ан}^{-1}.$$

Мож PRB02 упр. условие в разн. буре $w \frac{\partial m}{\partial x} - d m = 0$

$$\text{Exch. torque не eq. для поверхности } \vec{T}_{ex} = -\vec{M}_0 \times d \frac{\partial \vec{m}}{\partial n} = -d \vec{M}_0 \times \frac{\partial \vec{m}}{\partial n}$$

Магнитостр. torque $\int (\vec{M}_0 \times \vec{h}_s) dV, \quad V = S_{\text{бок}} \cdot \Delta x, \quad \Delta x$ — длина элемента $\vec{h}_s(x)$ в направлении S

$$\vec{T}_{mag} = \vec{M}_0 \times \vec{h}_s \cdot \Delta x, \quad \vec{T}_{ex} + \vec{T}_{mag} = 0$$

$$-\vec{M}_0 \times d \frac{\partial \vec{m}}{\partial n} + \vec{M}_0 \times \vec{h}_s \Delta x = 0, \quad \vec{M}_0 \times \frac{\partial \vec{m}}{\partial n} = \begin{vmatrix} i & j & k \\ 0 & M_s & 0 \\ \frac{\partial m_x}{\partial n} & \frac{\partial m_z}{\partial n} & 0 \end{vmatrix} = i M_s \frac{\partial m_z}{\partial n} - k M_s \frac{\partial m_x}{\partial n}$$

$$\vec{k}: \quad -d(-M_s) \frac{\partial m_x}{\partial n} - M_s h_s^x \Delta x = 0, \quad d \frac{\partial m_x}{\partial x} - h_s^x \Delta x = 0. \quad \text{если } \vec{n} = \hat{x}, \text{ то } m_x$$

если $\vec{n}$ — в направлении y

$h_s^x = -2\pi m_x$ в равнов. положении. но p . $\Delta x \approx L$ - граница 110x, 190 $h_s^x \neq 0$.

$$\alpha \frac{\partial m_x}{\partial n} + 2\pi L m_x = 0 \rightarrow \frac{\alpha}{w} \frac{\partial m_x}{\partial \xi} + 2\pi L m_x = 0$$

Выводим $h_m^x = h_s^x + h_v^x = [-2\pi + 0(p)] m_x + I_1(-\frac{1}{2}, p) \frac{\partial m_x}{\partial x} = \left| x \rightarrow \frac{z}{w} \right| =$

$$= -2\pi m_x + p(2\ln p - 1) \frac{\partial m_x}{\partial x} w$$

$$\alpha \frac{\partial m_x}{\partial n} + L [2\pi m_x + p(2\ln p - 1) \frac{\partial m_x}{\partial x}] = 0, \quad \left[\alpha + pL(2\ln \frac{1}{p} + 1) \right] \frac{\partial m_x}{\partial x} + 2\pi L m_x = 0$$

Pinning stripes $\rightarrow \frac{\partial m_x}{\partial x} + d(p, \alpha) m_x = 0, \quad d(p, \alpha) = \frac{2\pi L}{\alpha + pLw(2\ln \frac{1}{p} + 1)} = \frac{1}{w} \frac{2\pi}{\frac{\alpha}{Lw} + p(2\ln \frac{1}{p} + 1)}$ - pinning.

$$\alpha = L_{ex}^2, \quad L_{ex}^2 = \frac{2A}{M_s^2} \cdot \left(\frac{L_{ex}}{wL} \right) \text{ частота } \propto p \ln \frac{1}{p}$$

$$\frac{L_{ex}^2}{wL} = p \left(\frac{L_{ex}}{L} \right)^2 \text{ частота } \propto p \ln \frac{1}{p}. \text{ Т.к. } L \approx L_{ex} (L = (1-3)L_{ex} \text{ где } FM),$$

то $\ln \frac{1}{p} \gg \left(\frac{L_{ex}}{L} \right)^2$ при $p \rightarrow 0$, т.е. микромасштабная dominates за счет $w \gg L$.

Частота $\left(\frac{L_{ex}}{L} \right)^2 \propto [2\ln \frac{1}{p} + 1]$, если $p \sim 0.1$ и $L \ll L_{ex}$, то "exch." dominates (ultra-thin limit $L \rightarrow 0$, $w \gg L$ (measure dots))

$$d(p, \alpha) = 2\pi \frac{L}{\alpha} \text{ или в безразмер. units } d = 2\pi \frac{Lw}{\alpha} = 2\pi \frac{1}{p} \left(\frac{L}{L_{ex}} \right)^2 \ll 1$$

pinning upon gauge (несуб. иrogen).

Начиная с границы $h_v^x = I_1(\frac{1}{2}, p) \frac{\partial m_x}{\partial x}$ и потому берем $\Delta x \approx L$ $\int dx [\dots]$.

$$h_v^x(x) = \frac{2}{L} \int_{-w/2}^{w/2} dx' \frac{\partial m_x(x')}{\partial x'} \ln \frac{z^2 + L^2}{z^2} \approx \frac{2}{L} \frac{\partial m_x}{\partial x} \Big|_{x=\frac{w}{2}} \int_{-w/2}^{w/2} dx' \ln \frac{z^2 + L^2}{z^2} = \left| \xi = x - x', \quad dz = -dx' \right| =$$

$$= \frac{2}{L} m'_x(w/2) (-1) \int_{x+w/2}^{x-w/2} dz \ln \frac{z^2 + L^2}{z^2} = -\frac{2}{L} m'_x \cdot w^2 \int_{x+1/2}^{x-1/2} dz \ln \frac{z^2 + p^2}{z^2} = \left| g(z) = \frac{2}{p} \ln \frac{z^2}{z^2 + p^2} \right| =$$

$$= +\frac{2}{L} w^2 \frac{p}{2} m'_x \int_{x+1/2}^{x-1/2} dz \ln g(z), \quad I_1(\xi, p) = \int_{-1/2-\xi}^{1/2-\xi} dt t g(t) = \left[\frac{1}{2} t^2 g(t) - p \ln(t^2 + p^2) \right]_{-1/2-\xi}^{1/2-\xi}$$

$$\int_{x+1/2}^{x-1/2} dz \ln g(z) = \left| \xi = -t \right| = \int_{-1/2-x}^{1/2-x} dt t g(t) = I_1'(x, p), \quad I_1(-\xi) = [\dots]_{-1/2+\xi}^{1/2+\xi} = [\dots]_{1/2-\xi}^{1/2-\xi} = -I_1(\xi) - \text{нормализ}$$

$$h_v^x(x) = w m'_x I_1(x, p) \quad (!) \quad h_v^x(x=w/2) = w m'_x I_1(1/2, p)$$

$$I(1/2, p) = \left[\frac{1}{2} t^2 g(t) - p \ln(t^2 + p^2) \right]_{-1}^{1} = -p \ln p^2 - \left[\frac{1}{2} g(1) - p \ln(1 + p^2) \right] =$$

$$= -2p \ln p - \frac{1}{2} g(1) + 0(p^3) = -2p \ln p - \frac{1}{2} \frac{2}{p} \ln \frac{1}{1+p^2} = -2p \ln p + \frac{1}{p} \ln(1+p^2) =$$

Т.е. $h_v^x(w/2) = w m'_x(w/2) \cdot p(1 + 2\ln \frac{1}{p})$ - микромас.

Вывод: $h_m^x(w/2) = h_s^x(w/2) + h_v^x(w/2) = -2\pi m_x(w/2) + w p(1 + 2\ln \frac{1}{p}) \frac{\partial m_x}{\partial x} \Big|_{x=w/2}$
 $\rightarrow h_m^x(w/2) = 0$ в равнов. положении при условии PRB'02. Вер, см. 26/11/05.

11/12 $h_m^x(w/2) = 0$ condition because $\vec{M}_0 + \vec{h}_m(w/2) = 0$ - zero and zero y.p. y.c.
 u.g. y.p. 1.-A.

В нас на поверхности stripe газ
 брызг как surface charges $\sim m_x(w/2)$ так и volume charges $(\sim \frac{\partial m_x}{\partial x}(w/2))$.

$$h_v^x(x) = \frac{2}{L} \int_{-w/2}^{w/2} dx' \frac{\partial m_x(x')}{\partial x'} \ln \frac{z^2 + L^2}{z^2} = -\frac{1}{w} \int_{-w/2}^{w/2} dx' \frac{\partial m_x(x')}{\partial x'} g(z), \quad z = x - x'$$

$$h_s^x(x \rightarrow w/2) = -2\pi m_x(w/2), \quad h_m^x(w/2) = -2\pi m_x(w/2) - \frac{1}{w} \int_{-w/2}^{w/2} dx' g(z) \frac{\partial m_x(x')}{\partial x'} (w/2 - x')$$

Сравн. с разложением PRB'02

$$h_m = \lambda m(w/2) = I_0(w/2) m(w/2) + I_1(w/2) \frac{\partial m}{\partial x} \Big|_{x=w/2}, \quad I_s(x) = \int_{-\frac{w}{2}-x}^{\frac{w}{2}-x} dz g(z) z^s$$

$$I_s(x) = \int_{-w/2}^{w/2} dx' g(x-x') (x-x')^s$$

Видно, что для разложения совпадают (нужно знать h_v^x) если можно
 вывести $\frac{\partial m_x(x')}{\partial x'}$ из $\log \int dx'(\dots)$ без ее u/μ $x'=x$.

3/III 2000г. я показал $g(z) = -4\pi \left[\delta(z) + \frac{p}{2\pi} \frac{1}{z^2} + \dots \right], p \rightarrow 0$.

$$h_v^x(x) = - \int_{-w/2}^{w/2} dx' \frac{\partial m_x(x')}{\partial x'} (-4\pi) z \left[\delta(z) + \frac{p}{2\pi} \frac{1}{z^2} \right] = 2p \int_{-w/2}^{w/2} dx' \frac{\partial m_x(x')}{\partial x'} \frac{1}{(x-x')}$$

- видно, вынос $\frac{\partial m_x}{\partial x} \Big|_{x'=x}$ из под интеграла.

Bound. Conditions

10/ix Две работы of pinning give thin stripes:

$$d = \frac{2\pi}{\left(\frac{L_{ex}^2}{wL}\right) + p(2\ln(p/2) + 1)}$$

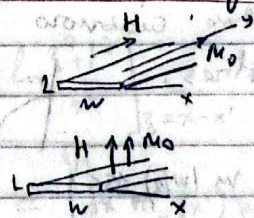
одомативные фан. газы PRB'02-stripes

$$d = \frac{2\pi}{p} \frac{1}{\left(\frac{L_{ex}^2}{L}\right) + 2\ln \frac{1}{p} + 1}, \quad L_{ex}^2 = \frac{2A}{M_s^2}, \quad \frac{\partial m_x}{\partial x} + d m_x = 0, \quad x \rightarrow \frac{x}{w}$$

Одним из способов измерения "d" можно найти экспериментально в FMR, BLS

H-||-plane: $\left(\frac{\omega}{\omega_M}\right)^2 = \left(\frac{\omega_H}{\omega_M} + 1 + \frac{\lambda}{4\pi}\right) \left(\frac{\omega_H}{\omega_M} - \frac{\lambda}{4\pi}\right)$

H-⊥-plane: $\left(\frac{\omega}{\omega_M}\right)^2 = \left(\frac{\omega_H}{\omega_M}\right) \left(\frac{\omega_H}{\omega_M} + 1 + \frac{\lambda}{4\pi}\right)$



$$\lambda_n = -4\pi + \pi^2 p(2n+1), \quad n=0, 1, 2, \dots$$

СФ $m_n(x) \sim A_n \sin x$, $x_n = \frac{\pi n}{w} \left(1 - \frac{2}{d}\right)$, $n=1, 2, \dots$ Если учесть за

основным FMR/BLS режимом, то $H_{ex} = \frac{2A}{M_s} x_n^2$ преобразуется мало и
 pinning в $\omega_n(H)$ -eqs. Выходит рефрез H_d^{nn} - магн. фл. гит. поле

и $\lambda_n = \frac{2}{p} \int_{-w/2}^{w/2} dx \ln \left(\frac{x^2}{x^2 + p^2} \right) m_n(x)$ - более точно рассматривать λ_n .

Для H || yz (i-plane) $H_d^{nn} = 0$ и $\omega_n(H)$ определяется только λ_n .

$$\lambda_n = \frac{\int_{-1/2}^{1/2} dx dx' g_{22}(x-x') m_n(x) m_n(x')}{\int_{-1/2}^{1/2} m_n^2(x) dx} \text{ или иначе } \lambda_n = \int_{-1/2}^{1/2} dx g_{22}(x) m_n(x)$$

$$\lambda_n m_n(x) = \int_{-1/2}^{1/2} dx' g_{22}(x-x') m_n(x'), \quad g_{22}(z) = \frac{2}{p} \ln \frac{z^2}{z^2 + p^2} \text{ для stripe.}$$

Уравнение сф $m_n(x) = \cos x_n x$, $x_n = \frac{\pi n}{w} (1 - \frac{z}{d})$ ато забавно от $d \rightarrow \lambda = d(d)$.

Моменту на наблюдая new b. conditions by FMR?

FMR res. field осн. нука $H_{res}(P)$ при $L_{ex} = 18 \text{ nm}$, $L = 18 \text{ nm}$, мембрана $p = \frac{L}{w}$ ($M_s = 700 \text{ G}$)

P	0.009	0.018	0.036	0.072
$H_{res}(P)$, kOe	0.932	0.855	0.714	0.478
width w, μm	2.0	1.0	0.5	0.25
$H_{res}(P)$, kOe сф осн. нука d(P)	0.932	0.854	0.711	0.486
$H_{res}^d(P)$, kOe		11.924	11.496	11.054
$H_{res}^d(P)$, kOe сф осн. нука d(P)		11.925	11.498	11.059

разные H_{res} из-за new d"
можно зачеркнуть при $w < 0.3 \mu\text{m}$
если $H \parallel Oy$ - in-plane

разные H_{res} из-за new d"
оно мало при H_x game
при $w = 0.25 \mu\text{m}$.

→ Резонанс: моды гистерезиса/потери new pinning d" (spin-exchange)

на FMR в stripe надо $H \parallel \text{long side}$, p мембрана (w-но-мембрана),

$M_s, (\frac{L_{ex}}{L})$ - мембрана. Обмен где $w = 0.5 - 1 \mu\text{m}$ дает nonferrous в pinning dipolar
 $d(p) = \frac{2\pi}{p(2\ln \frac{1}{p} + 1)}$, которое $\sim 10\%$, но сфв H_{res} оно max $\leq 10 \text{ Oe}$.

File "SW - single - stripe 2. mod".

→ Hammer Base. То же самое: материал (M_s, L_{ex}) и мембрана L?

Если мембрана L, то $(\frac{L_{ex}}{L})^2$ возростает и $\frac{d(0) - d(L_{ex})}{d(0)}$ возростает, то

сф pinning $d(0) \approx \frac{2\pi}{p[\dots]}$ тоже возростает и замечное

увеличение отнюдь не от pinning ($p \rightarrow 0$) на 10-20% из-за $(\frac{L_{ex}}{L})$ надо
повлиять на H_{res} .

$$\Delta m(w/2) = m(w/2) \rightarrow 0$$

увеличение $m(w/2) \ll 1$ - game на 100% не разбавляется на H_{res} .

Надо меньше материал (M_s, L_{ex}).

Но $\frac{L_{ex}}{\sqrt{A\mu_0}} = 5-6 \text{ nm} \approx \text{const} \Rightarrow$ надо уменьшать M_s (к $\mu\text{m Co}$).

Альтернатива: уменьшение ширины stripe w до 0.1-0.2 μm .

Нужно определить $h_s^x(x)$ и $h_v^x(x)$ стрижки по области $\Delta x = L$ близи поверхности. Стрижка $x = w/2$.
 Это как раз "boundary conditions" конечно тип. - менов. близи поверхности.

$$I_0(x) = \int_{-\frac{1}{2}-x}^{\frac{1}{2}-x} d\xi g(\xi) = \left[\xi g(\xi) - \frac{1}{2} \ln \xi \right]_{-\frac{1}{2}-x}^{\frac{1}{2}-x}, \quad I_1(x) = \int_{-\frac{1}{2}-x}^{\frac{1}{2}-x} d\xi \xi g(\xi) = \left[\frac{1}{2} \xi^2 g(\xi) - p \ln(\xi^2 + p^2) \right]_{-\frac{1}{2}-x}^{\frac{1}{2}-x}$$

$I_0(x)$ и $I_1(x)$ - особые (24/III 012), т.к. $I_2(x) = \int_{-\frac{1}{2}-x}^{\frac{1}{2}-x} d\xi \xi^2 g(\xi)$ и при $s \geq 2$ можно так -
 считать по "р" $\forall x$ и суживаемости и при $|x| = 1/2$ нет. $g(\xi) = \frac{2}{p} \ln \left(\frac{\xi^2}{\xi^2 + p^2} \right)$.

См. 6/IV 012. 0 magnetostatic bound. conditions from bulg. - максимумы на газе близи в.с.)

$$h_s^x(x) = 2\pi m_x \left(\frac{w}{2} \right) \theta(x - \frac{1}{2}) + 4m_x \left(\frac{w}{2} \right) \left[\frac{1}{2} \frac{\xi}{p} \ln \frac{\xi^2 + p^2}{\xi^2} + \arctg \frac{\xi}{p} \right]_{\xi = \frac{w}{2} - x}$$

$$h_s^x(x) = 2\pi m_x \theta(x - \frac{1}{2}) + 4m_x \left[-\frac{1}{4} \xi g(\xi) + \arctg \frac{\xi}{p} \right]_{\xi = \frac{w}{2} - x} \quad \text{— поле от } m_x(w/2) \text{ поверхн. заряда}$$

— поле от поверхн. заряда $m_x(-1/2)$ тоже включается, т.к. $|x - \frac{w}{2}| \leq L$.

$$J_s = \int_{-\frac{w}{2}-L}^{\frac{w}{2}} dx h_s^x(x) = w \int_{-\frac{w}{2}-L}^{\frac{w}{2}} dx h_s^x(x) = -2\pi m_x L + 4wm_x \int_{-\frac{w}{2}-L}^{\frac{w}{2}} dx \left[-\frac{1}{4} \xi g(\xi) + \arctg \frac{\xi}{p} \right]_{\xi = \frac{1}{2} - x}$$

$$= \left| \begin{matrix} \xi = \frac{1}{2} - x, & x = 1/2, & \xi = 0 \\ d\xi = -dx, & x = \frac{1}{2} - p, & \xi = p \end{matrix} \right| = -2\pi m_x L + 4wm_x \int_0^p d\xi \left[-\frac{1}{4} \xi g(\xi) + \arctg \frac{\xi}{p} \right] =$$

$$= -2\pi m_x L + 4wm_x \left(-\frac{1}{4} \right) \left[\frac{1}{2} \xi^2 g(\xi) - p \ln(\xi^2 + p^2) \right]_0^p + 4wm_x \int_0^p d\xi \arctg \frac{\xi}{p} =$$

$$= -2\pi m_x L + 4wm_x \left(-\frac{1}{4} \right) \left[\dots \right] + 4wm_x \left[\xi \arctg \frac{\xi}{p} - \frac{p}{2} \ln(p^2 + \xi^2) \right]_0^p =$$

$$= -2\pi m_x L + (-1)wm_x \left[\frac{1}{2} p^2 g(p) - p \ln(2p^2) + p \ln p^2 \right] + 4wm_x \left[p \arctg 1 - \frac{p}{2} \ln(2p^2) + \frac{p}{2} \ln p^2 \right] =$$

$$= -2\pi m_x L - wm_x \left[\frac{1}{2} p^2 \frac{2}{p} \ln \frac{1}{2} + p \ln \frac{1}{2} \right] + 4wm_x \left[p \frac{\pi}{4} + \frac{1}{2} p \ln \frac{1}{2} \right] =$$

$$= -2\pi m_x L + 4m_x w \cdot p \frac{\pi}{4} = -2\pi m_x L + \pi m_x L = -\pi m_x L$$

$$\int_{-\frac{w}{2}-L}^{\frac{w}{2}} dx h_s^x(x) = -\pi m_x L, \quad m_x = m_x(w/2)$$

$$J_v = \int_{-\frac{w}{2}-L}^{\frac{w}{2}} dx h_v^x(x) = \int_{-\frac{w}{2}-L}^{\frac{w}{2}} dx \frac{2}{L} \int_{-\frac{w}{2}-L}^{\frac{w}{2}} dx' \left[\frac{\partial m_x(x')}{\partial x'} \xi \ln \frac{\xi^2 + L^2}{\xi^2} \right]_{\xi = x - x'}$$

$$= \left| \begin{matrix} \xi = x - x' \\ d\xi = -dx' \end{matrix} \right| = \frac{2}{L} \int_{-\frac{w}{2}-L}^{\frac{w}{2}} dx' m'_x(x') \int_{-\frac{w}{2}-L-x'}^{\frac{w}{2}-x'} d\xi \xi \ln \frac{\xi^2 + L^2}{\xi^2} = \int_{-\frac{w}{2}-L}^{\frac{w}{2}} dx' m'_x(x') w(-1) \int_{-\frac{w}{2}-p-x'}^{\frac{w}{2}-x'} d\xi \xi g(\xi) =$$

$$= (-w) \int_{-\frac{w}{2}-L}^{\frac{w}{2}} dx' m'_x(x') \left[\frac{1}{2} \xi^2 g(\xi) - p \ln(\xi^2 + p^2) \right]_{\xi = \frac{w}{2}-x'-p}^{\frac{w}{2}-x'} =$$

$$= (-w) \int_{-\frac{w}{2}}^{\frac{w}{2}} dx' \frac{\partial m_x(x')}{\partial x'} \frac{1}{p} \left[\xi^2 \ln \xi^2 - (\xi^2 + p^2) \ln(\xi^2 + p^2) \right]_{\xi = \frac{w}{2}-x'-p}^{\frac{w}{2}-x'} = \left| \begin{matrix} \frac{1}{2} - x' = y \\ -dx' = dy \end{matrix} \right| =$$

$$= \left(\frac{-w}{p} \right) \int_{-1}^1 dy (-1) \frac{\partial m_x(\frac{1}{2}-y)}{\partial y} \left[\xi^2 \ln \xi^2 - (\xi^2 + p^2) \ln(\xi^2 + p^2) \right]_{y-p}^y$$

$$J_v = \frac{w}{p} \int_0^1 dy \frac{\partial m_x(y)}{\partial y} \left[y^2 \ln y^2 - (y^2 + p^2) \ln(y^2 + p^2) - (y-p)^2 \ln(y-p)^2 + \{(y-p)^2 + p^2\} \ln\{(y-p)^2 + p^2\} \right]$$

Нормализация по (p/y) , т.е. берем $y=0$.

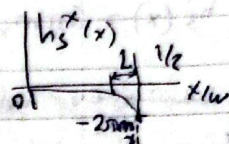
$$\text{Еще надо } F(z) \Big|_{z=\frac{1}{2}-x'} = \left[\frac{1}{2} z^2 g(z) - p \ln(z^2 + p^2) \right]_{z=\frac{1}{2}-x'} = F(\frac{1}{2}-x') - F(\frac{1}{2}-x'p) =$$

$$= F(y) - F(y-p) \approx p F'(y) = p [3g(y)]_{y=\frac{1}{2}} = p y g(y) \quad \text{--- так как } F''(y) \text{ не имеет конечных значений}$$

$$J_v = (-w) \int_{\frac{1}{2}-L}^{\frac{1}{2}} dx' \frac{\partial m_x(x')}{\partial x'} p y g(y) \Big|_{y=\frac{1}{2}-x'} = \int_{\frac{1}{2}-L}^{\frac{1}{2}} dx' h_v^x(x)$$

$$h_v^x(x) = (-1) \int_{\frac{1}{2}-L}^{\frac{1}{2}} dx' \frac{\partial m_x(x')}{\partial x'} 3g(z) \Big|_{z=\frac{1}{2}-x'}, \text{ т.е. } J_v = L h_v^x(\frac{1}{2}).$$

$$J_s = \int_{\frac{1}{2}-L}^{\frac{1}{2}} dx' h_s^x(x) = \frac{1}{2} h_s^x(\frac{1}{2}) L, \quad h_s^x(\frac{1}{2}) = -2\pi m_x(\frac{1}{2})$$



$$\text{Но } h_v^x(\frac{1}{2}) = - \int_{\frac{1}{2}-L}^{\frac{1}{2}} dx' \frac{\partial m_x(x')}{\partial x'} 3g(z) \Big|_{z=\frac{1}{2}-x'} \text{ --- нужно вычислить.}$$

$$\frac{\partial m_x(x')}{\partial x'} = \mu(x') = \mu(\frac{1}{2}) + \mu'(\frac{1}{2})(x' - \frac{1}{2}) + \dots = \sum_s \mu^{[s]}(\frac{1}{2}) (x' - \frac{1}{2})^s \frac{1}{s!}$$

$$h_v^x(\frac{1}{2}) = - \int_{\frac{1}{2}-L}^{\frac{1}{2}} dx' \sum_{s=0}^{\infty} \mu^{[s]}(\frac{1}{2}) \frac{1}{s!} (x' - \frac{1}{2})^s 3g(z) \Big|_{z=\frac{1}{2}-x'} = \left| z = \frac{1}{2} - x' \right| = \left| dz = -dx' \right| = \int_{\frac{1}{2}}^0 dz \sum_{s=0}^{\infty} \mu^{[s]}(\frac{1}{2}) \frac{(-1)^s}{s!} z^s 3g(z) =$$

$$= - \int_0^1 dz \sum_{s=0}^{\infty} \mu^{[s]}(\frac{1}{2}) \frac{(-1)^s}{s!} z^{s+1} g(z) = - \sum_{s=0}^{\infty} \mu^{[s]}(\frac{1}{2}) \frac{(-1)^s}{s!} \int_0^1 dz z^{s+1} g(z) =$$

$$= - \sum_{s=0}^{\infty} \frac{(-1)^s}{s!} \mu^{[s]}(\frac{1}{2}) I_{s+1}(-\frac{1}{2}) = - \mu(\frac{1}{2}) I_1(-\frac{1}{2}) + \sum_{s=1}^{\infty} \frac{(-1)^s}{s!} \mu^{[s]}(\frac{1}{2}) I_{s+1}(-\frac{1}{2})$$

$$\rightarrow h_v^x(\frac{1}{2}) = - \left(\frac{\partial m_x}{\partial x} \right)_{x=\frac{1}{2}} \cdot I_1(-\frac{1}{2}) + \sum_{s=1}^{\infty} \frac{(-1)^s}{s!} \mu^{[s]}(\frac{1}{2}) I_{s+1}(-\frac{1}{2})$$

Но при $p \rightarrow 0$ имеем $I_0 \sim I_1 \sim p \ln p$, $I_2, I_3, \dots, I_s \sim p$ --- малые поправки.

$$I_1(-\frac{1}{2}) = -I_1(\frac{1}{2}) = -[-2p \ln p + p] = -p [2 \ln \frac{1}{p} + 1]$$

$$J_v = L h_v^x(\frac{1}{2}) = L \left(\frac{\partial m_x}{\partial x} \right)_{x=\frac{1}{2}} \cdot p [2 \ln \frac{1}{p} + 1] \text{ --- grand?}$$

Пр. условие при $x=1/2$: $h_m(x) = h_s^x(x) + h_v^x(x)$

$$\frac{d}{dx} \frac{\partial m_x}{\partial x} - \int_{\frac{1}{2}-L}^{\frac{1}{2}} dx h_m(x) = 0, \quad \frac{d}{dx} \frac{\partial m_x}{\partial x} - J_s - J_v = 0$$

$$\frac{d}{dx} \frac{\partial m_x}{\partial x} + \pi m_x L + w L p [2 \ln \frac{1}{p} + 1] \frac{\partial m_x}{\partial x} = 0, \quad \text{ан. 10/1x}$$

2-е условие --- pinning

$$d(p, d) = \frac{\pi}{p} \frac{1}{(Lx/L)^2 + 2 \ln \frac{1}{p} + 1}$$

pinning при $y \rightarrow 0$ --- не имеет смысла. --- поверхность stripe

6. 2-й параметр --- $h_v(\frac{1}{2}), h_s(\frac{1}{2})$. Т.е. $\int_{\frac{1}{2}-L}^{\frac{1}{2}} dx h_m(x)$ --- условие pinning из-за $h(x)$

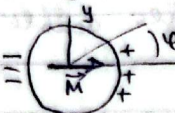
Dogman u. otcinab. Referat report na PRL-paper by Martens et. al. no reversal B rings (оформление, наименование, название).
 Упреждающее чтение рукописи paper в Science no упомин. yacobson.

Нормировка $R_c(L, d)$ d. vortex $\rightarrow$ d. single-domain transition in tri-layer cylinders F/N/F где $\varphi_{\text{max}} \approx \pi/2$ и $H=0$. Double-vortex statics

 - double SD state  - double vortex state c opposite vorticity (signs). (DV)

DV under manipulation. Энергия = $2 \times$ (single vortex). Т.е. w/m $H=0$ vortices are in cylinder center u маневры. энергия coupling of v. cores nonzero spheres w/m $R \approx 0.5-1 \mu m$.

Вектор double SD state не имеет $\text{div } \vec{m}$, б.е. безмгновенное за счет side surface charges $m_p \neq 0$ w/m $p=R$.

  $m_p(\varphi) = \pm M_s \cos \varphi$ где dot c $\vec{M} = \pm M_s \vec{i}$. $m_p(\varphi) = -M_s \cos \varphi$

$$W_{\text{int}}(S) = \int dS_1 dS_2 \frac{\partial(r_1) \partial(r_2)}{|r_1 - r_2|}, \quad r_1 \in S_1, r_2 \in S_2, S_i - \text{долики}, \text{ но берем } p=R \text{ i-то dot.}$$

$$\partial(\vec{r}) = m_p(p=R, \varphi, z) = m_p(\varphi), \text{ or } z'' \text{ поворачиваем, т.е. } L \sim L_{\text{ex}}.$$

См. 23/VI, где дана проблема power W_{int} для double-v. dots.

$$W_{\text{int}}(S) = \int dS dS' \frac{\partial(r) \partial(r')}{|r - r'|} = \int_{S_1} dz R d\varphi \int_{S_2} dz' R d\varphi' \frac{(M_s \cos \varphi)(-M_s \cos \varphi')}{|r - r'|} =$$

$$= -R^2 M_s^2 \int_{-L/2}^{L/2} dz \int_{-L/2}^{L/2} dz' \int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi' \frac{\cos \varphi \cos \varphi'}{|r - r'|} = -R^2 M_s^2 \int dz dz' \int d\varphi d\varphi' \cos \varphi \cos \varphi' \int dk e^{-k|z-z'|} \times$$

$$+ \sum_{\mu} J_{\mu}(kR) J_{\mu}(kR) e^{i\mu(\varphi - \varphi')}$$

$$J_{\mu} = \int_0^{2\pi} d\varphi e^{i\mu\varphi} \cos \varphi = \frac{1}{2} \int d\varphi e^{i\mu\varphi} [e^{i\varphi} + e^{-i\varphi}] = \pi (\delta_{\mu,1} + \delta_{\mu,-1}), \quad |J_{\mu}|^2 = \pi^2 (\delta_{\mu,1} + \delta_{\mu,-1})$$

- не асим. or $|J_{\mu}|^2$ or 23/VI.

SD-diff $W_m^{\text{int}}(S)$ energy source из $W_m^{\text{int}}(S)$ 23/VI и посто. заменим $S^2 \rightarrow (-1)$ w/m $M_{S1} = M_{S2}, L_1 = L_2$ - одинаковые член.

$$\text{SD}_{11} - \text{SD}_{11} \quad W_{\text{int}}(S) = -2\pi^2 R^2 M_s^2 \int \frac{dk}{k^2} e^{-kd} (1 - e^{-kL})^2 J_1^2(kR), \quad V_m = 2 \cdot \pi R^2 L.$$

$$W_{\text{int}} = \frac{1}{M_s^2 V_m} W_{\text{int}}(S) = -\frac{\pi}{\beta} \int_0^{\infty} \frac{dt}{t^2} e^{-td} (1 - e^{-\beta t})^2 J_1^2(t) = -2\pi F_{\text{int}}(\beta, d) \text{ см. 23/VI.}$$

Норматив. энергия of double SD state c $(\pm M_x)$ opposite:

$$W_m(S) = \bar{W}_m(S_1) \cdot \frac{V_m}{2} + \bar{W}_m(S_2) \cdot \frac{V_m}{2} + W_{\text{int}}(S) = \bar{W}_m(S) \cdot V_m + W_{\text{int}}(S)$$

$$\rightarrow \frac{1}{M_s^2 V_m} W_m(S) = W_m(S) + W_{\text{int}}(S) = 2\pi F_1(\beta) - 2\pi F_{\text{int}}(\beta, d) = 2\pi [F_1(\beta) - F_{\text{int}}(\beta, d)]$$

$$F_1(\beta) = \int dt/t f(t) J_1^2(t) - \text{парам. ф. } M_x(\beta) \text{ single SD dot}$$

Double Vortex $W(D.V.) = W(V_1) \cdot \frac{V_m}{2} + W(V_2) \cdot \frac{V_m}{2} + W_m^{int} \cdot V_m = W(V) \cdot V_m + W_m^{int} \cdot V_m$
 $W(V) = W_0 + W_m$ - normal energy single-vortex w/o $H=0$,
 $W_m^{int} = W_m^{int}(p, d)$ - normal magnetostatic coupling of vortex cores or $22/\mu$
 w/o $H=0$ $W_m^{int} \ll W(V)$, zero begins w/o $R = 0.5 - 1 \mu m$.

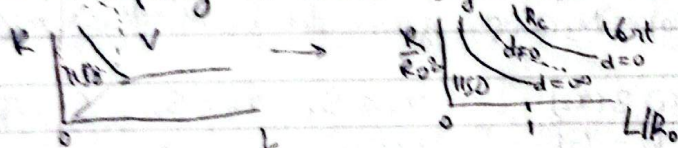
$$W(D.V.) = \frac{1}{V_m} W(D.V.) = W(V) + W_m^{int} \propto W(V). \quad W(d.SD) = W_m(s) + W_m^{int}(s)$$

Классическая модель неформы Double-Vortex $\leftrightarrow$ double SD state
 $R_c(L, d)$ энергетическая радиус-функция — $W(D.V.) = W(d.SD)$.

Note: Уменьшение $W_m^{int}(s) < 0$ для d.SD state, то $R_c(L, d)$ будет меньше $R_c(L)$ — для isolated disk ($d \rightarrow \infty$).

$$W_{vort}(R, L) = 2\pi [F_1(\beta) - F_{int}(\beta, d)] - \text{нормальная энергия} \quad F_{int}(\beta, d) = \frac{1}{2\beta} \int_0^\beta \frac{e^{-td}}{t^2} (1 - e^{-\beta t})^2 J_1^2(t) dt$$

M. Coupling in double-layer disk configuration gives double S.D. state ($\pm M_x$).



Для $L \geq R_0$ с учетом геометрии: $W(V) = \left(\frac{R_0}{R}\right)^2 \left[\frac{\pi}{3} - \ln \left\{ \xi_1 \frac{L^{1/3} R_0^{2/3}}{R} \right\} \right]$, $\xi_1 = 0.68$.

$\beta = \frac{L}{R} \approx d = \frac{L}{R} \ll 1$, то $F_{int}(\beta, d) \ll F_{int}(\beta, d)$ поэтому пренебрежем β и d .

$$F_{int}(\beta, d) = \frac{1}{2\beta} \int_0^\beta \frac{e^{-td}}{t^2} (1 - e^{-\beta t})^2 J_1^2(t) dt = \left| \beta \ll 1, 1 - e^{-\beta t} = \beta t \right| =$$

$$= \frac{1}{2\beta} \int_0^\beta \frac{e^{-td}}{t^2} \beta^2 t^2 J_1^2(t) dt = \frac{1}{2} \beta \int_0^\beta e^{-td} J_1^2(t) dt - \text{выражение равно } d \neq 0.$$

$$2\pi F_1(\beta) = N_s(\beta) = 2\beta \left[\ln \frac{8}{\beta} - \frac{1}{2} \right].$$

$$W_{vort}(R, \beta) = \left(\frac{R_0}{R}\right)^2 \left[\frac{\pi}{3} - \ln \left\{ \xi_1 \beta^{1/3} \left(\frac{R_0}{R}\right)^{2/3} \right\} \right], \quad R_0^2 = \frac{2A}{M_s^2}.$$

Находим $R_c(\beta)$, считаем $R_c(\beta)$ от $L = \beta R_c(\beta)$. Получим "Double-Vortex model".

→ Нормальная радиус-функция $R_c(L, d)$ увеличивается с L .

Boundary Conditions

Отсюда papers no "Boundary Conditions" и surface anisotropy.

ГЛАВ. ПРАВО: J.C. Kittel, Phys. Rev. **110**, 1295 (1958) — strong pinning just $\frac{ab}{a_0} k \ll 1$

+ (2) G.T. Rado, J.R. Waertman, J. Phys. Chem. Solids **11**, 315 (1959) —

$$\frac{2A}{M_s} \vec{i}_z \times \frac{\partial \vec{m}}{\partial n} + \vec{T}_s = 0, \quad \vec{M} = M_s \vec{i}_z + \vec{m}, \quad \vec{T}_s = -\vec{m} \times \vec{h}_s$$

$$\frac{2A}{M_s^2} \vec{M} \times \frac{\partial \vec{m}}{\partial n} + \vec{T}_s = 0, \quad \vec{T}_s = \vec{m} \times \nabla_n E_s = \vec{m} \times \vec{h}_s, \quad \vec{h}_s = -\nabla_n E_s, \quad \vec{u} = \frac{\vec{M}}{M_s}.$$

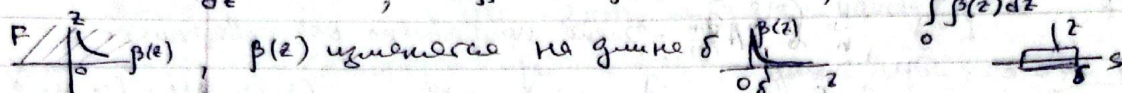
$\vec{T}_s$ — это поперечные торques упругого обмена (Ks взаимодействие). Круге не вращается о магнитном. Вращение в поперечном. Энергия E_s (erg/cm²)

Условие $\vec{u} \times [\nabla_n E_s - 2A \frac{\partial \vec{u}}{\partial n}] = 0$ из Rado PRB'82 возникает из расч. мого $\vec{u}$ в области $\delta \rightarrow 0$ 

- + ③ Akhiezer et al. "Spin waves", 1968 гласит, что в тонком слое, но с вращением, то Rado-м. возникнет эффект, если непрерывность потока энергии через поверхность S.

Хотя мого интерпр. по конфигурации $\perp S$ также не совсем ясно.

$z=0 \quad d \frac{\partial \vec{m}}{\partial z} - \vec{m} = 0$, $\vec{h}_{eff} = \vec{h} - \beta(z) \vec{m} \propto \nabla^2 \vec{m}$, $d = \frac{\delta}{\int_0^\delta \beta(z) dz}$ - inverse pinning.



У меня $\delta \approx 2$ толщина слоя.

Условие непрерывности $d \frac{\partial \vec{m}}{\partial z} - \vec{m} = 0$:

a) $\omega \ll \gamma M_s \beta(0)$. $h_a(z) = -\beta(z) \vec{m}$, у меня $h_s^x(x) \approx -2\pi m_x(x)$, $\beta(0) = 2\pi$

$\frac{\omega}{\gamma} \ll 2\pi M_s$, $\omega \ll \frac{\omega_M}{2}$ - не очень хорошо выполняется

b) $2k^2 \ll \beta(0)$, $k^2 \ll \frac{2\pi}{\delta}$, $k \ll \frac{\sqrt{2\pi}}{\sqrt{\delta}} = \frac{\sqrt{2\pi}}{L_{ex}} = \frac{2.5}{20 \cdot 10^{-7} \text{ cm}} \ll 10^6 \text{ cm}^{-1}$

Для нашей stripe mode $k_x \approx \frac{\pi}{w} = \frac{\pi}{10^{-4} \text{ cm}} = 3 \cdot 10^4 \text{ cm}^{-1}$ - это условие выполняется

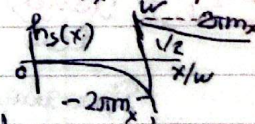
c) $\beta(0) \gg 1$, $2\pi \gg 1$ - хорошо выполняется.

$h_m^x(x) = h_s^x(x) + h_v^x(x)$, $h_m^x(x = \frac{w}{2}) = -2\pi m_x \pm p(2\ln \frac{1}{p} + 1) w \left(\frac{\partial m_x}{\partial x} \right)_{x=w/2}$

$h_m^x(w/2) \approx -2\pi m_x \pm p(2\ln \frac{1}{p} + 1) (k_x w) m_x = [-2\pi \pm p(2\ln \frac{1}{p} + 1) n\pi] m_x$, n - какой мого $k_x \approx \frac{n\pi}{w}$

$\beta(0) = \beta(x=w/2) = -2\pi \pm p(2\ln \frac{1}{p} + 1) n\pi = -2\pi + O(p \ln p)$

-2π берется из $h_s^x(x \rightarrow \frac{w}{2} = 0) = -2\pi m_x(w/2)$ и от L не зависит.



Все условия для р. ур. $d \frac{\partial \vec{m}}{\partial z} - \vec{m} = 0$ выполняются, кроме $\omega \ll \frac{1}{2} \omega_M$.

Все полностью на том, что $\int_0^\delta \beta(z) dz \rightarrow 0$ при $\delta \rightarrow 0$ или сходимости к 0 "негустее" $\sim \delta$.

Значит $\sim \omega$ в р. условиях неограничен, но возникает от $\sim L$ так же как $\langle h_m(x) \rangle_{x=w/2} \sim L$. Можно взять $\delta \ll L$? Akhiezer берет $\int_0^\delta \beta(z) dz = \beta(0)$

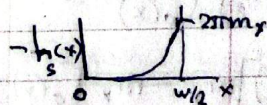
р. условие берется Rado-Weertman'59:

$\frac{2A}{M_s^2} \vec{M} \times \frac{\partial \vec{M}}{\partial n} + (\vec{M} \times \vec{h}) dx = 0$, $\vec{T}_s = \int_\delta dx (\vec{M} \times \vec{h})$ при $\delta \rightarrow 0$ берется пополю.

$\vec{T}_s = -\vec{M} \times \nabla_n E_s$ - у Rado-Weertman.

$E(s) = E_s \delta(x - w/2)$ - мого δ $\int_\delta dx E(s) = E_s$

Если $E(s) = E_s f(x)$, $\int_\delta dx E(s) = E_s \int_\delta dx f(x)$, $f(x)$ имеет максимум при $x = \frac{w}{2}$



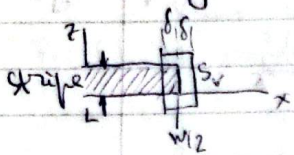
Уменьшение $h_v^x(x)$ от x не так важно, т.к. $h_v^x(x) \sim p$ мало ($p \ll 1$).

B PRB'02-84 xue y. ycu. zamuchu gne $m_z(x)$: $\frac{\partial m_z}{\partial z} \pm dm_z = 0$

$$\lambda_x m_x(x') = \int dx' g_{xx}(x, x') m_x(x') - \text{yp. na } m_x(x)$$

Ho $g_{xx}(x, x') = -4\pi \delta(x - x') - g_{zz}(x, x')$ gne stripe u

$\lambda_z m_x(x) = \int dx' g_{zz}(x - x') m_x(x')$, т.е. $m_x(x)$ ygoвeсb. paнoнy
 oне uнe. yp. kak u $m_z(x)$ e $\lambda_z = -4\pi - \lambda_x \Rightarrow$ eи uнe y. ycu. uнo $m_z(x)$.



$\frac{\partial}{\partial n}$, $\vec{n}$ b $2A \frac{\partial \vec{m}}{\partial n}$ - uнe uаuаbueно oт uоbepнocтu
 т.е. $\frac{\partial}{\partial n} = -\frac{\partial}{\partial x}$ uлu $x = w/2$. $\int dV M \cdot \vec{\Delta} \vec{M} = \int ds \vec{M} \times \frac{\partial \vec{M}}{\partial n}$
 - S_v бoлbшa тaк, uнo $\vec{n}$ uнe uаuаbueно oт uоbepнocтu. $V = S_v \cdot 2\delta$.

$$h_v^x(x) = - \int_{x-1/2}^{1/2} dx' \frac{\partial m_x(x')}{\partial x'} g(z) \Big|_{z=x-x'} = - \int_{-1/2}^{1/2} dx' m(x') g(z) \Big|_{z=x-x'} = \Big|_{x-x'=z} =$$

$$= \int_{x-1/2}^{x+1/2} dz M(x-z) g(z) = \Big|_{z=-u} = - \int_{-1/2-x}^{1/2-x} du M(x+u) u g(u) = - \int_{-1/2-x}^{1/2-x} dz M(x+z) g(z)$$

$I_1(x) = \int_{-1/2-x}^{1/2-x} dz g(z)$ u бoлbшe $I_5(x) = \int_{-1/2-x}^{1/2-x} dz z^5 g(z)$, $g(z) = \frac{2}{p} \ln \frac{z^2}{z^2 + p^2}$

$$M(x+z) = M(x) + M'(x)z + \dots = \sum_{s=0}^{\infty} \frac{1}{s!} M^{(s)}(x) z^s \frac{1}{s!}, \quad M(x) = \frac{\partial m_x(x)}{\partial x}$$

$$h_v^x(x) = - \int dz \sum_{s=0}^{\infty} \frac{1}{s!} M^{(s)}(x) z^s g(z) = - \sum_{s=0}^{\infty} \frac{1}{s!} M^{(s)}(x) \int dz z^{s+1} g(z) = - \sum_{s=0}^{\infty} \frac{1}{s!} M^{(s)}(x) \frac{I_{s+1}}{s+1}(x)$$

$h_v^x(x) = - \frac{\partial m_x(x)}{\partial x} I_1(x) - \sum_{s=1}^{\infty} \frac{1}{s!} \frac{\partial^{s+1} m_x(x)}{\partial x^{s+1}} \frac{I_{s+1}}{s+1}(x)$ u бoлbшe $I_s(x, p) = f(x)$
 uлu $p \ll 1$.

d. field stripe
or volume charges

Ecли $m_x(x) \sim \cos kx$, $\frac{\partial^{s+1} m_x}{\partial x^{s+1}} \sim k^{s+1} m_x \sim k^s \frac{\partial m_x}{\partial x}$, $k^s = (\pi n)^s$, n - кoлeч
 Дae uаuаbueно $I_1(x) \sim \frac{(\pi n)^s}{s!} I_{s+1}(x)$ uлu $p \ll 1$.

$$h_s^x(x) = 2\pi m_x \theta(x - \frac{1}{2}) + 4m_x \left[\frac{1}{2} \frac{z}{p} \ln \frac{z^2 + p^2}{z^2} + \arctg \frac{z}{p} \right] = 2\pi m_x \theta(x - \frac{1}{2}) + 4m_x \left[\frac{1}{4} g(z) + \arctg \frac{z}{p} \right]$$

u зaгeн $m_x(1/2)$ $z = \frac{1}{2} - x$

$$h_s^x(x) = 2\pi \theta(x - \frac{1}{2}) m_x - m_x I_0(x) \frac{1}{2}, \quad x \ll 1/2 \dots$$

$$I_0(x) = \left[z g(z) - 4 \arctg \frac{z}{p} \right]_{1/2-x}^{1/2-x}$$

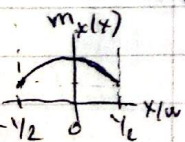
uзue зaгeн $I_0(x)$ бeзгeнeнe c
 uоbepн. зaгeнeнe $m_x(1/2)$ uлu $x = 1/2$.

$$h_m^x(x) = \int_{-1/2}^{1/2} dx' g_{xx}(x, x') m_x(x') = -4\pi m_x(x) - \int_{-1/2}^{1/2} dx' g_{zz}(x, x') m_x(x'), \quad x \in (-\frac{1}{2}, \frac{1}{2})$$

Дae uнe. S_v uоbу (uеuеи) uаcь uаuаbueно $h_m^x(x)$ бeзгeнeнe
 c uоbepн. зaгeнeнe $m_x(1/2) = m_x(-1/2)$, yаcь uаuаbueно $m_x(x) = \text{const} \equiv m_x(1/2)$

$$h_s^x(x) = -4\pi m_x(x) - m_x(1/2) \int_{-1/2}^{1/2} dx' g_{zz}(x, x') = -4\pi m_x(x) - m_x(1/2) I_0(x)$$

$$h_s^x(x) = (+) 2\pi \theta(x + \frac{1}{2}) m_x + 2\pi \theta(x - \frac{1}{2}) m_x - m_x I_0(x) = 2\pi [\theta(x + \frac{1}{2}) + \theta(x - \frac{1}{2})] m_x - m_x I_0(x)$$



$-I_0(x)/v_T$
 $h_s^x(x) = 2\pi \left[-\theta(x+\frac{1}{2}) + \theta(x-\frac{1}{2}) \right] m_x - I_0(x) m_x$, $m_x = m_x(V_2) = m_x(-1/2)$
 где m_x — это $m_x(\pm V_2)$.
 Ранее было $\hat{h}_s^x(x) = 2\pi \theta(x-1/2) m_x - [3g(z) - 4\alpha z \operatorname{ctg} \frac{z}{p}] \cdot m_x$, $z = 1/2 - x$
 где только от $m_x(V_2)$.

Если $x \in (-1/2, V_2)$ $h_s^x(x) = -[4\pi + I_0(x)] m_x$,
 было $\hat{h}_s^x(x) = -2\pi m_x - [3g(z) - 4\alpha z \operatorname{ctg} \frac{z}{p}] \cdot m_x$, $z = 1/2 - x$

То $h_s(x)$ совпадает при $x \neq 1/2$, где при $x=0$ они близки.

Если тогда $m_x(x)$ — переменно, $m_x(-1/2) = -m_x(V_2)$ — то сходнее,
 но вблизи поверхности $x=1/2$ $\hat{h}_s^x(x)$ очень точно.

В $h_s^x(x)$ разложение $I_1(x)$ доминирует, $\frac{I_1(x,p)}{I_2(x,p)} \Big|_{x=1/2} = \operatorname{ctg} p \approx 10$ где $p=0.0001$.

Ближе конкурент где $I_1(x)$ есть $I_2(x)$.

Но с уменьшением "p" степень локализации $I_1(x)$ и фактически не изменяется.
 Факт "Bound-Condit. mod".

Разложение $h_v^x(x)$ по $I_s(x)$ — локальное, но у нас $m^{(s+1)}(x) I_{s+1}(x)$ с большими "s" — плата за предсказание некоррелированной interaction в локальном приближении.

Замечание и угадка $\rightarrow J_v = \int_{1/2-p}^{1/2} dx h_v^x(x) \approx p h_v^x(V_2)$ от $13/\pi x$ — спорное не доказано. (16/1x)

$\hat{I}_1(x, p \rightarrow 0) = -2p \ln \left(\frac{1/2-x}{1/2+x} \right)$. $\int_{1/2-p}^{1/2} I_1(x) dx = (-2p) \int_{1/2-p}^{1/2} \left[\ln(1/2-x) - \ln(1/2+x) \right] dx =$
 $= (-2p) \left[-(1/2-x) \ln(1/2-x) + (1/2-x) - (1/2+x) \ln(1/2+x) + (1/2+x) \right]_{1/2-p}^{1/2} = (-2p) [1 + p \ln p - p +$
 $+ (1-p) \ln(1-p) - (1-p)] = (-2p) [p \ln p + (1-p) \ln(1-p)] = (-2p) [p \ln p + (1-p)(-p)] =$
 $= (-2p) [p \ln p - p] = -2p^2 [\ln p - 1] = 2p^2 [\ln \frac{1}{p} + 1]$

В PRB'02 было $I_1(V_2) = -I_1(-1/2) = p(2\ln \frac{1}{p} + 1)$ или $\int_{1/2-p}^{1/2} dx I_1(x) = I_1(V_2)p = p^2 [2\ln \frac{1}{p} + 1]$

$\hat{I}_1(x, p \rightarrow 0) = -2p \ln \left(\frac{1/2-x}{1/2+x} \right)$ — log-расхождение при $x=1/2$, тогда как $I_1(\frac{1}{2}) = p(2\ln \frac{1}{p} + 1)$ — конечно.
 Но при $p \rightarrow 0$ разложение $\hat{I}_1(x)$ и $I_1(x)$ — очень мало.

$I_s(x, p \rightarrow 0) = -\frac{2p}{(s-1)!} \left[(1/2-x)^{s-1} - (-1)^{s-1} (1/2+x)^{s-1} \right]$, $I_s(\frac{1}{2}, p \rightarrow 0) = 2p \frac{(-1)^{s-1}}{(s-1)!} \sim p$

$\rightarrow h_v^x(V_2) = -\frac{\partial m_x(V_2)}{\partial x} p(2\ln \frac{1}{p} + 1) - \sum_{s=1} \frac{1}{s!} \frac{(-1)^s}{s} (2p) m_x^{(s+1)}(V_2)$

Члены в $\sum$ есть $\sim \frac{1}{s!}$ и есть все $O(p)$, т.е. при $p \rightarrow 0$ можно считать $p \ln \frac{1}{p}$.

Таким образом показывается, что где $p=0.001:0.1$ в разложении $h_v^x(x)$ с $I_1(x)$ может конкурировать только $I_2(x)$. $I_1(x)/I_2(x) = (5 \pm 10)$ и растет при $p \rightarrow 0$.

$$f(\beta t) = 1 - \frac{1}{\beta t} (1 - e^{-\beta t}), \quad J(\beta, \rho) = \int_0^\infty dt f(\beta t) J_0(t) J_1(\rho t), \quad \rho \in \text{eg. R.}$$

$$J(\beta, \rho) = \int_0^\infty dt J_0(t) J_1(\rho t) - \frac{1}{\beta} \int_0^\infty \frac{dt}{t} (1 - e^{-\beta t}) J_0(t) J_1(\rho t), \quad \int_0^\infty dt J_0(t) J_1(\rho t) = \begin{cases} \frac{1}{\rho}, & \rho > 1 \\ \frac{1}{2}, & \rho = 1 \\ 0, & \rho < 1 \end{cases}$$

$$J_1(\beta) = \int_0^\infty \frac{dt}{t} (1 - e^{-\beta t}) J_0(t) J_1(\rho t) \quad \text{— см. V.L. Luke "Integrals of Bessel functions"}$$

$$\frac{dJ_1(\beta)}{d\beta} = \int_0^\infty dt e^{-\beta t} J_0(t) J_1(\rho t)$$

$$J_1(\beta) = \int_0^\infty \frac{dt}{t} (1 - e^{-\beta t}) J_0(t) J_1(t) = I(1, 0, -1) \Big|_{\beta=0} - I(1, 0, 1) \Big|_{\beta}$$

$$I(1, 0, -1) = \int_0^\infty \frac{dt}{t} e^{-\beta t} J_0(t) J_1(t) = \frac{2}{\pi} \frac{E(k)}{k} - \frac{\beta}{2}, \quad k^2 = \frac{4ab}{\beta^2 + (a+b)^2} = \frac{4}{\beta^2 + 4} = \frac{1}{1 + \beta^2/4}$$

$$I(1, 0, -1) = \frac{2}{\pi} \sqrt{1 + \beta^2/4} E\left(\frac{1}{\sqrt{1 + \beta^2/4}}\right) - \frac{1}{2} \beta$$

$$E(k) = 1 + \frac{1}{2} k^2 \left(\ln \frac{4}{k} - \frac{1}{2} \right) + O(k^4), \quad k'^2 = 1 - k^2 = 1 - \frac{1}{1 + \beta^2/4} = \frac{\beta^2/4}{1 + \beta^2/4} \approx \frac{\beta^2}{4}$$

$$I(1, 0, -1) = \frac{2}{\pi} \sqrt{1 + \beta^2/4} \left[1 + \frac{1}{2} \frac{\beta^2}{4} \left(\ln \frac{8}{\beta} - \frac{1}{2} \right) \right] - \frac{1}{2} \beta$$

$$J_1(\beta, \rho=1) = \frac{2}{\pi} - \frac{2}{\pi} (1 + \beta^2/8) \left[1 + \frac{\beta^2}{8} \left(\ln \frac{8}{\beta} - \frac{1}{2} \right) \right] + \frac{1}{2} \beta = \frac{2}{\pi} - \frac{2}{\pi} \left[1 + \frac{\beta^2}{8} \left(\ln \frac{8}{\beta} - \frac{1}{2} \right) + \frac{\beta^2}{8} \right] + \frac{1}{2} \beta$$

$$= \frac{1}{2} \beta + \left(-\frac{2}{\pi} \right) \frac{\beta^2}{8} \left(\ln \frac{8}{\beta} + \frac{1}{2} \right)$$

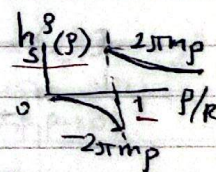
$$J(\beta, \rho) = \frac{1}{2} - \frac{1}{\beta} \left[\frac{1}{2} \beta - \frac{\beta^2}{4\pi} \left(\ln \frac{8}{\beta} + \frac{1}{2} \right) \right] = 0 + \frac{\beta}{4\pi} \left(\ln \frac{8}{\beta} + \frac{1}{2} \right)$$

$$h_s^s(\rho=1) = 4\pi m_p(1) J(\beta, \rho=1) = 4\pi m_p \frac{\beta}{4\pi} \left(\ln \frac{8}{\beta} + \frac{1}{2} \right) = m_p \cdot \beta \left(\ln \frac{8}{\beta} + \frac{1}{2} \right)$$

$$\text{то } J(\beta, \rho) = 0 - \frac{1}{\beta} \left[\frac{\beta}{2} - \frac{\beta^2}{4\pi} (\dots) \right] = -\frac{1}{2} + \frac{\beta}{4\pi} \left(\ln \frac{8}{\beta} + \frac{1}{2} \right) \quad \text{— потому что } W_m(s), \beta \ll 1.$$

$$h_s^s(\rho \rightarrow 1-0) = 4\pi m_p(1) \left[-\frac{1}{2} + \frac{\beta}{4\pi} (\dots) \right] = -2\pi m_p + m_p \beta \left(\ln \frac{8}{\beta} + \frac{1}{2} \right)$$

$$h_s^s(\rho \rightarrow 1-0) = -2\pi m_p(1) + m_p O(\beta \ln \beta) \quad \text{— где учтем}$$



Како бугенска г. wave or volume charges уз $h_p(\rho)$ аналог. stripes.

$$h_p(\rho) = h_s^s(\rho) + h_v^s(\rho)$$

$$h_p(\rho) = \int_0^1 d\rho' \rho' g(\rho, \rho') m_p(\rho') = \int_0^1 d\rho' \rho' g(\rho, \rho') \left[m_p(\rho) + \frac{\partial m_p(\rho)}{\partial \rho} (\rho' - \rho) + \dots \right]$$

Бугенска (аналог. stripe) $h_s^s(\rho) = m_p(\rho) \int_0^1 d\rho' \rho' g(\rho, \rho')$ — surface u/m $\rho \rightarrow 1-0$

$$h_v^s(\rho) = \frac{\partial m_p(\rho)}{\partial \rho} \int_0^1 d\rho' \rho' g(\rho, \rho') (\rho' - \rho) \quad \text{— volume бугенска.}$$

$$\int_0^1 d\rho' \rho' g(\rho, \rho') = 4\pi J(\beta, \rho), \quad J(\beta, \rho) = \int dt f(\beta t) J_0(t) J_1(\rho t) \quad \text{— бугенска.}$$

$$\int_0^1 d\rho' \rho'^2 g(\rho, \rho') = (-4\pi) \int d\rho' \rho'^2 \int dk k f(kL) J_1(k\rho) J_1(k\rho') = (-4\pi) \int dk k f(kL) J_1(k\rho) a(k)$$

$$a(k) = \int_0^1 d\rho \rho^2 J_1(k\rho) = \frac{1}{k^3} \int_0^k dx x^2 J_1(x) = \frac{1}{k^3} x^2 J_2(x) \Big|_0^k = \frac{1}{k} J_2(k)$$

$$J'_V(p, p) = \int_0^1 ds' p'^2 g(p, p') = (-4\pi) \int dt f(p t) J_1(t p) J_2(t) \quad p = 1 + (p-1)$$

$$\int_0^1 ds' p' g(p, p') (p' - p) = -4\pi \int dt f(p t) J_1(t p) J_2(t) - p 4\pi \int dt f(p t) J_1(t p) J_0(t) =$$

$$= -4\pi \int dt f(p t) J_1(t p) [J_0(t) + J_2(t)] - 4\pi (p-1) \int dt f(p t) J_1(t p) J_0(t)$$

$$J_{\nu-1}(x) + J_{\nu+1}(x) = \frac{2\nu}{x} J_\nu(x), \quad J_0(t) + J_2(t) = \frac{2}{t} J_1(t), \quad -2J'_V = J_{V+1} - J_{V-1}$$

$$J_V(p, p) = \int_0^1 ds' p' g(p, p') (p' - p) = -8\pi \int \frac{dt}{t} f(p t) J_1(t p) J_1(t) - (p-1) 2\pi(p, p)$$

$$P(p, p) = \int \frac{dt}{t} f(p t) J_1(t) J_1(p t) = \int \frac{dt}{t} \left[1 - \frac{1}{p t} (1 - e^{-p t}) \right] J_1(t) J_1(p t) =$$

$$= \int \frac{dt}{t} J_1(t) J_1(p t) - \frac{1}{p} \int \frac{dt}{t^2} (1 - e^{-p t}) J_1(t) J_1(p t) \quad \text{hormonal anamurazeky xoro vlu } p=1.$$

$$\int_0^\infty dt t^{-1} J_1(bx) J_1(ax) = \frac{1}{2} \frac{1}{b} \Gamma(1) \frac{a}{\Gamma(2)}, \quad a < b. \quad \Gamma(z) = \Gamma(z+1) = z \Gamma(z) = 1, \quad \Gamma(1) = 1$$

$$\int \frac{dt}{t} J_1(t) J_1(p t) = \frac{1}{2} p, \quad p < 1, \quad \int \frac{dt}{t} J_1^2(t) = \frac{1}{2}, \quad p=1 \text{ - nepod.}$$

$$\rightarrow Q(p, p) = \int \frac{dt}{t^2} (1 - e^{-p t}) J_1(t) J_1(p t), \quad \frac{dQ}{dp} = \int \frac{dt}{t} e^{-p t} J_1(t) J_1(p t)$$

$$\frac{dQ(p, p=1)}{dp} = \int dt t^{-1} e^{-p t} J_1^2(t) = I(1, 1, -1) = \frac{\beta}{\pi k} E(k) - \frac{\beta k}{2\pi} (2 + \beta^2) K(k) + \frac{1}{2}$$

$$k^2 = \frac{1}{1 + \beta^2/4} \quad k'^2 = 1 - k^2 = \frac{\beta^2/4}{1 + \beta^2/4} \approx \beta^2/4, \quad k' = \beta/2, \quad \frac{1}{\sqrt{1 + \beta^2/4}} = \frac{1}{2} [1 - O(\beta^2)]$$

$$\left. \frac{dQ(p)}{dp} \right|_{p=1} = \frac{\beta}{\pi} \frac{1}{\sqrt{1 + \beta^2/4}} E\left(\frac{1}{\sqrt{1 + \beta^2/4}}\right) - \frac{\beta}{2\pi} \frac{1}{\sqrt{1 + \beta^2/4}} \frac{2}{\beta} \left(1 + \frac{\beta^2}{4}\right) K\left(\frac{1}{\sqrt{1 + \beta^2/4}}\right) + \frac{1}{2} =$$

$$= \frac{\beta}{\pi} \sqrt{1 + \beta^2/4} \left[E(k) - K(k) \right] + \frac{1}{2}, \quad k \rightarrow 1, \quad \beta \rightarrow 0$$

$$E(k) - K(k) = 1 + \frac{k'^2}{2} \left(\ln \frac{4}{k'} - \frac{1}{2} \right) - \ln \frac{4}{k'} - \frac{1}{4} k'^2 \left(\ln \frac{4}{k'} - 1 \right) + O(k'^4 \ln k') \approx$$

$$\approx 1 - \ln \frac{4}{k'} + \frac{1}{4} k'^2 \ln \frac{4}{k'} + O(k'^4 \ln k') = 1 - \ln \frac{8}{\beta} + \frac{1}{4} \frac{\beta^2}{4} \ln \frac{8}{\beta}$$

$$\frac{dQ(p)}{dp} = \frac{1}{\pi} \beta \left(1 + \frac{\beta^2}{8} \right) \left[1 - \ln \frac{8}{\beta} + O(\beta^2 \ln \beta) \right] + \frac{1}{2} \approx \frac{1}{\pi} \beta \left[1 - \ln \frac{8}{\beta} \right] + \frac{1}{2} + O(\beta^3 \ln \beta)$$

$$Q(p, p=1) = \frac{1}{\pi} \int dp \beta \left[1 - \ln \frac{8}{\beta} \right] + \frac{\beta}{2} + C, \quad \int dx x \ln x = x^2 \left[\frac{1}{2} \ln x - \frac{1}{4} \right] = \frac{x^2}{2} (\ln x - \frac{1}{2})$$

$$Q(p, 1) = \frac{1}{\pi} \frac{\beta^2}{2} + \frac{1}{\pi} \int dp \beta \left[\ln \beta - \ln 8 \right] + \frac{\beta}{2} + C = \frac{1}{2\pi} \beta^2 + \frac{1}{\pi} \left[\frac{\beta^2}{2} (\ln \beta - \frac{1}{2}) - \ln 8 \cdot \frac{\beta^2}{2} \right] + \frac{\beta}{2} + C$$

$$C=0, \text{ т.к. } Q(0, 1)=0, \quad Q(p, 1) = \frac{\beta^2}{2\pi} \left[1 + \ln \beta - \frac{1}{2} - \ln 8 \right] + \frac{\beta}{2} = \frac{\beta^2}{2\pi} \left(\ln \frac{\beta}{8} + \frac{1}{2} \right) + \frac{\beta}{2}$$

$$J_V(p, p=1) = -8\pi P(p, 1) = (-8\pi) \left[\frac{1}{2} - \frac{1}{\beta} Q(p, 1) \right] = (-8\pi) \left[\frac{1}{2} - \frac{\beta}{2\pi} \left(\ln \frac{\beta}{8} + \frac{1}{2} \right) - \frac{1}{2} \right] =$$

$$J_V(p, p=1) = -4\beta \left(\ln \frac{\beta}{8} + \frac{1}{2} \right)$$

$$\rightarrow h_V^p(p \neq 1) = \frac{\partial m_p(p)}{\partial p} J_V(p, p) - \text{gives none of volume charges } m_p$$

~ c term. ~ $\beta^3 \ln \beta$.

(21/12) Lecture paper by D. Mills, PRB '04 - SW in long wires.

Uniaxial FMR - mag of thin 1-magnetized stripes.

SW stripes Φ - mag stripe $m_n(x) = \cos x_n x$, $x_n = \frac{\pi n}{w} [1 - \frac{2}{d}]$

Corr. typ. - method (p. 212) uniaxial mag FMR $I_n \sim |M_n|^2$, $M_n = \int_{-w/2}^{w/2} dx m_n(x)$,
 где $m_n(x)$ нормирована на 1:

$$M_n = \int dx A_n \cos(x_n x) = A_n \frac{1}{x_n} (+1) \sin x_n x \Big|_{-w/2}^{w/2} = A_n \frac{2}{x_n} \sin x_n \frac{w}{2}, \quad n=1,3,5,\dots$$

где $\cos x_n x$ - mag

$$\int m_n^2 dx = A_n^2 \int dx \cos^2 x_n x = A_n^2 \frac{1}{2} \int dx (1 + \cos 2x_n x) = A_n^2 \left[\frac{w}{2} + \frac{1}{2} \frac{(1)}{2x_n} \sin 2x_n x \Big|_{-w/2}^{w/2} \right] =$$

$$= A_n^2 \frac{w}{2} \left[1 + \frac{1}{2x_n w} \sin 2x_n w \right] = A_n^2 \frac{w}{2} \left[1 + \frac{1}{x_n w} \sin x_n w \right] = 1$$

$$A_n^2 = \left(\frac{w}{2} \right)^{-1} \frac{1}{1 + \frac{\sin x_n w}{x_n w}} \quad \sin x_n w = \sin \pi n (1 - \frac{2}{d}) = \sin \pi n \cos \frac{2\pi n}{d} - \cos \pi n \sin \frac{2\pi n}{d} =$$

$$= (-1)^{n+1} \sin \frac{2\pi n}{d} \approx \frac{2\pi n}{d}, \quad n \text{ - нечет.}$$

$$\text{PRB '02: } \frac{2\pi}{d} = p \left[\frac{1}{p} \cdot 2 + 1 \right] = p \left[2 \frac{1}{p} + 1 \right] \ll 1, \quad \text{т.к. } p = \frac{20 \text{ nm}}{1000 \text{ nm}} \approx 0,02$$

$$M_n = -A_n \frac{2}{x_n} \sin x_n \frac{w}{2}$$

$$\sin x_n \frac{w}{2} = \sin \frac{\pi n}{2} (1 - \frac{2}{d}) = \sin \frac{\pi n}{2} \cos \frac{\pi n}{d} - \cos \frac{\pi n}{2} \sin \frac{\pi n}{d} =$$

$$= (-1)^n \cos \frac{\pi n}{d} = (-1)^n \left[1 - \frac{1}{2} \left(\frac{\pi n}{d} \right)^2 \right]$$

$$I_n \sim |M_n|^2 = 4 A_n^2 \frac{1}{x_n^2} \sin^2 x_n \frac{w}{2} = 4 \frac{1}{w} \frac{1}{1 + \frac{2\pi n}{d} \cdot \frac{\pi n (1 - \frac{2}{d})}{2}} \cdot \frac{1}{x_n^2} \left[1 - \frac{1}{2} \left(\frac{\pi n}{d} \right)^2 \right]$$

$$I_n \sim \frac{8}{w} \frac{1}{1 + \frac{2}{d} (1 - \frac{2}{d})} \frac{w^2}{\pi^2 n^2 (1 - \frac{2}{d})^2} \left[1 - \frac{1}{2} \left(\frac{\pi n}{d} \right)^2 \right] \approx \frac{8w}{\pi^2} \frac{1}{n^2} \frac{\cos^2 \frac{\pi n}{d}}{(1 - \frac{2}{d})}$$

т.к. $\text{pinning } d(p) \gg 1$, то $I_n = |M_n|^2 \sim \frac{w}{n^2}$ - uniaxial SW-mag stripes
 с M_0 110y или M_0 110z, $n=1,3,5,\dots$

Тогда $I_n \sim \left| \int dx dy dz m_n(x) \right|^2 \sim L^2 \left| \int dx m_n(x) \right|^2$, т.е. $I_n \sim \frac{wL^2}{n^2}$

Еще представим $\vec{m}(x) = \sum C_n \vec{m}_n(x)$, то [гип. - метод]

$$C_n = C \omega^2 \frac{\int \vec{h}(x) \cdot \vec{m}_n^*(x) dx}{\omega^2 - \omega_n'^2 + 2i\omega_n' \omega_n''}, \quad \omega_n = \omega_n' + i\omega_n'' - \text{св. частота } n\text{-мод.}$$

Безы резонанса $C_n(\omega \approx \omega_n') = C \frac{\omega_n'}{2i\omega_n''} \int \vec{h}(x) \cdot \vec{m}_n(x) dx$, $\vec{h}(x)$ - uniform like FMR, $\vec{h} \parallel \text{b-axis}$.
 т.е. $C_n \propto m_n^x$; $\vec{h} \parallel 110x$

$\omega_n'' = \alpha_n \omega_n' \approx \alpha \omega_n'$ с тем damping не зависит от номера моды

$$C_n(\omega \approx \omega_n') = \frac{C}{2i} \frac{1}{\alpha} \frac{\vec{h}_x}{\omega_n'} \int dV m_n^x(x), \quad I_n = |C_n|^2, \quad C - \text{нормировка } m_n(x)$$

$$I_n(\omega \approx \omega_n') = \frac{1}{4} \frac{1}{\alpha^2} (\vec{h}_x)^2 L^2 \left| \int dx m_n(x) \right|^2 = \frac{1}{4\alpha^2} (\vec{h}_x)^2 L |M_n|^2 \rightarrow I_n \sim \frac{(wL^2)(\vec{h}_x)^2}{n^2 \alpha^2}$$

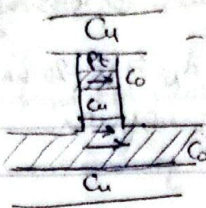
uniaxial SW-modes of stripes с $\vec{M}_0 \parallel \text{axis}$ или $\vec{M}_0 \parallel \text{axis}$

Две групп. резонансы stripe $I_n \sim \frac{1}{n^2}$

Однако $I_2 \approx \times$ резонанс ($n=1$) имеет $I_3/I_1 = 1/9$.
 uniaxial SW-mag w/ FMR
 6 stripes, 110x

→ Seminar Gernot Bauer. "Magneto-electronics." Spin pumping / transfer

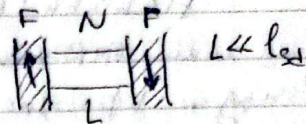
Writing in MRAM - are very common now. read by GMR.



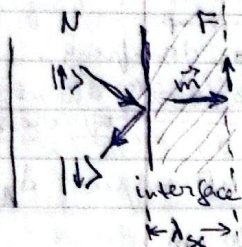
- nanopillar (Kiselev, 2003)

Spin-diffus. length $l_{SD} = \sqrt{D\tau_{sf}}$

τ_{sf} - spin flip time.



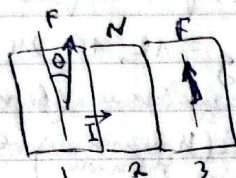
Spin accum. F - main regular number N - main (2 cross polarized e⁻)



$$L(\theta = \pi/2) = \frac{N}{2\pi} (\mu_{\uparrow} - \mu_{\downarrow})$$

- Slonczewski-torque.

- process e⁻ na voloknomen N/F u jumbogit u torque b F.



$$R(\theta) = I_{\uparrow\downarrow}^c / \Delta V$$

$I_{\uparrow\downarrow}^{c,s}$ - charge or spin currents -

- on jege uos bo mameoT pacmT.

Character of F/N/F structure

$$\vec{L} = -\frac{\hbar}{2e} \vec{I}_{\uparrow\downarrow}^s$$

- Slonczewski torque

Viazhkin R(θ) - measurements (2004)

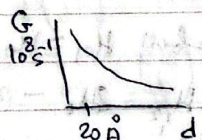
Dynamics: transverse ($\perp \vec{M}$) spin current Cozzges torque na $\vec{M}$

Spin pumping (Buttiker) : $I_{\text{pump}} = g_{\uparrow\downarrow} \vec{m} \times \frac{d\vec{m}}{dt} \frac{\hbar}{2\pi}$

$g_{\uparrow\downarrow}$ gas bueg u b d⁴ u on jegeu. $g_{\uparrow\downarrow} \vec{m} \times \vec{S} \times \vec{m}$ - men Cuorebaos

1/τ_{sf} maso b Cu, no 1/τ_{sf} beuus b Pt-spacer.

Mizubami (2001)



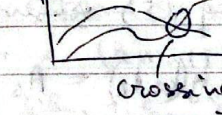
$$G(d) = G_0 + \frac{(g_{MB}) G_{\uparrow\downarrow}}{e^2 d} - P_y / Pt$$

- y Cuoreba mameoT pacmT

$H_{1,2}(\omega)$

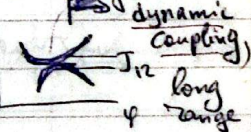


$H_{1,2}(\omega)$



crossing

$H_{1,2}$



dynamic coupling long range

B oacm crossing ≈ oqnatalosa FMR - cu. Heinrich (2003).

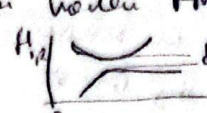
→ Cu. mas padom no yu. Crossing zacos FMR in exchange -

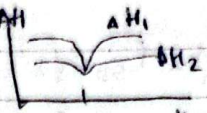
- coupled beayer u 3φ. vospromenen damping'a H1 u H2 - neg -

No damping b oacm crossing $H_1(\omega)$ u $H_2(\omega)$ nepenopmyl'sca u bez spin-transfer/pumping effects in sandwich F/N/F with coupled layers (Yone - 92). YFM 36, 875-878 (1991). Dyn. exch. coupling to non-local damping.

→ Buguno dynamic coupling u 3φ spin-momentum pumping J_{12} ecto Heinrich et al. PRL 90, 187601 (2003)

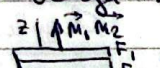
note $d_N < l_{SD}$ - spin-diffus. length, $l_{SD} \sim 100$ nm.

Здесь рассуждения, что H_1^0 и H_2^0 - изол. слоев разуме
и $H_1(\omega)$, $H_2(\omega)$ - уравне затухающ волн FMR $F_1/N/F_2$
непресекание. В области crossing H_1  разбегутся
 $\delta H \sim J_{12}$ если game где $d_N \sim 10^2 - 20$ nm.

Наконец 'PR' оз из неприя
успехи теории FMR микров. безне уравнов
crossing. Упрощен! упр crossing $H_1(\omega)$ и $H_2(\omega)$  $H_2 - H_1$, умно
близк 6 ат из-за spin pumping, т.к. spin-currents компенсируются
 H_1 - "optic" peak берга умно, т.к. $d_1 = d_0 + d_{spin-pump}$
 δH_2 "упреждение" темно d_0 - "intrinsic" Гундберга damping.
 $d_{spin-pump} = \delta H g^2 / (8\pi MV) \sim 1/d_F$. Кон. ФРМ'91 game $d_{12}^{(0)}$ and narrow 1
after crossing.

→ Double-Stripe FMR База на $FeN/Ag/FeN$ stripes нафн ФМР упр $d_{Ag} =$
 $= 6, 12, 18$ nm и на ∞ - микр. с $d_{Ag} = 12, 18, 24, 30$ nm.
Упр $H \parallel Oy$ - long side of d. stripe - одно гб микр, H микр. - огу микр.

Две ∞ - микр. F/N/F одно берга 2 микр. (H микр.) H_1 микр. H_2 микр. $d_{Ag} \leq 24$ nm.

Согласно ФТ'93  k_2 - SWR, если сучи угловыми

$H_{01} = H_{02}$ (норм ФМР $k_2=0$), но $k_2 \neq 0$ $k_2 \neq 0$
сдвиг $I \sim (H_{01} - H_{02})^2 = 0$. Упр $H_{01} = H_{02}$ - огу микр. сучи, если
только uniform (FMR, $k_2=0$) peak.

Упр Exch. Coupling $\Phi = - \int dS A_s \vec{M}_1 \vec{M}_2$, $\alpha = A_s / d v_1 v_2$

норм FMR ($k_2 d \ll 1$) $2H_{1,2} = H_{01} + H_{02} - \alpha \mu_{\pm} \pm \sqrt{(H_{01} - H_{02} + \alpha \mu_{\pm})^2 + 4 \alpha^2 \mu_{\pm}^2 / k_2^2}$

$\mu_{\pm} = \mu_1 \pm \mu_2$, $M_i = M_i v_i$. Если сучи огу микр. $H_{01} = H_{02}$
 $\sqrt{\dots} = \sqrt{\alpha^2 [\mu_{\pm}^2 + 4 \mu_1 \mu_2]} = \alpha \mu_{\pm}$ и $2H_{1,2} = 2H_0 - \alpha \mu_{\pm} \pm \alpha \mu_{\pm} = \begin{cases} 2H_0 \\ 2H_0 - 2\alpha \mu_{\pm} \end{cases} ?$
 $H_2 = H_0$
 $H_1 = H_0 - \alpha M_s$

Упр микр. упр Eq. (5) сдвиг ФТ'93:

$$D_1 k_1 D_2 k_2 \lg(k_1 d_1) \lg(k_2 d_2) = A_s [D_1 M_1 k_1 \lg(k_1 d_1) + D_2 M_2 k_2 \lg(k_2 d_2)]$$

Две огу микр. сучи $D_1 = D_2$, $M_1 = M_2$, $f_1 = f_2 \Rightarrow k_1 = k_2 = k$

$$D^2 k^2 \lg^2(k d) = 2 A_s D M_s k \lg(k d) \Rightarrow D k \lg(k d) = 2 A_s M_s$$

а) если $k=0$, $\frac{\omega}{\gamma} = H - 4\pi M_s$ ум $H_0 = \frac{\omega}{\gamma} + 4\pi M_s$ - ФМР, $I_0 \neq 0$

б) если $k \neq 0$, умно $D k \lg(k d) = 2 A_s M_s$, если

$$\text{сучи } k d \ll 1, \quad \pm k^2 d^2 = 2 A_s M_s, \quad k^2 = \pm \frac{2 A_s M_s}{d^2} \text{ ум } (H_0 - H) = \pm \frac{2 A_s M_s}{d}$$

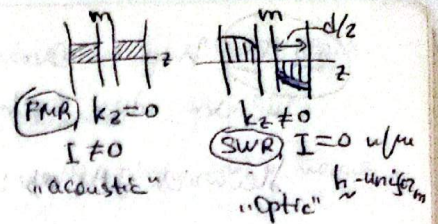
$H_1 = H_0 - \frac{2 A_s}{d} M_s$ - СР $k_2 \neq 0$, $k_1 d \ll 1$. Но СР $k_2 \neq 0$

умно $I=0$ упр FMR.

→ Barbog: неосл. на $A_s (I_2) \neq 0$ упр FMR 2 - микр. $H_{01} = H_{02}$ если
double stripe только огу микр.

Cur. analyzing FMR in double-strips or 24-28/V.

Примера: h_z и h_x могут SW в нуле с одинак. частотой:
"Acoustic" и "Optic" modes при $A_s > 0$ (FM coupling)
(sym.) (antysym.)



$$k_1^2 (\text{"optic" mode}) = \frac{4A_s M_s}{Dd}, \quad \frac{d}{2} - \text{толщина FM слоя.}$$

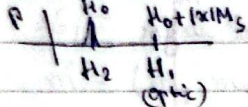
т.е. SW могут быть F/N/F с угловыми F-состояниями "at least" где:

с нулевым P | H_0 H_1 H_2 H , но только symmetrical могут с $H_z = H_0$ резонанс
возбуждаются при FMR (h_z -uniform).

$$\frac{1}{f} \omega(k) = H_2 - 4\pi M_s + Dk^2, \quad \text{FMR } H_0 = \frac{\omega}{\gamma} + 4\pi M_s, \quad \text{SWR } H_1 = \frac{\omega}{\gamma} + 4\pi M_s - Dk_1^2 < H_0$$

Due AFM coupling in (0,0)-phase ($A_s < 0$); $k_1^2 = -\frac{4M_s}{Dd} |A_s|$ и $k_1 = i k_s$, $k_s = \left(\frac{4M_s |A_s|}{Dd} \right)^{1/2}$

$$H_2 (\text{SWR}) = H_0 - Dk_1^2 = H_0 + Dk_s^2 > H_0 \quad (\text{FMR})$$



→ Т.е. при нулевой phase interlayer coupling A_s и моды не смешиваются (но $k, d \ll 1$ то есть даже для SW моды) FMR возбуждаются только симметричные - $\int dz m(z) \neq 0$
и "acoustic" моды с $k_z = 0$. "Optic" mode FMR не возбуждается из-за ее антисимметрии (antisymmetric).

→ Если же interlayer coupling long-range как dyn. coupling в Heinrich - PRL'03, то не имеет смысла FMR моды где угловыми числами в

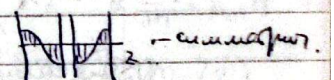
F/N/F - sandwich film: нуль углов. - "acoustic".

$$k_1^2 d^2 = \frac{4A_s M_s d}{D} = \left| D = \frac{2A}{M_s} \right| = \frac{4A_s}{2A} M_s^2 d = \frac{4A_s d}{L_{ex}^2} \ll 1, \quad A_s \ll \frac{L_{ex}^2}{4d} \approx \frac{(20 \text{ nm})^2}{4 \cdot 10 \text{ nm}} = 10 \text{ nm}$$

$$J_{12} = \frac{A_s}{d}, \quad J_{12} \ll \frac{1}{4} \left(\frac{L_{ex}}{d} \right)^2 - \text{не смешиваются если } d < 10 \text{ nm},$$

$$\delta H = H_2 - H_1 = \frac{4A_s M_s}{d} = 2M_s \left(\frac{A_s}{d_1} \right) \sim 10^2 \text{ Oe} \text{ соответствует } \frac{A_s}{d_1} = 2J_{12} \sim 0.1 \text{ и } J_{12} \ll \frac{1}{4} \left(\frac{L_{ex}}{d} \right)^2 \text{ при } d \leq L_{ex}.$$

Note: Due F/N/F reference при FMR не имеет значения (нулевая) $\int dz m(z) \neq 0$



Но это невозможно $I_3 < I_1$, т.к. $\int dz m(z) = 0$.

У Бауэра же $I_3 \approx I_1$ для F/N/F film, т.е. SW действительно не возбуждается.

→ Бауэр не признает crossing $F_1/N/F_2$ $H_1(\omega) \approx H_2(\omega)$ и $\delta H \sim J_{12}$ с FMR в F/N/F - пленке с углов. числами.

F/N/F FMR → Углы: направление облучения FMR спектра $F_1/N/F_2$ ($\subset F_1 \neq F_2$)
и layers излучает заблуждение $J_{dyn}(d)$ из "optic" peak. J_{dyn} no?

Gerit Bauer: J_{dyn} это действительно не exch. coupling, а гомогенизация
близко к ΔH из-за g-фактора (spin pumping). J_{dyn} в основном от A_s (static)
и возбуждается не в направлении $H_1(\omega)$ и $H_2(\omega)$ при crossing ак симметрично (!)

Escaneado con CamScanner

Hamilton's JAP's: $M_2 \frac{\partial \vec{u}_1}{\partial t} = -[M_1 \times \vec{u}_1^{\text{rot}}] + \frac{G_1}{r_{12}^3} [M_1 \times \frac{\partial \vec{u}_1}{\partial t}] +$

Neglect $g_1^0 = g_2^0$ gen. lorsque $\delta y \ll A$. $\vec{T}_1 = \frac{1}{\omega_1} \left[\vec{m}_1 \times \frac{d\vec{m}_1}{dt} = \vec{m}_2 \times \frac{d\vec{m}_2}{dt} \right] \vec{g}^{10}$

Due in plane response $\vec{M}_{\text{eff}} \parallel \vec{Oz} = \vec{T} = \frac{\hbar \gamma}{4\pi d_1} \frac{d}{dt} (\vec{m}_1 - \vec{m}_2)$

T.e. $\vec{T}_1 \sim \hat{n} \times \frac{\partial}{\partial t} (\vec{r}_1 - \vec{r}_0) \neq 0$ because gas "picks" mode $\vec{r}_1 - \vec{r}_0 = \vec{r}_2 \neq 0$.

16. optical mode number Intensity vs ω crossing.

10/10/10

2) може ли в односторонней структуре $A/N/F$ не быть proper FAK-интер

DB-Rechner eing. zu Heinrich unter für FMR unter 6 R/Au/R film

Boulog: a local am-F/F states give supersymmetric interpretation.

-> того же направления к ин-новациям $F_1/N/F_2$ с забегом по времени

29/11 Waveman tested paper RB'02 - Bouyt no vortex dynamics. Ref. Report
 Non generative geometry. $L=80\text{ nm}$ after $R=80\text{ nm}$ - $\square$ k. no other
 Bouyt non-geometric for interpretation. Zaburavets $M_p(\epsilon)$ - for other modes.
 No other non-geometric analysis as it is.

Bound. Conditions  Many times "exchange-dipolar" boundary conditions are assumed. Cu. 10/ix: $\vec{T}_{ex} + \vec{T}_m = 0$, $\vec{T}_{ex} = -\vec{M}_0 \times d \frac{\partial \vec{m}}{\partial n}$, $\vec{M} = \vec{M}_0 + \vec{m}$ - just SW. $\frac{1}{S} \vec{T}_m = \int_{S \in S} d\vec{p} \vec{M}_0 \times \vec{h}_m \approx \vec{M}_0 \times \vec{h}_m \Delta p$, $\Delta p \ll L$ - otherwise nonhomogeneous m. norm.

 $\vec{M}_0 \parallel \hat{z}$, saturated cylinder in $\vec{H} \parallel \hat{z}$ T.E. $\vec{M}_0$ is linear vector - hence M_0
 $\vec{M}_0 \times \vec{X} = \begin{vmatrix} \hat{s} & \hat{\varphi} & \hat{z} \\ 0 & 0 & M_0 \\ X_p & X_\varphi & X_z \end{vmatrix} = \hat{\varphi} M_0 X_p - \hat{p} M_0 X_\varphi \Rightarrow \vec{M}_0 \times \left[-d \frac{\partial \vec{m}}{\partial n} + \vec{h}_m \Delta p \right] = 0$ (circular sym.)

For cylinder diameter $\ll X_p = 0 \cup X_\varphi = 0$. $\begin{cases} X_p = -d \frac{\partial m_p}{\partial n} + h_m^p \Delta p = 0 \\ X_\varphi = -d \frac{\partial m_\varphi}{\partial n} + h_m^\varphi \Delta p = 0 \end{cases}$

Assume $h_m^p = h_s^p + h_v^p$ since non-zero

16/ix: just because from cylinder $p=R$ assumption, independent exchange.

$\vec{m} = (m_p, m_\varphi, 0)$ $h_m^p = \int d^2 p' [G_{pp} m_p + G_{p\varphi} m_\varphi + G_{pz} m_z]$

Myself $\vec{m} = f(p)$ - radial sym. modes. Use here $G_{2p}(p, p')$ assumption: $\int d\varphi' G_{p\varphi}(p, \varphi') = 0$, $\int dz' dz'' G_{zz}(p, p', zz') = 0 \rightarrow h_m^p(z) = \int dz' \int d^2 p' G_{pp}(p, p') m_p(p')$

Derive $\frac{1}{L} \int dz \rightarrow h_m^p(p) = \frac{1}{L} \int dz dz' \int d\varphi' \int d^2 p' G_{pp}(p, p', \varphi', zz') m_p(p') = \int d^2 p' g_{pp}(p, p') m_p(p')$

$g_{pp}(p, p') = \frac{1}{L} \int dz dz' \int d\varphi' G_{pp}(p, p', \varphi', zz')$ - calculation of φ boundary conditions, since $m_p = f(p)$ then.

$h_m^\varphi = \int dz' \int d^2 p' [G_{\varphi p} m_p + G_{\varphi\varphi} m_\varphi + G_{\varphi z} m_z] = 0$ (just because $\vec{m}(p)$ is radial sym. modes)

with $G_{\varphi p} = 0$ "no φ " "no φ "

Many times from cylinder: $-d \frac{\partial m_p}{\partial p} + h_m^p \Delta p = 0$, $\frac{\partial m_\varphi}{\partial p} = 0$ with $p=R$, $\vec{M}_0 \parallel \hat{z}$ - saturated cyl. 1SD

 $\vec{M}_0 \parallel \hat{\varphi}$, vortex. $\vec{M}_0 \times \vec{X} = \begin{vmatrix} \hat{s} & \hat{\varphi} & \hat{z} \\ 0 & M_0 & 0 \\ X_p & X_\varphi & X_z \end{vmatrix} = \hat{p} M_0 X_z - \hat{z} M_0 X_p \Rightarrow \begin{cases} X_p = -d \frac{\partial m_p}{\partial n} + h_m^p \Delta p = 0 \\ X_z = -d \frac{\partial m_z}{\partial n} + h_m^z \Delta p = 0 \end{cases}$

$h_m^z(p) = \int dz' \int d^2 p' [G_{zp} m_p + G_{z\varphi} m_\varphi + G_{zz} m_z] = \int dz' \int d^2 p' G_{zz}(p, p') m_z(p')$

$h_m^z(p) = \frac{1}{L} \int dz dz' \int d\varphi' \int d^2 p' G_{zz}(p, p', \varphi', zz') m_z(p') = \int d^2 p' g_{zz}(p, p') m_z(p')$

Many times from cylinder: $-d \frac{\partial m_p}{\partial p} + h_m^p \Delta p = 0$, $-d \frac{\partial m_z}{\partial p} + h_m^z \Delta p = 0$, $\vec{M}_0 \parallel \hat{\varphi}$ vortex b.c.

Due stripes $\vec{M}_0 \parallel \hat{y}$ (same as before) since 10/ix $\vec{M}_0 \times \vec{X} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & M_0 & 0 \\ X_x & X_y & X_z \end{vmatrix} = \hat{i} M_0 X_z - \hat{k} M_0 X_x$

$\begin{cases} X_x = -d \frac{\partial m_x}{\partial x} + h_m^x \Delta x = 0 \\ X_z = -d \frac{\partial m_z}{\partial x} + h_m^z \Delta x = 0 \end{cases}$

$h_m^x = \int d^3 x' [G_{xx} m_x + G_{xy} m_y + G_{xz} m_z] = \int d^3 x' g_{xx}(x, x') m_x(x')$

$h_m^z = \int d^3 x' [G_{zx} m_x + G_{zy} m_y + G_{zz} m_z] = \int d^3 x' g_{zz}(x, x') m_z(x')$

For $m_{x,z}(x)$ same equations.

Consider 16/12 $h_m^p(p, \pi, 0) = [-2\pi + \alpha(p)] m_p + 4p(l_0 \frac{p}{2} + \frac{1}{2}) \frac{\partial m_p}{\partial p}$

$$0 = -\alpha \frac{\partial m_p}{\partial p} + h_m^p \Delta p + |\Delta p - l| = [-2\pi + \alpha(p)] m_p L + L 4p(l_0 \frac{p}{2} + \frac{1}{2}) \frac{\partial m_p}{\partial p} - \frac{\alpha}{R} \frac{\partial m_p}{\partial p}$$

$$= [-2\pi + \alpha(p)] m_p L + \frac{\partial m_p}{\partial p} [4pL(l_0 \frac{p}{2} + \frac{1}{2}) - \frac{\alpha}{R}], \quad p \text{ eq. R}$$

$$\frac{\partial m_p}{\partial p} + \alpha m_p = 0, \quad d(R, l) = \frac{2\pi L}{4pL(l_0 \frac{p}{2} + \frac{1}{2}) + \frac{\alpha}{R}} = \frac{2\pi R}{4p(l_0 \frac{p}{2} + \frac{1}{2}) + \frac{1}{R}}$$

$$\frac{L^2}{Rl} = \left(\frac{p}{l}\right)^2, \quad L^2 = d$$

$$d(p, l) = \frac{2\pi}{p \left[4(l_0 \frac{p}{2} + \frac{1}{2}) + \left(\frac{l}{l}\right)^2 \right]}$$

pinning due to magnetic $M_0 \cdot D_x$
magnon $p = R$
vortex

Surface anisotropy K_s

→ Eine neue Surface, welche die 2. Anisotropie anisotropie.

Con. f.p. - Mervob $E_s = -\frac{K_s}{M_s^2} (\vec{M} \cdot \vec{n})^2$, $\vec{n}$ - normal vector OAH

$$\vec{H}_s = -\frac{\partial E_s}{\partial \vec{M}} = + \frac{2K_s}{M_s^2} (\vec{M} \cdot \vec{n}) \vec{n}, \quad \vec{T}_s = \int \vec{M} \times \vec{H}_s dx, \quad x \perp \text{ surface } S$$

$$\text{longer } \vec{T}_s = \vec{M} \times \int dx \vec{H}_s = \vec{M} \times \int dx \frac{2K_s}{M_s^2} (\vec{M} \cdot \vec{n}) \vec{n} = \vec{M} \times \vec{h}_s, \quad \vec{T}_{sa} = \frac{2K_s}{M_s^2} (\vec{M} \cdot \vec{n}) \vec{n}, \quad K_s = \int_0^{2\pi} K_s(x) dx$$

$$0 = \frac{1}{S} \sum \vec{T} = \frac{\vec{T}_{ex}}{S} + \frac{\vec{T}_m}{S} + \frac{\vec{T}_s}{S} = -\vec{M} \times \alpha \frac{\partial \vec{M}}{\partial n} + \vec{M} \times \int dx \vec{h}_m + \vec{M} \times \vec{h}_{sa}, \quad \vec{T}_{sa} = \int dx \vec{H}_s$$

$$\text{p. y. y. y. y. } \vec{M} \times \left[-\alpha \frac{\partial \vec{M}}{\partial n} + \int dx \vec{h}_m + \vec{h}_{sa} \right] = 0, \quad \vec{h}_s = \int h_m dx + \vec{h}_{sa}$$

$$\text{Due stripe } \vec{n} \parallel \vec{Ox} \quad \vec{h}_s^x = \int dx h_m^x + h_s = h_m^x L + h_{sa}$$

$$h_s^x = (h_m^x + h_{sa}) L + h_{sa} = -2\pi m_x L + p(2lp - 1) \frac{\partial m_x}{\partial x} + \frac{2K_s}{M_s^2} m_x = h_m^x + h_{sa} + O(plp)$$

$$\text{Gibrium } h_s^x = -2\pi m_x L, \quad h_{sa}^x = \frac{2K_s}{M_s^2} m_x, \quad h_s^x \gg h_{sa}^x \text{ wenn } 2\pi L \gg \frac{2K_s}{M_s^2}$$

$$\frac{K_s}{M_s^2 L} \ll 1, \quad \text{tunnen } K_m = \pi L M_s^2 = 3 \cdot 20 \cdot 10^{-7} \text{ cm} \cdot 10^6 \frac{\text{erg}}{\text{cm}^2} \approx 6 \frac{\text{erg}}{\text{cm}^2} \sim 10 \frac{\text{erg}}{\text{cm}^2}$$

T.e. wenn $K_s \ll K_m \approx 10 \frac{\text{erg}}{\text{cm}^2}$, so haben anisotropie immer unbedeutend.

Wenn L ne zero nard ($L \geq 10 \mu\text{m}$), so K_s norm beiga norm beiga.

Due soft magnetic dots beiga $K_s \ll \pi L M_s^2$.

$$\text{Due uniaxial } E_s = -\frac{K_s}{M_s^2} (\vec{M} \cdot \vec{p})^2 \rightarrow \vec{h}_{sa} = \frac{2K_s}{M_s^2} (\vec{M} \cdot \vec{p}) \vec{p}$$

$$h_{sa}^p = \frac{2K_s}{M_s^2} m_p, \quad h_{sa}^p = \left(\frac{2K_s}{M_s^2} \right) m_p, \text{ wenn } m_p \text{ - dynamic in component.}$$

$$\text{e.g. } h_m^p = -2\pi m_p + O(plp).$$

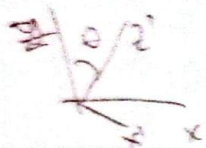
→ However, no due stripe c Mollgumme, no due uniaxial yzer K_s beiger u. zume 2π → 2π [1 - $\frac{K_s}{\pi M_s^2 L}$] b. tunnen d(p) und d(p) pinning nard.

→ However, no due d(p), d(p) nard paggen na "u" um R coeber. due nard pinning b. cm⁻¹ (standard). 2π u. 2π d(p) d(p) nard.

$$-\begin{vmatrix} \hat{x}' & \hat{y}' & \hat{z}' \\ 0 & 0 & M_z \\ m_x & m_y & 0 \end{vmatrix} = -\hat{y}' M_z m_x + \dots$$

$$\begin{vmatrix} x' & y' & z' \\ 0 & 0 & m_y \\ h_m^x & h_m^y & h_m^z \end{vmatrix} = \hat{y}' M_y h_m^x - \hat{x}' h_m^y M_y$$

$$h_z^z = \sum_{\beta} N_z^{\beta} M_{\beta}^0 = N_z^z M_{zz}^0$$



$$a_{ij} = \frac{\partial x_i}{\partial x_j}$$

$$A_i' = \sum a_{ij} A_j$$

$$\begin{pmatrix} M_x' \\ M_y' \\ M_z' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

$$A'_{ij} = \sum_{kl} a_{ik} a_{jl} A^{kl}$$

$$N_z^z = (A')^{zz} = \sum_{kl} a_{zk} a_{zl} A^{kl}, \quad A^{kl} = A^{kk} \delta_{kl}$$

$$\begin{aligned} \sum_k (a_{zk})^2 A^{kk} &= (a_{zx})^2 A_{xx} + (a_{zy})^2 A_{yy} + (a_{zz})^2 A_{zz} = \\ &= \sin^2 \theta N_{xx} + \cos^2 \theta N_{zz} \end{aligned}$$

$$2N_x + N_z = 4\pi$$

$$N_x = \frac{1}{2}(4\pi - N_z)$$

$$N_z^z = N_x \sin^2 \theta + N_z \cos^2 \theta = \frac{1}{2}(4\pi - N_z) \sin^2 \theta + N_z \cos^2 \theta = 2\pi \sin^2 \theta + \frac{N_z}{2} [2\cos^2 \theta - \sin^2 \theta]$$

$$2\pi + \frac{h_z^z}{N_z} = -N_z^z + 2\pi = 2\pi(1 - \sin^2 \theta) - \frac{N_z}{2} [2\cos^2 \theta - \sin^2 \theta] =$$

$$= 2\pi \cos^2 \theta - \frac{N_z}{2} [\cos^2 \theta + \cos 2\theta] = \frac{1}{2}(4\pi - N_z) \cos^2 \theta - \frac{N_z}{2} \cos 2\theta$$

$O(p)$

3 bonus Seagate's agbavar: Waller anasomr. nasensa na nojabar (no ero cubes). Uazar emy, too ne otkazubarov, no Seagate's compensation mmo yepofus namucame patent application. Penurb b sume. 2 weeks.

Namucame rekva Boundary Conditions - paper.

→ Division Meeting: Bonjocan gumancupobennu na 2005 y 24/ix
upregosensero DOE - review b Jan. 2005 (mucur. arazari go 15 Nov. -
- 260% - mamer, no xomr ucceagobars u $\leq 40\%$ too cgeann u ony ducubarem).

A. Slavin Duckycame co Curbin cogepmanne paper "Bound. Conditions".
Downag A. Curbin na MTI-Seminar: "theory of SW in restricted geom."
- b ocnobnem upregosono u nonoferno magnetized stripes u
senmoro rectang. elements.

→ Bugur Curbinu ero amunap, odicogennu rek. pestr 25-26/ix
u namucame paper on Boundary Conditions.

Green Card Appl. <http://uscis.gov> → Application Procedures to Become Permanent Resident

① Form I-485 Application to Register Permanent Residence 27/ix
ecso ofpazex Slavina

② Form G-325A Biographic Info (biogpast 14-79 ecum) + Copies of Passport
ecso ofpazex Slavina

③ I-693 Medical Exam. Sheet(?)

④ 3 Color Photo (ne nozame 30 gna)

⑤ Form I-864 Affidavit of Support — b paze na nago eam ecso Employment
Petition

⑥ Form I-765 Authorization of Employment -? km I-797?

⑦ Form I-94 (Arrival/Departure Record)

Namucame Green Card Application no Grene 27-28/ix

Form I-140 (Immigrant Petition for Alien Worker) — upmomenennu, too
co-cabeller Application no qapme EB1-EB (extraordinary ability)

Bugur b Human Resources: a garmen rak non-Regular Employee 29/ix
nojabar EB1-EB application can (ono ne esb employment based), gouzaz
no a esb Extraordinary (ab. Inter. + Lett.), Model Labor Letter ne gam (proungubugyannu u
nocebas. u agbavaru. confidential)

FBI-EA u. H1-B status extension applications ne uformaberas
8/20/99, sam 8/22/99 negabars na H1-B extension - kogo gkezaso.
200 perm. residence application FBI-EA negana. H1-B status number
genishobars noma Perm. Resident - Status ne uheny, no ≤ 6 m.

Yegars beca(?) Cincere papers cited my work u
v. some support letters b. gars support letters y b. gars y gars, korg, ne
m. ne cited.

→ Kto uz ybecarux y gars y gars y gars y gars (15 Handover)
cited

✓ ① M. Grimsditch — cited PRL'02, PRB'02-v. M(H), PRB'02-m. coupl., IE³TM-01-v. M.
JAP'00, PRB'02-v. M. dyn., PRB'03, PRB'02-b. cond., PRB'03

variant gars
letter

✓ ② C.A. Ross — cited APL'99-3 p. 1, APL'00-6 p. 1

✓ ③ P.A. Growell — cited PRL'02-3 p. 1, PRB'02-b. cond., JAP'02, PRB'03
APL'01, PRB'02-v. dyn., JMM'02-MPD

✓ ④ T. Shingo — cited PRL'02, ~~IE³TM-01~~, PRB'01, JAP'00-2 p. 1, APL'01
address y Shinga

⑤ F. Nizzoli — cited PRL'02, ~~IE³TM-01~~, JAP'00-2 p. 1, PRB'01, APL'01

✓ ⑥ R. Skomski — cited PRL'02-2 p. 1, JAP'02 v. TM, PRB'02 m. coupl., JAP'00

✓ ⑦ D.J. Sellmeyer — cited PRL'02, JAP'00

✓ ⑧ V. Melushko — cited PRB'02 v. M(H), PRB'02 m. coupl., APL'01, IE³TM-01,
PL-01

⑨ J. Kirschner — cited PRL'02, JAP'02 v. TM, PRB'02 v. M(H), PRB'02 v. dyn.

✓ ⑩ R. Hertel — cited PRL'02, JAP'02 v. TM, PRB'02 v. M(H), PRB'02 v. dyn.

✓ ⑪ J. Fidler — cited PRL'02, JAP'02 v. TM, PRB'02 v. dyn.

✓ ⑫ T. Schreff — cited PRL'02, JAP'02 v. TM, PRB'02 v. dyn.

⑬ S.D. Bader — cited APL'99, IE³TM-01, PRB'02 m. coupl.

⑭ H. Kronmüller — cited PRB'02 v. M(H), PRB'02 v. dyn.

✓ ⑮ U. Krey — cited PRB'02 v. M(H), JAP'00, PRB'02 v. dyn., JMM'02-MPD

- (16) B. Dieny — cited APL'99, APL'00
- (17) P. Moch — cited PRB'02 b. cond. - 2paga, Univ. Paris - Nord 13
- (18) D. Weiss — cited PRB'02 v.M(H) - 2paga
- (19) G. Bertotti — cited PRL'02, PRB'03, ien.it
(I. Mayergoyz)
- (20) J.A.C. Bland — cited PRL'02
- (21) D.L. Mills — cited PRB'02 b. cond.
- (22) T. Rasing — cited Appl. Ph. B'02 - 2paga
- (23) M.R. Freeman — cited PRL'02, PRB'03
- (24) J. Zweck — cited vortex papers - Univ. Regensburg
- (25) B. Hillebrands — cited many SW papers
- (26) B. Ivanov — cited many vortex papers & JAP-00
- (27) Y. Otani — cited vortex dyn. papers & v. statics
- (28) H. Hoffmann — cited vortex papers - Univ. Regensburg
- (29) C. Back — cited PRL - Univ. Regensburg
- (30) D. Pescia — cited JAP'00
- (31) M. Pardavi-Horvath — cited PRL'02
- (32) K. Dunadjele — cited APL'99
- (33) M.R. Ibarra — cited JMMM'95 — Univ. Zaragoza
- (34) F. Grandjean — cited JMMM'95 - 3paga - Belgium
- (35) I. Mayergoyz — cited PRL'02, PRB'03, isaak@eng.umd.edu
- (36) I.K. Schuller — cited APL'99, APL'2000
- (37) S. Demokritov — cited SW papers

Green Card

Passover was Green Card Application. Began 6 known Res. :

- 1) if no Regular Employee $\rightarrow$ no application no cert employ. - based
- 2) HR no representative non-empl. - based applications via ESI-EA
- 3) given original Appl. as reqd, whether or not refer approved
- 4) because 6 appl. for position with no reply no more consideration
a new one made as given, no application - then.

Coccoloba crassa 30 by 20 mm, green, unspotted, like a
crassula, etc. & some cited.

→ наличие запросов на использование с интерпретацией матрицы то;

- 1) Ross, Demetrius - в первом случае нулевой - нулевой, Ross в первом случае
- 2) Гиньо, Шеллер, Шонда, Губбиди, Митри? в первом Губбиди и Митри и Гиньо
- 3) Меллешко, Гриндич, Гривелл - в первом случае в первом Меллешко и Гривелл

a. Morcas (2)

4/E Info. of Senior Res. Fellowship in U.K. (ca. 21/1) - Roy Chantrell.

Deadline of applications - Nov. 11, 2009

Eng. Phys. Society Research Council (EPSRC) - www.epsrc.ac.uk

Senior Res Fellowship (SRF), ≤ 3 B neg, deadline - Nov. 10, 2004 -

- few applications in 2005.

Q: Are Fellowships — better or 5 nos. Wages? 480

T. Shima Seminar. Rept island films (Rept

? Yben
Nago your
water position
in U.K.
2008 negative.

$K_{11} \perp$ que $x \approx 0,38$ b Fe_xPt_{1-x} b (001) orientation.

Cost - $x = 0.5$ of L10-phase

Baruane pada waktu c. magnezium (MgO dan Pt)

1- β -c - max glue $t = 10 \text{ nm}$ of PET films + hexo

назад с воспомт.

$H_c = 50-60 \text{ kJ/m}^3 \text{ at } T=300 \text{ K}$ (earlier $T=4.5 \text{ K}$)

Captain u notat Bader'y bon comments/Changes x DOE-review.

S/25 Работав над "Green Card" application (список кто может и где находиться).

6/X Craba knows changes to DOE-review proposal no. "Nanomagnetics" section "Spin Dynamics". Base of nanomagnetics much more extensive, also by a factor, to include double-vertex cases. Much more precise citations. Doabur & O. S. - stripes, DVR, new b. conditions & new eq. of $\vec{M}$ -motion

(7/8) Oshpobova cover letter to C.I.S.
 Hamman reject - examine re. paper by Kpyruek et al. no
 SW b ML - cylindrical wires (JAP).

Conf. fee - go 15/8 11/8 Nov. Fax sub
 Oshpobova AOL - go 14/8 - "Larysaguer"

8/8 Zanonenne Form I-140 GC application. Regent. nucen Marcos'a
 u Gubbio, nucen Nizzoli. Growell u Nucen Letter o citations.

11/8 Zanonenne Form I-140. Nucen go - ob os oshpobovann u employment.
 Nucen okay of AOL u Nucen registration by Fax. Nucen oshpobova cyl. dots
 Hamman plan of future work que MTI (no u Nucen V. Vinokur).

AML (12/8) Oshpobova Conferences o Nucen Nucen X-ray (gucen Growell, Ross etc.)
 u "nanomagnetism"

Plans:

Bonus b DOE-plan u Bader:

- 1) FMR in mag. stripes
- 2) resonance of domain walls in GPT, RPT
- 3) M(H), dynamics of double-vortex F/N/F
- 4) New bound. conditions
- 5) New eq. of motion c(M) (const. - Nuc.)
- 6) RPT/Rh vak Nucen
- 7) Mar. Nucen - Nucen Nucen no SW

Bonus b FWP MTI u Vinokur:

- 1) MPD u mag. ring
- 2) Hamman (t) que vortex
- 3) Nucen u "breathing" vortex modes

Zanon:

- 4) RPT - K₁(T) Nucen re bonus
- 5) Spin-pumping effects in FMR of F/N/F - re bonus
- 6) RPT/Magnet M(H) Nucen H, T < T_c - re bonus

Oshpobova 1998-2004 papers b "Yenerax Nucen Nucen" u oshpobova papers
 b "Nature", "Science" no small mag. elements and magnetization switching
 que b. conditions paper.

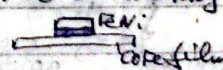
Oshpobova Nucen Cover Letter, GC-application u Nucen, Nucen.
 Hamman Introduction to Bound. Conditions paper.

13/8

Werne papers Feldtkeller'65 (Bloch lines $\rightarrow$ vortex $m_z(p)$)

Thiele APR'04 o pump-probe registration RRh AF $\rightarrow$ F transition

Demidov APR'04 o Nucen Nucen SW Nucen RRh rect. element b Nucen Nucen



Vortex

Nucen no Vortex - profile:

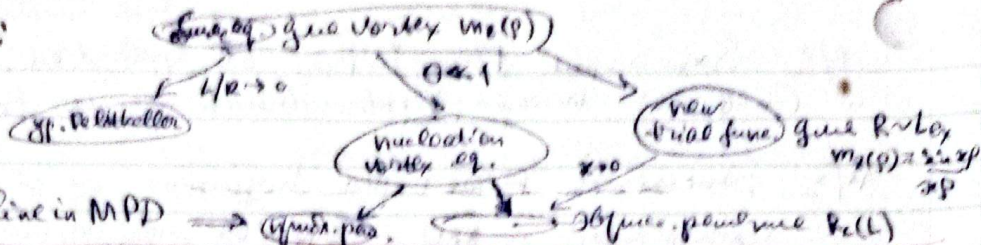
- 1) Ansatz Feldtkeller'65 $m_z(p) = e^{-2p^2/p^2}$ Nucen Nucen (L/p) Nucen Nucen - Nucen Nucen, $L \rightarrow 0$
- 2) Ansatz Usav'93 $m_z(p) = (b^2 - p^2)/(b^2 + p^2)$ - Nucen Nucen, i.e. Nucen
 Nucen Nucen L, R, Nucen Nucen $L \geq 2L_{ex}$, u Nucen Nucen Nucen, BP soliton/skyrmon
- 3) Nucen JAP'04 Nucen Nucen Nucen $m_z(p)$ 2D-vortex, a ansatz $m_z(p) = \frac{\sin 2p}{2p}$
 Nucen Nucen Nucen $L/L_{ex} \geq 2$ u Nucen Nucen $R \geq L_{ex}$.

Vortex- γp -e Feldkeller

$$\theta''(p) + \frac{1}{p} \theta'(p) + \left(x^2 - \frac{1}{p^2}\right) \sin\theta(p) \cos\theta(p) = 0, \quad 1/p \rightarrow 0$$

Ansatz nehmen & Bzge $\cos\theta(p) = u(p) e^{-2\beta^2 p^2}$, w. $u(p)$ wählen $p \rightarrow 0$.

New Vortex-Gus-JAPV4 :



$$(\cos\theta)' = -\sin\theta \cdot \theta', \quad (\cos\theta)'' = -\cos\theta (\theta')^2 - \sin\theta \theta''$$

$$\theta' = -\frac{1}{\sin\theta} (\cos\theta)' = -\frac{1}{\sin\theta} [u' e^{-2\beta^2 p^2} + u (-4\beta^2 p) e^{-2\beta^2 p^2}] = -\frac{e^{-2\beta^2 p^2}}{\sin\theta} [u' - 4\beta^2 p u]$$

$$\sin\theta \theta'' = -\cos\theta (\theta')^2 - (\cos\theta)'', \quad (\cos\theta)'' = -4\beta^2 p e^{-2\beta^2 p^2} [u' - 4\beta^2 p u] + e^{-2\beta^2 p^2} [u'' - 4\beta^2 u - 4\beta^2 p u']$$

$$0 = \sin\theta \theta'' + \sin\theta \theta' \frac{1}{p} + \left(x^2 - \frac{1}{p^2}\right) \sin^2\theta \cos\theta =$$

$$= -e^{-2\beta^2 p^2} \left\{ -4\beta^2 p (u' - 4\beta^2 p u) + u'' - 4\beta^2 u - 4\beta^2 p u' \right\} + \frac{1}{p} (-1) e^{-2\beta^2 p^2} [u' - 4\beta^2 p u] + \left(x^2 - \frac{1}{p^2}\right) \sin^2\theta u e^{-2\beta^2 p^2} - u e^{-2\beta^2 p^2} \frac{1}{\sin^2\theta} e^{-4\beta^2 p^2} (u' - 4\beta^2 p u)^2 = 0$$

$$4\beta^2 p (u' - 4\beta^2 p u) - u'' + 4\beta^2 u + 4\beta^2 p u' - \frac{1}{p} (u' - 4\beta^2 p u) + \left(x^2 - \frac{1}{p^2}\right) u \sin^2\theta - \frac{u}{\sin^2\theta} e^{-4\beta^2 p^2} (u' - 4\beta^2 p u)^2 = 0, \quad \sin^2\theta = 1 - u^2 e^{-4\beta^2 p^2}$$

$$-u'' + u' \left[8\beta^2 p - \frac{1}{p} \right] + 8\beta^2 (1 - 2\beta^2 p^2) u + \left(x^2 - \frac{1}{p^2}\right) u \sin^2\theta - \frac{u}{\sin^2\theta} e^{-4\beta^2 p^2} (u' - 4\beta^2 p u)^2 = 0$$

γp -e na $u(p)$. Gr. y.ubine $u(p=0)=1$, n. e. $\cos\theta(0)=u(0)=1$

$$0 = (\cos\theta)'_{p=0} = e^{-2\beta^2 p^2} [u' - 4\beta^2 p u]_{p=0} = u'(0) = 0$$

T. e. $u(0)=1, u'(0)=0$ n. y.ubine na $u(p)$.

$$-u'' + \frac{1}{p} [8\beta^2 p^2 - 1] u' + 8\beta^2 (1 - 2\beta^2 p^2) u + \left(x^2 - \frac{1}{p^2}\right) \sin^2\theta \cdot u - \frac{u}{\sin^2\theta} e^{-4\beta^2 p^2} (u' - 4\beta^2 p u)^2 = 0$$

n. ansatz $\cos\theta(p) = u(p) e^{-2\beta^2 p^2}$ & g.ze Feldkeller. γp -e na $u(p)$

β^2 unter cuben "core" radius.
feld. ansatz n. x. n. $\beta p \gg 1 \rightarrow$

a) Es sei $\beta p \gg 1$, so $\sin^2\theta = 1$ u $-u'' + 8\beta^2 p u - 16\beta^4 p^2 u + x^2 u = 0$

$$-u'' + (x^2 + 8\beta^2 p - 16\beta^4 p^2) u = 0 \Rightarrow -u'' + [2x^2 - (x - 4\beta^2 p)^2] u = 0$$

$$-u'' + (x^2 - 16\beta^4 p^2) u = 0 \text{ - na feld. r. y.ubine } u(p) \text{ - hier}$$

b) $\beta p \rightarrow 0$ $-u'' - \frac{1}{p} u' + 2\beta^2 u - \frac{1}{p^2} u(1-u^2) - \frac{u^3}{1-u^2} 16\beta^4 p^2 = 0$

Corpus, so $u(p)=1$ na ab. p.ubine $\Rightarrow$ u.ubine z.ubine.

Feldtkeller $\Theta(p) = \arccos[u(p)e^{-2p^2 p^2}] \approx u(p) \approx 1$ if $p \rightarrow 0$

Us Ginzburg's approach $\Theta(p) \sim J_1(x_0 p)$, no nonexchange interaction
 Feldtkeller, remember $\Theta(p) = J_1(x_0 p)$ asymptotically correct if $p \rightarrow 0$
 $\Theta \ll 1$.

Here proportionally if $p \rightarrow 0$ $\Theta(p) = \arccos F(p) = \sqrt{1-F^2} = (1-u^2 e^{-2p^2 p^2})^{1/2} \sim p$,
 same as asymptotically correct $\Theta(p) \sim J_1(x_0 p) \sim p$.

Исследования Шиньо, Ding (PRL barrier).
 Gabbiotti's paper on J. Phys. Cond. Mat., vortex core in
 2D media (!). Double-writer

Decega & Krüster: one parameter $\chi(d, \text{space}, d)$ for F/N/F disk
 & antiferromagnetic $M_1(z)$ & $M_2(z)$ if $\vec{r}$ (opposite vorticity)
 & under $\chi(d)$ for parallel magnetic M_1, M_2 .

Remember $M(H)$ for various F/N/F disk sizes by MOKE, &
 various magnetic properties. modes ($n=1$ - uniform), MFM-like,
 SQUID/VSM-like i.e. give some good evidence arrays of c. dots.

- for $M_x(t)$ - translation mode give
 constant core; 2) for $|M_x(\omega)|^2$ two groups eigenfrequencies
 3) for $M_x(\omega)$ give more if $\omega > \omega_0$ (single disk) - more narrow
 4) if $H_z(t)$ & increase "breathing" mode of single disk

→ Note: nonuniform translation mode frequencies give F/N/F cylinder
 & magnetostatic coupling → splitting of ω_0 single disk.

Vortex
 $m_z(p)$

$$J_V(z) = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k}}{2^{2k} k! \Gamma(V+k+1)} \quad J_1(z) = \frac{z}{2} \left[1 - \frac{z^2}{2^{2 \cdot 2}} + \dots \right]$$

$\Gamma(n) = (n-1)!$
 $\Gamma(1) = \Gamma(2) = 1, \Gamma(1/2) = \sqrt{\pi}$

For a vortex if $\theta \ll 1, m_z = \cos \theta$: $\theta'' + \frac{1}{p} \theta' + (\alpha^2 - \frac{1}{p^2}) \theta = 0 \rightarrow \theta(p) = C J_1(x_0 p)$
 where $\alpha = \frac{4\pi}{a} = \frac{4\pi}{L_x} = \frac{1}{l_x}$, i.e. $x_0 = p/l_x$. Local max-va if $p \rightarrow 0$.

For a π or 2π vortex: $\theta'' + \frac{1}{p} \theta' + (\alpha^2 - \frac{1}{p^2}) \sin \theta \cos \theta = 0$ under asympt.

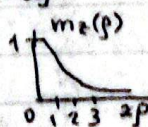
remember $\left\{ \begin{aligned} \theta(p) &= \alpha x_0 (1 - \frac{1}{8} \alpha^2 p^2), \quad x_0 = p/l_x \ll 1 \\ m_z(p) &= 1 - \alpha^2 x_0^2 \end{aligned} \right\}$ and $\left\{ \begin{aligned} \theta(p) &= \frac{\pi}{2} - \frac{b}{2} \sqrt{\frac{\pi}{2p_x}} e^{-p^2}, \quad x_0 \gg 1 \\ m_z(p) &= b \frac{1}{\sqrt{x_0}} e^{-p^2} \end{aligned} \right\}$

For asymptotically small nonuniform in Kosevich JETP '83.

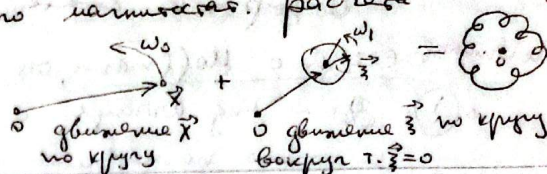
Maximum abscissa $l_x = (A/2\pi m_z^2)^{1/2}$ or $l_x = (A/k_1)^{1/2}$.

Orebugno $\theta(p) = C J_1(x_0 p)$ gives $\theta(p) = C x_0 [1 - \frac{1}{8} \alpha^2 p^2]$ & corr. & Kosevich '83.

→ Ansatz Feldtkeller & Usav's gives better agreement if $p \rightarrow 0$, but better
 if $p/l_x \gg 1$. Rel. $\cos \theta(p) = e^{-2p^2 p^2}$ - rather common choice.



Escaneado con CamScanner



vertices - асимптоты - сумма двух в. пр. движения с расстояни a_0 и a_1

Fourier Spectrum

Passer nag Bound. Conditions paper.

$$18/\bar{x}$$

The parallel alignment case (compares 17/2) & Kristen ran a y meme
gave TM - reason of vortex movement. The antinaturalism (AF)
alignment - inorganic ne cross scheme: & Kristen - open hypothesis link $m_x(w)$

Too a bugue Krusten b. rucu. pravece (?)

→ Vortex papers Zagorodny PRL'04, Caputo-J. Phys. A'03, Kovalov - Eur. Ph. J. '03

19-20

None to Review papers. Запросы Т-140, разномыслие по пунктам, рассмотрен-
- воров к оценке. Запрос Синьо - инициал Letter by Rex.

2) write Cover Letter re overdue Seagate I-29 application & ban appl. "outstanding res."; 3) write commentary box Attachments; 4) second page offer letter

"outstanding res."; 3) проверить корректность всех Attachments; 4) second page
offer letter

~~5) Юношка из I-140~~

for two vertices, two $\vec{m}_1(\vec{p})$, $\vec{m}_2(\vec{p})$ of shifted vertices gave
 all negative side-free charges / two-vertex vax 6 JAP'02 and PRB'02.

+ (vol.1 - core1) + (vol.2 - core1) + (core1 - core2) n/m arbitrary $\bar{X}_1, \bar{X}_2$.
 Но $X_{1,2}/R \ll 1$ and un. presumed. Полагая $\bar{p}_1, \bar{p}_2$ vortex'ов огунаевои.

$$f(z) = \frac{1}{c} [iz + a(1-z^2)]$$

$$w(z, \bar{z}) = \begin{cases} f(z), & |z| \leq 1 \\ \frac{\bar{f(z)}}{1 \overline{f(z)}}, & |z| > 1 \end{cases}$$

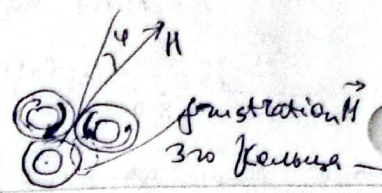
$S = a$ - vertex center shift. $f(z)$ maps $\mathbb{D}(r, y)$ monomorphically onto y Mertens et al.

21/8

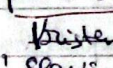
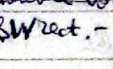
непринятая Form I-140, уведомление и confab GC - petition.
Обзор сек. членов PRB, PRL, JAP, APL.

Hoffmann, Becega & Hoffmann, on irregularities of magnets:

- 1) Ring arrays & 3 rings in unit cell
- монеты не упорядочены и адсорбция $M(H) \propto H^2$
- 2)  Cylindrical double-layer dot & F/AF coupling -
- circular spectroscopy of vortex upon reversal:  vortex reversal
- non-vortex reversal как пин-ин $H(c)$
направленные браз. поле



План чистых падов (go 6/11 Oct. - MMM):

- 1) Внесение paper в conditions to "Science" и patent & Sengate go 29/8
- 2) магнетизм MMM гомоген - neutral no FMR-stripe,  double-vortex-oval,  double-vortex-poster
- 3) MPD, $M(H)$ double-vortex & double-vortex transl. mode
- 4) браз. в Jacksonville (K., A., A.) и переупорядочение 2 JMMM - PRB - papers

22/8

Double-vortex dynamics

Вращательная "translation" mode в double-vortex F/N/R disk.

В рамках Met-JMMM'02 уведомление $(\vec{m} \cdot \vec{n})_r = 0$:

$$f(z) = \frac{1}{c} \left[iz + \frac{1}{2} (a - \bar{a} z^2) \right], \quad c = R/r - \text{dimensionless core radius}$$

$$\frac{E_{ex}}{4\pi M^2 V} = \frac{1}{4\pi} \left(\frac{L_e}{R} \right)^2 \left[2 - \ln \frac{2c}{1 + \sqrt{1 - a^2}} \right], \quad \text{нпм } a=0 \text{ обогреть к неогр.}$$

Рассуждение: $a^2 \rightarrow |a|^2$ згеев, $a = a' + ia''$. В JMMM'02 a'' - браз.

В абз. уведомлении $W_m^{rel}(a) = \frac{1}{2} a^2 \frac{1}{\beta} F_1(\beta) \sim a^2$, $a^2 \rightarrow |a|^2$.

В Gus-JAP'02 $s = i\hat{a}$ - core shift, $\hat{a}$ - браз. и $f(z) = \frac{1}{c} [iz + \hat{a}(1 - z^2)]$ - on paper.
 $\hat{a} = \frac{a}{2}$, $\vec{s} = \frac{\vec{a}}{R} \Rightarrow$ наго обогреть $\vec{s} \propto a = a' + ia''$. Тран. гравитация?

Note: magnetic surface anisotropy of disk нпм $p=R$ (5/11)

Surface energy

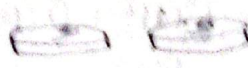


$$\frac{W_m^s}{(\pi R^2 L)} = F(\beta) m_p^2(R), \quad \text{где } F(\beta) = 2\beta \left[\ln \frac{8}{\beta} + \frac{3}{2} \right], \quad \beta = L/R \ll 1 \quad (7/11)$$

$$\text{В Ivanov-APL'02: } \frac{1}{(\pi R^2 L)} W_m^s = \frac{2\pi L^2 R I_m}{\pi R^2 L} m_p^2 \ln \frac{4}{\beta} = 2 I_m \beta \ln \frac{4}{\beta} \cdot m_p^2, \quad \beta \ll 1, \quad I_m = 2, m=0$$

Тогда $\frac{1}{V} W_m^s = 2\beta \left[\ln \frac{8}{\beta} + \frac{3}{2} \right] m_p^2$, не совс. с $\frac{1}{V} W_m^s$ - Ivanov-APL'02 - т.е.
в Ivanov'02 - не берется W_m^s .

→ "pinning" в Ivanov-APL-02 и Ivana JAP-04 $\Delta = d = \frac{RL}{2\pi L^2} \ln \left(\frac{4R}{L} \right)$ не совпадает
с формулой Science $d = \frac{2\pi}{\beta} \left[4 \left(\ln \frac{8}{\beta} - \frac{1}{2} \right) + \left(\frac{L_e}{L} \right)^2 \right]^{-1}$. и замена не берется (!?)

B. phase of gyre  coupled disks, vector in TModes
[Simulacra PRB'03] the gyration or directions, in gyration or polarizations P_i of vortices.

→ Shibata '03: Same but with general non-regular "rigid" vortex model

if $\beta = (\mu_1 \mu_2) \neq 0$ vector gyration coupling of disks

Y Shibata '03: $W_{int} = C_1 C_2 [\eta_x S_{1x} S_{2x} - \eta_y S_{1y} S_{2y}]$, $\vec{S}_i = \frac{\vec{X}_i}{R} - v \cdot \text{shift}$.

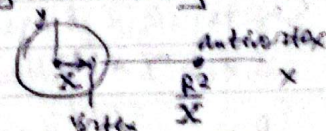
Y none none $W_{int} = \lambda (\vec{X}_1 - \vec{X}_2)^2 = \lambda R^2 [S_1^2 + S_2^2 - 2 \vec{S}_1 \cdot \vec{S}_2] =$
 $= \lambda R^2 [\vec{S}_1^2 + \vec{S}_2^2 - 2 (S_{1x} S_{2x} + S_{1y} S_{2y})]$ two nags up to be sure.

non-regular / broken $W_{int}^{(1)}, W_{int}^{(2)}$.

Before we y Shibata '03 $\eta_x = -\eta_y = \eta$ a bit because it "isotropic" model
 can y none. loc, m. 18/8

One nags: vortex-antivortex [Ansatz $\frac{z-a}{1-\bar{a}z}$ by PRB'01-Gus]

$$\Phi(x, y) = \frac{\pi}{2} + \arctan \frac{y}{x-X} - \arctan \frac{y}{x-\frac{R^2}{X}}$$



→ $\Phi(x, y) = \arctan \frac{y}{X} + \arctan \left(\frac{y-Y}{x-X} \right) + \arctan \left(\frac{y-\bar{Y}}{x-\bar{X}} \right)$ $\left\{ \begin{array}{l} "+" \text{ coord. v.-v. nags } \Rightarrow \vec{\omega} = (\hat{m} \cdot \hat{n}) \hat{k} \\ "-" \text{ coord. v.-a. nags } \Rightarrow \frac{\partial \Phi}{\partial t} = 0 \end{array} \right.$
Korow, m. 6/11 2008
 $X = X \frac{R^2}{X^2 + Y^2}$, $\bar{Y} = Y \frac{R^2}{X^2 + Y^2}$ - coordinates of image vortex or antivortex. "conformal" model

$X = R \cos \omega_0 t$ & $Y = R \sin \omega_0 t$ [Verker Eur. Ph. J. '03], ω_0 - vacuum TM mode.

B. cons. long the $\arctan \frac{Y}{X} = \arctan \left(\frac{\sin(\omega_0 t + \phi)}{\cos(\omega_0 t + \phi)} \right) = \omega_0 t + \phi_0 = \phi_0$ & $\phi_0 = \pm \frac{\pi}{2}$

$w(z, \bar{z}) = \frac{1}{2} \theta \cdot e^{i\Phi(x, y)}$, $\theta = \theta(x, y)$, $m_z = \cos \theta$, $m_x + i m_y = \sin \theta e^{i\Phi}$.

Pr.-Pr. 1.624: $\arctan x + \arctan y = \begin{cases} \arctan \frac{x+y}{1-xy}, & xy < 1 \\ \pi \text{ sign}(x) + \arctan \frac{x+y}{1-xy}, & xy > 1 \end{cases}$

Vortex-Vortex $\Phi(x, y) = \frac{\pi}{2} + \arctan \left(\frac{y-Y}{x-X} \right) + \arctan \left(\frac{y-\bar{Y}}{x-\bar{X}} \right) =$
 $= \phi_0 + \arctan \left\{ \left[\frac{y-Y}{x-X} + \frac{y-\bar{Y}}{x-\bar{X}} \right] \left[1 - \frac{y-Y}{x-X} \cdot \frac{y-\bar{Y}}{x-\bar{X}} \right]^{-1} \right\}$

Hydro Znach $m_x \sim \cos \Phi$ & $m_y \sim \sin \Phi$.

$\cos(\arctan A) = \frac{1}{\sqrt{1+A^2}}$, $\sin(\arctan A) = \frac{A}{\sqrt{1+A^2}}$

Two-vortex $c(\hat{m} \cdot \hat{n}) = 0$ pacifegennne $\vec{m}(\vec{r})$ Zageenna $f(z) = \frac{1}{c} [iz + \frac{1}{2}(a - \bar{a}z^2)]$

$f(z) = 0$ gacs uol-gunnarbi of vortex lines.

$+ \bar{a}z^2 - 2iz - a = 0$, $z_{1,2} = \frac{1}{2\bar{a}} [2i \pm \sqrt{-4 + 4(-a)\bar{a}}] = \frac{1}{2\bar{a}} [2i \pm \sqrt{4|a|^2 - 4}]$

$z_{1,2} = \frac{1}{\bar{a}} [i \pm \sqrt{|a|^2 - 1}] = \frac{a}{|a|^2} [i \pm \sqrt{|a|^2 - 1}] = \frac{ia}{|a|^2} [1 \pm \sqrt{1 - |a|^2}]$

$$|a|^2 \ll 1 \text{ - мал. параметр } \sqrt{1-|a|^2} = 1 - \frac{1}{2}|a|^2 + O(|a|^4), \quad |a| \rightarrow 0$$

$$z_1 = \frac{ia}{|a|^2} \left[1 + 1 - \frac{1}{2}|a|^2 \right] = 2i \frac{a}{|a|^2} - \frac{1}{2}ia \text{ - image vortex, } |z_1| \approx \frac{2}{|a|} \gg 1 \text{ - upsp line goes}$$

$$z_2 = \frac{ia}{|a|^2} \left[1 - 1 + \frac{1}{2}|a|^2 \right] = \frac{1}{2}ia, \quad z_2 = s_x + is_y = \frac{1}{2}|a|e^{i(\varphi_a + \pi/2)}, \quad a = |a|e^{i\varphi_a}$$

$$\vec{s} \perp \vec{a}, \text{ t.e. vortex shift } \vec{s} \perp \vec{a}, \quad s_x = \frac{1}{2}|a| \cos(\varphi_a + \pi/2), \quad s_y = \frac{1}{2}|a| \sin(\varphi_a + \pi/2), \quad \vec{s} = \vec{r}/R.$$

$$f(z) = \frac{1}{2c} [\bar{a}z^2 - 2iz - a] = -\frac{1}{2c} (z - z_1)(z - z_2)$$

$$z_1 z_2 = \frac{ia}{|a|^2} \left[\frac{1 + \sqrt{1-|a|^2}}{a + b} \right] \frac{ia}{|a|^2} \left[\frac{1 - \sqrt{1-|a|^2}}{a - b} \right] = -\frac{a^2}{|a|^4} [1 - (1-|a|^2)] = -\frac{a^2}{|a|^4} |a|^2 = -\frac{a^2}{|a|^2}$$

$$z_1 z_2 = -\frac{a^2}{|a|^2} = \frac{(ia)^2}{|a|^2}, \quad z_1 \bar{z}_2 = \frac{ia(-i\bar{a})}{|a|^4} [1 + \sqrt{1-|a|^2}][1 - \sqrt{1-|a|^2}] = \frac{1}{|a|^2} |a|^2 = 1$$

T.e. yesobue na nenomene volupob vortex u image-vortex (z_2) (z_1) $(z_1 \bar{z}_2 = 1)$

$$z_2 = s_x + is_y, \quad z_1 = \frac{1}{\bar{z}_2} = \frac{1}{s_x - is_y} = \frac{s_x + is_y}{\sqrt{s_x^2 + s_y^2}}, \quad z_2 = (s_x, s_y) \Rightarrow z_1 = \left(\frac{s_x}{\sqrt{s_x^2 + s_y^2}}, \frac{s_y}{\sqrt{s_x^2 + s_y^2}} \right)$$

Здесь $z, \bar{z}$ в eq. R, where $z_1 \bar{z}_2 = R^2$.

$$\rightarrow z_2 = X + iY \Rightarrow z_1 = \frac{R^2}{\bar{z}_2} = \frac{R^2}{\sqrt{X^2 + Y^2}} (X, Y) \text{ - преобразование инверсии.}$$

$$\text{Barren } s_x + is_y = \frac{1}{2}i(a_x + ia_y) \text{ um } (\vec{s} = \frac{1}{2}i\vec{a}), \text{ но верно в мекл. приближении по } |a| \ll 1. \text{ (см. 28/VI 011.)}$$

Concl. near 200-баррелам

$$\text{div } \vec{m} = 2 \text{Re} \frac{\partial m}{\partial z} = 4 \text{Re} \frac{\partial}{\partial z} \left[\frac{w(z)}{1 + |w|^2} \right], \quad m_+ = m_x + im_y$$

$$\text{B. Imml'oz: } \text{div } \vec{m} = -\frac{a}{R} \cos \varphi + O(a^2), \quad a \text{ - параметр. - это преобразование бинагома "core"}$$

$$\text{Pone "core" } |w|=1, \quad w(z) = \frac{f(z)}{|f(z)|} = \sqrt{\frac{f(z)}{\bar{f}(z)}} \quad \frac{\partial w}{\partial z} = \frac{1}{2|f(z)|} \frac{\partial f(z)}{\partial z}$$

$$f(z) = \frac{1}{c} \left[iz + \frac{1}{2}(a - \bar{a}z^2) \right], \quad \frac{\partial f(z)}{\partial z} = \frac{1}{2} [i - \bar{a}z], \quad \text{см. 29/VI 011.}$$

$$\text{div } \vec{m} = 4 \text{Re} \frac{\partial}{\partial z} \left[\frac{w(z)}{1 + |w|^2} \right] = 2 \text{Re} \frac{\partial}{\partial z} w(z) = 2 \text{Re} \frac{1}{2|f(z)|} \frac{\partial f(z)}{\partial z} = \frac{1}{|f(z)|} \text{Re} \frac{\partial f}{\partial z}$$

$$c^2 |f(z)|^2 = [iz + \frac{1}{2}(a - \bar{a}z^2)][-i\bar{z} + \frac{1}{2}(\bar{a} - a\bar{z}^2)] = |z|^2 + O(a) = \rho^2 + O(a)$$

$$\text{div } \vec{m}(\vec{s}) = \frac{1}{|f(z)|} \text{Re} \frac{\partial f}{\partial z} = \frac{1}{|f(z)|} \text{Re} \frac{1}{c} (i - \bar{a}z) = -\frac{1}{p} \text{Re}(\bar{a}z)$$

$$\bar{a}z = (a_x - ia_y)(x + iy) = a_x x + a_y y + i(a_x y - a_y x)$$

$$\rightarrow \text{div } \vec{m}(\vec{s}) = -\frac{1}{p} \text{Re}(\bar{a}z) = -\frac{1}{p} (a_x x + a_y y) = -[a_x \cos \varphi + a_y \sin \varphi]$$

- сдвинутый заряд от shifted vortex в нуле $\vec{s} = \frac{1}{2}i\vec{a}$ $(\vec{s} = \frac{1}{2}i\vec{a})$

$$L \frac{1}{d_s} \frac{1}{2} W_{\text{int}}^{\text{vex-vex}} = \frac{M_s^2}{(R^2)} \int_{V_1} d^3 r \int_{V_2} d^3 r' \frac{\text{div } \vec{m}(r) \text{div } \vec{m}(r')}{|r - r'|}$$

$$\text{div } \vec{m}(r) \text{div } \vec{m}(r') = [a_{x1} \cos \varphi + a_{y1} \sin \varphi] [a_{x2} \cos \varphi' + a_{y2} \sin \varphi'], \quad \vec{s}_j = \frac{1}{2}i\vec{a}_j, \quad j=1,2$$

$$\frac{1}{|r - r'|} = \int dk e^{-k|r - r'|} J_0(k|\vec{s} - \vec{s}'|) = \int dk e^{-k|r - r'|} \sum_j e^{i\mu(\varphi - \varphi')} J_1(k\rho) J_1(k\rho')$$

$$\int_0^{2\pi} d\varphi \cos \varphi e^{i n \varphi} = \pi (\delta_{n,1} + \delta_{n,-1}), \quad \int_0^{2\pi} d\varphi \sin \varphi e^{i n \varphi} = \frac{\pi}{i} [\delta_{n,-1} - \delta_{n,1}]$$

$$W_{int}^{vol-vol} = \frac{M_s^2}{(R^2)} \int d\mathbf{z} \int d\mathbf{p} \int d\varphi \int d\mathbf{z}' \int d\mathbf{p}' \int d\varphi' \int d\mathbf{k} e^{-k|\mathbf{z}-\mathbf{z}'|} \sum_n e^{i n (\varphi - \varphi')} J_n(k\rho) J_n(k\rho')$$

$$\times [a_{x1} \cos \varphi + a_{y1} \sin \varphi] [a_{x2} \cos \varphi' + a_{y2} \sin \varphi'] =$$

$$= \frac{M_s^2}{(R^2)} \int d\mathbf{k} \int d\mathbf{z} d\mathbf{z}' e^{-k|\mathbf{z}-\mathbf{z}'|} \int d\mathbf{p} \int d\mathbf{p}' \sum_n J_n(k\rho) J_n(k\rho') \int d\varphi e^{i n \varphi} [a_{x1} \cos \varphi + a_{y1} \sin \varphi] \int d\varphi' e^{i n \varphi'} [a_{x2} \cos \varphi' + a_{y2} \sin \varphi']$$

$$= \frac{M_s^2}{R^2} (\dots) \sum_n J_n^2(k\rho) [a_{x1} (\delta_{n,1} + \delta_{n,-1}) + a_{y1} \frac{1}{i} (\delta_{n,-1} - \delta_{n,1})] J_n(k\rho') [a_{x2} (\delta_{n,1} + \delta_{n,-1}) + a_{y2} \frac{1}{i} (\delta_{n,-1} - \delta_{n,1})]$$

$$= \frac{\pi M_s^2}{R^2} (\dots) \sum_n J_n(k\rho) J_n(k\rho') [a_{x1} a_{x2} (\delta_{n,1} + \delta_{n,-1}) + \frac{1}{i} (a_{x1} a_{y2} + a_{y1} a_{x2}) (-\delta_{n,1} + \delta_{n,-1}) + a_{y1} a_{y2} (\delta_{n,1} + \delta_{n,-1})]$$

$$W_{int}^{vol-vol} = 2\pi^2 \frac{M_s^2}{R^2} (\vec{a}_1 \cdot \vec{a}_2) \int d\mathbf{k} \int d\mathbf{z} d\mathbf{z}' e^{-k|\mathbf{z}-\mathbf{z}'|} \int d\mathbf{p} \int d\mathbf{p}' J_1(k\rho) J_1(k\rho')$$

$$f_v(k) = \int_{-L/2}^{L/2} dz \int_{-L/2}^{L/2} dz' e^{-k|\mathbf{z}-\mathbf{z}'|} = |z > z'| = \int_{-L/2}^{L/2} dz e^{-kz} \int_{-L/2}^{L/2} dz' e^{kz'} = -\frac{1}{k} e^{-kz} \left|_{-L/2}^{L/2} - \frac{1}{k} e^{kz} \right|_{-L/2}^{-L/2} =$$

$$= -\frac{1}{k^2} [e^{-k(L/2)} - e^{-kL/2}] [e^{-kL/2} - e^{-k(L/2)}] = \frac{1}{k^2} e^{-kd} (1 - e^{-kL})^2$$

$$\rightarrow W_{int}^{vol-vol} = 2\pi^2 \frac{M_s^2}{R^2} (\vec{a}_1 \cdot \vec{a}_2) \int d\mathbf{k} f_v(k) \left[\int d\mathbf{p} \int d\mathbf{p}' J_1(k\rho) \right]^2$$

$$\int d\mathbf{k} f_v(k) \left[\int d\mathbf{p} \int d\mathbf{p}' J_1(k\rho) \right]^2 = \int \frac{d\mathbf{k}}{k^2} e^{-kd} (1 - e^{-kL})^2 \left[\int d\mathbf{p} \int d\mathbf{p}' J_1(k\rho) \right]^2 = |t = kR| =$$

$$= R^5 \int \frac{dt}{t^2} e^{-td} (1 - e^{-\beta t})^2 \left[\int d\mathbf{x} \int d\mathbf{x}' J_1(t\rho) \right]^2$$

$$W_{int}^{vol-vol}(\beta, d) = 2\pi^2 M_s^2 R^3 (\vec{a}_1 \cdot \vec{a}_2) F(\beta, d), \quad F(\beta, d) = \int_0^\infty \frac{dt}{t^2} e^{-td} (1 - e^{-\beta t})^2 I_v^2(t),$$

$$I_v(t) = \int d\mathbf{x} \int d\mathbf{x}' J_1(t\rho)$$

$$\beta = L/R, \quad d \rightarrow d/R$$

$$\vec{a}_1 \cdot \vec{a}_2 = \frac{2}{i} \vec{S}_1 \cdot \frac{2}{i} \vec{S}_2 = -4 (\vec{S}_1 \cdot \vec{S}_2) - \text{normal.}$$

$$S_x + i S_y = \frac{1}{2} i a = \frac{1}{2} e^{i\pi/2} (a_x + i a_y), \quad S_x = -\frac{1}{2} a_y, \quad S_y = \frac{1}{2} a_x$$

$$(\vec{a}_1 \cdot \vec{a}_2) = a_{x1} a_{x2} + a_{y1} a_{y2} = 4 S_{y1} S_{y2} + 4 S_{x1} S_{x2} = 4 (\vec{S}_1 \cdot \vec{S}_2)$$

$$\rightarrow W_{int}^{vol-vol}(\beta, d) = 8\pi^2 M_s^2 R^3 F(\beta, d) (\vec{S}_1 \cdot \vec{S}_2), \quad \vec{S}_{1,2} = \frac{1}{R} \vec{X}_{1,2} - \text{shift of 1,2 vortex from the dot center}$$

$$G = 2\pi q p M_s L / \delta \rightarrow \omega_0 = \frac{x}{G} = |W = \frac{1}{2} x X^2| = \frac{x \delta}{2\pi M_s L}, \quad [x] = \text{erg/cm}^2, [X] = \text{cm}$$

$$W(\vec{X}_1, \vec{X}_2) = W_1(\vec{X}_1^2) + W_2(\vec{X}_2^2) + W_{int}(\vec{X}_1, \vec{X}_2) - \text{noninter. energies of shifted vortices} \quad F/N/F \text{ dot vortex - vortex}$$

$$\rightarrow W(\vec{X}_1, \vec{X}_2) = \frac{1}{2} x_1 X_1^2 + \frac{1}{2} x_2 X_2^2 + 8\pi^2 M_s^2 R F(\beta, d) (\vec{X}_1 \cdot \vec{X}_2) \quad \text{! Interaction } \sim (\vec{X}_1 \cdot \vec{X}_2)$$

$$\text{Corrao JAP'02} \quad x = \frac{(2/3)^2}{J(0)} \pi L M_s^2 = \frac{4}{9} \pi L M_s^2 g [4\pi F_v(\beta) - \frac{1}{2} (R_0/R)^2] = 4\pi L M_s^2 [4\pi F_v(\beta) - \frac{1}{2} (R_0/R)^2]$$

$$F_v(\beta) = \int \frac{dt}{t} f(\beta t) \left[\int d\mathbf{p} \int d\mathbf{p}' J_1(t\rho) \right]^2 - \text{free volume charges.}$$

$$W(\vec{X}_1, \vec{X}_2) = \frac{1}{2} x_1 X_1^2 + \frac{1}{2} x_2 X_2^2 + \mu (\vec{X}_1 \cdot \vec{X}_2), \quad \mu = 8\pi^2 M_s^2 R F(\beta, d) > 0$$

Заданная $W(\vec{X}_1, \vec{X}_2)$ в буге or 17/E:

→ $W(\vec{X}_1, \vec{X}_2) = \frac{1}{2}(x+\mu)X_1^2 + \frac{1}{2}(x+\mu)X_2^2 - \frac{\mu}{2}(\vec{X}_1 - \vec{X}_2)^2$, энергия нормируется
 when force $|\vec{X}_1 - \vec{X}_2|, \mu > 0$.

17/E same phenomenon $W(\vec{X}_1, \vec{X}_2) = \frac{1}{2}xX_1^2 + \frac{1}{2}xX_2^2 + \lambda(\vec{X}_1 - \vec{X}_2)^2$

Заданы ω_0, ω_1 где $\vec{X} = \frac{1}{2}(\vec{X}_1 + \vec{X}_2) = \vec{z} = (\vec{X}_1 - \vec{X}_2)$ - вектор координат,
 то сам $x \rightarrow (x+\mu)$, $\lambda \rightarrow -\frac{1}{2}\mu$.

$\omega_0 = \frac{x}{G} \rightarrow \omega_0 = \frac{(x+\mu)}{G}$ и $\omega_1 = \frac{(x+\mu)}{G} \rightarrow \omega_1 = \frac{(x-\mu)}{G}$, $\mu > 0$

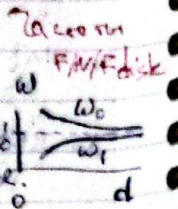
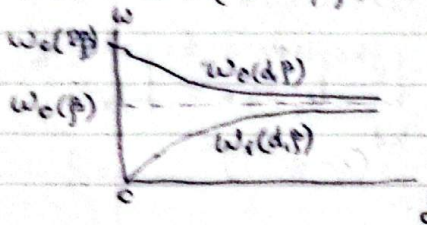


Fig 2-11 $\vec{X}$ - вектор центра $\omega_0 = \frac{(x+\mu)}{G}$, а $\vec{z}$ - вектор $\omega_1 = \frac{(x-\mu)}{G}$

Уменьшение объема volume changes interdicted in coupling.
 Coupling vol.-core и core-core interaction.

Note: $\omega_0(d \rightarrow 0, \beta) = \omega_0(2\beta)$, т.к. $\omega_{ind} = 0$ если ω_{core} given c. radius 2L.



$\omega_0^{ind}(\beta) = \frac{x}{G} = \frac{4\pi L M_S^2 \cdot 4\pi F_V(\beta)}{2\pi M_S L} = 8\pi \beta M_S F_V(\beta)$

$F(\beta, d) = \int_0^d e^{-\beta t} (1 - e^{-\beta t})^2 I_V^2(t) dt$
 $F_V(\beta) = \int_0^\infty f(\beta t) I_V^2(t) dt$, $f(x) = 1 - \frac{1}{x}(1 - e^{-x})$

$x = 16\pi^2 L M_S^2 F_V(\beta) \approx 16\pi^2 M_S^2 L \frac{1}{2}\beta \int_0^\infty dt I_V^2(t)$, $\beta \rightarrow 0$.

$\chi = 8\pi^2 M_S^2 R F(\beta, d) \xrightarrow{d \rightarrow 0} 8\pi^2 M_S^2 R \int_0^\infty dt (1 - e^{-\beta t})^2 I_V^2(t) \xrightarrow{\beta \rightarrow 0} 8\pi^2 M_S^2 R \beta^2 \int_0^\infty dt I_V^2(t)$

тогда можно $\beta \rightarrow 0$ или $\int_0^\infty dt I_V^2(t)$ считать.

Буга $\chi = \chi(d=0)$ when $\beta \ll 1$, (номер 200 rate $\beta \beta$), $\Rightarrow \omega_1(d) \xrightarrow{d \rightarrow 0}$

25/2 Group-meeting.

Tri-layer Base impact in nitrogen clean new PEEM measurements
 (Bepko) of double-layer F/N/F dots of cylindrical and elliptical shape.

Remanent state curves забавно as d_F, d_N и номер dots
 single-vortex, double-vortex, 1, 2, 3-domain in upper layer

также браться к F/N/F or F/N/F PEEM и буга. Dynamics

Remanent state curves забавно as d_F, d_N и номер dots

→ Kristen measurement $M_x(t), M_y(t)$ of F/N/F dots measuring
 in narrow low-frequency $\omega_1 \leq 0.1$ GHz Fmode gives 40/1/40 nm dots
 c R = 250 nm, $M_S = 380$ G.

Nonlinear, nonlocal dynamics of P/N/F dots core curves

Забавно as dot chirality. Use measure of $(\vec{X}_1 + \vec{X}_2) \cdot (\vec{X}_1 - \vec{X}_2)$

негативно as chirality. No chirality $C_1 = C_2$ (C-chirality) неасп.

неасп. in measure $|M_x(\omega)|^2$ и $|M_y(\omega)|^2$ R to 350-500 nm.

→ Summary by V. Rase "Growth of TMR media structure Co/AlO₃/CoAl"

Disc 40/40 nm disk		$C_1 C_2 = -1$	$C_1 C_2 = 1$	$P_1 P_2 = +1$	Annular disk TM-modes in F/N/F disk
~ 0.1 GHz	ω_1	$(X_1 + X_2)$	$(X_1 + X_2)$		
~ 1.0 GHz	ω_0	$(X_1 - X_2)$	$(X_1 + X_2)$		D Vortex dynam.

$$\frac{\kappa}{G} = 8\pi f M_s F_v(p), \quad \frac{M}{G} = \frac{8\pi^2 M_s^2 R F(p,d)}{2\pi M_s L \frac{1}{p}} = 4\pi f M_s \frac{1}{p} F(p,d)$$

→ h/w of p and d.

$$\omega_0(p,d) = \frac{1}{G} [\alpha(p) + \mu(p,d)] \quad \omega_1(p,d) = \frac{1}{G} [\alpha(p) - \mu(p,d)]$$

$$\omega_0(p,d) = 4\pi f M_s \left[2F_v(p) + \frac{1}{p} F(p,d) \right], \quad \omega_1(p,d) = 4\pi f M_s \left[2F_v(p) - \frac{1}{p} F(p,d) \right]$$

h/w of p and d.

$$2F_v(p) - \frac{1}{p} F(p,d=0) = 2 \int_0^1 dt f(t) I_v^2(t) - \frac{1}{p} \int_0^1 dt (1-e^{-pt})^2 I_v^2(t) =$$

$$= \int_0^1 dt I_v^2(t) \left[2f(pt) - \frac{1}{pt} (1-e^{-pt})^2 \right] = \int_0^1 dt I_v^2(t) \left[2 - \frac{2}{pt} (1-e^{-pt}) - \frac{1}{pt} (1+2e^{-pt}+e^{-2pt}) \right] =$$

$$= \int_0^1 dt I_v^2(t) \left[2 - \frac{3}{pt} + \frac{1}{pt} e^{-pt} (1+e^{-pt}) \right] \neq 0 \text{ even } \beta \ll 1.$$

$$2F_v(p) + \frac{1}{p} F(p,d=0) = \int_0^1 dt I_v^2(t) \left[2 - \frac{1}{pt} + \frac{1}{pt} e^{-2pt} \right] = 2 \int_0^1 dt I_v^2(t) \left[1 - \frac{1}{2pt} + \frac{1}{2pt} e^{-2pt} \right] =$$

$$= 2 \int_0^1 dt I_v^2(t) f(2pt), \quad \text{i.e. } \omega_0(p,d=0) = 8\pi f M_s \int_0^1 dt I_v^2(t) f(2pt) = 8\pi f M_s F_v(2p) = \omega_0^{\text{isol}}(2p),$$

($2F_v(2p)$)

уточнее: для малых β , т.е. $\omega(p,d=0)$ совпадает с $\omega_0^{\text{isol}}(2p)$ где существуют чиральности.

Хорошо $\omega_0(p,d=0) = \omega_0^{\text{isol}}(2p)$, но $\alpha(p) \neq \mu(p,d)$ и $\beta \ll 1$.

$\alpha(p) = \mu(p,d=0)$ с точностью до β^2 . Т.е. $\omega_1(p,d=0) = 0 + O(\beta^3)$ ($O(\beta^2)$ for me, 22/01)

Calculations of micromagnetic simulations by Kristen (C1C2=1):

40/40 nm, R=250 nm, $M_s=800$ G

Maier - $\omega_1 = 0.174$ GHz, $\omega_0 = 1.285$ GHz

Kristen - $\omega_1 \approx 0.35$ GHz, $\omega_0 \approx 1.1$ GHz.

ω_0 me xofenno cocr. c Kristen ω_0 (C1C2=-1) ≈ 1.20 GHz, no

ω_1 - me $\gg \omega_1$ - Kristen (C1C2=-1) $\approx 0.05-0.07$ GHz.

→ Barboz: ω_0 anant. eq. Gouy-Lorentz c xofenno xofenno que C1C2=±1, (uog) a bar c ω_1 - uog - cocr. cocr., opurue or simulations beuue.

Магнетизм барьеров намагниченности  X_2  X_1 que me c C1C2=±1?
 - 4200 Wex yueuueuue de u cocr. cocr. ω_0, ω_1 .

→ Hago uoxfeneu core-trajectories $X_i(t), Y_i(t)$ que cocr. cocr.

2. Vortex dynamics u Vortex-barrier

Becege c Kristen double v. dynamics. Ei yueuueuue R, Ax u yueuue core-trajectories. (26/1)
Becege c Ding o v. barrier PPL. He pemuue ryga cocr. Поверхности $\Delta E_s(R) \propto \tau(R)$.

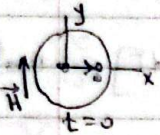
Обобщенное уравнение (17/2) of vortex movement in F/N/F disk

$$\vec{X}(t) = \frac{1}{2} [\vec{X}_1(t) + \vec{X}_2(t)] \quad \vec{Z}(t) = \vec{X}_1(t) - \vec{X}_2(t)$$

$$\vec{X}(t) = \vec{A} \cos \omega_0 t + \vec{B} \sin \omega_0 t \quad \vec{Z}(t) = \vec{a} \cos \omega_1 t + \vec{b} \sin \omega_1 t$$

$$\vec{G} \times \dot{\vec{X}} = \kappa \vec{X}, \quad \dot{\vec{X}} = \vec{\omega} \times \vec{X}, \quad \vec{G} \times \vec{\omega} \times \vec{X} = \vec{\omega} (\vec{G} \cdot \vec{X}) - \vec{X} (\vec{G} \cdot \vec{\omega}) = -\vec{X} G_2 \omega_2 = \kappa \vec{X}$$

$$\omega_0 = -\omega_2 = \kappa / G_2$$

① $C_1 C_2 = +1$  $\vec{X}(0) = \vec{A} = A_x \vec{i}$, $\omega_0 = \frac{2\pi}{T_0}$, $\vec{X}(t = T_0/4) = \vec{A} \cos \frac{2\pi T_0}{T_0} + \vec{B} \sin \frac{\pi}{2} = \vec{B} = B_y \vec{j}$ $\vec{X}(t) = A_x \vec{i} \cos \omega_0 t + B_y \vec{j} \sin \omega_0 t$ $(\vec{A} \cdot \vec{B}) = 0$

$$0 = \vec{Z}(0) = \vec{a} = 0, \quad \vec{Z}(t = T_0/4) = \vec{b} \quad \vec{Z}(t) = \vec{b} \sin \omega_1 t$$

$$\vec{X}_1(t) = \vec{X}(t) + \frac{1}{2} \vec{Z}(t) = A_x \vec{i} \cos \omega_0 t + B_y \vec{j} \sin \omega_0 t + \frac{1}{2} \vec{b} \sin \omega_1 t$$

$$\vec{X}_2(t) = \vec{X}(t) - \frac{1}{2} \vec{Z}(t) = A_x \vec{i} \cos \omega_0 t + B_y \vec{j} \sin \omega_0 t - \frac{1}{2} \vec{b} \sin \omega_1 t$$

Смещено $3\omega_0 / \sqrt{11}$ от $\vec{H}$. где $(\vec{m} \cdot \vec{H})_z = 0$ Vortex model $\langle \vec{m} \rangle_v = \frac{1}{3} \vec{a}$, $\vec{a} = a_x + i a_y = a' + i a''$

is from $f(z) = \frac{1}{2} [iz + \frac{1}{2}(a - \bar{a}z^2)] \dots$ и $\vec{S} = \frac{\vec{X}}{R} = \frac{1}{2} i \vec{a}$ направление на $\pi/2$

$\langle \vec{m} \rangle \parallel \vec{H}$. $H_y > 0$ case. $S_x < 0$ when $C = +1$
 $S_x > 0$ when $C = -1$

$$\langle \vec{m} \rangle \cdot \vec{H} = \frac{2}{3} |\vec{S}|$$

$$\langle \vec{m} \rangle = \hat{z} \times \vec{S} \left(\frac{2}{3} \right) (-1) \text{sign}(C)$$

C - chirality, direction of $\vec{m}$ rotation of vortex

$$\hat{z} \times \vec{S} = \vec{j} S_x - \vec{i} S_y, \quad \langle m_y \rangle \sim S_x, \quad \langle m_x \rangle = \frac{2}{3} S_x (-1) \text{sign}(C)$$

Disjunct: $\langle \vec{m}_1 \rangle_v = \hat{z} \times \frac{\vec{X}_1}{R} \left(-\frac{2}{3} \right) \text{sign}(C_1)$, $\langle \vec{m}_2 \rangle_v = \hat{z} \times \frac{\vec{X}_2}{R} \left(-\frac{2}{3} \right) \text{sign}(C_2)$

$$\langle \vec{m} \rangle_v = \langle \vec{m}_1 \rangle_v + \langle \vec{m}_2 \rangle_v = \left(-\frac{2}{3} \right) \frac{1}{R} \hat{z} \times [\vec{X}_1 \text{sign}(C_1) + \vec{X}_2 \text{sign}(C_2)]$$

• Если $C_1 C_2 = +1$ - одинаковые sign(C), то $\langle \vec{m} \rangle_v = \left(-\frac{2}{3} \right) \text{sign}(C) \frac{1}{R} \hat{z} \times [\vec{X}_1 + \vec{X}_2] \sim [\vec{X}_1 + \vec{X}_2]$
 - направление $\langle \vec{m} \rangle(t)$ совпадает с направлением $(\vec{X}_1 + \vec{X}_2)$ и ω_0 .

• Если $C_1 C_2 = -1$ $\langle \vec{m} \rangle_v = \left(-\frac{2}{3} \right) \text{sign}(C_1) \frac{1}{R} \hat{z} \times [\vec{X}_1 - \vec{X}_2] \sim [\vec{X}_1 - \vec{X}_2]$
 - направление $\langle \vec{m} \rangle_v$ совпадает с направлением $(\vec{X}_1 - \vec{X}_2)$ и ω_1 .

Важно, что $\omega_1 > \omega_0$ для $C_1 C_2 = -1$

? Нам же важно high- ω_0 микс где $\langle \vec{m} \rangle_v$ и ω_0 и ω_1 - микс где $\langle \vec{m} \rangle_v$ и ω_1 и ω_0 - микс где $C_1 C_2 = +1$, см. ⑥ or 25/2.

Но где $C_1 C_2 = -1$ на FFT видно что $\langle \vec{m} \rangle_v$ goes high- ω_0 микс, а не low- ω_1 микс, как ожидалось. Потому $\langle \vec{m} \rangle_v \sim (\vec{X}_1 - \vec{X}_2) \subset \omega_1$, $\omega_1 > \omega_0$.

$$\vec{Z}(t) = \vec{b} \sin \omega_1 t, \quad \text{на } 17/2 \text{ берем } \begin{cases} z_x = -z_0 \cos(\omega_1 t + \varphi) \\ z_y = z_0 \sin(\omega_1 t + \varphi) \end{cases}, \quad z_x^2 + z_y^2 = z_0^2$$

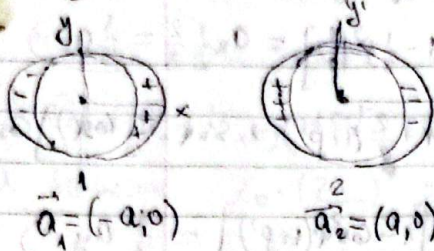
$$\vec{Z} = -i \vec{z}_0 \cos(\omega_1 t + \varphi) + \vec{j} z_0 \sin(\omega_1 t + \varphi) = -i \vec{z}_0 [\cos \omega_1 t \cos \varphi - \sin \omega_1 t \sin \varphi] + \vec{j} z_0 [\sin \omega_1 t \cos \varphi + \cos \omega_1 t \sin \varphi]$$

$$= z_0 [-i \cos \varphi + \vec{j} \sin \varphi] \cos \omega_1 t + z_0 [\vec{i} \sin \varphi + \vec{j} \cos \varphi] \sin \omega_1 t \Rightarrow \vec{Z}(0) = z_0 [-i \cos \varphi + \vec{j} \sin \varphi] \neq 0$$

Будем считать $\vec{Z}(0) = 0$, тогда $\vec{Z}(0) \neq \vec{Z}(0) = \vec{a}$ и $(\vec{a} \cdot \vec{b}) = 0$ где ω_1 - микс.

$\vec{r}(t) = \vec{a} \cos \omega t + \vec{b} \sin \omega t$, где $\vec{a} = z_0 [-i \cos \varphi + j \sin \varphi]$, $\vec{b} = z_0 [i \sin \varphi + j \cos \varphi]$, φ - угловая координата

$\begin{cases} \vec{X}_1(t) = A i \cos \omega t + B j \sin \omega t + \frac{1}{2} \vec{a} \cos \omega t + \frac{1}{2} \vec{b} \sin \omega t \\ \vec{X}_2(t) = A i \cos \omega t + B j \sin \omega t - \frac{1}{2} \vec{a} \cos \omega t - \frac{1}{2} \vec{b} \sin \omega t \end{cases}$ - координаты вращающихся в 1, 2 направлениях от центра dot.



$W_{int} = |M|(\vec{a}_1, \vec{a}_2) = -|M|a^2$ $C_1 C_2 = -1$

$W_{int} = -\frac{M}{2} |\vec{X}_1 - \vec{X}_2|^2$ - энергия взаимодействия вращающихся в 1, 2 направлениях от центра dot.

$\vec{S} = (S_x, S_y) = \frac{1}{2} i (a' + i a'') = \frac{1}{2} (-a_y, a_x)$. $\vec{a} = 2 \hat{z} \times \vec{S} = 2 \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ a_x S_y & 0 & (-1) \end{vmatrix} = [2j S_x - 2i S_y] \quad (-1)$
 $a_x = +2S_y, -a_y = 2S_x \Rightarrow \vec{a} = (-2) \hat{z} \times \vec{S}$ (27/5)

Берем, что true "core" $\langle \vec{m} \rangle = \frac{1}{3} \vec{a}$? и где чиральность C ?

$m_x + i m_y = w(z, \bar{z}) = \frac{f(z)}{|f(z)|}$, $f(z) = \frac{i}{c} z + \frac{1}{2c} (a - \bar{a} z^2)$, $a = a_x + i a_y$

$\bar{a} z^2 = (a_x - i a_y)(x + i y)^2 = (a_x - i a_y)(x^2 - y^2 + 2i x y) = a_x(x^2 - y^2) + 2a_y x y - i a_y(x^2 - y^2) + 2i a_x x y$

$m_x = \text{Re} w(z, \bar{z}) = \frac{1}{|f|} \text{Re} f(z) = \frac{1}{c} \text{Re} \left[i z + \frac{a}{2} - \frac{\bar{a}}{2} z^2 \right] = \frac{1}{c} \left[-y + \frac{1}{2} a_x - \frac{1}{2} \{ a_x(x^2 - y^2) + 2a_y x y \} \right] \frac{1}{|f|}$

$|f|^2 = f f^* = \frac{1}{c^2} \left[i z + \frac{1}{2} (a - \bar{a} z^2) \right] \left[-i \bar{z} + \frac{1}{2} (\bar{a} - a \bar{z}^2) \right] = \frac{1}{c^2} \left[|z|^2 + \frac{1}{2} i z \bar{a} - \frac{i}{2} \bar{z} a - \frac{1}{2} a \bar{a} \bar{z}^2 + \frac{1}{2} \bar{a} a z^2 + \frac{1}{4} (a - \bar{a} z^2)(\bar{a} - a \bar{z}^2) \right]$

$z \bar{a} - a \bar{z} = (x + i y)(a_x - i a_y) - (a_x + i a_y)(x - i y) = a_x x - i a_y x + i a_x y + a_y y - a_x x + i a_x y - i a_y x - a_y y = 2i(a_x y - a_y x)$
 $|a - \bar{a} z^2|^2 = O(a^2)$

$c^2 |f|^2 = \rho^2 - (a_x y - a_y x) - \rho^2 (a_x y - a_y x) + O(a^2) = \rho^2 - (1 + \rho^2)(a_x y - a_y x) + O(a^2)$ (28)

$\begin{cases} m_x(\vec{\rho}) = \frac{1}{c|f|} \left[-y + \frac{1}{2} a_x - \frac{1}{2} \{ a_x(x^2 - y^2) + 2a_y x y \} \right] \\ m_y(\vec{\rho}) = \frac{1}{|f|} \left[\text{Im} f(z) \right] = \frac{1}{c|f|} \left[x + \frac{1}{2} a_y - \frac{1}{2} \{ 2a_x x y - a_y(x^2 - y^2) \} \right] \end{cases}$

Т.к. $\langle \vec{m} \rangle$ и $\text{div} \vec{m}$ не сопадают, значит $\sim a^0$ не однородно ($a=0$ - centered vortex), т.е. гомогенно $\langle \vec{m} \rangle$ и $\text{div} \vec{m}$ зависят от a .

$m_x(\vec{\rho}) \Rightarrow c|f| = \sqrt{\rho^2 - (1 + \rho^2)(a_x y - a_y x)} = \rho \sqrt{1 + \frac{1}{\rho^2}(1 + \rho^2)(a_x \sin \varphi - a_y \cos \varphi)} = \rho \left[1 - \frac{1}{\rho} (1 + \rho^2)(a_x \sin \varphi - a_y \cos \varphi) \right]$

$m_x(\vec{\rho}) = \frac{1}{c|f|_0} [g_0 a] + \frac{c^{-1}}{|f|_0} [\dots]_{a=0} = \frac{1}{\rho} \left[-y + \frac{1}{2} a_x - \frac{1}{2} \rho^2 \{ a_x \cos 2\varphi + a_y \sin 2\varphi \} \right] + \frac{1}{\rho} \left[1 + \frac{2}{\rho} (1 + \rho^2)(a_x \sin \varphi - a_y \cos \varphi) \right] (-1) \rho \sin \varphi$ - канон. true "core" с точ. го. a .

Получим "core" $m_x(\vec{\rho}) = \text{Re} w = \text{Re} f(z) = \frac{1}{c} \left[-y + \frac{1}{2} a_x - \frac{1}{2} \rho^2 \{ a_x \cos 2\varphi + a_y \sin 2\varphi \} \right]$

$\langle m_x \rangle = \frac{1}{\pi} \int_0^{2\pi} \int_0^\infty \rho d\rho d\varphi m_x(\vec{\rho}) = \frac{1}{\pi} \int_0^{2\pi} \int_0^\infty \rho d\rho [\dots] + [\text{true "core"}] = [\text{интеграл по всему полю}] + [\text{soliton "core"}] - [\text{meron "core"}]$
 Это и есть $|a| \gg c$. Как 28/март, что-то не так с формулой.

Image vortex $z_1 = \frac{ia}{|a|^2} [c + 1 - \frac{1}{2}|a|^2] = \frac{ia}{|a|^2} (c+1) - \frac{1}{2}ia = \left\{ \frac{2ia}{|a|^2}, c=+1 \right\}$
 $\left\{ -\frac{1}{2}ia, c=-1 \right\}$

$z_2 = \frac{ia}{|a|^2} [c - 1 + \frac{1}{2}|a|^2] = \frac{ia}{|a|^2} (c-1) + \frac{1}{2}ia = \left\{ \frac{1}{2}ia, c=+1 \right\}$
 $\left\{ -\frac{2ia}{|a|^2}, c=-1 \right\}$

Image u op. vortex uenclousie uocanu u/u ceno zona C.

Ocnobon' vortex centered at $z_0 = \frac{1}{2}iaC$, a image vortex at $z_{im} = \frac{2ia}{|a|^2}C$

u bounen. $z_0 \cdot (z_{im})^* = \frac{1}{2}iaC \cdot \frac{2(-i)a}{|a|^2}C = 1$, $z_0 \bar{z}_{im} = 1$ van 22/8.

Pakut Orfegakuyoban u onfaban Wen Lin Soagete. patent application on Composite F/Metamag. media.

28/8

Co-dots

beega c Ding o new exp. data on Co-nanodots: Sm nanofiber images of Co-dots in SD, 2-, 3-domain, C, S and vortex state. Zano uocno ormanu goin uerexogor b C-state, 3-domain state, a no vortex state - u3-ya gepponob.

uocpau u uocanu to Ding vortex barrier calcs. $\Delta E_{SD \rightarrow V}(K, L) = \beta(K, L, T) = \frac{\Delta E_V}{kT}$.
 File "Vortex-Barrier1.med"

→ uocanu b uerac u3 ANL:

- 1) Met-Gus - Leaf PRB'04 - published
- 2) Gus - RFA/ARh-PRB'04 - published
- 3) Gus-Nov - VortMPD-JAP'04 - published (Novosad)
- 4) Gus-Gus - SW-mathieu - J.Ph.C.Mat.'04 - published
- 5) Ding-Gus - Co-vort. barrier - PRL (Bader)
- 6) Gus-Scholtz-Nov - Vortex rad. modes - PRB (Novosad)
- 7) Bajer-Gus - SW rad. dots - PRB
- 8) Gus-Nov - FMR stripe - JAP (Novosad)
- 9) Hoffmann-Gus - Bro y(w) - JAP (Bader, Hoffmann)
- 10) Chubykalo-Gus - FFA/ARh-numer. - JAP
- 11) Buchanan-Gus - Dobb-vort. - JAP (Novosad)
- 12) Mrazek-Gus - FePt-K₁ - PRL, resubm. to EuroPh. Lett.
- 13) Gus-Slavin - B. condit. - Science, nanocena, no ne uocanu
- 14) Gus-Buch - Dvort Dynam - APL

T.2. 3e Feb. - Oct. 2004 uocanu b uerac 11 papers, u3 sum 4 published yore, b M-x Novosad co-author, b 6-papers eca xoc ogun co-author u3 Mag. Films Group. MSD.

MMM-gonagor / FMR stripe

Deranne slides gne MMM-FMR stripe gonaga, uocanu uerac file to Kris. u Val. Gbenwuo uocno exp. slides. Dano gne FMR-sample #1 case u Reference-spectrum u/u m FMR-spectrum (in-plane). Cu. (S#1) fitting or 23/11. Dno (S#1) uer af. uenaga, eca uocno FMR-ueag. Reference (as-film) uenaga

29/8

Double vortex dynamics

Double-vortex dynamics

$$\chi(z) = \frac{1}{2} [iz\bar{c} + \frac{1}{2}(a - a\bar{a}^2)] \rightarrow \text{div} \vec{m} = \frac{1}{|f(z)|} \text{Re} \frac{\partial f}{\partial z} = \frac{1}{|f|} \text{Re} \frac{1}{2} [c\bar{c} - \bar{a}a] =$$

$$= -\frac{1}{c|f|} \text{Re}(a\bar{z}) = -\frac{1}{c} (a_x \cos \theta + \sin \theta a_y), \text{ cm}^{-2} / \text{K}. \quad \text{div} \vec{m} = -\frac{1}{2} (a_x \cos \theta + a_y \sin \theta) + O(a^2)$$

Since $\text{div} \vec{m}$ suggests a magnetic monopole, we can use the following to find $\vec{m}$.

Vortex centered at $z_0 = \frac{1}{2} a\bar{c}$. $\frac{1}{2}(\chi + i\psi) = \frac{1}{2} c i (a_x + i a_y) = \frac{1}{2} c (-a_y + i a_x)$

$$s_x + i s_y = \frac{1}{2} c (-a_y + i a_x), \quad s_x = -\frac{1}{2} c a_y, \quad s_y = \frac{1}{2} c a_x.$$

$$(\vec{a}_1 \cdot \vec{a}_2) = a_{1x} a_{2x} + a_{1y} a_{2y} = (2c_1 s_{y1})(2c_2 s_{y2}) + (-2c_1 s_{x1})(-2c_2 s_{x2}) =$$

$$= 4c_1 c_2 (s_{1x} s_{2x} + s_{1y} s_{2y}) = 4c_1 c_2 (\vec{s}_1 \cdot \vec{s}_2), \quad \text{independent of } c_1, c_2 \text{ and } \mu \text{ (see 23/12)}$$

$$W(\vec{x}_1, \vec{x}_2) = \frac{1}{2} x_1^2 + \frac{1}{2} x_2^2 + 8\pi^2 M_s^2 R F(\beta, d) (\vec{x}_1 \cdot \vec{x}_2) c_1 c_2, \quad \text{r.p.}$$

μ or 23/12 has a minimum at $\mu = 8\pi^2 M_s^2 R F(\beta, d) c_1 c_2$. - know you know

largest $\mu > 0$ is the value of $c_1 c_2$.

→ В частности, при $c_1 c_2 = -1$, $\mu < 0$ и $\omega_0 = \frac{1}{G} (x_1 + \mu) < \omega_1 = \frac{1}{G} (x_1 - \mu)$, $\omega_0 < \omega_1$

т.е. ω_0 ($\vec{x}_1 + \vec{x}_2$) - более малая, а частота ω_1 ($\vec{x}_1 - \vec{x}_2$) - большая, $\langle \vec{m} \rangle \sim (\vec{x}_1 - \vec{x}_2)$

коррелирует с $\omega_1 > \omega_0$ частотой, что и было показано в симуляциях.

$$\langle \vec{m} \rangle_j = -\frac{2}{3} \hat{z} \times \vec{s}_j \cdot \vec{c}_j, \quad \langle \vec{m} \rangle_0 = \frac{1}{2} \langle \vec{m} \rangle_1 + \frac{1}{2} \langle \vec{m} \rangle_2 \sim (c_1 \vec{x}_1 + c_2 \vec{x}_2) \text{ коррелирует с } c_1 c_2$$

Application to Nebraska (Teymbel).

3/11

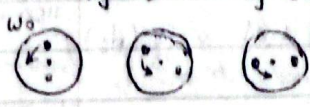
Discussion of double-vortex & Kristen. Wisconsin, Bare.

Kristen's numerical simulations of double-vortex dynamics:

1) configuration of v. cores & axes $x_1(t), y_1(t)$ и $x_2(t), y_2(t)$

$x_1(t) \approx x_2(t), y_1(t) \approx y_2(t)$ сближаются по фазе на 180° - разная фаза

$x_j(t) \approx y_j(t)$ сближаются по фазе на 90° → circular motion

ω_0  - zero ω_0 - high freq. mode runs $(\vec{x}_1 - \vec{x}_2)$ где $c_1 c_2 = -1$
и частота $\omega_1 = 1/2 \omega_0 \approx 0.5 \text{ GHz}$

2) To zero $(\vec{x}_1 - \vec{x}_2)$ сближаются по фазе на 180° - high-freq. mode не происходит

$x_1(t), x_2(t)$ - сближаются по фазе на 180° , а $(\vec{x}_1 - \vec{x}_2)(t)$ - сближаются по фазе на 90° - circular motion

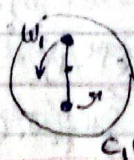
3) $(x_1 + x_2) \approx (y_1 + y_2)(t)$ сближаются по фазе на 90° - circular motion

и частота ω_1 - $\omega_1 \approx 0.05 \text{ GHz}$ (!)

То есть где $c_1 c_2 = -1$ $\omega_0 \approx \omega_1$ происходит смешивание,

частота ω_0 - $(\vec{x}_1 + \vec{x}_2)$ меньше, а частота ω_1 - $(\vec{x}_1 - \vec{x}_2)$ - больше, $\mu < 0, \omega_0 < \omega_1$

One $c_1 c_2 = +1, \mu > 0$ и $\omega_0 > \omega_1$



(C) $\vec{r} = \vec{r}_1 - \vec{r}_2$, $\vec{r}(t) = \vec{a} \cos \omega t + \vec{b} \sin \omega t$, $\vec{r}(0) = \vec{a} = 0$ - find trajectory
 $\vec{r}_1 = \vec{r} + \vec{r}_2$ $\vec{r}_2(1/4) = -1$ $\vec{r}_2(1/4) = 0$ $\vec{r}_1(0) = -1 = a_y$
 $\omega_1 = 2\pi/\tau_1$

$\vec{r}(t) = -i \sin \omega t \vec{i} - j \cos \omega t \vec{j}$

$\vec{r} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$ $X(0) = 0, Y(0) = -1 \Rightarrow \vec{r}(t) = -i \sin \omega t \vec{i} - j \cos \omega t \vec{j}$
 $X(1/4) = -1, Y(1/4) = 0$

Annular disk structure $\vec{r}$ core ≈ 76 nm, a disk $\vec{r}$ core ≈ 6 nm
 Give $40/1/40$ nm Py/Ni/Py dots $c, 2R = 1 \mu m$ (Kristen).

$\vec{r}(t) = 76 (i \sin \omega t \vec{i} + j \cos \omega t \vec{j})$ $\vec{r} = 6 (i \sin \omega t \vec{i} + j \cos \omega t \vec{j})$

$\omega/2\pi = 0.51$ GHz $\omega_0 \approx 0.025$ GHz $\frac{\omega}{2\pi} = \frac{1}{T} = f$
 B dot: $\vec{r}_1 = \vec{r} + \frac{1}{2}\vec{r} = 6(i \sin \omega t \vec{i} + j \cos \omega t \vec{j}) + 38(i \sin \omega t \vec{i} + j \cos \omega t \vec{j})$
 B dot: $\vec{r}_2 = \vec{r} - \frac{1}{2}\vec{r} = 6(i \sin \omega t \vec{i} + j \cos \omega t \vec{j}) - 38(i \sin \omega t \vec{i} + j \cos \omega t \vec{j})$
 (Core position)

Core size $\tau_1 = \frac{2\pi}{\omega_1} = \frac{1}{0.51} \text{ ns} \approx 2 \text{ ns}$ $\tau_2 = \frac{2\pi}{\omega_2} = \frac{1}{0.025} \text{ ns} \approx 40 \text{ ns}$, $T_1 \approx 2 \text{ ns}$
 $\vec{r}_1 = (x_1, y_1)$ $\vec{r}_2 = (x_2, y_2)$

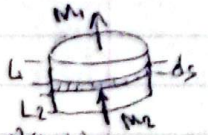
Observation of double vortex annular disk $\vec{r}$ under double frequency ω_1 .
 Two core loops in $\vec{r}$ $C_1 C_2 = +1$ i.e. core trajectories of same direction $\frac{1}{2}(x_1 + x_2)$ would be ω_0 - vortex, $\omega_0 > \omega_1$ $C_1 C_2 = -1$ u $C_1 C_2 = +1$,
 u/vu equilibrium phase $(C_1 C_2)$ under core annular disk cores na trajectories.

Heaven, then trajectory $\vec{r}(0)$? Any reason in one?

Microscopy MCD - karyotype of the vortex trajectories in double layer dot - oxygen up c Kristen.

Cur. MM - d.vortex slides prepared by Kristen. Bonjour $R_p(L)$ under
 gas d.vortex dots? $\mu_0 \mu_m \beta_c$ or $\mu_m \mu_m \beta_c$ or 1.81 u $\beta_c(ds)$.

Web-site Duncanson, comp manager Robert Hoff:
 Za schreibende site MSD oberer Dave "lehman@aul.ger" -
 Hamcaro eng. Texas up time text u m pdf, a karyotype *jpg



Double-vortex MPD 11 ± 1 SD nqpe-rog
 $V_{mag} = R^2(L_1 L_2) \pi$ - Normalized frequency of magnetostatic. Coupling of two cylinders in 1 state - SD. Two random cylinder geometry.
 Give blue vortex cores or $22/\sqrt{1}$, $m_2(p) = \frac{b^2 - p^2}{b^2 + p^2}$. B eqs or $22/\sqrt{1}$
 hago up the normalized $m_2(p) = 1$. Torque $m_{1,2}(p) = p_{1,2}$, $\beta_j = \pm 1$.

$I_n(k) = M_{sn} \int_0^R dp p m_{2n}(p) J_0(kp) = M_{sn} p_n \int_0^R dp p J_0(kp) = M_{sn} p_n \frac{R}{k} J_1(kR)$

$W_m^{int} = -\frac{g}{2} (2\pi)^2 M_s^2 p_1 p_2 \int dk \frac{R^2}{k^2} J_1^2(kR) e^{-kd} (e^{kt} - 1)^2 = -4\pi^2 M_s^2 p_1 p_2 R^3 \int dt J_1^2(t) e^{-kd} (1 - e^{kt})^2$
 $W_m^{int}(\beta, d) = \frac{M_s^2}{V_{mag}} W_m^{int} = -2\pi (M_s^2) p_1 p_2 \frac{1}{\beta} \int_0^\infty dt \frac{1}{t^2} J_1^2(t) e^{-td} (1 - e^{kt})^2$ - $\mu_0 \mu_m \beta_c$ u $\mu_m \mu_m \beta_c$ dots

$$W_m(\perp SD) = W_{m1} + W_{m2} + W_m^{int}, \quad w_m = \frac{1}{V_m} W_m = \frac{2W_{m1}}{V_m} + w_m^{int} \text{ одинаковые dots.}$$

$$w_m^\perp(\beta, d) = w_m^0(\beta) + w_m^{int}(\beta, d) - \text{energy of d. dot } \perp \text{ SD state.}$$

(14/12) → Energy double-dot in antiparallel in-plane SD state (↑↓) etc. **F/N/F**

$$w_m'' = w_{m0}'' + w_m^{int}(\beta, d) = 2\pi [F_1(\beta) - F_{int}(\beta, d)], \quad w_m = \frac{1}{4\pi^2 V_m} W_m.$$

$$\text{Згач } F_{int}(\beta, d) = \frac{1}{2\beta} \int dt \frac{J^2(t)}{t^2} e^{-td} (1 - e^{-\beta t})^2.$$

Energy bias analogical of  state. $w_m^{int}(\beta, d) = -2\pi \frac{p_1 p_2}{\beta} \int dt \frac{J^2(t)}{t} e^{-td} (1 - e^{-\beta t})^2$

$$\text{или } w_m^{int}(\beta, d) = -4\pi p_1 p_2 F_{int}(\beta, d)$$

Energy $\perp$ SD u $\parallel$ d. SD states (абсолютно п/м $\beta_c(d)$ у) $w_m''(\beta, d) = w_m^\perp(\beta, d)$
 $p_1 p_2 = 1$

$$2\pi [F_1(\beta) - F_{int}(\beta, d)] = w_m^\perp(\beta) - 4\pi F_{int}(\beta, d)$$

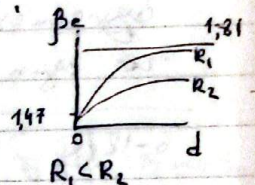
$$w_m^\perp(\beta) = \frac{1}{2} N_{zz}(\beta), \quad 2N_{xx} + N_{zz} = 4\pi, \quad N_{zz} = 4\pi - 2N_{xx}, \quad N_{xx} = 2w_m^{\parallel} = 4\pi F_1(\beta)$$

$$2\pi F_1 - w_m^\perp = \frac{1}{2} N_{xx} - \frac{1}{2} N_{zz} = \frac{1}{2} N_{xx} - \frac{1}{2} (4\pi - 2N_{xx}) = \frac{3}{2} N_{xx} - 2\pi = \frac{3}{2} (N_{xx} - \frac{4\pi}{3})$$

$$\frac{3}{2} [N_{xx}(\beta) - \frac{4\pi}{3}] = -2\pi F_{int}(\beta, d), \quad N_{xx} = 4\pi F_1(\beta), \quad \text{или } (3F_1(\beta) + F_{int}(\beta, d)) = 1$$

ура на $\beta_c(d)$ где  ↔  непересекаются SD states.

d - interdisk spacer thickness бер. R, $d = \frac{d_s}{R} = \left(\frac{d_s}{R_0}\right) \left(\frac{R_0}{R}\right)$



File "Double-vortex.med".

→ $\beta_c(d)$ values at R, no renormalization to system
 unrepres. [1.47, 1.81] где $\forall d$ и $\forall R$.

Балог: нормирование $\beta_c(d)$ где F/N/F cylinder абсолюте
 2-й курс МПД абсолютный непересек  ↔  SD states.

Семинар J. Sord (Barcelona, Spain) "Perpendicular exchange bias in MLS"

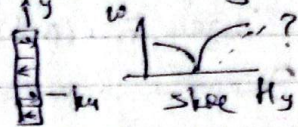
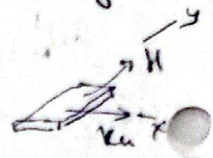
Рассмотреть эквивалентность $M(H)$ of $\frac{\sum_{i=1}^N \mu_i}{\sum_{i=1}^N \mu_i}$ и микроскопический
 магнетизм. Упрощенная генерализация, на основе $E_v = -J_v \vec{u}_1 \vec{u}_2$

Преподна МММ'а oral talks: Jiaqi Ji, Chang, Hoffmann, Buchanan, Gushenko, Hoffmann-Chang описан конденсированное
 Hex-bias (unidirect-anisotropy) и shape anisotropy in FeNi/JmMn
 stripes. Вспомогательная $\vec{H}_{\text{eff}} = \vec{H}_{\text{EB}} + \vec{H}_{\text{ext}}$ и взаимодействие

$H_{\text{eff}}, Q_{\text{eff}}(J_{12}, K_u)$ в $M(H)$ of Stoner-Wahl. model, 2-й crazy guess
 без микроскопической версии $M(H)$ of FeNi stripes на AF-матрице.
 Заметно, что N-reversal в stripes еще более non-uniform.

5/IV Dargunban FMR-stripe MIM - talk in slides que SW-vec. poster co Crabun - Gubbiotti.

Osaznyane e Baren do e Marcos paper no reversal
 6 m. stripes (long rad. elements) e $K_u \perp$ guinnai oen.
 3a reversal observe SW mago e $H_y \neq 0$, ee H_y coarbercober
 ofazobanno stripe domain structure upu $H_y \rightarrow 0$
 In analytic eqs no exampunens, no
 agerbanow combe (ta Wigen, ramblunep).



Bare $\rightarrow$ Osazny. e Baren "nanjumenus", koropoe a cozgno b nogryp-
 ne abaneco Camen kbanpmanjebannow ee menden o June-
 orvax no "D.Vortex" u "Bio-disk" ifanvax. Mai D.vortex
 orvax (Wint-core, Wint side charges u yo) Bana ne venovzoban
 a no Bio-disk mureo geasb ne upocan (mgen' exenpumenes)
 F-rad'n. woue no a bmarvax b ho abnechenne)

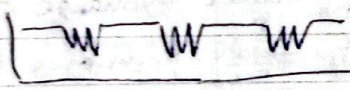
$\rightarrow$ Becega e Kristen o ee nocegnut. Simulations of Double-vortex
 dynamics: upanve pgyvaxem qua $C_1, C_2 = \pm 1$, buera-venne
 no qon v. core gbusenue, a boopai - ver.

MMM'04

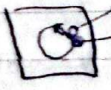
6-12/XI Noezgna e A.u.A. na MMM'04 b Jacksonville, Florida.

I H. Stoll (Germany) nazyvnu unepemue pgyvaxem
 no π -xmed vortex core switching as function $H_0(t)$ in-plane field
 amplitude. **Vortex-Core reversal in in-plane mw-field (H. Stoll)**

1)   - V-Core-trajectories, Haxnonens no otnom. kocen
 of square element
 noce core reversal

2) $\frac{dF_{tot}}{dt}$  - zupnu vortex demonstrates 3-oscillations
 abzanvne e rendupelbannow SW

$E_{ex} \propto \cos t$, a $E_{zcm} = E_m$ - amono ouceupvax na coab. Dacore
 of TM-mode

3)      
 $p=+1$ $p=-1$ $p=+1$ $p=-1$ $p=-1$
 repelannu SW switching core zabepmen

4) E_{cm} guenbannu p noynvno of in-plane field H_0 no
 polarization of core $p=-1$ switches back to vicrognany $p=+1$,
 koropoe coab. renbunni noynvno

II Dargun no m. dynamics b mag. rings.

(16/XI) Paper on mag. conditions paper.

Pinning rect. stripe $d = \frac{2\pi [1 - K_s / (\pi M^2 L)]}{\beta [1 + 2 \ln \frac{1}{\beta} + \frac{L^2 x}{L^2}]}$

Pinning circ. cylinder $d = \frac{2\pi [1 - K_s / (\pi M^2 L)]}{\beta [4 (\ln \frac{8}{\beta} - \frac{1}{2}) + \frac{L^2 x}{L^2}]}$

Obvious properties: $d = \frac{2\pi [1 - K_s / (\pi M^2 L)]}{\beta [a + b \ln \frac{1}{\beta} + \frac{L^2 x}{L^2}]}$, $\rho = \frac{L}{w}$ and $\rho = \frac{L}{R}$

1) One stripe $a=1, b=2$

2) One cylinder $4 (\ln \frac{8}{\beta} - \frac{1}{2}) = 4 (\ln 8 + \ln \frac{1}{\beta}) - 2 = 4 \ln \frac{1}{\beta} + 4 \ln 8 - 2$

$\ln 8 = \ln 2^3 = 3 \ln 2$, $4 \ln 8 - 2 = 12 \ln 2 - 2 = 2 (6 \ln 2 - 1)$

i.e. $a = 2 (6 \ln 2 - 1)$, $b = 4$. $a \approx 6.32$.

D. vortex dynamics F/N/F disk

(17/XI) No. 6309 no vortex in. Mischenko & Bane. Harmonic D vortex Dynamics paper. Pole-free model

$\langle \vec{m} \rangle_v = -\frac{2}{3} C \hat{z} \times \vec{S}$, $\vec{S} = \frac{\vec{X}}{R}$. $\langle \vec{m} \rangle_v = \frac{1}{3} \vec{a}$, $S_x = -\frac{1}{2} C a_y$, $S_y = \frac{1}{2} C a_x$

$-\frac{2}{3} C \hat{z} \times \vec{S} = -\frac{2}{3} C \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ S_x & S_y & 0 \end{vmatrix} = -\frac{2}{3} C [\vec{j} S_x - \vec{i} S_y] = -\frac{2}{3} C [\vec{j} (-\frac{1}{2} C a_y) - \vec{i} (\frac{1}{2} C a_x)] = \frac{1}{3} \vec{a}$

$\vec{a} = (-2) C \hat{z} \times \vec{S}$

$\langle \vec{m}_j \rangle = -\frac{2}{3} C_j \hat{z} \times \vec{S}_j$, $\langle \vec{m} \rangle_v = \frac{1}{V} \int d^3 r \vec{m} = \frac{1}{V} \int_{V_1} d^3 r \vec{m}_1 + \frac{1}{V} \int_{V_2} d^3 r \vec{m}_2$ $V = V_1 + V_2$ properly defined

Even (good) b F/N/F dot ogunakobu, so $\langle \vec{m} \rangle_v = \frac{1}{2} \langle \vec{m}_1 \rangle_{V_1} + \frac{1}{2} \langle \vec{m}_2 \rangle_{V_2}$
 $V_1 = V_2 = V/2$

$\langle \vec{m} \rangle_v = \frac{1}{2} (-\frac{2}{3}) C_1 \hat{z} \times \vec{S}_1 + \frac{1}{2} (-\frac{2}{3}) C_2 \hat{z} \times \vec{S}_2 = -\frac{1}{3} \hat{z} \times [C_1 \vec{X}_1 + C_2 \vec{X}_2] \frac{1}{R}$

the two $p_1 = p_2$ - ogunakobu uarepuzhuni(?)

Вспомогательная уД of $\vec{X}_1, \vec{X}_2$ at 17/XI где было $p_1 = p_2$ and $p_1 p_2 = 1$.

Case Torga monno Jmab $\vec{G}_1 = \vec{G}_2 = \vec{G}$. $\vec{G}_j = -\hat{z} 2\pi p_j \frac{LM_s}{\delta}$, where L, M_s, δ ogunakobu

$p_1 p_2 = -1$ even $p_1 p_2 = -1$, where $p_1 = 1, p_2 = -1$, so $\vec{G}_1 = \vec{G}, \vec{G}_2 = -\vec{G}$

$\left\{ \begin{aligned} \vec{G} + \frac{d\vec{X}_1}{dt} &= \kappa' \vec{X}_1 + \frac{\partial W_{int}}{\partial \vec{X}_1} \\ -\vec{G} + \frac{d\vec{X}_2}{dt} &= \kappa' \vec{X}_2 + \frac{\partial W_{int}}{\partial \vec{X}_2} \end{aligned} \right\} \Rightarrow \left\{ \begin{aligned} \vec{G} + \frac{d\vec{X}}{dt} &= 2\kappa' \vec{X} + \left[\frac{\partial W_{int}}{\partial \vec{X}_1} + \frac{\partial W_{int}}{\partial \vec{X}_2} \right] \\ 2\vec{G} \times \frac{d\vec{X}}{dt} &= \kappa' \vec{Z} + \left[\frac{\partial W_{int}}{\partial \vec{X}_1} - \frac{\partial W_{int}}{\partial \vec{X}_2} \right] \end{aligned} \right\}$ $\vec{X} = \frac{1}{2} (\vec{X}_1 + \vec{X}_2)$
 $\vec{Z} = \vec{X}_1 - \vec{X}_2$

So $W_{int}(\vec{X}_1, \vec{X}_2) = W_{int}(\vec{X}_1 - \vec{X}_2)$ $\Rightarrow \frac{\partial W_{int}}{\partial \vec{X}_1} + \frac{\partial W_{int}}{\partial \vec{X}_2} = 0$

$$W_{int} = \lambda \vec{z}^2, \quad \frac{\partial W_{int}}{\partial \vec{z}_1} = \frac{\partial W_{int}}{\partial \vec{z}_2} = 2\lambda \vec{z} - (-1)2\lambda \vec{z} = 4\lambda \vec{z}$$

$$\left\{ \begin{aligned} \vec{G} \times \frac{d\vec{z}}{dt} &= 2\alpha' \vec{X} \\ 2\vec{G} \times \frac{d\vec{X}}{dt} &= \alpha' \vec{z} + 4\lambda \vec{z} \end{aligned} \right\} \quad \vec{G} \times \frac{d\vec{z}}{dt} = 2\alpha' \frac{d\vec{X}}{dt} \quad \vec{G} \times \vec{G} \times \vec{X} = \vec{G}(\vec{G} \cdot \vec{X}) - (\vec{G} \cdot \vec{G})\vec{X} = \vec{G}(\vec{G} \cdot \vec{X}) - G^2 \vec{X} = -G^2 \vec{X}$$

$$2\vec{G} \times \vec{G} \times \frac{d\vec{X}}{dt} = (\alpha' + 4\lambda) \vec{z} \times \vec{G} = (\alpha' + 4\lambda) \vec{G} \times \vec{z}, \quad -2G^2 \frac{d\vec{X}}{dt} = (\alpha' + 4\lambda) \vec{G} \times \vec{z}$$

$$\vec{G} \times \frac{d\vec{z}}{dt} = 2\alpha' \frac{d\vec{X}}{dt} = 2\alpha' \frac{(-1)}{2G^2} (\alpha' + 4\lambda) \vec{G} \times \vec{z}, \quad \frac{d^2 \vec{z}}{dt^2} = -\frac{\alpha'}{G^2} (\alpha' + 4\lambda) \vec{z},$$

$$\text{r.e. } \omega_z^2 = \frac{\alpha'}{G^2} (\alpha' + 4\lambda)$$

$$W_{tot} = \frac{1}{2} \alpha X_1^2 + \frac{1}{2} \alpha X_2^2 + \mu (\vec{X}_1 \cdot \vec{X}_2) = \frac{1}{2} (\alpha + \mu) X_1^2 + \frac{1}{2} (\alpha + \mu) X_2^2 - \frac{1}{2} \mu (\vec{X}_1 - \vec{X}_2)^2$$

$$\text{r.e. } \alpha' = \alpha + \mu, \quad \lambda = -\frac{1}{2} \mu, \quad \omega_z = 23/\bar{z}$$

$$\omega_z^2 = \frac{1}{G^2} (\alpha + \mu) (\alpha + \mu - 2\mu) = \frac{1}{G^2} (\alpha + \mu) (\alpha - \mu) \rightarrow \omega_z = \frac{1}{G} \sqrt{(\alpha + \mu)(\alpha - \mu)} \quad p_1 p_2 = -1$$

Т.к. $\mu \sim \epsilon, \epsilon_2$, то частота $\vec{z}$ -моды ω_z не зависит от знака ϵ, ϵ_2 !

$$2\vec{G} \times \frac{d\vec{X}}{dt} = (\alpha' + 4\lambda) \frac{d\vec{z}}{dt}, \quad \vec{G} \times \vec{G} \times \frac{d\vec{z}}{dt} = \vec{G}(\vec{G} \cdot \frac{d\vec{z}}{dt}) - \frac{d\vec{z}}{dt}(\vec{G} \cdot \vec{G}) = -G^2 \frac{d\vec{z}}{dt}$$

$$\vec{G} \times \vec{G} \times \frac{d\vec{z}}{dt} = 2\alpha' \vec{G} \times \vec{X} = -G^2 \frac{d\vec{z}}{dt}, \quad \frac{d\vec{z}}{dt} = -\frac{2\alpha'}{G^2} \vec{G} \times \vec{X}$$

$$2\vec{G} \times \frac{d^2 \vec{X}}{dt^2} = (\alpha' + 4\lambda) \frac{d^2 \vec{z}}{dt^2} = (\alpha' + 4\lambda) \frac{(-2\alpha')}{G^2} \vec{G} \times \vec{X} \rightarrow \frac{d^2 \vec{X}}{dt^2} = -\frac{\alpha'(\alpha' + 4\lambda)}{G^2} \vec{X}$$

$$\rightarrow \omega_X^2 = \frac{1}{G^2} \alpha'(\alpha' + 4\lambda) = \omega_z^2 \quad \text{but zero ga!} \quad \text{В случае } p_1 p_2 = -1$$

направление моды $\vec{z}$ зависит (или нет) от знака μ , но не зависит от знака ϵ, ϵ_2 . Проверим. $\vec{X}_1$ и $\vec{X}_2$ — главные оси эллипса $\vec{X}$ — главный вектор вращения. $\vec{X} \cdot \vec{z}$ равен 0, т.е. $\perp \vec{G}$, т.к. $\vec{G} \parallel \vec{z}$.

УД $\left\{ \begin{aligned} \vec{G} \times \frac{d\vec{z}}{dt} &= 2(\alpha + \mu) \vec{X} \\ \vec{G} \times \frac{d\vec{X}}{dt} &= \frac{1}{2}(\alpha - \mu) \vec{z} \end{aligned} \right\}$ — упр. и не канонизировать
но канонизировать $\vec{X} = \frac{1}{2}(\vec{X}_1 + \vec{X}_2)$, $\vec{z} = \vec{X}_1 - \vec{X}_2$
в случае $p_1 p_2 = 1$ case.

$$W(X_1, X_2) = \frac{1}{2} \alpha X_1^2 + \frac{1}{2} \alpha X_2^2 + \mu (\vec{X}_1 \cdot \vec{X}_2)$$

$$\left\{ \begin{aligned} \vec{G} \times \frac{d\vec{X}_1}{dt} &= \alpha \vec{X}_1 + \mu \vec{X}_2 \\ -\vec{G} \times \frac{d\vec{X}_2}{dt} &= \alpha \vec{X}_2 + \mu \vec{X}_1 \end{aligned} \right\} \quad \vec{G} \times \vec{X}_1 = \alpha \vec{X}_1 + \mu \vec{X}_2, \quad \vec{G} \times \vec{G} \times \vec{X}_2 = -G^2 \vec{X}_2 = -\vec{G} \times [\alpha \vec{X}_2 + \mu \vec{X}_1]$$

$$\vec{G} \times \vec{G} \times \vec{X}_1 = -G^2 \vec{X}_1 = \vec{G} \times [\alpha \vec{X}_1 + \mu \vec{X}_2], \quad \ddot{\vec{X}}_1 = -\frac{\alpha}{G^2} [\alpha \vec{X}_1 + \mu \vec{X}_2] + \frac{\mu}{G^2} [\alpha \vec{X}_2 + \mu \vec{X}_1]$$

$$\ddot{\vec{X}}_1 = -\frac{(\alpha^2 - \mu^2)}{G^2} \vec{X}_1 \Rightarrow \omega_{X_1}^2 = \frac{1}{G^2} (\alpha^2 - \mu^2)$$

$$\vec{G} \times \ddot{\vec{X}}_2 = -\alpha \ddot{\vec{X}}_2 - \mu \ddot{\vec{X}}_1 = -\alpha \frac{1}{G^2} \vec{G} \times [\alpha \vec{X}_2 + \mu \vec{X}_1] - \mu \frac{(-1)}{G^2} \vec{G} \times [\alpha \vec{X}_1 + \mu \vec{X}_2]$$

$$\ddot{\vec{X}}_2 = -\frac{\alpha}{G^2} [\alpha \vec{X}_2 + \mu \vec{X}_1] + \frac{\mu}{G^2} [\alpha \vec{X}_1 + \mu \vec{X}_2] = -\frac{1}{G^2} (\alpha^2 - \mu^2) \vec{X}_2 \Rightarrow \omega_{X_2}^2 = \frac{1}{G^2} (\alpha^2 - \mu^2)$$

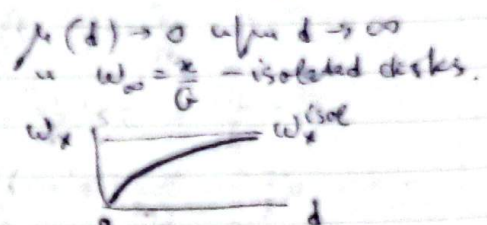
T.e. $\omega_{\pm}^2 = \omega_x^2 = \omega_y^2 = \omega_z^2 = \frac{1}{G^2}(\alpha^2 - \lambda^2)$ Zacrosy
cena gde
 $\frac{d\vec{x}_1}{dt^2} = -\omega_x^2 \vec{x}_1$ $\frac{d\vec{x}_2}{dt^2} = -\omega_y^2 \vec{x}_2$ P1P2 = -1 u afektsionabno
chello
gibnitsye burzong

Do osnove ac angloa p1p2 = 1, korga

$\omega_0 = \omega_x = \frac{1}{G}(\alpha + \lambda)$ $\omega_1 = \omega_z = \frac{1}{G}(\alpha - \lambda)$ Zacrosy gde

Aradiguo

$\omega_x^2 (P1P2 = -1) = \omega_y^2 \omega_z^2 (P1P2 = +1)$



1 gde, P1P2 = +1



2 gde, P1P2 = -1

Maximalnaya eigefrequency $\omega_x^2 (P1P2 = -1) = \frac{1}{G^2}(\alpha^2 - \lambda^2)$ u $\mu(d \rightarrow 0)$ Soglasno
maksimu

T.e. $\lambda(d \rightarrow 0) \approx \alpha$

Chani 03

Chani PAB'03 que $\xleftrightarrow{P1} \xleftrightarrow{P2}$ does gas Zacrosy $(-\lambda_x = \lambda_y)$
 $\lambda = 0$

$\omega_{P1P2} = \omega_0 \sqrt{(1 + P1P2\lambda)(1 + \lambda)}$ $\omega_{P1P2}^* = \omega_0 \sqrt{(1 - P1P2\lambda)(1 - \lambda)}$

Coord. $\vec{x}_1(0) = (0, C_1H)$ $\vec{x}_2(0) = (0, C_2H)$ (coord. var. $\vec{x}_1(0) = (0, C_1H)$
y.c. $\vec{x}_2(0) = (0, C_2H)$)
 $H = H_y$ - cosgde $x_1(0), x_2(0)$ H - uniform H - step function

$W_{int} = C_1 C_2 (\eta_x a_{1x} a_{2x} - \eta_y a_{1y} a_{2y})$ $\lambda_{x,y} = \frac{1}{\alpha} \eta_{x,y}$ $\omega_0 = \frac{\alpha}{G}$

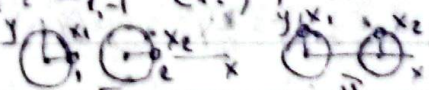
$\eta_y = -\eta_x$ $\lambda_y = -\lambda_x \Rightarrow W_{int} = C_1 C_2 \eta_x (\vec{a}_1 \cdot \vec{a}_2)$ $\eta_x = \eta$ - anan?
masy 1/41

Zacrosy Chani que $P1P2 = +1$ $\omega_{1,1} = \omega_0(1 + \lambda)$ $\omega_{1,1}^* = \omega_0(1 - \lambda)$

Zacrosy Chani que $P1P2 = -1$ $\omega_{1,-1} = \omega_0 \sqrt{1 - \lambda^2}$ $\omega_{1,-1}^* = \omega_0 \sqrt{1 - \lambda^2}$

T.e. u Chani $\omega_{1,-1} = \omega_{1,-1}^*$ - Zacrosy burzongenno tozno esli $\eta_y = \eta_x$

Do y Chani 2003 $\omega_{1,-1} \neq \omega_{1,-1}^*$ (!?), no monno obzracheno

tozno upu $\eta_y \neq \eta_x$ (!)  bego

m. coupling, gde b' chuyannu I u II paznani, do kax

ebi $\frac{1}{\mu_1} \frac{1}{\mu_2} \times I \rightarrow \frac{1}{\mu_1} \frac{1}{\mu_2} \times II$ m. coupling gby gnuvni c $\vec{p}_1 \parallel 0x$ u $\vec{p}_2 \perp 0x$

y kax $W_{int} \sim (\vec{x}_1 \cdot \vec{x}_2) \sim (\vec{a}_1 \cdot \vec{a}_2)$ u z. za kumel'vinnogo
 pachetennye gde. Umenno $W_{int} \sim (\vec{x}_1 \cdot \vec{x}_2)$ vozbaumu

Obes $X = \frac{1}{2}(x_1 + x_2)$ u $Z = x_1 - x_2$ - odobu, voogunaru. y Chani
 ω -splitting/shift iznachenno kax, kax kax m. coupling pigoz
 chesnyy gde obzracheno kax (ko chabn. c F/N/F dot y'men)

Работа по теме: Double-Vortex Dynamics.

Анализ DOE Proposal Bader, B. и другие Refs.

вспомогательные JAP-04 - V.M.P.D. и J.Ph.C. Malin - SW, каровские
доклады к FY'05.

19/11

Важная информация: эксперимент Фадина по Vortex TM-модели.

Фадина - эксперимент TM-модели

$$\omega_0/2\pi = 275 \text{ MHz } (2R=1 \mu\text{m}) \text{ и } 162 \text{ MHz } (2R=2 \mu\text{m}), L=40 \text{ nm}.$$

РН: - 6 бинарных циклов.

См. также JAP-02 Фадина, Vortex TM-модель: $\omega_0(p) = 8\pi J M_s F_v(p)$,

$$F_v(p) = \int dt \int \frac{d\mathbf{r}}{r} I_v^2(t), I_v(t) = \int d\mathbf{x} \times \mathbf{J}_t(\mathbf{x}, t).$$

$$p_1 = \frac{40 \text{ nm}}{500 \text{ nm}} = 0.08 \text{ и } p_2 = \frac{40 \text{ nm}}{1 \mu\text{m}} = 0.04, \frac{1}{2} \left(\frac{R_0}{R} \right)^2 = \frac{1.3 \cdot 10^{-3}}{2} \approx 0.65 \cdot 10^{-3} \ll 4\pi F_v(0.08) = 0.083$$

Оценочные моменты: где-то здесь.

Calc. $\frac{\omega_0(0.08)}{2\pi} = 0.39 \text{ GHz}$ $\frac{1}{2\pi} \omega_0(0.04) = 0.20 \text{ GHz}$ $\mu H M_s = 800 \text{ G}$, $f = 295 \frac{\text{GHz}}{\text{keV}}$

→ В работе Фадина и Бадера с коллегами о модели вращательного движения 1995 г.
(сначала, но потом у него не так, но была работа "D-Vortex" и Bio-disk, etc)
Бадер сказал, что Бадина не может сделать такие расчеты
(не имеет необходимого программного обеспечения, как он и PI-инженер)
и тогда я сам попробовал с ними по этому вопросу (пока не опубликовано)

2. Vortex Zagarodny: PRL'04: "Bryskennye" модели of Vortex Dynamics

Vortex dynamics

Упр-е Thiele не описывает switching $m_z(0)$ -vortex в circular
- polarized in-plane field и cycloidal trajectory core ($M \frac{d^2 \mathbf{r}}{dt^2} = \text{force}$).

Уже: модель для switching $m_z(0)$ vortex в экв. H. Stoll - MM'04
показывает именно вращательное in-plane field и с расчетом ω
 $\approx \omega_{TM}$ of vortex, а не $\omega \gg \omega_{TM}$ как у Zagarodny'04

Closed orbit безмассовая $\mu H M = \langle m_z \rangle = \text{const}$ и $\psi(t) = 0$ или $\psi = 2\pi$,
Zagarodny'04 не в фазе. Упр-е Thiele замкнутый Eq.(12) $\mu H M = \text{const}, \psi = 0$.

→ Бабос Zagarodny'04: $\mu H M$ на самом деле circular polarized in-plane
m.w. field и damping - vortex траектория и не вращательная

$$\frac{dR}{dt} = 0, \frac{dM}{dt} = 0, \frac{d\phi}{dt} + \frac{d\psi}{dt} = \omega, \vec{r} = (R, \phi) - \text{core position, } \omega - \text{rotation H}$$

Затем мы делаем с численным решением (R, ϕ) и (M, ψ)
используя уравнения vortex'a. $M(t) \sim l^2(t)$, $l(t)$ - v. core radius, $\psi(t)$ - фазы m_z и ψ -фаза SW.

Note → Метод эфф. Lagrangian замедленного времени $\mu H M$, $\psi = 0$, $H = 0$.

Обыкновенные + Океанов Био-particles (nm Au) + ee
 vortex E-barrier.

24/XI

Kristen upturns micromag. data on w of $P_1 P_2 = -1$ case D. Vortex.

Due $P_1 P_2 = +1$, $C_1 C_2 = -1$ Kristen Inu. micrograph

$2R = d_{\text{min}}$	1000	750	500
β	0.08	0.104	0.16
Calc	0.125	0.128	1.28
Simul Q10, f_1	0.51 ± 0.01	0.68 ± 0.02	1.20 ± 0.1
GH ₂ , f_2	0.021	0.039	0.025 ± 0.005
Calc	0.052	0.087	0.173

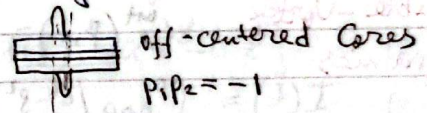
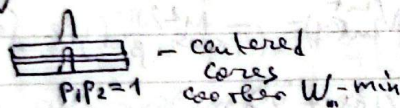
Disk $F/N/F = 40/1/40 \text{ nm}$
 $\tau, \epsilon, L = \text{const}, D = \text{variable}.$
 $d_s = 1 \text{ nm}$

$w_n = F(\beta, d) = F\left(\frac{L}{R}, \frac{d_s}{R}\right)$ since $L = \text{const}$ и $\frac{d_s}{R} = \beta \frac{d_s}{L}$ и $w_n(\beta)$.

→ Ruzhko, two w_n - calc better same as w_n - simulated.

w_x (same, micro) gonna upturns coverage $\ll w_0(2\beta)$ isolated disk give $40/1/40 \text{ nm}$ F/N/F disk. No too no talk, t.e. usd Ruzhko JAP-02, and new-Kristen paper micromag. of $w_0(\beta)$ - nebylno. Chagan Kristen unob smadky.

Kristen simulated, two give $P_1 P_2 = -1$ vortex-core trajectory замкнувшаяся. Уфимов, бугров, бо бугрови-бугров v. cores пазухи чашб.



Due $P_1 P_2 = -1$ any real particles core positions $X_1^0, X_2^0 \neq 0$, t.e. w_m^{vol} .
 They're $X_j^0 = 0$, no $w_m^{\text{core}} > 0$ they're particles cores и $X_j^0 \neq 0$.
 $w_m^{\text{core}} \ll w_m^{\text{vol}}$, no w_m^{core} have the cores trajectories up $P_1 P_2 = -1$.

Наран + Океанов paper Vortex-barrier в чашке.

(25-27/XI)

Уфимов генератор сун! Дун. сунма зубавенно continuous paper accumulation up $N \rightarrow \infty$ (N -times micromag. area), t.e. contin. paper cores give in-plane size. Due out-plane sizes ($\sim 1-5 \text{ nm}$), $N_{\text{max}} = \frac{L}{a}$ - небыло (а-немаз пленки) и ген. paper авто самозащита or конъюнкцион, где маюк тоньше dots ген. paper беген, конъюнкцион репер ретров. Конъюнкцион paper (micromag.) онебен на ма. ностенна $\Phi(\vec{r})$ и т.п. гадане $\frac{\partial \Phi}{\partial n_1} - \frac{\partial \Phi}{\partial n_2} = 3$, $\Phi(+0) = \Phi(-0)$.

Group Meeting: Posters give DOE-Review in Jan. '05.

28/XI

Hamman и нонан to Sam Abstract of my poster "Spin Dynamics", Begey Kristen.

30/11 → Becega e Ruihua Chang o Fe-nanostripes/islands ka Pt-referenax Wuerana una coorbas. paper draft. Zenne ero. Fe/Pt(111)

→ Becega e Kristen u Karen - Vortex Dynamics paper.

- 1) nocunat Abstract ka APS
- 2) oduguna e Bader (11/11/00) sta upegues nocunatu b PRL
- 3) bouclunat nocunay single-disk wo b OOMMF (Nov) u LG solver (Kristen) pazumati.
- 4) gononunat novo Fig. 3 $W_n(\beta)$ unipolar. vorunam, up date Figs.
- 5) nocunat nocunay que $p_1 p_2 = -1$ core trajectories abt. Zenneam, vorupue euse u franzesice
- 6) Kristen nocunay figura Macenno, novo que $p_1 p_2 = -1$ eob sanovo agna racunad.



? Yp-e Thiele negociarimo?

Hammer Abstracts Vortex Dynamics " " Boundary Conditions "

Inucaa bzainogenicosue v. cores b creax P/N/F disk, eciu $\vec{X}_1 \neq \vec{X}_2 \neq 0$, var fene. M. Coupling cuadru, no buquino basunay que gozunat v. trajectory que $p_1 p_2 = -1$.

M. Coupling

22/11 a nocunat zepunat m. coupling of centered cores:

Double-Vortex Dynamics

$$W_m^{int}(\beta, d) = \frac{1}{V_{mag}} W_m^{int}(\beta, d) = -M_s^2 \frac{2\pi}{\beta} p_1 p_2 \int_0^\infty dt e^{-t\beta} (1 - e^{-\beta t})^2 I^2(t)$$

$$I(t) = \int_0^c dp p \left(\frac{c^2 - p^2}{c^2 + p^2} \right) J_0(tp) \quad \text{Two core-core interaction..}$$

Due vortex'ab b pazunat creax unecunay zepunat ym $\vec{X}_1, \vec{X}_2$ u ogunacubue eciu ($L_1 = L_2 = L, M_{s1} = M_{s2}$):

$$W_m^{int}(\beta, d) = \frac{1}{V_{mag}} W_m^{int} = -M_s^2 \frac{2\pi}{\beta} \int_0^\infty dt e^{-t\beta} (1 - e^{-\beta t}) I_1(t) I_2(t)$$

$$I_n(t) = \int_0^c dp p \cos \theta_n(p) J_0(tp) \quad \text{yba ne rak upocuo, r. k.}$$

$M_z^n(p)$ yba ne effer axial symmetric, bego $\vec{X}_1 \neq 0, \vec{X}_2 \neq 0$.

$$W_m^{int} = \int dp dp' \int d\vec{r} d\vec{r}' \frac{M_{1z}(\vec{r}) M_{2z}(\vec{r}')}{|\vec{r} - \vec{r}'|}, \quad M_{2z} \text{ zabunat } \vec{r} \text{ "p"}$$

Gopazgo zabunat, zen W_m^{int} (core-core), nocunat exagubac

W_m^{int} (core-volume). W_m^{int} (core-volume) nocunat $\neq 0$ que

neceat. cytar $\vec{X}_1, \vec{X}_2 \neq 0$ a gubunayue vortex'ab.

Stago zoo yba, mare zabunat. Speerayun $p_1 p_2 = -1$ cytar ne efferimo.

Corr. 22/2 $\text{div} \vec{m}(\vec{r}) = - (a_x \cos \varphi + a_y \sin \varphi)$ give $f(z) = \frac{1}{c} [izC + \frac{1}{2}(a - \bar{a}z^2)]$

$$\text{div} \vec{m} = - \frac{1}{\rho} \text{Re}(\bar{a}z) = - \frac{1}{\rho} (\vec{a} \cdot \vec{r})$$

$$m_z(\vec{r}) = \frac{1 - |\vec{r}|^2}{1 + |\vec{r}|^2}, \quad w(z\bar{z}) = f(z) \text{ by } \mu \text{ "core"}. \quad \text{Vortex center } z_0 = \frac{1}{2}iCa$$

$$c^2 |\vec{r}|^2 = c^2 |f|^2 = c^2 f \bar{f} = [izC + \frac{1}{2}(a - \bar{a}z^2)] [-i\bar{z}C + \frac{1}{2}(\bar{a} - a\bar{z}^2)] =$$

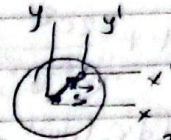
$$= |z|^2 + \frac{1}{2}iC(1+|z|^2)(\bar{a}z - a\bar{z}) + \frac{1}{4}(a - \bar{a}z^2)(\bar{a} - a\bar{z}^2) =$$

$$= \rho^2 + \frac{1}{2}iC(1+\rho^2)(\bar{a}z - a\bar{z}) + \frac{1}{4}[\rho^4 - a^2\bar{z}^2 - \bar{a}^2z^2 + |a|^2|z|^4]$$

(ρ, φ) - the same as before
 $\rho=0$ - vortex core

$$\vec{a} = -2C \hat{z} \times \vec{S}$$

Shifted Vortex centered in point $z = \frac{\vec{X}}{\rho} = \vec{S}$



$$m_z(\vec{r}') = \frac{c^2 - \rho'^2}{c^2 + \rho'^2}$$

$$x' = x - S_x, \quad y' = y - S_y, \quad \vec{r}' = \vec{r} - \vec{S}, \quad \rho'^2 = (\vec{r} - \vec{S})^2 = \rho^2 + S^2 - 2\vec{S} \cdot \vec{r}$$

$$m_z(\vec{r}) = \frac{c^2 - \rho'^2}{c^2 + \rho'^2}, \quad \frac{1}{2}i(\bar{a}z - a\bar{z}) = - (a_x y - a_y x) = - (\vec{a} \times \vec{r})_z$$

$$\bar{a}z = (a_x + ia_y)(x + iy) = a_x x + a_y y + i(a_x y - a_y x), \quad \text{Re}(\bar{a}z) = \vec{a} \cdot \vec{r}$$

$$\text{Im}(\bar{a}z) = (\vec{a} \times \vec{r})_z$$

$$\frac{1}{2}iC(1+\rho^2)(\bar{a}z - a\bar{z}) = -C(1+\rho^2)(\vec{a} \times \vec{r})_z$$

$$\vec{a} \times \vec{r} = -2C \hat{z} \times \vec{S} \times \vec{r} = -2C [\vec{S}(\hat{z} \cdot \vec{r}) - \vec{r}(\hat{z} \cdot \vec{S})] = 0$$

$$\vec{r} \times \vec{a} = (2C) \vec{r} \times \hat{z} \times \vec{S} = (-2C) [\hat{z}(\vec{r} \cdot \vec{S}) - \vec{S}(\vec{r} \cdot \hat{z})] = -2C(\vec{r} \cdot \vec{S})\hat{z}$$

$$(\vec{r} \times \vec{a})_z = -2C(\vec{r} \cdot \vec{S})$$

$$c^2 |\vec{r}|^2 = c^2 |f|^2 = \rho^2 - C(1+\rho^2)(\vec{a} \times \vec{r})_z + \frac{1}{4}[|a|^2(1+\rho^4) - a^2\bar{z}^2 - \bar{a}^2z^2] \quad (1)$$

$$f(z) = \frac{1}{c}iz' \rightarrow c^2 |f|^2 = |z'|^2 = \rho'^2 = \rho^2 - 2\vec{S} \cdot \vec{r} + S^2 \quad (2)$$

(1) и (2) show the equivalence even for vortex core rigid, beq

(2) unknown $f(z') = \frac{1}{c}iz'$ - rigid vortex without measurement on $\vec{S}$.

(1) - shifted vortex с измерением параметров. (1) - без измерения $\vec{m}(\vec{r}) = 0$ model.

$$c^2 |f|^2 = \rho^2 - C(1+\rho^2)2C(\vec{r} \cdot \vec{S}) + O(a^2) = \rho^2 - 2(1+\rho^2)(\vec{r} \cdot \vec{S}) + O(a^2)$$

$$a^2\bar{z}^2 + \bar{a}^2z^2 = (a\bar{z} + \bar{a}z)^2 - 2|a|^2|z|^2 = (2\text{Re}(\bar{a}z))^2 - 2|a|^2|z|^2 = 4(\vec{a} \cdot \vec{r})^2 - 2|a|^2\rho^2$$

$$\vec{a} \cdot \vec{r} = a_x x + a_y y = (2S_y C)x + (-2S_x C)y = 2C[xS_y - yS_x] = 2C(\vec{r} \times \vec{S})_z$$

$$c^2 |f|^2 = \rho^2 - 2(1+\rho^2)(\vec{r} \cdot \vec{S}) + \frac{1}{4}[|a|^2(1+\rho^4) - 4(\vec{a} \cdot \vec{r})^2 + 2|a|^2\rho^2]$$

$$c^2 |f|^2 = \rho^2 - 2(1+\rho^2)(\vec{r} \cdot \vec{S}) - (\vec{a} \cdot \vec{r})^2 + \frac{1}{4}|a|^2(1+\rho^2)^2, \quad |a|^2 = 4|\vec{S}|^2$$

$$c^2 |f|^2 = [\rho^2 - (1+\rho^2)\vec{S}^2] - (\vec{a} \cdot \vec{r})^2$$

$$\rightarrow m_z(\vec{r}) = \frac{c^2 - [\rho^2 - 2(1+\rho^2)(\vec{r} \cdot \vec{S}) + S^2(1+\rho^2) - (\vec{a} \cdot \vec{r})^2]}{c^2 + [\rho^2 - 2(1+\rho^2)(\vec{r} \cdot \vec{S}) + S^2(1+\rho^2) - (\vec{a} \cdot \vec{r})^2]}$$

shifted vortex

$m_z(\rho, \varphi)$ of shifted vortex

$$c f(z) = \frac{1}{c} [izC + \frac{1}{2}(a - \bar{a}z^2)]$$

Pole-free model

$m_z(\rho, \varphi) = 0$ gauge theory "core"-shifted vortex.

Рассмотрим $\frac{1}{r^3} \cos \varphi$, т.е. нелокальную. $\int d\varphi e^{i\mu\varphi} m_2(\vec{r}, \varphi)$.
 То же самое с $\mu=0, 1, 2$. Матрица $W_{int} \sim (\vec{S}_1 \cdot \vec{S}_2)$.

Для $W_{int}(\text{core-vol})$ рассмотрим матрицу $m_2(\vec{r})$, $\text{div } \vec{m} = -\frac{1}{\rho}(\vec{a} \cdot \vec{r}) \sim a$. $\text{div } \vec{m}$ имеет $\mu=\pm 1$ компоненту.

$$m_2(\vec{r}, \varphi) = \frac{c^2 - \rho^2 + d}{c^2 + \rho^2 - d} = \frac{1}{c^2 \rho^2} \frac{1}{1 - \frac{d}{c^2 \rho^2}} [c^2 - \rho^2 + d] = \frac{1}{c^2 + \rho^2} \left[1 + \frac{d}{c^2 + \rho^2} \right] (c^2 - \rho^2) \left[1 + \frac{d}{c^2 + \rho^2} \right] =$$

$$= \frac{c^2 - \rho^2}{c^2 + \rho^2} \left[1 + \frac{d}{c^2 + \rho^2} + \frac{d}{c^2 - \rho^2} \right] = m_2^{(0)} \left[1 + d \frac{2c^2}{(c^2 - \rho^2)^2} \right]$$

$$m_2(\vec{r}, \varphi) = m_2^{(0)} \left[1 + \frac{2c^2}{(c^2 - \rho^2)^2} 2(1 + \rho^2) (\vec{r} \cdot \vec{S}) \right] = m_2^{(0)}(\rho) + \frac{4c^2(1 + \rho^2)}{(c^2 + \rho^2)^2} (\vec{r} \cdot \vec{S})$$

$$W_{int}(\text{core-vol}) \sim \int d\varphi \int d\varphi' \text{div } \vec{m}(\vec{r}) \cdot m_2(\vec{r}') e^{i\mu(\varphi - \varphi')} =$$

$$= \int d\varphi \text{div } \vec{m}(\vec{r}) e^{i\mu\varphi} \int d\varphi' m_2(\vec{r}') e^{-i\mu\varphi'}$$

$$\int d\varphi \text{div } \vec{m} e^{i\mu\varphi} = - \int d\varphi [a_x \cos \varphi + a_y \sin \varphi] e^{i\mu\varphi} = -\pi [a_x (\delta_{\mu,1} + \delta_{\mu,-1}) + \frac{a_y}{i} (\delta_{\mu,1} - \delta_{\mu,-1})] =$$

$$= -\pi [a_x (\delta_{\mu,1} + \delta_{\mu,-1}) + i a_y (\delta_{\mu,1} - \delta_{\mu,-1})]$$

$$W_{int}(\text{core-vol}) \sim (-\pi) [a_x (\delta_{\mu,1} + \delta_{\mu,-1}) + i a_y (\delta_{\mu,1} - \delta_{\mu,-1})] \int d\varphi' e^{-i\mu\varphi'} m_2(\vec{r}') =$$

$$= (2\pi) \left[a_{x1} \cos \varphi' + \frac{i a_{y1}}{2} (e^{-i\varphi'} - e^{i\varphi'}) \right] m_2(\vec{r}') d\varphi' = \left| i (e^{-i\varphi} - e^{i\varphi}) = i \frac{e^{i\varphi} - e^{-i\varphi}}{2i} (-2i) = \right| =$$

$$= -2\pi [a_{x1} \cos \varphi' + a_{y1} \sin \varphi'] m_2(\vec{r}') d\varphi' =$$

$$= -2\pi \int d\varphi' [a_{x1} \cos \varphi' + a_{y1} \sin \varphi'] \left[m_2^{(0)}(\rho) + g(\rho) (\vec{r}' \cdot \vec{S}) \right] = -2\pi \int d\varphi' (\vec{a}_1 \cdot \vec{r}') (\vec{r}' \cdot \vec{S}) g(\rho)$$

$$\int d\varphi' (\vec{a}_1 \cdot \vec{r}') (\vec{r}' \cdot \vec{S}) = \rho \int d\varphi' [a_{x1} \cos \varphi' + a_{y1} \sin \varphi'] [S_{x2} \cos \varphi' + S_{y2} \sin \varphi'] =$$

$$= \rho \int_0^{2\pi} d\varphi [a_{x1} S_{x2} \cos^2 \varphi + a_{y1} S_{y2} \sin^2 \varphi] = \pi \rho (a_{x1} S_{x2} + a_{y1} S_{y2}) = \pi \rho (\vec{a}_1 \cdot \vec{S}_2) \neq 0$$

т.е. $\vec{a}_1$ и $\vec{S}_2$ не перпендикулярны. $W_{int}(\text{core-vol}) \neq 0$.

$$W_{int}(\text{core-core}) \sim \int d\varphi \int d\varphi' m_2(\vec{r}) m_2(\vec{r}') e^{i\mu(\varphi - \varphi')}$$

$$W_{int}(\text{core-vol}) \sim (\vec{a}_1 \cdot \vec{S}_2) = -2C_1 \hat{z} \times \vec{S}_1 \cdot \vec{S}_2, \quad W_{int}(\text{core-vol}) \sim (\vec{a}_2 \cdot \vec{S}_1)$$

$$\rightarrow W_{int}(\text{core-vol}) \sim [(\vec{a}_1 \cdot \vec{S}_2) + (\vec{a}_2 \cdot \vec{S}_1)] = -2[C_1 \hat{z} \times \vec{S}_1 \cdot \vec{S}_2 + C_2 \hat{z} \times \vec{S}_2 \cdot \vec{S}_1]$$

(не обобщаем $\vec{a}_1, \vec{a}_2$ на $(\vec{S}_1 \cdot \vec{S}_2)$ сразу).

1/xii

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b}), \quad (\vec{a} \times \vec{b}) \cdot \vec{c} = (\vec{b} \times \vec{c}) \cdot \vec{a} = (\vec{c} \times \vec{a}) \cdot \vec{b} =$$

$$\hat{z} \times \vec{S}_1 \cdot \vec{S}_2 = (\vec{S}_1 \times \vec{S}_2) \cdot \hat{z} \quad \hat{z} \times \vec{S}_2 \cdot \vec{S}_1 = (\vec{S}_2 \times \vec{S}_1) \cdot \hat{z} = -(\vec{S}_1 \times \vec{S}_2) \cdot \hat{z}$$

$$\rightarrow W_{int}(\text{core-vol}) \sim (-2)[C_1 \hat{z} \cdot (\vec{S}_1 \times \vec{S}_2) + C_2 \hat{z} \cdot (\vec{S}_2 \times \vec{S}_1)] = (-2)(C_1 - C_2) \hat{z} \cdot (\vec{S}_1 \times \vec{S}_2)$$

- это coupling только если $C_1 \neq C_2$ (!) $W_{int}(\text{core-vol})$.

$$W_{int} \text{ (core-val)} \sim (C_1 - C_2) \hat{z} \cdot (\vec{X}_1 \times \vec{X}_2) = AC (\vec{X}_2 \times \hat{z}) \cdot \vec{X}_1 = AC (\hat{z} \times \vec{X}_1) \cdot \vec{X}_2$$

$$\frac{\partial W_{int}}{\partial \vec{X}_1} \sim AC (\vec{X}_2 \times \hat{z}), \quad \frac{\partial W_{int}}{\partial \vec{X}_2} \sim AC (\hat{z} \times \vec{X}_1) \quad \text{— same meaning}$$

лучше назвать yD где $\vec{X}_1(t), \vec{X}_2(t)$ — vortex cores coordinates.

Нужно написать p_1, p_2 cores $m_{2j}(\vec{p}, \varphi) = p_j \frac{c^2 - [\dots]}{c^2 + [\dots]}$

Do upbeget $W_{int} \text{ (core-val)} = W_{int} \text{ (val-core)} + W_{int} \text{ (val-core)} \sim$

$$\sim [(\vec{a}_1 \cdot \vec{S}_2) p_2 + (\vec{a}_2 \cdot \vec{S}_1) p_1] = (-2) [C_1 p_2 \hat{z} \times \vec{S}_1 \cdot \vec{S}_2 + C_2 p_1 \hat{z} \times \vec{S}_2 \cdot \vec{S}_1] =$$

$$= (-2) (C_1 p_2 - C_2 p_1) \hat{z} \cdot (\vec{S}_1 \times \vec{S}_2) \quad \text{— upbeget. Dynamics F/N/Fdis}$$

Одновременно Dvort-dynamics manuscript e Baber (val, Kris)
 PRL — mono no kamcaco general Abstract u Introduction, a
 realme — no xbarac experiment,

Одновременно e Kristen cy'ane $p_1 p_2 = -1, C_1 C_2 = -1$ —

— immunoresponse σ response, кооперативное поведение.

Имеем results где $p_1 p_2 = +1, C_1 C_2 = +1$ — her action

растет, будем из-за того, что до non-metastable state,

a dynamics до не equil. dynamics, a имеем $C_1 C_2 = -1$.

- По энергетике
- ① $p_1 p_2 = +1, C_1 C_2 = -1$, ок. состояние, акая dynamics с ω_x, ω_z
 - ② $p_1 p_2 = -1, C_1 C_2 = -1$, некое поле по энергии, одна часть — элементарное
 - ③ $p_1 p_2 = +1, C_1 C_2 = +1$ — некая dynamics без собств. роста
 - ④ $p_1 p_2 = -1, C_1 C_2 = +1$ — самое энерг. невоинное состояние — все расчеты u simulations

Судя ② $p_1 p_2 = -1, C_1 C_2 = -1$ мне надо рассмотреть асимметрию, рассмотреть $W_{int} \text{ (core-val)} u W_{int} \text{ (core-core)}$.

$$W_{int} \text{ (core-core)} \sim \int d\vec{r} \int d\vec{r}' m_{21}(\vec{r}) m_{22}(\vec{r}') e^{i\mu(\vec{r}-\vec{r}')} \quad , \quad d \sim a, \beta \sim a^2$$

$$m_2(\vec{r}) = \frac{c^2 - p^2 + d - \beta}{c^2 + p^2 - d + \beta} = [c^2 - p^2 + d - \beta] \frac{1}{c^2 + p^2} \left[1 + \frac{(d-\beta)}{c^2 + p^2} + \frac{d^2}{(c^2 + p^2)^2} \right] =$$

$$= \frac{1}{c^2 + p^2} \left[c^2 - p^2 + m_2^{(0)} (d - \beta) + d^2 m_2^{(0)} \frac{1}{c^2 + p^2} + d + \frac{d(d-\beta)}{(c^2 + p^2)} - \beta \right] =$$

$$= \frac{1}{c^2 + p^2} \left[c^2 - p^2 + d(1 + m_2^{(0)}) - \beta(1 + m_2^{(0)}) + d^2 \frac{1}{c^2 + p^2} (1 + m_2^{(0)}) \right]$$

$$1 + m_2^{(0)} = 1 + \frac{c^2 - p^2}{c^2 + p^2} = \frac{2c^2}{c^2 + p^2} \quad \text{— можно упрощ. geo.}$$

$$m_z(\vec{p}) = m_z^{(0)}(p) + \frac{2c^2}{(c^2 + p^2)^2} \left[\alpha - \beta + \frac{d^2}{c^2 + p^2} \right], \quad \alpha = 2(1+p^2)(\vec{p} \cdot \vec{S})$$

$$m_z(\vec{p}) = m_z^{(0)}(p) + \frac{2c^2}{(c^2 + p^2)^2} \left[2(1+p^2)(\vec{p} \cdot \vec{S}) - s^2(1+p^2) + (\vec{a} \cdot \vec{p})^2 + \frac{4(1+p^2)^2}{c^2 + p^2} (\vec{p} \cdot \vec{S})^2 \right]$$

$$\vec{a} \cdot \vec{p} = 2c (\vec{S} \times \vec{S}) \cdot \hat{z} = 2c (\vec{S} \times \hat{z}) \cdot \vec{p} = 2c (\hat{z} \times \vec{p}) \cdot \vec{S}$$

$$m_z(\vec{p}) = m_z^{(0)}(p) + \frac{2c^2}{(c^2 + p^2)^2} \left[-s^2(1+p^2) + 2(1+p^2)(\vec{p} \cdot \vec{S}) + (\vec{a} \cdot \vec{p})^2 + \frac{4(1+p^2)^2}{c^2 + p^2} (\vec{p} \cdot \vec{S})^2 \right]$$

параметры $m_z(\vec{p})$ of shifted vortex no $\vec{a}$ and $\vec{S} = \vec{X}/R - f(z) = \frac{1}{c} [iz + \dots]_{\text{model}}$

$$m_z(\vec{p}) = m_z^{(0)}(p) + g_0(p) + g_1(p)(\vec{p} \cdot \vec{S}) + g_2(\vec{a} \cdot \vec{p})^2 + g_3(p)(\vec{p} \cdot \vec{S})^2$$

$$W_{\text{int}}(\text{core-core}) \sim \int d\varphi e^{i\mu\varphi} \int d\varphi' e^{-i\mu\varphi'} \left[f_0(p) + g_1(p)(\vec{p} \cdot \vec{S}_1) + g_2(\vec{a}_1 \cdot \vec{p})^2 + g_3(p)(\vec{p} \cdot \vec{S}_1)^2 \right] \times$$

$$\times \left[f_0(p') + g_1(p')(\vec{p}' \cdot \vec{S}_2) + g_2(\vec{a}_2 \cdot \vec{p}')^2 + g_3(p')(\vec{p}' \cdot \vec{S}_2)^2 \right]$$

$$\int d\varphi m_z(\vec{p}) e^{i\mu\varphi} = \int d\varphi e^{i\mu\varphi} \left[f_0 + g_1(\vec{p} \cdot \vec{S}) + g_2(\vec{a} \cdot \vec{p})^2 + g_3(\vec{p} \cdot \vec{S})^2 \right]$$

$$\int d\varphi e^{i\mu\varphi} f_0 = f_0 2\pi \delta_{\mu 0}, \quad \int d\varphi e^{i\mu\varphi} (\vec{p} \cdot \vec{S}) = p \int d\varphi e^{i\mu\varphi} (\cos\varphi S_x + \sin\varphi S_y) =$$

$$= \pi p \left[S_x (\delta_{\mu,1} + \delta_{\mu,-1}) + S_y \frac{1}{2i} (\delta_{\mu,-1} - \delta_{\mu,1}) \right]$$

$$\text{Thus } W_{\text{int}}(\text{core-volume}) \approx \int d\varphi d\varphi' (\vec{a}_1 \cdot \vec{p}) (\vec{p}' \cdot \vec{S}_2) e^{i\mu(\varphi - \varphi')} \sim (\vec{a}_1 \cdot \vec{S}_2)$$

$$W_{\text{int}}(\text{core-core}) \sim \int d\varphi d\varphi' e^{i\mu\varphi} e^{-i\mu\varphi'} \left[f_0(p) f_0(p') + g_1(p) g_1(p') (\vec{p} \cdot \vec{S}_1) (\vec{p}' \cdot \vec{S}_2) + \right.$$

$$\left. + f_0(p) g_2(p') (\vec{a}_2 \cdot \vec{p}')^2 + f_0(p) g_3(p') (\vec{p}' \cdot \vec{S}_2)^2 + f_0(p') g_2(p) (\vec{a}_1 \cdot \vec{p})^2 + f_0(p') g_3(p) (\vec{p} \cdot \vec{S}_1)^2 \right]$$

we consider vector $\vec{a}_j$ and $\vec{S}_j$. p_j - new variable.

But when considering $f_0(p) = m_z^{(0)}(p) + \frac{2c^2}{(c^2 + p^2)^2} (-s^2)(1+p^2)$ -

- gives nonrenormalizable α_j , corresponding to meson mass.

$$\rightarrow W_{\text{int}}(\text{core-core}) \sim \int d\varphi d\varphi' e^{i\mu(\varphi - \varphi')} g_1(p) g_1(p') (\vec{p} \cdot \vec{S}_1) (\vec{p}' \cdot \vec{S}_2) \sim (\vec{S}_1 \cdot \vec{S}_2)$$

Even $W_{\text{int}}(\text{core-core}) \sim (\vec{S}_1 \cdot \vec{S}_2) \sim (\vec{X}_1 \cdot \vec{X}_2)$, to find answer we substitute in $(\vec{X}_1 - \vec{X}_2)^2$ - then we have good finite energy spectrum simulated Kasten.

But also $W_{\text{int}}(\text{core-volume})$. Пусть $p_1 p_2 = -1$ и

$$W_{\text{int}}(\text{core-volume}) = \lambda \hat{z} \cdot (\vec{X}_1 \times \vec{X}_2). \quad \text{уравнение на это и аналог. уравнение cores?}$$

$$W_m(\vec{X}_1, \vec{X}_2) = \frac{1}{2}(\kappa + \mu) X_1^2 + \frac{1}{2}(\kappa + \mu) X_2^2 - \frac{\mu}{2} (\vec{X}_1 - \vec{X}_2)^2 + \lambda \hat{z} \cdot (\vec{X}_1 \times \vec{X}_2)$$

$$W_m(\vec{X}_1, \vec{X}_2) = \frac{1}{2} \kappa X_1^2 + \frac{1}{2} \kappa X_2^2 + \mu (\vec{X}_1 \cdot \vec{X}_2) + \lambda \hat{z} \cdot (\vec{X}_1 \times \vec{X}_2)$$

$$\left\{ \begin{aligned} \vec{G} \times \frac{d\vec{X}_1}{dt} &= \alpha \vec{X}_1 + \mu \vec{X}_2 + \lambda \vec{X}_2 \times \hat{z} \\ -\vec{G} \times \frac{d\vec{X}_2}{dt} &= \alpha \vec{X}_2 + \mu \vec{X}_1 + \lambda \hat{z} \times \vec{X}_1 \end{aligned} \right\} - \text{сн. 5D p.p.2 = -1 or 17/11}$$

$$\lambda = F(C_1, C_2, p, p_2)$$

$$\begin{aligned} \vec{G} \times \vec{X}_1 &= \alpha \vec{X}_1 + \mu \vec{X}_2 + \lambda \vec{X}_2 \times \hat{z}, \quad \vec{G} \times \vec{G} \times \vec{X}_1 = -G^2 \vec{X}_1 = \alpha \vec{G} \times \vec{X}_1 + \mu \vec{G} \times \vec{X}_2 + \lambda \vec{G} \times \vec{X}_2 \times \hat{z} \\ \vec{G} \times \vec{X}_2 \times \hat{z} &= \vec{X}_2 (\vec{G} \cdot \hat{z}) - \hat{z} (\vec{G} \cdot \vec{X}_2) = G_z \vec{X}_2, \quad -\vec{G} \times \vec{G} \times \vec{X}_2 = \alpha \vec{G} \times \vec{X}_2 + \mu \vec{G} \times \vec{X}_1 + \lambda \vec{G} \times \hat{z} \times \vec{X}_1 \\ \vec{G} \times \hat{z} \times \vec{X}_1 &= \hat{z} (\vec{G} \cdot \vec{X}_1) - \vec{X}_1 (\vec{G} \cdot \hat{z}) = -G_z \vec{X}_1 = -G^2 \vec{X}_2 \\ \vec{G} \times \vec{X}_1 &= \alpha \frac{(-1)}{G^2} [\alpha \vec{G} \times \vec{X}_1 + \mu \vec{G} \times \vec{X}_2 + \lambda \vec{G} \times \vec{X}_2 \times \hat{z}] + \mu \frac{1}{G^2} \vec{G} \times [\alpha \vec{X}_2 + \mu \vec{X}_1 + \lambda \hat{z} \times \vec{X}_1] + \\ &\quad - \frac{\lambda}{G^2} [\alpha \vec{X}_1 + \mu \vec{X}_2 + \lambda \vec{X}_2 \times \hat{z}] + \lambda \frac{\vec{G} \times [\alpha \vec{X}_2 + \mu \vec{X}_1 + \lambda \hat{z} \times \vec{X}_1]}{G^2} \times \hat{z} \\ \ddot{\vec{X}}_1 &= -\frac{\alpha}{G^2} [\alpha \vec{X}_1 + \mu \vec{X}_2 + \lambda \vec{X}_2 \times \hat{z}] + \frac{\mu}{G^2} [\alpha \vec{X}_2 + \mu \vec{X}_1 + \lambda \hat{z} \times \vec{X}_1] + \frac{\lambda}{G^2} [\alpha \vec{X}_2 + \mu \vec{X}_1 + \lambda \hat{z} \times \vec{X}_1] \times \hat{z} \\ \ddot{\vec{X}}_1 &= -\frac{1}{G^2} (\alpha^2 - \mu^2) \vec{X}_1 + \frac{\lambda}{G^2} [-\alpha \vec{X}_2 \times \hat{z} + \mu \hat{z} \times \vec{X}_1 + \alpha \vec{X}_2 \times \hat{z} + \mu \vec{X}_1 \times \hat{z} + \lambda \hat{z} \times \vec{X}_1 \times \hat{z}] \\ \hat{z} \times \vec{X}_1 \times \hat{z} &= \vec{X}_1 (\hat{z} \cdot \hat{z}) - \hat{z} (\hat{z} \cdot \vec{X}_1) = \vec{X}_1 \Rightarrow \ddot{\vec{X}}_1 = -\frac{1}{G^2} (\alpha^2 - \mu^2 - \lambda^2) \vec{X}_1 = -\omega_0^2 \vec{X}_1 \end{aligned}$$

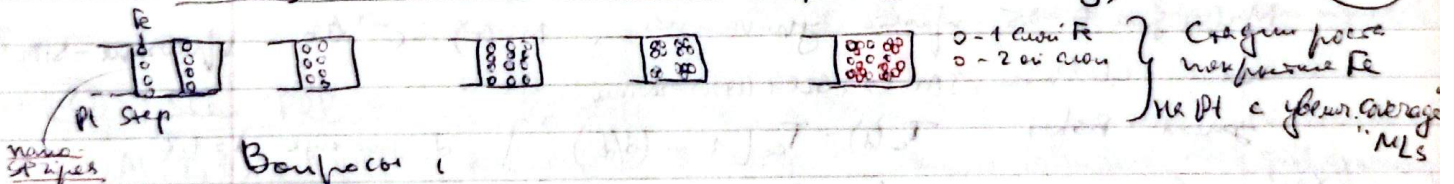
Т.е. частота затуха $\omega_0^2 = \frac{1}{G^2} (\alpha^2 - \mu^2 - \lambda^2)$ - огиба, а глупе -

уравнение, т.к. 5D две компоненты $\vec{X}_1(t)$ одинаковые.

Несомненно $\vec{X}_1 = (X_1, Y_1)$. Все 5D замкнута через $\vec{X}_1, \vec{X}_2$ - как векторы и нет анизотропии м. coupling rule $d_1 X_1 X_2 + d_2 Y_1 Y_2$ с $d_1 \neq d_2$.

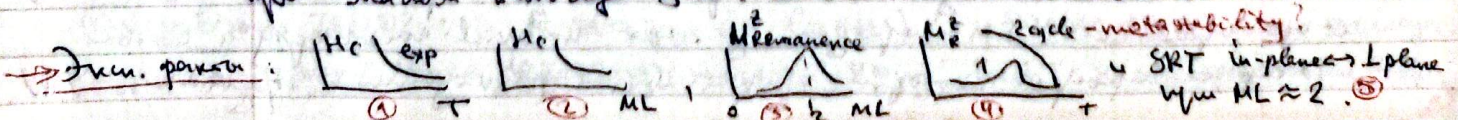
Fe/Pt(111) - SRT = F(ML of R) - Nanocheins / Nanostripes

Анализ Fe-nanostripes на Pt-поверх (Ruifeng Cheng) и сослуж. (2/11)



Вопросы:

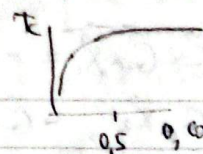
- 1) минимальная величина $M(H)$ без микромгнетизма - это микромгнетизм или superparamagnetism?
- 2) $\frac{2Ku(T)}{M_s} d_{SR} = 4\pi D_{eff} M_s$ - размерность осн. уравнения? JAP 69 5209 (1991)
или $d_{SR} \sim M_s^2(T)$, т.к. d_{SR} от T не зависит.
- 3) $K_c(T) \sim e^{-(dT/T_c)}$ это требует "pinning" или thermoinactivation? Сн. экв. фазы $K_c(T) \sim e^{-T}$ универсальн. сн. 1D Ising m. spin-blocks.
- 4) Все экв. $M(H)$ H_c, M_R , нанесенные в paper, не применимы в $H \perp$ surface?
- 5) Какое поле T-cycling (heating/cooling) в узком диапазоне M_R и H_c при малом "coverage" of Fe? Если м. нестационарные состояния M_R ?



Обзор мис. по ML-islands, stripes ^{of ferromagnetic} на нанострукт. медн. нити. и др.

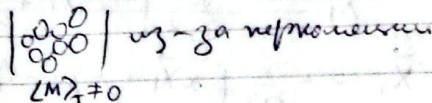
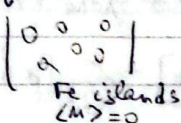
① Elmers - PRL 04: Fe (110) on W (110)

$$\frac{T_c(ML) - T_c(0)}{T_c(ML)} = \left(\frac{\theta}{\theta_0}\right)^{-\lambda}$$



$T_c(0) \approx M_2(t)$ зависит от 2D Ising.
 $T_c(\theta) \rightarrow 0$ при $\theta \rightarrow \theta_0$ и
 exp. найдено $\theta_0 = 0.07$ и
 ширина Fe-Stripe $w_0 = \theta_0 \cdot W \approx 0.8 \text{ nm}$

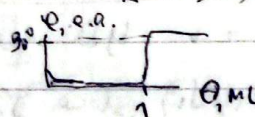
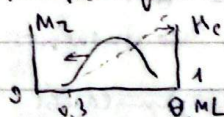
т.е. Fe stripes на W ферромагнитна вплоть до покрытия 0.8nm
 Fe stripe ферромагнитна при комнатной и при высоких $T > 500 \text{ K}$,
 а при $T = 300 \text{ K}$ Fe образует островки, которые SPM и содержат FM при более θ
 (MLR) за счет некомпенсации.



ОПН || $[1\bar{1}0]$ - и линия бинар. н.т., линия etal удерживается по ОПН stripe

+ ② D. Li - PRB 01: Fe на Pd (110)

FM при $\theta_{Fe} > ML \cdot 0.3$. Но при $\theta < 0.7$ M_z и $M_s(T) \sim e^{-T/T_0}$
 assume of 2D-FM systems.



опн direction

SRT of $\perp$ ОПН to in-plane ОПН $\sim$ coverage, $ML(\theta)$.

$\theta = 0.5$: M_z и M_s резко возрастают с T , а $H_c(T) = \text{const}$

$\theta = 0.7$: M_z, M_s, H_c - резко возрастают с T . $H_c(T)$ похоже на $g_{\text{an. media}}$.

$M_z(T)$ при $\theta = 0.5$ и более убывает, $M_z(T) \sim e^{-T/T_0}$. Вспомогательная - SPM?

набл. от Chien, JAP '69, S267 (1991) - review.

Зависит Fisher $T_c(d) = T_{c0} [1 - (d/d_0)^{-\lambda}]$, d - coverage, $\lambda = 1.2$

Хорошо совпадает. Critical stripe width где FM $w_0 = W \cdot d_0 \approx 6 \text{ \AA}$

+ ③ Gambardella 'Nature' 02 FM in 1D-chains Co on Pt

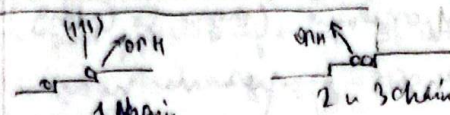
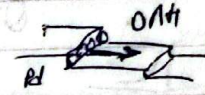
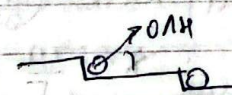
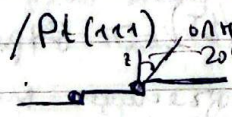
Nature 03 (K, ML Co-магнетизм)

SPM - recognize to 1D-chain too strong Co-magnetism (10-20 atoms),

иногда коопер. из-за exchange $\rightarrow$ goes to ferromagnetic $M(H)$ -exp. X-мед
 $E_{\text{an}} = 2.0 \text{ meV/atom}$ to 1D усилена $\rightarrow E_{\text{an}}$ (bi-chain или ML, bulk Co).

1D-Ising model (Landau) goes too усилена с $N > N_c = e^{2J/T}$, J-exch.
 взаимодействие на границе $\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow$

Увеличение концентрации железа и при конформации ферромагнитной $\sim 10^2$
 может проявиться FM-поведение за счет uniaxial anisotropy и coher.
 эффекта. Эффект, который проявляется при взаимодействии, образующих DW.

- ④ Gambardella et al. PRL'04 Co на Pt (997), cryogenic Pt(111)
 OAH change  $\frac{OAH \uparrow}{0.000 \text{ fcc} \rightarrow 0.000 \text{ fcc}} \frac{Co}{Pt}$
 mono chains $\approx 1 \text{ ML of Co}$
 Too goryzaro by $M_z(B, T)$, σ_{xx} & $M(H, T)$ - measurements
 Hydroor Coherent rotation & Superparamagnetism. $H_c(ML)$ $\approx 1000 \text{ Oe}$.
- ⑤ Dorantes-Davila PRL'98: ab-initio, MAE ybawr. & yuenbennenn
 unipolar & yuenor stripe.
- ⑥ Robles-PRB'00: Co wires на Pd(100) - ab initio calcs, m. coupling wires orob uua.
- ⑦ Berger - PRL'92: Co на Cu(111), OAH || stripe edge
- ⑧ H.J. Choi et al. PRL'99: Fe/Pd(001)
 OAH is in-plane & $\perp$ to stripe edges. $T_c(RML)$. 
 Step-induced mag. moment of Pd. $E_{an} \sim \text{step density}$.
- ⑨ Y.Z. Wu, PRB'03: Co/Cu(001)  $\gamma_{\text{interface}}$
 unipolar $2 \pi M_z^2$
- ⑩ A. Shick, PRB'04: Co/Pt(111) 
 Ab-initio calc. - tilted OAH or uopuauu K. uua.
 OAH-tilting $\approx 30^\circ$ due Fe/Pt(111) - JMM₂ in press.
- ⑪ Y.Z. Wu, PRB'02: Fe on Ag(001). Step-induced anisotropy uo uokawzobane
 bounze steps, uoouuuu E_{an} - strain mismatch.
- + ⑫ J. Shen 'PRB-97: Fe stripes on Cu(111).
 $M(T, t)$ - georce ouuennu na ouabe 1D Ising & Glauber block-spin dynamics.
 Eoou anouuouuouou bounennu uueeueuua $M(H, T, dH/dt)$. IE³ 32-1567 (1998).
 Taxue J. Shen - JAP'97 & Skomski - JAP'97 - u/uuennu Ising-model, penouu. y.
 Fitted $K_1 = 0.55 - 0.80 \frac{\text{MJ}}{\text{m}^2} \rightarrow K_1^{\text{bulk}} \frac{\text{B}}{\text{A}} 1710$ gna Sub-ML mag. films. $M_s \sim \frac{3H}{4T}$
 $\approx 0.05 \text{ MJ/m}^2$.
- + ⑬ J. Shen, J.Ph. Cond. Mat. - 03: Fe on Cu(111) - oozop. Nanostripes uobts.
 u/uu ouuennu $M(H)$ uouowzobane q. $M_z(H)$ uo Shen'PRB-97
 Sez baekuu gon. comments
- + ⑭ D. Sander. J.Ph. Cond. Mat. - 04: oozop 7MB, exp. b ououuau
 (16, R603)
- ⑮ G. Brown, JAP'02: Fe stripes on Cu(111). Paocor $M_z(T)$ uouo ouuauy.
 uo ouen. Mouuo Heisenberg ouuau. anisotropy, Eoh. J na uouueuou uennu bulk.

3/XII Organ to Leasing office Synergy room 200 Feb. 3 & Jananubano
renting contract in the very morning (organ to Cathy).

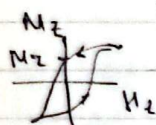
→ Nonpolar last version of Pro-sensor paper in Moscow ee Hoffmann
& very common in morning work in ferrofluids, zig arrays, $\frac{F}{AF}$ disk.

Shen' PRB'97 give nonpolar Ising model gas (02-01H)

$$\rightarrow \frac{M_z}{M_s} = \left[1 + \frac{2J}{T} (1-x^2) \right] x - \frac{\tau}{T} (1-x^2) \left[1 - \frac{4J\tau}{T} (1-4\tanh^2 \frac{J}{T}) \right], \quad x = \tanh \frac{h}{T}, \quad \Gamma = \Gamma_0 \exp\left(-\frac{K_v V_0}{kT}\right)$$

$\tau = \frac{1}{M_s} \frac{\partial H}{\partial t}$ - field sweep rate.

$$\frac{M_z}{M_s} = \frac{M_z(h=0)}{M_s} = -\frac{\tau}{T} \left[1 - \frac{4J\tau}{T} (1-4\tanh^2 \frac{J}{T}) \right] \neq 0 \text{ at } \tau \neq 0 \quad \tau = \frac{1}{M_s} \frac{\partial H}{\partial t} \neq 0$$



$$\tau < 0 \text{ const. } M_z > 0. \quad \frac{M_z}{(T)} = M_s / 4 \Gamma^{-1} \left[1 + \frac{4J|\tau|}{T} (1-4\tanh^2 \frac{J}{T}) \right]$$

Ordering $M_z(T) \sim |\tau| \Gamma^{-1} \sim \exp\left(\frac{K_v V_0}{4T}\right)$ - exp. nature of process in T

$M_z(H) = 0$ gives $H_c(T)$.

Ordering $H_c(T) \sim \tau \sim \left(\frac{\partial H_z}{\partial t}\right)$. $\left[1 + \frac{2J}{T} (1-x^2) \right] x - \frac{\tau}{T} (1-x^2) = 0, a = [1]$

$H_c(T), M_z(T)$ When $x = \tanh \frac{h}{T} \ll 1$; $x \approx \frac{\tau a}{T}$, $\frac{H_c(T)}{T} \approx |\tau| f(T) \Gamma_0^{-1} \exp\left(+\frac{K_v V_0}{kT}\right)$,

is Ising-Glauber $H_c(T) \sim T f(T) \exp\left(\frac{K_v V_0}{kT}\right) \sim \exp.$ nature of process in T.

~ $e^{T_0/T}$ Take in process magnetic ordering $M_z(T)$ D. Li - PRB'01 - Fe/Pd

• $H_c(T)$ - R. Cheng - manuscript Fe/Pt.

→ Shen-97 general paper, 200 $M_z(T, H, \tau)$ nonpolar
give small J in narrow hysteresis loop by Glauber dynamics
of 1D Ising model. Can factor in ordering in equilibrium
no ordering.

5/XII

Nano-chains & nano-stripes (ordering of 2/XII)

1+ (16) J. Hauschild, PRB-98: Fe stripes on W(110), 01H blocks. 1 stripes

Don't know, 200 FM-ordered in T_c magnetic by inter-
stripe dipolar coupling. Dimer spin-block $L \sim e^{T_0/T}$

Super-FM magnetic phase by m. coupling, then by 20
spin-freezing & magnetic Ising by nonpolar stripe.

1+ (17) Pratzner, PRL'01: Fe/W(110). Then, no ordering, 20 $\chi(T) \sim \frac{1}{T} e^{\Delta E/T}$

give nanostripes, 200 con. & 1D-Ising model in non-dimer. $\Delta E = 2J$ factor.
nonpolar spin-block $\uparrow \downarrow \uparrow \downarrow$. S. blocks prepared by $\Delta W < \delta < 1 \text{ nm}$
in energy 2J.

(18) Pratzner: PRB-02 u PRB-03: Fe stripes na W(110)

Wzrostowe / pacumowe Pratzner-PRL-01. $\chi(T) \sim e^{AE/T}$. Donosim. interstripe exchange uz - za najpoczniej Au na Pd. 1D-Ising nio bogiemie cinn - Snokob.

+ (19) L. Rego, PRB-01: mienime pienie yp. gbumenie $\langle S_i \rangle$ u $\langle S_i S_j \rangle$ Jue 2D-Ising model (racune $\approx$ ang'rai cin. (12) Shen PRB-97, Shen JAP-97) abno zannean 3D.

+ (20) J. Shen, PRB 56, 11134 (1997) - Bonoware paper na chais & Ban ML - uenore Fe na Cu (111). M_s fcc (Kurz) $\rightarrow$ bcc SRT. M_L w/mroga SRT of $\perp \rightarrow \parallel$ gbumenie ML) b fcc $\rightarrow$ bcc Transformation que Fe. Xare 2D percolation of stripes u continuous film magnetostates some gas buag $K_s/d = 9\pi N_{eff} M_s^2$.

Group meeting. Odyngiemie exp. yonuni Shima na ozgaru nano-spheres array u D-Vertex Dynamics. Kriston w/menana new simulations for $F_1/N/F_2$ disk e pazymen yonunenani F_1, F_2 azeb.

Delega e Kuchma: field sweep rate dh/dt yona uniaa belega ogunobium u $M(H)$ nezio b $H_{max} = 6$ koe uniaa za 15-20 min.

K. H. Fischer, J. A. Hertz. Spin glasses, Cambridge Uni. Press, 1991.

Bopactmow niohexoga
$$\frac{W(S_i \rightarrow -S_i)}{W(-S_i \rightarrow S_i)} = \frac{e^{-\beta E_i}}{e^{\beta E_i}} = \frac{1 - S_i \tanh \beta \hat{h}_i}{1 + S_i \tanh \beta \hat{h}_i}$$

Ansatz by Glauber: $W(S_i \rightarrow -S_i) = \frac{1}{2\tau_0} [1 - S_i \tanh \beta \hat{h}_i]$, $S_i = \pm 1$ Ising's spin

$$E_i = S_i \hat{h}_i, \hat{h}_i = h_i + \sum_j J_{ij} S_j$$

$$e^{-\beta E_i} = e^{-\beta S_i \hat{h}_i} = |S_i = +1| = e^{-\beta \hat{h}_i}, 1 - S_i \tanh \beta \hat{h}_i = 1 - \tanh \beta \hat{h}_i = \frac{e^{\beta \hat{h}_i} - e^{-\beta \hat{h}_i}}{e^{\beta \hat{h}_i} + e^{-\beta \hat{h}_i}} = \frac{1}{e^+ + e^-} [e^+ e^- - e^+ + e^-] = \frac{1}{e^+ + e^-} 2e^- = \frac{2}{e^+ + e^-} e^{-\beta \hat{h}_i} \text{ que } S_i = +1$$

$$e^{-\beta S_i \hat{h}_i} = |S_i = -1| = e^{\beta \hat{h}_i}, 1 + \tanh \beta \hat{h}_i = \frac{2}{e^+ + e^-} e^{\beta \hat{h}_i}$$

$$\langle S_i \rangle(t) = \sum_{\{S\}} S_i P(S_1, \dots, S_i, \dots, S_N, t)$$

$$\frac{d\langle S_i \rangle}{dt} = \sum_{\{S\}} S_i \frac{dP}{dt} = \sum_{\{S\}} S_i \left[- \sum_j W(S_j \rightarrow -S_j) P(\cdot S_j) + \sum_j W(-S_j \rightarrow S_j) P(\cdot -S_j) \right] =$$

$$= \frac{1}{2\tau_0} \sum_{\{S\}} S_i \left[- \sum_j (1 - S_j \tanh \beta \hat{h}_j) P(\cdot S_j) + \sum_j (1 + S_j \tanh \beta \hat{h}_j) P(\cdot -S_j) \right] =$$

$$= \frac{1}{2\tau_0} \sum_{\{S\}} S_i [\dots] = -\frac{1}{\tau_0} [\langle S_i \rangle - \langle \tanh \beta \hat{h}_i \rangle] - \text{yp-e nioysep gubemnie.}$$

$$\tau_0 \frac{d\langle S_i \rangle}{dt} = - [\langle S_i \rangle - \langle \tanh \hat{h}_i \rangle], \quad \hat{h}_i = h_i + \sum_j J_{ij} S_j$$

MFA: $\langle \tanh \hat{h}_i \rangle = \tanh \langle \hat{h}_i \rangle, \quad \langle \hat{h}_i \rangle = h_i + \sum_j J_{ij} \langle S_j \rangle$

$$-\tau_0 \frac{d\langle S_i \rangle}{dt} = \langle S_i \rangle - \tanh \langle \hat{h}_i \rangle - \text{уменьшо по eq. усложно}$$

Баша, но не дисс. Бадер, уагаз то он констатация на мое ипогу.

Намечание APL - Dvortex dyn. paper.

7/XII Nano-chains / nano-stripes Озлог же Fe/Pt(111) paper Ruitma.

(21) Vedmedenko, PRL '04: Fe/W(110) - гомогенные цепи в nano-stripes

боже не 1D chain, а замкнутые кольца, опущенные, вращаясь (170) вокруг осей. Обмен и упругие взаимодействия.

(22) Kawakami, PRL-96: Fe/Ag(001) ML-stripes имеют ген. in-plane m. anisotropy, вращаясь ~ stripe density. Fe/Ag(001) имеет $K_2(1)$, то упругие $\sim$ SRT $1 \rightarrow 11$ и μ ML ≈ 6 .

8/XII (23) Schmann, PRB-94: Co/Cu(001). $\chi(d) \sim (d_c - d)^{-2.4}$ - экстр. настраивать (d)

2D-percolation непереход Co в FM state и μ $d_c(\text{book}) \approx 1.3 \text{ ML}$. $t_m \approx 1 \text{ s}$.

(24) S. Hoje - PRB 2000: SML Co/Cu(110). Уменьша d_c - coverage гомогенно $K_1 \approx 0$, уменьшение relax.time τ и увеличение FM на микроскопическом уровне.

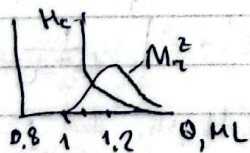
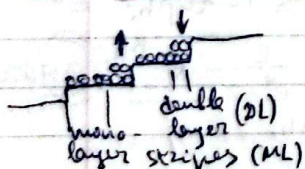
± (25) C. Coulon - PRB '04: Glauber dynamics 1D chain

$$H = -2J \sum_i \vec{S}_i \vec{S}_{i+1} - D \sum_i S_{iz}^2, \quad \tau(\tau) = \tau_{\text{block}}(\tau) e^{8JS^2/\tau} = \tau_0 e^{(8J+D)S^2/\tau}$$

Есть абн. $\chi(\tau)$ сущности chain-blocks, то и μ $T \rightarrow 0$ же $\chi(\tau) \sim \frac{1}{\tau} e^{UJ/T}$ как у Skonski '97

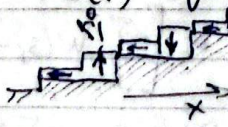
RJ Glauber, J. Math. Phys. 4, 294 (1963).

± (26) H. Elmers, PRB-99: Fe/W(110) - есть все осн. эксперим. paper Ruitma.



и μ $\theta > 1 \text{ ML}$ боимся DL-stripes с $\vec{M} \perp$ поверхности, они из-за dip. interact. взаимодействуют AF

ML-stripes имеют $\vec{M}$ in-plane. Т.е. с ростом θ , и μ $\theta_c = 1$ есть SRT $11 \rightarrow 1$ за счет DL-stripes отклонения. $H_c(\tau)$ и μ $\theta = 1.2 \text{ ML}$ тогда exponential с TT , и μ μ 10^3 раз на 50 K.



— картина $\theta(x)$ (по $\vec{M}$) микроскопическая картина взаимодействия ML.

DL представляет собой цепочку, и μ μ $\vec{M}$ in-plane (а.а.), и μ μ $\vec{M} \perp$ и с $H_2 \perp$ микр. взаимодействием как SD-SW particles с uniax. anisotropy, габариты близки H_c^2 . с ростом ML - coverage есть DL-stripe

Escaneado con CamScanner

Comments:

$H_2 \uparrow \quad \downarrow \downarrow \downarrow \quad \uparrow \uparrow \uparrow$ $\uparrow \uparrow \uparrow \uparrow$ - genuine DW b usenorka, just spin flipping,
 (E_1) (E_2) $E_2 < E_1$ за счет Zeeman, т.к. $E_{an} = E_{ex}$ отменяются.


 - rare "saddle" point (max E) between minima of reaction Icing,
 barrier is zbarrier or J , a random or H_1 u H_2 . Dissociation DW (10)
 gives thermo-activated $\propto e^{-E/T} \Rightarrow H_0 \propto e^{-E/T}$.

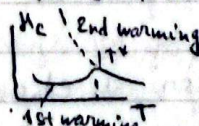
Bugrem 2-а тина DW: толщине $\sim 6\text{nm}$ и узкие $\sim 2\text{nm}$ в узких местах stripes.
Bugrem NL (in-plane) / DL ($\perp$ plane) - рефлекование stripes как у Elmers-99.

(28) Pietzsch, Science-01: Fe/W(110) stripes. Spin-polarized STM measurements reveal reversal stripes-DL in H₂. $\delta_{DL} \approx 6 \text{ nm}$. Rotation 360° - DL can rotate through 180° - DL reversal, which can be achieved by a magnetic field. AF - state of domains in DL-Fe is observed only upon cooling or Te & Tm. (equil.)

1. OAH w/м $\theta_{Fe} < 4.3 \text{ mV}$, w/c Fe-fcc с separation. исключаем: $t_{cr} = \frac{2K_c}{2B_{Fe} + 4\pi M^2}$
- крив. + толщина SRT $I \rightarrow II$. $\Delta E = E_{\perp} - E_{\parallel}$; $E_{\parallel}$, $E_{\perp}$ in-plane, out-of-plane strains

K_s (irrevers) changes c T-activated structure changes. $K_s(T) = K(T) + K_{irr}$

Transformation, neofascism.



- $H_c(T) \sim e^{T_0/T}$ where T_0 is 2nd warming.

$K(t)$ - 1st cycle - pacer $\rightarrow$ 1st warming $\rightarrow$ Re - overproof of T^* , work per island stops.

$\chi_c(\tau)$ - 2nd cycle exp - wagger $\sim T$, t.k. ocr problem yone barpocum, see exp. T-activation (Ising model)

③0 F.T. Hempel, Adv. Phys. - 98; обзор. Крит. показаний 2D и 3D-моделей. Анизотропия induced by steps. (SRT) — утвержд. т.ч. тесно связан со структурой, переходом в bulk-phase.

9/XII Ivanova Slavin, Thymal, Chantrell, Bader, Krüsten.
 Current version of b. conditions paper. Paper has
 structure & Figs. of D-Vortex dynamics paper - APL, now in Bader's.

→ Bozomovskoe oboznamenie zhen. Dvukhna Fe/Pt(111)

1) SRT $L \rightarrow \infty$ chezani c fcc (fcc) $\rightarrow$ bcc nepexogom Fe c $K_1 \neq 0$

2) $H_c(ML)$ nagenne (Elmers'99) oboznen. na osnove oshchisheniya u
 posra 2-ML stripes chezy 1st ML, 2-ML (OL) stripes umenya
 nepremennogo nulyony () - nomen na SW-razmeru c H_c vysokoi.
 c posra ML - gromozhnye DW bogo stripe raznomysheniya u $H_c \downarrow$.
 AF-ordering of 2-ML stripes uz. za interstripe m. coupling
 $M_z^2(ML) \propto \frac{1}{d_c ML}$ oshchisheniya SRT $L \rightarrow \infty$ c posra $\theta(ML)$.

3) $H_c(T)$ - exp. nagenne c posra T oshchisheniya neprobnoe dynamics D-Ising -
 - flipping of spin-blocks. $H_c(T) \sim e^{Kv_0/kT}$

4) $M_z(T)$ - neprobnoe oboznen c neprobnoe nepexogom fcc $\rightarrow$ bcc Fe.
 $H_c(T) \sim e^{Kv_0/kT}$ yshchisheniya c bozoro T-umka.

D-Vortex statics

10/XII Bozoga, Krüsten o cosoelenn D-Vortex-statics paper.
 Ei nomen nalyte posra cosoelenn in-plane SP $\uparrow$ state,
 korotse (PEEM-exp) c shchisheniya nepexogom D-Vortex State.
 In-plane SP state umenya bozoro oboznenie neprobnoe
 & double-layer P/M/F disk na oshchisheniya c isolated disk

Nano chains/stripes

Bozoga c Krüsten o nano chain/stripes Fe/Pt(111). oboznenie
 li nomen $H_c(T)$, $H_c(ML)$, $M_z(T)$, $M_z(ML)$ korotse nomen oshchisheniya
 u oshchisheniya nomen cbov zhen to paper.

Ushchisheniya SRT nomen $M_z \approx 2$ $L \rightarrow \infty$ nomen, $2Ks/d_c = 4\pi DM_s^2$ - oshchisheniya
 oshchisheniya ypr-e.
 Nomen changes to "Fe/Pt-stripes" paper.

Bound. conditions - stripe dip. field

Rezagovsheniya zhen paper u change of $h_s(x)$, $h_v(x)$ - bozoro
 - Section.

Bozoro nomen $\int_{-w/2}^{w/2} dx [h_s, h_v] \sim (h_s, h_v) \cdot L$ oshchisheniya, oshchisheniya (cu. B/X)
 $\langle h_s(x) \rangle_L = \frac{1}{2} h_s(w/2)L$, no $\langle h_v(x) \rangle_L = h_v(w/2)L$. Eshchisheniya umenya
 yshchisheniya no x & oshchisheniya $[w/2 \sim L, w/2]$ bozoro surface $x = w/2$ of stripe,
 to "Pinnig" d yshchisheniya & "2" paza! Eshchisheniya c PRBOZ-stripes,

где $m_x(x)$ не дано. т.е. PRB'02 unknown. $\int dx H_m(x) = L \cdot H_m(w/2)$

$$h_s^x = 2\pi m_x \theta(x-w/2) + m_x F(z/L), \quad F(z) = 2z \ln(1 + \frac{z^2}{L^2}) + 4 \operatorname{arctg} \frac{z}{L}, \quad z = x - w/2.$$

$$h_v^x(x) = (-1) \int_{-w/2}^{w/2} dx' \frac{\partial m_x(x')}{\partial x'} \left[2\pi \theta(z) - 2 \frac{z}{L} \ln\left(\frac{z^2 + L^2}{L^2}\right) - 4 \operatorname{arctg} \frac{z}{L} \right], \quad z = x - x'.$$

$$\begin{cases} h_s^x(x) = m_x(w/2) \left[-2\pi \theta(z) + F(z/L) \right]_{z=x-w/2}^{z=w/2-x}, & m_x(x) = m_x(w/2) \text{ берем из-за } \int(\dots) \\ h_v^x(x) = \int_{-w/2}^{w/2} dx' \frac{\partial m_x(x')}{\partial x'} \left[-2\pi \theta(z) + F(z/L) \right]_{z=x-x'}^{z=w/2-x}, & x \rightarrow \frac{w}{2} - 0, \quad z = x - x' > 0 \text{ не считаем.} \end{cases}$$

Напрям. $h_s(x)$ и $h_v(x)$ равны, но для $z \geq 0$.

$$h_s(w/2) = -2\pi m_x(w/2) \quad \text{н/м} \quad x \rightarrow \frac{w}{2} - 0. \quad h_v(x \rightarrow w/2 - 0) \text{ берем. считаем, где}$$

$$h_v(x) = m'_x(w/2) \int_{-w/2}^{w/2} dx' \left[-2\pi \theta(z) + F(z/L) \right]_{z=x-x'}^{z=w/2-x} \quad \text{берем } \frac{\partial m_x}{\partial x} \text{ из } \int(\dots).$$

$$h_v(w/2) = m'_x(w/2) \int_{-w/2}^{w/2} dx' \Phi(z) \Big|_{z=\frac{w}{2}-x'}^{z=\frac{w}{2}-x} = \left| \frac{w}{2} - x' = t \right| = m'_x(w/2) \int_0^w dt \Phi(t)$$

$$\Phi(t) = -2\pi \theta(t) + F(t/L). \quad \text{10/12} \text{ показано, что } \theta(t) \text{ так complex. с } \operatorname{arctg} t \Big|_{L \rightarrow 0}$$

$$h_v(w/2) = m'_x(w/2) \int_0^w dt \cdot \frac{2t}{L} \ln\left(\frac{t^2 + L^2}{L^2}\right) = m'_x(w/2) w \cdot I_1(V_2, p)$$

Не показано, что $\frac{\partial m_x(x')}{\partial x'}$ можно брать из-за $\int(\dots)$.

$$\text{Итого } \frac{\partial m_x(x')}{\partial x'} = \frac{\partial m_x(x)}{\partial x} + \frac{\partial^2 m_x(x)}{\partial x^2} (x' - x) + \dots, \quad \text{в PRB'02 было показано}$$

что можно $\frac{\partial^2 m_x}{\partial x^2}$ и выше можно игнорировать н/м $p \rightarrow 0$.

$$\begin{aligned} h_v(x) &= (-4) \int_{-w/2}^{w/2} dx' \frac{\partial m_x(x')}{\partial x'} \int_0^\infty dk \frac{f(kL)}{k} \sin k(x-x') = (-4) \frac{\partial m_x(x)}{\partial x} \int_{-w/2}^{w/2} dx' \int_0^\infty dk \frac{f(kL)}{k} \sin k(x-x') = \\ &= (-4) m'_x(x) \int_{-w/2}^{w/2} dx' \int_0^\infty dk f(kL) \left[\sin kx \cdot \cos kx' - \sin kx' \cdot \cos kx \right] = (-8) m'_x(x) \int_0^\infty \frac{dk}{k^2} f(kL) \sin kx \cdot \sin \frac{k}{2} \end{aligned}$$

$$h_v(w/2) = (-8) m'_x(w/2) \int_0^\infty \frac{dk}{k^2} f(kL) \sin^2 \frac{k}{2} = (-8) m'_x(w/2) w a(p), \quad a(p) = \int_0^\infty \frac{dk}{k^2} \sin^2 \frac{k}{2} \cdot f(kL)$$

$a(p)$ - как-то $\sim p \ln p$.

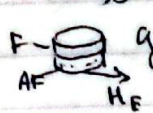
Regan's problem B. cond't - paper $\rightarrow$ 17/11 - deadline DOE posters. (13/11)

Group meeting: Shima - arrays of $\diamond \diamond \diamond$ Rept Zabus & He (book) = 60 kcal, coupling;

Kristen - noncommutative vortex - statics paper; Val. - nonrotational stripe-line experiment no single/trilayer gliders to find TM-vortex modes.

Mr. Odzop no homo-chains/strips gone paper Pukha.

F/AF disk $\rightarrow$ Hoffmann's previous exp. data - n/м гомогенное состояние (J. Sert) no

F-  glycerolium gas. Han, n elsewhere symmetrically $\hat{H}_{\text{ext}} \hat{H}_E$,
где $\hat{H}_E$ - exchange bias field from AF disk-layer.

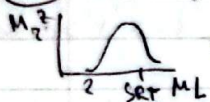
Nano-chains u nano-stripes, SRT



+ (31) Pappas, PRB-92 : exp. SRT $I \rightarrow II$ б $Fe/Ag(100)$ u $Fe/Cu(100)$ на
 прыкладзе таго ж у T . O_2 чыста II -state
 T -залежнасць Fe/Cu адрозніваецца, а Fe/Ag - не.
 Fe/Ag тэжэс $K_{\perp}$ у T -cycling.



+ (32) Allensbach - PRL-92 : exp. $M_z \approx 0$ б адрозніваецца SRT, $SRT = f(d, T)$.



- на y Рундма на Fe/Al . $M_s \neq 0$ у SRT уз stripe-domain struct.

SRT c T на T : I -SD state $\rightarrow$ nucleation $\rightarrow I$ -stripe domains ($\approx SRT$) $\rightarrow$ irregular shaped in-plane domains

(33) Sellmann, PRB-01 : $X/Co/Au(111)$, $X = Au, W$. Ec MPD

з $K_{\perp}$ у Fe/Co H_c^2 з Co на W c ML на y Рундма,
 на адрозніваецца не.

(34) Dürkop - JMMM-97 u Elmers - JM3-99 : $Fe/W(110)$ - exp. на W ,
 no absorption of O, H_2, O_2 б SRT $I \rightarrow II$ c W на W .

(35) Pappas - PRL-90 : $Fe/Cu(100)$. SRT не c $fcc \rightarrow bcc$

у SRT ($K_{\perp} \approx 0$) $\langle M \rangle = 0$ - Мермін-Вагнер-2D. у W на W : dynamic SPM u domains- (stripe-d)

+ (36) Sander - PRL-04 : $Ni/Cu(001)$. $d_{001}(II \rightarrow I) = 11 ML$. б W на W $2 ML$ u $(II \rightarrow I)$
 на W на W H_2 . H_2 у W на W $K_{\perp}$ (elastic) u W на W SRT $II \rightarrow I$.
 I u II -states W на W W на W $P(H_2)$.

14/11

Решена задача по $M(H)$ за 10-20 min $\leftarrow H_{max} = 6$ kG.
 т.е. частота $H(t) \sim \omega/2\pi = \frac{1}{10-20 \text{ min}} \approx \frac{1}{10 \cdot 60 \text{ s}} \sim 0.001 \text{ Hz}$ - очень малая

Field sweep rate $\frac{dH}{dt} = \frac{\Delta H}{\Delta t} = \frac{6 \text{ kG}}{5 \text{ min}} = \frac{6 \text{ kG}}{60 \text{ s}} \approx 100 \frac{\text{G}}{\text{s}}$ - не мало,
 т.к. H_{max} достаточно велико.

Рассм. MFA по Glauber - model (6/11):

$$-\tau \frac{d\langle S_i \rangle}{dt} = \langle S_i \rangle - \tanh \beta \langle \hat{h}_i \rangle, \quad \langle \hat{h}_i \rangle = h_i + \sum_j J_{ij} \langle S_j \rangle, \quad S_i = \pm 1 - \text{Ising spin}$$

$h_i = \mu H = h$ or yzua ne zabymu, ext. H_z - field.

$$\sum_j J_{ij} \langle S_j \rangle = \langle S_j \rangle \sum_j J_{ij} = \langle S_j \rangle zJ, \quad J - \text{если } j=i \pm 1, \quad z=2 - \text{если } j=i \pm 1, \text{ иначе } 0$$

Обозначим $\langle S_i \rangle = m$, $zJ \rightarrow J$, тогда yd:

$$-\tau \frac{dm}{dt} = m - \tanh \beta (h + Jm), \quad h = h(t) \text{ буга } \begin{cases} h(t) = h_0 \cos \omega t \\ h(t) = \tau \cdot t, \quad \tau = \frac{dh}{dt}, \quad h < h_0. \end{cases}$$

н/м $\beta h \ll 1$ yd номер один миссиссиппи, тхххх.

$$-\tau \dot{m} = m - \beta (h + Jm), \quad \dot{m} = -\frac{1}{\tau} [1 - \beta J] m + \frac{\beta}{\tau} h(t)$$

Для 2D-Ising $\beta J_0 = 0.44$ (J_0 - если б ух. рассматриваем) .. $d = 1 - \beta J$.

$$\dot{m} = -\frac{d}{\tau} m + \frac{\beta}{\tau} h, \quad \text{н/м } h(t) = h_0 \cos \omega t$$

$$m(t) = C_1 \sin \omega t + C_2 \cos \omega t, \quad C_1 \omega \cos \omega t - C_2 \omega \sin \omega t = -\frac{d}{\tau} [C_1 \sin \omega t + C_2 \cos \omega t] + \frac{\beta}{\tau} h_0 \cos \omega t$$

$$\cos \omega t: \quad \omega C_1 = -\frac{d}{\tau} C_2 + \frac{\beta}{\tau} h_0, \quad \sin \omega t: \quad -C_2 \omega = -\frac{d}{\tau} C_1$$

$$C_2 = \frac{d}{\omega \tau} C_1, \quad \omega C_1 = -\frac{d}{\tau} \frac{d}{\omega \tau} C_1 + \frac{\beta}{\tau} h_0, \quad C_1 \left[\omega + \frac{d^2}{\omega \tau^2} \right] = \frac{\beta}{\tau} h_0$$

$$C_1 = \frac{\beta}{\tau} h_0 \frac{1}{\omega + \frac{d^2}{\omega \tau^2}} = \beta h_0 \frac{\omega \tau}{(\omega \tau)^2 + d^2}, \quad C_2 = \frac{d}{\omega \tau} C_1 = \beta h_0 d \frac{1}{\omega^2 \tau^2 + d^2}$$

$$m(t) = \beta h_0 \frac{\omega \tau}{(\omega \tau)^2 + d^2} \sin \omega t + \beta h_0 d \frac{1}{\omega^2 \tau^2 + d^2} \cos \omega t = \beta h_0 \left[\frac{\omega \tau}{\omega^2 \tau^2 + d^2} \sin \omega t + \frac{d}{\omega^2 \tau^2 + d^2} \cos \omega t \right]$$

$$\sin \varphi = \frac{\omega \tau}{\sqrt{(\omega \tau)^2 + d^2}} \Rightarrow m(t) = \beta h_0 \frac{\cos(\omega t - \varphi)}{\sqrt{(\omega \tau)^2 + d^2}} \quad \text{н/м } h(t) = h_0 \cos \omega t.$$

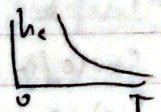
→ Quasi static H_c : $m(t) \sim \cos(\omega t - \varphi) = 0, \quad \omega t - \varphi = \pm \frac{\pi}{2}$

$$h_c = h(t) \Big|_{\omega t - \varphi = \pm \pi/2} = h_0 \cos(\varphi \pm \pi/2) = \pm h_0 \sin \varphi = \mp h_0 \frac{\omega \tau}{\sqrt{(\omega \tau)^2 + d^2}}$$

$$h_c = h_c(\omega \tau) = h_0 \frac{\omega \tau}{\sqrt{(\omega \tau)^2 + d^2}}, \quad d = 1 - \beta J - \text{dynamic coerc. force } H_c$$

Если $\omega \tau \ll d$, $h_c \approx \frac{h_0}{d} \omega \tau \sim \omega \tau$, $\tau = \tau_0 e^{K_B/kT}$ - spin-block relaxation time.

т.е. $h_c \sim \omega \tau_0 \exp(T_0/T)$ - exp. завис. $\leftarrow$ from T



Remanence: $h(t) = 0 \quad \omega t = \pm \pi/2, \quad m_z = m_0 \cos(\pm \pi/2 - \varphi) = m_0 \sin \varphi$

$$\rightarrow m_z = \frac{\beta h_0}{\sqrt{\omega^2 \tau^2 + d^2}} \frac{\omega \tau}{\sqrt{\omega^2 \tau^2 + d^2}} = \beta h_0 \frac{\omega \tau}{\omega^2 \tau^2 + d^2}, \quad m_z(T) \text{ some characteristic temperature, } \text{then } h_c(T), \text{ so } m_z(T) \sim e^{T_0/T} \text{ if } \omega \tau \ll 1.$$

$\ln(\rho J_m + \rho h)$, $\rho h \ll 1$ surface becomes. $\frac{\mu H}{kT} = \frac{10^3 \cdot 10^3 \cdot 10^3}{10^{16} \cdot 10^2 \cdot 10^3} = \frac{10^9}{10^{21}} = 10^{-12}$

for $\rho J_m \ll 1$ distance we have surface after $T \rightarrow 0$, $J \rightarrow 0$ becomes.

$\ln x = \ln x_0 + \frac{1}{x_0} \Delta$, $\ln(\rho J_m + \rho h) \approx \ln \rho J_m + \frac{1}{\rho J_m} \rho h$

4D: $\tau \frac{dm}{dt} = -m + \ln \rho J_m + \frac{1}{\rho J_m} \rho h(t)$ — source term.

$\ln \rho J_m = \rho J_m - \frac{1}{2}(\rho J)^2 m^2$, $\frac{1}{\rho J_m} = 1 - \ln^2 x = 1 - (\rho J)^2 m^2 + O(m^4)$

$\tau \frac{dm}{dt} = -(1 - \rho J)m - \frac{1}{2}(\rho J)^2 m^2 + [1 - (\rho J)^2 m^2] \rho h$

$\tau \frac{dm}{dt} = -(1 - \rho J)m - \frac{1}{2}(\rho J)^2 m^2 + \rho h$ — charge & dynamic Ginz-Landau (TDGL)

TDGL: $\tau \frac{dm}{dt} = -\frac{\delta F}{\delta m}$, $F(m) = \frac{1}{2}am^2 + \frac{1}{4}bm^4 - hm$, $a < 0$.

Bound. Conditions

$\vec{N}/F_n$

Ap. gauge.

$\vec{M} \times \left[\alpha \frac{\partial \vec{M}}{\partial n} + \vec{H}_m L \right] = 0$

Due stripe $\vec{M}_0 = M_s \vec{j}$, $\vec{m} = \vec{i} m_x + \vec{k} m_z$, $\vec{h}_m = h_x \vec{i} + h_z \vec{k}$ — dyn. d.p. field

$\vec{H}_m = \vec{H}_m^0 + \vec{h}_m$

$\vec{M} \times \left[\alpha \frac{\partial \vec{m}}{\partial n} + \vec{H}_m \right] = \vec{M}_0 \times \left[\alpha \frac{\partial \vec{m}}{\partial n} + (\vec{H}_m^0 + \vec{h}_m) \right] = \vec{M}_0 \times \alpha \frac{\partial \vec{m}}{\partial n} + \vec{M}_0 \times \vec{h}_m$

$M_s \alpha \left(\vec{i} \frac{\partial m_z}{\partial n} - \vec{k} \frac{\partial m_x}{\partial n} \right) + M_s (\vec{i} h_z - \vec{k} h_x) = 0$

$\vec{i}: \alpha \frac{\partial m_z}{\partial n} + L h_z = 0$

$\vec{k}: \alpha \frac{\partial m_x}{\partial n} + L h_x = 0$

(1) $\vec{n} = \vec{j}$, Side surface, $\alpha \frac{\partial m_z}{\partial x} + L h_z = 0$, $\alpha \frac{\partial m_x}{\partial x} + L h_x = 0$

(2) $\vec{n} = \vec{z}$, Face surface, $\alpha \left(\frac{\partial m_z}{\partial z} \right) + L h_z = 0$, $\alpha \left(\frac{\partial m_x}{\partial z} \right) + L h_x = 0 \Rightarrow h_x = 0, h_z = 0$, even for k_z .

$h_z = \int dx' G_{zz}(xx') m_z(x')$ and $h_x = \int G_{xx}(xx') m_x(x') dx'$, o.k.

due stripe have $\langle \cdot, \cdot \rangle_{y,z}$ nonzero, G_{xx} and $G_{zz} \neq 0$. $G_{xx}(xx') + G_{zz}(xx') = -4\pi \delta(x-x')$

$m_z \vec{k}$ Surface charges on $x = \pm w/2$ no charges, i.e. $(\vec{m} \cdot \vec{n}) = 0$
 $m_x(x) \vec{i}$ no charges & volume charges, i.e. $\text{div } \vec{m} = \frac{\partial m_x}{\partial x} = 0$
 $m_z \vec{k}$ charges surface charges nonzero on faces $z = 0, L$

$h_z^z(x) = \frac{1}{L} \int dx' \int dy' (-1) \frac{\partial}{\partial z} \int_{z'=0,L} \frac{m_z(x') dS'}{|x-x'|}$, $h_z^z(x) = \frac{1}{L} \int dx' \int dy' \int_{z'=0,L} \frac{m_z(x')}{|x-x'|} \frac{\partial}{\partial x'} \frac{\partial}{\partial z}$
 $h_z^z(x) = 0$ $\int dx' \frac{1}{|x-x'|} \Big|_0^L \sim \int dx' e^{-k|x-x'|} \Big|_0^L = 0$. $dS' = dx' dy'$ due $z=0, L$ no charges.

$h_z^z(x) = -\frac{1}{L} \int dy' \int dx' m_z(x') \frac{1}{|x-x'|} \Big|_0^L \neq 0$ and $h_z(x) = \int dx' G_{zz}(xx') m_z(x')$

$$\psi(r) = \int_{\text{Surf}} \frac{m_1(r') dS'}{|r-r'|} + \int_{\text{Space}} \frac{m_2(r') dS'}{|r-r'|} - \int dV' \frac{\text{div } \vec{m}(r')}{|r-r'|}$$

$$\vec{h}_s(r) = -\nabla \psi_s = -\nabla \int_{\text{Surf}} \frac{m_1(r') dS'}{|r-r'|} - \nabla \int \frac{m_2(r') dV'}{|r-r'|}$$

$$h_x(x) = \int dx' G_{xx}(xx') m_{1x}(x') \quad h_z(z) = \int dx' G_{zz}(xx') m_{2z}(x') \quad \text{where } L \sim \gamma, z,$$

Остаток бугно, тоо $h_x(x)$ ортогоналене тоооо $m_x(x)$ -component а
н.гапери чегане с m_2 (face surface charges $\sigma=0,1$) бугага б h_x не гави.

→ Говорим poster "M. Dynamics" гур DOE Review. Оган Бадер'а
"B. Conditions" paper на оргиб. Дуеке с Бадер DVortex-dyn. paper
(comments vor-voy Berkeley, неонеме) а Ринка paper на Fe/Pt(111)
-nanostripes. Бадер нобел'еги ное ипогеме на чег. тог (12 months)
а чеган, тоо paper-work гави. егави лауре (сеп. MSD). Ога
ногави ное на clearance (ногави Badge on Mon, Dec. 20) а нанимис б НК.

С топи б номиниру. Говорим DOE - Dyn. poster а DOE -
- highlights. Огнине poster на чегане а с Виген, Бери. (16/XII)

Changes to "Vortex Dynamics" paper с Werner. Ога оргиб 17/XII

Referee positive, но оги с гавиране.

С топи б инстале косп. . ноеи к Family Dr. б Edwards Guter (Napor.)

Оспадана терса Science paper "Bound. Conditions" с чегане (18-19/XII)
а "Vort. Dynamics" paper с Werner. Ночан revised text to co-authors.

→ Payson paper PRB'04 Mori-Saveliev на оуеане ФП б (20/XII)
Cyl. dot in in-plane field. Бегоа с P. Roy, ellipt. dots.

→ Бави нпине exp. data by Frank and Val.:

D. Vortex dynamics ① $\chi(w)$ нонгави мик (cusp) нпм $w/2\pi = 180$ MHz б tri-layer 40/40 nm
disk of FeNi с $D \approx 1 \mu m$. FeNi/Al/FeNi. Бупов мика (low-w) не буги.
Буги 300 $w/2\pi$ high-w мика уз APL DVortex Dyn. - гави
нпобеге.

Frank experiment ② Пагмепи FeNi dots - single-layer disks б нпегави exp Frank
двм $D = 2.2$ (2.3) μm а $D = 1.1 \mu m$ нпм $L = 40$ nm.

One single-disk двм exp. 19/XI : $\omega_0/2\pi$ (2R $\approx 1 \mu m$) = 275 MHz а 162 MHz (2R $\approx 2 \mu m$)

Тенепо $\beta_1 = \frac{40}{550}$ а $\beta_2 = \frac{40}{1100}$ неонеме β , тоо 19/XI, 200

нпобеге к гави. ω_0 -Calc. нпм $\gamma = 2.95$ GHz/kOe а $M_s = 800$ G:

$$\frac{\omega_{01}}{2\pi} (40/550) = 0.357 \text{ GHz} \quad \text{а} \quad \frac{\omega_{02}}{2\pi} (40/1100) = 0.184 \text{ GHz}.$$

$M_s = 700$ G гави $\omega_{01}/2\pi = 312$ MHz а $\omega_{02}/2\pi = 161$ MHz, Буги к гави. гави.

① Disk F/N/F disk $\approx 40/1/40 \text{ nm}$, $2R = 1000 \text{ nm}$
 Calc. $\omega_x = 52 \text{ MHz}$, $\omega_z = 728 \text{ MHz}$ $\delta \omega_{\text{Zeeman}} \omega_{\text{isol}} (2 \cdot 40 \text{ nm}) = 729 \text{ MHz}$. $\hbar \omega$
 $\rightarrow \Omega (AP_1 = -1) = 195 \text{ MHz}$. Т.е. расстройка $(P_1 P_2 = -1)$ -case $\delta \omega_{\text{Zeeman}}$ велико и
 несоответствующий $\omega_{\text{exp}} = 180 \text{ MHz}$. Показана $f/2\pi = 2.95 \frac{\text{GHz}}{\text{keV}}$, $M_s = 800 \text{ G}$.
 Небольшое изменение M_s даёт хорошее согласие с эксп.

21/03 B PRB'04 - Savitsky - critical slowdown. Vortex critical slowdown

$$\eta \frac{ds}{dt} = -\frac{\partial W}{\partial s}, \quad \psi = \frac{1}{s}, \quad s = \frac{1}{\psi} \text{ - vortex shift} \Rightarrow \left\{ \begin{array}{l} \frac{d\psi}{dt} = -\frac{1}{\psi} \delta\psi, \quad h < h_c \\ \frac{d\psi}{dt} = -\frac{2A}{\eta} \psi^5, \quad h > h_c \end{array} \right\}, \quad \delta\psi = \psi - \psi_{\text{eq}}$$

- "probe" ω_{exp} ψ - $\psi(H)$.

Relax. time $\tau = -\eta \frac{B^2}{A^3} = 512\eta \frac{B^2}{(h_c - h)^3}$ - estimate. $\psi_{\text{eq}} = \frac{1}{s_{\text{eq}}} = \sqrt{\frac{-A}{2B}} = \sqrt{\frac{h_c - h}{16B}}$

$$W(\psi) = W(0) + A(h) \psi^2 + B(h) \psi^4 + O(\psi^6), \quad \psi = \frac{1}{s}. \quad A(h) < 0 \text{ near } h_c, \quad B(h_c) > 0.$$

$\rightarrow$ 4D gauge s - reheating, η - friction, η - logarithm no divergence, $\tau \sim (h_c - h)^{-3}$ estimate.

Local dynamics near critical point - L.L. van Nieuwenhuis-Platen.

$$s = \frac{1}{\psi}, \quad ds = -\frac{d\psi}{\psi^2}, \quad \frac{\partial W}{\partial s} = \frac{\partial W}{\partial \psi} \frac{\partial \psi}{\partial s} = \frac{\partial W}{\partial \psi} (-\psi^2), \quad \eta \frac{d\psi}{\psi^2 dt} = -\frac{\partial W}{\partial \psi} (-\psi^2) \rightarrow \eta \frac{d\psi}{dt} = -\psi^4 \frac{\partial W}{\partial \psi}$$

$$h > h_c, \quad W(\psi) = W(0) + A\psi^2 + \dots$$

$$\eta \frac{d\psi}{dt} = -\psi^4 [2A\psi + \dots] = -2A\psi^5 \rightarrow \frac{d\psi}{dt} = -\frac{2A}{\eta} \psi^5 \text{ - как у Savitsky'04}$$

Но 4D gauge s governed double hysteresis, т.к. потенциал $W(s) = W(x)$ и $\psi \sim G$ -parameter.

$$W(\psi) = W(0) + A\psi^2 + B\psi^4, \quad \psi_{\text{eq}} = \sqrt{\frac{-A}{2B}} = \psi_0, \quad \delta\psi = \psi - \psi_0$$

$$\psi^2 = (\psi_0 + \delta\psi)^2 = \psi_0^2 + 2\psi_0 \delta\psi + \delta\psi^2, \quad \psi^4 = (\psi_0 + \delta\psi)^4 = \psi_0^4 + 4\psi_0^3 \delta\psi + 6\psi_0^2 \delta\psi^2 + \dots$$

$$\frac{\partial W}{\partial \psi} = 2A\psi + 4B\psi^3 = 2A(\psi_0 + \delta\psi) + 4B[\psi_0^3 + 3\psi_0^2 \delta\psi + \dots] = 2[A + 6B\psi_0^2] \delta\psi$$

$$[A + 6B\psi_0^2] = [A + 6B \frac{(-A)}{2B}] = -2A, \quad A(h) \sim (h_c - h).$$

$$\eta \frac{d\psi}{dt} = -\psi^4 \frac{\partial W}{\partial \psi} = -(\psi_0 + \delta\psi)^4 (-4A) \delta\psi = -\psi_0^4 (-4A) \delta\psi = -\frac{4A^2}{B^2} (-4A) \delta\psi = \frac{A^3}{B^2} \delta\psi$$

$$A(h) = \frac{1}{8}(h - h_c), \quad \eta \frac{d\psi}{dt} = \frac{A^3}{B^2} \delta\psi, \quad \tau = -\frac{B^2 \eta}{A^3} = -\frac{8^3 \eta B^2}{(h_c - h)^3} \rightarrow \text{как у Savitsky'04}$$

Всё $\tau \sim (h_c - h)^{-3}$ адекватно $s = \psi$ замена.

См. П.-П. Т.Ю. Физ. Конструкция. §101. Релаксация координаты
 магнитной системы т. ФП II.

ψ - Пондус-Хармоника. $\frac{d\eta}{dt} = -\gamma \frac{\partial \Omega}{\partial \eta}$ с исходными коэф. γ .

$$\text{тот } \Omega = \Omega_0 + d(T - T_c) \eta^2 + b \eta^4, \quad \eta_{\text{eq}} = \bar{\eta} = \sqrt{\frac{d(T_c - T)}{2b}}, \quad a = d(T - T_c)$$

$$\frac{dy}{dt} = -2\delta [ay + 2by^3], \text{ assuming } \delta y = y - \bar{y}.$$

$$ay + 2by^3 = a(\bar{y} + \delta y) + 2b(\bar{y}^3 + 3\bar{y}^2\delta y) = [a + 6b\bar{y}^2]\delta y = -2a\delta y$$

$$\frac{d(\delta y)}{dt} = -2\delta(-2a)\delta y = -4\delta^2 a (T_0 - T) \cdot \delta y = -\frac{1}{\tau_0} \delta y, \quad \left(\tau_0 = \frac{1}{4\delta^2 a (T_0 - T)} \right) \sim (T_0 - T)^{-1} \text{ for } T < T_c.$$

$$\text{If } T > T_c \quad \tau_0 = \frac{1}{2\delta^2 a (T - T_c)}.$$

$$\text{Ecm } \Omega = \int dV [\Omega_0 + ay^2 + by^4 + g(\nabla y)^2]. \text{ TO relaxation equation}$$

$$\frac{\partial(\delta y)}{\partial t} = - \left[\frac{\delta y}{\tau_0} - 2\delta g \nabla^2(\delta y) \right] \text{ using Pythagorean theorem } \frac{d(\delta y)}{dt} = -\frac{1}{\tau_k} \delta y, \quad \tau_k^{-1} = \tau_0^{-1} + 2\delta g k^2$$

Ecm case is more complicated, so

$$\frac{\partial(\delta y)}{\partial t} = -\frac{1}{\tau_0} \delta y + 2\delta g \nabla^2 \delta y + \delta h, \quad h(r,t) = h(\omega, \vec{k}) e^{i(\vec{k} \cdot \vec{r} - \omega t)} \rightarrow \delta y_r = f(\omega, \vec{k}) h(\omega, \vec{k}),$$

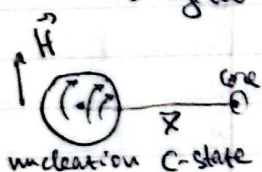
$$\text{we } f(\omega, \vec{k}) = \frac{\delta}{\tau_k^{-1} - i\omega} \text{ - micro-relax. susceptibility. } f(0, \vec{k}) = \delta \tau_k$$

Static, uniform $f(0,0) = \chi_0 = \delta \tau_0$, i.e. $\chi_0(T)$ and $\tau_0(T)$ - uniform susceptibilities, approaching infinity at $T \rightarrow T_c$.

Применяются эти теории Л. Халатника, также же как и теории ФТЛ Langau, не сходя к T_c .

$\vec{S} = \vec{X}/R$ - векторный параметр порядка, но Savel'ev'04 утверждает. Все relax. dynamics Savel'ev-Nori'04 это предполож. $\eta \frac{dS}{dt} = -\frac{\partial W}{\partial S} \text{ и } S = \frac{1}{\psi}$.
Ecm сразу параметр $\eta \frac{d\psi}{dt} = -\frac{\partial W}{\partial \psi}$, то можно $\tau(h) \sim (h_c - h)^{-1}$, т.е. все сходится к теории Л. - Халатника.

Можно $\frac{dy}{dt} = -2\delta [ay + 2by^3 - g \nabla^2 y]$ - modified time-dependent Ginzburg-Landau (TDGL).



$$\text{If } \eta \frac{dS}{dt} = -\frac{\partial W}{\partial S} \text{ и } \eta \frac{d\psi}{dt} = -\frac{\partial W}{\partial \psi} \text{ не верно, т.к. они предположительно}$$

то $W(\vec{S})$ или $W(\psi)$ - скалярная функция. Или параметр

$\vec{r}$ - (без nucleation не дает) в значение $\vec{S} = \vec{X}/R$ по отношению к $\vec{H}$.

Можно $W = W(\vec{S})$. $\vec{S}_0 \perp \vec{H}$ при equilibrium. Пусть $\vec{H} = (0, H)$.

$$\text{Или же было } \eta \frac{d\vec{S}}{dt} = -\frac{\partial W}{\partial \vec{S}}, \text{ так как relax. } \psi \text{ и } \psi \text{ не универсальны.}$$

$$\text{Com. 26/2 } \langle \vec{m} \rangle = -\frac{2}{3} C \hat{z} \times \vec{S} \text{ и } W(\vec{S}) = W(S, h=0) - \langle \vec{m} \rangle \cdot \vec{h}, \quad |\vec{S}| \leq 1.$$

$$W(\vec{S}) = W_0(S) + \frac{2}{3} C (\hat{z} \times \vec{S}) \cdot \vec{h}, \text{ коэф. энергия two-vortex } (\vec{m} \cdot \vec{h}) = 0 \text{ можно, но } \hat{z} \times \vec{S} \sim \langle \vec{m} \rangle \text{ без } \psi \text{ models.}$$

$$(\hat{z} \times \vec{S}) \cdot \vec{h} = (\vec{h} \times \hat{z}) \cdot \vec{S}, \quad \frac{\partial W}{\partial \vec{S}} = \frac{\partial W_0}{\partial S} \frac{\vec{S}}{S} + \frac{\partial}{\partial \vec{S}} [u(\vec{h} \times \hat{z}) \cdot \vec{S}] = W_0' \frac{\vec{S}}{S} + u \vec{h} \times \hat{z}$$

$$\hat{y} = \begin{pmatrix} y_x & 0 \\ 0 & y_y \end{pmatrix} \quad \hat{y} \frac{d\vec{S}}{dt} = W_0' \frac{\vec{S}}{S} + u \vec{h} \times \hat{z} \quad \text{ - } \psi \text{ где } S_x \text{ и } S_y \text{ зависят } \psi \text{ и } \psi \text{ не универсальны. } \theta \text{ zero } \langle \vec{m} \rangle \text{ и } \psi \text{ без } \psi \text{ model.}$$

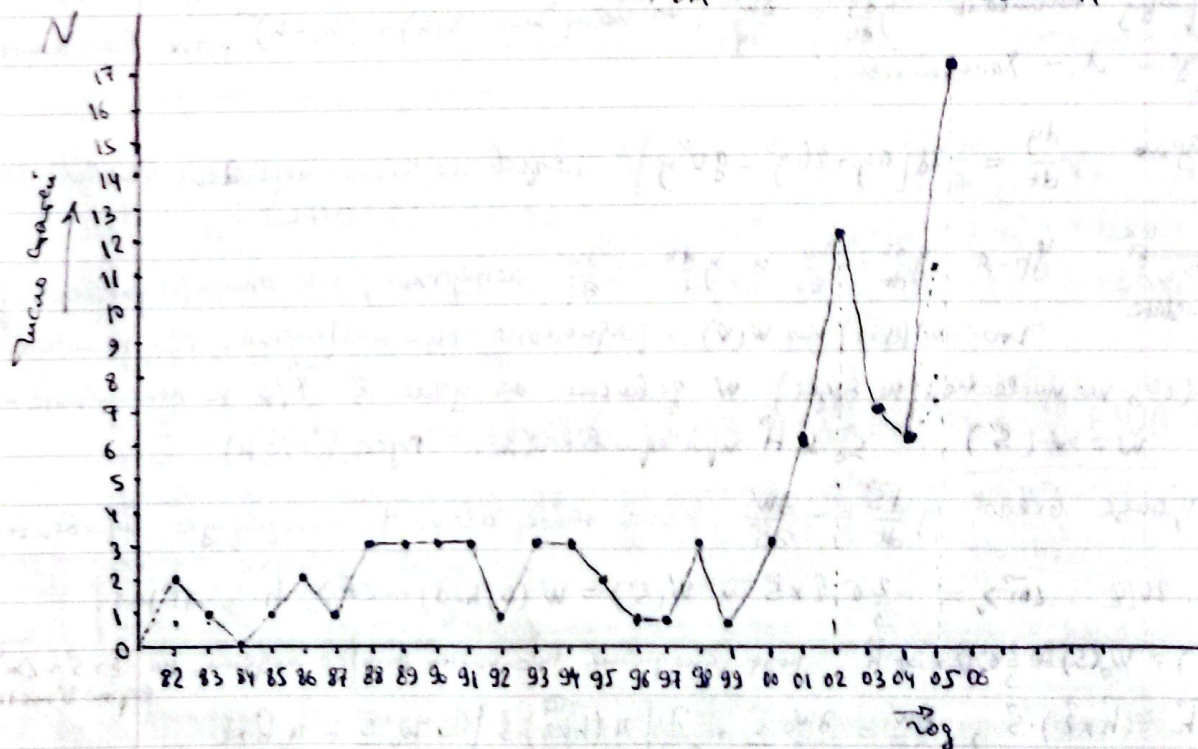
Escaneado con CamScanner

~~Badar~~ nepehan upabuz (uzmushkenniyto) B. Conditions - Seconce paper.

Referencia a Roy o Feli-Bo. $M(x, T)$ - proceso. Obtenen eny ufuncion de
Obra MFA hacer $M(x, T)$ que cubren con papel. Norm. Obra hacer
6 Dec. '03 u Progress Reports (progrmacion econ. Xisavai - Kehlfield).

August: A. & "Pine Ridge" apts (garden 8/26/81) & "Highcrest townhomes" (ne nonprob.)
 Pagan. "Band. Conditions" paper com. changes Baderia.

- ① 4 papers JAP-MMM - accepted, publ.
- ② Ding - PRL, accepted, publ.
- ③ Shale - PRB, accepted, publ.
- ④ Beyer - PRB and Bunk, accepted
- ⑤ Feit - EPL, accepted → published
- ⑥ F/N/F disk dyn. - APL subm., publ.
- ⑦ B. Goud. - Science, subm. → PRL, resubm. → PRB - resubm. → publ.
- ⑧ Feit - 2 (Amst) - ?
- ⑨ Feit/Reh - APL, subm.
- ⑩ Cheng - Rpt-Surveys - PRB, subm., publ.
- ⑪ Feit-Exch. simpl. - PRB, subm., kago noca. 6 goprai myssner
- ⑫ Dvortex statics - PRB?
- ⑬ Bio-disk - Memotch. AS?
- ⑭ Exp. - TM-vortex - PRB-Rsubm., publ.



25/XII

Мелан. Емне. пабори (ан. 14/XI)

- 0) Мелан. Арел веће јавно: до дејств. вирус, хипертермија, в. динамике + (-).
- 1) Cylinder. double-layer dot F/A/F: m. reversal, dynamics (Axel)
- 2) Vortex barrier micromagnetic simulation.
- 3) Bound. Conditions paper "Science" (Chabon, Bader?) References to PRL 25/12
- 4) F/N/F dot states: MPD, MCH. Paper to PRB (Kristen, Vol) BMC scale MPD version 12
- 5) F/N/F dot dynamics, TM modes give $AP2 = -1$ (J. J. M. T. P. Kristen)
- 6) Ring MPD/xo - paper for APL or JAP (numer. Bader)
- 7) Vortex modes: "breath", azimuthal u "central" $M_z(t)$ oscillations
- 8) FMR & F/N/F stripes u 1D stripe array (Bader)
- 9) Vortex core reversal in $H_x(w)$ - in plane field (X-MCP, H. Stoll)
- 10) Vortex relax. time through MSD → E-state transition (uexp. Nov. + PRB 04) y.d. $\psi = \frac{1}{2} \rightarrow \frac{1}{2} \sim (h-h)$
- 11) M(x,T) & $(FeNi)_{1-x}Hex$ - MFA + AF exchange (c Chantrell) - notes Fax
- 12) Fe/Pt - nano-strips $M_z, H_c = f(ML), M_z, H_c = F(T)$ (c Anilima) notes revised version to Ruiting
- 13) Permalloy. craters - JMMM, JAP
- 14) DOE review poster, highlights + viewgraphs notes 12/1
- 15) update Mag. Films Group u CNM - Nanomagnetism web-pages
- 16) Oreganopolis u ocrucis Seagate - patent.

Опразнување. бон/поси:

- 1) c A. k. б. јавно / сменено пријаву при мерење
- 2) AIG абсоцфрх. (приодбавено, уметовување суму) u остана пермис. Emission test & PRB.
- 3) Канон/мерења на нову рб.
- 4) Завојност наметну б Remington (сбер. Ренг) QMA. Reming ANL 1/12, raz OTAKH. cr. Man. 28/1
TVorm. c 20/1, тенап. QMA. 3/12 hogn. kb. u cgarze/hony. deposit
- 5) Hogn. komu nash. 1-94, d. appl. form to K. Liptak (ufoyu. Hl-status)
- 6) Hogn. k. nashguc na APS-meeting, Los Angeles Marrub?
- 7) Hogn. application and na Citi Grad. cards, Aope na Visa-Platin. Grad. c.
- 8) Zampocur W-2 & Seagate ja Jan. 04 (Charlotte).
- 9) Hogn. no see, Blue Grass - card u cardu o bopope Family Dr
- 10) Hogn. k. Hogn. Tax Return-04 (W-2 Seagate, % PNC Bank - Pittsb.) Pitts. → ANL travel
- 11) On. bgy b Remington, Hogn. nashguc, claim deposit usof.
- 12) Coob. of usmanenun agfoca, Bantidue, Citibank, AIG
- 13) Zampocur u Remington \$150. deposit to Naperville Electric Co.

26/xii ID $\ddot{\vec{X}} = -\omega^2 \vec{X}$ тина 2D - гармонич. осциллятор б
 D Vortex dynamics $U(\vec{r}) = U(\rho) = \alpha \rho^2 \rightarrow dX^2$ бобие не ознарае
 $p_1 p_2 = -1$ абстрактна криволинейна траектория (см. д.д., т. 1, §23)
 $\vec{X} = (X, Y)$. $X(t) = a \cos(\omega t + \alpha)$, $Y(t) = b \cos(\omega t + \beta)$ и крм

$\alpha - \beta \neq \pm \pi/2$ траектория не е крм $X^2 + Y^2 = \text{const}$.

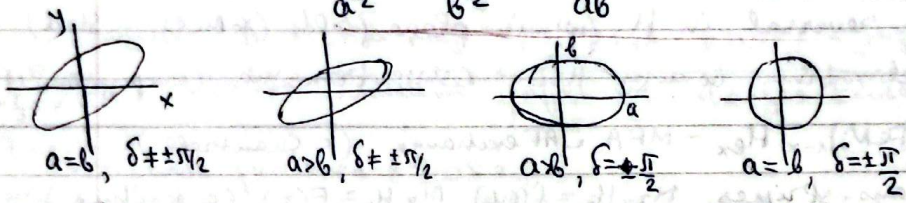
$$|\dot{\vec{X}}|^2 = |\dot{X}|^2 + |\dot{Y}|^2 = a^2 \omega^2 \sin^2(\omega t + \alpha) + b^2 \omega^2 \sin^2(\omega t + \beta)$$

Ем $a = b$ и $\alpha - \beta = \pm \pi/2$, то $|\dot{\vec{X}}|^2 = a^2 \omega^2 [\sin^2(\omega t + \alpha) + \cos^2(\omega t + \alpha)] = a^2 \omega^2 = \text{const}$,

т.е. $|\dot{\vec{X}}|^2 = \text{const}$, не зависи от t , траектория - крм.

Вобщее траектория елине:

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} - \frac{2XY}{ab} \cos \delta = \sin^2 \delta, \quad \delta = \beta - \alpha.$$

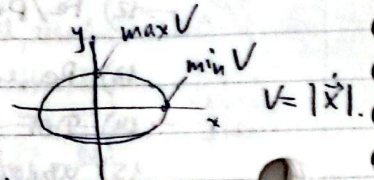


- бсга елине крме $a=b$, $\delta = \pm \pi/2$.

Крм $\delta \neq \pm \pi/2$ елине елине и некрм, $\varphi = \omega t + \alpha$.

$$|\dot{\vec{X}}|^2 = \omega^2 [a^2 \sin^2 \varphi + b^2 \sin^2(\varphi + \delta)]$$

зависит от "t".



В узлах неск елине елине min, б минимал - max.

$V(t) = |\dot{\vec{X}}|$ елине. немогетелен $F(t)$ с ω и 2ω .

Note: $W_{int}(\vec{X}_1, \vec{X}_2) = a_0 + a_1 (\vec{X}_1 \cdot \vec{X}_2) + a_2 \hat{z} \cdot (\vec{X}_1 \times \vec{X}_2)$ - симетричне magn.
 vol-vol core-core core-vol coupling of F/N/F disk

Core-core int. неперомпугер a_1 , кореме основане от vol-vol interaction и $\Rightarrow$ неважно. Но core-vol interaction може боре важно гие траектории, т.к. имеем груго симетричне не еравн. с $(\vec{X}_1 \cdot \vec{X}_2)$. $W_{int} = \mu (\vec{X}_1 \cdot \vec{X}_2)$

т.к. $V(t) = |\dot{\vec{X}}|$ не елине const, то каго ббеге б 2D не $M \ddot{\vec{X}}$.

$$\left\{ \begin{aligned} M \ddot{\vec{X}}_1 + \vec{G} \times \dot{\vec{X}}_1 &= \alpha \vec{X}_1 + \frac{\partial W_{int}}{\partial \vec{X}_1} \\ M \ddot{\vec{X}}_2 - \vec{G} \times \dot{\vec{X}}_2 &= \alpha \vec{X}_2 + \frac{\partial W_{int}}{\partial \vec{X}_2} \end{aligned} \right\} \rightarrow \left\{ \begin{aligned} M \ddot{\vec{X}}_1 + \vec{G} \times \dot{\vec{X}}_1 &= \alpha \vec{X}_1 + \mu \vec{X}_2 \\ M \ddot{\vec{X}}_2 - \vec{G} \times \dot{\vec{X}}_2 &= \alpha \vec{X}_2 + \mu \vec{X}_1 \end{aligned} \right\}$$

Може ли преобразе компонентите по нормална ос? $\vec{G}$, негнат.

Оне неможе габие бм. тина core-vol. interaction $W_{int} (c.v.) = \lambda \hat{z} \cdot (\vec{X}_1 \times \vec{X}_2)$ (см. 1/211)

$$\left\{ \begin{aligned} M \ddot{\vec{X}}_1 + \vec{G} \times \dot{\vec{X}}_1 &= \alpha \vec{X}_1 + \mu \vec{X}_2 + \lambda \vec{X}_2 \times \hat{z} \\ M \ddot{\vec{X}}_2 - \vec{G} \times \dot{\vec{X}}_2 &= \alpha \vec{X}_2 + \mu \vec{X}_1 + \lambda \hat{z} \times \vec{X}_1 \end{aligned} \right\}$$

- то камо елине елине 2D гие $p_1 p_2 = -1$ case.

$$M \ddot{\vec{X}}_1 + \vec{G} \times \dot{\vec{X}}_1 = \alpha \vec{X}_1 + \mu \vec{X}_2 + \lambda \vec{X}_2 \times \hat{z} \quad M \ddot{\vec{X}}_1 + \vec{G} \times \frac{1}{\mu} [-\vec{G} \times \dot{\vec{X}}_1 + \alpha \vec{X}_1 + \mu \vec{X}_2 + \lambda \vec{X}_2 \times \hat{z}] =$$

$$= \alpha \vec{X}_1 + \mu \vec{X}_2 + \lambda \vec{X}_2 \times \hat{z}, \quad \vec{G} \times \vec{G} \times \dot{\vec{X}}_1 = -G^2 \dot{\vec{X}}_1$$

Беге империор е негнат - камо по нормална ос, б ббеге са

$$\begin{aligned}
 M \ddot{\vec{X}}_1 + \frac{1}{M} \vec{G} \times [\kappa \vec{Y}_1 + \mu \vec{X}_2 + \lambda \vec{Y}_2 \times \hat{z}] &= -\frac{G^2}{M} \dot{\vec{X}}_1 + \kappa \dot{\vec{X}}_1 + \mu \dot{\vec{X}}_2 + \lambda \dot{\vec{X}}_2 \times \hat{z} \\
 M \ddot{\vec{X}}_2 + \frac{1}{M} \vec{G} \times [\kappa \dot{\vec{X}}_1 + \mu \dot{\vec{X}}_2 + \lambda \dot{\vec{X}}_2 \times \hat{z}] &= -\frac{G^2}{M} \dot{\vec{X}}_2 + \kappa \dot{\vec{X}}_1 + \mu \dot{\vec{X}}_2 + \lambda \dot{\vec{X}}_2 \times \hat{z} \\
 M \ddot{\vec{X}}_1 + \frac{1}{M} \vec{G} \times [\kappa \dot{\vec{X}}_1 + \mu \dot{\vec{X}}_2 + \lambda \dot{\vec{X}}_2 \times \hat{z}] &= (\kappa - \frac{G^2}{M}) \frac{1}{M} [-\vec{G} \times \dot{\vec{X}}_1 + \kappa \vec{Y}_1 + \mu \vec{X}_2 + \lambda \vec{X}_2 \times \hat{z}] + \\
 &+ \mu \frac{1}{M} [\vec{G} \times \dot{\vec{X}}_2 + \kappa \vec{X}_2 + \mu \vec{Y}_1 + \lambda \hat{z} \times \vec{X}_1] + \lambda \frac{1}{M} [\vec{G} \times \dot{\vec{X}}_2 + \kappa \vec{X}_2 + \mu \vec{Y}_1 + \lambda \hat{z} \times \vec{X}_1] \times \hat{z} - \text{meow}
 \end{aligned}$$

используем то $\dot{\vec{X}}_j$ в чл. уг.

$$\begin{aligned}
 \vec{G} \times \vec{G} \times \dot{\vec{X}}_1 &= -G^2 \dot{\vec{X}}_1 = \vec{G} \times [-M \ddot{\vec{X}}_1 + \kappa \vec{X}_1 + \mu \vec{X}_2 + \lambda \vec{X}_2 \times \hat{z}] \\
 \vec{G} \times \vec{G} \times \dot{\vec{X}}_2 &= -G^2 \dot{\vec{X}}_2 = \vec{G} \times [M \ddot{\vec{X}}_2 - \kappa \vec{X}_2 - \mu \vec{Y}_1 - \lambda \hat{z} \times \vec{X}_1] \\
 M \ddot{\vec{X}}_1 + \frac{1}{M} \vec{G} \times \kappa \frac{\vec{G}}{(-G^2)} \times [-M \ddot{\vec{X}}_1 + \kappa \vec{X}_1 + \mu \vec{X}_2 + \lambda \vec{X}_2 \times \hat{z}] &+ \frac{1}{M} \vec{G} \times \mu \frac{\vec{G}}{(-G^2)} \times [M \ddot{\vec{X}}_2 - \kappa \vec{X}_2 - \mu \vec{Y}_1 - \lambda \hat{z} \times \vec{X}_1] + \\
 + \frac{1}{M} \vec{G} \times \lambda \frac{\vec{G}}{(-G^2)} \times [M \ddot{\vec{X}}_2 - \kappa \vec{X}_2 - \mu \vec{Y}_1 - \lambda \hat{z} \times \vec{X}_1] \times \hat{z} &= -\frac{G^2}{M} \ddot{\vec{X}}_1 + \kappa \ddot{\vec{X}}_1 + \mu \ddot{\vec{X}}_2 + \lambda \ddot{\vec{X}}_2 \times \hat{z} \\
 M \ddot{\vec{X}}_1 + \frac{\kappa}{M} [-M \ddot{\vec{X}}_1 + \kappa \vec{X}_1 + \mu \vec{X}_2 + \lambda \vec{X}_2 \times \hat{z}] &+ \frac{\mu}{M} [M \ddot{\vec{X}}_2 - \kappa \vec{X}_2 - \mu \vec{Y}_1 - \lambda \hat{z} \times \vec{X}_1] + \\
 + \frac{\lambda}{M} [M \ddot{\vec{X}}_2 - \kappa \vec{X}_2 - \mu \vec{Y}_1 - \lambda \hat{z} \times \vec{X}_1] \times \hat{z} &= -\frac{G^2}{M} \ddot{\vec{X}}_1 + \kappa \ddot{\vec{X}}_1 + \mu \ddot{\vec{X}}_2 + \lambda \ddot{\vec{X}}_2 \times \hat{z} \\
 M \ddot{\vec{X}}_1 - \kappa \ddot{\vec{X}}_1 + \frac{\kappa^2}{M} \vec{X}_1 - \frac{\mu^2}{M} \vec{X}_1 - \frac{\lambda \mu}{M} \hat{z} \times \vec{X}_1 - \frac{\lambda \mu}{M} \vec{Y}_1 \times \hat{z} - \frac{\lambda^2}{M} \hat{z} \times \vec{X}_1 \times \hat{z} &= -\frac{G^2}{M} \ddot{\vec{X}}_1 + \kappa \ddot{\vec{X}}_1 \\
 \hat{z} \times \vec{X}_1 \times \hat{z} &= \vec{X}_1 (\hat{z} \cdot \hat{z}) - \hat{z} (\vec{X}_1 \cdot \hat{z}) = \vec{X}_1 \\
 M \ddot{\vec{X}}_1 - 2\kappa \ddot{\vec{X}}_1 + \frac{G^2}{M} \ddot{\vec{X}}_1 + \frac{\kappa^2}{M} \vec{X}_1 - \frac{\mu^2}{M} \vec{X}_1 - \frac{\lambda^2}{M} \vec{X}_1 &= 0 \\
 M^2 \ddot{\vec{X}}_1 + (G^2 - 2\kappa M) \ddot{\vec{X}}_1 + (\kappa^2 - \mu^2 - \lambda^2) \vec{X}_1 &= 0, \quad \omega_0^2 = \frac{1}{G^2} (\kappa^2 - \mu^2 - \lambda^2) - \text{across} \\
 &\quad 1/\sqrt{M} \text{ if } M=0.
 \end{aligned}$$

if $M=0$ we have $\ddot{\vec{X}}_1 + \omega_0^2 \vec{X}_1 = 0$ or $1/\sqrt{M}$ - берем.

УД в системе м. Coupling case $F/M/F$ $\mu=\lambda=0$

$$\begin{aligned}
 M \ddot{\vec{X}}_1 + \vec{G} \times \vec{Y}_1 &= \kappa \vec{X}_1 \rightarrow M \ddot{\vec{X}}_1 + \vec{G} \times \dot{\vec{X}}_1 = \kappa \ddot{\vec{X}}_1, \quad M \ddot{\vec{X}}_1 = -\vec{G} \times \dot{\vec{X}}_1 + \kappa \ddot{\vec{X}}_1 \\
 M \ddot{\vec{X}}_1 + \vec{G} \times \frac{1}{M} [-\vec{G} \times \dot{\vec{X}}_1 + \kappa \ddot{\vec{X}}_1] &= \kappa \ddot{\vec{X}}_1, \quad M \ddot{\vec{X}}_1 + \frac{G^2}{M} \ddot{\vec{X}}_1 + \frac{\kappa}{M} \vec{G} \times \dot{\vec{X}}_1 = \kappa \ddot{\vec{X}}_1 \\
 M \ddot{\vec{X}}_1 + \frac{G^2}{M} \ddot{\vec{X}}_1 + \frac{\kappa}{M} [\kappa \vec{Y}_1 - M \ddot{\vec{X}}_1] &= \kappa \ddot{\vec{X}}_1 \rightarrow M^2 \ddot{\vec{X}}_1 + (G^2 - 2\kappa M) \ddot{\vec{X}}_1 + \kappa^2 \vec{X}_1 = 0 \quad \text{isolated layer}
 \end{aligned}$$

предположим $\vec{X}_1(t) = \vec{X}(\omega) e^{i\omega t}$, we have $\ddot{\vec{X}}_1 = -\omega^2 \vec{X}_1$, $\ddot{\vec{X}}_1 = \omega^4 \vec{X}_1$, в УД $\ddot{\vec{X}}_1 \parallel \vec{X}_1$.

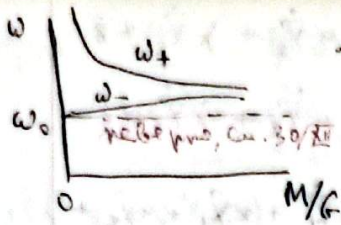
$$\omega^4 M^2 + (2\kappa M - G^2) \omega^2 + (\kappa^2 - \mu^2 - \lambda^2) = 0$$

$$\left(\frac{M}{G}\right)^2 \omega^4 - (1 - \frac{2\kappa M}{G^2}) \omega^2 + \omega_0^2 = 0, \quad \omega_{1,2}^2 = \frac{1}{2} \frac{G^2}{M^2} \left[1 - \frac{2\kappa M}{G^2} \pm \sqrt{\left(1 - \frac{2\kappa M}{G^2}\right)^2 - 4 \frac{\mu^2}{G^2} \omega_0^2} \right]$$

$$-b \pm \sqrt{b^2 - 4 \frac{\mu^2}{G^2} \omega_0^2} = -b \pm b \sqrt{1 - 4 \frac{\mu^2}{G^2 b^2} \omega_0^2} \approx -b \pm b \left[1 - \frac{2 \frac{\mu^2}{G^2 b^2} \omega_0^2}{2} \right] = -b \pm b \left[1 - \frac{2 \mu^2}{G^2 b^2} \omega_0^2 \right], \quad b < 0$$

$$\omega_+^2 = \frac{1}{2} \frac{G^2}{M^2} [-b + \sqrt{\dots}] = \frac{1}{2} \frac{G^2}{M^2} [2|b| - \frac{2 \mu^2}{G^2 |b|} \omega_0^2] = \frac{G^2 |b|}{M^2} \left[1 - \frac{\mu^2}{G^2 b^2} \omega_0^2 \right] \xrightarrow{M \rightarrow 0} \infty$$

$$\omega_-^2 = \frac{1}{2} \frac{G^2}{M^2} [-b - \sqrt{\dots}] = \frac{1}{2} \frac{G^2}{M^2} \frac{2 \mu^2}{G^2 |b|} \omega_0^2 = \frac{1}{|b|} \omega_0^2, \quad \omega_-^2 = \frac{1}{(1 - \frac{2\kappa M}{G^2})} \omega_0^2 \xrightarrow{M \rightarrow 0} \omega_0^2, \quad G < 0? \quad \text{проблема с } O(M)$$



— vortex F-disk um 1st disk B F/U/F trilayer-disk
um yzere zena $M\ddot{\vec{x}}_1$ B 5D.

M - nago cruceo ogeuono kak $\int d^3\vec{r} m(\vec{r})$ of vortex static.

Nago B 5D $M \rightarrow -M$, zoob um $\vec{r} = 0, \lambda, \mu = 0$
um $\vec{r} = 0, \lambda, \mu = 0$ $M\ddot{\vec{x}} = -2\ddot{\vec{x}}$

5D $\left(\frac{M}{G}\right)^2 \ddot{\vec{x}}_1 + \left(1 - \frac{2M}{G^2}\right) \ddot{\vec{x}}_1 + \omega_0^2 \vec{x}_1 = 0$ — nureo ne robofure o trajectory.

Nago cprone lagrangian u nureo trajectory B (p, φ) coordinates kak B A. A. T. 1.

$L = K - W, W = W(\vec{x}_1, \vec{x}_2) = W_1 + W_2 + W_{int}(\vec{x}_1, \vec{x}_2)$

5D $\frac{d}{dt} \frac{\partial L}{\partial \dot{\vec{x}}_j} - \frac{\partial L}{\partial \vec{x}_j} = 0, \vec{x}_1: M\ddot{\vec{x}}_1 + \vec{G} \times \dot{\vec{x}}_1 = \frac{\partial W}{\partial \vec{x}_1}, \frac{d}{dt} \frac{\partial L}{\partial \dot{\vec{x}}_1} = M\ddot{\vec{x}}_1 + \vec{G} \times \dot{\vec{x}}_1$

$\frac{\partial L}{\partial \dot{\vec{x}}_1} = M\dot{\vec{x}}_1 + \vec{G} \times \vec{x}_1, L(\dot{\vec{x}}_1) = \frac{1}{2} M(\dot{\vec{x}}_1)^2 + (\vec{G} \times \vec{x}_1) \cdot \dot{\vec{x}}_1 + C$

$\rightarrow \hat{L}(\dot{\vec{x}}_1, \dot{\vec{x}}_2) = \frac{1}{2} M(\dot{\vec{x}}_1)^2 + \frac{1}{2} M(\dot{\vec{x}}_2)^2 + (\vec{G} \times \vec{x}_1) \cdot \dot{\vec{x}}_1 - (\vec{G} \times \vec{x}_2) \cdot \dot{\vec{x}}_2 - W(\vec{x}_1, \vec{x}_2)$

zoob abt. umureoi zennit. nreopure (nreopure) :

1) $W_{int}(\text{core-val}) = \lambda \hat{z} \cdot (\vec{x}_1 \times \vec{x}_2)$, ne obagennice k $F(\vec{x}_1 - \vec{x}_2)$?

2) zennit $\vec{G} \times \dot{\vec{x}}_j$, gonume otkonume (onume) or classical mechanics (planetary motion) ?
(nagodeu cme noreu $\vec{r}(\vec{r} \times \vec{B})$)

29/7/11

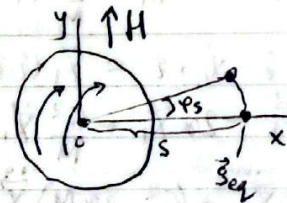
Vortex critical slow-down (cm. 21/XI)

umureu Saveliev - Nori - PRB'04.

S - Nori umreobum mozo zennit of shifted zidiv.

um

$w(s) = -\frac{1}{2} \left(\frac{R_0}{R}\right)^2 \ln\left(1 - \frac{1}{s^2}\right) - \frac{4}{3\pi} G(s) \cdot h \cos \varphi_s + w_m(s)$



um static approach min $w(s)$ oobet. $\varphi_s = 0$ ($\vec{s} \perp \vec{H}$) u $s = s_0 = s_{eq}$.

umureu noreu $\vec{\psi} = \frac{\vec{s}}{|\vec{s}|^2}$ — beopure, $\varphi_\psi = \varphi_s$ — umureu Nori'gu.

$G(s) = (s^2 + 1)E(1/s) - (s^2 - 1)K(1/s)$. Banne um: zabucenost $w(s) \sim \cos \varphi_s$

φ_s boofure roneo k umureu $\cos \varphi_s$ (umureu),

$w(\vec{\psi}) = w(0) + A\psi^2 + B\psi^4$

$A = w_m^{(2)}(\beta) + \frac{1}{2} \frac{R_0^2}{R^2} + \frac{1}{8} h \cos \varphi_s$
 $B = w_m^{(4)}(\beta) + \frac{1}{4} \frac{R_0^2}{R^2} + \frac{1}{64} h \cos \varphi_s$ } $h \rightarrow h \cos \varphi_s$
y Nori'gu

$w_m(s) = w_m^{(0)} + w_m^{(2)}s^2 + w_m^{(4)}s^4 + \dots$

Tenepo $A(\varphi_s), B(\varphi_s)$. Pnceu. relaxation 5D gne $\psi = |\vec{\psi}|$ u $\varphi_\psi = \varphi_s$.

yp. 1. — Xaapure: $\eta \frac{d\varphi_s}{dt} = -\frac{\partial w}{\partial \varphi_s}, \eta \frac{d\psi}{dt} = -\frac{\partial w}{\partial \psi}$

$-\eta \frac{d\varphi_s}{dt} = \frac{\partial A}{\partial \varphi_s} \psi^2 + \frac{\partial B}{\partial \varphi_s} \psi^4 = -\frac{1}{8} h \sin \varphi_s \cdot \psi^2 - \frac{1}{64} h \sin \varphi_s \cdot \psi^4 = -\frac{1}{8^2} h \sin \varphi_s [8 + \psi^2] \psi^2$

$8\psi^2 + \psi^4 = |\psi = \psi_0 + \delta\psi| = [\psi_0^2 + 2\psi_0 \delta\psi] 8 + \psi_0^4 + 4\psi_0^3 \delta\psi \approx 8\psi_0^2 + \psi_0^4$ b um. umureu.

$-\eta \frac{d\psi}{dt} = 2A(\varphi_s)\psi + 4B(\varphi_s)\psi^3, A(\varphi_s) = A(0) + O(\varphi_s^2), B(\varphi_s) = B(0) + O(\varphi_s^2)$ b um. umureu.

$$\frac{\partial w}{\partial \psi} = 2 \left[A(0) + 6B(0)\psi^2 \right] \delta\psi = |\psi_0^2 = -\frac{A(0)}{2B(0)}| = 2 \left[A + 6B \frac{(-A)}{2B} \right] \delta\psi = -4A(0)\delta\psi$$

$$8\psi_0^2 + \psi_0^4 = \frac{(-A)}{2B} \left[8 + \frac{(-A)}{2B} \right] = -\frac{A}{4B^2} [16B - A] = 8\psi_0^2 = -4 \frac{A(0)}{B(0)}$$

$$-\eta\psi \frac{d\psi}{dt} = -\frac{1}{64} h \psi^3 (-4) \frac{A(0)}{B(0)}, \quad \eta\psi \frac{d\psi}{dt} = -\frac{1}{16} h \frac{A(0)}{B(0)} \cdot \psi^3 \quad - \text{знак?}, \text{ logo } A(0) < 0, \quad h < h_c$$

$$\rightarrow A(0) = \frac{1}{8} h + w_m^{(0)} + \frac{1}{2} \left(\frac{R_0}{R} \right)^2 = \frac{1}{8} (h - h_c), \quad h_c = 8 \left[-w_m^{(0)} - \frac{1}{2} \left(\frac{R_0}{R} \right)^2 \right], \quad w_m^{(0)} < 0.$$

$$W_h = -h + \frac{1}{8} h \psi^2 + \frac{1}{64} h \psi^4. \quad \text{Equil. } \psi_s = 0 \text{ supergen. no zero } \psi^0(!)$$

$$w(\vec{r}) = w(\psi=0, \psi_s) + A(\psi_s)\psi^2 + B(\psi_s)\psi^4, \quad w(0, \psi_s) = -h \cos \psi_s + w_m^{(0)}$$

$$\frac{\partial w}{\partial \psi_s} = h \sin \psi_s \Rightarrow -\eta\psi \frac{d\psi}{dt} = \frac{\partial w}{\partial \psi_s} = h \sin \psi_s + O(\psi^2)$$

$$\eta\psi \frac{d\psi}{dt} = -h\psi_s, \quad \frac{d\psi_s}{dt} = -\frac{1}{\tau_\psi} \psi_s, \quad \tau_\psi = \frac{\eta\psi}{h} \quad - \text{не under oscillation n/m } \Phi \Pi \Pi \quad h \rightarrow h_c = 0.$$

$$-\eta\psi \frac{d\psi}{dt} = h \sin \psi_s + \frac{\partial A}{\partial \psi_s} \psi^2 = h \sin \psi_s - \frac{1}{8} h \sin \psi_s \cdot \psi^2 \approx h \left[1 - \frac{1}{8} \frac{(-A)}{2B} \right] \psi_s = h \left[1 + \frac{A}{16B} \right] \psi_s$$

$$\frac{A(0)}{16B(0)} = \frac{1}{128} \frac{(h-h_c)}{B(0)}$$

$$\tau_\psi(h) = \frac{\eta\psi}{h \left[1 + \frac{1}{128B} (h-h_c) \right]}$$

не oscillation -
- бреша перемещения ψ
- при $\vec{s}$ и $\vec{\psi}$.

$$-\eta \frac{d\psi}{dt} = \frac{\partial w}{\partial \psi} = -4A(0)\delta\psi, \quad \tau = -\frac{\eta}{4A(0)} = \frac{2\eta}{(h_c-h)}, \quad \tau(h) = \frac{2\eta}{(h_c-h)} \quad - \text{бреша relaxation of } |\vec{\psi}| = 1/5.$$

Бубог: relax. yd fine grain $\vec{s} - \psi_s$ consequences or relax. yd
fine $|\vec{s}| = 1/|\vec{\psi}|$. Бреша relaxation of $|\vec{\psi}|$ $\tau \sim (h_c-h)^{-1}$ under observation
bug fine $\Phi \Pi \Pi$ region langay. Paradoxes $(h_c-h)^{-1}$ or $(h_c-h)^{-3}$
как у S.-Nori'04. Проблема: Nori hasn't yd fine "s", хана
направление не погрешка ах. $\psi = \psi_s$ - yd nago pacen. fine "psi".
 ψ_s не possible crit. slow-down, τ_ψ - конерно в точке $\Phi \Pi \Pi$.

D. Vortex dynamics F/N/F disk $\epsilon \mu_1 \mu_2 = -1$

$$\text{Larfaunman } \hat{L}(\vec{x}_1, \vec{x}_2) = \frac{1}{2} M (\dot{\vec{x}}_1)^2 + \frac{1}{2} M (\dot{\vec{x}}_2)^2 + (\vec{G} \times \vec{x}_1) \cdot \dot{\vec{x}}_1 - (\vec{G} \times \vec{x}_2) \cdot \dot{\vec{x}}_2 - W(\vec{x}_1, \vec{x}_2)$$

$$(\vec{G} \times \vec{x}_1) \cdot \dot{\vec{x}}_1 = (\vec{x}_1 \times \dot{\vec{x}}_1) \cdot \vec{G} = |\vec{G} - G\hat{z}| \dot{\phi} = -G(\vec{x}_1 \times \dot{\vec{x}}_1) \cdot \hat{z} = -G(\vec{x}_1 \times \dot{\vec{x}}_1)_z$$

$$(\vec{x}_1 \times \dot{\vec{x}}_1) \text{ under conservation momenta } (\vec{r} \times \vec{p} = \vec{L}).$$

$$\text{Берно м } \vec{L} = \sum \vec{L}_i = \sum (\vec{x}_i \times \dot{\vec{x}}_i) = \text{const. ?}$$

$$\text{Example, } (\vec{G} \times \vec{x}_1) \dot{\vec{x}}_1 + (\vec{G} \times \vec{x}_2) \dot{\vec{x}}_2 = (\vec{x}_1 \times \dot{\vec{x}}_1) \cdot \vec{G} + (\vec{x}_2 \times \dot{\vec{x}}_2) \cdot \vec{G} = [\vec{x}_1 \times \dot{\vec{x}}_1 + \vec{x}_2 \times \dot{\vec{x}}_2] \cdot \vec{G} = \vec{L} \cdot \vec{G} = \text{const}$$

fine $\mu_1 \mu_2 = \pm 1$ case. Ноуи, General Consequences & Papanicolaou.

$$\frac{d\vec{L}}{dt} = \frac{d}{dt} [\vec{r}_1 \times \dot{\vec{r}}_1 + \vec{r}_2 \times \dot{\vec{r}}_2] = (\dot{\vec{r}}_1 \times \dot{\vec{r}}_1) + (\dot{\vec{r}}_2 \times \dot{\vec{r}}_2) = 0 \quad \text{по векторным}$$

абсолютно жёсткая шаровая система, $\dot{\vec{r}}_j \parallel \vec{r}_j$, $\ddot{\vec{r}}_j = -\omega_j^2 \vec{r}_j$ ($M=0$)

$$P_1 P_2 = +1: \vec{r}_1 \times \ddot{\vec{r}}_1 = \vec{r}_1 \times \frac{1}{M} [-\vec{G} \times \dot{\vec{r}}_1 + \alpha \vec{r}_1 + \mu \vec{r}_2 + \lambda \vec{r}_2 \times \hat{z}] = \frac{1}{M} [-\vec{G}(\vec{r}_1 \cdot \dot{\vec{r}}_1) + \mu \vec{r}_1 \times \vec{r}_2 + \lambda \hat{z}(\vec{r}_1 \cdot \dot{\vec{r}}_2)]$$

$$\vec{r}_2 \times \ddot{\vec{r}}_2 = \vec{r}_2 \times \frac{1}{M} [\vec{G} \times \dot{\vec{r}}_2 + \alpha \vec{r}_2 + \mu \vec{r}_1 + \lambda \hat{z} \times \vec{r}_1] = \frac{1}{M} [\vec{G}(\vec{r}_2 \cdot \dot{\vec{r}}_2) + \mu \vec{r}_2 \times \vec{r}_1 + \lambda \hat{z}(\vec{r}_2 \cdot \dot{\vec{r}}_1)]$$

$$\frac{d\vec{L}}{dt} = (\vec{r}_1 \times \ddot{\vec{r}}_1) + (\vec{r}_2 \times \ddot{\vec{r}}_2) = \frac{1}{M} \vec{G} [(\vec{r}_2 \cdot \dot{\vec{r}}_2) - (\vec{r}_1 \cdot \dot{\vec{r}}_1)] = 0 \Rightarrow (\vec{r}_1 \cdot \dot{\vec{r}}_1) = (\vec{r}_2 \cdot \dot{\vec{r}}_2) \quad \text{где } P_1 P_2 = -1$$

Энергия системы

$$E = \sum_i \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L, \quad \text{где } \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$

$$\dot{x}_i \frac{\partial L}{\partial \dot{x}_i} + \dot{y}_i \frac{\partial L}{\partial \dot{y}_i} = \vec{r}_i \cdot \frac{\partial L}{\partial \dot{\vec{r}}_i}, \quad E = \sum_j \dot{\vec{r}}_j \cdot \frac{\partial L}{\partial \dot{\vec{r}}_j} - L$$

$$E = \dot{\vec{r}}_1 \cdot [M \dot{\vec{r}}_1 + \vec{G} \times \vec{r}_1] + \dot{\vec{r}}_2 \cdot [M \dot{\vec{r}}_2 + \vec{G} \times \vec{r}_2] - \left[\frac{1}{2} M \dot{x}_1^2 + \frac{1}{2} M \dot{x}_2^2 + (\vec{G} \times \vec{r}_1) \cdot \vec{r}_1 - (\vec{G} \times \vec{r}_2) \cdot \vec{r}_2 - W(\vec{r}_1, \vec{r}_2) \right] = \frac{1}{2} M (\dot{\vec{r}}_1)^2 + \frac{1}{2} M (\dot{\vec{r}}_2)^2 + W(\vec{r}_1, \vec{r}_2)$$

$$E = \frac{1}{2} M (\dot{\vec{r}}_1)^2 + \frac{1}{2} M (\dot{\vec{r}}_2)^2 + W(\vec{r}_1, \vec{r}_2) \quad \text{— энергия tri-layer F/N/F disk, не забывая про инварианты } \vec{G}_j \text{ (см. 1)}$$

Используем: инварианты $\vec{G}$ — векторы Лагранжа по $\dot{\vec{r}}_j$.

Используем также $\vec{X} = (M \vec{r}_1 + M \vec{r}_2) \frac{1}{M+M} = \frac{1}{2} (\vec{r}_1 + \vec{r}_2)$ и все случаи вложения

где $P_1 P_2 = +1$ (где с $\vec{X}$ совмещаются и можно $\vec{X} = 0$, см. координаты век в п. 1.1.3)

$$W(\vec{r}_1, \vec{r}_2) = \frac{1}{2} (\alpha + \mu) [x_1^2 + x_2^2] - \frac{1}{2} \mu (\vec{r}_1 - \vec{r}_2)^2 + \lambda \hat{z} \cdot (\vec{r}_1 \times \vec{r}_2)$$

$$\text{или } W(\vec{X}, \vec{z}) = (\alpha + \mu) [X^2 + \frac{1}{4} z^2] - \frac{1}{2} M z^2 + \lambda \hat{z} \cdot (\vec{z} \times \vec{X}), \quad \vec{z} = \vec{r}_1 - \vec{r}_2, \quad \vec{X} = \frac{1}{2} (\vec{r}_1 + \vec{r}_2)$$

$$W(\vec{X}, \vec{z}) = (\alpha + \mu) X^2 + \frac{1}{4} (\alpha - \mu) z^2 + \lambda \hat{z} \cdot (\vec{z} \times \vec{X})$$

$$\textcircled{1} P_1 P_2 = +1, \quad (\vec{G} \times \vec{r}_1) \cdot \dot{\vec{r}}_1 + (\vec{G} \times \vec{r}_2) \cdot \dot{\vec{r}}_2 = [(\vec{r}_1 \times \dot{\vec{r}}_1) + (\vec{r}_2 \times \dot{\vec{r}}_2)] \cdot \vec{G} = \vec{L} \cdot \vec{G} = \text{const}$$

$$\vec{L} = (\vec{X} + \frac{\vec{z}}{2}) \times (\dot{\vec{X}} + \frac{\dot{\vec{z}}}{2}) + (\vec{X} - \frac{\vec{z}}{2}) \times (\dot{\vec{X}} - \frac{\dot{\vec{z}}}{2}) = 2 \vec{X} \times \dot{\vec{X}} + \frac{1}{2} \vec{z} \times \dot{\vec{z}}$$

$$\Rightarrow \hat{L}(X, z) = \frac{1}{2} M [\dot{X}^2 + \frac{1}{4} \dot{z}^2] + 2 \vec{G} \cdot [\vec{X} \times \dot{\vec{X}} + \frac{1}{4} \vec{z} \times \dot{\vec{z}}] - (\alpha + \mu) X^2 - \frac{1}{4} (\alpha - \mu) z^2 - \lambda \hat{z} \cdot (\vec{z} \times \vec{X})$$

Без учета с λ выражения $\vec{L} = \text{const}$: $\hat{L}(X, z) = M \dot{X}^2 - (\alpha + \mu) X^2 + \frac{1}{4} M \dot{z}^2 - \frac{1}{4} (\alpha - \mu) z^2$ — переменные X и z разделимы, так как в $\vec{G}$ -вектор не входят.

Без учета с λ разделимы $\Rightarrow$ где $\vec{X}, \vec{z}$ ортогональны. $\hat{L}(X, z) = \hat{L}_1(X) + \hat{L}_2(z)$

$$\textcircled{2} P_1 P_2 = -1, \quad (\vec{G} \times \vec{r}_1) \dot{\vec{r}}_1 + (\vec{G} \times \vec{r}_2) \dot{\vec{r}}_2 = [\vec{r}_1 \times \dot{\vec{r}}_1 + \vec{r}_2 \times \dot{\vec{r}}_2] \cdot \vec{G}$$

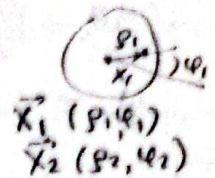
$$\vec{r}_1 \times \dot{\vec{r}}_1 - \vec{r}_2 \times \dot{\vec{r}}_2 = (X + \frac{z}{2}) \times (\dot{X} + \frac{\dot{z}}{2}) - (X - \frac{z}{2}) \times (\dot{X} - \frac{\dot{z}}{2}) = \vec{z} \times \dot{\vec{X}} + \vec{X} \times \dot{\vec{z}}$$

$$\Rightarrow \hat{L}(X, z) = M [\dot{X}^2 + \frac{1}{4} \dot{z}^2] + [\vec{z} \times \dot{\vec{X}} + \vec{X} \times \dot{\vec{z}}] \cdot \vec{G} - (\alpha + \mu) X^2 - \frac{1}{4} (\alpha - \mu) z^2 \quad \text{— переменные } X, z \text{ не разделимы}$$

$$W(x_1, x_2) = \frac{1}{2} \kappa (x_1^2 + x_2^2) + \mu (\vec{x}_1 \cdot \vec{x}_2), \text{ where } \lambda = 0.$$

Умножив $\vec{x}_2$: $\vec{x}_2 = \frac{1}{\mu} [M \ddot{\vec{x}}_1 + \vec{G} \times \dot{\vec{x}}_1 - \kappa \vec{x}_1]$ в уравн. 3D.

$$(\dot{\vec{x}}_1)^2 = (\dot{\rho}_1)^2 + \rho_1^2 \dot{\varphi}_1^2$$



$$W(x_1, x_2) = \frac{1}{2} \kappa (\rho_1^2 + \rho_2^2) + \mu \rho_1 \rho_2 \cos(\varphi_1 - \varphi_2), \text{ where } \varphi_{1,2} \text{ are constant.}$$

Умножив $(\dot{\vec{x}}_1)^2 = (\dot{\vec{x}}_2)^2$. $\vec{x}_2^2 = \frac{1}{\mu^2} [M^2 (\ddot{\vec{x}}_1)^2 + (\vec{G} \times \dot{\vec{x}}_1)^2 + \kappa^2 x_1^2 + 2M \dot{\vec{x}}_1 \cdot \vec{G} \times \dot{\vec{x}}_1 - 2\kappa M \dot{\vec{x}}_1 \cdot \vec{x}_1 - 2\kappa \vec{G} \times \dot{\vec{x}}_1 \cdot \vec{x}_1]$

$$M(\ddot{\vec{x}}_1)^2 = E - W(x_1, x_2) = E - \frac{1}{2} \kappa x_1^2 - \frac{1}{2} \frac{\kappa}{\mu^2} [\dots] - \vec{x}_1 \cdot [M \ddot{\vec{x}}_1 + \vec{G} \times \dot{\vec{x}}_1 - \kappa \vec{x}_1] -$$

— можно расст. как грав. уравнение для $\vec{x}_1(t)$. Очень сложное, не мн.

$$\text{Упроще } \left(\frac{M}{G}\right)^2 \ddot{\vec{x}}_1 + \left(1 - \frac{2\kappa M}{G^2}\right) \ddot{\vec{x}}_1 + \omega_0^2 \vec{x}_1 = 0.$$

Для упрощенного грав. 3D vortex'a:

30/XII

$$M \ddot{\vec{x}} + \vec{G} \times \dot{\vec{x}} = -\kappa \vec{x}, \text{ в 3D. 26/XII когда } M \rightarrow -M, \text{ то } \vec{x} \text{ имеет противоположный } \kappa \text{ парам. осей. } M \ddot{\vec{x}} = -\kappa \vec{x}, \kappa > 0 \text{ (или } \kappa \rightarrow -\kappa)$$

3D из $\frac{d}{dt} \frac{\partial \hat{L}}{\partial \dot{\vec{x}}_j} = \frac{\partial \hat{L}}{\partial \vec{x}_j}$ или $\frac{d}{dt} \frac{\partial \hat{L}}{\partial \dot{\vec{x}}_j} = -\frac{\partial W}{\partial \vec{x}_j}$ — не так, т.к. $\vec{G}$ -мем. в $\hat{L}$ зависит от $\vec{x}_j$

Лагранжиан 26/XII не верный (!). Изучим.

→ 3D эквивалентно гармон. осциллятору с затуханием в маг. поле $\vec{H} \parallel OZ$. (Сила Лоренца).

$$\vec{G} \times \dot{\vec{x}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & G_z \\ \dot{x} & \dot{y} & 0 \end{vmatrix} = \vec{j} G_z \dot{x} - \vec{i} G_z \dot{y}$$

$$\begin{cases} M \ddot{x} - G_z \dot{y} = -\kappa x \\ M \ddot{y} + G_z \dot{x} = -\kappa y \end{cases} \quad \begin{cases} z = x + iy, \quad M \ddot{z} - G_z (\dot{y} - i \dot{x}) = -\kappa z \\ M \ddot{z} + i G_z \dot{z} + \kappa z = 0 \end{cases}$$

— вместо одного векторного 3D возмущим одно комплексное (Л.А. т.2, §21).

$$z(t) = z_0 e^{i\omega t} \rightarrow -M\omega^2 + i G_z \omega + \kappa = 0$$

$$+M\omega^2 + G_z \omega - \kappa = 0, \quad \omega_{1,2} = \frac{1}{2M} [-G_z \pm \sqrt{G_z^2 + 4M\kappa}], \quad G_z < 0.$$

Мног 26/XII грав 3D $M^2 \ddot{\vec{x}} + (G^2 + 2\kappa M) \ddot{\vec{x}} + \kappa^2 \vec{x} = 0$ (мног хопем две coupled disks)

$$\omega_{\pm}^2 = \frac{1}{2} \frac{G^2}{M^2} \left[1 + \frac{2\kappa M}{G^2} \pm \sqrt{\left(1 + \frac{2\kappa M}{G^2}\right)^2 - \frac{4M^2 \kappa^2}{G^4}} \right] = \frac{1}{2} \frac{G^2}{M^2} \left[1 + \frac{2\kappa M}{G^2} \pm \sqrt{1 + 4 \frac{\kappa M}{G^2}} \right]$$

$$\omega_{\pm}^2 (M \rightarrow 0) = \frac{1}{2} \frac{G^2}{M^2} \left[1 + \frac{2\kappa M}{G^2} \pm \left(1 + 2 \frac{\kappa M}{G^2} - \frac{4}{8} \left(\frac{\kappa M}{G^2}\right)^2 + \dots\right) \right], \quad \omega_+^2 = \frac{G^2}{M^2} + O(M^{-1})$$

$$\omega_-^2 = \frac{1}{2} \frac{G^2}{M^2} (1) 2 \frac{\kappa^2 M^2}{G^4} = + \frac{\kappa^2}{G^2} = +\omega_0^2, \quad (1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3, \quad x = \frac{4\kappa M}{G^2}$$

$$\omega_-^2 = \frac{1}{2} \frac{G^2}{M^2} \left[1 + \frac{x}{2} - \sqrt{1+x} \right] = \frac{1}{2} \frac{G^2}{M^2} \left[1 + \frac{x}{2} - \left(1 + \frac{x}{2} - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \dots\right) \right] = \frac{1}{2} \frac{G^2}{M^2} \left(\frac{1}{8}x^2 - \frac{1}{16}x^3 \right) =$$

$$= \frac{1}{2} \frac{G^2}{M^2} \frac{1}{8} x^2 \left(1 - \frac{x}{2}\right) = \frac{\kappa^2}{G^2} \left(1 - \frac{2\kappa M}{G^2}\right), \quad \omega_-^2 = \omega_0^2 \left(1 - \frac{2\kappa M}{G^2}\right) \text{ — отн. с 26/XII } \mu/\mu \text{ } M \rightarrow -M \text{ там.}$$

$$\omega_{1,2} = \frac{G_z}{2M} [-1 \pm \sqrt{1+x}] = \frac{G_z}{2M} \left[1 \pm \left(1 + \frac{x}{2} - \frac{1}{8}x^2\right) \right], \quad \omega_1 = \frac{G_z}{M} + O(M^0), \quad M \rightarrow 0.$$

$$\omega_2 = \frac{G_2}{2M} \left[\frac{x}{2} - \frac{x^2}{16} \right] (-1) = - \frac{G_2}{2M} \frac{x}{2} \left[1 - \frac{x}{4} \right] = - \frac{G_2}{2M} \frac{4Mx}{2G_2^2} \left[1 - \frac{Mx}{G_2^2} \right]$$

$$\omega_2 = - \frac{x}{G_2} \left[1 - \frac{Mx}{G_2^2} \right] \text{ const. c } \omega_1^2 = \frac{x^2}{G^2} \left(1 - \frac{2xM}{G^2} \right).$$

Тогда $\omega_2 > 0$ надо учитывать $G_2 < 0$ для вихря.

$$\omega_{1,2}^2 = \frac{G^2}{4M^2} \left[-1 \pm \sqrt{1+x} \right]^2 = \frac{G^2}{4M^2} \left[1 + 1 + x \mp 2\sqrt{1+x} \right] = \frac{G^2}{2M^2} \left[1 + \frac{x}{2} \pm \sqrt{1+x} \right]$$

$$\omega_{\pm}^2 = \frac{G^2}{2M^2} \left[1 + \frac{x}{2} \pm \sqrt{1+x} \right], \quad x = \frac{4xM}{G^2}. \text{ Тогда } \omega_{1,2}^2 \text{ и } \omega_{\pm}^2 \text{ совпадают.}$$

$z(t) = z_0 e^{i\omega t} = |z_0| e^{i(\omega t + \alpha)}$, $x = \operatorname{Re} z = |z_0| \cos(\omega t + \alpha)$, $y = \operatorname{Im} z = |z_0| \sin(\omega t + \alpha)$.
- угловое фаз $\pm \pi/2$, амплитуды равны $\Rightarrow z = x + iy$ описывает круг.
Хотя $z = x + iy$ некое выражение.

УД F/N/F disk $\left\{ \begin{aligned} -M\ddot{x}_1 + G_2 \dot{x}_1 &= \kappa \bar{x}_1 + \mu \bar{x}_2 + \lambda \bar{x}_2 \times \hat{z} \\ -M\ddot{x}_2 - G_2 \dot{x}_2 &= \kappa \bar{x}_2 + \mu \bar{x}_1 + \lambda \hat{z} \times \bar{x}_1 \end{aligned} \right\}$ $\hat{z} \times \bar{x} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & 0 \end{vmatrix} = \hat{j}x - \hat{i}y$
гла обобщенных координат $\vec{r}$ и $\vec{p}$ в каноническом $\vec{H}$ и $\hat{z}$.

$$\left\{ \begin{aligned} -M\ddot{x}_1 + G_2 \dot{x}_1 &= \kappa x_1 + \mu x_2 + \lambda y_2 & (1) \\ -M\ddot{y}_1 - G_2 \dot{y}_1 &= \kappa y_1 + \mu y_2 - \lambda x_2 & (2) \\ -M\ddot{x}_2 - G_2 \dot{x}_2 &= \kappa x_2 + \mu x_1 - \lambda y_1 & (3) \\ -M\ddot{y}_2 + G_2 \dot{y}_2 &= \kappa y_2 + \mu y_1 + \lambda x_1 & (4) \end{aligned} \right. \quad \begin{aligned} (1)+(2) \cdot i: -M\ddot{z}_1 - G_2 i \dot{z}_1 &= \kappa z_1 + \mu z_2 - i \lambda z_2 \\ (3)+(4) \cdot i: -M\ddot{z}_2 + G_2 i \dot{z}_2 &= \kappa z_2 + \mu z_1 + i \lambda z_1 \end{aligned}$$

УД F/N/F disk $\left\{ \begin{aligned} +M\ddot{z}_1 + i G_2 \dot{z}_1 + \kappa z_1 &= -(\mu - i\lambda) z_2 \\ M\ddot{z}_2 - i G_2 \dot{z}_2 + \kappa z_2 &= -(\mu + i\lambda) z_1 \end{aligned} \right\} \quad (1) \quad (2)$
(P1P2 = -1) $\rightarrow$ Если $\lambda = 0$ и $G_2 \rightarrow -G_2$ в (2), то $x = \frac{1}{2}(z_1 + z_2)$ и $z = z_1 - z_2$ канонические для УД (P1P2 = +1).

Тогда для (P1P2 = +1) $\left\{ \begin{aligned} M\ddot{x} + i G_2 \dot{x} + \kappa x &= -\mu x \\ M\ddot{z} + i G_2 \dot{z} + \kappa z &= \mu z \end{aligned} \right\} \quad \begin{aligned} \omega_x &= \frac{1}{G}(\kappa + \mu) \text{ и } \mu M = 0 \\ \omega_z &= \frac{1}{G}(\kappa - \mu), \text{ P1P2 = +1 case. } (\mu < 0). \end{aligned}$

1) Для P1P2 = +1 только λ (параметры симметрии) меняет значения x и z .
Но $\lambda \ll \mu$ и мы можем пренебречь в 1м. членом.

2) Для P1P2 = -1 также λ , симметрии параметры, а он не меняет.

$\rightarrow$ Лагранжиан $\hat{L}(x_1, x_2) = a(\vec{G} \times \vec{x}_1) \cdot \dot{\vec{x}}_1 - b(\vec{G} \times \vec{x}_2) \cdot \dot{\vec{x}}_2 + \hat{L}_0$
 $(\vec{G} \times \vec{x}) \cdot \dot{\vec{x}} = (\vec{x} \times \dot{\vec{x}}) \cdot \vec{G} = (\dot{\vec{x}} \times \vec{G}) \cdot \vec{x}$, $L = a(\vec{G} \times \vec{x}) \cdot \dot{\vec{x}} + L_0$

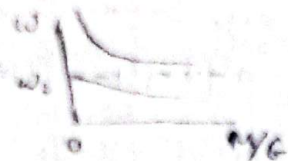
$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\vec{x}}} - \frac{\partial L}{\partial \vec{x}} = 0, \quad \frac{d}{dt} a(\vec{G} \times \vec{x}) - a(\dot{\vec{x}} \times \vec{G}) + (\text{т.ч. } L_0) = 2a(\vec{G} \times \dot{\vec{x}}) + (L_0) = \vec{G} \times \dot{\vec{x}} + (L_0)$$

т.е. $a = 1/2$, $b = 1/2$ и лагранжиан (P1P2 = -1):

$$\hat{L}(\vec{x}_1, \vec{x}_2) = \frac{1}{2} M (\dot{\vec{x}}_1)^2 + \frac{1}{2} M (\dot{\vec{x}}_2)^2 + \frac{1}{2} (\vec{G} \times \vec{x}_1) \cdot \dot{\vec{x}}_1 - \frac{1}{2} (\vec{G} \times \vec{x}_2) \cdot \dot{\vec{x}}_2 - W(\vec{x}_1, \vec{x}_2)$$

Всё энергии сохраняется $E = \sum_j \dot{\vec{x}}_j \frac{\partial L}{\partial \dot{\vec{x}}_j} - L = \frac{1}{2} M (\dot{\vec{x}}_1)^2 + \frac{1}{2} M (\dot{\vec{x}}_2)^2 + W(\vec{x}_1, \vec{x}_2)$
в аналогичном выражении по $\vec{x}_j$.

31/10) 1) we have vector $(p_1, p_2) = (-1)$ y uosmalo ad. theory $\omega = \omega_0 [1 - \frac{\mu x}{G^2}] + O(\mu^2)$



$$\begin{aligned} (2) &: M \ddot{\bar{z}}_2 + i G_2 \dot{\bar{z}}_2 + \kappa \bar{z}_2 = -\bar{v} \bar{z}_1 \\ (1) &: M \ddot{\bar{z}}_1 + i G_2 \dot{\bar{z}}_1 + \kappa \bar{z}_1 = -\bar{v} \bar{z}_2 \end{aligned} \quad , \quad \nu = \mu + i\lambda, \quad p_1 p_2 = -1.$$

begin $X = z_1 + \bar{z}_2, \quad Z = z_1 - \bar{z}_2$
$$\begin{cases} M \ddot{X} + i G_2 \dot{X} + \kappa X = -\bar{v} \bar{X} \\ M \ddot{Z} + i G_2 \dot{Z} + \kappa Z = \bar{v} \bar{Z} \end{cases}$$
 - y p-a gus $\bar{X}, \bar{Z}$

procedemos con el sistema en forma $p_1 p_2 = +1(1)$

$$\bar{v} \bar{Z} = (\mu - i\lambda)(Z - i\bar{Z}) = \mu Z - \lambda \bar{Z} + i[-\lambda Z - \mu \bar{Z}], \quad i G_2 \dot{Z} = G_2 (i\dot{Z} - \dot{\bar{Z}})$$

$$\begin{cases} M \ddot{Z}' + G_2 \dot{Z}'' + \kappa Z' = \mu Z' - \lambda \bar{Z}'' \\ M \ddot{Z}'' + G_2 \dot{Z}' + \kappa Z'' = -\lambda Z' - \mu \bar{Z}'' \end{cases} \quad , \quad Z = Z' + i Z''$$

Bugno, $z(t) = z_0 e^{i\omega t} = |z_0| e^{i(\omega t + \phi)} = |z_0| [\cos(\omega t + \phi) + i \sin(\omega t + \phi)]$

$Z' = |z_0| \cos(\omega t + \phi), \quad Z'' = |z_0| \sin(\omega t + \phi)$ no es problema yD gus (Z', Z'')

$Z(t) = a e^{i\omega t} + b e^{-i\omega t}$ - nos quedamos b $M \ddot{Z} + i G_2 \dot{Z} + \kappa Z = \bar{v} \bar{Z}$

$$\begin{aligned} -\omega^2 M [a e^{i\omega t} + b e^{-i\omega t}] + i G_2 [i\omega a e^{i\omega t} - i\omega b e^{-i\omega t}] + \kappa [a e^{i\omega t} + b e^{-i\omega t}] &= \\ = \bar{v} [\bar{a} e^{-i\omega t} + \bar{b} e^{i\omega t}] \end{aligned}$$

$$\begin{aligned} e^{i\omega t}: & \quad -\omega^2 M a - \omega a G_2 + \kappa a = \bar{v} \bar{b} \\ e^{-i\omega t}: & \quad -\omega^2 M b + \omega b G_2 + \kappa b = \bar{v} \bar{a} \end{aligned} \quad \begin{cases} [\kappa - \omega G_2 - \omega^2 M] a = \bar{v} \bar{b} \\ [\kappa + \omega G_2 - \omega^2 M] b = \bar{v} \bar{a} \end{cases}$$

$$\bar{v} \bar{b} = [\kappa - \omega G_2 - \omega^2 M] \bar{a}, \quad [\kappa + \omega G_2 - \omega^2 M] \frac{1}{\bar{v}} [\kappa - \omega G_2 - \omega^2 M] \bar{a} = \bar{v} \bar{a}$$

$$[(\kappa - \omega^2 M)^2 - \omega^2 G_2^2] = |\nu|^2 - \text{con } \mu \text{ y } \lambda \text{ uicler gba } \text{ban. } \omega_{\pm}^2$$

si buscamos $\omega_{\pm}$ en forma, resulta $b = \frac{1}{\bar{v}} [\kappa - \omega G_2 - \omega^2 M] \bar{a}, \quad \omega = \omega_{\pm}$ gba $\omega_{\pm}^2 = \frac{1}{2} [\kappa \pm \sqrt{\kappa^2 - \omega^2 G_2^2 - \omega^4 M^2}]$

$$Z(t) = a e^{i\omega t} + [\kappa - \omega G_2 - \omega^2 M] \frac{1}{\bar{v}} \bar{a} e^{-i\omega t}, \quad \nu = \mu + i\lambda, \quad \frac{1}{\bar{v}} = \frac{\bar{v}}{|\nu|^2}$$

$$[\kappa - \omega G_2 - \omega^2 M] \frac{1}{\bar{v}} = \bar{v} [\kappa + \omega G_2 - \omega^2 M]^{-1}, \quad Z(t) = a e^{i\omega t} + \bar{v} \bar{a} e^{-i\omega t}$$

$$\bar{a} \bar{v} = (a' - i a'') (\mu - i\lambda) = \mu a' - \lambda a'' - i(\lambda a' + \mu a'')$$

$$Z = z_1 - \bar{z}_2 = x_1 + i y_1 - (x_2 - i y_2) = x_1 - x_2 + i(y_1 + y_2), \quad a = |a| e^{i\phi_0}, \quad \nu = |\nu| e^{i\psi_0}$$

$$Z(t) = |a| e^{i(\omega t + \phi_0)} + \bar{v} \bar{a} e^{-i(\omega t + \phi_0 + \psi_0)} \quad |a| |\bar{v}|$$

$$Z(t) = |a| [\cos(\omega t + \phi_0) + i \sin(\omega t + \phi_0)] + \bar{v} \bar{a} [\cos(\omega t + \phi_0 + \psi_0) - i \sin(\omega t + \phi_0 + \psi_0)]$$

$$\begin{cases} \frac{Z'(t)}{|a|} = \cos(\omega t + \phi_0) + \bar{v} |\bar{a}| \cos(\omega t + \phi_0 + \psi_0) \\ \frac{1}{|a|} Z''(t) = \sin(\omega t + \phi_0) - \bar{v} |\bar{a}| \sin(\omega t + \phi_0 + \psi_0) \end{cases} \quad \text{debemos } Z'^2 + Z''^2 = \text{const} (?)$$

hacer $\phi_0 = 0$, $\frac{1}{a^2} Z'^2 + \frac{1}{a^2} Z''^2 = \cos^2 \omega t + \sin^2 \omega t + \bar{v}^2 |\bar{a}|^2 [\cos^2(\omega t + \phi_0 + \psi_0) + \sin^2(\omega t + \phi_0 + \psi_0)] +$

Escaneado con CamScanner

$$\sin \delta = \frac{c \sin \phi}{\sqrt{1 + 2 \cos \phi \cdot c + c^2}}$$

$$\cos \omega t + c \cos(\omega t + \phi) = \sqrt{1 + 2c \cos \phi + c^2} [\cos d \cdot \cos \omega t - \sin d \cdot \sin \omega t] = (\sqrt{1 + 2c \cos \phi + c^2}) \cdot \cos(\omega t + d)$$

$$\sin \omega t - C \sin(\omega t + \varphi) = \sin \omega t - C \sin \omega t \cdot \cos \varphi - C \cos \omega t \cdot \sin \varphi =$$

$$(1 - c \cos \varphi) \sin \omega t - c \sin \varphi \cdot \cos \omega t, \rightarrow \cos \beta = \frac{-c \sin \varphi}{\sqrt{1 - 2c \cos \varphi + c^2}}, \sin \beta = \frac{+c \cos \varphi + 1}{\sqrt{1 - 2c \cos \varphi + c^2}}$$

$$\sin \omega t - c \sin(\omega t + \phi) = \left[-\sin \beta \sin \omega t + \cos \beta \cos \omega t \right] = \cos(\omega t + \beta) \cdot \sqrt{1 - 2c \cos \phi + c^2}$$

$$\left\{ \begin{array}{l} \frac{1}{a} z'(t) = \sqrt{1+2c \cos \varphi + c^2} \cos(\omega t + \alpha) \\ \frac{1}{a} z''(t) = \sqrt{1-2c \cos \varphi + c^2} \cos(\omega t + \beta) \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} z'(t) = A \cos(\omega t + \alpha) \\ z''(t) = B \cos(\omega t + \beta) \end{array} \right\}$$

- с. 49, в. 22 описывается из Л.П. т. 1, § 23.

$$A = a \sqrt{1 + 2c \cos \varphi + c^2}, \quad B = a \sqrt{1 - 2c \cos \varphi + c^2}, \quad c = f/v, \quad \varphi = \varphi_v.$$

Упр. 6. Спелусопиу $\frac{\xi'^2}{A^2} + \frac{\xi''^2}{B^2} - \frac{2\xi'\xi''}{AB} \cos \delta = \sin^2 \delta$, $\delta = \beta - \alpha$.

$A=B$ только если $\cos \varphi_0 = 0$, $\varphi_0 = \pm \pi/2$ или $c = f|V| = 0$, $|V| = 0$, $\mu = 0$ (ker $\lambda = 0$)
 $\sqrt{\lambda} = \mu + i\lambda$, $\varphi_0 = \arctg \frac{\lambda}{\mu} \neq \pm \pi/2$, т.к. $\lambda < \mu$.

Wp $|0\rangle = 0$, $\sin \alpha = 0$, $\cos \beta = 0$, i.e. $\beta - \alpha = \pm \pi/2$ u $A = B \Rightarrow k/\mu g$. The zero-point

Фазы $a = \varphi_a$ не зависят от начального условия. Можно $\omega t \rightarrow \omega t + \varphi_a$, но на траекторию это не влияет, т.к. она задана лишь ωt и $A, B = F(\omega, N1, \varphi_1)$.

Note: Замечено, что при увеличении n , coupling слаб F/N/F

1. e. $|\vec{V}| \neq 0$. Масса M мусора увеличивается $|\vec{V}| \neq \text{const}$

и/или Эконом. движение.

$$\begin{aligned} \cos \delta &= \cos(\beta - \alpha) = \cos \alpha \cos \beta + \sin \alpha \sin \beta = [(1 + c \cos \varphi)(-c \sin \varphi) + \\ &+ c \sin \varphi \cdot (c \cos \varphi - 1)] \frac{1}{\sqrt{c^2 - 1}} = \frac{1}{\sqrt{c^2 - 1}} [-c \sin \varphi - c^2 \cos \varphi \sin \varphi + c^2 \sin \varphi \cos \varphi - c \sin \varphi] = \\ &= \frac{1}{\sqrt{c^2 - 1}} (-2)c \sin \varphi, \quad \sqrt{c^2 - 1} = \sqrt{(1 + c \cos \varphi + c^2)(1 - c \cos \varphi + c^2)} = \sqrt{(1 + c^2)^2 - c^2 \cos^2 \varphi} \end{aligned}$$

$$\cos \delta = - \frac{2c \sin \varphi}{\sqrt{(1+c^2)^2 - c^2 \cos^2 \varphi}}, \quad \cos \delta \sim |V| \sin \varphi_0 - \text{Naxson formula on degeneracy interlayer coupling } nD''.$$

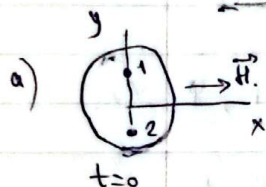
X'(t). $\eta_{\mu\nu} \varphi_\nu \rightarrow \varphi_\nu + \pi$, $A \rightarrow B$, $B \rightarrow A$, $C \rightarrow -C$, $\cos \alpha \rightarrow \frac{1 - \cos \varphi}{\sqrt{-}} = -\sin \beta$, $\sin \alpha \rightarrow \frac{-\cos \beta}{\sqrt{-}} = \cos \beta$
 $\cos \beta \rightarrow \frac{C \sin \varphi}{\sqrt{+}} = \sin \alpha$, $\sin \beta \rightarrow \frac{-\cos \alpha - 1}{\sqrt{+}} = -\cos \alpha$, $d \rightarrow \frac{\pi}{2} + \beta$, $\beta \rightarrow -\frac{\pi}{2} + \alpha$.

$$\begin{cases} X'(t) = B \cos(\omega t + \frac{\pi}{2} + \beta) = -B \sin(\omega t + \beta) \\ X''(t) = A \cos(\omega t + \alpha - \frac{\pi}{2}) = A \sin(\omega t + \alpha) \end{cases}$$

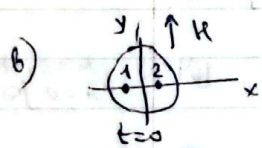
$$\begin{aligned} 2X_1(t) = Z'(t) + X'(t) &= A \cos(\omega t + \alpha) - B \sin(\omega t + \beta) = A [\cos \omega t \cos \alpha - \sin \omega t \sin \alpha] - B [\sin \omega t \cos \beta + \cos \omega t \sin \beta] = [A \cos \alpha - B \sin \beta] \cos \omega t + \\ &+ (-1) [A \sin \alpha + B \cos \beta] \sin \omega t = a [1 + \cos \varphi \cdot c + 1 - c \cos \varphi] \cos \omega t + \\ &+ (-1) a [c \sin \varphi - c \sin \varphi] \sin \omega t = 2a \cos \omega t \end{aligned}$$

$$\begin{aligned} 2Y_1(t) = Z''(t) + X''(t) &= B \cos(\omega t + \beta) + A \sin(\omega t + \alpha) = B [\cos \omega t \cos \beta - \sin \omega t \sin \beta] + \\ &+ A [\sin \omega t \cos \alpha + \cos \omega t \sin \alpha] = \cos \omega t [B \cos \beta + A \sin \alpha] + \sin \omega t [-B \sin \beta + A \cos \alpha] = \\ &= \cos \omega t \cdot [-c \sin \varphi + c \sin \varphi] a + a \sin \omega t \cdot [1 - c \cos \varphi + 1 + c \cos \varphi] a = 2a \sin \omega t - ? \end{aligned}$$

→ $X_1(t) = a \cos \omega t$, $Y_1(t) = a \sin \omega t$ — описано движение кр. п. в. Нужно.



Var. y. c. y. c. $Y_1(0) = -Y_2(0) \neq 0$, $X_1(0) = X_2(0) = 0$. Нужно
 $Z'(0) = 0$, $Z''(0) = 0$, $X'(0) = 0$, $X''(0) = Y_1(0) - Y_2(0) = 2Y_1(0) \neq 0$
 X-модуль возмущения.



Нужно, $X_1(0) = -X_2(0) \neq 0$, $Y_1(0) = Y_2(0) = 0$
 $Z'(0) = X_1(0) - X_2(0) = 2X_1(0) \neq 0$, $Z''(0) = 0$, $X'(0) = 0$, $X''(0) = 0$
 Z-модуль возмущения.

→ Т.е. Z и X-модуль несут возмущения одновременно.

Нужно $Z'(0) = A \cos \alpha \neq 0$, $Z''(0) = B \cos \beta = 0$, $\cos(\varphi_0 + \beta) = 0$ или $B = 0$ (?)

$X'(0) = -B \sin(\varphi_0 + \beta) = 0$, $X''(0) = A \sin(\varphi_0 + \alpha) = 0$, $A \cos(\varphi_0 + \alpha) = \max_{(min)}$

$$W(\vec{X}_1, \vec{X}_2) = \frac{1}{2} (x + \mu) (x_1^2 + x_2^2) - \frac{1}{2} \int |\vec{X}_1 - \vec{X}_2|^2 + \lambda \hat{z} \cdot (\vec{X}_1 \times \vec{X}_2), \quad \mu = 1/\mu_1 c, c_2 = -\mu_1 < 0$$

Vortex cores и μ — комбинация Z и X .

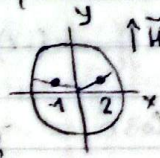
Нужно $\lambda = 0$, $\varphi_1 = 0$, $B^2 = a |1 - c|$, $c = f H = [\kappa + \omega G - M \omega^2]^{-1} 1/\mu_1$

Может $c = 1$? $1/\mu_1 = \kappa + \omega G - M \omega^2$, $\omega = \omega_-$ — eigen frequency.

$$\omega_- (M \omega) = \frac{1}{|G|} \sqrt{x^2 - \mu^2}, \quad \kappa + \omega_- G = \kappa \pm \sqrt{x^2 - \mu^2} \neq 1/\mu_1 \text{ если } \kappa \neq 1/\mu_1.$$

$1/\mu_1 \neq [\kappa - \sqrt{x^2 - \mu^2}]^2$, $B \neq 0$, тогда var. y. c. $Z'(0) \neq 0$, $\alpha = 0$, $\alpha = 0$.

$$W(\vec{X}_1, \vec{X}_2) = \frac{1}{2} (x - 1/\mu_1) (x_1^2 + x_2^2) + \frac{1}{2} 1/\mu_1 |\vec{X}_1 - \vec{X}_2|^2 \Rightarrow$$



Нужно $\lambda = 0$, $\varphi_1 = 0$, $B^2 = a |1 - c|$, $c = f H = [\kappa + \omega G - M \omega^2]^{-1} 1/\mu_1$

$Z'(0) = 2X_1(0) \neq 0$, $Z''(0) = Y_1(0) + Y_2(0) = 2Y_1(0) \neq 0$, $X'(0) = 0$, $X''(0) = 0$

$Z'(0) = A \cos(\varphi_0 + \alpha) = \max_{(min)}$, $Z''(0) = B \cos(\varphi_0 + \beta) \neq 0$, $X'(0) = -B \sin(\varphi_0 + \beta) \approx 0$

Возмущения Z -модуль, в основном!

$X''(0) = A \sin(\varphi_0 + \alpha) \approx 0$

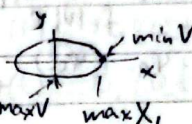
$$W_{\text{int}}(\text{core-val}) = \lambda \hat{z} \cdot (\vec{X}_1 \times \vec{X}_2) = \lambda \hat{z} \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & 0 \\ x_2 & y_2 & 0 \end{vmatrix} = \lambda \hat{z} \cdot (-\vec{e}_1 x_2 y_1 + \vec{e}_2 x_1 y_2) = \lambda (x_1 y_2 - x_2 y_1)$$

$$W_{\text{int}}(c-v) = \lambda [x_1 y_2 + x_1 y_1] = \lambda x_1(\omega) [y_1 + y_2] = -\lambda |1| \text{ забавно or } \text{Sign}(1).$$

$$\text{Диаметр } \omega \text{ (при } \lambda=0) \quad \frac{A}{B} = \frac{1+c}{1-c} = \varepsilon$$

$$\text{Гор. } \mu \text{ кристалл } (40/140 \text{ nm}, D=1000 \text{ nm}, \beta = 4\%/500 = 0.08) \approx \frac{62}{26} = \frac{31}{13} \approx 2.4$$

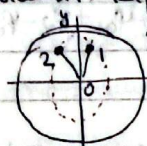
$$1+c = \varepsilon(1-c), \quad 1-\varepsilon = -c(1+\varepsilon), \quad c = \frac{\varepsilon-1}{\varepsilon+1} \approx \frac{2.4-1}{2.4+1} = \frac{1.4}{3.4} \approx 0.4$$

Важно, что max $X_1(t)$ совпадает min $V(t)$  т.е. core задегнессе

в узлах затем движется и увеличивается

в минимуме. Это совпадает. Диаметр. Кармане, движение "core".

У кривых (снова на 90°)



$$y_1(0) = y_2(0) \approx 270 \text{ nm}$$

$$x_1(0) = 20 \text{ nm}, \quad x_2(0) = -50 \text{ nm}$$

т.е. simulation дает $z'(0) = x_1(0) - x_2(0) = 70 \text{ nm}$, $z''(0) = y_1 + y_2 = 540 \text{ nm} (!)$

$$x'(0) = x_1 + x_2 = -30 \text{ nm}, \quad x''(0) = y_1 - y_2 = 0 (!) - \text{т.е. } \text{возбуждается,}$$

в основном, z - мюга.

Уз-за zero $x'(0) = x_1(0) + x_2(0) \neq 0$? Уз-за λ ?

Замеча 1 ≥ 2 дает $\lambda = 0$ или $\lambda \sim (c_1 - c_2)$. У инвариантно и/ли реверси замеч.

Замеч вытекает $\parallel \vec{Oy} \parallel \vec{H}$.

У кристалл от расщепления в $\frac{60}{28} \approx 2$ раза, т.е. simulated $\left(\frac{A}{B}\right)^{-1} \approx \frac{2.62}{26} \approx 4.7$,

$B \approx 5A$, замеч вытекает в паре $\parallel \vec{Oy} \parallel \vec{H}$.

$$\varphi_0 = \pi, \quad \frac{B}{A} = \frac{1+c}{1-c} > 1, \quad c \approx \frac{1/\mu}{[\alpha + \omega G]} = \frac{1/\mu}{\alpha + \sqrt{\alpha^2 - \mu^2}} \approx 1 \text{ где } \beta \ll 1, \quad \alpha \approx 1/\mu + O(\beta^2)$$

$$\text{Simul. } c = \frac{\varepsilon-1}{\varepsilon+1} = \frac{4.7-1}{4.7+1} = \frac{3.7}{5.7} \approx 0.65 \text{ где } \beta = 0.08.$$

$$z'(0) = A \cos \alpha = a(1 + \cos \varphi_0 \cdot c) = a(1-c), \quad z''(0) = B \cos \beta = +a \cdot c \sin \varphi_0, \quad \varphi_0 = \pi/2$$

$$z'(0) = A \cos(\varphi_0 + d) = -A \sin d = -a \sin \varphi_0 \cdot c - \text{мало, } \ll z''(0), \text{ т.к. } \varphi_0 \approx \pi.$$

$$x'(0) = -B \sin(\varphi_0 + \beta) = -B \cos \beta = -a c \sin \varphi_0 - \text{мало.}$$

$$x''(0) = A \sin(\varphi_0 + d) = A \cos d = a(1-c)$$

$$\rightarrow \text{Тогда } z''(0) = B \cos(\varphi_0 + \beta) = -B \sin \beta = -a [c \cos \varphi_0 - 1] = a(1+c).$$

$$\text{Итак } \begin{cases} z'(t) = |a| [\cos(\omega t + \varphi_0) + f|\nu| \cos(\omega t + \varphi_0 + \varphi_0)] \\ z''(t) = |a| [\sin(\omega t + \varphi_0) - f|\nu| \sin(\omega t + \varphi_0 + \varphi_0)] \end{cases}$$

Здесь $\omega = \omega_{\pm}$, $f = f(\omega) = f(\omega_{\pm})$. $\vec{z}(t) = z'(t) + i z''(t)$ и/ли $\omega = \omega_{-}$, $\omega = \omega_{+}$ -

- это оговорка фазового. перемены 3D где $z(t)$, есть $z_{+}(t)$ и $z_{-}(t)$.

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$$z_+(t) = a_+ e^{i\omega_+ t} + f(\omega_+) \bar{a}_+ \bar{v} e^{-i\omega_+ t}, \quad z_-(t) = a_- e^{i\omega_- t} + f(\omega_-) \bar{a}_- \bar{v} e^{-i\omega_- t}$$

Общие решение ЛОД: $z(t) = c_1 z_+(t) + c_2 z_-(t)$, $c_{1,2}$ - константы.

$$z(t) = c_1 [a_+ e^{i\omega_+ t} + f_+ \bar{a}_+ \bar{v} e^{-i\omega_+ t}] + c_2 [a_- e^{i\omega_- t} + f_- \bar{a}_- \bar{v} e^{-i\omega_- t}] =$$

$$= |a_+| |c_1| \left[e^{i(\omega_+ t + \varphi_{a_+} + \varphi_{c_1})} + f_+ |v| e^{-i(\omega_+ t + \varphi_{a_+} - \varphi_{c_1} + \varphi_v)} \right] + |a_-| |c_2| \left[e^{i(\omega_- t + \varphi_{a_-} + \varphi_{c_2})} + f_- |v| e^{-i(\omega_- t + \varphi_{a_-} - \varphi_{c_2} + \varphi_v)} \right]$$

Без ограничений, естественно нормуем $|c_1| = |c_2| = 1$, $c_{1,2} \rightarrow c_{\pm}$. $g = |a_-|/|a_+| > 0$.

$$\frac{1}{|a_+|} z(t) = e^{i(\omega_+ t + \varphi_{a_+} + \varphi_{c_+})} + f_+ |v| e^{-i(\omega_+ t + \varphi_{a_+} + \varphi_v - \varphi_{c_+})} +$$

$$+ g \left[e^{i(\omega_- t + \varphi_{a_-} + \varphi_{c_-})} + f_- |v| e^{-i(\omega_- t + \varphi_{a_-} + \varphi_v - \varphi_{c_-})} \right]$$

$$\frac{1}{|a_+|} z'(t) = \cos(\omega_+ t + \varphi_{a_+} + \varphi_{c_+}) + f_+ |v| \cos(\omega_+ t + \varphi_{a_+} + \varphi_v - \varphi_{c_+}) + g \cos(\omega_- t + \varphi_{a_-} + \varphi_{c_-}) +$$

$$+ g f_- |v| \cos(\omega_- t + \varphi_{a_-} + \varphi_v - \varphi_{c_-})$$

$$\frac{1}{|a_+|} z''(t) = \sin(\omega_+ t + \varphi_{a_+} + \varphi_{c_+}) - f_+ |v| \sin(\omega_+ t + \varphi_{a_+} + \varphi_v - \varphi_{c_+}) + g \sin(\omega_- t + \varphi_{a_-} + \varphi_{c_-}) -$$

$$- g f_- |v| \sin(\omega_- t + \varphi_{a_-} + \varphi_v - \varphi_{c_-})$$

$2X_1(t) = z' + X' = [X' \text{ сум. от } z' \text{ заменив } \varphi_v \rightarrow \varphi_v + \pi \text{ и } g \text{ на } -g \text{ (конвергенция)}]$

$$\frac{1}{|a_+|} z_+(t) = e^{i(\omega_+ t + \varphi_{a_+}^3)} + f_+ |v| e^{-i(\omega_+ t + \varphi_{a_+}^3 + \varphi_v)} \quad - \text{одно из произвольных решений.}$$

$$\frac{1}{|a_+|} z'_+(t) - \frac{1}{|a_+|} X_+(t) = e^{i(\omega_+ t + \varphi_{a_+}^x)} + f_+ |v| e^{-i(\omega_+ t + \varphi_{a_+}^x + \varphi_v)}$$

$$2X_1(t) = z'_+(t) + X'_-(t) = |a_3| \left[\cos(\omega_+ t + \varphi_{a_+}^3 + \varphi_{c_+}^3) + f_+ |v| \cos(\omega_+ t + \varphi_{a_+}^3 + \varphi_v - \varphi_{c_+}^3) + \right.$$

$$+ g f_- \cos(\omega_- t + \varphi_{a_-}^3 + \varphi_{c_-}^3) + g f_- |v| \cos(\omega_- t + \varphi_{a_-}^3 + \varphi_v - \varphi_{c_-}^3) \left. \right] + |a_x| \left[\cos(\omega_+ t + \varphi_{a_+}^x + \varphi_{c_+}^x) - \right.$$

$$\left. - f_+ |v| \cos(\omega_+ t + \varphi_{a_+}^x + \varphi_v - \varphi_{c_+}^x) + g_x \cos(\omega_- t + \varphi_{a_-}^x + \varphi_{c_-}^x) - g_x f_- |v| \cos(\omega_- t + \varphi_{a_-}^x + \varphi_v - \varphi_{c_-}^x) \right]$$

$$\rightarrow 2X_1(t) = |a_3| \cos(\omega_+ t + \varphi_{a_+}^3) + |a_x| \cos(\omega_+ t + \varphi_{a_+}^x + \varphi_{c_+}^x) +$$

$$+ f_+ |v| \left[|a_3| \cos(\omega_+ t + \varphi_{a_+}^3 + \varphi_v - \varphi_{c_+}^3) - |a_x| \cos(\omega_+ t + \varphi_{a_+}^x + \varphi_v - \varphi_{c_+}^x) \right] +$$

$$+ |a_3| g f_- \cos(\omega_- t + \varphi_{a_-}^3 + \varphi_{c_-}^3) + |a_x| g_x \cos(\omega_- t + \varphi_{a_-}^x + \varphi_{c_-}^x) +$$

$$+ f_- |v| \left[|a_3| g f_- \cos(\omega_- t + \varphi_{a_-}^3 + \varphi_v - \varphi_{c_-}^3) - |a_x| g_x \cos(\omega_- t + \varphi_{a_-}^x + \varphi_v - \varphi_{c_-}^x) \right]$$

Даже если $\omega_+ \gg \omega_-$ и тогда можно $\omega_- \rightarrow 0$ ($\omega_- = 0$), то

$$\rightarrow 2X_1(t) = |a_3| \cos(\omega t + \varphi_a^3) + |a_x| \cos(\omega t + \varphi_a^x) + f |v| \left[|a_3| \cos(\omega t + \varphi_a^3 + \varphi_v) - \right.$$

$$\left. - |a_x| \cos(\omega t + \varphi_a^x + \varphi_v) \right],$$

т.е. a_+ , a_- - произвольные константы и

ведь $|a_x| \neq |a_3|$, $\varphi_a^x + \varphi_a^3$ как параметр $3/4\pi$ и т.д.

Но в принципе замечу, будем, мы-же называть ω_+ и ω_- .

$$\xi(t) = |a_z| \left[e^{i(\omega t + \varphi_0^z)} + f|v| e^{-i(\omega t + \varphi_0^z + \varphi_v)} \right]$$

$$X(t) = |a_x| \left[e^{i(\omega t + \varphi_0^x)} + f|v| e^{-i(\omega t + \varphi_0^x + \varphi_v + \pi)} \right]$$

$$\rightarrow 2y_1(t) = \xi'' + X'' = |a_z| \sin(\omega t + \varphi_0^z) + |a_x| \sin(\omega t + \varphi_0^x) + f|v| \left[-|a_z| \sin(\omega t + \varphi_0^z + \varphi_v) + |a_x| \sin(\omega t + \varphi_0^x + \varphi_v) \right]$$

$\vec{X}_1(t)$ спектрограмма отрезков времени, т.к.

$$\left\{ \begin{aligned} X_1(t) &= A_1 \cos(\omega t + \varphi_{x1}) \\ Y_1(t) &= B_1 \cos(\omega t + \varphi_{y1}) \end{aligned} \right\} \text{ со значениями } A_1, B_1, \varphi_{x1}, \varphi_{y1} = \hat{f}(\varphi_0^x, \varphi_0^z, |a_{x3}|, |a_{z3}|)$$

Две волны вращаются относительно $\left\{ \begin{aligned} X_1(t) &= A_1(\omega_+ t) \cos(\omega_+ t + \varphi_{x1}) \\ Y_1(t) &= B_1(\omega_+ t) \cos(\omega_+ t + \varphi_{y1}) \end{aligned} \right\} \omega_+ < \omega_+$

$$A_1(t) \sim \cos \omega_+ t, B_1(t) \sim \cos \omega_+ t, \text{ например.}$$

Частота Кристиан Бугер $\omega_+/2\pi \approx 250 \text{ MHz}$, $\omega_-/2\pi \approx 35 \text{ MHz}$.

ω_+ — частота. ω_- — частота. ω_+ — частота. ω_- — частота.

Данная величина $D = 1000 \text{ nm}$, $40/1/40 \text{ nm}$ FeNi.

Но $\omega_+ \sim \omega_0$ — частота ω_+ , ω_- — частота. Скорость массы vortex M/G .
 M/G — частота. M/G — частота. M/G — частота.

Решение $X_1(t), Y_1(t)$ зависит от многих параметров, где ω_+ — частота. ω_- — частота. ω_+ — частота. ω_- — частота.

$$\frac{WM}{G} = \left| G = 2\pi L M_s / \delta \right|^{-1} = \omega \frac{JRL \delta}{\gamma^2 \delta 2\pi L M_s} = \frac{\omega}{\gamma^2 \delta} \left(\frac{R}{M_s} \right) \approx \frac{0.1 \text{ GHz}}{2.4 \text{ GHz}} \frac{500 \text{ nm}}{10 \text{ nm}} \approx 0.04 \cdot 50 \approx 2 (!)$$

$$M \text{ в } \text{YD } M \ddot{x} + \gamma x = M_s H, [x]_{\text{cm}}, M \omega^2 x = \frac{1}{V} W, M \omega^2 \approx \gamma^2 \delta, \delta = \pi (A/k_1)^{1/2}$$

$$\text{У нас } \text{YD } M \ddot{x} + \dots + \gamma x = -\frac{\partial W}{\partial x}, \text{ если } x = \frac{X}{R}, \text{ то } [M \ddot{x}] = W = \text{erg}, \frac{M}{V} \ddot{x} = \frac{W}{V} = M_s H$$

$$\frac{MR}{V} = M_w, M = \frac{V}{R} M_w = \pi R L M_w = \frac{\pi R L}{\gamma^2 \delta} \cdot f M_s (\text{FeNi}) = \frac{1}{4\pi} 30 \text{ GHz} \approx 2.4 \text{ GHz}$$

$$\delta \approx \sqrt{\frac{A}{2\pi M_s}} = \frac{1}{4\pi} \text{ cm} \sim 5 \text{ nm}$$

Итак получается, что $\frac{WM}{G} \sim 1$, если массу vortex определять по m_w .

Но если M -vortex имеет значение $\pi p_2 = -1$, она имеет и значение $\pi p_2 = +1$,
 и значение $\pi p_2 = 0$. Значит ω_+, ω_- . Рассчитаем.

Найдем M -vortex по $\vec{M}(\vec{r})$ на основе работ Ухарова.

УД где ω_+ частота:

$$\left\{ \begin{aligned} X_1(t) &= A_1^+ \cos(\omega_+ t + \varphi_1^+) + A_1^- \cos(\omega_- t + \varphi_1^-) \\ Y_1(t) &= B_1^+ \sin(\omega_+ t + \varphi_1^+) + B_1^- \sin(\omega_- t + \varphi_1^-) \end{aligned} \right.$$

Корр. $A_1^{\pm}, B_1^{\pm}, \varphi_1^{\pm}, \varphi_2^{\pm}$ — параметры, связанные с частотой $F/N/F$ disk.

У нас $4 \times 2 = 8$ параметров где 4 — частота. Значит 2 — частота.

$$X_1(t) = \cos \omega_+ t + g \cos(\omega_- t + \varphi) = \cos \omega_+ t + g \cos(\omega_+ t - \Delta \omega t + \varphi), \Delta \omega = \omega_+ - \omega_-$$

$$X_1(t) = \cos \omega_+ t + g [\cos \omega_+ t \cdot \cos(\Delta \omega t - \varphi) + \sin \omega_+ t \cdot \sin(\Delta \omega t - \varphi)] =$$

$$= [1 + g \cos(\Delta \omega t - \varphi)] \cos \omega_+ t + g \sin(\Delta \omega t - \varphi) \sin \omega_+ t$$

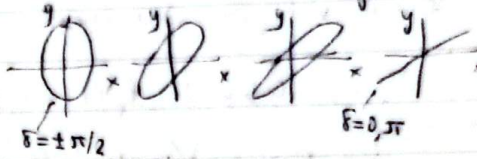
$$\frac{1 + g \cos(\Delta \omega t - \varphi)}{\sqrt{1 + 2g \cos(\Delta \omega t - \varphi) + g^2}} = \cos \alpha, \quad \frac{g \sin(\Delta \omega t - \varphi)}{\sqrt{1 + 2g \cos(\Delta \omega t - \varphi) + g^2}} = \sin \alpha$$

$$X_1(t) = \sqrt{1 + 2g \cos(\Delta \omega t - \varphi) + g^2} \cos(\omega_+ t - \alpha), \quad A_1^2(t) = 1 + 2g \cos(\Delta \omega t - \varphi) + g^2,$$

$$\max A_1 = 1 + g, \quad \min A_1 = 1 - g$$

У Кристина знаменне ноль параболі (мама і сестра, 200 coordinate bytes)

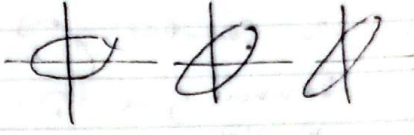
$$S(t) = \omega_+ t \quad \text{в } y \text{ - } u \quad \left\{ \begin{array}{l} x(t) = A \cos(\omega t + \alpha) \\ y(t) = B \cos(\omega t + \beta) \end{array} \right\} \quad \delta = \beta - \alpha$$

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB} \cos \delta = \sin^2 \delta$$


Евм. Взаєм. $\alpha = 0, \beta(t) = \omega_+ t, \delta = \beta = \omega_+ t, \pi$

— 200 coordinate bytes, 200 coordinate bytes.

Кого курс $A(\omega_+ t)$ и $B(\omega_+ t)$ — маємо.



$$\cos \alpha + \cos \beta = \cos \left[\frac{\alpha + \beta}{2} + \frac{\alpha - \beta}{2} \right] + \cos \left[\frac{\alpha + \beta}{2} - \frac{\alpha - \beta}{2} \right] = 2 \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}, \quad A_1^+ = A_1^- = A$$

$$\frac{1}{A} X_1(t) = \cos(\omega_+ t + \varphi_x^+) + \cos(\omega_+ t + \varphi_x^-) = 2 \cos \left[\frac{1}{2} (\Delta \omega t + \Delta \varphi) \right] \times$$

$$\times \cos \left[\frac{1}{2} ((\omega_+ + \omega_-)t + \varphi_x^+ + \varphi_x^-) \right]$$

$$\rightarrow X_1(t) = \underbrace{2A \cos \frac{1}{2} (\Delta \omega t + \varphi_x^+ - \varphi_x^-)}_{A(t)} \cdot \cos \frac{1}{2} ((\omega_+ + \omega_-)t + \varphi_x^+ + \varphi_x^-) \quad \text{— we remember the same way}$$

$$\frac{y_1(t)}{B} = \sin(\omega_+ t + \varphi_y^+) + \sin(\omega_+ t + \varphi_y^-) = \left\{ \begin{array}{l} \sin \left[\frac{\alpha + \beta}{2} + \frac{\alpha - \beta}{2} \right] + \sin \left[\frac{\alpha + \beta}{2} - \frac{\alpha - \beta}{2} \right] = \\ = \sin(t) \cos(-) + \cos(t) \sin(-) + \sin(t) \cos(-) - \\ - \cos(t) \sin(-) = 2 \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2} \end{array} \right.$$

$$\rightarrow y_1(t) = \underbrace{2B \cos \frac{1}{2} (\Delta \omega t + \varphi_y^+ - \varphi_y^-)}_{B(t)} \sin \frac{1}{2} ((\omega_+ + \omega_-)t + \varphi_y^+ + \varphi_y^-), \quad B_1^+ = B_1^- = B$$

Note: Бомбардировка модифицированных спектров (including rotating ellips by Kristen) — не упрощается. Кого курс $X_1(t), y_1(t)$, модифицированный δ (4D 2-го порядка). File "Double-Vect-Dynamics1.mxd"

3/1 Занесение I-539 в журнал, Аннотация. Кого-то в FEN: HO file (Chant). Обработка/анализирование текста, B. Conditions paper game "Science".

4/1 Озвучивание I-539 занесение. и копии I-94, passports to Karen Liptak. Обзор мн. и мн.м. Ресурсов. Проблема "Bound. Conditions" paper. В архиве за 1991.

6/I B AVL nozga, ingeneiro novo, no zona, no maragum.

7/I Passa nag DOE-poster "Complex Dynamics of Micromagnets" " DOE-viewgraph. Nicoma Chantrell, Weller, Freeman e Zamporini to send reference letters to Nebraska Univ. (E. Teymbal).

8/I Passa nag DOE viewgraph u Fig.1 e "Bound. Conditions" paper.

10/I Nicoma to Boie (Nebraska), Wlodkowski, Liptak (HR), Chantrell (AM-10, ref. letter), Scholtz (PRB-vdynamics).

Passa nag DOE poster, high light u view-graph. horum version to Kris. Val, Sam.

→ Sub-group meeting: Zaimur. dots (P. Roy), packaga o Dvortex dynamics

give $p_i p_j = +1$ u $p_i p_j = -1$ case (huma naca M vortex, nozga nozga bura u fura na u single-disk experiment u nozga anal. eq. for ω u micromag. simulations).

Rufina u nicoma PRB Fe/Pt nanostripes paper e meo bura.

11/I OZgorum. Zaimur e Blue Cross (Fam. Dr. choice - Kansas e Naperville).

Kanacame Dvortex statics paper. MPD - mura: Dvort $\geq d$, SD(11) nozga

14/12/07, $d \cdot SD(11) \geq d \cdot SD(1) - 3/21$ 047. Bie no pabensky Zaimur qez.

12/I OZgorum, oZgorum, gome. Cover Letter, bura Referees u nozga "B. Conditions" paper to Science.

DOE posters OZgorum / changes DOE poster, highlights, graph view on "Complex M. dynamics" u nozga to M. Metz.

→ Beaga e Bader o neagubom coZpmanu poster Marcosa u M. excitations in nano-particles. Neozobomno yobemg. Marcos, 200 go ero na. puzosel ne dno, method, theory que oZgorum SW b nano-particles, u 200 or name "edge" or "end" mode in rect. dots.

13/I Donamen / uZmenen Fe/Pt - nanostripes paper e Rufina.

AnaZgorum "Mino" publications 2005-2004.

Donamen - Marcos ero DOE-poster (oZgorum ero nozgorum u fura).

→ Muzum. Zaimur: naon frequency splitting ω_{im} of azimuthal vortex modes.

U fura 2000 mura. distribution should rotate e $\Delta\omega = \omega_{im} - \omega_{im} \sim 1$ MHz.

No na bozgorum $\pm m$ azimuthal modes? Puzuma $\Delta\omega$ - v. topolog. field.

SW/C. Bayer uZmenen oZgorum na SW-PRB paper. Nozgorum, no nuzma coZgorum / uZmenen teura. Dvortex - exp.

→ Bura coZgorum, 200 Frank que 40/140 nm Fe dot bura 2nd peak u fura $\omega_2 = 485$ MHz (!)

14/12 Задача (обзор) касаясь no Vortex dynamics: боковые
Vortex mass M. *Vortex dynamics: collective coordinates approach*

Учебный paper by Völkel et al. PRB 50, 12711 (1994) оге paper. 2 coupled
vortex's в обобщен. $\vec{G}$ -тензор и $\hat{M}$ -тензор нагваронаноме no
homopar vortex об. too uprague gno $\odot\odot$ dynamics 2-vortices
b oghon dot.

$$\text{yD } 1 \dots 1. \quad \frac{d\vec{S}}{dt} = \vec{S} \times \left(-\frac{\partial \mathcal{H}}{\partial \vec{S}} \right), \quad \vec{S}^2 = \text{const}, \quad \frac{\partial \vec{S}^2}{\partial x_i} = 0$$

$$-\vec{S} + \frac{d\vec{S}}{dt} = +\vec{S} + \vec{S} \times \frac{\partial \mathcal{H}}{\partial \vec{S}} = \vec{S} \left(\vec{S} \cdot \frac{\partial \mathcal{H}}{\partial \vec{S}} \right) - \frac{\partial \mathcal{H}}{\partial \vec{S}} S^2, \quad \vec{S}(\vec{r}, t) = \vec{S}[\vec{r} - \vec{X}(t), \vec{V}(t)]$$

$$-\frac{\partial \vec{S}}{\partial x_i} + \frac{d\vec{S}}{dt} = \frac{\partial \vec{S}}{\partial x_i} + \vec{S} \times \frac{\partial \mathcal{H}}{\partial \vec{S}} = \vec{S} \left(\frac{\partial \vec{S}}{\partial x_i} \cdot \frac{\partial \mathcal{H}}{\partial \vec{S}} \right) - \frac{\partial \mathcal{H}}{\partial \vec{S}} \left(\vec{S} \cdot \frac{\partial \vec{S}}{\partial x_i} \right) = \vec{S} \left(\frac{\partial \vec{S}}{\partial x_i} \cdot \frac{\partial \mathcal{H}}{\partial \vec{S}} \right)$$

$$-\vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_i} \times \frac{d\vec{S}}{dt} \right) = \left(\frac{\partial \mathcal{H}}{\partial \vec{S}} \cdot \frac{\partial \vec{S}}{\partial x_i} \right) S^2 = S^2 \frac{\partial \mathcal{H}}{\partial x_i} = -S^2 \frac{\partial \mathcal{H}}{\partial x_i}$$

$$\frac{d\vec{S}}{dt} = \frac{\partial \vec{S}}{\partial x_j} \frac{dx_j}{dt} + \frac{\partial \vec{S}}{\partial v_j} \frac{dv_j}{dt} = -\frac{\partial \vec{S}}{\partial x_j} \frac{dX_j}{dt} + \frac{\partial \vec{S}}{\partial v_j} \frac{dV_j}{dt}$$

$$\vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_i} \times \frac{d\vec{S}}{dt} \right) = +\vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_i} \times \left[-\frac{\partial \vec{S}}{\partial x_j} \frac{dX_j}{dt} + \frac{\partial \vec{S}}{\partial v_j} \frac{dV_j}{dt} \right] \right) = -\vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_i} \times \frac{\partial \vec{S}}{\partial x_j} \right) \frac{dX_j}{dt} +$$

$$+ \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_i} \times \frac{\partial \vec{S}}{\partial v_j} \right) \frac{dV_j}{dt} = S^2 \frac{\partial \mathcal{H}}{\partial x_i}$$

$$-S^2 \frac{\partial \mathcal{H}}{\partial x_i} - \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_i} \times \frac{\partial \vec{S}}{\partial x_j} \right) V_j = -\vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_i} \times \frac{\partial \vec{S}}{\partial v_j} \right) \frac{dV_j}{dt} \quad \int d^2 \vec{r} (\dots)$$

$$-\frac{\partial \mathcal{H}}{\partial x_i} - \frac{1}{S^2} \int d^2 \vec{r} \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_i} \times \frac{\partial \vec{S}}{\partial x_j} \right) V_j = -\frac{1}{S^2} \int d^2 \vec{r} \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_i} \times \frac{\partial \vec{S}}{\partial v_j} \right) \frac{dV_j}{dt}$$

$$\text{Обозначим } \vec{F} = -\frac{\partial \mathcal{H}}{\partial \vec{X}}, \quad \hat{G}_{ij} = -\frac{1}{S^2} \int d^2 \vec{r} \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_i} \times \frac{\partial \vec{S}}{\partial x_j} \right) - \text{w/poreror},$$

$$M_{ij} = -\frac{1}{S^2} \int d^2 \vec{r} \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_i} \times \frac{\partial \vec{S}}{\partial v_j} \right) - \text{тензор масс.}$$

$$\text{Тогда yD b var. nepeuenur } \boxed{F_i + \hat{G}_{ij} V_j = M_{ij} \frac{dV_j}{dt}} - \text{обозн. ype Thiele.}$$

$$\text{Если } \vec{r} = (x_1, x_2) - 2D \text{ вектор, } \frac{\partial \vec{S}}{\partial x_i} \parallel \text{ xoy плоскости и тензор } G_{12}, G_{21} \neq 0$$

$$\hat{G}_{12} = -S^2 \int d^2 \vec{r} \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_1} \times \frac{\partial \vec{S}}{\partial x_2} \right) = -\hat{G}_{21}. \quad F_j^i = \hat{G}_{ij} V_j$$

$$F_1^2 = G_{12} V_2, \quad F_2^1 = G_{21} V_1 = -G_{12} V_1, \quad \vec{F}^2 = G_{12} (V_2, -V_1) \perp \vec{V}(V_1, V_2)$$

$$\vec{F}_g = \vec{G} \times \vec{V}, \quad \vec{F}_g = \begin{vmatrix} i & j & k \\ 0 & 0 & G_3 \\ V_1 & V_2 & 0 \end{vmatrix} = i G_3 V_2 - j G_3 V_1, \quad \begin{matrix} G_{12} V_2 = -G_3 V_2 \\ -G_{12} V_1 = G_3 V_1 \end{matrix} \Rightarrow G_3 = -G_{12}$$

$$\vec{G} - \text{w/poreror}, \quad \vec{G} = (0, 0, G_3) \text{ и } \hat{G}_{ij} V_j = (\vec{F}_g)_i = (\vec{G} \times \vec{V})_i$$

$$\vec{G} = -G_{12} \hat{z} = \hat{z} S^{-2} \int d^2 \vec{r} \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_1} \times \frac{\partial \vec{S}}{\partial x_2} \right) \Rightarrow \vec{F} + \vec{G} \times \vec{V} = \hat{e}_i M_{ij} \frac{dV_j}{dt} - \text{yD}$$

Для нахождения тензора масс надо знать $\vec{S} = \vec{S}(\vec{r}, \vec{V})$.

В мин. приближении по X_j гравитацию считать $\hat{G}_{ij}, M_{ij}$ при $\vec{X}=0, \vec{V}=0$

$$\hat{G}_{ij} = -s^{-2} \int d^2 \vec{x} \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_i} \times \frac{\partial \vec{S}}{\partial x_j} \right)_0, \quad \vec{S} = \vec{S}(x_i, 0), \quad \vec{V} \text{ не берем.}$$

Т.е. минимальное значение энергии при статической конфигурации вихря $\vec{S}(x, 0)$.

$$M_{ij} = -s^{-2} \int d^2 \vec{x} \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x_i} \times \frac{\partial \vec{S}}{\partial x_j} \right)_0 \text{ и надо знать } \vec{S}(\vec{V}) \text{ хотя } \vec{V} \text{ не берем.}$$

$$\vec{S}(\vec{x}, t) = \vec{S}[\vec{x} - \vec{X}(t), \vec{V}(t)] = \vec{S}_0(\vec{x}) + \vec{S}_1(\vec{x}, \vec{V}), \quad \vec{S}_0(\vec{x}) = \vec{S}(x, 0) - \text{static vortex (centered)},$$

$$\text{а } \vec{S}_1 \sim \vec{V} - \text{малая поправка. } \vec{S}(\vec{x}, t) = \vec{S}(\vec{x}, 0) + \left(\frac{\partial \vec{S}}{\partial v_j} \right)_{v_j=0} v_j$$

$$\vec{S}_1(x, v) - \text{изменение вихря при движении. } \frac{\vec{S}}{S} \rightarrow \vec{m}.$$

Для "side-charge free" model shifted dynamic vortex (30/11/01): $m_z(p, \varphi) = m_z^{(0)}(p) + \frac{4c^2(1+p^2)}{(c^2+p^2)^2} (\vec{p} \cdot \vec{X}) \frac{1}{R}$

$$\vec{m}(\vec{p}, t) = \vec{m}^{(0)}(p) + \left(\frac{\partial \vec{m}}{\partial v_j} \right)_{v_j=0} v_j$$

$$m_z(p, \varphi) = m_z^{(0)}(p) + \frac{4c^2(1+p^2)}{(c^2+p^2)^2} (\vec{p} \cdot \vec{X}) \frac{1}{R} \quad \left. \begin{array}{l} \text{Получен, и сам и сам есть ген.} \\ \text{поправка } \sim \vec{X}. \end{array} \right\}$$

→ Гипотеза: предполагается, что $\vec{m}(\vec{p})$ - "side charge free" distribution от энергии от $\vec{m}_0(p)$ - static vortex и $\delta \vec{m} = \vec{m}(\vec{p}) - \vec{m}_0(p) \sim \vec{V}$.

Поскольку $M_{ij} = M \delta_{ij}$ пока считаем.

$$yD \quad M \ddot{\vec{X}} = \vec{G} \times \dot{\vec{X}} + \vec{F} = \vec{G} \times \dot{\vec{X}} - \frac{\partial \mathcal{H}}{\partial \dot{\vec{X}}} = \vec{G} \times \dot{\vec{X}} - \alpha \dot{\vec{X}}, \quad \alpha \dot{\vec{X}} = \vec{G} \times \dot{\vec{X}} - M \ddot{\vec{X}}$$

$$\delta m_z(p, \varphi) = \frac{4c^2}{R} \frac{(1+p^2)}{(c^2+p^2)^2} (\vec{p} \cdot \vec{X}) = \frac{4c^2}{\alpha R} \frac{(1+p^2)}{(c^2+p^2)^2} \vec{p} \cdot [\vec{G} \times \dot{\vec{X}} - M \ddot{\vec{X}}], \quad \vec{p} \cdot (\vec{G} \times \dot{\vec{X}}) = (\vec{p} \times \vec{G}) \cdot \dot{\vec{X}}$$

$$m_z(p, \varphi) = m_z^{(0)}(p) + \delta m_z(p, \varphi) = m_z^{(0)}(p) + \frac{4c^2}{\alpha R} \frac{(1+p^2)}{(c^2+p^2)^2} (\vec{p} \times \vec{G}) \cdot \dot{\vec{X}} + (\text{член } c M \ddot{\vec{X}}) + O(\dot{\vec{X}}^2)$$

$$\frac{\partial m_z}{\partial \dot{\vec{X}}} = \frac{4c^2}{\alpha R} \frac{1+p^2}{(c^2+p^2)^2} (\vec{p} \times \vec{G}) - \text{член } c M \ddot{\vec{X}} \text{ и при } \dot{\vec{X}}, \ddot{\vec{X}} \rightarrow 0.$$

$$f(z) = \frac{1}{c} \left[iz\bar{c} + \frac{1}{2}(a - \bar{a}z^2) \right], \quad m_x + im_y = \frac{2W(z\bar{z})}{1+|W|^2}, \quad W(z\bar{z}) = \begin{cases} f(z), & |f| \leq 1 \\ f(z)/|f|, & |f| > 1 \end{cases}$$

Выводим new-core: $c^2|W|^2 = c^2|f(z)|^2 = p^2 - 2(1+p^2)(\vec{p} \cdot \vec{X}) \frac{1}{R} + O(a^2)$

$$m_x = \frac{2}{1+|W|^2} \text{Re } f(z), \quad \text{Re } f(z) = \frac{1}{2} f(z) + \frac{1}{2} \bar{f}(z) = \frac{1}{2c} \left[iz\bar{c} + \frac{a}{2} - \frac{\bar{a}}{2} z^2 \right] + \frac{1}{2c} \left[-i\bar{z}\bar{c} + \frac{\bar{a}}{2} - a\bar{z}^2 \right]$$

$$2c \text{Re } f(z) = i\bar{c}(z - \bar{z}) + \frac{a}{2} + \frac{\bar{a}}{2} - \frac{1}{2}\bar{a}z^2 - \frac{1}{2}a\bar{z}^2 = i\bar{c}(2iy) + a' + O(z^2)$$

$$\bar{a}z^2 = (a' - ia'')(x+iy)^2 = (a' - ia'')(x^2 - y^2 + 2ixy) = a'(x^2 - y^2) + a''2xy + i[a''(y^2 - x^2) + 2xya']$$

$$-\frac{1}{2}(\bar{a}z^2 + a\bar{z}^2) = -\frac{1}{2}2\text{Re}(\bar{a}z^2) = -a'(x^2 - y^2) - a''2xy$$

$$2c \text{Re } f(z) = -2cy + a' - a'(x^2 - y^2) - 2a''xy$$

$$m_x = \frac{c^2}{c^2 + [p^2 - 2(1+p^2)(\vec{p} \cdot \vec{S})]} \frac{1}{c} [-2cy + a' - a'(x^2 - y^2) - 2a''xy], \quad \begin{cases} s_x = -\frac{1}{2}ca'' \\ s_y = \frac{1}{2}ca' \end{cases} \vec{s} = \frac{\vec{X}}{R}$$

$$(m_x) = \frac{c^2}{c^2 + p^2 - a} = \frac{c^2}{c^2 + p^2} \frac{1}{1 - \frac{a}{c^2 + p^2}} = \frac{c^2}{c^2 + p^2} \left[1 + \frac{a}{c^2 + p^2} \right]$$

$$m_x = \frac{c^2}{c^2 + p^2} \left[1 + \frac{2(1+p^2)(\vec{p} \cdot \vec{s})}{c^2 + p^2} \right] \left[-2cy + a' - a'(x^2 - y^2) - 2a''xy \right] =$$

$$= \frac{c}{c^2 + p^2} \left[-2cy - 2cy \frac{2(1+p^2)(\vec{p} \cdot \vec{s})}{c^2 + p^2} + a' - a'(x^2 - y^2) - 2a''xy \right] =$$

$$= \frac{c}{c^2 + p^2} \left[-2y - 2y \frac{2(1+p^2)(\vec{p} \cdot \vec{s})}{c^2 + p^2} + 2s_y - 2s_y(x^2 - y^2) + 4s_x xy \right]$$

$$\rightarrow m_x(p, \varphi) = \frac{2c}{c^2 + p^2} \left[-y - y \frac{2(1+p^2)(\vec{p} \cdot \vec{s})}{c^2 + p^2} + 2s_y(1 - x^2 + y^2) + 2s_x xy \right] =$$

$$= \frac{2c}{c^2 + p^2} \left[-p \sin \varphi - p \sin \varphi \frac{2(1+p^2)}{c^2 + p^2} (\vec{p} \cdot \vec{s}) + 2s_y(1 - p^2 \cos 2\varphi) + 2s_x p^2 \sin 2\varphi \right] -$$

- Bragg shifted core.

Cnapyren shifted core: $m_x = \text{Re } w(z\bar{z}) = \text{Re} \frac{f(z)}{|f(z)|} = \frac{1}{\sqrt{p^2 - 2(1+p^2)(\vec{p} \cdot \vec{s})}} \frac{1}{2} [-2cy + a' - a'(x^2 - y^2) - 2a''xy]$

$$\frac{1}{\sqrt{p^2 - a}} = \frac{1}{p} \frac{1}{\sqrt{1 - a/p^2}} = \frac{1}{p} \frac{1}{1 - \frac{a}{2p^2}} = \frac{1}{p} \left(1 + \frac{a}{2p^2} \right)$$

$$m_x = \frac{1}{p} \left(1 + \frac{a}{2p^2} \right) \frac{1}{2} [-2cy + a' - a'(x^2 - y^2) - 2a''xy] = \frac{1}{p} \frac{1}{2} [-2cy + a' - a'(x^2 - y^2) - 2a''xy - 2cy \frac{a}{2p^2}]$$

$$m_x = c \left[-\sin \varphi + \frac{1}{p} s_y (1 - p^2 \cos 2\varphi) + \frac{1}{p} s_x p^2 \sin 2\varphi - \sin \varphi \frac{(1+p^2)(\vec{p} \cdot \vec{s})}{(c^2 + p^2)p^2} \right]$$

$$m_x(p, \varphi) = c \left[-\sin \varphi + \frac{1}{p} (1 - p^2 \cos 2\varphi) s_y + p \sin 2\varphi s_x - \frac{1}{p^2} \sin \varphi (1+p^2) (\vec{p} \cdot \vec{s}) \right] - \text{Bragg core}$$

$$xX = (\vec{G} \times \vec{X})_x - \dot{X}M, \quad xy = (\vec{G} \times \vec{X})_y - M\dot{y}, \quad \vec{G} \times \vec{V} = -\vec{i} G_z V_y + \vec{j} G_z V_x$$

$$\delta m_x = \frac{2c}{c^2 + p^2} \left[-p \sin \varphi \cdot \frac{2(1+p^2)}{c^2 + p^2} \frac{1}{2R} \vec{p} \cdot (\vec{G} \times \vec{X}) + \frac{1}{R} G_z V_x (1 - p^2 \cos 2\varphi) - \frac{1}{2R} G_z V_y p^2 \sin 2\varphi \right] -$$

Bragg core

$$M_{ij} = -s^{-2} \int d^2 \vec{x} \vec{s} \cdot \left(\frac{\partial \vec{s}}{\partial x_i} \times \frac{\partial \vec{s}}{\partial x_j} \right) = - \int d^2 \vec{x} \vec{m} \cdot \left(\frac{\partial \vec{m}}{\partial x_i} \times \frac{\partial \vec{m}}{\partial x_j} \right)$$

$$\frac{\partial \vec{m}}{\partial x_i} \times \frac{\partial \vec{m}}{\partial x_j} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial m_x}{\partial x_i} & \frac{\partial m_y}{\partial x_i} & 0 \\ \frac{\partial m_x}{\partial x_j} & \frac{\partial m_y}{\partial x_j} & 0 \end{vmatrix} = \vec{k} \left[-\frac{\partial m_x}{\partial x_i} \frac{\partial m_y}{\partial x_j} \right]$$

Vortex mass

B npegeabnem M_{ij} opez $\vec{s}$ common and komonens. n

Due $\Phi = \tan^{-1}(s_y/s_x)$ n $s_z = m$ nphase

$$\rightarrow M_{ij} = - \int d^2 \vec{x} \left[\frac{\partial \Phi}{\partial x_i} \frac{\partial s_z}{\partial x_j} - \frac{\partial \Phi}{\partial x_j} \frac{\partial s_z}{\partial x_i} \right], \quad w(z\bar{z}) = \frac{\theta}{2} e^{i\Phi(x,y)}$$

Con. 22/2002. $\Phi(x,y) = \Phi_0 + \arctg \left(\frac{y-\bar{y}}{x-\bar{x}} \right) + \arctg \left(\frac{y-\bar{y}}{x-\bar{x}} \right), \quad \begin{cases} \bar{x} = x \frac{R^2}{\sqrt{x^2+y^2}} \\ \bar{y} = y \frac{R^2}{\sqrt{x^2+y^2}} \end{cases}$

$\bar{x}, \bar{y}$ - coordinates of image vertex.

$$z, \bar{z} = (x + iy)(\bar{x} - i\bar{y}) = x\bar{x} + y\bar{y} + i(y\bar{x} - \bar{y}x) = \rightarrow$$

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$$= \frac{x^2 R^2}{\sqrt{(\dots)}} + \frac{y^2 R^2}{\sqrt{(\dots)}} + i \frac{yx R^2}{\sqrt{(\dots)}} - i \frac{xy R^2}{\sqrt{(\dots)}} = R^2 \frac{x^2 + y^2}{\sqrt{x^2 + y^2}} \neq 1. \quad \text{Note: } \lim_{\vec{r} \rightarrow 0} \frac{\partial \Phi}{\partial x}, \frac{\partial \Phi}{\partial y} \text{ v.m. } \vec{r} = 0.$$

Busquemos luego $\bar{X} = x \frac{R^2}{x^2 + y^2}$, $\bar{Y} = y \frac{R^2}{x^2 + y^2} \Rightarrow z_1 \bar{z}_2 = R^2$.

$$z_1 = \frac{1}{2} i a, \quad z_2 = \frac{2 i a}{|a|^2} \rightarrow z_1 \bar{z}_2 = \frac{1}{2} i a (-2 i) \frac{a}{|a|^2} = 1 \quad \text{b. u. elegancia, c. } R^2 = 1,$$

t.e. $\vec{a}, \vec{a}' = \vec{X}/R$ B og. R - la que ya gora.

$$s = \frac{1}{2} a \quad z_1 = s, \quad z_2 = \frac{1}{z_1} = \frac{1}{s} = \frac{1}{s_x - i s_y} = \frac{s_x + i s_y}{s_x^2 + s_y^2}, \quad z_2 = \frac{2 i a}{|a|^2} = \frac{4 s}{|2 s|^2} = \frac{s}{|s|^2}$$

$$z_1 = s = \frac{1}{R} (X + i Y), \quad z_2 = \frac{s_x + i s_y}{|s|^2} = R^2 \frac{X + i Y}{x^2 + y^2} = \frac{1}{R} (\bar{X} + i \bar{Y}), \quad \frac{\bar{X}}{R} = \frac{s_x}{|s|^2}, \quad \frac{\bar{Y}}{R} = \frac{s_y}{|s|^2}$$

$$\arctg\left(\frac{y - \bar{y}}{x - \bar{x}}\right) = \arctg\left(\frac{y - s_y/|s|^2}{x - s_x/|s|^2}\right)$$

$$\frac{\partial \Phi}{\partial x} = \frac{\partial}{\partial x} \left[\Phi_0 + \arctg\left(\frac{y - \bar{y}}{x - \bar{x}}\right) + \arctg\left(\frac{y - \bar{y}}{x - \bar{x}}\right) \right] = \frac{1}{1 + \left(\frac{y - \bar{y}}{x - \bar{x}}\right)^2} \frac{(y - \bar{y}) (-1)}{(x - \bar{x})^2} +$$

$$+ \frac{1}{1 + \left(\frac{y - \bar{y}}{x - \bar{x}}\right)^2} \frac{(y - \bar{y}) (-1)}{(x - \bar{x})^2} = - \frac{(y - \bar{y})}{(x - \bar{x})^2 + (y - \bar{y})^2} - \frac{(y - \bar{y})}{(x - \bar{x})^2 + (y - \bar{y})^2}$$

$$\frac{\partial \Phi}{\partial y} = \frac{1}{1 + \left(\frac{y - \bar{y}}{x - \bar{x}}\right)^2} \frac{1}{x - \bar{x}} + [\dots] = \frac{(x - \bar{x})}{(x - \bar{x})^2 + (y - \bar{y})^2} + \frac{(x - \bar{x})}{(x - \bar{x})^2 + (y - \bar{y})^2}, \quad \begin{matrix} X, Y \rightarrow 0 \\ \bar{X}, \bar{Y} \rightarrow \infty \end{matrix}$$

$$\frac{(y - \bar{y})}{(x - \bar{x})^2 + (y - \bar{y})^2} \approx - \frac{\bar{y}}{\bar{X}^2 + \bar{Y}^2}, \quad \bar{X}^2 + \bar{Y}^2 = R^4 \frac{x^2 + y^2}{(x^2 + y^2)^2} = \frac{R^4}{x^2 + y^2}$$

$$\frac{\bar{y}}{\bar{X}^2 + \bar{Y}^2} = \frac{1}{R^4} (x^2 + y^2) \frac{y}{R^2} \frac{R^2}{x^2 + y^2} = \frac{1}{R^2} y, \quad \frac{\bar{x}}{\bar{X}^2 + \bar{Y}^2} = \frac{1}{R^2} x$$

$$\left(\frac{\partial \Phi}{\partial x}\right)_0 = - \frac{y}{x^2 + y^2} + \frac{\bar{y}}{\bar{X}^2 + \bar{Y}^2} = - \frac{y}{x^2 + y^2} + \frac{1}{R^2} y \approx - \frac{y}{x^2 + y^2} \quad \left| \quad \vec{X} = \frac{1}{\kappa} [\vec{G} \times \vec{V} - M \vec{V}] \right.$$

$$\left(\frac{\partial \Phi}{\partial y}\right)_0 = \frac{x}{x^2 + y^2} - \frac{\bar{x}}{\bar{X}^2 + \bar{Y}^2} = \frac{x}{x^2 + y^2} - \frac{1}{R^2} x \approx \frac{x}{x^2 + y^2}$$



$$\frac{\partial \Phi}{\partial \vec{V}} = \frac{\partial \Phi}{\partial \vec{X}} \cdot \frac{\partial \vec{X}}{\partial \vec{V}} = \sum_j \frac{\partial \Phi}{\partial X_j} \frac{\partial X_j}{\partial V_i} \quad \begin{cases} X = \frac{1}{\kappa} (\vec{G} \times \vec{V})_x = - \frac{1}{\kappa} G_z V_y \\ Y = \frac{1}{\kappa} (\vec{G} \times \vec{V})_y = \frac{1}{\kappa} G_z V_x \end{cases}$$

$$\frac{\partial \Phi}{\partial V_x} = \frac{\partial \Phi}{\partial X} \frac{\partial X}{\partial V_x} + \frac{\partial \Phi}{\partial Y} \frac{\partial Y}{\partial V_x} = \frac{\partial \Phi}{\partial Y} \frac{1}{\kappa} G_z \approx - \left(\frac{\partial \Phi}{\partial y}\right)_0 \frac{G_z}{\kappa}, \quad \text{r.u. } \arctg\left(\frac{y - \bar{y}}{x - \bar{x}}\right) \text{ ne gaer bueaga v.m. } X, Y \rightarrow 0.$$

$$\frac{\partial \Phi}{\partial V_y} = \frac{\partial \Phi}{\partial X} \frac{\partial X}{\partial V_y} + \frac{\partial \Phi}{\partial Y} \frac{\partial Y}{\partial V_y} = \frac{\partial \Phi}{\partial X} (-1) \frac{G_z}{\kappa} = + \left(\frac{\partial \Phi}{\partial x}\right)_0 \frac{G_z}{\kappa}, \quad \frac{\partial m_z}{\partial \vec{X}} = - \frac{\partial m_z}{\partial \vec{Y}}$$

$$\frac{\partial m_z}{\partial \vec{X}} = \frac{4 c^2 (1 + \rho^2)}{(c^2 + \rho^2)^2} \frac{1}{\kappa} \vec{\rho}, \quad \frac{\partial m_z}{\partial \vec{V}} = \frac{4 c^2 (1 + \rho^2)}{\kappa R (c^2 + \rho^2)^2} (\vec{\rho} \times \vec{G}), \quad \frac{\partial m_z}{\partial \vec{X}} = f(\rho) \vec{\rho}, \quad \frac{\partial m_z}{\partial \vec{V}} = \frac{f(\rho)}{\kappa} (\vec{\rho} \times \vec{G})$$

$$M_{xx} = - \int d^2 \vec{r} \left[\frac{\partial \Phi}{\partial x} \frac{\partial m_z}{\partial V_x} - \frac{\partial \Phi}{\partial V_x} \frac{\partial m_z}{\partial x} \right] = - \int d^2 \vec{r} \left[- \frac{y}{\rho^2} \frac{f(\rho)}{\kappa} (\vec{\rho} \times \vec{G})_x - \left(- \frac{G_z}{\kappa}\right) \frac{x}{\rho^2} f(\rho) \vec{\rho}_x \right] =$$

$$= - \int d^2 \vec{r} \frac{1}{\kappa} \frac{f(\rho)}{\rho^2} [- y G_z y - x^2 G_z] = \frac{G_z}{\kappa} \int d^2 \vec{r} \frac{f(\rho)}{\rho^2} [x^2 + y^2] = \frac{G_z}{\kappa} \int d^2 \vec{r} f(\rho)$$

$$M_{xx} = \frac{G_z}{\pi} 2\pi \int_0^1 d\rho \rho f(\rho), \quad f(\rho) = \frac{4c^2}{R} \frac{1+\rho^2}{(c^2+\rho^2)^2}$$

$$M_{xy} = - \int d^2 \vec{\rho} \left[\frac{\partial \Phi}{\partial x} \frac{\partial m_z}{\partial y} - \frac{\partial \Phi}{\partial y} \frac{\partial m_z}{\partial x} \right] = - \int d^2 \vec{\rho} \left[-\frac{y}{\rho^2} \frac{f(\rho)}{\pi} (\vec{\rho} \times \vec{G})_y - \frac{G_z}{\pi} \left(-\frac{y}{\rho^2} \right) (-1) f(\rho) x \right] =$$

$$= - \frac{G_z}{\pi} \int d^2 \vec{\rho} \frac{1}{\rho^2} f(\rho) [xy - xy] = 0$$

$$M_{yx} = - \int d^2 \vec{\rho} \left[\frac{\partial \Phi}{\partial y} \frac{\partial m_z}{\partial x} - \frac{\partial \Phi}{\partial x} \frac{\partial m_z}{\partial y} \right] = - \int d^2 \vec{\rho} \left[\frac{x}{\rho^2} \frac{f(\rho)}{\pi} (\vec{\rho} \times \vec{G})_x + \frac{G_z}{\pi} \frac{x}{\rho^2} (-1) f(\rho) y \right] = 0$$

$$M_{yy} = - \int d^2 \vec{\rho} \left[\frac{\partial \Phi}{\partial y} \frac{\partial m_z}{\partial y} - \frac{\partial \Phi}{\partial y} \frac{\partial m_z}{\partial y} \right] = - \int d^2 \vec{\rho} \left[\frac{x}{\rho^2} \frac{f(\rho)}{\pi} (\vec{\rho} \times \vec{G})_y - \frac{G_z}{\pi} \left(-\frac{y}{\rho^2} \right) (-1) f(\rho) y \right] =$$

$$= \frac{G_z}{\pi} \int d^2 \vec{\rho} [x^2 + y^2] \frac{1}{\rho^2} f(\rho) = \frac{G_z}{\pi} 2\pi \int_0^1 d\rho \rho f(\rho)$$

$$M_{xx} = M_{yy} \quad \text{и} \quad M_{xy} = M_{yx} = 0, \quad \text{т.е.} \quad \hat{M} \text{ - диагональ и } M_{ij} = M \delta_{ij}, \quad i, j = x, y.$$

$$J = \int_0^1 d\rho \rho \frac{(1+\rho^2)}{(c^2+\rho^2)^2} = \frac{1}{2} \int_0^1 dt \frac{(1+t)}{(c^2+t)^2} = \frac{1}{2} \int_0^1 dt \frac{1-c^2+t+c^2}{(c^2+t)^2} = \frac{(1-c^2)}{2} \int_0^1 \frac{dt}{(c^2+t)^2} +$$

$$+ \frac{1}{2} \int_0^1 \frac{dt}{c^2+t} = \frac{1}{2} (1-c^2) \frac{(-1)}{c^2+t} \Big|_0^1 + \frac{1}{2} \ln(c^2+t) \Big|_0^1 = -\frac{1}{2} (1-c^2) \left[\frac{1}{c^2+1} - \frac{1}{c^2} \right] + \frac{1}{2} \ln \left(\frac{c^2+1}{c^2} \right) =$$

$$= -\frac{1}{2} (1-c^2) \frac{(-1)}{c^2(c^2+1)} + \frac{1}{2} \ln \left(1 + \frac{1}{c^2} \right) = \frac{1}{2} \frac{1}{c^2} \left(\frac{1-c^2}{1+c^2} \right) + \frac{1}{2} \ln \left(1 + \frac{1}{c^2} \right), \quad c = \frac{b}{R} \text{ - equil. radius core}$$

$$\rightarrow M = \frac{G_z}{\pi} 2\pi \frac{4c^2}{R} \int_0^1 d\rho \rho \frac{(1+\rho^2)}{(c^2+\rho^2)^2} = \frac{1}{\pi} G_z \frac{2\pi}{R} 4c^2 \left[\frac{1}{2} \frac{1-c^2}{1+c^2} + \frac{1}{2} \ln \left(1 + \frac{1}{c^2} \right) \right] =$$

$$= \frac{4\pi G_z}{\pi R} \left[\frac{1-c^2}{1+c^2} + c^2 \ln \left(1 + \frac{1}{c^2} \right) \right], \quad \text{и} \quad \mu c^2 \ll 1 \quad \left(M = \frac{4\pi G_z}{\pi R} + O(c^2) \right) \quad \text{маленький вихрь}$$

$$M \ddot{\vec{X}} - \vec{G} \times \dot{\vec{X}} + \kappa \vec{X} = 0, \quad \frac{\omega M}{G_z} = \omega \frac{4\pi}{\pi R}, \quad W = \frac{1}{2} \kappa X^2, \quad [W] = \frac{\text{erg}}{\text{cm}^3} \cdot \text{cm}^3$$

$$M \omega^2 = \kappa, \quad [M] = \frac{[\kappa]}{[\omega^2]} = \frac{\text{erg} \cdot \text{cm}^2}{\text{cm}^2} = \text{g}.$$

$$M = \frac{4\pi \cdot 2\pi L M_s}{\gamma} \frac{1}{\pi R} = \frac{8\pi^2 L}{R} \frac{M_s}{\gamma \pi}, \quad [M] = \frac{M_s^2}{\gamma M_s \cdot \kappa} = \frac{\text{erg} \cdot \text{cm}^2}{\text{cm}^3 \cdot \text{Hz} \cdot \text{erg}} = \frac{\text{sec}}{\text{cm}} - ?$$

Вспомогательная \$M_{ij}\$ надо задавать нулевыми.

$$\frac{d\vec{s}}{dt} = \vec{s} \times \left(-\frac{\partial \mathcal{H}}{\partial \vec{s}} \right), \quad \vec{s} = \vec{m} \frac{M_s}{\gamma} \quad \text{координат. вл. - нулевой спин.}$$

$$\vec{s} = \frac{M_s V}{\gamma} \vec{m}, \quad \vec{m} = \vec{m} / M_s. \quad \text{Правильно паучи и правильность M - малый вихрь.}$$

17/I \$\rightarrow\$ Bezege c Bader u Rukina o Re - nanostripe paper.

Definieren text Refs. Paccagnan Bader of M. Pazzere F/N/F disk.

\$\rightarrow\$ Bezege c Bader o: g CNM nano - publications 2003 at - 2004 Dec.

(отослать на DOE - rev. camera + задавать accepted, in press papers or Janice), b) об update Publ. - site Namen rpyunt (DOE - rev. list + Janice + weboScience) за 2001 - 2004.

Нормировка Kristen Turekno Name \$m_p(t)|_{p=R}\$ где F/N/F um F - disk.

Импульс Орбита, Гуды, Навиле (normal changes to FePL-2(r) manuscript).

B JAP'02 a uncorroborated:

$$\vec{G} = \frac{M_s L}{f} \int d^3 \vec{x} \sin \theta (\nabla \Phi \times \nabla \theta), \quad \vec{G} = -G \hat{z}, \quad G = 2\pi p q \frac{L M_s}{f}, \quad \vec{G} \times \vec{X} = \frac{\partial W}{\partial \vec{X}}$$

$$[W] - \text{energy}, \quad W = \frac{1}{2} \kappa X^2, \quad \omega_0 = \frac{\kappa}{G}, \quad [\omega_0] = \frac{\text{erg}}{\text{cm}^2 \cdot \text{cm} \cdot G \cdot G} = \text{Hz}$$

$$\text{Mysim-gu gals def. } \vec{G} = \int d^2 \vec{x} (\nabla \Phi \times \nabla S^z) = -S \int d^2 \vec{x} (\nabla \Phi \times \nabla \theta) \sin \theta, \quad \text{t.e. } \frac{S^z}{S} = \cos \theta.$$

$$G_{ij} = \int d^2 \vec{x} \left[\frac{\partial \Phi}{\partial x_i} \frac{\partial S^z}{\partial x_j} - \frac{\partial \Phi}{\partial x_j} \frac{\partial S^z}{\partial x_i} \right], \quad M_{ij} = - \int d^2 \vec{x} \left[\frac{\partial \Phi}{\partial x_i} \frac{\partial S^z}{\partial x_j} - \frac{\partial \Phi}{\partial x_j} \frac{\partial S^z}{\partial x_i} \right]$$

Vortex mass

$$\frac{d\vec{S}}{dt} = \vec{S} \times \left(-\frac{\partial \mathcal{H}}{\partial \vec{S}} \right) \text{ crabn. c. yd } \frac{M_s}{f} \frac{d\vec{m}}{dt} = \vec{m} \times \left(-\frac{\partial \mathcal{H}}{\partial \vec{m}} \right), \quad \vec{m} = \frac{\vec{S}}{M_s}, \quad [\mathcal{H}] = \frac{\text{erg}}{\text{cm}^3}$$

$$\text{Ito ufm } \int d^2 \vec{x} \rightarrow \int d^3 \vec{x} (\dots) 14/1 \quad \int d^3 \vec{x} \frac{\partial \mathcal{H}}{\partial x_i} = V \frac{\partial \mathcal{H}}{\partial x_i} = \frac{\partial W}{\partial x_i}, \quad [W] = \text{erg}$$

$$\int d^3 \vec{x} = L \int d^2 \vec{x} \text{ gne 2D - zagarn.}$$

T. e. b M or 16/1 nago zolaburo uncorroborated $\left(\frac{M_s L}{f} \right)$.

$$m_z(p, \varphi) = m_z^{(0)}(p) + 4c^2 \frac{(1+p^2)}{(c^2+p^2)^2} \left(\vec{p} \cdot \frac{\vec{X}}{R} \right), \quad p, c \text{ b eg. } R, \quad x, y \text{ b eg. } R \text{ z gca.}$$

$$\frac{\partial m_z}{\partial \vec{x}} = -\frac{\partial m_z}{\partial \vec{x}} = \frac{4c^2}{R} \frac{(1+p^2)}{(c^2+p^2)^2} \vec{p}$$

$$M = \frac{4\pi G_z}{\kappa} \left(\frac{M_s L}{f} \right) R^n, \quad [M] = \left[\frac{G}{\kappa} \right] \left[\frac{M_s L}{f} \right] [R^n] = \frac{[G]}{[W]}, \quad \text{t.e. } R^n = 1, n=0.$$

$$M = 4\pi \frac{G_z M_s L}{\kappa f}, \quad M = 4\pi \cdot 2\pi p q \frac{M_s L}{f} \frac{M_s L}{\kappa f} = 8\pi^2 p q \frac{1}{\kappa} \frac{M_s^2 L^2}{f^2}$$

$$p=1, q=1 \Rightarrow M = \frac{8\pi^2}{\kappa} \frac{M_s^2 L^2}{f^2}, \quad [M] = \frac{\text{cm}^2}{\text{erg}} \frac{\text{erg} \cdot \text{cm}^2}{\text{cm}^3 \cdot \text{cm}^2 \cdot \text{Hz}^2} = \frac{\text{cm} \cdot \text{cm}^2}{\text{Hz}^2} = \frac{\text{erg}}{\text{cm}^2 \cdot \text{Hz}^2}$$

$$[G] = \frac{\text{cm} G^2}{\text{Hz}} = \frac{\text{erg}}{\text{cm}^2 \cdot \text{Hz}^2}, \quad [\kappa] = \frac{\text{erg}}{\text{cm}^2}, \quad M \vec{X} - \vec{G} \times \vec{X} + \kappa \vec{X} = 0.$$

$$\text{Kuznetsov's harmonic mass vortex: } \omega \frac{M}{G} = \omega \frac{8\pi^2}{\kappa} \frac{M_s^2 L^2}{f^2} \frac{f}{2\pi L M_s} = \omega 4\pi \frac{M_s L}{\kappa f} = 4\pi \frac{\omega}{f M_s} \frac{M_s^2 L}{\kappa}$$

$$\text{Note: } \kappa = 16\pi^2 L M_s^2 F_V(\beta), \quad \omega \frac{M}{G} = 4\pi \frac{\omega}{f M_s} \frac{M_s^2 L}{16\pi^2 L M_s^2 F_V(\beta)} = \frac{\omega}{\omega_M} \frac{1}{F_V(\beta)} = 2 \frac{\omega}{\omega_0(\beta)} \text{ t.e.}$$

$$\text{(single-disk) mac. wen banen, } \omega_0(\beta) = \frac{\kappa}{G} \text{ - racora TM ufm } M=0.$$

Vortex mass

$$\Rightarrow M = \frac{8\pi^2}{\kappa} \frac{M_s^2 L^2}{f^2} = \frac{8\pi^2}{16\pi^2 L M_s^2 F_V(\beta)} \frac{M_s^2 L^2}{f^2} = \frac{1}{2} \frac{L}{f^2 F_V(\beta)} \text{ - macra vortex in single disk. } \frac{\text{cm} \cdot 8-9/\text{m}}{10/\text{m}}$$

$$M_{\text{DW}} = \frac{1}{f^2 \delta} \text{ - gne cfaburama. } M(\beta, L) = \frac{1}{2} \frac{L}{f^2 F_V(\beta)}, \quad M(\beta, L) \sim \frac{L}{F_V(\beta)} \sim \frac{L}{\beta} \sim R \text{ ufm } \beta \ll 1.$$

$$\text{Zacora TM (30/10)} \quad \omega_{1,2} = \frac{1}{2M} [-G_z \pm \sqrt{G_z^2 + 4M\kappa}] = -\frac{G_z}{2M} [1 \mp \sqrt{1 + \frac{4M\kappa}{G_z^2}}], \quad \text{acm } G_z > 0$$

$$\frac{4M\kappa}{G_z^2} = 4 \frac{M}{G} \frac{\kappa}{G} = 4 \frac{1}{\omega_M F_V(\beta)}, \quad \omega_0 = 2 \frac{1}{\omega_0} \omega_0 = 8, \quad \omega_{1,2} = -\frac{G_z}{2M} [1 \mp 3], \quad \omega_1 = \frac{G_z}{M}, \quad \omega_2 = -2 \frac{G_z}{M}.$$

$$\frac{M}{G} \frac{\kappa}{G} = \frac{M}{G} \omega_0 = 2, \quad \frac{G}{M} = \frac{1}{2} \omega_0, \quad \omega_1 = \frac{1}{2} \omega_0, \quad \omega_2 = -\omega_0. \text{ t.e. b panax } (\vec{m} \cdot \vec{n})_s = 0 \text{ model gbe racora } \omega_{1,2}.$$

Thinly coated, mega gravimetric $\omega_1 = \frac{1}{2} \omega_0$?

$M = \frac{1}{2} \frac{L}{\int^2 E(r)} \cdot$ Mass density $M(1/I) = \frac{\pi}{8} \frac{RL}{\int^2} \sim \frac{\pi}{8} R$ - geometric formula.

UD 30/11/04. $\begin{cases} M\omega^2 x - G_1 \omega y = -\kappa x \\ M\omega^2 y + G_2 \omega x = -\kappa y \end{cases} \rightarrow \begin{cases} (M\omega^2 + \kappa)x = G\omega y \\ (M\omega^2 + \kappa)y = -G\omega x \end{cases} \rightarrow \begin{cases} (M\omega^2 + \kappa)x - G\omega y = 0 \\ G\omega x + (M\omega^2 + \kappa)y = 0 \end{cases}$

x -arbitrary, $y = \frac{1}{G\omega} (M\omega^2 + \kappa)x$, $y_1 = \frac{\kappa}{G\omega} (M\omega^2 + \kappa) = \frac{2}{G\omega_0} (M\omega_0^2 + \kappa)x$, $\omega_0 = \frac{\kappa}{G}$

$\frac{1}{4} M\omega_0^2 + \kappa = \frac{1}{4} M \frac{\kappa^2}{G^2} + \kappa = \kappa \left(\frac{1}{4} \frac{M\kappa}{G^2} + 1 \right) = \kappa \left(\frac{1}{4} 2 + 1 \right) = \frac{3}{2} \kappa$

$y_1 = \frac{2}{\kappa} \frac{3}{2} \kappa x = 3x$. 2) $y_2 = \frac{\kappa}{G\omega_2} (M\omega_2^2 + \kappa) = \frac{\kappa}{-G\omega_0} (M\omega_0^2 + \kappa) = -\frac{\kappa}{\kappa} \left(\frac{M\kappa^2}{G^2} + \kappa \right) = -3x$?

Butterfly, two ω_1 and ω_2 - eigenmodes coexist. 2 modes. Frequency? Her.

Since $-M\omega^2$ nego $y_1 = \frac{\kappa}{G\omega_1} (-M\omega_1^2 + \kappa) = \frac{\kappa^2}{\kappa} \left(-\frac{M}{4} \omega_0^2 + \kappa \right) = \frac{2}{\kappa} \left(-\frac{1}{4} 2 + 1 \right) \kappa x = x$

2) $y_2 = \frac{\kappa}{G\omega_2} (-M\omega_2^2 + \kappa) = -\frac{\kappa}{G\omega_0} (-M\omega_0^2 + \kappa) = -\frac{\kappa}{\kappa} (-2 + 1) \kappa x = x$ - etc

Frequency up/down, amplitude x and y about $\omega_{1,2}$ values.

Lower $\omega_1 = \frac{1}{2} \omega_0$ re frequency/brw./brw. in-plane static field?

Due F/N/F disk case, $M^2 \omega^4 - (2\kappa M + G^2) \omega^2 + (\kappa^2 - \mu^2 \lambda^2) = 0$

use $(\kappa - \omega^2 M)^2 - \omega^2 G^2 = 1V^2$, $\omega^4 M^2 - (G^2 + 2\kappa M) \omega^2 + \kappa^2 - 1V^2 = 0$, $V = \mu + i\lambda$.

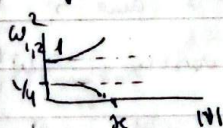
$\frac{2\kappa M}{G^2} = 4$ within $(\vec{m} \cdot \vec{n})_s = 0$ model. $\frac{M^2 \omega^4}{G^2} - 5\omega^2 + \frac{1}{G^2} (\kappa^2 - V^2) = 0$.

$\omega_{1,2}^2 = \frac{1}{2} \frac{G^2}{M^2} \left[5 \pm \sqrt{\frac{36}{25} - 4 \frac{M^2}{G^2} \frac{(\kappa^2 - V^2)}{G^2}} \right]$, $\frac{M\kappa}{G^2} = 2$, $\frac{G}{M} = \frac{\omega_0}{2}$, $\frac{M}{G^2} = \frac{2}{\kappa}$

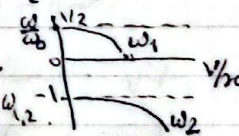
$\omega_{1,2}^2 = \frac{1}{2} \frac{\omega_0^2}{4} \left[5 \pm \sqrt{\frac{36}{25} - 4 \cdot 2^2 + 4 \frac{M^2}{G^4} V^2} \right] = \frac{1}{8} \omega_0^2 \left[5 \pm \sqrt{9 + 4 \frac{M^2}{G^4} V^2} \right]$

$P_1 P_2 = -1 \rightarrow \omega_{1,2}^2 = \frac{1}{8} \omega_0^2 \left[5 \pm \sqrt{9 + 16 \frac{V^2}{\kappa^2}} \right]$. $V = 0$ coupling $V=0$; $\omega_1 = \frac{1}{2} \omega_0$, $\omega_2 = -\omega_0$.

M. coupling per gravimetric factor



$P_1 P_2 = -1$ Case. Frequency $\omega_{1,2}$.



Due $P_1 P_2 = +1$: $x) M\ddot{x} + iG_2 \dot{x} + (\kappa + \mu)x = 0$, $-M\omega^2 - G_2 \omega + (\kappa + \mu) = 0$.

$M\omega^2 + G_2 \omega - (\kappa + \mu) = 0$, $\omega_{1,2}^* = \frac{1}{2M} [-G_2 \pm \sqrt{G_2^2 + 4M(\kappa + \mu)}]$

$\omega_{1,2}^* = \frac{1}{2} \left[-\frac{\omega_0}{2} \pm \frac{\omega_0}{2} \sqrt{1 + 4 \frac{M}{G^2} (\kappa + \mu)} \right] = -\frac{1}{4} \omega_0 \left[1 \pm \sqrt{9 + \frac{8M}{\kappa}} \right]$

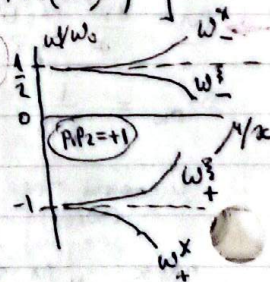
3) $M\ddot{z} + iG_2 \dot{z} + (\kappa - \mu)z = 0$, $\omega_{1,2}^* = -\frac{1}{4} \omega_0 \left[1 \pm \sqrt{9 - \frac{8M}{\kappa}} \right]$

B APL-submitted a Omas $\omega_{1,2}^* = \frac{1}{G} (\kappa \pm \mu) = \omega_0 (1 \pm \frac{\mu}{\kappa})$

no more, no grime before/after $\omega_{1,2}^*$. $M \sim C_1 C_2 < 0$.

$\omega_+^* \omega_+^* = \frac{1}{16} \omega_0^2 \left[1 + \sqrt{9 + \frac{8M}{\kappa}} \right] \left[1 + \sqrt{9 - \frac{8M}{\kappa}} \right]$, $(\omega_+^*)^2 = \frac{1}{4} \omega_0^2 \left[10 + \frac{8M}{\kappa} + 2\sqrt{9 + \frac{8M}{\kappa}} \right]$

$\omega_{1,2}^* \approx \omega_{APL} \approx \omega_{new}$. $\omega_{\pm}^* = \omega_{\pm}^*$ - gyration? 3 cases ω_n .



18/1 Re-nanostripes paper: Pashina, Vasyunovskiy, changes by Bader.
 E-nano-stripes Babunskaya: specific growth of spin block or T. Vasyunovskiy $H_c(T) \sim 1/T$
 Shukhailo-PRB-92 $N(T) = N_0 \exp(T_0/T)$, $T_0 \sim wT_c(0)$.

$H_c(T) \approx H_0 w T_0 \exp(NK_1/T)$, N - grown correlated spin-block.

Giamberelli - Nature '02: finite 1D Ising chain with a number of a FM coupled spins $N > N_c(T) = \exp(2J/kT)$. The J - exchange energy, oxygen peroxide via Giamberelli [Ref. to A. A., T. S., no name in am].

Kop. pyromagnetic $\Gamma(R) = \langle S_0 S_R \rangle - \langle S_0 \rangle \langle S_R \rangle$, co. Yeamans, 1992, r.s.

One 1D Ising ($u=0$) $\Gamma(R) = (\tanh \beta J)^R$ when $H=0$, $\mathcal{H} = -J \sum S_i S_{i+1} - H \sum S_i$.

$\Gamma(R) = \left[e^{-\ln(\frac{1}{\tanh \beta J})} \right]^R = e^{-R/\xi}$, $\xi^{-1} = -\ln(\tanh \beta J)$ - Koppen gamma.

$\tanh x = x - \frac{1}{3}x^3 + \dots$ when $x \ll 1$. $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1 - e^{-2x}}{1 + e^{-2x}} = (1 - e^{-2x})(1 - e^{-2x}) = 1 - 2e^{-2x}$, $x \gg 1$

$\xi^{-1} = -\ln \tanh \beta J \approx -\ln [1 - 2e^{-2\beta J}] \approx -[-2e^{-2\beta J}] = 2e^{-2\beta J}$

T.e. Koppen gamma 1D-Ising $\xi(T) = \frac{1}{2} e^{2J/kT}$ at 0.3. no/moga parameter when $kT \ll J$ or $H=0$.

$kT \approx 0.8$ meV when $T=10$ K. T_c (D/A stripes) ~ 200 K. $J \sim 200$ K.

no/moga $\xi(T) \sim e^{2J/kT}$ xopomo when $T \ll 200$ K, a mago $H_c(T) \forall T$, [40, 200 K].

$\forall T, H$ 6 parameter 1D-Ising

$\xi^{-1} = -\ln \left[\frac{e^{\beta J} \cosh \beta H - \sqrt{e^{2\beta J} \sinh^2 \beta H + e^{2\beta J}}}{e^{\beta J} \cosh \beta H + \sqrt{e^{2\beta J} \sinh^2 \beta H + e^{2\beta J}}} \right] = -\ln \left[\frac{1 - e^{\beta J} \frac{1}{\cosh \beta H} \sqrt{1 - \dots}}{1 + e^{\beta J} \frac{1}{\cosh \beta H} \sqrt{1 - \dots}} \right] =$

$= \ln \frac{1+z}{1-z} = 2 \operatorname{Arth} z = 2 \operatorname{Arth} \left(\frac{e^{\beta J}}{\cosh \beta H} \sqrt{e^{2\beta J} \sinh^2 \beta H + e^{2\beta J}} \right) = \xi^{-1}(T, H)$.

$\beta H \rightarrow \beta h = \beta \mu H \ll 1$ and $\cosh \beta h = 1 + O(\beta^2 h^2)$, $\sinh^2(\beta h) = O(\beta^2 h^2)$ -

no/moga 6 parameter no $\beta h \ll 1$ and parameter (can N/χ 0.42) oxygen

$\xi^{-1}(T) = 2 \operatorname{Arth}(e^{-2\beta J}) = -\ln(\tanh \beta J)$ and H vanishes.

$\xi(T) = N(T)$ under current becomes smaller. observed along spin chain and parameter spin block of 1D-Ising model.

$N(T) = \left[\ln \frac{1}{\tanh \beta J} \right]^{-1} \rightarrow H_c(T) \sim wT = wT_0 \exp \left[\frac{N(T)K_1}{kT} \right]$

1D-Ising $\langle S \rangle = \frac{e^{\beta J} \sinh \beta h}{\sqrt{e^{2\beta J} \sinh^2 \beta h + e^{2\beta J}}}$, r.e. $\sqrt{1 - \dots} = \frac{1}{\langle S \rangle} e^{\beta J} \sinh \beta h$ and $\xi^{-1}(T, H) = 2 \operatorname{Arth} \left[\frac{\tanh \beta h}{\langle S \rangle} \right]$ - Krasulo (equilibrium magnetization) r.e. $\langle S \rangle$ - equil. magnetization.

$H_c(T) = H_0 w T_0 \exp \left[\frac{K_1}{kT} \ln^{-1}(1/\tanh \beta J) \right]$ - try p-uno mago unomogobare gne fitting exp. data Pashina.

Coercivity of nanostripe - $(H_c(T) = H_0 \omega \tau_0 \exp[\frac{T_K}{T} \ln^{-1}(\coth \frac{T_E}{T})])$, $T_K = \frac{K_1}{k_B}$, $T_E = \frac{J}{k_B}$

$H_c(T)$ — число асимптотически забуксованных или злых узлов. $\eta_{\text{заморожено}}$:
 T_K и T_e . 1D-Ising dynamics model.

$T_e = 100 \text{ K}$ const. $J = kT_e = 86 \text{ meV}$. Hago shchto $J \sim 10 \frac{\text{meV}}{\text{at Fe}}$.

$$T_K = \frac{1}{k} K_1 \sim 1k \text{ const. } K_1 \sim 1 \text{ meV. } 1k = 8,6 \cdot 10^{-5} \text{ eV} = 8,6 \cdot 10^{-2} \text{ meV.}$$

Note, $N=15$ carb. $\beta J = 1.7$, $\frac{T_c}{T} = 1.7$, $T \sim \frac{100 \text{ K}}{1.7} \approx 60 \text{ K}$.
[Gambardello]

В мюон PRB-клина обозначенных $J \rightarrow J_0$ и J_0 - обсерв. интервал
всего в том числе Fe. По мюонной генерации N_x номер > 1 , $J_0 = N_x J_{Fe-Fe}$.

ТFe-Fe минералов не имеет, тогда рассмотрим только $T_k = K_1/K_B$

Cu. Shick et al.

Описание new Eq. для $H_c(T)$ для $Pu_{1-x}Ni_x$ и u/p atom ee
 glass new fitting для K_1 и K_1/N_x (meV/Fe atom).

13/5 Разбор заверенных Referees к SW-PRB paper с Bayer.

Регенерирование / восстановление DOE-порт "M. dynamics".

Barman proofs "FMR"-JAP u "Bio-X"-JAP papers.

Oxygeneum c. Baren changes to Dvortex-statics paper.

Macca vortex

Vortex mass

$$M = \frac{1}{2} \frac{L}{\delta^2 F_v(\beta)}$$

$$F_v(\beta) = \beta \cdot F_w(\beta), \text{ where } F_w(\beta) = 0.09 - 0.08$$

and since $0 < \beta < 0.1$

Am. 10/11

$$[M] = \frac{279.5}{\text{cm}^2} = g.$$

это др. неверно

$$= \frac{5 \cdot 10^3 \cdot 10^7 \text{ cm} \cdot \text{Å}^2}{2(2\pi)^2 \text{ g} \cdot 10^{12} \text{ Hz}} = \frac{5}{2(2\pi)^2 \text{ g}} 10^{-16} \text{ g} = 7,04 \cdot 10^{-3} 10^{-16} \text{ g} \approx 7 \cdot 10^{-19} \text{ g} \quad (7 \cdot 10^{-22} \text{ kg})$$

$m_2 = 9 \cdot 10^{-28} \text{ g}$ - масса фотона.

Знаки в рамке! Засоби 17/I
дубини 4m?

Для $P_1 P_2 = -1$ — только одна точка? Истина.

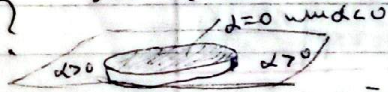
20/I Slavin: necessary Bound. Conditions" ^{B. conditions} ^{/ SW w/ spin-moment transfer} ^{b PRL}

указан на возможные dynamics - дискретности $L, R \approx 10 \text{ nm}$.

Норми, папер Киселов - Nature, беззастовична су б неопитна уму

уфронту не розв. Норме це визначає SW без pinning

200 coniac. c. kummen olababulum pinning c. y. melababulum pagne / rob dot



- некая величина на обе стороны $d=0$ (current)

- cylinder pore diameters $\sim 10-100 \text{ nm}$ = mainly $\ll \lambda$

Bezugsgrößen: $SW = W = W_{FMR}$ u. normierung. Bringen einen d = 0 (exp.)

Как по сравнению с другими. В 1D-среде локализация SW
получить легко, но в 2D - ее в принципе нет.

Поискане на Fe-nanostripes by Rukina. **Fe-nanostripes** (2/1)

$$H_c(T) = H_0 \exp\left[\frac{T_K}{T} \ln^{-1}\left(\cosh \frac{T_c}{T}\right)\right], \quad N(T) = \ln^{-1}\left(\cosh \frac{T_c}{T}\right) - \text{функция намагниченности Fe-nanostripes}$$

Ум $kT \ll J$, $T_c \gg T$, $N(T) = \frac{1}{2} \exp(2J/kT) = \frac{1}{2} \exp\left(2 \frac{T_c}{T}\right)$.

$$H_c(T) = H_0 \exp\left(\frac{T_K}{T} N(T)\right), \quad \ln \frac{H_c}{H_0} = \frac{T_K}{T} N(T), \quad \ln \ln \frac{H_c}{H_0} = \ln \frac{T_K}{T} + \ln N(T)$$

$$\ln N(T) = \ln\left[\frac{1}{\ln(\cosh(T_c/T))}\right] = -\ln \ln(\cosh(T_c/T))$$

Ум $N = \text{const}$ не проблема: $\ln H_c(T) = \ln H_0 + N \frac{T_K}{T} = F(1/T)$, максимум $\rightarrow T_K N$, мин. $\rightarrow H_0$.

Units: $1 \text{ erg} = 6.2415 \cdot 10^{11} \text{ eV}$, $1 \text{ eV} = 1.6022 \cdot 10^{-12} \text{ erg}$, $1 \text{ K} = 8.6174 \cdot 10^{-5} \text{ eV} \approx 8.62 \cdot 10^{-2} \text{ meV}$.

$$\frac{d}{dT} \ln \frac{H_c}{H_0} = T_K \frac{d}{dT} \left(\frac{N}{T}\right) - \text{уменьшение } H_0. \quad \frac{H_c'}{H_c} = T_K f(T, T_c), \quad f(T, T_c) = \frac{d}{dT} \left(\frac{N(T)}{T}\right)$$

Оценим $T_K \sim 0.1 T_c$. Если $T \gg T_c$, $N(T) = \ln^{-1}(T/T_c) \rightarrow H_c(T) \approx H_0 \exp\left[\frac{T_K}{T \ln(T/T_c)}\right]$

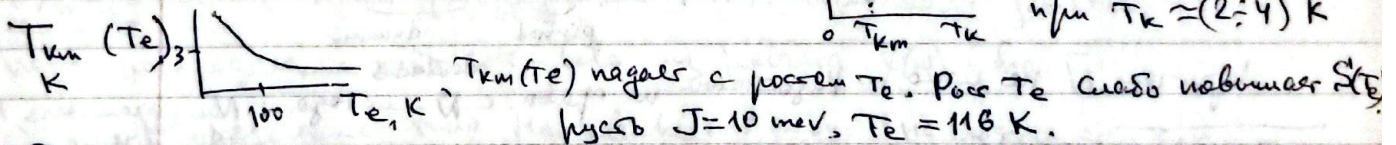
$$\frac{T_K}{T \ln T_c} \sim 0.1 \frac{T_c}{T \ln T_c} \ll 1, \text{ если } T \gg T_c, \quad H_c(T \gg T_c) \approx H_0 + O(T_K/T).$$

$$H_c(T \gg T_c) \approx H_0 \left[1 + \frac{T_K}{T \ln T_c}\right], \quad H_0 = H_c(T \rightarrow \infty). \text{ Т.е. } H_0 \text{ найдем не надо, просто берем } H_0 = H_c(T_{\text{max}}) - \text{experiment.}$$

Note: $H_0(0.7 \text{ ML}) = 37 \text{ Oe}$, $H_0(1.1 \text{ ML}) = 42 \text{ Oe}$ — значения для конкретной толщины.

Ф. квадрат. сумма $S(T_K, T_c) = \sum_j [H_c(T_K, T_c, T_j) - H_{\text{exp}}(T_j)]^2$.

Упредположим $T_c \approx 100 \text{ K}$ — параметр $S(T_K)$ имеет минимум при $T_K \approx (2-4) \text{ K}$.



Для $\theta = 1.1 \text{ ML}$ — найдем $T_{Km} = 1.50 \text{ K}$ или $K_1 = 0.129 \text{ meV}$

Для $\theta = 0.7 \text{ ML}$ — $T_{Km} = 1.74 \text{ K}$ или $K_1 = 0.150 \text{ meV}$ — $S(T_{Km})$ минимум.

Это значение K_1 на Fe segment импurity N_x .

→ В среднем случае, с $N = 15$ даме выходило $K_1 \approx \text{const}$ для $\theta = 0.7 - 1.1 \text{ ML}$.

Уменьшив J (и $T_c = J/k_B$) можно получить больше значение K_1 .

$J \gg K_1$ ($10 \text{ meV} \gg 0.15 \text{ meV}$) требует о состоянии, можем (апоно-объектно) достигнуть критического уровня — spin domains прыжков $\uparrow \downarrow$ в поле антагонизма.

→ Note: old fitting gives 1.1 ML gives $T_{Km} = 11.3 \text{ K} \Rightarrow$ на порядок больше K_1 , но малую сумму $S(T_{Km})$.

Минимум $S(T_K, T_c) \rightarrow \min$ дает T_{Km} — неверную форму кривой $H_c(T)$ и при $T_c = 116 \text{ K}$, соответств. мин high-T и low-T режимам.

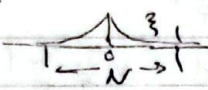
(15) G. Brown et al. JAP 91 7056 (2002) ооооо. Fe/Cu(111) $J_{FeFe} = 2 \text{ meV}$ обз. анал.

$M_z(t)$ - exp. Xorda fitting $M_z(t) \sim \exp$. gus 64×64 p. on $Li(111)$ gus $J = 7.5 \text{ meV}$.

Due nanostripes p. na $Li(111)$ $J \leq 2 \text{ meV} (!)$.

gus fitting: $H_c(T)$

J nomer doro (non-segment exchange) $\ll J_1$ (buzgus segment-spin block exchange), i.e. spin-blocks mogyt zashchislyt svoystvo vprezhdeniye. A ne voso gusmurekko, vsk y mnd.

Bozmoro, omene gusna dyn. domain $N(T) = \xi(T)$ - kop. gusne a voshennu.  Lyzme $N = (2 \pm 3) \xi$. Bepno mnd $N \sim \xi$.

tozga $H_c(T) = H_0 \exp \left[\frac{T_K}{T} N_0 \ln^{-1} \left(\coth \frac{T_0}{T} \right) \right]$, cam $N(T) = N_0 \xi(T)$

Too vozbeut y mndomro $T_0 = J/k_B$, ne gusmurekko $T_K = K_1/k_B$.

Bepu $J = 1 \text{ meV}$, $T_0 = 11.6 \text{ K}$ Bepo, mogyt $N_0 K_1 = k_B (226.7 \text{ K}) = 19.5 \text{ meV}$ y fitting.

Cam game $N_0 = 10$, $K_1 \approx 2 \text{ meV}$ - vproso mndomro (vsk y Gambar delta-02 b 12-6) T_K^{ef} chain

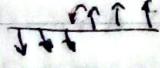
Vpu zom H_c  corxue $H_c^{fit}(T) \sim H_c^{exp}(T)$ gubomro mnd, gusmurekko Lyzme $N(T) = \bar{N} = 15$ - method. vpu mnd. pame.

→ Bibliog: new $H_c(T) = H_0 \exp \left[\frac{T_K}{T} N_0 \ln^{-1} \left(\coth \frac{T_0}{T} \right) \right]$ tozga Lyzme omenebaet E-nanostripe zashchennu, zom vproso $H_c(T) = H_0 \exp [T_0/T]$. U mndomro gusmurekko J u K_1 (puz. puzgumom) too ne mnd.

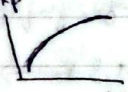
Min Ombka fitting $H_c = H_0 \exp (T_0/T)$ b e^{-e^2} poz mndomro $H_c = H_0 \exp \left[\frac{T_K}{T} \xi(T) \right]$.

Kompozom. Baymans: $N(T) = \underbrace{\bar{N}}_{\text{pinned}} + \underbrace{N_0 \xi(T)}_{\text{dynamic domains}}$, $\xi(T) = \ln^{-1} \left(\coth \frac{T_0}{T} \right)$

Vpu zom $\bar{N}$ ne mnd. Hegosaron no qabn. c $\bar{N}$ - mndomro: New parameters, N_0 u J gus fitting.

B mndy $\bar{N}$ - mndomro robozup paper Elmers-PRB'99 - stripes nepremenno cerenue (r.e. s pinning , mndomro. obzobomro ym mndomro) u Pietzsch-PRL-00 c gbyua mndomro DW (6nm u 2nm mndomro) b stripes. Torga bozmoro gbyume DW  $\tau \sim e^{\Delta E/T}$, ΔE - nepoboyt ognu omnd.

Nonpodyem $N(T) = \bar{N} + N_0 \ln^{-1} \left(\coth \frac{T_0}{T} \right)$ c $N_0 = 2$ u $\bar{N} = 10-15$

Ze xam fitting:  - ne eob vpmoe, kak doro dvi cam $H_c = H_0 e^{T_0/T}$.

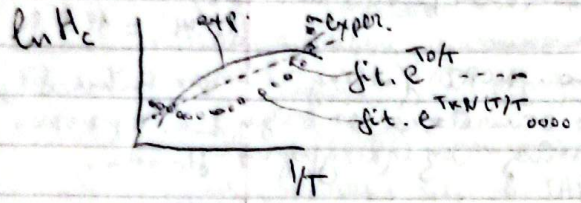
$\ln H_c(T) = \ln H_0 + \frac{T_K}{T} N(T)$, slope $\ln H_c^{exp} = f(1/T)$ mndomro c (pocem $\frac{1}{T} (!)$,

ko $N(T)$ pocem c pocem T^{-1} (mndomro T) - mndomro mndomro.

→ Кислотный $J = 2 \text{ meV}$, $T_c = 23.6 \text{ K}$, $(N_0 = 1)$ измерен $\begin{cases} T_K = 11.79 \text{ K}, & K_1 = 1.016 \text{ meV}, 0.7 \mu\text{V} \\ T_K = 10.34 \text{ K}, & K_1 = 0.831 \text{ meV}, 0.1 \mu\text{V} \end{cases}$
или параметр погрешки $H_c^{\text{exp}}(T)$.

Если погрешка или $N_0 = 0$ (т.е. $\exp(T_0/T)$), то $\begin{cases} T_K = 11.32 \text{ K}, & K_1 = 0.935 \text{ meV}, 0.7 \mu\text{V} \\ T_K = 11.31 \text{ K}, & K_1 = 0.935 \text{ meV}, 0.1 \mu\text{V} \end{cases}$
параметры T_K близкие или равные, каребо погрешки.

→ Т.е. если $N = 15$ в $N(T)$ получается, только тогда погрешка $H_c^{\text{exp}}(T)$ хороша.



Exper. $\ln H_c(T)$ лучше всего погрешка by
 $\ln H_c = \ln H_0 + \frac{T_0}{T}$ - или при N .

$\ln H_c = \ln H_0 + \frac{T_K}{T} N(T)$ лучше fitting или $N(T)$

$\ln H_c/H_0 = (T_0/T)^\alpha$ с $\alpha < 1$ хорошо погрешка бы.

$(T_0/T)^\alpha = \frac{1}{T} \frac{T_0^\alpha}{T^{\alpha-1}} = \frac{1}{T} (\frac{1}{T})^{\alpha-1} T_0^\alpha \Rightarrow N^*(T) \sim (\frac{1}{T})^{\alpha-1}$ убыв. ф-ция $(\frac{1}{T})$, или $N^*(T) \sim T^{1-\alpha}$ $\alpha < 1$.

Такой fitting можно сделать, но какой физ. смысл T_0 тогда?

$H_c(T) \sim T(T)$, безонне распределение $F(T)$ или $F(\Delta E)$ можно бы дать, в принципе, $T_{\text{eff}} \sim \exp((T_0/T)^\alpha)$, иначе fitting $H_c^{\text{exp}}(T)$.

$H_c(T) \sim \frac{\omega T}{\sqrt{\omega^2 T^2 + \alpha^2}} = |\omega \ll 1| \sim \frac{1}{\alpha} \omega T$, см. 14/211, где $\alpha = 1 - \beta J$. $H_c(T) = \frac{H_0}{\alpha} e^{\frac{T_K}{T} N}$

$\ln H_c = \ln H_0 + \frac{T_K}{T} N(T) - \ln \alpha(T)$, $\alpha(T)$ где-то погрешка $H_c(T)$ с T .

Но $\alpha(T)$ соеенная ф-ция и номер быль выведена в T_0 , $T = T_0 e^{\Delta E/T}$.

$\ln T_0 = \beta \Delta E = E$ итер распределение $F(E)$. Возможно ли $\int E \cdot E F(E) = E^2$?

Восм. распределение типа Cole-Cole из параметра $\chi(\omega)$ - bio.

Распределение типа Havriliak 6-8/24 от. не подходит, т.к.

там погреш. $\frac{1}{1+i\omega\tau}$, а не $\frac{\omega\tau}{\sqrt{\alpha^2 + \omega^2\tau^2}}$ как для H_c .

$\chi(\omega) = \chi_0 \frac{1}{[(i\omega\tau_0)^p + 1]^\delta}$, $\chi''(\omega)$ - номер где-то необходимое, т.к. усреднение по ω (или T) .. $\omega\tau_0 \rightarrow z$, T_0 - центр распределения $\ln \frac{T}{T_0}$.

$\chi''(\omega) \sim \chi(\omega) - \bar{\chi}(\omega) = \frac{1}{[(iz)^p + 1]^\delta} - \frac{1}{[(-iz)^p + 1]^\delta} \sim (z)^\delta \sim T_0^\delta$

Может в $H_c \sim \frac{\omega\tau}{\sqrt{\omega^2\tau^2 + \alpha^2}}$, $\omega\tau$ надо усреднить. Нет, т.к. $\omega \sim 10^3 \text{ Hz}$, $T = T_0 e^{\frac{\Delta E}{T}} \sim 10^{-10} \text{ s}$ или $T > 40 \text{ K}$.

Упомянул B. Conditions paper not format PRL, ссылка со ссылкой.

Намечено /revision & Anal. calcs. в D-Vortex - Statics paper. Номинал to Kristen.

Намечено new theor. section в Bi-heterophases paper. Discuss с Rukha $H_c(T)$ - погрешка.

(25/I) Дополнительная "B. Conditions" paper in Sov. Lett. и журнал B PRL
 - LA 10602.

Fe-Nanostripes

Непомнящих Discuss. Section of paper with Ruitma on Fe/Pt-nanostripes
 Чувствуем now MCAD logarithm $H_c(T) = H_0 e^{T_0/T}$ c $\ln H_c = \ln H_0 + \frac{T_0}{T}$ - Origin
 и c Johnson Table 1 by Ruitma ($N=15$).

exp.	θ	0.7 ML	1.1 ML
MCAD $T_0 = NT_0$		169.8 K	169.7 K
Origin T_0, K		$247.2 K \pm 22.9$	$230.1 K \pm 16.4$
Orig-Ruitma T_0		158 K	163 K

$\ln H_0 = 3.026 (0.7 ML)$ и $\ln H_0 = 2.933 (1.1 ML)$
 ± 0.252
 Origin $H_0 (0.7 ML) = 20.6$ Oe и $H_0 (1.1 ML) = 18.9$ Oe
 ± 0.180

Мои MCAD fitting лучше Ruitma-fit, но отсутствует в Origin Lin. regression fitting и 3-го порядка fit.

Мои Origin fitting $\ln H_c = \ln H_0 + \frac{T_0}{T} = f_2(T)$ и Ruitma-Origin fitting $H_c = H_0 e^{T_0/T}$
 gave better T_0 и H_0 . Оса fitting $H_c(T)$ и Johnson equation совмещ. с $H_c^{exp}(T)$.
 Оса fitting лучше Ruitma H_0, T_0 , т.к. K_1 уменьшается с T_0 и не зависит от H_0 .
 Лучше с Ruitma. Обсуждаю с член changes to Fe/Pt-nanostripes paper
 и Johnson regression Table 1, добавлю N_x и T_0 и K_1 .

(26/I) DOE Review - Dornag G. Galtree / Dornag, Bader, Hoffmann, Novosad / мои poster.

Работы / comments и B.A. Ivanov, C.E. Zaspel, PRL 94, 027205 (2005)
 "High-frequency vortex modes in Vortex-Gate Nanomagnets"

Notes: **Ivanov-PRL'05 Vortex Dynamics**

- 1) $H_z^m = -4\pi M_z$. Too narrow, оса fitting и Johnson H_z ,
 vortex dot vortex и vortex в I-saturated state, см. now APL'04 c Kakazei.
 $H_z^m = f(\vec{B}, L/R)$
- 2) Due to the splitting of $\omega_m(\pm m)$ - modes because of H_z . Splitting
 в rings = 0 без H_z , т.к. там нет splitting и 2-го порядка. "core"
 (но в I-saturated dots/rings splitting есть?)
- 3) В работе Ivanov (CP) где работа Zaspel и Novosad. Обсуждаю
 и Johnson $g_0(k) = J_0(k) + 2J_1(k)$, $J = m$ с $H_z = 0$.
 Но это и Johnson забывает, что Johnson. Johnson, $R \approx 200-500$ nm, где и Novosad в PRB-Werner, что $\mu_p(p) = J_1(kp)$ и $m=0$,
 в осях и $g_0(k) = J_0(k) + 2J_1(k)$ и Novosad (2006).
- 4) Значит $\omega_m \sim \sqrt{4R}$ где high- ω мод и Novosad в PRB-02,
 и Novosad Novosad в PRB-Werner (сво архив - reprint). Убавлю Zaspel от
 двух работ. Где осях на Novosad?
- 5) Novosad Novosad, что $\omega_m \sim (4R)$ как в Novosad JAP-02, но Novosad.
 Novosad Novosad T_0 - работа $\Rightarrow$ splitting 4 modes. Novosad $\omega_{\pm 1}$ работа и Novosad
 Novosad с Novosad M.

→ (6) Paper eigenfrequencies основан на в. conditions Eq. (13).

Но они неверные! там не выведена магнитостатическая волна (charges, surface $\gg$ each contribution due to surface R - zone).

Иванов и его соавторы, утверждая $\mu_p(r) \sim g(r)$ из физ. ур. поперечности зарядов, а не поперечности $g(r) \sim g_0(r)$ отсюда PoL charges и т.д.

(7) Расчеты Ивана и расчеты магнитных элементов от магнитных элементов ФГ, и к асимпт. расчет $\cos(\omega t - m\phi)$ убавка (хем $G_m(r, r') = \int d\phi \int d\phi' u_m(\phi) G(r, r') u_m(\phi')$, $u_m = \cos(\omega t - m\phi)$ $\sin(\omega t - m\phi)$), где $G(r, r')$ магнитостатическая ФГ и т.д.

Эту хем. я привожу еще, где saturated dots в JAP'00, APL'04 и PRB'Werner где vertex.

→ Вывод: (4) на волнах абсорб. по (1), (3), (6), особенно (6) gives wrong surface waves как же cylinder так и ring.

$$\text{Гр. условие Eq. (13)} \rightarrow \frac{dg}{dz} + I_m \frac{L}{4\pi l_0^2} g \int_0^R r Z F(r, R) = 0, z=R - \text{Ivanov PRL'05}$$

$$l_0^2 = C/(M_s^2 \omega^2), C = 2A. I_m = 2$$

$$\frac{dg}{dz} + \frac{L}{2\pi l_0^2} \ln\left(\frac{4R}{L}\right) g(R) = 0, z=R - \text{Ivanov APL'02}$$

$I_m = 1, m=0$

→ I. e. pinning parameter Ivanov $\Lambda = \frac{RL}{2\pi l_0^2} \ln\left(\frac{4R}{L}\right)$

$$\text{Мои pinning parameter для cylinder } d = \frac{2\pi}{\beta \left[a + b \ln \frac{1}{\beta} + \frac{L_0^2}{L^2} \right]}, \quad \begin{matrix} a = 2(6\ln 2 - 1) \\ b = 4 \\ L_0^2 = 2A/M_s^2 \end{matrix}$$

$$\text{Ординату } L_0^2 = 4\pi l_0^2$$

$$\text{Вывод: vol. charges बनाय } d \text{ wrong! } d = \frac{2\pi L^2}{\beta L_0^2} = \frac{2\pi RL}{4\pi l_0^2} = \frac{RL}{l_0^2} \gg 1, l_0 \approx 5 \text{ nm}$$

$$\Lambda = \frac{RL}{l_0^2} \frac{1}{2\pi} \ln\left(\frac{4}{\beta}\right) - \text{основание пропорции } \frac{1}{2\pi} \ln\left(\frac{4}{\beta}\right) = 0.5 - 0.7 \text{ где } \beta = 0.05 \div 0.2$$

$$a + b \ln \frac{1}{\beta} = 2(6\ln 2 - 1) + 4 \ln \frac{1}{\beta} = 6.32 + 4 \ln \frac{1}{\beta} = 18.3 \div 12.8 \text{ где } \beta = 0.05 \div 0.2$$

$$\frac{L_0^2}{L^2} \approx \left(\frac{20 \text{ nm}}{L}\right)^2 < 1 \text{ если } L > 20 \text{ nm. Т.е. each बनाय } \ll (\text{v. magnetostatics})$$

$$\text{где } \beta \approx 0.1, L \sim 20 \text{ nm тонких.}$$

$$\text{Мои } d(\beta) = 2.46 \div 6.87 \text{ где } \beta = 0.05 \div 0.2. \text{ Ивана } \Lambda \sim \frac{RL}{l_0^2} = \frac{RL}{(5 \text{ nm})^2} = \frac{160 (!)}{1} \text{ где } \begin{matrix} R = 200 \text{ nm} \\ L = 20 \text{ nm} \end{matrix}$$

! Pinning Ивана $\Lambda \gg d(\beta)$ - это как раз, на поверхности cylinder.

Это неубедительно, ведь Иван основан на в. экв. Ивана в знаменателе d , неубедительна - Ивана, это Ивана и заблуждение pinning parameter.

$$\rightarrow \text{Ивана } \mu_p = -RLM_s F(\beta, R) m_p(R) \leftarrow \text{проверка}$$

$$\text{Вывод: Ивана } \text{показан } \Lambda - \text{pinning } \text{показан как } \Lambda = \frac{E_{\text{surf. mag.}} R}{A} \text{ в } g(r) \text{ R.W.}$$

27/5 Сославские Смена papers по "Neomagnetism" 30 Oct. 2003 - Dec. 2004. Домашня Винокур, Нарман, Белобородов, Матвеев - DOE Review. Kristen - Vort Stat.

22/I Vortex. Seagate patent appl. (Vortex dynamics)
 "Nanomagnetism" (file Janice). Duckyane
 - APS by Crowell - 3 min. govt - double ver
 Duckyane c Konan 1D, 2D prez. napexog
 Passozan o mag. vortex, ML u Double-
 - Беседа с Сабурман: как составить Bay
 Odygum recent papers Trans-Trans. u Bues
 Reman namcaro yuozno paper no Ver

$$L = 40 \text{ nm}$$

$$2R = 1,100 \text{ nm}$$

$$J = 1,76 \cdot 10^{11} \frac{1}{\text{T} \cdot \text{s}}$$

$$M_s = 750 \text{ G}$$

Pierre 23



$$\frac{\omega}{2\pi} = 275 \text{ MHz}$$

23/I nogumca kb. konfaks na AMLI - apartu
 31/I Revised Bayer-PRB-SW paper, Ser
 nocan to Glavin u Bayer.

Odyor Fe-nanostripes paper c yfabrcu S. Bader. OK to publish.

1/II Kefeezg na kb. - town-home AMLI at River Run. Zamnam 150 \$ deposit "Nap-Elect"

2/II Konfaks u nocan JAP - SW stripes paper. Konfaks u
 re-submitted PRL - b. conditions paper.

Usumbar namcaro, too no Nebraska - application a bomen
 to nybrae 20 uz dnce > 100 applications, no b 6-ky gne interview
 ne bomen.
 Pucame Oucane, Charlotte, Angfero, Rent.com. Zkazas rlu.

3/II Cgova kb. na Remington. Review nanomimexae papers.
 - Dobroqumca, too over za bozy me bueabes 10/II (za 11/XII 3/II) -
 oagum ho amary, too mure no bumen uz deposit.

4/II Pegaryupobame D. Vortex-PRB Statics paper c Kristen.
 Cuenca on-line home-address b Bank-One u Citi-Bank.
 Zampro Seagate accounting for W-2 form (Ranthun).

5/II - no mery. Haran mcaro Dvortex Dynamics (p, p2 = -1) paper.
 6/II Hamec qumca L. Thompson (Canada). Vortex Dynamics in Easy Plane ferromagnets

7/II Cgpa. namum quotes on-line at 21st Century, "Progressive", "State Farm" u
 Odygum c Cuebman Seagate - patent. no vlep.

Seminar yugum: 1) Kristen maza, too $AE = E(C_{r1}=1) - E(C_{r2}=-1)$ bobe
 ne 20% xan maza b APL-subm., a ~ 1% a $\frac{AE}{1/2k} \frac{1}{N}$ yombees c ybenu. maza
 ower (yuenum cell-see); 2) Pierre zammelee mca. padrecau v. dynamics
 b single-layer glinders ($R \sim 50 \text{ nm}$), nopydu emy noqpuuo upreum
 agumdes $m_r(\theta)$ gne upobepu abey upamr. yabebui

8/II Kefemene c Baei no nobozy PRL - vortexdyn - Demokritov's
 u maza pabos. Baye b base farm. Zbame b AIG (new cpa. - 5354) u Rev. Card
 Hamec J. Slonczewski - 85 paper. A. mrova. Citi-Bank.

9/II Review of new papers. Becega & Baran & Ivanax:

- ① Baran & Egger describe new approach for short vortex-TM dynamics paper. Exp. Frank + Micromag. Simulations + theory TM, APL and PRB - liquids (single layer disk).
- V.mass ② A. Egger first author (Evan & Kristen) new approach short paper. No vortex mass b cyl. dot.
- ③ Hammer long paper no DVortex ($p/p_s = \pm 1$) dynamics, but theory, experiment, simulations & Baran, Kristen + Eric Koro. F/N/F disk. Fun?
- ④ Pierre Ray nonpolarized normal $m(g)$ -cylinder $R \approx 50$ nm give b. conditions "vortex barrier".
- ⑤ P. Ray shorter paper no (9/II) DVortex elliptical dot.
- ⑥ X, FMR/BLS - ring b HL, DWR b MLS $\in$ Hall u v. dynamics u circular h(w) - na dygyma. F/MR b F/N/F ~~disk~~ stripes ne genera, r. k. Chomene u general BLS theory (Kostylev et al). Vort. barrier $H_{n,an}(T)$ - exp?
- ⑦ B Nandluch - govniye Kristen osalam min DVortex dynamics, cecomam na neme, ceg. govny, koropm i cegere b ANL soon (Nandluch, MS colloq. theory)
 $\downarrow$
 cegam gov. b ygyne que APS conf.

Theme papers by Perzlmeier et al, PRL 94 (2005) 057202 - "BLS/Kerr in F/N-Squares in vortex state" u Vavassari et al, APL 86 072507 (2005) - "A Magn. resistance of single magnetic vortices" (zammerboham namy F/N/F vortex u gno GMR).

Theme papers by J.C. Slonczewski - 85 (Conf. 84) u A.B. Borisov no "spiral" u "target" - solitons give 2D-ferromagnet (∞ -film).

→ Baran upines PRB - submitted paper by Zaspel - Grouwell no vortex dynamics, stopmo koropm one upnam b PRL '05.

T. e. zgea nobropna ta me amodka c ynam. y cabmim u gr. (an. 26/I).

→ Seminar talk by A. Bhattacharaya - bimenne E-nome na R (conf. presub.) u mar. oberstva (H_c) of La_{0.6}Mn_{0.4}O films. Ogor ryzum results, namoro cheux.

Water Remintgen: zbonna Kristen. nobepgna meter-water reading 109.2 na 10/II u nepmog 11/I - 3/II dygyl cross \$36.38. Dobalub coga \$56.77 za 11/II - 10/I - \$56.77 neytmu \$93.15. Cornacuse na barec \$93.15 u deposit (\$450.). Deposit rest dygyls noman me b ser. us gien c gne vbezga 3/II 051.

Vortex dynamics Osmechene exp. Fradin no TM-vortex mode b single-layer cylindrical dot (an. 19/II, 20/II 042.). Exp. $\frac{\omega_0}{2\pi} = 275 \text{ MHz}$ ($2R = 1.1 \mu\text{m}$), $\frac{\omega_0}{2\pi} = 162 \text{ MHz}$ ($2R = 2.2 \mu\text{m}$)
 Kristen nobepgna $M_s(\text{exp}) = 700 \text{ G} \rightarrow \frac{\omega_{01}}{2\pi} = 312 \text{ MHz}$ u $\frac{\omega_{02}}{2\pi} = 161 \text{ MHz}$ - calculated b xopowem cornacuse c exp.

11/II

Связь параметров related to V. Mass.

1) Single-layer disk, bearing $\hat{M}$ is TM-disk с массой — max масса
геоплант (Larson, ...)

2) $\hat{M}$ -параметр — универсальный в DVortex P/M/P disk и универсальное exp
(данные в масса $\hat{M}$ — универсальны во всех планетарных системах?) и симметрии

Масса vortex в single-layer disk.

Vortex mass

① Slowdownski-85 : $\hat{L} = \int dxdy \mathcal{L}$, $\mathcal{L} = -\Phi \cos \theta - \mathcal{H}$, $\delta \mathcal{L} = \cos^2 \theta + (\partial \theta)^2 + \sin^2 \theta (\partial \Phi)^2$
(2D case plane)

Универсальное $\mathcal{L}_u = K_1 \cos^2 \theta + A [(\partial \theta)^2 + \sin^2 \theta (\partial \Phi)^2] = \frac{K_1}{A} [\cos^2 \theta + \delta^2 (\partial \theta)^2 + \delta^2 \sin^2 \theta (\partial \Phi)^2]$, $\delta^2 = \frac{A}{K_1}$
 $\vec{x} \rightarrow \frac{\vec{x}}{\delta}$. $E = \int dxdy dz \delta \mathcal{L}_{vol} = \int dxdy (L \delta \mathcal{L}_{vol})$, $\delta \mathcal{L}_{surf} (2D) = L \delta \mathcal{L}_{vol}$ где 2D — 3D.

$$\frac{1}{K_1 L} \delta \mathcal{L}_u (surf.) = \delta \mathcal{L} (slanc.)$$

$$L_{vol}^{slanc} = -\frac{M_s}{\delta} \int dxdy dz \Phi \cos \theta, \quad L^{slanc} (slanc) = - \int dxdy \Phi \cos \theta \Rightarrow L_u = L (slanc) \cdot \frac{M_s L}{\delta} -$$

— кин. энергия, y slanc. t — безразмерное.

$$\hat{L}_u = -\frac{M_s L}{\delta} \int dxdy \Phi \cos \theta = -\frac{M_s L}{\delta} \delta^2 \int d\bar{x} d\bar{y} \frac{\partial \Phi}{\partial t} \cos \theta, \quad (\bar{x}, \bar{y}) = \frac{1}{\delta} (x, y)$$

$$\tau = \delta M_s t, \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} \cdot \delta M_s, \quad \frac{\partial}{\partial x} = \frac{1}{\delta} \frac{\partial}{\partial \bar{x}}$$

$$\hat{L}_u = -\frac{M_s L}{\delta} \delta^2 \delta M_s \int d\bar{x} d\bar{y} \frac{\partial \Phi}{\partial \tau} \cos \theta, \quad \hat{L} (slanc) = - \int d\bar{x} d\bar{y} \frac{\partial \Phi}{\partial \tau} \cos \theta, \quad \hat{L}_u = + M_s^2 \delta^2 L \cdot \hat{L} (slanc.)$$

$$W_u = \int dxdy dz [K_1 \cos^2 \theta + A(\partial \theta)^2 + \dots] = K_1 L \delta^2 \int d\bar{x} d\bar{y} [\cos^2 \theta + (\frac{\partial \theta}{\partial \bar{x}})^2 + \dots] = K_1 L \delta^2 W (slanc.)$$

$$W_u = K_1 L \delta^2 W (slanc.), \quad W (slanc.) = \int d\bar{x} d\bar{y} [\cos^2 \theta + (\frac{\partial \theta}{\partial \bar{x}})^2 + \dots]$$

$$W_u = \int dxdy dz \mathcal{H}_{vol} = \int dxdy (L \mathcal{H}_{vol}) = K_1 L \delta^2 \int d\bar{x} d\bar{y} \mathcal{H} (slanc.)$$

$$-\delta \mathcal{L}_u (surf.) = L \delta \mathcal{L}_u (vol.) = K_1 L \delta^2 \mathcal{H} (slanc.), \quad \hat{L}_u \mathcal{H} (slanc.) \text{ — безразмерное}$$

$$\hat{L}_u (surf.) = L \hat{L}_u (vol.) = M_s^2 L \delta^2 \mathcal{L} (slanc.) - \text{кинетическая энергия, безразмерная}$$

$$T = \frac{1}{2} \dot{\vec{r}}_i \hat{M}_{ij} \dot{\vec{r}}_j \rightarrow T_{single disk} = \frac{1}{2} \hat{M}_{\alpha\beta} \dot{x}_\alpha \dot{x}_\beta, \quad \text{где } i=1, j=1.$$

$$4T = \pi \ln R (\dot{\vec{r}}_i)^2 - \pi \ln r_r (\dot{\vec{r}}_i)^2 = \pi \ln \frac{R}{r_r} (\dot{\vec{r}})^2 - \text{масса гравитационная}$$

no $d, \beta = x, y$. $M_{\alpha\beta} = M \delta_{\alpha\beta}$

$$T = \frac{\pi}{4} \ln \frac{R}{r_r} \left(\frac{d\bar{x}}{d\tau} \right)^2 - \text{кин. энергия Slowdownski,}$$

$$T_u = M_s^2 \delta^2 L \cdot T (slanc.) = M_s^2 \delta^2 L \cdot \frac{\pi}{4} \ln \frac{R}{r_r} \frac{1}{\delta^2} \frac{1}{\delta^2 M_s^2} \left(\frac{d\bar{x}}{d\tau} \right)^2 = \frac{L}{\delta^2} \frac{\pi}{4} \ln \frac{R}{r_r} \left(\frac{d\bar{x}}{d\tau} \right)^2$$

$$T_u = \frac{1}{2} M \left(\frac{d\bar{x}}{d\tau} \right)^2 \Rightarrow M = \frac{\pi}{2} \frac{L}{\delta^2} \ln \frac{R}{r_r} - \text{масса vortex по Slowdownski-85,}$$

где R — радиус, где δu расщепляется на вращение (разрыв скорости), r_r — радиус "core"

M - Slonczewski noxoma na novo $M = \frac{1}{2} \frac{L}{d^2} \frac{1}{F_v(p)}$ or $17/1$
 $(M = (P/2) \frac{L}{d} \ln R/r)$

No fiz. context $R \gg 1$ " $\ln R/r \sim 1$, i.e. $M \gg M(\text{Slon})$
 $F_v(p) \ll 1$, i.e. $p \ll 1$

→ Paralog: new mass vortex $\gg$ mass single-vortex 2D easy plane Slonczewski 85

Note: в новом явлении использована "side-charge-free" model с глубина vortices,
 бие и смагнута гора, где Φ - поле непрямая линия.

У Slonczewski $\cos\theta(r) = \sum p_i u(r-r_i) + p(r)$, где $u(r) = \cos\theta(r)$ is static
 vortex profile, $p(r)$ - уменьшение профиля vortex из-за SW или SW барьер.
 $p(r) \approx -\frac{\dot{x}_i}{2\pi} \sin\beta_i \sim (\dot{x})$, i.e. уменьшение vortex профиля из-за SW
глубина $(\dot{x} \neq 0)$ и масса $\sin \sim \dot{v}$ or $17/1$ физическая разница.

One 2-vortex pole-free model $\cos\theta(r) = p_1 u(r-r_1) + p_2 u(r-r_2) + p(r)$

$\cos\theta(r) = p_1 u(r-r_1) + p(r)$, i.e. image vortex p_2 генерирует поле горы или SW
rigid v. $r_2/R \ll 1$.

Т.е. р-а Слончевского $p(r) = m_z^{\text{dyn}}(r) - m_z^{\text{rigid v.}}(r)$
масса vortex $M \sim p(r)$.

shifted pole-free
vortex

shifted rigid vortex

② L. Thompson-04 -thesis гипотеза vortex mass 2D easy plane ferrimagnet

$$M_v = \frac{\pi q^2 r_v^2}{J a^4} \ln \frac{R_s}{r_v}, \quad E = -\frac{1}{2} \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j + \sum_i K S_{iz}^2$$

J - обмен (i,i+1) - сумма сочетаний, $r_v^2 = \frac{J}{2K}$, a - нефизическая радиус сочетания - R_s ,
 q - коэффициент зависимости "vorticity". масса v. от обмена поле "core".

Это похоже на выражение Wyllah.

Граничное условие $\frac{d\vec{P}}{dt} = \vec{F}_g$ сочетания $-\frac{e}{c} \vec{A}$ или $\vec{P} = \vec{P} - \frac{e}{c} \vec{A}$, т.е.
независимый сочетания поле $\vec{B} = \text{rot} \vec{A}$. $\oint \vec{B} \cdot d\vec{S} = \Phi_0$ - квант поток.

$$\hat{L}(\text{Thomp}) = S \int \frac{d^2r}{a^2} \left[-\Phi \cos\theta - \frac{1}{2} S a^2 J (\nabla\theta)^2 + \dots \right] \quad E_A = S \int \frac{d^2r}{a^2} \left[\frac{1}{2} S J a^2 \frac{1}{r^2} \sin^2\theta \right]$$

$$\hat{L}_u = -\frac{M_s L}{\delta} \int dx dy \Phi \cos\theta - \int dx dy L \left[K_1 \cos^2\theta + A (\nabla\theta)^2 + \dots \right]$$

$$\frac{S}{a^2} \rightarrow \frac{M_s L}{\delta}, \quad \frac{1}{2} S^2 a^2 J \Rightarrow AL, \quad LK_1 \rightarrow \frac{1}{2} S^2 J \frac{1}{r_v^2}, \quad \frac{1}{2} S^2 J \rightarrow AL, \quad r_v^2 \rightarrow \frac{A}{K_1}$$

$$\frac{r_v^2}{J a^4} = \frac{1}{a^2} \frac{r_v^2}{J a^2} = \frac{M_s L}{\delta} \frac{r_v^2}{S J a^2} = \frac{M_s L}{\delta} \frac{A S}{K_1 + 2AL} \frac{S^0}{a^2} = \frac{M_s}{\delta 2K_1} \frac{M_s L}{\delta} = \frac{M_s^2 L}{2K_1 \delta^2}$$

T.e. масса Thompson & more obviously in paper models

$$\rightarrow M(\text{Thomp}) = \frac{\pi \tau_v^2}{4a^4} \ln \frac{R}{\tau_v} = \pi \frac{M_s^2 L}{2K_1 \delta^2} \ln \frac{R}{\delta}, \quad \delta^2 = \frac{A}{K_1} = \frac{J}{2K}$$

$$M(\text{mass}) = \frac{1}{2} \frac{L}{\delta^2} \frac{1}{F_v(\beta)} \quad \text{сравн. с} \quad M(\text{Thomp}) = \frac{\pi M_s^2 L}{2K_1 \delta^2} \ln \frac{R}{\delta}$$

Due thin cylinder $K_1 (\text{thin}) \approx 2\pi M_s^2 \rightarrow M(\text{Thomp}) = \frac{1}{4} \frac{L}{\delta^2} \ln \frac{R}{\delta}$

Очевидно $M(\text{mass}) \gg M(\text{Thompson})$, т.е. $F(\beta) \ll 1$, а $\ln \frac{R}{\delta} \gg 1$ due Submicron dots.

T.e. $\frac{M(\text{mass})}{M(\text{Thomp})} = \frac{2}{\ln \frac{R}{\delta} \cdot F_v(\beta)} \approx \frac{2(R/L)c}{\ln \frac{R}{\delta}} \gg 1 \quad \text{when } \frac{R}{L} \gg 1, \quad F_v(\beta) \approx c\beta \quad \beta \ll 1, c \approx 1$

Булбог: наша масса pole-free model known usually $\gg$

$\gg$ mass 2D easy plane model paper Thompson-04

Title: "Giant magnetic vortex mass in finite cylindrical nanodot"

③ Wyss-PRB'96 gear v. mass $M = \frac{\pi q^2}{45a^2(1-\lambda)} \ln \frac{R}{a_0}$, а-нашог параметр a_0 - cutoff radius $\approx 0.2a$.

XYZ-exch. 2D model $\rightarrow \mathcal{H} = -J \sum_{n,a} [S_n^x S_{n+a}^x + S_n^y S_{n+a}^y + \lambda S_n^z S_{n+a}^z]$, $\tau_v^2 = \frac{1}{4} \frac{\lambda}{(1-\lambda)}$

$$M = \frac{\pi q^2 \tau_v^2}{Ja^2 \lambda} \ln \frac{R}{a_0} \rightarrow M = \frac{\pi q^2 \tau_v^2}{Ja^2 \lambda} \ln \frac{R}{a_0} \quad \text{с параметром } \tau_v, \lambda \approx 1$$

Тоо булганга караганда с $M(\text{Thomp})$ караганда, егер $\lambda \lesssim 1$.

13/11 Намунае Zangwill "Vortex-Mass" paper

Reg. CV a letter from Dim. Nuccini to Yves Rocher, ter. Rebate a Rent. Com.

⑭/11 Group meeting. Ouyang's Намунае rep./exp. stripe line/modeling paper no vortex dynamics (TM) b single-layer. Обсужден, тоо гомоно док гле

укажем $w \pm 1$, CCW = CW-motion a замкнул. Диск. (Pierre) b stating a.c. field h

Many more copies of papers no topol. excitations:

1) I.A. Ovidts et al. Ph. St. Sol. a104, 113 (1987)

2) A.B. Borisov et al. Physica D31, 49 (1988)

3) A.B. Borisov et al. Physica D99 (1996), 18.

$\rightarrow$ Generalize Vortex Mass paper:

1) generalize, too "side-charge-free" $\vec{m}(z)$ aka. $\propto$ parameter β 1-1.

2) give $F/N/F$ gauge up to some mass m . no P_j, G_j gauge factor

3) piece. mass-coupling M_{ij} ($i,j=1,2$ - none gauge) give $F/N/F$ gauge

(15/11) Pierre Wuytack preprint by Shoh-OS a boyfriend. nobody one
 Robert, 2nd TM nanotech boggyquest in-plane hv.

Korea ugras F/AF-vortex reversal ugras gae Axel.

Search no same Vortex dynamics: named these lagged Trimper'79 paper.

→ Thomas Winlin (Sagade patent) o choux yubunex o patent annare.

Dejuna Saving o Bank One na chae name (Beneficiary Lara u Anton - ugrasjust).

(16/11) Bane ugrasuna "new" thing: TM vortex resonance o cpepur.
 Zaczuse. Kazan luy ugras nanogeo lro zuchepunexuobunso, nobun

a ugrasun. Wpukun. nobun nes, no pacres cuonunoi.

S/F dot → Dejuna luy c Loptin nan project: double-layer structure

Superconductor - ferromagnet (weak, cnynei T_c)  . Ugras -

seg $H=0$ SC gopunupyes, c ybeur. H,

u nobunex Hex of weak ferromagnet, gepponcamenay u nobunexes u

conpounenue pego pacres c H. Unanogobes que H-sensor, mag. memory.

Vortex mass Com. 17/11/01. Gyrovector. $\hat{G}_{ij} = \int d^3r \hat{g}_{ij}(r)$, Gyrotensor $\vec{G} = \int d^3r \vec{g}(r)$

$$g_{ij} = \frac{M_s}{\delta} \vec{m} \cdot \left(\frac{\partial \vec{m}}{\partial x_i} \times \frac{\partial \vec{m}}{\partial x_j} \right), \quad 2D\text{-zagara temuo } g_{ij} \neq 0 \quad \vec{g} = g_z \hat{z} = -\frac{\hat{z} M_s}{\delta} \vec{m} \cdot \left(\frac{\partial \vec{m}}{\partial x} \times \frac{\partial \vec{m}}{\partial y} \right)$$

($g_z = -g_{xy}$)

$$\vec{g} = g_z \hat{z} = -\frac{\hat{z} M_s}{\delta} \frac{1}{p} \vec{m} \cdot \left(\frac{\partial \vec{m}}{\partial p} \times \frac{\partial \vec{m}}{\partial \varphi} \right), \quad \vec{g} = -\frac{\hat{z} M_s}{\delta} \vec{m} \cdot 2i \left(\frac{\partial \vec{m}}{\partial z} \times \frac{\partial \vec{m}}{\partial \bar{z}} \right)$$

$$\vec{g} = \frac{M_s}{\delta} \sin \theta (\nabla \Phi \times \nabla \Theta) \quad c \quad w(z, \bar{z}) = \exp \left[\frac{i}{2} \Theta(x, y) \right] e^{i \Phi(x, y)}, \quad \Theta, \Phi - \text{cpepur. yubun } \vec{m}.$$

$$\rightarrow \text{Khabulo: } w(z) = \frac{\prod_l (z - z_l)^{q_l}}{1} = \exp \left(\ln \prod_l (\dots) \right) = \exp \left[\sum_l q_l \ln(z - z_l) \right] =$$

$$= \exp \left[\sum_l q_l \operatorname{Re} \ln(z - z_l) + i \sum_l q_l \operatorname{Im} \ln(z - z_l) \right] = \exp \left[\frac{\Theta}{2} \right] \cdot e^{i \Phi(x, y)}$$

$$\rightarrow \Phi(x, y) = \sum_l q_l \operatorname{Im} \ln(z - z_l) = \sum_l q_l \cdot \operatorname{arctg} \left(\frac{y - y_l}{x - x_l} \right) \quad \text{u ne nago nuwane}$$

Yr-a gae $\Phi(x, y), \Theta(x, y)$ penas kae Trimper'79. Jo choba 2-vortex models.

$$\exp \left[\frac{\Theta}{2} \right] = \exp \left[\sum_l q_l \operatorname{Re} \ln(z - z_l) \right] = \prod_l \left[(x - x_l)^2 + (y - y_l)^2 \right]^{-q_l/2}$$

$$\text{BAP'02} \quad \vec{G} = \frac{M_s}{\delta} L \int d^2r \sin \theta (\nabla \Phi \times \nabla \Theta), \quad \vec{G} = -G \hat{z}, \quad G = \left(\frac{M_s L}{\delta} \right) 2\pi q P.$$

Tance ne ugrasunue $\vec{G}$ o Slonezewski, JMMM12, 108 (1979), Huber PRB82, JAP'82

(17/11) Pabosa nay Vortex-Mass paper. Bouys u ANL lawyers: nobunex
 nobunex "Assignment" gae Sagade Patent.

Pegunufubunue Bio-Vortex dick paper gae Nanotech'05 Conf.

(18/11) Задача: написать и защитить Убали Assignment/Declaration no S. patent.

APL coordination, 2nd Dvortex-Dyn. paper no 2-nd paragraph (1 referee no asked),

ногрозобин и нечетно the paper to ArXiv - CondMatt database.

Vortex dynamics → Belega & Krut'ev in Dvortex dynamics: обобщение $C_1, C_2 = +1$ и $P_1, P_2 = +1$ case
 core reversal, obnory caseb za Bpeme cragene $H \times(t)$ field
 or 100 de go 0.

Belega & Baner: APL-D. paper, grant F/SC & Vinokur - не заупреж. B exp.,

DV-dynamics F/N/F, vort. dynamics in ellipt. dot (Pierre Currier),

Расска ной Vortex - Mass paper. **Vortex mass / Topologic field**

- **Ного** :
- ① обобщ. total. field, unimpro, moment unimpro; показав
 орбит. total. field or dipolar field
 - ② показав, no side-charges-free model also $\approx$ полемие $\gamma p. A-A$.
 или констант "до полемие."
 - ③ показав, no tensor. wave + v. mass gap splitting $\omega \pm m$ - eigenfrequencies

$$2D \text{ vortex } M \ddot{\vec{X}} - \vec{G} \times \dot{\vec{X}} + \kappa \vec{X} = 0$$

$$2D \text{ згроз. вектор } \vec{G} \text{ в м. поле } m \ddot{\vec{X}} - \frac{e}{c} (\vec{V} \times \vec{H}) = -\frac{\partial W}{\partial \vec{X}}, \quad \frac{d}{dt} \left[m \dot{\vec{X}} - \frac{e}{c} (\vec{X} \times \vec{H}) \right] = -\frac{\partial W}{\partial \dot{\vec{X}}}$$

$\vec{G} = -\frac{e}{c} \vec{H}$ $\frac{1}{c} \vec{H}$

См. Шела-04. Ного иносочн инфобектор.

$$y \text{ Shela-04 gyrovector } \vec{G}_s = Q \frac{2\pi \hbar}{a^2} \hat{z} \quad \text{и} \quad Q = \frac{1}{2} q p, \quad \vec{L} = \frac{S}{a^2} \int d^2 r (1 - \cos \theta) \hat{e} - E(\theta, \varphi)$$

$$\text{Показав } \frac{\hbar S}{a^2} \rightarrow \frac{M_s L}{\delta}$$

$$\text{норм. } \hat{Q} = \frac{q}{\pi} \sin \theta_0 \frac{d\theta_0}{d\varphi}, \quad Q = \frac{1}{4\pi} \int d^2 r \hat{Q}(r) = \frac{q}{4\pi} 2\pi \int_0^{\pi} \frac{1}{\rho} \sin \theta_0 \frac{d\theta_0}{d\varphi} = \frac{1}{2} q (-1) \cos \theta_0(r) \Big|_0^{\pi} =$$

$$= \frac{1}{2} q (-1) [\cos \frac{\pi}{2} - \cos 0] = \frac{1}{2} q p, \quad \cos \theta_0(0) = p$$

$$\vec{A}(p) = -\frac{q}{\pi} \cos \theta_0 \hat{\varphi}, \quad \vec{B} = \text{rot} \vec{A} = \begin{vmatrix} \hat{\rho} & \hat{\varphi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{1}{\rho} \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ 0 & A_\varphi & 0 \end{vmatrix} = \hat{z} \frac{\partial A_\varphi}{\partial \rho} = \hat{z} (-q) \frac{\partial}{\partial \rho} \left(\frac{\cos \theta_0}{\rho} \right) = -q \hat{z} \left[-\frac{1}{\rho^2} \cos \theta_0 - \frac{\sin \theta_0}{\rho} \frac{d\theta_0}{d\rho} \right]$$

$$\neq \vec{B} = \hat{z} \frac{q \sin \theta_0}{\rho} \frac{d\theta_0}{d\rho}$$

$\vec{A}(p)$ - константн и $\vec{B} = \text{rot} \vec{A}$ не эквивалент $\vec{B} = \hat{Q} \hat{z}$.

$$\text{Может } \vec{A} = -\frac{q}{\pi} (1 - \cos \theta_0) \hat{\varphi}, \quad \vec{B} = \hat{z} \frac{\partial A_\varphi}{\partial \rho} = \hat{z} (-q) \left[-\frac{1}{\rho^2} (1 - \cos \theta_0) + \frac{1}{\rho} \sin \theta_0 \frac{d\theta_0}{d\rho} \right] \neq \hat{Q} \hat{z}$$

Komineas-96: gyrovector density $\gamma_i = -\frac{1}{2} \epsilon_{ijk} (\partial_j \vec{m} \times \partial_k \vec{m}) \cdot \vec{m}$

$$\text{div } \vec{\gamma} = 0 \Rightarrow \vec{\gamma} = \text{rot} \vec{a}, \quad \vec{\gamma} = \nabla \Pi \times \nabla \Phi, \quad \Pi = \cos \theta$$

Вектор-потенциал $\vec{a}$ имеет форму $\vec{a} = \Pi \nabla \Phi$.

$$\vec{\gamma} = \text{rot} \vec{a} = \text{rot} (\Pi \nabla \Phi) = \text{rot} (\nabla \Phi) \cdot \Pi + \nabla \Pi \times \nabla \Phi = \nabla \Pi \times \nabla \Phi$$

$$\text{Для vortex } \Phi = q\varphi + \Phi_0, \quad \nabla \Phi = \left(\frac{\partial}{\partial \rho}, \frac{1}{\rho} \frac{\partial}{\partial \varphi} \right) \Phi = \frac{1}{\rho} \frac{\partial \Phi}{\partial \varphi} \hat{\varphi} = \frac{q}{\rho} \hat{\varphi}$$

Поэтому $\vec{a} = \cos \theta \cdot \frac{q}{\rho} \hat{\varphi}$, но невопрос Shela'04 с тем, go знае
 Я использовал неверное свр. $\text{rot} \vec{A}$ в числ. координатах.

$\text{Haga } \vec{B} = \text{rot } \vec{A} = \frac{1}{r} \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & A_\theta & A_\phi \end{vmatrix} = \frac{1}{r} \frac{\partial}{\partial r} (r A_\phi) \hat{\theta} = \frac{1}{r} \frac{\partial}{\partial r} (r A_\phi) \hat{\theta}$
 $\text{div } \vec{F} = \frac{1}{r} \frac{\partial}{\partial r} (r F_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta F_\theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (F_\phi \sin \theta)$

T.e. como lo dice Komineas'96: $\vec{A}(r) = -\frac{q}{r} \cos \theta_0 \hat{\phi}$ u $A_\phi(r)$ converge a 0 como $r \rightarrow 0$.

$B_z = \frac{q}{r} \sin \theta_0 \frac{d\theta_0}{dr}$

 Tangente como $\text{rot}(\text{grad } \Phi) = 0$
 $\text{div}(\text{rot } \vec{F}) = 0$

T-na Círculo $\oint_C \vec{A} d\vec{l} = \int_V d^3r (\text{rot } \vec{A})_z = \int_V d^3r B_z \neq 0$

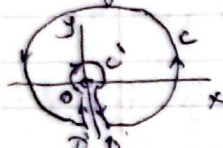


$\oint_C \vec{A} d\vec{l} = \oint_{\phi=0}^{\phi=2\pi} A_\phi r d\phi = r \int_0^{2\pi} d\phi \left(-\frac{q}{r}\right) \cos \theta_0 = -2\pi q \cos \theta_0(r)$, como $r \rightarrow 0$

$\Phi = \int d^3r B_z = \int d^3r \int d\phi \frac{q}{r} \sin \theta_0 \frac{d\theta_0}{dr} = 2\pi q \int d\theta_0 \sin \theta_0 = 2\pi q [\cos \theta_0(r)]_{r_2}^0 = 2\pi q [\cos \theta_0(0) - \cos \theta_0(r_2)] = 2\pi q [1 - \cos \theta_0(r_2)]$, como $r_2 \gg b$ - core radius.

Por lo tanto $\Phi = 2\pi q$. De $\oint_C \vec{A} d\vec{l} = -2\pi q \frac{\cos \theta_0(r_1)}{r_1} r_1 = 0$, como $r_1 \gg b$.

W como se ve $\oint_C \vec{A} d\vec{l} \neq \int_V d^3r B_z$ como C - curva $r = r_1 < r_2 \gg b$.



porque $r_1 \rightarrow 0$

$\oint_{C'} \vec{A} d\vec{l} = \oint_C \vec{A} d\vec{l} + \int_C + \int_{C'}$ $\oint_C r d\phi (-\frac{q}{r}) \cos \theta_0 = (-q) 2\pi \cos \theta_0(r)$. $\int_C + \int_{C'} = 0$.
 $\oint_{C'} \vec{A} d\vec{l} = -2\pi q \cos \theta_0(r_1) + 2\pi q \cos \theta_0(r_2) = 2\pi q [\cos \theta_0(r_2) - \cos \theta_0(r_1)]$

T.e. $r \rightarrow 0$, $\theta_0(r) = (1-r)\frac{\pi}{2}$, $\cos \theta_0(r) = r \Rightarrow \oint_{C'} \vec{A} d\vec{l} = 2\pi q$

u como $\oint_{C'} \vec{A} d\vec{l} = \int_{r=r_1}^0 d^3r B_z = \Phi = 2\pi q$

$\rightarrow$ U3 - za simetría $\vec{A}(r)$ como $\oint_C$ no converge a 0 como $r \rightarrow 0$.

Komineas'96 dice: $(\vec{X})_z = \frac{1}{r} \frac{\partial m_z}{\partial r} = \frac{1}{r} \frac{\partial \cos \theta}{\partial r} = -\frac{\sin \theta}{r} \frac{d\theta}{dr}$ u $\vec{A} = \frac{q}{r} \cos \theta \hat{\phi}$.

$\rightarrow$ A. - A. tom 2: $\vec{p} = \frac{e}{c} \vec{v} \times \vec{H}$ que depende de $\vec{H}$ en un uniform mag. field $\vec{H}$.

$\omega = \frac{eH}{mc}$ - frecuencia, $\vec{r}$ vector.

Oscilaciones de $\vec{x}$ en $\vec{H}$: $\ddot{\vec{x}} + \omega_0^2 \vec{x} = \frac{e}{mc} \vec{x} \times \vec{H}$ u $\vec{x}$ vector

condiciones de $\vec{x} \perp \vec{H}$: $\omega_{\pm} = \sqrt{\omega_0^2 + \frac{1}{4} \left(\frac{eH}{mc}\right)^2} \pm \frac{eH}{2mc}$, $\omega_{\pm} = \omega_0 \pm \frac{eH}{2mc}$ u $\vec{x}$ vector.

Note $B_z(r)$ es $\vec{B}$ -perpendicular, $\vec{A}(r)$ no depende de r , no $\vec{A} \sim \vec{H}$ uniform vector.

$\vec{H} = -\frac{c}{e} \vec{G} = -\frac{c}{e} \hat{z} (-1) \frac{M_0 L}{r} 2\pi q r = \frac{c}{e} \frac{M_0 L}{r} 2\pi q r \hat{z}$

¿Es en $\vec{H}$ mismo campo que $\vec{B} = \text{rot } \vec{A}$? $\vec{B}_z = \frac{1}{\pi b^2} \int d^3r B_z = \frac{1}{\pi b^2} 2\pi q r$

Hay que ver si $\vec{H}$ coincide con $\vec{B}$ como se muestra, e - vector. ¿Por qué?

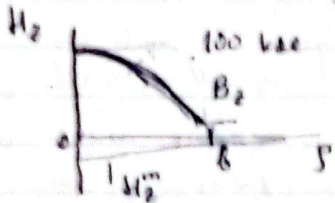
$$\gamma = \frac{q}{2mc}, \text{ where } q = e, \text{ } \Phi_0 = \frac{hc}{2e} = 2 \cdot 10^{-7} \text{ G} \cdot \text{cm}^2.$$

Distance between vortices $\bar{B}_2 = \frac{\Phi_0}{\pi a^2}$, $\Phi_0 = \frac{\pi \hbar c}{e} \psi(0)$ and $\Phi_0 = 2\pi \left(\frac{\hbar c}{e}\right)$

$\Phi_0 = 2\pi$ in SI units, $\hbar = c = e = 1$.

$$b = (0.6-1) R_0 \approx 10-20 \text{ nm}, \quad \bar{B}_2 = \frac{4 \cdot 10^{-7} \text{ G} \cdot \text{cm}^2}{\pi (1.2)^2 \cdot 10^{-12} \text{ cm}^2} \approx 10^5 \text{ G} \approx 100 \text{ kG(1)}$$

Magnetostatic field  $\langle H_2^m(r=0, t) \rangle_2 \sim M_2 \sim 1 \text{ kOe}$.



Topologic field
- магнетостатическое поле $B_2(r)$ вращается и $H_2^m(r)$ магнетостатическое поле вращается
вокруг z-оси в вихре.

$$B_2(r) = \hat{Q} = \frac{q}{r} \sin \theta_0 \frac{d\theta_0}{dr}, \quad \frac{d\theta_0}{dr} = -\frac{q}{r^2} \sin \theta_0 \text{ from Uskov's ansatz } \sin \theta_0 = \frac{2br}{b^2 + r^2}$$

$$B_2(r) = -\left(\frac{d\theta_0}{dr}\right)^2 = -\frac{q^2}{r^2} \sin^2 \theta_0 = -\frac{q^2}{r^2} \frac{4b^2 r^2}{(b^2 + r^2)^2} = -q^2 \frac{4b^2}{(b^2 + r^2)^2}, \quad r \leq b, \quad q=1$$

$$B_2(0) = -\frac{4}{b^2}, \quad B_2(r=b) = -\frac{1}{b^2}, \quad B_2(r) = -q^2 \frac{4b^2}{(b^2 + r^2)^2}$$

$$\int_{r \leq b} d^2 r B_2(r) = 2\pi \Phi_0$$

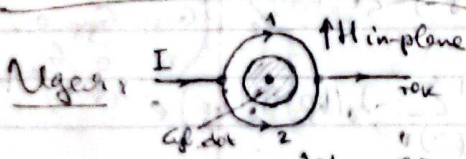
$$B_2(r) = B_2(0) \frac{b^4}{(b^2 + r^2)^2}, \quad \int d^2 r B_2 = B_2(0) b^4 \int_0^b dr \int_0^{2\pi} d\phi \frac{1}{(b^2 + r^2)^2} =$$

$$= B_2(0) 2\pi b^4 \frac{1}{2} \int_0^b \frac{1}{(b^2 + t)^2} dt = B_2(0) 2\pi b^4 \frac{1}{2} (-1) \frac{1}{(b^2 + t)} \Big|_0^b = B_2(0) \pi b^4 \left[\frac{1}{b^2} - \frac{1}{2b^2} \right] =$$

$$= B_2(0) \frac{1}{2} \pi b^2 = 2\pi \Phi_0 = \Phi_0. \quad \left(B_2(0) = \frac{2}{\pi b^2} \Phi_0, \quad B_2(r) = B_2(0) \frac{b^4}{(b^2 + r^2)^2} \right)$$

магнетостатическое поле в вихре в угл. дт

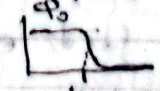
$$\bar{B}_2 = \frac{\Phi_0}{\pi b^2}$$

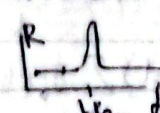


In-plane field H вращается в. осе z дт.

when core is dot $\Phi = \Phi_0$, interference pattern is circular

when core is line $\Phi \neq 0$ - interference pattern is not

in the moment of annihilation Φ  Φ passes to Φ_0 so 0 is

interference pattern is not  - interference pattern is not

$$\text{At } H_2 \text{ core } H_2^m \text{ appears not, i.e., } \Delta \Phi \sim H_2^m \cdot \pi b^2 \sim 10^3 \text{ G} \cdot 3 \cdot 10^{-12} \text{ cm}^2 \sim 10^{-9} \text{ G} \cdot \text{cm}^2 \sim 0.01 \Phi_0$$

$\Delta \Phi \sim 0.01 \Phi_0$ - increase flux in vortex core.

T.e. $R(H)$ abs. energy level - ionization core $B_2 \sim 2$ (non-zero charge).
H20: radius dot $D = 2R \leq l_e$ - length of electron.

(21/11) Group meeting. Sam spoke about the position on the 11th.

holographic method (Seagate patent) is over for now.

Next vortex experiment
 Holographic Bose condensates in Ah-Born plasma
 of vortex state in anode Holog. in experiment:
 1) holographic vortex in R-configuration. ϕ is given in $\mathbb{R}^2$
 2) non-vortex TM motion of H_2 , a holographic, two H_2
 dyes under microscope in a vortex TM.

Vortex TM - $f(\omega)$
 The vortex harmonic $\hat{x}(\omega) = \frac{\langle \vec{m} \rangle}{\hbar}$ is the translation mode of disk.
 - large shape $x''(\omega)$ gives damping "d" in $\mathbb{R}^2$ and $d = d(L, R)$.
 $d(L, R)$ is the distance between the two magnetic domains
 in the L, R plane, and the two-magnon Bragg frequency.
 Oleg's paper by Komineas-PBB: gradient of cylind. domain
 in small cylindrical dot. Gradient is $K_1/2\pi M_s^2 \geq 2$, i.e. the left dots.

Vortex dynamics: Mass

(22/11) Data. Zeros of the vortex mass. The vortex is static (rigid) vortex, $\vec{m} \propto \vec{x}$. $\frac{\partial m_z}{\partial x} = \frac{\partial \Phi}{\partial x}$ is given by mass M .

Masses. PT symmetry $G_{\alpha\beta}(\vec{r}, \vec{r}') \rightarrow \Gamma_{\alpha\beta}(\vec{r}, \vec{r}') = -(\nabla_{\vec{r}})_\alpha (\nabla_{\vec{r}'})_\beta \frac{1}{|\vec{r} - \vec{r}'|}$
 Due to the symmetry from static vortex $\theta = \theta_0(\varphi) + \psi$, $\mu_\beta = \mu$ SW

$$\vec{\frac{M}{M_s}} = \mu \hat{\rho} + (\sin \theta_0 + \psi \cos \theta_0) \hat{\varphi} + (\cos \theta_0 - \psi \sin \theta_0) \hat{z}$$

Static vortex $\vec{\frac{M_0}{M_s}} = \sin \theta_0 \hat{\varphi} + \cos \theta_0 \hat{z}$, dyn. part $\vec{\frac{m}{M_s}} = \mu \hat{\rho} + \psi \cos \theta_0 \hat{\varphi} - \psi \sin \theta_0 \hat{z}$

Dyn. $\vec{m} (m_\rho, m_\varphi, m_z)$ - see components.
 $G_{\alpha\beta}(\vec{r}, \vec{r}') = \langle \Gamma_{\alpha\beta}(\vec{r}, \vec{r}') \rangle_z = \begin{pmatrix} G_{\rho\rho} & G_{\rho\varphi} & 0 \\ G_{\varphi\rho} & G_{\varphi\varphi} & 0 \\ 0 & 0 & G_{zz} \end{pmatrix}$
 $\sim \frac{1}{|\vec{r} - \vec{r}'|}$, $G_{zz} = 0$.

$$\frac{1}{|\vec{r} - \vec{r}'|} = \int d\vec{k} e^{-k|\vec{r} - \vec{r}'|} J_0(k|\vec{r} - \vec{r}'|) = \int d\vec{k} e^{-k|\vec{r} - \vec{r}'|} \sum_{(\vec{k})} J_0(k\rho) J_0(k\rho') e^{i\psi(\varphi - \varphi')}$$

Due to $d_\beta = \rho_\beta \varphi$: $G_{\alpha\beta}(\vec{r}, \vec{r}') = -(\nabla_{\vec{r}})_\alpha (\nabla_{\vec{r}'})_\beta \langle \frac{1}{|\vec{r} - \vec{r}'|} \rangle_z$, $\langle \dots \rangle_z = \frac{1}{L} \int_0^L dz dz' (\dots)$

$$\langle e^{-k|\vec{r} - \vec{r}'|} \rangle_z = \frac{1}{L} \int dz dz' e^{-k|\vec{r} - \vec{r}'|} = \frac{2}{k} f(kL), \quad f(x) = 1 - \frac{1}{x}(1 - e^{-x})$$

$$G_{\alpha\beta}(\vec{r}, \vec{r}') = -(\nabla_{\vec{r}})_\alpha (\nabla_{\vec{r}'})_\beta \int d\vec{k} \frac{2}{k} f(kL) \sum_{\vec{k}} J_0(k\rho) J_0(k\rho') e^{i\psi(\varphi - \varphi')}$$

$\vec{\tau} = (\frac{\partial}{\partial \rho}, \frac{1}{\rho} \frac{\partial}{\partial \varphi}, \frac{\partial}{\partial z})$, $h_\alpha(\vec{r}) = \int d\vec{r}' G_{\alpha\beta}(\vec{r}, \vec{r}') m_\beta(\vec{r}') = \text{gen. coord.}$

By Oleg's Ivanov: $\begin{cases} \psi(\vec{r}, t) = f(\varphi) \cos(m\varphi - \omega t) \\ \mu(\vec{r}, t) = g(\varphi) \sin(m\varphi - \omega t) \end{cases}$ Oleg's unreplicated transl. mode $c = m = \pm 1$.

$$m_p = \mu, \quad m_\varphi = \cos\theta_0 \mu, \quad m_z = -\sin\theta_0 \mu. \quad \theta_0 = \theta_0(\varphi)$$

T. e. Azimutal. Dann $m(\vec{r}, \varphi, t) \sim \cos(m\varphi - \omega t)$ u. $m \sim \sin(m\varphi - \omega t)$.

$$G_{pp}(\vec{r}\vec{r}') = -\frac{\partial^2}{\partial p \partial p'} \int dk \frac{1}{k} f(kL) \sum_v J_0(kp) J_0(kp') e^{iv(\varphi - \varphi')}$$

$$G_{p\varphi}(\vec{r}\vec{r}') = -\frac{\partial}{\partial p} \frac{\partial}{\partial p'} \int dk \frac{1}{k} f(kL) \sum_v v J_0(kp) J_0(kp') e^{iv(\varphi - \varphi')}$$

$$G_{\varphi p}(\vec{r}\vec{r}') = -\frac{i}{p} \frac{\partial}{\partial p} \int dk \frac{1}{k} f(kL) \sum_v v J_0(kp) J_0(kp') e^{iv(\varphi - \varphi')}$$

$$G_{\varphi\varphi}(\vec{r}\vec{r}') = -\frac{1}{pp'} \int dk \frac{1}{k} f(kL) \sum_v v^2 J_0(kp) J_0(kp') e^{iv(\varphi - \varphi')}$$

$$I_c^{vm}(\varphi) = \int_0^{2\pi} d\varphi' e^{iv(\varphi - \varphi')} \cos(m\varphi' - \omega t), \quad I_s^{vm}(\varphi, t) = \int_0^{2\pi} d\varphi' e^{iv(\varphi - \varphi')} \sin(m\varphi' - \omega t)$$

$$I_c^{vm} = e^{iv\varphi} \int d\varphi' e^{-iv\varphi'} \frac{1}{2} \left[e^{i(m\varphi' - \omega t)} + e^{-i(m\varphi' - \omega t)} \right] = \frac{1}{2} e^{iv\varphi} \int_0^{2\pi} d\varphi' \left[e^{-i\omega t} \delta_{v,m} + e^{i\omega t} \delta_{v,-m} \right]$$

$$I_s^{vm} = e^{iv\varphi} \frac{1}{2i} \int d\varphi' e^{-iv\varphi'} \left[e^{i(m\varphi' - \omega t)} - e^{-i(m\varphi' - \omega t)} \right] = \frac{\pi}{i} e^{iv\varphi} \left[e^{-i\omega t} \delta_{v,m} - e^{i\omega t} \delta_{v,-m} \right]$$

$$\begin{cases} h_p(\vec{r}, t) = \int d^2\vec{r}' \left[G_{pp}(\vec{r}\vec{r}') m_p(\vec{r}') + G_{p\varphi} m_\varphi(\vec{r}') \right], & m_r(\vec{r}, t), \quad G_{pz} = 0 \\ h_\varphi(\vec{r}, t) = \int d^2\vec{r}' \left[G_{\varphi p}(\vec{r}\vec{r}') m_p(\vec{r}') + G_{\varphi\varphi} m_\varphi(\vec{r}') \right], & h_z(\vec{r}, t) = \int d^2\vec{r}' G_{zz}(\vec{r}\vec{r}') m_z(\vec{r}') \end{cases}$$

• Due unterf. prob, kogerm. $m_\varphi, m_z \sim \ell(\vec{r}, t) \sim \cos(m\varphi - \omega t)$

$$\begin{aligned} \rightarrow \sum_v v^n a_v(\vec{r}\vec{r}') I_c^{vm}(\varphi, t) &= \sum_v v^n a_v \pi e^{iv\varphi} \left[e^{-i\omega t} \delta_{v,m} + e^{i\omega t} \delta_{v,-m} \right] = \\ &= \pi \left[e^{-i\omega t} m^n a_m e^{im\varphi} + e^{i\omega t} (-1)^n m^n e^{-im\varphi} a_{-m} \right] = \left| a_m = J_m J_m, a_{-m} = a_m \right| = \\ &= \pi m^n a_m \left[e^{i(m\varphi - \omega t)} + (-1)^n e^{-i(m\varphi - \omega t)} \right] = 2\pi m^n a_m \times \begin{cases} \cos(m\varphi - \omega t), & n=0,2 \\ i \sin(m\varphi - \omega t), & n=1 \end{cases} \end{aligned}$$

• Due unterf. prob, kogerm. $m_p = \mu(\vec{r}, t) \sim \sin(m\varphi - \omega t)$

$$\begin{aligned} \rightarrow \sum_v v^n a_v I_s^{vm} &= \frac{\pi}{i} \sum_v v^n a_v e^{iv\varphi} \left[e^{-i\omega t} \delta_{v,m} - e^{i\omega t} \delta_{v,-m} \right] = \frac{\pi}{i} \left[m^n a_m e^{im\varphi} e^{-i\omega t} - \right. \\ &\left. - (-1)^n m^n a_{-m} e^{-im\varphi} e^{i\omega t} \right] = \frac{\pi}{i} m^n a_m \left[e^{i(m\varphi - \omega t)} - (-1)^n e^{-i(m\varphi - \omega t)} \right] = \frac{2\pi}{i} m^n a_m \times \begin{cases} \sin(m\varphi - \omega t), & n=1 \\ i \cos(m\varphi - \omega t), & n=2 \end{cases} \end{aligned}$$

• Due $G_{pp}, G_{\varphi\varphi}, G_{zz}$ $a_m = a_{-m}$, $a_m = J_m(kp) J_m(kp')$ ergebnis.

• Da $G_{\varphi p}, G_{p\varphi}$: $a_v = \frac{\partial J_v(kp)}{\partial p} J_v(kp')$, $J_v'(x) = \frac{1}{2} [J_{v-1} - J_{v+1}]$

$$J_{-v}' = \frac{1}{2} [J_{-v-1} - J_{-v+1}] = \frac{1}{2} [-J_{v+1} + J_{v-1}] = J_v' \quad \text{weil } J_v = (-1)^v J_{-v}$$

$$\text{Nun } J_{-v}' = \frac{1}{2} [(-1)^{v+1} J_{v+1} - (-1)^{v-1} J_{v-1}] = \frac{1}{2} (-1)^v [-J_{v+1} + J_{v-1}] = (-1)^v J_v' \quad \text{weil } J_v = (-1)^v J_{-v}.$$

$$d\vec{p}=z: \quad \Gamma_{zz}(r, r') = -\frac{\partial}{\partial z} \frac{\partial}{\partial z'} \frac{1}{|r-r'|} \quad \text{u.} \quad G_{zz}(\vec{r}, \vec{r}') = \frac{1}{L} \int_0^L \int_0^L dz dz' (-1) \frac{\partial}{\partial z} \frac{\partial}{\partial z'} \frac{1}{|r-r'|} = -\frac{1}{L} \frac{1}{|r-r'|} \Big|_0^L \Big|_0^L$$

• Ungehe v^n $n=0$ geht G_{pp}, G_{zz} , $n=1$ geht $G_{p\varphi}, G_{\varphi p}$ u. $n=2$ geht $G_{\varphi\varphi}$.

F/A Vortex Dzyaloshinskii & Axel AF/F vortex. One nomination paper to Nature Mat, it was rejected. Zagaria:

- ① observ. gr. fluctuations $H_{lin}(\theta)$ gr. based vortex
- ② observ. wave $H_{lin}(\theta)$ curren. v. nucleation mode at "C" & "S"

observ. Axel F/N/F trans. modes gr. $C_{12}=-1, P_{12}=+1$ & nongr. Frank-data single layer disk.

Vortex mass

Чрез формулу v. mass надо вывести неправильно & "rigid" vortex $\sim \vec{x}$.

- ① Slonczewski JMMM-79 номинал упростили, но v. mass не ввели, ббес $\vec{v}$ (средний магнит DW).
- ② Niforov-Sonin - JETP'83 ббес разделение $m = m_0 + m_1$, $\Phi = \Phi_0 + \Phi_1$, $(m = m_2)$ ^{static} замечая сущее что неправо m_1, Φ_1 и пишем ee
 что 2D e. plane magnet что $\theta_0(z), z \rightarrow 0$ & $z \rightarrow \infty$. Maccy vortex не ввожу
 $m_1, \Phi_1 \sim \vec{v}$
- ③ Slonczewski 85 ббес $\Phi = \Phi_v + \psi$, $\cos \theta = \cos \theta_0 + \beta(z)$, ^(rigid v.) рассуждая $\psi(r, t), \beta(r, z)$ как SW reference, таме $\psi, \beta \sim \vec{v}$ ббес vortex mass, same mass-tensor for coupled vortices

V. mass gauge given $\cos\theta = \frac{m_z(\vec{r}-\vec{X})}{r} + p(r)$, $p(r) \sim \vec{r}$
 rigid rot.

$$m_z(\vec{r}-\vec{X}) = m_z(r) - \vec{X} \cdot \nabla_r m_z(r)$$

T. e. V. mass no Slonczewski superconductor approximation real $m_z(\vec{r}, t)$ or (moving vortex) $m_z(\vec{r}-\vec{X})$ of moving rigid vortex. I approximation, no be vortex emergence as gaugeless. Phase in phase $-\vec{X} \cdot \nabla_r m_z$

4) Goulet-PRB'89 approximation phaseless Nijmegen-Sonin $m = m_0 + m_1$ $\Phi = \Phi_0 + \Phi_1$
 give 2D XXZ-model u given phase go $E \propto \delta^2$ zeros, i.e. no vortex v. mass, no also as no vortex.

5) Ivanov-Steph. PL-89 u.s. $\theta(\vec{r}) = \theta_0(|\vec{r}-\vec{r}(t)|) + \ell(\vec{r}, t)$ $\Phi = \omega t + \Phi_0(|\vec{r}-\vec{r}(t)|) + \psi(\vec{r}, t)$ $\left. \begin{array}{l} \text{f. b.} \\ \text{superconductor v. mass, current } \vec{j} \text{ is only 2D ferromagnet, u x as seen in } \end{array} \right\}$
 approximation vortex, u x as seen in approximation vortex.

6) Wyllie (book'94) no vortex v. mass give 2D XXZ-model u (PRB-96) approximation as u.s. ω_m .

Due phase in SV mag. vortex ground state u u.s. $\theta = \theta_0 + \ell$, μ_p .

$\Phi = \arctg(m_y/m_x)$ no vortex state u.s. μ_p .

$\cos\theta = \cos(\theta_0 + \ell) = \cos\theta_0 - \sin\theta_0 \cdot \ell = m_0(r) + m_1$, no $m_0(r)$ gives static vortex, a no. q.m. current given state shifted rigid vortex.

V. mass no vortex u.s. $\left(\frac{\partial m_z}{\partial \vec{v}}\right)_{\vec{X}, \vec{v}=0}$, $\left(\frac{\partial \Phi}{\partial \vec{v}}\right)_{\vec{X}=0, \vec{v}=0}$, i.e. no vortex

$$m_z(\vec{v}) = m_z(0) + \vec{A} \cdot \vec{v} \quad \Phi(\vec{v}) = \Phi(0) + \vec{B} \cdot \vec{v}, \quad \text{components } \vec{A} \text{ u } \vec{B}.$$

In no vortex dot $\vec{v}=0$ u.s. $\vec{X}=0$, i.e. static vortex is always centered, no no equilibrium position.

$$m_z = \cos\theta, \quad m_x + im_y = \sin\theta e^{i\Phi}$$

T. e. u.s. (PT) vortex, u.s. u.s. phase u.s. (m_z, Φ) , a s (m_p, m_d, m_z) , Bege PT u.s. u.s. G_{dp} , $dp = g, q, z$.

$$\vec{m} = \vec{m}_0 + \delta\vec{m}, \quad \delta m_z = -\sin\theta_0 \cdot \ell, \quad \delta m_p = \cos\theta_0 \cdot \ell, \quad \delta m_p = \mu_p = -\sin\theta_0 \cdot \psi$$

$$\text{u.s. } \theta = \theta_0 + \ell, \quad \Phi = \Phi_0 + \psi, \quad m_z = m_z^0 + m_1, \quad m_1 = -\sin\theta_0 \cdot \ell$$

30/11/2001. I u.s. u.s. radial modes $\ell = \ell(r)$, $\psi = \psi(r) \rightarrow$ PRB'05.

Com. 16/11/2002. static dipolar vortex field, u.s. $\vec{H}_m^0(r) = (0, 0, H_m^z(r))$ - только $H_m^z(r)$ $\rightarrow$ $H_m^z(r)$

$$H_m^z(r) = \int_0^R d\rho' \rho' g_{zz}(\rho, \rho') M_z^0(\rho'), \quad g_{zz}(\rho, \rho') = -\frac{4\pi}{L} \int_0^L dk [1 - e^{-kL}] J_0(k\rho) J_0(k\rho')$$

Static $H_m^p(r, t)$ - component vector no 2 u uuzogaz vpr $L \rightarrow \infty$
 Vremoz. def. que $\Phi \Gamma \hat{\Gamma}(r, t)$, $\hat{G}(p, p') = \frac{1}{L} \int d\vec{r} d\vec{r}' \hat{\Gamma}(r, r')$, $\hat{g}(p, p') = \int d\vec{r} \hat{G}(p, p')$

Assumption: number 1.-1. eq. que $\frac{d\theta}{dt}$ u $\frac{d\phi}{dt}$ u no. nareem

(*) $\theta(\vec{r}, t) = \theta_0(\vec{r} - \vec{X}) + \ell(\vec{r}, t)$, $\phi(\vec{r}, t) = \omega t + \phi_0(\vec{r} - \vec{X}) + \psi(\vec{r}, t)$, $\vec{X} = \vec{X}(t)$, $\vec{r}$ core uenpr.

$$\frac{d\theta}{dt} = \nabla_r \theta(\vec{r}) \cdot \frac{d(\vec{r} - \vec{X})}{dt} + \frac{d\ell}{dt} = -\nabla \theta \cdot \dot{\vec{X}} + \frac{d\ell}{dt}, \quad \frac{d\phi}{dt} = \omega - \nabla \phi_0 \cdot \dot{\vec{X}} + \frac{d\psi}{dt} = \text{res.}$$

Formee uuen $\theta(\vec{r}, t) = \theta[\vec{r} - \vec{X}(t)]$, $\phi(\vec{r}, t) = \phi[\vec{r} - \vec{X}(t)]$

$\frac{d\theta}{dt} = -\nabla_r \theta \cdot \dot{\vec{X}}$, $\frac{d\phi}{dt} = -\nabla_r \phi \cdot \dot{\vec{X}} + \frac{\partial \phi}{\partial t}$, $\text{Zarem nogesabu. (*) u c torn.}$

go uuenpr no $\dot{\vec{X}}$, ℓ , ψ uenob: $\frac{d\theta}{dt} = -\nabla_r \theta_0(\vec{r}) \cdot \dot{\vec{X}}$, $\frac{d\phi}{dt} = -\nabla_r \phi_0(\vec{r}) \cdot \dot{\vec{X}} + \omega$

UD $\frac{M_s}{\delta} \frac{d\phi}{dt} = \frac{\delta \mathcal{H}}{\delta m}$, $\frac{M_s}{\delta} \frac{dm}{dt} = -\frac{\delta \mathcal{H}}{\delta \phi}$, $m = \cos \theta$. $\frac{\delta \mathcal{H}}{\delta m}, \frac{\delta \mathcal{H}}{\delta \phi}$ uuenpruzem $\ell \ll 1, \psi \ll 1$
 $\subset \theta = \theta_0 + \ell, \phi = \omega t + \phi_0 + \psi$.

Uuenpruzem uuenpruzem: $\left\{ \begin{array}{l} \frac{M_s}{\delta} [-\nabla_r \phi_0 \cdot \dot{\vec{X}} + \omega] = \frac{\partial (\delta \mathcal{H})}{\partial \phi} \ell + \frac{\partial (\delta \mathcal{H})}{\partial \psi} \psi \\ \frac{M_s}{\delta} (-\sin \theta) (-\nabla_r \theta_0 \cdot \dot{\vec{X}}) = \text{uuenpruzem } (\ell, \psi) \end{array} \right\}$

$\frac{\delta \mathcal{H}}{\delta m} = -\frac{1}{\sin \theta} \frac{\delta \mathcal{H}}{\delta \theta}$, $\frac{M_s}{\delta} (-\sin \theta) \frac{d\phi}{dt} = \frac{\delta \mathcal{H}}{\delta \theta}$, $\sin \theta \frac{d\phi}{dt} = \sin(\theta_0 + \ell) [-\nabla \phi_0 \cdot \dot{\vec{X}} + \omega] =$
 $= [\sin \theta_0 + \cos \theta_0 \cdot \ell] [\omega - \nabla \phi_0 \cdot \dot{\vec{X}}] = \omega \sin \theta_0 + \omega \cos \theta_0 \cdot \ell - \sin \theta_0 \nabla \phi_0 \cdot \dot{\vec{X}}$
 (comparative)

uuenpruzem uuenpruzem $\left\{ \begin{array}{l} a \ell + b \psi = \vec{e} \cdot \dot{\vec{X}} \\ c \ell + d \psi = \vec{f} \cdot \dot{\vec{X}} \end{array} \right\}$, $\text{uuenpruzem } (\ell \sim \dot{\vec{X}}, \psi \sim \dot{\vec{X}})$.

Note: b uen. no $\dot{\vec{X}}, \dot{\vec{X}}, \ell, \psi$ uuenpruzem gozazemo uuenpruzem $\theta_0(\vec{r}), \phi_0(\vec{r})$.
 $\theta_0(\vec{r} - \vec{X}) \cdot \dot{\vec{X}} = [\theta_0(\vec{r}) - \nabla_r \theta_0(\vec{r}) \cdot \vec{X}] \dot{\vec{X}} \approx \theta_0(\vec{r}) \cdot \dot{\vec{X}}$

uuenpruzem (ℓ) uuenpruzem no $\dot{\vec{X}}, \dot{\vec{X}} \rightarrow$ UD uuenpruzem no $\dot{\vec{X}}, \dot{\vec{X}}$.

25/11 Zafersop. na APS-05 meeting. uuenpruzem e-mail b PRC o tax return interest. Vortex mass / UD

(L. 30/10/01) $\vec{p} \cdot \vec{p} = 1$. $-\frac{m d\vec{m}}{\delta dt} = \vec{m} \times \vec{H}_{ef}$, $\text{quay } \vec{p} \cdot \vec{p} \text{ uuenpruzem, o.v. } \vec{G}_{\vec{p}} \text{ b uuenpruzem.}$

uuenpruzem $\vec{m} = \vec{m}_0 + \vec{\mu}$. $\vec{H}_{ef} = \vec{H}_{ex} + \vec{H}_m = \vec{H}_{ex}^0 + \vec{H}_m^0 + \vec{h}_{ex} + \vec{h}_m$.

$\vec{m} \times \vec{H}_{ef} = (\vec{m}_0 + \vec{\mu}) \times (\vec{H}_{ex}^0 + \vec{H}_m^0 + \vec{h}_{ex} + \vec{h}_m) = \vec{\mu} \times (\vec{H}_{ex}^0 + \vec{H}_m^0) + \vec{m}_0 \times (\vec{h}_{ex} + \vec{h}_m)$

$\rightarrow \vec{\mu} = \delta \vec{m} = (\mu_x, \mu_y, \mu_z) = (-\sin \theta_0 \psi, \cos \theta_0 \ell, -\sin \theta_0 \ell)$

$H_m^p(\vec{p}, \vec{p}) = \int d^3 \vec{p}' [G_{pp} m_p + G_{p\psi} m_\psi]$, $H_m^p(\vec{p}) = \int d^3 \vec{p}' [G_{p\psi} m_p + G_{p\psi} m_\psi]$, $H_m^z(\vec{p}) = \int d^3 \vec{p}' G_{zz} m_z$

Ground state vortex under $m_F^0 = 0$, $\vec{m}^0 = (0, m_x^0, m_z^0)$. For vortex $\vec{m}^0$ is in the xy plane.

$$(1) -\frac{M_0}{d} \frac{dm_F}{dt} = m_x H_x^0 - m_z H_z^0 = (exh.) + (m_x^0 / \mu_F) (H_m^{z0} + h_m^z) - (m_z^0 / \mu_F) (H_m^{x0} + h_m^x) =$$

$$= (exh.) + \mu_F H_m^{z0} + m_x^0 h_m^z - \mu_F H_m^{x0} - m_z^0 h_m^x \quad \vec{m}^0 = \{(\rho)\} \text{ rotated, radial sign.}$$

$$(2) \frac{M_0}{d} \frac{dm_F}{dt} = (exh.) + m_x H_m^x - m_z H_m^z = (exh.) + (m_x^0 / \mu_F) (H_m^{x0} + h_m^x) - \mu_F H_m^{z0} = (exh.) +$$

$$+ \mu_F H_m^{x0} + m_x^0 h_m^x - \mu_F H_m^{z0}$$

$$(3) -\frac{M_0}{d} \frac{dm_F}{dt} = (exh.) + \mu_F H_m^x - m_x H_m^z = (exh.) + \mu_F H_m^{x0} - m_x^0 h_m^z - \mu_F H_m^{z0}$$

$H_m^{x0} = \int d\varphi [G_{\varphi x} m_x^0] = 0$, $H_m^{z0} = \int d\varphi [G_{\varphi z} m_z^0] = 0$, $H_m^{z0} \neq 0$ - b. any
polarization. Can state vortex.

$$\left\{ \begin{aligned} -\frac{M_0}{d} \frac{dm_F}{dt} &= (exh.) + H_m^{z0} \mu_F + m_x^0 h_m^z - m_z^0 h_m^x \\ -\frac{M_0}{d} \frac{dm_F}{dt} &= (exh.) + m_x^0 h_m^z - \mu_F H_m^{z0} \end{aligned} \right\} \quad -\frac{1}{d} \frac{dm_F}{dt} = (exh.) - m_x^0 h_m^z$$

$\vec{m} = (m_x, m_y, m_z) = (\sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta)$. Components: $\dot{m}_x m_z^0 - \dot{m}_z m_x^0 = \dot{\varphi}$ (30/10/01)

$$m_x + im_y = \sin\theta e^{i\varphi} = e^{i\varphi} (m_y + im_x), \quad m_y + im_x = \sin\theta e^{i(\varphi-\pi/2)} \rightarrow \begin{cases} m_y = \sin\theta \cos(\varphi-\pi/2) \\ m_x = \sin\theta \sin(\varphi-\pi/2) \end{cases}$$

$$\left\{ \begin{aligned} \dot{m}_y &= \frac{d}{dt} [\sin\theta \cos(\varphi-\pi/2)] = \cos\theta \dot{\theta} \cos(\varphi-\pi/2) - \sin\theta \sin(\varphi-\pi/2) \dot{\varphi} \\ \dot{m}_x &= \frac{d}{dt} [\sin\theta \sin(\varphi-\pi/2)] = \cos\theta \dot{\theta} \sin(\varphi-\pi/2) + \sin\theta \cos(\varphi-\pi/2) \dot{\varphi} \\ \dot{m}_z &= \frac{d}{dt} \cos\theta = -\sin\theta \dot{\theta} \end{aligned} \right. \quad \text{Static } \varphi = \varphi_0 = C \frac{\pi}{2} + \varphi - \text{true vortex } C = \pm 1.$$

$\cos(\varphi_0 - \varphi) = \cos(C \frac{\pi}{2}) = 0$, no gyration $\varphi_0 = 0$.

$$\text{vortex } \dot{m}_y = -\sin\theta \cdot C \dot{\varphi}, \quad \dot{m}_x = \cos\theta \dot{\theta} C, \quad \dot{m}_z = -\sin\theta \dot{\theta}$$

$$C \dot{m}_x \cos\theta - \dot{m}_z \sin\theta = [C^2 \cos^2\theta + \sin^2\theta] \dot{\theta} = \dot{\theta}$$

$$\text{vortex } \dot{m}_y \cos(\varphi-\pi/2) + \dot{m}_x \sin(\varphi-\pi/2) = \cos\theta \dot{\theta}, \quad \dot{m}_y \sin(\varphi-\pi/2) - \dot{m}_x \cos(\varphi-\pi/2) = -\sin\theta \dot{\varphi}$$

- force of vortex. Note, no no $\dot{\theta}$ having any influence.

$$\text{vortex } \left(\frac{M_0}{d} \right) \dot{\varphi} = 1. \quad -\dot{m}_y C \cos\theta - (-\dot{m}_z \sin\theta) = -\dot{\theta} = C \cos\theta [(exh.)_y + m_x^0 h_m^z - \mu_F H_m^{z0}] =$$

$$- \sin\theta [(exh.)_x - m_z^0 h_m^x] = [C \cos\theta (exh.)_y - \sin\theta (exh.)_x] + C (m_z^0)^2 h_m^z - C \cos\theta H_m^{z0} \mu_F +$$

$$+ \sin\theta m_x^0 h_m^x, \quad \vec{m}^0 = (0, \sin\theta_0, \cos\theta_0)$$

$$-\dot{\theta} = [C \cos\theta (exh.)_y - \sin\theta (exh.)_x] + [C \cos^2\theta_0 + \sin^2\theta_0] h_m^z - C \cos\theta H_m^{z0} \mu_F$$

Point C - chirality phenomenon, non-zero work $C = \pm 1$. Chirality appears just after?

$$-\dot{\theta} = [C \cos\theta (exh.)_y - \sin\theta (exh.)_x] + h_m^z - \cos\theta_0 H_m^{z0} \mu_F \quad \text{calc. c. } \vec{m}^0 \text{ or } 30/10/01 \text{ (good)}$$

$$-\dot{\theta} = [exh.]_{\varphi} + M_0 \int d\varphi [G_{\varphi\varphi} \mu_F + G_{\varphi\varphi} \mu_F] - \cos\theta_0 H_m^{z0} \mu_F$$

усп. 10/11 $G_{pp} \mu_p + G_{pe} \mu_e = G_{pp} \mu_p + G_{pe} (\vec{r} \cdot \vec{r}') \cos \theta_0(r') \cdot \ell(\vec{r}')$

(1) $-\dot{\vec{\theta}} = [\text{ex.}]_{\vec{e}_2} + M_s \int d^2 \vec{r}' [G_{pp}(\vec{r} \cdot \vec{r}') \mu_p(\vec{r}') + G_{pe}(\vec{r} \cdot \vec{r}') \cos \theta_0(r') \cdot \ell(\vec{r}')] - \cos \theta_0 H_m^{20} \mu_p(\vec{r})$

Вспомог. уД где $\vec{m}_p, m_p = m_p^0 + \mu_p = \mu_p = -\sin \theta_0 \cdot \psi$. ψ - см. задача к Ф.

$-\dot{m}_p = (\text{exch.})_p + H_m^{20} \mu_p + m_p^0 h_m^2 - m_p^0 h_m^4$

(2) $-\dot{m}_p = (\text{ex.})_p + \cos \theta_0 H_m^{20} \psi(\vec{r}) + \sin \theta_0 \int d^2 \vec{r}' G_{ee}(\vec{r} \cdot \vec{r}') (-\sin \theta_0(r')) \ell(\vec{r}') M_s -$

$-\cos \theta_0 \int d^2 \vec{r}' [G_{ep}(\vec{r} \cdot \vec{r}') \mu_p(\vec{r}') + G_{pe}(\vec{r} \cdot \vec{r}') \cos \theta_0(r') \ell(\vec{r}')] M_s$ - это уД отмен.

в уД $\vec{r}_p$ 30/11 2011. ген. уравнения маг. тока с G_{ep}, G_{pe} , второе = 0 где $\vec{r}_p$ маг. маг.

В маг. осознать, где vortex $\dot{m}_p = -\sin \theta \cdot \dot{\Phi}$.

(Exch.) так же будет влезать 30/11 011. где падающие маг. - вprobe на по paper Ivanov - 98 PRB. Но здесь есть другая задача из $\frac{\partial}{\partial \varphi}, \frac{\partial^2}{\partial \varphi^2}$. Kosevich-90.

См. Kosevich-90 где Isotrope ferromagnet ($\frac{1}{\rho_0} = \frac{\rho}{1} = 0$):

$$\begin{cases} -\frac{1}{8\lambda M_s} \sin \theta \frac{d\Phi}{dt} = \nabla^2 \theta - \frac{1}{\rho^2} \sin \theta \cos \theta, & \lambda = 2A/M_s^2, \quad \Phi_0 = f(\varphi), \quad \Phi_0 = C\frac{\pi}{2} + \varphi \\ \frac{1}{8\lambda M_s} \sin \theta \frac{d\theta}{dt} = \sin^2 \theta \cdot \nabla^2 \Phi + \sin 2\theta (\nabla \theta \cdot \nabla \Phi) \end{cases}$$
 см. 31/11 011. $(\nabla \Phi)^2 = \frac{1}{\rho^2}$

$\nabla \Phi = \nabla [\Phi_0 + \psi] = \hat{r} \frac{\partial \Phi}{\partial r} + \hat{\varphi} \frac{1}{\rho} \frac{\partial \Phi}{\partial \varphi} = \hat{r} \frac{\partial \psi}{\partial r} + \hat{\varphi} \frac{1}{\rho} \frac{\partial \psi}{\partial \varphi} + \hat{\varphi} \frac{1}{\rho} \frac{\partial \Phi_0}{\partial \varphi}$

$\sin^2 \theta (\nabla^2 \Phi) = \sin^2 (\theta_0 + \psi) = [\sin \theta_0 + \cos \theta_0 \psi]^2 \nabla^2 (\Phi_0 + \psi) = [\sin^2 \theta_0 + \sin 2\theta_0 \psi] \nabla^2 (\Phi_0 + \psi) =$
 $= \sin^2 \theta_0 \cdot \nabla^2 \psi + \psi \sin 2\theta_0 \nabla^2 \Phi_0 = \sin^2 \theta_0 \nabla^2 \psi$

$\nabla \theta \cdot \nabla \Phi = (\hat{r} \frac{\partial \theta}{\partial r} + \hat{\varphi} \frac{1}{\rho} \frac{\partial \theta}{\partial \varphi}) \cdot (\hat{r} \frac{\partial \psi}{\partial r} + \hat{\varphi} \frac{1}{\rho} \frac{\partial \psi}{\partial \varphi}) = \frac{\partial \theta}{\partial r} \frac{\partial \psi}{\partial r} + \frac{1}{\rho^2} \frac{\partial \theta}{\partial \varphi} \frac{\partial \psi}{\partial \varphi} = \frac{\partial \theta_0}{\partial r} \frac{\partial \psi}{\partial r} + \frac{1}{\rho^2} \frac{\partial \psi}{\partial \varphi} \frac{\partial \Phi_0}{\partial \varphi} \sim \psi$

$$\begin{cases} -\frac{1}{8\lambda M_s} \sin \theta \dot{\Phi} = \nabla^2 \theta_0 - \frac{1}{\rho^2} \sin \theta_0 \cos \theta_0 + \nabla^2 \psi - \frac{1}{\rho^2} \cos 2\theta_0 \cdot \psi \\ \frac{1}{8\lambda M_s} \sin \theta_0 \dot{\theta} = \sin^2 \theta_0 \cdot \nabla^2 \psi + \sin 2\theta_0 \left(\frac{\partial \theta_0}{\partial r} \frac{\partial \psi}{\partial r} + \frac{1}{\rho^2} \frac{\partial \psi}{\partial \varphi} \right) \end{cases}$$

$$\begin{cases} -\frac{1}{8\lambda M_s} \sin \theta \cdot \dot{\Phi} = \nabla^2 \psi - \frac{1}{\rho^2} \cos 2\theta_0 \cdot \psi \\ \frac{1}{8\lambda M_s} \dot{\theta} = \sin \theta_0 \nabla^2 \psi + 2 \cos \theta_0 \left(\frac{\partial \theta_0}{\partial r} \frac{\partial \psi}{\partial r} + \frac{1}{\rho^2} \frac{\partial \psi}{\partial \varphi} \right) \end{cases}$$

$-\nabla^2 \mu_p = \nabla^2 (\psi \sin \theta_0) = \sin \theta_0 \nabla^2 \psi + 2 \nabla (\sin \theta_0) \nabla \psi + \psi \nabla^2 (\sin \theta_0)$

$\sin \theta_0 \nabla^2 \psi + 2 \cos \theta_0 \nabla \theta_0 \cdot \nabla \psi = -\nabla^2 \mu_p - \psi \nabla^2 (\sin \theta_0) = -\nabla^2 \mu_p + \frac{\mu_p}{\sin \theta_0} \nabla^2 (\sin^2 \theta_0)$

Вprobe, что то же $\Phi, \dot{\theta}$ не имеют к ψ - 2 (8-9) Ivanov-PRB-98.

$\rightarrow \mu_p$ - а задача (8-9) PRB 98 решается к ψ^2

$$\begin{cases} [-\nabla^2 + V_1(r)] \psi + \frac{2 \cos \theta_0}{\rho^2} \frac{\partial \mu}{\partial \varphi} = -\frac{1}{c\tau} \frac{\partial \mu}{\partial t} \\ [-\nabla^2 + V_2(r)] \mu - \frac{2 \cos \theta_0}{\rho^2} \frac{\partial \psi}{\partial \varphi} = \frac{1}{c\tau} \frac{\partial \psi}{\partial t} \end{cases}$$
 $V_1(r) = \left(\frac{1}{\rho^2} - \frac{1}{\tau^2} \right) \cos 2\theta_0$
 $V_2(r) = \left(\frac{1}{\rho^2} - \frac{1}{\tau^2} \right) \cos^2 \theta_0 - \left(\frac{d\theta_0}{dr} \right)^2$

$$C_{2V} = \frac{A_0}{2} \sqrt{\frac{\lambda}{1-\lambda}} \quad 2JS A_0 \sqrt{1-\lambda} = JS A_0^2 \sqrt{\lambda} \sim \alpha M_S. \quad A_0 \text{ макс. пенетри.}$$

Для zero anisotropy полагая $\lambda = 1$ и $\frac{1}{r^2} \sim \sqrt{1-\lambda} = 0$, zero function $V_1(p), V_2(p)$
 μ (уравнения) $= -\mu_p$.

Exch. terms yd covn. с полами и полами до $31/\pi$ отн. За невысказан $\frac{\partial \mu}{\partial \varphi}$.

Проблема. Slonczewski '79 (если опуст. член $\vec{H}_{\text{ex}}$ через θ, φ):

$$\begin{cases} \frac{M_S}{\delta} \dot{\Phi} \sin \theta = A (\nabla \Phi)^2 \sin 2\theta - 2A \nabla^2 \theta, & A = \frac{1}{2} \alpha M_S^2 \\ -\frac{M_S}{\delta} \dot{\theta} \sin \theta = -2A (\nabla^2 \Phi) \sin^2 \theta - 2A (\nabla \Phi) (\nabla \theta) \sin 2\theta \end{cases}$$

$$\begin{cases} \frac{M_S}{\delta 2A} \sin \theta \dot{\Phi} = \frac{1}{2} (\nabla \Phi)^2 \sin 2\theta - \nabla^2 \theta \\ \frac{M_S}{\delta 2A} \sin \theta \dot{\theta} = (\nabla^2 \Phi) \sin^2 \theta + (\nabla \Phi \cdot \nabla \theta) \sin 2\theta \end{cases} \quad \begin{cases} \Phi = \omega t + \frac{\Phi_0}{2} + \varphi + \psi(p, t) \\ \theta = \theta_0(p) + \ell(p, t) \end{cases}$$

$$\nabla \Phi = \hat{p} \frac{\partial \Phi}{\partial p} + \hat{\varphi} \frac{1}{p} \frac{\partial \Phi}{\partial \varphi}, \quad (\nabla \Phi)^2 = \left(\frac{\partial \Phi}{\partial p} \right)^2 + \frac{1}{p^2} \left(\frac{\partial \Phi}{\partial \varphi} \right)^2 = \left(\frac{\partial \psi}{\partial p} \right)^2 + \frac{1}{p^2} \left(\frac{\partial \Phi_0}{\partial \varphi} + \frac{\partial \psi}{\partial \varphi} \right)^2 \approx \frac{1}{p^2} \left[1 + 2 \frac{\partial \psi}{\partial \varphi} \right] + O(p^2)$$

$$\begin{cases} \frac{M_S}{\delta 2A} \sin \theta \dot{\Phi} = \frac{1}{2} \sin 2\theta \cdot \frac{1}{p^2} \left[1 + 2 \frac{\partial \psi}{\partial \varphi} \right] - \nabla^2 \theta = \frac{1}{2p^2} \sin 2\theta + \frac{1}{p^2} \sin 2\theta_0 \frac{\partial \psi}{\partial \varphi} - \nabla^2 \theta \\ \frac{M_S}{\delta 2A} \sin \theta \dot{\theta} = (\nabla^2 \psi) \sin^2 \theta_0 + \sin 2\theta_0 \left[\theta'_0 \frac{\partial \psi}{\partial p} + \frac{1}{p^2} \frac{\partial \ell}{\partial \varphi} \right] \end{cases}$$

$$\begin{cases} \frac{1}{\delta \alpha M_S} \sin \theta \dot{\Phi} = -\nabla^2 \ell + \frac{1}{p^2} \cos 2\theta_0 \cdot \ell + \frac{1}{p^2} \sin 2\theta_0 \cdot \frac{\partial \psi}{\partial \varphi} \\ \frac{1}{\delta \alpha M_S} \dot{\theta} = (\nabla^2 \psi) \sin \theta_0 + 2 \cos \theta_0 \left(\frac{\partial \theta_0}{\partial p} \frac{\partial \psi}{\partial p} \right) + 2 \cos \theta_0 \cdot \frac{1}{p^2} \frac{\partial \ell}{\partial \varphi} \end{cases} \quad \mu_p = -\sin \theta_0 \cdot \psi$$

$$\begin{aligned} (*) & \left\{ \frac{1}{\delta \alpha M_S} \sin \theta \dot{\Phi} = -\nabla^2 \ell + \frac{1}{p^2} \cos 2\theta_0 \cdot \ell - \frac{2 \cos \theta_0}{p^2} \frac{\partial \mu_p}{\partial \varphi} \right\} \\ (**) & \left\{ \frac{1}{\delta \alpha M_S} \dot{\theta} = -\nabla^2 \mu_p + \frac{\mu_p}{\sin \theta_0} \nabla^2 (\sin \theta_0) + \frac{2 \cos \theta_0}{p^2} \frac{\partial \ell}{\partial \varphi} \right\} \end{aligned}$$

$$\nabla^2 (\sin^2 \theta_0) = \left[\frac{\partial^2}{\partial p^2} + \frac{1}{p} \frac{\partial}{\partial p} + \frac{1}{p^2} \frac{\partial^2}{\partial \varphi^2} \right] \sin^2 \theta_0 = \frac{\partial}{\partial p} (2 \sin \theta_0 \cos \theta_0 \theta'_0) + \frac{1}{p} (2 \sin \theta_0 \cos \theta_0 \theta''_0) =$$

$$= \frac{\partial}{\partial p} (\sin 2\theta_0 \theta'_0) + \frac{\sin 2\theta_0}{p} \theta''_0 = 2 \cos 2\theta_0 (\theta'_0)^2 + \sin 2\theta_0 \theta''_0 + \frac{\sin 2\theta_0}{p} \theta'_0$$

$$\nabla^2 (\sin \theta_0) = \frac{\partial}{\partial p} (\cos \theta_0 \theta'_0) + \frac{1}{p} (\cos \theta_0 \theta''_0) = -\sin \theta_0 (\theta'_0)^2 + \cos \theta_0 (\theta''_0 + \frac{1}{p} \theta'_0)$$

$$\rightarrow \text{Если } \theta''_0 + \frac{1}{p} \theta'_0 = \frac{1}{p^2} \sin \theta_0 \cos \theta_0 \text{ + только если } \theta_0(p) \text{ - реш. уравн. } \tan \frac{\theta_0}{2} = \frac{p}{b}, p \leq b.$$

$$\text{Тогда } (**) \frac{1}{\delta \alpha M_S} \dot{\theta} = -\nabla^2 \mu_p + \left[\theta_0'^2 + \frac{\cos^2 \theta_0}{p^2} \right] \mu_p + \frac{2 \cos \theta_0}{p^2} \frac{\partial \ell}{\partial \varphi} \text{ совн. с ур. (9)}$$

Ivanov - PRB-98. Т.е. ур. в уравнения (9) верно только при ansatz μ_{exch} или ПЛН ansatz $\theta''_0 + \frac{1}{p} \theta'_0 = -(\alpha^2 - \frac{1}{p^2}) \sin \theta_0 \cos \theta_0$ в своем объеме. Также и ПЛН. Также. Если есть магнитостатика ПЛН-ansatz (с $\alpha=0$ или $\neq 0$) в общем неверен.

Ур. (*) совпадает с ур. (8) Ivanov-98 с учетом $\sin \theta_0 \dot{\Phi} = j$. Но верже знак δ "грешки".

Аналогично по теореме, в.к. $\vec{y}_D = \frac{1}{\delta} \frac{d\vec{m}}{dt} = \vec{m} \times \vec{\Pi}_{\text{ог}}$, с тем же самым результатом.

$$(c) -\frac{1}{j} \frac{d m_p}{dt} = 2 M_s \left[-\nabla^2 \ell + \frac{\cos 2\theta_0}{r^2} \ell - \frac{2 \cos \theta_0}{r^2} \frac{\partial \ell}{\partial \varphi} \right] + \cos \theta_0 H_m^{(2)} \ell(\vec{r}) + M_s \sin \theta_0 (-i) \left\{ d^2 \vec{r}' \cdot \vec{G}_{\vec{r}\vec{r}'} \sin \theta_0(r') \ell(\vec{r}') - M_s \cos \theta_0 \int d^2 \vec{r}' \left[G_{\varphi\varphi} \ell(\vec{r}') + G_{\varphi\varphi} \cos \theta_0(r') \ell(\vec{r}') \right] \right\}$$

26/11 The $\psi(\beta, \varphi)$, $\mu(\beta, \varphi)$ and the average value $\bar{\mu}$ are independent of β and φ .
- $\bar{\mu} = \mu(\beta, \varphi)$, $\mu(\beta, \varphi) = \mu(\beta, \varphi)$.

4 Ivanov-Pl's 82' $\omega_{010} = 8 \text{ d Ms}$ current 3D under bug home under/under/under

$$\begin{aligned}\nabla\Phi_0 &= \frac{\hat{r}}{r} \\ \nabla\Theta_0 &= \hat{\phi}\Theta_0' \\ \Theta_0' &\rightarrow 0\end{aligned}$$

Glove.-79

здесь $\vec{H}$ - магнитное поле. (!)

$$\vec{m} = m/\mu_s = (m_p, m_e, m_t) = (\sin\theta \cos(\phi-\psi), \sin\theta \sin(\phi-\psi), \cos\theta)$$

$$\frac{\partial \vec{p}}{\partial \phi} = \frac{\partial}{\partial \phi} [\hat{p} m_p + \hat{\phi} m_\phi + \hat{z} m_z] = \hat{p} \sin \theta (-1) \sin(\phi - \psi) + \hat{\phi} \sin \theta \cos(\phi - \psi) + \hat{z} 0$$

$$\vec{H} \cdot \frac{\partial \vec{m}}{\partial \theta} = H_p \frac{\partial m_p}{\partial \theta} + H_y \frac{\partial m_y}{\partial \theta} + H_z \frac{\partial m_z}{\partial \theta} = H_y \cos \theta \cos(\varphi - \varphi) + H_y \cos \theta \sin(\varphi - \varphi) + H_z (-\sin \theta)$$

Due vortex $\Phi = \psi + c\frac{\pi}{2} + \varphi$, $\Theta = \Theta_0 + \ell$. $\begin{cases} \cos(\Phi - \varphi) = \cos(\psi + c\frac{\pi}{2}) \approx -c \cdot \psi \\ \sin(\Phi - \varphi) = \sin(\psi + c\frac{\pi}{2}) = c, c = +1 \end{cases}$

$$\vec{H} \cdot \frac{\partial \vec{m}}{\partial \Theta} = H_p \cos(\Theta_0 + \ell) (-\psi) + H_q \cos(\Theta_0 + \ell) - \sin(\Theta_0 + \ell) H_z =$$

$$= -\psi \cos \Theta_0 H_p^0 + (H_p^0 + h_m^q) [\cos \Theta_0 - \sin \Theta_0 \ell] - (H_z^0 + h_m^z) [\sin \Theta_0 + \cos \Theta_0 \ell] =$$

$$= (0) - \psi \cos \Theta_0 H_p^0 + \cos \Theta_0 h_m^q - \sin \Theta_0 H_p^0 \ell - H_z^0 \cos \Theta_0 \ell - h_m^z \sin \Theta_0 = |H_{p,0} = 0| =$$

$$= \cos \Theta_0 h_m^q - h_m^z \sin \Theta_0 - H_z^0 \cos \Theta_0 \ell. \quad -\frac{dmp}{dt} = \frac{d}{dt} (\sin \Theta_0 \psi) = \sin \Theta_0 \frac{d\Phi}{dt}$$

Slow: $\frac{1}{\gamma} \sin \Theta \frac{d\Phi}{dt} = (\text{exch.}) - \frac{1}{M_s} \vec{H} \cdot \frac{\partial \vec{m}}{\partial \Theta} = (\text{exch.}) - \cos \Theta_0 h_m^q + h_m^z \sin \Theta_0 + H_z^0 \cos \Theta_0 \ell$

Acc: $-\frac{1}{\gamma} \frac{dmp}{dt} = \frac{1}{\gamma} \sin \Theta_0 \frac{d\Phi}{dt} = (\text{exch.}) + \cos \Theta_0 H_m^{z0} \ell - M_s \sin \Theta_0 (d^2 \vec{p}' \cdot \vec{G}_{zz} \sin \Theta_0 \ell - M_s \cos \Theta_0 h_m^q \frac{1}{M_s})$

$$-\frac{1}{\gamma} \frac{dmp}{dt} = (\text{exch.}) + H_m^{z0} \mu_\varphi + m_\varphi^0 h_m^z - m_z^0 h_m^q = \sin \Theta_0 h_m^z - \cos \Theta_0 h_m^q + H_m^{z0} \cos \Theta_0 \ell + (\text{ex.})$$

Важно не забывать, что это уравнение, так что надо уД, где $\frac{dmp}{dt}$ считается со Slowen'ski-79.

Slow: $\vec{H} \cdot \frac{\partial \vec{m}}{\partial \Phi} = H_p \frac{\partial mp}{\partial \Phi} + H_q \frac{\partial mq}{\partial \Phi} + H_z \frac{\partial mz}{\partial \Phi} = H_p (-1) \sin \Theta \sin(\Phi - \varphi) + H_q \sin \Theta \cos(\Phi - \varphi) =$

$$= -h_p^q \sin \Theta_0 + h_m^q \sin \Theta (-\psi) \approx -h_p^q \sin \Theta_0$$

Acc: $\frac{1}{\gamma} \frac{d\Theta}{dt} = (\text{exch.}) + h_m^p - \cos \Theta_0 H_m^{z0} \mu_p$, где $\sim H_m^{z0}$ в уД $\hat{\Theta}$ связан с const. загл. $\hat{\Theta}$.

Slow: $+\frac{1}{\gamma} \sin \Theta \frac{d\Theta}{dt} = (\text{exch.}) + \vec{H} \cdot \frac{\partial \vec{m}}{\partial \Phi} = (\text{exch.}) - h_m^p \sin \Theta_0 \Rightarrow \frac{1}{\gamma} \frac{d\Theta}{dt} = (\text{ex.}) - h_m^p$

Важно: все уД, где $\hat{\Theta}$ имеет значение. Если правильно знака и сопоставить. Если $\sim H_m^{z0}$, которого нет у Slowen'ski-79. Разрешение с Eq.(1).

А знак может быть у нас в уравнении уД (1) $25/\pi$. с тем же $\sim H_m^{z0}$ - считаем, что у Slowen-79 у нас нет, т.к. $\frac{\partial m_z}{\partial \Phi} = 0$. Заменяем знак на противоположный, т.е. в (1):

Вспомогательные уравнения:

$$(1) \frac{1}{\gamma} \dot{\Theta} = \alpha \left[-\nabla^2 \mu + \frac{\nabla^2 (\sin \Theta_0)}{\sin \Theta_0} \mu + \frac{2 \cos \Theta_0}{\rho^2} \frac{\partial \ell}{\partial \varphi} \right] + \cos \Theta_0 \frac{H_m^{z0}}{M_s} \mu - \int d^2 \vec{p}' \left[G_{pp} \mu(\vec{p}') + G_{pq} \cos \Theta_0 \ell(\vec{p}') \right]$$

$$(2) \frac{1}{\gamma} \dot{mp} = \alpha \left[-\nabla^2 \ell + \frac{\cos 2\Theta_0}{\rho^2} \ell - \frac{2 \cos \Theta_0}{\rho^2} \frac{\partial \mu}{\partial \varphi} \right] + \cos \Theta_0 \frac{H_m^{z0}}{M_s} \ell - \sin \Theta_0 \int d^2 \vec{p}' G_{zz} \sin \Theta_0(\vec{p}') \ell(\vec{p}') -$$

$$\ell = \ell(\vec{p}, \varphi), \mu = \mu(\vec{p}, \varphi).$$

→ Два уравнения уД: ① оператор $\hat{\Theta} = -\nabla \Theta \cdot \vec{\hat{x}}$, $\hat{\Phi} = -\nabla \Phi \cdot \vec{\hat{x}}$, $\Theta, \Phi = f(\vec{p} - \vec{x}(t)) \rightarrow \text{transl. mode}$

② $\ell(\vec{p}) = f(\vec{p}) \cos(m\varphi - \omega t)$, $\mu(\vec{p}) = g(\vec{p}) \sin(m\varphi - \omega t) \sim 24/\pi$, Fourier

Transf. no. t" и угадыв. от $\varphi \rightarrow$ guess pariet SW ℓ, μ over vortex ground state.

→ Непосредственно подставим (1)+(2) в уравнение $\dot{\Theta} = -\nabla \Theta \cdot \vec{\hat{x}}$, $\dot{\Phi} = -\nabla \Phi \cdot \vec{\hat{x}}$, т.е. найдем precession solution $\ell, \mu \sim \vec{\hat{x}}$. Это guess vortex mass и упр-е для $\vec{\hat{x}}(t)$. Тогда ωt (как Ivanov-89) в Φ становится не нужно. Nikiiforov'83 вт в Φ бумажка, но на уД, m, Φ , это не бумажка, а Gourea'89 вт универсальный базис.

$$\nabla \theta_0 = \left(\hat{r} \frac{\partial}{\partial r} + \hat{\phi} \frac{1}{r} \frac{\partial}{\partial \phi} \right) \theta_0(r) = \hat{r} \frac{\partial \theta_0}{\partial r} = \hat{r} \theta_0', \quad \nabla \Phi_0 = \hat{\phi} \frac{1}{r} \frac{\partial \Phi_0}{\partial \phi} = \hat{\phi} \frac{1}{r}, \quad \Phi = \Phi_0 + \psi$$

$$m_p(\vec{r}, t) = m_p[\vec{r} - \vec{X}(t)], \quad \dot{m}_p = \frac{\partial m_p(\vec{r})}{\partial \vec{r}} \frac{\partial (\vec{r} - \vec{X})}{\partial t} = -\nabla m_p \cdot \vec{X}$$

$$\dot{m}_p = \frac{d}{dt} m_p = \frac{d}{dt} \mu_p = \frac{d}{dt} (-\sin \theta_0 \psi) = -\sin \theta_0 \frac{d\psi}{dt} = -\sin \theta_0 \frac{d\Phi}{dt} = -\sin \theta_0 [-\nabla \Phi_0 \cdot \vec{X}]$$

$$\dot{m}_p = \sin \theta_0 \nabla \Phi_0 \cdot \vec{X}, \quad \dot{\theta} = -\nabla \theta_0 \cdot \vec{X}$$

$$\nabla^2 (\sin \theta_0) = -\sin \theta_0 (\theta_0')^2 + \cos \theta_0 (\theta_0'' + \frac{1}{r} \theta_0')$$

$$\text{Тогда } \theta_0'' + \frac{1}{r} \theta_0' = \frac{1}{r^2} \sin \theta_0 \cos \theta_0 + \frac{1}{r} \sin \theta_0 (d\vec{r} \vec{r}' g_{zz}(\vec{r}, \vec{r}') \cos \theta_0(r)) = \frac{1}{r^2} \sin \theta_0 \cos \theta_0 + \frac{\sin \theta_0}{r} \frac{H_m^{20}}{M_s}$$

$$H_m^{20}(r) = M_s (d\vec{r} \vec{r}' G_{zz}(\vec{r}, \vec{r}')) m_z^0(r') = M_s (d\vec{r} \vec{r}' g_{zz}(\vec{r}, \vec{r}') \cos \theta_0(r'))$$

$$\Delta \frac{\nabla^2 (\sin \theta_0)}{\sin \theta_0} + \cos \theta_0 \frac{1}{M_s} H_m^{20} = -\Delta (\theta_0')^2 + \frac{\cos \theta_0}{\sin \theta_0} (\theta_0'' + \frac{1}{r} \theta_0') \Delta + \cos \theta_0 \frac{1}{M_s} H_m^{20} =$$

$$= -(\theta_0')^2 \Delta + \Delta \frac{\cos \theta_0}{\sin \theta_0} \left[\theta_0'' + \frac{1}{r} \theta_0' + \frac{1}{\Delta} \sin \theta_0 H_m^{20} \right] \quad \text{Важно заметить, что если бы exch. и магнитосфера были разных знаков.}$$

Разобьемся с знаками.

можно в духе Nishikawa '83 решить числ. уД. при $r \rightarrow 0, \cos \theta_0 \rightarrow 1$ - вблизи core, $r > b, \cos \theta_0 = 0$ - вне "core".

Ansatz $\psi(\vec{r}) = V a(r) \cos(m\varphi - \varphi_0), \quad \mu(\vec{r}) = V b(r) \sin(m\varphi - \varphi_0), \quad V = |\vec{X}|,$

φ_0 связан с направлением $\vec{X} = \vec{v}, m = \pm 1$.

$m_1(\vec{r}) = m_2(\vec{r}) = -\sin \theta_0 \psi(\vec{r}), \quad \varphi_0 = \omega t(?)$ тогда $\cos(\pm \varphi - \omega t)$ соответствует вращению.

УД выполняется после числов. ansatz.

Вспомогательная 14/1 гала $\mu_z(\vec{r}) \sim (\hat{r} \times \vec{G}) \cdot \vec{X} \sim (\hat{\phi} \cdot \vec{X}) \sim V_\phi$

$$\Phi = -\left(\frac{\partial \Psi}{\partial y}\right)_0 \frac{G_z}{\kappa} V_x + \left(\frac{\partial \Psi}{\partial x}\right)_0 \frac{G_z}{\kappa} V_y + \Phi_0 = \Phi_0 + \frac{1}{\kappa} \vec{G} \cdot (\nabla \Psi \times \vec{v}) \sim (\hat{\phi} \times \vec{G}) \cdot \vec{v} \sim (\hat{\phi} \cdot \vec{X}), \quad \Phi \sim V_\phi$$

то вычислим предельного.

$$V_r + iV_\phi = e^{-i\varphi} (V_x + iV_y) = e^{-i\varphi} v (\cos \varphi_0 + i \sin \varphi_0) = v e^{i(\varphi_0 - \varphi)}, \quad \begin{aligned} v_r &= v \cos(\varphi - \varphi_0) \\ -v_\phi &= v \sin(\varphi - \varphi_0) \end{aligned}$$

(27/11)

Еще добавлю уД - Smeetski'79, то

Vortex mass

$$\frac{1}{\delta m_s} \dot{\theta} = \Delta \left[-\nabla^2 \mu + \frac{\nabla^2 (\sin \theta_0)}{\sin \theta_0} \mu + \frac{2 \cos \theta_0}{r^2} \frac{\partial \psi}{\partial \phi} \right] - h_p^m$$

$$\nabla^2 (\sin \theta_0) = -\sin \theta_0 (\theta_0')^2 + \cos \theta_0 (\theta_0'' + \frac{1}{r} \theta_0'), \quad \theta_0(r) \text{ решено числ. заранее.}$$

В числ. решено $\dot{\theta} = 0$ задан, аббревиатура, $\Phi = 0$ для $(\nabla \Phi)^2 \sin 2\theta - 2\nabla^2 \theta = 0$ - в exch. уравнении.

$$(\nabla \Phi_0)^2 = \left(\hat{\phi} \frac{1}{r} \right)^2 = \frac{1}{r^2}, \quad \nabla^2 \theta_0 = \frac{1}{r^2} \sin \theta_0 \cos \theta_0, \quad \nabla^2 \theta_0 = \theta_0'' + \frac{1}{r} \theta_0' - \text{exch.}$$

$$\nabla^2 (\sin \theta_0) = -\sin \theta_0 (\theta_0')^2 + \cos \theta_0 \frac{1}{r^2} \sin \theta_0 \cos \theta_0 = \sin \theta_0 \left[-(\theta_0')^2 + \frac{\cos^2 \theta_0}{r^2} \right]$$

$$\text{С учетом же магнитосферы } \Phi = 0 \rightarrow \frac{\delta W}{\delta \theta} = A (\nabla \Phi)^2 \sin 2\theta - 2A \nabla^2 \theta - \vec{H} \cdot \frac{\partial \vec{M}}{\partial \theta} = 0$$

$$\frac{1}{M_s} \vec{H} \frac{\partial \vec{M}}{\partial \theta} = H_\phi \cos \theta \cos(\varphi - \varphi_0) + H_\phi \cos \theta \sin(\varphi - \varphi_0) + H_z (-\sin \theta) = -H_z \sin \theta, \quad \text{т.к. } H_{\phi, \varphi} = 0$$

$$A \frac{1}{r^2} \sin 2\theta_0 - 2A \nabla^2 \theta_0 + M_s H_z^0 \sin \theta_0 = 0, \quad \nabla^2 \theta_0 = \frac{1}{r^2} \sin \theta_0 \cos \theta_0 + \frac{M_s}{2A} H_z^0 \sin \theta_0, \quad \frac{M_s}{2A} = \frac{1}{\Delta m_s}$$

$$\int d^2 \vec{r}' \sin \theta_0(\vec{r}') G_{zz}(\vec{r}') = \int d\vec{r}' \sin \theta_0 \int d\vec{r} \sum_{\vec{k}} \sum_{\vec{l}} [e^{-i\vec{k}\vec{l}}] J_0 J_l e^{i\vec{v}(\vec{r}-\vec{r}')} v a(\vec{r}') \cos(m\varphi - \varphi_0) =$$

$$= \frac{2v}{L} \int d\vec{r}' \sin \theta_0 a(\vec{r}') \int d\vec{k} [\dots] \sum_{\vec{p}} J_0 J_p I_c^{pm} = \frac{2v}{L} \int d\vec{r}' \sin \theta_0 a(\vec{r}') \int d\vec{k} [\dots] 2\pi J_0 J_0 \cos(m\varphi - \varphi_0) =$$

$$= \frac{4\pi v}{L} \int d\vec{r}' \sin \theta_0 a(\vec{r}') \int d\vec{k} [e^{-i\vec{k}\vec{l}}] J_0 J_0 \cos(m\varphi - \varphi_0)$$

$$\int d^2 \vec{r}' G_{\varphi\varphi}(\vec{r}') = \int d\vec{r}' d\vec{r}' \left(-2i \frac{1}{p} \frac{\partial}{\partial \vec{r}'}\right) \int d\vec{k} f \sum_{\vec{p}} v J_0 J_p e^{i\vec{v}(\vec{r}-\vec{r}')} v b(\vec{r}') \sin(m\varphi - \varphi_0) =$$

$$= -2iv \int d\vec{r}' b(\vec{r}') \int d\vec{k} f \sum_{\vec{p}} v \left(\frac{1}{p} \frac{\partial}{\partial \vec{r}'} J_0 J_p\right) I_c^{pm} = -2iv \int d\vec{r}' b(\vec{r}') \int d\vec{k} f \frac{2\pi m}{L} (\dots) \cos(m\varphi - \varphi_0) =$$

$$= -4\pi m v \cos(m\varphi - \varphi_0) \int d\vec{r}' b(\vec{r}') \int d\vec{k} f \left[\frac{1}{p} \frac{\partial}{\partial \vec{r}'} J_m J_m\right]$$

$$\int d^2 \vec{r}' G_{\varphi\varphi} \cos \theta_0(\vec{r}') = \int d\vec{r}' \cos \theta_0 \int d\vec{r}' \left(-\frac{2}{p p'}\right) \int d\vec{k} f \sum_{\vec{p}} v^2 J_0 J_p e^{i\vec{v}(\vec{r}-\vec{r}')} v a(\vec{r}') \cos(m\varphi - \varphi_0) =$$

$$= -2v \int d\vec{r}' \cos \theta_0 a(\vec{r}') \int d\vec{k} f \sum_{\vec{p}} v^2 \left(\frac{1}{p p'} J_0 J_p\right) I_c^{pm} = -2v \int d\vec{r}' \cos \theta_0 a(\vec{r}') \int d\vec{k} f 2\pi m^2 \left(\frac{1}{p p'} J_m J_m\right) \cos(m\varphi - \varphi_0) =$$

$$= -4\pi m^2 v \cos(m\varphi - \varphi_0) \int d\vec{r}' \cos \theta_0 a(\vec{r}') \int d\vec{k} f \left[\frac{1}{p p'} J_m J_m\right]$$

$$-\frac{1}{dM_S} \dot{m}_p = -\nabla_p^2 \varphi + \frac{m^2}{p^2} \varphi + \frac{\cos 2\theta_0}{p^2} \varphi - \frac{2\cos \theta_0}{p^2} m v b \cos(m\varphi - \varphi_0) + \frac{H_m^2}{dM_S} \cos \theta_0 \varphi -$$

$$-\frac{1}{2} \sin \theta_0 \frac{4\pi v}{L} \cos(m\varphi - \varphi_0) \int d\vec{r}' a(\vec{r}') \int d\vec{k} [e^{-i\vec{k}\vec{l}}] J_0 J_0 - \frac{1}{2} \cos \theta_0 (-1) 4\pi m v \cos(m\varphi - \varphi_0) \int d\vec{r}' b(\vec{r}') \int d\vec{k} f \left[\frac{J_m J_m}{p} \frac{\partial}{\partial \vec{r}'}\right]$$

$$-\frac{1}{2} \cos \theta_0 (-1) 4\pi m^2 v \cos(m\varphi - \varphi_0) \int d\vec{r}' \cos \theta_0 a(\vec{r}') \int d\vec{k} f \left[\frac{1}{p p'} J_m J_m\right]$$

→ luego $\dot{m}_p = \frac{\sin \theta_0}{p} v_p = \frac{\sin \theta_0}{p} v \cos(m\varphi - \varphi_0)$, $m = \pm 1$.

$$-\frac{1}{dM_S} \frac{\sin \theta_0}{p} = -\nabla_p^2 a + \frac{m^2}{p^2} a + \frac{\cos 2\theta_0}{p^2} a - \frac{2m \cos \theta_0}{p^2} b + \frac{H_m^2}{dM_S} \cos \theta_0 a - \frac{4\pi \sin \theta_0}{dL} \int d\vec{r}' a(\vec{r}') \int d\vec{k} [\dots] J_0 J_0 +$$

$$+ \cos \theta_0 \frac{4\pi m}{d} \int d\vec{r}' b(\vec{r}') \int d\vec{k} f \left[\frac{J_m J_m}{p} \frac{\partial}{\partial \vec{r}'}\right] + \cos \theta_0 \frac{4\pi m^2}{d} \int d\vec{r}' \cos \theta_0 a(\vec{r}') \int d\vec{k} f \left[\frac{1}{p p'} J_m J_m\right]$$

→ $d_{zz}(\vec{r}) = \frac{1}{L} \int d\vec{k} [e^{-i\vec{k}\vec{l}}] J_0(k\rho) J_0(k\rho')$, $d_{\varphi\varphi}(\vec{r}) = \int d\vec{k} f(k) \frac{J_1(k\rho) \partial J_1(k\rho')}{p^2}$, $d_{\varphi\varphi}(\vec{r}) = \int d\vec{k} f(k) J_1(k\rho) J_1(k\rho')$

(*) $-\nabla_p^2 a + \frac{2\cos^2 \theta_0}{p^2} a - \frac{2m \cos \theta_0}{p^2} b + \frac{H_m^2}{dM_S} \cos \theta_0 a - \frac{4\pi \sin \theta_0}{d} \int d\vec{r}' d_{zz}(\vec{r}') a(\vec{r}') \cdot \frac{\sin \theta_0(\vec{r}')}{p} +$

+ $\frac{4\pi \cos \theta_0}{d} m \int d\vec{r}' d_{\varphi\varphi}(\vec{r}') b(\vec{r}') + \frac{4\pi \cos \theta_0}{d} \int d\vec{r}' \cos \theta_0(\vec{r}') d_{\varphi\varphi}(\vec{r}') a(\vec{r}') = -\frac{1}{dM_S} \frac{\sin \theta_0}{p}$

(*) u (**) u es lo mismo y_p a que $a(p)$, $b(p)$. $d_{zz}(\vec{r}) = (4\pi)^{-1} d_{zz}(\vec{r}) \xrightarrow{L \rightarrow 0} -\frac{1}{p} \delta(\vec{r}-\vec{r}')$

Group meeting: - tener V. dyn. TM - paper Bane re u/jegrebu.

→ todo muy bien. Bujica: leame yfime u/poke en exp. - X-MCD/PEEM con

laser field-pulse repetition is 125 MHz. h/roco TM mequencia →

generamos samples na F/N/F um F/AF cyl. dots que TM-measurements.

um bozfyngar $\omega_n \gg \omega_0 = 125 \text{ MHz}$, no vale boucunore kerem, t.v. h_n is uniform and in plane. h_1 curvose (10 kG) re u/juniorre. Nopyrna Pierre mequencia $\vec{m}_{dyn}(\vec{r})$ cyl. dot u u/poke en $\vec{m}_{dyn} \sim \vec{r}$.

(2)(*) γ_p - a gauge $a(p)$, $b(p)$ gauge vortex dynamical magn. profile u. v. mass.

Markus. $\Phi \sim d_{\rho} \rho(p, \theta)$ - elementary gauge vortex $m = \pm 1$ modes.

Решение (2)(*) можно использовать если а) $\rho \rightarrow 0$, б) $\rho \gg b$, в) core

(сравно 14/1 side-charges free model) for $\delta m_z(\vec{r}) = \frac{4c^2}{(2R)} \frac{1+\rho^2}{(c^2+\rho^2)^2} (\vec{r} \times \vec{G}) \cdot \vec{x}$, $\rho \in \text{eq. R}$

if $\rho \gg c$ $\delta m_z(\vec{r}) \sim \frac{(1+\rho^2)\rho}{(c^2+\rho^2)^2} \approx \frac{1+\rho^2}{\rho^3} \sim \frac{R^2+\rho^2}{\rho^3} \sim \frac{1}{\rho}$ because gauge as "core".

$\Phi = \Phi_0 + \psi$, $\psi(\vec{r}) \sim \frac{1}{\rho}$. $\mu_p(\vec{r}) = -\sin\theta_0(\rho) \psi(\vec{r}) \sim \frac{1}{\rho}$, т.е. gauge or "core" function $\propto$ gauge m_p .

I Полюс $\cos\theta_0(\rho) = 0$ в (2), (2*), т.е. $\rho > b$ outside.

$$(2) \quad -\nabla_{\vec{r}}^2 b + \frac{4\pi}{2} \int d\vec{s}' d\vec{s}'' b(\vec{s}') + \frac{4\pi}{2} m \int d\vec{s}' \cos\theta_0(\vec{s}') d\vec{s}'' a(\vec{s}'') = -\frac{1}{4\pi M_s} \theta_0'$$

$$(2*) \quad -\nabla_{\vec{r}}^2 a - \frac{4\pi}{2} \sin\theta_0 \int d\vec{s}' d\vec{s}'' d_{zz} a(\vec{s}') = -\frac{1}{4\pi M_s} \frac{\sin\theta_0}{\rho}$$

$$\frac{1}{R} \rightarrow d_{zz}(\vec{s}'') \rightarrow -\frac{1}{\rho} \delta(\vec{r}-\vec{s}'') : \quad -\nabla_{\vec{r}}^2 a(\rho) + \frac{4\pi}{2} \sin\theta_0 a(\rho) = -\frac{1}{4\pi M_s} \frac{\sin\theta_0}{\rho} - \text{H.D.Y. func. because.}$$

$$\text{В пределе можно считать } \frac{4\pi}{2} \sin\theta_0 a(\rho) \approx -\frac{1}{4\pi M_s} \frac{\sin\theta_0}{\rho}, \quad a(\rho) = -\frac{1}{4\pi M_s} \frac{1}{\rho} = -\frac{1}{\omega_m} \frac{1}{\rho}$$

$$(2*): \quad \int d\vec{s}' d\vec{s}'' d_{\rho\rho}(\vec{s}'') b(\vec{s}') = \frac{m}{\omega_m} \int d\vec{s}' \frac{\cos\theta_0(\vec{s}')}{\rho'} d\vec{s}'' b(\vec{s}'') = -\frac{1}{\omega_m} \theta_0', \quad \theta_0' \approx 0 \text{ в "core"}$$

$$\rightarrow \int_0^R d\vec{s}' d\vec{s}'' d_{\rho\rho}(\vec{s}'') b(\vec{s}') = \frac{m}{\omega_m} \int_0^R d\vec{s}' \frac{\cos\theta_0(\vec{s}')}{\rho'} d\vec{s}'' b(\vec{s}'') = F(\rho) - \text{unperf. } \gamma_p \text{ на } b(\rho).$$

$$\delta m_z(\vec{r}) = -\sin\theta_0(\rho) \ell(\vec{r}), \quad \ell(\vec{r}) = v \cos(m\varphi - \varphi_0) \left(-\frac{1}{\omega_m}\right) \frac{1}{\rho} \text{ и } \delta m_z(\vec{r}) \sim \frac{1}{\rho}.$$

$$d_{\rho\rho}(\vec{s}', \vec{s}'') = \left(\frac{d\vec{k}}{k} f(k) \right) \frac{\partial J_1(k\rho)}{\partial \rho} \frac{\partial J_1(k\rho')}{\partial \rho'}. \text{ where } k \text{ continuous, } \rho, \rho' \text{ no } \rho \text{ because } \rightarrow \text{ "free" pole.}$$

$$\mu_p(\vec{r}) = v \sin(m\varphi - \varphi_0) \cdot b(\rho). \text{ Because } \mu_p(R) \approx 0 \text{ and } b(R) = 0 \text{ как в "pole-free" model.}$$

$$F(\rho) = \int d\vec{s}' \frac{\cos\theta_0(\vec{s}')}{\rho'} d\vec{s}'' b(\vec{s}''), \quad d\vec{s}'' b(\vec{s}'') = \left(\frac{d\vec{k}}{k} f(k) \right) \frac{\partial J_1(k\rho)}{\partial \rho} J_1(k\rho'), \quad F(0) \sim d_{\rho\rho}(0, \vec{s}') \neq 0.$$

$$\text{Mass vortex } M_{ij} = - \int d\vec{r} \left[\frac{\partial \Phi}{\partial x_i} \frac{\partial m_z}{\partial x_j} - \frac{\partial \Phi}{\partial x_j} \frac{\partial m_z}{\partial x_i} \right], \quad \begin{cases} \delta m_z = -\sin\theta_0 \cdot \ell(\vec{r}, \varphi) \\ \Phi = \Phi_0 + \psi(\vec{r}, \varphi) \\ \mu_p = -\sin\theta_0 \cdot \psi(\vec{r}, \varphi) \end{cases}$$

$$d\Phi = d\psi = d\left(-\frac{1}{\sin\theta_0} \mu_p\right), \quad dm_z = -d(\sin\theta_0 \ell)$$

$$\text{Vortex core } m=1, \text{ core } v_p = v \cos(\varphi - \varphi_0), \quad v_\varphi = -v \sin(\varphi - \varphi_0)$$

$$\ell(\vec{r}) = a(\rho) v \cos(\varphi - \varphi_0) = a(\rho) v_p. \text{ Core } 2\pi/\ell \text{ where } \text{core } a(\rho), b(\rho)$$

$$\text{Core } v_p = v \sin(m\varphi - \varphi_0), \quad v_\varphi = v \cos(m\varphi - \varphi_0) \text{ с } \text{core.}$$

$$\text{Core } v_p = i v_\varphi = v e^{i(m\varphi - \varphi_0)} = -i(v_p + i v_\varphi) = -i e^{i\varphi} (v_p + i v_\varphi)$$

$$v_p + i v_\varphi = v (\cos\varphi + i \sin\varphi) = v e^{i\varphi} = e^{i\varphi} (v_p + i v_\varphi) = e^{i\varphi} i (v_\varphi - i v_p) = e^{i(\varphi + \pi/2)} v e^{-i(m\varphi - \varphi_0)}$$

$$\text{For } m=1 \quad v_p + i v_\varphi = v e^{i(\varphi_0 + \pi/2)}, \quad v_p = v \cos(\varphi_0 + \pi/2), \quad v_\varphi = v \sin(\varphi_0 + \pi/2)$$

$$\ell(\vec{r}) = a(\rho) v_\varphi, \quad \mu_p(\vec{r}) = b v \sin(m\varphi - \varphi_0) = b(\rho) v_p$$

$$M_{\rho\rho} = - \int d\vec{r} \left[\frac{\partial \Phi}{\partial \rho} \frac{\partial m_z}{\partial \rho} - \frac{\partial \Phi}{\partial \rho} \frac{\partial m_z}{\partial \rho} \right] = \int d\vec{r} \frac{\partial \psi}{\partial \rho} \frac{\partial \cos\theta_0}{\partial \rho} = \int d\vec{r} \frac{\partial \psi}{\partial \rho} \frac{\partial}{\partial \rho} (-\sin\theta_0) \theta_0' =$$

$$= \int d\vec{r} \theta_0' \frac{\partial \mu_p}{\partial \rho} = \int d\vec{r} \theta_0' b(\rho) = 2\pi \int d\vec{r} \theta_0'(\rho) b(\rho)$$

$$M_{\varphi\varphi} = - \int d^2\vec{r} \left[\frac{\partial \Phi}{\partial \varphi} \frac{\partial m_z}{\partial \varphi} - \frac{\partial \Phi}{\partial \varphi} \frac{\partial m_z}{\partial \varphi} \right] = 0, \quad M_{\varphi\varphi} = - \int d^2\vec{r} \left[\frac{\partial \Phi}{\partial \varphi} \frac{\partial m_z}{\partial \varphi} - \frac{\partial \Phi}{\partial \varphi} \frac{\partial m_z}{\partial \varphi} \right] = 0$$

$$M_{\varphi\varphi} = - \int d^2\vec{r} \left[\frac{\partial \Phi}{\partial \varphi} \frac{\partial m_z}{\partial \varphi} - \frac{\partial \Phi}{\partial \varphi} \frac{\partial m_z}{\partial \varphi} \right] = - \int d^2\vec{r} \left[1 \cdot (-\sin\theta_0) a - 0 \right] = \int d^2\vec{r} \frac{\sin\theta_0}{\rho} a(\rho)$$

Т.е. угловая масса $M_{\varphi\varphi} = \int d^2\vec{r} \theta'_0 b(\rho)$, $M_{\varphi\varphi} = \int d^2\vec{r} \frac{\sin\theta_0}{\rho} a(\rho) \neq 0$. **Vortex mass**

Видно, что подынтегр. ф-ии θ'_0 и $\frac{\sin\theta_0}{\rho}$ — это просто функции (независимые)
 — если уравнения для $a(\rho)$, $b(\rho)$.

Note: Если независимые системы для $a(\rho)$, $b(\rho)$ имеют решение, то для
 временной однород. системы для ортогональных неогор. значений θ'_0 и $\frac{\sin\theta_0}{\rho}$.

и наоборот, независимые решения не ось ортогональны к θ'_0 и $\frac{\sin\theta_0}{\rho}$.

$\Rightarrow M_{\varphi\varphi}, M_{\varphi\varphi} \neq 0$. $M_{\varphi\varphi}$ определяет "core" region, где $\theta'_0 \neq 0$, $M_{\varphi\varphi}$ — в основном "core" +
 вне "core"

$$a(\rho) = -\frac{1}{\omega_m} \frac{1}{\rho} \Rightarrow M_{\varphi\varphi} = \int d^2\vec{r} \frac{\sin\theta_0}{\rho} \left(-\frac{1}{\omega_m}\right) \frac{1}{\rho} = -\frac{2\pi}{\omega_m} \int d\rho \frac{\sin\theta_0(\rho)}{\rho}, \quad \text{"условия, определяющие"} \quad \text{знак } \rho.$$

$$M_{\varphi\varphi} = \frac{2\pi}{\omega_m} \left[\int_0^b d\rho \frac{1}{\rho} \frac{2b\rho}{b^2 + \rho^2} + \int_b^R d\rho \frac{1}{\rho} \right] = \frac{2\pi}{\omega_m} \left[\int_0^1 dt \frac{2}{1+t^2} + \ln \frac{R}{b} \right] = \frac{2\pi}{\omega_m} \left[2 \arctan t \Big|_0^1 + \ln \frac{R}{b} \right]$$

$$M_{\varphi\varphi} = \frac{2\pi}{\omega_m} \left[\frac{\pi}{2} + \ln \frac{R}{b} \right] \text{ — криво. (в безразм. ед.)}. \quad \boxed{M_{\varphi\varphi} = \frac{M_s L}{\delta} \frac{2\pi}{\omega_m} \left[\frac{\pi}{2} + \ln \frac{R}{b} \right]} \quad \text{Vortex mass}$$

$$M_{\varphi\varphi} = \frac{1}{2} \frac{L}{\delta^2} \left[\frac{\pi}{2} + \ln \frac{R}{b} \right] \sim \frac{L}{\delta^2} \quad \text{см. Ginzburg 2010}$$

Связь генер. и калибровочных компонент тензоров (10/11 стр., $\varphi = \varphi' + \alpha$):

$$\left\{ \begin{aligned} G_{xx} &= G_{\varphi\varphi} \cos^2\varphi - (G_{\varphi\psi} + G_{\psi\varphi}) \sin\varphi \cos\varphi + \sin^2\varphi G_{\psi\psi} \\ G_{xy} &= (G_{\varphi\varphi} - G_{\psi\psi}) \cos\varphi \sin\varphi + G_{\varphi\psi} \cos^2\varphi - G_{\psi\varphi} \sin^2\varphi \\ G_{yx} &= (G_{\varphi\varphi} - G_{\psi\psi}) \cos\varphi \sin\varphi - G_{\varphi\psi} \sin^2\varphi + G_{\psi\varphi} \cos^2\varphi \\ G_{yy} &= G_{\varphi\varphi} \sin^2\varphi + (G_{\varphi\psi} + G_{\psi\varphi}) \cos\varphi \sin\varphi + G_{\psi\psi} \cos^2\varphi \end{aligned} \right\} \quad \text{— преобраз. Арфкен}$$

Т.к. $M_{\varphi\varphi} = M_{\psi\psi} = 0$, то $\hat{M}$ был квадратичным ($G_{xy} = G_{yx} = 0$, $M_{xy} = M_{yx} = 0$)

в генер. представл. надо $M_{\varphi\varphi} = M_{\psi\psi}$. $\hat{M}_{ij}^* = M \delta_{ij}$, $M^* = \frac{M_s L}{\delta} \frac{2\pi}{\omega_m} \left[\frac{\pi}{2} + \ln \frac{R}{b} \right]$

17/1 pole-free ansatz gives $M = \frac{1}{2} \frac{L}{\delta^2 F(\beta)}$, где $F(\beta) = \frac{\pi}{2} + \ln \frac{R}{b}$, $M^* = \frac{1}{2} \frac{L}{\delta^2} \left[\frac{\pi}{2} + \ln \frac{R}{b} \right]$.

Видно $M \gg M^*$ если $\beta \ll 1$.

Но "core" мал, осн. вклад в $M_{\varphi\varphi}$ дает область $\rho \geq b$, где $\cos\theta_0 = 0$. Можно считать $\cos\theta_0 = 0$, а интеграл $\int d^2\vec{r} \delta(\rho - \rho') \sim \delta(\rho - \rho')$. Надо $\nabla_\rho^2 a$ сохраним.

$$\kappa^2 = \frac{4\pi}{\delta} : -\nabla_\rho^2 a + \frac{4\pi}{\delta} \sin\theta_0 a(\rho) = -\frac{1}{\delta M_s} \frac{\sin\theta_0}{\rho}, \quad -\nabla_\rho^2 a + \kappa^2 a = -\frac{\kappa^2}{\omega_m} \frac{1}{\rho}, \quad a(\rho) \text{ — найти.}$$

$$\nabla_\rho^2 = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho}, \quad \boxed{a'' + \frac{1}{\rho} a' - \kappa^2 a = -\frac{\kappa^2}{\omega_m} \frac{1}{\rho}}$$

Modified φ -вещие уравнения. $z^2 w'' + z w' - (z^2 + \nu^2) w = 0$, $w(z) = I_\nu(z)$.

10DY имеет решение $a_0(\rho) = I_0(\kappa\rho)$. Такое решение АН DY

$$a_i(\rho) = a_0 + \rho^{\alpha} \ln \rho + \rho^\beta, \quad \kappa\rho = x. \quad a'' + \frac{1}{x} a' - a = \frac{\kappa}{\omega_m} \frac{1}{x}, \quad a_1 = x^\alpha \ln x + x^\beta$$

$$a'_1 = \alpha x^{\alpha-1} \ln x + x^{\alpha-1} + \beta x^{\beta-1}, \quad a''_1 = \alpha(\alpha-1) x^{\alpha-2} \ln x + \alpha x^{\alpha-2} + \beta(\beta-1) x^{\beta-2}$$

$p > b$: $Q(p) = C_1 I_0(xp) + C_2 K_0(xp) + a_1(p)$, $a_1(p)$ - "зачем" переменная ЛНД.

$a_1(p)$ можно найти методом Гаусса, но лучше использовать метод Гаусса

47 $a_1(p) = \int_{-\infty}^{\infty} d\vec{s} G(\vec{r}, \vec{s}) \frac{C}{p}$, $a_1(x) = \int d\vec{s} G(\vec{r}, \vec{s}) \frac{\pi}{\omega_M} \left(\frac{1}{\vec{s}} \right)$, $C = \frac{\pi}{\omega_M}$

(1/11) ЛНД: $y'' + \frac{1}{x} y' - y = \frac{C}{x}$, $\text{but } y'' + X_1(x) y' + X_2(x) y = V(x)$, $V(x) = \frac{C}{x}$, $X_1(x) = \frac{1}{x}$
 $y(x) = C_1 y_1(x) + C_2 y_2(x) + \frac{1}{B} \int_0^x V(\xi) e^{\int \xi d\xi} [y_1(\xi) y_2(x) - y_1(x) y_2(\xi)] d\xi$

$y_1(x) = \frac{1}{B} \int d\xi \frac{C}{\xi} \xi [I_0(\xi) K_0(x) - I_0(x) K_0(\xi)] = \frac{C}{B} \int d\xi [I_0(\xi) K_0(x) - I_0(x) K_0(\xi)]$

То есть у нас. Переменная ЛНД $y'' + X_1 y' + X_2 y = V(x)$

→ $y(x) = \int_{x_0}^x d\xi \frac{V(\xi)}{W(\xi)} [y_1(\xi) y_2(x) - y_1(x) y_2(\xi)]$, где y_1, y_2 - переменные ЛНД

а $W(x) = y_1(x) y_2'(x) - y_2(x) y_1'(x)$ - Wronskian. Для ЛНД $y'' + \frac{1}{x} y' - y = 0$

- y_1 - это модифицированная Бессель-функция. $W(x) = -\frac{1}{x}$, $B = -1$ (проверьте).

Общая переменная ЛНД: $y(x) = C_1 I_0(x) + C_2 K_0(x) - C \int_0^x d\xi [I_0(\xi) K_0(x) - I_0(x) K_0(\xi)]$, $C = \frac{\pi}{\omega_M}$, $x = xp$

→ $Q(p) = C_1 I_0(xp) + C_2 K_0(xp) - \frac{\pi}{\omega_M} \int_0^{xp} d\xi [I_0(\xi) K_0(xp) - I_0(xp) K_0(\xi)]$

→ $M_{\text{eff}} = \int d^2 \vec{p} \frac{\sin \theta_0}{p} Q(p) = 2\pi \int_0^R dp \sin \theta_0(p) Q(p) = \left(\frac{\sin \theta_0}{\cos \theta_0} \right) + 2\pi \int_0^R dp Q(p)$

$\frac{1}{x^2} = \frac{1}{4\pi} = \frac{2A}{4\pi \mu_s^2} = \frac{L_{ex}^2}{4\pi} = l_{ex}^2$, $\frac{1}{x} = \frac{L_{ex}}{\sqrt{4\pi}} = l_{ex} \approx 5 \text{ nm}$, $xp = \frac{p}{l_{ex}} \gg 1$ в "core"
 $y_1(x) \rightarrow a_1(p) = -C \int d\xi [I_0(\xi) K_0(xp) - I_0(xp) K_0(\xi)]$ $\pi b = \frac{b}{l_{ex}} > 2$

Гр. Ф. $\int_0^{xp} d\xi I_0(\xi) = 2 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} I_{2n+1}(xp)$, $K_0(z) = \int_0^{\infty} e^{-z \cosh t} dt$

$K_0(z) = -\ln \frac{z}{2} \cdot I_0(z) + \sum_{k=0}^{\infty} \frac{2^k}{2^k (k!)^2} \psi(k+1)$, $K_0(z) = \left(\frac{\pi}{2z} \right)^{1/2} e^{-z} \left[1 - \frac{1}{8z} + O(1/z^2) \right]$

$I_0(z) \approx \frac{1}{\sqrt{2\pi z}} e^z \left[1 + \frac{1}{8z} + O(1/z^2) \right] + O\left(\frac{1}{\sqrt{z}} e^{-z} \right)$, $I_0(z) = \sum_{k=0}^{\infty} \frac{z^{2k}}{2^{2k} (k!)^2}$

$\int_0^{xp} d\xi I_0(\xi) = 2 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} I_{2n+1}(xp) \approx \frac{2}{\sqrt{2\pi z}} e^z \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left[1 - \frac{(n^2-1)}{8z} + \dots \right]_{n=2n+1}$

- так как при $z = xp$ - функция убывает.

$I_0(\xi) K_0(xp) - I_0(xp) K_0(\xi) = \frac{1}{\sqrt{2\pi \xi}} e^{\xi} \left[1 + \frac{1}{8\xi} \right] \frac{\pi^{1/2}}{\sqrt{2xp}} e^{-xp} \left[1 - \frac{1}{8xp} \right] - \frac{e^{xp}}{\sqrt{2\pi xp}} \left[1 + \frac{1}{8xp} \right] \frac{\pi^{1/2}}{\sqrt{2\xi}} e^{-\xi} \left[1 - \frac{1}{8\xi} \right]$
 $\approx \frac{1}{2\sqrt{3xp}} [e^{\xi-xp} - e^{xp-\xi}] = \frac{\text{sh}(\xi-xp)}{\sqrt{3xp}}$

Т.к. в "core" ($p > b$) $xp \gg 1$, то член $C_2 K_0(xp)$ не дает вклада в $Q(p)$ и M_{eff}
 можно считать $C_2 = 0$, $\ell(\vec{p}) = v(\omega(\vec{p}) - \omega_0) a(p)$, $\sin_2(\vec{p}) = -\sin \theta_0(p) \ell(\vec{p})$

$a(p)$ определяет m_z -dynamic component, a strong pinning causes. где $\sin_p = \mu_p$ (вспомогательная)

Откуда брать C_1 ? Если мы будем забирать от C_1 .

$$\gamma_+ = \int \frac{dz}{\sqrt{z}} e^z = \left| \begin{matrix} z=t^2, dz=2tdt \\ dz=2\sqrt{z}dt \end{matrix} \right| = 2 \int dt e^{t^2}, \quad \gamma_- = \int \frac{dz}{\sqrt{z}} e^{-z} = 2 \int dt e^{-t^2}$$

$$a(\rho) = C_1 I_0(x\rho) - \frac{x}{\omega_m} \int_0^{\rho} dz [I_0(z) K_0(x\rho) - I_0(x\rho) K_0(z)]$$

Но физ. смысл нам надо только решить, решение которого, а именно $a(\rho)$ и $b(\rho)$, т.е. $a(\rho) \sim \frac{x}{\omega_m} \frac{\sin \theta_0}{\rho}$. Тогда C_1 (2) будет не зависеть.

$$a(\rho) = -\frac{x}{\omega_m} \int_0^{\rho} dz [I_0(z) K_0(x\rho) - I_0(x\rho) K_0(z)] \rightarrow M_{\text{ф}} = 2\pi \int_0^{\rho} dz \sin \theta_0(z) dz$$

масса фотона

или физически $a(\rho) = -\frac{x}{\omega_m} \int_0^{\rho} dz \frac{\sin(z-x\rho)}{\sqrt{z} x\rho}$ — интерференция и другие эффекты.

Выводка координаты PRB - V. дж. и другие, которая в PNC.

$\frac{2}{m}$

Алгоритм: Преобразование координат тензоров или другие координаты.

Начиная от координат (ρ, φ) к декартовым координатам (x, y)

Для компар. тензоров 2-го ранга $A'_{ij} = \sum_{kl} \frac{\partial x_i}{\partial x_k} \frac{\partial x_j}{\partial x_l} A^{kl}$, $x'_i = x'_i(x_k)$

$$\begin{cases} x = \rho \cos \varphi \\ y = \rho \sin \varphi \end{cases} \quad \frac{\partial x}{\partial \rho} = \cos \varphi, \frac{\partial x}{\partial \varphi} = -\rho \sin \varphi, \frac{\partial y}{\partial \rho} = \sin \varphi, \frac{\partial y}{\partial \varphi} = \rho \cos \varphi$$

$$\begin{matrix} x(x, y), & x = x'_i = x'_i(\rho, \varphi) \\ y(x, y), & y = x'_i = x'_i(\rho, \varphi) \end{matrix} \quad A'^{xx} = \sum_{kl} \frac{\partial x_i}{\partial x_k} \frac{\partial x_i}{\partial x_l} A^{kl} = \frac{\partial x}{\partial \rho} \frac{\partial x}{\partial \rho} A^{\rho\rho} + \frac{\partial x}{\partial \rho} \frac{\partial x}{\partial \varphi} A^{\rho\varphi} +$$

$$+ \frac{\partial x}{\partial \varphi} \frac{\partial x}{\partial \rho} A^{\varphi\rho} + \frac{\partial x}{\partial \varphi} \frac{\partial x}{\partial \varphi} A^{\varphi\varphi} = \cos^2 \varphi A^{\rho\rho} - \rho \sin \varphi \cos \varphi [A^{\rho\varphi} + A^{\varphi\rho}] + \rho^2 \sin^2 \varphi A^{\varphi\varphi}$$

Вывод $\frac{\partial}{\partial \varphi} \rightarrow \frac{1}{\rho} \frac{\partial}{\partial \varphi}$, где $\rho = \rho$ — координата.

$$A'^{xy} = \cos \varphi \sin \varphi A^{\rho\rho} + \cos^2 \varphi A^{\rho\varphi} - \sin^2 \varphi A^{\varphi\rho} - \sin \varphi \cos \varphi A^{\varphi\varphi}, \quad A'^{yy} = \dots$$

$$A'^{yy} = \sin^2 \varphi A^{\rho\rho} + \sin \varphi \cos \varphi (A^{\rho\varphi} + A^{\varphi\rho}) + \cos^2 \varphi A^{\varphi\varphi}$$

То есть с преобраз. $(G_{\rho\rho}, \rho\rho) \rightarrow G_{xx,yy}$ от $2\pi/\pi$ координат и другие преобраз. преобразование тензоров (компонент). И при $A^{\rho\rho} = A^{\varphi\varphi} = 0$, $\begin{cases} A'^{xx} = \cos^2 \varphi A^{\rho\rho} + \sin^2 \varphi A^{\varphi\varphi} \\ A'^{yy} = \sin^2 \varphi A^{\rho\rho} + \cos^2 \varphi A^{\varphi\varphi} \end{cases}$

Нужно еще показать, что M_{ij} имеет смысл как $2\pi/\pi$, откуда x_i, y_i — координаты.

Преобразование $A'^{xx} = A'^{yy}$ дает $A^{\rho\rho} = A^{\varphi\varphi}$.

$$F(z) = \int dz K_0(z) = \int dz \int_0^\infty dt e^{-zcht} = - \int_0^\infty dt \frac{e^{-zcht}}{cht} - \text{нормировка по } \frac{1}{z} \text{ не нужна.}$$

Для $\int dz I_0(z)$ — замечать аналогично интеграл. $K_0(x) = \int_0^\infty \frac{\cos(xt) dt}{\sqrt{1+t^2}}$

$$\text{Для } I_0 \text{ есть } I_0(x) = \frac{1}{\pi} \int_0^\pi d\theta \ln(x \cos \theta). \quad I_0(x) = \frac{1}{\pi} \int_0^\pi e^{x \cos \theta} d\theta$$

Вспомогательные: $I_0(x) K_0'(x) - I_0'(x) K_0(x) = -\frac{1}{x}$, $I_0(x) K_{n+1}(x) + I_{n+1}(x) K_0(x) = \frac{1}{x}$

или $x \rightarrow 0$ $K_0(x) = -\ln x - \gamma + \ln 2 + \dots$

Еще раз вспомним $I_0(z), K_0(z)$, то надо $J_{\pm} = \int \frac{e^{\pm iz}}{\sqrt{z}} dz$
 $\int_0^{\infty} \frac{e^{-iz}}{\sqrt{z}} dz = \sqrt{\frac{\pi}{i}} \Phi(\sqrt{iz})$ или $\int_0^{\infty} dx \frac{e^{-ix}}{x^{\nu+1}} = \frac{1}{\Gamma(\nu)} \gamma(\nu, iu), \nu > 0$.

$\Phi(x) = \frac{1}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ универсальная функция. $\Phi(x) = 1 - \frac{1}{\pi} e^{-x^2} \left[\frac{\gamma(\nu/2)}{x^{\nu/2}} - \dots \right], x \gg 1$

$\Phi(xy) = \frac{2y}{\sqrt{\pi}} \int_0^x dt e^{-t^2 y^2}$ x - номер Бэра минимума.

Еще минимумами обладают (то "core" есть, и здесь $\theta_0 = \pi/2$) - есть, то
 для $\cos(x)$ $a'' + \frac{1}{p} a' = -\frac{1}{dM_S p}$, $a' = y$, $y' + \frac{1}{x} y = -\frac{C}{x}$ или $\cos$ - $y_0 = -C$
 иначе $y_0 = \frac{C_2}{x} C$
 $a(p) = -Cp + C_1$ и $M_{ex} \sim \int dp a(p) \sim R^2$

→ тогда из-за неустойчивости, т.е. $b \rightarrow \infty$, $\frac{1}{p}$ $\frac{1}{p}$ $\frac{1}{p}$ и $\cos \theta_0 = 0$
 для минимума нет.

Note: Формально предел $\frac{1}{R} \rightarrow 0$ ведет к ПН анизотропии, но dyn. dip. fields $\rightarrow 0$.
 Но у нас M_{ex} при $L/R \rightarrow 0$ ($\cos \theta_0$ - область) $\sim e^R$ - расхождение с экспериментом.

Также $I_0 K_0 - K_0 I_0 = \frac{\text{sh}(z-p)}{\sqrt{z-p}}$ довольно точная замена где p - $a(p)$.

Но $a(p) \sim e^p$ и $M_{ex} \sim e^R$ - точно.

Иногда рассуждают о $a(p)$ с нуля. Ищутся соответствия с ПН-ке.

3/III

Зам., зам., зам., зам. на переср. минимума. Bio-nanoparticles

См. также by S.-H. Chang "Bio-Magnetic Nanoparticles".

Обзор, в основном, цитирует работы, с упором на эксперимент. Оптим. расс. $\chi''(\omega)$
 и mag. vortices $\sim 10\%$ below.

Mag. vortices → Небольшое универсальное (где работа с Hoffmann): magnetic properties of
 magn. vortices. - Мелким $M(H)$ и $M(T)$ на расср. вращательных
 в эдго враща. Иллюстрация рисунка.

Vortex mass: $f(p) = \int_0^p dz \frac{\text{sh}(z-p)}{\sqrt{z-p}}$, $f'(p) = 1 \cdot \frac{\text{sh}(p-p)}{p} + \int_0^p dz \frac{\partial}{\partial p} \frac{\text{sh}(z-p)}{\sqrt{z-p}} = \int_0^p dz \frac{1}{\sqrt{z}} \left(-\frac{1}{2} \frac{1}{p^{3/2}} \text{sh}(z-p) - \frac{1}{p^{1/2}} \text{ch}(z-p) \right)$
 $f'(p) = - \int_0^p dz \frac{1}{\sqrt{z-p}} \left[\text{ch}(z-p) + \frac{1}{2} \frac{1}{p} \text{sh}(z-p) \right] \approx - \int_0^p dz \frac{\text{ch}(z-p)}{\sqrt{z-p}}, p \gg 1$

$F(p) = \int dz [I_0(z) K_0(p) - I_0(p) K_0(z)]$, $F'(p) = \int dz [I_0'(p) K_0(p) - I_0(p) K_0'(z)] =$

$= \int dz [-I_0(z) K_1(p) - I_1(p) K_0(z)] = - \int dz [I_0(z) K_1(p) + I_1(p) K_0(z)]$ $I_0 K_1 + K_0 I_1(x) = \frac{1}{x}$

$F''(p) = -\frac{1}{p} - \int dz [I_0(z) K_1'(p) + I_1'(p) K_0(z)] = -\frac{1}{p} - \int dz \left[I_0 \left(-\frac{1}{p} K_1 - K_0 \right) + \right.$
 $\left. K_0(z) \left(-\frac{1}{p} I_1 + I_0 \right) \right]$
 $F''(p) + \frac{1}{p} F' - F = -1/p$, т.е. $F(p)$ удовлетв. ЛНДУ.

Остается проверить расср. $a(p)$ через с. о. минимума $\Phi(x)$ или Γ -функцию $\gamma(\nu, \mu)$.

$$f(p) = \int_{p_0}^p \frac{e^{i\sqrt{z}p} - e^{i\sqrt{z}p_0}}{2\sqrt{z}p} dz = \frac{1}{2\sqrt{p}} e^{-p} \int_{p_0}^p \frac{dz}{\sqrt{z}} - \frac{1}{2\sqrt{p_0}} e^{p_0} \int_{p_0}^p \frac{dz}{\sqrt{z}} = \frac{1}{2\sqrt{p}} e^{-p} \sqrt{\pi} \Phi(i\sqrt{p}) -$$

$$- \frac{1}{2\sqrt{p_0}} e^{p_0} \sqrt{\pi} \Phi(\sqrt{p_0}) - \text{аналогично } p_0 = 0.$$

$$\Phi(\sqrt{p}) = 1 - \frac{1}{\pi} e^{-p} \frac{\Gamma(\gamma/2)}{\sqrt{p}} \quad \text{erfc}(z) = 1 - \Phi(z), \quad p \gg 1. \quad \Gamma(\gamma/2) = \sqrt{\pi}$$

$$1 - \Phi(z) = \frac{1}{\sqrt{\pi} z} e^{-z^2}, \quad \Phi(z) = 1 - \frac{1}{\sqrt{\pi} z} e^{-z^2}, \quad \Phi(i\sqrt{p}) = 1 - \frac{1}{\sqrt{\pi} i\sqrt{p}} e^{-p}$$

$$f(p) = \frac{1}{2\sqrt{p}} e^{-p} \frac{\sqrt{\pi}}{i} \left[1 - \frac{1}{i\sqrt{\pi} p} e^p \right] - \frac{1}{2\sqrt{p_0}} e^{p_0} \sqrt{\pi} \left[1 - \frac{1}{\sqrt{\pi} p_0} e^{-p_0} \right] = \frac{1}{p} - \frac{\sqrt{\pi}}{2\sqrt{p}} e^{-p} + O(e^{-p}), \text{ если } p_0 = 0$$

$$f(p) = \int_{p_0}^p \frac{\text{sh}(z-p)}{\sqrt{z}p} dz \rightarrow \int_{u_0}^u \frac{e^{-x}}{\sqrt{x}} dx = \int_0^u + \int_{u_0}^0 = \int_0^u - \int_0^{u_0} = [\Phi(\sqrt{u}) - \Phi(\sqrt{u_0})] \sqrt{\frac{\pi}{2}}$$

$$f(p) = \int_{p_0}^p \frac{\text{sh}(z-p)}{\sqrt{z}p} dz = \frac{1}{p} - \frac{\sqrt{\pi}}{2\sqrt{p}} e^{-p} - \frac{1}{p_0} + \frac{\sqrt{\pi}}{2\sqrt{p_0}} = \frac{\sqrt{\pi}}{2\sqrt{p}} e^{-p} [\Phi(i\sqrt{p}) - \Phi(i\sqrt{p_0})] -$$

$$- \frac{\sqrt{\pi}}{2\sqrt{p_0}} e^{p_0} [\Phi(\sqrt{p_0}) - \Phi(\sqrt{p_0})] = \frac{\sqrt{\pi}}{2\sqrt{p}} e^{-p} \left[-\frac{1}{i\sqrt{\pi} p} e^p + \frac{1}{i\sqrt{\pi} p_0} e^{p_0} \right] - \frac{\sqrt{\pi}}{2\sqrt{p_0}} e^{p_0} \left[-\frac{1}{\sqrt{\pi} p_0} e^{-p_0} + \frac{1}{\sqrt{\pi} p_0} e^{p_0} \right] =$$

$$= \frac{1}{2p} - \frac{1}{2} \frac{1}{\sqrt{p p_0}} e^{-p+p_0} + \frac{1}{2p} - \frac{1}{2\sqrt{p p_0}} e^{p-p_0} = \frac{1}{p} - \frac{1}{\sqrt{p p_0}} \text{ch}(p-p_0), \quad p, p_0 \gg 1.$$

$$\rightarrow f(p) = \int_{p_0}^p \frac{\text{sh}(z-p)}{\sqrt{z}p} dz = \frac{1}{p} - \frac{1}{\sqrt{p p_0}} \text{ch}(p-p_0)$$

$$a(p) = -\frac{x}{\omega_m} f(xp) = -\frac{x}{\omega_m} \left[\frac{1}{xp} - \frac{1}{x\sqrt{p p_0}} \text{ch} x(p-p_0) \right] = \frac{1}{\omega_m} \left[\frac{\text{ch} x(p-p_0)}{\sqrt{p p_0}} - \frac{1}{p} \right]$$

$$a(p) = \frac{1}{\omega_m} \left[\frac{\text{ch} x(p-p_0)}{\sqrt{p p_0}} - \frac{1}{p} \right]$$

при $p, p_0 \gg 1$. Ранее было $a(p) = -\frac{1}{\omega_m} \frac{1}{p}$ в первом приближении.

$$a'' + \frac{1}{p} a' - x^2 a(p) = \frac{x^2}{\omega_m} \frac{1}{p}, \quad x^2 = \frac{4\pi}{\epsilon} = \frac{1}{\epsilon_x^2}$$

$\rightarrow M_{\text{vortex}} = \int_0^R \frac{\sin \theta_0(p)}{p} a(p) dp = 2\pi \int_{p_0}^R dp a(p)$ — так же exp-расчет с верхним пределом (R).
Вне core — брэгг $p_0 = b$ по условию. $x b = b/\epsilon_x > 2$.

Но тогда $b(\epsilon_x) = (0.6 \pm 1) \epsilon_x = (0.6 \pm 1) \sqrt{\pi} \epsilon_x = (2.1 \pm 3.5) \epsilon_x$, т.е. $b = \pi b = (2.1 \pm 3.5) \gg 1$.

Т.к. реальный численный $b_m > b_{\text{us}}$, то $\frac{b}{\epsilon_x} \gg 1$ с помощью соотношения в смысле разномыслия $K_0(b/\epsilon_x) \sim I_0(b/\epsilon_x)$. Ошибка $\sim \frac{1}{8(b/\epsilon_x)} \sim \frac{1}{8(3)} \sim 4\%$ если считать ϵ_x в разном. $I_0(z), K_0(z)$ по $z \gg 1$.

$$M_{\text{vortex}} = \frac{M_s L}{\delta} \cdot 2\pi \int_0^R dp a(p) = \frac{M_s L}{\delta} \frac{2\pi}{\omega_m} \int_{p_0}^R dp \left[\frac{\text{ch} x(p-p_0)}{\sqrt{p p_0}} - \frac{1}{p} \right] = \frac{L}{\delta^2} \int_{p_0}^R dp \left[\frac{\text{ch} x(p-p_0)}{\sqrt{p p_0}} - \frac{1}{p} \right]$$

$$f_1(R) = \int_{p_0}^R \frac{\text{ch}(p-p_0)x}{\sqrt{p}} dp = \frac{1}{2} \int_{p_0}^R \frac{1}{\sqrt{p}} [e^{x(p-p_0)} + e^{-x(p-p_0)}] dp = \frac{1}{2} e^{-x p_0} \int_{p_0}^R \frac{e^{xp}}{\sqrt{p}} dp + \frac{1}{2} e^{x p_0} \int_{p_0}^R \frac{e^{-xp}}{\sqrt{p}} dp =$$

$$= \frac{1}{2} e^{-x p_0} \left[\Phi(i\sqrt{xR}) - \Phi(i\sqrt{x p_0}) \right] \frac{\sqrt{\pi}}{i\sqrt{x}} + \frac{1}{2} e^{x p_0} \left[\Phi(\sqrt{xR}) - \Phi(\sqrt{x p_0}) \right] \frac{\sqrt{\pi}}{\sqrt{x}} =$$

$$= \frac{1}{2} e^{-x p_0} \frac{\sqrt{\pi}}{i\sqrt{x}} \left[-\frac{1}{i\sqrt{\pi} \sqrt{xR}} e^{xR} + \frac{1}{i\sqrt{\pi} \sqrt{x p_0}} e^{x p_0} \right] + \frac{1}{2} e^{x p_0} \frac{\sqrt{\pi}}{\sqrt{x}} \left[-\frac{1}{\sqrt{\pi} \sqrt{xR}} e^{-xR} + \frac{1}{\sqrt{\pi} \sqrt{x p_0}} e^{-x p_0} \right] =$$

$$= \frac{1}{2} \frac{1}{\sqrt{xR}} e^{x(R-p_0)} - \frac{1}{2} \frac{1}{\sqrt{x p_0}} - \frac{1}{2} \frac{1}{\sqrt{xR}} e^{-x(R-p_0)} + \frac{1}{2} \frac{1}{\sqrt{x p_0}} = \frac{1}{\sqrt{xR}} \text{sh} x(R-p_0) \quad p_0 = b \text{ в первом приближении, иначе еще sh}.$$

$$M_{\text{vortex}} = \frac{L}{2\delta^2} \left[\frac{1}{p_0} f_1(R) - \ln \frac{R}{p_0} \right] = \frac{L}{2\delta^2} \left[\frac{\text{sh} x(R-p_0)}{\sqrt{xR p_0}} - \ln \frac{R}{p_0} \right], \quad \ln \frac{R}{p_0} \ll \text{sh} xR, \quad R \gg p_0$$

$$M_{\text{vortex}} = \frac{1}{2} \frac{L}{\delta^2} \frac{\text{sh}(xR)}{\sqrt{xR p_0}} - \text{вне "core" брэгг } b \text{ vortex mass. Взяли } L \ll R. \quad p_0 \approx b \text{ — "core" width}$$

→ Проблема $a(p) \rightarrow M_{\text{eff}}$ парес, $M_{\text{eff}} \rightarrow M_{xx}$ востребована.

F/AF vortex

4/III

Задача и comments к Soft-Hoffmann paper "Exch. Biased Vortices"
F/AF Vortex dots — reversal.

Наша гонимая на MSD семинар "Vortex Dynamics"

- F-dot
- 1) Vortex barrier — relax. dynamics (PRL-Ding)
 - 2) Vortex TM — high-freq. modes (PRB-Scholz) — ofus, uogroz. Cylindrical, square u elliptic dot — TM.
 - 3) B. conditions — gene vort. dynamics (PRL-subm.) + V. Statics (PRB-02, JAP-04)
 - 4) Vortex mass — pacusenevne TM-mod/TM-exp. Frank u high-w (Gt) — moz (exp. Biemell u Zhu)
- F/N/F-dot
- 5) D. vortex Statics (JAP, PRB-subm.) — gene obozpa
 - 6) D. vortex dynamics bez mass (APL-subm.)
 $C_1 C_2 = -1, P_1 P_2 = 1$
 - 7) D. vortex dynamics c M ($C_1 C_2 = -1, P_1 P_2 = -1$ u $C_1 C_2 = +1$)

+ obozpa negabnyx vortex-dynamics paper (Grimsditch, Ivanov, Zivieri, Buess, Baust u gp)

F/AF vortex

Axel

Monemo u Han(θ), $\theta = \hat{T} \hat{H}_E$ uz F/AF reversal obozpa yzuebei za-

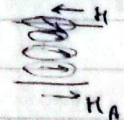
6/III buevovos energy barrier gene vortex $\Delta E(\theta, H)$?

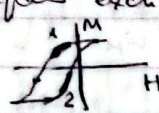
AF/F interaction zto na none $\hat{H}_E$, a znerime $-(d^2 \vec{p} / \vec{M}(\vec{p}) \cdot \hat{H}_E < \text{neogno-}$
pognoy puevpege... namamuzemostu $\vec{M}(\vec{p})$ u amozopomeu $\hat{H}_E = H_E \vec{e}$.

Moner nosovuy Han F/AF vortex dake zabuev ot θ ? Bege ufm
v. annihilation $M(\vec{p})$ cyuecbevno neogpepogua.

Tornee $E_{\text{ex}}(\text{AF/F})$ — zto surface energy $\sim \delta(z)$ u znerime na manay,
xosb $H_E \sim \frac{E_{\text{ex}}}{df}$ moner dube manym.

Hoffmann Zagara Vortex reversal c E_{ex} (interface AF/F) $\sim \delta(z)$ noxoma na
in-plane "exchange spring". M. reversal neognozogen no torumne
nammeesce c nobernom, ozgabane "spring" b soft magn. layer.



Pono uniaxial anisotropy hard layer u puev exch. bias field $\hat{H}_E$ (tornee ex. b. energy).
nepeamumebame AF — chue ner.  ufm $H \downarrow$, b osv. 1 syer $\approx$ vortex
reversal, a ufm $H \uparrow$ (osv. 2) $\approx$ coherent reversal u u coherent otmuzn interface $z=0$
u vortex btmuzn $z=L$ reversal.



$F = F_1$ (ferromag) + F_2 (AF) + F_{12} , F_2 (AF) ne uzmeesce, AF — chui

gaer noxko $F_{12} = E_{\text{ex}} \sim \delta(z)$ — Surface exchange. Interface F/AF plane — $z=0$.

$E_{\text{ex}} = -\vec{M} \vec{H}_E \delta(z)$. Пучеб $\vec{H}_E \parallel \hat{x}$

Bound. condition na interface ($z=0$) $\vec{M} \times [-d \frac{\partial \vec{M}}{\partial n} + \vec{H}_E] = 0$

Due vortex reversal gocevemo "rigid" vortex model, rge $\vec{M} = (0, M_\phi, M_z)$ b chuev usq. fuzer
x'o'y' c yuevpeu O' b yuevpe vortex "core".

Due core-reversal model $\vec{M} = (M_x, M_y, 0)$.

Therefore core contribution rigid v. model takes $\vec{M}(M_x, M_y, 0)$ to ensure $\chi \neq 0$,
 c using $\vec{M} \cdot \vec{M}' = 0$ and $\vec{M} \cdot \vec{M}' = 0$.

$$\vec{M} \times \vec{M}' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & 0 \\ M'_x & M'_y & 0 \end{vmatrix} = \hat{k} (M_x M'_y - M_y M'_x) = \hat{k} \left[M_x \frac{\partial M_y}{\partial n} - M_y \frac{\partial M_x}{\partial n} \right], \quad \frac{\partial}{\partial n} = \pm \frac{\partial}{\partial z}$$

$$\vec{M} \times \vec{H}_E = -k M_y H_E, \quad \text{с.м. } H_E \parallel \hat{i} \text{ (ox)}$$

$$-2 \vec{M} \times \frac{\partial \vec{M}}{\partial n} + \vec{M} \times \vec{H}_E = -2 \hat{k} \left[M_x \frac{\partial M_y}{\partial n} - M_y \frac{\partial M_x}{\partial n} \right] - k M_y H_E = 0$$



$$z=0 \quad \left[M_x \frac{\partial M_y}{\partial n} - M_y \frac{\partial M_x}{\partial n} - M_y H_E = 0 \right] \text{ - b. conditions. Tak. } \frac{\partial \vec{M}}{\partial z} \neq 0 \text{ гонимая из-за } H_E \neq 0$$

1) $H_E \rightarrow 0$, тогда можно $\frac{\partial M_x}{\partial z} \Big|_{z=0} = 0 \rightarrow$ observe vortex $\vec{M} = (0, M_0, M_z)$, $M_0 = f(\rho)$

2) $H_E \gg M_s$, $M_y|_{z=0} = 0$ - SD state на поверхности $z=0$ $\vec{M} = (M_x, 0, 0)$, $\vec{M} \parallel \vec{H}_E$.

$H_E \sim M_s$ - vortex (w/m $z=L$) the force perpendicular w/m surface $z=0$ m.

А с.м. L - толщина F-слоя мала, so vortex located near surface
 создающего с.м.

Note: убавляем H_E создаем surface vortex (H_E analogous H_x -external),
 но H_E гравитирует локально w/m $z=0$ m, otherwise $\vec{M}$ or H_x - uniform.

На поверхности $z=L$ имеем $\frac{\partial \vec{M}}{\partial z} = 0$.

Физ. разумно предполагать "rigid vortex" $\vec{M}(\vec{r})$ с учетом $\vec{X} = f(z)$.

$\vec{X} = (0, y) \perp \vec{H}_E$ $y(z) \sim \langle M_x \rangle = f(z)$ $y(0) < R$ - shifted vortex
 $y(0) > R$ - vortex core due to $\vec{M}$, C-state, or SD state

$y(z) \sim H_E$, т.к. w/m $H_E = 0$ тогда $y(z) = 0$ - centered vortex.

Задача сводится к нахождению угла θ по $y(z)$ из ур. 1.1. + b. conditions.

$y(z)$ - задает 3D distribution of $\vec{M}(\vec{r}, z)$ - vortex like curve.

Rigid v. model: $f(z) = \frac{z}{c}$, $w(z) = \begin{cases} f(z), & |f(z)| < 1 \\ f(z)/|f(z)|, & |f(z)| > 1 \end{cases}$, $\vec{s} = \frac{\vec{X}}{R} = (0, y) \perp \hat{R}$

$$m_z = \frac{1 - |w|^2}{1 + |w|^2}, \quad m_x + i m_y = \frac{2w}{1 + |w|^2} \text{ - задает ось } \vec{M} \text{ и } y(z).$$

$$\text{Удобно } \vec{m} = \vec{M}/M_s = (\sin \theta \cos \Phi, \sin \theta \sin \Phi, \cos \theta), \quad w = \tan \frac{\theta}{2} \cdot e^{i\Phi}$$

$$\rightarrow \text{Ур. 1.1. } \begin{cases} A(\nabla \Phi)^2 \sin^2 \theta - 2A \nabla^2 \theta - \vec{H}_m \cdot \frac{\partial \vec{M}}{\partial \theta} = 0 \\ 2A(\nabla^2 \Phi) \sin^2 \theta + 2A(\nabla \Phi \cdot \nabla \theta) \sin 2\theta + \vec{H}_m \cdot \frac{\partial \vec{M}}{\partial \Phi} = 0 \end{cases} \text{ см. 26/II, } H_E \text{ входит в } \Phi \text{ и не входит в bulk-UD}$$

Если обл. "core" с $\cos \theta \neq 0$ предполагать, то $\vec{H}_m \cdot \frac{\partial \vec{M}}{\partial \theta} \approx -H_m^2 \sin \theta$,

$$\vec{H}_m \cdot \frac{\partial \vec{M}}{\partial \Phi} = -H_m^2 \sin \theta \sin(\Phi - \varphi) + H_m^2 \sin \theta \cos(\Phi - \varphi) \quad \Phi(x, y) = \frac{\pi}{2} + \arctan \left(\frac{y - y_0}{x - x_0} \right)$$

$m_z = \cos \theta \approx 0$ для "core", $\sin 2\theta = 2 \sin \theta \cos \theta \approx 0$, $\sin^2 \theta \approx 1$, $\sin \theta \approx 1$. $\theta(x, y) \approx \pi/2$

$$\begin{cases} 2A \nabla^2 \theta - M_s H_m^2 = 0 & \text{ - ур-е center vortex } \\ 2A \nabla^2 \Phi - H_m^2 M_s \sin(\Phi - \varphi) + H_m^2 M_s \cos(\Phi - \varphi) = 0 \end{cases} \text{ (1) - вынужд. комплексное } \text{ (2) - задает ур-е на } \Phi(x, y, z) \text{ - задает } y(z).$$

Вектор $\vec{H}$ - none ext.

$\rightarrow$ $\Psi(r, z) = \Phi(r, z, t) = \pi/2 + \arctg(\frac{y-y(z)}{x})$ where $\{y(z) = \text{gaps}$
 $\Psi(r, z) = Y(z)$. We use Ψ as a new coordinate - convenient coordinate,
 a linear $y/R \approx 1$ near resonance (no vortex).

$\rightarrow$ Hence convenient to solve $W_m(s), W_{ex}(s)$ in PRB'02, $s = y/R$
 Torus & $W_{ex}(s)$ for a vortex $\sim \frac{\partial s}{\partial z}$ (shifted vortex)

$W[Y(z)] = \int dz [W_{ex}(s) + W_m(s)], \quad (s = s(z))$

Local $W_m(z \rightarrow 0)$ approx
 we remember u, v for $s = s(z)$.

$W_{ex} = A \int d^2p (\nabla \Phi)^2 = \left(\frac{\partial \Phi}{\partial x} \right)^2 + A \int dV \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right]$

Consider two cases: $w = 0$ gives upper & lower vortices. Focus on W_{ex} and
 our vortex $y(z)$ & $W_{ex}(0)$. $(\partial w / \partial z) \approx 0$.

$\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 = \left(\frac{\partial}{\partial z} \cos \Phi \right)^2 + \left(\frac{\partial}{\partial z} \sin \Phi \right)^2 = \sin^2 \Phi (\Phi_z')^2 + \cos^2 \Phi (\Phi_z')^2 = (\Phi_z')^2$

$\frac{\partial \Phi}{\partial z} = \frac{\partial}{\partial z} \arctg\left(\frac{y-y(z)}{x}\right) = \frac{1}{1 + (y-y(z))^2/x^2} \cdot (-1) \frac{\partial y}{\partial z} = -\frac{x}{x^2 + (y-y(z))^2} \frac{\partial y}{\partial z}$

$W_{ex} = A \int dV \left(\frac{\partial \Phi}{\partial z} \right)^2 + (\dots)_{s=y(z)} = A \int dV \frac{x^2}{[x^2 + (y-y(z))^2]^2} (y')^2 + (\dots)$

$\int d^2p \frac{x^2}{[x^2 + y^2 - 2yY + Y^2]^2} = \int_0^R \rho d\rho \int_0^{2\pi} d\varphi \frac{\rho^2 \cos^2 \varphi}{[\rho^2 - 2\rho \sin \varphi Y + Y^2]^2} = F(Y)$ - see figure

$W_{ex}(s) = (\dots)_{s=s(z)} + A \int dz F(Y) (y')^2 = (\dots) + AR^2 \int dz F(s) (s')^2$, since
 $s = s(z), s' = \partial s / \partial z, s = y/R$.

$W = \int dz \left[AR^2 F(s) (s')^2 + M_s^2 V \frac{1}{L} \frac{R_0^2}{2} \ln(1-s^2) + \frac{1}{L} M_s^2 V \frac{2}{\pi} s^2 \sum_n F_n(\beta) I_n^2(s) \right]$ - in PRB'02

$V = \pi R^2 L, \quad AR^2 = \frac{2A}{M_s^2} \frac{1}{2} M_s^2 R^2 = R_0^2 \frac{1}{2} M_s^2 \frac{V}{\pi L}$

$\rightarrow \frac{W}{M_s^2 V} = \frac{1}{L} \int dz \left[\frac{1}{2\pi} R_0^2 F(s) \left(\frac{\partial s}{\partial z} \right)^2 + \frac{1}{2} \left(\frac{R_0}{R} \right)^2 \ln(1-s^2) + \frac{2}{\pi} s^2 \sum_n F_n I_n^2(s) \right]_{s=s(z)}$ - of course!

$W_{ex} = W_{ex}(0) + \frac{1}{2} (R_0/R)^2 \ln(1-s^2); \quad W_m(s) = \frac{2}{\pi} s^2 \sum_n F_n I_n^2(s)$ - vortex energy
 2 - vortices appear at eq. minimum $z \rightarrow z/L, \quad \tilde{z} = z/L$.

$\frac{\partial}{\partial z} = \frac{\partial}{\partial(z/L)} = \frac{1}{L} \frac{\partial}{\partial \tilde{z}}$

$\rightarrow \frac{1}{M_s^2 V} W[s(z)] = \frac{1}{L} \int dz \left[\frac{1}{2\pi} \left(\frac{R_0}{L} \right)^2 F(s) (s')^2 + \frac{1}{2} \left(\frac{R_0}{R} \right)^2 \ln(1-s^2) + W_m(s) \right]$

$\rightarrow$ gaps remain on y - e na $s(z)$ but $\frac{\partial}{\partial s} \frac{\partial W}{\partial s} - \frac{\partial W}{\partial s} = 0$ - y - e $\tilde{z}$ - e $\tilde{z}$

B min. upper minimum $F(0) = \int d^2p \int d\varphi \frac{\rho^2 \cos^2 \varphi}{\rho^4} = \int_0^1 d\rho \int_0^{2\pi} d\varphi \cos^2 \varphi = \pi \ln \frac{1}{c}, \quad c = \frac{b}{R}$

$\frac{1}{M_s^2 V} W = \frac{1}{L} \int dz \left[\frac{1}{2} \left(\frac{R_0}{L} \right)^2 \ln \frac{1}{c} (s')^2 - \frac{1}{2} \left(\frac{R_0}{R} \right)^2 s^2 + 2\pi F_1 s^2 \right], \quad a(R, \beta) = 2\pi F_1(\beta) - \frac{1}{2} \frac{R_0^2}{R^2}$

$\frac{1}{M_s^2 V} W = \frac{2}{L} \int dz \left[\frac{1}{2} (s')^2 + \frac{1}{2} \lambda^2 s^2 \right], \quad \lambda = \left(\frac{R_0}{L} \right)^2 \ln \frac{1}{c}, \quad \lambda^2 = 2 \frac{a}{\gamma} = f(R; L), \quad \lambda^2 > 0$

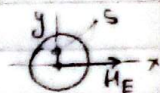
Лагранж. $\mathcal{L} = \int dz \left[\frac{1}{2} (\dot{s})^2 - \frac{1}{2} \lambda^2 s^2 \right] \rightarrow \frac{d}{dz} \frac{\partial \mathcal{L}}{\partial \dot{s}} - \frac{\partial \mathcal{L}}{\partial s} = 0 \rightarrow s'' + \lambda^2 s = 0$ — небольшая
завис.
от z (III)
 $\xi = \frac{z}{L}$ $s(z) = C_1 \cos \lambda z + C_2 \sin \lambda z$. B. Cond. $s'(1) = 0$, $s'(0) = ?$

$$M_y \frac{\partial M_y}{\partial z} - M_z \frac{\partial M_z}{\partial z} = M_s^2 \left[\cos \Phi \frac{\partial \sin \Phi}{\partial z} - \sin \Phi \frac{\partial \cos \Phi}{\partial z} \right] = M_s^2 \left[\cos^2 \Phi \frac{\partial \Phi}{\partial z} + \sin^2 \Phi \frac{\partial \Phi}{\partial z} \right] = M_s^2 \frac{\partial \Phi}{\partial z}$$

$$M_y H_E = M_s \sin \Phi H_E, \quad M_s^2 \frac{\partial \Phi}{\partial z} = M_s H_E \sin \Phi, \quad \frac{\partial \Phi}{\partial z} = \left(\frac{H_E}{M_s} \right) \sin \Phi \text{ — на } z=0.$$

$$\Phi = \pi/2 + \arctg \left(\frac{y-s}{x} \right), \quad \sin \Phi = \sin \left(\pi/2 + \arctg \left(\frac{y-s}{x} \right) \right) = \cos \left(\arctg \left(\frac{y-s}{x} \right) \right)$$

$$\frac{\partial \Phi}{\partial z} = - \frac{x}{x^2 + (y-s)^2} \frac{\partial s}{\partial z} R, \quad \arctg \frac{y-s}{x} = \theta, \quad \theta = 2 \arctg a, \quad \cos \theta = \cos(2 \arctg a)$$



$$\arctg \left(\frac{y-s}{x} \right) = \arctg \frac{y}{x} + \frac{1}{1 + \left(\frac{y-s}{x} \right)^2} \left(-\frac{s}{x} \right) = \arctg \frac{y}{x} - \frac{s}{x^2 + y^2} = \varphi - \alpha s$$

$$\cos(\varphi - \alpha s) = \cos \varphi \cos \alpha s + \sin \varphi \sin \alpha s = \cos \varphi + \frac{x}{x^2 + y^2} s \cdot \sin \varphi$$

$$\rightarrow \text{мн. ур. градиент} \quad - \frac{x}{x^2 + y^2} \frac{\partial s}{\partial z} = \frac{H_E}{M_s} \left[\cos \varphi + \frac{x \cdot \sin \varphi}{x^2 + y^2} s \right] \text{ — согласуем по } x, y!$$

$$- \frac{1}{s} \cos \varphi \cdot \frac{\partial s}{\partial z} = \frac{H_E}{M_s} \left[\cos \varphi + \frac{1}{s} \cos \varphi \sin \varphi s \right], \quad - \frac{1}{s} \frac{\partial s}{\partial z} = \frac{H_E}{M_s} \left[1 + \frac{1}{s} \sin \varphi s \right] \quad z=0$$

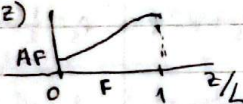
$$\int dz \frac{1}{s} = \int ds \frac{1}{s} = 2 \pi R, \quad \int dz \frac{1}{s} (1) = \pi R^2, \quad - 2 \pi R \frac{\partial s}{\partial z} = \frac{H_E \pi R^2}{M_s}, \quad \frac{\partial s}{\partial z} = - \frac{1}{2} \frac{H_E}{M_s R} \quad \text{при } z=0$$

$$\frac{\partial s(z)}{\partial z} = - \lambda C_1 \sin \lambda z + C_2 \lambda \cos \lambda z, \quad \frac{\partial s(z=1)}{\partial z} = 0 = \lambda [-C_1 \sin \lambda + C_2 \cos \lambda] = 0, \quad \frac{C_2}{C_1} = \tan \lambda$$

$$s(z) = C \cos(\lambda(1-z)) \text{ — нет.} \quad s(z) = C \cos(\lambda(1-z))$$

$$\frac{\partial s}{\partial z} = \frac{1}{L} \frac{\partial s}{\partial \xi} = \frac{C}{L} \sin \lambda(1-\xi) (-\lambda) = - \frac{R}{2} \frac{H_E}{M_s}, \quad \frac{C}{L} \lambda \sin \lambda = \frac{R}{2} \frac{H_E}{M_s} \rightarrow C = \frac{1}{2} R L \frac{H_E}{M_s \lambda \sin \lambda}$$

$$\rightarrow \text{Решение мн. по } s(z) = \frac{1}{R} y(z) \text{ ур. зазора.} \quad s(z) = \frac{H_E R L}{2 M_s \lambda \sin \lambda} \cos \lambda(1-z), \quad \xi = \frac{z}{L}, \quad s \ll 1$$



Проблема.
 То же $\vec{H}=0$.

Hoffmann

APL-Divortex — еще на печать.

(7/III)

Group-meeting. Becega e Pierre как у moving vortex

profile $\vec{m}(z,t)$ извлечь info о vortex Mass и проблеме (non-0 model).

Обсуждение с Becega new — exp. paper по vortex TM и работе по

V. mess. Kristen упоминала movie of Divortex motion ($C_1 C_2 = -1$, $A P_2 = +1$).

Becega e Kristen ee тачи. проблема F/AF reversal: S-double vortex

state (1) — вращательная упр reversal, это e coherent reversal т.е. $\cos$ на $\frac{1}{2}$ — $\frac{1}{2}$

F/AF reversal Becega e Axel! рассказан о $H_E(z)$ motion of F/AF dot reversal.

Non-uniform bias $H_E(z)$ gives non-uniform $M(z)$ distribution!

1) тогда зазора, похожа на "exch" in-plane spring, reversal starts

e наблюдается и всегда есть сине vortex и coherent reversal

где модно урна $H_E H_{ext}$. Модель $\vec{X}(z)$ — core-установка

2) берем H_L и H_I of $M(H)$ модель для асимметричности.

3) проблема $M(z,H)$ модель можно в X-MCD, PEEM (сложно),

уменьше меньше $L, R, H_E(\tau)$ и следует за top-layer ($z=L$) — MFM и

$\langle m(z,H) \rangle$. преобразование $\vec{X}(x,y)$ может быть и непрерывной.

Dado A/B del parcer bome e $\vec{X}(0) \parallel \vec{Oy}$ - vortex core position too give
 Modulus $\|\vec{X}\|_{\vec{Ox}}$. Usando $\vec{X}(x, y)$ univo gfo nonoverlapp uou b vert. dynamics.

$\frac{8}{m}$ bome e Pierre o dyn. vortex profile. (ignores c num):

Vortex dynamics u v. mass (v. dyn. profile)

1) paguine numerical $m_z(\vec{r})$ e "qud vortex u $\vec{r} = (\vec{m}, \vec{n}) = 0$ model
 T.E. naceuano kopoma uouano pole-free model give dynamical shifted center,

2) notuano $m_z(\vec{r}) \rightarrow m_z(\vec{r}), \Phi(\vec{r})$ u ex gabuene uou de
 Ouppoen vortex core $\vec{X}$. $\{d^2\vec{r}/dt^2 \mid \frac{\partial m_z}{\partial \vec{r}} \frac{\partial \Phi}{\partial \vec{r}} \dots\}$ gfo uoua vortex mass.

Kogo notuano $m_z(\vec{r}, \vec{e})$ que pole-free two vortex model.

Pierre que det e $2R = 1100 \text{ nm}$, $L = 40 \text{ nm}$ - notuano univo $\vec{L} \times \vec{r}$ uou gabuene
 dyn. vortex center $X = 27 \text{ nm}$, $Y = 14 \text{ nm}$. $\arctg \frac{Y}{X} = 0.4784$

$f(z) = \frac{1}{c} [izC + \frac{1}{2}(a - \bar{a}z^2)]$, $w(z\bar{z}) = f(z)$ bome "core". Pole-free model
 (cu. 30/11/04).

$$m_z(\vec{r}) = \frac{1 - |w|^2}{1 + |w|^2} = \frac{1 - |f|^2}{1 + |f|^2}, \quad \vec{a} = -2C \hat{z} \times \vec{S}, \quad \vec{S} = \vec{X}/R, \quad |a|^2 = 4|f|^2.$$

$$(\vec{S} \cdot \vec{a})^2 = [\vec{S} \cdot (\hat{z} \times \vec{S}) (-2C)]^2 = [(\hat{z} \times \vec{S}) \cdot \vec{S}]^2 4C^2 = \dots \quad \vec{a} \cdot \vec{S} = 2C(\vec{S} \times \vec{S})_z, \quad \vec{S}, C, \vec{S} \text{ b egr.}$$

$$c^2|f|^2 = \rho^2 - 2(1+\rho^2)(\vec{S} \cdot \vec{S}) + \frac{1}{4}(|a|^2(1+\rho^2)^2 - (\vec{a} \cdot \vec{S})^2) =$$

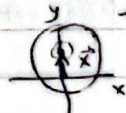
$$= \rho^2 - 2(1+\rho^2)(\vec{S} \cdot \vec{S}) + S^2(1+\rho^2)^2 - 4[(\vec{S} \times \vec{S})_z]^2 \quad \vec{S} \cdot \vec{S} = \frac{x^2}{R^2} + \frac{y^2}{R^2}$$

$$(\vec{S} \times \vec{S})_z = xS_y - yS_x.$$

$$[(\vec{S} \times \vec{S})_z]^2 = x^2S_y^2 + y^2S_x^2 - 2xyS_xS_y. \text{ Bez op. obynuan notuano } S_x = 0.$$

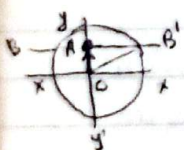
$$\begin{cases} S_x = S \cos \varphi_s \\ S_y = S \sin \varphi_s \end{cases} \quad c^2|f|^2 = \rho^2 - 2(1+\rho^2)S \cos(\varphi - \varphi_s) + S^2(1+\rho^2)^2 - 4[\cos^2 \varphi \sin^2 \varphi_s + \sin^2 \varphi \cos^2 \varphi_s - \frac{1}{2} \sin 2\varphi \sin 2\varphi_s] \rho^2 S^2$$

$$\text{Dado } S_x = 0, \varphi_s = \pi/2: \quad c^2|f|^2 = \rho^2 - 2(1+\rho^2)S \sin \varphi + S^2(1+\rho^2)^2 - 4 \cos^2 \varphi \cdot \rho^2 S^2$$



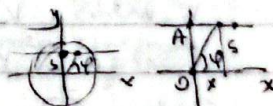
$$m_z(\vec{r}) = \frac{c^2 - [\rho^2 - 2(1+\rho^2)S \sin \varphi + S^2(1+\rho^2)^2 - 4\rho^2 S^2 \cos^2 \varphi]}{c^2 + [\rho^2 - 2(1+\rho^2)S \sin \varphi + S^2(1+\rho^2)^2 - 4\rho^2 S^2 \cos^2 \varphi]} - \text{b uoune kopg.}$$

c uoune kopg. e uoune kopg. b uoune kopg. gfoa, $\vec{X} \parallel \vec{y}$.



a) Corene vortex uoune notuano yoy', $\varphi = \pi/2$, $\rho = y$

$$m_z(y) = \frac{c^2 - [y^2 - 2(1+y^2)S + S^2(1+y^2)^2]}{c^2 + [y^2 - 2(1+y^2)S + S^2(1+y^2)^2]}, \quad (x=0)$$



b) Corene notuano BAB' $\perp \vec{S}$. $y = S$. $\frac{S}{x} = \tan \varphi$, $\rho^2 = x^2 + y^2 = S^2 \tan^2 \varphi + S^2 = \frac{S^2}{\sin^2 \varphi}$, $\rho = \frac{S}{\sin \varphi}$

$$x = S \cot \varphi, \quad \rho^2 = x^2 + S^2, \quad \rho^2 \cos^2 \varphi = \rho^2 - \rho^2 \sin^2 \varphi = x^2$$

$$m_z(x) = \frac{c^2 - [x^2 + S^2 - 2(1+x^2+S^2)S + S^2(1+S^2+x^2)^2 - 4S^2x^2]}{c^2 + [x^2 + S^2 - 2(1+x^2+S^2)S + S^2(1+S^2+x^2)^2 - 4S^2x^2]}$$

Kopg. uou v. core que pole-free model (cu. 22/2/04).

$$f(z) = 0 \text{ u } m_z(\vec{r} = \vec{e}_z) = 1. \quad \vec{S} = \frac{1}{2}ia, \quad |S| = \frac{1}{2}|a| \rightarrow z_2 = \frac{2Si}{4S^2} [1 - \sqrt{1 - 4S^2}] = \frac{1}{2S} [1 - \sqrt{1 - 4S^2}] i$$

Uoune $\vec{S} \parallel \vec{Oy}$, $\vec{a} \parallel \vec{Ox}$. $|a| < 1$, $|S| < 1/2$.

Toune uou $S < 1$, $z_2 \approx S + O(S^3)$. T.E. uoune kopg. $S = \frac{1}{2}ia$ uou $f(z) = \frac{1}{c} [iz + \frac{a}{2} - \bar{a}z^2]$

De abynuan, b abynuan, e uoune kopg. core center, uou, uou $m_z(z_2) = 1$. Paguine $(z_2 - S) \leq 1\%$ cu $S \leq 0.1$.

$$\rightarrow S(\text{Pierre}) = \frac{1}{550 \text{ km}} \sqrt{(11)^2 + (11)^2} \text{ km} = 0.0553 \text{ u} \text{ vlogramu } M_2(x), M_2(y)$$

dyn vortex pole-free model (Lennard-Jones) core is bounded by surface charges and vortices

Kago nepism b auc. voprosu c voprosu b r. z. - max $m_2(\rho)$ - dan ammeljane / uvelenennja oreljugee.

$$m_2(y) = \frac{c^2 - [y - s(1+y^2)]^2}{c^2 + [\dots]^2}$$

Dynamics → Deformed vortex
Vertex mass

Rigid vort. $f(z) = \frac{1}{2}(z-s) \rightarrow c^2 |s|^2 = \rho^2 - 2\vec{s} \cdot \vec{\rho} + \rho^2 = (\rho^2 - \vec{s}^2)^2$

→ pole free model $\rightarrow c^2 |s|^2 = \rho^2 - 2(1+\rho^2)(\vec{\rho} \cdot \vec{s}) + s^2(1+\rho^2)^2 - 4[(\vec{\rho} \times \vec{s})_z]^2$

Vertex core $m_2(\rho) = \frac{1-|s|^2}{1+|s|^2}$ paxoson uju $\vec{\rho} \approx \vec{s}$, nosony que pole-free model

$$c^2 |s|^2 = \rho^2 - 2(\vec{s} \cdot \vec{\rho}) + s^2 - 2\rho^2(\vec{\rho} \cdot \vec{s}) + s^2\rho^2(2+\rho^2) - 4[(\vec{\rho} \times \vec{s})_z]^2 = (\vec{\rho} - \vec{s})^2 + O(s^4)$$

Due $s < 0.1$ pole-free model naxoson covnages que $m_2(\rho)$ c rigid v. model.

No cam $\rho \sim 1$, to pazumme mogleni canobusce beuno, Rigid v. model unner rousko surface charges $m_2(\rho \approx 1)$, a pole-free model - rousko volume charges - bue "core".

Nauce osonne $m_2(\rho)$ pole-free model or rigid v. vobur o r. z. to massa vertex

$$\sim \int d^2\vec{\rho} \left[\frac{\partial m_2}{\partial x} \frac{\partial \Phi}{\partial y} - \frac{\partial \Phi}{\partial x} \frac{\partial m_2}{\partial y} \right] \text{ voburoge b anobno uz-ja "bue-core" bueage.}$$

→ b r. z. m. paxos Pierre uvelennje $m_2(\rho)$ dynamic vertex to cfabennu c $m_2(\rho)$ rigid - gobosno dounne, r. e. m. m. "core" u no doun. ok paxos b gans dounne bueage b v. mass.

(Zneme m. z. to m. m. M dyge dounne, zu M - mass vertex g/m)

pole-free model paxos or IS/I?

No M - pole-free mod. gans pazumme TM rousko, r. e. bue no beunne.

Osonnne pole-free (side charges free) model $\equiv$ z-model, $z = (\vec{m} \cdot \vec{n})_z = 0$ v. mass pole-free model

→ Due z-model (zo/zi ovi.) : $m_2(s, \varphi) = m_2^{(0)}(\rho) + f(\rho)(\vec{\rho} \cdot \vec{x})$, $f(\rho) = \frac{4c^2(1+\rho^2)}{R(c^2+\rho^2)^2}$, ρ beg. R.

Ca. 16/I : $\frac{\partial m_2}{\partial x} = f(\rho)\vec{\rho}$, $\frac{\partial m_2}{\partial y} = \frac{\partial m_2}{\partial x} \cdot \frac{\partial \vec{x}}{\partial y} = \frac{1}{x} f(\rho)(\vec{\rho} \times \vec{G})$ - na anobe $\vec{x} = \frac{1}{x} [\vec{G} \times \vec{V} - M\vec{V}]$

Funorege, to $\frac{\partial m_2}{\partial y}$, $\frac{\partial \Phi}{\partial y}$ nosony naitu uz yd $\vec{x} = \frac{1}{x} [\vec{G} \times \vec{V} - M\vec{V}]$ voburoge bueage u anobno - covnages.

Def. $M_{ij} = - \int_{S_M} d^2\vec{\rho} \left[\frac{\partial \Phi}{\partial x_i} \frac{\partial m_2}{\partial y_j} - \frac{\partial \Phi}{\partial y_j} \frac{\partial m_2}{\partial x_i} \right]$ u que z-model $m_2 = 0$ bue "core" $\Rightarrow$ an. m. $S_M = \text{core}$

No 16/I a onnne M_{ij} gans $\int_{S_M} d^2\vec{\rho} = (\text{bue dot})$, r. e. bueage za paxos z-model.

$\frac{\partial m_2}{\partial x}, \frac{\partial m_2}{\partial y} \sim f(\rho) \left\{ \begin{matrix} s(\rho) \\ \frac{1}{s^2} \end{matrix} \right\}$ - bueage "xboe" zabunnen $m_2(\rho)$ bue core, to anobno bue z-model. tra bueage gans pazumme M.

Coporo b paxos z-model $\frac{\partial m_2}{\partial x}, \frac{\partial m_2}{\partial y} = \left\{ \begin{matrix} f(\rho), \rho < c \\ 0, \rho > c \end{matrix} \right\}$, r. e. $\int_0^c d\rho$, a no $\int_0^c d\rho$ rax 16/I

$$J_c = \int_0^c d\rho \frac{4(1+\rho^2)}{(c^2+\rho^2)^2} = \frac{1}{2}(1-c^2)(-1) \frac{1}{c^2+t} \Big|_0^c + \frac{1}{2} \ln(c^2+t) \Big|_0^c = -\frac{1}{2}(1-c^2) \left[\frac{1}{2c^2} - \frac{1}{c^2} \right] + \frac{1}{2} \ln 2 = \frac{1}{4} \frac{1-c^2}{c^2} + \frac{1}{2} \ln 2$$

$$M_c = \frac{G}{\kappa} 2\pi \int_0^c d\rho \rho f(\rho) = \frac{2\pi G}{\kappa} \frac{4c^2}{R} J_c = \frac{4\pi G}{\kappa R} \left[\frac{1}{2}(1-c^2) + c^2 \ln 2 \right] \approx \frac{2\pi G}{\kappa R}, \text{ r. k. } c = b/R \ll 1.$$

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- $|w_+|$ — боковая
составляющая
вектора w

УД W_{3/2}in - $K\vec{X} + \vec{G} \times \vec{X} = M\ddot{\vec{X}}$, $\vec{G} = 2\pi\rho\vec{q}$ - УД-W_{3/2}in как у меня JAP'02, но
знаю $G_2 > 0$ (в JAP'02, $G_2 < 0$). Как $\vec{m}$ $(\vec{m} \cdot \vec{z}) = K$

* In case JAP 2 CCW ($\omega > 0$) $\frac{d\theta}{dt} = \omega$ $\Rightarrow$ vortex $\text{guz } p=1$ — $\Rightarrow$ vortex $\text{no } T_-$ (large amplitude) mode $m=-1$

4D JAP'02 $\vec{G} \times \vec{X} = \alpha \vec{X}$, w/ $\vec{G} = -\left(2\pi\mu \frac{LM_S}{\hbar}\right) \hat{z}$, $G_z < 0$ in TM under CCW motion

Угловая скорость $\vec{\omega}$ и углов. перем. $\vec{\varphi}$ $\left(\begin{smallmatrix} m \rightarrow -m \\ \text{ccw} \rightarrow \text{cw} \end{smallmatrix} \right)$ и $\vec{\omega} \rightarrow \omega (m \rightarrow +1 \text{ и } \text{ccw})$. Полярная

Coherent wave (w.r. TM-CCW JAP'02). No interference $|w-1| < |w+1|$ large amp. small amplitude $e^{i(m\phi - \omega t)}$
 $\rightarrow m = +1, \omega > 0$
 Coherent CCW wave

→ Boillog: low-frequency TM-mode ω_- is surface plasmon ($m=+1$, CCW-motion)

Это и есть битовая ТМ, которую Bane, Kristan etc. считают идеальной и которая была на эксперименте Grawell-03 и Frank (strip-line). $\langle m \rangle \sim \vec{X}(t)$ верно и $\Rightarrow$ каждое.

Torga mode TM - mode $\omega_1 = \frac{1}{2} \omega_0 (\sqrt{5} - 1) > 0$ exists because large amplitude of TM.

а $\omega_2 = -\frac{1}{2}\omega_0(\sqrt{5}+1) < 0$ (св) тн-може с малки амплитуды.

$$\omega_{1,2}(M) \cdot \omega_1(M=0) = \omega_0, \quad \omega_2(M \rightarrow 0) = -\frac{G}{M}, \quad \omega_{1,2} = \pm \sqrt{\frac{X}{M}}, \quad M \rightarrow \infty$$

Мені партер M vertex дає M_v $\omega_{1,2}(M_v) = \omega_{1,2} - \text{н/поверхності}$.

Но мы фиксируем конкретный процесс и получаем L, R dot.

Model: $\frac{G}{M} = \omega_0$, $\omega_0 = \frac{\alpha}{G}$, r.e. $G^2 = \alpha M$ u $M_v = \frac{G^2}{\alpha}$ $\frac{\alpha}{M} = \frac{\alpha^2}{G^2} = \omega_0^2$ - V. mass (the four-mass)
u3 - thiele eq.

T.e. для $(M=0, G)$ и $(M, G=0)$ получим ТМ волны для $\omega_0 = \frac{\pi}{G}$ и $\omega_0 = \sqrt{\frac{\pi}{M}}$ - одна и та же ω_0 . Для $(M \neq 0, G \neq 0)$ получим волны ТМ $\omega_1 = \omega_0 \frac{1}{2}(\sqrt{5}-1)$.

М, можно было решить иначе тем я, и $\{ \vec{p} \} [\frac{\partial}{\partial x} \frac{\partial}{\partial v} \dots]$, где w_1 и w_2 — волны, но micromagnetic simulations не возбуждают small-amplitude motion w_2 .

→ Moment in M_v caused by $M_v \approx \frac{W}{v^2}$ (W - Inertia dot c vortex, v - in-gn velocity)?

Мож. $M \sim \frac{L}{F_2(\beta)} \sim R$ (2.44). Это же $M \sim R$ в рамках WySin-96 increases of XZ-model,
с парамет. $m_p(R) = 0$.

→ Kan osmose T_1 u T_2 mogu na spatial distribution? Cu. Wynn-Volkel, PAB'96 - mehanika.

(a) где T_+ мого ($\omega_+ > \omega_-$) $\psi(p)$

размещено сим. и есть вне "core"

The left graph plots $\psi(p)$ against p . It shows a symmetric curve starting from a negative value at $p=0$, crossing the axis, and peaking at T_+ before decaying towards $1/R$ as p increases. The right graph plots $\psi(p) = -\sin \theta \cdot \phi(p)$ against p . It shows an antisymmetric curve that is zero at $p=0$ and reaches a peak labeled T_+ as p increases.

⑥ где T - модуль (является ТМ) - регулярный элемент не, но $\chi(p)$, $\mu(p)$ можно локализовать вблизи "core". Т.е. T_+ не локализуется на core (это асим. СВ)
 T_- - локализована на v-core - для ТМ

Note: Ivanov-PRIOS neregularizovan gde TM u omceles na splitting $\Delta\omega = \omega_- - \omega_+$

no 6 Zaspel-PRB-submitted game revolves, tho pr. условия предлагается только

сигналу ТМ - Т-. Промодульоване. Все гнє $L = 50 \text{ км}$, $R_{\text{вн}} =$ $\begin{cases} R(\text{Iv. '05}) = 200 \text{ км}, & R(\text{заг.}) = 250 \text{ км} \\ \omega_{\text{м}}(\text{Iv.}) = 28.4 \text{ ГГц}, & \omega_{\text{м}}(\text{заг.}) = 30 \text{ ГГц} \end{cases}$
Назвем небольшое измен. R и M_2 сделало вторую ТМ поду невозможной?

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TM vortex Т.е. можно говорить high-frequency TM (ω_+) некая особенность, из-за ее малой амплитуды или в finite dots с $R \geq 200$ nm ее вообще нет?

Если T_+ - мода с.с.б., ее можно возбудить с.с.б. Circular-polarized a.c. field.

Нормировать это нормалью импульса.

Какой частоты для TM $\omega_0(L/R)$ с M-расчетом, где $\omega_{tm} = 0.62 \omega_0$ —

— это моды с.с.б. или инф. ферми-вращатель vortex Mass $M = \frac{1}{4} \frac{L}{\delta^2 F_0(p)}$

Vortex mass Note: моды можно считать для подхожа для расчёта M, если предполагать, что $\vec{X} \sim \vec{G} \times \vec{v}$ ^{of shifted vortex} — это динамическая величина.

Но $M \sim \frac{1}{\kappa}$, $W = \frac{1}{2} \kappa X^2$ и κ , а следовательно M будет зависеть от вида моды.

"Pole-free" model дает разницу κ (соглас. с дисперсионным по ω_{tm}) и удовлетворяет уравнению граничного условия для тонкого диска $m_R(R) = 0$.

Т.е. Pole-free model с $(\vec{m} \cdot \vec{n})_z = 0$ физически разумна и выделена.

С группой скорости, M может быть найдена из прямого решения ур-я Л.-А.: аналитическое $(\delta \vec{m} \cdot \vec{v})$ или численное, если использовать определение M через $\frac{\partial \Phi}{\partial x}, \frac{\partial \Phi}{\partial y}, \frac{\partial m_z}{\partial x}, \frac{\partial m_z}{\partial y}$.